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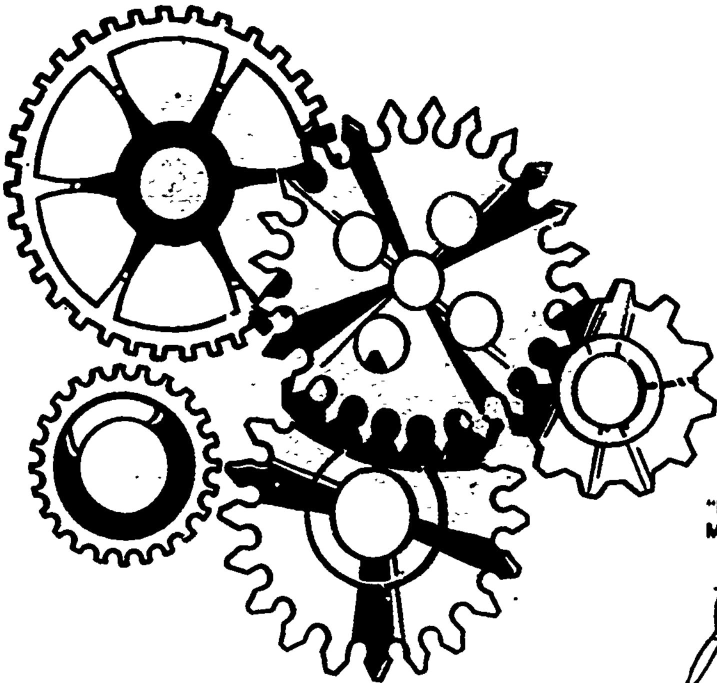
ABSTRACT

This manual contains materials for a numeracy course for adult industrial workers. In addition to assessment tests, seven units are provided. Unit topics are whole numbers; fractions; decimals; percents, median, and range; measurement and signed numbers; ratio/proportion and introduction to algebra; and computer literacy using algebra software. Materials within each unit include objectives, dictionary (words and definitions), some explanation, drill sheets (exercises), and a unit review. Unit tests are included. A final section of the manual contains suggested instructional strategies for each unit and some general suggestions for using the manual. (YLB)

Math/Measurement for Upgrading Skills of Industrial Hourly Workers

Math Manual

by
Joan McMahon



Gearing Up for the Future

Project #98-0044
Pennsylvania Department of Education

Northampton Community College

Math/Measurement Literacy for Upgrading Skills of Industrial Hourly Workers

M A T H   M A N U A L

by

Joan L. McMahon
Curriculum Developer

August 31, 1990

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Harrisburg, Pennsylvania

Adult Literacy Department
Northampton Community College
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Disclaimer

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ASSESSMENT TESTS

Name ____________________ **Date** ____________________

Please do the work in the space provided.
Place your answers on the answer sheet. Unless otherwise requested, express all decimal answers to two (2) decimal places and reduce all fractions to lowest terms.

Write the number.

1. twenty-three ________

2. nine thousand, two hundred, sixty-eight ________

Write the word.

3. 47 ____________________

4. 1,091 ____________________

How many thousands, hundreds, tens, and ones?

5. 4,679 ____ thousands, ____ hundreds, ____ tens, ____ ones

6. 9,067 ____ thousands, ____ hundreds, ____ tens, _____ ones

Circle the largest number.

7. 1,020 1,021 1,121

8. 999 9,099 9,909

Put these numbers in order. Write the largest first.

9. 5,089 _____
   5,090 _____
   5,900 _____
   5,908 _____

10. 1,010 _____
    1,001 _____
    1,101 _____
    1,111 _____

Add or subtract.

11. $\begin{array}{r} 50 \\ +\ 30 \\ \hline \end{array}$

12. $\begin{array}{r} 78 \\ -\ 42 \\ \hline \end{array}$

13. $\begin{array}{r} 113 \\ +\ 245 \\ \hline \end{array}$

14. $\begin{array}{r} 746 \\ -\ 301 \\ \hline \end{array}$

15. There were 34 yes votes and 62 no votes. How many voted at all? ________

16. A dealer had 529 cars in stock. 302 cars were sold. How many were left? ________

Subtract.

17.
```
  3,146
- 1,908
_______
```

18.
```
  560
-  98
_____
```

19.
```
  7,005
-   418
_______
```

Add.

20. 414 + 36 + 389 + 20 = ________________

Write the numbers.

21. Fifty thousand, four hundred ninety-eight

____

22. Eight hundred million, seven thousand, four hundred seven

____

Find the Answer.

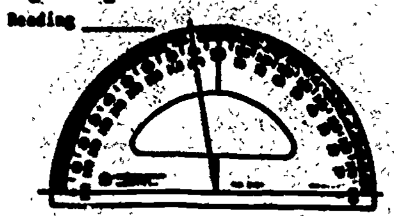
23.
```
  9 0, 8 5 5
-    2, 4 1 1
____________
```

24. $58 + \frac{(13 + 32 - 9)}{3} - 17 =$

25. Find the average of 77, 68, 93, 55, and 82.

____

26. Write the proper protractor reading shown below.



**Estimate to the nearest ten thousand.
Circle the correct answer.**

27. 4,978,040 people voted. 1,561,820 voted for Scott. How many voted for someone else?

6,420,000 3,420,000 2,360,000

Multiply.

28. $4 \times 9 =$ ______

29. $6 \times 5 + 4 - 2 \times 3 =$

Divide.

30. $81 \div 9 =$ ______

31. $14,877 \div 1102 =$ ______
(Express remainder two ways - fraction and decimal)

32. $6 \div 6 =$ ______

Solve this problem.

33. Eduardo gave Spanish lessons. He charged $7 a lesson for each student. There were 9 students in his class. How much did he make for each class?

____

Multiply.

34.
$$\begin{array}{r} 87 \\ \times\ 9 \\ \hline \end{array}$$

35.
$$\begin{array}{r} 345 \\ \times\ 8 \\ \hline \end{array}$$

Divide. Show if any numbers are left over.

36. $3\overline{)398}$

37. $37\overline{)8,650}$

Find the answer.

38. Mr. Gomez worked 236 days last year. He made $11,800. How much did he make each day? __________

Add or subtract.

39. $8\frac{9}{11} + \frac{5}{11}$

40. $35\frac{3}{14} - 16\frac{11}{14}$

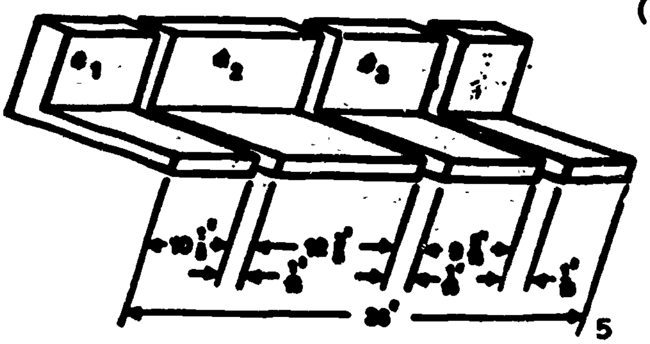
41. $25\frac{5}{8} - 9\frac{3}{4}$

42. $37\frac{3}{8} - 19\frac{5}{6}$

43. Express $\frac{237}{33}$ as a mixed number.

44. Three pieces are cut from the length of angle iron shown in the following sketch. What decimal fraction of the original length of angle iron (36 in.) is the length of angle iron remaining? Include all $\frac{1}{16}$ in. cuts in the computations.

(Note! in. = ")



5

Solve.

45. If a carpenter needs nails longer than 1 3/8-inch, should she use 1 3/4-inch or 1 5/16-inch nails?

____

46. An electrician needs 20 3/4 feet of wiring for one job and 32 3/8 feet for another job. How many feet of wiring are needed for both jobs?

____

47. Patrick needs 1 3/4 cups of sugar for a cake and 1 3/4 cups for the frosting. If he has 3 cups of sugar, does he have enough? If not, how much more does he need?

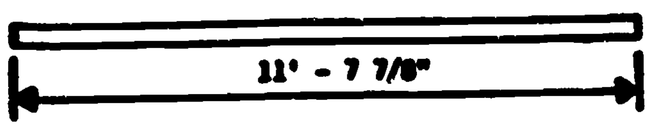
____

Multiply or divide.

48. $3\frac{1}{2} \div 1\frac{3}{4}$

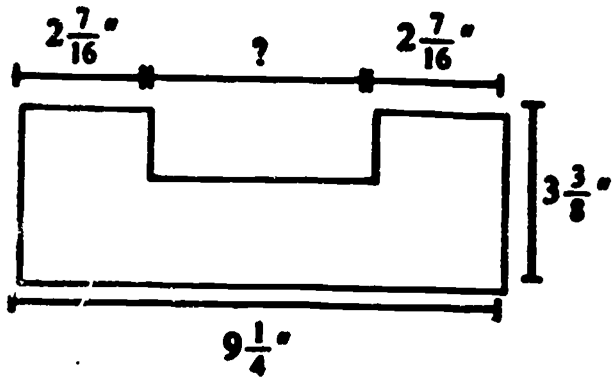
49. $1\frac{1}{5} \times 2\frac{1}{4}$

50. Find the center of the object below.



Solve.

51. Find the missing dimension.

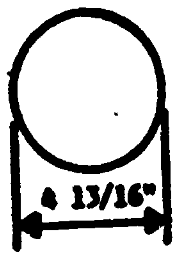


____

Add or Subtract

52. Add .9 and .3

53. Find the center of the circle below.



Multiply or Divide.

54. .0 4 1
x .0 3

55. 34) .0 8 1 6

56. .6) 5 .4 7 2

57. .025) 2 3 7. 5

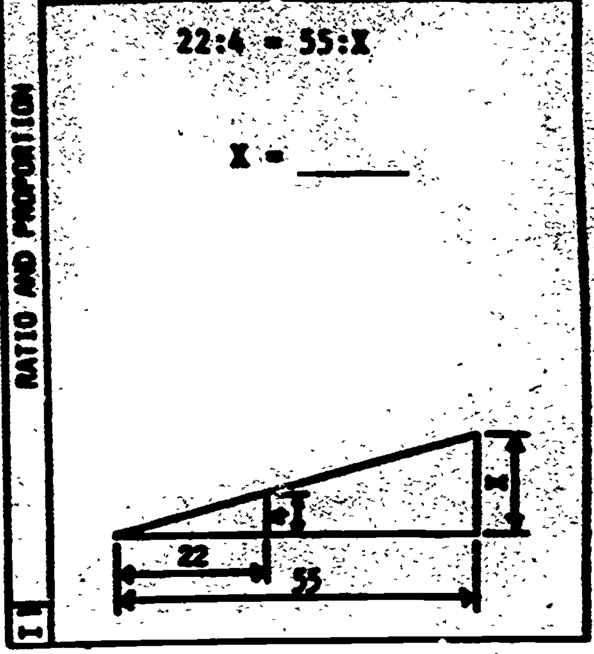
Solve.

58. 9 is _____ percent of 45?

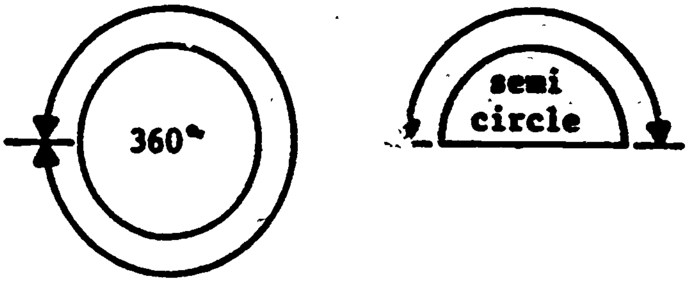
59. What is 5% of 900? _____

60. 80% of what number is 60? _____

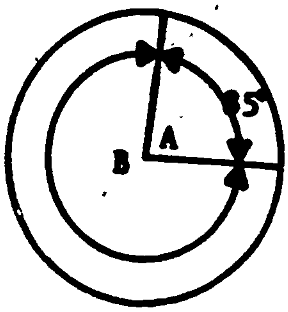
61.



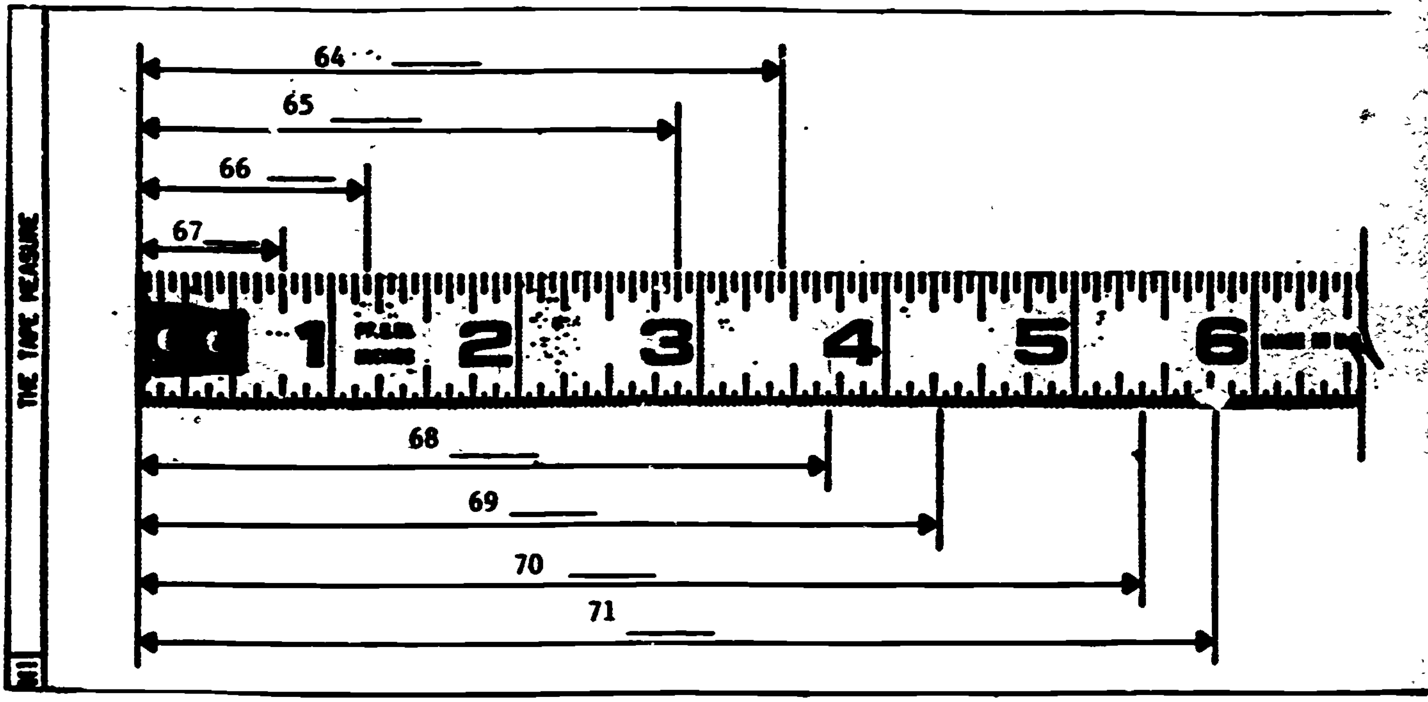
62. A circle is made up of 360° therefore, a semi circle is made up of ________________ degrees.



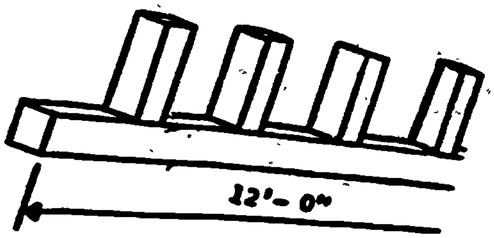
63. In the circle shown below angle A is 85°, therefore, angle B is ______________ degrees.



64 - 71. Below is an American (English) tape measure. Fill in the missing dimension below.



72. You are given the materials and a print of the object on the right. Your job is to lay out the center-lines for the 4 up-right objects to 1/16th + of an inch accuracy so that the spaces are the same size.



Method One -- Solve by using all feet.

Step 1. Draw a sketch of the object and dimension.

Note: There are 4 uprights, but there are 5 spaces.

Step 2. Divide by the number of spaces. (Carry the decimal 3 places if necessary.)

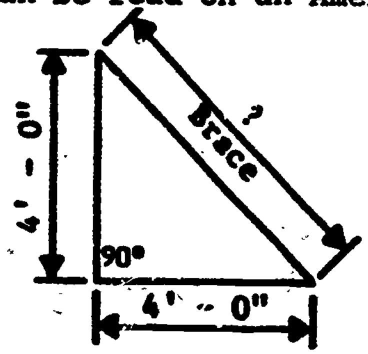
Step 3. Convert the decimal so you can read the dimension on an American (English) tape measure. Multiply by the desired denominator.

Step 4. Make a list of each separate dimension.

73. Find 1/2 of 4'9" = ______________.

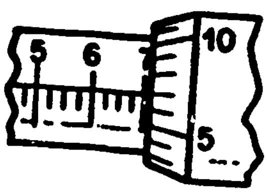
74. Find 1/2 of 8 lb. 5 oz. __________.

75. Find the length of the brace on the right.
Convert answers so it can be read on an American
(English) tape measure.



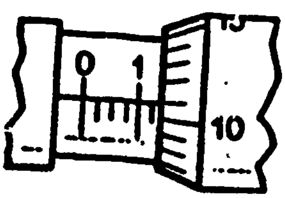
76. Read the settings on the English micrometer scales shown below (0.001").

a.



a. __________

b.



b. __________

ANSWER SHEET

1. ____________
2. ____________
3. ____________
4. ____________
5. ____________
6. ____________
7. ____________
8. ____________
9. ____________
10. ____________
11. ____________
12. ____________
13. ____________
14. ____________
15. ____________
16. ____________
17. ____________
18. ____________
19. ____________
20. ____________
21. ____________
22. ____________
23. ____________
24. ____________
25. ____________
26. ____________
27. ____________
28. ____________
29. ____________
30. ____________
31. ____________
32. ____________
33. ____________
34. ____________
35. ____________
36. ____________
37. ____________
38. ____________
39. ____________
40. ____________
41. ____________
42. ____________
43. ____________
44. ____________
45. ____________
46. ____________
47. ____________
48. ____________
49. ____________
50. ____________
51. ____________
52. ____________

53. ______________

54. ______________

55. ______________

56. ______________

57. ______________

58. ______________

59. ______________

60. ______________

61. ______________

62. ______________

63. ______________

64. ______________

65. ______________

66. ______________

67. ______________

68. ______________

69. ______________

70. ______________

71. ______________

72. ______________

73. ______________

74. ______________

75. ______________

76. ______________

Name ______________________________ Date ____________________

Please do the work in the space provided.
Place your answers on the answer sheet. Unless otherwise requested, express all decimal answers to two (2) decimal places and reduce all fractions to lowest terms.

Subtract

1.
```
   7,005
 -   418
 ________
```

Add

2. 414 + 36 + 389 + 20 = ____________________

Find the Answer

3.
```
   90,855
 -  2,411
 _________
```

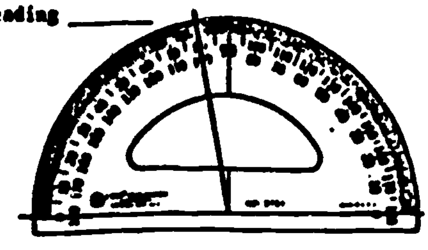
4. $58 + \left(\frac{13 + 32 - 9}{3}\right) - 17 =$

5. Find the average of 77, 68, 93, 55, and 82.

____

6. Write the proper protractor reading shown below.

Reading ______



Estimate to the nearest ten thousand.
Circle the correct answer

7. 4,978,040 people voted. 1,561,820 voted for Scott. How many voted for someone else?

6,420,000 3,420,000 2,360,000

Multiply

8. 6 x 5 + 4 - 2 x 3 = ________________

Divide

9. 14,877 ÷ 1,102 = ____________
(Express remainder two ways - fraction and decimal)

Multiply

10.
```
  87
x  9
____
```

11.
```
  3,405
x    37
_______
```

Divide - Show if any numbers are left over

12. 3) 398

13. 37) 8,650

Find the answer

14. Mr. Gomez worked 236 days last year. He made $11,800. How much did he make each day?

____

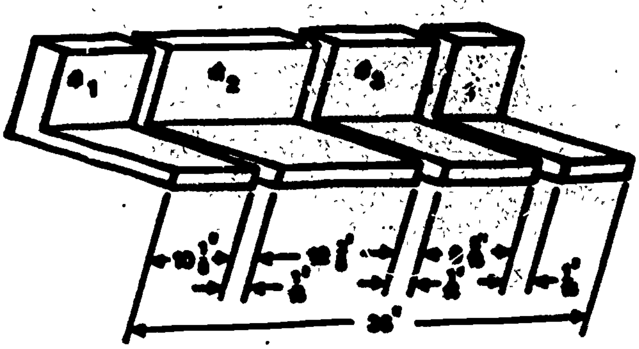
Add or Subtract

15.
```
   8  9
     11
+   5
     11
_______
```

16.
```
   35  3
      14
-  16 11
      14
________
```

19. Express $\frac{237}{33}$ as a mixed number. ________________

20. Three pieces are cut from the length of angle iron shown in the following sketch. What decimal fraction of the original length of angle iron (36 in.) is the length of angle iron remaining?
Include all $\frac{1}{16}$ in. cuts in tha computations.
(NOTE! in. = ")



Solve

21. If a carpenter needs nails longer than 1 3/8-inch, should she use 1 3/4-inch or 1 5/16-inch nails?

____

22. An electrician needs 20 3/4 feet of wiring for one job and 32 3/8 feet for another job. How many feet of wiring are needed for both jobs?

____

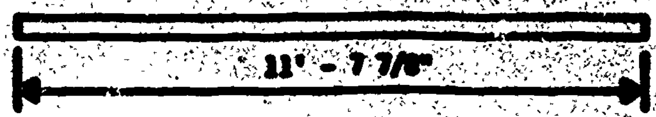
23. Patrick needs 1 3/4 cups of sugar for a cake and 1 3/4 cups for the frosting. If he has 3 cups of sugar, does he have enough? If not, how much more does he need?

____

Multiply or divide

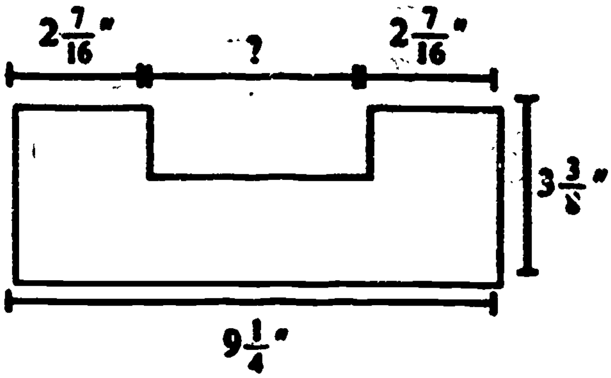
24. $3\frac{1}{2} \div 1\frac{3}{4}$ 25. $1\frac{1}{5} \times 2\frac{1}{4}$

26. Find the center of the object below.



Solve

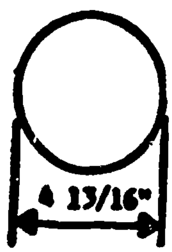
27. Find the missing dimension.



Add or subtract

28. Add .9 and .3

29. Find the center of the circle below.



Multiply or Divide

30.

```
  .041
x .03
______
```

31.

```
34 ) .0816
```

32. .6) 5.472

33. .025) 237.5

Solve

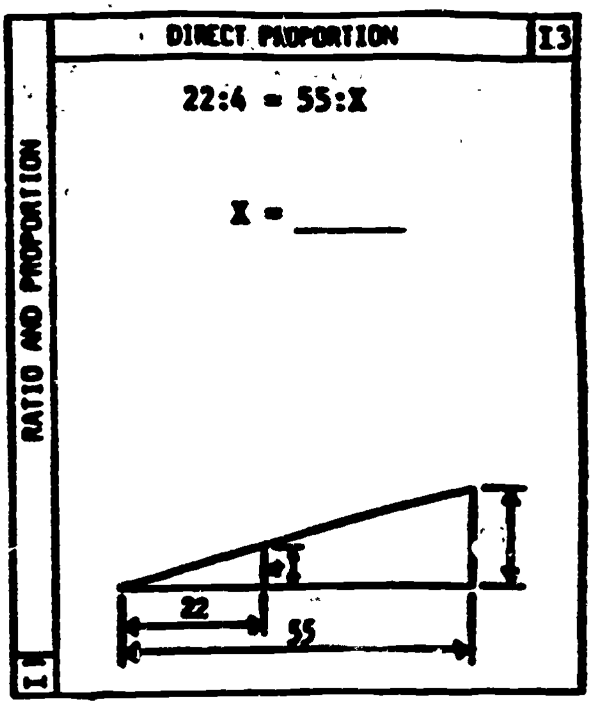
34. 9 is percent of 45? ______

35. What is 5% of 900? ______

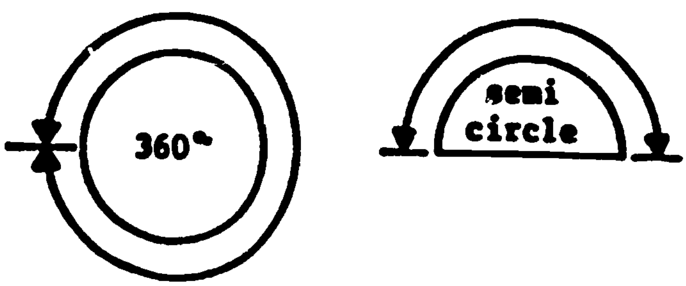
36. 80% of what number is 60?

______

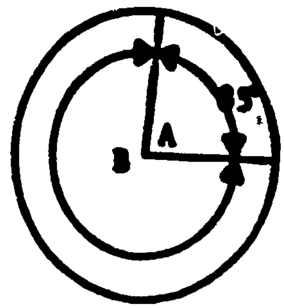
37.



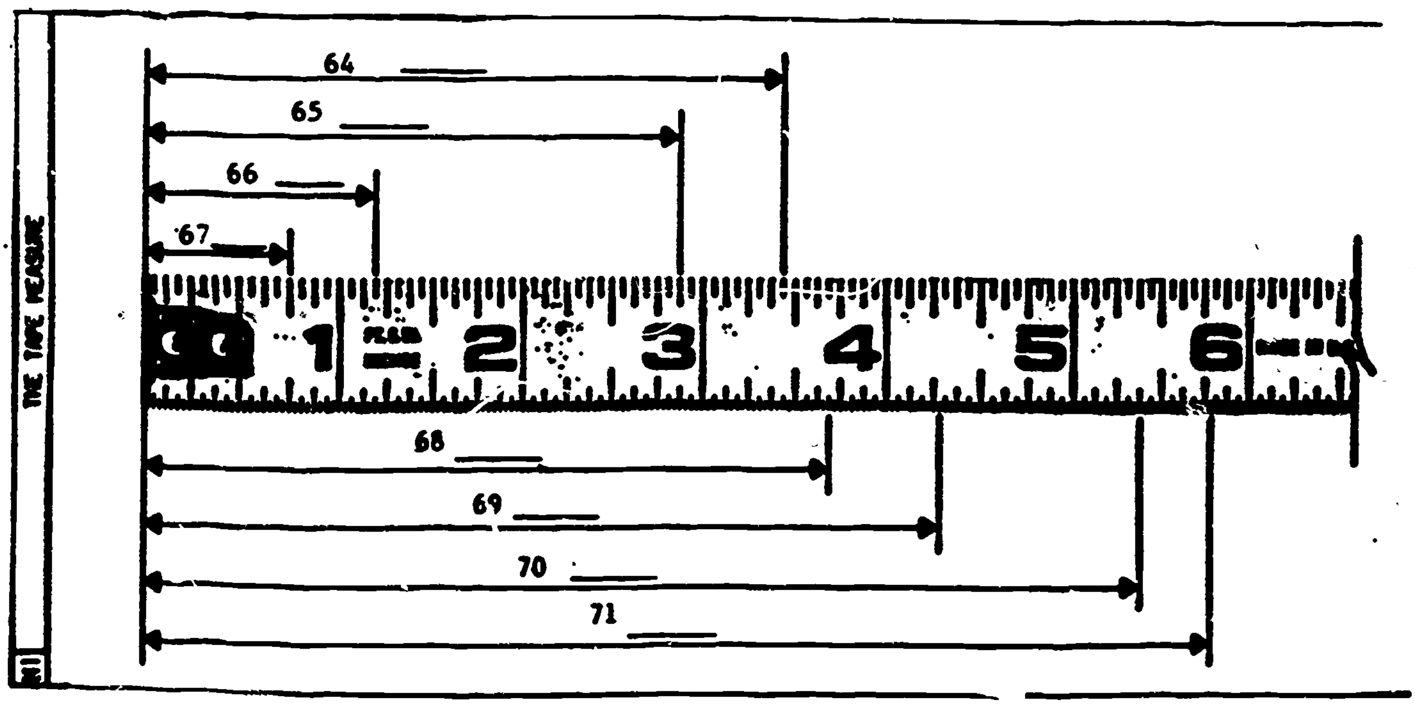
38. A circle is made up of 360° therefore, a semi circle is made up of ______ degrees.



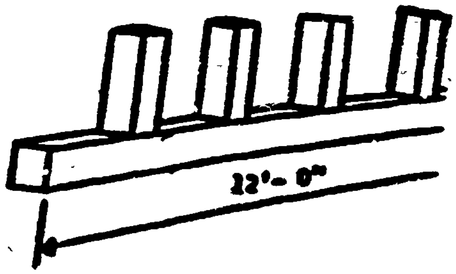
39. In the circle shown below angle A is 85°, therefore, angle B is ______________degrees.



40 - 47. Below is an American (English) tape measure. Fill in the missing dimension below.



48. You are given the materials and a print of the object on the right. Your job is to lay out the center-lines for the 4 upright objects to 1/16th + of an inch accuracy so that the spaces are the same size.



Method One --- Solve by using all feet :

Step 1. Draw a sketch of the object and dimension.

Note: There are 4 uprights, but there are 5 spaces.

Step 2. Divide by the number of spaces. (Carry the decimal 3 places if necessary.)

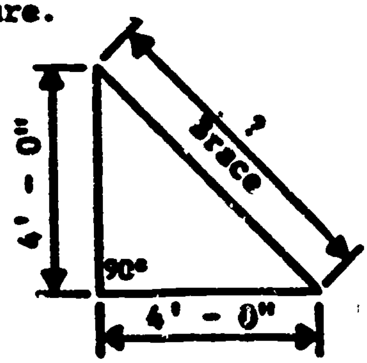
Step 3. Convert the decimal so you can read the dimension on an American (English) tape measure. Multiply by the desired denominator.

Step 4. Make a list of each separate dimension.

49. Find 1/2 of 4'9" = ____________________

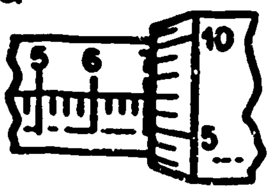
50. Find 1/2 of 8 lb. 5 oz. ________________

51. Find the length of the brace on the right. Convert answers so it can be read on an American (English) tape measure.



52. Read the settings on the English micrometer scales shown below (0.001").

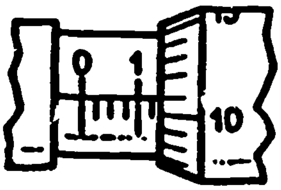
a.



a. ____________________

b. ____________________

b.



Round these numbers to the nearest hundreds

53. 5,364

54. 4,321

Round these numbers to two decimal places

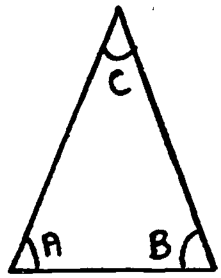
55. 3.14159

56. 73.84793

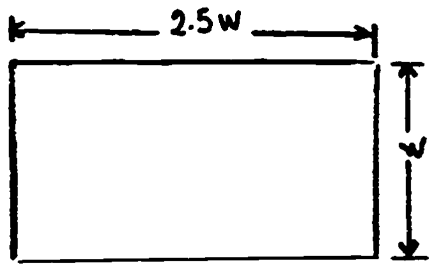
__________ __________

57. 3.64 x 10 = ________________

58. In an isosceles triangle, two angles are equal. In some isosceles triangle, Figure below, angles A and B are equal. Angle C is 15.8 more than angle A. Find the three angles.



59. A storage room is 2.5 times longer than its wide. See Fig. below. The perimeter is 28 m. Find the length and width.



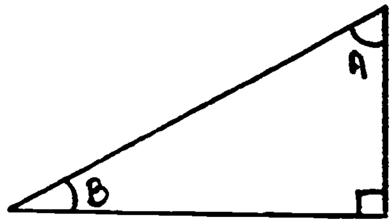
60. The total cost of an electrical shop and the lot was $84,800. The building costs $830 more than eight times the cost of the lot. Find the cost of the building and of the lot.

____

61. A conductor 65 ft long is cut into two pieces. One piece is 5 ft longer than one-half the other. How long is each piece?

____

62. **The sum of the three angles in any triangle is always 180 , regardless of the size and shape. In a right triangle (one 90 angle), one acute angle is twice the order, Fig. below. What are the two acute angles?**



Found the total surface area of the following objects. Round as neccesary.

63. A sphere with a diameter of 3.00 in.

____

64. A rectangle solid with dimensions : 44 cm ; 35.35 cm ; and 10.0 cm.

____

Change to noted unit of measure

65. 3.5 ft = ____________ in.

66. 25 m = ____________ cm.

67. 0.0003 kg. = ____________ g.

ANSWER SHEET

1. ______	23. ______	45. ______
2. ______	24. ______	46. ______
3. ______	25. ______	47. ______
4. ______	26. ______	48. ______
5. ______	27. ______	49. ______
6. ______	28. ______	50. ______
7. ______	29. ______	51. ______
8. ______	30. ______	52. ______
9. ______	31. ______	53. ______
10. ______	32. ______	54. ______
11. ______	33. ______	55. ______
12. ______	34. ______	56. ______
13. ______	35. ______	57. ______
14. ______	36. ______	58. ______
15. ______	37. ______	59. ______
16. ______	38. ______	60. ______
17. ______	39. ______	61. ______
18. ______	40. ______	62. ______
19. ______	41. ______	63. ______
20. ______	42. ______	64. ______
21. ______	43. ______	65. ______
22. ______	44. ______	66. ______
		67. ______

UNIT 1: WHOLE NUMBERS

OBJECTIVES

After studying this unit, you will be able to:

- add, subtract, multiply, and divide whole numbers
- understand the meaning of a whole number or unit
- be able to pick out key words in written problems
- understand the order of operations in solving problems
- solve word problems

Dictionary: Look for the following helpful words which will determine the type of problem.

		Arithmetic Sign or Operation
1.	total	+
2.	sum	+
3.	remaining	-
4.	difference	-
5.	increased by	+
6.	decreased by	-
7.	more than	+
8.	less than	-
9.	added to	+
10.	plus	+
11.	minus	-
12.	subtracted from	-
13.	calculate	perform operations indicated by words or signs
14.	solve	calculate

15.	find	calculate
16.	answer units	report answer in same measure or name as applied to the numbers in the problems
17.	measure	inches, pounds, acres, devices, gallons, etc.

