ED 320 915 TM 014 894

AUTHOR	Flores, Penelope V.
TITLE	How Dick and Jane Perform Differently in Geometry: Test Results on Reasoning, Visualization, Transformation, Applications, and Coordinates.
PUB DATE	Apr 90
NOTE	37p.; Paper presented at the Annual Meeting of the American Educational Research Association (Boston, MA, April 16-20, 1990).
PUB TYPE	Reports - Research/Technical (143) -- Speeches/Conference Papers (150)
EDRS PRICE	MF01/PC02 Plus Postage.
DESCRIPTORS	Comparative Testing; *Geometry; High Schools; *High School Students; Item Bias; Mathematical Applications; Mathematics Curriculum; *Mathematics Tests; *Performance; Secondary School Mathematics; *Sex Differences; Standardized Tests; *Test Results; Thinking Skills; Visualization
IDENTIFIERS	University of Chicago Math Project

ABSTRACT

The effects of an experimental geometry course on gender differences in standardized test performance were studied. The experimental approach involved: (1) reading new concepts and materials followed by class discussion; (2) relating applications to real-life situations; and (3) conducting a spiraling review of concepts throughout the year. Subjects were 810 students (49% were males, and 51% were females) in grades 9 through 12. Matched pairs, which each included one class using the University of Chicago Math Project textbook, "Geometry," and one class using a traditional textbook, were compared in terms of students' performance on a standardized test. The experimental classes reduced the gender gap by improving girls' test scores. On a content-specific test, girls in the experimental classes significantly reversed the trend of boys' dominance on applications and coordinates. Girls remained on a par with boys on items of visualization in three dimensions and on transformations. On the open-ended proof writing test, girls in the experimental classes performed as well as did boys. In traditional classes, a negligible gap remained, favoring boys. Four tables provide study data, and 10 graphs illustrate the trends. Sample test items are included. (SLD)

How Dick and Jane Perform Differently in Geometry: Test Results on Reasoning, Visualization, Transformation, Applications, and Coordinates

Penelope V. Flores
San Francisco State University

ABSTRACT[1]

This paper shows that interactions arise between gender and instruction that make geometry performance on tests effective for females but not for males thus revealing that the magnitude of gender differences could be a function of the context of the learning situation in Geometry.

In an experimental study of 810 geometry students at 9th and 10th grades in 13 schools located in 5 states across the country using a matched-pair research design, findings show that on a standardized test, the experimental classes reduced the gender gap on student performance by improving girls' test scores.

On a content-specific test, girls in the experimental classes have significantly reversed the general trend of boy's dominance on <u>Applications</u> and <u>Coordinates</u>. Girls remain on par with boys on items in <u>Visualizations in three dimensions</u>, and <u>Transformations</u>.

On the open-ended proof writing test, girls in the experimental classes appear to have again significantly reversed some contemporary math findings (except the Senk and Usiskin study) of results favoring boys. Girls in the experimental classes on average appear to depart from the stereotype of not being as good as boys in geometry proof/reasoning skill.

Results on girls' test performance in the traditional geometry classes not only substantiated contemporary research findings of declining gender differentials in mathematics but as this study shows, the differences in geometry appear to have been reduced to zero.

---

[1] Paper presented at the 1990 meeting of the American Educational Research Association, April 16-20, Boston, MA.

How Dick and Jane Perform Differently in Geometry: Test Results on Reasoning, Visualization, Transformation, Applications, and Coordinates[1]

Penelope V. Flores
San Francisco State University

Do mathematics test achievement differentials persist between males and females? Many studies have examined this question and found declining gender differences (Friedman 1989, Linn and Hyde 1989, Maccoby and Jaklin 1974). Before 1974 there were significant gender differences (Hyde, 1981), but after 1974 the cognitive gender differences have shrunk to negligible levels (Linn & Hyde, 1989).

At the elementary grades, no differences are found (Callahan and Clements 1984, Dossey, Mullis, Lindquist, and Chambers 1988, McKay 1979, Siegel 1968). The studies that showed differences favored the females (Brandon, Newton, and Hammond 1985, Hawn, Ellet, and Des Jardines 1981, Potter and Levy 1968, Shipman 1972). Boys outperform girls on the problem solving subtests in the 2nd grade, but differences declined in 5th and disappeared by 8th grade (Lewis and Hoover 1986). The picture is mixed at junior high school. There are small sex differences (in favor of females) reported by Tsai and Walberg (1979) and in favor of boys (Hillton and Berglund 1974). No difference are reported by others (Cicirelli 1967, Connor and Serbin 1985). Of the gifted youth, substantial differences are noted favoring males (Benbow and Stanley 1982, Weiner 1984).

Strong evidence that differential coursework accounts for considerable amount of sex difference (Pallas and Alexander 1983, Wise, Steel, and McDonald 1979). The explanation of differential coursework can not be offered as an explanation at junior high so the argument cannot be applied. Further studies of the general population show that differential course taking does not fully account for the sex difference in mathematical tasks (Armstrong 1981, Friedman 1987, Ramist and Arbeiter 1986).

Sex role socialization studies (Fennema and Sherman 1977) showed that females are more likely to disagree that math is a

---

[1] Paper presented at the 1990 meeting of the American Educational Research Association, Boston, MA. I am grateful to the University of Chicago School Math Project for the use of the data.

male domain, but females are less convinced than males that mathematics would be useful to them personally. Sex role socialization which is just beginning at junior high school may account for gender differences at this grade level, although this argument was challenged by (Eccles and Jacobs 1986).

There is a connection between the kind of classroom environment and gender differences in terms of teacher behavior and student learning (Fennema and Peterson 1985). Results of other studies indicate that teachers who encourage "autonomous learning behaviors" produce greater gains (Koehler 1986).

Gender differentials are reported on spatial skills (Sherman 1967) which speculates that this might function not only as an intervening variable but also that it had an experiential basis. Some evidence indicates that spatial visualization does not account for differences in mathematics performance (Armstrong 1981, Fennema and Sherman 1977, Fennema and Tartre 1985, Pattison and Grieve 1984). Spatial visualization plays a differentiating role for the sexes and is a significant predictor of mathematics success for girls than for boys (Fennema and Tartre 1985, Smith 1964, Weiner 1984), while opposite results are also reported (Connor and Serbin 1985, Very 1967). In the area of spatial relations, especially in mental rotations, Linn and Hyde (1990) believe that there is a decline which may be attributed to increased female participation in sports.

A study of gender differentials on spatial relations can be informed through the study of Geometry. Geometry is more comprehensive than spatial relations and mental rotations. A geometry course includes mathematics and the ability to visualize, make transformations, do mental rotations, configure shapes that tessellate, compute coordinates, and apply theorems and properties of mathematics into a sequence of logical proofs. On the ability to write proofs, Senk and Usiskin's (1984) large-scale nation-wide study is particularly illuminating. They found that girls and boys perform equally well even on complex mathematical tasks in Geometry Proof writing when both in-class and out-of-class exposure to tasks is equal.

Given that girls enter geometry classes with certain attitude and access differentials generated by social pressures related to mathematics learning, it becomes important to examine emergent trends of the equal opportunity to learn if we want to gain more understanding of the mathematics gender issue.

Research Issue

We are interested in the proposition: In an experimental geometry course where new concepts and material are learned through reading the text first before being discussed in class, where applications to the formulations learned are related to real life situations, where concepts learned appear in a spiralling review throughout the year, then we will expect to find no significant gender differentials in geometry on a standardized test as well as on a content-specific test of an open-ended proof question type.

In contrast when geometry is learned in the typical lecture cum discussion method in traditional environments of mathematics learning, then we will expect to find gender differentials due to the social context of instruction.

