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ABSTRACT

Ongoing work toward developing a learning environment that will perform real-time diagnoses of students' difficulties in solving mathematical word problems is described. The learning environment designed consists of a microworld and expert modules. The microworld (or toolbox) is a collection of mouse-driven interfaces that facilitate a transition between manipulative models of arithmetic word problems (such as those using physical blocks) to the more abstract symbolic models (such as an open-sentence equation). The expert module is composed of mathematical and linguistic problem solving models. A linguistic or reading expert first generates a conceptual representation of the actions and sets in a word problem, and the mathematical expert then attempts to assign part-whole roles to the conceptualized sets and arrive at a numerical model. The focus is on the expert reading model. A conceptual analyzer, EDUCE (Explaining Discourse Understanding with Conceptual Expectations), reads English word problems and generates conceptual representations of the quantities and actions in the word problem. By altering its linguistic abilities, the learning environment will perform real-time diagnoses of students' erroneous solutions and give the teacher hypotheses of potential misunderstandings. In its instructional role, EDUCE is designed to explain the role of certain words and phrases in a word problem. In its cognitive modeling role, it helps explain why some problems are harder than others. Four tables are included. (SLD)

Understanding the Role of Linguistic Processes in the Solution of Arithmetic Word Problems

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Abstract: Mathematical word probleme are challenging. Students find them hard to solve, text book writers find them difficult to present, and teachers consider word problems not only a challenge to teach but often an overly complex task given a class of students. Recent work in cognition has provided insights into the tacit mathematical processes involved when students solve word problems; however, little evidence is available concerning the word problem solver's linguistic processes. In an attempt to synthesize the linguistic and mathematical skills needed in a word problem solving model, this work aims at uncovering the tacit reading tasks which cause some word problems to be more difficult to solve than others. A conceptual analyzer reads English word problems and generates conceptual representations of the word problem, where the quality of the representation is dependent on the model's current level of reading expertise. By altering its linguistic abilities, a learning environment will perform real-time diagnosis of students' erroneous solutions and provide the classroom teacher with hypotheses of potential misunderstandings. The implication for instruction is that students' errors may have more to do with their "reading" of the problem than with a deficient mathematical understanding.

1. Introduction

Recent research on children's cognition and problem solving has provided detailed insight into the multiple knowledge sources involved when students solve arithmetic word problems (Carpenter, *et al.*, 1988; Cummins *et al.*, 1988). Solving word problems requires interaction with multiple representations of knowledge, beginning with the crucial mapping between the words in the problem and an internal conceptualization of entities such as the actors, objects, and actions in the problem. This initial process of reading and developing conceptualizations is the focus of this paper. Of interest are the high cognitive demands which word problems place on young readers. That is, word problem solvers are expected to: (a) interpret the unique mathematical sense of words, e.g., *some* means a set with an unknown quantity; and (b) map complex linguistic constructions onto mathematical relationships, e.g., "*have more than*" involves a difference between two sets.

Our specific objective in modeling the reading process is to expose the often overlooked but crucial reading comprehension strategies essential in word problem solutions. Prior models of arithmetic word problem solving (Cummins, *et al.*, 1988; Kintsch and Greeno, 1985; Briars and Larkin, 1984; Riley, *et al.*, 1983) have provided detailed insight into the mathematical processes involved in word problem solutions; however, little evidence is available concerning the word-problem-solver's linguistic processes. In particular, our reading model both appreciates the role which linguistic misunderstandings play in incorrect word problem solutions and monitors the amount of cognitive processing involved during reading. By modeling the solution process from the beginning, i.e., from the linguistic statement of the problem, we are in a better position to hypothesize why some problems are more difficult to solve than others. The implication for instruction is that students' errors may be related to their inability to conceptualize a problem or their inability to comprehend problems with high cognitive processing loads.