Section 1: Using a Calculator (based on available model)

1-A Instruction for addition (+): 13 + 98 =

1-B Examples

1-B.1 25 + 39 = ________

1-B.2 107 + 1425 = ________

1-B.3 111 + 17 + 25,739 = ________

1-C Instruction for subtraction (-): 98 - 13 =

1-D Examples

1-D.1 129 - 35 = ________

1-D.2 203 - 193 = ________

1-D.3 1195 - 50 - 165 = ________

DRILL SHEET

UNIT 1: USING CALCULATOR - ADDITION AND SUBTRACTION

1D-1	3 + 79 =	______
1D-2	11 + 97 =	______
1D-3	14 + 35 =	______
1D-4	33 + 33 =	______
1D-5	79 - 14 =	______
1D-6	55 - 52 =	______
1D-7	83 - 14 =	______
1D-8	22 - 6 =	______
1D-9	44 + 33 =	______
1D-10	25 + 18 =	______
1D-11	44 + 13 - 11 =	______
1D-12	58 + 58 - 47 =	______
1D-13	47 - 3 + 4 =	______
1D-14	29 + 29 - 23 =	______
1D-15	33 - 19 - 11 =	______
1D-16	1,001 + 1,275 =	______
1D-17	1,975 + 10,345 =	______
1D-18	11,989 - 8,763 =	______
1D-19	5,040 - 3,009 =	______
1D-20	25,876 + 23,999 =	______
1D-21	33,011 - 17,666 =	______
1D-22	38,848 + 160 =	______
1D-23	38,848 - 1,606 =	______
1D-24	74,929 - 119 =	______
1D-25	73,919 - 108 =	______

Section 2: Using a Calculator

2-A Instruction for multiplication (x): 5 x 16 =

1. set decimal indicator at zero
2. enter 5
3. press [x] key
4. enter 16
5. press [=] key; display shows 80

2-B Examples

2-B.1	14 x 11 =	______
2-B.2	179 x 555 =	______
2-B.3	139 x 93 =	______

2-C Instruction for division (÷): 75 - 5 =

1. set decimal indicator at zero
2. enter 75
3. press [÷] key
4. enter 5
5. press [=] key; display shows 15

2-D Examples

2-D.1	555 ÷ 15 =	______
2-D.2	40 ÷ 8 =	______
2-D.3	900 ÷ 50 =	______

DRILL SHEET

UNIT 1: USING CALCULATOR - MULTIPLICATION AND DIVISION

2D-1	17 x 3 =	______
2D-2	11 x 19 =	______
2D-3	107 x 4 =	______
2D-4	905 ÷ 5 =	______
2D-5	168 ÷ 4 =	______
2D-6	238 ÷ 2 =	______
2D-7	919 x 804 =	______
2D-8	75 x 75 =	______
2D-9	1000 ÷ 25 =	______
2D-10	500 ÷ 125 =	______
2D-11	63 x 44 x 2 =	______
2D-12	11 x 97 x 8 =	______
2D-13	5555 ÷ 11 =	______
2D-14	16 ÷ 4 =	______
2D-15	108 ÷ 4 =	______
2D-16	12 x 3 x 48 =	______
2D-17	798 ÷ 14 =	______
2D-18	999 ÷ 333 =	______
2D-19	126 ÷ 3 =	______
2D-20	9 x 15 =	______
2D-21	3088 ÷ 4 =	______
2D-22	108 ÷ 9 =	______
2D-23	2727 ÷ 3 =	______
2D-24	74 x 11 =	______
2D-25	555 x 232 =	______

Section 3: Place Value

A. You are a clerk at a retail store. All you have is dimes and pennies. How do you give someone 43 cents change using the fewest number of coins?

B. 43 = 10 10 10 10 1 1 1

28 =

C. 43 =

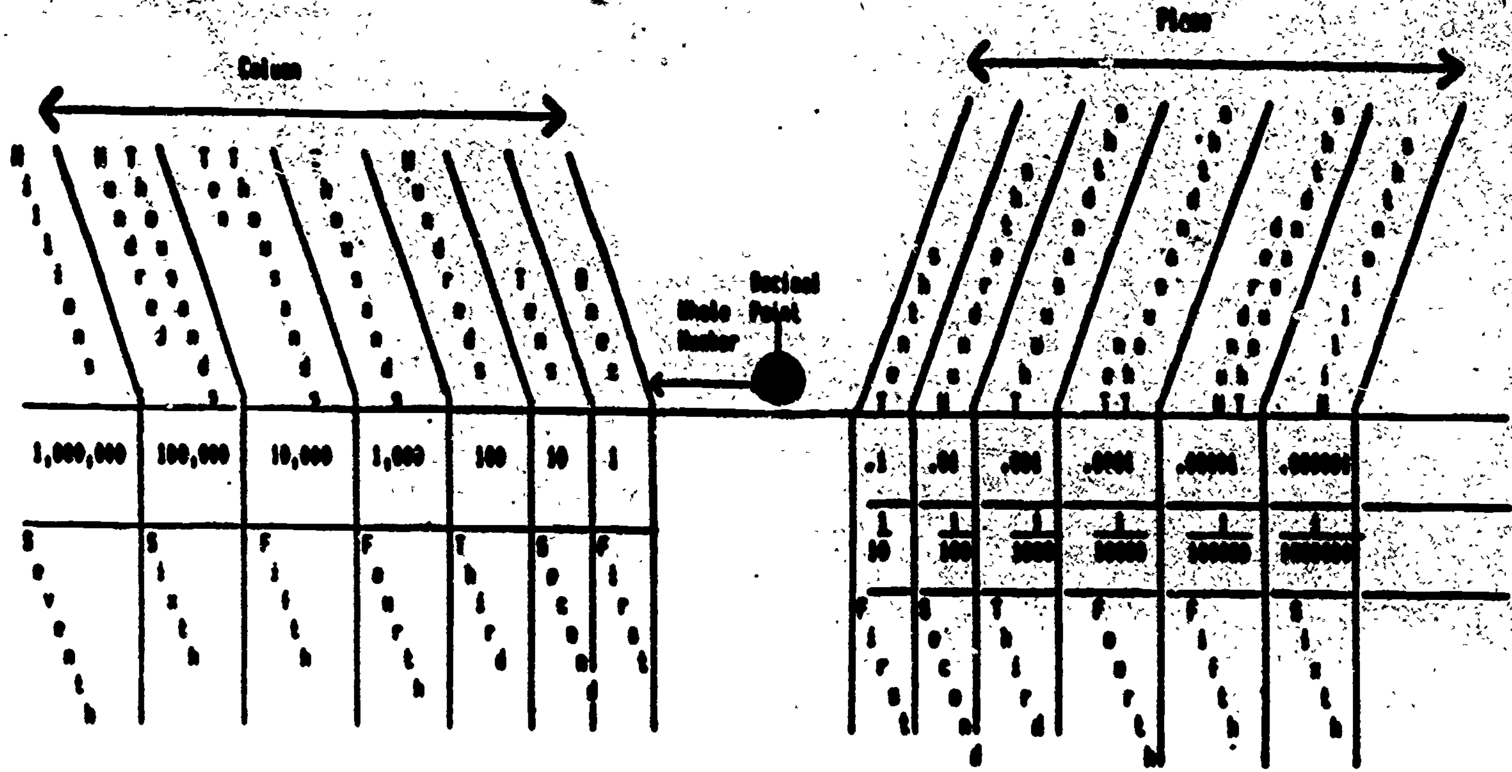
+ 28 =

OR

D. Decimal System Place Value

1. One hundred twenty-three
2. Two thousand thirty-five
3. Thirty thousand seven
4. Sixteen thousand five hundred

	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones
1.					1	2	3
2.							
3.							
4.							



Whole numbers are to the left of the decimal point, and numbers less than one whole (1) are to the right of the decimal point. (In written number form, the word <u>and</u> represents the decimal point.)

Using the chart above as a guide, write the following numbers:

1. Thirty thousand seven ____________________

2. One hundred twenty thousand one and thirty seven hundredths ____________________

3. Two hundred two and five thousandths ____________________

4. Seven and forty-five ten thousandths ____________________

5. Two million six hundred thousand forty-one ____________________

Section 4: What is a whole?

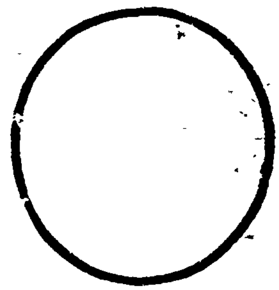


Fig. 1.4

The circle above could represent a <u>whole</u> grapefruit, or a <u>whole</u> dollar. It could represent anything that was complete, meaning nothing leftover or missing.

We use integers which are the numerals we associate with counting to make up whole numbers. These are 1,2,3,4...

Word Problems - Addition & Subtraction

Earlier in Lessons 1 and 2 we learned how to add, subtract, multiply, and divide using a calculator. We will now learn that there are words that instruct us what to do in arithmetic operations.

Examples

1. Bob's hours for the past week are listed below:

   a. What is the total number of hours he worked?

   ____

   b. If any hours over 40 hours per week are overtime, how many hours of overtime did he work?

Monday:	7
Tuesday:	10
Wednesday:	6
Thursday:	12
Friday:	11

   ____

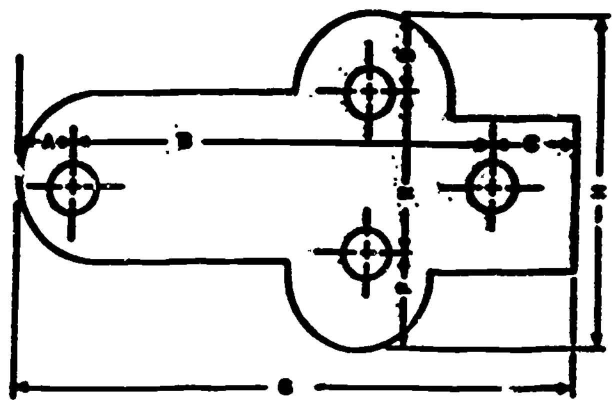
2. Sue's yearly salary at the XYZ Company was $26,700. When the company relocated, Sue accepted a position with the ABC Company for

$1,900 less than she was making at XYZ. How much is she making at ABC?

____

3. Shown below is a board used to manufacture micro-electronic devices. The letters, A through H, represent dimensions on the board; the dimension for each letter is measured between lines touched by the arrowheads.

In this exercise, 12 different sets of dimensions are given with 2 dimensions missing in each set. Solve for the missing dimensions (in millimeters); place answers in blank spaces: * Helpful if a straight edge is used under each line.

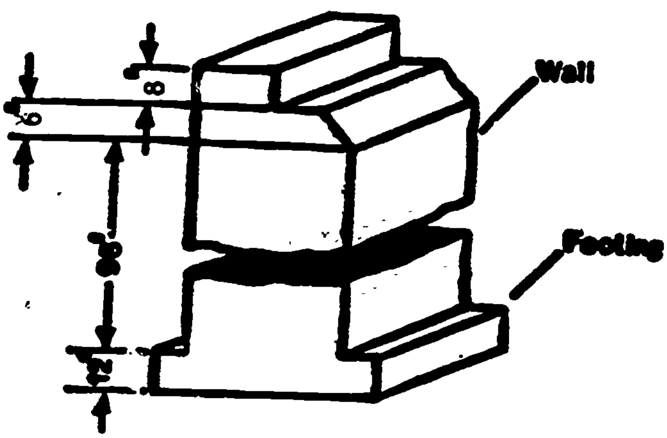


	A	B	C	D	E	F	G	H
1.	3	15	4	5	7	6		
2.	24	67	27	22	38	27		
3.	107	312	98	98	127	79		
4.	206	781	196	184	307	129		
5.	10	40		7	14		63	27
6.	92	238		79	143		400	300
7.	22		19	30		29	80	140
8.	103		98		48	33	387	101
9.	237		196	41	103		701	172
10.		39	29		47	29	100	101
11.		96	68	47		48	210	190
12.		207	109	117	290		400	500

DRILL SHEET

UNIT 1: WORD PROBLEMS - ADDITION, SUBTRACTION, MULTIPLICATION & DIVISION

3D-1 A section of a concrete wall and footing for a clean room are shown in the figure below. What is the total height of the wall and footing?

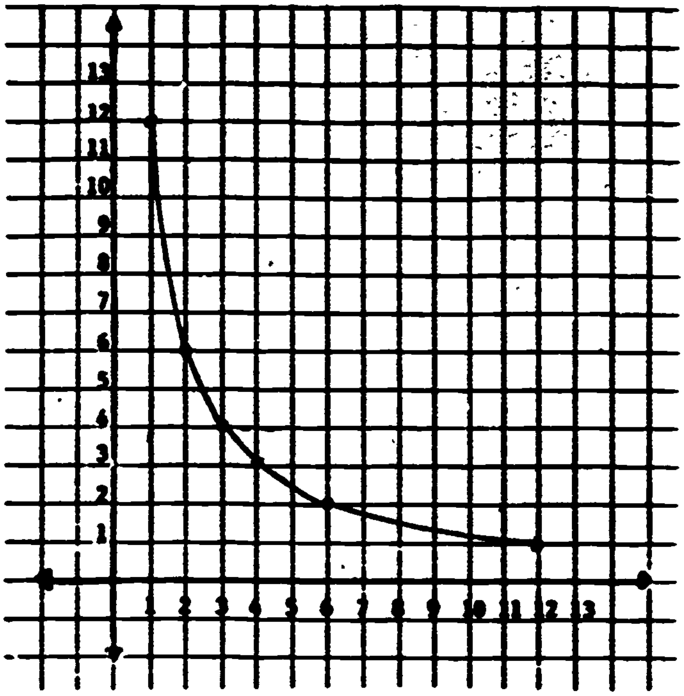


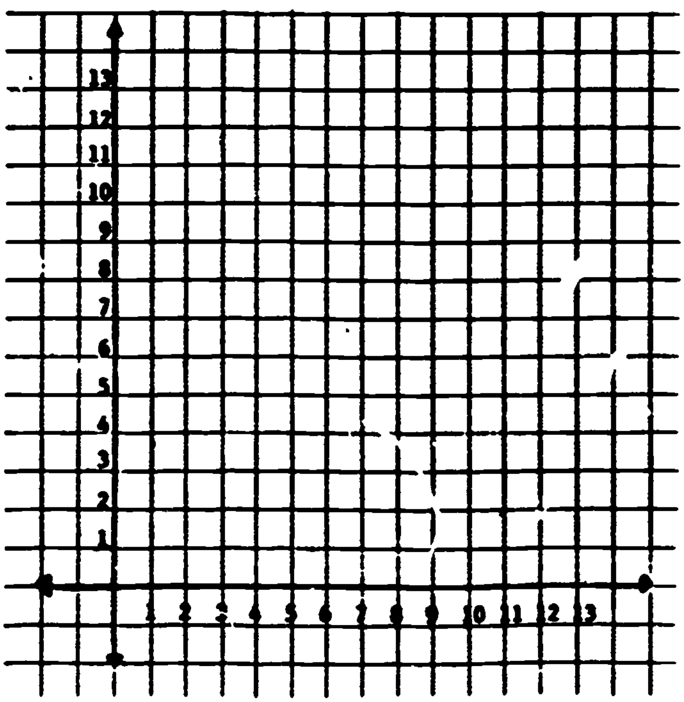
____

3D-2 During the first week of April, the following number of wafers were cut from silicon rods: 5,570 wafers on Monday; 7,855 wafers on Tuesday; 7,236 wafers on Wednesday; 6,867 wafers on Thursday; and 6,643 wafers on Friday. During the following week, 4,050 more wafers were cut than during the first week. Find the total wafers cut during the first two weeks of April.

____

3D-3 The figure below shows a graph constructed for the product 12. Construct a graph for the product 24. Will it have the same shape?





3D-4 A duplicating machine operator at ALPO makes copies of printed material for various departments. A log is kept to record the number of copies made for each department. The monthly log is shown in Fig. 1-2:

	NUMBER OF COPIES			
	WEEK 1	WEEK 2	WEEK 3	WEEK 4
DEPARTMENT				
Production	853	712	956	1088
Eng. & Design	1050	936	277	732
Personnel	2756	1935	2080	993
Accounting	830	0	344	130
Sales	1202	555	3859	2444
Data Processing	85	53	0	187
Purchasing	1932	1637	767	845
Inspection	177	286	53	0
Rec. & Shipping	538	613	423	778

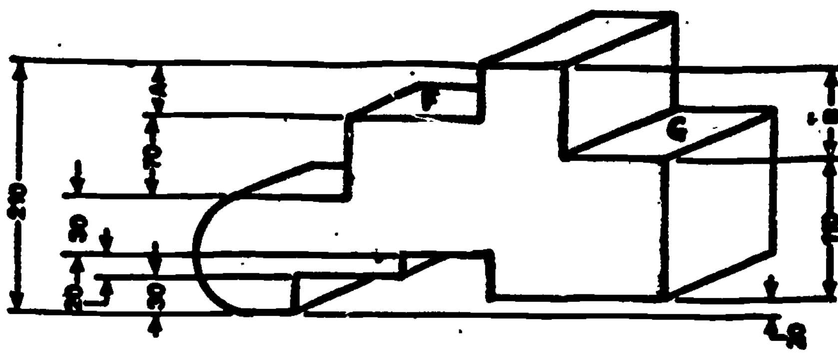
Fig. 1-2

a. Find the total number of copies made in week 1 _____
b. Find the total number of copies made in week 2 _____
c. Find the total number of copies made in week 3 _____
d. Find the total number of copies made in week 4 _____
e. Find the total number of copies made for the month: _____

3D-5 ALPO was charged $465 for repairs to the company shuttle bus. The charges for labor are $196. Paint and materials cost $67 and replacement parts cost $110. How much profit is made by the outside contractor?

_____

3D-6 The tool and die department must know dimensions A and B in order to finish grind surfaces F and G of the component for a testing machine shown in the figure below. All dimensions are in millimeters:



a. Solve for A; report answer in millimeters______

b. Solve for B; report answer in millimeters______

3D-7 The cafeteria baker makes a batch of cookie mix which weighs 48 pounds. Pastry flour and other ingredients are used to make the mix. The weights of the other ingredients are 13 pounds of almond paste, 12 pounds of margarine, 5 pounds of egg whites, and 7 pounds of granulated sugar. How many pounds of pastry flour are used? ______

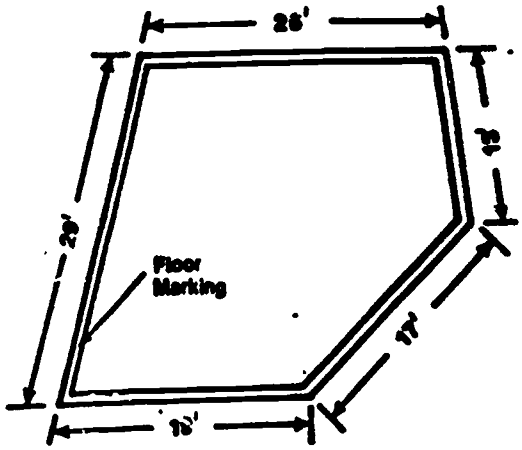
3D-8 Sue March went to town and bought a skirt for $22, and a hat for $7. How much money did she receive in change from two $20 bills? ______

3D-9 During a basketball game, seven players scored the following points: 12, 9, 21, 16, 14, 17, and 35. If these were the only scores for the team, what was the team's final score? ______

3D-10 There are three machines available to saw-cut silicon rods. Machine A cuts 13 inches per minute, Machine B cuts 8 inches per minute, and Machine C cuts 11 inches per minute. How many inches of silicon can be cut in 1 minute if all three machines are working? ______

3D-11 If four containers in a storage area have a capacity of 12 gal., 27 gal., 55 gal., and 21 gal., could 100 gal. of acid be stored in those containers? ______

3D-12 How many feet (') of floor marking are needed to enclose the inspection area illustrated in the following figure:



______

3D-13 During a one-month period, the maintenance department installed the following numbers of switch outlets: 23, 14, 36, 27, 19, 21, 34, and 28. What is the total number of outlet boxes installed?

______

3D-14 The credit union clerk received the following amounts from employees: $85, $125, $137, $96, and $109. How much total money was received?

______

3D-15 The shipping department air freighted the following amounts of micro-electronic components: 7,286 lb., 8,106 lb., 7,832 lb., and 8,215 lb. What was the total weight shipped? ______

3D-16 The following pounds of solder were purchased by ALPO during a one-month period: 40, 75, 125, 70, 150, 80, 95, 110, and 60. What is the total number of pounds of solder purchased that month?

______

3D-17 **Three lengths of tubing for packaging are needed: 15 in., 18 in., and 10 in. Can these pieces be cut from a length of tubing 42 in. long?** ______

3D-18 **An ALPO pipefitter needs 23 pieces of pipe. Each piece must be 443 centimeters (cm) long. What is the length of pipe needed?** ______

3D-19 **How many reams of paper are required by a printer who has 12 jobs to do for ALPO, each of which requires 32 reams of paper?** ______

3D-20 **If the maintenance contractor has 15 employees who are paid $185 each per week, what is the weekly payroll for the employees?** ______

3D-21 **A truck can haul 400 fixtures. How many fixtures can be moved by the truck in 15 trips?** ______

3D-21 **Thirty-five rolls of electrical cable were ordered, each contains 1025 ft. What is the total number of feet of cable on the rolls?** ______

3D-22 **An ALPO construction estimator earns $2600 per month. How much is earned in 6 months?** ______

3D-23 **How many resistors are needed to build 30 TV's if each TV requires 305 resistors?** ______

3D-24 **An assembly line can produce 23,090 parts a day. If the assembly line has no problems, how many parts can be produced in 20 working days?** ______

3D-25 **What is the weekly payroll if 48 employees each earn $218 a week?** ______

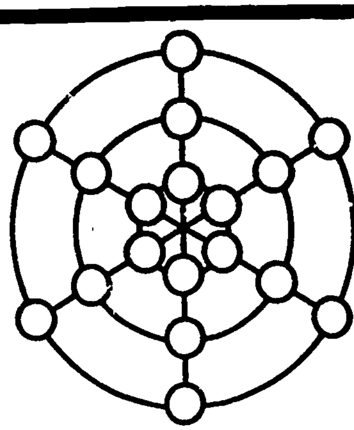
3D-26 **In one ALPO building where 46 outlets are being installed, 1472 ft. of cable are used. What is the average number of feet used per outlet?** ______

3D-27 **Twelve water tanks are constructed in a welding shop at a total contract price of $14,940. What is the price per tank?** ______

3D-28 Fifteen packages of conduit are purchased. The total weight of the shipment is 3120 lb. What is the weight per package?

3D-29 Now try this!

Arrange numbers in the figure below so that the sum around each circle and along each straight line is the same.



3D-30 With experience, the distributive property can be used for more difficult mental computations.

Thought Process

35 x 102 = 35 x (100 + 2)
= (35 x 100) + (35 x 2)
= 3500 = 70
= 3570

Thought Process

45 x 98 = 45 x (100 - 2)
= (45 x 100) - (45 x 2)
= 4500 - 90
= 4410

Can you find these products mentally?

15 x 103   101 x 92   18 x 99

Section 5: Order of Operations

1. First, do all operations within grouping symbols. Grouping symbols are parentheses (), brackets [], and braces {}.

2. Next, do multiplication and division operations in order from left to right.

3. Last, do addition and subtraction operations in order from left to right.

* Reminder that • is a sign for multiplication also.

Examples

1. Find the value of

	(15 + 6) x 3 - 28 - 7	=	
	21 x 3 - 28 - 7	=	
	63 - 28 ÷ 7	=	
	63 - 4	=	59
2.	9 + 12 - 5	=	
	21 - 5	=	16
3.	35 + 30 ÷ 5	=	
	35 + 6	=	41
4.	(35 + 30) - 5	=	
	65 - 5	=	13
5.	(10 x 8) - (5 • 4)	=	
	80 - 20	=	4
6.	(240 ÷ 80) • 15 - 3	=	
	3 x 15 - 3	=	
	45 - 3	=	15

DRILL SHEET

UNIT 1: ORDER OF OPERATIONS

4D-1	12 + 4 ÷ 2	= ______
4D-2	26 ÷ 2 x 8	= ______
4D-3	6 x 5 + 4 - 2 x 3	= ______
4D-4	10 ÷ 5 + 10 - 2 x 4	= ______
4D-5	96 ÷ (3 x 2) + 4 x (3 x 5)	= ______
4D-6	[6 + (3 - 1) x 2]	= ______
4D-7	86 - [5 x (16 + 7 - 2) - 3]	= ______
4D-8	4 ÷ 2 x 5 x 9 ÷ 3	= ______
4D-9	(26 + 2) - (5 + 1)	= ______
4D-10	4 x 5 + (6 - 4) ÷ 2	= ______
4D-11	18 ÷ 2 x (4 + 3)	= ______
4D-12	24 ÷ 3 x (4 + 1)	= ______
4D-13	8 x 3 ÷ 2 + 6 x 7	= ______
4D-14	(410 x 3) - 87	= ______
4D-15	27 ÷ 3 + 99 x (6 - 5)	= ______
4D-16	97 - 3 x 11 + 8	= ______
4D-17	[7 x (11 x 3) - 8 x (7 + 3)]	= ______
4D-18	{19 + [3 x (9 + 4) - 9 - 3] - 11}	= ______
4D-19	{27 ÷ 3 x 2 x [27 - 6 x 4] + 27}	= ______
4D-20	{27 + [(11 - 9) - 2 x 7 x (6 + 5)] ÷ 11}	= ______

DRILL SHEET

UNIT 1: PRACTICAL APPLICATIONS REQUIRING ORDER OF OPERATIONS OF ARITHMETIC EXPRESSIONS

5D-1 An engine is used by ALPO's receiving department to lift heavy crates on the loading dock. The horsepower needed to lift crates can be found as follows:

horsepower = (weight of crate x distance lifted - time) ÷ 550

Find the horsepower needed to lift each of the crates listed in the following table:

CRATE	WEIGHT OF CRATE	DISTANCE LIFTED	TIME	HORSEPOWER
A	660	10	4	
B	1100	12	6	
C	1650	14	7	
D	3300	8	16	
E	2640	10	8	

5D-2 The accounting department computes the annual depreciation of each piece of tooling, equipment, and machinery in ALPO. From a detailed itemized list, the accounting department groups all items together that have the same life expectancy (number of years of usefulness) as shown in the following table:

GROUP	COST	FINAL VALUE	NUMBER OF YEARS OF USEFULNESS	ANNUAL DEPRECIATION
a. Tooling	$14,500	$ 1,200	5 years	
b. Equipment	$28,350	$ 3,750	6 years	
c. Equipment	$17,900	$ 2,040	10 years	
d. Machinery	$67,700	$ 7,940	8 years	
e. Machinery	$80,300	$10,600	10 years	

TOTAL ANNUAL DEPRECIATION ______

Find the annual depreciation for each group and the total depreciation of all tooling, equipment, and machinery, as follows:

Annual Depreciation = (cost - final value) ÷ number of years of usefulness

5D-3 * **A general rule is used in the health field to determine a child's dosage of medicine; it is as follows:**

$$\text{Child's dosage} = \text{age of child} \div \text{age of child} + 12 \times \text{average adult dosage}$$

What dose (number of milligrams) of morphine sulfate should be given to a 3 year-old child if the adult dose is 10 milligrams?

____

* **It should be noted that the weight of a child plays a large role in the dosage of medicine.**

UNIT 1: REVIEW

-1R1- 362 + 1,491 + 73 + 29,248 ________

-1R2- 4,793 - 404 ________

-1R3- 7,878 x 403 ________

-1R4- 1755 ÷ 27 ________

-1R5- $58 + \frac{13 + 32 - 9}{3} - 17$ ________

-1R6- A printing shop prints 9,600 ALPO letterheads on Monday, 11,760 on Tuesday, 13,354 on Wednesday, 8,846 on Thursday, and 12,215 on Friday. How many letterheads are printed during the week?

____

-1R7- Five machines at ALPO produce the same product. Each machine has a counter which records the number of parts produced. Counter readings for the beginning and end of one week's production are shown in the following table:

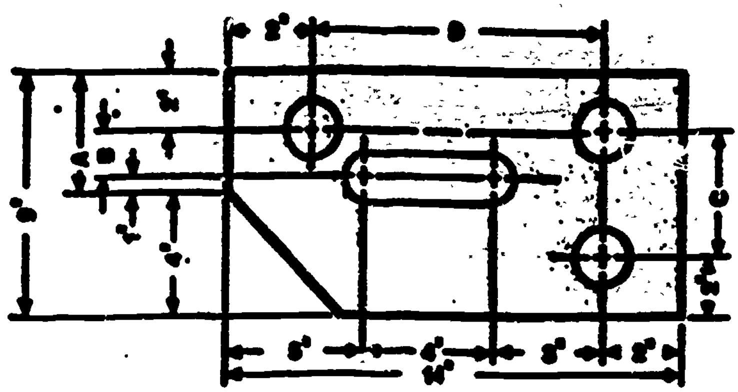
	Machine 1	Machine 2	Machine 3	Machine 4	Machine 5
Counter Reading Beginning of Wk	18,925	14,382	8,408	36,604	903
Counter Reading End of Week	47,763	41,206	36,441	70,335	28,027
Number of Parts Produced					

a. How many parts does each machine produce during the week?

b. What is the total weekly production?

____

-1R8- In order to make the photostat fixture shown below, a tool and die maker must determine dimensions A, B, C, and D.



a. Dimension A ________

b. Dimension B ________

c. Dimension C ________

d. Dimension D ________

-1R9- ALPO's Pioneer Store orders 18 cartons of hand calculators. Each carton contains 64 calculators. They pay $9.00 per calculator, and then sell each at $7.00 profit. What is the total money collected when all the calculators are sold?

____

-1R10- If one piece of shipping tubing is 111 in. long, how much will be left if you cut a 79-in. piece off the tubing?

____

-1R11- Joe weighs 236 lb. now. Joe weighed 294 lb. two years ago. How many pounds did Joe lose during the past two years?

____

-1R12- The cooling system of a car will hold 24 qt. of water and antifreeze. If the cooling system is full and contains 2 gal. of antifreeze, how many quarts of water are in the cooling system? (1 gal. = 4 qts.)

____

-1R13- If your church social group made 4,000 hoagies for a benefit and distributed 2,641 of them, could they fill an order for 1,540 more?

______

-1R14- A ream of paper contains 500 sheets. How many sheets are contained in 23 reams of paper?

______

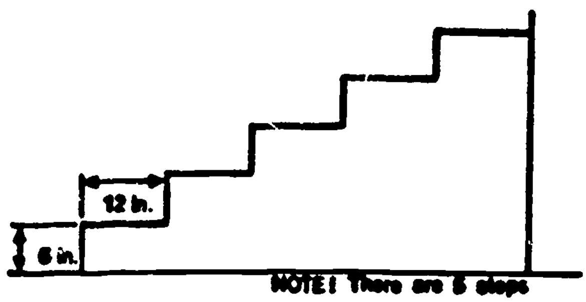
-1R15- ALPO has 17 cars with six-cylinder engines that need the spark plugs replaced. If the plugs come in packages of eight, how many packages do they purchase to handle this job?

______

-1R16- A part-time computer operator earned $1,755 in 13 weeks. How much did he earn per week?

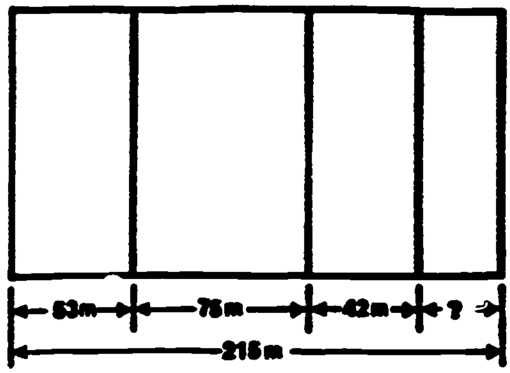
______

-1R17- How long a piece of carpet (in inches) should you buy for a runner for the steps shown in the figure below:



______

-1R18- Find the missing measure in the following figure:



-1R19- For her annual salary, a furnace operator received the following monthly pays: $1096; $1117; $1067; $1742; $1896; $2056; $2308; $1483; $1597; $1223; $1074; and $1029. What was the furnace operator's annual salary? ________

-1R20- If a machine setter earns $23,400 annually, what is the monthly salary? ________

-1R21- [21 ÷ (7 x 3) + (4 x 5) + 25] = ________

-1R22- (720 ÷ 360) + (180 x 2) - 1 = ________

UNIT 2: COMMON FRACTIONS

OBJECTIVES

After completing this unit, you will be able to:

- understand the meaning of a fraction, or part of the whole
- define the terms numerator, denominator, improper fraction, proper fraction, and mixed number
- write equivalent fractions
- express fractions in lowest terms
- understand the meaning of a mixed number
- express fractions as mixed numbers
- express mixed numbers as fractions
- determine lowest common denominator
- add fractions, mixed numbers, and whole numbers
- subtract fractions, mixed numbers, and whole numbers
- multiply fractions, mixed numbers, and whole numbers
- divide fractions, mixed numbers, and whole numbers
- follow order of operations involving fractions
- solve word problems involving fractions

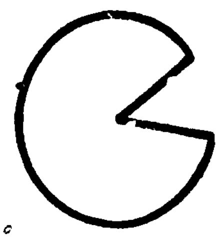
Dictionary:

		Meaning
1.	Fraction	part of the whole
2.	Numerator	number in a fraction above divisor line; for parts of the denominator
3.	Denominator	number in a fraction below divisor line; for how many parts make up the whole

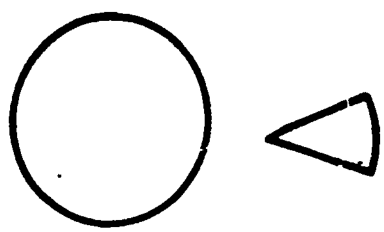
4.	Terms	numerator and denominator in a common fraction
5.	Common	familiar.
6.	Common Fraction	one whole number divided by another whole number
7.	Proper Fraction	less than a whole or one
8.	Improper Fraction	more than a whole or one
9.	Mixed Number	a whole number with a proper fraction
10.	Equivalent	identical or same

Section 1: Concept of a Fraction

1-A What is a fraction?



Note in the above illustration that a part, slice, or fraction of the whole circle or "pie" is missing.



Now note in the above illustrations that there is a whole circle or "pie" on the left and a part or slice of the same size circle or pie on the right. This slice is a fraction of the whole remaining, or "leftover."

Thus, the subject of fractions is nothing more than the study of parts or portions that make up the whole number.

A whole number, as defined earlier in Unit 1, means "nothing leftover." For example when you divide any number by itself, the result will <u>always</u> equal one.

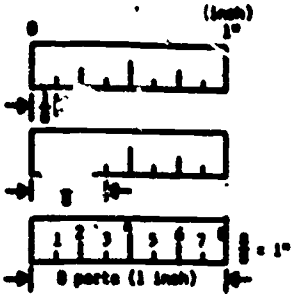
Examples

$\frac{11}{11} = 1;$ $\frac{135}{135} = 1;$ $\frac{63}{63} = 1;$ $\frac{1025}{1025} = 1$

1-B When we divide one whole number by another whole number, we create a <u>common fraction</u>.

Every common fraction consists of two parts: a whole number above the division sign, called the <u>numerator</u>; and a whole number below the division sign, called the <u>denominator</u>.

The <u>denominator</u> indicates the fractional parts that make up the whole. When we speak of a steel rule or ruler or yardstick having an inch with quarter (1/4), eighth (1/8), sixteenth (1/16), or thirty-second (1/32) graduations, we are indicating the number of parts of these that make up the whole: $\frac{4}{4}$; $\frac{8}{8}$; $\frac{16}{16}$; $\frac{32}{32}$.



There are two types of common fractions, proper and improper. In a proper fraction, the numerator (upper number) is always less, i.e., smaller than the denominator (lower number). In an improper fraction, the numerator is always greater, i.e., larger than the denominator.

