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ABSTRACT

The roles of cognition and metacognition were examined in the mathematical problem-solving behaviors of students as they worked in small groups. As an outcome, a framework that links the literature of cognitive science and mathematical problem solving was developed for protocol analysis of mathematical problem solving. Within this framework, each behavior is categorized by heuristic episode and cognitive level. Data were obtained from videotapes of six small groups of seventh-grade students (N 27), who attended a middle school in Queens (New York City), as they worked together to solve a mathematical problem. Three coders viewed each tape in 1-minute intervals. Watching one or two students in the group, the coders assigned one of the following episodic categories and cognitive levels: (1) read (cognitive); (2) understand (metacognitive); (3) analyze (metacognitive); (4) plan (metacognitive); (5) explore (cognitive or metacognitive); (6) implement (cognitive or metacognitive); (7) verify (cognitive or metacognitive); (8) watch (undetermined cognitive level); and (9) listen (undetermined cognitive level). Explicitly delineating the role of metacognition and cognition within the heuristics of problem solving gives researchers and mathematics teachers a tool to evaluate problem solving. Implications for cooperative group processes and classroom assessment are discussed. Three figures and one table contain the study data. The framework that was developed is outlined. (Author/SLD)

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Protocol Analysis of Group Problem Solving in Mathematics: A Cognitive-Metacognitive Framework for Assessment

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Abstract

Recent summaries of studies investigating mathematical problem solving suggest that a primary source of difficulty may lie in students' inabilities to actively monitor and subsequently regulate their cognitive processes during the solution of a mathematical problem. The purpose of this study was to expand these findings by examining the role of cognition and metacognition in the mathematical problem solving behaviors of students as they work in small groups. As an outcome, a framework was developed for protocol analysis of mathematical problem solving that links the literature of cognitive science and mathematical problem solving. Within this framework each behavior is categorized by heuristic episode and cognitive level.

The data were obtained from the videotapes of six small groups of seventh grade students as they worked together to solve a mathematical problem. Three coders viewed each tape in one minute intervals. They each watched one or two students in the group and assigned one of the following episodic categories and appropriate cognitive levels: READ (Cognitive), UNDERSTAND (Metacognitive), ANALYZE (Metacognitive), PLAN (Metacognitive), EXPLORE (Either Cognitive or Metacognitve), IMPLEMENT (Cognitive or Metacognitive), VERIFY (Cognitive or Metacognitive) and WATCH and LISTEN (Undetermined cognitive level). By explicitly delineating the role of metacognition and cognition within the mathematician's heuristics of problem solving, researchers and mathematics teachers have a conceptual tool with which to evaluate mathematical problem solving. The Framework is also discussed in terms of its implications for cooperative group processes and classroom assessment.

Protocol Analysis of Group Problem Solving in Mathematics: A Cognitive-Metacognitive Framework

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Objectives and Background

This exploratory study was designed to investigate the heuristic and cognitive processes that occur when seventh-grade students work in small groups to solve a mathematical problem. As an outcome of the study a new framework for protocol analysis of mathematical problem solving was developed. The purpose of this framework is to clearly identify the role of metacognition and cognition within the heuristic framework of mathematical problem solving.

Researchers have begun to identify the importance of metacognition in successful task performance in a variety of domains (Flavell, 1981; Baker & Brown, 1982). The notion of the importance of metacognition has recently been recognized by researchers in mathematical problem solving (e.g. Garofalo & Lester, 1985; Schoenfeld, 1983, 1987; Silver, 1985, 1987). Schoenfeld (1987) identifies self-regulation as a metacognitive behavior that is crucial for successful problem solving in mathematics. His work has suggested that monitoring and self-regulating behaviors are characteristic of the behaviors of expert mathematicians as they solve mathematical problems. In contrast, inexperienced problem solvers lack such metacognitive behaviors and are therefore impeded in their problem-solving endeavors. These results suggest that instructional settings should be designed that will foster such metacognitive behaviors. Small-group problem solving is a setting that appears to be conducive for involving students in metalevel discussions and problem analysis (Silver, 1987; Schoenfeld, 1987). The purpose of this research is to take an exploratory view of the problem-solving behaviors, both cognitive and metacognitive, of small groups of students working cooperatively to solve a mathematical problem.