This study of linguistic processing is one component of a larger interdisciplinary research project to develop an interactive learning environment for students experiencing difficulties with word problems. The learning environment is composed of two main modules: the microworld and the experts. The microworld (or *tool-box*) is a collection of mouse-driven interfaces which facilitate a transition from manipulative models of arithmetic word problems, such as those which use physical blocks, to the more abstract symbolic models such as an open-sentence equation. The expert module is composed of mathematical and linguistic problem solving models. A linguistic or reading expert first generates a conceptual representation of the actions and sets in a word problem and the mathematical expert then attempts to assign part-whole roles to the conceptualized sets and arrive at a numerical answer. The next section briefly introduces the microworld component of our prototype learning environment and the remaining sections focus on the expert reading model.

2. The Tool Box

Our microworld of interactive graphics tools has received considerable attention in our prototype implementations.[1] In line with the goal of enabling students to construct new meanings from concrete situations (NCTM, 1989), the development of new tools has focused on environments which allow students to manipulate objects, i.e., icons, via the mouse in increasingly abstract representations. Young students (K-2) who have yet to acquire schemata for representing word problems rely heavily on physical manipulatives to model the problem (Carpenter, 1985; Briars and Larkin, 1984; Riley, *et al.*, 1983).

In particular, the *Make-Set* tool allows students to "pick" object-icons and collect them together in certain areas of the screen. This tool facilitates a transition from physical "on the table" objects to more abstract mathematical symbols (Langford, 1986) by providing a transition between symbols, e.g., from four individual soda-can-icons to the more abstract representation of one soda-can-icon with four dots (::), to one soda-can-icon with the number (4), and eventually to a number 4-icon. Actual screen printouts of students using this tool are included in LeBlanc and Lapadula (1990).

By supplying a visual means of transition from concrete to more abstract concepts, this tool also serves our goal of preparing students for extension of their basic

[1] The overall prototype system executes on both SUN and VAX workstations. The graphical interface is written in C and runs under the X-window system (v11). The reading expert to be discussed in the next section is written in Common Lisp. Specific tools are described more fully in LeBlanc and Lapadula (1990).

mathematical knowledge to somewhat more difficult problems. Seeing soda cans as dots or as a number may lead the student to similarly see concepts less easily visualized, such as distance or time, as dots or as a number, subject to the same mathematical operations as soda cans. Of particular interest at this stage in our design is to observe students using these tools and to isolate those times when students seem capable (and willing!) to move beyond solutions involving physical manipulatives or icons which still look physical to more abstract symbol systems.

Another new tool, the *Equation Builder*, allows students to "build" equations; that is, students select operands and arithmetic operators to construct mathematical relationships, including higher-level algebraic relationships. This tool serves as a follow-up to the *Make-Set* tool in the sense that the equation model facilitates representations which can not be performed with manipulative modeling, for example, representing a transfer of a set of *some* objects out of an existing set of objects. Designs for future tools include environments which will facilitate the use of part-whole strategies, including a tool which allows students to identify which of the manipulated sets occupy the "roles" of the parts and whole.

3. EDUCE - The Reader

EDUCE (Explaining Discourse Understanding with Conceptual Expectations) is an expectation-based conceptual analyzer (LeBlanc and Russell, 1989) adapted from the work of Schank and Riesbeck (1981) and developed according to principles found to be distinct for word-problem-solving. Instead of using already encoded problem schemas or propositions as do other cognitive models of arithmetic word problem solving, the parser "reads" the English word problem and produces a conceptual representation of the quantities and actions in the word problem. This ability to "read" the word problem serves a dual role in our research efforts. In an instructional role, EDUCE is designed to explain the role of certain words and phrases in a word problem as well as hypothesize about student misconceptions, e.g., an invalid "clue-word" use of the word *altogether* such as "altogether means add." In its cognitive modeling role, EDUCE is providing insight into why some word problems are harder to solve than others. The next two sections introduce this dual capacity more fully.

3.1. Diagnosing Potential Misconceptions

The ability to hypothesize about students' linguistic and mathematical misconceptions is a main focus in the development of the learning environment. Such diagnostic capability demands that our problem solving models perform at varying levels of expertise, from expert to novice, in an attempt to determine *why* a student arrived at an incorrect answer.

