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ABSTRACT

This document provides research-based information to help school district personnel select appropriate mathematics education programs for their limited English proficient (LEP) elementary school students. A review of the mathematics education literature is discussed in the context of the reform movement in school mathematics. Two instructional prc.:ams for effectively teaching mathematics to LEP students, Active Mathematics Teaching (AMT) and Cognitively Guided Instruction (CGI), are discussed in detail. Examples, using addition and subtraction problems, illustrate each program. Since teaching math in a student's native language may be more effective than limiting instruction to English, Spanish translations of examples are also included to demonstrate how simple presentation of problems can facilitate their solving. Recommendations are woven throughout the text and each section ends with a list of additional recommendations for teaching mathematics to LEP students. The following final recommendations are suggested: (1) choose and use manipulatives carefully; (2) manipulatives should support discussion about mathematics, not replace it; (3) activities should emphasize the mathematics content; (4) the National Council of Teachers of Mathematics' document, "Curriculum and Evaluation Standards," provides specific recommendations for content that should be emphasized as well as deemphasized; (5) AMT has been proven effective for conveying large amounts of basic information that is well organized; (6) CGI shows promise for developing problem solving skills, higher order thinking, and enhancing student confidence; and (7) mathematics is too important for students' futures to be reduced to computations or omitted entirely. Five figures are included. A list of 72 references is appended. (FMW)

Teaching Mathematics With Understanding To Limited English Proficient Students

Walter G. Secada
Deborah A. Carey

ERIC®
Clearinghouse on Urban Education

TEACHING MATHEMATICS WITH UNDERSTANDING TO LIMITED ENGLISH PROFICIENT STUDENTS

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INTRODUCTION

This document provides research-based information to help school district personnel select appropriate mathematics education programs for their limited English proficient (LEP) students. It moves from research to practice with a strong emphasis on the end users: managers of bilingual education and/or English as a second language programs, teachers of mathematics for LEP students, principals, administrators, and others who are concerned about the mathematics education of LEP students.

Drawing mainly from mathematics education research, the discussion of that literature is placed within the larger contexts of the reform movement in school mathematics (National Council of Teachers of Mathematics [NCTM], 1989a; 1989b; National Research Council [NRC], 1989) and of two instructional programs for teaching mathematics: Active Mathematics Teaching (Good & Grouws, 1979; Good, Grouws, & Ebmeier, 1983) and Cognitively Guided Instruction (Carpenter, Fennema, Peterson, Chiang, & Loef, 1990; Fennema & Carpenter, 1988). After the mathematics education literature in each of these areas is reviewed, issues involving the education of LEP students are discussed. Recommendations are woven throughout the text, and each section ends with a list of additional recommendations for the mathematics education of LEP students.

Within the constraints of limited space, it was not possible to cover all the relevant aspects of the research literature, nor every issue of importance. Thus, this monograph seeks depth rather than breadth of coverage of mathematics and/or bilingual education research. Most examples are from the research on addition and subtraction. In part, this is because most LEP students are enrolled in the primary grades, and, thus, the content that was selected should be of interest to many readers. This detailed treatment of limited content will, hopefully, encourage readers to compare and contrast Active Mathematics Teaching (AMT) with Cognitively Guided Instruction (CGI). Also, it will facilitate decisions about using either or both approaches in some combination. The general principles of AMT and CGI transfer across mathematical content; it is the examples that are constrained.

There are many different kinds of programs for LEP students. They include maintenance bilingual education, late and early exit transitional bilingual education, English as a second language, and sheltered English programs—to name but a few. Also, there are many different kinds of teachers who work with LEP students. There is the teacher who speaks only English who may find herself in a classroom where LEP children speak many different languages, or there may be the bilingual, biliterate teacher whose classroom contains LEP children with a native language background that matches her own language proficiencies. The reader will need to evaluate for himself or herself how well each example applies personally.

Note that while the pronoun "she" is used when referring to a teacher, and "he" when referring to a student when the gender of either is indeterminate, there is no intent to stereotype roles through such usage.

SOME WORKING ASSUMPTIONS

Some assumptions on what it means to be limited English proficient (LEP) and to learn mathematics, on good practice in the teaching of mathematics, on understanding mathematics, on mathematics as more than computations, and on strategic mathematics teaching guided the development of this monograph. These working assumptions are discussed below.

BEING LIMITED ENGLISH PROFICIENT AND LEARNING MATHEMATICS

In some ways, this monograph challenges some misconceptions of what it means to be limited English proficient and to learn mathematics. Researchers and practitioners often assume that because a child's English proficiency is less than desirable the child either cannot do or cannot learn mathematics beyond basic computations. For example, many transitional programs first move their middle and high school students into all-English mathematics classrooms on grounds that students will master basic computational skills there. But those courses are general mathematics or some variant; they usually are limited only to basic skills and they spell the end of students' mathematics course taking. Hence, this practice effectively excludes children from access to the more advanced portions of the mathematics curriculum.

More desirable options are to place students in courses that include more advanced content and that lead into later course taking even though such courses may require the modification of instruction to accommodate a student. Alternatively, if access to more advanced mathematical content is possible only if the student remains in a bilingual setting, then the student's transfer into an all-English setting should be delayed. Content that is learned, after all, transfers across languages.

There is quite a bit of research linking level of English language proficiency to mathematics achievement (see reviews by De Avila, 1988; and by De Avila & Duncan, 1981). However, these links might not occur naturally. Rather, they might be forged by school practices. In a study of first grade Hispanic LEP children (i.e., children whose schooling had just begun), Secada (in press-a) found minimal relationship between children's fluency, either in English or in Spanish, and their solving of arithmetic word problems in English or in Spanish. Fluency was measured by the story retelling portion of the Language Assessment Scales, Pre-School Version (Duncan & De Avila, 1986, 1987).

Moreover, there is ample research which acknowledges the challenges posed by children's varying levels of language proficiency, but which posits that students identified as limited English proficient are actually in the process of steadily developing proficiency in both their native language and English (Hakuta, 1986). There are cognitive benefits to being bilingual—benefits that are linguistic in nature (Diaz, 1983), but also that are being documented in mathematics (Duran, 1988; Secada, in press-a). These findings support educational practices that aim to help students achieve competence in both of their languages and that provide access to mathematics that is not constrained because of purported language deficits.

Good Practice

We have assumed that what research documents as good practice in the teaching of mathematics for monolingual English speaking populations can be adapted for LEP students, provided that practice is informed by what is known about their educational needs in general. An example of such an effort is the *Significant Bilingual Instructional Features Study* (Tikunoff, 1985) wherein the principles of direct instruction were used to identify the characteristics of good instructional practices in bilingual classrooms.

Also, LEP students can be taught much content in their native languages on the assumption that the knowledge will transfer to the English language as the students' proficiency in English increases (Hakuta, 1986). Hence, bilingual teachers who use native language approaches for teaching mathematics should engage their students in worthwhile mathematics; the knowledge that students develop eventually will transfer.

Learning Mathematics with Understanding

The mathematics education research literature strongly suggests that the best teaching practices are those that assess what students understand in a range of mathematical problem settings and then develop those understandings to their mathematical end points.

The most important outcome for mathematics instruction is the student's learning with understanding. If something is not taught so that it can be learned with understanding, then instruction should be changed so that understanding can take place; or, teaching that content should be postponed; or, if that content can't be taught meaningfully, then it shouldn't be taught at all.

Mathematical understanding means more than that students simply display what they have just been taught. Mathematical understanding means that students can link what they are learning to previous knowledge that they already (should) have. Understanding means that students can explain why they believe something is true in a way that is sensible to someone else. Students may be somewhat tentative or even unsure of their explanations. But, as they try to explain what they mean and to make sense of what they are doing, students who understand a concept or how they solved a particular problem should become more sure of themselves, or they should realize the limits of their understanding. Finally, students with understanding become confident in their abilities to apply the mathematics that they know in new settings and to make sense of those settings using that knowledge.

Understanding for LEP students is problematic on a variety of grounds. First, students' explanations may be tentative not only because of content mastery, but also because of the language used in the discussion. Even when using their native languages, many bilingual students may have difficulty expressing their thoughts in as sophisticated a manner as teachers might like. This is because many teachers (and researchers) confound how people use mathematical language with actual knowledge of mathematics. People who sound like they know what they are talking about are judged to have knowledge, while those who don't express themselves well are judged not to have such knowledge. However, if the best practices begin where students are academically, then teachers of LEP students need to begin not only with what students understand but also with how they can express their understandings. Further, teachers should help students develop both mathematical understanding and its communication (Tikunoff, 1985; NCTM, 1989a, 1989b).

Understanding develops from what is already known and it develops over time—sometimes slowly, yet sometimes in a flash of insight. For example, all children enter school with a broad range of understandings about numbers, counting, and addition and subtraction as evidenced through their solving of word and non-verbal problems (Carpenter & Moser, 1982, 1983, 1984; Fuson, 1988; Secada, in press-a). These are informal understandings, based on intuitions, and, often, they are not completely interrelated. Instruction should build upon and develop this knowledge that children bring with them to school. Doing so results in children's learning mathematics and in their becoming more confident about their abilities. Below, we describe one effort to accomplish this: Cognitively Guided Instruction (CGI).

Unfortunately, beginning in first grade, the mathematics curriculum ignores the rich store of knowledge that children bring to school (Carpenter, 1985; Putnam, Prawat, & Reineke, 1990). Precisely because instruction fails to build upon such knowledge, children come to divorce school mathematics from the real world mathematics that they already know. They learn that mathematics is little more than unrelated facts, algorithms, and tricks that must be memorized for tests and can be forgotten as quickly as possible with no practical consequence (Carpenter, 1985).

Similarly, instruction for students from culturally diverse backgrounds often does not take account of the everyday sources of their informal knowledge, i.e., the knowledge that they bring from home. Many writers have warned about the dangers of such cultural discontinuities in educational practices (Moll,1990; Stanic, in press; Tikunoff, 1985). Hence, teachers of LEP students not only need to focus on their students' informal understandings, but they also need to seek those understandings in ways and in settings that are other than commonly supposed.

MATHEMATICS IS MORE THAN COMPUTATIONS

Basic computational skills are to mathematics what learning the alphabet is to reading, or what learning the notes is to playing and writing music. They are a beginning, but not enough.

Since most adults in the United States do their computations on five dollar pocket calculators, the overemphasis placed on computations throughout elementary school and in remedial mathematics is questionable. It makes little sense to withhold from children something that is an integral part of their cultural heritage, especially when the use of a calculator for all routine computations could make so much more mathematics available to our students (NCTM, 1989a).

Memorized number facts and calculations do have a place in the curriculum. But, if we teach so that students understand how numbers are interrelated and how they can be used to solve problems, the memorized number facts will follow (Carpenter et al., 1990). People who understand how numbers are related among themselves (i.e., they have what is called number sense) also have memorized a store of basic facts that are well organized and useful; people who don't understand do not.

