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ABSTRACT

The first document in this set is a final report titled "Preparation for the Mathematics GED Test: A Computer Based Program," which describes a project to develop a General Educational Development (GED) mathematics preparation program for the adult learner at the 9-12 grade level. The other two documents are a teacher's guide and a student workbook, which together with a computer software package comprise the GED mathematics program for adult learners at the 9-12 grade level. The teacher's guide includes program purpose and instructions for the teacher, instructions for the student, answer key for the student workbook problems, and diagnostic examination with an answer key. The student workbook contains explanation and examples as well as exercises for each of the five units of the program. Each unit consists of one to five chapters. Units are: (1) number relationships (whole numbers, fractions, decimals and percent); (2) measurement (length and height measurement, weight measurement, liquid measurement, metric system); (3) data analysis; (4) algebra (arithmetic operations using integers, algebraic equations, inequalities, equations in two variables); and (5) geometry (geometric terms, geometric shapes, volume, coordinate geometry, word problems). (YLB)

GED Math-A Computer Assisted Mathematics Curriculum

CONTENTS:

1 Student Workbook
1 Teacher's Guide
~~1~~ Computer Software Disk(IIc, IIe and IIGS compatible)

Developed by:

ADULT EDUCATION SERVICES
Dr. Robert W. Zellers, Project Director
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Johnstown, PA 15905

These materials are a result of an adult education project which was supported in whole or part by the United States Department of Education. However, the opinions expressed herein do not necessarily reflect the position or policy of the United States Department of Education or the Pennsylvania Department of Education and no official endorsement should be inferred. The project products are a result of a Section 353 grant funded under the Adult Education Act, Amendments of 1988 (P. L. 100-297) administered through the Pennsylvania Department of Education, Bureau of Vocational and Adult Education, Division of Adult Basic and Literacy Education, Harrisburg, PA. 17126-0333.

FINAL REPORT

PREPARATION FOR THE MATHEMATICS GED TEST -A COMPUTER BASED PROGRAM-

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PREPARATION FOR THE MATHEMATICS GED TEST

-A COMPUTER BASED PROGRAM-

TABLE OF CONTENTS

I. ABSTRACT

PREPARATION FOR THE MATHEMATICS GED TEST -A COMPUTER BASED PROGRAM-

The project developed a GED mathematics preparation program for the adult learner which has the necessary software for computer based instruction. The program is of a contemporary and comprehensive nature which provides the adult learner with a positive attitude and solid understanding of mathematics. The target audience is adult learners at the 9 to 12 grade level. The product is a GED mathematics program which includes: a computer software package, a student's manual, and a teacher's guide.

II. INTRODUCTION

Let's be honest. Are we worried about the $2.4 trillion national debt? Does it bother us that our leaders want to spend another trillion on Star Wars. No, because we don't know what a trillion is. It's simply "a lot" to a generation of math students who can't tell a square root from a binomial. This situation leads to "muddled personal decisions" and "misinformed government policies." If you don't understand math concepts you can't react to news about megaton warheads or discussion about the probability of contracting a disease. Many people take a perverse pride in being deficient in math. In fact, unlike other failings which are hidden, mathematical illiteracy is often flaunted. According to Shirley Frye, president of the National Council of Teachers of Mathematics, we must do a better job of communicating the need to be mathematically literate. John Paulos states in his recent book, *Innumeracy: Mathematical Illiteracy and its Consequences,* that to prepare today's students for the future we must take advantage of the technology of today, specifically calculators and computers.

This issue pertains directly to adult education programs. January, 1988 marked the beginning of a year of transition from the use of the previous versions of the Tests of General Educational Development (GED) to the newly revised forms. A study of the performance of nearly 12,000 adult examinees who took the new GED Tests between January and June, 1988 has been conducted by the GED Testing Service. In the preliminary findings on two of the tests, Writing Skills and Mathematics, the performance of the adult examinees was significantly lower than the performance of high school seniors. On the Mathematics Test, seniors correctly answered 70% of the items, while adult examinees correctly answered 58% of these items. Knowing which areas of test performance are relatively low should help adult educators to determine what skills and content knowledge should be targeted for special instructional emphasis. In all five areas of the Mathematics Test, the adult examinees scored lower than the high school seniors. Specifically, the results were as follows: Measurement (seniors-72% adults-64%), Number Relationships (seniors-66% adults-48%), Data Analysis (seniors-76% adults 70%), Algebra (seniors-73% adults-60%), and Geometry (seniors-62% adults-44%). The source of these statistics was the General Educational Development Testing Service, a division of the American Council on Education.

Furthermore, fewer people have been taking and passing the new GED test as compared to the old one. There have been some changes made in the context, format, and cognitive levels of the GED Mathematics Test. To cite two specific examples, there is five percent more algebra content on the revised test and content that relates to computer awareness is in many of the tests.

The purpose of the project was to develop a computer based GED math preparation program for the adult learner which has the necessary software and instructional materials. It was the intention of this project to create a sound, practical, and sequential curriculum in mathematics which can be used with a computer. The proposer believes by integrating the computer with this type of a content that these materials will offer a high interest and motivational feature which may enhance the learning outcome. We believe that this approach will improve the previously mentioned GED Mathematics Test scores. It was the intention of the proposer to create specific problems and activities in the areas where the adult examinees are not doing well.

With the tremendous increase in the number of personal computers presently being utilized in all areas of education, it seems ineffective not to make use of the available technology. The proposer believes that the use of computers is beginning to take a firm foothold in the field of adult basic education in Pennsylvania. Many ABE instructors and administrators have their own computer or have access to the use of a computer. Furthermore, a recent Pennsylvania 310 project produced a publication entitled "Computers and Software: A Utilization Survey-1987", which has put forth some strong evidence that supports our belief. The investigator surveyed all of the ABE instruction centers in Pennsylvania with an eighty five percent response rate.The study stated in its final report the following relevant findings:

-80% of the instructors had a positive overall reaction after using the computers for a period of time.
-82% of the students had a positive over-all reaction toward computer aided instruction.
-89% of the instructors felt the overall educational value of using computers for instructional purposes was very effective.
-65% of the instructors used computers daily and 26% used them weekly.
-57% of the administrators used the computer for administrative applications.

Our line of reasoning is twofold. First, we know computers are fast and efficient learning tools and secondly, the adult education community in Pennsylvania has them available and wants to make use of them. Another added bonus to this learning program is the fact that the student will become familiar with the use of a computer and, by working through the materials, will develop a basic understanding of computer literacy. This is especially important since the new GED Mathematics Test has initiated computer awareness into the content of the test. Furthermore, this will equip the adult learner with valuable carrying over skills for the workplace which rapidly is becoming surrounded by computers and is expecting these kinds of skills from its employees.

Many students find math a frightening subject and many educators secretly share this view. Our message has to be that math is more than computation, more than memorizing rules. It's investigation, it's exploring, testing things out, finding short cuts, and posing problems as well as solving them. Most importantly, the students' interest level will be increased if they realize what they learn will really be of service to them. We sincerely believe that our materials will vastly improve both the "image" and "outcome" of math instruction. The curriculum covers the 9 to 12 grade levels.

We surveyed and investigated what is presently available in (1) the commercial markets, (2) the past 309/310 projects (both in Pennsylvania and nationally)and, (3) the resources of AdvancE and other clearinghouses, and we feel that our approach will be innovative and possibly a unique approach to the subject. This proposer developed a mathematics curriculum for the 9 to 12 grade level that includes:

- a computer software program
- a student's manual
- a teacher's guide

III. OBJECTIVES

The overall goal of the proposer was to create a computer based mathematics curriculum which would benefit GED students in the adult education programs throughout the state of Pennsylvania. The primary objectives of the project were as follows:

1. The project will create a computer based mathematics preparation program which utilize methods and materials directed toward the improvement of math scores on the GED Examination .

2. The project will create a computer based mathematics preparation program which utilize methods that are suitable for both large and small group instruction and one-to-one individualization and will lead to improvement in their GED Examination scores.

3. The project will create a computer based mathematics preparation program which will include the latest current level of computer technology.

4. The project will create a computer based mathematics preparation program which will be of a high interest level, comprehensive, and contemporary and will provide them with the knowledge to improve their GED Examination scores.

5. The project will create a GED mathematics preparation program which will provide the student with numerous survival skills in the area of computer literacy.

General Design

The project produced a GED mathematics preparation program for the 9 -12 grade level which is both practical and of high interest to the adult education student. The specific intention was to place the mathematics content matter into an instructional format which is sequential, motivational and possesses a workplace knowledge base.

Competency in the program will be determined by set percentage skill levels throughout the segments of the program. A comprehensive mastery exam is included in the final portion of the curriculum program. All of the exercises, activities and materials in the program are contemporarily based and constructed with this in mind so the student develops a personal interest in the material and this hopefully will breed an internal self motivation which is so essential in successful learning experiences.

We comprehensively addressed new concepts and content areas which are appropriate for the 9 - 12 grade level. We believe that this particular set of materials will provide relevant and needed content preparation for successful completion of the new GED examination. We have closely followed the development of the new instrument and addressed all of the necessary elements.

Location

The project was established at the offices of Adult Education Services and the University of Pittsburgh at Johnstown. Dr. Zellers, a faculty member at the institution, had the school's full support in this undertaking. The project staff also made use of the Greater Johnstown Vocational Technical School's adult education program both in terms of its personnel and some of its resource materials. The GJVTS is conveniently located within walking distance of the university campus. A long standing working relationship exists between the institutions as the result of cooperative efforts in past adult education projects. The project staff also had the comprehensive printing resources of UPJ at their ready disposal as well as the resources of the Regional Computer and Resource Center, which is located on the campus. The proposer also utilized the facilities of AVIS (Audio Visual Instructional Services) and the materials available in the Mathematics Learning Laboratory. The proposer had full access to the Education Division's Curriculum Materials Center which has a large collection of adult education related materials. The University of Pittsburgh at Johnstown has always been a strong supporter of adult education in the region and is well known for its leadership, affiliation and commitment to quality educational programs and projects.

Methods and Materials

One important aspect of teaching mathematics is that of generating student motivation. Present research suggests that computers can significantly increase motivation, and we did this through the judicial use of various innovative aspects of the computer. This methodology ensured that a workbook like program was not created, but rather a dynamic educational software program which will be educationally sound and will aid the students' learning in a manner which could not be achieved with a traditional textbook or workbook approach.

Our program requires no prior computer knowledge or skill. The user (student) selects concepts for study from a list of topics (menu) which is presented on the screen. All instructions for using the program are available in clear, concise statements at the beginning of the program, and are also available for review at critical locations throughout the lesson. Since the Apple II C, Apple II E, and Apple II GS are the most widely purchased and used computers in the classroom, our software is designed to be compatible with these machines. The "Computers and Software: A Utilization Survey-1987" study reported that 82% of the computers at ABE instructional sites in Pennsylvania were Apple II C's and E's. Transfer of this program to other applications also will be possible.

We elected to use the floppy disk technology rather than a tape or hard disk storage system. This decision was made because the floppy disk will operate more reliably and will allow us to use [illegible]m files, an important consideration for student record keeping. Furthermore, the floppy disk is much less expensive and more common in the classroom than is the hard disk. Since the the program will not be copyrighted, it is possible to transfer the program to a tape or hard disk. It will also be possible for individuals with basic programming skills to make unique modifications to the program to meet specific local needs.

We believe that our methods resulted in a effective learning package for adult education students wishing to improve their skills in mathematics so they may successfully pass the GED Mathematics Test.

V. PROJECT PERSONNEL

Adult Education Services is an organization made up of adult educators in west central Pennsylvania which provides program, curriculum development, evaluation, instruction and administrative services to adult education agencies at the local, state and federal level. The group consists of public school teachers, ABE instructors, curriculum specialists, administrators, and college professors, all of whom have have substantial training and experience in adult basic education. Their major purpose is to provide professional expertise to interested organizations by effectively combining their human resources and sharing their facilities. They have been able to successfully produce quality adult education materials and projects for the past decade.

This project was coordinated by Dr. Robert Zellers of the University of Pittsburgh at Johnstown who has been an Associate Professor in the school's Education Division for the past twenty years. Many of his previous 309/310 projects are used throughout the state in ABE programs and some of his materials have been circulated on a national basis. One of his recent projects has been chosen by the United States Office of Education to be distributed to all fifty states. He has directed numerous previous 309/310 projects and is a knowledgeable and competent adult educator. He supervised the project staff and was responsible for the quality of the end product. His direction of the project was done on a gratis basis. The project staffing consisted of the following positions:

<u>Computer Programmer</u>- This position involved designing the overall framework for the computer related instructional segment. The individual worked closely with the project staff members to develop a quality instructional format. This individual designed the operational software so that it is correlated and integrated with the mathematics preparation program. This individual placed the materials on the diskette and developed the technical aspects of the operation. This person was skilled in computer programming and mathematics. The individual devoted approximately 290 hours to the project.

<u>Curriculum Developer</u>- This position involved the creation and writing of the instructional program. This individual researched, designed and constructed the curriculum design for the program. This individual was responsible for creating the student activities and the teacher materials. This person was skilled in adult education, mathematics and curriculum design. This individual devoted approximately 260 hours to the project.

<u>Mathematics Content Specialist</u>- This individual developed the content for the textual materials and the computer screen displays. This individual was responsible for the review and editing of the project materials. This individual was trained and educated in the field of mathematics and has a strong involvement in adult basic education. This individual devoted approximately 260 hours to the project.

<u>Production/Materials Specialist</u>- This person handled various aspects of the creation of the project materials. This person also performed the field testing, evaluation, assembly, production

and served as editor of the project materials. This person developed the instructional format for the project's printed products. This individual was skilled in graphics, formatting, instructional design and materials production. This person also did some of the computer programmer's responsibilities. This individual devoted over 400 hours to the project.

The project positions were classified on a part-time basis in order to acquire the best available individuals who are currently working in the field as adult educators. This was done in order to ensure quality, to maintain cost effectiveness and to keep the salary costs at a reasonable level. All project staff members were selected on their commitment to project excellence and only individuals with adult education experience and the appropriate skills and competencies were chosen for the specific positions.

VI. EVALUATION

The evaluation segment of the project was done in various deliberate stages. As the materials were developed in draft form they were carefully scrutinized by the project director in conjunction with the curriculum developers. The materials also were shared with various adult education specialists and adult education teachers to solicit their comments and suggestions on the materials regarding applicability, relevance, interest, content level and method. Based on this critical evaluation, the materials were appropriately modified and development continued. At this point the materials were field tested with local adult education programs for trial use with students. Upon modification the materials was placed in final form.

An evaluation design also was implemented which measured assessment of individual staff and group staff accomplishments and provided for accurate and complete records of project activities. A specific and significant part of the evaluation phase was the use of an evaluation plan which provides an operational framework for measuring program objectives in correlation with the project activities. This model lists the project's goals and objectives on a monthly basis and then makes certain they are accomplished in a timely, proper and complete manner. It is a rigorous, continuous and comprehensive approach which permitted the project to maintain close control of the quality of the process and helped to assure completion of the intended outcomes. It was the primary objective of the proposer to create a 9 - 12 grade level Mathematics preparation program which will be readily implementable in the field so we made every effort, through a persistent and ongoing program evaluation, to ensure the quality of the program. Furthermore, the project cooperated with all state and federal evaluations.

<u>Time Schedule</u>

The project operated from July 1, 1989, to June 30, 1990. The development, evaluation, writing, field testing, and production of the materials took place during that time period. Listed below is the time and activity design for the project.

Phase I	July, August, September October, 1989	Research, design, and develop draft forms of the materials.
Phase II	November,December January, 1989, 1990	Further development of materials and more evaluation and field testing.
Phase III	February, 1990	Modification of materials and more evaluation and field testing.

Phase IV	March, April, May, 1990	Development and evaluation of computer software package. Evaluation and field testing.
Phase V	June, 1990	Assembly and production of materials.

VII. DISSEMINATION

The project actively sought the cooperation of all local agencies, organizations and institutions which could be of assistance. Linkage and coordination was initiated with numerous and varied regional entities. Examples of this are our discussions and meetings with the following: Ken Wallick, Adult Education Services network, local ABE teachers, local ABE program directors, and our discussions with personnel at AdvancE.

Significant and noteworthy documents and materials will be shared at the various state adult education meetings and the appropriate conferences. AdvancE will receive 50 copies of the entire set of materials. There is no " lock" on the disks. This means the local adult education programs will be able to borrow the disks from AdvancE and copy them. Therefore, for less than $2.00 (the cost of a blank disk) the local teacher or administrator will have a significant and major set of materials. We sincerely believe that the adult education professionals throughout the state will have full and open access to the materials. Furthermore, all appropriate materials, upon receiving consent of the division, will be forwarded to the National Diffusion Network, the Center for Educational Improvement, and the Education Research Information Center.

VIII. CONCLUSIONS AND RECOMMENDATION

The project proceeded throughout the year as planned. The development of the textual materials took longer than expected. The coverage of the mathematics material was extensive. As usual the act of programming some of the mathematical problems and configurations was difficult. However, we believe the end product turned out very well. The computer program and accompanying materials should prove quite helpful to students preparing for the GED examination.

We are pleased and proud of our final product. If anyone has comments or questions about the project you should contact the individual listed below:

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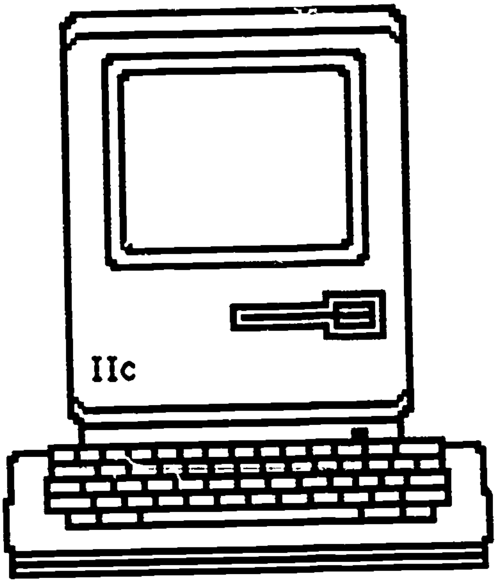
GED MATH
A COMPUTER ASSISTED MATHEMATICS CURRICULUM



TEACHER'S GUIDE

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Project Title: Preparation for the Mathematics GED Test-A Computer Based Program

Contract Number: 98-9041 (1989-90)

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GED MATH
A COMPUTER ASSISTED MATHEMATICS CURRICULUM

TEACHER'S GUIDE

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GED MATH
A COMPUTER ASSISTED
MATHEMATICS CURRICULUM

Table of Contents

I. TO THE TEACHER

Included in this Teacher's Guide is the following: Program purpose and instructions for the teacher, Instructions for the student, Answer key for the student workbook problems, diagnostic exam with an answer key and the program software disk.

The purpose of this project was to develop an GED mathematics curriculum for the adult basic education student (9-12 grade level) which would have the necessary software for computer assisted instruction. It was the intention of this project to create a sound, practical, and high interest curriculum in mathematics which could be used with a computer. The material is presented in such a way that the student will realize that what he/she learns will actually be beneficial in his/her daily living. It was mathematics instruction.

The project produced an mathematics program which is both practical and of high interest to the adult education student. The specific intention was to place the mathematics content matter into an instructional format which would be sequential and motivational. The approach was a functional one which had as its goal to produce a learner who has a solid understanding of the mathematics, enabling him/her to be an effective and contributing member of our society. All of the exercises, activities, materials, and computer software in the program are contemporarily based and constructed with the student in mind so that he/she will develop a personal interest in the material. This will encourage an internal self motivation which is so essential in successful learning experiences.

The program requires no prior computer knowledge or skill. The user (student) selects concepts for study from a list of topics (menu) which are presented on the screen. All instructions for using the program are available to the user in clear, concise statements at the beginning of the program, and they are available for review at critical places throughout the lessons.

Interactive text was used to maximize the student's involvement with the learning process. Student responses are required frequently throughout the program and, based upon the response, the program branches out to meet the individual needs of each student. In other words, not all students will receive identical tutorials. In this way, there will be a true aspect of individualization to this learning program. The student responses include both narrative type input as well as standard, test type items. By using a combination of different formats, the student can effectively be monitored as to his/her understanding of the various mathematical topics.

You are provided with various kinds of materials for use with this computer assisted mathematics program. The first item is a software program (diskette) which the student will be able to use by himself/herself with the computer. The diskette will run on an Apple IIc, Apple IIe, or Apple IIGS computer. It should be single or double drive and have a monitor or a television with an R.F. modulator.

The floppy disk technology is used rather than a tape or hard disk storage system. This decision was made because the floppy disk operates much faster and more reliably than a tape and allows random access files to be easily used - an important consideration for student record keeping. Further, the floppy disk is much less expensive and more common in the classroom than is the hard disk. Since the diskette is not copy protected, it is possible to transfer the program to a tape or to a hard disk. It is also possible for individuals with BASIC programming skills to make modifications to the program to meet specific local needs.

Please be advised that this program covers only the essentials of GED mathematics. Many choices had to be made and some items had to be omitted because of computer space limitations. However, we believe we have covered the most important aspects of the content area in a clear and concise fashion so that the adult basic education student can begin to feel more comfortable with mathematics without being overwhelmed or alienated.

In summary, this project has resulted in an effective learning package for the ABE student who wants to improve his/her skills in mathematics. This approach uses several standard features of a personal computer to increase student motivation, learning, and retention. The program maximizes student interaction and it requires a minimum amount of computer hardware (equipment). The program is menu driven and very user friendly so that no prior experience with a computer will be necessary. Through this program, learning mathematics can be fun!

II. TO THE STUDENT

The intent of this program is to improve your mathematics skills so that you are better equipped to function in the adult world. The skills that you are about to acquire will also help to increase your chances of getting the job that you desire. The program developers believe that with your interest in improving your mathematics skills and with your sustained effort, you can indeed make progress in the way you present yourself in the everyday world. The materials are designed to help you pass the GED Mathematics Exam

The program materials include a computer diskette and a workbook which covers various areas of GED mathematics. The mathematics is divided into the following areas:

I. Number Relationships
II. Measurement
III. Data Analysis
IV. Algebra
V. Geometry

The program is not meant to be a complete study of mathematics. Instead, it is one which stresses the essentials of mathematics for improvement of your basic skills. Each lesson will help you to understand mathematical concepts through explanations, examples, and exercises. The workbook, which accompanies the computer program, has exercises which will further your mastery and knowledge of mathematics.

As you begin each lesson on the computer, study the explanations and examples, then do the exercises. Do not hesitate to repeat the lesson several times. The extra practice will help you to master the material. When you are finished with each lesson, go to the same lesson in your workbook for more practice. Have your teacher or tutor correct the exercises in the workbook and help you to correct any errors that you may have made.

This program should help you to improve your mathematics skills in a short time. Good luck!

III. ANSWER KEY FOR WORKBOOK EXERCISES

Unit I

Chapter 1

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1. 90
2. 3
3. 50,000
4. 400
5. d

Section II - Page 4

1. 4,420
2. 66
3. 115
4. 191
5. 111

Section IIIa - Page 8

1. 124
2. 160
3. 4 R 2

Section IV - Page 12

1.	a.	4.	c	1.	538 acres	4.	56 feet
2.	e.	5.	d.	2.	278 miles	5.	3 pieces of candy
3.	b.			3.	760 students		

Chapter 2

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1. 15
2. $\frac{2}{5}$
3. $\frac{4}{13}$
4. $\frac{80}{100}$
5. yes, yes, no
6. yes
7. $\frac{2}{3}$ $\frac{8}{13}$ $\frac{1}{3}$ $\frac{1}{4}$

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1. No, Yes, Yes, Yes, No
2. $2\frac{1}{6}$ 2 $2\frac{2}{4}$
3. $\frac{7}{4}$ $\frac{37}{7}$ $\frac{15}{4}$ $\frac{83}{8}$

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2. 4 4. 9

Section V - Page 26

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2. $\frac{14}{3}$ or $4\frac{2}{3}$ 5. $\frac{1}{14}$

3. $\frac{19}{24}$ 6. $\frac{5}{12}$

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1. $7\frac{3}{4}$ 2. $8\frac{2}{3}$ 3. $8\frac{9}{10}$

Section V - Page 29

1. $1\frac{2}{7}$

2. $3\frac{1}{20}$

3. $6\frac{5}{6}$

Section VI - Page 30

1. $\frac{10}{40} = \frac{1}{4}$ 3. $\frac{3}{24} = \frac{1}{8}$

2. $\frac{15}{32}$ 4. $\frac{8}{21}$

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1. 10, 2, 1, $\frac{6}{1}$, 6

2. 11, 45, 1, 9, $\frac{99}{170}$

3. 35
4. $12\frac{1}{4}$

5. $15\frac{1}{2}$

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1. $\frac{55}{69}$

2. $1\frac{1}{5}$

3. $3\frac{1}{6}$

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1. $\frac{70}{1}$

2. $\frac{20}{100} = \frac{1}{5}$

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1. 24 games
2. 16 women
3. a.
4. b
5. b

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2. $\frac{6}{12} = \frac{1}{2}$

3. $\frac{12}{15} = \frac{4}{5}$

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3. c
4. a

1. a
2. c
3. d
4. b

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1. 14.6
2. 14.32
3. 195
4. 15

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1.	.07	6.	b
2.	.5	7.	c
3.	.005	8.	c
4.	.53	9.	c
5.	1.015	10.	b

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2.	4.1	5.	1,049.1
3.	14.13	6.	78.19

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1.	c	4.	d
2.	d	5.	b
3.	c		

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1.	c	5.	b
2.	b	6.	a
3.	c	7.	b
4.	a	8.	b

Section V - Pages 56, 57

1.	$\frac{1}{2}$	4.	$4\frac{1}{4}$	7.	.75	10.	$.66\frac{2}{3}$
2.	$1\frac{3}{10}$	5.	$\frac{1}{3}$	8.	.5	11.	.2
3.	$\frac{43}{100}$	6.	$\frac{1}{16}$	9.	$.27\frac{3}{11}$	12.	$83\frac{1}{3}$

Section VI - Page 58

1.	18%	6.	$\frac{10}{100} = \frac{1}{10}$
2.	.24	6.	3.5
3.	24%	7.	$2\frac{1}{3}\%$
4.	$\frac{13}{100}$		

Page 59

1.	.24	4.	1.21
2.	.0004	5.	3.56
3.	.091		

Page 60

1.	110%	4.	300%
2.	30%	5.	.4%
3.	$4\frac{1}{4}\%$	6.	$\frac{3}{8}\%$

Page 62

1.	$166\frac{2}{3}\%$	4.	300%
2.	$55\frac{5}{9}\%$	5.	50%
3.	240%		

Page 63

1.	31.80	5.	1.25
2.	1.2	6.	100
3.	30	7.	1.05
4.	110	8.	0

Page 66

1.	c	3.	b	5.	c
2.	a	4.	a	6.	d

Unit II

Chapter 1

Section Ia - Page 68

1.	12	3.	36
2.	3	4.	1,760

Section II - Page 69

1.	1,520	4.	4,000
2.	6,276	5.	.017
3.	$4\frac{1}{6}$	6.	15,600

Section III - Pages 70, 71

1.	485 in.	5.	.2 meters
2.	4 yd. 1 ft.	6.	12 ft. 11 in.
3.	1 yd. 2 ft. 10 in.	7.	6 meters of timber
4.	3 ft. or 36 in.		

Chapter 2

Section II - Page 73

1.	3	4.	63.2
2.	4,510	5.	5.1
3.	.019		

Section III - Pages 74, 75

1.	2 lbs. 6 oz.	5.	143 grams
2.	11 lbs. 1 oz.	6.	.006 and .00113 kilograms
3.	12 lbs. 13 oz.	7.	2,650 grams
4.	185 lbs.		

Chapter 3

Section II - Page 78

1.	12	5.	320 c
2.	40	6.	.5
3.	6 cups	7.	2.5
4.	5k		

Section III - Page 80

1.	64 ounces	5.	1400 bottles
2.	1.4 pints or $1\frac{2}{5}$ pints	6.	4 gall. 1 qt. 14 oz. of gas
3.	$1\frac{5}{8}$ cup	7.	104 ounces
4.	$\frac{1}{4}$ liter	8.	10.241

Chapter 4

Section III - Page 86

1.	1,000	4.	780
2.	4.25	5.	.017
3.	1,000 = 100,000		

Section IV - Page 88

1.	37,000	3.	1,000	5.	54.3
2.	.264	4.	10,000		

Unit 3

Chapter 1

Section Ia - Page 89, 90

1.	a. 1988		2.	a. Bill
	b. 2.54			b. Mary
	c. Tom			c. Bill
	d. Karen, 1984-1986			d. John & Bill
	e. Mary & Joe, 1988			e. $220

Section Ib - Pages 91, 92

1. a. Ohio d. 9%
   b. Iowa e. North Dakota & Virginia
   c. 10.5%

2. a. $28 c. $70
   b. Tom

Page 93

3. a. December 10°
   b. 72°

Page 94

4. a. $210 d. $128
   b. December
   c. $670

Page 95

5. a. 13.9% c. 32.3%
   b. 24.2% d. 15.9%

Section II - Pages 96, 97

1.	53.5	5.	395.2
2.	248	6.	52.2
3.	302	7.	$101.3
4.	41.06	8.	145

Section III - Page 99

1.	1:3; 1/3; 1 to 3	5.	5:3, 5/3; 5 to 3
2.	1:2; 1/2; 1 to 2	6.	3:5; 3/5; 3 to 5
3.	5:2; 5/2; 5 to 2	7.	8:9
4.	1:2; 1/2; 1 to 2	8.	7:6

Section III - Pages 100, 101

1.	15	4.	\$9.60
2.	4.8 eggs	5.	13.87 hrs.
3.	\$102.50		

Pages 101, 102

1.	$\frac{1}{1000}$	4.	$\frac{1}{4}$
2.	$\frac{1}{6}$	5.	$\frac{5}{16}$
3.	$\frac{3}{15}$		

Unit IV

Chapter I

Section II - Page 106

1.	11	5.	0
2.	-3	6.	$-\frac{1}{3}$
3.	-7	7.	$\frac{-5}{7}$
4.	-12		

Pages 106, 107

1.	-2	4.	-11
2.	4	5.	$\frac{5}{5}$ or 1
3.	2		

Section III - Page 108

1.	24	4.	-4
2.	-18	5.	-5
3.	20		

Section IV - Pages 110, 111

1.	5	4.	-2
2.	-154	5.	-14
3.	-20		

Chapter 2

Section I - Page 112

1.	$x + 15$	4.	$6x$
2.	$x + 6$	5.	$x - 3$
3.	$4x$	6.	$\frac{x}{7}$ or $x+ 7$

Section II - Pages 114, 115

1.	$x=7$	3.	$x=0$
2.	$x=3$		

Section III - Pages 116, 117

1.	15 and 16	4.	75, 290
2.	5	5.	$185
3.	12, 8	6.	$6,162.50, $3,837.50

Section IV - Pages 121, 122

1.	6"	4.	6"
2.	260 miles	5.	$t=3$
3.	12 miles		

Chapter 3

Section II - Page 125

1.	$x \leq 12$	4.	$x > 5$
2.	$x > \frac{3}{2}$	5.	$x < -1$
3.	A	6.	B

Chapter 4

Section I - Page 130

A = (2, 1)
B = (-1, -1)
C = (2, -2)
D = (-3, 3)

Section II - Page 133

1.	yes	4.	yes
2.	no	5.	no
3.	yes		

Section IV - Page 139

1.	(2, 0)	1.	(0, 4)
2.	(6, 0)	2.	(0, 2)

Section V - Page 141

1. $3x - 4 = y$ 3. $\frac{-3}{2}x + 1$
2. $y = -x + 5$

1. $m = 3$ y-intercept = (0,04) 3. $m = \frac{-3}{2}$ y-intercept = (0,1)
2. $m = -1$ y-intercept = (0, 5)

1. $m = \frac{2}{2}$ or 1 3. $m = 6$
2. $m = 2$ 4. $m = 0$

1. yes 3. no
2. yes

Pages 143, 144, 145

1.	right angle	5.	reflex angle	9.	reflex
2.	acute angle	6.	90°	10.	straight
3.	obtuse angle	7.	acute	11.	acute
4.	straight angle	8.	reflex	12.	obtuse
				13.	90°-180°, obtuse
				14.	acute; 1-90°
				15.	straight; 180°

Unit 5

Chapter 1

Section Ia

1.	e	2.	g
2.	b	6.	d
3.	f	7.	a
4.	c		

Section Ib - Page 146

1.	a	7.	h
2.	f	8.	i
3.	b	9.	j
4.	d	10.	k
5.	e	11.	c
6.	g		

Pages 146, 147, 148, 149

1.	22°, complementary	6.	98°; corresponding
2.	46°, supplementary	7.	103°, alternate exterior angles
3.	52°, vertical	8.	81°, alternate interior angles
4.	48° and 138°	9.	92° and 88°
5.	127°	10.	145°, 145°

Section IIa. - Pages 150, 151

1.	23 in. 90°	5.	441 sq. ft.
2.	parallelogram, 12 in.	6.	601 yds.
3.	148 in.	7.	60 sq. ft.
4.	168 yds.	8.	2 ft.

Section IIb - Page 152

1.	15.7	4.	11,304 sq. ft.
2.	27.13	5.	25.12 sq. in.
3.	3,768 ft.		

Section IIc - Page 153

1.	b	5.	f
2.	a	6.	d
3.	c	7.	g
4.	e		

Pages 153-157

1.	68°, isosceles	9.	8 in.
2.	60°, equilateral	10.	4,650 sq. yd.
3.	58°	11.	yes
4.	82°	12.	no
5.	13°	13.	20 in.
6.	21 in.	14.	3 ft.
7.	18 in.	15.	5.3
8.	28 sq. ft.		

Section III - Pages 158, 159

1.	90 cubic ft.	5.	2355 cu. ft.
2.	120 m^3	6.	3,925 m^3
3.	9 ft.	7.	11 in.
4.	27 cu. in.		

Section IV - Pages 161 - 166

2.	6 and 3	5.	8	8.	5	11.	9.9	14.	(2.5, 6)
3.	1	6.	2	9.	6.4	12.	11.4	15.	(3, -4.5)
4.	8	7.	4	10.	6.4	13.	(1,0)		

Section V - Pages 167 - 169

1. $176.80
2. 1140 sq. ft.
3. 17.8 ft.
4. 400 sq. ft.
5. 10.2 ft.

IV. DIAGNOSTIC EXAM

1. In the following numeral: 843 what does the 4 represent?

   a) 4 groups of 100
   b) 40
   c) 4 ones
   d) none of the above

2. Add 321 and 89.

   a) 111,010
   b) 1,100
   c) 410
   d) 41

3. Subtract 16 from 345.

   a) 329
   b) 321
   c) 331
   d) 229

4. 24 x 4 =

   a) 68
   b) 88
   c) 816
   d) 96

5. Which is the correct method to solve

$$\begin{array}{r} 741 \\ \underline{\times\ 76} \end{array}$$

a) $\begin{array}{r} 741 \\ \underline{\times\ 76} \\ 4246 \\ \underline{+4987} \end{array}$ b) $\begin{array}{r} 741 \\ \underline{\times\ 76} \\ 4446 \\ \underline{+\ 5187\ } \end{array}$ c) $\begin{array}{r} 741 \\ \underline{\times\ 76} \\ 42246 \\ \underline{+\ 49287\ } \end{array}$ d) $\begin{array}{r} 741 \\ \underline{\times\ 76} \\ 4446 \\ \underline{+\ 5187} \end{array}$

6. Solve 98 + 14 =

   a) 6
   b) 7
   c) 7 R 1
   d) 92

7. The local library has 4321 fiction books and 431 non-fiction. How many books does the library have in all?

a) 3880
b) 4752
c) 4110
d) none of the above

8. Angela took 5 tests. Her scores were 75, 92, 88, 86, and 94. What was her average score?

a) 87
b) 78
c) 88
d) 94

9. The Smiths wanted to erect a fence around their new swimming pool. They will need 26 feet on all 4 sides. Which operation will you use to solve the problem?

a) 26 + 4
b) 26 + 4
c) 26 x 4
d) 26 + 26 + 4

10. Melissa wanted to make enough money to buy a new sewing machine. If the sewing machine costs $324.00, how many hours must Melissa work to earn enough money? What , if any, information is needed to solve this problem?

a) The amount of money Melissa earns per hour
b) Where Melissa works
c) The size of the sewing machine
d) No other information is needed.

11. Which set of fractions are equivalent?

a) 3/4 and 3/8
b) 2/4 and 1/2
c) 1/8 and 8/1
d) 3/3 and 1/9

12. Which fraction is in lowest terms?

a) 9/12
b) 8/16
c) 3/4
d) 3/2

13. Change 14/3 to a mixed fraction.

a) 14/3
b) 14 1/3
c) 4 2/3
d) 3/2

14. Which statement is true?

a) In general, the lower the numerator, the smaller the fraction.
b) If the numerators are the same, the fraction with a larger denominator is larger.
c) If the numerators are smaller.
d) None of the above are true.

15. Which of the following comparisons are true?

a) 4/3 > 3/2
b) 1/8 > 1/4
c) 1/4 < 1/8
d) 1/8 > 1/16

16. Subtract 3/4 from 1.

a) 1 3/4
b) 1/4
c) 3/4
d) 1

17. Add 4 3/5 and 12 3/5.

a) 17
b) 17 1/5
c) 16 3/5
d) 8

18. Find the product of 3/5 and 1/2.

a) 3/10
b) 4/7
c) 15/5
d) 6/5

19. Divide 3/4 by 1/2.

a) 3/2
b) 3/8
c) 6/4
d) 8/3

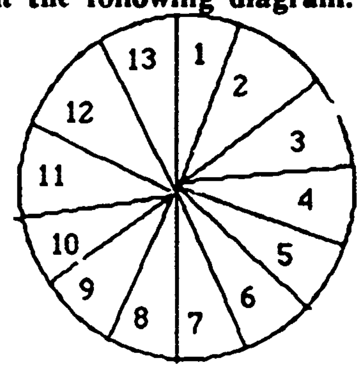
20. Compare the ratio. 16/24 = ?/3

a) 8
b) 3
c) 16
d) 2

21. While traveling across the state, Joe drove 8 hours each day. How many total hours did he drive in 4 days. Which of the following will solve this problem using a ratio

a) 8/4 = ?/4
b) 8/1 = 4/?
c) 8/1 = ?/4
d) 4/8 = ?/4

22. Look at the following diagram.



What is your probability of spinning a number greater than 2?

a) 1/13
b) 11/13
c) 2/11
d) 2/13

23. Choose the correct value for the underlined digit. 0.<u>4</u>3

a) 4
b) 4 tenths
c) 40
d) 4 hundredths

24. Round 21.73 to the nearest tenth.

a) 22
b) 21
c) 21.7
d) 21.8

25. Subtract

$$\begin{array}{r} 91.4 \\ -\ \ 4.9 \\ \hline \end{array}$$

a) 86.3
b) 98.3
c) 8.65
d) 21.8

26. Find the product of 4.623 and .01.

a) .04623
b) 4.623
c) 462.3
d) .4623

27. Choose the correct answer for 9.3 ÷ 3.1 .

a) 3
b) 0.3
c) 0.31
d) 0.03

28. Write 3/4 as a decimal.

a) .30
b) .34
c) .75
d) 7.5

29. Change 3/4 to a percent.

a) .75%
b) 75%
c) 750%
d) 7.5%

30. Express 431% as a mixed number.

a) 43 1/100
b) 431
c) 4 31/100
d) 431/100

31. Jonathan sold 36 raffle tickets for his club's fund-raiser. Mary sold 85. A total of 720 tickets were sold. What percent of the raffle tickets did Jonathan sell? How would you solve this?

a) (36 ÷ 720) x 100
b) (720 ÷ 36) x 100
c) 36 ÷ 720
d) none of the above

32. How would you find the number of inches in 36 feet?

a) 4 x 12
b) 36 x 3
c) 36 x 12
d) none of these

33. Sally had 4 yards 3 inches of material to make a tablecloth. She used only 3 yd. 1 ft.. How much material is left?

a) 2 ft. 3 in.
b) 1 yd.
c) 1 ft.
d) 36 in.