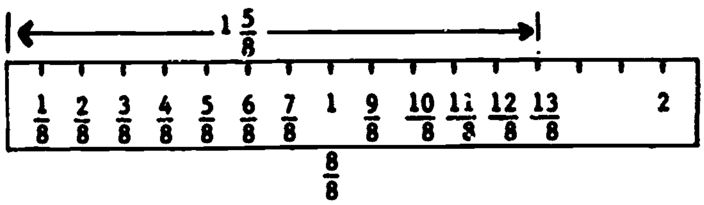
1-C Examples

proper	improper
$\frac{1}{4}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{9}{16}$, $\frac{17}{32}$	$\frac{5}{4}$, $\frac{9}{8}$, $\frac{7}{6}$, $\frac{19}{16}$, $\frac{37}{32}$

As you can see from the preceding examples, proper fractions are less than a whole, and improper fractions are more than a whole, (or greater than 1).

Note: One reason for reducing an improper fraction such as $\frac{13}{8}$, is that measuring-instrument parts (increments) are usually written and located in reduced form.

Example:



[It is easier to find 1 $\frac{5}{8}$ on a tape measure than to count out thirteen $\frac{1}{8}$ parts.]

DRILL SHEET

UNIT 2: PROPER AND IMPROPER FRACTIONS

Complete the following:

Proper Fractions		Improper Fractions	
1D-1	$\frac{\quad}{2}$	1D-16	$\frac{\quad}{2}$
1D-2	$\frac{\quad}{4}$	1D-17	$\frac{\quad}{4}$
1D-3	$\frac{\quad}{8}$	1D-18	$\frac{\quad}{8}$
1D-4	$\frac{\quad}{16}$	1D-19	$\frac{\quad}{16}$
1D-5	$\frac{\quad}{32}$	1D-20	$\frac{\quad}{32}$
1D-6	$\frac{\quad}{6}$	1D-21	$\frac{\quad}{6}$
1D-7	$\frac{\quad}{9}$	1D-22	$\frac{\quad}{9}$
1D-8	$\frac{\quad}{12}$	1D-23	$\frac{\quad}{12}$
1D-9	$\frac{\quad}{15}$	1D-24	$\frac{\quad}{15}$
1D-10	$\frac{\quad}{64}$	1D-25	$\frac{\quad}{64}$
1D-11	$\frac{\quad}{50}$	1D-26	$\frac{\quad}{50}$
1D-12	$\frac{\quad}{27}$	1D-27	$\frac{\quad}{27}$
1D-13	$\frac{\quad}{23}$	1D-28	$\frac{\quad}{23}$
1D-1	$\frac{\quad}{25}$	1D-29	$\frac{\quad}{25}$
1D-15	$\frac{\quad}{17}$	1D-30	$\frac{\quad}{17}$

Section 2: Mixed Numbers

When dividing the numerator of an improper fraction by the denominator (i.e., the upper number by the lower number), the result will be some whole number and a fractional part remaining or "leftover".

For example $\frac{37}{32}$ = 1 whole and a remainder of $\frac{5}{32}$,

or $\frac{32}{32} = 1 + \frac{5}{32}$, written as $1\frac{5}{32}$;

this is a <u>mixed</u> <u>number</u>.

2-A Examples

1. $\frac{47}{16} = 2\frac{15}{16}$

<u>solution</u>
$\frac{16}{16} = 1$; $\frac{16}{16} = 1$; remainder $\frac{15}{16}$
adding:
$1 + 1 + \frac{15}{16} = 2\frac{15}{16}$

2. $\frac{73}{64} = 1\frac{9}{64}$

<u>solution</u>
$\frac{64}{64} = 1$; remainder $\frac{9}{64}$
adding:
$1 + \frac{9}{64} = 1\frac{9}{64}$

3. $\frac{123}{32} = 3\frac{27}{32}$

<u>solution</u>
$\frac{32}{32} = 1$, $\frac{32}{32} = 1$, $\frac{32}{32} = 1$;
remainder $\frac{27}{32}$
adding:
$1 + 1 + 1 + \frac{27}{32} = 3\frac{27}{32}$

DRILL SHEET

UNIT 2: EXPRESSING IMPROPER FRACTIONS AS MIXED NUMBERS

Convert each improper fraction to a mixed number:

2D-1	$\frac{67}{64} =$	2D-16	$\frac{7}{3} =$
2D-2	$\frac{128}{9} =$	2D-17	$\frac{11}{4} =$
2D-3	$\frac{31}{15} =$	2D-18	$\frac{15}{9} =$
2D-4	$\frac{9}{5} =$	2D-19	$\frac{23}{21} =$
2D-5	$\frac{13}{11} =$	2D-20	$\frac{13}{9} =$
2D-6	$\frac{77}{55} =$	2D-21	$\frac{99}{77} =$
2D-7	$\frac{22}{19} =$	2D-22	$\frac{103}{67} =$
2D-8	$\frac{75}{9} =$	2D-23	$\frac{28}{15} =$
2D-9	$\frac{111}{103} =$	2D-24	$\frac{743}{23} =$
2D-10	$\frac{87}{44} =$	2D-25	$\frac{1142}{115} =$
2D-11	$\frac{93}{91} =$	2D-26	$\frac{2291}{303} =$
2D-12	$\frac{65}{17} =$	2D-27	$\frac{171}{55} =$
2D-13	$\frac{23}{5} =$	2D-28	$\frac{27}{4} =$
2D-14	$\frac{14}{3} =$	2D-29	$\frac{2745}{179} =$
2D-15	$\frac{33}{19} =$	2D-30	$\frac{14107}{2439} =$

Section 3: Equivalent Fractions

In order to solve common fraction problems, it is necessary to be able to express any of these as an equivalent fraction. This means the numbers will change but the value will not.

3-A Examples

1. Express $\frac{23}{7} = \frac{?}{28}$ as an equivalent fraction.

solution: Step 1 Set up a multiplier fraction, e.g.,

$$\frac{23}{7} \times \frac{(\ \ )}{(\ \ )} = \frac{\ \ }{28}$$

Step 2 divide 28 by 7 = 4

Step 3 substitute 4 in the numerator and the denominator

$$\frac{23}{7} \times \frac{(4)}{(4)} = \frac{92}{28}\text{, Answer}$$

Note! Since $\frac{4}{4} = 1$, and $1 \times \frac{23}{7} = \frac{23}{7}$, therefore the value of $\frac{23}{7}$ does not change, although the resultant numbers changed to $\frac{92}{28}$.

2. Express $\frac{15}{32} = \frac{?}{160}$ as an equivalent fraction.

solution: Step 1 Set up a multiplier fraction.

Step 2 divide 160 by 32 = 5

Step 3 substitute 5 in the numerator and denominator

$$\frac{15}{32} \times \frac{(5)}{(5)} = \frac{75}{160}\text{, Answer}$$

Note! Since $\frac{5}{5} = 1$, and $1 \times \frac{15}{32} = \frac{15}{32}$, the value of $\frac{15}{32}$ does not change, although the resultant numbers changed to $\frac{75}{160}$.

DRILL SHEET

UNIT 2: EQUIVALENT FRACTIONS

Express each common fraction as an equivalent fraction:

3D-1	$\frac{17}{13} = \frac{\quad}{52}$	3D-13	$\frac{3}{3} = \frac{\quad}{9}$
3D-2	$\frac{26}{15} = \frac{\quad}{60}$	3D-14	$\frac{4}{9} = \frac{\quad}{27}$
3D-3	$\frac{7}{71} = \frac{\quad}{142}$	3D-15	$\frac{1}{2} = \frac{\quad}{16}$
3D-4	$\frac{9}{18} = \frac{\quad}{54}$	3D-16	$\frac{3}{16} = \frac{\quad}{64}$
3D-5	$\frac{22}{37} = \frac{\quad}{111}$	3D-17	$\frac{7}{8} = \frac{\quad}{32}$
3D-6	$\frac{14}{97} = \frac{\quad}{194}$	3D-18	$\frac{9}{32} = \frac{\quad}{64}$
3D-7	$\frac{6}{7} = \frac{\quad}{21}$	3D-19	$\frac{1}{64} = \frac{\quad}{128}$
3D-8	$\frac{11}{3} = \frac{\quad}{9}$	3D-20	$\frac{5}{8} = \frac{\quad}{40}$
3D-9	$\frac{4}{15} = \frac{\quad}{45}$	3D-21	$\frac{7}{16} = \frac{\quad}{48}$
3D-10	$\frac{6}{11} = \frac{\quad}{55}$	3D-22	$\frac{14}{17} = \frac{\quad}{51}$
3D-11	$\frac{23}{25} = \frac{\quad}{75}$	3D-23	$\frac{9}{19} = \frac{\quad}{57}$
3D-12	$\frac{19}{3} = \frac{\quad}{15}$	3D-24	$\frac{6}{13} = \frac{\quad}{52}$
		3D-25	$\frac{5}{32} = \frac{\quad}{96}$

Section 4: Expressing Fractions In Lowest Terms

Unless specifically instructed not to, common fractions are to be reduced to their lowest terms.

4-A Examples

1. Express $\frac{148}{36}$ as a fraction in lowest terms.

solution: Step 1 Determine one common number that can be divided into 148 and 36 so that the results are whole numbers, e.g.,

$148 \div 4 = 37$, and $36 \div 4 = 9$, thus

$\frac{148}{36} = \frac{37}{9}$, Answer

Step 2 Continue this procedure until no further common number can be divided into both the numerator and denominator so that the result is a whole number.

2. $\frac{3}{6} = 3 \div 3 = 1$, and $6 \div 3 = 2$, thus $\frac{3}{6} =$

$\frac{1}{2}$, Answer

3. $\frac{5}{30} = 5 \div 5 = 1$, and $30 \div 5 = 6$, thus $\frac{5}{30} =$

$\frac{1}{6}$, Answer

4. $\frac{6}{16} = 6 \div 2 = 3$, and $16 \div 2 = 8$, thus $\frac{6}{16} = \frac{3}{8}$

$\frac{3}{8}$, Answer

5. $\frac{180}{15} = 180 \div 15 = 12$, and $15 \div 15 = 1$, thus

$\frac{180}{15} = \frac{12}{1}$ or 12, Answer

6. $\frac{21}{49} = 21 \div 7 = 3$, and $49 \div 7 = 7$, thus $\frac{21}{49} =$

$\frac{3}{7}$, Answer

DRILL SHEET

UNIT 2: EXPRESSING FRACTIONS IN LOWEST TERMS

Express each common fraction in lowest terms:

4D-1 $\frac{9}{27} =$

4D-2 $\frac{12}{16} =$

4D-3 $\frac{24}{32} =$

4D-4 $\frac{4}{8} =$

4D-5 $\frac{14}{8} =$

4D-6 $\frac{21}{9} =$

4D-7 $\frac{30}{16} =$

4D-8 $\frac{28}{8} =$

4D-9 $\frac{14}{12} =$

4D-10 $\frac{32}{96} =$

4D-11 $\frac{48}{96} =$

4D-12 $\frac{32}{64} =$

4D-13 $\frac{22}{16} =$

4D-14 $\frac{12}{8} =$

4D-15 $\frac{16}{8} =$

4D-16 $\frac{32}{16} =$

4D-17 $\frac{24}{8} =$

4D-18 $\frac{8}{24} =$

4D-19 $\frac{16}{32} =$

4D-20 $\frac{51}{17} =$

4D-21 $\frac{17}{51} =$

4D-22 $\frac{28}{56} =$

4D-23 $\frac{64}{128} =$

4D-24 $\frac{96}{4} =$

4D-25 $\frac{48}{3} =$

Section 5: Expressing Mixed Numbers As Fractions

In order to be able to multiply and divide mixed numbers, you must be able to express these as common fractions.

Express each mixed number as a common fraction.

5-A Examples

1. $1\frac{31}{32} = \frac{63}{32}$

solution: Step 1 multiply the whole number by the denominator, or $1 \times 32 = 32$

Step 2 add the numerator to the product resulting in Step 1, or $32 + 31 = 63$

Step 3 place the result obtained in Step 2 as the numerator over the original denominator, or, $\frac{63}{32}$, so that

$1\frac{31}{32} = \frac{63}{32}$, Answer

2. $3\frac{7}{8} = \frac{31}{8}$

solution: Step 1 multiply the whole number by the denominator, or $3 \times 8 = 24$

Step 2 add the numerator to the product resulting in Step 1, or $7 + 24 = 31$

Step 3 place the result obtained in Step 2 over the original denominator, or $\frac{31}{8}$, so that $3\frac{7}{8} = \frac{31}{8}$, Answer

3. $15\frac{13}{16} = \frac{253}{16}$

solution: Step 1 multiply the whole number by the denominator, or $15 \times 16 = 240$

Step 2 add the numerator to the product resulting in Step 1, or $240 + 13 = 253$

Step 3 place the result obtained in Step 2 as the numerator over the original denominator, or $\frac{253}{16}$, so that

$15\frac{13}{16} = \frac{253}{16}$, Answer

Note: Two of the most common applications of complex fractions in trade and industry are:

1) Converting a fraction part of a denominator number to a decimal.

Examples: $1\frac{3}{4}$ inches is $1\frac{3}{4}$ parts of a foot or

$\frac{1\frac{3}{4}}{12}$, a complex fraction.

$12\overline{)1.75}$ = .1458 feet as decimal. $2\frac{1}{8}$ ounces is $2\frac{1}{8}$

16th parts of a pound or $\frac{2\frac{1}{8}}{16}$, a complex fraction.

$16\overline{)2.125}$ = .1328 pound as a decimal.

2) Changing percents to decimals.

Examples: $\frac{1}{8}$% is $\frac{\frac{1}{8}}{100}$ of 100 parts or $\frac{1}{8}$, a complex fraction.

$100\overline{).125}$ = .00125 as a decimal.

$15\frac{1}{2}$% is $15\frac{1}{2}$ of a 100 parts or $\frac{15\frac{1}{2}}{100}$, a complex fraction.

$100\overline{)15.5}$ = .155 as a decimal.

DRILL SHEET

UNIT 2: EXPRESSING MIXED NUMBERS AS FRACTIONS

Express each mixed number as a common fraction:

5D-1 $1\frac{3}{8} =$

5D-2 $2\frac{11}{16} =$

5D-3 $13\frac{3}{8} =$

5D-4 $7\frac{1}{2} =$

5D-5 $13\frac{13}{32} =$

5D-6 $14\frac{1}{8} =$

5D-7 $3\frac{7}{13} =$

5D-8 $7\frac{3}{11} =$

5D-9 $3\frac{2}{7} =$

5D-10 $97\frac{1}{9} =$

5D-11 $105\frac{3}{5} =$

5D-12 $22\frac{7}{9} =$

5D-13 $33\frac{31}{32} =$

5D-14 $19\frac{13}{16} =$

5D-15 $11\frac{2}{3} =$

5D-16 $24\frac{4}{13} =$

5D-17 $7\frac{9}{11} =$

5D-18 $13\frac{13}{14} =$

5D-19 $6\frac{15}{16} =$

5D-20 $7\frac{3}{8} =$

5D-21 $5\frac{3}{4} =$

5D-22 $6\frac{1}{2} =$

5D-23 $3\frac{17}{32} =$

5D-24 $22\frac{9}{16} =$

5D-25 $8\frac{4}{9} =$

Section 6: Lowest Common Denominator

To be able to add and subtract common fractions, you must determine the lowest common denominator.

6-A Examples

1. Determine the lowest common denominator (LCD) for the fractions $\frac{1}{4}$, $\frac{3}{16}$, $\frac{5}{8}$

solution: Step 1 since the LCD can never be less than the largest denominator, it has to be at least as large as 16

Step 2 the other two denominators, 4 and 8, must divide into 16 a <u>whole</u> number of times, or $\frac{16}{4} = 4$, and $\frac{16}{8} = 2$, thus <u>16 is the LCD</u> for these three fractions

2. Determine the lowest common denominator (LCD) for the fractions $\frac{3}{11}$, $\frac{1}{4}$, $\frac{1}{5}$

solution: Step 1 since the LCD can never be less than the largest denominator, it has to be at least as large as 11

Step 2 the other two denominators 4 and 5, must divide into 11 a whole number of times; let's check $\frac{11}{4} = 2\frac{3}{4}$, and $\frac{11}{5} = 2\frac{1}{5}$. As you can see, each division results in a whole number, plus a remainder. Thus, 11 cannot be the LCD.

Step 3 we must involve all three denominators; to do this, multiply $11 \times 4 = 44$, and then $44 \times 5 = 220$.

Check: $\frac{220}{11} = 20$

$\frac{220}{4} = 55$

$\frac{220}{5} = 44$, thus <u>220 is the LCD</u> for these three fractions.

<u>Note</u>! Some times, the apparent LCD resulting from multiplying all the

denominators can be reduced further by dividing by 2, or 3, or 5, etc. to meet the requirements of LCD. This will be illustrated in Example 3.

3. Determine the lowest common denominator (LCD) for the fractions $\frac{1}{15}, \frac{1}{3}, \frac{1}{7}$

solution: Step 1 $15 \times 3 = 45$, and $45 \times 7 = 315$

Step 2 but 315 is not the LCD, since it can be reduced by dividing by 3, or

$$\frac{315}{3} = 105$$

Step 3 $\frac{105}{15} = 7$

$\frac{105}{3} = 35$

$\frac{105}{7} = 15$, thus <u>105 is the LCD</u>

NOTE! A simple method to determine the lowest common denominator (LCD) is as follows:

4. Determine LCD for $\frac{2}{21}, \frac{3}{14}$ = ________

solution: Step 1 Multiply the denominators, or $21 \times 14 = 294$

Step 2 Divide the result in Step 1 by the highest common factor for 21 and 14, which is 7; and $\frac{294}{7} = 42$, <u>the LCD</u>.

Now to simplify things:

Steps to follow where determining the Lowest Common Denominator (L.C.D.):

1. Look at your largest number - see if that will work

   e.g. $\frac{1}{2}$, $\frac{3}{4}$

2. If numbers (denominators) are even and odd and the largest number does not work - then go through the multiplies of the largest number

   e. g. $\frac{1}{3}$, $\frac{3}{8}$, $\frac{1}{6}$ --- 8 , 16, "24"

3. If numbers are all odd, use prime number factoring system - using just <u>prime</u>* numbers

   e.g. $\frac{1}{3}$, $\frac{1}{7}$, $\frac{1}{9}$

   3 X 1   7 X 1   3 X 3

   3 X 3 X 7 = 63

4. If numbers are all prime, then multiply them together

   e.g. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{7}$

   2 X 3 X 7 = 42

* A Prime Number is a number larger than 1 that can be evenly divided only by itself and 1. The number 2 is the first prime; it can be evenly divided only by 2 and 1.

You can pick out prime numbers from a list of natural numbers.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

First, circle 2, the first prime. Then cross out all of the multiples of 2 because they are not primes, such as 4, 6, 8, 10, 12,... Next, circle the next number which is 3 and cross out its multiples such as, 6, 9, 12, 15, 18,... This process is continued with succeeding numbers not previously crossed out.

Use of the sieve is not as laborious as it looks. A number less than 100 that is not a prime must have a factor less than 10, because 10 x 10 = 100. Thus, sieving out multiples of the primes through 7 will identify all primes from 2-100.

Can you extend the process to find all the primes from 2-200? (You only have to sieve out primes less than 15, because 15 x 15 = 225.)

DRILL SHEET

UNIT 2: LOWEST COMMON DENOMINATORS

Determine the lowest common denominator for each set of fractions.

6D-1 $\frac{1}{4}, \frac{3}{16}, \frac{5}{8}$ = ______

6D-2 $\frac{3}{4}, \frac{1}{3}, \frac{7}{8}$ = ______

6D-3 $\frac{3}{8}, \frac{7}{16}, \frac{3}{32}$ = ______

6D-4 $\frac{1}{5}\ \frac{1}{4}, \frac{1}{2}$ = ______

6D-5 $\frac{1}{4}, \frac{3}{8}, \frac{1}{5}$ = ______

6D-6 $\frac{1}{6}, \frac{1}{12}, \frac{3}{24}$ = ______

6D-7 $\frac{9}{16}, \frac{7}{8}, \frac{3}{32}$ = ______

6D-8 $\frac{2}{5}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}$ = ______

6D-9 $\frac{3}{12}, \frac{1}{8}, \frac{1}{4}, \frac{3}{16}$ = ______

6D-10 $\frac{2}{9}, \frac{1}{27}, \frac{3}{54}$ = ______

6D-11 $\frac{17}{32}, \frac{15}{16}, \frac{19}{64}$ = ______

6D-12 $\frac{1}{9}, \frac{1}{3}, \frac{26}{27}$ = ______

6D-13 $\frac{6}{7}, \frac{11}{14}, \frac{27}{28}$ = ______

6D-14 $\frac{1}{27}, \frac{3}{108}, \frac{7}{9}$ = ______

6D-15 $\frac{29}{64}, \frac{31}{32}, \frac{15}{16}$ = ______

6D-16	$\frac{1}{11}, \frac{3}{22}, \frac{5}{66}$	=	______
6D-17	$\frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \frac{31}{32}$	=	______
6D-18	$\frac{63}{64}, \frac{15}{16}, \frac{31}{32}$	=	______
6D-19	$\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}$	=	______
6D-20	$\frac{2}{7}, \frac{1}{3}, \frac{3}{5}$	=	______
6D-21	$\frac{23}{32}, \frac{7}{16}, \frac{11}{64}, \frac{3}{4}$	=	______
6D-22	$\frac{2}{3}, \frac{7}{17}, \frac{49}{51}$	=	______
6D-23	$\frac{1}{4}, \frac{1}{8}, \frac{1}{32}, \frac{1}{64}, \frac{1}{16}$	=	______
6D-24	$\frac{3}{16}, \frac{7}{48}, \frac{7}{8}, \frac{1}{2}, \frac{3}{4}$	=	______
6D-25	$\frac{47}{48}, \frac{15}{16}, \frac{31}{32}, \frac{95}{96}$	=	______

Section 7: Adding Common Fractions

7-A Examples

1. Add the following fractions:

$\frac{1}{8} + \frac{19}{32} + \frac{7}{16} + \frac{3}{4} =$ ______

solution: Step 1 arrange fractions in vertical fashion

$$\begin{array}{r} \frac{1}{8} \\[1ex] \frac{19}{32} \\[1ex] \frac{7}{16} \\[1ex] +\ \frac{3}{4} \\ \hline \end{array}$$

Step 2 determine lowest common denominator, LCD

$$\begin{array}{rl} \frac{1}{8} & = \\[1ex] \frac{19}{32} & = \frac{\quad}{32} \qquad \text{LCD} \\[1ex] \frac{7}{16} & = \\[1ex] +\ \frac{3}{4} & = \\ \hline \end{array}$$

Step 3 use principle of equivalent fractions to obtain new form of numerators

$$\frac{1}{8} \times \frac{(4)}{(4)} = \frac{4}{32}$$

$$\frac{19}{32} \times \frac{(1)}{(1)} = \frac{19}{32}$$

$$\frac{7}{16} \times \frac{(2)}{(2)} = \frac{14}{32}$$

$$+\ \frac{3}{4} \times \frac{(8)}{(8)} = \frac{24}{32}$$

Step 4 add the numerators and place total over the common denominator

$$\begin{array}{r} 4 \\ 19 \\ 14 \\ +\ \underline{24} \\ 61, \end{array} \qquad \frac{61}{32}$$

Step 5 reduce the improper fraction to a mixed number (always to lowest terms)

$\frac{61}{32} = 1\,\frac{29}{32}$, Answer

2. $\frac{1}{3} + \frac{3}{8} + \frac{5}{16} + \frac{1}{4} + \frac{37}{48} =$ ___

solution: Step 1

$$\frac{1}{3}$$

$$\frac{3}{8}$$

$$\frac{5}{16}$$

$$\frac{1}{4}$$

$$+\ \frac{37}{48}$$

Step 2

$\frac{1}{3} =$

$\frac{3}{8} =$

$\frac{5}{16} =$

$\frac{1}{4} =$

$+\ \frac{37}{48} = \frac{\quad}{48}$ LCD

Step 3

$\frac{1}{3} \times \frac{(16)}{(16)} = \frac{16}{48}$

$\frac{3}{8} \times \frac{(6)}{(6)} = \frac{18}{48}$

$\frac{5}{16} \times \frac{(3)}{(3)} = \frac{15}{48}$

$\frac{1}{4} \times \frac{(12)}{(12)} = \frac{12}{48}$

$+\ \frac{37}{48} \times \frac{(1)}{(1)} = \frac{37}{48}$

Step 4

16
18
15
12
+ 37
98, $\frac{98}{48}$

Step 5 $\frac{98}{48} = 2\ \frac{2}{48}$, or $2\ \frac{1}{24}$, Answer

DRILL SHEET

UNIT 2: ADDING FRACTIONS

Add the following fractions and reduce the final answer to lowest terms.

7D-1 $\frac{1}{4} + \frac{3}{8} =$ ________

7D-2 $\frac{5}{16} + \frac{9}{32} =$ ________

7D-3 $\frac{1}{2} + \frac{3}{7} + \frac{2}{3} =$ ________

7D-4 $\frac{5}{8} + \frac{3}{16} + \frac{3}{32} + \frac{1}{64} =$ ________

7D-5 $\frac{15}{16} + \frac{7}{8} + \frac{1}{2} + \frac{3}{4} =$ ________

7D-6 $\frac{1}{7} + \frac{3}{8} + \frac{93}{112} =$ ________

7D-7 $\frac{7}{16} + \frac{3}{8} + \frac{63}{64} + \frac{3}{4} + \frac{19}{32} =$ ________

7D-8 $\frac{1}{5} + \frac{3}{10} + \frac{11}{15} + \frac{17}{30} =$ ________

7D-9 $\frac{7}{25} + \frac{43}{50} + \frac{83}{100} =$ ________

7D-10 $\frac{6}{11} + \frac{2}{33} + \frac{17}{99} =$ ________

7D-11 $\frac{14}{17} + \frac{11}{17} + \frac{9}{17} + \frac{15}{17} =$ ________

7D-12 $\frac{3}{10} + \frac{1}{5} + \frac{3}{4} =$ ________

7D-13 $\frac{2}{3} + \frac{7}{9} + \frac{11}{12} =$ ________

7D-14 $\frac{9}{32} + \frac{1}{4} + \frac{5}{24} =$ ________

7D-15 $\frac{1}{2} + \frac{5}{6} + \frac{11}{12} =$ ________

7D-16 $\frac{14}{15} + \frac{3}{10} + \frac{29}{30} =$ ________

7D-17 $\frac{1}{5} + \frac{1}{7} + \frac{1}{4} =$ ______

7D-18 $\frac{6}{7} + \frac{1}{3} + \frac{1}{2} =$ ______

Section 8: Adding Fractions, Mixed Numbers, and Whole Numbers

Add fractions, mixed numbers, and whole numbers. Reduce answers to lowest terms.

8-A Examples

1. $3\frac{3}{16} + 9 + 14\frac{27}{64} =$ ________

solution: Step 1

$$\begin{array}{r} 3\frac{3}{16} \\ 9 \\ +\ 14\frac{27}{64} \\ \hline \end{array}$$

Step 2

$$\begin{array}{rcr} 3\frac{3}{16} & = & 3\frac{12}{64} \\ 9 & = & 9 \\ +\ 14\frac{27}{64} & = & 14\frac{27}{64} \\ \hline & & 26\frac{39}{64} \end{array}$$

Answer

2. $23\frac{1}{8} + 36\frac{7}{32} =$ ________

solution: Step 1

$$\begin{array}{r} 23\frac{1}{8} \\ +\ 36\frac{7}{32} \\ \hline \end{array}$$

Step 2

$$\begin{array}{rcr} 23\frac{1}{8} & = & 23\frac{4}{32} \\ +\ 36\frac{7}{32} & = & 36\frac{7}{32} \\ \hline & & 59\frac{11}{32} \end{array}$$

Answer

3. $6\frac{13}{16} + 73 + 27\frac{3}{4} + 19\frac{17}{32} =$ ______

solution: Step 1

$$\begin{array}{r} 6\frac{13}{16} \\ 73\phantom{\frac{13}{16}} \\ 27\ \frac{3}{4} \\ +\ 19\frac{17}{32} \\ \hline \end{array}$$

Step 2

$$\begin{array}{rcr} 6\frac{13}{16} & = & 6\frac{26}{32} \\ 73\phantom{\frac{13}{16}} & = & 73\phantom{\frac{26}{32}} \\ 27\ \frac{3}{4} & = & 27\frac{24}{32} \\ +\ 19\frac{17}{32} & = & 19\frac{17}{32} \\ \hline & & 125\frac{67}{32} \end{array}$$

Note! $\frac{67}{32}$ is an improper fraction, i.e., greater than a whole $\left(\frac{32}{32} = 1\right)$.

Step 3

$$\begin{array}{rcr} 125\frac{67}{32} & = & 125\phantom{\frac{3}{32}} \\ & & 2\ \frac{3}{32} \\ & & +\ \ \ \ \ \\ \hline & & 127\ \frac{3}{32} \end{array}$$

Answer

DRILL SHEET

UNIT 2: ADDING FRACTIONS, MIXED NUMBERS, AND WHOLE NUMBERS

Add and reduce answers to lowest terms.

8D-1 $7 + \frac{7}{8} =$ ________

8D-2 $10\frac{1}{16} + \frac{7}{16} =$ ________

8D-3 $4\frac{3}{16} + 12\frac{3}{8} =$ ________

8D-4 $8 + \frac{7}{4} + \frac{8}{3} + \frac{7}{2} =$ ________

8D-5 $9\frac{1}{6} + 8\frac{5}{9} + 7 =$ ________

8D-6 $14\frac{3}{4} + 30\frac{1}{2} + 4 =$ ________

8D-7 $7\frac{2}{3} + 9\frac{3}{4} + 11\frac{1}{2} =$ ________

8D-8 $3\frac{3}{4} + 5\frac{4}{7} + 2\frac{9}{14} + 7\frac{1}{2} =$ ________

8D-9. $37\frac{31}{32} + 19\frac{15}{16} + 11\frac{3}{4} + 4\frac{1}{2} =$ ________

8D-10 $29 + 11\frac{5}{6} + 37 + 3\frac{7}{12} =$ ________

8D-11 $132\frac{1}{32} + 3 + 11\frac{23}{32} + 19\frac{7}{32} =$ ________

8D-12 $16 + 165 + 3\frac{1}{64} =$ ________

8D-13 $\frac{23}{32} + 23\frac{23}{32} + 11\frac{13}{16} =$ ________

8D-14 $17\frac{11}{64} + 9\frac{57}{64} + 3\frac{1}{16} =$ ______

8D-15 $23\frac{9}{11} + 17\frac{29}{33} + 4 =$ ______

8D-16 $6\frac{5}{13} + 13\frac{27}{39} + 65 =$ ______

8D-17 $16\frac{11}{15} + 3\frac{23}{30} + 7\frac{1}{5} =$ ______

8D-18 $\frac{13}{17} + 9\frac{11}{51} + 63 =$ ______

8D-19 $22 + 23 + 169\frac{11}{64} =$ ______

8D-20 $\frac{17}{35} + 99\frac{3}{7} + 6\frac{19}{35} =$ ______

8D-21 $11 + \frac{1}{9} + 33\frac{1}{3} =$ ______

8D-22 $37 + 37 + 37 =$ ______

8D-23 $65 + 37\frac{1}{37} + 69\frac{36}{37} =$ ______

8D-24 $125\frac{11}{25} + 6\frac{4}{5} + 71 =$ ______

8D-25 $33\frac{3}{19} + 16\frac{55}{57} =$ ______

Section 9: Subtracting Fractions

Subtract fractions. Reduce answers to lowest terms.

9-A Examples

1. $\frac{13}{15} - \frac{9}{20} =$ ________

solution: Step 1 arrange in vertical form

$$\begin{array}{r} \frac{13}{15} \\ -\ \frac{9}{20} \\ \hline \end{array}$$

Step 2 determine lowest common denominator (LCD)

$$\begin{array}{r} \frac{13}{15} = \frac{52}{60} \\ -\ \frac{9}{20} = \frac{27}{60} \\ \hline \end{array}$$

Step 3 subtract

$$\begin{array}{r} \frac{52}{60} \\ -\ \frac{27}{60} \\ \hline \frac{25}{60} = \frac{5}{12} \end{array}$$ Answer

2. $\frac{19}{20} - \frac{4}{5} =$ ________

solution: Step 1

$$\begin{array}{r} \frac{19}{20} \\ -\ \frac{4}{5} \end{array}$$

Step 2

$$\begin{array}{r} \frac{19}{20} = \frac{19}{20} \\ -\ \frac{4}{5} = \frac{16}{20} \\ \hline \end{array}$$

Step 3 $\frac{19}{20}$

$$\begin{array}{r} \frac{16}{20} \\ - \ \frac{16}{20} \end{array}$$

$$\begin{array}{r} \frac{19}{20} \\ -\ \frac{16}{20} \\ \hline \frac{3}{20} \end{array} \quad \text{Answer}$$

3. $67 - 33\frac{4}{13} =$ ________

solution: Step 1

$$\begin{array}{r} 67 \\ -\ 33\frac{4}{13} \\ \hline \end{array}$$

Step 2

$$\begin{array}{rcl} \overset{66}{67} & = & 66\frac{13}{13} \\ -\ 33\frac{4}{13} & = & 33\frac{4}{13} \\ \hline \end{array}$$

Note above that 1 whole $\left(\frac{13}{13}\right)$ was "borrowed" from 67 in order to provide a number from which to subtract $\frac{4}{13}$.

solution: Step 3

$$\begin{array}{r} 66\frac{13}{13} \\ -\ 33\frac{4}{13} \\ \hline 33\frac{9}{13} \end{array} \quad \text{Answer}$$

DRILL SHEET

UNIT 2: SUBTRACTING FRACTIONS

Subtract and reduce to lowest terms.

9D-1 $\frac{7}{10} - \frac{3}{10} =$ ______

9D-2 $\frac{9}{16} - \frac{1}{16} =$ ______

9D-3 $\frac{15}{16} - \frac{2}{3} =$ ______

9D-4 $\frac{19}{20} - \frac{3}{5} =$ ______

9D-5 $\frac{1}{3} - \frac{3}{16} =$ ______

9D-6 $\frac{5}{8} - \frac{1}{6} =$ ______

9D-7 $\frac{12}{13} - \frac{1}{3} =$ ______

9D-8 $\frac{57}{64} - \frac{1}{2} =$ ______

9D-9 $\frac{31}{32} - \frac{9}{16} =$ ______

9D-10 $\frac{23}{25} - \frac{3}{10} =$ ______

9D-11 $\frac{1}{9} - \frac{1}{18} =$ ______

9D-12 $\frac{3}{4} - \frac{17}{32} =$ ______

9D-13 $\frac{9}{16} - \frac{7}{16} =$ ______

9D-14 $\frac{13}{16} - \frac{9}{16} =$ ______

9D-15 $\frac{31}{32} - \frac{11}{32} =$ ______

9D-16 $\frac{15}{39} - \frac{1}{13} =$ ______

9D-17 $\frac{16}{51} - \frac{3}{17} =$ ______

9D-18 $\frac{29}{48} - \frac{1}{4} =$ ______

9D-19 $\frac{22}{33} - \frac{2}{11} =$ ______

9D-20 $\frac{15}{32} - \frac{9}{64} =$ ______

9D-21 $\frac{7}{8} - \frac{7}{16} =$ ______

9D-22 $\frac{14}{27} - \frac{3}{54} =$ ______

9D-23 $\frac{9}{32} - \frac{9}{64} =$ ______

9D-24 $\frac{17}{25} - \frac{3}{5} =$ ______

9D-25 $\frac{1}{16} - \frac{1}{64} =$ ______

Section 10: Subtracting Fractions, Mixed Numbers, and Whole Numbers

Subtract and reduce to lowest terms.