Subjects and Procedures

The subjects for this study included 27 seventh grade students who attended an urban middle school in the borough of Queens in New York. The students were selected from three average-ability mathematics classes that were taught by two teachers. In each of the three classes two groups of students were randomly selected for observation and videotaping (six groups in all). Each group contained students differing in ability, sex, race and ethnic background.

On the day of the study, the students were instructed by their teacher to sit with their groups and solve the following problem by working together with their group members: "A banker must make change of one dollar using 50 coins. She must use at least one quarter, one dime, one nickel, and one penny. How many of each coin must she use to do this?" The problem-solving session lasted between 15 and 20 minutes.

The framework for the protocol analysis of the group problem solving was adapted from Schoenfeld (1983). He devised a scheme of parsing protocols into episodes and executive decision points. This framework was used as a starting point from which to analyze the role of metacognition and cognition in the mathematical problem-solving behaviors of the students as they worked in their small groups. The new framework differs from Schoenfeld's by the explicit delineation of the roles of metacognition and cognition within the mathematician's heuristics of problem solving.

The data were obtained from the videotapes of the six small groups of students as they worked together to solve the problem. Three coders viewed each tape in one minute intervals. They each watched one or two students in the group and assigned one of the following episodic categories and appropriate cognitive levels: READ (Cognitive), UNDERSTAND (Metacognitive), ANALYZE (Metacognitive), PLAN (Metacognitive), EXPLORE (Cognitive or Metacognitive), VERIFY (Cognitive or Metacognitive) and WATCH & LISTEN (Undetermined cognitive level). The differentiation made between cognitive and metacognitive behaviors were in keeping with the perspectives of cognitive researchers (e.g., Flavell, 1981; Baker & Brown, 1982). Our working distinctions of Cognitive and Metacognitive were from Garofalo & Lester (1985). Simply stated, "cognition is involved in doing, whereas metacognition is involved in choosing and planning what to do and monitoring what is being done."

Results and Discussion

The results of this study are discussed in terms of four major categories: Metacognition, Cognition, Watch and Listen, and the Recursive Nature of the Problem-Solving Process. Aside from the episode of WATCH & LISTEN, each of the episodes were assigned a level of cognition using Garofalo & Lester's distinction of metacognition and cognition. The lack of verbalization during episodes categorized as WATCH & LISTEN (assigned when students were silently giving their attention to the ideas or work of others) made it impossible to infer a level of cognition. Nonetheless, this category may still be an important dimension in the process of problem solving in a small-group setting. Recursion in problem solving is discussed below. Table 1 shows the percent distribution of Cognitive, Metacognitive, and Watch and Listen behaviors for the six problem-solving groups. Figures 1, 2 and 3 present the diagram of the behavior of groups 1A, 3A and 3B (three groups with contrasting problem-solving behaviors).

1. Metacognition

The percent distribution of metacognitive behaviors in Table 1 shows that all of the groups exhibited metacognitive behaviors. Note that the only group that did not solve the problem, Group 1A, had the lowest percentage of episodes at the metacognitive level and the highest percentage of episodes at the cognitive level. This unsuccessful group also had the lowest percentage of metacognitive behaviors during the exploratory phase (3.6). This was in contrast to Group 3A which had the highest percentage of metacognitive behaviors during the exploratory phase (25.6). These results suggest the importance of metacognitive processes in small-group mathematical problem solving.