34. Which has more, a bottle with 33 ounces or one with 2 pounds 6 ounces?

a) Neither, they are equal.
b) 33 ounces is more than 2 pounds 6 oz.
c) 33 ounces is less than 2 pounds 6 oz.
d) Can't be compared

35. How many ounces of water must be added to a 16 oz. can of frozen apple juice concentrate to make 1/2 gallon of juice?

a) 64 oz.
b) 34 oz.
c) 48 oz.
d) 32 oz.

36. What is the absolute value of -24?

a) -24
b) 2.4
c) -2.4
d) 24

37. $-7 + 3 =$

a) 10
b) -10
c) -4
d) 4

38. $-4 - 11 =$

a) -15
b) 15
c) -7
d) 7

39. $(-4)(5) =$

a) 20
b) -20
c) 9
d) -9

40. $-21/-3 =$

a) 7
b) -7
c) 21/-3
d) none of these answers

41. $8 \div \frac{-4}{5} =$

a) $8 \times \frac{-5}{4}$

b) $8 \times \frac{5}{4}$

c) $8 \div \frac{5}{4}$

d) $8 \div \frac{-5}{4}$

42. Solve $8 + 3 \times 2 - 1 =$

a) 21
b) 13
c) 12
d) 23

43. Solve $10x - 3 = 4x + 1$ $x = ?$

a) $\frac{2}{3}$
b) 4
c) 2
d) $\frac{3}{2}$

44. The sum of 3 numbers is 47. The second number is twice the first and the third is twice the sum of the first two plus 2. Find the 3 numbers.

a) 4, 8, 26
b) 3, 6, 20
c) 5, 10, 32
d) 4, 9, 32

45. A train traveled 315 miles in 9 hours. How fast (what rate) was the train traveling?

a) 40 mph
b) 30 mph
c) 20 mph
d) 35 mph

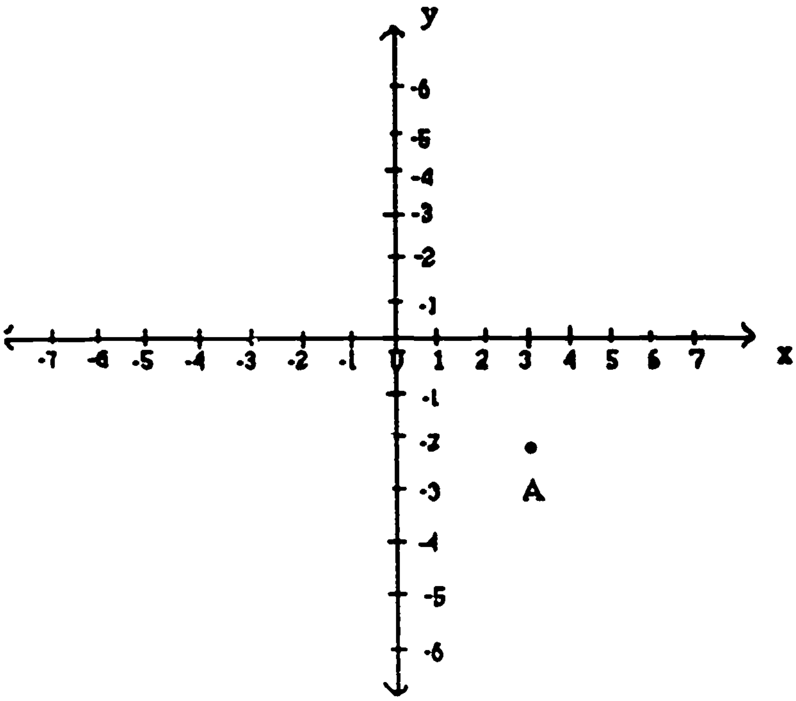
46. A car travels at 55 mph. 3 hours later a helicopter leaves traveling at 175 mph. Assuming that both can travel in a straight line, how soon will the helicopter catch up to the car?

a) $4\frac{1}{3}$ hrs.
b) 3 hrs.
c) 4 hrs.
d) 9 hrs.

47. Solve $2x + 4 < x + 3$ for x

a) $x > 1$
b) $x < -1$
c) $x = 1$
d) $x > -1$

48.



A can be represented by the ordered pair _____.

a) (3, 2)
b) (-2, 3)
c) (3, -2)
d) (-3, -2)

49. Find the slope of a line containing the ordered pairs (1,3) and (2,2).

a) 1
b) 3
c) -1
d) -3

50. Change the following equation to slope-intercept form. $3x + y = 4$

a) $y = 3x + 4$
b) $y = -3x + 4$
c) $3x = y + 4$
d) $3x = -y + 4$

Section II
Data Analysis

51.

average weekly income			
name	1984	1986	1988
Joe	$220	$226	$228
Kim	$230	$230	$236
Mary	$450	$458	$460
Sam	$356	$362	$370

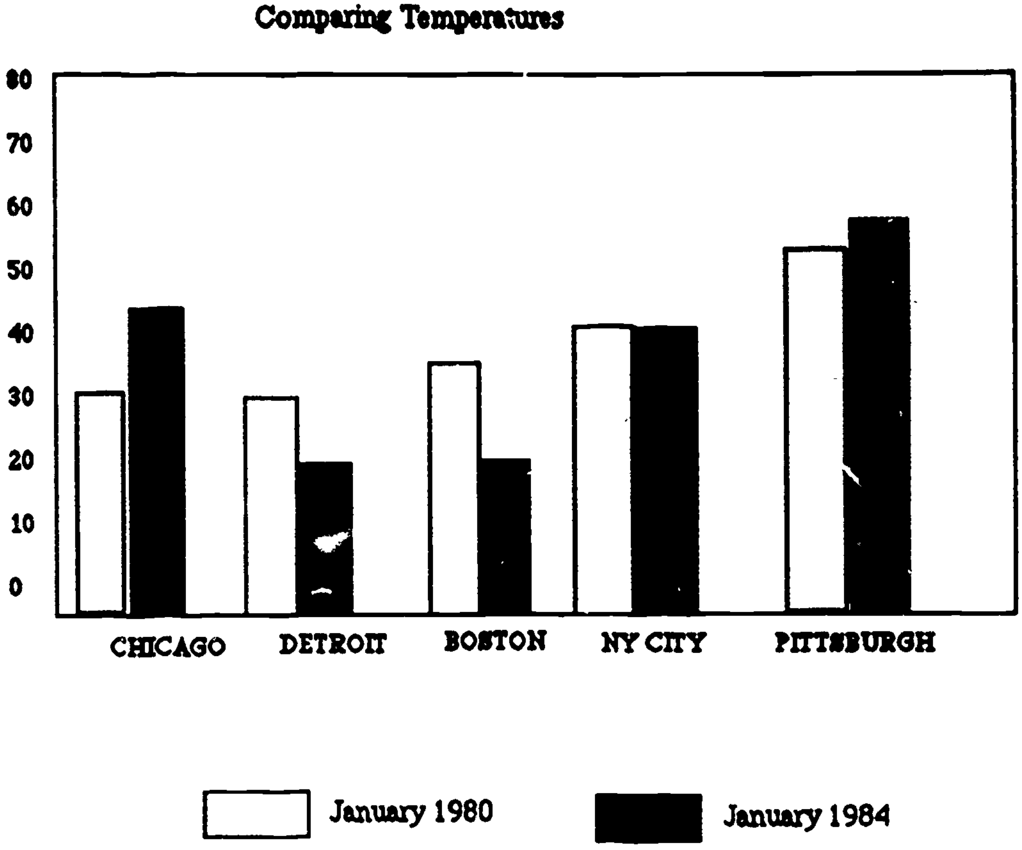
a. Who had the highest weekly earning in 1988? ______
-refer to unit 3: section I

b. Who had the lowest weekly earning in 1986? ______
-refer to unit 3

c. Who had the greatest wage increase and the lowest from 1984 to 1988 and what is the amount? ______
-refer to unit 3

d. What is Sam's weekly income in 1986? ______
-refer to unit 3

52.



a. What was the temperature in Chicago and New York City in 1980?_________

-refer to unit 3, section 1

b. What city had the highest temperature in 1984 and what was it?

____

-refer to unit 3

c. What cities had the lowest and highest temperature in 1984?

____

-refer to unit 3

53. What is the mean high so far in July if the first 6 days were: 80°, 83°, 84°, 80°, 87°, 88°? ______
-refer to unit 3, section II

54. Jill's test scores are 25, 36, 30, 33, 38. What is the average score?

________ -refer to unit 3, section II

55. Mary's test scores are also 25, 36, 30, 33, 38. What is the median?

________ -refer to unit 3, section II

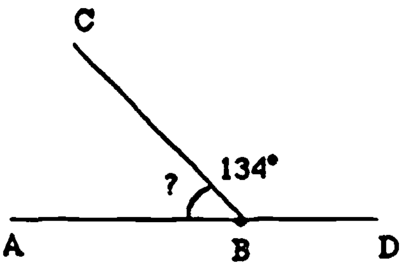
56. A recipe ~~calls for 2 cups~~ of sugar and 4 eggs. What is the ration of eggs to sugar in lowest terms? ______
-refer to unit 3, section III

57. Mrs. Millin's recipe calls for 3 eggs to make 24 biscuits. How many eggs would be need to make 40 biscuits? ______
-refer to unit 3, section III

58. There are 5 cherry, 2 grape, and 3 strawberry flavored candies left. What is the probability of Suzy selecting a grape candy? ______
-refer to unit 3, section III

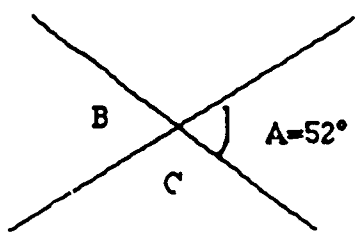
Section III
Geometry

59. What is the measure of ∠ABC? _____



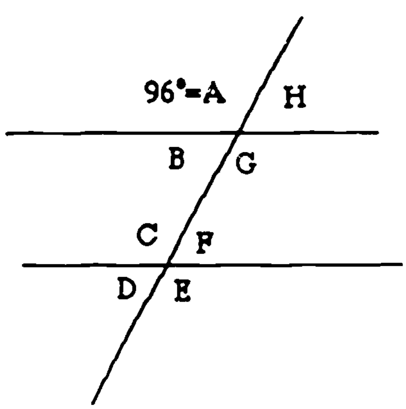
-refer to unit 5, section IB

60. What is the r.easure of ∠B and ∠C? ____ ______



-refer to unit 5, section IB

61. What is the measure of ∠E and ∠D? _____; ________



-refer to unit 5, section IB

62. The perimeter of a square building is 3,000 meters. How many meter long is each side of the building?______
-refer to unit 5, section IIA

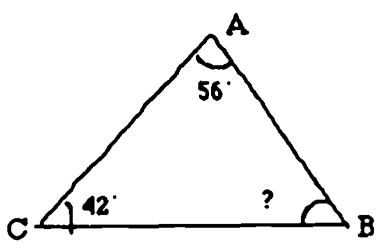
63. What is the area of a rectangular box with the length 4 ft. and width 6 ft.?______

-refer to unit 5, section IIA

64. Tom and Bill split a large pizza with a radius of 9 inches. How many square inches did they each get? ________
-refer to unit 5, section IIB

65. What is the diameter of a circle with an area of 69 sq.in.? ______
-refer to unit 5, section IIB

66. Find $\angle B$. ______



-refer to unit 5, section IIC

67. What is the perimeter of a triangle with the sides of 6 in., 7 in., and 10 in.? ________

-refer to unit 5, section IIC

68. What is the area of triangle with a height of 7 in. and base of 5 in.?

____

-refer to unit 5, section IIC

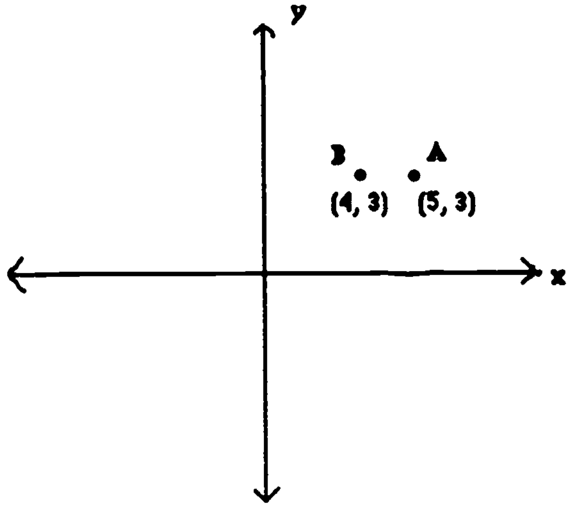
69. Mr. Jones went for a walk and when he left his house he went 5 miles north and 8 miles west. How far was he from his house? ________
-refer to unit 5, section IIC

70. What is the volume of a rectangular container 6 in. by 7 in. by 7.5 in.?

____

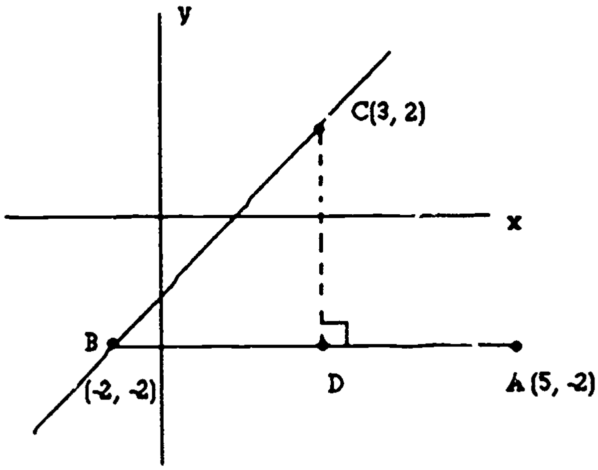
-refer to unit 5, section III

71. Plot these two points and tell the distance between the two. A(5,3) and B(4,3) _____



-refer to unit 5, section IV

72. What is the perpendicular distance from C to line AB?



-refer to Unit 5, section IV.

73. **What is the distance between points (5, 3) and (-2, -4)?**

-refer to Unit 5, section IV.

74. **What is the midpoint of a line joining points (3, 5) and (1, 8)?**
-refer to Unit 5, section IV.

75. **What is the midpoint of a line joining points (-2, 6) and (4, -3)?**
-refer to Unit 5, section IV

76. **What is the length of a house with the area of 400 sq. ft. and the height 8 ft.?**

______

- refer to chapter 5

77. **Jim wants to carpet a triangular room 5 yds. by 8 yds. What is the area?______**

-refer to chapter 5

78. **John is an architect and on his blueprint is a triangular figure that forms a right angle. The two sides are 2 in. and 3 in. What is the hypotenuse? ______**

-refer to chapter 5

V. ANSWER KEY FOR DIAGNOSTIC EXAM

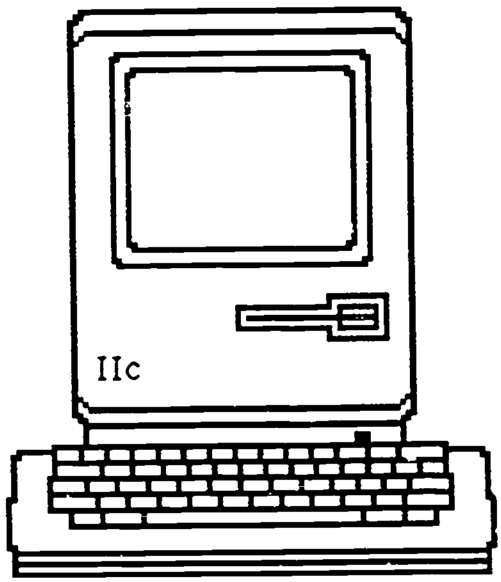
1.	b	16.	b	31.	a	46.	a
2.	c	17.	b	32.	c	47.	b
3.	a	18.	a	33.	a	48.	c
4.	d	19.	a	34.	c	49.	c
5.	b	20.	d	35.	c	50.	b
6.	b	21.	c	36.	d	51.	a. Mary b. Joe
7.	b	22.	b	37.	c		c. greatest-Sam $14
8.	a	23.	b	38.	a		lowest-Kim $6
9.	c	24.	c	39.	b		d. $362
10.	a	25.	d	40.	a	52.	a. 32° and 40°
11.	b	26.	a	41.	a		b. Pittsburgh 60°
12.	c	27.	a	42.	b		c. Detroit & Pittsburgh
13.	c	28.	c	43.	a	53.	83.6°
14.	c	29.	b	44.	c	54.	32.4
15.	d	30.	c	45.	d	55.	33

56.	2:1	71.	1
57.	5 eggs	72.	4
58.	$\frac{1}{5}$	73.	9.9
59.	46°	74.	(2; 6.5)
60.	52°, 128°	75.	(1, 1.5)
61.	96°, 84°	76.	50 ft.
62.	24 sq. ft.	77.	20 sq. yds.
63.	24 sq. ft.	78.	3.60
64.	127.17 sq. in.		
65.	9.4 in.		
66.	82°		
67.	23 in. or 1 ft. 11 in.		
68.	17.5 sq. in.		
69.	9.4 miles		
70.	315 cu. in.		

Question Reference Chart

QUESTIONS	REFER TO:
1-10	Unit I. Chapter 1
11-22	Unit I, Chapter 2
23-31	Unit I, Chapter 3
32 & 33	Unit II, Chapter 1
34	Unit II, Chapter 2
35	Unit II, Chapter 3
36-42	Unit IV, Chapter 1
43-46	Unit IV, Chapter 2
47	Unit IV, Chapter 3
48-50	Unit IV, Chapter 4
51 & 52	Unit III, Section I
53-55	Unit III, Section II
56-58	Unit III, Section III
59-61	Unit V, Section I
62-63	Unit V, Section II
64-65	Unit V, Section II-B
66-69	Unit V, Section II-C
70	Unit V, Section III
71-77	Unit V, Section IV
78	Unit V, Chapter 5

GED MATH
A COMPUTER ASSISTED MATHEMATICS CURRICULUM



STUDENT WORKBOOK

These materials are a result of an adult education project which was supported in whole or part by the United States Department of Education. However, the opinions expressed herein do not necessarily reflect the position or policy of the United States Department of Education or the Pennsylvania Department of Education and no official endorsement should be inferred. The project products are a result of a Section 353 grant funded under the Adult Education Act, Amendments of 1988 (P. L. 100-297) administered through the Pennsylvania Department of Education, Bureau of Vocational and Adult Education, Division of Adult Basic and Literacy Education, Harrisburg, PA. 17126-0333.

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GED MATH
A COMPUTER ASSISTED MATHEMATICS CURRICULUM

STUDENT WORKBOOK

Developed by:

Dr. Robert W. Zellers
Donna M. Zellers
Rob A. Eckenrod

TO THE STUDENT

The intent of this program is to improve your mathematics skills so that you are better equipped to function in the adult world. The skills that you are about to acquire will also help to increase your chances of getting the job that you desire. The program developers believe that with your interest in improving your mathematics skills and with your sustained effort, you can indeed make progress in the way you present yourself in the everyday world. The materials are designed to help you pass the GED Mathematics Exam

The program materials include a computer diskette and a workbook which covers various areas of GED mathematics. The mathematics is divided into the following areas:

I. Number Relationships
II. Measurement
III. Data Analysis
IV. Algebra
V. Geometry

The program is not meant to be a complete study of mathematics. Instead, it is one which stresses the essentials of mathematics for improvement of your basic skills. Each lesson will help you to understand mathematical concepts through explanations, examples, and exercises. The workbook, which accompanies the computer program, has exercises which will further your mastery and knowledge of mathematics.

As you begin each lesson on the computer, study the explanations and examples, then do the exercises. Do not hesitate to repeat the lesson several times. The extra practice will help you to master the material. When you are finished with each lesson, go to the same lesson in your workbook for more practice. Have your teacher or tutor correct the exercises in the workbook and help you to correct any errors that you may have made.

This program should help you to improve your mathematics skills in a short time. Good luck!

GED MATH
A COMPUTER ASSISTED
MATHEMATICS CURRICULUM

Student Workbook

TOPIC **PAGE**

Unit I - Chapter 1 •Whole Number Operations•

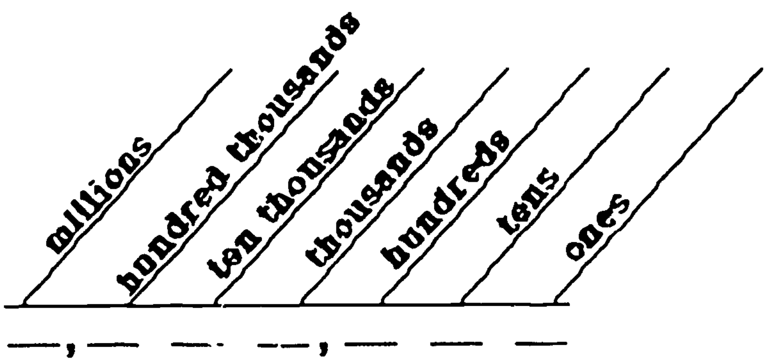
Section I Introduction to Whole Numbers

In order to work with whole numbers, you must first understand our numeration system.

Our system is called a "base 10" system because any number of objects can be represented by using only 10 digits, 0 through 9. This is possible because of "place value."

Each digit represents a face value as well as a place value. The digit's position in the numeral determines its place value. For example, the 5 in the numeral 52 is in the "tens" place, therefore it represents 5 groups of 10 or 5 x 10.

Place Value Chart



Using the above chart, what place value is represented by the 7 in 7,430? (7). Right, 7 is in the thousands place.
This represents 7 groups of 1,000 or 7,000.

Notice the "zero" in the ones column. Even though there are no ones, the zero must be placed in this column as a place holder. Without the zero, the number would be 743.

Is 743 the same as 7430? Yes or No

"Zero" is very important in our numeration system as a place holder.

Let's evaluate the following number: 67,032

There are 2 ones
3 tens
0 hundreds
7 thousands
6 ten thousands

the 6 represents 6 x 10,000 or 60,000
the 7 represents 7 x 1,000 or 7,000
the 0 represents 0 x 100 or 0
the 3 represents 3 x 10 or 30
the 2 represents 2 x 1 or 2

In the following numeral, <u>8321</u>, what does the 3 represent?

a. 3 groups of 10
b. 3 groups of 100
c. 3 groups of 1,000
d. 30

Right, the correct answer is <u>b</u>. The 3 represents 3 groups of 100.

Now, what is the <u>value</u> of 3 in 8321?

a. 3
b. 300
c. 30
d. 3000

Right, the value of 3 in 8321 is 300!

In the following set of practice problems, you will be asked to determine the value of the underlined digit.

<u>Practice Problems</u>

1. 46,0<u>9</u>1 (90)
2. 62<u>3</u> (3)
3. <u>5</u>0,433 (50,000)
4. <u>4</u>01 (400)
5. How is the 0 used in the numeral 602?

a. 0 groups of ten
b. 0 groups of hundreds
c. a place holder
d. a & c

Section II Addition and Subtraction With Whole Numbers

When you combine two or more numbers, the answer is called the <u>sum</u>.
To add 732 and 4,091:

1. First, line up the ones under ones, tens under tens, hundreds under hundreds, etc.

```
   732
+4 091
```

2. Add each column separately

```
   1
   732
+4 091
 4,823
```

Notice that in the "tens" column, 3 + 9 = 12. When the answer in any column is a 2 digit number, put the digit on the right (in this case a 2) under the column and <u>carry</u> the other number (in this case a 1). Notice that the 1 was added to the 7 in the hundreds column.

Sometimes you are asked to find the <u>difference</u> between two numbers. This means you must subtract one number from another.
For example, find the difference between 391 and 48.

1. Subtract the smaller number from the larger number, lining up the numbers according to place value.

```
 391
- 48
```

2. Subtract each column beginning with the <u>ones</u> column.

3. Sometimes, you must <u>borrow</u> in order to subtract.

```
  3 9 1
-   4 8
```

•you can't subtract 8 from 1
•therefore you need more <u>ones</u> in the top number
•to get more "ones" you may borrow one group of 10 from the tens column

8 11

$3\cancel{9}\cancel{1}$
$-\ 48$

•now you have 11 in the ones column and 8 in the tens column
•continue to subtract from right to left

```
 8 11
 3 9 1
-  4 8
 343
```

Practice Problems
Addition and Subtraction

Read each problem carefully to determine which operation you are going to perform. Remember, finding the sum, or combining numbers means addition; and finding the difference indicates subtraction.

1. Find the sum of 4399 and 21 (4420)

Hint: you are adding the 2 numbers

Hint: line up columns according to place value

```
  4399
+   21
```

2. Combine 62 and 4 (66)

3. Add 99 and 16 (115)

Hint: Remember, when you have a 2 digit answer in one column, carry the digit on the left into the next column.

4. What is the difference between 231 and 40? (191)

Hint: Difference means subtract the smaller number from the larger number.

Hint: Remember to line up columns according to place value.

```
  231
-  40
```

You can't subtract 4 from 3. You must borrow from the hundreds column

```
  1 13
  2 3 1
-   4 0
```

5. Subtract 241 from 352 (111)

Section III Multiplication and Division with Whole Numbers

The answer to a multiplication problem is called the product. The numbers you are multiplying are the multiplicand and the multiplier.

For example: 832 x 35

1. To multiply two numbers, place the large number on top and smaller number underneath, carefully lining up according to place value.

   8 3 2 -- multiplicand
   x 3 5 -- multiplier

2. To solve, you must first find partial products by multiplying each digit in the multiplicand by each digit of the multiplier.

   First, multiply 8 3 2 by 5

1 1 8 3 2 x 5 4 1 5 0	As with addition, when the answer in one column is 2 digits, carry the digit on the left.

1 1 8 3 2 x 5 4 1 5 0	5 x 2 = 0 place 0 in ones, carry 1 to tens column 5 x 3 = 15 + 1 (add 1 carried from ones) product of 5 x 3 Place 6 in tens column carry 1 to hundreds Multiply 5 x 8 = 40 add 1 that was carried 41

4. 4160 is a partial product
   Now you must complete the problem by multiplying 30 x 832.

   8 3 2
   x 3 0
   2 4, 9 6 0 -- partial product

5. Add the partial products

   8 3 2
   x 3 5
   4 1 5 0 -- partial product
   + 2 4, 9 6 0 -- partial product
   2 9 1 2 0

Division is indicated by the symbols ÷ or $\overline{)\quad}$

The answer to a division problem is called the <u>quotient</u>.

<u>Example</u>

divisor

$6\overline{)204}$

dividend

OR

dividend

$204 \div 6$

divisor

The number being divided is the <u>dividend</u>.
The number dividing is the <u>divisor</u>.

1. First, ask yourself how many times does 6 divide into 2. none. $\begin{array}{r} 0 \\ 6\overline{)204} \end{array}$

2. Does 6 divide into 20? yes.

$$\begin{array}{rr} & 34 \\ & 6\overline{)204} \\ 3 \times 6 = & \underline{-18} \\ & 24 \\ 4 \times 6 = & \underline{24} \\ & 0 \end{array}$$

There are 3 groups of 6 in 20 with 2 remaining.
Subtract 18 from 20
bring down the 4

does 6 divide into 24? Yes
4 times, place 4

3. <u>To check</u> multiply answer by divisor

34	quotient
x 6	divisor
204	dividend

If the division x quotient = dividend, the answer is correct.

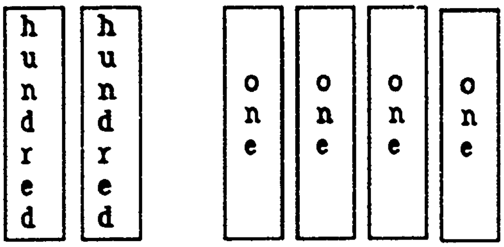
More detailed explanation, press y for yes. (Section IIa)
Not needed, press Return.

Section IIIa

Divide 6 into 204

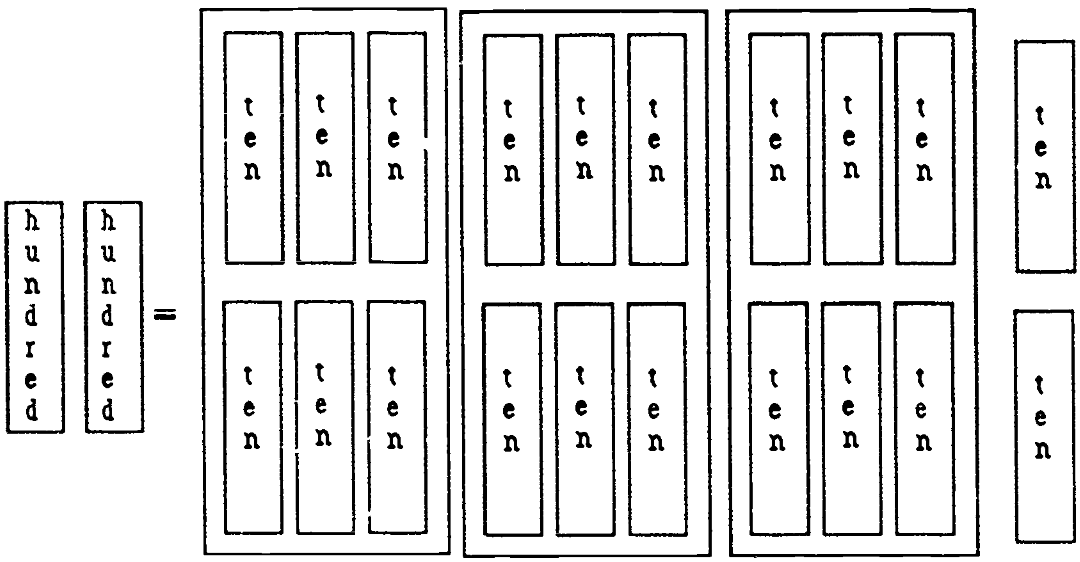
Think of 204 as money

2 one hundred dollar bills
0 ten dollars
4 one dollar bills



1. Can you divide the 2 hundred dollar bills into groups of 6 without changing them to ten dollar bills? No.

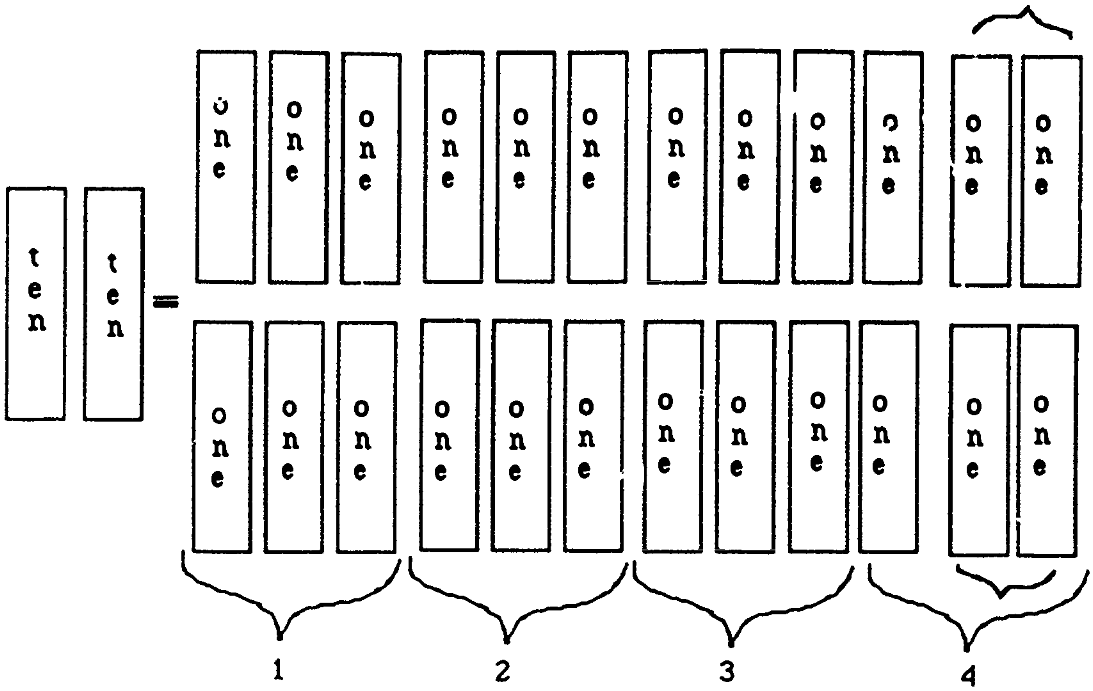
$$\begin{array}{r} 0 \\ 6\overline{)204} \end{array}$$



2. Now can you place the ten dollar bills into groups of 6? Yes. How many? 3
   How many are left? 2 tens.

$$\begin{array}{rr} & 03 \\ & 6\overline{)204} \\ 3 \times 6 = & \underline{-\ 18} \\ & 2 \text{ remaining} \end{array}$$

3. Now, can you divide the 2 ten dollar bills into groups of 6 without changing them into ones? No.



	34	Add the 4 ones you already have to the twenty ones.
	6) 204	You now have 24 one dollar bills.
	- 18	Can you divide these 24 into piles of 6? Yes.
	24	
group of 6	- 24	(Then show them being circled into 4 piles.)
	none left	

Practice Problems in Multiplication & Division

Read each problem carefully to determine which operation to use. Remember, the product is the answer to a multiplication problem and the quotient is the answer to a division problem.

1. Find the product of 31 and 4.
2. Multiply 16 and 10.
3. Divide 14 by 3

Prompt 1: Set up properly. $3\overline{)14}$

Prompt 2 : Does 3 divide into 1? No. $\begin{array}{r} 0 \\ 3\overline{)14} \end{array}$

into 14? Yes. $\begin{array}{r} 4 \\ 3\overline{)14} \end{array}$

$4 \times 3 =$ $\underline{-12}$

How many remaining? 2

Hint 3: The answer is given as 4 R 2

R stands for remainder

(In a later lesson, you will learn to express the remainder differently).

4\. 321 ÷ 4 = (80 R 1)

Prompt 1: Set up properly. $4\overline{)321}$

Prompt 2 : $\begin{array}{r} 80 \\ 4\overline{)321} \\ \underline{32} \\ 1 \end{array}$ - can 4 be divided into 1?

5\. 631
x 14 (9834)

Hint 1: find partial products $\begin{array}{r} 631 \\ \underline{4} \end{array}$ and $\begin{array}{r} 631 \\ \underline{10} \end{array}$

Hint 2: Add partial products

6\. $31\overline{)6634}$ (214)

Hint 1: Does 31 divide into 6? (No)

Hint 2: into 66? (Yes)

How many times? (2)

$\begin{array}{r} 02 \\ 31\overline{)6634} \\ 2 \times 31 \ \underline{-62} \end{array}$

Section IV Word Problems With Whole Numbers

There are many practical applications for whole number operations. The following word problems are examples. You will be using addition, subtraction, multiplication and division to solve these problems. Remember to look for "key" phrases or words to help you determine which operation to use.

total
in all
sum
combine

> usually indicate addition

difference
how many more
how much less
how much larger than

> usually indicate subtraction

product multiplication

distributed evenly
evenly divided
quotient

> division

averages involve addition and division

Also remember the 5 steps of problem solving:

1. What is being asked for?
2. What information do you need to solve the problem?
3. Which arithmetic operation will you need to use?
4. D the arithmetic and check your work.
5. Does your answer make sense?

SAMPLE
PROBLEMS:

1. John's average reading rate is 185 words per minute. How many words does he read in 10 minutes?

   To solve, multiply 185 by 10.
   10 x 185 = 1850 words

2. The township library has 3,421 non-fiction books and 6,304 fiction books. How many books does the library have <u>in all</u>?

   "In all" indicates addition.

$$\begin{array}{r} 3421 \\ \underline{+6304} \\ 9725 \end{array}$$ books

3. The seating capacity at a football stadium is 60,000. 48,321 fans bought tickets for the game. How many seats are left?

"How many are left" indicates subtraction.

$$\begin{array}{r} 60{,}000 \\ -\ 48{,}321 \\ \hline 11{,}679 \end{array} \text{ seats}$$

4. What is Joan's average test score if her scores are 85, 97, 74, 92.

To find the average, first you add all scores together.

$$\begin{array}{r} 85 \\ 97 \\ 74 \\ 92 \\ \hline 348 \end{array}$$

Then, divide the total by the number of tests.

$$\begin{array}{r} 87 \\ 4\overline{)348} \\ \underline{32} \\ 28 \\ \underline{28} \end{array}$$

The average score is 87.

PROBLEMS:
Name the operation you will use to solve:

1. John Stewart's farm is 325 acres. He is considering the purchase of the adjoining property consisting of 213 acres. If he combines these two properties, how many acres will he own all together?
   a) addition b) subtraction c) multiplication d) division e) addition and division

2. Monica drove 300 miles on Tuesday, 215 miles on Wednesday and 319 miles on Thursday. What is the average number of miles she drove each day?
   a) + b) - c) x d) + e) + and +

3. Mark received 653 votes during a recent school election. 1413 students voted. How many students did <u>not</u> vote for Mark?
   a) + b) - c) x d) + e) + and +

4. The Brubakers want to fence an area which is 14 feet on all 4 sides. How many feet of fence will they need?
   a) + b) - c) x d) + e) + and +

5. Miss Michaels, the first grade teacher, has 23 students in her class and 69 pieces of bubble gum. If she divides the gum equally between the students, how many pieces will each get?
   a) + b) - c) x d) + e) + and +

Now, you will solve each of the problems. Go back to question 1.

1.
2.
Hint: Add total miles driven; divide by number of days.
3.
4.
5.

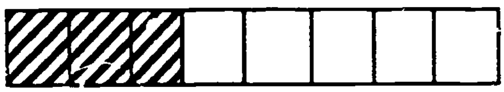
Unit I - Chapter 2 •Fractions•

This chapter will introduce, or refamiliarize, you with fractions. You will learn exactly what a fraction represents and how to compare fractions. The following sections will cover addition, subtraction, multiplication and division of fractions.

Section I What is a Fraction?

When a unit is divided into equal parts, the number expressing the relation of one or more parts to the total number of equal parts in the whole unit is called a <u>fraction</u>.

Although at first glance, this definition seems complicated, the following picture will illustrate the simplicity of this concept.



This bar is divided into 8 <u>equal</u> parts. 3 parts are shaded. The fraction which represents the shaded parts is:

$$\frac{3}{8}$$

3 is the <u>numerator</u>   8 is the <u>denominator</u>

The <u>denominator</u> tells how many equal parts the whole is divided into.
The bar is divided into <u>8</u> parts.
The <u>numerator</u> tells how many parts are represented.
The <u>3</u> represents the 3 parts of the bar that are shaded.

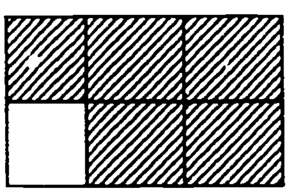
Another way to think of a fraction is as the part of a group.

A C C B A D F C

This is a group of <u>8</u> letters. 3 of the letters are C's.

$\frac{3}{8}$ 3 is the number of C's
8 is the total number of letters in the group

Now you try one:



This rectangle is divided into how many equal parts? (6)
Right! There are 6 parts.
How many parts are shaded? (5)
Right! There are 5 parts shaded. Type this as a fraction. $\frac{5}{6}$
Great!

(numerator)

5 number of parts represented
6 number of parts in whole

(denominator)

(If wrong, show starting with the denominator)

NOTE: Fractions with the same numerator and denominator equal 1.

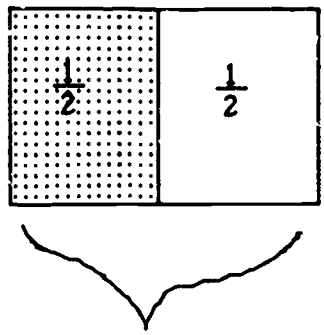
4 - number represented
4 - number of parts in the whole

$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{4}$	$\frac{1}{4}$

Section II Equivalent Fractions

Fractions representing the same number are equivalent fractions.
As in the previous section, the following diagram will explain equivalent fractions.

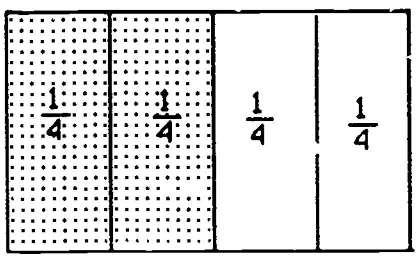
(A)



2 total parts

Of the 2 parts (the denominator) 1 part is shaded (the numerator). The fraction representing the shaded are is $\frac{1}{2}$

(B)



$\frac{2}{4}$ is the fractional representation of the shaded area.