10-A Examples

1. $33\frac{7}{10} - 17\frac{3}{4} =$ ______

solution: Step 1

$$\begin{array}{r} 33\ \frac{7}{10} \\ -\ 17\ \frac{3}{4} \\ \hline \end{array}$$

Step 2

$$\begin{array}{r} 33\ \frac{7}{10} = 33\ \frac{14}{20} \\ -\ 17\ \frac{3}{4} = 17\ \frac{15}{20} \\ \hline \end{array}$$

Note that 15 cannot be subtracted from 14; we need to borrow 1 whole $\frac{(20)}{(20)}$ from 33 and add to 14.

Step 3

$$\begin{array}{r} \overset{32}{33} = 32\ \frac{34}{20} \\ -\ 17\ \frac{15}{20} \\ \hline 15\ \frac{19}{20} \end{array}$$ Answer

2. $16\frac{11}{33} - 15\frac{4}{11} =$ ______

solution: Step 1 -

$$\begin{array}{r} 16\ \frac{11}{33} \\ -\ 15\ \frac{4}{11} \\ \hline \end{array}$$

Step 2

$$\begin{array}{rl} 16\,\frac{11}{33} & = 16\,\frac{11}{33} \\[6pt] -\;15\,\frac{4}{11} & = 15\,\frac{12}{33} \\ \hline \end{array}$$

Step 3

$$\begin{array}{r} \overset{15}{16}\,\frac{44}{33} \\[6pt] -\;15\,\frac{12}{33} \\ \hline \frac{32}{33} \end{array}$$

Answer

DRILL SHEET

UNIT 2: SUBTRACTING FRACTIONS, MIXED NUMBERS AND WHOLE NUMBERS

Subtract and reduce to lowest terms.

10D-1 $21 - \frac{9}{16} =$ ______

10D-2 $107 - \frac{5}{9} =$ ______

10D-3 $31 - 29\frac{7}{12} =$ ______

10D-4 $9\frac{3}{8} - \frac{1}{8} =$ ______

10D-5 $27\frac{31}{32} - \frac{7}{8} =$ ______

10D-6 $53\frac{17}{28} - 12\frac{3}{7} =$ ______

10D-7 $11\frac{1}{4} - \frac{13}{16} =$ ______

10D-8 $1\frac{13}{18} - \frac{15}{18} =$ ______

10D-9 $19\frac{7}{8} - 18\frac{13}{16} =$ ______

10D-10 $1\frac{27}{28} - \frac{1}{56} =$ ______

10D-11 $37\frac{19}{51} - 28\frac{14}{17} =$ ______

10D-12 $62\frac{6}{7} - 53\frac{3}{14} =$ ______

10D-13 $13 - \frac{27}{32} =$ ______

10D-14 $101 - 100\frac{31}{64} =$ ______

10D-15 $65\frac{9}{11} - 24\frac{3}{33} =$ ______

10D-16 $24\frac{1}{16} - 8\frac{1}{32} =$ ______

10D-17 $54 - 1\frac{59}{64} =$ ______

10D-18 $1\frac{63}{64} - \frac{17}{64} =$ ______

10D-19 $37\frac{9}{13} - 22\frac{7}{52} =$ ______

10D-20 $1\frac{3}{8} - \frac{5}{8} =$ ______

10D-21 $18\frac{9}{23} - 17\frac{9}{46} =$ ______

10D-22 $\frac{1}{4} - \frac{1}{64} =$ ______

10D- 3 $\frac{3}{4} - \frac{3}{8} =$ ______

10D-24 $16\frac{5}{6} - 13\frac{7}{12} =$ ______

10D-25 $22\frac{9}{10} - 19\frac{9}{15} =$ ______

DRILL SHEET

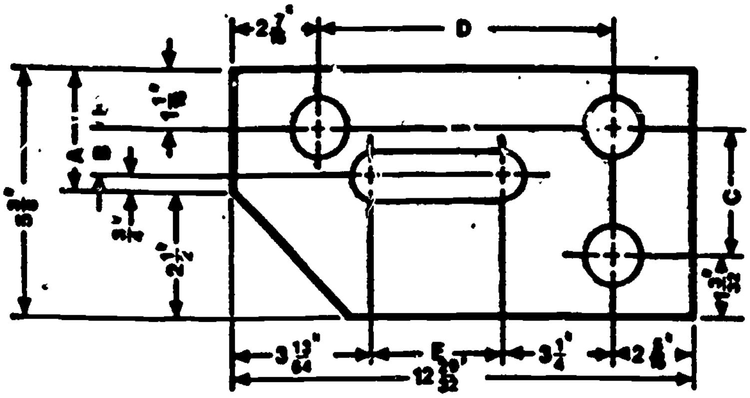
Section 11: Adding and Subtracting Fractions In Practical Applications

Perform the required operation and reduce to lowest terms.

11D-1 The cafeteria baker prepares a ALPO anniversary cake mix which weighs 100 pounds. The cake mix consists of shortening and other ingredients. The weights of the other ingredients are 20 1/2 pounds of flour, 29 3/4 pounds of sugar, 18 1/8 pounds of milk, 16 pounds of whole eggs, and a total of 5 1/4 pounds of flavoring, salt, and baking powder. How many pounds of shortening are used in the mix?

____

11D-2 Determine dimensions A,B,C,D, and E of the photoresist mask shown in the figure below (all dimensions are in.):



a.________

b.________

c.________

d.________

e.________

11D-3 Before starting a wiring job for the new switchboard in ALPO's guard shack, the electrician takes an inventory of materials and finds that 4,550 feet of BX cable are in stock. The following lengths of cable are removed from stock for the job: 275 1/4 feet; 48 feet; 56 1/2 feet; 212 3/4 feet; and 148 feet. Upon completion of the job, 87 1/4 feet are left over and returned to stock. How many feet of cable are in stock after completing the job?

____

11D-4 A truck is loaded at a structural steel supply house for a delivery to ALPO's construction site.

The order calls for 125 feet of channel iron which weighs 3 3/4 tons, 140 feet of I beam which weighs 4 3/10 tons, and 80 feet of angle iron which weighs 2 1/5 tons. The maximum legal tonnage permitted to be hauled by the truck is 9 1/2 tons. All of the channel iron and I beam are loaded. Only part of the angle iron is loaded so that the maximum legal tonnage is met but not exceeded. By how many tons of angle iron will the delivery be short of the order?

______

11D-5 In ALPO's warehouse, cartons move along a conveyor belt that is 423 1/2 feet long. Only 117 3/8 feet of the conveyor belt is at ground level; the rest is overhead. How much of the belt is overhead?

______

11D-6 What is the difference between a piece of clean room floor grating 1 7/16 inches and a piece 2 17/32 inches thick?

______

11D-7 A piece of round silicon rod is 7 3/4 inch in diameter. How much must be removed to get a diameter of 6 37/64 inch?

______

11D-8 At the beginning of trading, one share of Bell Atlantic stock was selling at 45 7/8 points. At the end of trading, the stock was selling at 42 1/4 points. How many points did it lose?

______

11D-9 An experienced machine setter can assemble his equipment in 6 1/3 hours. A learner can do the same job in 9 1/4 hours. How much faster is the experienced machine setter than the learner?

______

11D-10 ALPO's storage gas tank is 3/4 full of gasoline. If 1/8 is drawn off, what fraction of the tank of gas remains?

______

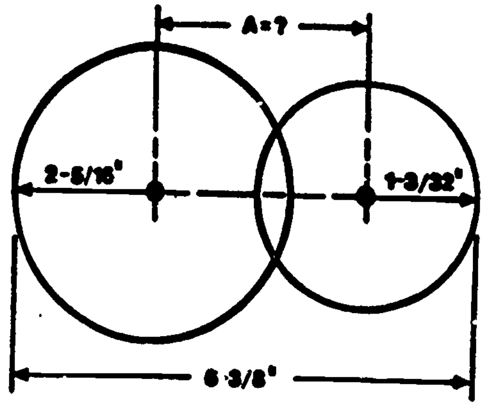
11D-11 Last month Bruce weighed 234 1/4 pounds. Now he weighs 227 5/16 pounds. How much weight has he lost?

______

11D-12 In one 8-hour working day, AL"O's mechanic spent 1 3/4 hours doing a tune-up, 2 1/2 hours doing a bra'.e job, and 3 1/3 hours replacing a clutch. How many hours did he have left to work on a broken axle?

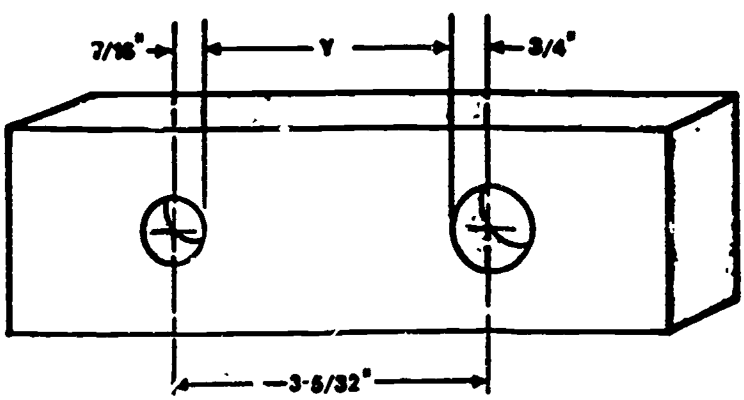
____

11D-13 Find dimension A in the following sketch?



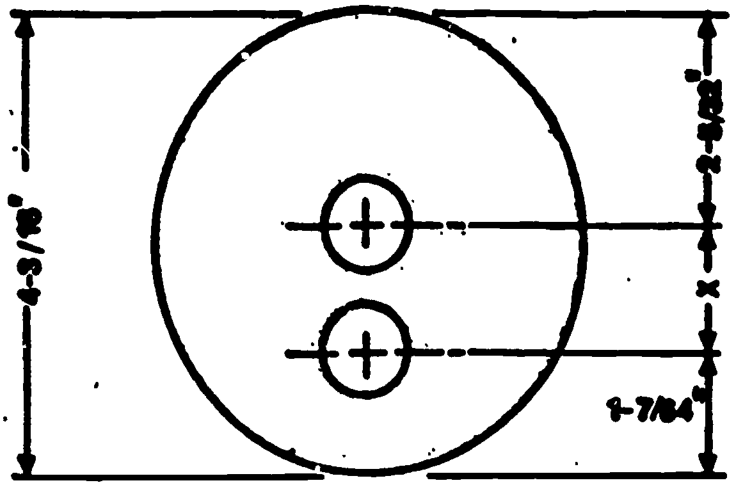
____

11D-14 Find dimension Y in the following sketch:



____

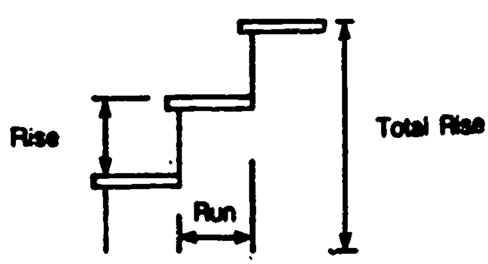
11D-15 Find the dimension X in the following sketch:



11D-16 Carpenters frequently use fractions and mixed numerals. When they build stairs, for example, the rise plus the run should be about 17 $\frac{1}{2}$ inches for each step. If the rise for a set of steps is 7 $\frac{3}{8}$ inches, what should the run be?

Solution: Rise + Run = 17 $\frac{1}{2}$

7 3/8 + Run = 17 $\frac{1}{2}$



Run = 17 $\frac{1}{2}$ - 7 $\frac{3}{8}$

Run = 17 $\frac{4}{8}$ - 7 $\frac{3}{8}$

Run = 10 $\frac{1}{8}$ inches

A fli: ht of stairs is to have a t tal rise of 8 $\frac{3}{4}$ i.et. How many steps will be needed if each rise if to be 7 $\frac{1}{2}$ inches?

Section 12: Multiplying Fractions

The multiplication of two fractions is defined as the product (from multiplication) of the numerators [illegible] the product (from multiplication) of the denominators.

12-A Examples

1. $\frac{3}{7} \times \frac{5}{8} = \frac{3 \times 5}{7 \times 8} = \frac{15}{56}$

2. $\frac{2}{3} \times \frac{2}{7} \times \frac{1}{5} = \frac{2 \times 2 \times 1}{3 \times 7 \times 5} = \frac{4}{105}$, Answer

12-B You can often reduce the amount of multiplication by "cancelling" or dividing one number in the numerator by one number in the denominator, provided the dividing number is the same.
Examples

1. $$\frac{14}{15} \times \frac{3}{49} = \frac{\overset{2}{14} \times \overset{1}{3}}{\underset{5}{15} \times \underset{7}{49}} = \frac{2}{35}\text{, Answer}$$

In the above example, the number 14 and 49 were divided by 7, i.e:, $\frac{14}{7} = 2$ and $\frac{49}{7} = 7$.

Similarly, the 3 and 15 were divided by 3. i.e.. $\frac{3}{3} = 1$ and $\frac{15}{3} = 5$.

2. $$\frac{7}{18} \times \frac{10}{12} \times \frac{4}{35} = \frac{\overset{1}{7} \times \overset{\overset{1}{5}}{10} \times \overset{1}{4}}{\underset{9}{18} \times \underset{3}{12} \times \underset{\underset{1}{5}}{35}} = \frac{1}{27}\text{, Answer}$$

Note!

In the above example, first we divided $\frac{7}{7} = 1$ and $\frac{35}{7} = 5$. Next, we divided $\frac{4}{4} = 1$ and $\frac{12}{4} = 3$. Then, we divided $\frac{10}{2} = 5$ and $\frac{18}{2} = 9$.

And last, we divided $\frac{5}{5} = 1$ and $\frac{5}{5} = 1$ since we must always reduce our final answer to lowest terms.

DRILL SHEET

UNIT 2: MULTIPLYING FRACTIONS

Multiply the fractions and reduce to lowest terms.

12D-1 $\frac{7}{4} \times \frac{4}{11} =$

12D-2 $\frac{7}{8} \times \frac{3}{5} \times \frac{1}{2} =$

12D-3 $\frac{7}{15} \times \frac{25}{9} =$

12D-4 $\frac{24}{35} \times \frac{5}{42} =$

12D-5 $\frac{36}{7} \times \frac{5}{12} =$

12D-6 $\frac{1}{8} \times \frac{1}{2} =$

12D-7 $\frac{3}{5} \times \frac{5}{8} =$

12D-8 $\frac{4}{5} \times \frac{7}{8} =$

12D-9 $\frac{5}{8} \times \frac{5}{16} =$

12D-10 $\frac{1}{6} \times \frac{11}{12} =$

12D-11 $\frac{5}{6} \times \frac{2}{5} =$

12D-12 $\frac{5}{7} \times \frac{14}{25} =$

12D-13 $\frac{1}{3} \times \frac{3}{4} \times \frac{8}{9} =$

12D-14 $\frac{19}{20} \times \frac{10}{11} \times \frac{2}{9} =$

12D-15 $\frac{11}{15} \times \frac{3}{4} \times \frac{9}{22} \times \frac{1}{2} =$

12D-16 $\frac{13}{16} \times \frac{4}{39} \times \frac{3}{5} \times \frac{25}{9} =$

12D-17 $\frac{17}{51} \times \frac{64}{9} \times \frac{81}{16} =$

12D-18 $\frac{125}{11} \times \frac{19}{6} \times \frac{55}{57} \times \frac{6}{25} =$

12D-19 $\frac{22}{9} \times \frac{17}{64} \times \frac{81}{57} \times \frac{128}{11} =$

12D-20 $\frac{108}{23} \times \frac{63}{9} \times \frac{92}{7} \times \frac{1}{3} =$

12D-21 $\frac{17}{64} \times \frac{3}{16} \times \frac{22}{68} =$

12D-22 $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} \times \frac{1}{6} =$

12D-23 $\frac{1}{9} \times \frac{1}{7} \times \frac{18}{3} \times \frac{21}{2} =$

12D-24 $\frac{14}{15} \times \frac{225}{64} =$

12D-25 $\frac{11}{32} \times \frac{9}{64} \times \frac{128}{55} \times \frac{96}{3} =$

Section 13: Multiplying Fractions, Mixed Numbers, and Whole Numbers

In order to multiply mixed numbers, you must first convert them to improper fractions, then multiply, and reduce to lowest terms.

13-A Examples

1. $$1\frac{2}{3} \times \frac{15}{17} = \frac{5}{\underset{1}{\cancel{3}}} \times \frac{\overset{5}{\cancel{15}}}{17} = \frac{25}{17} = 1\ \frac{8}{17}$$

2. $$6 \times \frac{5}{3} = \frac{\overset{2}{\cancel{6}}}{1} \times \frac{5}{\underset{1}{\cancel{3}}} = \frac{10}{1} \text{ or } 10$$

3. $$5\frac{1}{2} \times \frac{2}{13} \times 3\frac{1}{3} = \frac{11}{\underset{1}{\cancel{2}}} \times \frac{\overset{1}{\cancel{2}}}{13} \times \frac{10}{3} = \frac{110}{39} = 2\ \frac{32}{39}$$

<u>Note</u>! When dividing $\frac{110}{39}$ on the calculator, you obtain 2 as a whole number, plus a <u>decimal</u> remainder of 0.8205 etc. To obtain the <u>fractional</u> remainder, multiply the denominator, 39, by the decimal and you obtain 31.999 etc., or 32.

DRILL SHEET

UNIT 2: MULTIPLYING FRACTIONS, MIXED NUMBERS, AND WHOLE NUMBERS

Multiply and reduce to lowest terms.

13D-1 $5\frac{1}{7} \times 3\frac{1}{4} =$

13D-2 $4 \times 7\frac{3}{8} =$

13D-3 $6\frac{2}{3} \times 3\frac{3}{5} \times \frac{2}{9} =$

13D-4 $5\frac{1}{5} \times \frac{2}{13} \times 3\frac{1}{3} =$

13D-5 $7\frac{1}{3} \times 2\frac{2}{11} \times \frac{1}{4} =$

13D-6 $\frac{3}{8} \times 12 =$

13D-7 $10 \times \frac{7}{15} =$

13D-8 $\frac{5}{9} \times 21 =$

13D-9 $\frac{3}{4} \times 4\frac{1}{3} =$

13D-10 $3\frac{7}{9} \times \frac{3}{10} =$

13D-11 $\frac{7}{15} \times 9\frac{13}{32} =$

13D-12 $3\frac{1}{5} \times 6\frac{25}{32} =$

13D-13 $20\frac{2}{3} \times 5\frac{1}{8} =$

13D-14 $\frac{3}{8} \times 24 \times 5\frac{1}{2} =$

13D-15 $19 \times \frac{4}{7} \times 3\frac{3}{5} =$

13D-16 $3\frac{1}{2} \times 4\frac{1}{2} \times \frac{3}{8} =$

13D-17 $2\frac{1}{12} \times 4\frac{3}{4} =$

13D-18 $4\frac{1}{6} \times 3\frac{1}{5} \times 1\frac{3}{10} =$

13D-19 $1\frac{5}{16} \times 2\frac{2}{3} \times 3\frac{1}{7} =$

13D-20 $2\frac{1}{2} \times 4\frac{1}{8} \times 1\frac{1}{4} =$

Section 14: Dividing Fractions

To divide fractions (or mixed numbers), invert the divisor (the number or fraction that follows the division sign) and then multiply. Because of this, every division problem with fractions becomes a multiplication problem.

14-A Examples

1. $\frac{20}{9} \div \frac{5}{6}$ (which is the same as $\dfrac{\frac{20}{9}}{\frac{5}{6}}$ = ________.

solution: Step 1 invert the divisor, i.e., $\frac{5}{6}$ becomes $\frac{6}{5}$ (Note! $\frac{6}{5}$ is the <u>reciprocal</u> of $\frac{5}{6}$)

Step 2 multiply $\frac{\overset{4}{\cancel{20}}}{\underset{3}{\cancel{9}}} \times \frac{\overset{2}{\cancel{6}}}{\underset{1}{\cancel{5}}} = \frac{8}{3}$, or $2\,\frac{2}{3}$, Answer

2. $\frac{15}{4} \div \frac{3}{8} = \frac{\overset{5}{\cancel{15}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{1}{\cancel{3}}} = \frac{10}{1}$ or 10 , Answer

3. $\frac{4}{3} - 6$, which is the same as $\frac{4}{3} \div \frac{6}{1}$

since $\frac{6}{1} = 6$. $\frac{\overset{2}{\cancel{4}}}{3} \times \frac{1}{\underset{3}{\cancel{6}}} = \frac{2}{9}$

DRILL SHEET

UNIT 2: DIVIDING FRACTIONS

Divide the fractions and reduce to lowest terms.

14D-1 $\frac{20}{7} \div \frac{15}{2} =$

14D-2 $11 \div \frac{22}{5} =$

14D-3 $\frac{3}{8} \div 6 =$

14D-4 $\frac{3}{8} \div \frac{14}{9} =$

14D-5 $2\frac{1}{8} \div \frac{17}{18} =$

14D-6 $3\frac{3}{4} \div 2\frac{1}{5} =$

14D-7 $\dfrac{5\frac{1}{4}}{7} =$

14D-8 $\frac{5}{6} \div \frac{4}{6} =$

14D-9 $\frac{1}{12} \div \frac{5}{12} =$

14D-10 $\frac{3}{5} \div \frac{9}{10} =$

14D-11 $\dfrac{\frac{3}{10}}{\frac{11}{15}} =$

14D-12 $\frac{7}{9} \div \frac{3}{7} =$

14D-13 $\dfrac{\frac{5}{11}}{\frac{19}{44}} =$

14D-14 $\frac{14}{15} \div \frac{7}{25} =$

14D-15 $\frac{4}{15} \div \frac{8}{45} =$

14D-16 $\frac{6}{7} \div \frac{18}{23} =$

14D-17 $\dfrac{\frac{3}{6}}{\frac{9}{27}} =$

14D-18 $\frac{61}{64} \div \frac{5}{16} =$

14D-19 $\frac{23}{27} \div \frac{1}{9} =$

14D-20 $\frac{1}{4} \div \frac{1}{8} =$

14D-21 $\frac{3}{8} \div \frac{5}{16} =$

14D-22 $\frac{17}{32} \div \frac{51}{64} =$

14D-23 $\frac{17}{32} \div \frac{5}{64} =$

14D-24 $\frac{9}{16} \div \frac{5}{8} =$

14D-25 $\frac{1}{2} \div 2 =$

Section 15: Dividing Fractions, Mixed Numbers, and Whole Numbers

Divide and reduce to lowest terms.

15-A Examples

1. $3\frac{7}{8} \div 1\frac{15}{16} =$

solution: Step 1 convert the mixed numbers to improper fractions

$$\frac{31}{8} \div \frac{31}{16} =$$

Step 2 invert the divisor, $\frac{31}{16}$, and multiply

$$\frac{\overset{1}{31}}{\underset{1}{8}} \times \frac{\overset{2}{16}}{\underset{1}{31}} = \frac{2}{1} \text{ or } 2 \text{ , Answer}$$

2. $5\frac{2}{7} \div 3\frac{3}{14} = \frac{37}{7} \div \frac{45}{14} =$

solution:

$$\frac{37}{\underset{1}{7}} \times \frac{\overset{2}{14}}{45} = \frac{74}{45} \text{ or } 1\frac{29}{45} \text{ , Answer}$$

Note, dividing $\frac{74}{45}$ with the calculator, you obtain 1.6444 etc., which means there is 1 whole plus a decimal remainder, or part of a whole "leftover." Multiplying 0.644 x 45 = 28.98, or 29 for the fractional part or numerator.

DRILL SHEET

UNIT 2: DIVIDING FRACTIONS, MIXED NUMBERS, AND WHOLE NUMBERS

Divide and reduce to lowest terms.

15D-1 $11 \div \frac{22}{5} =$

15D-2 $\frac{3}{8} \div 6 =$

15D-3 $2\frac{1}{8} \div \frac{17}{18} =$

15D-4 $3\frac{3}{4} \div 2\frac{1}{5} =$

15D-5 $\dfrac{5\frac{1}{4}}{7} =$

15D-6 $14 \div \frac{2}{3} =$

15D-7 $\frac{2}{3} \div 14 =$

15D-8 $\frac{7}{9} \div 14 =$

15D-9 $\dfrac{\frac{32}{35}}{16} =$

15D-10 $\frac{7}{8} \div 5\frac{1}{16} =$

15D-11 $3\frac{2}{3} \div \frac{11}{24} =$

15D-12 $2\frac{19}{32} \div \frac{3}{64} =$

15D-13 $\dfrac{15\frac{7}{25}}{6} =$

15D-14 $3\frac{1}{2} \div 7\frac{5}{6} =$

15D-15 $12 \div \frac{3}{8} =$

15D-16 $\dfrac{14}{\frac{2}{3}} =$

15D-17 $\dfrac{1\frac{7}{8}}{\frac{3}{4}} =$

15D-18 $2\frac{1}{2} \div 1\frac{1}{4} =$

15D-19 $33 \div 1\frac{3}{8} =$

15D-20 $98 \div \frac{7}{64} =$

15D-21 $3000 \div 7\frac{1}{2} =$

15D-22 $1\frac{2}{3} \div 1\frac{2}{3} =$

15D-23 $3\frac{1}{45} \div \frac{7}{45} =$

15D-24 $\dfrac{1\frac{1}{16}}{1\frac{1}{64}} =$

15D-25 $\dfrac{25\frac{1}{16}}{25} =$

DRILL SHEET

UNIT 2: MULTIPLYING AND DIVIDING FRACTIONS IN PRACTICAL APPLICATIONS

Solve these word problems and reduce answers to lowest terms, and label answers with appropriate unit of measure:

16D-1 As the speed of an automobile increases, the amount of gasoline that is used also increases. The table shown below lists the gasoline mileage (miles per gallon) at various speeds for a particular 6-cylinder automobile. Determine the total number of gallons of gasoline used when the automobile travels the speeds and times indicated:

Miles per hour	Miles per gallon
30	30
35	27 1/2
40	25
45	23
50	21
55	20

a. 30 mi/hr for 1 1/3 hr ______

b. 40 mi/hr for 3 3/4 hr ______

c. 50 mi/hr for 7/10 hr ______

d. 55 mi/hr for 3 1/5 hr ______

16D-2 The purchasing department of ALPO purchases supplies for the maintenance department as itemized lists. One such list is shown following:

6 2/3 boxes of Item A at $3 1/4 per box
1/3 yard of Item B at $4 1/2 per yard
8 pieces of Item C at $15 3/4 per dozen pieces

Find the total cost of the items listed. ______

16D-3 Seven machine operators earn $79.84 each per day, 5 days a week. If each operator saves 3/16 of their earnings, how much will all seven operators save in 1 year? (Consider 52 paychecks per operator per year.)

______

16D-4 A certain machine can cut 6 3/8 in. of silicon rod in 1 minute. How many inches of the rod can this machine cut in 15 seconds.
(60 seconds = 1 minute)

____

16D-5 A new cinder block wall in the test set department is 23 1/2 blocks long and 14 blocks high. <u>With</u> the concrete joint (i.e., the mortar) a block measures 8 1/4 in. by 16 1/4 in. If the maintenance department is to paint this wall, how large an area should they plan to paint?
(Area = height x length)

____

16D-6 GMA's research center has 217 1/2 acres of land. If it is divided into five equal tracts of land, how many acres will each tract contain?

____

16D-7 How many pieces of candy can you buy with 18 cents if each piece costs 3/4 of a cent?

____

16D-8 Ten different orders were received for packaging chips from a piece of tubing that was 12-ft. long. How many whole pieces of tubing 3 9/16 in. long can be cut for each order? Each order must get the same amount. (12 in. = 1 foot)

____

16D-9 A certain piece of cardboard used for packaging devices weighs 2 1/8 oz. If a bundle of these pieces of cardboard weighs 21 1/4 lb., how many pieces are in a bundle? (16 oz. = 1 lb.)

____

16D-10 A service station manager bought 15 qt. of oil for $\$8\frac{1}{8}$. If he sells one quart for $\$\frac{3}{4}$, how much is his gross profit? (Express your answer in a fraction of a dollar.)

____

16D-11 What is the weight of 16 pieces of drill rod each 6 1/2 ft. long if drill rod of this kind and size weighs 5/8 lb. per foot?

____

16D-12 A certain alloy used for making terminal leads is made up of 3/8 copper, 1/4 lead, 1/16 zinc, and 5/16 tin by weight. We need 480 lb. of this alloy. How many pounds of each metal must we use?

copper	______	zinc	______
lead	______	tin	______

16D-13 The net weight of nails in a keg is 240 lb. The weight of each nail is 3/4 oz. How many nails are in the keg?

______

16D-14 Find the total weight of three castings weighing 76 1/2 lb. each, seven castings weighing 23 1/4 lb. each, and two castings weighing 23 1/8 lb. each.

______

16D-15 There are two corner latches on the cover in your clean room that are 4 feet apart. If it was decided to add two additional latches between the two corner latches (equally spaced), how far apart should they be?

______

Section 17: Order of Operations

This is identical to the procedure discussed in Unit 1 for whole numbers:

Do all operations within grouping symbols: parentheses ( ); brackets [ ]; and braces { } first. Within the parentheses, etc., do multiplication or division operations (whichever comes first) in order from left to right. Then do addition and subtraction operations in order from left to right.

After completing the work in the grouping symbols, do multiplication or division operations (whichever comes first) in order from left to right.

Last, do addition and subtraction operations in order from left to right.

17-A Examples

1. $\left(\frac{5}{9} + \frac{2}{3}\right) \div \frac{1}{3} =$ ___

solution: Step 1

$$\frac{5}{9} = \frac{5}{9}$$

$$+\ \frac{2}{3} = \frac{6}{9}$$

$$\frac{11}{9}$$

Step 2 $\frac{11}{9} \div \frac{1}{3} = \frac{11}{\cancel{9}_{3}} \times \frac{\cancel{3}^{1}}{1} = \frac{11}{3}$ or $3\ \frac{2}{3}$ Answer

2. $\frac{5}{9} + \frac{2}{3} \div \frac{1}{3} =$ ___

solution: Step 1 $\frac{2}{3} \div \frac{1}{3} = \frac{2}{\cancel{3}_{1}} \times \frac{\cancel{3}^{1}}{1} = \frac{2}{1}$ or 2

Step 2 $\quad 2$

$$\begin{array}{r} + \quad \frac{5}{9} \\ \hline 2\frac{5}{9} \end{array} \quad \text{Answer}$$

3. $(1\frac{3}{4} + 3\frac{3}{8} \times 6) \div 18 + \frac{1}{2} =$

solution: Step 1 $\quad [1\frac{3}{4} + (3\frac{3}{8} \times 6)] = [1\frac{3}{4} + (\frac{27}{\cancel{8}_{4}} \times \cancel{6}^{3})]$

$= [1\frac{3}{4} + (\frac{27}{4} \times 3)] = [1\frac{3}{4} + \frac{81}{4} \text{ or } 20\frac{1}{4}]$

$$\begin{array}{r} = \quad 1\frac{3}{4} \\ + \quad 20\frac{1}{4} \\ \hline 21\frac{4}{4} \end{array} \text{ or } 22$$

Step 2 $\quad 22 \div 18 = \frac{11}{9} = 1\frac{2}{9}$

Step 3 $\quad 1\frac{2}{9} \times \frac{2}{2} = \quad 1\frac{4}{18}$

$$\begin{array}{lr} + \frac{1}{2} \times \frac{9}{8} = & \frac{9}{18} \\ \hline & 1\frac{13}{18}, \end{array} \quad \text{Answer}$$

4. $\dfrac{15}{2\frac{2}{3}} + \dfrac{25\frac{1}{5}}{4}$ (which is the same as $15 \div 2\frac{2}{3} + 25\frac{1}{5} \div 4$) =

solution: Step 1 $15 \div 2\frac{2}{3} = 15 \div \frac{8}{3} = 15 \times \frac{3}{8} = \frac{45}{8}$ or

$5\frac{5}{8}$

Step 2 $25\frac{1}{5} \div 4 = \frac{\overset{63}{\cancel{126}}}{5} \times \frac{1}{\cancel{4}_{2}} = \frac{63}{10}$ or $6\frac{3}{10}$

Step 3 $5\frac{5}{8} + 6\frac{3}{10} =$

$$\begin{array}{rcr} 5\frac{5}{8} & = & 5\frac{25}{40} \\ +\ 6\frac{3}{10} & = & 6\frac{12}{40} \\ \hline & & 11\frac{37}{40} \end{array}$$ Answer

DRILL SHEET

UNIT 2: ORDER OF OPERATIONS - FRACTIONS, WHOLE NUMBER, AND MIXED NUMBERS

17D-1 $\frac{7}{8} + \frac{5}{16} - \frac{3}{4} =$

17D-2 $(\frac{7}{8} \times 14) \div (\frac{3}{8} \times 6) =$

17D-3 $\dfrac{7\frac{1}{8} \times (6\frac{7}{8} - 4\frac{3}{16})}{4\frac{3}{4}} =$

17D-4 $\frac{13}{16} \div 4 \times 3 - \frac{1}{4} =$

17D-5 $44 \div 11 \times 4 - \frac{7}{8} =$

17D-6 $\dfrac{\frac{7}{8} + 1\frac{7}{16}}{8} =$

17D-7 $23\frac{1}{3} \div \frac{7}{8} + \frac{15}{16} =$

17D-8 $(\frac{15}{16} \div \frac{1}{3} + 2) \div (\frac{11}{32} \times 43 - 9) =$

17D-9 $\dfrac{77}{(10 - \frac{7}{16}) + (\frac{13}{32} \div \frac{1}{3})} =$

17D-10 $35 + \frac{1}{8} - 3 \div 13 \times 5 =$

UNIT 2: REVIEW

-2R1- $\frac{3}{4} + \frac{7}{8} + \frac{5}{16}$ ______

-2R2- $37\frac{7}{10} - 19\frac{3}{4}$ ______

-2R3- $\frac{3}{4} \times 12 \times 1\frac{5}{8}$ ______

-2R4- $5\frac{3}{5} \div \frac{15}{16}$ ______

-2R5- $8\frac{3}{4} \times (10 + 5\frac{1}{16}) \div (12 + \frac{2}{3})$ ______

-2R6- $(12 \times 8) \div (8 \times 3)$ ______

-2R7- $(\frac{7}{8} \times 4 - \frac{3}{4} + 5) \times 12\frac{3}{16}$ ______

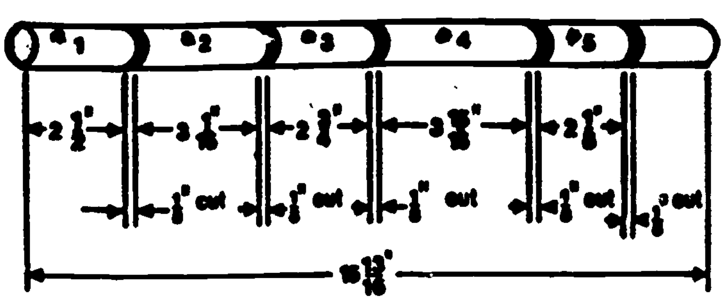
-2R8- Express $\frac{9}{16}$ as an equivalent fraction with a denominator of 64 ______

-2R9- Express $\frac{72}{128}$ in lowest terms ______

-2R10- Express $\frac{451}{64}$ as a mixed number ______

-2R11- Find the lowest common denominator for $\frac{7}{2}, \frac{2}{3}, \frac{1}{2}$ ______

-2R12- Five pieces are cut from the length of silicon rod shown. After the pieces are cut, the remaining length is discarded. What fractonal part is discarded? ______ in.