Methodology

Data Source. The subjects in this study are 810 geometry students from 13 schools located in 5 states (California, Michigan, Illinois, Ohio, and Pennsylvania). The schools included rural, suburban, and city schools. In the study 49% are males and 51% are females.

Research Design. The matched pair design was used in the study. The experimental design matches pairs of classes who are using the University of Chicago Math Project (UCSMP) Geometry with a class using the regular geometry textbook. The text used by majority of the comparison classes was Geometry published by Houghton and Mifflin.

The pairs are in one school except for one. In this instance no comparison pair could be obtained within the same school and so a school sharing similar socioeconomic characteristics within the district was used as the comparison class.

The pairs consisted of 9th, 10th, and mixed 10th/11th/12th grade classes with the majority at 10th. There were originally 45 classes where 22 of the classes were experimental. The sample was partitioned by treatment (experimental and comparison). Subpopulations were partitioned by gender.

A pretest - Entering Geometry Student Test - was administered at the beginning of the school year. The mean differences were analyzed using the t-test statistic. Initial 18 pairs were examined based on how close the pretest mean scores were between the experimental and the comparison pairs in each school. On one site, a comparison class was not kept intact at the semester break resulting in changing the class composition substantially. In this school, the students were allowed to move between the experimental and comparison classes at the semester change. This event would not have been problematic. We could have included the individual student scores as long as the students were assigned to the classes within the treatment groups. For example, a student could have been assigned to another experimental class or another comparison class within the classes involved in the study. But the problem was that some of the Geometry classes where some students were programmed were not part of the overall pool of classes in the study.

Using the criteria described below, eight pairs were eliminated thus:

1. Pairs were eliminated if when looking only at students who took both pre and posttests, any pair whose <u>Entering Geometry Student Test</u> pretest scores yielded a p-value less than .05.

   Pairs were eliminated if the p value was less than .05 in estimating the effect size g,

   where g = (Mean Exp - Mean Comp)/standard deviation pooled.

   The operational definition of matched pairs as derived from the effect size is:

   good match = $|g| < .1$
   medium match = $.1 < |g| < .25$
   poor match = $|g| > .25$

2. Any member of a pair that dropped out of the study. For example, when the experimental teacher resigned in one school, his classes and those of the comparison classes were also eliminated.

3. Any pair about which additional information led us to believe that classes were not comparable. For example, discussions with teachers indicated classes were substantively different (honors 9th graders versus average ability 10th graders).

In cases where two experimental classes were matched with the same comparison class, the single best matched pair was selected, based on an examination of means, standard deviations, range, and distribution shape. In cases where the experimental and comparison teachers had multiple classes, the same matching procedure was applied, and only the best matched pairs were selected.

We ended up finally with ten matched pairs who had all the pre and posttests. This translated into 350 students where 190 (53%) were in the experimental classes and 163 (47%) were in the comparison classes. Of the sample population 177 (50%) were females and 176 (50%) were males.

Instruments

Pretest. A pretest was administered in the fall. The test is a 40-item geometry readiness test developed by the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) and the University of Chicago School Math Project (UCSMP). Half of the items were algebra skills used in geometry and the remaining 20 items were geometry readiness items.

Posttests. The posttest consisted of a standardized test and two content-specific tests. These were administered in the spring on three consecutive days of 40 minutes each. The standardized test had 40 items of standard geometry published by American Testronics. No calculators were allowed in this test.

The other tests consisted of Part 1 and 2 content-specific tests developed by UCSMP. Part 1 was an 11-item open-ended proof test. The first 5 items were of the multiple-choice type, while the remaining 6 items were open-ended. Of the six open-ended items, the first required the student to fill in four missing statements or reasons in a proof; the second required translation of a verbal statement into an appropriate figure, "given," and "to prove"; and the last four required the student to write complete proofs, ranging from easy to difficult, covering congruent and similar triangles, parallel lines, and quadrilaterals. Several pilot studies of the proof tests had been conducted to insure appropriate test length, clarity of instructions, and approximate balance of item difficulty and subject matter across forms, but no effort was made to make the forms comparable nor statistically equivalent. By making two different forms, performance on a greater number of proof items could be analyzed. The proof test forms were alternated among the students so that approximately half of the students in each class received each form (see samples of items in Appendix A).

Part 2 test was a 40-item content specific instrument. Individual items of this test were identified as belonging to subtests of specific geometry skills. These subtests focus on Reasoning/Proof, Transformation, Visualization/Three Dimensions, Applications, (which include computer and calculator applications), Coordinates, and Standard Knowledge (see sample items in Appendix B). Calculators were allowed on this test.

Scoring of the open-ended items involved five experienced high school mathematics teachers (3 females, 2 males). Proof items were graded on a scale of 0 to 4 based on general criteria developed internally among the scorers and the co-director of the

Project and based on published criteria developed by Malone, Douglas, Kissane, and Mortlock (1980) as given below:[2]

> 0-Student writes only the "given", or writes invalid or useless deductions.
>
> 1-Student writes at least one valid deduction and gives reason.
>
> 2-Student shows evidence of using a chain of reasoning, either by deducing about half of the proof and stopping or by writing a sequence of statements that is invalid only because it is based on faulty reasoning early in the steps.
>
> 3-Student writes a proof in which all steps follow logically, but in which there are errors in notation, vocabulary, or names of theorems.
>
> 4- Student writes a valid proof with at most one error in notation.

The highest possible score on the open-ended test was 5 [multiple choice items] + (6 * 4 [open-ended items]) = 29. Before grading each item, graders discussed the application of the general criteria to that item. Every item on each students' test was scored independently by a different pair of graders who had no access to the student's name, sex, or school. Interrater agreement ranged from 85 percent to 95 percent across the 12 items. Less than 2 percent of the scores of the pair of graders differed by more than one point. When the two grader's scores disagreed, a third independent blind reading was undertaken, and a median of the three scores was chosen as the item score.

Another instrument was the Student Opinion Survey which attempted to tap the attitude of students toward mathematics in general and geometry in particular. It also surveyed student attitude toward their text, their future mathematics plans and grades they expect to receive in geometry.

Teachers were requested to keep their classes intact at the second semester in order to maintain the integrity of the research design. However, in one school where computerized programming of students at second semester was used, a pair of classes was dropped from the study. A teacher questionnaire was administered at the time that the students in their class were taking their tests. This instrument asked about their attitude

---

[2] A score of 8 was included where students wrote nothing or left the page blank. This was to differentiate a score of 9 which indicated missing cases.

toward the course, and teacher perception about students' attitude towards the course.

The classrooms were observed once during the year and representative samples of students were interviewed. In addition all the experimental teachers were interviewed. In the final analysis, only those students who had pretest scores and all three posttests were included.

The mean differences between males and females on the Entering Geometry Test was obtained. This pretest score was used as a covariate in analyzing posttest scores. Two-way Ancova by treatment and gender with pretest scores are reported.

GEOMETRY ACHIEVEMENT RESULTS

The results of the matching procedure showing means and standard deviation are presented in Table 1. The results of the main effect of treatment and main effect of gender are presented in Tables 2 and 3. Results presented are those after adjustment for the pretest. The adjusted means take into account diffences between classes on the pretest at the beginning of the school year. Recall that in our matching procedure, there were some good matches, medium matches and poor matches. For instance, if the experimental class has a pretest mean that is smaller than the comparison group at the beginning of the year, then the posttest mean for experimental will be increased by an amount that depends on the pretest difference. In some cases the adjusted means was slightly larger than the unadjusted difference. The adjusted means allowed us to construct more sensitive tests of differences between classes since these means adjust for pre-existing differences between classes.