2. Cognition

The percent distribution of cognitive behaviors in Table 1 shows that all of the groups exhibited cognitive behaviors. Note the relatively high percentage of cognitive behaviors (44.5) of Group 1A during the exploratory phase. The students in this unsuccessful group seemed to be caught up in the doing of mathematics without sufficient thinking about what they were doing to keep their explorations on track. On an individual level, the videotapes show that some students seemed adept at metacognitive-type behaviors but appeared unable to follow through at the cognitive level. In several of the groups certain students took on a "monitoring" role. These students rarely picked up a pencil to do any of the work. They spent their time inspecting and commenting on the work of their peers. When the pencil was placed in their hands they seemed to be at a loss for how to execute their own ideas. The necessity for the appropriate interplay of cognitive and metacognitive behaviors is most apparent for successful problem solving.

3. Watch and Listen

When studying group problem solving the role of watching and listening cannot be underestimated. It was not possible to assign a cognitive level to watch and listen behaviors, although it clearly plays a major role in the group process. The degree of watching and listening behaviors of students may be the defining factor of whether students are engaged in a group process at all. In the group that did not solve the problem, (fig. 1) the students hardly listened to one another. Of all five groups, this group had the lowest percentage of watch and listen behaviors (Table 1). In contrast, in one of the other groups, 3B (fig. 3), most of the students were watching and listening while one person was doing the majority of the work. These examples point to the importance of the balance of watching and listening behaviors during the group problem-solving process.

4. Recursive Nature of Problem Solving

It was evident in this study, (figs. 1, 2 and 3), that the heuristic episodes occurred recursively. In all of the groups the students returned several times to different episodic categories. Most often they would return to the words of the problem to gain a clearer understanding of the problem. They could often be heard reminding one another of the conditions that had to be met in the solution of the problem. In fact, all of the groups returned several times to the episodic category, Understanding the Problem. None of the groups solved the problem using a strict linear approach such as: 1) Read the problem, 2) Understand what the problem is asking for, 3) Analyze the problem, 4) Plan how to solve the problem, 5) Implement the plan, and then 6) Verify the solution.

Conclusion

The analysis of problem-solving behavior in the small-group protocols provided justification for demarcating metacognitive processes from cognitive processes. This important distinction has implications both at the theoretical and practical level. The results of this study suggest that different processes serve different important functions and future research is needed to gain a better understanding of how interrelationships among processes affect the efficiency and effectiveness of problem solving. Information was also gained about the recursive nature of heuristic episodes. The limitations of this study are that the framework has been applied to only a small number of problem-solving groups and it has been used with only one problem. Based on the results, however, it seems reasonable to suggest that this framework shows promise as being a powerful tool for the future study of mathematical problem solving in small-group settings.

Implications for Classroom Assessment

The implications for research and development in context-based classroom-level assessment are several. The current research in metacognitive and cognitive processes is hampered by limited accessibility to indicators of these processes in natural settings. One implication of the present study is that within a small-group setting the interactions among students allow for the elicitation of these processes. The processes are thus more accessible to observation and recording for assessment purposes.

A second implication is that the episodic categories presented here have meaning from both a mathematical problem-solving perspective and from instructional and psychological research perspectives. For the teacher, the analyses of problem-solving behaviors can provide diagnostic information regarding students' strengths and weaknesses in applying different cognitive processes inherent in the heuristic episodes. Such information can be used by teachers for prescriptive instruction. For the researcher, the analyses of the problem-solving behaviors can provide information regarding the balance of cognitive, metacognitive, and watch and listen behaviors that are most favorable for productive group problem solving.

While the cognitive-metacognitive framework reported here requires further validation, the data suggest that it has potential for being a tool for classroom-level assessment of mathematical problem solving in a small-group setting.