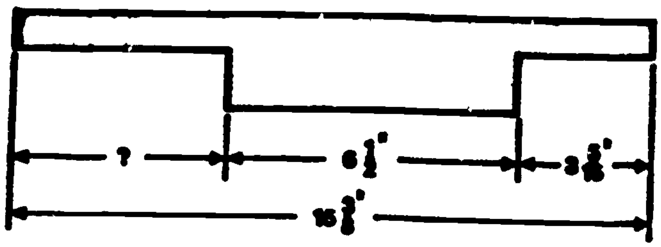


-2R13- What is the thickness of a chemical room countertop made of $\frac{7}{8}$ in. plywood and $\frac{1}{16}$ in. Formica? ______

-2R14- Three pieces of steel plate for a curing furnace base are welded together. What is the total thickness if the pieces are $\frac{1}{2}$ ft., $\frac{7}{16}$ ft., and $\frac{29}{32}$ ft.? ______

-2R15- If 4 $\frac{3}{8}$ gallons of water are used to dilute 7 $\frac{1}{4}$ gallons of acid, how many gallons are in the mixture? ______

-2R16- Find the missing length (?) in the following sketch:



______ in.

-2R17- An engineering technician needed 10 pieces of wire each 3 $\frac{5}{8}$ in. long. What length will be needed for the wire pieces? ______

-2R18- A fuel tank that holds 75 liters of fuel is $\frac{1}{4}$ full. How many liters of fuel are in the tank? ______

-2R19- A developer subdivided 5 $\frac{1}{4}$ acres into lots each containing $\frac{7}{10}$ of an acre. How many lots were made? ______

-2R20- A stack of $\frac{5}{8}$ in. thick plywood is 21 $\frac{7}{8}$ in. high. How many sheets of plywood are in the stack?

-2R21- An order of aluminum trays costs $145, including a $10 shipping fee. If the price of 1 lb. of aluminum is $3, how many pounds of aluminum are in the order?

-2R22- If 7 $\frac{1}{2}$ kilowatts (KW) of power are distributed equally over 5 resistors, what is the average number of kilowatts per resistor?

-2R23- To convert from a Celsius temperature reading to a Fahrenheit temperature reading, we follow the procedure:

Fahrenheit (F) = $\frac{9}{5}$ x Celsius (C) + 32

Find F, when C = 35

-2R24- To convert from a Fahrenheit temperature reading to a Celsius temperature reading, we follow the procedure:

Celsius = $\frac{5}{9}$ x (Fahrenheit - 32)

Find C, when F = 98 $\frac{3}{5}$

UNIT 3: DECIMALS

OBJECTIVES

After completing this unit, you will be able to:

- understand the meaning of a decimal or part of the whole
- understand the meaning of a decimal place, or the order of position to the right of the decimal point
- understand "rounding" decimals
- express common fractions as decimals
- express decimals as common fractions
- add and subtract decimals
- multiply and divide decimals
- follow order of operations involving decimals
- solve word problems involving decimals

Dictionary:	Meaning
1. Decimal	fraction whose denominator is 10 or some power of 10
2. Decimal point	dot or period
3. Decimal place	position of number to right of decimal point
4. Rounding, or rounding off	reporting a number to a specific place after applying 5 and over or under 5 rule
5. Find	calculate; solve

Section 1: Concept of a Decimal

What is a decimal?

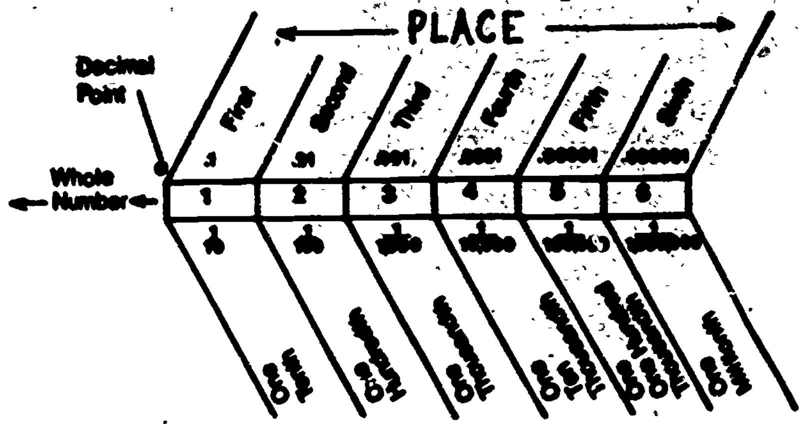
We have already learned in Units 1 and 2 that there are whole numbers and parts of whole numbers which we called common fractions. We also learned in Unit 2 that when we divide the numerator of a proper fraction by the denominator on a calculator we obtain a number to the right of a decimal point (actually a dot) -- this is a decimal of the whole. The whole number is always to the left of the decimal point, and the decimal is to the right of the decimal point.

For example, divide $\frac{7}{8}$ and you obtain 0.875 (when the decimal point indicator on the calculator is positioned at 3).

If you divide $\frac{8}{8}$, you obtain 1.000 (with the decimal point indicator at 3). So 0.875 is a decimal of 1.000, and it is another way of expressing the common fraction $\frac{7}{8}$.

Section 2: Decimal Place

Illustrated below is the decimal point indicator:



In the above illustration, whole numbers are written to the left of the decimal point, and decimals are written to the right of the decimal point.

The decimal place refers to the position of a number (let's say it is 1 as above) to the right of the decimal point; these places are shown as 1, 2, 3, 4, 5, and 6, and are spoken as first, second, third, fourth, fifth, and sixth. (We can go on and on to the right with additional decimal places, but for our applications, the sixth is sufficient.)

(Also shown in the illustration are the corresponding common fractions for the decimals. Note that the number of zeros for any common fraction is the same as the place, i.e., for $\frac{1}{1000}$ there are 3 zeros and this is in the third place.)

2-A Examples

In the decimal 0.175463, the numeral 3 is in the sixth place. Indicate the "place" for the following:

4 ______

7 ______

6 ______

5 ______

1 ______

Section 3: Rounding Off Decimal Numbers

You will often be instructed to give an answer or a result which is correct to a certain number of decimal places. If you have a result of 47.264725 and want it "rounded" to three places, your answer is 47.265; rounded to two places, it is 47.26; rounded to one place, 47.3.

3-A Procedure for rounding off an answer to a given number of decimal places:

1. Set the decimal point indicator of the calculator one digit beyond the number of places the final result is to be rounded.

   Example - If a final answer is to be reported to the second place, set the indicator at 3. (Remember, regardless how many numbers that follow the third place, you disregard all of them and concentrate on the third place only for rounding off.)

2. If the number you are concentrating on is equal to 5 or more (6, 7, 8, 9), add 1 to the number before it. (For our discussion in Step 1, it is added to the number in the second place.)

3. If the number you are concentrating on is 4 or less (3, 2, 1), do not change the number before it. (For our discussion in Step 1, the number in the second place remains unchanged.)

3-B Examples

1. Round 0.73862 to 3 decimal places.

solution: Step 1 Set decimal indicator on calculator at 4.

Step 2 Concentrate on the number 6 which is in the fourth place.

Step 3 Since 6 is more than 5, we add 1 to the number in the third place, and our final answer is 0.739 (to 3 places, as requested).

2. Round 3.805 to 2 places.

solution: Step 1 Set decimal indicator at 3.

Step 2 Concentrate on the number 5 which is in the third place.

Step 3 Since this number is equal to 5, we add 1 to the number in the second place, and our final <u>answer is 3.81</u>.

3. Round 0.784 to 2 places.

solution: Step 1 Set decimal indicator at 3.

Step 2 Concentrate on the number in the third place.

Step 3 Since 4 is less than 5, we <u>do not</u> change the second place number, and our final <u>answer is 0.78</u>.

<u>Note</u>! Now that we have embarked into decimal numbers there is a certain language and procedure we follow:

<u>First</u>, in speaking (orally) it is important to emphasize the decimal point. For example, the number 17.565 is spoken as "seventeen point five six five."

<u>Second</u>, when we write a decimal (with no whole number to the left of the decimal point), it is important to place a zero to the left of the decimal point. For example, 0.765. This removes any misunderstanding about the number being a <u>decimal</u>.

DRILL SHEET

UNIT 3: ROUNDING DECIMALS

Round the numbers to the indicated decimal places (shown in parentheses):

1D-1	0.0855 (2)	=	1D-11	7.829 (1)	=
1D-2	0.0063 (2)	=	1D-12	14.003 (1)	=
1D-3	0.057 (1)	=	1D-13	0.05645 (3)	=
1D-4	76.8999 (3)	=	1D-14	0.05645 (4)	=
1D-5	139.0062 (2)	=	1D-15	9.37335 (3)	=
1D-6	33.01997 (4)	=	1D-16	0.37335 (4)	=
1D-7	1.927 (3)	=	1D-17	0.37335 (2)	=
1D-8	1.9254 (3)	=	1D-18	33.01997 (2)	=
1D-9	0.0015 (3)	=	1D-19	33.01997 (3)	=
1D-10	13.33 (1)	=	1D-20	13.0039 (3)	=

Section 4: Expressing Common Fractions As Decimals

To express a common fraction as a decimal, divide the numerator by the denominator.

4-A Examples

1. $\frac{7}{8}$ (or $7 \div 8$) = 0.875 (to 3 places), Answer

2. $\frac{1}{16}$ (or $1 \div 16$) = 0.0625 (to 4 places), Answer

3. $\frac{23}{24}$ (or $23 \div 24$) = 0.958 (to 3 places), Answer

4. $\frac{1}{4}$ (or $1 \div 4$) = 0.25 (to 2 places), Answer

5. $\frac{1}{10}$ (or $1 \div 10$) = 0.1 (to 1 place), Answer

6. $\frac{63}{64}$ (or $63 \div 64$) = 0.984 (to 3 places), Answer

7. $\frac{63}{64}$ (or $63 \div 64$) = 0.9844 (to 4 places), Answer

8. $\frac{11}{16}$ (or $11 \div 16$) = 0.688 (to 3 places), Answer

9. $\frac{37}{64}$ (or $37 \div 64$) = 0.578 (to 3 places), Answer

10. $\frac{37}{64}$ (or $37 \div 64$) = 0.5781 (to 4 places), Answer

11. $\frac{37}{64}$ (or $37 \div 64$) = 0.57813 (to 5 places), Answer

12. $\frac{37}{64}$ (or $37 \div 64$) = 0.578125 (to 6 places), Answer

DRILL SHEET

UNIT 3: EXPRESSING COMMON FRACTIONS AS DECIMALS

Express the following as decimals to the places indicated in parentheses:

2D-1	$\frac{2}{9}$ =	________	(3)	2D-11	$\frac{19}{6}$[illegible] =	________	(4)
2D-2	$\frac{19}{29}$ =	________	(3)	2D-12	$\frac{31}{32}$ =	________	(4)
2D-3	$\frac{5}{12}$ =	________	(3)	2D-13	$\frac{3}{8}$ =	________	(2)
2D-4	$\frac{17}{21}$ =	________	(3)	2D-14	$\frac{17}{64}$ =	________	(3)
2D-5	$\frac{1}{32}$ =	________	(4)	2D-15	$\frac{9}{32}$ =	________	(3)
2D-6	$\frac{1}{8}$ =	________	(3)	2D-16	$\frac{3}{64}$ =	________	(4)
2D-7	$\frac{1}{4}$ =	________	(2)	2D-17	$\frac{17}{1000}$ =	________	(2)
2D-8	$\frac{1}{2}$ =	________	(3)	2D-18	$\frac{11}{500}$ =	________	(3)
2D-9	$\frac{9}{16}$ =	________	(4)	2D-19	$\frac{13}{75}$ =	________	(4)
2D-10	$\frac{3}{4}$ =	________	(1)	2D-20	$\frac{13}{16}$ =	________	(4)

* Note: A functional number whose quotient repeats a particular sequence of number is called a **repeating or periodic decimal.** A periodic decimal is often indicated by a bar over the digits that repeat.

For example: $\frac{3}{7} = .\overline{428571}$

Section 5: Expressing Decimals As Common Fractions

To express a decimal as a common fraction follow a four-step procedure. For example, to convert 0.065:

solution: Step 1 - Determine the place of the last digit, 5; it is in the third place.

Step 2 - Since the third place is thousandths, the denominator will be $\frac{\quad}{1000}$.

Step 3 - Disregard all zeros between the decimal point and the first numeral to the right; then write the number 65 in the numerator, as $\frac{65}{1000}$.

Step 4 - Reduce the common fraction to lowest terms, so that $\frac{\overset{13}{\cancel{65}}}{\underset{200}{\cancel{1000}}} = \frac{13}{200}$, Answer,

since the numerator and denominator can be divided by the common number 5.

5-A Examples

1. 0.8 = ____

solution: Step 1 - The 8 is the first place.

Step 2 - The denominator is $\frac{\quad}{10}$.

Step 3 - The numerator is 8, and the common fraction is $\frac{8}{10}$.

Step 4 - Reduce to lowest terms, so that $\frac{\overset{4}{\cancel{8}}}{\underset{5}{\cancel{10}}} = \frac{4}{5}$, Answer, since the numerator and denominator can be divided by 2.

2. 0.006 = ____

solution: Step 1 - The 6 is in the third place.

Step 2 - The denominator is $\frac{\quad}{1000}$.

Step 3 - The numerator is 6, and the common fraction is $\frac{6}{1000}$.

Step 4 - Reduce to lowest terms, so that

$$\frac{\overset{3}{\cancel{6}}}{\underset{500}{\cancel{1000}}} = \frac{3}{500}$$

, Answer, since the numerator and denominator can be divided by 2.

DRILL SHEET
UNIT 3: EXPRESSING DECIMALS AS COMMON FRACTIONS

Express the following as common fractions (reduce to lowest terms).

3D-1	0.375 =		3D-11	0.3125 =	
3D-2	0.062 =		3D-12	0.625 =	
3D-3	0.335 =		3D-13	0.8125 =	
3D-4	0.4375 =		3D-14	0.500 =	
3D-5	0.4375 =		3D-15	0.125 =	
3D-6	0.2 =		3D-16	0.09375 =	
3D-7	0.12 =		3D-17	0.1875 =	
3D-8	0.235 =		3D-18	0.25 =	
3D-9	0.93 =		3D-19	0.750 =	
3D-10	0.75 =		3D-20	0.313 =	

Section 6: Adding and Subtracting Decimals

The most important step in decimal calculations is to set the decimal point indicator for the proper place. Earlier we learned to set the decimal point indicator one digit to the right of the place we are instructed to report the final result when we are asked to round an answer.

In the absence of any instructions, for reporting a result to a specific decimal place, we must report the entire number resulting from the arithmetic operation. This will be illustrated in the following examples:

6-A Examples of addition

1. 147.935 + 0.13 = ______

solution: Step 1 - Observe that the third decimal place is the farthest place, or position from the decimal point for a numeral in either number. Because of this, for addition and subtraction problems on our calculator, set the decimal indicator at 3 (we were not given any instructions to round off).

Step 2 - Enter 147, then press the [ ] key, and then enter 935.

Step 3 - Press [+] key.

Step 4 - Press [ ] key, then enter 13. (Note, although it is important to write a zero before the decimal, it is not necessary to enter the zero in the calculator.) Display shows 148.065, Answer.

2. 17 + 0.0134 + 91.02 + 139.001 = ______

solution: Step 1 - Enter 17 - you do not have to enter the decimal point for any whole number.

Step 2 - Press [+] key.

Step 3 - Press [ ] key and enter 0134.

Step 4 - Press [+] key.

Step 5 - Enter 91, then press [ ] key, and then enter 02.

Step 6 - Press [+] key.

Step 7 - Enter 139, then press [ ] key, and then enter 001.

Step 8 - Press [+] key; display shows 247.0344, Answer

3. $0.0732 + \frac{7}{16} + 0.2323 =$ ______

solution: Step 1 - Set decimal indicator at 4.

Step 2 - Press [ ] key, then enter 0732.

Step 3 - Press [+] key.

Step 4 - Since it is impossible to enter a common fraction in the calculator, we must first convert $\frac{7}{16}$ to a decimal; in dividing 7 by 16, we obtain 0.4375. Press [ ] key, then enter 4375.

Step 5 - Press [+] key.

Step 6 - Press [ ] key, then enter 2323.

Step 7 - Press [+] key; display shows 0.7430.

6-B Examples of subtraction

1. 44.6 - 27.368 = ______

solution: Step 1 - Observe that the third decimal place is the farthest place, or position from the decimal point for a numeral in either number. Because of this, set the decimal indicator at 3 (again, we were not given any instructions to round off).

Step 2 - Enter 44, then press [ ] key, then enter 6. (Note! The calculator automatically displays two zeros following the 6, thus providing a 3-place number for subtraction.)

Step 3 - Press [+] key

Step 4 - Enter 27, then press [ ] key, then enter 368.

Step 5 - Press [-] key; display shows 17.232, Answer

2. $46\frac{3}{10} - 33.912 =$ ______

solution: Step 1 - Set decimal indicator at 3.

Step 2 - Convert the common fraction $\frac{3}{10}$ of the mixed number $46\frac{3}{10}$ to a decimal; thus, $\frac{3}{10}$ becomes 0.3. Enter 46, then press [ ] key, then enter 3.

Step 3 - Press [+] key.

Step 4 - Enter 33, then press [ ] key, then enter 912.

Step 5 - Press [-] key; display shows 12.388, Answer

DRILL SHEET

UNIT 3: ADDING AND SUBTRACTING DECIMALS

4D-1 $0.317 + 0.029 =$

4D-2 $8.036 + 16 + 0.7 =$

4D-3 $0.877 - 0.304 =$

4D-4 $10.002 - 0.1999 =$

4D-5 $83.712 + 0.056 + 35 =$

4D-6 $3.062 - 1.956 =$

4D-7 $12.002 + 0.018 + 0.003 + 0.017 =$

4D-8 $305.1 + 43.95 + 0.014 + \frac{1}{8} =$

4D-9 $0.009 - 0.0086 =$

4D-10 $26.009 - 25.999 =$

4D-11 $175 - 173.909 =$

4D-12 $23\frac{1}{8} - 0.375 =$

4D-13 $144 - 11.883 =$

4D-14 $11.883 - 0.144 =$

4D-15 $12.01 + 11.003 + 0.0175 =$

4D-16 $0.3134 - 0.179 =$

4D-17 $0.347 + 1\frac{7}{8} + 0.5 + 3.1414 =$

4D-18 $11.09 + 1.109 + 0.1109 =$

4D-19 $39\frac{15}{16} - 17.9435 =$

4D-20 $63 + 63.63 + 0.63 =$

4D-21 $1179 - 1011.9755 =$

4D-22 $12 + 5.005 + 0.1 =$

4D-23 $0.095 - 0.091 =$

4D-24 116.0157 - 16.01 =

4D-25 33 + 33.93 + 0.93 + 0.935 =

4D-26 67.05 - 37.095 =

4D-27 $22\frac{1}{16} + 0.0625 + 0.8750 =$

4D-28 77.9703 - 64.848 =

4D-29 $18 - 17\frac{61}{64} =$

4D-30 25 + 11.88 + 0.0033 =

Section 7: Multiplying and Dividing Decimals

7-A Multiplying Decimals

When multiplying decimals, it is important to follow instructions for the decimal place when reporting the final answer. Otherwise, you must set the decimal place indicator so that the complete result is displayed.

Examples

1. 60.412 x 0.53 = ______

solution: Step 1 Since there are no instructions for the decimal place, you must count the total number of decimal places for both numbers--it is 5. Therefore, set the decimal indicator at 5.

Step 2 Enter 60, then press [ ] key, then enter 412.

Step 3 Press [x] key.

Step 4 Press [ ] key, then enter 53.

Step 5 Press [=] key; display shows 32.01836, Answer

2. 60.412 x 0.59 = ______

(round to 2 places)

solution: Step 1 We are instructed to round to 2 places, so we set the decimal indicator at 3.

Step 2 Enter 60, press [ ] key, then enter 412.

Step 3 Press [x] key.

Step 4 Press [ ] key, then enter 59.

Step 5 Press [=] key; display shows 35.643; since the third place numeral is less than 5, the number 4 remains unchanged and our <u>final answer is 35.64</u>.

3. $1\frac{9}{16}$ x 0.33 x 4.27 = ______

(round to 4 places)

Section 7: Multiplying and Dividing Decimals

7-A Multiplying Decimals

When multiplying decimals, it is important to follow instructions for the decimal place when reporting the final answer. Otherwise, you must set the decimal place indicator so that the complete result is displayed.

Examples

1. 60.412 x 0.53 = ______

solution: Step 1 Since there are no instructions for the decimal place, you must count the total number of decimal places for both numbers--it is 5. Therefore, set the decimal indicator at 5.

Step 2 Enter 60, then press [ ] key, then enter 412.

Step 3 Press [x] key.

Step 4 Press [ ] key, then enter 53.

Step 5 Press [=] key; display shows 32.01836, Answer

2. 60.412 x 0.59 = ______

(round to 2 places)

solution: Step 1 We are instructed to round to 2 places, so we set the decimal indicator at 3.

Step 2 Enter 60, press [ ] key, then enter 412.

Step 3 Press [x] key.

Step 4 Press [ ] key, then enter 59.

Step 5 Press [=] key; display shows 35.643; since the third place numeral is less than 5, the number 4 remains unchanged and our <u>final answer is 35.64</u>.

3. $1\frac{9}{16} \times 0.33 \times 4.27 =$ ______

(round to 4 places)

solution: Step 1 Set decimal indicator at 5.

Step 2 Enter 1, then press [ ], then enter 5625 (decimal of $\frac{9}{16} = 0.5625$).

Step 3 Press [x] key.

Step 4 Press [ ] key, then enter 33.

Step 5 Press [=] key; display shows 0.51563.

Step 6 Press [x] key.

Step 7 Enter 4, then press [ ] key, then enter 27.

Step 8 Press [=] key; display shows 2.20174; when rounded to 4 places, the <u>answer is 2.2017</u>.

7-B Dividing Decimals

When dividing decimals, it is important to follow instructions for decimal place when reporting the final answer. Otherwise, you set the decimal indicator to the right as far as it can go on our calculator, or at 6.

Examples

1. 0.338 ÷ 0.52 = ______

solution: Step 1 Since there are no instructions for decimal place in the final answer, set the decimal indicator at 6.

Step 2 Press [.] key, and enter 338.

Step 3 Press [ ] key.

Step 4 Press [.] key, then enter 52.

Step 5 Press [=] key; display shows 0.650000.

Since all the zeros following the 5 are not significant numbers, we drop them and show the answer as 0.65. To test this, multiply 40 by 0.650000, and multiply 40 by 0.65. The <u>answer is the same, 26</u>.

2. 1.7594 ÷ 6.03

(round to 3 places)

solution:

Step 1 Set decimal indicator at 4.

Step 2 Enter 1, then press [ ] key, then enter 7594.

Step 3 Press [÷] key.

Step 4 Enter 6, then press [ ] key, then enter 03.

Step 5 Press [=] key; display shows 0.2918; since the fourth place numeral is more than 5, increase the 1 to 2; the final answer is 0.292.

DRILL SHEET

UNIT 3: MULTIPLYING AND DIVIDING DECIMALS

Perform the operations indicated and round to the places shown in parentheses:

5D-1	0.9×0.5 (4) =
5D-2	0.8×29 (3) =
5D-3	0.63×0.16 (4) =
5D-4	$7.22 \times \frac{3}{8}$ (3) =
5D-5	0.025×0.09 (4) =
5D-6	$0.42 \times 11 \times 0.4$ (4) =
5D-7	$0.009 \times 120 \times 6.7$ (4) =
5D-8	$0.8 \div 0.2$ (3) =
5D-9	$6.3 \div 0.3$ (3) =
5D-10	$1.44 \div 0.08$ (3) =
5D-11	$18.750 \div \frac{3}{4}$ (3) =
5D-12	$0.002 \div 0.91$ (3) =
5D-13	$153.73 \div 14.27$ (4) =
5D-14	$0.0084 \div 3.094$ (4) =
5D-15	$3876.5 \div 5.125$ (3) =
5D-16	$12.1 \times 0.761 \times 0.0035$ (4) =
5D-17	$0.0035 \div 17$ (3) =
5D-18	$1.7901 \div 40$ (1) =
5D-19	$35.001 \div 7.2$ (2) =
5D-20	$\frac{3}{16} \times 1.704 \times 1\frac{1}{4}$ (4) =
5D-21	$19\frac{7}{8} \div 0.0115$ (3) =

5D-22 $0.0115 \div 19\frac{7}{8}$ (3) =

5D-23 $270.05 \div 11.953$ (4) =

5D-24 60 x 0.44 x 11.1 x 9.7 (4) =

5D-25 Thunder and lighting occur at the same time. Light travels almost instantly, and sound travels about .206 miles per second. To find how far away a storm is, follow these steps.

1. When you see the flash, count the number of seconds it takes before you hear the thunder.
2. Multiply the number of seconds the thunder took to reach you by .206 in order to find the distance in miles.

For example, if it takes 5 seconds for thunder to reach you after you see the lightning, how far away is the storm?

Distance = .206 x Seconds It Takes Thunder to Reach You
= .206 x 5
= 1.030 Miles

The storm is about a mile away.

How far is the storm if it takes 8 seconds for thunder to reach you?

Section 8: Order of Operations Involving Decimals

The rules to follow with decimals are exactly the same as discussed for whole numbers and common fractions. We repeat them here:

Do all operations within grouping symbols: parentheses ( ); brackets [ ]; and braces { } first. Within the parentheses, etc., do multiplication or division operations (whichever comes first) in order from left to right. Then do addition and subtraction operations in order from left to right.

After completing the work in the grouping symbols, do multiplication or division operations (whichever comes first) in order from left to right.

Last, do addition and subtraction operations in order from left to right.

8-A Examples

1. 9.03 + 2.75 x 0.9 = _______

   (round to 2 places)

solution: Step 1 Set decimal indicator at 3.

Step 2 Since multiplication appears in the problem, we do this first, and our result is 2.475.

Step 3 Add 2.475 to 9.03; display shows 11.505; rounded to 2 places, our final <u>answer</u> <u>is</u> <u>11.51</u>.

2. 12.4 x (13.88 - 0.07 x 0.5) = _______

   (round to 2 places)

solution: Step 1 Set decimal indicator at 3.

Step 2 We do the operations within the parentheses first, and within we do the multiplication first; our result is 0.035.

Step 3 Subtract 0.035 from 13.88; our result is 13.845.

Step 4 Multiply 13.845 by 12.4; our result is 171.678; rounded to 2 places, our final <u>answer</u> <u>is</u> <u>171.68</u>.

DRILL SHEET

UNIT 3: ORDER OF OPERATIONS INVOLVING DECIMALS

Perform the indicated operations and report final answers to the decimal places shown in parentheses:

6D-1 1.31 x 6 - 8.2 ÷ 3.4 (2) =

6D-2 67 + 13 - 9 x 4 ÷ 3 (2) =

6D-3 24 ÷ 6.7 x 3.9 (2) =

6D-4 42.090 x 3.045 + 99 ÷ 3 x 4 (3) =

6D-5 0.009 ÷ 3 x 11.11 + 0.047 (3) =

6D-6 25 + 11 x 5 ÷ 5.1 (1) =

6D-7 64.08 - 19.009 ÷ 4 (3) =

6D-8 134.7 x 0.01 ÷ 0.5 (2) =

6D-9 0.02 + 0.002 x 0.002 ÷ 0.002 (3) =

6D-10 1.24 + 1.35 + 9.05 ÷ 0.15 (3) =

6D-11 12 + 32.09 - 9.04 ÷ 2 x 4.1 (4) =

6D-12 $14\frac{15}{16} - 13 + 4.7 \times \frac{3}{8} \div \frac{1}{2}$ (2) =

6D-13 0.07 - .004 x 13.019 ÷ 7.9 (3) =

6D-14 14.7 ÷ 2 x 3 + 8 ÷ 4.2 (2) =

6D-15 0.019 x 0.1 ÷ 0.01 (2) =

6D-16 $1\frac{7}{32} + 19\frac{3}{32} + 0.195 - 0.79$ (3) =

6D-17 $\frac{11}{32} \div 3.5 \times 4 + 19.04 - \frac{3}{16}$ (3) =

6D-18 10.9 x 10.9 ÷ 10.9 + 10.9 - 10.9 (3) =

6D-19 0.525 x 0.505 x 0.525 ÷ 0.505 (3) =

6D-20 113.013 + 0.013 x 9.08 (3) =

6D-21 The cost of electricity is based on the number of kilowatt hours. A kilowatt is 1000 watts, and a kilowatt hour is the amount of electricity used by a 1000-watt appliance in one hour. To

find the number of kilowatt hours used by an appliance, use this formula:
kw = (Watts ÷ 1000) x No. of Hours.

How much does it cost to run a 210-watt television for 60 hours a month if the cost of electricity is \$.0525 per kilowatt hour?

kw = (Watts ÷ 1000) x No. of hours

= (210 ÷ 1000) x 60

= .210 x 60

= 12.6 kilowatt hours

To find the cost, multiply the number of kilowatt hours by the cost per watt hour.

12.6 x .0525 = \$.66150 = \$.66

The cost is about \$.66.

Find the cost to operate:

1. A 100-watt bulb for 500 hours.

2. A 5750-watt clothes dryer for 20 hours.

UNIT 3: REVIEW

-3R1- Write each of the word statements as a decimal number:

a. Forty-four and four tenths ________

b. Sixty point six-six ________

c. One hundred one point zero-one ________

d. Two hundred twenty-two and two hundredths ________

-3R2- In the decimal 0.93758, indicate the place for the following:

a. 3 ________ d. 8 ________

b. 5 ________ e. 9 ________

c. 7 ________

-3R3- Round off to one place:

a. 2.36 ________ c. 17.14 ________

b. 15.1905 ________ d. 139.07 ________

-3R4- Round off to three places:

a. 0.1786 ________ c. 1.3050 ________

b. 15.1905 ________ d. 33.0303 ________

-3R5- Express these common fractions as decimals to 4 places:

a. $\frac{11}{16}$ ________ c. $\frac{17}{1000}$ ________

b. $\frac{17}{24}$ ________ d. $\frac{39}{64}$ ________

-3R6- Express these decimals as common fractions:

a. 0.3125 ________ d. 0.890625 ________

b. 0.390625 ________ e. 0.28125 ________

c. 0.65625 ________

-3R7- During the rainy season, a weather station recorded the following rainfall for one week:

Sunday	3.57 in.
Monday	2.78 in.
Tuesday	2.89 in.
Wednesday	3.86 in.
Thursday	1.98 in.
Friday	4.19 in.
Saturday	3.88 in.

What is the total rainfall for the week? ______

-3R8- Ed is a repairman. His wages are $416.45 per week before any deductions. His deductions are:

Federal Tax	$82.98
F.I.C.A.	12.58
Union dues	4.87
Credit union	25.00
Health plan	6.50

What is his take home pay? ______

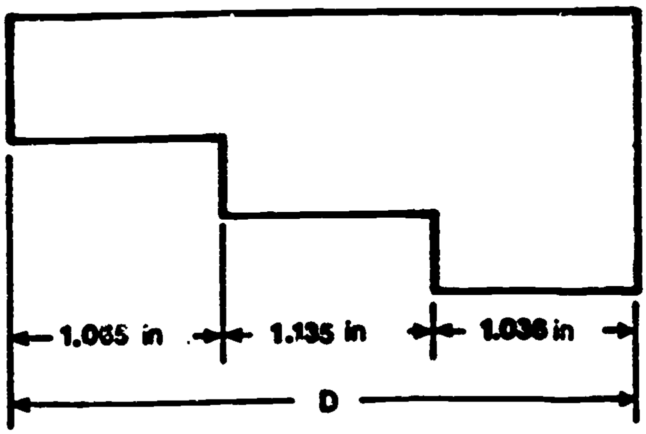
-3R9- A machine setter uses four shims with thicknesses of:

0.0625 in.; 0.0500 in.; 0.1375 in.; 0.0025 in.

What is the total thickness of the four shims?

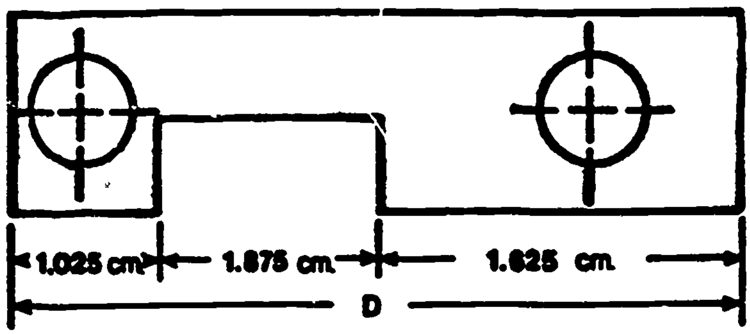
______

-3R10- a. Find the dimension marked D in the following two sketches:



______

b. Find the dimension marked D in the following sketch:



____

-3R11- From the sum of 5.75 and 2.45 subtract the sum of 1.54 and 3.77.

____

-3R12- How much is left when 66.887 is taken from 87.992?

____

-3R13- 1.007 x 2.08 x 3.19 (to 3 places) _______

-3R14- A shop drawing shows a part to be 1.75 in. long. If the drawing is enlarged 1.5 times, how long will the part be on the drawing?

____

-3R15- Type M 3-in. copper water tubing weighs 2.68 pounds per foot. How much does 25.8 feet weigh?

____

-3R16- A mechanic determines the total cost of a repair job as $560. Labor costs are $350. What decimal of the total cost is the labor cost?

____

-3R17- An inspector checks 175 pieces of a lot containing 615 pieces. What decimal of the lot has not been inspected?

____

UNIT 4: PERCENT, AVERAGE, MEDIAN and RANGE

OBJECTIVES

After completing this unit, you will be able to:

- express percents as decimal or common fractions
- express decimal and common fractions as percents
- determine percentage, percent (rate), and base
- solve word problems involving percents
- determine the arithmetic average ($\bar{X}$)
- determine the median
- determine the range (R)

Dictionary:

	Meaning
1. Percent (%)	of each hundred, or per hundred
2. Base	the number of parts making up the whole unit (100%)
3. Percentage	the fractional part which is less than or more than the base
4. Rate	the number to multiply the base by to obtain percentage

Section 1: Concept of a Percent

What is a percent?

Percent is one of the best ways to compare one quantity to another quantity. The word "percent" means in the hundred, or of each hundred.

A percent is designated by the symbol %. Thus, 10% means 10 percent, or 10 out of one hundred.

Section 2: Expressing Percents as Decimal and Common Fractions

Since a percent cannot be used to perform arithmetic operations, we must convert it to a form that will enable us to do this.

<u>"All percents must be divided by 100 to convert them to decimals."</u>

2-A Examples

1. Express $37\frac{1}{2}\%$ as a decimal fraction.

solution: Step 1 Percent means of each hundred, or the second place, set the decimal indicator one place beyond, or at 3.

Step 2 $37\frac{1}{2}\% = 37.5\%$

Step 3 $\frac{37.5\%}{100} = 0.375$ Answer

Note that the instant we divide by 100, the % sign "disappears."

2. Express 33.6% as a decimal.

solution: Step 1 Set decimal indicator at 3.

Step 2 $\frac{33.6\%}{100} = 0.336$ Answer

3. Express 147.9% as a decimal.

solution: Step 1 $\frac{147.9\%}{100} = 1.479$ Answer

4. Express 323.2% as a decimal.

solution: Step 1 $\frac{323.2\%}{100} = 3.232$ Answer

2-B Examples

1. Express $37\frac{1}{2}\%$ as a common fraction.

solution: Step 1 Convert percent to decimal, or $37\frac{1}{2}\% = 0.375$

Step 2 Since 0.375 is in the third decimal place, set up a fraction with 1000 in the denominator as $\frac{\quad}{1000}$, and place the numerator as $\frac{375}{1000}$

Step 3 Reduce to lowest terms: divide the numerator and denominator by 125, so $\frac{\overset{3}{375}}{\underset{8}{1000}} = \frac{3}{8}$, Answer

2. Express 33% as a common fraction.

solution: Step 1 Convert percent to decimal; $\frac{33\%}{100} = 0.330$, or 0.33

Step 2 Since 0.33 is in the second decimal place, set up a fraction with 100 in the denominator as $\frac{\quad}{100}$, and place the number 33 (not the decimal) in the numerator as $\frac{33}{100}$

Step 3 Since we cannot reduce to lower terms, our answer is $\frac{33}{100}$

3. Express 221.5% as a common fraction.

solution: Step 1 Convert percent to decimal; $\frac{221.5\%}{100} = 2.215$

Our result thus far is a mixed decimal number, i.e., a whole number 2 and a decimal fraction 0.215. We must convert the 0.215 to a common fraction.