Beyond computations, there are many other worthwhile topics for coverage in school mathematics. These topics include number and spatial sense, geometry, measurement, and chance (NCTM, 1989a). How programs and teachers might make trade-offs in what content is covered for LEP students is discussed in greater depth below.

TEACHING MATHEMATICS STRATEGICALLY

It is better to teach a few important topics, and to teach them well, than it is to try to teach too much and fail to teach anything well. Porter (1989), and Porter, Floden, Freeman, Schmidt, and Schwille (1988), have documented how elementary school teachers spend much of their time reviewing mathematics material that should have been taught the previous year. Then, they rush through or skip whole chapters and units in an effort to cover the course before the end of the term or school year. By default, students revisit the same old content and only minimumly delve into new and important material. It would be better to decide what the important content is and to spend time teaching that content well (see Putnam, Prawat, & Reineke, 1990).

For bilingual and other teachers of LEP students, strategic teaching becomes even more important. As noted earlier, the first all-English course into which transitional programs place middle and high school LEP students is often general mathematics (or some variant), a course that is focused on basic computations. LEP students deserve to have access to more advanced mathematics—and to the careers that such knowledge makes available.

Recently, there has been increased emphasis on content based instruction in English as a Second Language (ESL; Chamot & O'Malley, 1988; Crandall, Dale, Rhodes, & Spanos, 1987, in press; Dale & Cuevas, 1987; Spanos, Rhodes, Dale, & Crandall, 1988). Hence, ESL teachers often find themselves asked to help students develop mathematical language in their courses. In numerous workshops involving the teaching of mathematics, one of us has faced ESL teachers, many of whom have students for at most an hour a week, arguing that they do not have the luxury of working on student understanding. Rather, they wanted to know how to remedy their students' computational errors, and how to use key words or to devise similar tricks for helping students solve word problems. In one half to one hour a week, a teacher cannot remediate what has been insufficiently taught during the balance of that week. Thus, teachers in this situation should determine the critical classroom tasks that their students need to understand in order to learn mathematics, and they should work on those. At a minimum, students receiving content based ESL instruction should learn how to use the glossary and index at the back of their mathematics books, how to pre-read the text for what will be covered the next day or week, how to monitor their own understanding of a mathematics lesson, and how to ask questions of the mathematics teacher when they get lost. These strategies are seldom taught explicitly in mathematics courses, although most successful students pick them up.

Sheltered-English mathematics classes often must take on additional learning objectives related to language development of LEP students. However, usually there is not enough time to both cover the regular mathematics curriculum adequately and do justice to language development. In such settings, teachers feel increasing pressure to spend time on just a few very basic lessons where both content and language can be worked on. Then, in a scenario that is worse than what is described above (Porter et al., 1988), teachers must skip even more sections of the mathematics course or they must rush to cover that material at the end of the year. In either case, neither the teaching of mathematics nor the development of language gets done well.

Depending on the kind of program they are teaching in, bilingual mathematics teachers need to ensure not only that they cover the content of their lessons, but also that it transfers properly to the all-English classroom. Unfortunately however, except for the fact that bilingual teachers who teach in delayed exit programs will tend to use more of the child's native language than those who teach in early exit programs, bilingual teachers differ very little among themselves—either in their goals or in their instructional methods (Ramirez, 1986). Bilingual teachers in late exit or dual language maintenance settings should attend to children's development of mathematical concepts and understandings as well as ways of using mathematics for communication via their native languages. Alternatively, bilingual teachers in early exit settings need to ensure that conceptual understandings are strong enough in the child's native language to transfer across languages and to help students develop their communication skills in English (Tikunoff, 1985).

By no means are any of the above teaching tasks easy. Their complexity is due to the fact that mathematics instruction for LEP students must meet multiple goals and objectives, although even the original learning objectives are difficult enough to meet in the limited time teachers have. Hence, mathematics teachers of LEP students find themselves overwhelmed with what is expected of them, and often they will meet just those goals that are easiest for their children and let the others go. Instead, teachers should try to meet these multiple demands strategically. They might combine some goals; they should decide which are the important goals, work on those, and let the others go. Yet, throughout the teaching process, the focus should be on developing understanding, on developing more than basic computational skills, and finally, on attempting fewer learning tasks, but doing them well.

CURRICULUM

A common misconception is that the official school curriculum is what actually gets taught to all students. In fact, teachers are probably the single most important decision-makers for determining the mathematics content to which students actually are exposed. When a teacher skips content because she believes that some of the material might be too difficult for her students, she has made a decision involving curriculum. When students spend days reviewing lessons covered during previous years or earlier during that same year, and then rush through the book at the end of the year, that is curriculum. When a teacher follows (or opts not to follow) her teacher guides in assigning homework, chooses materials and manipulatives to support her teaching, or poses a real world problem, she is determining the mathematics that her students receive. Hence, a teacher's first question to ask is, "What mathematics content should I teach?"

CURRENT PRACTICE

In elementary school, most mathematics is taught through what is known as the spiralling curriculum. The same topics are covered over the course of three or more years, although, theoretically, more in-depth each time it is presented. Content moves up within the book, so that what was new the first year becomes review by the third. Unfortunately, Porter (1989, Porter et al., 1988) has shown how this organization of the content leads to the same low level content being covered year after year. New or more challenging content that goes beyond basic computational skills development is seldom taught even when it appears in the text because it appears at the end when there is little time remaining to deal with it deeply. This "underachieving curriculum" has been blamed for the United States' abysmal performance on international comparisons of mathematics achievement (McKnight, Crosswhite, Dossey, Kifer, Swafford, Travers, & Cooney, 1987).

Not only do these flaws in the mathematics curriculum prevent students from learning all that they might personally benefit from learning, but they also inhibit our society's ability to meet technical, economic, and defense needs into the next century. Moreover, many people are concerned about the costs to society if our schools are unable to produce students who are sufficiently mathematically literate to participate in our most cherished democratic institutions and in our rapidly changing economic system (Secada, 1990, in press-b).

Thus, educators of LEP students should not be satisfied with a mathematics curriculum like the one native English speakers are exposed to, for its adequacy is questionable. Rather, curriculum for LEP students should be developed so that they can experience a more effective mathematics education.

REFORM MOVEMENT CHANGES FOR SCHOOL MATHEMATICS

Existing knowledge about how children learn mathematics can inform recommendations for teaching them successfully (Romberg & Carpenter, 1986). Further, demographers, economists, scientists, and mathematics educators have made projections about the sort of world that today's students will live in that permit some educated conjectures about what sorts of mathematics all students—mainstream as well as LEP—will need in order to participate in that world (Johnston & Packer, 1987; National Alliance for Business [NAB] 1986a, 1986b; NRC, 1989). Thus, the mathematics curriculum is being changed so that it is more genuine and useful for all students (NCTM, 1989a; NRC, 1989; Putnam et al., 1990; Romberg & Stewart, 1987).

In one of the major documents of the current reform movement, the National Council of Teachers of Mathematics (1989a, p. 5) outlines a vision based on five broad goals for all students in mathematics:

(1) that they learn to value mathematics;

(2) that they become confident in their ability to do mathematics;

(3) that they become mathematical problem solvers;

(4) that they learn to communicate mathematically; and

(5) that they learn to reason mathematically.

In achieving these goals, students should be exposed to and examine a variety of situations in which mathematics is useful and makes sense. Students should make and validate conjectures about the situations that they are studying. They should apply what they have learned in new settings. They should solve problems. And finally, students should talk with one another and write using mathematical language and symbols.

To help developers of mathematics curricula write texts that will enable students to achieve these five broad goals, the NCTM's (1989a) *Curriculum and Evaluation Standards for School Mathematics* lists criteria for mathematics curricula in grades K 4, 5-8, and 9-12, and also for evaluation of the curriculum and student achievement. Cutting across grade levels are four facets of mathematics: that it is problem solving, that it involves interpersonal communication, that reasoning is crucial to learning mathematics, and that mathematical knowledge is interconnected. Other criteria for curriculum development are focused on specific content strands: numbers, operations, fractions and decimals, measurement, geometry and spatial sense, probability and statistics, algebra, patterns and relationships. For evaluation, the *Curriculum and Evaluation Standards* note that the assessment of student learning should be aligned to the curriculum and should rely on multiple sources of information. Also, assessment should include problem solving, communication, reasoning, concepts, procedures, and dispositions. Evaluation of school mathematics, according to the *Curriculum Standards*, should include program design, curriculum and instructional resources, and finally, instruction itself.

As expressed by NCTM, "all students, regardless of their language or cultural background, must have access to the full range of mathematics courses offered. Their patterns of enrollment should not differ substantially from those of the total student population. . . . [If] unacceptable patterns emerge, an evaluation should identify the barriers creating the situation and recommend action" (1989a, pp. 239-240). One of the principle barriers to full participation in mathematics is the early and inappropriate placement of students in all-English mathematics classes where the focus is on the development of basic skills to the detriment of other content. It is better to keep students in bilingual settings where they can engage in worthwhile mathematics.

The *Curriculum and Evaluation Standards* (NCTM, 1989a) also provide some very specific recommendations about aspects of school mathematics that should be emphasized and others that should be deemphasized.[1] First, the *Standards* emphasize new content. Number theory, discrete mathematics, probability and statistics, geometry, and measurement are critical content if students are to participate in the world of the next century. They need to become part of the content that all children are exposed to.

Second, all content should be situated within meaningful contexts. These settings should call for students to solve problems, to conjecture, to reason, to validate and prove

[1] For a more detailed presentation of these recommendations, see NCTM, 1989a, pp. 20-21. 70-73, 126-127, 191.

their conjectures, and to communicate. Mathematics should become a social task and something that students do, as oppose to absorb. For students of limited English proficiency, bilingual settings may prove the best for them to engage in such social interactions. Alternatively, schools that cannot use native language approaches will need to adapt their all-English settings so that all students can participate in the social interactions through which mathematics will be learned. Moreover, such settings need to reflect the diverse ways in which students have learned to communicate, the tasks they consider meaningful, and how they go about attempting to do those tasks (Cole & Griffin, 1987; Moll, 1990; Secada, 1990; Tikunoff, 1985).

Third, content that is outdated or is not meaningful should be deemphasized. Outdated content includes complex computations, such as adding long columns of multi-digit numbers or doing long division, where use of paper and pencil is required. There are more efficient tools, like calculators, for doing these computations, and they should be used.

Non-meaningful content to deemphasize includes the use of key words for solving word problems. Key words work only for some problems, not for all of them. That they work so well for current curricula reflects the impoverishment of those curricula; it does not prove that there is anything special about key words. Moreover, the use of key words short-circuits children's natural tendency of trying to figure out problem situations (see, e.g., Carpenter & Moser, 1983), and it communicates that mathematics is little more than a bunch of unrelated rules that are applied in a mindless manner.

Fourth, superficial rules should be eliminated for they work only for limited content and children misapply them in other settings. For example, primary school teachers often tell children that one cannot take away a larger from a smaller number. But, if it is 8 degrees right now, and the weather bureau calls for an overnight drop of 12 degrees, what will the temperature be tomorrow? You can "take away" a larger from a smaller number; what you get is a negative number.