You can see that $\frac{1}{2}$ and $\frac{2}{4}$ represent sections of the same size.

$\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions.

Rule: When the numerator and denominator of a fraction are divided or multiplied by the same number, the result is a fraction of the same value.

Compare: $\frac{1}{2}$ and $\frac{2}{4}$

$$\frac{1 \times 2}{2 \times 2} = \frac{2}{4}$$

$\frac{1}{2}$ and $\frac{2}{4}$ are equivalent

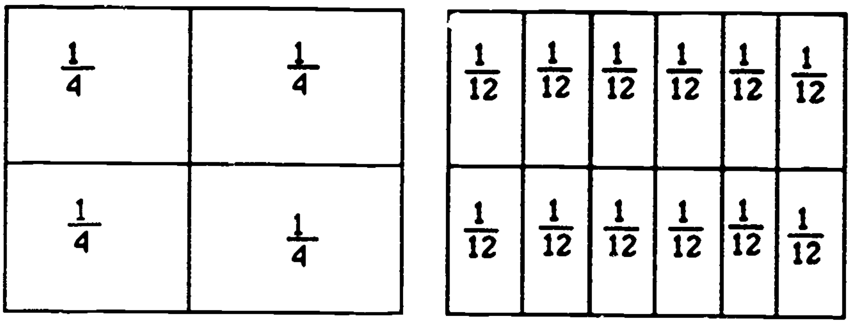
Look at this fraction: $\frac{3}{4}$

Let's multiply the numerator <u>and</u> denominat y 3.

$$\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

$\frac{3}{4}$ and $\frac{9}{12}$ are equivalent.

Proof:



(A) (B)

(Superimpose diagram B on diagram A to show equivalence).

Given two fractions, you can determine if they are equivalent by cross multiplying.
<u>Example</u>:

$\frac{1}{2} \quad \frac{2}{4}$ $\qquad$ $2 \times 2 = 4$
$1 \times 4 = 4$

If both products are the same, the fractions are equivalent.

Try this; find the <u>cross product</u> of the following fractions:

$\frac{3}{5} \quad \frac{6}{10}$ $\qquad$ $5 \times 6 = ?$ (30)
$3 \times 10 = ?$ (30)

Are $\frac{3}{5}$ and $\frac{6}{10}$ equivalent? (Yes)

$\frac{2}{3} \quad \frac{8}{9}$ $\qquad$ $3 \times 8 = ?$ (24)
$2 \times 9 = ?$ (18)

Are $\frac{2}{3}$ and $\frac{8}{9}$ equivalent? (No)

Although there are many ways to represent the same fraction - $\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{6}{12}$ -- there is only one fraction which expresses the number in <u>lowest terms</u>.

A fraction is in <u>lowest terms</u> when its numerator and denominator cannot be divided exactly by the same number.

<u>Example</u>: $\frac{6}{12}$ is not in lowest terms. 6 and 12 can <u>both</u> be divided exactly.

$\frac{6 \div 3 = 2}{12 \div 3 = 4}$

But is $\frac{2}{4}$ in lowest terms? (No) 2 and 4 can <u>both</u> be divided exactly by 2

$\frac{2 \div 2 = 1}{4 \div 2 = 2}$

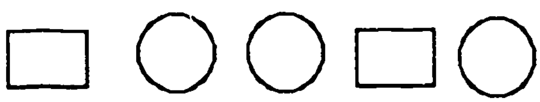
Is $\frac{1}{2}$ in lowest terms? (Yes) 1 and 2 cannot be divided exactly by the same number.

Change $\frac{6}{24}$ to lowest terms. $\frac{6 \div (6) = (1)}{24 \div (6) = (4)}$

PRACTICE PROBLEMS for section I. II

1. In the fraction $\frac{8}{15}$, which number is the denominator?

2. What fraction of the following group of objects are squares?



<u>Hint</u>: Total number of objects? (5)

3. In Bill's desk there are 13 pencils. 4 of these pencils are red. What fraction of the pencils are red?

4. A local volunteer organization has 100 raffle tickets to sell. 20 tickets are as yet unsold. What fraction of the tickets are sold?

<u>Hint</u>: What is being asked for? Unsold or (Sold)
<u>Hint</u>: Total # of tickets? (100)

5. Are these fractions equivalent?

$\frac{1}{2}$ and $\frac{2}{4}$

$\frac{3}{5}$ and $\frac{6}{10}$ Show each set separately.

$\frac{2}{4}$ and $\frac{4}{12}$

6. Mrs. Smith baked 24 brownies. She gave 18 brownies to the neighbor. Is the fraction she gave away equivalent to $\frac{3}{4}$?

<u>Hint</u>: Are $\frac{18}{24}$ and $\frac{3}{4}$ equivalent?

Hint: Either find cross products 3 x 24 and 4 x 18 or determine if 18 and 24 were divided by the same number. $\frac{18 \div (6)}{24 \div (6)} = \frac{3}{4}$

7. Change the following fractions to lowest terms.

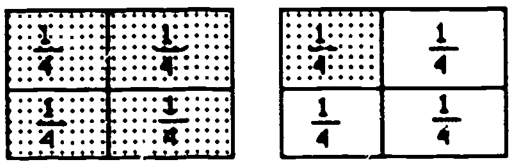
$\frac{6}{9} =$ $\quad \frac{24}{39} =$ $\quad \frac{1}{3} =$ $\quad \frac{4}{16} =$

Section III Improper Fractions and Mixed Numbers

If the numerator is equal to or larger than the denominator, the fraction is improper.

Examples: $\frac{5}{4}$ $\frac{3}{3}$ $\frac{24}{12}$

An improper fraction can be changed to a mixed number by dividing the numerator by the denominator.



$\frac{5}{4}$ are shaded

$$\text{denominator} - 4\overline{)\,5\,}\ \text{-numerator} \quad (\text{quotient } 1)$$

$\underline{-4}$

1 - write remainder as a fraction

The remainder becomes a numerator.
The denominator is the divisor.

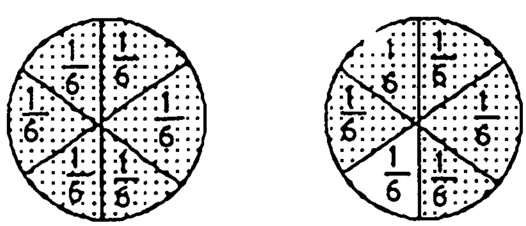
$\frac{1}{4}$

$\frac{5}{4} = 1\frac{1}{4}$

$1\frac{1}{4}$ is a mixed number because it contains a whole number and a fraction.

Now, change a mixed number to an improper fraction. $1\frac{5}{6} = ?$

Picture $1\frac{5}{6}$ as pieces of pie.



$1 = \frac{6}{6}$ $\quad$ 6

Count the shaded pieces.

$\frac{11}{6}$ are shaded. $\frac{11}{6}$ is an improper fraction. $\quad 1\frac{5}{6} = \frac{11}{6}$

There is a short cut for changing mixed numbers to improper fractions.

Example: $1\frac{5}{6}$ Multiply the whole number by the denominator.

$6 \times 1 = 6$

Add the numerator $6 + 5 = 11$. Place this number over the denominator. $\frac{11}{6}$

Try changing $3\frac{3}{4}$ to an improper fraction. $3\frac{3}{4}$

Denominator x whole number? $4 \times 3 = 12$.
Add the product to numerator of the fraction. $12 + 3 = 15$.
Place the sum over the denominator. $\frac{15}{4}$

Problems:

1. Are these fractions improper?
   $\frac{2}{5}$ $\frac{6}{3}$ $\frac{3}{2}$ $\frac{12}{11}$ $\frac{4}{4}$ $\frac{2}{4}$

2. Change each improper fraction to a mixed number.

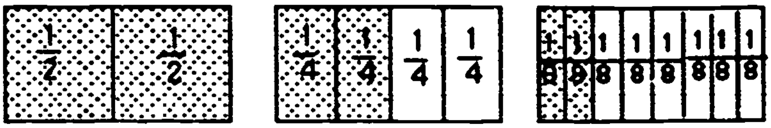
   $\frac{13}{6} =$ $\frac{4}{2}$ $\frac{10}{4}$

3. Change each mixed number to an improper fraction.
   $1\frac{3}{4} =$ $5\frac{2}{7} =$ $3\frac{3}{4} =$ $10\frac{3}{8} =$

Section IV Comparing and Ordering Fractions

In order to compare or order fractions, you must be able to determine which fraction is larger or which is smaller.

Fractions with the same numerator and different denominators are easy to compare.



$\frac{2}{2}$ $\frac{2}{4}$ $\frac{2}{8}$

It is easy to see that $\frac{2}{8}$ is smaller than $\frac{2}{4}$. It is also easy to see that $\frac{2}{4}$ is smaller than $\frac{2}{2}$. Notice that the numerator for these three fractions are is the same --2-- but the denominators change. The larger the denominator, the smaller the fraction.

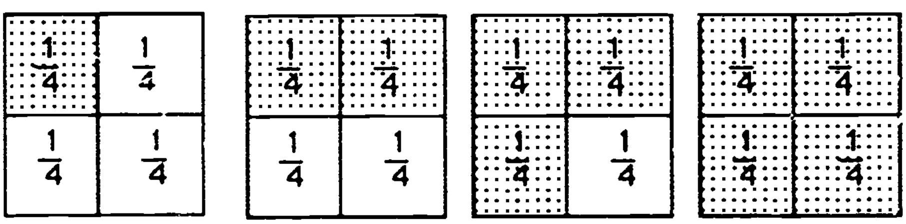
Of these fractions, which is largest? $(\frac{1}{2})$

$\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{12}$

Which is smallest? $(\frac{1}{12})$

Rule: If numerators are the same, the fraction with the greater denominator is smaller.

Fractions with the same denominator and different numerator are also easy to compare and order.



$\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ $\frac{4}{4}$ or 1

If the denominators are the same, the fraction with the greater numerator is larger.

$\frac{2}{4}$ is more than $\frac{1}{4}$
$\frac{3}{4}$ is more than $\frac{2}{4}$
$\frac{1}{4}$ is less than $\frac{3}{4}$

There are symbols which can be used when comparing fractions. You are already familiar with one of these " = " means equal to. There are two more symbols which you will be using to compare fractions.

> means "is greater than" < means "is less than"

Examples:

$\frac{2}{4} > \frac{1}{4}$ $\frac{2}{4}$ is more than $\frac{1}{4}$

$\frac{3}{4} > \frac{2}{4}$ $\frac{3}{4}$ is more than $\frac{2}{4}$

$\frac{1}{4} < \frac{3}{4}$ $\frac{1}{4}$ is less than $\frac{3}{4}$

As noted, ordering and comparing fractions with the same denominator is easy. The fraction with the largest numerator is largest? So how do you compare the fractions $\frac{8}{9}$ and $\frac{4}{5}$??

The first step is to find a common denominator. Use the following steps to find a common denominator.

1. Try the largest denominator. 9
   Can the other denominator be divided into this number exactly? $9 \div 5$ (no)

2. If not, try multiples of the largest denominator $2 \times 9 = 18$ Can 18 be divided exactly by the other denominator, 5? (no)
   $3 \times 9 = 27$ $27 \div 5$? no
   $4 \times 9 = 36$ $36 \div 5$? no
   $5 \times 9 = 45$ $45 \div 5$? yes

   45 is the common denominator for these fractions.

3. Multiplying the 2 denominator will always yield a common denominator but not always the lowest common denominator.

   The lowest common denominator (L.C.D.) is the smallest number that can be divided exactly by the denominator of all the fractions.

The next step in comparing $\frac{8}{9}$ and $\frac{4}{5}$ is to change each fraction to an equivalent fraction with the denominator of 45. $\frac{8}{9} = \frac{?}{45}$ and $\frac{4}{5} = \frac{}{45}$

When you change a fraction to an equivalent fraction with a larger denominator, you are changing the fraction to a higher term. To change $\frac{8}{9}$ to $\frac{}{45}$:

1. First divide the new denominator, 45, by the original denominator, 9.
   $45 \div 9 = 5$

2. Then multiply the numerator of the given fraction $\frac{8}{9}$ by that quotient. $8 \times 5 = 40$.

$\frac{8}{9} = \frac{40}{45}$ ----- $45 \div 9 = 5$ 5×8 40

Now you know that $\frac{8}{9} = \frac{40}{45}$, but you still need to change $\frac{4}{5}$ to an equivalent fraction with the denominator of 45.

You try this. $\frac{4}{5} = \frac{}{45}$

Step 1: Divide new denominator by original denominator (45 ÷ 5 = 9)
Step 2: Multiply numerator by original fraction by this quotient (4 x 9 = 36)

$\frac{4}{5} = \frac{?}{45}$ (36)

So $\frac{8}{9} = \frac{40}{45}$ Which is larger? $\frac{40}{45}$

$\frac{4}{5} = \frac{36}{45}$

Express the comparison between these fractions using > or <.

$\frac{8}{9} > \frac{4}{5}$ or $\frac{4}{5} < \frac{8}{9}$

<u>Practice Problems:</u>

Change each of these fractions to an equivalent fraction having the specified denominator.

1. $\frac{1}{2} = \frac{}{4}$ 2. $\frac{2}{3} = \frac{}{6}$ 3. $\frac{3}{4} = \frac{}{16}$ 4. $\frac{}{5} = \frac{}{15}$

Compare the following fractions by finding their common denominators. Express answer using >, < symbols.

1. $\frac{11}{20}\ \frac{2}{5}$ $\frac{11}{20} > \frac{2}{5}$ or $\frac{2}{5} < \frac{11}{20}$

<u>Hint</u>: Find common denominator. Try largest denominator first.

2. $\frac{5}{8}\ \frac{3}{7}$ $\frac{5}{8} > \frac{3}{7}$ or $\frac{3}{7} < \frac{5}{8}$

<u>Hint</u>: Common denominator is 8 x 7 or 56.
<u>Hint</u>: $\frac{5}{8} = \frac{?}{56}$ $\frac{3}{7} = \frac{?}{56}$ (24)

3. $\frac{2}{9}\ \frac{3}{15}$ $\frac{3}{15} > \frac{2}{9}$ or $\frac{2}{9} < \frac{3}{15}$

HINT: Be sure fractions are in <u>lowest terms</u> before finding the common denominator. (The smaller the denominator, the easier they are to manipulate).

<u>Hint</u>: $\frac{3}{15}$ is not in lowest terms. Can 3 and 15 be divided exactly by the same number? (Yes)
What? (3) <u>3</u> ÷ 3 = <u>1</u>

$$15 \div 3 = 5$$

Hint: Now find the common denominators for $\frac{2}{9}$ and $\frac{1}{3}$.

To order fractions you must first find a common denominator for all of the fractions, then order according to numerators. Remember, if the denominators are the same, the fractions get larger as the numerator increases.

Example: Order the following fractions from largest to smallest.

$$\frac{1}{4} \quad \frac{3}{8} \quad \frac{11}{24} \quad \frac{5}{6}$$

1. Look at the largest denominator, 24.

2. Will all denominators divide evenly into 24?

   $24 \div 4 = 6 \quad 24 \div 24 = 1$
   $8 \div 4 = 2 \quad 24 \div 6 = 4$ YES
   Then 24 is the common denominator.

3. Change each fraction to a fraction with the denominator of 24.
   $\frac{1}{4} = \frac{6}{24} \quad \frac{3}{8} = \frac{9}{24} \quad \frac{11}{24} = \frac{11}{24} \quad \frac{5}{6} = \frac{20}{24}$

4. Arrange in order (in this case largest to smallest.

   $\frac{20}{24} \; \frac{11}{24} \; \frac{9}{24} \; \frac{6}{24}$

5. Now, write them in order in lowest terms.

   $\frac{5}{6} \; \frac{11}{24} \; \frac{3}{8} \; \frac{1}{4}$

Problems:

Order the following sets of fractions from smallest to largest.

1. $\frac{1}{9} \; \frac{2}{3} \; \frac{11}{36} \; \frac{3}{4}$ answer-

Hint: Find common denominator (36)
Hint: $\frac{1}{9} = \frac{4}{36} \quad \frac{2}{3} = \frac{24}{36} \quad \frac{3}{4} = \frac{27}{36}$

2. $\frac{3}{5} \; \frac{5}{50} \; \frac{14}{25} \; \frac{20}{100}$

or

HINT· First, reduce all fractions to lowest terms!

Hint: Fractions in lowest terms are: $\frac{3}{5}\ \frac{1}{10}\ \frac{14}{25}\ \frac{1}{5}$

Hint: Find common denominator of fractions in lowest terms. Can all denominators divide evenly into 25?? (no)
into 50?? (yes)

Hint: $\frac{3}{5}=\frac{30}{50}\quad \frac{1}{10}=\frac{5}{50}\quad \frac{14}{25}=\frac{28}{50}\quad \frac{1}{5}=\frac{20}{100}$

Section V Adding and Subtracting Fractions

To add or subtract fractions, you must find a common denominator. Then, you add or subtract the numerators.

Examples: $\frac{1}{2}+\frac{1}{2}=\frac{2}{2}$ (1+1)

In most problems, you should simplify your answer. $\frac{2}{2}=1$
1 is the answer in simplest form or lowest terms.

$\frac{3}{4}+\frac{1}{12}=?$ Common denominator? (12) $\frac{3}{4}=\frac{?}{12}$ (9)

$\frac{9}{12}+\frac{1}{12}=\frac{?}{12}$ (9 + 1 = 10)

Is $\frac{10}{12}$ in simplest form? (no)

$\frac{10}{12}=\frac{5}{6}$ $\frac{3}{4}-\frac{1}{2}=?$

Common denominator? (4) $\frac{3}{4}-\frac{2}{4}=\frac{\ }{4}$ subtract numerator 3-2=1

$\frac{3}{4}-\frac{2}{4}=\frac{1}{4}$

Practice Problems:

Express answers in lowest terms and improper fractions as mixed numbers.

1. $\frac{1}{12}+\frac{4}{12}=?$

2. $\frac{8}{9}+\frac{2}{3}=?$

3. $\frac{3}{8}+\frac{5}{12}=?$

Hint: Common denominator? (24)
Hint: Rewrite using common denominator.
$\frac{9}{24}+\frac{10}{24}$

4. $\frac{4}{5}-\frac{2}{5}=?$

5. $\frac{3}{14}-\frac{1}{7}=?$

6. $\frac{3}{4}-\frac{1}{3}=?$

<u>Hint</u>: Common denominator? (12)

Sometimes you will be adding and subtracting mixed numbers.

<u>Example</u>: $2\frac{3}{4} + 4\frac{1}{4}$

<u>Step 1</u>: Write problem vertically lining up whole numbers and fractions.

$$\begin{array}{r} 2\frac{3}{4} \\ +\,4\frac{1}{4} \\ \hline \end{array}$$

<u>Step 2</u>: Rewrite using common denominator if necessary. 4 is the common denominator.

<u>Step 3</u>: Add fractions; add whole numbers.

$$\begin{array}{r} 2\frac{3}{4} \\ +\,4\frac{1}{4} \\ \hline 6\frac{4}{4} \end{array}$$

<u>Step 4</u>: Simplify $6\frac{4}{4} =$ (7)

<u>Practice Problems</u>:

1. $\begin{array}{r} 4\frac{1}{2} = 4\frac{2}{4} \\ 3\frac{1}{4} = +\,3\frac{1}{4} \\ \hline \end{array}$ 2. $\begin{array}{r} 5\frac{1}{3} \\ +\;3\frac{1}{3} \\ = \underline{\quad\quad} \end{array}$ 3. $\begin{array}{r} 6\frac{1}{5} = 6\frac{2}{10} \\ +\;2\frac{7}{10} = +2\frac{7}{10} \\ \hline \end{array}$

The steps for subtracting fractions are practically the same as for adding.

<u>Step 1</u> Write vertically

<u>Step 2</u> Rewrite using common denominator

<u>Step 3</u> Subtract fractions; subtract whole numbers

<u>Step 4</u> Simplify

$$\begin{array}{rcr} 5\frac{2}{3} & = & 5\frac{4}{6} \\ -\;2\frac{1}{2} & = & 2\frac{3}{6} \\ \hline & & 3\frac{1}{6} \end{array}$$

Sometimes, the upper fraction is not larger than the lower fraction.

<u>Example</u>: $\begin{array}{r} 13\frac{1}{5} \\ -\ 4\frac{3}{4} \\ \hline \end{array}$

Rewrite with equivalent fractions.

$$\begin{array}{rcl} 13\frac{1}{5} & = & 13\frac{4}{20} \\ -\ \ 4\frac{3}{4} & = & -\ 4\frac{15}{20} \\ \hline \end{array}$$

Notice, the next step is to subtract the lower fraction from the upper fraction $\frac{4}{20} - \frac{15}{20}$
As the problem is written, you can't do this. As in subtraction using whole numbers, sometimes you must borrow from the column to the left. $\begin{array}{r} 13\frac{4}{20} \\ -\ 4\frac{15}{20} \\ \hline \end{array}$

Borrow 1 from 13

$$\begin{array}{r|l} 13 & \frac{24}{20} \\ -\ 4 & \frac{15}{20} \\ \hline \end{array} \quad = \quad \begin{array}{r|l} \overset{12}{\cancel{13}} & 1\ \frac{4}{20} \\ -\ 4 & \ \ \frac{15}{20} \\ \hline \end{array}$$

Change the mixed fraction $1\frac{4}{20}$ to an improper fraction.

$$\begin{array}{r|l} \overset{12}{\cancel{13}} & 1\ \frac{24}{20} \\ -\ 4 & \ \ \frac{15}{20} \\ \hline \end{array}$$

You <u>can</u> subtract 15 from 24.

Complete the problem: $\begin{array}{r|l} 12 & 1\ \frac{24}{20} \\ -\ 4 & \ \ \frac{15}{20} \\ \hline \multicolumn{2}{c}{(8\ \frac{9}{20})} \end{array}$

Is $8\frac{9}{20}$ in lowest terms? (yes)

Practice Problems:

1. $3\frac{4}{7}$
   $-\ 2\frac{2}{7}$
   ______

2. $14\frac{3}{20} = 14\frac{3}{20}$
   $-\ 11\frac{1}{10} = -\ 11\frac{2}{20}$
   ______ ______

3. $9\frac{3}{4} = 9\frac{9}{12}$
   $-\ 2\frac{11}{12} = -\ 2\frac{11}{12}$
   ______ ______

Hint: You will hav o borrow to complete the subtraction in the fractions column.

3. (continued) $9\frac{9}{12} = 8\ 1\frac{9}{12}$ $\quad 8\frac{21}{12}$
   $-\ 2\frac{11}{12} = -\ 2\frac{11}{12}$ $\quad -\ 2\frac{11}{12}$
   ______ ______ ______
   $6\frac{10}{12}$

Hint: Ch ie $1\frac{9}{12}$ to improper fraction.

Hint: $6\frac{10}{12}$ in lowest terms? (no) $6\frac{10}{12} = 6\frac{5}{6}$

Section VI Multiplying and Dividing Fractions

To multiply two fractions, you must multiply the numerator by the numerator and the denominator by the denominator.

$$\frac{3}{5} \times \frac{2}{3} = \frac{3 \times 2}{5 \times 3} = \frac{6}{15}$$

Reduce answer to lowest terms.

Practice Problems:

1. $\frac{2}{5} \times \frac{5}{8} = \frac{2 \times 5 = ?}{5 \times 8 = ?}$

2. $\frac{3}{4} \times \frac{5}{8} = \frac{3 \times 5 =}{4 \times 8 =}$

3. $\frac{3}{8} \times \frac{1}{3} =$

4. $\frac{4}{7} \times \frac{2}{3} =$

$\boxed{\textit{Shortcut}}$

To simplify the multiplication of fractions, you may divide any top number into any bottom number or any bottom number into any top number.

Example: $\frac{2}{5} \times \frac{10}{13}$

Notice that 5 (a bottom number) divides evenly into 10 (a top number).

$$\frac{2}{\underset{1}{\cancel{5}}} \times \frac{\overset{2}{\cancel{10}}}{13} \quad \text{Think: } 10 \div 5 = 2$$

Think: $5 \div 5 = 1$

Now, multiply across $\quad \frac{2}{\underset{1}{\cancel{5}}} \times \frac{\overset{2}{\cancel{10}}}{13} = \frac{2 \times 2 =}{1 \times 13 =} \frac{4}{13}$

Notice how much simpler the multiplication is. It is much easier to multiply 1 x 13 than 5 x 13.

Sometimes, you can divide more than one way.

Example:

$$\frac{3}{\cancel{7}_{1}} \times \frac{\cancel{21}^{3}}{36}$$

Think: 7 into 7 is 1
7 into 21 is 3

Now look at the other 2 numbers, 3 and 26. They have a common divisor as well. 3 divides evenly into 3 and 36.

$$\frac{\cancel{3}^{1}}{\cancel{7}_{1}} \times \frac{\cancel{21}^{3}}{\cancel{36}_{12}}$$

Multiply accross $1 \times 3 = \underline{3}$
$1 \times 12 = 12$

Reduce $\frac{3}{12} = \frac{1}{4}$

<u>Practice Problems</u>:

1. $\frac{4}{5} \times \frac{2}{4}$

2. $\frac{3}{4} \times \frac{2}{9}$

Notice, that when you use this shortcut, your answer is often in lowest terms.

To multiply a fraction by a whole number, change the whole numb.r to an improper fraction and multiply across.

<u>Example</u>: $5 \times \frac{3}{8} =$

Change 5 to improper fraction by placing a fraction bar under the 5 and inserting a 1 as the denominator.

$5 = \frac{5}{1}$ 5 becomes the numerator. Multiply across.

$$\frac{5}{1} \times \frac{3}{8} = \frac{15}{8} \text{ Reduce } \frac{15}{8} = 1\frac{7}{8}$$

To multiply mixed numbers, change to improper fractions and multiply across.

<u>Example</u>: $2\frac{1}{3} \times 3\frac{1}{5}$

$2\frac{1}{3} = \frac{7}{3}$

$3\frac{1}{5} = \frac{16}{5}$ $\frac{7}{3} \times \frac{16}{5} = \frac{12}{15}$ Reduce $\frac{112}{15}$ to $7\frac{7}{15}$

Practice Problems:
Complete the following problems. Reduce answers to lowest terms. Use the shortcut when possible.

1. $10\frac{3}{5} = \frac{\quad}{10} \times \frac{3}{5} =$

2. $2\frac{1}{5} \times 4\frac{5}{10} = \frac{\quad}{5} \times \frac{\quad}{10}$

3. $8\frac{5}{9} \times 4\frac{1}{11} =$

Hint: $\frac{77}{9} \times \frac{45}{11}$ Can you use the shortcut? (yes)

4. $5\frac{1}{4} \times 2\frac{1}{3} =$

5. $7 \times 2\frac{3}{14} =$

Hint: $\frac{7}{1} \times \frac{31}{14}$

Hint: Can you use the shortcut? (yes)

Reduce to lowest terms: $\frac{31}{2} = 15\frac{1}{2}$

Now that you can multiply fractions, division is easy.

Step 1: Write the problem. (Remember, with division, you must place fractions in proper order. With multiplication, order doesn't matter).

Step 2: Rewrite all mixed numbers and whole numbers as improper fractions.

Step 3: Invert the divisor and change the sign to x.

Step 4: Multiply across.

Step 5: Reduce if necessary.

Example: $\frac{3}{4} \div \frac{1}{3}$

Step 1: $\frac{1}{3}$ is the divisor. The problem is not the same as $\frac{1}{3} \div \frac{3}{4}$; be careful to write problem in correct order.

Step 2: There are no mixed numbers or whole numbers.

Step 3: $\frac{3}{4} \times \frac{3}{1}$

Step 4: $\frac{3}{4} \times \frac{3}{1} = \frac{9}{4}$

Step 5: $\frac{9}{4} = 2\frac{1}{4}$

Note: To invert a fraction, you make the denominator the numerator a I the numerator th denominator. $\frac{3}{4}$ inverted is $\frac{4}{3}$

Example:

Step 1: Write problem $8 \div 2\frac{2}{5}$

Step 2: Rewrite whole & mixed numbers $\frac{8}{1} \div \frac{12}{5}$

Step 3: Invert divisor; change to mult. sign. $\frac{8}{1} \times \frac{5}{12}$

Step 4: Multiply across $\frac{8}{1} \times \frac{5}{12} = \frac{40}{12}$

Step 5: Reduce $\frac{40}{12} = 3\frac{1}{3}$

*Easy way to remember rules for division: invert the division and then follow the rules for multiplication.

Practice Problems:

1. $3\frac{2}{3} \div 4\frac{3}{5} =$ $\frac{11}{3} \div \frac{23}{5}$ $=$. $\frac{11}{3} \times \frac{5}{23}$

2. $3 \div 2\frac{1}{2} =$

Hint: $\frac{3}{1} \div \frac{5}{2}$

Hint: Invert division and multiply

3. $4\frac{3}{4} \div 1\frac{1}{2} =$

Hint: $\frac{19}{4} \div \frac{3}{2}$

Hint: Invert divisor and multiply

Section VII Ratios, Proportions and Probability

Fractions can also be used to express a ratio. A ratio is another way to compare numbers.

Example:

You can buy 2 pounds of potatoes for 32 cents. What is the ratio of potatoes to cents?

$$\frac{\text{pounds of potatoes}}{\text{cents}} = \frac{2}{32} = \frac{1}{16}$$

Notice that the answer is reduced to lowest terms, $\frac{1}{16}$. However, you should not change an improper fraction to a mixed number when expressing a ratio.

Note: The numbers in a ratio must be written in the order asked for. In this case, it was pounds to cents. Potatoes was asked for first so it becomes the numerator.

Example:

24 people are planning to attend a football game. They will be riding in 6 cars. What is the ratio of people to cars?

$$\frac{\text{People}}{\text{cars}} = \frac{24}{6} = \frac{4}{1}$$

If the answer had been $\frac{1}{4}$ it would be wrong, because that would be the ratio of cars to people.

Sometimes, you will not be given all the information needed to write the ratio.
For example: There are 24 students in the class. 8 of the students are girls. What is the ratio of boys to total number of students?

First, you need to discover how many boys are in the class. To get this number, subtract the number of girls from the number of students. 24 - 8 = 16. There are 16 boys.
Therefore, the ratio of boys to total students is: $\frac{16}{24}$ or $\frac{2}{3}$

There are two other ways this ratio could be expressed: 2 to 3 or 2:3
If you see a ratio expressed in either of these ways, you can rewrite it as a fraction by placing the first number on top (the numerator). $\frac{2}{3}$

Practice Problems: (Solve and reduce to lowest terms).

1. Mary can type 70 words in 1 minute. What is the ratio of words to minutes?

2. There are 100 questions on a science exam. 20 are true/false and the rest are multiple choice.
   a) What is the ratio of true/false to the total number of questions?

   b) What is the ratio of multiple choice to the total number of questions? $\frac{4}{5}$

Hint: Subtract t/f from the total.

c) Write a ratio comparing the number of true/false questions to the number of multiple choice questions.

$\frac{1}{4}$

A proportion is made up of 2 equivalent ratios. To determine if 2 ratios are proportions, cross multiply.

Example: $\frac{3}{4}$ and $\frac{9}{12}$ $4 \times 9 = 36$; $3 \times 12 = 36$

The produces are = so the ratios are proportions.

This sho'ıld b- very familiar to you. You used this same method n a previous section to determine if two fractions are equivalent.

Many times a word problem can be written as a proportion. In many cases, one of the numbers will be needed to write the proportion. Finding this missing number will provide the answer to the problem.

Example:

Write a ratio equivalent to $\frac{2}{3}$ with 6 as the denominator.

$\frac{2}{3} = \frac{?}{6}$

To find the missing numerator, find the cross product of 2 x 6 and divide by the remaining number. (3) $2 \times 6 = 12$; $12 \div 3 = 4$ $\frac{2}{3} = \frac{4}{6}$

Find the missing term in $\frac{4}{5} = \frac{8}{10}$

Find cross product of 5 and 8 (40).

Divide 40 by 4 (10).

10 is the missing term. $\frac{4}{5} = \frac{8}{10}$

The key to using a proportion to solve a work problem is to set up the problem correctly. You must be sure the numbers are in the right order.

Example:

While traveling across the country, Susan drives 8 hours each day. How many total hours will Susan drive in 14 days?

First: Set up a ratio of Susan's hours driven in one day. $\frac{\text{hours}}{\text{days}} = \frac{8}{1}$

Second: Set up an equivalent ratio (a proportion) comparing how many hours she will drive in 14 days. $\frac{\text{hours}}{\text{days}} = \frac{8}{1} = \frac{?}{14}$

Notice that you don't have a number to place in the top of the 2nd ratio. Finding this number will solve the problem.

Third: a) cross multiply 8 and 14.
b) divide this product by the remaining number. 112 ÷ 1 = 112.

Complete the proportion: $\frac{8}{1} = \frac{112}{14}$

Practice Problems:
Solve the following problems by setting up a proportion and solving for the missing number.

1. Eric's Little League team won 3 games out of every 4 they played. They played 36 games this season. How many games did they win?

Hint: Set up ratio of games won to games played.
$\frac{\text{Games won}}{\text{games played}} = \frac{3}{?}\ (4)$

Hint: Now set up a proportion of games won for the total number played.
$\frac{\text{games won}}{\text{games played}} = \frac{3}{4} = \frac{?}{(36)}$

2. At the local high school, there are 3 male teachers for every 2 females. If there are 24 male teachers, how many female teachers are there?

Hint: Set up ratio of men to women $(\frac{3}{2})$

Hint: Write a proportion using the information you have.
$\frac{\text{men}}{\text{women}} = \frac{3}{2} = \frac{(24)}{?}$

3. A recipe calls for 2 cups of milk for every 5 cups of flour. If 15 cups of flour are used, which expression below shows a proportion which can be used to discover how many cups of milk are needed?

a. $\frac{2}{5}=\frac{?}{15}$ b. $\frac{5}{2} = \frac{?}{15}$ c. $\frac{2}{5} = \frac{15}{?}$ c. $\frac{5}{2} = \frac{10}{?}$ d. $\frac{5}{2} = \frac{10}{?}$

4. Kelly can type 53 words in 1 minute. Which expression represents the number of words Kelly types in 15 minutes?

a. (53 x 1)/15
b. (53 x 15)/1
c. (15 x 1)/53
d. (53 + 1) x 15

5. Tom figures that it takes him 3 hours to nail 16 boards together to form the frame for a wall. How many hours will it take to nail 48 boards together?

a. (3x16)/48
b. $\frac{3}{16}=\frac{?}{48}$
c. (16 x 48)/3
d. (3 + 48)/16

Probability

How could you determine the probability of rolling a 6 on the throw of a dice?

First of all, you must determine the number of all possible outcomes. There are 6 possible outcomes in the roll of a dice. Then, you must determine the number of favorable outcomes that are possible. How many times does 6 appear on the dice? (1) So, there is a 1 in 6 chance that you will roll a 6.

This probability can be expressed as the fraction $\frac{1}{6}$.

$$\text{probability} = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

Let's look at another problem. Suppose there are 500 raffle tickets being sold by the local high school. If you bought 10 tickets, what is your probability of winning?

$$\frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{\qquad}{500}$$

Since 500 tickets were sold, there are 500 possible winning tickets.

$$\frac{\text{favorable}}{\text{possible}} = \frac{10}{500}$$

Since you bought 10 tickets, you have 10 chances to win out of the 500 tickets.
Reduce the probability to lowest terms.

$$\frac{10}{500} = \frac{1}{50}$$

So your chance of winning are 1 in 50.

Practice Problems:
Use this situation to solve the following 2 problems.

The local grocery store ran a special sale on cans of vegetables. Although they were sold at a very low cost, they had one defect. None of them had labels. The grocery store had them sorted into stacks of green beans, corn and peas. Joan bought 3 cans of green beans, 6 cans of corn and 4 cans of peas, but when she got them home, she had mixed them up.

1. What is the probability that the first can she opens will be green beans?

Hint: Find the total number of cans purchased. 3 cans of beans
6 cans of corn
4 cans of peas
13 is the total number of possible outcomes

Hint: How many possible favorable outcomes are there? (3) Right, there are 3 cans of beans.

Hint: Write this probability as a fraction.

2. Joan opened the first can she chose and found that it was a can of peas. The next day, she opened another can. What is the probability that this can will contain corn?

Hint: Find the total number of possible outcomes. Remember, 1 can is gone. (12)

Hint: How many cans of corn are there? (6)

Hint: $\frac{\text{favorable outcomes}}{\text{total possible outcomes}} = \frac{(6)}{(12)}$

Hint: Reduce probability to lowest terms

Right, there is a 1 in 2 change that the next can opened will be corn!

3. John has a bag containing 15 apples. 3 of these apples are rotten. If he reaches into the bag without looking, what is the probability that he will select a good apple?

Hint: possible outcomes? (15)

Hint: possible favorable outcomes? (12)
Right, since there are 15 total apples and 3 are rotten, then 12 are good.

Hint: Write the probability and reduce.

Unit I - Chapter 3 •Decimals and Percents•

Section I Introduction to Decimals

As you learned in the beginning of the unit, our numeration system is a "base 10" system. A number's value is determined by its position.

Example:

hundreds tens ones
3 4 3

The value of 4 is 40 because it is in the "tens" place.

In the chapter on fractions, you learned to express a number less than 1 as a fraction. A number less than 1 can also be expressed using place value.

Example:

decimal
325.4

All numbers to the left of the decimal represent whole numbers. All numbers to the right of the decimal represent a part of a whole.

The 4 in this number represents 4 tenths of a whole.

Just as each place to the left of the decimal represents a value, so does each place to the right of the decimal.

thousands hundreds tens ones . tenths hundredths thousandths ten thousandths

___ ___ ___ ___ • ___ ___ ___ ___

These values are based on groups of ten.
10 ones = 1 group of 10
10 tens = 1 group of 100
10 tenths = 1 whole or 1
10 hundredths = 1 tenth

Note: 10 tenths can also be written as $\frac{10}{10}$. You know that any fraction with the same numerator and denominator equals 1.

All decimals can be written as fractions with the denominator as some multiple of 10.

Example $0.4 = 4 \text{ tenths} = \frac{4}{10}$

$0.35 = 35 \text{ hundredths} = \frac{35}{100}$

$0.041 = 41 \text{ thousandths} = \frac{41}{1000}$

A decimal is read as a whole number with the place name of the last number to the right.

A mixed decimal contains a whole number and a decimal.

Example: 4.31
To read this, say the number to the left of the decimal. Say "and" to represent the decimal point. Then read the number to the right of the decimal using the place name of the last digit.

"4 and 31 hundredths"

Practice Problems:
Choose the correct value for the underlined digit.

1. 2<u>5</u>4

   a. 5 tenths b. 50 c. 500 d. 5

2. 0.<u>5</u>3

   a. 50 b. 5 tens c. 5 tenths d. 5

3. 3.45<u>6</u>

   a. 6 b. 6 x 100 c. 6 thousandths d. 6000

4. <u>1</u>4.9

   a. 1 x 10 b. 1 c. 1 tenth d. 100

Choose the correct way to read each number.

1. 0.1

   a. one tenth b. one ten c. 1 d. one hundredth

2. 4.5

   a. forty-five b. four and 5 c. four and 5 tenths d. 4 and 5 hundredths

3. 315.04

   a. three hundred fifteen and 4 tenths
   b. three hundred fifteen and 4
   c. three hundred fifteen 4 hundredths
   d. three hundred fifteen and 4 hundredths

4. 46.001

a. forty-six and 1 hundredth
b. forty-six and 1 thousandth
c. forty-six
d. forty-six and 1 tenth

Section II Rounding and Comparing Decimals

In many problems, you will be asked to round the decimal. To round a decimal, you write it with fewer places. You can do this when your answer does not need to be exact.

Example: Round 3.46 to the nearest tenth

What digit is in the tenth place? (4)
To round off to the nearest tenth, you examine the number to the right of the tenth place. This number is 6.
If the number is less than 5, you eliminate it. If the number is more, you add 1 to the number in the tenths place. Since 6 is more than 5, change the number to read 3.5.