For each effect, the results are reported first on the standardized test (American Testronics), and second on Geometry Part 2, and last on Geometry Part 1. Geometry Part 2 results of the analysis is partioned further by subtests: Proof or reasoning, Applications, Visualization, Transformation, and Coordinates. The analysis on Geometry Part 1, Form A (multiple choice, open-ended) and Form B (multiple choice, open-ended) are reported last.

Tables 2 and 3 show overall main effects of treatment and gender. Results of the interaction test are also presented through means, standard deviation, and p values for all three posttests. Examining the results of the interaction test before reporting the main effects of treatment contributes to better understanding of the data. For example, if we find no difference overall between the treatment on a particular posttest means but do not test for interactions between treatment and gender first, we may miss important findings about the variable which may mask important relationships.

In obtaining the significance level in the interaction test, we used the Bonferroni method where the critical p value used is .05/6 = .008; the reason being that when we examine four groups which has six comparisons, we should gain more precision when we divide the .05 level of significance by six. For instance, the two dimensions of treatment 1) experimental, 2) comparison, and gender can have four subgroups: namely 3) experimental and male 4) experimental and female 5) comparison and male, and 6) comparison and female. These four groups can have a maximum of six pair-wise comparisons. Therefore, in order to avoid Type 1 error and get around the problem of incorrectly rejecting a true null hypothesis, applying our method guarantees that we are certain we do not commit such an error.

Having carefully designed all the controls by treatment in a meticulous process of matching pairs of classes and applying a critical value for level of significance, then and only then were we ready to examine the gender issue in depth among Geometry high school students at 9th/10th grades.

Findings

Pretest Scores

Analysis of Entering Geometry Student Test results presented in Table 1 shows no significant differences between the matched pairs on the overall test. There is no significant difference between the treatment groups by subtests on the CDASSG and the UCSMP pregeometry/algebra items. All pairs of classes in the study start out equal at the beginning of the year.

[Table 1 around here].

Posttest scores

<u>Standardized test</u>. Table 2 shows the main effect of gender on all posttests after adjustment for the pretest.

[Table 2 around here].

Results of the analysis yield no significant interaction overall (mean for males = 13.65, sd = .41, mean for females = 12.30, sd = .41). On standardized tests in general, no difference is found between treatment groups, i.e., girls compare favorably with boys. Within the comparison classes, males in general outperform the females on the standardized test but not significantly (p = .01) Within the experimental classes the treatment by gender interaction test yields a value of .532 suggesting that girls' and boys' test performance are not much different on the standardized test. In other words girls are performing at par with, not below par with boys. (For a representation of the interaction of tretment and gender on the standardized test based on Table 2, refer to Figure 1a).

<u>Geometry Part 2. Content-specific test</u> (40 items). On the Geometry Part 2 test (proof/reasoning, applications, visualization, transformation, and coordinates) overall results show no interaction on the posttest and no significant main effect of gender (mean for females = 10.67, sd = 43, mean for males = 10.22, sd = .43, p = .457). Within the experimental classes however, the picture changes. Females outscore their male counterparts but not significantly with a mean score of 12.59, sd = .58 and males with a mean score of 11.51, sd = .59. An overall comparison of the experimental and comparison classes in the treatment show significantly lower mean scores for the comparison classes.

Notice that within the comparison classes, females consistently score lower than males, whereas within the experimental classes it is the other way around: females consistently outperform the males slightly, although the difference is not statistically significant.

The question remains: where do males and females differ significantly on the geometry posttest? Is it on proof/reasoning, spatial relations, transformation, visualization, application skills, coordinates? Table 3 presents the analysis of Geometry Part 2 showing the breakdown by subtests. On the proof/reasoning items, within the experimental classes, females tend to score slightly higher than the boys, but not significantly (p = .011), while within the comparison classes there is no statistical difference between genders (p = .969).

On the application items, the main effect of gender is again not significant (females = 1.82, sd = .12, males = 1.49, sd = .12, p =.053) Within the comparison classes boys and girls tend to have comparable mean scores, but within the experimental group the females tend to score higher than their counterparts on this subtest (females = 2.31, sd = .16, males = 1.71, sd = .16, p = .009).

No significant interactions are noted on the visualization and transformation subtests. There are no effects of gender either. On average, females tend to be up to par with the males on these two subtests. Contemporary research reports that girls are less proficient than males on spatial or three dimensional skills and that these differences are declining through the years. The results of our analysis show not only declining differences but that indeed the differences may have been reduced to zero.

A significant interaction is noted on the coordinates subtest but only within the experimental group. Females within this group tend to benefit significantly from the course (females = 1.5, sd = .10, males = .75, sd = 10, p = .004) Males tend to have higher mean scores than females within the comparison classes, but the difference is not statistically significant. (See plot of the interaction terms in Figures 2e to 2e for proof/reasoning, applications, coordinates, visualization and transformation).[3]

[Figures 2a to 2e around here.]

Geometry Part 1. Proof writing test; multiple-choice, open-ended type (11 items). The results of Geometry Part 1 interaction test are mixed (refer to Table 2). Overall significant interaction is found on the open-ended format of the test.

On Form A, multiple choice, no significant interaction is noted. Females in general outscore males on items dealing with the algebra skills needed to do geometry problems (females = 1.98, sd = .15, males = 1.53, sd = .15, p = .004) On Form B open-ended, a significant interaction is noted suggesting that within group differences must be examined to get a better view of the overall results. Which group is significantly contributing to this observed interaction between treatment and gender? The data indicate they are the females within the experimental classes where they have a mean score of 2.52, sd = .21 while their male counterparts yielded a mean score of 1.44, sd = .41, p = .0001.

The same trend is observed on Form B, multiple choice. The females in the experimental classes are significantly different from their male counterparts. We found that girls appear to be significantly outperforming the boys within the experimental classes but not within the comparison classes. Again on Form A open-ended, and on Form B open-ended, the females within the

---

[3] The graphs show the pattern of interaction and should not be compared especially since units are not the same. For interaction coefficients refer to Table 3.

experimental group seem to be pulling up the mean scores. For example, on Form A open-ended, girls in the experimental group outperformed the boys significantly (females = 2.52, sd = .58, males = 1.57, sd = .21, p = .0001). Females within the comparison classes appear to have lower mean scores than the boys, although the difference is not statistically different from zero. (see Figure 1b to 1e for the interaction plots based on Table 1)

Summary[4]

1. On the standardized test, it seems that the experimental classes reduced the gender gap on student performance by improving girls' test scores.

2. On the content-specific test by subtests, the experimental treatment appears to have significantly reversed the general trend of boy's dominance in the two subtests, namely Applications and Coordinates. Girls seem not to lag behind boys in spatial test items covered in Visualizations in three dimensions, and Transformations. In short girls and boys remained on par with each other on these tests.

3. On the open-ended proof test, the experimental class appears to have again significantly reversed the contemporary math findings of results favoring boys (Senk and Usiskin study being the exception). Girls in the experimental classes on the average appear to depart from the stereotype of not being as good as boys in geometry proof/reasoning skills.

4. Results on girls' test performance in the traditional geometry classes not only substantiated the contemporary research findings that the gender differential in mathematics as it applies to spatial relations is declining, but as this study shows the differences had been reduced to zero, and in the experimental classes, the girls had outperformed the boys in two out of five geometry subtests. However, in the traditional geometry classes the negligible gap remains favoring boys.

---

[4] Care should be exercised that the language used is not interpreted as sexist.