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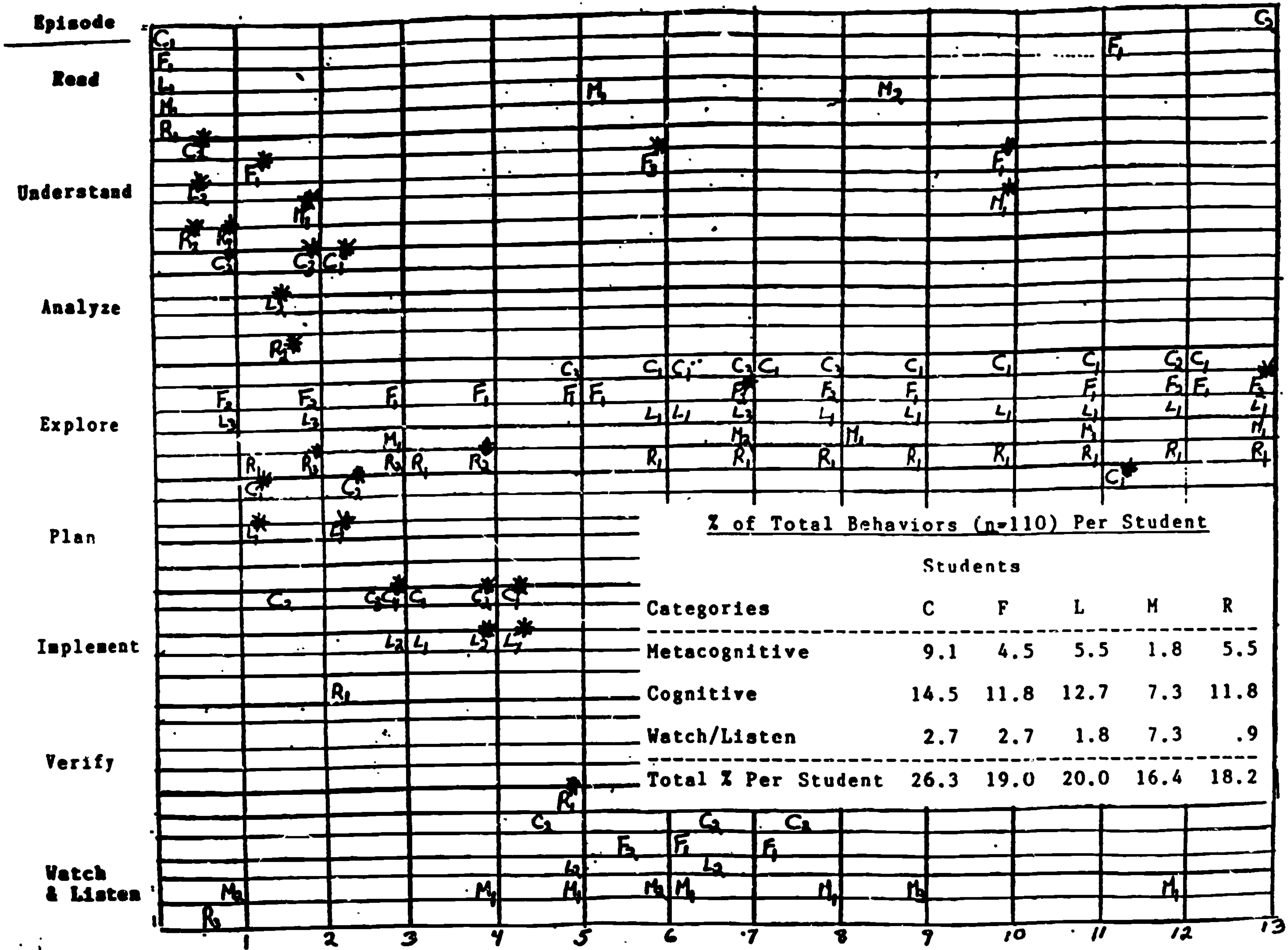
Table 1

Percent Distribution of Cognitive, Metacognitive, and Watch and Listen Behaviors by Problem-Solving Group