Try 35.1235 to the nearest hundredth.
What digit is in the hundredths place? (2)
What digit is to the right of the 2? (3)
Is this less than 5? (yes)
Then the 2 in the hundredths stays the same and you eliminate all the digits to the right.
35.1235 to the nearest hundredths is 35.12.

Sometimes you are told to "round to one decimal place." This is the same as rounding to the nearest 10th.

Practice Problems:

1. Round 14.631 to the nearest tenth.

2. Round 14.317 to 2 decimal places.

Hint: This is the same as rounding to the nearest hundredth.

3. Round to the nearest whole number 194.62.

Hint: Again, you check the number to the right of the number you are rounding to.

Try that again.

4. Round to the nearest whole number 15.32

Hint: Is the 3 greater than or = to 5? (no)

To compare numbers containing decimals, you write both numbers with the same number of decimal places.

Extra zeros at the end of a decimal number does not change its value.

Example: 0.1 = 0.10 = 0.100 = .1000

You can see that this is true by writing these numbers in fractional form.

$$.1 = \frac{1}{10}$$

$$.10 = \frac{10}{100}$$

$$.100 = \frac{100}{1000}$$

$$.1000 = \frac{1000}{10000}$$

CAUTION: You can only add zeros after the last digit in a decimal.

$1.01 \neq 1.1$ ($\neq$ means is not =)

As fractions, you can see that $\frac{1}{100}$ and $\frac{1}{10}$ are not the same. $\frac{1}{100}$ is in lowest terms.

Example: Which is larger? 0.15 or 0.2?

Step 1: Change both numbers to same number of decimal places.

0.15
0.20

Step 2: 0.20 > 0.15 (Remember the symbols: > means greater than; < less than)

Order these decimals from smallest to largest.

0.53 0.411 0.412 0.402 0.4013

Step 1: Write all same number of decimal places. Since .4013 has 4 decimal places (more than any of the other numbers). All of the numbers should be written with 4 decimal places.

0.53 = 0.5300
0.411 = 0.4110
0.412 = 0.4120
0.402 = 0.4020
0.4013 = 0.4013

Step 2: Order from smallest to largest.

0.4013 0.4020 0.4110 0.4120 0.5300

Problems for Section II

Write each of the following as a decimal.

1. $\frac{7}{100}$

2. $\frac{5}{10}$

3. $\frac{5}{1000}$

4. $\frac{53}{100}$

5. $1\frac{15}{1000}$

Choose the best answer for the following:

6. 1.654 rounded to the nearest hundredth

   a. 1.655
   b. 1.65
   c. 1.6
   d. 1.7

7. 5.49 to the nearest whole number

   a. 5.5
   b. 10
   c. 5
   d. 6

8. 516.015 to 2 decimal places

   a. 516.01
   b. 516.0150
   c. 516.02
   d. 516.1

9. What value is represented by the hundredths place in 1541.072?

   a. 70
   b. 500
   c. 7 hundredths
   d. 5 x 100

10. Arrange in order from smallest to largest.

A	B	C	D	E
1.05	1.1	1.6	1.61	1.2

a. A E D C B
b. A B E C D
c. B A E C D
d. C D B E A

Section III. Addition and Subtraction of Decimals

When adding and subtracting decimals, follow these rules:

1. **Line up decimals with point under point.**
2. **Give each number the same amount of decimal places by adding zeros.**
3. **Complete the addition or subtraction.**
4. **Bring decimal point straight down in the answer.**

Example:

Add 4.5 to 3.41

```
  4.50 (add zero)
+ 3.41
  7.91
```

(Bring decimal point straight down in answer)

Subtract .19 from 1.05

Be careful to place larger number on top.

```
  0 9 15
  1.05
- 0.19
   .86
```

Complete subtraction, borrowing if necessary. Bring decimal point straight down.

Subtract 1.9 from 3.95

```
  3.95
- 1.90  Add zero so both numbers have same amount of decimal places.
  2.05
```

Complete the operation. Bring decimal straight down.

Practice Problems:

1. 1.62 + 0.5 =

2. 5.3 - 1.2 =

3. 14.43 - 0.3 =

Hint: Add zeros to give each number same number of decimal places:

```
  14.43
-   .30
```

4. 12 - 0.64 =

Hint: Add zeros so both numbers have same number of decimal places.

```
  12.(00)
-  0.64
```

5. Michael's odometer reading on his car was 15,093.6 when he left for vacation. When he got home again, the odometer reading was 16,142.7. How many miles did Michael drive while on vacation?

6. Mrs. Steward spent $14.75 on fruit, $34.15 on meat, $20.79 on cleaning products and $8.50 on paper supplies. How much money did Mrs. Stewart spend on this shopping trip?

Hint: Notice that you add money in the same way you add other decimals.

	tenths (dimes)	hundredths (pennies)
$____ .	____	____

Section IV -- Multiplication and Division of Decimals

Multiplication

Step 1: Write as multiplication problem with longer number on top.

```
Ex. 4.654
  x   3.5
```

There is no need to line up the decimals.

Step 2: Complete the multiplication as if both were whole numbers.

```
Ex.   4.654
    x   3.5
      23270
     13962
     162890
```

Step 3: To correctly place the decimal point in the answer, count the number of decimal places in the original problem.

```
 4.654 -- 3 places
x  3.5 -- 1 place
```

There are 4 total decimal places in this problem.

Step 4: The answer will contain the same number of decimal places as the original problem.
Ex. In this problem, the answer will contain 4 decimal places. Count over 4 decimal places from the right. The decimal will be after the 6.

```
16.2890
   4321
```

Example:
If you bought 3 shirts at $8.50 per shirt, how much money did you spend?

```
 $8.50 --  2 decimal places
 x   3 -- +0 decimal places
$25.50     2 decimal places
```

Example:
Find the product of 0.07 and 0.002.

```
 0.002 --  3 decimal places
x 0.07 -- +2 decimal places
.00014     5 decimal places
```

Notice that zeros were added so that the decimal could be placed 5 decimals to the right of the last digit.

Practice Problems:
Choose the correct answer for the following:

1. 9.4 x 3 =

a. 2.82 b. 282 c. 28.2 d. .282

2. 4.63 x 0.04 =

   a. 1.852 b. 18.52 c. 185.2 d. .1852

3. 0.03 x 0.04 =

   a. .12 b. .012 c. .0012 d. .00012

4. 0.7 x 0.0005 =

   a. 0.0035 b. 0.35 c. 0.035 d. 0.00035

5. 331 x 0.6 =

   a. 1986 b. 198.6 c. 19.86 d. .1986

Division

Dividing a decimal by a whole number
Example: 4.6 ÷ 2

Step 1: Set up as a division problem using the division frame $2\overline{)4.6}$

Step 2: Place decimal point in the answer directly above decimal point in the dividend.

$2\overline{)4.6} \rightarrow$ dividend

$$\begin{array}{r} 2.3 \\ 2\overline{)4.6} \\ -4 \\ \hline 06 \\ -6 \\ \hline 0 \end{array}$$

Step 3: Divide as usual

Ex. 3 ÷ 6 The divisor is larger than the dividend.

Step 1: $6\overline{)3}$

Step 2: Write the dividend (the number under the division frame) with a decimal point and zeros.

$$6\overline{)3.0}$$

Step 3: Set decimal point in the answer directly above the decimal point in the dividend.

Step 4: Divide as usual.

$$\begin{array}{r} .5 \\ 6\overline{)3.0} \\ \underline{30} \\ 0 \end{array}$$

Step 5: You may have to continue adding zeros to continue the division if the answer is not even (there is a remainder).

Example: 14 ÷ 30

Step 1:
$30\overline{)14}$

Step 2, 3:

$30\overline{)14.0}$

Step 4:

```
      0 .4
30 ⟌ 14.0
     120
      20
```

Step 5:

```
      0 .4666
30 ⟌ 14.0000
     120
      200
     -180
       200
      -180
        200
      - 180
         20
```

Notice that no matter how many zeros you add, the decimal is repeated. A common way to write the answer with a repeating decimal is to round it off. You will be told what place to round to.

Example: 0.467 is rounded to the hundredth place.

Another way to indicate a repeating decimal is to place a bar over the part of the decimal that repeats.

Example: $0.4\overline{6}$

Note: In Step 5 when you are adding zeros, many times the answer will eventually work out evenly. If not, simple add enough zeros to carry out the division to the number of decimal places needed.

Example: 4 + 17 to nearest hundredth

Step 1:

```
17 ⟌ 4
```

Step 2, 3:

```
       .
17 ⟌ 4.
```

Step 4, 5:

```
       .
17 ⟌ 4.000
    -3 4
        60
      - 51
         90
       - 85
          5
```

Notice that the division is carried out one place to the right of the desired answer. This is so you know whether to round down or to round up.

0.235 to nearest hundredth
0.24 indicates rounding up in hundredth place

Dividing by a decimal.

Example: $5 \div .4$

Step 1: Set up as division using division frame.

$.4\overline{)5}$

Step 2: Move divisor a whole number by moving the decimal point as far to the right as possible.

$.4.\overline{)5}$

What you have actually done to move the decimal point 1 place to the right in the division is you multiplied by 10.

$$\begin{array}{r} .4 \\ \underline{\times 10} \\ 4.0 \end{array}$$

Step 3: Now you must multiply the dividend by the same number by which you multiplied the divisor. $10 \times 5 = 50$

$.4.\overline{)5.0.}$

RULE: You move the decimal point in the dividend to the right the same number of places you moved the decimal point in the divisor. You may need to add zero in the dividend.

Step 4: Divide as usual

$$\begin{array}{r} .4\overline{)5.0.0} \\ 4 \\ \hline 10 \\ -8 \\ \hline 20 \\ 20 \\ \hline \end{array}$$

$5 \div .4 = \boxed{12.5}$

Example: $22.75 \div 32.5$

Step 1: $32.5\overline{)22.75}$

Step 2: $32.5.\overline{)22.75}$ Move 1 to right in dividor, move 1 to right in dividend.

Step 3: $32.5.\overline{)22.7.5}$ Place decimal point in answer directly above decimal point in dividend

Step 4:

$$\begin{array}{r} 0.7 \\ 325.\overline{)227.5} \\ -2275 \\ \hline 0 \end{array}$$

$22.75 \div 32.5 = 0.7$

Practice Problems:
Choose the correct answer.

1. $5\overline{)\,.20}$

a. 0.4 b. 0.40 c. 0.04 d. 4

2. $6\overline{)\,3.6}$

a. 6 b. 0.6 c. 0.06 d. 0.006

3. $3\overline{)\,2}$ to 2 decimal places

a. 0.66 b. 6.67 c. 0.67 d. 6.66

4. $8\overline{)\,3}$

a. 0.375 b. 3.75 c. 375 d. .0375

5. $.25\overline{)\,1}$ Choose the correct division problem. (t

a. $25\overline{)\,1}$ b. $25\overline{)\,100}$ c. $25\overline{)\,1,00}$ d. $.25\overline{)\,100}$

6. Solve $3.1\overline{)\,9.3}$

a. 3 b. 0.3 c. 0.03 d. 0.31

7. $7.3\overline{)\,2.92}$

a. 4 b. 0.4 c. 0.04 d. 4.4

8. .75 + .4 to nearest tenth

a. 1.8 b. 1.9 c. 2 d. 1.87

Section V Changing Fractions to Decimals and Decimals to Fractions

Remember that fractions are really division problems.

$\frac{3}{4}$ means 3 divided by 4

$\frac{12}{2}$ means 12 divided by 2

Step 1: To change a fraction to a decimal, set the fraction up as a division problem with the denominator as the divisor.

Example:

$$\frac{3}{4} = 4\overline{)3}$$

Step 2:

Carry out the division.

$$\begin{array}{r} .75 \\ 4\overline{)3.00} \\ -28 \\ \hline 20 \\ -20 \\ \hline \end{array}$$

$\frac{3}{4} = 0.75$

Step 1: To change a decimal to a fraction, write the decimal with its proper denominator.
-Remember, decimals are actually ways to write fractions with denominators of 10, 100, 100 etc.

$.1 = \frac{1}{10}$ $.01 = \frac{1}{100}$ $.001 = \frac{1}{1000}$

Example: $.75 = \frac{75}{100}$

Step 2: Reduce to lowest term. $\frac{75}{100} = \frac{3}{4}$

Sometimes a decimal will contain a fraction as part of it. $.16\frac{2}{3}$

Step 1: Set up fraction with proper denominator.

$$.16\tfrac{2}{3} = \frac{16\tfrac{2}{3}}{100}$$

Step 2: Multiply the numerator and the denominator by the denominator of the small fraction in the numerator.

$$\frac{16\frac{2}{3}}{100}\,\frac{\times\ 3}{\times\ 3} = \frac{\overset{(3\times16)+3\,(\frac{2}{3})}{48+2}}{300} = \frac{50}{300}$$

Step 3: Reduce fraction to lowest term $\frac{50}{300} = \frac{1}{6}$

To check, change $\frac{1}{6}$ to a decimal.

Step 1: $6\overline{)1}$

Step 2,3:

$$\begin{array}{r} .16\frac{4}{6} \\ 6\overline{)1.00} \\ \underline{6} \\ 40 \\ \underline{36} \\ 4 \end{array}$$

Write remainder as a fraction with the divisor as the denominator.

Reduce fraction $.16\frac{4}{6} = .16\frac{2}{3}$

Practice Problems:
Change to fractions and simplify.

1. $0.5 =$
2. $1.3 =$
3. $0.43 =$
4. $4.25 =$
5. $.33\frac{1}{3} =$

Hint: Set up as fraction with $33\frac{1}{3}$ as the numerator. $\dfrac{33\frac{1}{3}}{(100)}$

Hint: Mult. both numerator and denominator by the denominator of the small fraction.

$$\frac{33\frac{1}{3}}{100} \begin{array}{l} \times\ (3) = \\ \times\ (3) = \end{array}$$

6. $0.06\frac{1}{4} =$

Hint: $\dfrac{6\frac{1}{4}}{100}$

Hint:

$\frac{6\frac{1}{4}}{100}$ x
x

Change to decimals. Some decimals will be written with 2 decimal places and an exact fraction. (ex: $\frac{2}{3} = 0.66\frac{2}{3}$)

7. $\frac{3}{4} =$

8. $\frac{1}{2} =$

9. $\frac{3}{11} =$

10. $\frac{2}{3} =$

11. $\frac{1}{5} =$

12. $\frac{5}{6} =$

Section VI Percent

You deal with the concept of percent (%) everyday. It is common to see a department store advertising a sale with 30% off the original price. Every time you borrow money from the bank, you pay the bank a percent of that money in interest. Likewise, the bank pays you a percentage of interest on money in your savings account. Your GED exam will be scored according to the percent you answered correctly.

As you can see, learning to work with percent can be very helpful.

Percent is a way of comparing values to 100. In your study of fractions, you found that $\frac{25}{100}$ is read as 25 hundredths. You learned that 25 hundredths could also be written as a decimal .25.

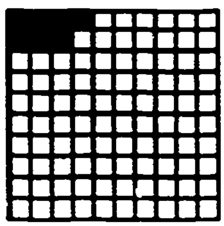
Now you will learn a new way to express hundredths -- PERCENT .25 = 25%.

So: $\frac{25}{100} = 0.25 = 25\%$

Example: Express .07 as a percent.

You know that .07 is equal to $\frac{7}{100}$ and that it represents 7 parts of 100.

Remember that % is a way of comparing a value to 100. In the grid to the right, you see that 7 of 100 parts are shaded. This equals 7%.



(Make grid and divide into 100 parts)
Shade 7 parts

Example: Express $\frac{100}{100}$ as a percent.

NOTE: $\frac{100}{100}$ as percent.

If 100 parts of 100 are shaded, then 100% is shaded. (Show grid with all spaces shaded.)

Practice Problems:

1. Write eighteen hundredths as a percent.
2. Write 24 hundredths as a decimal.
3. Write 24 hundredths as a percent.
4. Write 13% as a fraction.
5. Write 10% as a fraction.
6. How many hundredths are in 3.5%?
7. Express $2\frac{1}{3}$ hundredths as a %.

Change % to decimals.

Example: Change 14% to a decimal.

Step 1: Rewrite the given number omitting %. 14

Step 2: Move decimal 2 places to the left. .14

Example: 13.5%

Step 1: 13.5

Step 2: .135 = 0.135

Example: .2%

Step 1: .2

Step 2: .002 = 0.002

Practice Problems:
Change to decimals

1. 24% =
2. .04% =
3. 9.1% =
4. 121% =
5. 356% =

Change decimals to percents

Step 1: Rewrite given number.

Step 2: Move decimal 2 places to the right.
(Do not write the decimal point if after moving it, it is located at the end of the number.)

Step 3: Write the percent sign (%)

Example: .36

Step 1: .36

Step 2: 36. = 36

Step 3: 36%

Example: 8.4

Step 1: 8.4

Step 2: 840. (Note that sometimes add a zero to move the decimal 2 places).

Step 3: 840%

Example: $00\frac{2}{5}$

Step 1: $.00\frac{2}{5}$

Step 2: $00.\frac{2}{5}$

Step 3: $\frac{2}{5}\%$

Practice Problems:
Change to percents:

1. 1.1 =

2. .3 =

3. $.04\frac{1}{4} =$

4. 3 =

5. .004 =

6. $.00\frac{3}{8} =$

Changing percents to fractions or mixed numbers.
Example: Change 25% to a fraction.

Step 1: Make a fraction by writing the given number as a fraction with the denominator of 100: $\frac{25}{100}$

Step 2: Reduce fraction to lowest terms $\frac{25}{100} = \frac{1}{4}$

$25\% = \frac{1}{4}$

HINT: If the fraction has a numerator of 100 or more, the answer will contain a whole number.

Example: Change 425% to a fraction.

Step 1: $\frac{425}{100}$

Step 2: $4\frac{1}{4}$

Changing Common Fraction to Percents

Example: Change $\frac{24}{40}$ to a percent

Step 1: Divide numerator by denominator, finding the quotient to 2 decimal places.

$$40\overline{)24.00}$$
$$.60$$
$$\underline{24.0}$$

Step 2: Change the decimal to a % by moving the decimal 2 places to the right and add the percent sign. .60% = 60%

Example: $4\frac{2}{9}$

Step 1: $4\frac{2}{9} = \frac{38}{9}$

$$
\begin{array}{r}
4.22\frac{2}{9} \\
9\overline{)\,38.0} \\
\underline{36} \\
20 \\
\underline{18} \\
20 \\
\underline{18} \\
2
\end{array}
$$

Change to fraction with devisor as denominator.

Step 2: $4.\underbrace{22}_{12}\frac{2}{9} = 422\frac{2}{9}\%$

Practice Problems:
Change to percents

1. $\frac{5}{3} =$

Hint: First find decimal equivalent to 2 places $(1.66\frac{2}{3})$

2. $\frac{5}{9} =$

3. $\frac{12}{5} =$

4. $\frac{9}{3} =$

5. $\frac{3}{6} =$

Finding a percent of a number

Example: What is 30% of $150.??

Step 1: Change % to a decimal. 30% = .30

Step 2: Multiply the given number by this decimal

$$
\begin{array}{r}
\$150 \\
\underline{\times\ .30} \\
\$45.00
\end{array}
$$

Example: Find 25% of 175

Step 1: 25% = (.25)

Step 2:
$$\begin{array}{r} 175 \\ \underline{x.25} \\ 875 \\ \underline{+350} \\ 43.75 \end{array}$$

25% of 175 = 43.75

Practice Problems:

1. Find 15% of 2:2
2. Find 12% of 10
3. Find 200% of 15
4. Find 1105 of 100
5. Find 5% of 25
6. Find $12\frac{1}{2}\%$ of 800

Hint: $12\frac{1}{2}\%$ to a decimal $(.12\frac{1}{2})$

Hint: First mult. 800 x 12= 9600
Then mult. $\frac{1}{2} \times 800 = 400$

Hint: 9600 and 400 are your partial products

$$\begin{array}{r} 800 \\ \underline{x.12\frac{1}{2}} \\ 12x80=9600 \\ \frac{1}{2}x800=\underline{400} \\ 100.00 \end{array}$$

7. Find $3\frac{1}{2}\%$ of 30

Hint: Partial products of $.03\frac{1}{2}$ and 30
3 x 30 = (90)
$\frac{1}{\cancel{2}_1} \times \cancel{30}$ (15)

8. Find 0% of 10 =

Word Problems with Percent

On the GED, you will be asked to find what % one number is of another number and to find a number when only a % of it is known. Just as you used proportions to solve problems involving fractions, you can also set up a proportion to solve problems involving percent.

Remember, when using proportions to solve a problem, part of the problem will always be unknown. Solve for the unknown portion using the rules for cross multiplying.

Finding a missing percent

Example: 18 is what % of 36?

Step 1: Set up a proportion. You know that you are trying to find a value compared to 100, $\frac{?}{100}$ which is the same as 18 compared to 36, $\frac{18}{36}$.

$$\frac{18}{36} = \frac{?}{100}$$

Step 2: Solve the proportion by cross multiplying.

$$\frac{18}{36} \quad \frac{?}{100} \qquad 18 \times 100 = 1800$$

Divide by the remaining number.

$$36 \overline{)1800} = 50$$
$$\underline{1800}$$

Since $\frac{18}{36} = \frac{50}{100}$ then 18 is 50% of 36.

Finding a missing total

Example: John used 8% of his money for electricity. He spent $75 for electricity. How much money did John have before he paid the electric bill?

Step 1: Set up a proportion.

One of the ratios is $\frac{8}{100}$ - $\frac{\text{\% of income spent on electric}}{\text{total income}}$

Another ratio is $\frac{75}{?}$ - $\frac{\text{amount spent for electricity}}{\text{total income}}$

$\frac{8}{100} = \frac{75}{?}$ - $\frac{\text{amount spent}}{\text{total income}}$

Step 2: Cross multiply and divide to solve.

$$\frac{8}{100} = \frac{75}{?} = \frac{100 \times \$75}{8} = \frac{\$7500}{8} = \$940$$

So 100% (or all) of John's income is $940.

Find a missing part

Example: Mary earned 6% interest on her $5000. How much interest did she earn?

Step 1: Set up a proportion.

$$\frac{(6)}{100} = \frac{?}{(5000)} = \frac{\text{interest}}{\text{total money}}$$

Step 2: Solve using rule for cross multiplying.

$$\frac{6}{100} \quad \frac{?}{5000} = \frac{6 \times 5000}{100} =$$

$$= 300$$

The missing part is 300 so Mary earned $300 in interest.

Two Part Problems

A problem of this type requires more than one operation (addition, subtraction, multiplication, division) to solve it.

Example: Look at the previous problem.
"Mary earned 6% interest on $5000." But suppose that instead of asking how much interest did Mary earn, the question is "how much money will she have with interest?"

The problem now requires 2 steps.

1. Figure out how much interest she earned.
2. Add the interest to her original amount of $.

Step 1: You already know that Mary earned $300 in interest.

$$\frac{6}{100} = \frac{300}{5000}$$

Step 2: Add the interest to the original amount of money. $5000 + $300 = **$5300.**

Example: Andrew sold 45 raffle tickets for his club's fund raiser. Elizabeth sold 75. Altogether, the club members sold 1500 tickets. How many % more raffle tickets did Elizabeth sell than Andrew?

Step 1: Find what % each sold be setting up a proportion.

Andrew

$$\frac{45}{1500} = \frac{?}{100} \quad \frac{\text{tickets sold}}{\text{total tickets}}$$

$$\frac{45}{1500} = \frac{3}{100}$$ Andrew sold 3%

Elizabeth

$$\frac{75}{1500} = \frac{?}{100} \quad \frac{\text{tickets sold}}{\text{total tickets}}$$

$$\frac{75}{1500} = \frac{5}{100}$$ Elizabeth sold 5%

Step 2: The question asked how many % More raffle tickets did Elizabeth sell than Andrew.

5% - 3% = 2%

Elizabeth sold 2% more than Andrew.

Practice Problems:
Choose the correct answer for each question.

1. The local university had an enrollment of 3200 in 1986. There was an increase of 15% in 1987. How many students were enrolled in 1987?

   a. 3200 students
   b. 2720 students
   c. 3680 students
   d. 3750 students

2. Mindy bought a winter coat for $120. This price reflected a 20% discount. What was the original price of the coat?

   Hint: $\frac{20}{100} \neq \frac{120}{?}$ $120 is not the amount of the discount (20%) but rather the remaining % or 80% of the original cost.

   a. $150
   b. $6000
   c. $175
   d. $96

3. A survey discovered that 56% of the household in an area containing 3422 households received delivery of the local paper. What number of households do not receive the paper?

   a. approx. 1916 households
   b. approx. 1506 households
   c. approx. 1711 households
   d. none of the above

Hint: There are 2 ways to solve this problem.
1. Subtract the % of homes that receive the paper from 100% to find what % of homes don't receive the paper.
2. Find how many households receive the paper (56% of 3422) and subtract that number from the total number of households.

4. The price of apples increased 15% to a price of \$1.15 per pound. Grapes are now \$1.48 per pound after a 9% increase. Which of the following statements is true?

   a. grapes were originally more expensive than apples
   b. apples were originally more expensive than grapes
   c. apples and grapes cost the same before the increase
   d. the % of increase for grapes is greater than the % of increase for apples

Hint: The apples at \$1.15 per pound reflect the original price of the apples, 100% plus the 15% increase. So \$1.15 equals 115% of the original price.

Hint: Set up proportion of: $\frac{\text{cost with increase}}{\text{original cost}} = \frac{\$1.15}{?} = \frac{115}{100}$

Hint: Cross multiply and divide to solve for the original price.

$$\frac{\text{cost of increase}}{\text{original cost}} = \frac{\$1.48}{?} = \frac{(109)}{100} = 100\% + 9\% \text{ increase}$$

5. Joe bought a new television for \$285. This was 85% of the original price. How much money to the nearest dollar did he save?

   a. \$33
   b. \$60
   c. \$50
   d. \$45

Hint: First determine the original cost of the television

$$\frac{85}{100} = \frac{285}{?}$$

Hint: Subtract sale price from original price

6. A discount store always sells their lawn furniture at a 15% reduction from the original price. At the end of the season, they had a clearance sale which reduced the sale price by another 10%. What was the clearance price of a picnic table that originally sold for \$135? Round off to the nearest cent (or hundredth).

Hint: Find reduced price first, then find 10% of the reduced price.

   a. \$101.25
   b. \$121.50
   c. \$114.75
   d. \$103.26

Unit II - Chapter 1 •Length and Height Measure•

Section I Introduction

To complete the following sections on measurement, you will need to know the following equivalencies in standard and metric.

1. 1 mile = _____ feet (5,280)
2. 1 yard = _____ feet (3)
3. 1 foot = _____ inches (12
4. 1 kilometer = _____ meters (1,000)
5. 1 hectometer = _____ meters (100)
6. 1 dekameter = _____ meters (10)
7. 1 decimeter = _____ meters (.1)
8. 1 centimeter = _____ meters (.01)
9. 1 millimeter = _____ meters (.001)

Correct answers are in parentheses. If a student missed any answers in 1-3 the computer should go to a review section (section Ia).

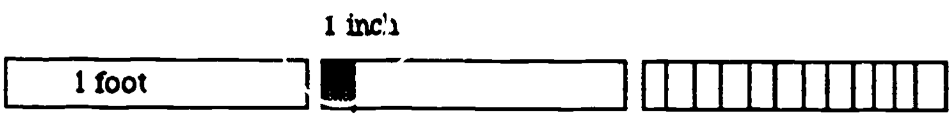
If a student missed a question in 4-9 then the following Hint should appear on screen:

Following problems in this section on length and height measure involve conversions within the metric system.

If you need to review the section on metrics, type YES. If you want to continue, press RETURN.

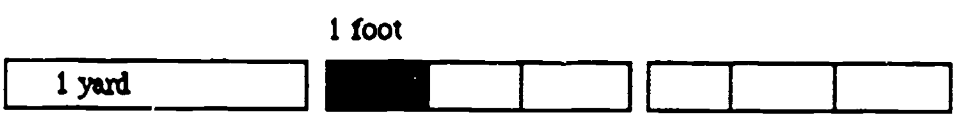
Section Ia. Practice Problems

1.



How many inches in 1 foot?

2.



How many feet in 1 yard?

3. How many inches in 1 yard?

Hint: There are 12 inches in 1 foot and 3 feet in one yard so therefore there is 36 inches in 1 yard.

4. How many yards in 1 mile?

Hint: Remember there are 3 feet in 1 yard and there are 5,280 feet in 1 mile.

Section II Converting Units Within the Same System

In the following problems you will practice converting units of measures within the same system. You will need not convert between standard and metric measurements. (Hints given if student gives a wrong answer).

1. How many yards in 4,560 feet?

Hint: Since you are changing from a smaller unit to a larger one, you divide. There are 3 ft. in 1 yard. So you would divide 4,560 by 3.

2. How many inches in 523 feet?

Hint: This time you are changing from a larger to smaller unit so you should multiply. Since there are 12 in. in one foot, you multiply 12 by 523.

3. Convert 50 inches into feet.

Hint: Remember there are 12 inches in 1 foot and you are changing from a smaller to larger unit.

$$\begin{array}{r} 4\frac{1}{6} \\ 12\,\overline{)\,50} \\ \underline{48} \\ 2 \end{array}$$

4. Convert 40 meters to centimeters

Hint: By knowing the prefixes of metric equivalents that tells you to move the decimal point 2 places to the right since meters are larger than centimeters (40.00 = 4000.)

5. Convert 17 millimeters to meters

Hint: This time the decimal is moved 3 places to the LEFT since a millimeter is smaller than a meter.
(17 = .0 1 7. = .017)

6. Convert 156 hectometers to meters

Hint: Move decimal to the RIGHT since you are going from a larger to smaller unit.

Section III Arithmetic Operations

In order to complete the problems in this section, you will need to add, subtract, multiply, and divide using units of length/height measure. Some of the problems will require you to convert units of measure to complete the problem.

Example: Add 23.6 meters and 50 centimeters.

Step 1: Change 50 centimeters to meters so you will be working with the same units of measure.

50 centimeters = .5 meters

Step 2: add 23.6 meters
+ .5 meters
24.1 meters

Example:
Multiply 2 feet 3 inches by 6

Step 1: Multiply feet and inches separately
2 feet x 6 = 12 feet
3 inches x 6 = 18 inches

Step 2: Add the totals = 12 ft 18 inc.

Step 3: Since 18 inches is more than 1 foot convert to feet and inches. 18 in. = 1 ft. 6 in. This should give you 13 ft. 6 inches.

Step 4: Since 13 feet is more than 1 yard, convert feet and yards
13 ft = 4 yds. 1 foot
The final answer is 4 yd 1 ft 6 inches

Before you begin the following set of problems, remember the 5 steps of problem solving:

1. What is being asked for?
2. What information do you need to solve the problem?
3. Which arithmetic operation will you need to use?
4. Do the arithmetic and check your work.
5. Does your answer make sense?

Problems:

1. The Smiths' new house is 40 ft 5 in long. How many inches long is the house?

Hint: There are 12 in. in one foot. You must multiply 12 x 40 = 480 inches. Don't forget the other 5 inches to make the final answer 485 inches.

2. Joe has one table 2 yd. 2 ft. long and another 1 yd. 2 ft. How long are the two tables put together?

Hint: Add the two measures to get 3 yd. 4 ft. Since there are 3 feet in 1 yard that can be changed.

3. Sue had 5 yd. 1 ft. 3 in. of material to make a dress. She only used 3 yd. 1 ft. 5 in. How much material doe she have left?

Hint: To solve this problem you must subtract.

$$\begin{array}{r} \overset{1}{\cancel{5}}\text{ yd. } \overset{\overset{3}{0}}{\cancel{1}}\text{ ft. } \overset{15}{\cancel{3}}\text{ in.} \\ -\ 3\text{ yd. } 1\text{ ft. } 5\text{ in.} \\ \hline 1\text{ yd. } 2\text{ ft. } 10\text{ in.} \end{array}$$

Begin subtracting the smaller unit first. You will have to borrow 12 inches from 1 foot. Next you will have to borrow 3 ft. from 1 yard.

4. Mr. Jones bought plank that was 7 feet 8 in. long. If he cuts the plank into 4 equal pieces, how long will each piece be?

Hint: Divide 7 ft. 8 in. by 4

$$4\overline{)7\text{ ft } 8\text{ in}}$$
$$\begin{array}{rr} \underline{4} & \underline{+36} \\ 3 & 44 \end{array}$$

$$\begin{array}{r} 11 \\ 4\overline{)44} \\ \underline{-44} \\ 0 \end{array}$$

After dividing 7 by 4 you have 3 feet left over that should be changed to in.

5. Tom is 2 meters tall and Bill is .0018 kilometers tall. By how much meters does their height differ?

Hint: Before doing any computation you must change the measurements to the same unit. .0018 kilometers = 1.8 meters - Then subtract the differences.

6. Mary bought 5 pieces of felt. Each piece was 2 feet 7 inches long. How much felt did she buy?

2 ft. x 5 = 10 ft.
7 in. x 5 = 35 in.
10 feet 35 inches

7. A carpenter ordered .3 dekameters of timber. How much timber in meters did he order if it was doubled?

Hint: First change .3 dekameters to meters = 3 meters
Then multiply by 2.

REVIEW
Things to remember:

Standard:

1. When going from a smaller to larger unit, divide.
2. When changing from a larger to smaller unit, multiply.

Metric:

1. When going from smaller to larger unit, move decimal to the LEFT.
2 When going from larger to smaller unit, move decimal to RIGHT.

Unit II - Chapter 2 •Weight and Measure•

Section I Equivalencies

You must know the following equivalencies to complete the following sections.

1. 1 ton =	pounds (2,000)
2. 1 pound =	ounces (16)
3. 1 kilogram =	grams (1,000)
4. 1 hectogram =	grams (100)
5. 1 dekagram =	grams (10)
6. 1 decigram =	grams (.1)
7. 1 centigram =	grams (.01)
8. 1 milligram =	grams (.001)

After the student understands these equivalencies, the student can choose to continue to the next section.

Section 2 Converting Units

The following problems will give you practice in converting units of measure within the same system.

1. How many pounds in 48 ounces ?

HINT: Changing from a smaller to larger unit tells you to divide 16 ounces in 1 pound.

2. Convert 2 ton 510 pounds to pounds.

HINT: To change from a larger to a smaller unit multiply 2,000 by 2 since there are 2,000 pounds in one ton. Then add the 510 pounds.

3. Convert 19 milligrams to grams.

HINT: Changing from a smaller to larger unit tells you to move the decimal to the LEFT.

4. Convert 632 grams to dekagrams.

HINT: Decimal should be moved to the left one place because a dekagram is 10X that of a gram.

5. Convert 510 grams to hectograms.

HINT: This time decimal moved to the left two places because a hectogram is 100X that of a gram.

Section III Arithmetic Operations

The following are word problems in which you will need to do various arithmetic procedures using weight measures.

EXAMPLE:

Add 60.2 grams and 5.3 decigrams.

Step 1: Change 5.3 decigrams to grams so you can add the same unit.

5.3 decigrams = .53 grams

Step 2: Add.

$$\begin{array}{r} 60.20 \text{ grams} \\ + \quad .53 \text{ grams} \\ \hline 60.73 \text{ grams} \end{array}$$

EXAMPLE:

Subtract 10 lb, 11 oz from 1Ton, 10 lb, 10 oz.

Step 1:

$$\begin{array}{rrrr} & 1 \text{ Ton} & 10 \text{ lb} & 10 \text{ oz} \\ - & & 10 \text{ lb} & 11 \text{ oz} \\ \hline & & 1{,}999 \text{ lb} & 15 \text{ oz} \end{array}$$

1. Which has more, a bottle with 33 ounces or one with 2 pounds 6 ounces ?

   Hint: First change 2 pounds 6 ounces to ounces. Remember there are 16 ounces in one pound. 2 pounds equals 32 ounces. Then add the other 6 ounces which gives you 38 ounces.

2. Sue's package weighs 4 lb. 8 oz. and Janet's package weighs 6 lb. 9 oz. If John carries both packages, what is the total weight?

   Hint: Add 4 lb. 8 oz. + 6 lb. 9 oz. = 10 lb. 17 oz.
   Since there are 16 oz. in 1 pound, that can be changed to 11 lb. 1 oz.

3. Joe's wheelbarrow weighs 42 lbs. 5 oz. After filling his wheelbarrow with leaves it weighed 45 lb. 2 oz. How much do the leaves weigh?

   Hint: Subtract:

$$\begin{array}{rr} 44 & 18 \\ \cancel{45} \text{ lb.} & \cancel{2} \text{ oz.} \\ - 42 \text{ lb.} & 5 \text{ oz.} \\ \hline 2 \text{ lb.} & 13 \text{ oz.} \end{array}$$

(Borrow 1 pound and change it to 16 ounces so you can subtract.

4. Joe carries 23 pound 2 ounce stone and he makes 8 trips. How much does he carry all together?

Hint: multiply each unit

23 pounds x 8 = 184 pounds

2 ounces x 8 = 16 ounces

16 ounces equals one pound so the answer is 185 pounds.

5. Kathy's candy bar weighs .1 kilograms. Jane's gob weighs .07 kilograms. Sue's piece of candy weighs 43 grams. How much does Kathy and Sue's candy weigh together in grams?

   Hint: First change .1 kilograms to 100 grams. Then add 100 grams + 43 grams and the answer is 143 grams.

6. A recipe calls for 12 grams of baking powder and 2.26 grams of salt. If I only wanted to make half a batch, how much baking powder and salt will I need in kilogram?

   Hint: divide by 2: 12 ÷ 2 grams = 6 grams - 2.26 ÷ 2 = 1.13 grams

   Then change to kilograms:

   6 grams = .006 kilograms - baking powder

   1.13 grams = .00113 kilograms salt

7. A book weighs 1.45 kilograms and another weighs 1.2 kilograms. How much is weight of both books in grams?

   Hint: Add 1.45 kg.
   + 1.20 kg.
   2.65 kg.

   Remember to always line up the decimal points when doing calculations. Change to grams: 2.65 kg = 2,650 grams.

Unit II - Chapter 3 •Liquid Measure•

Section I ____ Introduction

To complete the following sections on liquid measure, you must know the following equivalencies:

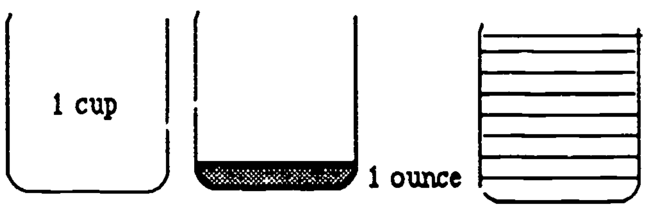
1. 1 pint = _____ ounces (16)
2. 1 cup = _____ ounces (8)
3. 1 quart = _____ pints (2)
4. 1 gallon = _____ quarts (4)
5. 1 liter = _____ milliliters (1000)
6. 1 liter = _____ centiliters (100)
7. 1 liter = _____ deciliters (10)

(Correct answers are in parentheses)

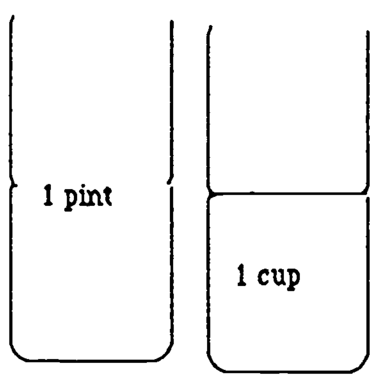
If a student missed an answer in 1-4, the computer should go to the review section labeled Section Ia. If a student missed a question in 5, 6 or 7 then the following Hint should appear on the monitor.

> The following problems in this section on LIQUID MEAURE involve conversions within the metric system. If you need to review the section on metrics, type YES. If you wish to continue with LIQUID MEASURE, press RETURN.

Liquid Measure: Section Ia.