In the context of a problem solving session, EDUCE reads each problem that will be posed to the student and generates a conceptual representation of the quantities and actions in the word problem where the "quality" of the representation is dependent on the level of linguistic expertise being used to simulate solution performance. In the instructional context, teachers often associate incorrect student answers with mathematical inabilities or mistakes, for example, "Oh, he must have added wrong." In an effort to highlight the crucial role of reading in the solution process, EDUCE models multiple levels of reading expertise by (1) varying its internal "meaning" for certain critical words (e.g., *some*, *more*) and (2) varying its linguistic processing capabilities (e.g., the capacity of its short-term memory, its ability to handle anaphoric or pronominal reference). For example, given the following sentence:

Jake had some soda cans.

EDUCE can access two meanings for the word *some* from its lexicon and therefore produce alternate conceptual meanings of the sentence. The two different simplified representations are shown below:

(1) STATE: possession
OBJECT: (physical (-animate) .... (quantity: *unknown*))
LOCATION/POSSESSOR: (physical (+human) (reference: Jake))

(2) STATE: possession
OBJECT: (physical (-animate) .... (type: some))
LOCATION/POSSESSOR: (physical (+human) (reference: Jake))

The first interpretation (1) is, of course, what we consider to be the correct one, that is, Jake possesses an *unknown* number of soda cans. Prior to instruction, however, many children do not recognize the mathematical sense of the word *some* (Langford, 1986; Briars and Larkin, 1984) and instead treat *some* as an adjective (Cummins, *et al.*, 1988). That is, "*some* cans" is often treated like "*red* cans." Because the reading model has access to both senses of the word *some*, diagnosis of student errors involves an attempt to detect a student's use of one sense over the other. This involves more than testing students with problems which do and do not include the word *some*. For instance, the following word problem involving a "set of *some*" can be solved manipulatively without a correct understanding of the word *some*.

Jake found 6 soda cans near the road. He gave some cans to Chris. Now Jake has 2 cans. How many cans did he give to Chris?

In this problem, students read the first sentence and create a set of six cans. Assuming a deficient understanding of the word *some*, students ignore the second sentence, that is, the cans transferred to Chris are not treated as a *set* of cans since no quantity is mentioned. Upon reading that Jake now has two cans, students typically separate out two cans from the original group of six. When asked how many Jake gave to Chris, students are faced with two groups of cans: a group of two belonging to Jake and a separate group of four .... "Hey, that must be the answer! Four!"

While such problems can be solved without a correct interpretation of the word *some*, others cannot, e.g., a transfer of possession problem where the initial set is unknown.

Jake found some soda cans. He gave 2 cans to Chris. Now Jake has 5 cans. How many cans did Jake have in the beginning?

The difficulty here is of course that students do not interpret Jake's initial group of *some* cans as a set and thus Jake has no cans to give to Chris. Students often create a set of two cans for Chris, a set of five cans for Jake, and answer "five" when asked about Jake's cans in the beginning, since Jake's set currently has five cans. The implication for instruction is that students who can not solve these initial-set-unknown problems: (1) may have a deficient part-whole strategy; i.e., they cannot recognize subsets (parts) and the superset (whole); and/or (2) may have a misunderstanding of the word *some*.

EDUCE maintains multiple meanings for other critical words (e.g., *more, less, altogether*) which in turn can affect the conceptualizations of complex phrases (e.g., *have-more-than, how-many-less-than*). By varying the meanings of certain critical words, EDUCE will produce a dynamic range of conceptual (mis)understandings which in turn contribute to incorrect answers and thus generate hypotheses of potential comprehension errors.