Step 2 Since 0.215 is in the third place, set up a fraction with 1000 in the denominator and 215 in the numerator as $\frac{215}{1000}$; the result thus far is

$$2\ \frac{215}{1000}.$$

Step 3 Reduce the common fraction to lowest terms;

$$\frac{\overset{43}{\cancel{215}}}{\underset{200}{\cancel{1000}}} = \frac{43}{200};\ \underline{\text{answer is}}\ 2\ \frac{43}{200}$$

2-C Type 1

Examples

1. What percent of 60 is 9?

(60 is the base, or whole; 9 is a fractional part or percentage of the base; we are asked to find the r [illegible])

solution: Step 1 [illegible] decimal indicator at 3.

Step 2 Set up a fraction with the base, 60, as the denominator and the fractional part (percentage), 9, as the numerator and divide, $\frac{9}{60}$ = 0.150, or 0.15

Step 3 Convert decimal to percent: 0.15 x 100 = 15.000 or 15%, Answer

You will note from example 1 and the others that follow, in Type 2 percent problems, the base always follows the word "of," and you will always be given two numbers, not percen[illegible] to solve the problem. The second number will be the numerator.

2. What percent of 140 is 105?

solution: Step 1 Set decimal indicator at 3.

Step 2 Set up fraction with 140, the base, as the denominator and 105, the fractional part (percentage) as the numerator and divide: $\frac{105}{140}$ = 0.750, or 0.75

Step 3 Convert decimal to percent:

0.75 x 100 = 75.000, or 75%, Answer

3. What percent of 8 is 25?

solution: Step 1 Set decimal indicator at 3.

Step 2 Note that 8 follow the word "of" which makes it the base, or denominator and 25 is the numerator; set up as a fraction and divide: $\frac{25}{8}$ = 3.125

Step 3 Convert decimal to percent:
3.125 x 100 = 312.500, or 312.5%,
Answer.

DRILL SHEET

UNIT 4: PERCENT PROBLEMS (TYPE 1)

Solve for each. Round answers to 2 decimal places:

1D-1 What percent of 87 is 95?

1D-2 What percent of 63 is 44.2?

1D-3 What percent of 19.4 is 8?

1D-4 What percent of 298 is 420?

1D-5 What percent of 40 is 13.1?

1D-6 What percent of $\frac{5}{8}$ is $\frac{3}{4}$?

1D-7 What percent of $\frac{11}{16}$ is $\frac{3}{8}$?

1D-8 What percent of $\frac{16}{16}$ is 4?

1D-9 What percent of 20 is 35?

1D-10 What percent of 116 is 118?

1D-11 What percent of $\frac{11}{32}$ is 6?

1D-12 What percent of 125 is 25?

1D-13 What percent of 25 is 125?

1D-14 What percent of 1.7 is $\frac{1}{17}$?

1D-15 What percent of $\frac{7}{8}$ is $\frac{3}{16}$?

1D-16 What percent of 75 is 70?

1D-17 What percent of 3 is 1?

1D-18 What percent of 1 is 3?

1D-19 What percent of 0.015 is 0.25?

1D-20 What percent of $\frac{19}{32}$ is $\frac{9}{16}$?

2-D Type 2

Examples

1. 9 is 15% of what number?
(9 is the fractional part, or percentage; 15% is the rate; the base, or whole we must solve for)

solution: Step 1 Set decimal indicator at 3.

Step 2 Write a fraction with 9 as the numerator and a question mark as the denominator: $\frac{9}{?}$

Step 3 Set up all Type 3 percent problems as follows: $\frac{9}{?} = 15\%$

Step 4 Convert the percent, 15%, to a decimal: $\frac{15\%}{100} = 0.150$ or 0.15

Step 5 Replace the question mark in Step 3 with 0.15 and divide:
$\frac{9}{0.15} = 60$, Answer

Note! In Type 2 percent problems, the wording "of what number"? will always be the number or quantity we must solve for, and it is replaced as a question mark in the denominator.

DRILL SHEET

UNIT 4: PRACTICAL APPLICATIONS OF PERCENTS

Solve for each. Round answers to 2 decimal places:

2D-1 Jim's gross pay of $30,100 was 10% less than last years' gross pay. What was last year's gross pay?

______

2D-2 In BIC-4 department, 4,185 usable devices were produced. If 7% of the parts produced were defective, how many parts were produced altogether?

______

2D-3 If an error of ± 3% is acceptable and the diameter is supposed to be 2.25", how much could the diameter vary from this value?

______

2D-4 A certain alloy is 78% tin. If a piece of this alloy weighs 15 pounds, what is the weight of the tin in this alloy?

______

2D-5 The pitch of a gear was 3% larger than it should have been. If the pitch was 1.8025", what should it have been?

______

2D-6 An ALPO carpenter estimates that a job requires 550 board feet of lumber, which includes 15% allowance for waste. How many board feet are allowed for waste?

______

2D-7 If $6\frac{1}{2}$ acres of a 15-acre orchard are harvested, what percent of the orchard is harvested?

______

2D-8 A machine produces 76 pieces when operating at 80% of its capacity. How many pieces can be produced when the machine is operating at full capacity?

______

2D-9 It is estimated that a company's earnings next year will be 125% of this year's earnings. If the company earned $650,000 this year, what are next year's estimated earnings?

______

2D-10 A baker prepares a 130-pound batch of bread dough and uses 120 pounds of the dough. What percent of the batch is used?

______

2D-11 In an electrical circuit, a certain resistor takes 16% of the total voltage. How many volts are taken by the resistor in a 230-volt circuit?

______

2D-12 A welder orders 150 square meters of steel plate. If 85 square meters are delivered, what percent of the order is received?

______

2D-13 A laboratory technician prepares 85% of a total required amount of solution. If the total amount of solution required is 2.8 liters, how many liters are prepared?

______

2D-14 A mason lays 72 feet of sidewalk which represents 40% of the total job. What is the total length, in feet, of the completed sidewalk?

______

2D-15 Before stretching, a spring measures 5.84 centimeters. The spring is stretched an additional 1.22 centimeters. What percent of the original length is the amount of stretch?

______

2D-16 By installing new machinery, a firm increases production by 25%. An average of 1,800 units are produced per day with the new machinery. Determine the average daily production before installation of the new machinery.

______

D-17 A retailer purchased merchandise at wholesale cost for $1,050. The wholesale cost is 35% less than the retail price. What is the retail price?

______

2D-18 A building contractor has 4,250 feet of 2" x 4" lumber at the start of a job. At the end of the first week of the job, 32% of the lumber is used. At the end of the second week 40% of the stock that remained at the end of the first week is used. How many feet of lumber remain unused at the end of the second week?

2D-19 A resistor is rated at 1,600 ohms with a tolerance of ± 4%. The resistor is checked and found to have an actual resistance of 1,510 ohms. By how many ohms is the resistor below the acceptable resistance low limit? ________

2D-20 Brazing solder contains 51.2% copper, 48.3% zinc, 0.1% iron, and lead. How many pounds of lead are required to make 2,000 pounds of solder?

____

2D-21 The manager of a clothing store computes semi-annual monthly profits as follows: July, $6,250; August, $5,700; September, $7,100; October, $6,000; November, $5,200; and December, $11,050. What percent of the entire semi-annual profit is the December profit?

____

2D-22 A landscaper estimates that 72 cubic meters of topsoil is required for a job. After the job is completed, it is found that the amount of topsoil estimated was 20% more than what was actually used. How many cubic meters of topsoil were used on the job?

____

2D-23 In a given volume of solution, 50 milliliters of acid makes up 20% of the solution. For the same volume solution, how many milliliters of acid are required to produce a 28% acid solution?

____

2D-24 On Monday, a manufacturing firm produced a total of 1,100 units with 3% of the units defective. On Tuesday, the firm produced a total of 1,000 units with $5\frac{1}{2}$% of the units defective. How many more acceptable units were produced on Monday than on Tuesday?

____

2D-25 A length of copper wire measures 20 feet $6\frac{1}{2}$ inches before being heated. When heated, the wire measures 20 feet 7 inches. What is the
4
percent of increase in length?

____

2D-26 This table shows the standing for one year of the Eastern Division of the National Conference of the NFL. Each team in the division is ranked according to games won. The percent is usually written as a three-place decimal.

St. Louis' Pct. = $\frac{\text{Games Won}}{\text{Games Played}}$

$\frac{n}{100} = \frac{10}{13}$

14n = 1100

n = .786

Team	Games Won	Games Lost	Pct.
St. Louis	10	3	.769
Dallas	9	4	.692
Washington	11	2	?
New York	7	6	?
Philadelphia	?	?	.357

Can you find the missing parts of the table? Remember that all teams play the same number oi games in a season.

Fraction	Decimal	Percent
$\frac{1}{20}$	.05	5%
$\frac{1}{8}$	.10	10%
$\frac{1}{8}$	$.12\frac{1}{2}$	$12\frac{1}{2}$%
$\frac{1}{6}$	$.16\frac{2}{3}$	$16\frac{2}{3}$%
$\frac{1}{5}$	.20	20%
$\frac{1}{4}$	.25	25%
$\frac{3}{10}$	.30	30%

$\frac{1}{3}$	$.33\frac{1}{3}$	$33\frac{1}{3}\%$
$\frac{3}{8}$	$.37\frac{1}{2}$	$37\frac{1}{2}\%$
$\frac{2}{5}$	.40	40%
$\frac{1}{2}$	.50	50%
$\frac{3}{5}$	.60	60%
$\frac{5}{8}$	$.62\frac{1}{2}$	$62\frac{1}{2}\%$
$\frac{2}{3}$	$.66\frac{2}{3}$	$66\frac{2}{3}\%$
$\frac{7}{10}$	.70	70%
$\frac{3}{4}$	.75	75%
$\frac{4}{5}$	.80	80%
$\frac{5}{6}$	$.83\frac{1}{3}$	$83\frac{1}{3}\%$
$\frac{7}{8}$	$.87\frac{1}{3}$	$87\frac{1}{3}\%$
$\frac{9}{10}$	.90	90%

Section 3: Arithmetic Average ($\bar{X}$)

Dictionary:	Meaning
1. Average ($\bar{X}$)	arithmetic average, i.e., sum of all numbers divided by the number count
2. Median	middle number
3. Range (R)	difference between the smallest and largest values
4. Graph	a drawn (plotted) representation of past data or statistics
5. Axes (X and Y)	the two boundaries that make up a graph
6. Chart (or charting)	a pre-printed form with boundaries to plot ongoing data
7. Plot (or plotting)	placing data points in reference to the X and Y axes

Sometimes we need to use approximate numbers for one reason or another. One commonly used approximation is averaging. Averages are often used for comparisons, such as comparing the average mileage different cars get per gallon of gasoline.

Averages used for our purpose at ALPO will mean arithmetic average. (It is also referred to as the mean in other work places.) Average is designated by the symbol X.

3-A Examples

1. Determine the average for this set of numbers:

   1036, 1002, 1019, 1056, and 1011 = ______

solution: Step 1 Since we are dealing with whole numbers only, set the decimal indicator at 1 (one beyond, or the tenth place).

At this point, your attention is called to a key on the calculator that has not been used in the past; it is the [N] key at the top of the row of three keys on the right. This is a tally key.

Section 4: Median

Another type of approximation is called median which means "in the middle". It differs from the average ($\bar{X}$) in that the numbers being studied are never added; instead we arrange the numbers.

4-A Examples

1. Determine the median for this set of numbers:

   23.6, 22.8, 23.0, 24.5, 25.3, 24.3, 23.2, and 24.5.

solution:

Step 1 Arrange the numbers in order from smallest to largest in vertical fashion as follows:

22.8
23.0
23.2
23.6
24.3
24.5
24.5
25.3

Step 2 Count the total in the listing; it is 8, an <u>even</u> number.

Step 3 Since there is no exact middle, or median for an even numbered listing, we select the two innermost numbers from top and bottom - 23.6 is fourth from the top and 24.3 is fourth from the bottom - add these two (with decimal indicator at 2); display shows 47.90.

Step 4 Since we need the average for the two numbers added in Step 3, we divide by 2; display shows 23.95.

Step 5 The <u>median</u> rounded to 1 place is 24.0, Answer.

2. Determine the median for this set of numbers:

3.9, 3.5, 4.2, 3.0, 3.8, 4.1, 3.7.

solution: Step 1 Arrange the numbers in order from smallest to largest in vertical fashion as follows:

3.0
3.5
3.7
3.8
3.9
4.1
4.2

Step 2 Count the total in the listing; it is 7, an odd number.

Step 3 The exact middle, or median, for a listing of 7 is 4 from the top or 4 from the bottom; the median is 3.8, Answer.

The rule for determining the median is as follows:

a) if there is an odd numbered listing, determine the exact middle in counting from the top or bottom;

b) if there is an even numbered listing, determine the innermost two, one from the top and one from the bottom; divide by two and round off if necessary.

DRILL SHEET

UNIT 4: DETERMINING MEDIANS

Determine the medians for each set of numbers, rounding to the place shown in parentheses:

3D-1 0.098, 0.100, 0.103, 0.097, 0.105, 0.102, 0.096 (3)

3D-2 143, 149, 150, 144, 146, 148, 147, 145 (0)

3D-3 $\frac{5}{16}, \frac{7}{16}, \frac{9}{16}, \frac{3}{16}, \frac{13}{16}, \frac{11}{16}, \frac{15}{16}$ (4)

3D-4 1.1, 1.9, 2.0, 1.3, 1.5, 1.7, 1.6, 1.4 (1)

3D-5 1198, 1190, 1199, 1195, 1194, 1193, 1196 (0)

3D-6 7, 9, 3, 1, 8, 5, 6, 4 (0)

3D-7 21.12, 21.16, 21.17, 21.09, 21.15, 21.14, 21.18 (2)

3D-8 0.0035, 0.0029, 0.0034, 0.0032, 0.0031 (4)

3D-9 17.9, 17.9, 17.7 (1)

3D-10 35.012, 35.019, 35.017, 35.016 (3)

Section 5: Range (R)

Range (R) is the spread or difference between the smallest value and the largest value.

Before we discussed medians, and you learned that in order to determine the median, it was necessary to arrange the numbers in a vertical listing from smallest to largest. This same procedure could be followed for determining the range (R).

5-A Examples

1. Determine the range (R) for the following numbers:

   13, 11, 12, 10, 12, 14, 13, 11

solution: Step 1 Arrange in vertical fashion;

10
11
11
12
12
13
13
14

Step 2 We see that the largest number is 14 and the smallest is 10; to obtain the difference between these two, subtract 10 from 14, this equals 4 which is the range (R).

Note! In order to minimize errors, especially when working with decimal fractions, always enter the larger number in the calculator, press [+] key, enter the smaller number and press [-] key; the display will show the difference, which is the range (R).

UNIT 4: REVIEW

Report answers to two decimal places.

-4R1- Express 109.6% as a decimal fraction. ________

-4R2- Express 37 $\frac{1}{2}$% as a common fraction. ________

-4R3- Express 201.5% as a common fraction. ________

-4R4- Express 0.85% as a decimal fraction. ________

-4R5- Write 0.91 as a percent. ________

-4R6- Express $\frac{5}{16}$ as a percent. ________

-4R7- What is 7% of 140? ________

-4R8- 60 is 80% of what number? ________

-4R9- What is 109 $\frac{3}{4}$% of 75? ________

-4R10- What percent of 130 is 105? ________

-4R11- What percent of $\frac{17}{16}$ is $\frac{17}{32}$? ________

-4R12- What percent of 302 is 402? ________

-4R13- 800 parts is 140% of what number of parts? ________

-4R14- $\frac{11}{32}$ is 40% of what number? ________

-4R15- Determine the range, average, and median for the following sets of numbers:

a. 6, 5, 9, 4, 8, 11, 10, 7

$\bar{X}$ ________
R ________
Median ________

b. 0.0098, 0.0087, 0.0092, 0.0087, 0.0098, 0.0082, 0.0093

$\bar{X}$ ________
R ________
Median ________

-4R16- The shipping department's number of orders processed for 1985 was as follows:

JAN	505	JUL	500
FEB	575	AUG	545
MAR	480	SEP	565
APR	600	OCT	615
MAY	495	NOV	625
JUN	615	DEC	600

a. Plot a line graph which follows this page.

b. Label the X and Y axes.

UNIT 5: CONCE. OF MEASURE & SIGNED NUMBERS

OBJECTIVES:

- Understanding Conversions with English units of linear measure
- Understanding the Metric Table
- Understanding the Metric Relationship and Conversions

Dictionary:

	Meaning
1. Linear Measure	Measure that pertains to lines or length
2. Metric	International System of Units of Measurement

Section 1: What is measure?

You have been using measure, or more precisely, units of measure your entire life. For example, you purchased a new car for 15,000 <u>dollars</u>, or you had 400 <u>square feet</u> of carpeting installed; or you purchased 3 <u>dozen</u> eggs; or you bought 1 <u>gallon</u> of milk and 3 <u>pints</u> of cottage cheese; or you worked 44 <u>hours</u> last week at ALPO. All of the underscored are <u>units of measure</u>.

If we would not identify an item or quantity with a unit of measure, there would be chaos because all we would have would be numbers.

<u>IMPORTANT:</u> You will <u>always</u> attach a unit of measure to answers you report in this math module. The only exception will be an exercise involving numbers only.

Keep in mind that it is acceptable to use abbreviations for units of measure. However, the abbreviations must be <u>exactly</u> as shown.

Section 2: Linear Measure

2A - Equivalent English units of linear measure

Examples

1. Express 76.5 inches as feet

solution: Step 1 Since inches and feet imply linear (straight line) measurement, select the appropriate conversion table, which is Table A on page P10.

Observe from Table A that 1 foot = 12 inches, or 12 inches = 1 foot.

Step 2 Set up every conversion as follows:

unit began with X conversion = desired unit in answer

for our Example 1, since there is 1 decimal place in our number, set the decimal indicator at 2, then set up as 75.5 in x $\frac{1 \text{ ft}}{12 \text{ in}}$ = ft

Step 3 Earlier we learned when multiplying fractions, we were able to divide the numerator and denominator by some common number. This same principle applies to units of measure; common units in the numerator and denominator can be divided or cancelled.

Thus, in our example we divide 1 in by 1 in, or $\frac{1 \text{ in}}{1 \text{ in}}$ = 1 and the inch units are cancelled, so 76.5 in x $\frac{1 \text{ ft}}{12 \text{ in}}$ = ft,

or $\frac{76.5 \times 1 \text{ ft}}{12}$ = 6.38 ft,

and rounded to 1 decimal place our answer is 6.4 ft.

To repeat, first check with the appropriate conversion table; second set up problem with unit of measure you began with on the left, times the conversion, equals the unit of measure you desire in the answer;

third cancel out the unwanted unit of measure so that only the unit of measure desired remains in the numerator; **fourth** multiply or divide the numbers to the left of the equal sign and place the result with the remaining unit of measure to the right of the equal sign as your answer.

2. Express 6.375 feet as inches.

solution: Step 1 First, set decimal indicator on calculator at 4 (one beyond the 3 decimal places given in our problem). Second, we will use Table A as our conversion table.

Step 2 Set up conversion as follows:

$$6.375 \text{ ft} \times \frac{12 \text{ in}}{1 \text{ ft}} = \qquad \text{in.}$$

Step 3 Cancel (divide) the feet (ft) and multiply the numbers:

$$6.375 \cancel{\text{ft}} \times \frac{12 \text{ in}}{1 \cancel{\text{ft}}} = 76.5000 \text{ in},$$

or 76.5 in, Answer.

3. Express $\frac{7}{18}$ mile as feet

solution: Step 1 Convert common fraction, $\frac{7}{18}$, to decimal fraction: $\frac{7}{18} = 0.3889$, or

0.389 (to 3 places).

Step 2 From Table A, note that 1 mile = 5,280 feet. Set up conversion:

$$0.389 \text{ mi} \times \frac{5{,}280 \text{ ft}}{1 \text{ mi}} = \qquad \text{ft}$$

Step 3

$$0.389 \cancel{\text{mi}} \times \frac{5{,}280 \text{ ft}}{1 \cancel{\text{mi}}} = 2{,}053.9200 \text{ ft},$$

or 2,053.92 ft, Answer.

4. Express 37.5 yards as feet and inches

solution: Step 1 When asked to report the answer in multiple units of measure, convert to the larger unit first, as follows:

$$37.5\ \cancel{yd} \times \frac{3\ ft}{1\ \cancel{yd}} = 112.50\ ft$$

Our preliminary result states that we now have 112 whole feet and a fraction, or 0.5 ft.

Step 2 Second, convert the fractional part of a foot to inches, as requested:

$$0.5\ \cancel{ft} \times \frac{12\ in}{1\ \cancel{ft}} = 6\ in$$

Step 3 Third, combine the two units of measure as 112 ft 6 in, Answer.

DRILL SHEET

UNIT 5: EQUIVALENT ENGLISH UNITS OF LINEAR MEASURE

Express each length as indicated; round answers to 3 decimal places:

1D-1	$25 \frac{1}{2}$ inches as feet =
1D-2	16.25 feet as yards =
1D-3	3960 feet as miles =
1D-4	78 inches as feet and inches =
1D-5	47 feet as yards and feet =
1D-6	$7 \frac{1}{4}$ feet as inches =
1D-7	$12 \frac{2}{3}$ yards as feet =
1D-8	0.6 mile as feet =
1D-9	$2 \frac{1}{4}$ yards as inches =
1D-10	$5 \frac{1}{2}$ yards as feet and inches =
1D-11	$\frac{1}{12}$ mile as yards and feet =
1D-12	6.2 yards as feet and inches =
1D-13	17.35 miles as rods and yards =
1D-14	193.5 feet as rods =
1D-15	$\frac{3}{5}$ mile as feet =

Section 3: Arithmetic operations with English compound numbers

Examples:

Perform the indicated arithmetic operation; express the answer in the same units of measure as those given in the exercise. Regroup the answer when necessary.

1. 15 ft 8 in + 6 ft 5 in =

solution: Step 1 Set up as an addition problem in vertical fashion and add:

$$\begin{array}{rr} 15 \text{ ft} & 8 \text{ in} \\ + \ 6 \text{ ft} & 5 \text{ in} \\ \hline 21 \text{ ft} & 13 \text{ in} \end{array}$$

Step 2 Since 13 inches is more than 1 foot (12 in), we regroup by subtracting 12 inches from 13 and add 1 foot to 21.

Step 3

$$\begin{array}{rr} 15 \text{ ft} & 8 \text{ in} \\ + \ 6 \text{ ft} & 5 \text{ in} \\ \hline 21 \text{ ft} & 13 \text{ in} \\ + \ 1 \text{ ft} & -12 \text{ in} \\ \hline 22 \text{ ft} & 1 \text{ in} \end{array}$$, Answer

2. 14 yd $\frac{1}{2}$ ft - 11 yd 2 ft =

solution: Step 1 Set up as a subtraction problem in vertical fashion:

$$\begin{array}{rr} 14 \text{ yd} & \frac{1}{2} \text{ ft} \\ - \ 11 \text{ yd} & 2 \text{ ft} \\ \hline \end{array}$$

Step 2 Borrow 1 yard from 14 and convert to 3 feet, then add to $\frac{1}{2}$ ft:

$$\begin{array}{rr} \overset{13}{14} \text{ yd} & 3\frac{1}{2} \text{ ft} \\ - \ 11 \text{ yd} & 2 \text{ ft} \\ \hline 2 \text{ yd} & 1\frac{1}{2} \text{ ft} \end{array}$$, Answer

PREFIX. (X)	LENGTH METRIC	ENGLISH	CAPACITY METRIC	ENGLISH	WEIGHT METRIC	ENGLISH
kilo x 1000	kilometer (km)	1 km = .6214 mi = 198.84 rd = 1093.6 yd = 3280.8 ft	kiloliter (kl)		kilogram (kg)	1 kg = 2.205 lb = 35.28 oz
hecto x 100	hectometer (hm)	1 hm = .6214 mi = 19.884 rd = 109.36 yd = 328.08 ft	hectoliter (hl)		hectogram (hg)	1 hg = .2205 lb = 3.528 oz
deka x 10	dekameter (dam)	1 dam = 1.988 rd = 10.936 yd = 32.808 ft	dekaliter (dal)	1 dal = 2.642 gal = 10.570 qt = 21.14 pt	dekagram (dag)	1 dag = .2205 lb = .3528 oz
1	1 meter (m)	1 m = 1.0936 yd = 3.281 ft = 39.37 in (.9144 m = 1 yd)	1 liter (l)	1 l = .2642 gal = .2114 pt (.946 L = 1 qt)	1 gram (g)	1 gm = .03528 oz. (28.348 gm = 1 oz) (454 gm = 1 lb)
deci x 0.1	decimeter (dm)	1 dm = .3281 ft = 3.937 in (254 dm = 1 in)	deciliter (dl)	1 dl = .1057 qt = .2114 pt = 3.382 oz	decigram (dg)	
centi x 0.01	centimeter (cm)	1 cm = .3937 in (2,54 cm = 1 in)	centiliter (cl)	1 cl = .02114 pt = .3302 oz	centigram (cg)	
milli x 0.001	millimeter (mm)	1 mm = .03937 in (25.4 mm = 1 in)	milliliter (ml)	1 ml = .03382 oz (29.563 ml = 1 oz)	milligram (mg)	
1 cu. ft	= 1728 cu in.		1.48 gal = 1 ft 1 gal = 231 in			

1 metric ton	= 1.102 ton
1000 kg	= .984 long ton
	= 2205 lb

Examples

1. Express 0.378 meter as millimeters

   Referring to metric table on the preceding page we note that

   10 x 10 = 100, and
   100 x 10 = 1000

   This tells us there are 1000 millimeters in 1 meter, or that 1 meter is equal to 1000 millimeters.

   Step 2 Set up conversion:

   $$0.378\,\cancel{m} \times \frac{1000\ mm}{1\,\cancel{m}} = 378\ mm\text{, Answer}$$

2. Express 472.5 m as km

solution Step 1 From the left column of Table B, note that millimeter is positioned higher than kilometer, thus kilometer is larger.

Place left-hand marker above meter in left-hand column. Place right-hand marker below kilometer in right-hand column. There are three 10's between the two markers; multiplying these we get 1000 m = 1 km (or 1 km = 1000 m).

Step 2 Set up conversion:

$$472.5\,\cancel{m} \times \frac{1\ km}{1000\,\cancel{m}} = 0.4725\ km\text{, Answer}$$

DRILL SHEET

UNIT 5: EQUIVALENT METRIC UNITS OF LINEAR MEASURE

Express each value in the unit indicated:

2D-1 30 mm as cm =

2D-2 8 cm as mm =

2D-3 2460 mm as m =

2D-4 23 m as cm =

2D-5 650 m as km =

2D-6 0.8 km as m =

2D-7 0.75 m as mm =

2D-8 12.2 cm as mm =

2D-9 372.5 m as km =

2D-10 1935 mm as dam =

2D-11 795 dm as hm =

2D-12 3975 mm as km =

2D-13 425 cm as dam =

2D-14 234.5 cm as hm =

2D-15 122.5 dam as km =

Section 4: Arithmetic operations with metric lengths

Example

1. 49.8 cm + 14.3 dm + 77.75 mm = _____ cm

We see that there are three different metric units of linear measure to be added, and the answer is to be reported in one of these. Therefore, two have to be converted.

solution: Step 1 Convert dm to cm;

$$14.3\ \cancel{dm} \times \frac{10\ cm}{1\ \cancel{dm}} = 143\ cm$$

Step 2 Convert mm to cm;

$$77.75\ \cancel{mm} \times \frac{1\ cm}{10\ \cancel{mm}} = 7.775\ cm$$

Step 3 Add all three cm's;

```
  49.8   cm
 143     cm
+  7.775 cm
-----------
 200.575 cm, Answer
```

2. 728.8 cm - 4.1 m = _____ cm

solution: Step 1 Convert m to cm;

$$4.1\ \cancel{m} \times \frac{100\ cm}{1\ \cancel{m}} = 410\ cm$$

Step 2 Subtract;

```
 728.8 cm
-410   cm
---------
 318.8 cm, Answer
```

3. 23 x 3.3 m = _____ m

solution: Step 1 Since the unit is the same from beginning of our problem to the answer, no conversion is needed; proceed as with usual multiplication:

```
 3.3 m
x 23
--------
 75.9 m, Answer
```

4. 8.64 dm - 6 = _____ dm

solution: Step 1 Again, there is no conversion of units required; proceed as with usual division:

$$\frac{8.64 \text{ dm}}{6} = 1.44 \text{ dm, Answer}$$

DRILL SHEET

UNIT 5: EXPRESSING ENGLISH UNITS AS METRIC UNITS OF LINEAR MEASURE AND METRIC UNITS AS ENGLISH UNITS OF LINEAR MEASURE

Report answers to decimal places indicated in parentheses:

3D-1	0.4 m as in =	________	(3)
3D-2	8 in as mm =	________	(2)
3D-3	12 mm as in =	________	(3)
3D-4	23 in as cm =	________	(2)
3D-5	4.5 m as ft =	________	(3)
3D-6	5.8 ft as m =	________	(2)
3D-7	0.18 km as ft =	________	(3)
3D-8	3.4 mi as km =	________	(2)
3D-9	0.75 m as ft =	________	(3)
3D-10	2.7 ft as cm =	________	(2)
3D-11	286 km as mi =	________	(3)
3D-12	71 in as m =	________	(2)
3D-13	4.6 m as yd =	________	(3)
3D-14	3872 ft as km =	________	(2)

3D-15	78 cm as ft =	____________	(3)
3D-16	$\frac{3}{4}$ in as mm =	____________	(2)
3D-17	9.8 cm as in =	____________	(3)
3D-18	8 ft 2 $\frac{1}{4}$ in as m =	____________	(2)
3D-19	19.3 yd as cm =	____________	(3)
3D-20	$\frac{3}{16}$ mi as m =	____________	(3)
3D-21	234 mm as yd =	____________	(2)
3D-22	0.18 km as yd =	____________	(3)
3D-23	13,984 ft as km =	____________	(2)
3D-24	$\frac{3}{11}$ mi as m =	____________	(3)
3D-25	0.055 mi as m =	____________	(2)
3D-26	542 L as kL =	____________	
3D-27	84 oz as gal =	____________	
3D-28	3.17 hL as L =	____________	
3D-29	0.2 gal as pt =	____________	
3D-30	0.06 L as mL =	____________	
3D-31	1.4 qt as oz =	____________	

3D-32 19000 mL as L = ____________

3D-33 1710 dL as hL = ____________

3D-34 25000 cL as daL = ____________

3D-35 1 hL as kL = ____________

3D-36 125 mL as dL = ____________

Section 5: Equivalent English-metric capacity measures

Examples

1. Express 2.5 ounces as milliliters. (Round answer to 3 places.)

solution: Step 1 Use conversion Table

Step 2 Set up conversion:

$$2.5 \text{ oz} \times \frac{29.563 \text{ mL}}{1 \text{ oz}} = \qquad \text{mL}$$

Step 3 Cancel ounces and multiply numbers:

$$2.5 \cancel{\text{oz}} \times \frac{29.563 \text{ mL}}{1 \cancel{\text{oz}}} = 73.908 \text{ mL, Answer}$$

2. Express 73.91 milliliters as ounces. (Round answer to 3 places.)

solution: Step 1 Use conversion Table

Step 2 Set up conversion:

$$73.91 \text{ mL} \times \frac{0.0338 \text{ oz}}{1 \text{ mL}} = \qquad \text{oz}$$

Step 3 Cancel milliliters and multiply numbers:

$$73.91 \cancel{\text{mL}} \times \frac{0.0338 \text{ oz}}{1 \cancel{\text{mL}}} = 2.498 \text{ oz, Answer}$$

DRILL SHEET

UNIT 5: EQUIVALENT ENGLISH-METRIC CAPACITY MEASURES

Express answers in units indicated; round answers to 2 decimal places:

4D-1 10.4 L as qt =

4D-2 4.2 oz as mL =

4D-3 8 gal as L =

4D-4 75 mL as oz =

4D-5 1.75 qt as L =

4D-6 513 L as gal =

4D-7 0.67 L as pt =

4D-8 21 oz as L =

4D-9 175 mL as gal =

4D-10 360 oz as L =

4D-11 305 mL as oz =

4D-12 64 L as qt =

4D-13 0.319 gal as mL =

4D-14 725 oz as L =

4D-15 0.75 qt as mL =

Section 6: Weight Measure

Equivalent English and metric units of weight measure

Examples

1. Express 200 ounces as pounds

solution:

Step 1 Set up conversion:

$$200 \text{ oz} \times \frac{1 \text{ lb}}{16 \text{ oz}} = \quad \text{lb}$$

Step 2 Cancel ounces and divide numbers:

$$200 \cancel{\text{oz}} \times \frac{1 \text{ lb}}{16 \cancel{\text{oz}}} = 12.5 \text{ lb, Answer}$$

2. Express 5400 grams as kilograms

solution: Step 1 Use conversion Table

Note from table that kilogram is farther down than gram; therefore kilogram is larger. Using left hand, right hand rule, there are three 10's in left-hand column, or 1 kg = 1000 grams.