Discussions

We have shown in this paper that interactions arise between gender and instruction. These interactions show that certain learning environments make geometry learning effective for women learners. It also shows that the magnitude of gender differences is so clearly a function of context or learning situation.

The experimental classes used the University of Chicago School Math Project (UCSMP) text. The material is characterized by presenting every single lesson with Skills, Properties of mathematics, Understanding the underpinnings of each question, and presenting the material through a variety of Representation, not only by mathematical symbols. UCSMP used the acronym SPUR for this approach. Experimental teachers were asked to assign the reading that begins each lesson first and then for the students to do the questions covering the reading, and to apply their understanding of the concept in a few exercises that determine whether or not they can apply their new knowledge. UCSMP calls this strategy CARE or the acronym for Covering the reading, Applications to real life situations, Review, and for challengeing projects, the Exploration questions.

What seems different between the two groups is the teaching strategy and use of the text as a learning-to-learn tool. The responsibility for learning resides within each individual student in the experimental group. Students in the experimental classes are expected to come to their geometry classes having read the text first, having assessed their comprehension of the text, and having solved a couple of problems on their own. The observed classes normally start the day's work with discussions with the teacher and among themselves of the variety of approaches used in the solution to problems encountered in doing their homework. Homework time as indicated in the Student Opinion Survey for both groups averaged 45 minutes per day. Therefore time spent on homework did not differ. Lecture method in teaching geometry was exercised at a minimum in the experimental classes as revealed in the teacher questionnaire and classroom observations. In contrast, classroom observations and the results of the teacher questionnaire of the comparison classes reveal the lecture method with the teacher showing how-to- methods of doing proofs still predominate in the teaching of Geometry.

Pace was an important ingredient in the experimental classes. Each lesson was a class period's work. Teachers were asked to follow the prescribed pace. For those who thought that more time was needed to review, it was shown early on in the year that the exercises provided in each lesson reviewed earlier material in many different applications of the concept (see

Appendix D for a sample lesson from UCSMP Geometry). The lesson-a-day pace ensured that more opportunity to cover material was built-in.

Reading the text was an issue confronted early on in the project. There were some who thought that the reading level in the UCSMP text was difficult for their students. The student survey and the teacher questionnaire attempted to get information about this aspect. We addressed this issue by undertaking a reading level test using the Fry formula. Random sample pages of both texts were analyzed. One hundred words were picked out each from 4 sampled pages. The number of sentences within the hundred words and the number of syllables obtained within the sentences were computed and plotted in a grid.[5] The results of the readability test showed that UCSMP reading levels were at the 5th to 9th grade level with one sample at 5th, and at 7th, 8th and 9th grade levels respectively for the remaining samples. The comparison readability test resulted in one sample page at 7th, one at 8th and the two remaining sample pages at 9th grade levels.[6] It seems that because reading math is seldom done; students were reacting to the math reading done, not the difficulty of the reading material in math. Girls are seen as better in verbal ability including reading than boys, but as contemporary research studies have shown, this gender gap has also declined in years (Linn and Hyde 1989).

If an important goal in teaching is to increase students' participation and performance in mathematics courses, classroom environments designed to show students how to think mathematically, to use the text differently as an instrumental tool for learning (i.e. to read the explanations of the text first before asking the teacher to expound on the subject matter, to think about applications to problems, devise algorithms in order to apply their understanding to real life situations), then this type of curricula may be the wave of the future that will promote equal success opportunity among students in geometry or mathematics in general.[7] The process, in the long run, may prove to be gender blind.

---

[5] See Edward Fry's formula, Rutgers University Reading Center.

[6] For a complete description of the readability test, see Flores, P.V., Hedges, L.V., & Stodolsky, S. Geometry Two-Year National Field Study, (in progress), University of Chicago School Math Project.

[7] See UCSMP Evaluation studies in Transition Mathematics, Algebra, Advanced Algebra, Geometry, and Functions and Statistics with Computers, University of Chicago.

BIBLIOGRAPHY

Aiken, L. (1986). Sex differences in mathematical ability: A review of the literature. Educational Research Quarterly, 10, 25-35.

Armstrong, J. M. (1981). Achievement and participation of women in mathematics: Results of two national surveys. Journal of Research in Mathematics Education, 12, 356-372.

Benbow, C. P., & Stanley, J. C. (1982). Consequences in high school and college of sex differences in mathematical reasoning ability: a longitudinal perspective. American Educational Research Journal, 19, 598-622.

Brandon, P. R., Newton, B. J., & Hammond, O. W. 1987). Children's mathematics achievement in Hawaii: Sex differences favoring girls. American Educational Research Journal, 24, 437-476.

Callahan, L.G. & Clements, D. H. (1984). Sex differences in rote-counting ability on entry to first grade: Some observations. Journal for Research in Mathematics Education, 15, 378-382.

Carpenter, T. P., Lindquist, M. M., Matthews, W., & Silver, E. A. (1983). Results of the NAEP mathematics assessment: Secondary school. Mathematics Teacher, 76, 652-659.

Cicirelli V. G. (1967) Sibling constellation, creativity, IQ, and academic achievement. Child Development, 38, 481-490.

Connor, J. M., & Serbin, L. A. (1985). Visual-spatial skill: Is it important on mathematics? Can it be taught? In S. F. Chipman, L. R. Brush, & D. M. Wilson (Eds.), Women and mathematics: Balancing the equation. (pp. 151-174). Hillsdale, NJ: Lawrence Erlbaum Associates.

Dossey, J. A. , Mullis, I. V. S., Lindquist, M. M., & Chambers, D. L. (1988). The mathematics report card: Are we measuring up? Trends and achievement based on the 1986 National Assessment. Princeton, NJ: Educational Testing Service.

Eccles, J. S., & Jacobs, J. E. (1986). Social forces shape math attitudes and performance. Signs: Journal of Women in Culture and Society. 11, 367-380.

Fennema, E. & Peterson, P. (1985). Autonomous learning behavior: a possible explanation of gender-related differences in mathematics. In L.C. Wilkinson & C.B. Marret (Eds.). Gender influences in classroom interaction (pp. 17-35). Orlando, FL: Academic Press.

Fennema, E. & Sherman, J. (1977). Sex-related differences in mathematics achievement: Spatial visualization and affective factors. American Educational Research Journal. 14, 51-71.

Fennema, E. & Sherman, J. (1978). Sex-related differences in mathematics achievement and related factors: A further study. Journal for Research in Mathematics Education. 9, 189-203.

Fennema E. & Tartre, L. A. (1985). The use of spatial visualization in mathematics by girls and boys. Journal for Research in Mathematics Education, 16, 184-206.

Flores, P. V., Hedges, L. V. & Stodolsky, S. (1990). Geometry Two-Year National Field Study. University of Chicago School Math Project. Chicago (in progress).

Friedman, L. (1987). Gender differences on ACT tests. Unpublished research report, University of Chicago, Chicago.

Hawn, H. C., Ellet, C. D., & Des Jardines, L. (1981, April) Differences in mathematics achievement between males and females in grades 1-3. Paper presented at the annual meeting of the Eastern Educational Research Association, Philadelphia.

Hilton, T. L., & Berglund, G. W. (1974). Sex differences in mathematics achievement--a longitudinal study. Journal of Research in Science Teaching, 22, 141-152.

Hyde, J. S. (1981). How large are cognitive gender differences? A meta-analysis usinf w and d. American Psychologist, 36, 892-901.

Koehler, M. C. S. (1986). Effective mathematics teaching and sex- elated differences in algebra one classes (Doctoral dissertation, University of Wisconsin-Madison, 1984). Dissertation Abstracts International, 46: 919A-920A.