Finally, curriculum should dictate assessment, not the reverse. The tyranny of achievement testing on what gets taught to students has been well documented (Silver, in press). Too often, teachers teach to a test. This is especially true in compensatory programs where program evaluation revolves around achievement test scores. This practice needs to be changed.

CURRICULUM FOR LEP STUDENTS

In addition to proficiency in English, low academic achievement is implied in most state and federal rules and regulations that define LEP status. LEP students are placed in compensatory programs that are intended to remedy their purported language and academic deficiencies. Cole and Griffin (1987, pp. 4-5) characterized the mathematics education that students in compensatory education programs receive:

> Administrators and teachers in districts with large minority populations are often under considerable pressure to reduce dropout rates and increase achievement test scores. . . . It is not surprising that educators of minority students are pressured to "do the basics" better and to leave innovative educational practices to others. However, a continued imbalance in the educational mandates that guide the education of minorities and of white middle-class children deepens the problem: as schools serving minority children focus their resources on increasing the use of well-known methods for drilling the basics, they decrease the opportunities for those children to participate in the higher level activities that are needed to excel in mathematics and science.

Hence, there is a very real danger that LEP students will be omitted from participation in a more meaningful mathematics curriculum that should result from the current mathematics education reform movement. In part, this is because of the stress on program evaluations that use standardized achievement tests. Such tests are overloaded with basic computational items and they contain very little advanced content.

Also, LEP students are in danger of bering left out of the current mathematics reform efforts because many people believe that LEP students cannot engage in the mathematics that their English proficient peers are capable of. The good news, however, is that research is beginning to document that LEP students are, in fact, capable of accomplishing many of those same learning tasks. For example, Secada (in press-a) found that first grade Hispanic LEP children can solve addition and subtraction problems that are similar to those that monolingual children can solve (Carpenter & Moser, 1983). LEP first grade children showed a sensitivity to the semantic structures of those problems similar to that shown by their English proficient peers.

De Avila, Duncan, and Navarrete (1987) have developed an activities based mathematics and science curriculum, *Finding Out/Descubrimiento*. Its successful development with LEP Hispanic students demonstrated that bilingual children can participate in the rapid give-and-take that characterizes cooperative group structures. Similarly, in another study involving junior and senior high school LEP students, Rosebery, Warren, and Conant (1990) and Warren, Bruce, and Rosebery (1988) have shown how speakers of Haitian Creole can develop academic and linguistic competence while actually doing science. *Cheche Konnen,* as this project is known, involves students whose academic and linguistic skills placed them near the bottom of their school's achievement. Hence, there is a slowly developing picture of LEP students suggesting that they should not be automatically excluded from efforts to improve the mathematics curriculum.

Another issue in mathematics curriculum for LEP students revolves around what is known as mathematical language (Crandall, et al., 1987, in press; Cuevas, 1984; Dale & Cuevas, 1987; Mestre, 1988; Pimm, 1987; Spanos, et al., 1988). Many educators of LEP students believe that mathematics contains specific language, unique terms and symbols, and methods of expression that occur when people engage in mathematical discourse. Possibly the most detailed analysis of mathematical language is that of Crandall and her colleagues who have found syntactic, semantic, and pragmatic features in mathematical discourse (cf. Spanos et al., 1988, pp. 226-227).

Chamot and O'Malley (1988); Crandall, Dale, Rhodes, and Spanos (1987), and Dale and Cuevas (1987), have recommended the development of mathematical language as a specific focus of classes involving LEP students. Mathematical terms, expressions, symbols, and ways of communicating (locutions) can be integrated into the ongoing flow of the class. For example, a teacher might ask students to describe or to write about some mathematical problem they are working on. During the process, she could point out inconsistencies and the need for clarity as her students struggle to say what they mean. Lampert (1988) has argued that this is how students should "reinvent meaning."

Teachers, especially those of LEP students, may be tempted to shortcut this process and instead to focus on key words or on vocabulary as "signalling" (Dale & Cuevas, 1987, p. 13) certain operations. We would strongly recommend against such practices. As indicated in the preceding section, key words work only because the current mathematics curriculum has been impoverished (i.e., they work for a limited set of problems). Reliance on key words will lead, inexorably, to misconceptions. For example, the terms "altogether" and "left" usually are taught as being key words that signal the operations of addition and subtraction, respectively. Yet consider the following word problems which many first and second graders—even LEP students—understand and can solve:

Thomas has 15 cars *altogether*. Of his cars, 9 are red and the rest are blue. How many of Thomas' cars are blue?

Mary had some balloons. She gave away 9, and now, she has 6 *left*. How many balloons did Mary have to start with?

Anyone who applies the key word method to these problems will get them wrong, because they would add 15 and 9 for the first problem and they would subtract 6 from 9 for the second.

Note that although these words do not signal specific operations, they do help describe how the numbers and their underlying sets are related to each other. In the first problem, "altogether" indicates that the 15 refers to the whole set, (i.e., to all of the cars that Mary has). Hence, 9 refers to part of that set, and the child needs to determine the size of the remaining part of the whole. The relation being described are between a set and its parts. Out of these deeper relations, the operations of addition and subtraction are derived. Interestingly, first grade children can articulate these and other relationships as they explain how they solve problems as they do. Key words, unfortunately, short-circuit students' reliance on those understandings. It is better to rely on students' understanding of mathematical language and to have them use that language to communicate among themselves and with their teachers.

A final curriculum issue for LEP students concerns the need for students to actually take mathematics courses. Learning of mathematics is dependent on the taking of courses (Oakes, 1990; Myers & Milne, 1988; Rock, Ekstrom, Goertz, & Pollack, 1986; West, Miller, & Diodato, 1985). In its own right, course taking is important. One cannot take geometry without taking algebra. Yet unfortunately, linguistic minorities receive little encouragement to persevere in taking courses. Moreover, minority females receive even less encouragement than do their male counterparts (MacCourquodale, 1988), even though, of course, they will benefit from knowledge of mathematics in the same ways as males do.

RECOMMENDATIONS

- Select content for LEP students strategically and based on mathematically relevant criteria. Recommendations from the *Curriculum and Evaluation Standards* (NCTM, 1989a) should be of assistance here.

- Don't water down the content of LEP students' mathematics courses. Though historical precedent may fuel pressure to simplify the content or to avoid more complex topics, such practices do not help LEP students.

- Find mathematically complex situations that students understand and provide them with access to the more advanced parts of the mathematics curriculum. Examples of such activities can be found in the *Curriculum and Evaluation Standards* (NCTM, 1989), in many mathematics methods books, in *Finding Out/ Descubrimiento* (De Avila et al., 1987), and in professional reference materials. An intriguing set of mathematics materials is being produced in the Netherlands; one example of this work which is becoming more widely available in this country is *Shadow and Depth* (Rijksuniversiteit, 1980). Older materials that are less readily available are the *Nuffield Project Materials* (1975).

- Don't lose sight of the fact that children and older students can create interesting and very meaningful problem solving situations from their own experiences. Many of these mathematics problems are likely to be drawn from the students' home backgrounds. Other problems may arise because students have common in-class experiences and they wish to pursue a particular question to its logical conclusion. The latter seems to have occurred in *Cheche Konnen* (Rosebery et al., 1990; Warren et al., 1988).

- Combine as many objectives as possible into single activities to meet both mathematics and language development objectives. Not every feature of mathematical language must be taught explicitly. Many, if not most, of those features can be derived by students as they engage in mathematical activities and, subsequently, when they need to communicate the results of those activities without ambiguity.

- When combining mathematics and language development objectives, determine whether the primary purpose is mathematics or language development. This should facilitate lesson planning.

- Avoid superficial attention to mathematical language at all costs. Key words and many rules have limited applications; they will confuse and harm students later on in their mathematics courses. If students generate their own rules, ask them to demonstrate when those rules are correct.

- Be aware of impending changes in mathematics education in the school district. Better yet, be the person to inform the district about changes occurring elsewhere. Then, ask to be involved in the committees and task forces which review and implement those changes. Whenever you see or hear something which will exclude your district's LEP population, ask about the assumptions that undergird the decision and try to change it. For example, if students must be taking advanced mathematics courses to have access to computer software and programming, ask if there might not be software that makes advanced mathematics more accessible to all students. There is. Spreadsheet programs can be used to model some very sophisticated processes; young children of all ability levels understand and can learn from Logo; the *Geometric Supposer* can be used to develop students' skills in making and proving conjectures.

- Encourage LEP students to try to understand what they are doing and why they are doing it. Pull-out teachers or aide who see students for a limited amount of time can help students understand how to read their texts and how to monitor their own learning.

- Encourage students to persevere in taking mathematics courses. Without mathematics courses, they will rapidly find themselves locked out of many later life employment opportunities (for more suggestions on this point, see Beane, 1985; also see, NRC, 1989).

ACTIVE MATHEMATICS TEACHING

One of the best known methods for teaching mathematics has grown out of the process-product research on teacher behaviors: Active Mathematics Teaching (AMT). Developed by Thomas Good and Doug Grouws, AMT is a form of what is known as direct instruction (Chambers, 1987; Good & Grouws, 1979; Good, et al., 1983).[2]

Direct instruction works well for conveying large amounts of highly structured materials to students who are just beginning to learn a subject. Most of the research on direct instruction has been basic skills oriented and has involved elementary school students or at-risk students who are engaged and on-task (see reviews by Brophy & Good, 1986, and Good & Brophy, 1989). The Significant Bilingual Instructional Features Study (Tikunoff, 1985) drew heavily from the research on direct instruction and especially from the work of Good and Grouws (1979; Good, et al., 1983).

CHARACTERISTICS OF ACTIVE MATHEMATICS TEACHING

AMT prescribes a highly structured sequence of teaching behaviors organized around a mathematics lesson. In a 45-minute mathematics lesson, a teacher should spend 8-10 minutes on review, 20-25 minutes on developing new content, and, at most, 10-15 minutes on individualized seatwork. Homework is assigned to supplement seatwork.

The AMT lesson begins with an 8-minute review of the previous day's homework, its concepts and skills. During this time, students should be given several mental computation exercises for review and to get them engaged and on-task. The teacher should provide immediate feedback on right or wrong answers. On the first Monday of each month, reviews should be longer (about 20 minutes) and should focus on the skills and concepts covered the previous month.

The key to an AMT lesson is its 20-minute development portion. Prerequisite skills and concepts should be checked. The teacher should provide process explanations, illustrations, and demonstrations. Students should be checked for comprehension frequently

[2] Programs wishing to receive training in Active Mathematics Teaching should contact Professor Douglas Grouws, Center for Study of the Behavioral Sciences, University of Missouri, Columbia, MO 65211.

during this time. The teacher should help students understand the material by using manipulatives (e.g. unifix cubes, base ten blocks, geometric solids) and concrete examples. During the lesson's development portion, the teacher should vary the pacing of the material in order to be sure that the students understand the examples and explanations provided.