How many ounces are in 1 cup? (8)

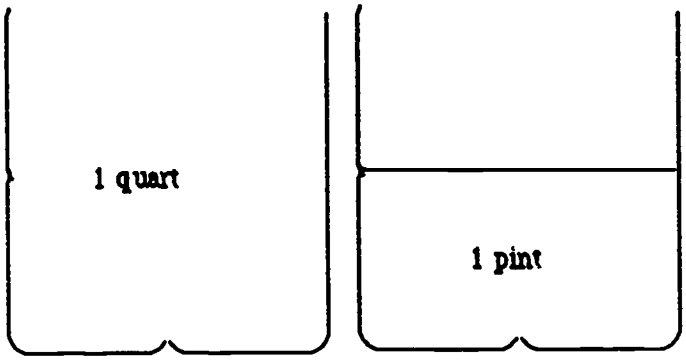


How many cups are in 1 pint? (2)

So how many <u>ounces</u> are in 1 pint? (16)

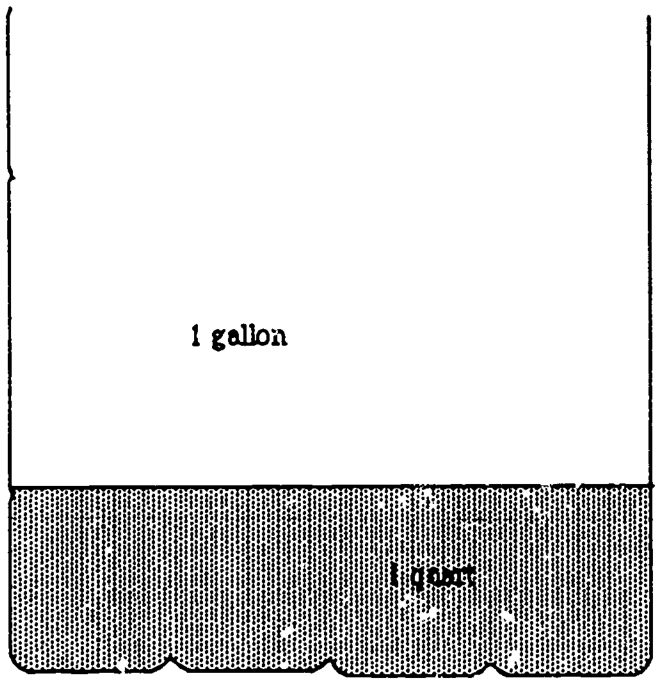
(If incorrect answer is given, the following hint should appear on the monitor):

<u>HINT</u>: There are 8 ounces in 1 cup. How many cups are in 1 pint? (2)
So how many ounces are in 1 pint? (16)



How many pints are in 1 quart? (2)
How many <u>ounces</u> are in 1 quart? (32)

<u>HINT</u>: Remember, there are 16 ounces in 1 pint and there are 2 pints in 1 quart.



How many quarts are in 1 gallon? (4)

How many pints are in 1 gallon? (8)

<u>HINT</u>: Remember, there are 2 pints in 1 quart and 4 quarts in 1 gallon.

Section II Converting Units of Liquid Measure Within the Same System

In the following problems, you will practice converting units of measure within the same system. You will not need to convert between Metric and Standard units of measure. (Hints are given if student gives wrong answer).

1. How many quarts are in 3 gallons? _____ (12)

Hint: How many quarts are in 1 gallon? (4) So how many quarts are in 3 gallons? (12)

4 qts. x 3 = 12 qts.

2. How many ounces are in 5 cups? (40)

Hint: There are 8 ounces in 1 cup.

3. Convert $1\frac{1}{2}$ quarts into cups. (6 cups)

Hint: If there are 4 cups in each quart, how many cups are in $\frac{1}{2}$ quart? (2)

4. Convert 5000 liters to kiloliter. (5k)

Hint: 1000 liters = 1 kiloliter

5. How many centiliters are in 3.2 liters? (320 c)

Hint: Remember, conversions within the metric system are simple. You just need to move the decimal point.

6. 500 ml = (0.5) liters?

Hint: You know that 1000 ml = 1 liter.
Therefore, 500 ml is less than 1 liter.

7. Convert 25 ml to centiliters. 25 ml = (2.5) cl.

Hint: How many militers are in 1 centiliter? (10) Right, there are 10 ml in 1 cl.

NOTE: Additional hints are given only after student gives the wrong answer. The student has the opportunity to complete the problem after each hint.

Section III Arithmetic Operations

In the following problems, you will be asked to add, subtract, multiply and divide using units of liquid measure. In some of the problems, you will need to convert units of measure to complete the problem.

Example:
Add 5.3 liters and 50 centiliters.

Step 1: Change 50 centiliters to liters so you will be working with the same units of measure.
50 centiliters = .5 liters

Step 2: Add 5.3 liters
+ .5 liters
5.8 liters

Example:
Multiply 2 cups 3 ounces by 6.

Step 1: Multiply cups and ounces separately.
2 cups x 6 = 12 cups
3 ounces x 6 = 18 ounces

Step 2: Add the totals
12 c 0 ounces
+ 0c 18 ounces
12 c 18 ounces

Step 3: Since 18 ounces is more than 1 cup, convert to cups and ounces.
18 oz. = 1 cup 2 ounces.

Step 4: Add 12 cups
+ 1 cups 2 ounces
13 cups 2 ounces

Example: Subtract 43 centiliters from 5.2 liters.
Express the answer in liters.

Step 1: Since your answer is to be expressed in liters, convert the 43 cl to liters.
43 cl = .43 l

Step 2: 5.2 liters 5.20 liters
- .43 liters - .43 liters
4.77 liters

Before you begin the following set of practice problems, remember the 5 steps of problem solving.

1. What is being asked for?
2. What information do you need to solve the problem?
3. Which arithmetic operation will you need to use?
4. Do the arithmetic and check you work.
5. Does your answer make sense?

Practice Problems

1. Steve poured a 12 ounce glass of milk from a full 1/2 gallon container. How many ounces of milk are left in the container?

Hint: Since the answer is to be expressed as ounces, your first step is to convert 1/2 gallon to ounces. 1/2 gallon =

2. Mary made 3 quarts of strawberry jelly. She plans to give each of her 5 friends a jar. How many pints can she give to each friend?

Hint: How many pints are in 3 quarts? (6)

3. A recipe calls for 3/4 c oil, 6 oz. milk and 1/2 c water. How many cups of liquid are used in this recipe? Simplify your answer.

Hint: Since the answer is to be expressed as cups, convert 6 oz. to a fraction of 1 cup.
$\frac{6}{16} = \frac{3}{8}$ cup

Hint: Add $\frac{3}{4}$c, $\frac{1}{2}$c, $\frac{3}{8}$c

(Remember to find a common denominator when adding fractions).

4. If 2 liters of soda are divided equally between 8 people, how many liters will each get? Express your answer as a fraction.

5. A pharmacy received a shipment of .35 kl of penicillin. To separate this into .25 l bottles, how many bottles will be needed?

Hint: Change to same unit of measure. .35 kl = (350 l)

Hint: Divide 350 l by .25 liters.

6. Jack took 4 containers to the local gas station. He knows that each container holds exactly 3 qt. 14 ounces. How much gasoline will he need to fill these containers. Express your answer in gallons, quarts & ounces.

Hint: Multiply each unit separately and add totals.
3 qt. x 4 = 12 qts. 14 oz. x 4 = 56 ounces

12 qt. 56 ounces

Hint: Simplify: Can qts. be changed to gallons? (yes)
12 qt. = 4 gallon

Hint: Now, you have 4 gallons 56 ounces. (Can you simplify further?) (yes)
56 ounces = 1 qt. 14 oz.

7. How many ounces of water must be added to a 24 oz. can of frozen orange concentrate to make 1 gallon of orange juice?

Hint: How many ounces are in 1 gallon? (128)

Hint: How many ounces of concentrate do you already have? (24)

Hint: How many more ounces of liquid do you need to make 1 gallon?

8. The directions on a container of weed killer call for 12 cl of weed killer per 500 cl of water.. How many l of mixture will this make if the mixture is doubled? (10.24l)

Hint: How is the answer to be expressed? liter or centiliters?

Hint: Add 12 cl and convert to liters

$$\begin{array}{r} 12 \text{ cl} \\ \underline{+500 \text{ cl}} \\ 512 \text{ cl} \end{array} = 5.12 \text{ liters}$$

Hint: Is 5.12 l the answer? (No)

Hint: The problem says the mixture is doubled.

$$\begin{array}{r} 5.12\text{l} \\ \underline{\text{x} \quad 2} \end{array}$$

Unit II - Chapter 4 •The Metric System•

Section I Introduction

The metric system is a standard unit of measurement based upon powers of ten. The system is based upon a unit of length called a meter. A meter is approximately 39" long - a little more than one yard. The meter is the basic unit of length in the metric system. The gram is the basic unit of weight and the liter is the basic unit of capacity.

Different units of length, weight or capacity are obtained by combining a prefix with a base unit (meter, gram or liter). These are the prefixes, their values and their symbols:

	prefix	value	symbol
	kilo	1000 (one thousand)	k
*	hecto	100 (one hundred)	h
*	deka	10 (ten)	da
*	deci	0.1 (one tenth)	d
	centi	0.01 (one hundredth)	c
	milli	0.001 (one thousandth)	m

*=not commonly used

Section II Units of Length

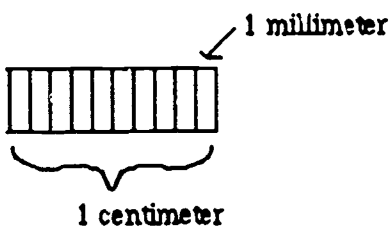
The relationships among metric units are based upon powers of ten. If you don't try to convert the metric system into the English system, the metric system is much easier to use. However, at first, it may be able to picture in your mind what these units represent.

1 meter is approx. 1 yard
1 centimeter is approx. the width of your little finger
1 millimeter is approx. as thick as a piece of paper.

Examine the following picture of a meter.
(Divide equally into 100 sections and 1000 sections)

Notice that the meter is divided into 100 centimeters.
100 centimeters = 1 meter
1 centimeter = .01 meter or one hundredth of a meter.

Now, notice that each centimeter is also divided into 10 equal parts. (Show meter divided into centimeters).
-divide each centimeter into millimeter
-enlarge one section of a centimeter



There are 10 millimeters in 1 centimeter.
1 meter (m) = 1000 millimeters (mm)
1 meter (m) = 100 centimeters (cm)
1 millimeter (mm) = .001 meters (one thousandth of a meter)
1 centimeter (cm) = .01 meter (one hundredth of a meter)

Now suppose you lined up 1000 meters end to end. To represent 1000 meters, use the prefix "kilo" which represents 1000. 1000 meters = 1 kilometer (km)

Look at the following chart showing the prefixes and base units combined.

kilometer = 1000 meters = km
* hectometer = 100 meters = hm
* dekameter = 10 meters = dam
* decimeter = 0.1 meters = dm
centimeter = 0.01 meters = cm
millimeter = 0.001 meters = mm

*-not commonly used

Change each of the following to meters:

Example: 5 cm = ? m (.05)

If 1 cm = .01 meter then 5 cm = .05 meters or five hundredths of a meter since a meter is divided into 100 centimeters.

1. 15 cm = _____m (.15)
2. 1 cm = _____m (.01)
3. 6 cm = _____m (.06)
4. 1 km = _____m (1000)
5. 10 km = _____m (10,000)
6. 534 mm = _____m (0.534)

Hints for these problems may be supplied by showing pictures of a meter divided into mm and cm.

Notice that in the preceding problems, the decimal point is all that moves. The numbers remain the same. We simply move the decimal point to the left or right depending on the unit.

It is possible to convert units using the following chart. Count the number of places from one unit to the other and move the decimal point that many places in the same direction.

1000	100	10	1	.1	.01	.001
kilo,	hecto,	deka,	meter,	deci,	centi,	milli

over to right 1 place

Example: .43 cm = ? mm 4.3 (to right one place)

Note: Show each step on chart for each of the following examples.

5 meters = ? mm (5000mm)
(to right 3 places so decimal moves right 3 places)

432 mm = ? dm (4.32)
(left 2 places)

Each should be shown using the chart.

12 m = ? km (.015)
(left 3)

Try this! Use the chart to help.

kilo, hecto, deka, meter, deci, centi, milli

1. Change each of the following to decimeters.

439m = ? dm (4390) Hint. move decimal 1 place right.

42 cm = ?dm (4.2)
9,626mm = ?dm (96.26) Hint: deci is? places to left? (2)

2. Change each of the following to meters.

   5km = ?m <u>Hint</u>: Remember, when you need to move the decimal to the right of a whole number, you must add zeros. 5 km = 5000.

   37cm = ?m (.37m)

   9.647mm = ?m (9.647m)

3. Change each of the following to kilometers.

   533m = ?km

   9,647mm = ?km <u>Hint</u>: How many places to the left is kilo from milli? (6)

   14 m = ?km <u>Hint</u>: Remember, you may need to add zeros on the left of the number to move decimal to the left. .014

Section III Measure of Weight

In the introduction to the metric system, you were told that a gram is the basic unit of measure for weight. Just as you added prefixes to meters to obtain other units, so you will add prefixes to the base unit gram to obtain other units for weight measurement.

unit	symbol	value
kilogram	kg	=1000 grams
hectogram	kg	=100 grams
dekagram	dag	=10 grams
gram	g	=1 gram
decigram	dg	=0.1 gram
centigram	cg	=0.01 gram
milligram	mg	=0.001 gram

To change from one metric unit to another, use the same procedure described in Section II.

Use this chart:

kilo - hecto - deka - gram - deci - centi - milli

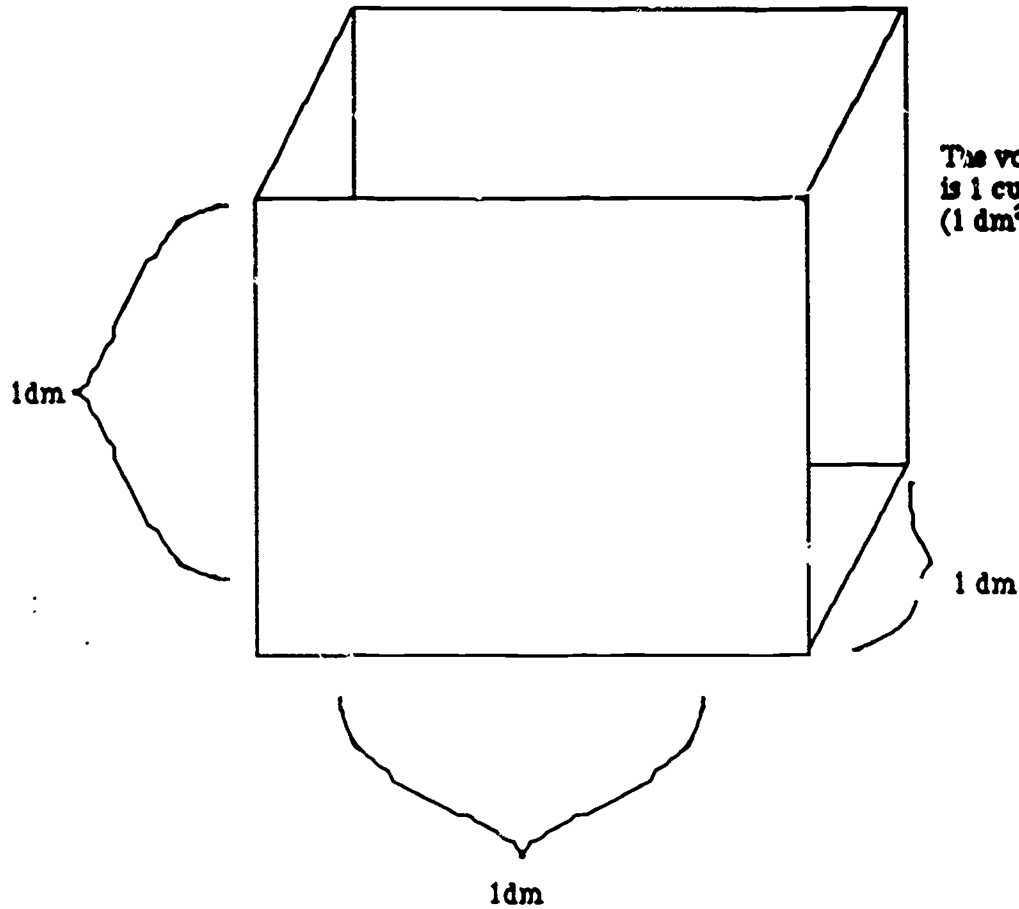
Complete each of the following:

1. ?mg = 1 g Hint: You are changing from grams to mg. Will you move the decimal left or right? (right)
2. ?g = 425cg
3. 1kg = ?g = ?cg
4. 78 cg = ? mg
5. 17g = ? kg

Section IV Measure of Capacity (Volume)

The liter is the basic unit of measurement for volume. A liter of liquid is approx. equal to one quart. The term liter is not technically an official part of the metric system. It is simply a common name for a cubic decimeter (dm^3).

1 liter (L) = $1 dm^3$



The volume of this box is 1 cubic decimeter. ($1\ dm^3$) or 1 L.

The metric prefixes used with linear measure can also be used with the liter. Symbols and conversions work the same way they did for length.

Unit	Symbol	Value
kiloliter	kL	1000 liters
hectoliter	hL	100 liters
dekaliter	dL	10 liters
liter	L	1 liter
deciliter	dL	0.1 liter
centiliter	cL	0.01 liter
milliliter	mL	0.001 liter

Convert each of the following:

Use this chart to help you.

kilo - hecto - deka - liter - deci - centi - milli

1. 37L = ?mL
2. 264mL = ?L
3. 1 kL = ?L
4. 10kL = ?L
5. 543mL = ?cL

Unit III - Chapter 1 •Data Analysis•

Data analysis covers various sections ranging from reading tables and graphs to ratios. Reading tables and graphs will be covered first.

Section Ia

1.

Average QPA

Name	1988	1986	1984	1982
Jeff	2.74	2.63	2.76	2.58
Karen	3.33	3.23	3.00	3.02
Mary	2.12	1.99	1.78	3.02
Joe	2.12	2.43	2.54	2.56
Tom	3.96	3.95	3.98	3.96

Before you begin answering the questions you should read the title which tells you what the table is about. The rest of the information is arranged in columns (down) and rows (across).

a. Which year did Mary have her highest QPA?

Hint: Find Mary's name and go across to find the highest number, go up the column to see what year it reads.

b. What was Joe's QPA in 1984?

Hint: Find the column heading 1984 and go down until you are across from Joe's name and that is your answer.

c. Who had the highest QPA in 1986?

Hint: Find the column labeled 1986 and find the largest number. Once you've found it go across to find the name.

d. Who had the greatest increase in QPA and in what year?

Hint: The key word is increase. Be sure you look for the ones who have increased and not decreased. Karen had the greatest increase because she went from a 3.00 in 1984 to a 3.23 in 1986.

e. What two people had the same QPA and in what year?

Hint: Mary and Joe both had a QPA of 2.12 and by looking at the heading of that row it tells you in 1988.

2.

Average Weekly Earnings

Name	1986	1985	1984
Bill	$456	$450	$435
Janice	$420	$400	$390
John	$238	$232	$226
Mary	$238	$230	$226

a. Who had the highest weekly earnings in 1986?

Hint: Find the column 1986 and find the highest number, then go across and you find the answer, Bill.

c. Who had the lowest weekly earnings in 1985?

Hint: Look across the row for 1985 and see that Mary had a weekly earning of $230.

c. Who had the greatest rise in weekly earning from 1984 to 1985?

Hint: Subtract the 1984 earnings from the 1985 earnings for each person. You should find that Bill had the greatest rise of $15.

d. Who had the least amount of increase from 1985 to 1986?

Hint: Subtract the 1985 earnings from the 1986 earnings for each person. You should find that both John and Bill had the least amount of increase of $6.

e. Between each person, what was the greatest dollar difference in earnings in 1985?

Hint: Find the column 1985 and subtract the lowest weekly earning from the highest weekly earning to get $220.

Section Ib

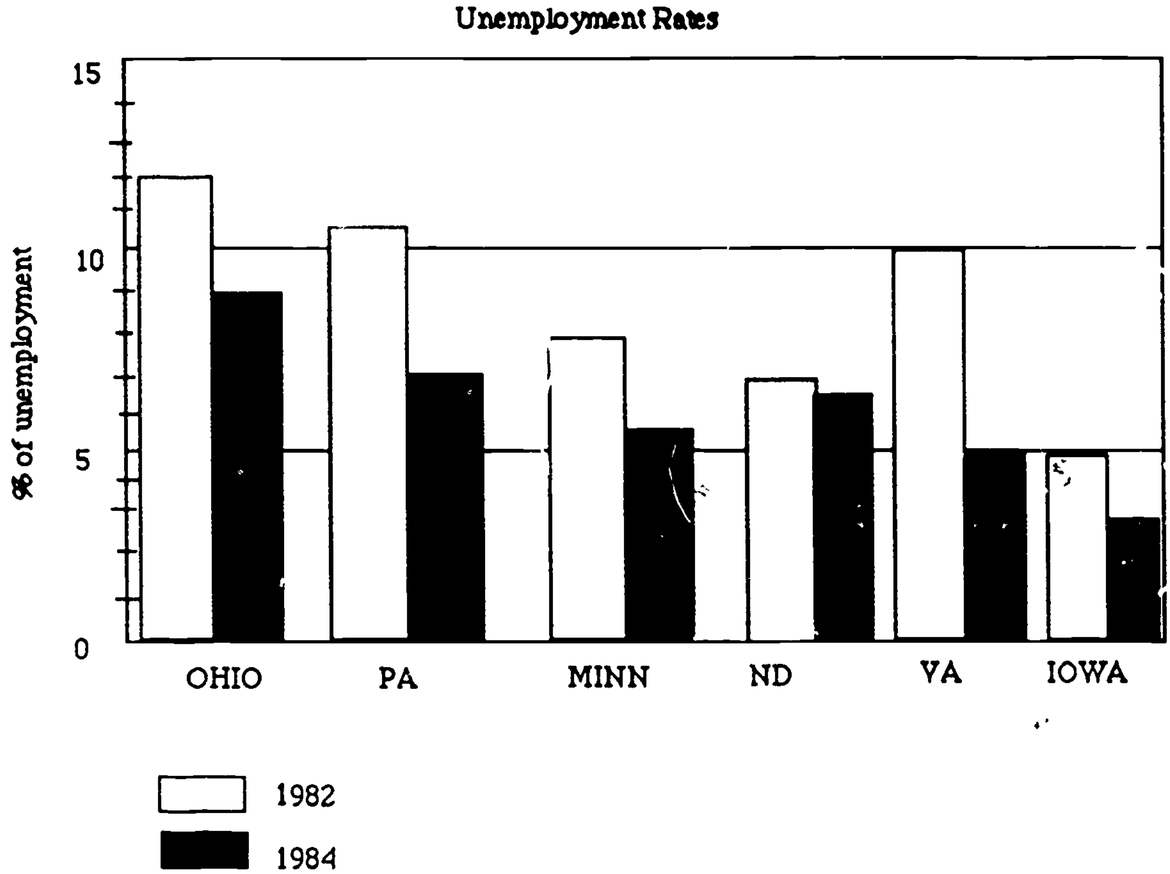
A graph is an easy way to compare information in picture form. There are three types of graphs: bar, line, and circle.

A bar graph compares numbers by using bars of different lengths to represent the numbers.

Four steps to take in reading a graph:

1. Read the title.
2. Read all headings on columns and rows so you know what is being compared.
3. Look at the numbers to see how they change.
4. If the graph uses color or symbols, look for a key to tell you what they mean.

1.



a. Which state had the highest unemployment rate in 1982?

Hint: The key tells you that 1982 is white. See which white area is the most and you will see that is Ohio.

b. Which state had the lowest unemployment rate in 1984?

Hint: 1984 is represented by back and lowest is Iowa.

c. What is the unemployment rate in PA in 1982?

Hint: Find PA and you will see that the white area, representing 1982, is in between 10% and 11% to give an answer of 10.5% or 10 1/2%.

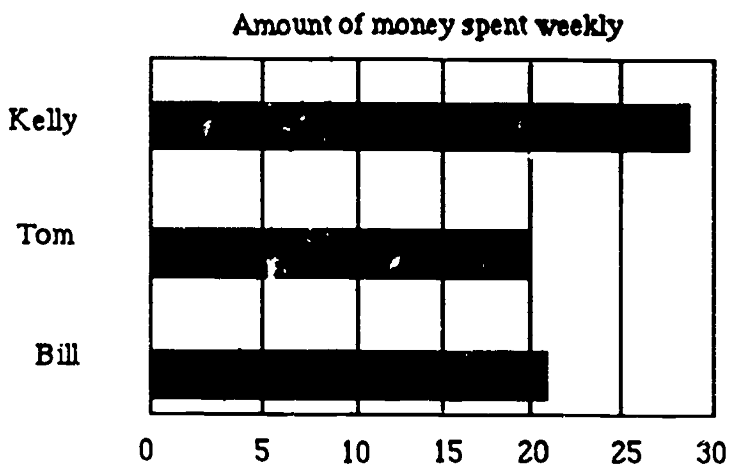
d. What is the unemployment rate in Ohio in 1984?

Hint: The black section representing the unemployment rate in Ohio shows 9%.

e. Which state had the least and most amount of change in unemployme ite?

Hint: The bars show the least amount of change in ND and the most in VA

2.



a. How much does Kelly spend weekly?

Hint: The bar shows she spends approx. $28 since it is slightly below 30.

b. Who spends the least amount?

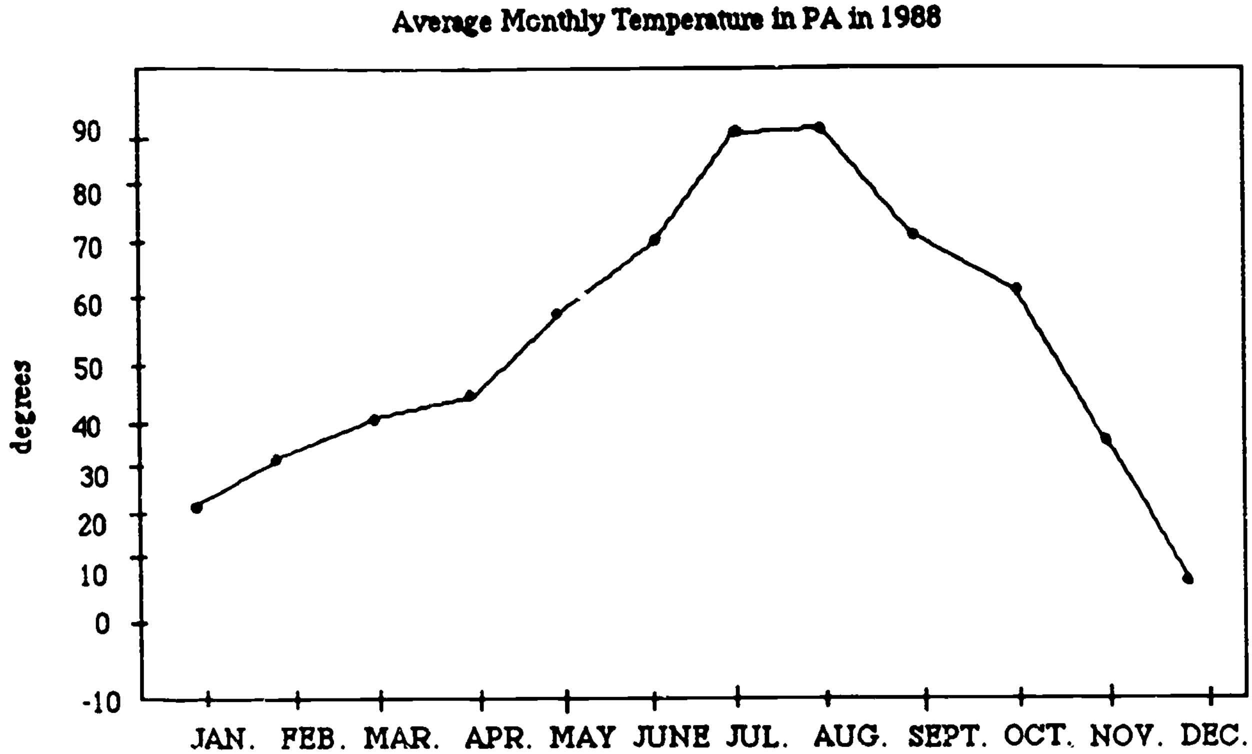
Hint: The shortest bar is the one representing Tom.

c. How much money do all 3 of them spend weekly?

Hint: First see how much each of them spend separately - Kelly-$28; Tom-$20; Bill-$21. Then add the 3 numbers together to get $70.

A line graph is a graph that shows change, over a period of time, by a line that connects points on the graph.

3.



a. Which month was the coldest and what was the temperature?

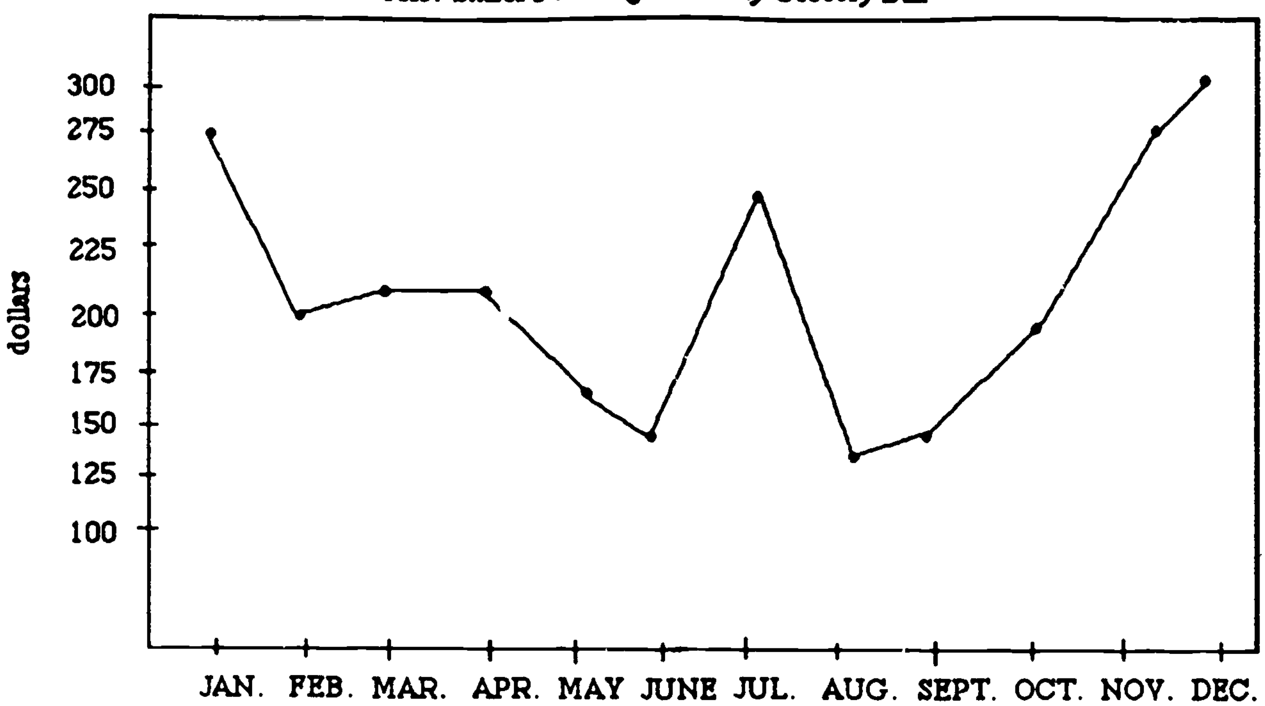
Hint: The lowest point was in the month of December and by going across you will see that it is 10°.

b. What was the average temp. in June?

Hint: First find the point that represents June and go across and see that it is a little above 70° to get 72°.

4.

Mrs. Smith's Average Monthly Grocery Bill



4.

a. How much does Mrs. Smith spend on groceries in April?

Hint: Find the point that represents April. The number should be a little above 200 but no more than 225. Therefore, approximately $210.

b. When does Mrs. Smith spend the most on groceries?

Hint: Find the highest point and go down to find the month December.

c. How much does Mrs. Smith Spend altogether in Jan., Aug., and Nov.?

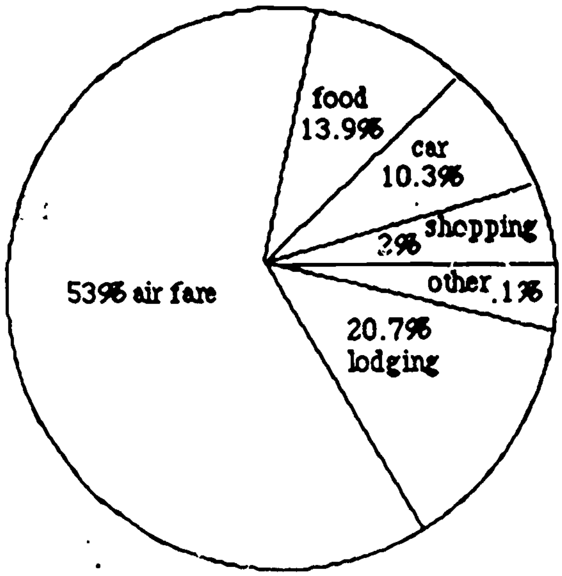
Hint: First find out how much she spends in Jan., Aug., and Nov. by moving up to the point and then by moving over to find the number. Jan.-$275; Aug., $125; Nov , $270. Then add them together to get $670.

d. How much does Mrs. Smith save from July to August?

Hint: Another way to put this is what is the difference. Find the amount spent in July and Aug. and then subtract. $253 - $125 = $128.

A circle graph looks like a pie cut into sections. The entire pie represents 100% and the slices represent part of the whole.

Ann's Travel Budget



5.

a. What percentage of Ann's travel was spent on food?

Hint: By looking at the graph it tells you 13.9% was spent of food.

b. What percent of Ann's trip was car and food?

Hint: Find how much she spent on car and how much she spent on food and add. (10.3 + 13.9 = 24.2%)

c. How much more does Ann spend on air fare than lodging?

Hint: Subtract the amount of lodging from air fare. (53.0 - 20.7 = 32.3%)

d. If Ann didn't go shopping and used that amount for food, how much would she have for food?

Hint: Find amount for shopping and add it to the amount for food. (13.9+2.0=15.9%)

Section II Mean and Median

The MEAN is the number you get when you add many different numbers together and divide the total by the number of items you added (AVERAGE).

Examples: Find the mean

1. 82, 36, 47, 49

Hint:
$$\begin{array}{r} 82 \\ 36 \\ 47 \\ +\ 49 \\ \hline 214 \end{array} \qquad 214 \div 4 = 53.5$$

2. 126, 360, 258

Hint:
$$\begin{array}{r} 126 \\ 360 \\ +\ 258 \\ \hline 744 \end{array}$$

3. 27, 138, 350, 5, 990

Hint:
$$\begin{array}{r} 27 \\ 138 \\ 350 \\ 5 \\ +\ 990 \\ \hline 1510 \end{array}$$

4. 23.7, 45.7, 53.8

Hint:
$$\begin{array}{r} 23.7 \\ 45.7 \\ +\ 53.8 \\ \hline 123.2 \end{array}$$

5. 723, 824.3, 26, 7.8

Hint:
$$\begin{array}{r} 723 \\ 824.3 \\ 26 \\ +\ 7.8 \\ \hline 1581.1 \end{array}$$

NOTE: If a student has a problem with these questions, the student can type "yes" for further MEAN questions, otherwise, type "return" to continue this section.

6. The first 6 days in October were the highs of 53°, 55°, 63°, 39°, and 51°. What is the mean highs so far in October?

Hint: Add all the temps.

```
  53
  55
  63
  42
  39
+ 51
 303        Divide by 6  (303 ÷ 6 = 50.5)
```

7. Mary's bills were $117 in January, $100 in February, and $87 in March. What was the average amount of Mary's bills? ($101.3)

Hint: Add the 3 monthly bills:

```
    $117
    $100
+   $ 87
    $304     Divide by 3  (304.0 ÷ 3 = 101.3)
```

8. Mr. Brown's bowling scores were 138, 106, 178, 170, 142, 133, 151. What was his average bowling score? (145)

Hint: Add Mr. Brown's scores:

```
   138
   106
   178
   170
   142
   133
 + 151
                          145.4
                       7 ) 1018
                           7
                           31
                           28
                            38
                            35
  1018     Divide by 7:      3
```

The MEDIAN is the central number in a set of numbers written in order of size.

Examples: Find the median

1. 36, 45, 56 (45)

Hint: The middle number is 45.

2. 11, 19, 25, 28, 37 (25)

Hint: The middle number is 25.

3. 270, 360, 471, 530, 725, 998, 1000 (530)

Hint: The middle number is 530.

NOTE: Again, students will have the opportunity to study more median problems if they don't understand by typing "yes" otherwise "return."

4. The scores of a girls' basketball team were 38, 80, 60, 73, 42, 44, 56, 82, 58. What is the median? (58)

Hint: First put all the scores down from smallest to largest: 38, 42, 44, 56, 58, 60, 73, 80, 82. The central number is 58.

5. Mary's scores on her test were 20, 25, 17, 21, 15. What is her median score? (20)

Hint: Arrange the scores from smallest to largest and then find the middle number.
15, 17, 20, 21, 25

6. Here are 5 runners on the track team. Find their mean and median height. (67.4 in) (67 in)

Tom	66 in.
Joe	69 in.
Mark	65 in.
David	70 in.
Mike	67 in.

Hint: Mean: Add the heights and divide by 5.

$$\begin{array}{r} 66 \\ 69 \\ 65 \\ 70 \\ +\ 67 \\ \hline 337 \end{array}$$

Divide 337 + 5 = 67.4 in.

Median: Arrange from smallest to largest. 65, 66, 67, 69, 70

Section III Ratios, Proportion and Probability

A RATIO is a comparison between two numbers. It can be written with a colon, fraction, or word to.

Example: Write the following with colon, fraction, and "to." Reduce to lowest terms when possible.

1. 1 woman to 3 men
2. 3 girls to 6 toys
3. 5 inches to 2 inches
4. 7 right to 14 wrong
5. 5 on to 3 off

6. In a shopping mall, Mary bought clothes in 3 out of the 5 women's clothing stores. Write this as a ratio.

Hint: Remember that the numbers in a ratio are written in the order in which they come in the sentence.

7. On a test Janice got 16 questions right and 2 wrong. Express the ratio of the number of questions right to the total number of questions.

Hint: First add the number right to number wrong (16+2=18). The ratio of the number right to total can be written 16:18. Then reduce ratio to lowest terms. 8:9

8. Jim is 42, Sally is 36, and Sue is 38. What is the ratio of Jim's age to Sally's age?

Hint: Can be written as 42:36. Then reduce to 7:6.

• A PROPORTION is a statement that two ratios are equal and can be written as two equal fractions. For example: 3:4 and 6:8 can be written as 3/4 and 6/8. These are equal fractions because you get 3/4 if 6/8 is reduced. If you write the statement 3/4 = 6/8, you have written a proportion.

1. The ratio of boys to girls at the party was 5 to 3. If there were 9 girls at the party, how many boys were there?

Hint: The best way to solve these word problems is to use a grid.

5	?	boys
3	9	girls

Multiply the diagonals and divide by the unused number. 5 x 9 = 45 and 45 ÷ 3 = 15.

2. A recipe calls for 3 eggs and this makes 25 cookies. If you wanted to make 40 cookies, how many eggs would you need?

Hint: First set up the ratios on a grid.

3	?	eggs
25	40	cookies

Multiply diagonals: 3 x 40 = 120
Divide: 120 ÷ 25 = 4.8

3. Janice went to the sweater sale and bought 2 sweaters for $41. How much would it cost for 5 sweaters?

Hint: Put information you know on the grid.

$41	?	Price
2	5	Sweaters

Multiply diagonals: 41 x 5 = 205
Divide by 2: 205 ÷ 2 = 102.5

4. During a sale, socks were priced at 3 pr. for $3.60. How much would 8 pair of socks cost?

Hint: Put information on grid.

3	8	Socks
\$3.60	?	Price

Multiply: 8 x 3.60 = 28.8
Divide:

$$\begin{array}{r} 9.60 \\ 3\,\overline{)\,28.8} \\ \underline{27} \\ 18 \\ \underline{18} \end{array}$$

5. Mr. Green drove 512 miles in 9 hours. He then had to stop for gas. At this rate, how many hours will it take him to drive 789 miles? (13.87 hours)

<u>Hint</u>: Use the grid for your information.