Lewis, J. C., & Hoover, H. D. (1983, April). Differential prediction of academic achievement in elementary and junior high school by sex. Paper presented at the annual meeting of the American Education Research Association, San Francisco.

Linn, M. C. & Hyde, J. S. (1989). Gender, mathematics, and science. Educational Researcher, 14, 51-71.

Maccoby, E. E., & Jacklin, C. N. (1974). The Psychology of sex differences. Standford, CA: Stanford University Press.

McKay, L. R. (1979). Sex bias in tests of mathematical aptitude (Doctoral dissertation, University of Texas at Austin, 1978). Dissertation Abstracts International, 39, 4103A.

Malone, J. A., Douglas, G. A., Kissane, B. V., & Mortlock, R. (1980) Measuring problem-solving ability. Problem-solving in School Mathematics S. Krulik & R. E. Reys (Eds.). Reston, VA: National Council of Teachers of Mathematics.

Pallas, S. M., & Alexander, K. L. (1983). Sex differences in quantitative SAT performance: new evidence on the differential coursework hypothesis. American Educational Research Journal, 20, 165-182.

Potter, M. C. & Levy, E. (1968). Spatial enumeration without counting. Child Development, 39, 265-272.

Pattison, P., & Grieve, N. (1984). Do spatial skills contribute to sex differences in different types of mathematical problems? Journal of Educational Psychology, 76, 678-689.

Peterson, P. L., & Fennema, E. (1985). Effective teaching, student engagement in classroom activities, and sex-related differences in learning mathematics. American Educational Research Journal, 22, 309-335.

Ramist, L., & Arbeiter, S. (1986). Profiles of college-bound seniors: 1985, New York: College Entrance Examination Board.

Senk, S., & Usiskin, Z. (1983). Geometry and proof writing: a new view of sex differences in mathematics ability. American Journal of Education, 91, 187-201.

Sherman, I. M. (1967). The problem of sex differences in space perception and aspects of intellectual functioning. Psychological Review, 74, 290-299.

Shipman, V. C. (1972). Disadvantaged children and their first school experiences. Princeton, NJ: Educational Testing Service Head Start Longitudinal Study Report PR-72-18.

Siegel, L. S. (1968). The development of the ability to process information. Journal of Experimental Child Psychology, 6, 368-383.

Smith, I. M. (1964). Spatial ability: Its educational and social significance. London: University of London Press, Ltd.

Swafford, J. O. (1980). Sex differences in first-year algebra. Journal for Research in Mathematics Education, 11, 335-346.

Tsai, S. L. & Walberg, H. J. (1979). Mathematics achievement and attitude productivity in junior high school. Journal of Educational Research, 76, 267-272.

Very, P. S. (1967). Differential factor structures in mathematical ability. Genetic Psychology Monographs, 75, 169-207.

Weiner, N. C. (1984). Cognitive aptitudes, personality variables, and gender difference effects on mathematical achievement for mathematically gifted students (Doctoral dissertation, Arizona State University, 1983.) Dissertation Abstracts International, 44, A.

Wise, L., Steel, L. & MacDonald, C. (1979). Origins and career consequences of differences in high school mathematics achievement. Palo Alto, Ca: American Institute for Research (ERIC Document Reproduction Services No. ED 180 846).

TABLES

TABLE 1

Analyses of Entering Geometry Test and Its Subtests

		MAIN EFFECTS			
		N	Mean	SD	p-Value
OVERALL	UCSMP	190	15.76	6.44	
	Comparison	168	15.07	6.10	.2912
CDASSG	UCSMP	190	9.76	3.39	
	Comparison	168	9.36	3.60	.2729
	NOexp	272	9.22		
UCSMP PRE-GEOMETRY	UCSMP	190	2.73	1.82	
	Comparison	168	2.71	1.71	.9257
ALGEBRA SCALES	UCSMP	190	3.26	2.28	
	Comparison	168	2.99	2.08	.2342

DRAFT

TABLE 2

Effects of Gender on All Posttests after Adjustment for Entering Geometry Test

TESTS		MAIN EFFECTS					INTERACTIONS							
		N	Mean	SD	p-Value		N	Mean	SD	1	2	3	4	p-Value
AMERICAN TESTRONICS	UCSMP	190	12.99	0.42		1. (UC-Feml)	97	12.74	0.59	.	.532	.282	.108	
	Comparison	163	12.96	0.43	.9517	2. (UC-Male)	93	13.24	0.57	.532	.	.095	.320	
	Female	177	12.30	0.41		3. (C-Feml)	80	11.85	0.61	.282	.095	.	.010	
	Male	176	13.65	0.41	.0211	4. (C-Male)	83	14.07	0.60	.108	.320	.010	.	.1439
PART TWO	UCSMP	190	12.06	0.41		1. (UC-Feml)	97	12.59	0.58	.	.193	.000	.000	
	Comparison	163	8.84	0.44	.0001	2. (UC-Male)	93	11.51	0.59	.193	.	.002	.003	
	Female	177	10.67	0.43		3. (C-Feml)	80	8.76	0.63	.000	.002	.	.843	
	Male	176	10.22	0.43	.4565	4. (C-Male)	83	8.93	0.62	.000	.003	.843	.	.3008
PART ONE MULTI (A)	UCSMP	87	1.98	0.15		1. (UC-Feml)	44	2.52	0.21	.	.000	.048	.004	
	Comparison	72	1.76	0.17	.3046	2. (UC-Male)	43	1.44	0.21	.000	.	.193	.528	
	Female	75	2.20	0.16		3. (C-Feml)	31	1.87	0.25	.048	.193	.	.476	
	Male	84	1.53	0.15	.0036	4. (C-Male)	41	1.63	0.22	.004	.528	.476	.	.0604
PART ONE MULTI (B)	UCSMP	82	1.85	0.14		1. (UC-Feml)	40	2.13	0.20	.	.048	.205	.552	
	Comparison	74	1.85	0.15	.9779	2. (UC-Male)	42	1.57	0.19	.048	.	.496	.193	
	Female	78	1.94	0.14		3. (C-Feml)	38	1.76	0.20	.205	.496	.	.536	
	Male	78	1.76	0.14	.3566	4. (C-Male)	36	1.94	0.21	.552	.193	.536	.	.0699
PART ONE OPEN (A)	UCSMP	70	6.07	0.68		1. (UC-Feml)	34	8.03	0.97	.	.005	.407	.738	
	Comparison	62	7.19	0.73	.2601	2. (UC-Male)	36	4.11	0.94	.005	.	.063	.011	
	Female	61	7.42	0.73		3. (C-Feml)	27	6.81	1.09	.407	.063	.	.603	
	Male	71	5.84	0.67	.1138	4. (C-Male)	35	7.57	0.96	.738	.011	.603	.	.0201
PART ONE OPEN (B)	UCSMP	65	6.19	0.53		1. (UC-Feml)	31	8.03	0.77	.	.001	.000	.001	
	Comparison	58	3.90	0.56	.0037	2. (UC-Male)	34	4.35	0.74	.001	.	.365	.941	
	Female	59	5.69	0.56		3. (C-Feml)	28	3.36	0.81	.000	.365	.	.342	
	Male	64	4.59	0.54	.0961	4. (C-Male)	30	4.43	0.78	.001	.941	.342	.	.0027

TABLE 3

Effects of Gender on Subtests, Part-Two after Adjustment for Entering Geometry Test