Before presenting individual seatwork assignments, the teacher should pose a series of brief product-oriented questions to students. These questions should be taken directly from the lesson's main points, and the teacher should expand on right or wrong answers with her own process explanations. At this point, the pace of the lesson should be brisk to keep students on-task and to signal a transition to the seatwork portion of the lesson.

Also as a transition to the seatwork portion of the lesson, the teacher should monitor students' work and assess comprehension by having the students do some controlled practice activities that extend the concepts discussed in the lesson. To maintain student on-task behavior, the teacher should check students after every one or two problems. This limits their chance to practice errors that will have to be corrected later. Controlled practice also provides an easy transition to individual seatwork.

The seatwork portion of the lesson should last about 15 minutes. Its purpose is for students to engage in successful practice of the concepts and skills introduced in the lesson. The teacher should have determined which students were having difficulty understanding the lesson during the controlled practice, and now should work with those students individually.

Students should not spend too much time on individualized seatwork. In a study of 31 high school mathematics teachers, Gersten, Gall, Grace, Erickson, and Stieber (1987) found that the most effective teachers tended to spend more time explaining and demonstrating materials (i.e., on development), to involve more students with their questioning during the transition to individualized seatwork, to spend less time on individual seatwork, and to spend less time working with individual students. Students seem to need the conceptual supports that teachers provide them in AMT, and hence, the more time they are left to their own devices (i.e., when they are working alone or unattended), the less they get from the lesson.

Peterson, Janicki, and Swing (1981), and Slavin (1989), have found that when cooperative groups replace individual seatwork, student achievement in mathematics increases. This suggests that students can support each other's learning after the lesson has first been presented by the teacher.

Finally, the lesson should end with a homework assignment that includes some review problems to maintain skills, and some problems that extend the seatwork portion of the lesson. The homework comprises the content for the next day's review.

SAMPLE INSTRUCTIONAL SEQUENCES

AMT is most efficient when transmitting a well organized body of knowledge to students. This knowledge should be hierarchically ordered, so that the teacher can review prerequisite knowledge and skills as the basis for developing the later knowledge.

For example, a teacher of LEP students in kindergarten or first grade may determine that the children know very little about numbers, addition, and subtraction. She may decide to implement a series of lessons designed to move them beyond knowing how to count small sets of objects to understanding addition and subtraction situations and to writing number sentences. Possible instructional sequences comprise Figures 1 and 2.

Either sequence could be implemented during a few days or over the course of a few months, depending on how much children know about numbers, addition, and/or subtraction. Some children enter school with very limited knowledge, and for them, mastering these sequences might take very long. The teacher must decide whether or not to use a child's native language depending on the program's characteristics, the child's command of that language, and her own proficiency in the child's native language.

The teacher should begin each lesson by reviewing the previous activity and then either continue the same activity with a broad range of numbers or introduce the more advanced activity in the next step of the sequence.

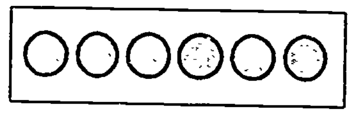
The sequence in Figure 1 contains characteristics that are helpful for young children who are still using objects. Tasks 1.C, 1.D, 2.B, 2.C, 2.D.i, and 2.D.ii all link numerical and mathematical symbols to real world referents. Tasks 1.E, 1.F, 2.D.i, and 2.D.ii actively move children away from the use of objects—a step that is often missing from activities that rely on the use of manipulatives—and allow them to develop mathematical language in context. Note how "altogether" is introduced as a synonym for

Figure 1

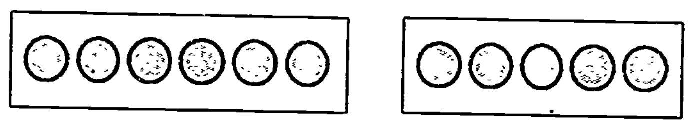
AN INSTRUCTIONAL SEQUENCE FOR DEVELOPING ADDITION

Assume that children know how to count small sets. Teacher should place stickers of dots or other objects on long row cards. Row length and numerosity (i.e., how many stickers there are on a row) should vary. Neatly, on 3 x 5 index card, write numerals corresponding to row numerosity. Note that many different numbers should be used.

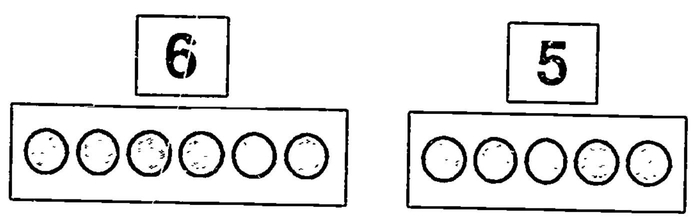
TASK 1.A: At this level, the teacher simply should put out cards with the rows of dots (array cards), and ask the children to determine how many dots there are on that card. She may, or may not, put out the corresponding numeral card.



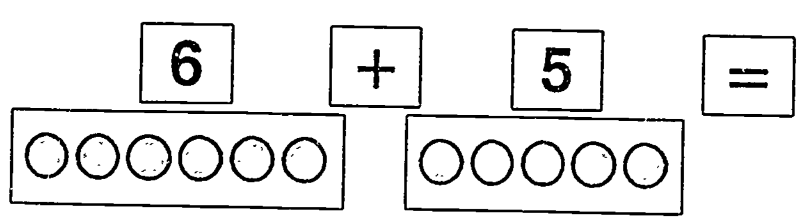
TASK 1.B: The teacher should put out an array card and tell the children how many dots there are. Children will often want to check and they should be encouraged to do so. The second array card should be put out, with the teacher noting how many dots there are. Finally, the teacher should end this sequence by asking children to determine "how many dots there are on both cards" or "in all." Over time, the term "altogether" may be used as a synonym for "in all."



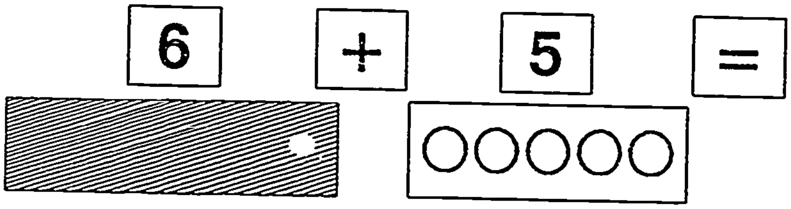
TASK 1.C: As she puts out each array card, the teacher should put out the corresponding numeral cards. She should introduce the numeral cards as being there to help the students remember how many dots there are. That way, the children won't have to count the dots over again.



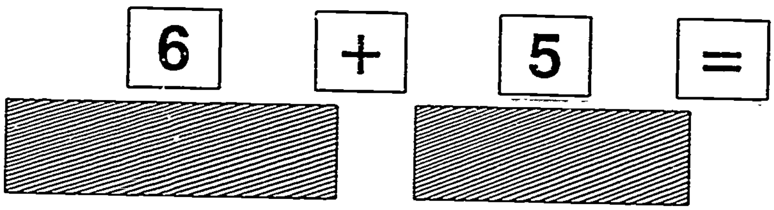
TASK 1.D: At this point, the teacher should introduce the addition and equality signs. "This sign is called the plus sign. When we want to add 6 and 5, we usually say 6 plus 5. This sign means plus."



TASK 1.E: At this point, the teacher hides the first array card from sight. After showing the first row of dots and its corresponding numeral card, she might say, "But now, I'm going to hide these dots from you," and turn the array card over. "Remember, there are 6 dots here. This card (point to numeral card) can remind you of that."



TASK 1.F: Now, the second array card gets hidden from sight. "Now, I'm going to make these problems even trickier for you. I'm hiding all the dots from you. Remember, there are 6 here, and 5 here. These cards can help you remember that."



TASK 1.G: At this point, remove the array cards totally. "Since I'm hiding the dots, we really don't need these here, do we? Let's just put them away."

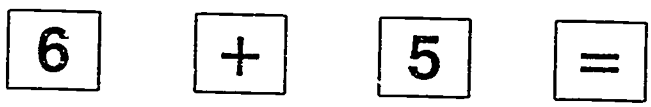
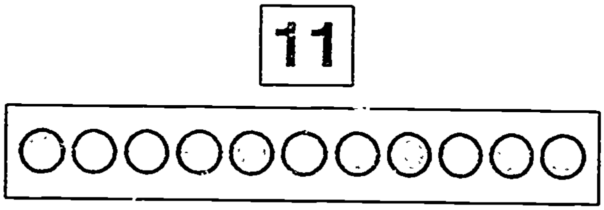


Figure 2

AN INSTRUCTIONAL SEQUENCE FOR DEVELOPING SUBTRACTION

TASK 2.A: Put out a single array card. Put corresponding numeral card over the middle of the array and remind student of its purpose.

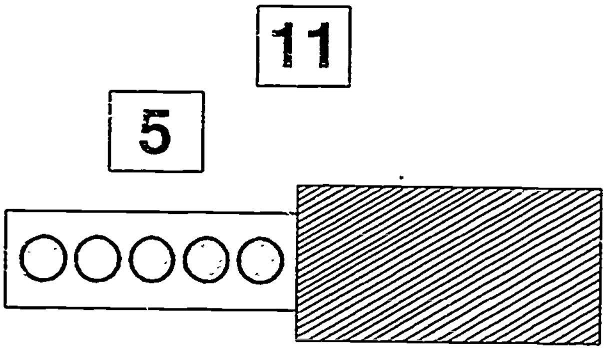


TASK 2.B: After array card has been set out, tell the child, "But now, I'm going to hide some of these dots from you." Turn the array card toward yourself, and cover a subset. Place partially covered card in front of child, and continue: "Now there are 5 dots showing. See, this card here tells you that." Place corresponding numeral card over the middle of the visible part of the array as in picture.

"Remember, I had 11 dots (point to 11 numeral card), and I hid some from you. Now, there are 5 dots showing (point to 5 numeral card). How many did I hide from you?" Point to covered section of the array.

When doing repeated trials of this task, vary not only the number of dots you show, but also the sides that you hide. Children who cannot figure out how to do this problem will either count make believe dots across the cover of the array, or they will guess and answer, or they will simply say "I don't know."

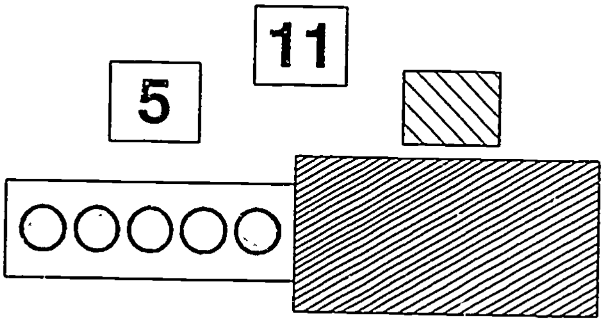
To help children get an idea of this problem, *hide* just 1, then 2, and then increasing numbers of dots. Also, try *showing* just 1, then 2, and then an increasing number of dots.