Groups	1A	1B	2A	2B	3A	3B
Total Number of Behaviors Coded	110	67	62	44	86	73
	Percent of Behaviors Coded in Each Category					
Metacognitive						
Understanding the Problem	8.2	17.9	21.0	11.4	11.6	8.2
Analyze	4.5	8.9	8.1	2.3	1.2	4.1
Explore*	3.6	11.9	16.1	15.9	25.6	15.1
Plan	4.5	4.5	4.8	0.0	2.3	0.0
Implement*	4.5	3.0	0.0	0.0	2.3	1.4
Verify*	.9	1.5	1.6	9.1	3.5	0.0
Total Percent of Behaviors that are Metacognitive	26.3	47.7	51.6	38.7	46.5	28.8
Cognitive						
Read	8.2	6.0	12.9	18.2	4.7	6.8
Explore	44.4	13.4	14.5	18.2	15.1	11.0
Implement	5.5	0.0	0.0	0.0	3.5	2.7
Verify	0.0	4.5	4.8	4.5	4.7	2.7
Total Percent of Behaviors that are Cognitive	58.2	23.9	32.2	40.9	28.0	23.2
Undetermined Cognitive Level						
Watch & Listen	15.5	28.4	16.1	20.5	25.6	47.9

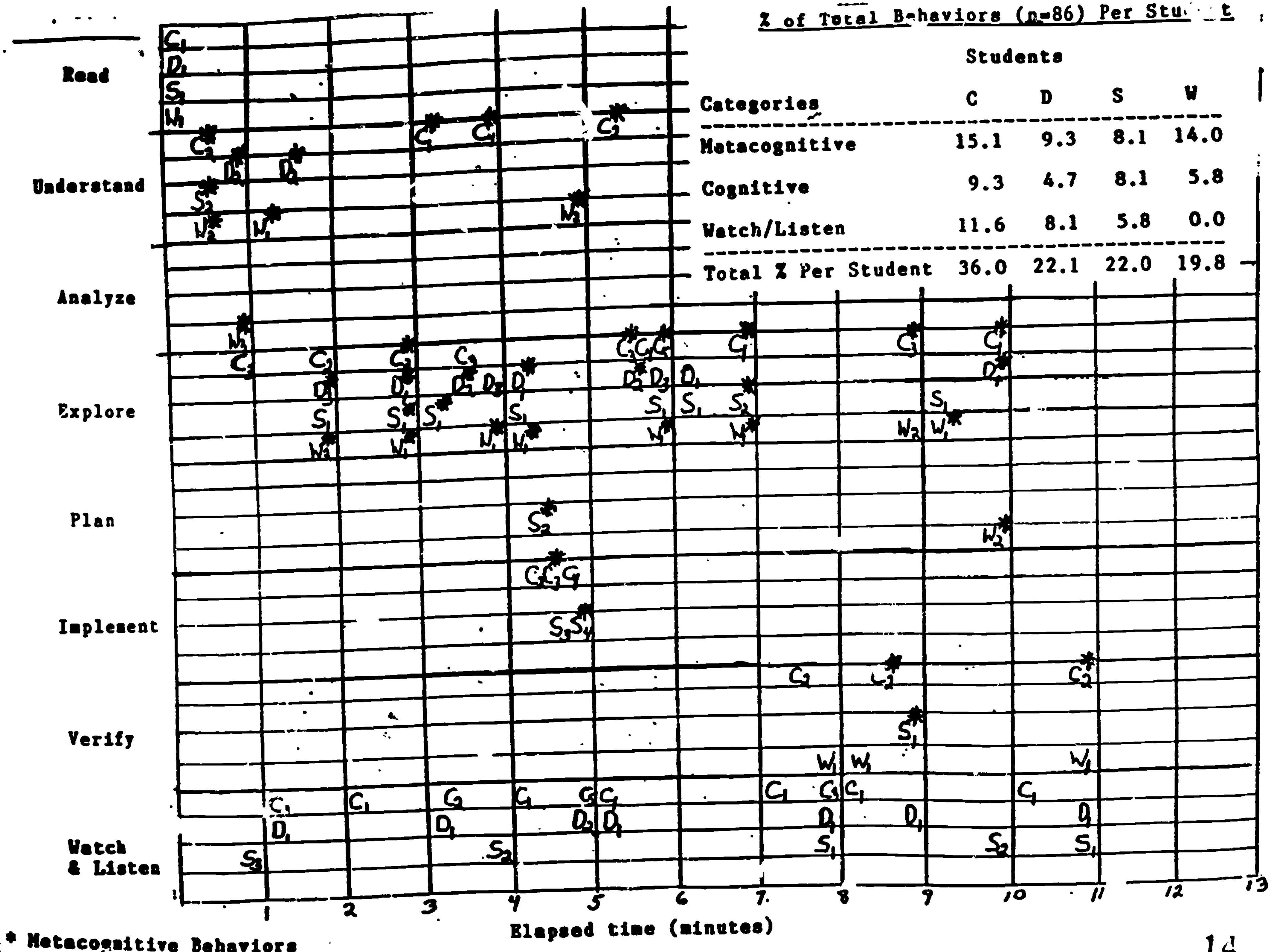


% of Total Behaviors (n=110) Per Student

Categories	Students C	F	L	M	R
Metacognitive	9.1	4.5	5.5	1.8	5.5
Cognitive	14.5	11.8	12.7	7.3	11.8
Watch/Listen	2.7	2.7	1.8	7.3	.9
Total % Per Student	26.3	19.0	20.0	16.4	18.2