		MAIN EFFECTS					INTERACTIONS							
		N	Mean	SD	p-Value		N	Mean	SD	1	2	3	4	p-Value
PROOF	UCSMP	190	4.51	0.18		1. (UC-Feml)	97	4.97	0.26	.				
	Comparison	163	3.25	0.20	.0001	2. (UC-Male)	93	4.04	0.26	.012	.			
	Female	177	4.10	0.19		3. (C-Feml)	80	3.24	0.28	.000	.037	.		
	Male	176	3.65	0.19	.0912	4. (C-Male)	83	3.25	0.28	.000	.038	.969	.	.0807
APPLI	UCSMP	190	2.01	0.11		1. (UC-Feml)	97	2.31	0.16	.				
	Comparison	163	1.30	0.22	.0001	2. (UC-Male)	93	1.71	0.16	.009	.			
	Female	177	1.82	0.12		3. (C-Feml)	80	1.33	0.17	.000	.107	.		
	Male	176	1.49	0.12	.0531	4. (C-Male)	83	1.28	0.17	.000	.068	.845	.	.0992
VISUAL	UCSMP	190	2.61	0.11		1. (UC-Feml)	97	2.78	0.15	.				
	Comparison	163	1.78	0.12	.0001	2. (UC-Male)	93	2.44	0.15	.110	.			
	Female	177	2.26	0.11		3. (C-Feml)	80	1.74	0.16	.000	.002	.		
	Male	176	2.13	0.11	.4081	4. (C-Male)	83	1.82	0.16	.000	.006	.724	.	.1787
TRANS	UCSMP	190	1.66	0.09		1. (UC-Feml)	97	1.69	0.12	.				
	Comparison	163	1.20	0.09	.0003	2. (UC-Male)	93	1.63	0.12	.744	.			
	Female	177	1.46	0.09		3. (C-Feml)	80	1.23	0.13	.010	.024	.		
	Male	176	1.41	0.09	.6914	4. (C-Male)	83	1.18	0.13	.004	.012	.812	.	.9621
COORD	UCSMP	190	0.95	0.07		1. (UC-Feml)	97	1.15	0.10	.				
	Comparison	163	0.68	0.08	.0081	2. (UC-Male)	93	0.75	0.10	.004	.			
	Female	177	0.90	0.07		3. (C-Feml)	80	0.64	0.11	.000	.433	.		
	Male	176	0.74	0.07	.3243	4. (C-Male)	83	0.72	0.11	.003	.838	.572	.	.0183

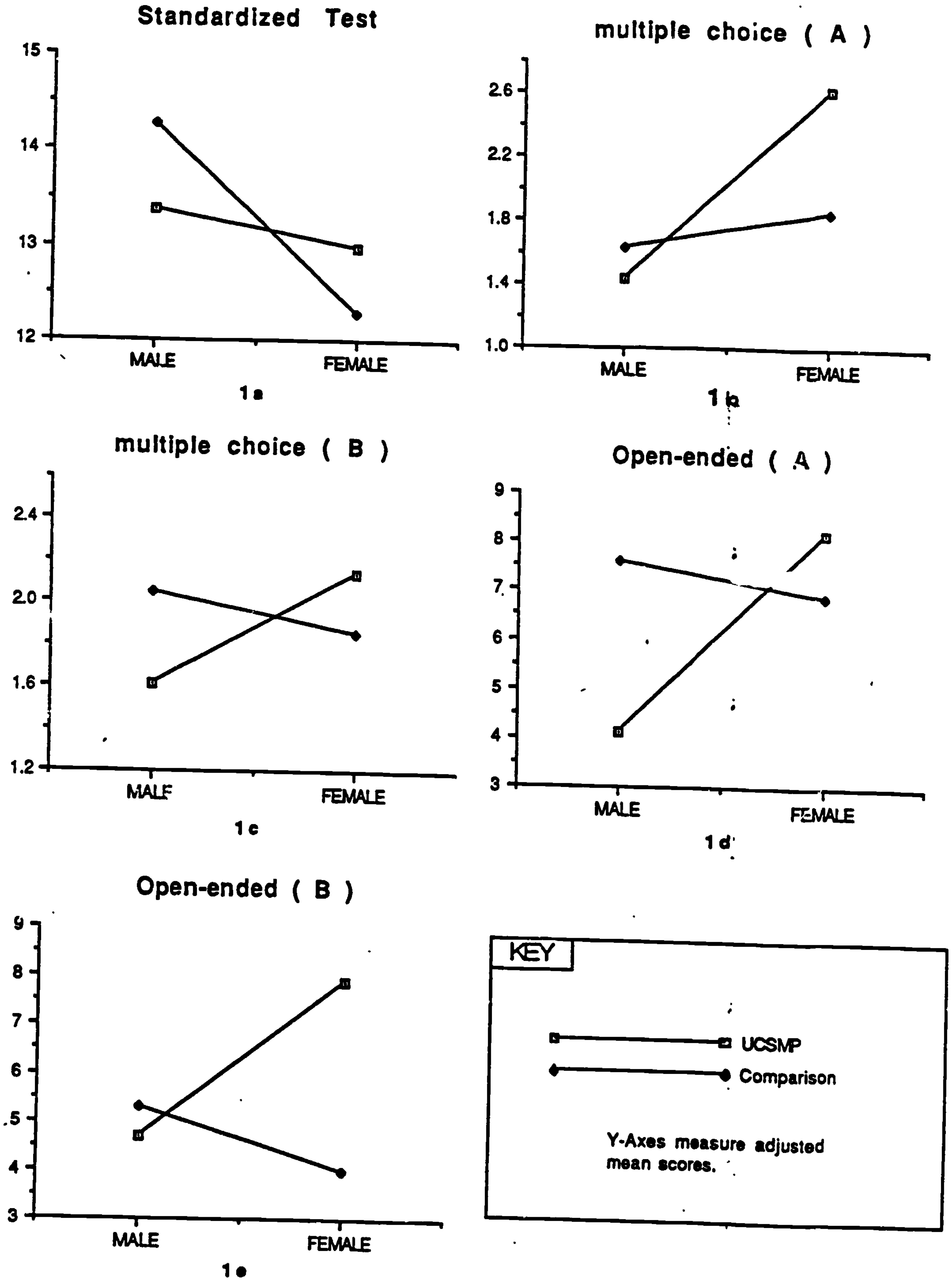
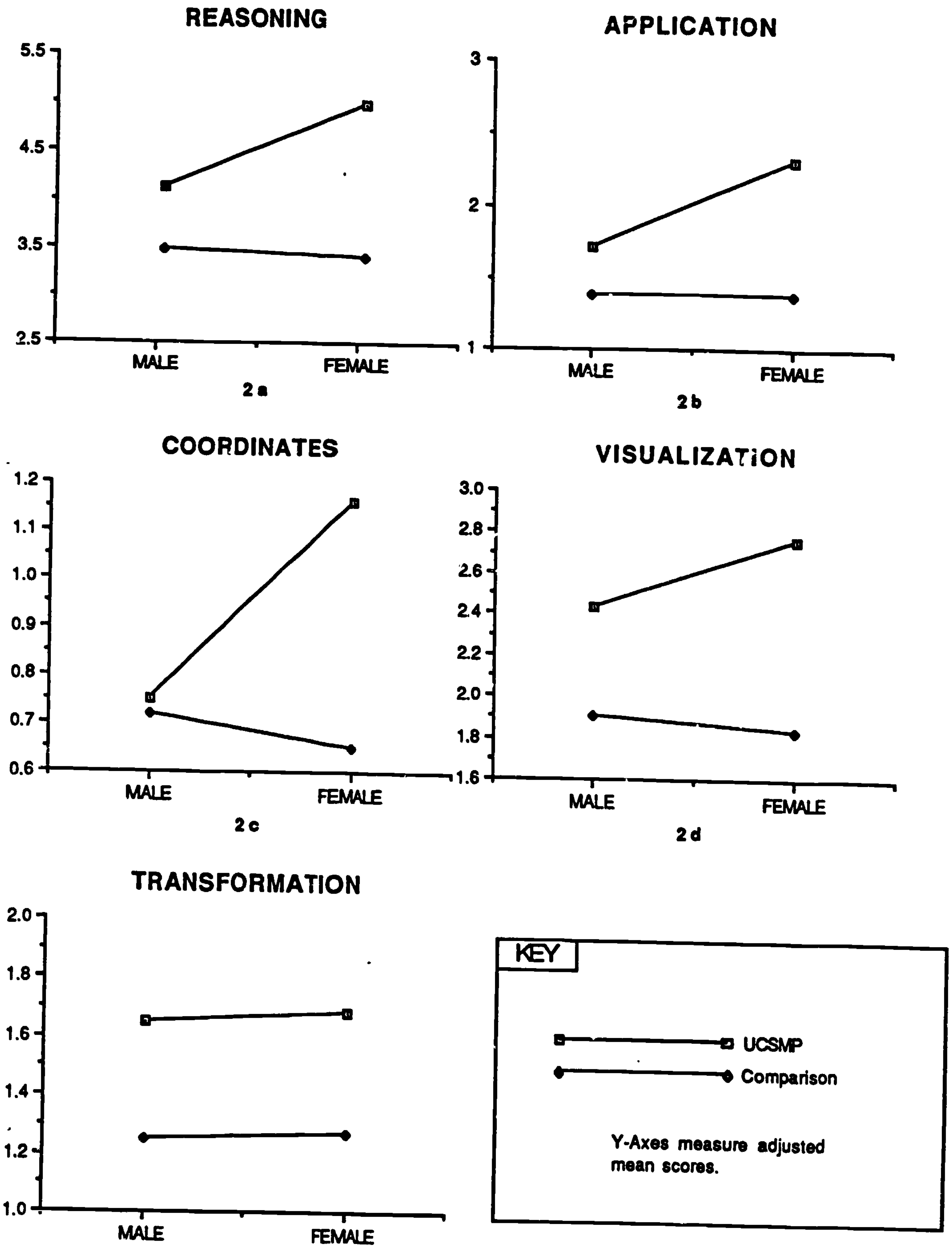
Standardized Test
15
14
13
12
MALE
FEMALE
1a
multiple choice (A)
2.6
2.2
1.8
1.4
1.0
MALE
FEMALE
1b
multiple choice (B)
2.4
2.0
1.6
1.2
MALE
FEMALE
1c
Open-ended (A)
9
8
7
6
5
4
3
MALE
FEMALE
1d
Open-ended (B)
9
8
7
6
5
4
3
MALE
FEMALE
1e
KEY
UCSMP
Comparison
Y-Axes measure adjusted mean scores.