512	789	miles
9	?	hours

Multiply: 9 x 789 = 7101
Divide: 7101 ÷ 512 = 13.87

A PROBABILITY is a ratio written in fraction form. It is the likelihood that something will happen.

<u>Example</u>: Probability = $\frac{\text{possible ways for something to happen}}{\text{total number of possible outcomes}}$

1. Jim has a lock on his locker at the gym. He has forgotten what the 3 digit combination is so he decides to try 831. What is the probability that his locker will open?

<u>Hint</u>: There are 1,000 possible combinations (10 first digit 0-9, 10 second and 10 third; 10 x 10 x 10 = 1000) and Jim has 1 possible way so the probability is 1/1,000 that 831 will work.

2. Tim rolled the die, what is the probability of him getting a number of 3?

<u>Hint</u>: A die has 6 sides and his change of getting a 3 is only 1.

3. Mrs. Klon's second grade class has 15 girls and 10 boys. If she randomly selects a student to be the leader, what is the probability of her choosing a girl?

Hint: The total number of outcomes is boys plus girls ($15 + 10 = 25$). Since the question asks for the probability of girls that is 15. $\frac{15}{25}$ which can be reduced to $\frac{3}{5}$.

4. Amy has 2 red sweaters, 1 purple sweater, 3 yellow sweaters and 2 gray sweaters. She can't decide which one to wear so if she selected a sweater at random, what is the probability it would be red?

Hint: The total outcome is 8 ($2 + 1 + 3 + 2$). Since there are 2 red sweaters there are 2 possible ways the sweaters could be chosen. $\frac{2}{8} = \frac{1}{4}$.

5. There are 5 cherry, 3 grape, 1 orange, 6 lime and 1 strawberry flavored lollipops left. What is the probability of Joey randomly selecting a cherry lollipop?

Hint: Total number of outcomes (add all flavors together) is 16. There are 5 cherry left so the answer is 5/16 and this cannot be reduced.

Unit IV - Chapter 1 •Arithmetic Operation Using Integers•

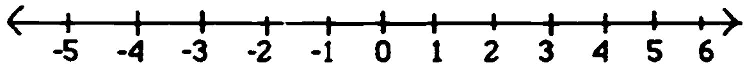
Section 1 The Set of Integers

The set of numbers you used in the unit, Number Relationships, included zero and positive numbers. Whole Numbers -- 0, 1, 2, 3 . . .

In algebra, you will extend this set to include negative numbers, numbers less then zero. This is called the set of integers. Integers -- . . . -3, -2, -1, 0, 1, 2, 3 . . .

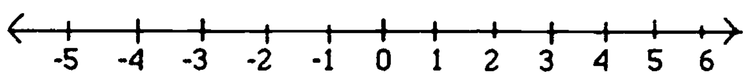
The set of integers include all positive numbers, all negative numbers and zero.

Integers can be represented using a <u>number line</u>.



Just as the ellipses (. . .) included in set notation for the Integers mean that there are an infinite number of positive and negative numbers, the arrows on the end of the number line indicate that the number line can be extended indefinitely.

Each number has an <u>absolute value</u>. The absolute value of a number is that number's distance from zero on the number line.



<u>Example:</u> Absolute value of 3 = 3 since 3 is 3 places away from zero.
The absolute value of a number is indicated by 2 bars - / /

<u>Example</u>: /3/ = 3 Reads - the absolute value of 3 = 3.

/-2/ = 2 The absolute value of -2 = 2 since -2 is 2 places from the zero on the number line.

Absolute value is always positive.

Try these:

/5/ = (5)
/-5/ = (5)
/321/ = (321)
/-3/= (3)

Great! Now you are ready to add and subtract integers.

Section II Addition and Subtraction of Integers

Rule 1: When adding signed numbers (positive and negative numbers) add the numbers without regard to the sign and use the common sign.

Example: $3 + 4 = 7$

Notice that this is not written $+3 +4 = +7$

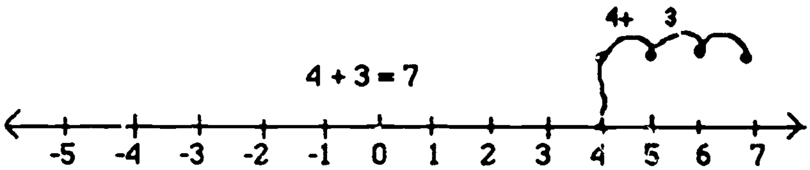
Any number written without a sign is considered positive.

Since 3 and 4 are both positive, the answer is positive.

Example: $-4 + -3 = -7$

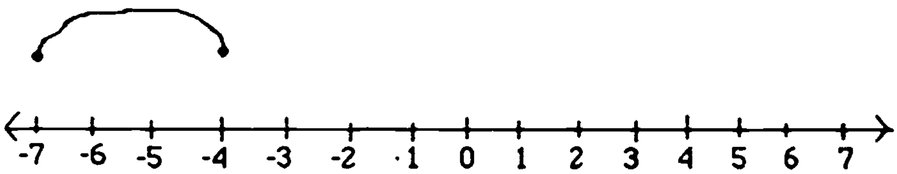
Add 4 and 3. Write the answer using the common sign (-).

Look at these examples using a number line.



Starting at zero, move 4 places to the right. To add 3 move 3 more places to the right. This is the answer. Remember, move to the right for positive numbers.

$-4 - 3 = -7$

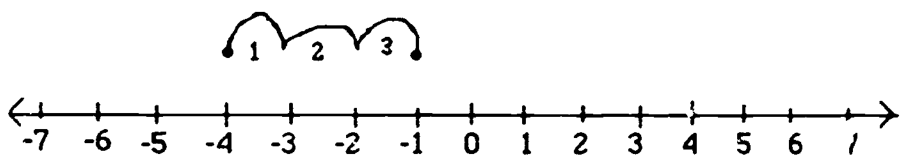


Find -4 by moving 4 places to the left of the zero. Move 3 more places to the left. The answer is -7. Remember, move to the left for negative numbers.

Sometimes, you will be asked to add numbers with different signs.

Example: $-4 + 3 = ?$

First, using the number line:



Find -4 on the number line. Since 3 is positive, move 3 places to the right. The answer is -1. $-4 + 3 = -1$.

Obviously you will not be able to use a number line when adding large numbers. There is a rule when adding numbers with unlike signs.

<u>Rule 2</u>: To add numbers with different signs, subtract the smaller number from the larger number and use the sign of the larger number.

-4 + 3 = 7

Subtract 3 from 4. 4 - 3 = 1. Use the sign of the larger number in the original problem. -1

Another way to write this rule is:

Subtract the absolute value of the smaller number from the absolute value of the larger number. Write the answer using the sign of the larger number.

-4 + 3 = /4/ - /3/ = 4-3

<u>Practice Problems</u>:

Find the following sums:

1. +7 + +4 =
2. -7 + +4 =
3. -13 + +6 =
4. -9 + -3 =
5. -27 + +27 =
6. -2/3 + +1/3 =
7. -3/7 + -2/7 =

*If there is a problem with the last two, screen should display a reminder that the rules are the same for adding signed fractions. If a review is needed for arithmetic operations with fractions, refer to Unit I, Chapter 2.

Now you are ready to <u>subtract</u> integers. Subtraction is defined as adding the negative. This makes subtraction easy since you already know how to add signed numbers.

<u>Rule</u> To subtract one signed number from another, change the sign of the number being subtracted and follow the rules for addition.

<u>Example</u>: -11 - 4 =

First, change the sign of the number being subtracted. (4) becomes (-4).
Now add (-11) and (-4) using addition rules. (-11) + (-4) = (-15).

Since you now have 2 numbers with like signs, add & use the common sign.

EASY!!!!!

Now try these:

1. -6 - -4 =
<u>Hint</u>: -6 + ? (4)

2. +8 - +4 =
<u>Hint</u>: +8 + ? (-4)

3. 12 - 10 =
<u>Hint</u>: Numbers without signs are considered <u>positive</u>.
<u>Hint</u>: (+12) - (+10) =
<u>Hint</u>: (+12) + (-10) =

4. $14 - 25 =$

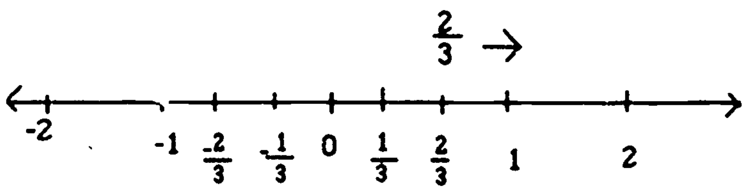
Hint: Rewrite if you need to as: $(+14) - (+25)$ then follow the rule for subtraction.

5. $\frac{4}{5} - (-\frac{1}{5}) =$

Hint: Rewrite using subtraction rule: $+4/5 + 1/5 = 5/5$

If you are having trouble on the GED with addition or subtraction of integers, draw a number line to help you picture the problem.

Example: $\frac{2}{3} - (-\frac{1}{3})$



Find $\frac{2}{3}$ on number line. Follow subtraction rule: $\frac{2}{3} + (+\frac{1}{3})$

Section III Multiplication and Division of Integers

There are two rules for multiplication and division of integers. If you remember these two rules you will have no problem.

Rule 1: If the signs are the same, the answer is positive.

Rule 2: If the signs are different, the answers are negative.

Examples of rule 1:

$4 \times 3 = 12$ Also written as $(4)\,(3) = 12$

Since 4 and 3 are both positive, the answer is positive.

NOTE: Parentheses around the numbers with no addition or subtraction sign between them indicate multiplication. $(3)\,(6) = 3 \times 6$

$(-3)\,(-12) = 36$

Both 3 and 12 are negative so the answer is positive.

Examples of rule 2:

$(-3)\,(4) = -12$

Signs are different so the answer is negative.

$(-6) \div (3) = -2$

The signs of 6 and 3 are different so the answer is negative

$(7)\,(\frac{-3}{5}) = \frac{-21}{5}$

What if the problem is $(7) \div (-\frac{3}{5})$?

Remember that division is the same as multiplication by the reciprocal. The reciprocal of $\frac{-35}{3}$ is? $(\frac{-5}{3})$

The problem is now $(7) \times \frac{-5}{3} = \frac{-35}{3}$

Find the following products and quotients:

1) $6 \times 4 =$
2) $(-6) \times 3 =$
3) $(5)\,(4) =$
4) $(-12) \div (3) =$
5) $\frac{-15}{3} =$

Hint: Fraction bar indicates division or $(-15) \div (3)$

6) $\frac{-21}{-7} =$

Before you go on to the next section, let's review the rules for addition, subtraction, multiplication, and division of signed numbers (Integers).

Fill in the missing words:

Addition:

Rule 1: When adding signed numbers add the numbers without regard to the signs and use the (common) sign.

Rule 2: To add numbers with different signs, subtract the (smaller) number from the (larger) number and use the sign of the (larger) number.

Subtraction:

Rule 1: To subtract one signed number from another, change the (sign) of the number being (subtracted) and follow the rules for (addition).

Multiplication and Division:

Rule 1: If the signs are the same, the answer is (positive).

Rule 2: If the signs are different, the answer is (negative).

Section IV Order of Operations

In many algebra problems, you will be asked to do more than one operation.

Example: 4 + (3x4) -2

To solve, you must follow the: Order of Operations Rule

1. Do the work in the parentheses first.
2. Multiply and divide in order through the expression.
3. Last, add and subtract through the expression.

So: 4 + (3x4) -2 =

Inside parentheses first = 4 + (12) - 2
Look for mult. or div. - there isn't any so: Add and Subtract: 4 + 12 - 2=
(16) - 2 = 14.

Let's look at another example:

6 - (4x3 + (-2)) =
6 - (12 + (-2)) =

Notice that the order of operation is followed inside the ( ) -- multiplication first, then addition. so 6 - (10) = -4.

If you don't follow the order of operations rule, your answer won't be the same. For example, if you simply completed each operation from left to right, your answer would be:

6 - (4x3 + (-2) =
2 x 3 + (-2) =
6 + (-2) = 4 This is not the right answer.

Let's review the order of operations.

1. Parentheses first.
2. Multiplication and Division.
3. Addition and Subtraction.

NOTE if there is a fr on bar work on numerator and denominator separately. Remember, the order of operations also apply within the parentheses.

Now, solve the following expressions:

1. (-3) + 4 x 2 =

Hint: Mult. first: -3 + 4x2 =
-3 + 8 =

2. (10+4) (-3 - 4x2) =

Hint: ( ) first: (14) (-3 -?) (8)

3. (3x -2) - (2x7) =

Hint: (?) - (?) -- (-6) - (14) =

4. $\frac{-2 + 4}{(6) + (6)} =$

Hint: Remember, when there is a fraction bar, work on numerator and denominator separately.

Hint: $\frac{\text{numerator}}{\text{denominator}} = \frac{-2 + 4}{(+6 + (-6)} = \frac{(2)}{-1}$ or (-2)

5. $\frac{(10+4)(-3-2)}{4 - (-1)} = \frac{-70}{5}$ or

Hint: numerator first (10+4) (-3 -2) =
() first = (14) (?) = (-5)
(14) (-5) = (-70)

Hint: Denominator 4 - (-1) = (?) 5

Unit IV - Chapter 2 •Algebraic Equations•

Section I Variables

In algebra, letters are often used to stand for numbers that we want to find. These letters are called variables. Any letter can be used but x and n are the most common.

You will be learning to write algebraic expressions using variables. An algebraic expression is simply an expression that includes variables and numbers instead of words.

Example: A number decreased by 5.

Let x = the number. So x - 5 is an algebraic expression.

Example	Expression
Four plus a number	$4 + x$
Eight less than a number	$x - 8$
Four times a number	$4x$ (means $(4)(x)$)
A number divided by three	$n/3$ or $\frac{n}{3}$ or $n \div 3$
Six divided by a number	$\frac{6}{n}$
The product of three and a number	3b (remember, any letter can represent the variable or unknown)

Rewrite the following as algebraic expressions. Use x to represent the unknown.

1. A number increased by fifteen
2. The sum of a number and six
3. The product of four and a number
4. Six times a number
5. Three less than a number
6. A number divided by 7

Section II   Equations

An equation is a statement that two amounts are equal. An equation can be written with words or with symbols. (Just as an expression can be written with words or symbols).

In algebra, most equations will contain a variable.

Examples: $4 + x = 1$
$3 - x = 0$
$10x = 40$
$\frac{15}{x} = 3$

To solve an equation you will find the number which makes the equation true.

Sometimes, you will be able to see the answer immediately.

Example: $4 + x = 5$ You know that $4 + 1 = 5$ so $x = 1$. 1 is the solution to this equation.

But sometimes, the answer is not this obvious.

Example: $x + 15 = 74$.

The object is to get the variable alone on one side of the equation. x = some number.

Rule 1: Whatever operation you perform on one side of the equation must be performed on the other side of the equation.

Rule 2: Addition Property of Equations
The variable's value does not change if you add the same number to both sides of the equation.

Look at $x + 15 = 74$. Add (-15) to both sides.

$$\begin{array}{rcl} x + 15 & = & 74 \\ \underline{+ \quad (-15)} & \underline{=} & \underline{+(-15)} \\ x & = & 59 \end{array}$$

59 is the solution.

To check the solution, substitute it for the variable. $x + 15 = 74$; $59 + 15 = 74$.

Look at this equation: $5x = 20$ You can solve this equation using the Multiplication Property of Equations. The variable's value does not change if both sides are multiplied (or divided) by the same number.

$5x = 20$

$$\frac{\overset{1}{\cancel{5}}x}{\underset{1}{\cancel{5}}} = \frac{20}{5}$$ Both sides divided by 5

$1x = 4$ 4 is the solution
Simplify: $x = 4$

NOTE: When the number 1 appears before the variable, 1x, the variable is written by itself since any number multiplied by 1 is itself.

Sometimes, you will have to combine similar (like) terms before solving an equation.

Example: $4x + 9 - 2 + x = 32$

4x 9 x and x are terms or expressions being added or subtracted.

-Terms consisting of numbers only (9, 12) are similar.
-Terms containing variables (letters) must have the exact same variables to be considered similar. 4x and x are similar terms.

Consider 3x and 3xy. These are not similar. 2y and 17y - these are similar.

To solve $4x + 9 - 2 + x = 32$.
Combine terms: $5x + 7 = 32$
Add -7: $5x + 7 - 7 = 32 - 7$
$5x = 25$

Divide by 5 $\frac{5x}{5} = \frac{25}{5}$ $x = 5$

Are the following similar term? Yes/no

5xy, 5x (no)
3xyz, -xyz (yes)
1/3x, 5 (no)
1/3x, -2/5x (yes)
3, 4 (yes)

Examine the following equation:
Notice that it is solved by first combining similar terms and then by applying the addition and multiplication properties for equations.

$4x + 14 = 6x$
$3x + 14 - 6x = 6x - 6x$
Add 6x to both sides (you want to have x on one side of the equation)

$4x + 14 - 6x = 6x - 6x$	Combine like terms.
$14 - 2x = 0$	
$-14 + 14 - 2x = -14$	Add -14 to both sides.
$-2x = -14$	Combine terms.
$\frac{-2x}{-2} = \frac{-14}{-2}$	Divide both sides by (-2)
$x = 7$	Simplify.

Solve the equations:

1. $11x - 12 = 9x + 2$

Hint: Use addition property to get the variables on one side.

Add (-9x) to both sides.

Hint: Add 12 to both sides.
Hint: Divide by 2

2. $2x + 17 = 23 \qquad x =$

Hint: Add -17
Hint: Divide by 2

3. $3x + 4 = 3 + 4 \qquad x =$

Hint: Add -4 to both sides $(3x = 0 + x)$
Hint: Add -x to both sides $(3x - x = 0 + x - x)$
Hint: Combine terms $(2x = 0)$
Hint: Divide by 2 $\left(\frac{2x}{2} = \frac{0}{2}\right)$

Reminder: If the numerator of a fraction = 0, the fraction = 0.

Section III Solving Word Problems

Being able to translate a word problem into an algebraic equation is the key to solving word problems.

First, you must determine what you are solving for and let this be represented by the variable.

Then you must use the other relevant information to write an equation.

The final step is solving the equation to find the value of the variable.

Example: There are twice as many boys in Dr. Joseph's physics class as there are girls. There are 75 students in this class. How many of these students are girls?

Let x = the number of girls
You know that there are twice as many boys as girls so there are 2x number of boys. Since there are 75 students altogether the equation is $2x + x = 75$.

Solve: Combine terms $3x = 75$; Divide by 3 $x = 25$
So there are 25 girls in the class. How many boys? (50)

Try this one:
The sum of 3 numbers is 47. The second number is twice the first and the third number is twice the sum of the first two plus 2. Find the three numbers.

First, let x = the first number. Now represent the second number (2x). How will you represent the third number? A key word is sum of the first 2. Write the sum of the first and second as an algebraic expression. $(x + 2x)$

But the third number equals two more then double sum of the first two. Write an expression to represent this. $(2 (x + 2x) +2)$

Reevaluate the information you have so far.
First number = x
Second number = 2x
Third number = $2 (x+2x)+ 2$ or $6x + 2$

NOTE: Follow the order of operations to simplify to $6x + 2$.

You also know that the sum of all three numbers equals 47. Write the equation.
$(x) + (2x) + (6x + 2) = 47$

Solve: $9x + 2 = 47$ Combine terms.
$9x = 45$
$x = 5$

What does the first number = ? (5)
second number = ? (10)
third number = ? (32)
Hint: substitute 5 into the expression $6x + 2$

Now try the following problems on your own.

1. The sum of two consecutive numbers is 31. Find the numbers.

Hint: Let x = the first number.
Hint: You know the second number is 1 more than the first or x + 1

2. For every question that Bill missed on his Biology exam, he got 23 correct. There were 120 total questions on the exam. How many did Bill miss?

Hint: Let x = number missed
Hint: 23x = number correct
Hint: 23x + x = 120

3. The sum of two numbers is 20. Their difference is 4. What are the two numbers?

Hint: Let x = the larger number
Hint: x - 4 = the smaller number
Hint: x + x - 4 = 20

4. A furniture store sold 215 more recliners during a recent promotion than they had sold in the previous month. What was the total number sold for each month if 365 units were sold altogether?

Hint: Let x = number sold the previous month
Hint: x + x + 215 = 365

First month?
Second month?

5. The cost of a first class ticket to Pittsburgh is $50 more than an economy ticket. The cost of both tickets together is $420. How much does the economy ticket cost?

Hint: Let x - economy ticket; so x + 50 = first class ticket price.
Hint: x + x + 50 = 420

6. Meredith makes 2 investments with her $10,000 inheritance. The first investment is $2,325 more than the second. Find the amount of each investment.

Hint: Let x = first investment
Hint: x - 2325 = second investment
Hint: x + x - $2,325 = $10,000

First investment =
Second investment =

Section IV Using Formulas to Solve Algebraic Equations

A formula is a rule that is always true. You will learn to use many formulas to help you solve problems in algebra.

To use a formula you substitute values you know into the formula and solve for the unknown value.

Example: Distance = Rate x Time or $d = rt$

If we know the rate an object is moving and the time that it moves at this rate, we can find the distance traveled.

Example: A car travels at 40 mph for 3 hours. How far does it travel?
Using the formula $d = rt$: $r = 40$ mph; $t = 3$ hours
$d = rt$ $\quad d = (40)(3)$
$d = 120$ miles

Sometimes you will be given the distance and the time but not the rate.

Example: A train traveled 315 miles in 9 hours. How fast (what rate) was the train traveling?
Using $d = rt$
Substitute 315 for d; 9 for t
Rewrite $315 = r(9)$
Solve for r (Use property for multiplication in equation)
Divide by 9 $\frac{315}{9} = \frac{r(9)}{9}$ Simplify $35 = r$
So rate = 35 miles per hour.

Example: A car travels at 55 mph. 3 hours later, a helicopter leaves traveling at 175 mph How soon will the helicopter catch up to the car?

With distance problems, it helps to draw a table and fill in what you know.

	d	r	t
car	55x	55 mph	x
helic.	175(x-3)	175 mph	x-3

You know the rate at which each vehicle is traveling. Allow x to = time the car travels. Then x - 3 = time the helicopter travels.
(Since when the helicopter catches the car, they will have traveled the same distance but the helicopter will have spent 3 hours less in traveling time).

Write a formula for $d = rt$ for the car. $d = 55(x)$
$d = 175(x-3)$
Since the distances traveled are equal, the equations 55x and 175 (x-3) are equal also.
$55x = 175(x-3)$

First simplify: $55x = 175x - 525$
Add $-175x$ $\quad -120x = -525$
Divide by -120 $\quad x = 4\ 1/3$ hours.
So the helicopter will overtake the car in approximately 4 1/3 hours.

Sometimes, you will be asked to solve a problem involving leaving from the same point and traveling in opposite directions.

Example: A car leaves Johnstown traveling east at 45 miles per hour. Another car leaves at the same time traveling 55 mph but heading west. How long will it take for the two cars to be 220 miles apart?

Make a chart filling in the information you have. Let x = time.

	d	r	t
car 1	45x	45	x
car 2	55x	55	x

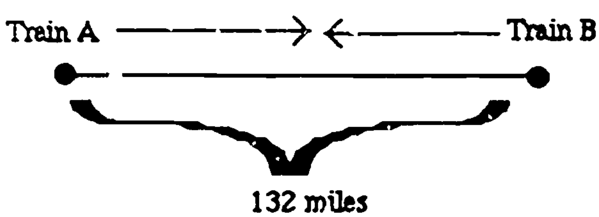
You also know that the total distance traveled by both cars is 220 miles, so
$45x + 55x = 220$.
Solve for x $\qquad 100x = 220$
$\qquad\qquad x = 2.2$ hours or 2 1/5 hours

The two cars will be 220 miles apart in 2 1/5 hours.

In another type of distance problem. 2 objects will leave from different points and travel toward one another.

Example: A train (A) leaves a station and travels north. Another train (B) leave a station 132 miles away and travels south. Train A is traveling 10 mph faster than train B. The two trains will meet in 2 hrs. How fast is each train traveling?

Look at a picture.



Trains A and B will meet somewhere along the 132 miles. They will not each travel the same distance since one train is traveling faster, but the total distance covered by both trains will be 134 miles.

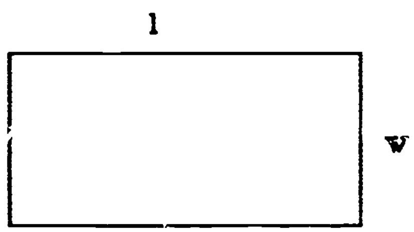
Set up a table:

	d	r	t
Train A	2x	x	2
Train B	2 (x-10)	x-10	2

Let x = rate for train A
x - 10 = rate for train B since B is traveling 10 mph slower than train A.
t = 2 hrs. for each train.
d = rt so Train A d= x2 or 2x
Train B d = 2 (x-10) or 2x -20

You know that the total distance traveled by A and B = 13 miles so:
2x + 2(x-10) = 132 miles
2x + 2x - 20 = 132
4x - 20 = 132
4x = 152
x = 38 mph
Train A travels 38 mph
Train B travels 38-10 mph or 28 mph.

You may be asked to solve a problem using the formula for finding perimeter of a rectangle. P = 2l + 2w or Perimeter = twice the length plus twice the width.



Example: If the perimeter of a rectangle equals 12" and the length is twice the width, what length are the sides?

Let x = width; 2 x = length.
Substitute what you know into P = 2l + 2w

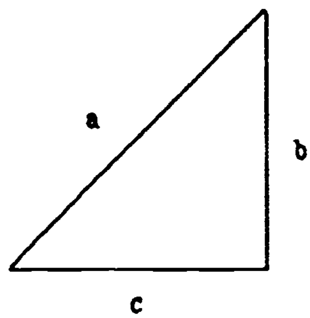
P = 12" so 12 = 2 (2x) + 2 (x)

Solve for x 12 = 4x + 2x
12 = 6x
2 = x

x = width = 2"
2x = length = 4"

Try solving these problems using the given formulas.

1. The perimeter of a triangle equals 18 inches. The formula used to find the perimeter of a triangle is P=a + b + c



If side a = 8" and is twice as long as side b, how long is side c?

Hint: Substitute what you know into the formula. P = a + b + c

P = 18"
a = 8"
b = 1/2a = 1/2 8 = 4"
c = x

18" = 8" + 4" + x
18 = 12 + x   x = 6"

2. Mary's car is traveling 65 mph. How far will she travel in 4 hours?
   d = rt

Hint: Which value are you solving for? (d), r or t?
Hint: Substitute: Let x = d   r = 65   t = 4
   x = 65 (4)

3. Mary walks at a certain pace for 2 hours. Elizabeth walks 2 mph faster than Mary. At the end of 2 hours Mary has walked 8 miles. How many miles will Elizabeth walk in 2 hours?

Hint: First discover how fast Mary walks. d = rt
   d = 8   r = x   t = 2
   8 = 2x
   4 = x   so Mary walks 4 mph

Hint: How fast does Elizabeth walk? (6 mph)
Hint: Solve for distance traveled by Elizabeth
   x = 6 (2)
   x = 12

4. The formula for the area of a rectangle is length times the width or A=lw. If the area of a rectangle is 216 inches, and the length is six times the width, how wide is the rectangle?

Hint: Substitute: $A = 216$ $W = x$ (the unknown variable) $l = 6x$ (6 times the width)

$216 = x(6x)$ (a variable times a variable (x)(x)
$216 = 6x^2$ $= x^2$ read x squared
$36 = x^2$

Hint: If $x^2 = 36$, what number times itself equals 36? (6)
Right, $6 \times 6 = 36$.

5. The interest formula is $I = p \times r \times t$ or $I = prt$.

I = interest
p = principal amount invested
r = rate of interest
t = time in years of investment

If I = \$1200 and the rate is 10% and the principal invested is \$4000, find the time of investment.

Hint: Substitute what you know, let the unknown be the variable.
I = \$1200 p = \$4000 r = 10% t = ?

Hint: $1200 = (4000)(10\%)(t)$
$1200 = (4000)(.10)(t)$
$1200 = 400(t)$
$\frac{1200}{400} = t$

*For review of % go to Unit I, Chapter 3, Section IV)

Unit IV - Chapter 3 •Inequalities•

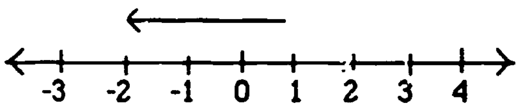
Section I Introduction to Inequalities

. An inequality means that two algebraic expressions are not equal. One expression is either greater than (represented by >) or less than (represented by <) then the other. Sometimes an expression is greater than or equal to the other expression ($\geq$), and sometimes it may be less than or equal to the other ($\leq$).

Examples:
$x > y$ read x is greater than y
$y < x$ read y is less than x
$A \geq B$ read A is greater than or equal to B
$B \leq A$ read B is less than or equal to A

An inequality may be represented on a number line.

Example: $-2 < 1$

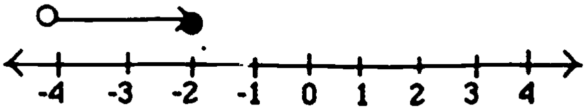


-2 is to the left of 1 on the number line so it is less than 1.

$1 > -2$ This true as well since 1 is to the right of -2 on the number line.

Section II Solving Inequalities

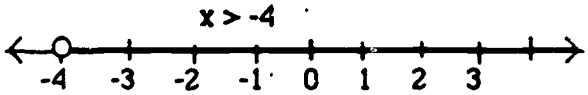
Consider the inequality $-2 > -4$



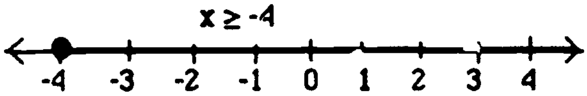
Since -2 is to the right of -4 it is greater than -4. But: $-3 > -4$, $-1 > -4$, $0 > -4$, $1 > -4$

All of the above inequalities are <u>also</u> true. In fact <u>all</u> values to the right of -4 on the number line are solutions to the inequality $x > -4$ This solution is represented on a number line with an open circle at -4 and a solid line on the number line.

Example:



If the solution or inequality is $x \geq -4$, then x can be replaced by -4 and the solution is true. $-4 \geq -4$. The values for x include -4 and all values to the right of -4 on the number line.

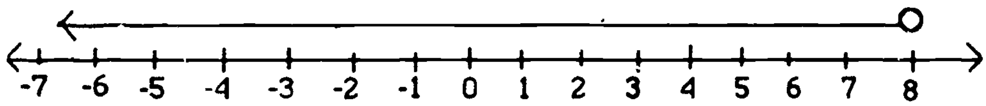


The solution to the inequality $x \geq -4$ is represented by a <u>closed</u> circle at -4 (since -4 is also = to -4) and a solid line over the rest of the number line to the right of -4.

To solve an inequality, you follow rules that are very similar to solving equations.

Example: $3x - 7 < 2x + 1$
Add -2x $3x - 2x - 7 < 2x - 2x + 1$
Combine terms $x - 7 < 1$
Add 7 $x - 7 + 7 < 1 + 7$
Combine terms $x < 8$

Graph $x < 8$



Example: $2x + 4 < x + 3$
Add -x $2x - x + 4 < x - x + 3$
Combine terms $x + 4 < 3$
Add -4 $x > -1$
*Notice that the inequality sign changed direction when a <u>negative</u> number was added to both sides.

Rule: Any time a negative number is added to both sides of an inequality, the inequality sign must be reversed.

Example: $x + 6 < 5x - 6$
Add -5x $x - 5x + 6 > 5x - 5x - 6$ (Inequality reverses)
Combine terms $-4x + 6 > -6$
Add -6 $-4x + 6 - 6 < -6 - 6$ (Inequality reverses)
Combine terms $-4x < -12$
Divide by -4 $x > 3$ (Inequality reverses)

Notice that when you added a negative number to both sides the inequality sign reversed (each time). Notice that when you divided by a negative number the inequality sign reverses.

Check your solution to be sure it is correct.
$x > 3$ Substitute 3 for x
$3 + 6 < 5(3) - 6$
$9 < 9$
Is this true? NO, 9 is not less than 9 so you know that the solution to the inequality must be greater than 3.
Therefore: $x > 3$ is correct.
Just to be sure, substitute 4 for x. $(4) + 6 < 5(4) - 6$
$10 < 14$
Yes, 10 is less than 14, so 4 is a solution to the inequality.

Solve: (Remember the rule for adding, multiplying and dividing using negatives.)

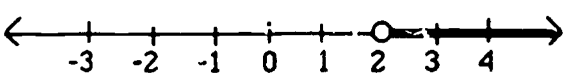
1. $4x + 12 \geq 60$

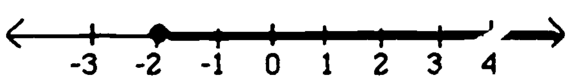
Hint: Add -12 $4x \leq 48$ Notice the inequality reverses.
Divide by 4 $x \leq 12$

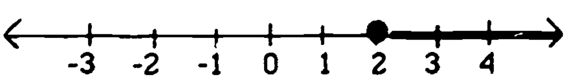
2. $5x > 12 - 3x$

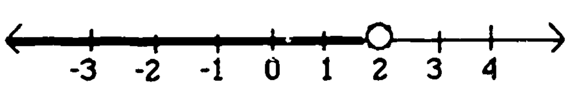
Hint: Add 3x $5x + 3x > 12$
Combine $8x > 12$
Divide by 8 $x > \frac{12}{8}$ or $\frac{3}{2}$

3. Graph the inequality $x > 2$. Which is correct?

A. 

B. 

C. 

D. 

4. $12x - 6 < 10x + 4$

Hint: Add $-10x$ $12x - 10x - 6 > 10x - 10x + 4$
$2x - 6 > 4$
Hint: Add 6 $2x > 10$
Divide by 2 $x > 5$

5. Five times a number is added to two. The result is greater than four times that same number added to 1. Find all of the numbers which make this statement true.

Hint: Rewrite as an inequality since more than one number will solve this statement.
Hint: $5x + 2$ (?>) $4x +$ (?1)
so $5x + 2 > 4x + 1$

Now solve the inequality:

Hint: $5x + 2 > 4x + 1$
Add $-4x$ $x + 2 < 1$
Add -2 $x < -1$

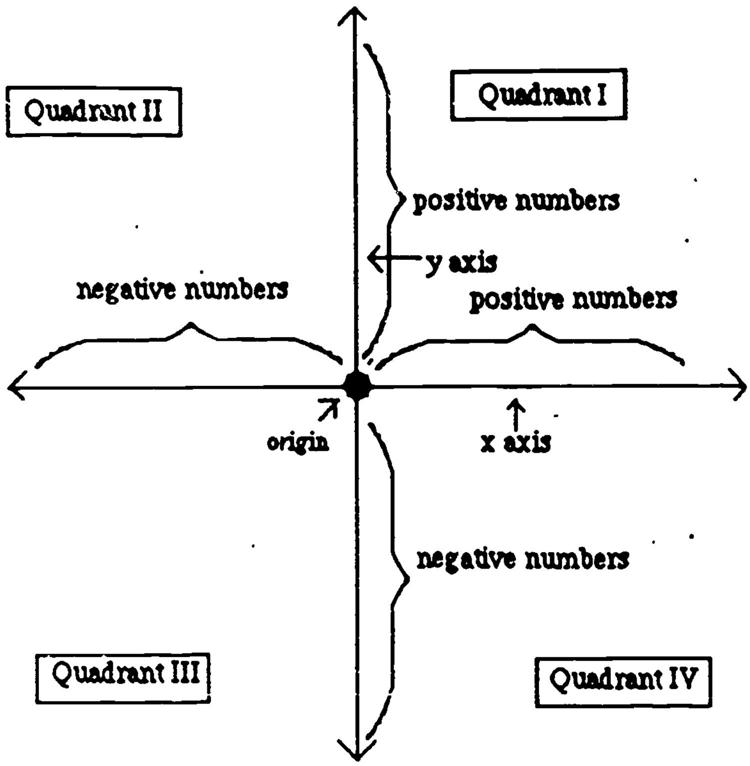
6. Two times a number added to one is less than three times the same number minus 2. Which inequality will solve this problem?

A. $2x + 1 > 3x + 2$
B. $2x + 1 < 3x - 2$
C. $3x - 2 < 3x + 1$
D. $2x - 2 < 3x + 1$

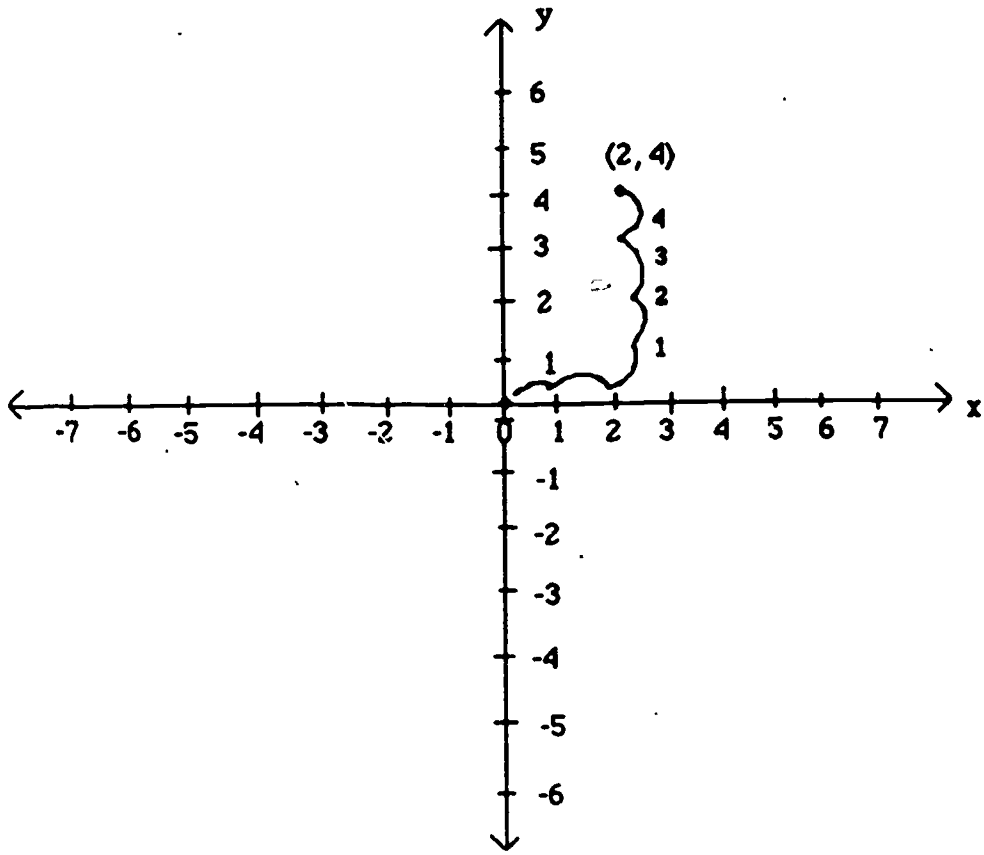
Unit IV - Chapter 4 •Equations With Two Variables•

Section I Graphs

Consider the following graph.



The horizontal line is the X-Axis.
The vertical line is the Y-Axis.
The point where the x-axis intersects with the y-axis is the ORIGIN (location of zero).
The graph is divided into 4 quadrants.
All positive numbers are found to the right of the origin on the x-axis and above the origin on the y-axis.
All negative numbers are found to the left of the origin on the x-axis and below the origin on the y-axis.