* = Metacognitive Behaviors

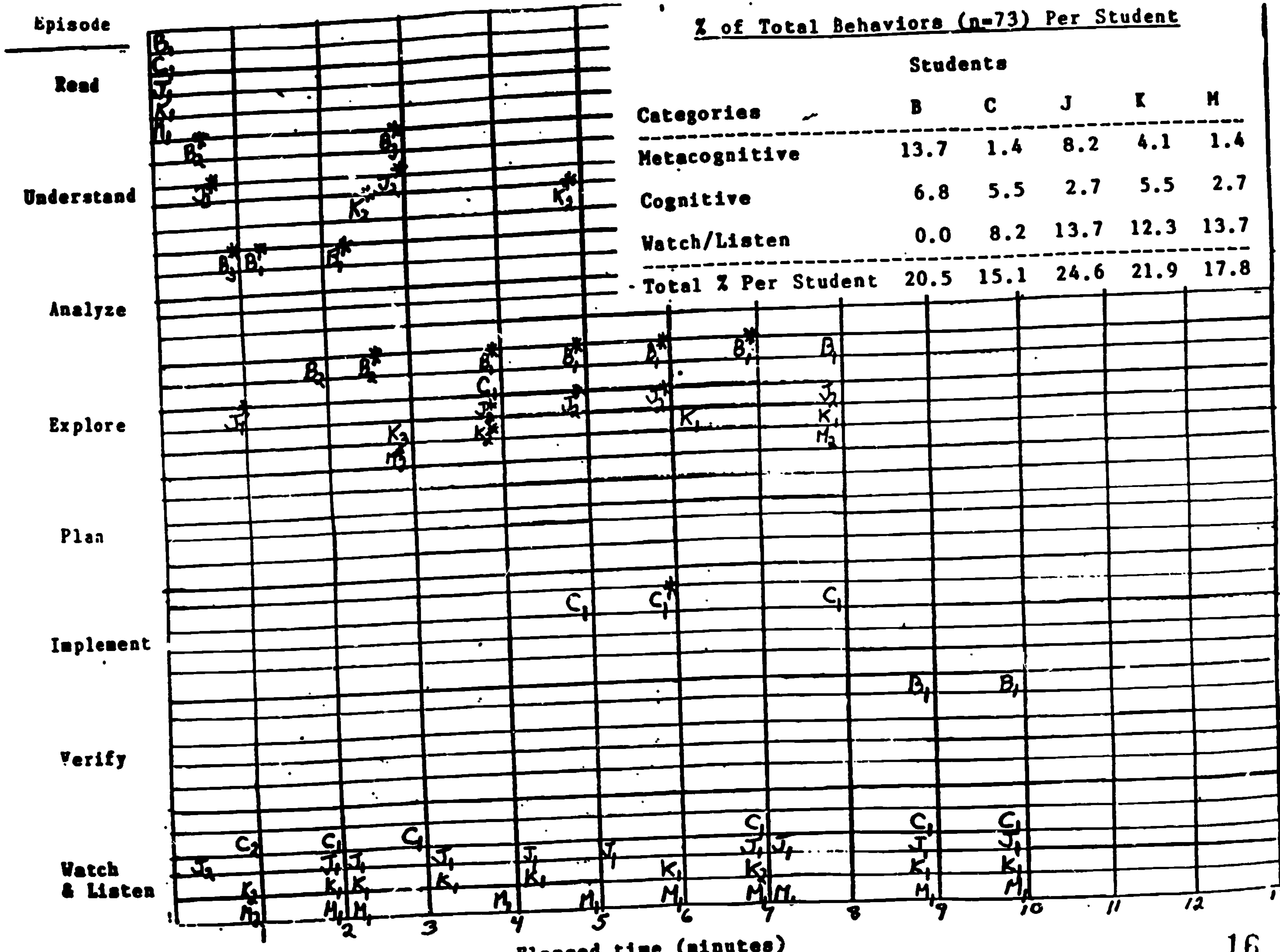
FIG. 1 Diagram of behaviors in Group 1A



% of Total Behaviors (n=86) Per Student

Categories	C	D	S	W
Metacognitive	15.1	9.3	8.1	14.0
Cognitive	9.3	4.7	8.1	5.8
Watch/Listen	11.6	8.1	5.8	0.0
Total % Per Student	36.0	22.1	22.0	19.8

FIG. 2 Diagram of behaviors in Group 3A



% of Total Behaviors (n=73) Per Student

Categories	B	C	J	K	M
	Students				
Metacognitive	13.7	1.4	8.2	4.1	1.4
Cognitive	6.8	5.5	2.7	5.5	2.7
Watch/Listen	0.0	8.2	13.7	12.3	13.7
Total % Per Student	20.5	15.1	24.6	21.9	17.8

* Metacognitive Behaviors

FIG. 3 Diagram of behaviors in Groups 3B

Cognitive-Metacognitive Framework for Protocol Analysis of Problem Solving in Mathematics

The following framework outlines the interactive relationship between metacognitive and cognitive processes in mathematical problem solving. The episodic categories are described both theoretically and empirically. The level or levels of cognition associated with each category are indicated as well. Note that during the course of problem solving these episodes need not occur in the order listed, may occur several times, and may indeed be bypassed completely.

Episode 1. READING THE PROBLEM (Cognitive)

Description: The student reads the problem.

Indicators: The student is observed as reading the problem or listening to someone else read the problem. The student may be reading the problem silently to him or herself or aloud to the group.

Episode 2. UNDERSTANDING THE PROBLEM (Metacognitive)

Description: The student considers domain-specific knowledge that is relevant to the problem. Domain-specific knowledge includes recognition of the linguistic, semantic and schematic attributes of the problem in his or her own words, and represents the problem in a different form.

Indicators: The student may be exhibiting any of the following behaviors:

(a) Restating the problem in his or her own words.

(b) Asking for clarification of the meaning of the problem.

(c) Representing the problem by writing the key facts or making a diagram or list.

(d) Reminding himself or herself or others of the requirements of the problem. Example: "Remember, we must use the exact number that is asked for in the problem."

(e) Stating or asking himself or herself whether he or she has done a similar problem in the past.

(f) Discussing the presence or absence of important pieces of information.

Episode 3. ANALYZING THE PROBLEM (Metacognitive)

Description: The student decomposes the problem into its basic elements and examines the implicit or explicit relations between the givens and goals of the problem.

Indicators: The student is engaging in an attempt to simplify or reformulate the problem. An attempt is made to select an appropriate perspective of the problem and reformulate it in those terms. Examples of statements that reflect such analysis is occurring are: "After you use all the given information it becomes an easy problem of addition." "Since the total is a multiple of five, I think the answer must be divisible by five."

Episode 4. PLANNING (Metacognitive)

Description: The student selects steps for solving the problem and a strategy for combining them that might potentially lead to problem solution if implemented. The student may also select a representation for the information in the problem. In addition, the student may assess the status of the problem solution and make decisions for change if necessary.