3.2. Monitoring the Reading Process

In addition to providing hypotheses of student's conceptual misunderstandings, EDUCE is also providing insight into the cognitive demands of reading arithmetic word problems. Evidence on text comprehension emphasizes the importance of monitoring the cognitive demands of linguistic processing, for example, the role of anaphoric reference (Obrien, 1987) and the importance of maintaining the most relevant information in short-term memory (Fletcher, 1986). Monitoring the process of *how* EDUCE generates a conceptual representation first involves isolating the tacit reading tasks essential to "understanding" sentences. Some of the tacit tasks the model is designed to perform as it "reads" a sentence include: lexical access, instantiating and remembering conceptual expectations (i.e., what concepts to *expect* later in the sentence), "merging" multiple concepts into one concept, and anaphoric search. The next two sections introduce how EDUCE monitors the reading process and how that monitoring is providing insights into the correlations between problem difficulty and linguistic processing.

3.2.1. An Example

A sample parse of the sentence,

Jake has 5 soda cans.

can help clarify the ideas in expectation-based analysis and how we monitor the reading load.

Upon reading the word *Jake*, EDUCE accesses that word's lexical entry and loads its conceptual meaning into short-term memory (STM):

CONCEPTS: (1) (physical (+human) (reference: Jake))

REQUESTS: nil

There are no expectations generated by the word *Jake*, i.e., the reading of this word does not request (or expect) any particular concepts to follow. The word *has* is then read and the conceptual entry for the word *has* is loaded into STM along with its associated requests:

CONCEPTS: (1) (physical (+human) (reference: Jake))
(2) STATE: possession
OBJECT: ?
LOCATION/POSSESSOR: ?

REQUESTS: (i) Who possesses? [search for a human earlier in the sentence]
(ii) What is possessed? [search for an object later on]

Before reading the next word, each of the requests associated with the word *has* is tested to see if it might be satisfied. The first request (i) is of course successful since a search finds the conceptual representation of Jake in STM (i.e., Jake's conceptualization is +human) and thus the "(1) Jake-concept" is merged into the possessor-slot of the "(2) possession-concept." The second request (ii) is unsuccessful since no conceptual-object currently resides in STM. Having read and completed the processing for the first two words, "*Jake has*," EDUCE has one concept and one outstanding request in STM:

CONCEPTS: (1) STATE: possession
OBJECT: ?
LOCATION/POSSESSOR: (physical (+human) (reference: Jake))

REQUESTS: (i) What is possessed?

The following table summarizes the monitoring of linguistic processing which has occurred during the comprehension of these first two words, "*Jake has*." The left column lists the linguistic variables (or tacit reading tasks) of interest and the right column lists the number of instances when this task occurred.

Summary of Linguistic Processing *Jake has...*	
Variable	**Instances**
maximum # of STM concepts	2
average # of STM concepts	1.5
number of requests posted	2
number of requests fired	1
number of *merged* concepts	1

Table 1

The next three words, "*5 soda cans*," form a noun group and are eventually merged into one "object-concept." The outstanding request from the word *has* which is expecting an object is now satisfied.

A final conceptualization for the sentence resides in STM:

CONCEPTS: (1) STATE: possession
OBJECT: (physical (-animate) (function: contain(object: soda)) (quantity: 5))
LOCATION/POSSESSOR: (physical (+human) (reference: Jake))

REQUESTS: nil

Table 2 summarizes the linguistic processing which occurred in order to arrive at such a conceptual understanding of the sentence.

Summary of Linguistic Processing *Jake has 5 soda cans.*	
Variable	**Instances**
maximum # of STM concepts	4
average # of STM concepts	2.4
number of requests posted	4
number of requests fired	4
number of *merged* concepts	3

Table 2

3.2.2. Pilot Studies

The current pilot experiments are in search of linguistic processing variables (e.g., the average number of concepts in STM, the number of conceptual requests posted) which correlate with the probability of word-problem-solution as a function of a problem's semantic structure and grade level (K-3). Previous empirical studies (Del Campo and Clements, 1987; Carpenter, 1985; Riley, *et al.*, 1983) have provided evidence of the probability of solution for word problems of varying semantic structures[2] and grade levels. Of interest is the potential for EDUCE to predict whether a problem will be difficult, i.e., whether a problem has a low probability of solution. The primary question at this point in our research is:

What linguistic constructions and processing cause some problems to be more difficult than others?