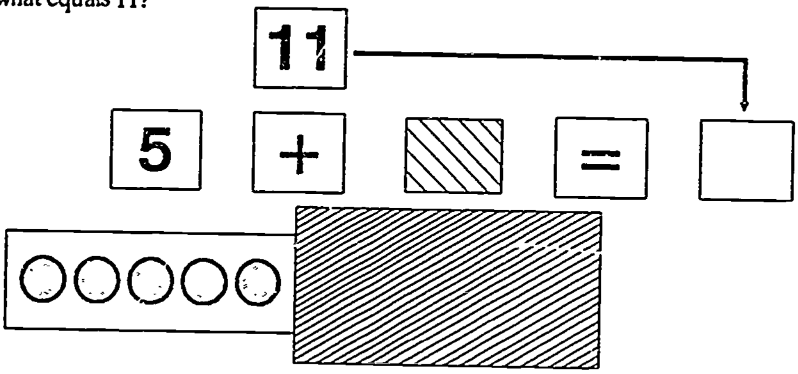
TASK 2.C: At this point, introduce a blank number card, and place it over the middle of the hidden array card. After following the script above, continue with, "See, this card means that we don't know how many were hidden. How many dots did I hide from you?"

Talk about the numeral cards in alternative ways. While pointing to the appropriate numeral cards, say something like, "We could think of this as being 5 plus those dots that I hid makes 11." Or, "We could think of this as 11 take away 5 that are still showing is how many I hid." End the discussion with, "How many did I hide?"

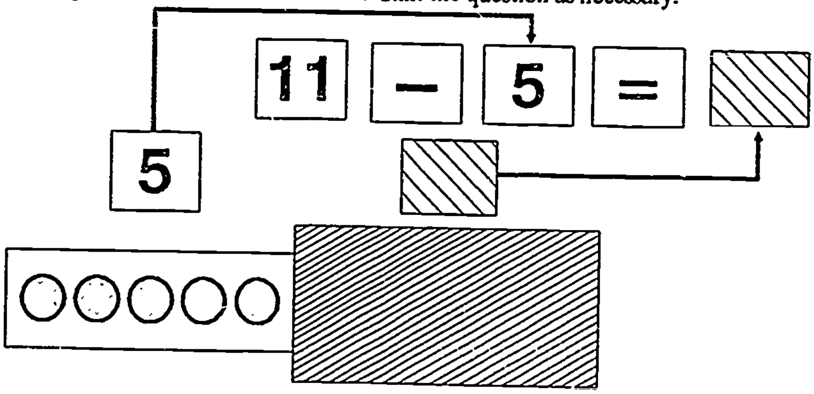
Assume that some terminology, such as plus and equals, should have been already introduced (Figure 1, Task 1.C). Other terminology, such as minus, could be introduced as a synonym for other terms. Develop as many different ways of talking about the problem as seems fruitful. Carefully select and expand on ways that lead to mathematically relevant terminology.



TASK 2.D.i: Using the language of missing addend addition--"5 plus some/how many equals 11"--use the plus and equal signs to make up a number sentence. Move the 11 numeral card to the correct place on the number sentence. Shift the question from "How many did I hide?" to something like "What number fits here?" and/or "5 plus what equals 11?"



TASK 2.D.ii: Here use the language of take away subtraction--"11 minus 5 equals how many?"--with the minus and equal signs. Move the 5 and blank numeral cards to their correct places in the number sentence. Shift the question as necessary.



NOTE: By shifting around which part of the array card gets hidden, one can generate many different kinds of number sentences. Also, different number sentences can be modified by talking about the situation in different ways. For example, "We could think of this as being 11 take away some is 5. How many did I take away?"

"in all." Neither are used as key words for addition; rather, they refer to union of both sets. The plus and equal signs are introduced after there has been talk about their meanings. Finally, the sequences are ordered so that each task logically follows the preceding and the teacher can link them in order.

Fuson has developed a tightly ordered sequence of activities that can move first grade children from simple addition and subtraction problems, like those found in Figures 1 and 2, to multi-digit problems (See Bell, Fuson, & Lesh, 1976; Fuson, 1989). Using a more extensive sequence than in Figures 1 and 2, Fuson and Secada (1986) were able to teach first grade children to add ten-digit numbers with regrouping.

A critical step in Fuson and Secada's extended sequence is helping children make the transition from counting-all to counting-on in addition. In Figure 1 a child would count-all to solve Task 1.C by counting the first and second sets, starting with the number 1 on through 11: "1, 2, 3,. . . 11." A child would solve the same problem by counting-on: starting to count from the number for the first set—6—without recounting that set: "6, 7, 8, 9, 10, 11."

Figure 3 provides a sequence of lessons to teach children to count-on. Secada, Fuson, and Hall (1983) found that most first graders can do Task 3.A. First graders who spontaneously answered Tasks 3.B and 3.C correctly, or who were taught those Tasks, tended to count-on without needing to practice the last step of counting-on, Task 3.D. It is included for the sake of completeness, however.

Again, note that the sequence in Figure 3 provides support with objects and that it develops mathematical language. Children learn to count-on using objects. The number for the first set also is referred to as the counting number that goes to the last dot in that set—which is how the set's numerosity is determined in the first place.

Moreover, the sequence outlined in Figure 3 dovetails into the instructional sequence outlined in Figure 1. Specifically, children who can count-on verbally in a given language and who understand Task 1.D (in Figure 1) can be taught to count-on for addition problems in that language. Similarly, Figure 2 dovetails into Figures 1 and 3. Many children who count-on (Figure 3) and who understand Task 1.E (Figure 1) also can solve Task 2.B (Figure 2).

Figure 3

AN INSTRUCTIONAL SEQUENCE
FOR TEACHING CHILDREN TO COUNT-ON

TASK 3.A: **VERBAL COUNTING-ON.** Ask the child to "start counting from the number 6, and keep counting until I tell you to stop." (Stop child after 4 or 5 numbers.)

NOTE: Most first grade children can count-on verbally for numbers less than 10. Children learn how to do this over the course of numerous counting experiences. If a child cannot count-on verbally, there are some activities that might foster this skill. For example, the child and someone else might alternate saying the counting numbers. Or, someone might say an arbitrary one or two numbers (4, 5) and the child should pick up the count from there (6, 7, ...). Do *not* drill a child on this competence.

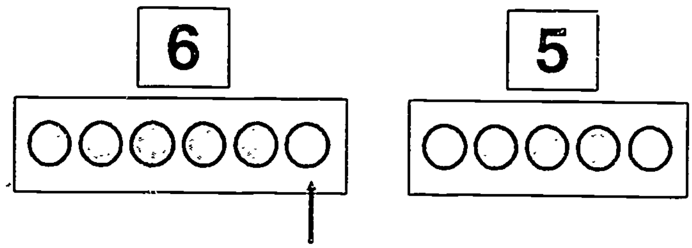
TASK 3.B: Set up the arrays as in Figure 1, Task 1.C. Remind the child why the numeral cards are in place. Then ask the child, "If you were to count all of these dots like this (make a sweeping left to right motion over the two arrays), then what number would this dot get?" Point to last dot of first array.

To have mastered Task 3.B, a child needs to understand that the number 6 is also the number that the last dot on that array will get. Hence, the child should respond 6, without having to count the dots on the array card. Children who have not mastered this task usually will fail to answer or they will count all of the dots of the first array card.

If a child does not have this competence, teach it by first allowing him to count the array and determine that the last object (dot) will get the number 6. "See, this card tells you how many dots there are here, and also, it tells you what number this dot will get. Let's try another one." Repeat 3 or 4 times, with different numbers.

If, after the third time, the child persists in counting the first array, then interrupt him. "Wait a minute. How many dots are there here?" (Response) "Since there are 6 dots, what number should this dot get?" (Response). "Okay, it should get 6, right. Now let's check it, just to be sure." Repeat this sequence, if needed, 3 or 4 times. Most first graders will catch on after the second or third trial.

Once the child has made the correct response, repeat the problem a couple of times to ensure mastery.



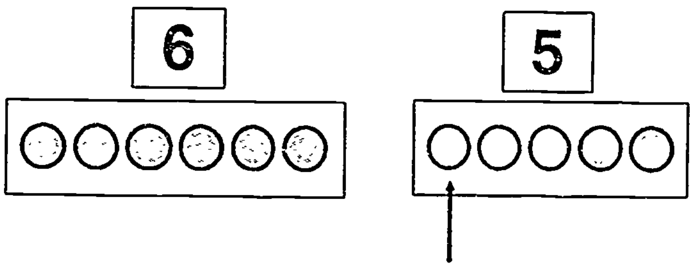
TASK 3.C: Set up the arrays as in Figure 1, Task 1.C, and remind the child of the purpose of the number card. Then ask the child, "If you were to count all of these dots like this (make a sweeping left to right motion over the two arrays), then what number would this dot get?" Point to *first* dot of second array. To have mastered Task 3.C, a child needs to understand that since the number 6 is the number that the last dot on the first array gets, the next number--i.e., 7--is the number that the first dot on the next array will get.

To demonstrate this competence, the child should respond 7, without having to count the array. Most children who fail this task will respond that the dot should get 1 or 5, or they will count all the dots from the very first to the seventh.

If a child does not have this competence, teach it by first allowing him to count from the first dot to that one. "Let's count them together." (Count the dots.) "Now, let's see. This card (first numeral card) tells you that there are 6 dots here, right?" (Response) So that means that this dot (point to dot of competence B) gets the number 6, and that *this* one (make exaggerated jump to dot of competence C) should get the number 7. Which it did, right?" (Response) "Let's try another one." Repeat 3 or 4 times, with different numbers.

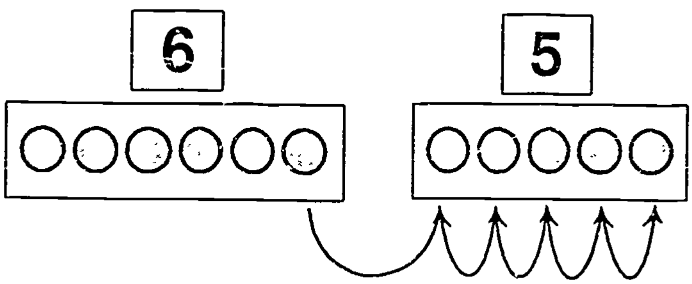
If, after the third time, the child persists in counting the first array, then interrupt him. "Wait a minute. How many dots are there here?" (Response) "Since there are 6 dots, what number should this dot (point to dot as per Task 3.B) get?" (Response) "And that means that this dot should get the number ___?" (Stress on the word "number" as if asking a question. Make an exaggerated jump to next dot.) (Response) "Okay, it should get 7, right. Now let's check it, just to be sure." Repeat this sequence, if needed, 3 or 4 times. Most first graders will catch on after the second or third trial.

Once the child has made the correct response, repeat the problem a couple of times to ensure mastery.