Step 2 Set up conversion:

$$5400 \text{ g} \times \frac{1 \text{ kg}}{1000 \text{ g}} = \quad \text{kg}$$

Step 3 Cancel grams and divide numbers:

$$5400 \cancel{\text{g}} \times \frac{1 \text{ kg}}{1000 \cancel{\text{g}}} = 5.4 \text{ kg, Answer}$$

DRILL SHEET

UNIT 5: EQUIVALENT ENGLISH AND METRIC UNITS OF WEIGHT MEASURE

Express each weight in the unit indicated; round answers to 2 decimal places:

5D-1 34 cz as lb =

5D-2 1880 g as kg =

5D-3 0.6 lb as oz =

5D-4 730 mg as g =

5D-5 48,000 lb as long tons =

5D-6 2.7 metric tons as kg =

5D-7 9700 lb as short tons =

5D-8 4.75 g as mg =

5D-9 0.66 short tons as lb =

5D-10 0.21 kg as g =

5D-11 1.08 long tons as lb =

5D-12 310,000 kg as metric tons =

5D-13 1445 dekagrams as metric tons =

5D-14 3.75 kilograms as hectograms =

5D-15 3.85 kilograms as dekagrams =

6-A Equivalent English-metric weight measures

Examples

1. Express 35 pounds as kilograms

solution: Step 1 Use conversion Table

Step 2 Set up conversion:

$$35 \text{ lb} \times \frac{0.454 \text{ kg}}{1 \text{ lb}} = \qquad \text{kg}$$

Step 3 Cancel pounds and multiply numbers:

$$35 \cancel{\text{lb}} \times \frac{0.454 \text{ kg}}{1 \cancel{\text{lb}}} = 15.89, \text{ Answer}$$

2. Express 15.89 kg as pounds. (Round answer to nearest whole pound.)

solution: Step 1 Use conversion Table

Step 2 Set up conversion:

$$15.89 \text{ kg} \times \frac{2.205 \text{ lb}}{1 \text{ kg}} = \qquad \text{lb}$$

Step 3 Cancel kilograms and multiply:

$$15.89 \cancel{\text{kg}} \times \frac{2.205 \text{ lb}}{1 \cancel{\text{kg}}} = 35 \text{ lb}, \text{ Answer}$$

DRILL SHEET

UNIT 5: EQUIVALENT ENGLISH-METRIC WEIGHT MEASURES

Express each weight in the unit indicated; round answers to 3 decimal places:

6D-1 154 kg as lb =

6D-2 0.9 oz as g =

6D-3 220 g as oz =

6D-4 964.8 lb as kg =

6D-5 3.07 metric tons as lb =

6D-6 40.4 short tons as metric tons =

6D-7 15,000 oz as kg =

6D-8 13,800 g as short tons =

6D-9 0.75 short tons as kg =

6D-10 11.02 t as lb =

SIGNED NUMBERS

OBJECTIVES

After completing this section, you will be able to:

- express word statements as signed numbers
- write signed number values using a number scale
- add, subtract, multiply and divide signed numbers
- solve combined operations of signed number expressions
- solve signed number problems

Dictionary:

	Term	Meaning
1.	Signed Number	a number, prefixed with a plus (+) or minus (-) sign which indicates direction from zero
2.	Positive Number	a number with no prefixed sign or with a plus (+) sign
3.	Negative Number	a number prefixed with a minus (-) sign
4.	Absolute Value	a number without its sign - it is technically the distance between the number and zero on the number line. The absolute value of 0 is 0. The absolute value of -7 is 7. The absolute value of 7 is 7.
5.	Number Scale	increasing/decreasing positive and negative numbers

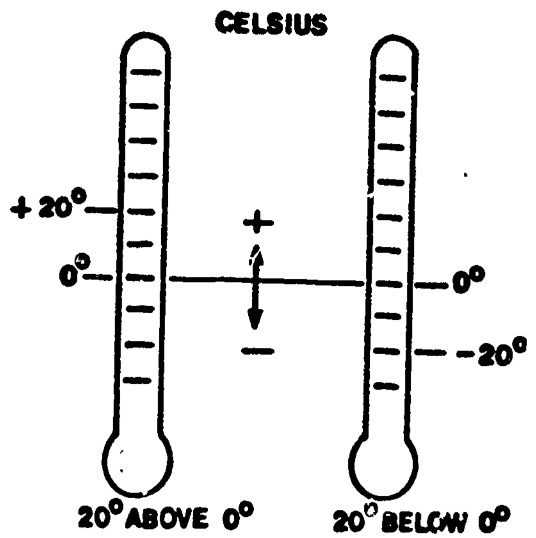
Section 1: Concept of a Signed Number

What is a signed number?

Signed numbers are used to indicate direction and distance from a reference point. Opposites such as up and down, left and right, north and south, and clockwise and counterclockwise, may be expressed using positive and negative signs. For example, 100 feet above sea level may be expressed as +100 feet, and 100 feet below sea level as -100 feet. Sea level in this case is the zero reference point.

In business applications, a profit of \$1000 is expressed as +\$1000, whereas a loss of \$1000 is expressed as -\$1000. Closing prices for stocks are indicated as up (+) or down (-) from the previous day's closing prices.

Shown below are two thermometers with the Celsius scale (C). On the left we read "20 degrees above zero", and we write +20 C. On the right we read "20 degrees below zero", and we write -20 C:



Signed numbers are used in programming operations for probing of wafers. From a reference point, the probe movements are expressed as + and - directions.

1-A Expressing word statements as signed numbers

Examples

1. A speed increase of 12 miles per hour is expressed as +12 mi/hr. Express a speed decrease of 8 miles per hour.

solution: Step 1 The key words are increase and decrease, one is the opposite of the other.

Step 2 Since an increase was expressed as +12 mi/hr, a decrease will be the opposite, or -8 mi/hr, Answer.

2. The reduction of a person's daily calorie intake by 400 calories is expressed as -400 calories. Express a daily intake increase of 350 calories.

solution: Step 1 Since a calorie decrease was expressed as -400 calories, the opposite of decrease or increase is expressed as +350 calories, Answer.

3. A company's assets of $73,600 are expressed as +$73,600. Express company liabilities of $48,000.

solution: Step 1 Since we expressed assets as +$73,600, the opposite of asset, a liability, is expressed as -$48,000, Answer.

Rules for Signed Numbers

Addition

<u>Like Signs</u>

- Add the absolute value of the numbers.
- Give the sum the same sign as the numbers.

<u>Unlike Signs</u>

- Subtract the number with the smaller absolute value from that of the larger.
- Give the answer the same sign of that of the number with the largest absolute value.

Subtraction

- Change the sign of the number being subtracted (subtrahend)
- Add the numbers using the rules for adding signed numbers

DRILL SHEET

EXPRESSING WORD STATEMENTS AS SIGNED NUMBERS

Express the answer to each word problem as a signed number:

7D-1 Traveling 50 kilometers west is expressed as -50 Km. How is traveling 75 kilometers east expressed?

______

7D-2 A wage increase of $25 is expressed as +$25. Express a wage decrease of $18.

______

7D-3 An increase of 30 pounds per square inch of pressure is expressed as +30 lb/sq in. Express a pressure decrease of 28 pounds per square inch.

______

7D-4 A circuit voltage loss of 7.5 volts is expressed as -7.5 volts. Express a voltage gain of 9 volts.

______

7D-5 A savings account deposit of $140 is expressed as +$140. Express a withdrawal of $280.

______

7D-6 A 0.75 percent contraction of a length of wire is expressed as -0.75%. Express a 1.2 percent expansion.

______

7D-7 A 15 pound weight gain is expressed as +15 lb. Express a weight loss of 10 pounds.

______

7D-8 In construction the height of a 25-foot roof is expressed as +25 feet. Express the depth of the 15 foot basement floor.

______

7D-9 A passing grade in school is expressed as +70. Express the failing grade of 55.

______

7D-10 The weight of participants undergoing dieting/exercise is recorded. How do you show:

(a) a loss of 5 lb? ______

(b) a gain of 6 lb? ______

(c) a loss of 3 lb? ______

(d) no change to 120 lb weight? ______

7D-11 Use signed numbers to show the following bank transactions:

(a) a deposit of $56 ______

(b) a withdrawal of $45 ______

(c) a withdrawal of $23 ______

(d) no change to $1500 in the account ______

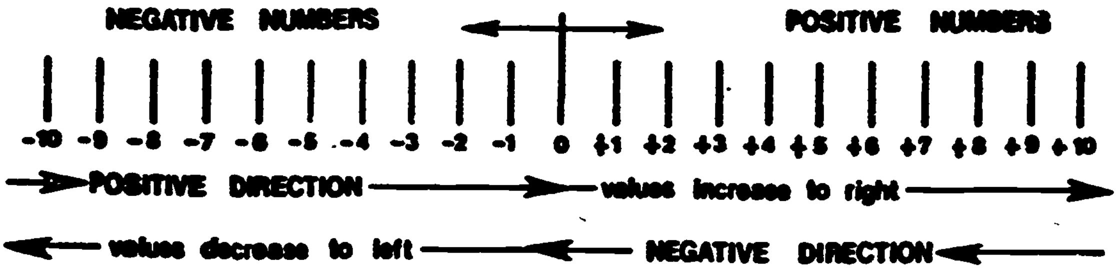
7D-12 Use signed numbers to show:

(a) a depth of 182 feet below sea level ______

(b) a loss of $185 on the stock market ______

Section 2: Number Scale

A number scale (illustrated below) shows the relationship between positive and negative numbers. The scale shows both distance and direction between numbers. Considering any number as a starting point and counting to a number to the right represents a positive (+) direction with numbers increasing in value. Counting to the left represents negative (-) direction with numbers decreasing in value:



Examples of usage of number scale:

Example 1. Starting at 0 and counting to the right to +5 represents 5 units in a positive (+) direction; +5 is 5 units greater than 0.

Example 2. Starting at 0 and counting to the left to -5 represents 5 units in a negative (-) direction; -5 is 5 units less than 0.

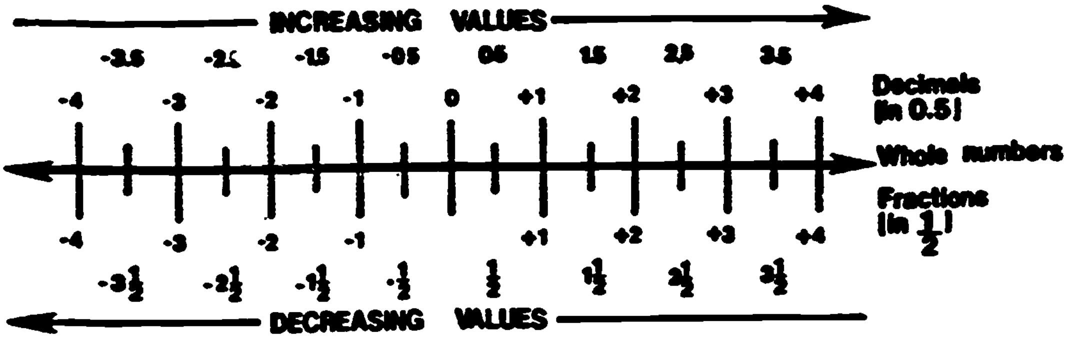
Example 3. Starting at -4 and counting to the right to +3 represents 7 units in a positive (+) direction; +3 is 7 units greater than -4.

Example 4. Starting at +3 and counting to the left to -4 represents 7 units in a negative (-) direction; -4 is 7 units less than +3.

Example 5. Starting at -2 and counting to the left to -10 represents 8 units in a negative (-) direction; -10 is 8 units less than -2.

Example 6. Starting at -9 and counting to the right to 0 represents 9 units in a positive (+) direction; 0 is 9 units greater than -9.

Decimals and fractions, because they represent parts of whole numbers or combinations of whole numbers and parts of whole numbers, can also be positive and negative. The number scale below illustrates decimal numbers and fractions along with whole numbers:



As with whole numbers, the positive sign (+) is usually not expressed, except for clarity or emphasis. The numbers $\frac{1}{2}$ and $+\frac{1}{2}$ are equivalent, just as 0.5 and -0.5 are equivalent.

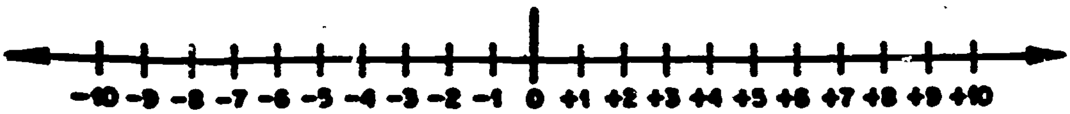
But the negative sign is always expressed as $(-\frac{1}{2})$, (-0.5), $(-3\frac{1}{2})$, (-3.5), and so on.

Zero, 0, has <u>no</u> sign.

DRILL SHEET

NUMBER SCALE

Using the number scale below, give the direction (+ or -) and the number of units counted going from the first to the second number:



8D-1 0 to +6 =

8D-2 0 to (-6) =

8D-3 (-2) to 0 =

8D-4 +2 to 0 =

8D-5 (-3) to +5 =

8D-6 (-7) to +1 =

8D-7 +8 to (-3) =

8D-8 +6 to (-6) =

8D-9 (-10) to (-4) =

8D-10 +9 to (-10) =

8D-11 +10 to (-10) =

8D-12 (-10) to +10 =

8D-13 +6 to (-5) =

8D-14 (-9) to +8 =

8D-15 (-3) to (-7) =

8D-16 (-9) to (-4) =

8D-17 +4 to +10 =

8D-18 +7 to +2 =

8D-19 (-4) to +7 =

8D-20 +6 to (-4) =

8D-21 **Place the following whole numbers, decimal numbers, and fractions on the number scale below:**

(a) 1

(b) (-1)

(c) 3

(d) $(-1\frac{1}{2})$

(e) (-6)

(f) $\frac{1}{2}$

(g) 2.5

(h) 2

(i) 4

(j) (-2)

(k) 6

(l) (-5)

(m) $4\frac{1}{2}$

(n) 5.5

(o) (-2.5)

(p) (-3)

(q) 0.5

(r) (-4)

(s) 5

(t) $(-4\frac{1}{2})$

(u) (-0.5)

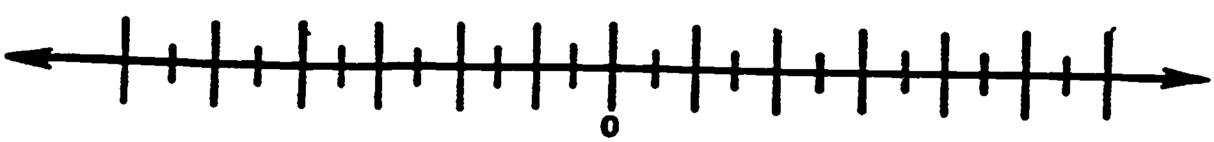
(v) $3\frac{1}{2}$

(w) $1\frac{1}{2}$

(x) (-5.5)

(y) $(-5\frac{1}{2})$

(z) $(-\frac{1}{2})$



8D-22 Select the greater of each of the two signed numbers and indicate the number of units by which it is greater:

(a) +3, (-2) =

(b) (-6), 0 =

(c) (-5), +1 =

(d) (-12), (-4) =

(e) (-28), (-73) =

(f) +18, +14 =

(g) (-18), (-14) =

(h) +10, (-12) =

(i) (-2.5), +2.5 =

(j) $(-86\ \frac{3}{4})$, 0 =

(k) +18.3, (-20.6) =

(l) $(-23\ \frac{1}{4})$, $(-15\ \frac{3}{8})$ =

(m) $+1\ \frac{1}{16}$, $(-1\ \frac{7}{8})$ =

(n) (-50.23), (-41.76) =

(o) $+\frac{3}{16}$, $(-\frac{9}{32})$ =

Section 3: Absolute Value

In order to solve signed number problems, it is important to understand the meaning of <u>absolute value</u>. The absolute value of a number is the number without any sign; it is indicated by placing the number between a pair of vertical bars (or lines).

The absolute values of +8 and (-8) are written as follows:

$$+8 = |8| \text{ and } (-8) = |8|$$

The absolute values of (-15) and +5 are written as follows:

$$(-15) = |15| \text{ and } +5 = |5|.$$

The absolute value of (-15) is 10 greater than the absolute value of +5.

DRILL SHEET

ABSOLUTE VALUE

Express each of the pairs of signed numbers as absolute values and report the difference (subtract the smaller absolute value from the larger absolute value):

9D-1	+15, (-10) =	______, ______, ______
9D-2	(-15), +10 =	______, ______, ______
9D-3	(-6), +2 =	______, ______, ______
9D-4	(-14), +14 =	______, ______, ______
9D-5	(-9), 0 =	______, ______, ______
9D-6	+9, 0 =	______, ______, ______
9D-7	(-23), +22 =	______, ______, ______
9D-8	+18, (-18) =	______, ______, ______
9D-9	+18, +18 =	______, ______, ______
9D-10	(-18), (-18) =	______, ______, ______
9D-11	$+6\frac{1}{4}$, $(-3\frac{3}{4})$ =	______, ______, ______
9D-12	$(-3\frac{1}{2})$, $(-12\frac{7}{8})$ =	______, ______, ______
9D-13	+12.7, (-9.8) =	______, ______, ______
9D-14	+10.54, (-12.46) =	______, ______, ______
9D-15	(-0.03), (-0.007) =	______, ______, ______

Section 4: Addition of Signed Numbers

<u>Procedure For Additon of Signed Numbers:</u>

1. For like signs, add the absolute values of the numbers and affix the same sign to the result:

   (+) + (+) = + (result)

   and

   (-) + (-) = - (result)

2. For unlike signs, find the difference between the absolute values of the numbers and affix the sign of the larger number to the result:

   (+) + (-) =

   or

   (-) + (+) =

   $\Big|\,\Big|\xleftrightarrow{\text{diff.}}\Big|\,\Big|$ = sign larger number (result)

4-A Addition of numbers with the same, or "like" signs.

Examples

1. Find the sum of two positive numbers

$(+5) + (+4) = \_\_\_\_$

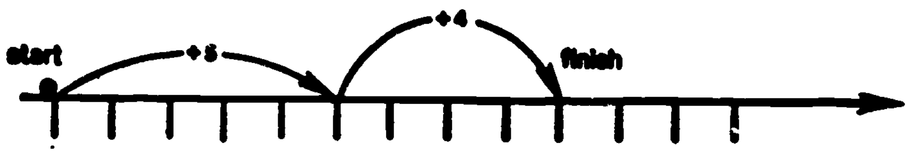
solution: Step 1 Add the absolute values of the two numbers:

$$\underbrace{|5| + |4|}_{\text{add}} = 9$$

Step 2 Since the signs for the two numbers are the same, or "like", affix this same sign to the answer as

$(+5) + (+4) = +9$, Answer

The preceding addition problem can be illustrated on the number scale below:



2. Find the sum of two negative numbers

$(-5) + (-4) = \_\_\_\_$

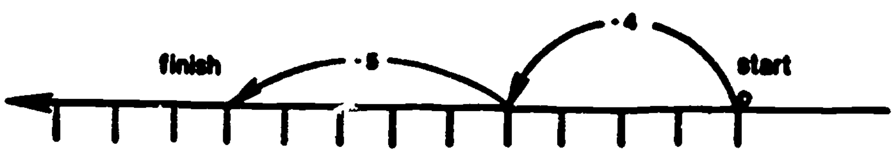
solution: Step 1 Add the absolute values of the two numbers:

$$\underbrace{|5| + |4|}_{\text{add}} = 9$$

Step 2 Since signs for the two numbers are like (both -), affix this same sign to the answer as

$(-5) + (-4) = -9$, Answer

The preceding addition problem can be illustrated on the number scale below:



4-B Addition of numbers with different or "unlike" signs

1. Find the sum of a positive and negative number

(+5) + (-3) = ____

solution: Step 1 Find the diff. between the absolute values of the two numbers:

$$|5| \overset{\text{diff.}}{\longleftrightarrow} |3| = 2$$

Note! To find the difference between any two numbers, enter the larger number in the calculator and press [+] key. Then enter the smaller number and press [-] key. For our example, the <u>difference</u> displayed is 2.

Step 2 Since the sign of the larger number in our problem was +, affix this sign to the answer as

(+5) + (-3) = +2, Answer

2. Find the sum of a negative and positive number

(-5) + (+3) = ____

solution: Step 1 Find the difference between the absolute values of the two numbers

$$|5| \overset{\text{diff.}}{\longleftrightarrow} |3| = 2$$

Step 2 Since the sign of the larger number in our problem was -, affix this sign to the answer as

(-5) + (+3) = -2, Answer

3. Find the sum of the following:

(+11) + (+7) + (-3) + (-9) + (+7) = _____

solution: Step 1 Add all positive numbers:

(+11) + (+7) + (+7) = +25

Step 2 Add all negative numbers:

(-3) + (-9) = -12

Step 3 Add the grouped results:

(+25) + (-12) = +13, Answer

DRILL SHEET

ADDITION OF SIGNED NUMBERS

Add the signed numbers as indicated:

10D-1 (+6) + (+9) =

10D-2 (+15) + (+8) =

10D-3 (+4) + (+20) =

10D-4 (+7) + (+18) + (+2) =

10D-5 0 + (+25) =

10D-6 (-12) + (-7) =

10D-7 (-8) + (-15) =

10D-8 0 + (-16) =

10D-9 (-14) + (-4) + (-11) =

10D-10 (-3) + (-6) + (-17) =

10D-11 (+12) + (-5) =

10D-12 (+18) + (-26) =

10D-13 (-20) + (+17) =

10D-14 (+46) + (-14) =

10D-15 (-23) + (+17) =

10D-16 (+25) + (-3) =

10D-17 (-25) + (+3) =

10D-18 (-18) + (-25) =

10D-19 (-4) + (-31) =

10D-20 (+27) + (-27) =

10D-21 (-15.3) + (- · 5) =

10D-22 (-15.3) + (+3.5) =

10D-23 (-16.4) + (· 2.7) =

10D-24 (+37.9) + (-40.4) =

10D-25 $(-9\ \frac{1}{4}) + (-3\ \frac{3}{4}) =$

10D-26 $(+18\ \frac{5}{8}) + (-21\ \frac{3}{4}) =$

10D-27 $(-13) + (-\frac{3}{16}) =$

10D-28 $(-13) + (+\frac{3}{16}) =$

10D-29 $(-4.25) + (-7) + (-3.22) =$

10D-30 $(+18.07) + (-17.64) =$

10D-31 $(+16) + (-4) + (-11) =$

10D-32 $(-21) + (-6) + (+14) + (+12) =$

10D-33 $(+30) + (-7) + (-8) + (+3) =$

10D-34 $(-10.2) + (-9) + (-7.6) + (+14.7) =$

10D-35 $(+8) + (+16.7) + (-4.1) + (+9.5) =$

10D-36 $(+1\ \frac{1}{4}) + (-2\ \frac{1}{2}) + (-\frac{3}{4}) + (+4\ \frac{1}{2}) =$

Section 5: Subtraction of Signed Numbers

Procedure For Subtraction Of Signed Numbers:

1. Re-arrange subtraction problem in vertical fashion:

   from (-7) - (+8) =

   to (-7)
- <u>(+8)</u>

2. Change sign of lower number (subtrahend) to opposite sign; and change function sign from subtraction to addition:

   from (-7)
- <u>(+8)</u>

   to (-7)
+ <u>(+8)</u>

3. Since <u>every</u> subtraction problem becomes an addition problem with the function sign change, re-arrange in horizontal fashion:

   from (-7)
+ <u>(+8)</u>

   to (-7) + (-8) =

4. Follow rules for addition to obtain answer.

5-A Subtraction of numbers with like signs

Examples

1. Subtract (+8) from (+5) = ____

solution: Step 1 Set up as subtraction problem in vertical fashion:

$$\begin{array}{r} (+5) \\ \underline{-(+8)} \end{array}$$

Step 2 Change sign of lower number (subtrahend) to the opposite sign, and change function sign from subtraction (-) to addition (+):

$$\begin{array}{r} (+5) \\ \underline{+(\overset{-}{\cancel{+}}8)} \end{array}$$

Step 3 As can be seen in Step 2, <u>every</u> subtraction problem with signed numbers becomes an addition problem. Since we are now adding, apply the rule for adding unlike signs, or

(+5) + (-8) =

$$|5| \xleftrightarrow{\text{diff.}} |8| = 3$$

Step 4 Affix sign of larger number to result as

(+5) + (-8) = -3, Answer

2. Subtract (-8) from (-5) = _____

solution: Step 1 Set up as subtraction problem in vertical fashion:

$$\begin{array}{r} (-5) \\ -\ \underline{(-8)} \end{array}$$

Step 2 Change sign of lower number to opposite sign, and, change function sign from subtraction to addition

$$\begin{array}{r} (-5) \\ +\ \underline{(+8)} \end{array}$$

Step 3 Add

$(-5) + (+8) =$

$|5| \xleftrightarrow{\text{diff.}} |8| = 3$

Step 4 Affix sign of larger number

$(-5) + (+8) = +3$, Answer

5-B Subtraction of numbers with unlike signs

Examples

1. Subtract (+8) from (-5) = _____

solution: Step 1

$$\begin{array}{r} (-5) \\ -\ \underline{(+8)} \end{array}$$

Step 2

$$\begin{array}{r} (-5) \\ +\ \underline{(-8)} \end{array}$$

Step 3 (-5) + (-8) = -13, Answer

2. Subtract (-8) from (+5) = _____

solution: Step 1

$$\begin{array}{r} (+5) \\ -\ \underline{(-8)} \end{array}$$

Step 2

$$\begin{array}{r} (+5) \\ +\ \underline{(+8)} \end{array}$$

Step 3 (+5) + (+8) = +13, Answer

DRILL SHEET

SUBTRACTION OF SIGNED NUMBERS

Subtract the signed numbers as indicated:

11D-1 $(-10) - (-8) =$

11D-2 $(+10) - (+8) =$

11D-3 $(+5) - (-13) =$

11D-4 $(+5) - (+13) =$

11D-5 $(-22) - (-14) =$

11D-6 $(+17) - (+8) =$

11D-7 $(+3) - (-19) =$

11D-8 $(+26) - (+31) =$

11D-9 $(+40) - (+40) =$

11D-10 $(-40) - (-40) =$

11D-11 $(-40) - (+40) =$

11D-12 $(-25) - 0 =$

11D-13 $0 - (+7) =$

11D-14 $0 - (-7) =$

11D-15 $(+36) - (+41) =$

UNIT 5 : REVIEW

-5R1- Express 42 inches as feet ______

-5R2- Add 5 ft 9 in and 8 ft 3 $\frac{3}{4}$ in ______

-5R3- Multiply 2 ft 8 $\frac{1}{2}$ in by 3 ______

-5R4- 17 yd 2 ft 10 in ÷ 4 ______

-5R5- 11 yd 1 ft - 7 yd 2 ft ______

-5R6- Express 616 meters as kilometers ______

-5R7- Find the sum of 4.6 cm and 17.8 mm in millimeters ______

-5R8- Express 12 mm to the nearer thousandth inch ______

-5R9- Write 1.6 cu ft as cubic inches ______

-5R10- Write 25 oz as pints (round to 2 decimal places) ______

-5R11- Express 2.3 L as milliliters ______

-5R12- Express 5.2 oz as milliliters (round to 3 decimal places) ______

-5R13- Write 0.17 kg as grams ______

-5R14- List each set of signed numbers in order of increasing value starting with the smallest value:

a. -5, -23, +8.0, +1, -18, +7.9 = __________

b. -7.5, 0, +7.5, -2.3, +0.5, -0.3 = __________

c. +21.3, 0, +20.6, -4.6, -7, -23.4 = __________

d. $+3\frac{1}{2}$, -3, $+6\frac{3}{4}$, $-6\frac{7}{8}$, $-6\frac{13}{16}$ = __________

Add or subtract the signed numbers as indicated:

-5R15- +14 + (-6) = __________

-5R16- 50 + (-23) = __________

-5R17- -37 - (-31) = __________

-5R18- -30.7 - (+5.5) = __________

-5R19- 15 + (-8) + (-15) + (-10) = __________

-5R20- -2.9 + 1.6 + 3.2 + 7.5 = __________

-5R22- $2\frac{1}{2} + (-\frac{3}{4}) + (-\frac{1}{8}) + \frac{3}{8}$ = __________

-5R23- (5+9) - (-3+6) = __________

-5R24- (-13-6) - (4+7) = __________

-5R25- (6.48-5.32) - (4-8.31) = __________

UNIT 6: RATIO/PROPORTION AND INTRODUCTION TO ALGEBRA

OBJECTIVE:

After completing this Unit you will be able to:

- Understand the concept of Ratio
- Understand the concept of proportion
- Understand the difference between "direct" and "indirect" proportions
- Solve each proportion for "X"

Dictionary:

	Meaning:
Ratio	The quotient of two numbers e.g. $\frac{6}{7}$ 6 : 7
Proportion	A statement where two ratios are equal $\frac{5}{7}$ $\frac{25}{35}$ 5 : 7 as 25 : 35
Direct Proportion	A proportion in which the order of the ratios is the same
Indirect Proportion	A proportion in which the order of the ratios is indirect

Unit 6: Ratio (Direct and Indirect Proportion)

Section 1: Ratio

A ratio is the quotient of two numbers, or comparison, of the __same kind.__ "Of the same kind", means both numbers are either abstract numbers, such as 30 and 10, or both numbers are expressed in the same units of measure, such as 20 feet and 30 feet.

The quotient of two numbers, a : b or a/b is sometimes referred to as a __ratio__ and read "the ratio of a to b". This is a convenient way to compare two numbers.

A statement that two ratios are equal, for example,

$\frac{2}{3} = \frac{4}{6}$ $\qquad$ $\frac{a}{b} = \frac{c}{d}$ is called a proportion and read "2 is to 3" as "4 is to 6"

The proportion a/b = c/d is often written in the form a : b = c : d which is read "a is to b as c is to d".

Which of the following proportions are true?

a) 3 : 4 = 21 : 28 $\qquad$ b) 20 : 35 = 65 : 91

c) 42 : 35 = 70 : 60 $\qquad$ d) 5 : 8 = 25 : 45

DRILL SHEET

Express each of the following as a ratio:

1D-1 1 in to 9 in ____________________

1D-2 10 cm to 50 cm ____________________

1D-3 5 to 6 ____________________

1D-4 6 mm to 12 mm ____________________

1D-5 4 kg to 20 kg ____________________

1D-6 9 to 10 ____________________

1D-7 20 to 6 ____________________

When you write a ratio in the form of a fraction, always write the fraction in its lowest terms. To say that a gear is "twenty to five" is unacceptable. Reduce the fraction $\frac{20}{5}$ to $\frac{4}{1}$ then say the ratio is "four to one". When writing answers to problems try to be clear and use correct terms.

Two important types are probability and percent ratios. The study of probability has become increasing important and will help you understand many statements about everyday affairs, such as weather predictions and odds on the World Series. For instance, probability provides the foundation upon which insurance rates are based.

Consider this:
A drawer contains 24 red socks and 8 blue socks. If you randomly select a sock from the drawer, what is the probability you will select a blue sock?

$$\text{Probability Ratio} = \frac{\text{Successful Ways}}{\text{All Possible Ways}}$$

$$= \frac{8}{32}$$

$$= \frac{1}{4}$$

The probability you will select a blue sock drawn at random is $\frac{1}{4}$. What does that mean?

The mechanical efficiency of a motor, steam engine, shop tool, and other mechanical or electrical devices that use and deliver useful energy is the ratio:

$$\text{Mechanical Efficiency (ME)} = \frac{\text{(Useful) output}}{\text{(Total) output}}$$

The <u>output</u> is the useful energy delivered <u>by</u> the machine, and the <u>input</u> is the amount of energy delivered to the machine. (The ratio is usually written as a percentage rather than a common fraction).

Example: An auto mechanic wants to find the efficiency of an automobile engine before a tune-up. From the engine specification chart he finds that the engine is rated to deliver 315 horsepower as 4400 r.p.m. When he tests the engine, he finds that it delivers only 270 horsepower at 4400 r.p.m. Find the efficiency.

Solution: $\text{Efficiency} = \frac{270 \text{ hp}}{315 \text{ hp}} = \frac{6}{7}$

Answer: The engine is producing six sevenths of its rated horsepower.

One of the most common examples of ratio used in everyday ma'ematics is pi (π) the 3.1416 ratio of the circumference of a circle to its diameter. It applies no matter whether the circle is measured in feet, inches, rods or whatever. Ratio even finds its way into law. A U.S. flag, for example, is to have a 1.9 ratio of length to width - whatever its size - its length should be 1.9 times its width.

Section II: Direct Proportion is one in which the order of the ratios is the same.
For example, small is to large as small is to large.

Example:
4 is to 16 as x is to 24
4 : 16 x : 24

$$\underbrace{4 : \overbrace{16 = x}^{\text{means}} : 24}_{\text{extremes}}$$

Make into an equation and calculate

$$16\ x = 4 \times 24$$
$$16\ x = 96$$
$$x = 6$$

* The product of the means equals the product of the extremes.

Solve each proportion for X

<u>Sample Problem</u>

a) $\frac{4}{5}$ x $\frac{X}{20}$

$(4)\ (20) = 5X$

$80 = 5X$

$X = 16$ - Answer

DRILL SHEET

2D-1 $\frac{3}{X} = \frac{1}{5}$

____

2D-2 $\frac{2}{7} = \frac{X}{28}$

____

2D-3 $\frac{X}{21} = \frac{5}{7}$

____

2D-4 $\frac{6}{11} = \frac{X}{22}$

____

2D-5 $\frac{3}{4} = \frac{X}{7}$

____

2D-6 $\frac{X}{21} = \frac{5}{7}$

____

2D-7 $\frac{12}{5} = \frac{7}{X}$

____

2D-8 $\frac{9}{2} = \frac{18}{X}$

____

2D- 9 $\frac{20}{30} = \frac{2}{X}$

____

2D-10 $\frac{45}{50} = \frac{X}{10}$

____

PRACTICAL APPLICATIONS

1. Two gears are in the ratio of 3:5 . If the larger gear has 40 teeth, how many teeth does the smaller gear have?

2. The power-to-weight ratio for a certain automobile engine is 7:8. If the engine weighs 400 lbs. How much power does it produce?

3. The odds of getting a pair of cards (two cards with the same face value) in a single deal in a power game are 3:4. (In four deals you can expect to get a pair of cards three times). How many pairs can you expect to get in 24 deals of the cards?

4. A road has a grade of 7 per cent which means that it rises 7 feet for every 100 feet measured horizontally. How much does the road rise in a mile? Figure the answer to the nearest foot?

5. A telephone pole 17 feet high casts a shadow 10 feet long. How tall is a church steeple which at the same time, casts a shadow 50 feet long?

6. A car travels at the rate of 50 miles per hour. How many feet per second is this?

Section 3: Indirect Proportion

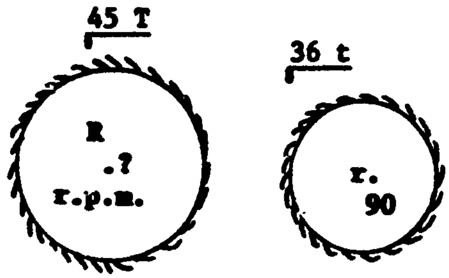
An indirect proportion is one in which the order of the ratios is indirect (inverse).
For example, large is to small, as small is to large.

In a proportion, the product means is equal to the product of the extremes.

* (Note that this rule is the same for both direct and indirect proportion).

1. Example: What are the r.p.m. of the big gear on the right

Use the formula T : t = r : P



INTRODUCTION TO ALGEBRA

OBJECTIVES:

After completing this unit, you will be able to :

- Understand the meaning of an algebraic equation
- Write a simple algebraic equation
- Solve equations
- Understand simultaneous equations

Dictionary:

	Meaning:
1. Algebra	A type of mathematics that uses numbers and letters that stand for numbers to solve problems.
2. Equation	An equation is a sentence that contains an equal sign (=); a statement that two amounts are equal.
3. Equivalent	Two equations are said to be equivalent if they have exactly the same solutions.
4. Variable	A lower case letter such as x, y, z, a, b, is commonly used to represent a member of a set of numbers.

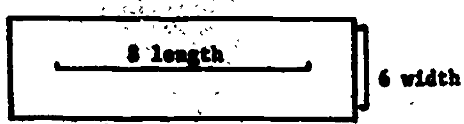
Section 1: Introduction to Algebra

Algebra is a part of mathematics that uses letters to represent numbers in the forms of equations and formulas calculated according to the rules of arithmetic.

A formula is a statement of a rule in the language of algebra. Rule: Area equals length times width.

Formula: $A = lw$

$A = 8 \times 6$



The set of relations existing among these known and unknown numbers which go to make up or specify a certain condition may then be expressed by means of the algebraic signs for additions, subtraction, powers, roots, etc., and the resulting form of statement of the condition will be an algebraic equation.

DRILL SHEET

3D-1 Write the formula for the rule: The resistance in an electrical circuit is equal to the voltage divided by the current.

____

3D-2 Write a formula for the rule:
The volume of a cube is equal to the cube of the length of a side.

____

3D-3 Write a formula for the rule:
The sum of the three interior angles of a triangle is equal to 180 .

____

Section 2: Solving Equations in Algebra

One of the most important methods for problem solving is the equation.

The mathematical statement $2x + 1 = 6$ is an equation. Solutions are numbers that make equations true statements.

Example: Show that 2 is a solution to the equation.

$3x + 4 = 10$

Substitute 2 for the x in the equation

$3(2) + 4 = 10$

In the equation $b = c - a$, solve for c.

1) Subtraction is indicated. Therefore get the unknown (c) alone by addition. Add a to both sides.

$$\begin{array}{rcl} b & = & c - a \\ +a & & +a \\ \hline a + b & = & c \quad o \\ a = b & = & c \quad \text{answer} \end{array}$$

Check by substituting the value of the unknown (c) and compute the equation.

$b = c - a$

$b = (a + b) - a$

$b = b$ Answer is OK because both sides are equal.