Figure 1



2a

2b

2c

2d

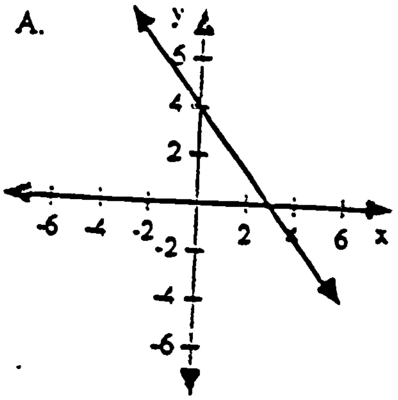
2e

Figure 2

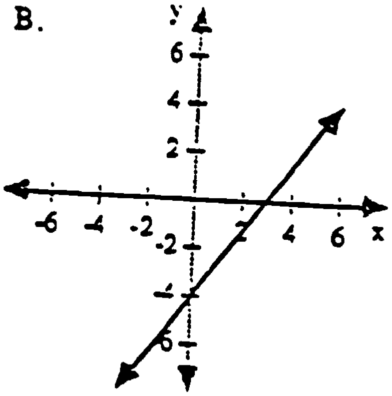
SAMPLE ITEMS ON THE CONTENT-SPECIFIC TEST
MULTIPLE CHOICE

____ 3. Which of the following is the graph of $y = 3x - 4$?

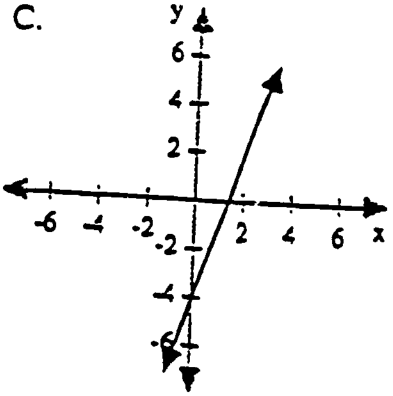
A.



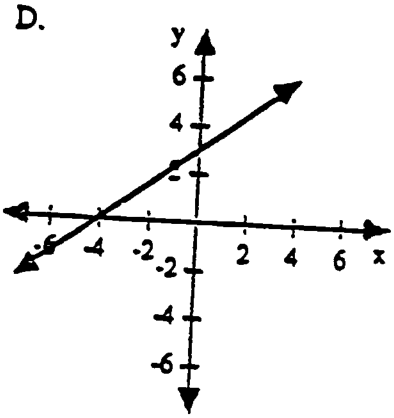
B.



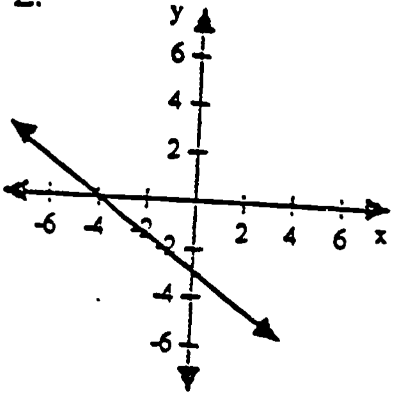
C.



D.



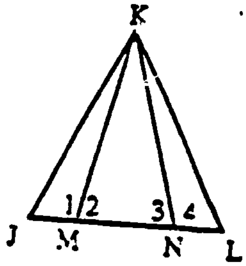
E.



To trisect an angle means to divide it into three parts of equal measure. In 1847, P.L. Wantzel *proved* that, in general, it is *impossible* to trisect angles using only a compass and an unmarked ruler. From his proof, what can you conclude?

A. In general, it is impossible to bisect angles using only a compass and an unmarked ruler.
B. In general, it is impossible to trisect angles using only a compass and a marked ruler.
C. In general, it is impossible to trisect angles using any drawing instruments.
D. It is still possible that in the future someone may find a general way to trisect angles using only a compass and an unmarked ruler.
E. No one will ever be able to find a general method for trisecting angles using only a compass and an unmarked ruler.

Given: ΔJKL at right with M and N between J and L; $\angle 2 \cong \angle 3$. Which statement *cannot* be concluded from the given information?



A. $\overline{KM} \cong \overline{KN}$
B. ΔKMN is isosceles.
C. $m\angle 1 + m\angle 2 = 180$
D. $\angle 1 \cong \angle 4$
E. $\angle J \cong \angle L$

The following statement is proved in most geometry books: "In an isosceles triangle, the base angles are congruent." What can be concluded from the proof?