TASK 3.D: COUNTING-ON. Set up arrays as in Figure 1, Task 1.C. Remind child why that number cards are in place. Then continue, "How many dots are there altogether. Remember, you don't need to count these dots (point to first array) because you already know there are 6 dots here. But you can if you need to. How many dots in all?" Repeat 2 or 3 times.

If child does not count-on (i.e., the child does not begin counting with the number 6), then interrupt. "Wait a minute. This card (point to numeral card for first array) tells you there are how many here?" (Response) "So this dot (Task 3.B) gets what number?" (Response) "And this dot (Task 3.C) gets what number?" (Response) "So we can count, 6, 7, 8, 9, 10, 11 (point to each dot as you count it). Do you understand?" (Response) "Let's try another one." Repeat 3 or 4 times.



Figures 1, 2, and 3 demonstrate the strengths of active mathematics teaching. AMT works best when there is a well organized body of material to be conveyed. Its organization allows the teacher to review content before developing new knowledge; hence she can help students link new information to what has been learned before. As can be seen by how Figures 1, 2, and 3 are related to each other, material that is well organized also allows the teacher to move instruction in many directions. Finally, well organized material helps the teacher maintain the pace of instruction. She can keep students on-task by varying the lesson's tempo, by reviewing and remediating material as necessary, or by going into new areas.

ISSUES FOR IMPLEMENTATION WITH LEP STUDENTS

Several issues need to be considered in implementing the AMT model with LEP students. Teachers need to decide whether to use individual, small group, or whole class instruction. Direct instruction and AMT originally were developed in whole class settings. Yet, subsequent experience shows that direct instruction can work effectively with small cooperative groups (Peterson et al., 1981; Slavin, 1989). Most of the class could be carried out by the small groups, although the developmental portion would entail whole class involvement.

Teachers should try to develop a language for doing the mathematics and integrate it into their lessons. For example, if a teacher is using cooperative groups and if two (or more) groups create their own unique ways of referring to the same thing, then the teacher should point out this linguistic idiosyncracy and help students understand how to use socially accepted conventions for mathematical discussions (see, e.g., Lampert, 1988). Communication between groups can become very complex if none of the terminology adopted by groups is mathematically acceptable. Yet also, if teachers take time to have their students explore what happens when everyone adopts his or her own conventions, student learning will become that much deeper.

In his description of effective bilingual teachers, Tikunoff (1985, p. 32) also described critical behaviors for all good direct instruction:

(1) communicate clearly by giving accurate directions, specifying tasks, and presenting new information with good explanations, outlines, summaries and reviews;

(2) obtain and keep student engagement by maintaining a task focus, pacing instruction appropriately, promoting student involvement, and communicating expectations for successful performance;

(3) monitor progress by reviewing work frequently and adjusting instruction to ensure student accuracy; and

(4) provide immediate feedback so that students know when they have been successful and/or are given information on how to achieve that success.

In order to achieve these four tasks, Tikunoff (1985, p. 34) recommended that teachers of LEP students:

(1) use both languages for instruction;

(2) integrate language skills development with academic skills development; and

(3) respond to and use information from the students' home culture in classroom management and in the content of their lessons.

In classes of mixed language levels, academic engagement of all students is particularly problematic. If the pace of instruction is too brisk, teachers risk losing some of their students; if it is too slow, they will lose others. Hence, teachers should monitor their pacing very closely and adjust it regularly. If a teacher moves at a pace that loses some LEP students, then she should be sure to visit those students during the seatwork phase of the lesson.

Mathematical language is important for communicating in mathematics classes. Even many English proficient students do not understand the meaning of terms like perimeter (distance around a figure). Hence, the development of such terminology—in context—is beneficial not just for LEP students, but for all students.

Teachers need to monitor what takes place during the individual seatwork portion of each lesson. As noted earlier, too much time on individual seatwork leads to lower achievement. There seem to be many reasons for this possibility. Some students might not understand the content of the lesson, and practice wrong processes. If the teacher does not catch these errors, students will reinforce improperly learned or misunderstood content. Time spent on properly developing the content helps ensure student understanding.

Also, with too much time for individualized seatwork, some students might engage in off-task behaviors, and, hence, not really work on the material. Once again, proper use of development time might help alleviate this problem.

Alternatively, teachers might replace individualized seatwork with cooperative group activities. Teachers should monitor such groups to ensure that LEP students do not find themselves isolated either because English dominant students have taken over the groups or because students have assigned their work to the most capable member of the group—thereby defeating the purpose of working together (Cole & Griffin, 1987).

ESL teachers, bilingual aides, and others who do not have self-contained classes but who are called upon to help LEP students with mathematics might also employ some Active Mathematics Teaching techniques. Even if time with a student is limited to for one-half hour, it is possible to design a tightly organized lesson that includes attention to pacing, on-task engagement, review of content, and other features of direct instruction. The purpose of each lesson should be communicated to the student.

RECOMMENDATIONS

- To use the direct instruction method, you need to organize the mathematics lesson in a sequential series of steps. Plan your sequences so that they can dovetail with each other.

- Plan the lessons for your unit to allow for variations in pacing. You should be able to backtrack, speed up, or go off in another direction if you see the need to develop another point.

- Integrate mathematical language in the context of your planned sequence.

- Try to ensure that your students have experiences for building upon their lessons. For example, a lesson on geometry involving the shapes of buildings will be more understandable if students have actually experienced the buildings whose shapes they are studying.

- Try to use multicultural referents and materials in your lessons. Since LEP students come from culturally diverse backgrounds, they are likely to have had many different kinds of experiences that could be related to mathematics.

- At the start of each lesson, tell your students very clearly what mathematics they will learn that day. At the end of the lesson, have them restate what they (should have) learned. Does what your students say they learned match what you indicated they would learn at the start of the day?

- Review prior knowledge so that students can tie their lesson to what they already know. If you find that your students have forgotten that information, review it in more detail.

- Spend the bulk of your time on the development portion of your lesson, not on seatwork. If you are spending very little time on development, then it is likely that the lesson is not being explained well enough or that you are not covering enough content.

- Be sure that during the course of the lesson you question every one of your students. Avoid questioning in a pattern (e.g., up one row and down the next) since students might tune you out until it is their turn.

- If you are teaching in English, be sure to monitor very carefully how well your LEP students are understanding the flow of the lesson and that they are on-task. If they do not understand what is happening, slow down. If you have someone else working with you, ask that person to explain what is happening either in the student's native language or in simplified English. If you are alone, and slowing down will lose much of the class, plan on spending some time with LEP students during the seatwork portion of your lesson.

- Regardless of the language used in the classroom, try to distinguish between a student's knowledge of the content and his communicative competence.

- If you are teaching in English, be sure to provide enough wait time for LEP students to answer questions. If a student struggles with an answer, but seems to be on target, expand the answer and ask the student to verify if that is what he means.

- If you are teaching in English, but your students prefer to answer in their native languages, allow them to do so. Then, ask if they might translate what they said into English, or ask if someone else would like to do that.

- Provide frequent and immediate feedback to your students so that they can monitor their understanding of the lesson.

- Do not spend too much time on individualized seatwork.

- Consider replacing individualized seatwork with students working in small cooperative groups.

- Seatwork and homework should have purposes. Use seatwork to build success. Use homework to extend the lesson and to supplement seatwork.

- Review homework. Correct seatwork. This helps you monitor student progress and it shows students that their work matters.

- Alert your students to what will be covered in class the next day. Encourage them to prepare for the topic. If your LEP students receive supplementary assistance, ask the people who work with them to preview the next day's activities with them.

COGNITIVELY GUIDED INSTRUCTION

Another promising approach for teaching mathematics is Cognitively Guided Instruction (CGI). Developed by Thomas Carpenter, Elizabeth Fennema, and Penelope Peterson, CGI focuses on students' thought processes while they solve mathematics problems (Carpenter & Fennema, 1988; Carpenter, Fennema, Peterson, & Carey, 1988; Carpenter, Fennema, Peterson, Chiang, & Loef, 1990).[3]

Cognitively Guided Instruction does not prescribe a program of teacher behaviors. Rather, CGI is based on four interlocking assumptions:

(1) Teachers should know how specific mathematical content (e.g., addition and subtraction) is organized in children's minds.

(2) Teachers should make mathematical problem solving the focus of their instruction of that content.

(3) Teachers should find out what their students are thinking about the content in question.

(4) Teachers should make instructional decisions (e.g., sequencing of topics) based on their knowledge of students' thinking.

The benefits of CGI are numerous. Students receive basic skills instruction in a problem solving context that is meaningful and that fosters higher order thinking skills. They become problem solvers, increasingly confident in their abilities to make sense out of new problems. Because teachers pitch problems to engage and stretch their students, the students are motivated to stay on-task. Teachers who use CGI have reported an increased sense of professional efficacy. Fennema (personal communication, May 1990) reports how one teacher observed that she always knew that she should listen to her students and that CGI provided her with a means of really listening to them—not only in math, but also in reading.

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[3] Programs that wish to receive training in Cognitively Guided Instruction should write to either Professor Thomas Carpenter or Professor Elizabeth Fennema, Longitudinal Study on the effects of Cognitively Guided Instruction, Wisconsin Center for Education Research, University of Wisconsin-Madison, 1025 West Johnson Street, Madison, WI 53706.

Moreover, CGI seems to help teachers be more equitable and accurate in judging their students' abilities, according to Fennema (personal communication, May, 1990). She found that, prior to going through the CGI staff development workshop, teachers were able to predict the problem solving strategies that their boys would use better than they could predict what their girls would do. After the workshop, CGI teachers were equally good in predicting how both boys and girls would solve arithmetic word problems.

PRINCIPLES OF CGI

The fundamental principle of CGI is that instructional decisions should be made by teachers based on their students' thinking. To understand how their students think, teachers should know how specific mathematical content is organized and how students acquire the concepts and skills of that content. Finally, teaching should focus on problem solving, problem solving processes, and student understanding.

The focus on problem solving does not diminish the importance of skill work or suggest that students should not have the opportunity to practice skills. On the contrary, students involved in CGI programs, where the focus was on problem solving, performed better on number fact recall than did students who had spent twice as much time practicing number facts (Carpenter, Fennema, Peterson, Chiang, & Loef, 1990).

ADDITION AND SUBTRACTION: AN EXAMPLE

Figure 4 presents eleven addition and subtraction word problems that are at the core of recent research on how primary school children learn mathematics (see Carpenter & Moser, 1982, 1983, 1984; Riley & Greeno, 1988; Riley, Greeno, & Heller, 1983) . These problems can be thought of as addition or subtraction since their solutions can be found either by adding 8+5 or by subtracting 13-5. However, primary school children consider each of these as distinct problem situations. The problem structures differ and the natural strategies that both bilingual and monolingual students use to solve these problems reflect those structures (Carpenter & Moser, 1983, 1984; Secada, in press-a).