DRILL SHEET

Use either the rule of addition or subtraction to find the solutions to these equations.

4D-1 $x + 20 = 60$

____

4D-2 $x - 10 = 100$

____

4D-3 $x - 2.6 = 4.7$

____

4D-4 Now try to use the rule of subtraction to solve for the variable.

a. $x + 10 = 20$

____

b. $b + 8 = 10$

____

c. $s + 1 = 11$

____

Section 3: Rule for Division in Equations

Divide both sides of an equation by the same quantity to keep the equation in balance.

$$4x = 16$$

Divide both sides by 4

$$\frac{4x}{4} = \frac{16}{4}$$

$$x = 4$$

Use the rule of division to solve for the variable.

1. $18t = 36$

   ____

2. $4t = 1.6$

   ____

3. $10a = 100$

   ____

4. $.5x = 10$

   ____

5. $\frac{1}{3}x = 9$

   ____

Express each word problem algebraically and solve.

1. The product of a number and 5 is 65.

2. A number divided by 3 is equal to 17.

3. A voltage (E) when doubles equals 628 V.

4. The quotient of a current (I) and 10 is D. 146.

5. A conductor is divided into four equal parts. Each part is 1.4 m long. Find the original length (L).

6. Write a problem to fit each equation:

a) $5\frac{1}{2} + n = 10\frac{1}{4}$

b) $\frac{n - 3}{3} = 3$

c) $(4 \times n) + 6 = 26$

7. Write an equation for each problem. The ALPO bus has enough room for 34 employees. If the driver started with a full load, stopped at R&D (Research and Development) to let of 18 employees, and picked up 9 employees going to the Training Room, how many employees were left on the bus?

8. Jerry came home from the Annual ALPO baseball game and was hungry. He found a note from his wife telling him to help himself to a piece of pie. Someone else had already eaten part of the pie, because only $\frac{3}{4}$ of it was left in the pan pan. After Jerry cut a piece for himself, only $\frac{1}{2}$ of the remained. What part of the whole pie did Jerry eat?

Section 4: Simultaneous Equations

Considering two equations: $x - y = 11$, the requirement that both equations are true for the same set of values for x and y defines them as SIMULTANEOUS equations. In simpler words -- only one value for x and one value for y will make these equations true.

Section 5: Solving Simultaneous Equations By Addition or Subtraction

<u>Addition</u>

Consider again the two equations: $x - y = 3$ and $x = y = 11$. If we think of them as balanced scales, we can show the following:

and

Therefore, if we take the left hand value or expression from one scale and add it to the left hand pan of the other; and, then do the same with the right hand value or expression, the following would result:

Note that the scale is scill in balance!!!

This can be shown only in equation form

$$\begin{array}{ll} & x - y = 3 \\ \text{plus} & x + y = 11 \end{array}$$

Combining or adding each side together, we see that $-y$ and $+y$ cancel each other, or:

$$2x = 14$$

$$x = \frac{14}{2} \text{ or } 7$$

Now substituting 7 for x in <u>either</u> equation and solving for y, we can readily show that y = 4.

<u>Subtraction</u>

Remembering the rule that if we subtract an equal amount from both sides of a balanced equation, the equation still balances. We can subtract either of these two equations from the other and solve for x and y. That is:

$$\begin{array}{r} x - y = 3 \\ -(x + y) = -11 \\ \hline \end{array} \qquad \text{subtracting}$$

$$\begin{array}{r} x - y = 3 \\ -(x + y = -11 \\ \hline 0 \quad -2y = -8 \end{array} \qquad \text{Therefore } y = \frac{-8}{-2} \text{ or } 4$$

And again, substituting 4 for y in either equation will result in x equaling 7.

The <u>main purpose</u> of deciding whether to subtract or add is to <u>eliminate one of the unknowns</u> so that a value can be found for the other. For example, in the set of equations following:

$$\begin{array}{l} 2x + y = 10 \\ 2x - 5y = 40 \end{array}$$

one can see that in order to eliminate an unknown, <u>X</u> and its numerical coefficient <u>2</u> are exactly the same in both equations. Therefore, it will be advantageous to subtract one from the other and then combine.

$$\begin{array}{r} 2x + y = 10 \\ -(2x - 5y) = -(40) \end{array} \qquad \begin{array}{r} 2x + y = 10 \\ -2x + 5y = -40 \\ \hline 0 + 6y = -30 \end{array} \quad y = -5$$

Substituting <u>-5</u> for y in one of the original equations; it can be seen that X = 7.5.

It follows then that in the set of equations:

$$\begin{array}{l} 5x + 2y = 20 \\ 3x - 2y = 12 \end{array}$$

one would simply add (combine) since +2y and -2y would drop out and x can easily be evaluated.

Multiplication/Addition/Subtraction

When solving simultaneous equations, it becomes necessary to multiply both sides of an equation (or sometimes both equations) by some quantity in order to eliminate one of the unknowns.

The main purpose is to get either one of the unknowns to have the same numerical coefficient but the opposite sign in order to eliminate it when combining.

Consider the equations:

$$2a - 4b = 21$$
$$3a - b = 11$$

One can multiply either the second equation by -4 to eliminate b; or multiply the first equation by 3 and the second by -2 to eliminate a.

Choosing the first method:

$2a - 4b = 21$	$2a - 4b = 21$
$-4(3a - b = 11)$	$-12a - 4b = -44$
combining (add)	$-10a = -23$
	$a = 2.3$
	$b = -4.1$

Steps in solving simultaneous equations by addition or subtraction.

1. Multiply one or both of the given equations, if necessary, by some factor, or factors, that will make the coefficients of one variable numerically equal.

2. Eliminate this variable by addition or subtraction.

3. Solve the resulting equation for one variable.

4. Find the value of the other variable by substituting the known value of one variable in one of the equations. (The second variable may be found in the same way as the first.)

DRILL SHEET

5D-1	3x	+	2y	=	8	5D-6	2x	-	3y	=	19
	x	-	4y	=	5		5x	+	4y	=	13
5D-2	4x	-	7y	=	-2	5D-7	5x	+	y	=	1
	3x	+	4y	=	17		x	+	2y	=	11
5D-3	2r	-	5s	=	13	5D-8	4m	-	3n	=	11
	7r	-	3s	=	2		7n	+	3m	=	-1
5D-4	3c	-	4d	=	-5	5D-9	4h	+	5k	=	-3
	7d	+	4c	=	-10		7k	=	3h	-8	
5D-5	7x	+	3y	=	9	5D-10	3y	-	7z	=	3
	7y	-	3x	=	-8		5z	-	7y	=	10
5D-11	3x	+	2y	=	8	5D-17	7x	-	10y	=	20
	7x	-	y	=	6		11x	-	15y	=	30

5D-12 $3x - 4y = 7$
$2x - 7y = -3$

5D-13 $3I + 2i = -9$
$4I - 7i = 1$

5D-14 $4R - 5r = 7$
$3R - 2R = 10$

5D-15 $5R - 6r = 7$
$9r + 4r = 10$

5D-16 $12x - 11y = 25$
$15y + 16x = 80$

5D-17 $21.3x - 9.5w = 6$
$9.5x - 21w = 0$

5D-18 $3E - 5e = 4$
$2E - 7e = -8$

5D-19 $24a - 15B = 40$
$15a - 24B = 40$

5D-20 $3a - 8b = 7$
$5a - 12b = 3$

5D-21 $2y = 7x + 6 = 0$
$5x + 3y - 1 = 0$

5D-22 $2.3x + 7.2y = 10$
$5.1x - 4.3y = 10$

5L-27 $3.2R - 1.5r = 25$
$2.1R - 2.5r = 5$

5D-24
$$\begin{aligned} 50I + 15i &= 80 \\ 15I + 25i &= 80 \end{aligned}$$

5D-25
$$\begin{aligned} ax + by &= e \\ cx + dy &= f \end{aligned}$$

5D-26
$$\begin{aligned} mx + ny &= 5c \\ nx - my &= 3b \end{aligned}$$

5D-28
$$\begin{aligned} 1.3R + 1.5r &= 6 \\ 1.5R + 1.3r &= 8 \end{aligned}$$

5D-29
$$\begin{aligned} bx - ky &= a \\ cx + ky &= n \end{aligned}$$

5D-30
$$\begin{aligned} 2ma - 3mb &= 4 \\ 5ma - 4mb &= 15 \end{aligned}$$

UNIT 6: REVIEW

Solve each of the equations for the variable shown.

6R-1 $Q = CV$, solve for C

6R-2 $R2 = Z2 - X2$, solve for Z

6R-3 $I = \frac{E}{Z}$, solve for Z

6R-4 $Xl = 2 \quad fL$, solve for L

6R-5 $R = \frac{P}{I2}$, solve for I

6R-6 $Xc = \frac{1}{2\pi fC}$, solve for C

6R-7 $R2 = Rt - R1 - Ra$, Solve for Rt

6R-8 $pf = \frac{R}{X}$, solve for R

6R-9 $P = \frac{120f}{N}$, solve for f

6R-10 $Ns = \frac{EsNp}{Ep}$, solve for Np

6R-11 $u = gmrp$, solve for rp

6R-12 $\frac{Zp}{Zs} = \frac{Np2}{Ns}$, solve for Zp

6R-13 Es Is = Ep Ip, solve for Ep

6R-14 $M = k\sqrt{L1\ L2}$, solve for k

6R-15 $C = \frac{0.088\quad 4kA(N-1)}{d}$, solve for A

UNIT 7: COMPUTER LITERACY USING ALGEBRA SOFTWARE

OBJECTIVES:

After completing this unit, you will be able to:

- Understand the basic functions of the computer
- Appreciate the computer as a valuable aid in the learning process therefore alleviating any technological fears.
- Understand that different computers have different ways of operating.

Dictionary:

	Meaning:
1. Monitor	a T.V. set - like display without a channel tuner. Input is from a "video" signal.
2. Disk Drive	a persphiral unit used to run programs of floppy disks.
3. Keyboard	the set of keys on an input device used for encoding characters by the depression of keys, as in a typewriter.
4. Hardware	the physical components of a computer system as opposed to the programs (software)

In this unit, students from both companies had one to two classes at the Math Computer Lab at the Community College. Students experienced working on both an Apple IIE and IBM computer.

Software consisted of:

- IBM Algebra I and Algebra II
- Preparing for Geometry and Algebra (P.G.A.)

Lesson 1: Measurement

1) Reading a Ruler
2) Units of Length
3) Units of Volume and Weight

Lesson 2: Introduction to Geometry

1) Terms in Geometry
2) Angles
3) Perimeter
4) Area and Volume

Lesson 3: Graphs

1) Miscellaneous Graphs
2) Bar Graphs or Bar Charts
3) Fractional Parts or Sets

Lesson 4: Integers

1) Absolute Value
2) Addition and Subtraction
3) Multiplication and Division

Lesson 5: Introduction to Algebra

1) Exponents and Square Roots
2) Scientific Notation
3) Operations with Exponents
4) Simplifying Expressions

Lesson 6: Mixed Topics

1) Money
2) Time and Calendar
3) Temperature
4) Roman Numerals
5) Sequences

Unit Examinations

MATHEMATICS FOR INDUSTRY

UNIT 1 EXAMINATION

Date ________________

Shift _______________

Name __________ ________________________

Grade ________ ______

Observe the instructions in solving the following problems:

1. Write the correct number for each word statement.

   a. Seven hundred thousand, seven hundred seven

   ____

   b. Four hundred thousand and ninety-two thousandths.

   ____

2. 4,793 - 404 = ____________

3. The sun is about 93,000,000 miles from the earth. Light travels at a speed of 186,000 miles per second.

   a. How many seconds does it take the light from the sun to reach the earth?

   ____

   b. How many minutes does it take the sunlight to reach the earth?

   ____

4. 7,867 x 403 = ____________

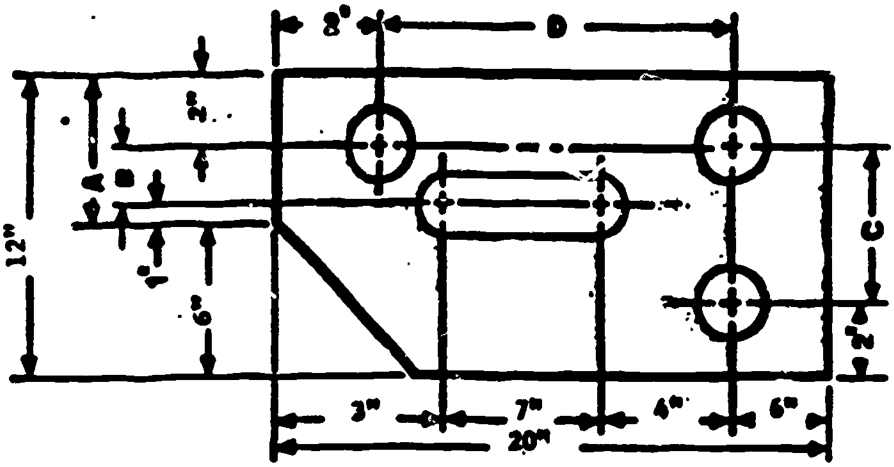
5. Divide 13,701 by 1101 ____________

6. $58 + \frac{13+32-9}{3} - 17 =$ ______

7. In estimating the time required to complete a job, a Section Chief determines that a total of 840 hours are needed. If four utility operators each work five days per week for seven hours per day, how many weeks are required to complete the job? ______

8. If a machine setter earns $23,820 annually, what is the monthly salary? ______

9. In order to make the fixture shown below, a machinist must determine dimensions A, B, C, and D; solve for dimensions A, B, C, and D:



A ______

B ______

C ______

D ______

10. Which total volume is greater, four drums containing 72, 45, 39, and 86 liters, or three drums containing 97, 115, and 74 liters? ____________

By how much is it greater? ____________

11. ALPO contracted to have its trucks repainted. If 17 trucks cost $159 each, and 43 trucks cost $267 each, how much money will ALPO be billed to have all the trucks painted? ____________

12. A technician has a silicon rod that totals 243 inches in length. If he must cut 9 equal pieces, how long will each piece be? ____________

13. 2,478 - 726 + 598 x 12 = ____________

14. 72 x 38 + 86,526 - 69 = ____________

15. A high-speed stamping machine can produce small flat parts at a rate of 96 per minute. If an order calls for 297,600 parts to be stamped, how many hours will it take to complete the job? (60 minutes = 1 hour) ____________

MATHEMATICS FOR INDUSTRY

UNIT 2 EXAMINATION

Date ______________

Shift ______________

Name ______________________________

Grade ______________

Observe the instructions in solving the following problems; always reduce to lowest terms:

1. $\frac{3}{4} + \frac{7}{8} + \frac{5}{16} =$ __________

2. $37\frac{7}{10} - 19\frac{3}{4} =$ __________

3. $\frac{3}{4} \times 12 \times 1\frac{5}{8} =$ __________

4. $5\frac{3}{5} - \frac{15}{16} =$ __________

5. $8\frac{1}{4} \times (6 + 5\frac{1}{16}) - (5 + \frac{3}{8}) =$ __________

6. Express $\frac{9}{10}$ as an equivalent fraction with a denominator of 60. __________

7. Express $\frac{72}{128}$ in lowest terms.

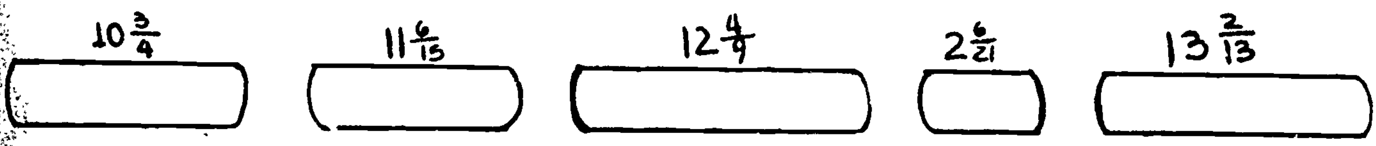
8. Express $3\frac{1}{4}$ as a fraction.

9. Express $\frac{451}{64}$ as a mixed number.

10. Find the lowest common denominator for $\frac{7}{12}$, $\frac{2}{3}$, $\frac{1}{2}$.

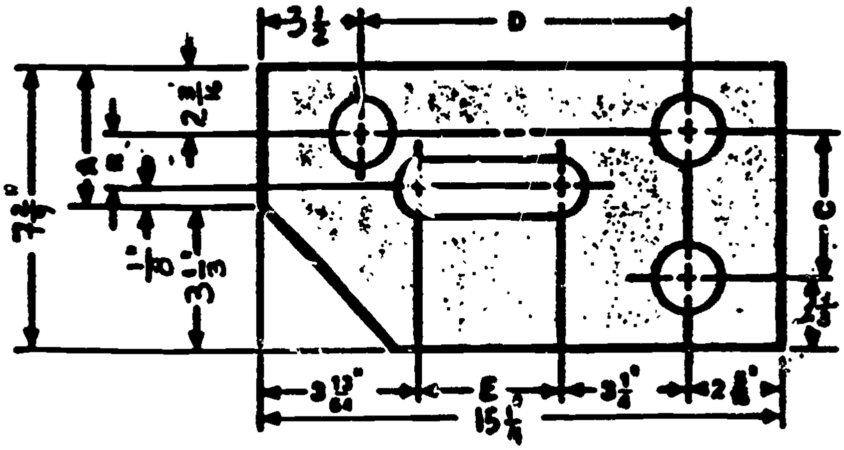
11. Find the total length of 12 pieces of wire each $9\frac{3}{16}$ inches long.

12. What would be the total length of the silicon rod formed by fusing together the five pieces of rod shown below:



13. An electrical wiring job requires the following lengths of BX cable: seven pieces each $6\frac{1}{2}$ inches long, four pieces each $34\frac{3}{4}$ inches long, and nine pieces each $19\frac{3}{8}$ inches long. What is the total length of cable needed. __________

14. A fixture is shown below. Find the indicated dimensions:



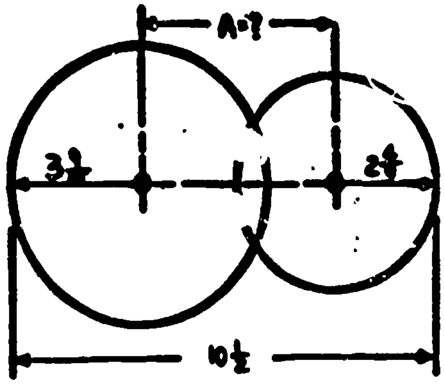
Dimension A = __________ Dimension D = __________

Dimension B = __________ Dimension E = __________

Dimension C = __________ Dimension F = __________

Dimension G = __________

15. Find the missing dimension A in the following sketch:



____

16. When aligning a motor, the following pieces of steel are used under the base: one piece $\frac{5}{16}$" thick; one piece $\frac{3}{8}$" thick; one piece $\frac{5}{32}$" thick; and one piece $\frac{5}{64}$" thick. What is the total thickness of all these pieces?

____

MATHEMATICS FOR INDUSTRY

UNIT 3 EXAMINATION

Date ______________

Shift ______________

Name ________________________________

Grade ______________

Observe the instructions in solving the following problems; always reduce to lowest terms or round the numbers to the indicated decimal places (shown in parenthesis).

Write each of the word statements as a decimal number.

1. **Five Hundred Four and Twenty-Two Hundredths** ____________.
2. **Two and Five Tenths** ____________.
3. **One Hundred Fifty and Three Thousandths** ____________.
4. 16.7091 (2) ____________.
5. 42.01997 (4) ____________.
6. .8999 (3) ____________.
7. 104.01997 (3) ____________.

Express these common fractions as decimals to four places.

8) $\frac{4}{9}$ ____________ 9) $\frac{11}{18}$ ____________

10) $\frac{41}{50}$ ____________ 11) $\frac{24}{27}$ ____________

Express these decimals as common fractions.

12) .21 ____________ 13) .0303 ____________

14) .625 ____________

15) From the sum of 22.825 and 35.099 subtract sum of 19.01 and .1258 ____________.

16) 22.08 x 19.23 x .092 = ____________.

17) **An assembler checks 630 parts for the total week. On Wednesday of that particular week, he checked 205. What decimal of the total checked is the amount checked on Wednesday? ______________________________.**

18) Complete the sequences

a) .14, .16, .18, _______, _______

b) .43, .49, .55, _______, _______

c) .56, .50, .44, _______, _______

d) .80, .40, .20, .10, _______, _______

MATHEMATICS FOR INDUSTRY

UNIT 4 EXAMINATION

Date ______________

Shift ______________

Name ______________________________

Grade ______________

Observe instructions in solving th' following problems:

1. Express 19.6% as a decimal fraction. ____________

2. Write 0.91 as a percent. ____________

3. What is 7% of 140? ____________

4. 60 is 80% of what number: ____________

5. Find 25% of 120. ____________

6. Express 218.7% as a decimal fraction (or mixed decimal). ____________

7. Express 109.8% as a common fraction (or mixed number). ____________

8. Convert 0.8125 to percent. (Round to 2 decimal places). ____________

9. What percent of 140 is 105? ____________

10. Determine the range (R), average (X), and median for the following sets of numbers:

    a. 6, 5, 9, 4, 8, 11, 10, 7

    R ____________

    X ____________

    Median ____________

    b. 0.0098, 0.0087, 0.0092, 0.0087, 0.0084, 0.0098

    R ____________

    X ____________

    Median ____________

MATHEMATICS FOR INDUSTRY

UNIT 5 EXAMINATION

Date ______________

Shift ______________

Name ______________________________________

Grade ______________

Observe instructions in solving the following problems:

1. Express 42 inches as feet. ____________

2. Multiply 2 ft 8 $\frac{1}{2}$ in by 3. ____________

3. Express 616 meters as kilometers. ____________

4. Find the sum of 4.6 cm and 17.8 mm in millimeters. ____________

5. Write 1.6 cu. ft. as cubic inches. ____________

6. Write 25 oz. as pints. (Round to 2 decimal places.) ____________

7. Express 2.3 L as milliliters. ____________

8. Express 5.2 oz. as milliliters. (Round to 3 decimal places.) ____________

9. Write 0.17 Kg as grams. ____________

10. Hot air passes through a duct at a rate of 500 cubic inches per second. Compute the number of cubic feet of hot air passing through the duct in one minute. (Round answer to 2 decimal places.) ____________

11. Add 61.9 cm + 17.9 dm + 74.3 mm + 0.08 m. ____________

12. Express 47.8 rods as yards and inches. ____________

13. Express 9.13 metric tons as lb. ____________

14. Select the greater of each of the two signed numbers and indicate the number of units by which it is greater:

a. (-51.79), (-39.35) = __________ ; __________

b. $(-29\frac{5}{16})$, $(-16\frac{5}{8})$ = __________ ; __________

15. Express each of the pairs of signed numbers as absolute values and report the difference:

a. +10.54, (-[illegible].46) = ________ ; ________ ; ________

b. (-0.08), (-0.009) = ________ ; ________ ; ________

16. List each of the following sets of numbers in order of increasing value, starting with the smallest value:

a. -17.6, +14.1, +2.3, -11.4, -7

____

b. What is the range of the above set of value?

17. a. 0, $-7\frac{1}{2}$, $-9\frac{5}{32}$, $-10\frac{1}{4}$, $-1\frac{7}{16}$

b. How far is it and what direction from the maximum to the minimum value?

____

MATHEMATICS FOR INDUSTRY

UNIT 6 EXAMINATION

Date ______________

Shift ______________

Name ________________________________

Grade ______________

Express the following as an algebraic expression.

1. Divide 25 by b ____________

2. Increase e by 12 ____________

3. Reduce y by 75 ____________

4. The square root of X plus 6.8 ____________

5. One half X minus four times y ____________

6. The cube root of h times the square of p ____________

7. Divide x by the product of 25 and y ____________

8. Take the square root of r, add s, and subtract the product of z and t:

____

9. In a series circuit the total resistance (Rt) of a circuit if equal to the sum of the three individual resistances R1, R2, R3. The circuit has a total resistance of 200 ohms. Express the resistance R1.

____

10. Impedance (Z) of a circuit is computed by adding the square of resistance (R) to the square of the reactance (X), then taking the square root of the sum. Express the circuit impedance.

____

11. The electrical power (P) dissipated by an incandescent lamp is directly proportional to the square of the voltage drop (E) across the lamp and inversely proportional to hte electric resistance (R) of the lamp.

____

Solve each of the equations for the value shown.

12. $Xl = 2\pi fL$, solve for L ____________________

13. $A = \frac{\pi D2}{4}$, solve for D ____________________

14. $Xc = \frac{1}{2\pi fC}$, solve for C ____________________

15. $M = k\sqrt{L1\ L2}$, Solve for L1 ____________________

Substitute numbers for letters and evaluate the following.

16. If X = 6 and y = 2 ____________________

a) 2xy - y = ____________________

b) (x + y) (x - y) = ____________________

c) $\frac{x + y}{x - y}$ = ____________________

d) 2x - x 3 = ____________________

e) 2x + xy - 4y = ____________________

Suggested Instructional Strategies/General Suggestions for Using Manual

Unit 1: Suggested Instructional Strategies/General Suggestions For Using Manual

WHOLE NUMBERS

1. Place Value

For this topic (every student, no matter what level can benefit)

Additional drill sheets with examples like the ones cited below should be provided for the students.

Students should be given a copy of the Numerical Periods Chart. (see Appendix)

Examples:

a. One Hundred Twenty - Three

b. Five Hundred One

c. Two Hundred Twenty-Five and Twenty-Five Thousandths **

* Usually in oral communication errors will be made "One Hundred and Twenty Five". Many students are not aware of the fact that 'and' is used only in place of the decimal point. (cite for example writing a check)

** Use English Numbering System Chart (which appears in Manual)

Additional drill (like the examples shown below) should also be provided

Write the words for each number listed below.

A. 225 ______________________________

B. 4035 ______________________________

C. 59 ______________________________

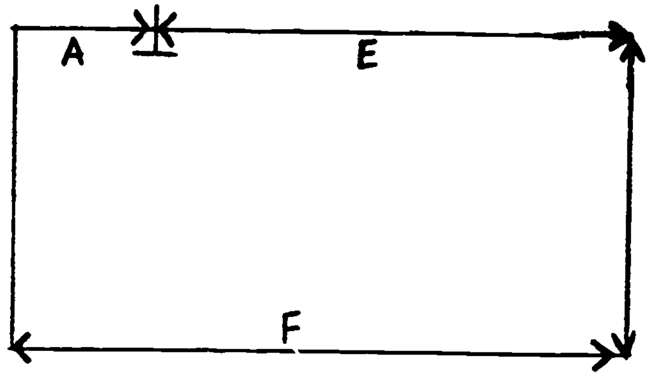
D. 199 ______________________________

2. "The Whole is Equal to the Sum of its Parts" - Using Graphics

Most effective if overhead projection is used.

Note: Some students will have a great deal of trouble interpreting diagrams. Additional examples should be given.

Examples:



F = 12 "   A" 2"   E = ?

3. Some students (this especially holds true when the class is not using the calculator (see Appendix for "Thoughts on Use of the Calculator") will need the aid of the Multiplication Chart (see Appendix).

Unit 2: Suggested Instructional Strategies/General Suggestions For Using Manual

COMMON FRACTIONS

1. Concept of a Fraction

This is a good time to emphasize that anytime in math when there is a line separating either a number, fraction, decimal, etc. from another number, fraction, decimal, etc. - it signifies that the numerator is divided by the denominator. e.g. $\frac{1}{6}$ can represent the fact that "1" cookie is being divided among "6" people; therefore, you will just receive a fraction of the cookie, or $\frac{1}{6}$ of it. 1 - 6. Re-emphasize this through-out the entire unit.

Allowing for differentiation among a heterogeneous group of students, more work should be done with lower -skilled students on the concept of a fraction. Using fraction strips (see Appendix) construct examples illustrating the concept of fraction. This can be a tie in with equivalent fractions. Example:

The maintenance Department at ALPO have decided to establish two teams. Each team will work the same amount of time each week. Team "A" decided to divide the 24 hour work day into 2 shifts, while "B" decided to divide the 24 hours into 4 shifts. (display a circle divided into 4 parts on transparency using an overhead projector). Superimpose a circle divided into halves onto the transparency divided into fourths.

2. Dictionary Terms - (lower-skilled group)

Since the terms numerator and denominator are frequently used, a good way to remember is "D" for down [$\frac{1}{6}$] the "6" is down - denominator; the upper number is therefore the numerator.

3. Lowest Common Denominator

With a lower to middle - skilled group of students, work through the steps illustrated in the curriculum guide. When working with a more advanced group, challenge with concentration on the prime number factoring system.

4. Addition of Common Fractions

Principle of equivalent fractions may or may not be used when obtaining new forms of numerators (may be more confusing than beneficial)

Lower-skilled students may be initially very confused with the different concepts of fractions. This might be a good time to quiz them on what they have already learned. (see Appendix) This method could and should be employed throughout the entire manual when the instructor feels it's necessary.

5. Adding Fractions, Mixed Numbers, and Whole Numbers

Students may experience some difficulty with transfer of improper fraction to whole number. Additional drill may be necessary.

6. Subtracting Fractions

Encourage students (using red pencils) to circle sign so as not to add when they should be subtracting.

Lower skilled students will experience difficulty with "borrowing" from whole number in order to be able to subtract the subtrahend from a suf cient amount.

7. Multiplication and Division of Fractions

Students will find the multiplication and division of fractions much easier than the addition and subtraction. When multiplying fractions, and "cancelling" demonstrate that a number can't be used more than once but, continue to cancel as long as there is a numerator and denominator that can be cancelled. e.g. $\frac{3}{164} \times \frac{4}{15} \times \frac{204}{6} = \frac{1}{6}$

Remind students that it doesn't always have to be diagonally worked - you must always be cancelling a <u>numerator</u> with a <u>denominator</u>.

Unit 3: Suggested Instructional Strategies/General Suggestions for Using Manual

DECIMALS

If using this manual in a heavy manufacturing environment where employees spend a great deal of time measuring with a micrometer, keep in mind that "shop talk" often confuses the reading of a decimal. Very often, .0065 is referred to as 6.5 and not sixty five thousandths, because they work in thousandths and often verbally abbreviate.

When expressing decimals as common fractions, you can have your students check their answer. e.g. 0.065

Disregard all zeros between the decimal point and the first number to the right - place the number "65" in the numerator's place - count how many numbers there were including the zeros - in this case "3" - that's the number of zeros you write and then add a 1 - 3 zeros plus a 1 = $\frac{65}{1000}$. When rounding, instruct students to wait to the end to round.

1. **Division of Decimals**

Special emphasis should be placed on the understanding of division of a "tenth" of a number yielding a whole number e.g. $.8 \div .4 = 2$

Students will have a hard time understanding why you have a whole number for an answer. Explanations should be given that there are "2" four tenths in every group of eight tenths. Its how many four tenths there are.

Unit 4: Suggested Instructional Strategies/General Suggestions For Using Manual

PERCENTS, MEDIAN and RANGE

1. Expressing Percents as Decimals and Fractions

When instructing students to express percents as decimals and fractions, show them how to move th. decimal point in accordance to ÷ or x. Two places to the right (X), as opposed to two places to the left (÷).

% ---- Decimal Fraction
Move decimal point two places to the left (÷ 100)
Decimal ---- % Fraction
Move decimal point two places to the right (x 100)

2. Type of Percent Problems

Students will definitely have own way of calculating Type I percent problems (perfectly alright) - will start to see the benefit of "formula" when working Type II and Type III problems.

Formula should be emphasized:

$$\boxed{\frac{\text{part}}{\text{whole}} = \frac{\text{rate}}{100}}$$

Particular attention should be pay to the comprehension of the word problems in the Practical Application of Percents section. For example, students should understand that in problem #1 that Jim's gross pay of 30,100 was 10% less than last year's gross pay or 90% of the whole.

When discussing Average, Median and Range be sure to note, especially in workplaces where additional training such as SPC (Statistical Process Control) will follow, that average and "mean" are synonymous.

Unit 5: Suggested Instructional Strategies/General Suggestions for Using Manual

CONCEPT OF MEASURE AND SIGNED NUMBERS

1. Numbers

A great deal of time should be spent on the Metric Chart. Students are often confused with the relationship and correlation from one unit of measure to another. This is a good time to reiterate the movement of the decimal point. e.g. one centimeter is equal to .3937 inches, while one decimeter is equal to 3.937 inches (the decimeter is .1 larger).

2. Signed Numbers

Concept of Signed Numbers should be illustrated vertically (as in the reading of thermometers) and also horizontally (as in the number line). Have students keep "Rules for Signed Numbers" handy when doing operations. This concept can be difficult and reinforcement is possible in the Computer Literacy unit.

Unit 6: Suggested Instructional Strategies/General Suggestions For Using Manual

RATIO AND INTRODUCTION TO ALGEBRA

Students have an automatic aversion just to the word "algebra". (even the best of them) Starting out very slowly, and discussing "unknowns" works best. Refer at this time to the many places that we do use Algebraic formula in order to solve for the unknown. Just in Unit 4 we used a formula to solve for the part, the whole or the rate.

Emphasize that when solving simultaneous equations a balance must always be maintained.

Students are often confused with the unknown not always on either the right hand side or either the left hand side of the equation.

Unit 7: Suggested Instructional Strategies/General Suggestions For Using Manual

COMPUTER LITERACY USING ALGEBRA SOFTWARE

Survey your students in order to find out the level of computer literacy. Students will usually fall in one of the following categories:

1. No computer experience
2. Limited experience
3. Some training and experience
4. Computer at home

Prior to computer class, group students according to the survey above (mixing different levels of literacy as much as possible). When grouping students at the individual computers (preferably two at the computer) have the student with the least amount of experience turn the computer and the monitor on, etc. while giving initial literacy instructions.

After students begin working on the software, content will become more important to them, and their uneasiness with the machine itself will star to dissipate.

A P P E N D I X

Thoughts on Use of the Calculator

The calculator can be used:

- To encourage students to be inquisitive with mathematical concepts

- To act as a flexible answer key to verify the results of computations

- As a resourceful tool which promotes student independence in problem solving

- To solve problems that have been too time consuming or impractical to be done with paper and pencil

NUMERICAL PERIODS CHART

MILLIONS			THOUSANDS			ONES		
hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Table for Basic Multiplication Facts

FRACTION EQUIVALENT STRIPS

ONE WHOLE
$\frac{1}{2}$ | $\frac{1}{2}$
$\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$
$\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$
$\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$
$\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$
$\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$
$\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$
$\frac{1}{9}$ | $\frac{1}{9}$ | $\frac{1}{9}$ | $\frac{1}{9}$ | $\frac{1}{9}$ | $\frac{1}{9}$ | $\frac{1}{9}$ | $\frac{1}{9}$ | $\frac{1}{9}$
$\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$

UNIT 2 QUIZ NAME____________________

1. In the fraction $\frac{2}{5}$, the 2 is the numerator or denominator?

____

2. In the fraction $\frac{2}{5}$, the 5 is the numerator or denominator?

____

Convert each improper fraction to a mixed number:

3. $\frac{67}{5}$ = ____________

5. $\frac{49}{4}$ = ____________

4. $\frac{92}{12}$ = ____________

6. $\frac{13}{2}$ = ____________

Express each common fraction as an equivalent fraction:

7. $\frac{2}{16} = \frac{\quad}{64}$

8. $\frac{2}{15}$ = ________

Express each common fraction in lowest terms:

9. $\frac{6}{36}$ = ________ 10. $\frac{15}{25}$ = ________ 11. $\frac{4}{8}$

Express these mixed numbers as fractions:

12. $2\frac{3}{8}$ = ________

13. $5\frac{4}{6}$ = ________

14. $2\frac{1}{2}$
$3\frac{1}{8}$
+ $1\frac{5}{6}$
____

15. $3\frac{1}{7}$
$16\frac{1}{2}$
+ $5\frac{2}{3}$
____