Indicators: The student describes an approach that he or she intends to use to solve the problem. This may be in the form of steps to be taken or strategies to be used. Examples of statements that reflect planning:

"Let's use the given information first and see what the problem looks like after that."

"Let's work backwards by estimating an answer and see how it must be adjusted to fit the problem."

"Let's draw a chart and fill in the numbers."

"Let's think of a different way to go about this."

"Let's check back to see where we went wrong."

Episode 5a. EXPLORING (Cognitive)

Description: The student executes a trial and error strategy in an attempt to reduce the descrepancy between the givens and the goals.

Indicators: The student engages in a variety of calculations without any apparent structure to the work. There is no visible sequence to the operations that the student is performing.

Episode 5b. EXPLORING (Metacognitive)

Description: The student monitors the progress of his or her or others' attempted actions thus far and decides whether to terminate or continue working though the operations. This differs from analysis in that it is less well structured and it is further removed from the original problem. If one comes across new information during exploration he or she may possibly return to analysis in the hope of using that information to better understand the problem.

Indicators:

(a) The student draws away from the problem to ask himself or herself or someone else what has been done during the exploration. Examples of such statements are:

"What are you doing?"

"What am I doing?"

(b) The student gives suggestions to other students about what to try next in the exploration. An example of such a comment is:

"It's getting too big, try it with one less."

(c) The student evaluates the status of the exploration. Examples of such statements are:

"This isn't getting us anywhere."

"I think that's the answer!"

Episode 6a. IMPLEMENTING (Cognitive)

Description: The student executes a strategy that grows out of his or her understanding, analysis and/or planning decisions and judgments. Unlike exploration, the student's actions are characterized by a quality of systematicity and deliberateness in transforming the givens into the goals of the problem.

Indicators: The student appears to be engaging in a coherent and well structured series of calculations. There is evidence of an orderly procedure.

Episode 6b. IMPLEMENTING (Metacognitive)

Description: The student engages in the same kind of metacognitive process as in the EXPLORING (metacognitive) phase of problem solving - monitoring the progress of his or her attempted actions. However, unlike the exploratory phase, the metacognitive decisions build on, check or revise those previously considered decisions. Furthermore, the student may consider a reallocation of his or her problem-solving resources given the time constraint within which the problem must be solved.

Indicators: During the implementation phase the student draws away from the work to check the state of the work; what has been done or where is it leading. Examples of statements reflecting this are:

"O.K. I used all the given conditions and now I will start adding what is left."

"Wait. You forgot to use the second point."

"This is taking too long. Try skipping the odd numbers."

Episode 7a. VERIFYING (Cognitive)

Description: The student evaluates the outcome of the work by a recalculation of its computational operations.

Indicators: The student redoes the computational operations he or she did before to check that it was done correctly.

Episode 7b. VERIFYING (Metacognitive)

Description: The student evaluates the solution of the problem by judging whether the outcome reflected adequate problem understanding, analysis, planning, and/or implementation. Should the student discover a discrepancy in this comparison search, he or she engages in new decision making for correcting the faulty metacognitive and/or cognitive processing that led to the incorrect solution. The ability to adjust one's thinking on the basis of evaluative information is another indication of self-regulatory competence. Should the evaluation of problem solution indicate an adequacy of or congruence with metacognitive and cognitive processing, the mental reiteration ends.

Indicators: After the student has decided that the solution or part of the solution has been obtained he or she may review the work in several ways:

(a) The student checks the solution process to see whether it makes sense. Statement example: "When we simplified the problem, did we use all of the given information?"

(b) The student checks to see if the solution satisfies the conditions of the problem. Statement example: "Does our answer satisfy both of the properties that were asked for?"

(c) The student explains to a groupmate how the solution was obtained. Statement example: "I knew it had to be a big number so I started with the largest numbers first."

Episode 8. WATCHING AND LISTENING (Uncategorized)

Description: This category only pertains to students who are working with other people. The student is attending to the ideas and work of others.

Indicators: The student appears to be listening to a group member who is talking or watching a group member who is writing.