The row headings refer to the actions of these problems. "Join" and "separate" problems in the first two rows include a direct or implied action on the quantities in the problem, where the joining or separating action takes place over time. For these problems there is an initial quantity, a change quantity, and a resultant quantity. The "part-part-

Figure 4

CLASSIFICATION OF ADDITION & SUBTRACTION WORD PROBLEMS

Join	**(Result Unknown)** Connie had 5 marbles. Jim gave her 8 more marbles. How many marbles does Connie have altogether?	**(Change Unknown)** Connie has 5 marbles. How many more marbles does she need to have 13 marbles altogether?	**(Start Unknown)** Connie had some marbles. Jim gave her 5 more marbles. Now she has 13 marbles. How many marbles did Connie have to start with?
Separate	**(Result Unknown)** Connie had 13 marbles. She gave 5 marbles to Jim. How many marbles does she have left?	**(Change Unknown)** Connie had 13 marbles. She gave some to Jim. Now she has 5 marbles left. How many marbles did Connie give to Jim?	**(Start Unknown)** Connie had some marbles. She gave 5 marbles to Jim. Now she has 8 marbles left. How many marbles did Connie have to start with?
Part-Part-Whole	**(Whole Unknown)** Connie has 5 red marbles and 8 blue marbles. How many marbles does she have?	**(Part Unknown)** Connie has 13 marbles. Five are red and the rest are blue. How many blue marbles does Connie have?	
Compare	**(Difference Unknown)** Connie has 13 marbles. Jim has 5 marbles. How many more marbles does Connie have than Jim?	**(Compare Quantity Unknown)** Jim has 5 marbles. Connie has 8 more than Jim. How many marbles does Connie have?	**(Referent Unknown)** Connie has 13 marbles. She has 5 more marbles than Jim. How many marbles does Jim have?

From: Carpenter and Moser (1983)

whole" and "compare" problems in the third and fourth rows involve a static relationship among the quantities.

Column headings refer to further distinctions among these problems based on which quantity is the unknown. Either the result, the change, or the start is unknown for the actions implied by join and separate problems. For the part-part-whole, the unknown quantity is either the whole set or one of its parts. Compare problems—which involve the comparison of two disjoint sets—have unknowns that are similar to join.

These distinctions among problem types are important when planning instruction because they affect how children try to solve them, and therefore, they affect problem difficulty as well. Children's problem solving processes fall into three broad categories: modeling, counting, and derived fact strategies. Children model problems when they use concrete objects to act out the action of the problem. For example, a child may solve the join/result unknown problem of Figure 4 by putting out a set of 5 blocks, putting out another set of 8, and counting the resultant set of 13 blocks. Alternatively, children might count without using objects in order to solve a problem. For example, a child might count-on from 8 to 13 when solving the join/result unknown problem. Finally, a child who uses derived fact strategies solves problems by using known number facts to derive the answer for a problem. For example, a child might know that 8+4=12, so that the answer for the join/result unknown problem is 1 more, i.e., 13.

For simply worded problems, difficulty is based on a combination of three things: the problem's semantic structure, the child's ability to understand the relationships conveyed through the problem, and how advanced the child's strategies are. For example, children who use direct modeling strategies will solve easy-to-model problems, such as the two result unknown problems (join and separate). Slightly more difficult for children who model are the part-part-whole/whole unknown problem, since it has no action; the compare problem since it has two sets; and the change unknown problems (join and separate) since they entail more complex relationships between the action and unknown quantity. The most difficult to model of the change problems are the start unknowns (join and separate). Children can model the part-part-whole/part unknown problems more easily than these.

When children use counting and derived facts strategies, they can solve even the most difficult-to-model problems, provided that they understand them. (For more information on the problem types and solution strategies see Carpenter, Carey, & Kouba, in press; Carpenter & Moser, 1983, 1984).

ASSESSMENT OF STUDENT'S THINKING

To make intelligent decisions about instruction, the teacher must assess what a child understands about a problem. Since ongoing assessment is integral to CGI, the distinction between assessment and instruction is a bit artificial. CGI assessment focuses on the processes by which students get an answer. It can be carried out individually, in small groups, or in whole class settings.

To start assessment, a teacher should provide students with counters (so that they can model problems if they wish), pose a problem, and if the student responds, follow up by asking: "How did you figure that out?" The teacher then uses additional questions, based on the student's responses, to help him clarify what he means. For example, if a student solves the join/change unknown problem of Figure 4 and says, "I counted," some good follow-up questions might be, "What number did you start counting with?", "What was the first number you said?", or "Show me how you counted."

Alternatively, if a student does not solve the problem correctly, the teacher has at least five options. First, pose a similar problem. If the teachers thinks that a student can solve this sort of problem, but seemed a bit confused, she could make some superficial changes in the problem—names, objects, number size—but keep it essentially the same. Also, she could suggest that the student use the objects "for help" and that "it's okay to count."

Second, a teacher might simplify the language of the problem even further. Many LEP and English proficient students find some of the extraneous story information a bit confusing. Figure 5 includes some examples of how word problems can be simplified.

Note that when posing or simplifying problems, teachers often modify their language without thinking about it. Most modifications seem trivial, but to a child they might not be. For example, something as seemingly innocuous as the placement of an "and" can add to a problem's linguistic complexity. Consider the following two variants of the join/start unknown problem, Figure 4:

Figure 5

WORD PROBLEMS IN ENGLISH, SPANISH, AND SIMPLIFIED

Problem Type	English	Spanish	Simplified
Separate, Result Unknown	Julie had 15 pencils, and she gave away 11 of them (pencils). How many pencils does Julie have now?	Julia tenía 15 lápices y luego regaló 11 de ellos (los lapices). ¿Cuántos lápices tiene ahora Julia?	Julie had 15 pencils. She gave away 11. How many does she have now?
Part-Part-Whole, Whole Unknown	Thomas has 4 blue crayons and 9 red crayons. How many crayons does he (Thomas) have in all (altogether)?	Tomás tiene 4 crayolas de color azul 7 y 9 rojas. ¿Cuántas crayolas tiene Tomás en total?	Thomas has 4 blues and 9 reds. How many is that in all?
Compare, Difference Unknown	Anne has 11 crayons, and Michael has 15 crayons. How many more crayons does Michael have than Anne?	Ana tiene 11 crayolas. Miguel tiene 15 (crayolas). ¿Cuántas crayolas más que Ana tiene Miguel?	Ana has 11 crayons. Michael has 15. Who has more? How many more?
Join, Change Unknown	Paul has 9 balloons. How many more balloons should Paul get in order to have 14 balloons?	Pablo tiene 9 globos. ¿Cuántos globos más debe obtener Pablo para que tenga 14 (globos)?	Paul has 9 balloons. He wants to have 14. How many (more) does he need?
Part-Part-Whole, Part Unknown	Robert has 14 toy cars in all (altogether). Six (6) of them (his toy cars) are blue and the rest are red. How many of Robert's toy cars are red?	Roberto tiene un total de 14 carritos de juguete. Seis (6) de sus carritos son rojos y el resto son azules.¿Cuantos do los carritos son azules?	Robert has 14 cars in all. Six are red. The rest are blue. How many are blue?
Join, Start Unknown	Rose had some blocks. She got 5 more (blocks) and now Rose has 13 blocks. How many blocks did she start with?	Rosa tenía algunos bloques. Luego recibío 5 (bloques) más y ahora Rosa tiene 13 bloques. ¿Cuántos bloques tuvo Rosa al principio?	Rose has some blocks. She got 5 more. Now, she has 13. How many did she start with?
Separate, Start Unknown	Cynthia had some candies. She gave away 6 candies, and now Cynthia has 9. How many candies did she have to start with?	Cindy tenía algunos dulces. Luego regaló 6 de los dulces y ahora tiene 9.¿Cuántos dulces tenía Cindy al principio?	Cynthia has some candies. She gives away 6. Now she has 9. How many did she start with?

Semantic categories from Carpenter and Moser (1983). Translations from Secada (1990b).

A. Connie had some marbles and Jim gave her 5 more marbles. Now she has 13 marbles. How many marbles did Connie have to start with?

B. Connie had some marbles. Jim gave her 5 more marbles, and now she has 13 marbles. How many marbles did Connie start with?

Variant A just seems more difficult than Variant B because of how it links two seemingly unrelated sentences. Variant B seems to link two more closely related actions. Informal observations during piloting with Hispanic LEP children, led to testers using variant B in the study by Secada (in press-a).

A third option, if children are having difficulty solving a word problem, is to add context to a similar problem. For example, a teacher could make the problem into a story about Bert and Ernie and use puppets to illustrate. Or, if the problem is a start unknown, one might use a closed bag and point to it when saying, "I have some ___ in here."

An additional way of adding familiar contexts to story problems is to refer to the children's home cultures and backgrounds. For example, a teacher of American Native children might wish to create problems around the theme of a Pow-Wow. Problems might be generated by talking about dancers as they enter or leave the competitions, prizes that are awarded, or any of the other activities that are a part of this occasion. After a few sample problems, children could generate their own.

An overarching story can also provide context. For example, a CGI teacher created the story of a friendly forest with its trees and animals. During the fall, squirrels saving nuts for the winter could provide grist for many story problems. In the Southwest, one would use a different setting.

A fourth option is to translate the problem into the students' native languages. For example, Figure 5 provides some English and Spanish translations of problems used by Secada (in press-a). Such translations would need to be carefully constructed to ensure that they don't become too difficult, that they use terms that the students understand, and that the problem structures remain the same across the languages. As a beginning rule of thumb, one should translate a sentence in one language for a sentence in the other. And then, words and phrases might be added to help smooth out how each problem "flows."

If all else fails, a teacher might try an easier problem that she is sure that the student will be able to solve. The point is to begin a conversation about the problem.

When using options 1 through 4, be sure to retain the semantic structures of the problems. At first, people will change a problem's structure, thinking they have simplified it. For example, changing a join/change unknown problem to something like "What is 13 take away 5?" transforms the problem so that it is not longer of that type.

To use these options, a teacher must understand the problem types. At first, teachers might write some story problems that they can share with each other and use as needed. Later on, students can help teachers develop a collection of problems that they have written for themselves.

When trying to assess what a student is thinking, teachers should focus discussion on how students figured out their solutions to the problems. There will be time during class for students to see if their answers were right or wrong. Initially, teachers find it very difficult not to correct a student's answer—especially when the student asks, "Did I get it right?" A good response is, "You are trying really hard. What I am interested in right now is how you figured this out."

During an individual conference, students deserve certain courtesies. They should be encouraged to explain what they did without being made to feel that their strategies are unworthy. Some rules to follow are these:

(1) Try to make the student feel comfortable talking about what he did.

(2) Start questioning with, "How did you figure that out?" Some children respond to "why" questions with, "Because I'm smart."

(3) Use your students' own responses in framing follow-up questions. If a student says, "I used the blocks." Ask him to "Show me what you did" or "...how you used them."

CLASSROOM STRATEGIES

Assessment and instruction are closely entwined during CGI lessons. The following vignette depicts how a teacher might incorporate assessment into instruction and might make some decisions based on what she is learning about her students. By asking students to talk about their solution strategies, the teacher also gives them an opportunity to share ideas and to develop a vocabulary for talking about word problems.