There are a variety of ways to restate this primary question, e.g., "What linguistic tasks create an overloading demand on students comprehension processes?"

We are currently updating EDUCE to keep track of the processing statistics as it parses the word problems. Our focus is to find two or more linguistic variables which correlate with solution probability. Traditionally, solution probabilities are correlated with the semantic structure of word problems. For example, *Compare* word problems involve a comparison of two quantities and are difficult for most students in the early (K-3) grades. The probabilities of solution for these problems are low relative to the

[2] Arithmetic word problems are traditionally classified according to their semantic structures, i.e., *Change* problems involve the action of a transfer of possession, *Combine* problems involve a static combination of quantities, and *Compare* problems involve a static difference of quantities.

problems with alternate semantic structures. From our early work, EDUCE is providing new hypotheses as to why this type of semantic structure is difficult, that is, reading word problems involving the complex linguistic construction "*have more than*" places a taxing cognitive demand on young readers. The two tables below reveal some differences in the amount of processing required for EDUCE to conceptualize a question from a *Change* word problem and a question from a *Compare* word problem.

Summary of Linguistic Processing *How many cans does Jake have now?*	
Variable	**Instances**
maximum # of STM concepts	3
average # of STM concepts	1.5
number of requests posted	7
number of requests fired	6
number of *merged* concepts	3

Summary of Linguistic Processing *How many more cans does Jake have than Chris?*	
Variable	**Instances**
maximum # of STM concepts	4
average # of STM concepts	2.0
number of requests posted	11
number of requests fired	8
number of *merged* concepts	4

The results to date are preliminary. The list of linguistic processing variables shown in the tables is our initial attempt to investigate the process of reading during word problem solution.[3] While EDUCE is not intended to model the students' reading comprehension in all aspects, it is providing a focus for future work.

In some ways, we already have a good intuitive feel for which linguistic variables will more highly correlate with problem difficulty. For example, we expect the average number of concepts in STM to be a better predictor of problem difficulty than, for instance, the number of anaphoric references.[4] We have verified that a considerable memory load is required for the predominately more difficult *Compare* problems, i.e., those which involve the complex linguistic construction *have-more-than*.

Of additional interest is how varying the level of reading expertise may affect the correlations across grade levels. In the preliminary experiments, EDUCE will always read at a constant "expert" level of reading knowledge and thus we expect to find higher correlations with say grade 3 probabilities of solutions than at the K-level. Varying EDUCE's ability to read may then alter our best-match situation. This is of course exactly the type of information that would benefit us most in terms of pedagogical implications. What EDUCE cannot do at one level may in fact be what children in grade 1, for instance, cannot do.

[3] For instance, in terms of conceptual requests being posted and fired, it may in fact be the case that some requests are "more costly" in terms of processing load than other requests, e.g., the action of merging two concepts together potentially involves more processing than recognizing a word as a single concept. Our initial assigning of scoring weights is somewhat *ad hoc* and the potential to alter the weights of some requests offers another potential manipulation while we are collecting data.

[4] Since the problems are all relatively short (e.g., 3 sentences) there exists little information to inhibit an easy anaphor/reference match. Future experiments may increase the distance between the anaphor and its reference in order to ensure that a long-term memory search is required.

4. Summary

In the last ten years, educational researchers have categorized arithmetic word problems according to their semantic structures and produced empirical evidence which correlates solution probability with these categories. Lacking in these results, however, is a definitive explanation as to *why* problems of certain semantic structures are difficult. In an attempt to more fully understand word problem solving, cognitive models have simulated problem solving procedures that children use; however, these models have largely ignored the process of reading. This work is an attempt to bridge the gap between the educational studies and past cognitive modeling work. Monitoring the role of linguistic processes focuses our research on the questions which ask *why* some problems are difficult.

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