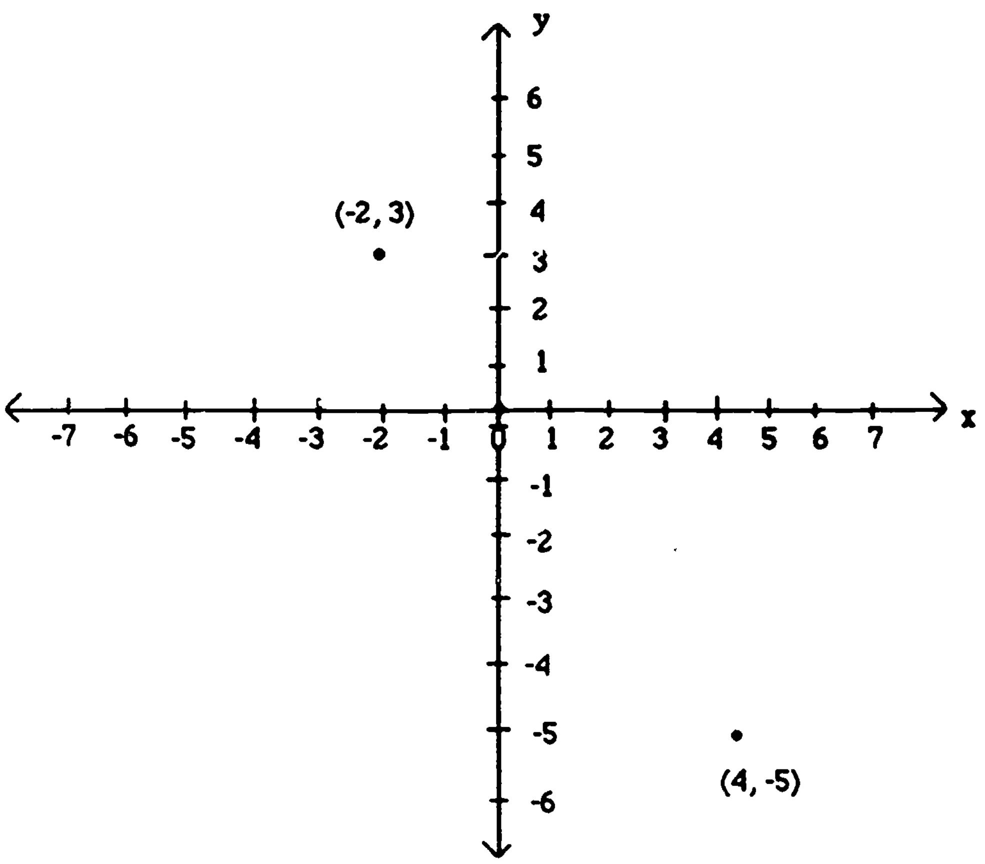
If you are given a point with an x value and a y value, it can by plotted on the graph. (x, y)

x = 2 -- the x-coordinate
y = 4 -- the y-coordinate Written (2, 4)

(2, 4) is called an ordered pair.

The x coordinate always comes first in an ordered pair. The y coordinate comes second.

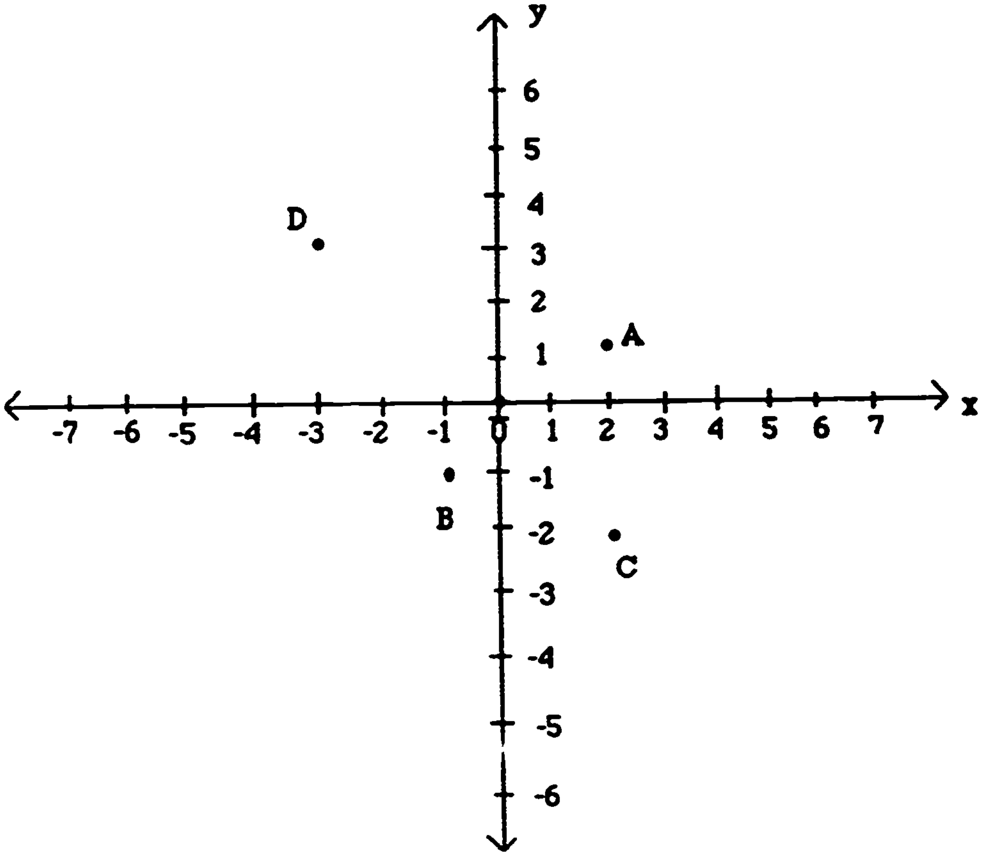
To plot (2, 4) first find 2 on x-axis. Move up above the 2 until you find the 4 on the y-axis. See example. You moved two places right and 4 places up.



Now plot (-2, 3) left 2 and up 3. Remember the x-value is always first.

Try (4, -5).

Now name the following ordered pairs.



A = (?, ?)
B = (?, ?)
C = (?, ?)
D = (?, ?)

Answers:

Section II Linear Equations

An equation in two variables can have many sections.

Example: $2x + y = 12$

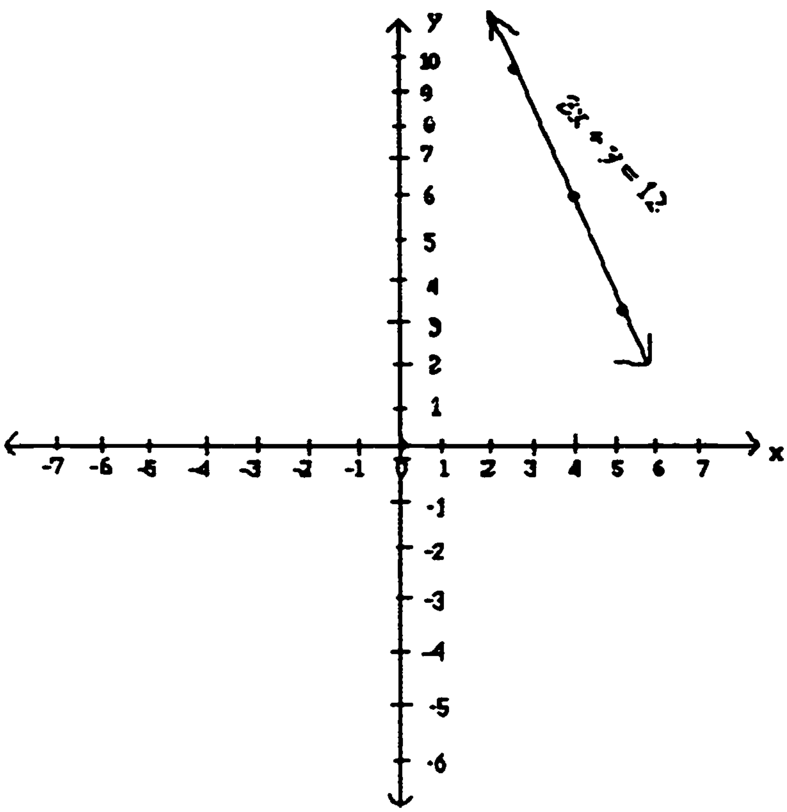
The solutions to this equation are written as ordered pairs. (x, y)

To find solutions to an equation with two values (a linear equation), choose a value for one value and substitute it into the equation. Solve for the other variable.

Suppose we let $x = 3$ in the equation $2x \div y = 12$.
We would substitute 3 for the x value. $2(3) + y = 12$
$6 + 7 = 12$

Solve for y:
Add -6 to both sides. $y = 6$

One possible solution is the ordered pair (3, 6) (x-value, y-value)



Plot (3, 6)

x value	y value
3	6
1	10
4	4

Now substitute $x = 1$
$2(1) + y = 12$
$2 + y = 12$
$y = 10$

$x = 4$
$2(4) + y = 12$
$8 + y = 12$
$y = 4$

Notice that when the ordered pairs (3, 6), (1, 10), and (4, 4) are plotted on the graph, they fall along a straight line. This line represents all possible solutions to the equation $2x + y = 12$. Every point on this line (which continues indefinitely in both directions) is a solution to the equation.

HINT: To graph the solution to a linear equation, you need to find only two solutions or ordered pairs because any two points determine a line.

Example: Graph the equation $x + y = 2$

Step 1: Make a table comparing x and y values.

x	y

Step 2: Choose two x values (any two numbers)

x	y
0	
-1	

Step 3: Substitute the x value into the equation and solve for y.

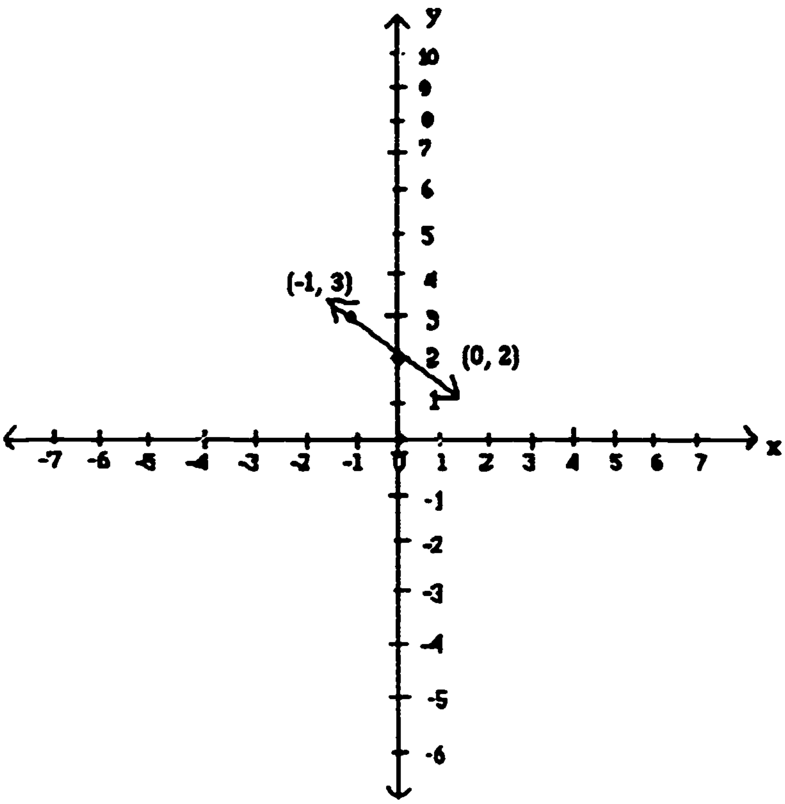
$0 + y = 2$
$y = 2$

$-1 + y = 2$
$y = 3$

x	y
0	2
-1	3

Step 4: Write the ordered pairs (x value, y value)
(0, 2) (-1, 3)

Step 5: Plot these two points.

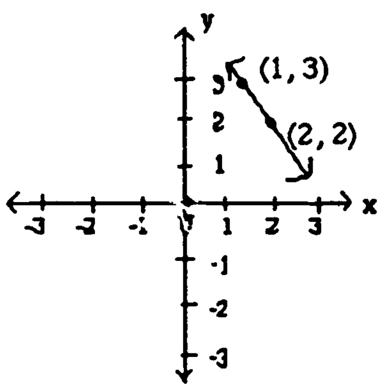


Step 6: Draw a straight line through these two points. The arrows on the ends of the line indicate that the line extends indefinitely in both directions. All points that fall along this line are solutions to the equation.

For each of the following, decide if the ordered pair is a solution of the equation.

1. $4x + y = 12$	(3, 0)	(yes)
2. $4x + y = 12$	(1, 16)	(no)
3. $2x + 3y = 15$	(0, 5)	(yes)
4. $2x - 3y = -2$	(-1/2, 1)	(yes)
5. $x + y = 16$	(-4, 14)	(no)

Example: $x + y = 4$

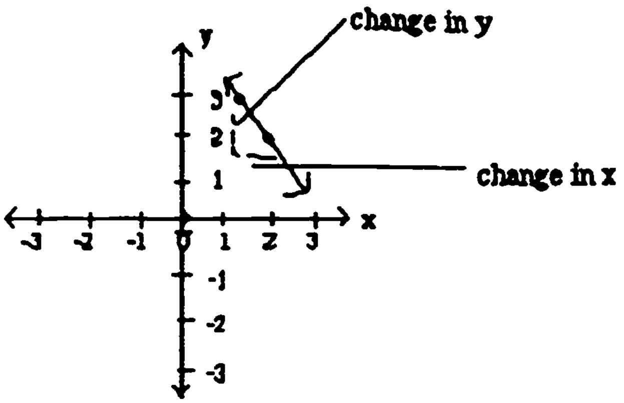


x	y
1	3
2	2

The ordered pairs you plotted to find this solution are (1, 3) (2, 2)
(x, y) (x, y)

$$\text{Slope} = \frac{\text{change in y}}{\text{change in x}} = \frac{3-2}{1-2} = \frac{1}{-1} \text{ or } -1$$

The slope of x + y = 4 is -1

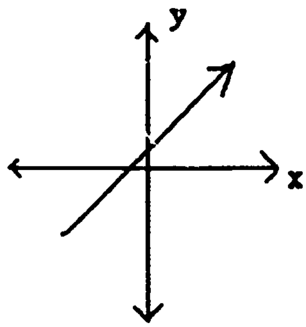


The horizontal dotted line represents the change in x. The vertical dotted line represents the change in y. Every time x decreases one value, x increases by 1.

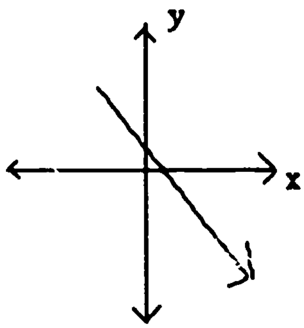
Section III Slope of a Line

The slope of a line is a measure of its steepness.

A positive slope indicates a rise from the left of the graph to the right.



A negative slope indicates a line that falls as it moves from left to right.



To find the slope of a line, choose any two points on the line. Then find the change in the y values of the two points and find the change in the x values.

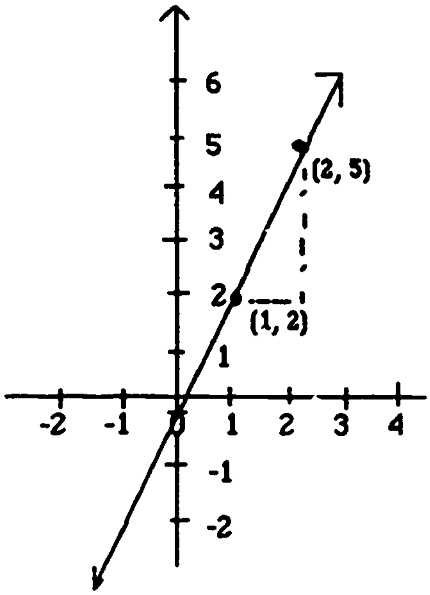
Let's look at one more example on a graph. $y = 3x - 1$

Substitute x values and solve for corresponding y values.

x	y
1	2
2	5

$x = 1$ $\quad y = 3(1) - 1$
$= 2$ $\quad (1, 2)$

$x = 2$ $\quad y = 3(2) - 1$
$= 5$ $\quad (2, 5)$



Slope is positive since it moves up from left to right.

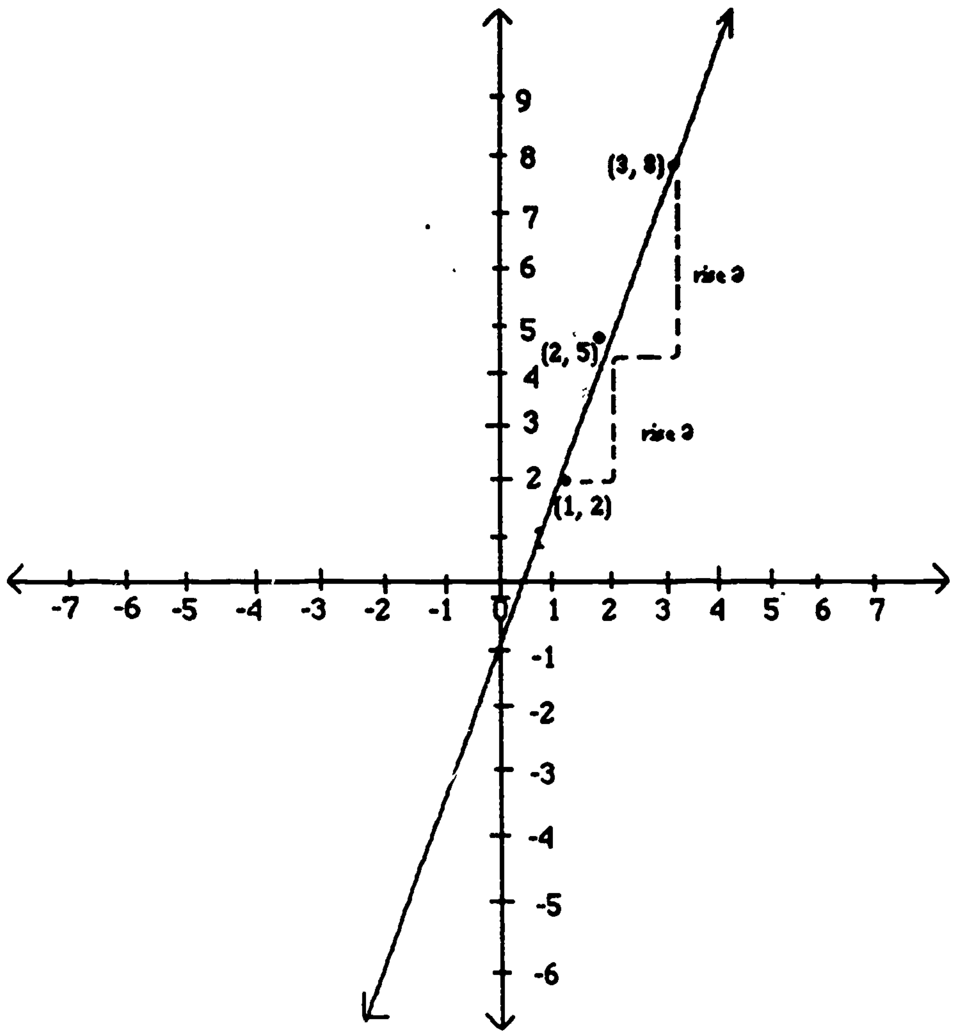
$$\begin{array}{cc} x\ y & x\ y \\ (1, 2) & (2, 5) \end{array}$$

$$\text{slope} = \frac{\text{change in y}}{\text{change in x}} = \frac{(2-5)}{1-2} = \frac{-3}{-1}$$

or $\frac{3}{2}$ corresponding rise in y value
increase of 1 in x value

For every increase of 1 in the x value there will be an increase of 3 in the y value.

Therefore, if $x = 3$ then $y = 8$.



The line y = 3x - 1 has a slope of 3.

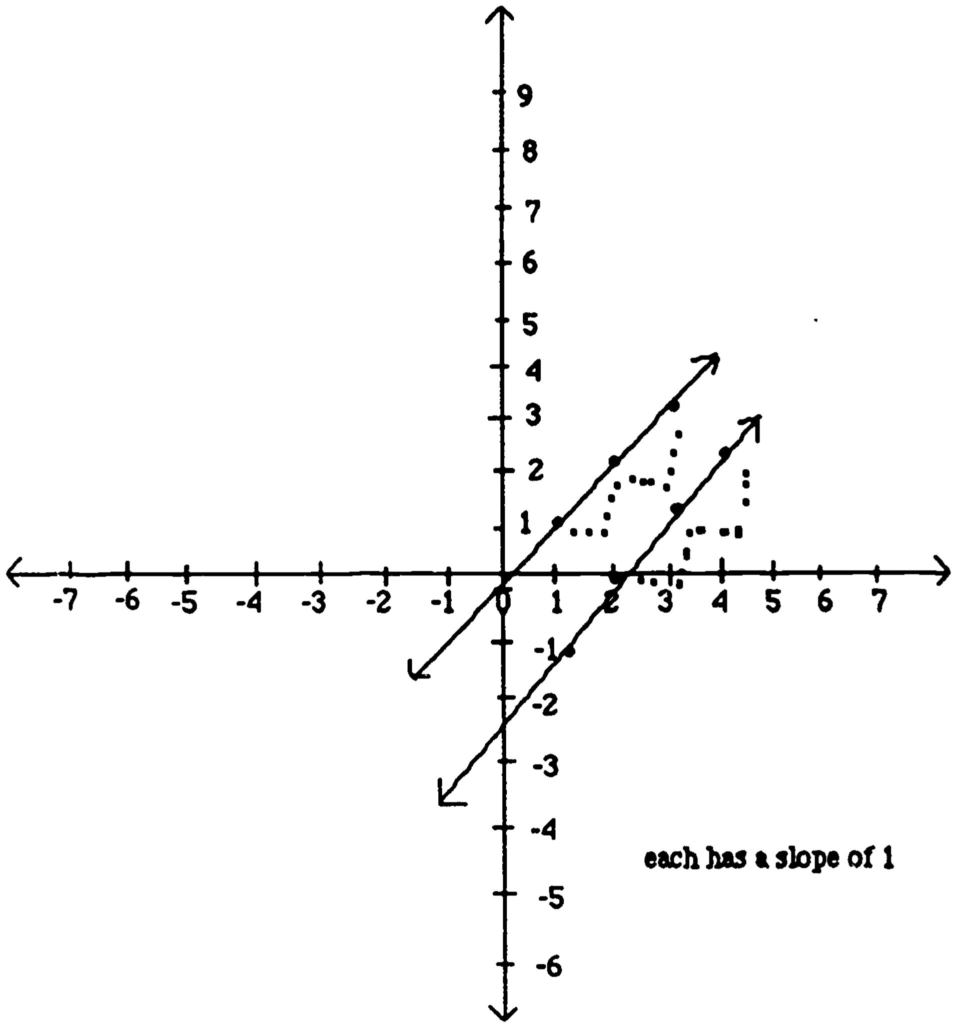
You can find the slope of a line without graphing if you are given any two points on the line.

<u>Example</u>: Given the points on a line (1, 4) (3, 8) Find the slope.

$$\text{slope} = \frac{\text{change in y}}{\text{change in x}} = \frac{8-4}{3-1} = \frac{4}{2} = \frac{2}{1}$$

This line has a slope of 2.
For every increase of 1 in the x value, there is a corresponding increase or rise of 2 in the y value.

Two lines with the <u>same</u> slope are parallel.

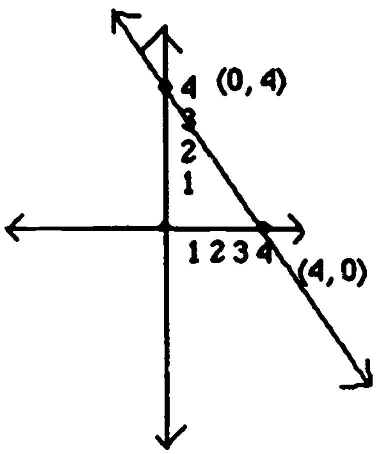

9
8
7
6
5
4
3
2
1
-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7
-1
-2
-3
-4
-5
-6
each has a slope of 1

Section IV Intercepts

The x intercept is the point where a line crosses the x axis.
The y intercept is the point where a line crosses the y axis.

To find the x intercept, use zero as the value for y.

Example: $x + y = 4$
$y = 0 \quad x + 0 = 4$
$x = 4$



(4, 0) is the x intercept

To find y intercept, use zero as the x value.
$x = 0 \quad 0 + y = 4$
$y = 4$

(0, 4) is the y intersept.

Find the x intercept of the following equations:

1. $2x + y = 4$
Hint: Set $y = 0$
$2x + 0 = 4$
$2x = 4$
$x = 2$

2. $x + 3y = 6$
Hint: $y = 0 \quad x + 3(0) = 6$
$x = 6$

Find the y intercept for the following:

1. $2x + y = 4$
Hint: $x = 0 \quad 2(0) + y = 4$
$y = 4$

2. $x + 3y = 6$
Hint: $x = 0 \quad 0 + 3y = 6$
$y = 2$

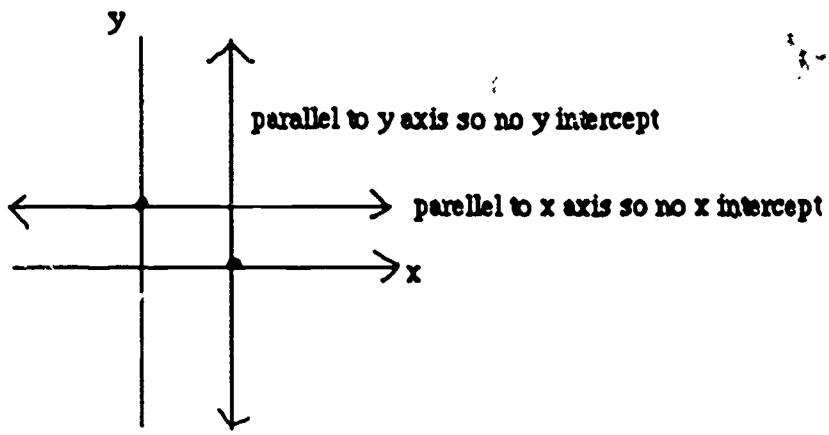
Section V Slope-Intercept Form

Now that you understand that the slope (usually represented by the letter m) of a line measures its steepness and tells the direction of the line.

If $m > 0$ increasing up from left to right
If $m < 0$ decreasing from left to right

You also know that the x intercept is the point where the line crosses the x axis and the y intercept is the point where the line crosses the y axis.

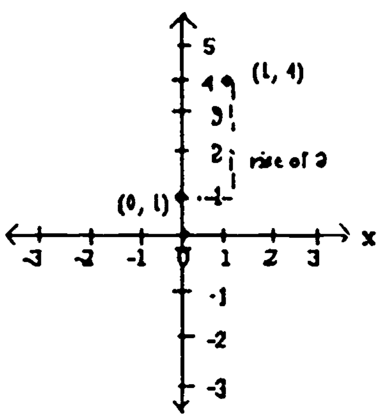
NOTE: A line // to the x axis will not have an x intercept. A line // to the y axis will not have a y intercept.



Now that you understand intercept and slope, you can learn to write a linear equation in a special form called the slope-intercept form.

$y = mx + b$
(slope) (y intercept)

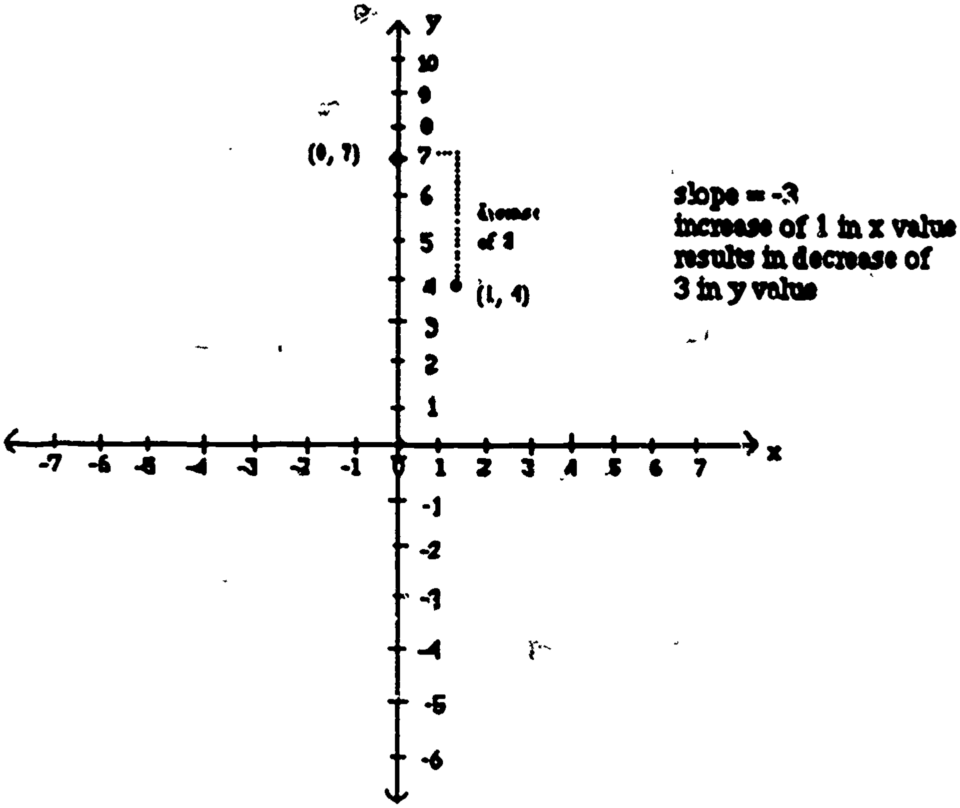
Example: Given the equation $y = 3x + 1$



m (slope) = 3
y intercept is (0, 1)

To change a linear equation into the slope-intercept form, you must rewrite it with the y on one side of the equation.

Example: $3x + y = 7$
Add $-3x$ $\quad y = -3x + 7$
(slope) (y intercept) (0, 7)



Change each of the following to slop-intercept form. $y = mx + b$

1. $3x - y = 4$
2. $x + y = 5$
3. $3x + 2y = 2$ (

Find the slope and y intersept of the following:

1. $3x - 4 = y$ $\quad m = (3)$ y -intersept =
2. $y = -x + 5$ $\quad m = (-1)$ y-intersept =
3. $y = -3/2x + 1$ $\quad m = (-3/2)$ y-intersept =

Find the slope of a line containing these points:

1. (4, 2) and (2, 1) $\quad m =$

Hint: $m = \frac{\text{change in y}}{\text{change in x}} \; \frac{3-1}{4-2} = \frac{2}{2} = 1$

2. (1, 3) and (2, 5) $\quad m =$

Hint: $m = \frac{5-3}{2-1} = \frac{2}{1} = 2$

3. (3, 5) and (2, -1) m =

4. (9, 0) and (6,) m =

Hint: $\frac{0-0}{9-6} = \frac{0}{3} = 0$

A line with a slope of zero is horizontal. It has no slope or zero shape.

You know that for two lines to be parallel, they must have the same slope. Determine which of these equations are parallel.

1. y = 3x + 4
   y = 3x + 2

Hint: m = 3 for both equations

2. 5y = 3 - 4x
   8x + 10y = 1

Hint: First rewrite equations in slope-intercept form (y = mx + b)

5y = 3 - 4x
y = 3/5 - 4/5x or y = -4/5x + 3/5 m = (-4/5)

8x + 10y = 1
10y = -8x + 1
y = -4/5x + 1 m = (-4/5)

3. y + 2 = 5 x
   5y + x = -15

Hint: y + 2 = 5 x
y = 5x - 2 m = (5)

5y + x = -15
5y = -x - 15
y = -x/5 - 3 or y = -1/5x - 3 m = (-1/5)

Unit V - Chapter 1 •Geometry•

Geometry is a section of mathematics that deals with measuring lines, angles, areas, and surfaces. The first item that must be covered is terms that you should be familiar with.

Section Ia.

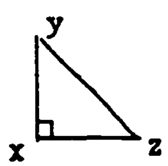
Match the following terms with the definition:

1. angle
2. degree
3. right angle
4. acute angle
5. obtuse angle
6. straight angle
7. reflex angle

a. an angle with more than 180° and less 360°
b. unit of measurement of an angle
c. an angle of less than 90°
d. angle with exactly 180°
e. two lines that extend from the same point
f. an angle of 90°
g. angle with more than 90° and less than 180°

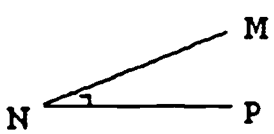
If a student does not get these questions right, section Ia will continue explaining these terms more. However, the student can type "return" to continue to the next section.

1. What kind of angle is angle x?



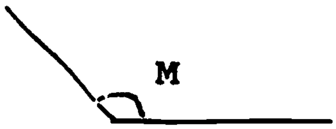
Hint: A small square on an angle tells you it is a right angle.

2. What kind of angle is angle MNP?



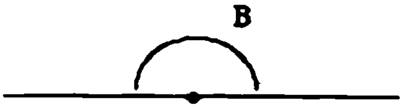
Hint: Since this angle is smaller than a right angle it is acute.

3. What kind of angle is angle M?



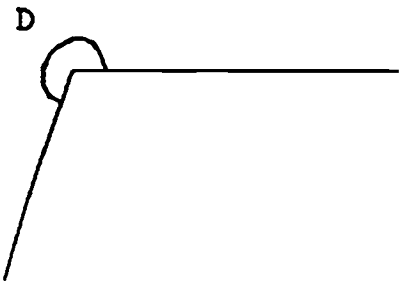
Hint: The angle is larger than a right angle but smaller than a straight. Angle M is an obtuse angle.

4. What kind of angle is angle B?



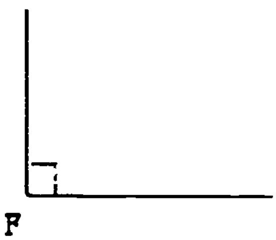
Hint: This is a straight angle because it looks like a straight line.

5. What kind of angle is angle D?



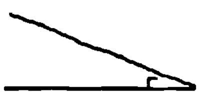
Hint: The sides of this angle is larger than a straight angle.

6. Determine the degree of angle F.



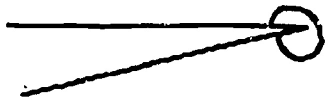
Hint: Since this is a right angle it is 90°.

7. Identify this angle picture?



Hint: This is an acute angle because it is smaller than a right angle.

8. Identify this angle picture.



Hint: This angle is larger than a straight angle so it is reflex.

9. What type of angle would be 280°?

Hint: This is a reflex angle because it's more than 180° and less than 360°.

10. What type of angle would be 180°?

Hint: A straight angle has exactly 180°.

11. What type of angle would be 20°?

Hint: An acute angle is less than 90°.

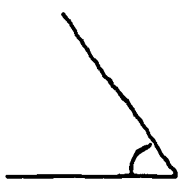
12. What type of angle would be 178°?

Hint: An obtuse angle is more than 90° and less than 180°.

13. Identify this angle and the degree.

Hint: The picture shows the sides of the angle larger than a right angle; obtuse. You do not know the exact degree but since it is an obtuse angle it is in between 90° and 180°.

14. Identify the angle and degree.



Hint: This is smaller than a right angle which tells you it is an acute angle and is anything less than 90°.

15. Identify the angle and degree.

Hint: A straight angle is like a line and exactly 180°.

Section Ib.

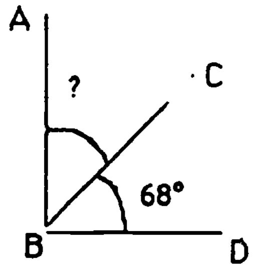
Match the following terms with a definition.

1. complementary angles
2. supplementary angles
3. vertical angles
4. opposite
5. adjacent
6. parallel
7. transversal
8. corresponding angles
9. alternate interior angles
10. alternate exterior angles
11. perpendicular

a. a pair of angles whose measures total 90°
b. angles that are across from each other when 2 line intersect and they are equal.
c. two lines that form right angles
d. angles that are across from each other
e. 2 angles next to each other
f. a pair of angles whose measures total 180°
g. 2 lines that run next to each other and never cross
h. a line that crosses 2 parallel lines
i. when transversal cuts 2 parallel lines the angles on same side and same direction equal
j. 2 angles on opposite sides of transversal and face inside parallel line are equal
k. 2 angles on opposite sides of transversal and face outside parallel line are equal

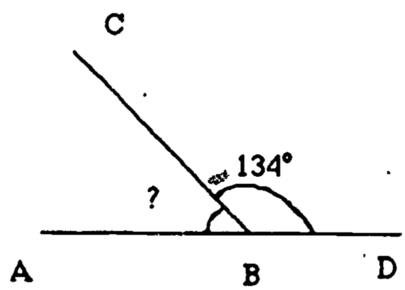
Again, if student does not understand these terms he/she can type "yes" for more instruction. Otherwise, by typing "return" the student can continue to the next section.

1. What is the measure of angle ABC and what are these angles called?



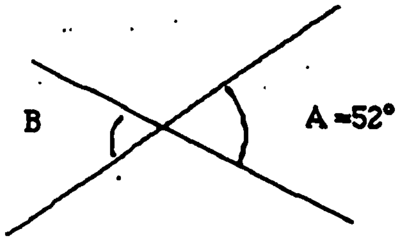
Hint: Since it is a right angle it tells you that they are complementary angles and together the angles equal 90°. Angle CBD or 68° - 90° = angle ABC or 22°.

2. What is the measure of ∠ ABC and what are these angles called?



Hint: Since this is a straight angle that tells you they are supplementary angles and together the angles equal 180°. ∠ CBD or 134° - 180° = ∠ ABC or 46°.

3. What is the measure of ∠ B and what are the angles called?

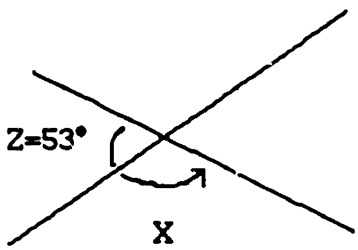


Hint: Angles across from each other are equal, vertical. Therefore, if ∠ A is 52° so is ∠ B.

4. What is the complement and supplement of an angle that measures 42°?

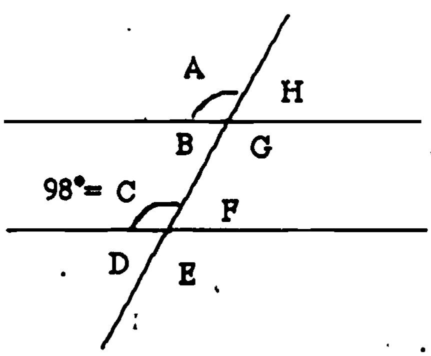
Hint: A complementary angle totals 90° so, 90-42 = 48°. A supplementary angle totals 180° so 180 - 42 = 138°.

5. Find the measure of angle X



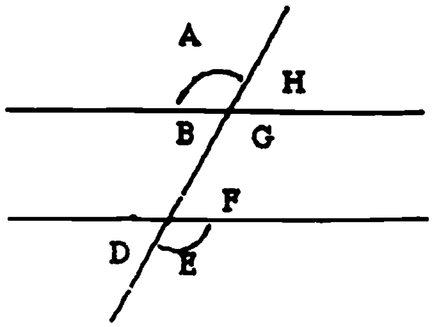
Hint: ∠ Z equals 53°. ∠Z and ∠X combined form a straight angle which is 180°. 180°- 53°=127°. ∠X = 127°

6. What is the measure of ∠A and what are these angles called?



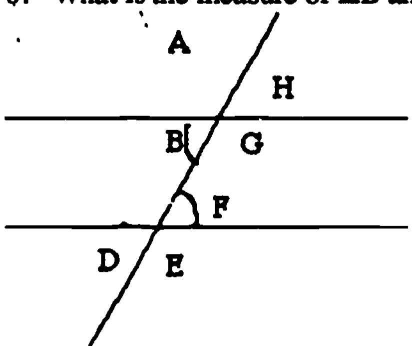
Hint: ∠A and ∠ C are corresponding angles and by definition they are equal. Therefore ∠A and ∠C and ∠A = 98°.

7. What is the measure of ∠A and what are these angles called?



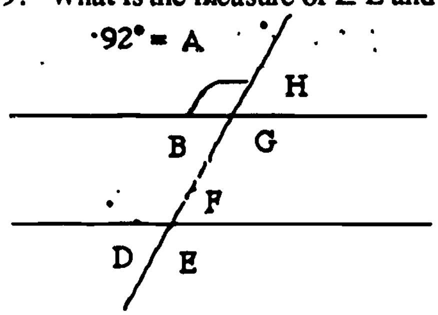
Hint: ∠A and ∠E are alternate exterior angles, therefore, ∠E = ∠A and ∠D = ∠H so ∠A = 103°.

8. What is the measure of ∠B and what are these angles called?



Hint: Angle B and angle F are alternate interior angles therefore angleB = angleF and angleC = angleG so angleB = 81°.