Teacher:	Nina had 8 stickers. She bought some more stickers. Now she has 12 stickers altogether. How many stickers did she buy?
Penny:	12. [Penny has one group of 12 chips on her desk.]
Sam:	That's what I got. [Sam also has one group of 12 on his desk.]
Teacher:	Penny, tell us how you figured that out.
Penny:	Well, first she had 8, so I counted 8 of these [chips]. Then she got some more and now she has 12. She got 12 [pointing to the whole group of chips].
Teacher:	Yes, she has 12 stickers altogether but the story already told us that. Let's listen again. [The teacher reads the story again.] Penny, what is the story asking us to find out?
Penny:	How many she bought.
Teacher:	How many stickers did Nina have to begin with?
Penny:	8.
Teacher:	First you said you counted out 8. Where is your group of 8?
Sam:	[Makes a group of 8 chips.] Here's 8.
Teacher:	Penny, you show me a group of 8, too. [The teacher models what Penny is doing by making a set of 8 on the overhead projector so the rest of the students can see Penny's modeling.] Is that how many she had altogether?
Penny:	No, she needs some more.
Teacher:	Sam, how many more would she need to have 9 stickers altogether?
Sam:	[Adds 1 more chip, hesitates, then begins to count them all.]
Teacher:	Sam, Nina had 8 stickers. [The teacher points to the set of 8, then points to a separate set of 1 that was added on.] How many more did I add so that she could have 9 [pointing to the set of 1 chip]?
Sam:	1?
Penny:	It's 1. 8, 9. [Penny adds another chip.]
Teacher:	How did you know that?

Penny: I just added on 1 more to 8 and got 9.

Teacher: If Nina had 8 stickers to begin with, how many more would you add on to make 12? [The teacher reconstructs the set of 8 chips on the overhead.]

Penny: Oh [Penny whispers the counting sequence "9, 10, 11, 12," keeping the second set separate from the set of 8, then counts the set she added on], 4.

Teacher: How many more stickers did Nina buy?

Penny: 4.

Teacher: Ok, let's try another problem. [The teacher gives the chil^d-^en a similar problem and focuses on Sam.] Pat had 7 shells in her bucket. Her brother gave her some more shells. Now Pat has 10 shells in her bucket. How many shells did her brother give her?

Sam: 10.

Penny: No, it's 3. See, 8, 9, 10 [pointing to a group of 3 chips on her desk].

The teacher began her questioning by asking how the problem had been figured out. At first, Penny and Sam did not fully understand the problem, so the teacher helped them focus on various of its parts: what they wanted to know, and what they already knew (the story already told us that). Then, she simplified the problem by asking how many would be needed to go from 8 stickers to 9 stickers. Penny caught on and extended her insight to the problem itself. Meanwhile, the teacher modeled what Penny was doing on an overhead projector so that the whole class could see. Afterwards, she posed another similar problem. At this point, Penny began to explain to Sam how to do it.

This teacher challenged her students by posing a problem that was slightly beyond their reach. She helped them attend to the details of the problem, and then she allowed them to engage in discussion among themselves about the problem and its solution. All the while, she was assessing what Sam and Penny understood about the problem. She used that knowledge to ask her next question or to point out the next fact.

As students and teachers become more comfortable with CGI, the give and take becomes easier. To encourage student discussion, teachers can:

(1) Point out disagreements and let students try to resolve them among themselves.

(2) Summarize results and introduce language that supports further discussion. First grade students in one CGI class discovered that the sum of two odd numbers is an even number. This became a "theorem" that was invoked in class discussion a few days later.

(3) Ask, "Did anyone do this a different way?" One CGI teacher encourages her students to come up with as many different ways as they can to solve a given problem.

(4) Allow students who are working together to solve a problem or to resolve a disagreement to go off and work without interruption. Have the students present their results when they are finished. This may itself engender further class discussion.

Students in some classrooms have helped to write problems. Since first grade students like large numbers, some of their problems reflected that. Not surprisingly, they were motivated to invent ways of solving their own problems. Thus, many first grade students had invented algorithms for doing multi-digit addition before the end of their first semester in school.

CGI teachers have used whole class settings; they have sent groups of students to work at problem centers; they have assigned individualized problem sets to students. Throughout, the focus has been on problems that challenge students and on students' discussion of their solutions.

ISSUES FOR IMPLEMENTATION WITH LEP STUDENTS

Teachers of LEP students need to be sure to include these students when implementing CGI. Students will communicate their ideas about mathematics in discussions with the teacher and with one another. Yet, such an environment takes time to develop, for LEP children cannot be expected to engage readily in conversations about their solution strategies if they have never done so.

One means of helping this environment to develop is to invite bilingual children to speak with one another in any language that they are comfortable using. If teachers insist that everything be explained first in English, many LEP students will not participate. In classes that enroll LEP children from a variety of language backgrounds, teachers will need to allow extra time for translations or for children to explain themselves in English.

Teachers should pose problems that LEP students can solve and specifically call on them to explain their solutions. After a student has explained himself in either English or his native language, the teacher needs to ensure that everyone has understood. The teacher might amplify the child's response, ask him to translate his response into the language that had not been used, or invite other students to translate among themselves and to discuss what their classmate has said.

Many LEP students receive little encouragement to speak about their ideas. Many girls are socialized to defer to boys. Hence, if teachers tend to call on students who answer first, LEP students and girls will often be left out of the conversation. Again, teachers need to reach out to these children and to be sure that they are included in the classroom's processes.

Some students might get impatient waiting for other children to answer. Since everyone should be learning rules for turn taking, they should be encouraged to listen to what everyone in the class has to say.

Teachers of LEP students in all-English classes may find their students struggling with new vocabulary in mathematics. Or, they may speak very little English in general. The same strategies that one would use for any other subject should be used here. Simplify language for both problems and questioning. Pose problems like those found on Figures 1 and 2. Expand on student responses. Recognize that some students pass through a silent period as they are first learning a second language. This is time when they are listening and learning the rules of discourse. Ask other bilingual students to use their native language skills with such students.

Generally, mathematical discourse in the classroom develops gradually. Since students develop mathematical language out of their own discourse, it is important for teachers to incorporate what their students say into the life of the classroom. As students take on additional responsibility for learning, teachers will need to monitor the situation and ensure that their LEP students are included in that discourse.

Finally, a practical concern of teachers is that parents may believe that mathematics consists only of paper and pencil computations, or that their children do not have time to waste on what may seem like games. So, to help parents better understand mathematics, and since primary school teachers often ask parents of LEP students to listen to their children read at night, teachers might send home some mathematics word problems that their children have written and solved in class. These and similar problems, then, would become that night's reading assignment. When parents see how their children invent solutions to the problems, they will better appreciate the efforts spent in class.

RECOMMENDATIONS

Many of the same recommendations as those for Active Mathematics Teaching apply here (see previous section). However, given the somewhat more open organization of Cognitively Guided Instruction, there are some additional recommendations that also should be considered.

- The opportunity for mathematical problem solving can occur throughout the school day. Rather than isolate mathematics to an assigned period of the day, take advantage of the problem solving situations that come up naturally.

- Whether or not you speak your children's native language(s), give students the opportunity to share their problem solving strategies in the classroom and within their groups in any languages that they wish to use. Monitor to ensure that everyone understands what someone has said.

- When engaged in whole class conversations, focus on students' understanding of what everyone else has said. Amplify responses and translate them into children's native languages, if possible, and into English so that everyone in the class can understand.

- Generally, there are many appropriate approaches to solving a given problem. Students become confident in their problem solving ability when they present their strategies and the strategies are discussed by the group. Children also become active participants in the discussion when they can contribute their thoughts on how they approached a problem solving task.

- During mathematics, students should be able to discuss how they are thinking about problems. As a result, students learn mathematics from one another as well as from the teacher. This validates their thinking and students begin to recognize that both the teacher and the students are a source of knowledge in the classroom.

- Take the time to ask students how they arrived at the answer to a problem. This can serve as an informal assessment and may help you decide what kind of problems and activities to include in your lesson. Also, this will allow you to build on the students' existing knowledge and carefully monitor any misconceptions that they might have about a particular concept.

- All students can engage in some aspect of the problem solving discussion because most problems can be solved by students with different levels of problem solving abilities. For example, with addition and subtraction word problems, students can solve problems by modeling, counting, using derived facts, or recalling facts. By recognizing and accepting a variety of solution strategies, you can present the same problem to a large group of students and the students can benefit from the explanations of others in the group.

- In order to create a problem solving environment, you need problem solving resources. Have the students write their own word problems, individually or in pairs. They can use vocabulary they are comfortable with, pictures, and drawings to complete their word problems. Children's literature is an excellent source of problem solving contexts from which students can generate word problems. Encourage children to use everyday experiences from home for creating word problems. Also, activities that all your children have engaged in can be a source of problems.

- Share the word problems that you write with other teachers. Create a bank of word problems that includes different translations as well as different English versions.

- The Cognitively Guided Instruction model provides specific information about children's thinking in the content of addition and subtraction. Less is known about how students learn some mathematical content than other mathematics. Hence, teachers need to understand the content to be covered and to work together to develop their own theories of how students approach various kinds of mathematics.

- When teachers of LEP students are teaching in English, they should modify the linguistic complexity of their problems to ensure their children's understanding.

- On the other hand, just because a student did not do well on an oral language proficiency scale does not mean he does not understand mathematics problems. The best thing to do is to pose problems and see for yourself.

- Monitor the class and specifically invite students to join. You may wish to prep some children with a problem or two so that they can feel some initial success. If they still don't join the discussion, they may not feel comfortable and need more time to learn the rules of participation. Give them that time.

SOME FINAL RECOMMENDATIONS

- Choose and use your manipulatives carefully. In too many classes, the task is not mathematics, but rather it is learning how to use the manipulative. It is better to introduce a few versatile manipulatives that can be used for a variety of mathematics lessons than to use too many manipulatives and to end up spending precious time teaching how to use each new one.

- Manipulatives should support discussion about mathematics, not replace it. Teachers and students should talk about the mathematics that they are doing. They should see how the manipulatives are illustrating that mathematics. Too often, students work quietly with manipulatives and do not have the opportunity to understand why they are doing what they are doing. For LEP students such a practice is potentially devastating, in terms of both mathematics learning and language development.

- If you don't see the mathematics in an activity, don't use it. Mathematics activities should be engaging and, if possible, enjoyable. But if all you see is the fun and not the mathematics, then use another activity.

- The NCTM *Curriculum and Evaluation Standards* provide specific recommendations for content that should be emphasized as well as deemphasized.

- Active Mathematics Teaching has been proven effective for conveying large amounts of basic information that is well organized.

- Cognitively Guided Instruction is promising for developing problem solving skills, higher order thinking, and enhancing student confidence.

- Finally, mathematics is too important for students' futures to be reduced to computations or to be omitted from student's education. Encourage students to persevere in taking mathematics courses. Choose your content and instructional approach strategically.

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