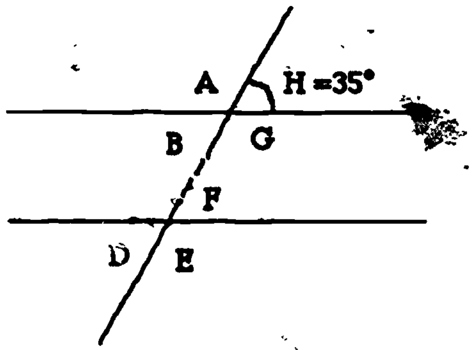
9. What is the measure of ∠ E and ∠ D?



Hint: Because ∠A = 92° so does ∠E because of alternate exterior angles. Then ∠E and ∠D combined form a straight angle which equals 180°. 92°-180°=88°.

10. What is the measure of ∠C?

∠E?



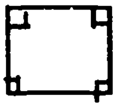
Hint: You know ∠D = ∠H because of alternate exterior. If ∠D = 35° then to find ∠C subtract 35 from 180 (straight angle) 180-35=145°. Finally ∠E also is 145° because of vertical angles.

Unit V - Chapter 2 •Geometry Shapes•

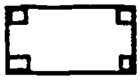
Section IIa. Quadrilaterals

Things to remember:
1. Quadrilaterals are four sided and angles total 360°.

   a. square - four equal sides - four right angles.



   b. rectangle - opposite sides are equal and four right angles.

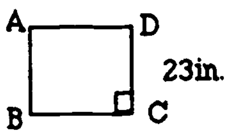


   c. parallelogram - opposite sides are equal and parallel.

   d. rhombus - four equal sides and opposite angles are equal.

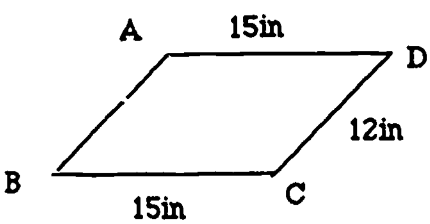
Do the following problems:

1. What is the length of AD and the measure of ∠B in this square?



Hint: Since this is a square and all sides of a square are equal, AD=23in. A square also has four right angles so ∠B = 90°.

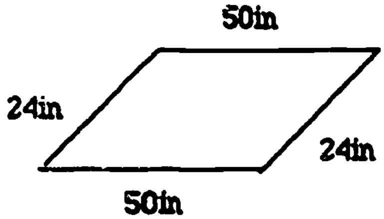
2. What kind of figure is ABCD and what is the length of AB?



AD parallel to BC
AB parallel to DC

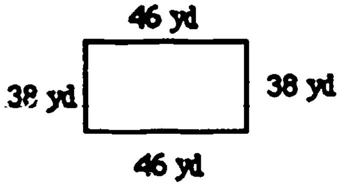
Hint: The figure is a parallelogram and since opposite sides are equal AB = 12 in.; same as DC.

3. What is the perimeter of this figure?



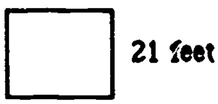
<u>Hint</u>: Remember that a perimeter of a figure is the distance around the outside. You should add all the sides: P = S+S+S+S or P=50+50+24+24 = 148 inc.

4. Mr. Grig wants to put a fence around his rectangular yard. The yard is 46 yards long and 38 yards wide. How many yards of fencing will Mr. Grig need?



<u>Hint</u>: It may help to have a picture. You know that the length is 46 yd and width is 38 yd. Since it tells you it is a rectangular shaped yard you know that opposite sides are equal. Now add the sides P=46+46+38+38 = 168 yds.

5. Mary wants to carpet her square room. She measured one side to be 21 feet. How many feet of carpeting will Mary need?



<u>Hint</u>: Since a square has all equal sides, each side is 21 feet. This time you are finding the area of a surface which is a number of square units inside a figure. A=lw or A=SXS; A=21x21=441sq. ft.

6. The perimeter of a square house is 2,404 yards. How many yards is each side of the house?

<u>Hint</u>: To find the length of each side you should divide 2,404, the perimeter, by 4 since there are 4 equal sides in a square. 2,404 + 4 = 601.

7. What is the area of a rectangle with the length 12 ft. and width 5 ft.?

<u>Hint</u>: A = wl so A = 12 x 5 = 60

8. The area of a parallelogram is 22 square feet. The height is 11 feet. What is the length in feet?

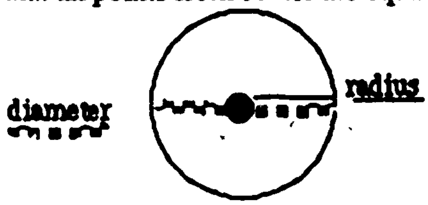
<u>Hint</u>: Use formula for area A=Lw - now fill in the numbers you do know.

22 = Lx 11
$\frac{22}{11}$ = L
Simplify - 2 =L

Section IIb Circles

Things to remember:

Circle - a closed curve that all points from center are equal distance.



Diameter - line through center of circle
Radius - line from center to outer edge
Circumference - perimeter around circle: $C=\pi d$; pi=3.14 multiplied by diameter
Area - amount of surface space inside the circle: $A = \pi r^2$; pi=3.14 mult. by doubled radius

Complete the following:

1. Find the circumference of a circle with the diameter of 5 in.

Hint: $C = \pi d$; fill in what you know: $C = 3.14 \times 5$ in. $= 15.7$

2. Find the circumference of a circle with the radius 4.32 in.

Hint: $C = \pi d$ - since radius is given double the radius to get the diameter.
$4.32 + 4.32 = 8.64$ diameter.
$C=3.14 \times 8.64 = 27.1296$ or 27.13

3. Kathy ran on a circular track that has a diameter of 300 feet. She ran 4 laps around the track. How many feet did she run?

Hint: $C = \pi d$; $C=3.14 \times 300 = 942$.
The distance around the track once is 942 ft. However, she ran 4 laps so multiply by 4. $942 \times 4 = 3{,}768$

4. Find the area of a circle with the diameter of 12 ft.

Hint: $A = \pi r^2$; Since radius is needed for the formula divide the diameter in half: $12 \div 2=6$- radius. $A=3.14 \times 6^2 = 3.14 \times (6 \times 6) = 3.14 \times 36 = A = 113.04$.
Notice that r^2 is not the same as diameter so be sure to put the radius in the formula.

5. Mary and Sue split an apple pie. The radius of the pie is 4 in. How many square inches did Mary & Sue each get?

Hint: $A = \pi r^2$; $A = 3.14 \times 4^2$; $A = 3.14 \times 16 = 50.24$ sq. in. altogether area of the pie. Don't forget to divide by 2 to find out how much they each got. $50.24 \div 2 = 25.12$.

Section IIc Triangles

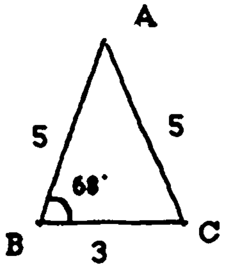
Terms to know:

1. equilateral triangle
2. isosceles triangle
3. congruent triangle
4. SAS requirement
5. ASA requirement
6. SSS requirement
7. Pythagorean theorem

a. triangle with 2 equal sides and angles opposite equal sides are equal
b. triangle with 3 equal sides and angles each measure 60°
c. two triangles that are exactly the same
d. congruent triangles of equal sides
e. congruent triangles with 2 sides and the angle in between equal
f. congruent triangles with 2 angles and the side in between equal
g. square of the hypotenuse (side opposite the right angle) equals the sum of the squares of the other two sides ($c^2 = a^2 + b^2$)

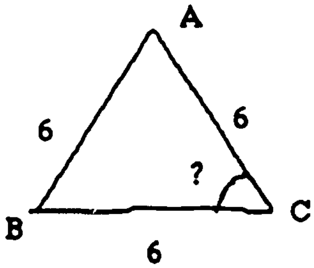
Complete the following:

1. What is the measure of $\angle C$ and what type of triangle is it?



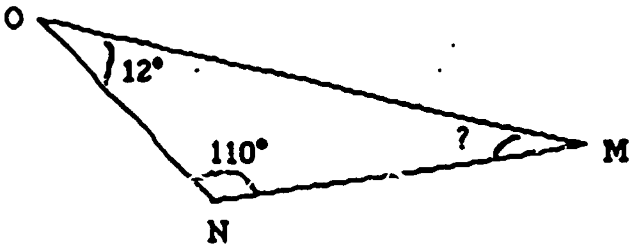
Hint: Because the triangle has 2 equal sides you know that it is an isosceles triangle and that $\angle B = \angle C$. Therefore $\angle C = 68°$

2. What is the measure of $\angle C$ and what type of triangle is it?



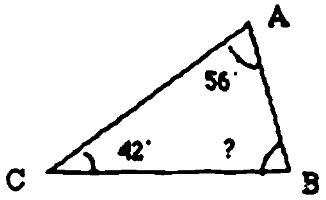
Hint: Since all 3 sides are equal it is an equilateral. By definition all the angles equal 60°.

3. Find the measure of ∠M



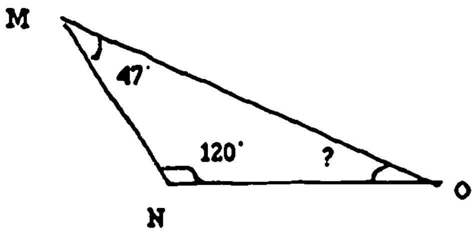
<u>Hint</u>: Angles of every triangle add up to 180°, not just equilateral triangles. First, add the angles you know. 12°+110° = 122°. Then subtract from 180°.
180°-122° = 58°; so ∠M = 58°

4. Find ∠B



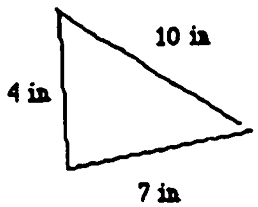
<u>Hint</u>: Add: 56° + 42° = 98°; Subtract: 180° = 98° = 82°.

5. Find ∠O



<u>Hint</u>: Add: 120° + 47° = 167°; Subtract: 180° - 167° = 13°

6. Find the perimeter of this triangle.

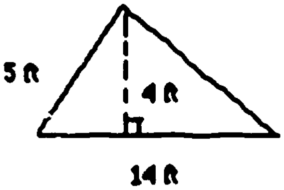


<u>Hint</u>: Add the 3 sides to find the perimeter: P = 4+7+10=21.

7. Find the perimeter.

Hint: Since 2 angles are 60° the other one must be 60° also which makes it an equilateral triangle which has 3 equal sides. P=6+6+6=18.

8. Find the area.



Hint: A = 1/2 bh; b=base and h=height.

A = 1/2 (14 x 4) = 1/2 56 = 28

Remember that the height is the perpendicular distance from any angle of triangle to opposite side.

9. A triangle with an area of 24 sq. in. and a height of 6 in. What is the base in inches?

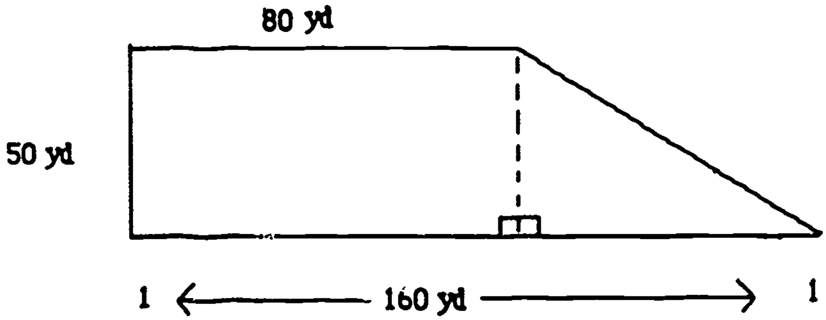
Hint: $A \frac{1}{2} bh$ Fill in what you know.

$A = \frac{1}{2} bh$

$24 = \frac{1}{2} b\, 6$

$\frac{24}{3} = \frac{3b}{3}$ $\quad 8 = b$

10. The figure below shows a lot that will be used for the local fair. What is the area of this lot?



Hint: First figure out the area of the rectangle. Remember A = lw
A = 50 x 80 = 4,000 sq. yd.
Next find the area of the triangle. The 106 yd side is the base of the rectangle/triangle.
Subtract the other side of the rectangle to get the base of the triangle. 106-80-26 yd.

The height of the triangle is 50 yd. Use the formula $A = \frac{1}{2}bh$

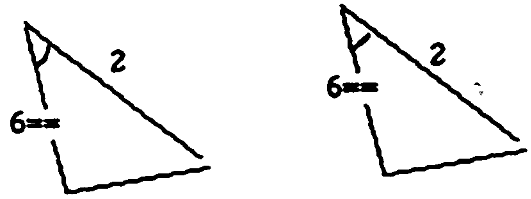
$A = \frac{1}{2} \times 26 \times 50$

$A = \frac{1}{2} 1300$

$A = 650$ sq. yd.

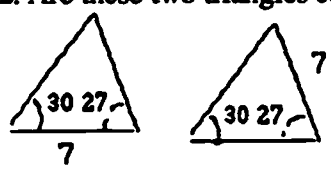
Now add to get the total: $650 + 4{,}000 = 4{,}650$ sq. yd.

11. Are these two triangles congruent?



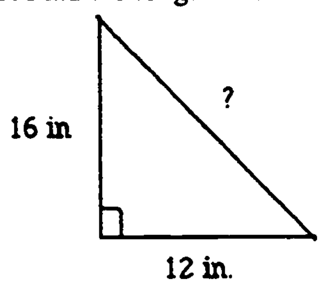
<u>Hint</u>: 2 sides and angle of one triangle is equal to 2 sides and corresponding angle of another. This is the SAS requirement.

12. Are these two triangles congruent?



<u>Hint</u>: The angles are equal and congruent however, even though the sides are equal, they don't correspond.

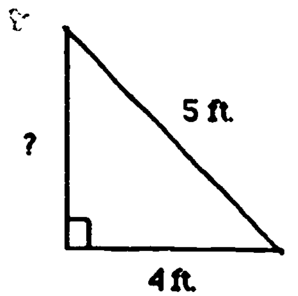
13. Find the length of the unknown side.



<u>Hint</u>: Fill into the formula $C^2 = a^2 + b^2$

$c\ 16^2 + 12^2$
$c^2 = 256 + 144$
$c^2 = 400$
$c = 20$

14. Find the missing side



<u>Hint</u>: $c^2 = a^2 + b^2$

$5^2 = 4^2 + b^2$

$25 \neq 16 + b^2$

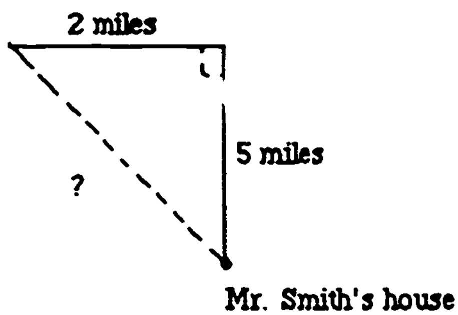
$\underline{-16 \quad -16}$

$9 = b^2$

$3 = b$

Remember c^2 is the hypotenuse which is the side opposite the right angle.

15. Mr. Smith walked to the grocery store. He left his house and walked 5 miles north and 2 miles due west. How far was he from home?



<u>Hint</u>: $c^2 = a^2 + b^2$

$c^2 = 10^2 + 4^2$

$c^2 = 100 + 16$

$c^2 = 116$

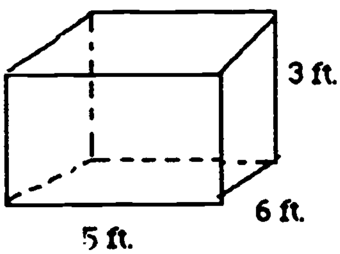
$c = 10.8$

Section III Volume

Volume is the number of cubic units of space a figure occupies and $V=lwh$; $V=s^3$; $V=\pi r^2h$

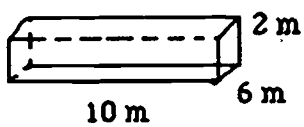
Complete the following:

1. Find the volume of this rectangle



Hint: Put the numbers into the formula. $V=lwh$ or $5 \times 6 \times 3 = 90$

2. Find the volume of this rectangle



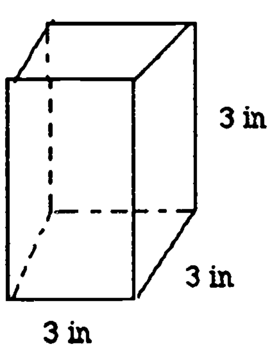
Hint: Put the numbers into the formula: $V=lwh$ or $10 \times 6 \times 2 = 120$

3. A rectangular van is 40 ft. long and 11 ft. high. The volume is 3,960 cu ft. What is the width?

Hint: Fill into the formula what you do know.

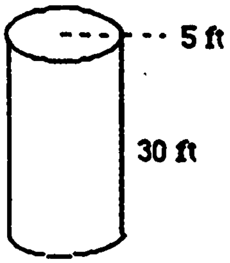
$$V = lwh$$
$$3{,}960 = 40 \times W \times 11$$
$$\frac{3{,}960}{440} = \frac{440}{440} w$$
$$9 = w$$

4. Find the volume of this cube.



Hint: Fill in the numbers to the formula. $V=S^2$ or $6 \times 6 \times 6 = 216$

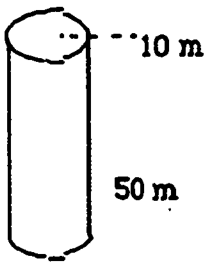
5. Find the volume of this cylinder.



<u>Hint</u>: Put into the formula what you know.

$V = \pi r^2 h$
$V=3.14 \times 5^2 \times 30$
$V = 3.14 \times 25 \times 30$
$V = 2{,}355$

6. Find the volume of this cylinder



<u>Hint</u>: First find the radius since the diameter is given. $r = 10 + 2 = 5$
Now fill in the formula.

$V = \pi r^2 h$
$V = 3.14 \times 5^2 \times 50$
$V = 3.14 \times 25 \times 50$
$V = 3.925$

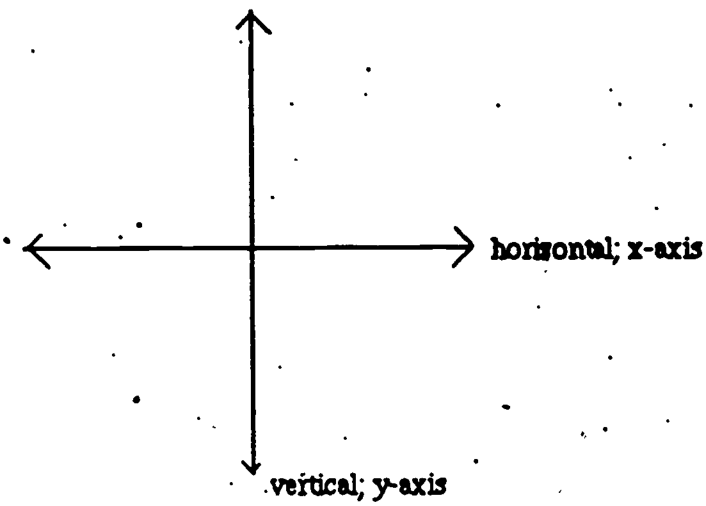
7. A cylindrical pipe has a diameter of 12 in. and the volume is 1243.33 cu. in. What is the height?

<u>Hint</u>: First find the radius. $r = d+2$ or $12 +2 = 6$
Then put into the formula what you do know.

$V = \pi r^2 h$
$1243.44 = 3.14 \times 6^2 \times h$
$1243.44 = 3.14 \times 36 \times h$
$\frac{1243.44}{113.04} = \frac{113.04h}{113.04}$
$11 = h$

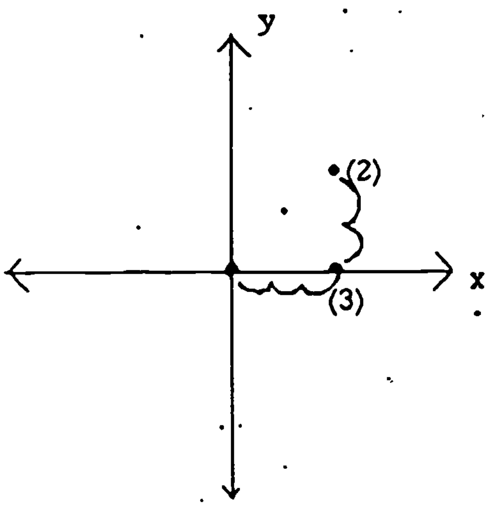
Section IV Plotting Ordered Pairs, Distance Between Points and Finding Midpoint

Review:



-This grid is used to help locate points.
-The first number of an ordered pair is an x-coordinate and the second is a y-coordinate.
-When starting to locate points begin at the origin which is where the axes cross.
-Positive numbers are the right on the x-axis and upper part of y-axis. Negative numbers are the left on the x-axis and lower part of y-axis.

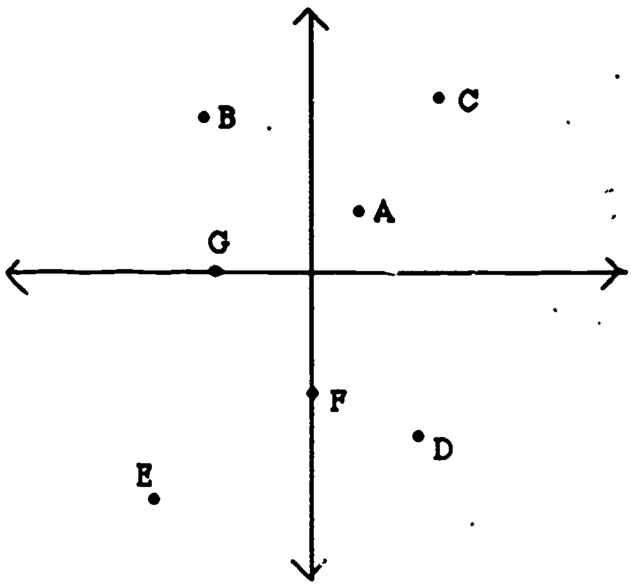
Example
Plot (3, 2)



Step 1: Start at the origin. The x-coordinate is 3 so move 3 spaces to the right (positive).
Step 2: The second number is 2 so move 2 spaces up (positive). Mark a dot at this point.

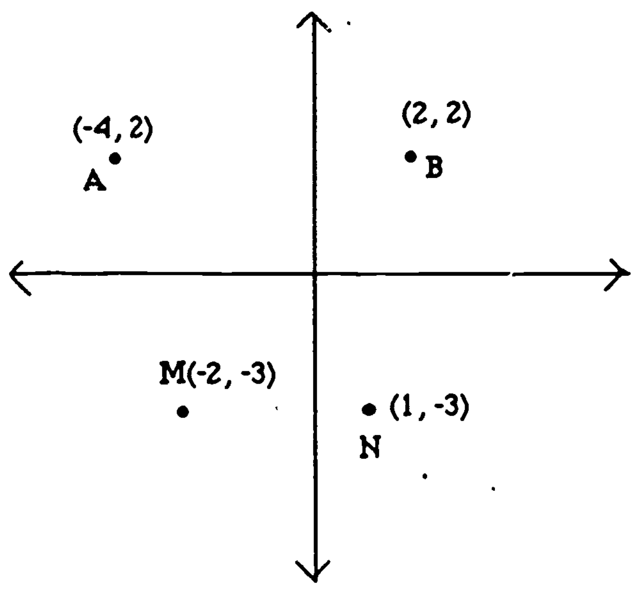
Complete the following:

1. Plot the following points: (1, 2)A, (-2, 4)B, (5, 3)C, (2, -3)D, (-3, -4)E, and (0, -2)F, (-2, 0)G



Hint: Remember on the x-axis (dealing with the first number) a positive number moves to the right and a negative number to the left. Also, the y-axis (second number) a positive number is up and the negative numbers go down.

2. What is the distance between point A and point B and the distance between point M and point N in this figure?



Hint: To solve, just count the spaces or units between the two points.

3. What is the distance between the ordered pairs? (5, 3) and (4, 3)?

Hint: The second number of the ordered pairs is the same so the line joining the points is parallel to the x-axis. Using the 5 and 4 the distance must be 5 - 4 or 1 unit.

4. What is the distance between (0, 7) and (0, -1)?

Hint: Using the y-coordinates 7 and -1 this time you must add the numbers since it is negative 1. 7 + 1 or 8 units.

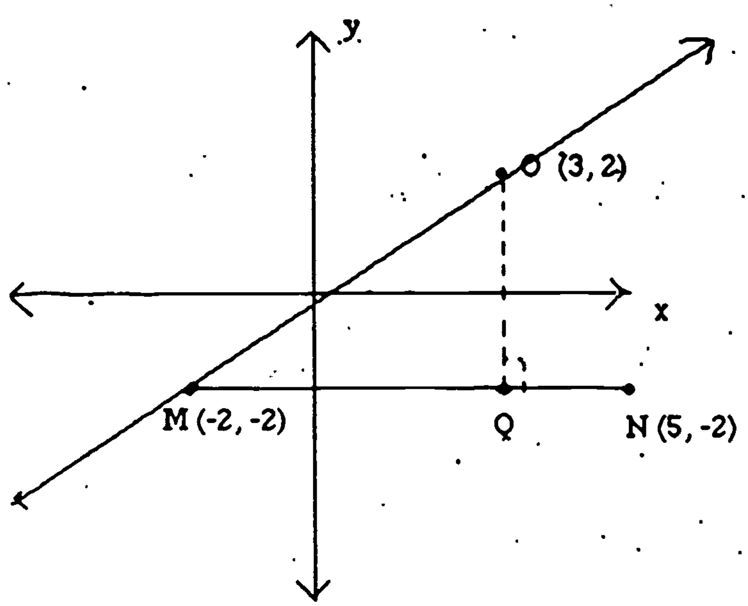
5. Find the distance between (3, 5) and (3, -3).

Hint: 5 + 3 or 8 units.

6. Find the distance between (3, 5) and (3, 3)

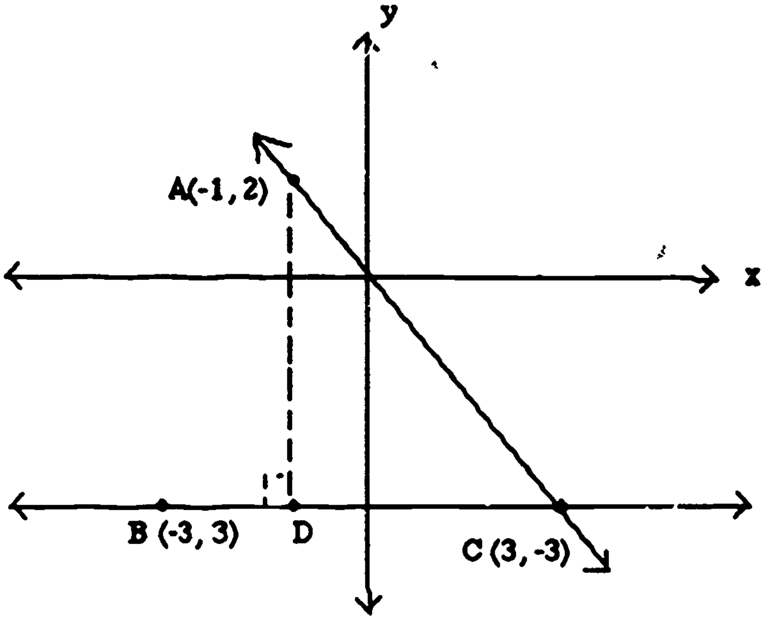
Hint: 5 - 3 or 2 units.

7. What is the perpendicular distance from 0 to line MN?



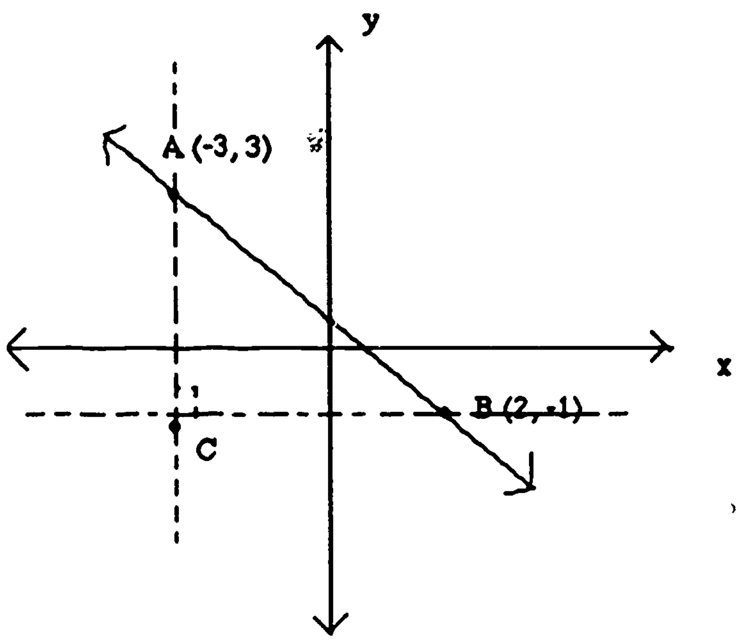
Hint: First draw a line perpendicular from point O to line MN. Label that point with a letter (Q). Next find the length of OQ. Point O is 2 units above the x-axis and point Q is 2 units below the x-axis; 2 + 2 = 4 units. Therefore the perpendicular distance from O to MN is 4 units.

8. Find the perpendicular distance from A to BC



Hint: Point D is (-1, -3) and A is (-1, 2) so the distance is 2 + 3 or 5 units.

9. What is the distance between points A and B?



Hint: Make a horizontal line through one point and a vertical line through the other. Label the point C (or any letter you want). Find the length of line AC and CB. AC=4 units and CB = 5 units. Now use the pythagorean theorem to find the hypotenuse or line AB.

$C^2 = a^2 + b^2$
$C^2 = 4^2 + 5^2$
$C^2 = 16 + 25$
$C^2 = 41$
$C = 6.4$

Another way of doing this problem is using the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

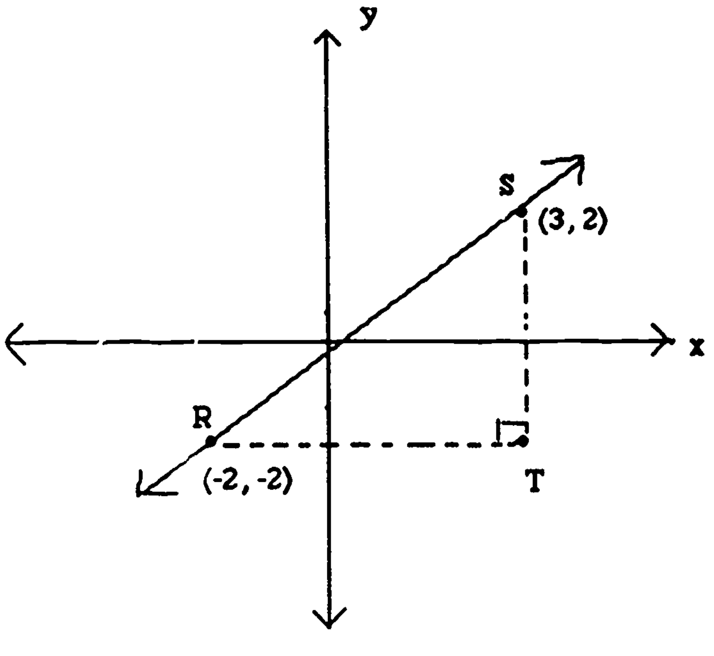
x_1 and x_2 are the first ordered pairs and y_1 and y_2 is the second. The first point (x_1, y_1) and the second (x_2, y_2).

$$d = \sqrt{(2 - -3)^2 + (-1 - 3)^2}$$

$$d = \sqrt{5^2 + 4^2}$$

$$d = \sqrt{41} \qquad d = 6.4$$

10. What is the distance between points S and R?



Hint: Make a horizontal and vertical line. Line RT = 5 units and line ST = 4 units.

$C^2 = a^2 + b^2$
$C^2 = 4^2 + 5^2$
$C^2 = 16 + 25$
$C^2 = 41$
$C = 6.4$ or

$$d = \sqrt{(x_2 - x.)^2 + (y_2 - y,)^2}$$

$$d = \sqrt{(2 - -3)^2 + (-1 - 2)^2}$$

$$d = \sqrt{5^2 + 4^2}$$

$$d = \sqrt{41} \qquad d = 6.4$$

11. What is the distance between point (5, 3) and (-2, -4)?

<u>Hint</u>: Use the distance formula or you could plot the numbers and the pythagorean theorem.

$$d = \sqrt{(x_2 - x.)^2 + (y_2 - y,)^2}$$

$$d = \sqrt{(-2 - 5)^2 + (-4 - 3)^2}$$

$$d = \sqrt{7^2 + 7^2}$$

$$d = \sqrt{49 + 49}$$

$$d = \sqrt{98} \qquad d = 9.9$$

12. What is the distance between point (6, 4) and (-1, -5)?

$$d = \sqrt{(x_2 - x.)^2 + (y_2 - y,)^2}$$

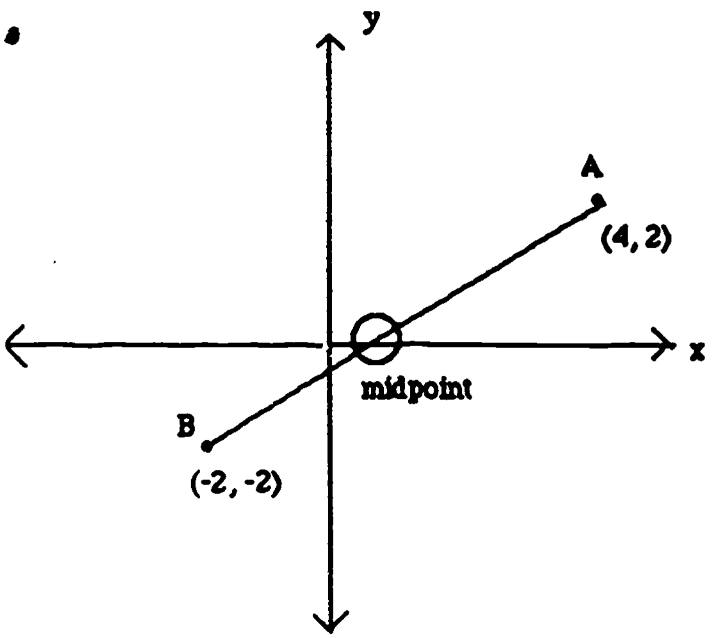
$$d = \sqrt{(-1 - 6)^2 + (-5 - 4)2}$$

$$d = \sqrt{7^2 + 9^2}$$

$$d = \sqrt{49 + 81}$$

$$d = \sqrt{130} \qquad d = 11.4$$

13. Find the ordered pair for the midpoint of line AB _______



Hint: First add the x-coordinates and divide by 2. 4 + (-2) = 2 + 2 = 1
The x-coordinate of the midpoint is 1. Next do the same with the y-coordinates and divide by 2. 2 + (-2) = 0 + 2 = 0
The y-coordinate of the midpoint is 0. The midpoint is the point (1, 0).

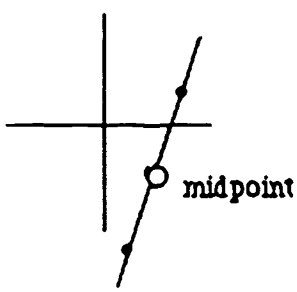
14. What is the midpoint of a line joining points (3, 5) and (2, 7)?

Hint: Add x-coordinates and divide by s: 3 + 2 = 5 + 2 = 2.5
Add y-coordinates and divide by 2: 5 + 7 = 12 + 2 = 6

15. What is the midpoint of a line joining points (2, -11) and (4, 2)? ____

Hint: 2 + 4 = 6 + 2 = 3 (x-coordinate)
-11 + 2 = -9 + 2 = -4.5 (y-coordinate)

A way to check is to plot the points:



Unit V - Chapter 5 •Word Problems•

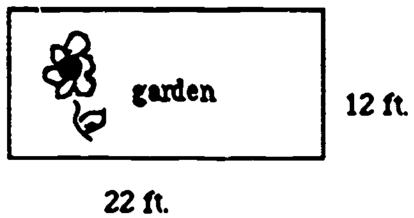
Section V   Word Problems

Before you begin the following set of problems, remember the five steps of problem solving:

1. What is being asked for?
2. What information do you need to solve the problem?
3. Which arithmetic operation or formula will you need to use?
4. Do the mat and check your work.
5. Does your answer make sense?

Do the following:

1. Mr. Jensen wants to put a fence around his rectangular garden. The fence costs $2.60 per foot. How much will this fence cost?



Hint: First find the perimeter:

$P = l + l + w + w$

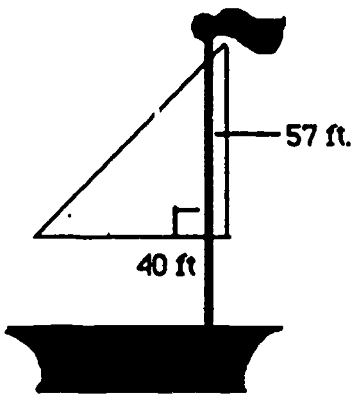
$P = 22 + 22 + 12 + 12$

$P = 68$ ft.

Now multiply the amount of ft. needed and the cost:

$2.60 \times 68 = \$176.80$

2. Jim's sail of his sailboat is shaped like a right triangle. What is the sail's area in square feet?



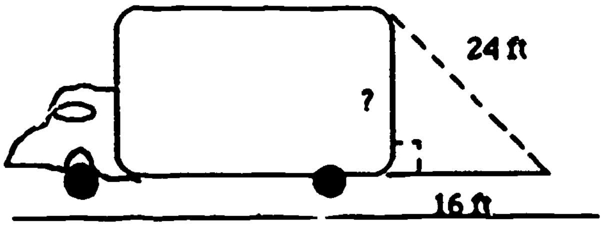
Hint: Use the area formula for a triangle.

$$A = \frac{1}{2} bh$$

$$A = \frac{1}{2} \times 40 \times 57$$

$A = 20 \times 57$
$A = 1140$

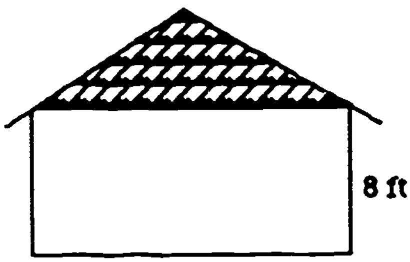
3. What is the height of this truck?



Hint: Use the pythagorean theorem.

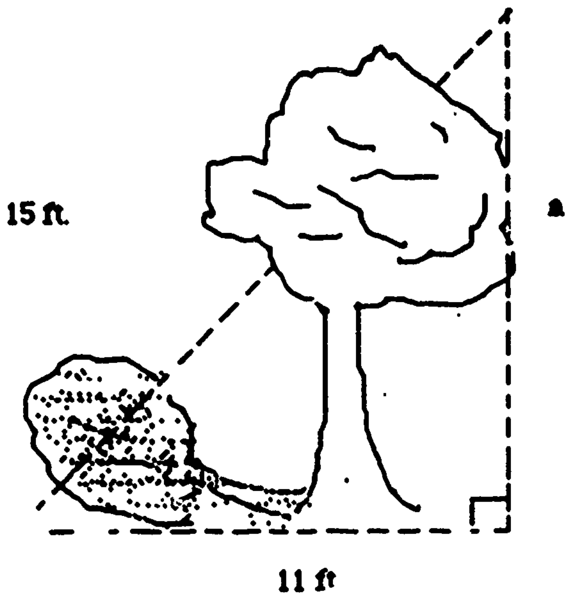
$c^2 = a^2 + b^2$
$24^2 = 16^2 + b^2$
$576 = 256 + b^2$
$320 = b^2$
$17.8 = b$

4. What is the area of the Kent's house?



Hint: Use the area of a rectangle formula.
$A = lw$
$A = 50 \times 8$
$A = 400$ sq. ft.

5. What is the height of the tree?



Hint: Use the pythagorean theorem.

$$c^2 = a^2 + b^2$$
$$15^2 = a^2 + 11^2$$
$$225 = a^2 + 121$$
$$104 = a^2$$
$$10.2 = a$$