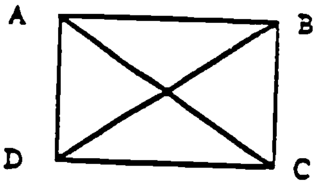
A. The statement holds for only one isosceles triangle.
B. The statement holds for most isosceles triangles, but we know it is not true for some isosceles triangles.
C. The statement holds for all isosceles triangles known today, but it is possible that later there may be some isosceles triangle discovered for which it does not hold.
The statement holds for all isosceles triangles.
E. none of the above

SAMPLE ITEMS ON THE CONTENT-SPECIFIC TEST
OPEN-ENDED

A rectangular field 50 yd wide and 100 yd long needs to be covered with sod. Each piece of sod is also rectangular — 1 ft wide and 3 ft long. How many pieces of sod are needed to cover the field? Show all work. Circle your answer.

A student had to prove on a test that the diagonals of a rectangle are congruent. Here is the student's "proof".

Given: ABCD is a rectangle.
Prove: $\overline{AC} \cong \overline{BD}$



1.	$\overline{AD} \cong \overline{BC}$; $\overline{AB} \cong \overline{DC}$	Opposite sides of a rectangle are congruent.
2.	$\angle ADC$ and $\angle ABC$ are right angles.	Definition of a rectangle
3.	$\angle ADC \cong \angle ABC$	All right angles are congruent.
4.	$\triangle ADC \cong \triangle CBA$	SAS Congruence
5.	$\overline{AC} \cong \overline{BD}$	Corresponding parts of congruent triangles are congruent.

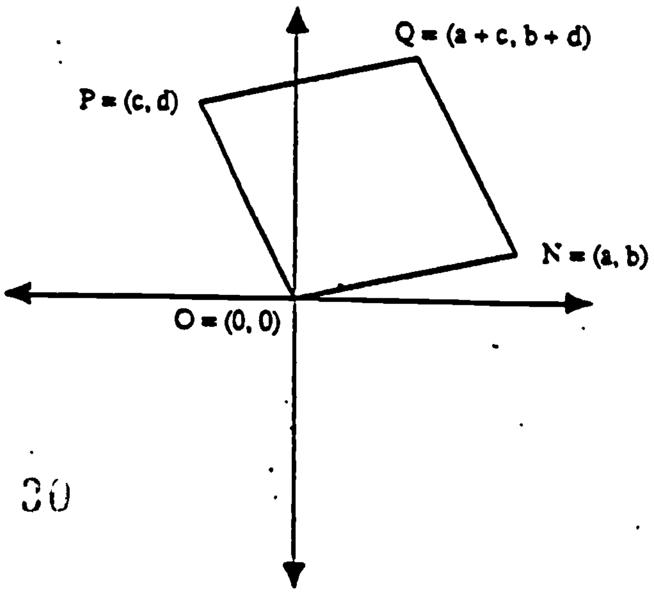
A. Which statement best describes your judgement of the student's solution? (Circle your response.)
(i) It is correct.
(ii) It is *not* correct.
(iii) I'm not sure whether it's correct or incorrect.

B. Why did you choose the response circled? That is, justify your answer to part A.

The terms "point", "line", and "plane" are usually taken as undefined terms in geometry. Explain (A) why this is done, and (B) whether it is necessary.

Given: Points N, O, P, and Q with coordinates as shown at right.

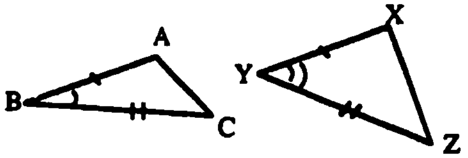
Prove: NOPQ is a parallelogram.



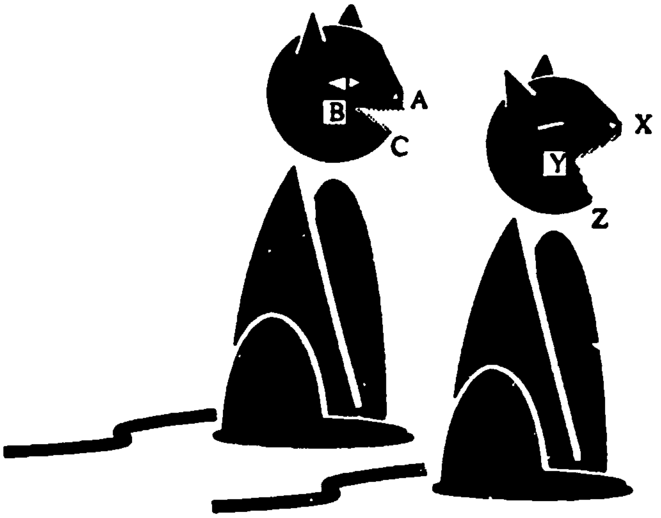
SAMPLE GEOMETRY LESSON
UCSMP TEXT

Lesson 7-8: The SAS Inequality

The SAS Congruence Theorem states that when two sides and the included angle of a triangle are congruent to corresponding parts of a second triangle, the triangles will be congruent. But what happens if the included angles are not congruent?



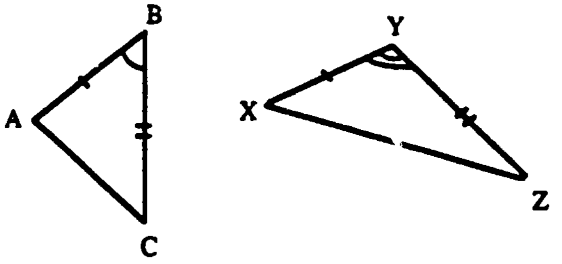
Percy, a sleepy Persian cat, is having a big yawn as shown below. As he starts his yawn, his mouth is not opened wide, but in the second picture his mouth is opened very wide. The geometric explanation for this everyday occurrence is as follows. The top and bottom of his jaw are the same in both pictures; thus AB = XY and BC = YZ. But since m∠XYZ is greater than m∠ABC, XZ > AC. This result generalizes as the *SAS Inequality*.



SAS Inequality Theorem:

If two sides of a triangle are congruent to two sides of a second triangle, and the measure of the included angle of the first triangle is less than the measure of the included angle of the second, then the third side of the first triangle is shorter than the third side of the second.

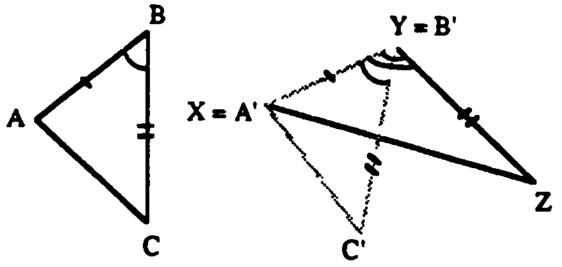
Proof A figure is drawn. Below it is stated the given and what to prove in terms of the figure.

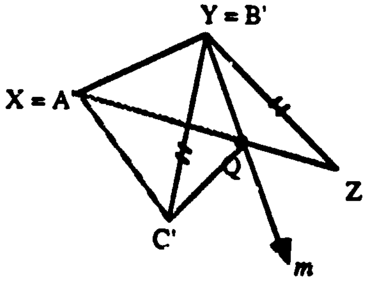


Given: AB = XY, BC = YZ, and $m\angle B < m\angle Y$.
Prove: AC < XZ

The method to use is not obvious. Since AB = XY, there is an isometry T with $T(\overline{AB}) = \overline{XY}$. $\Delta A'B'C'$ (in red) is $T(\Delta ABC)$. T is chosen so that C', the image of C, is on the same side of $\overline{XY}$ as Z. The result is shown below. Note that $\Delta C'YZ$ is isosceles since C'Y = ZY.

Since $m\angle A'B'C' < m\angle XYZ$, $\overrightarrow{YC'}$ lies in the interior of $\angle XYZ$. Below, *m*, the symmetry line of isosceles $\Delta C'YZ$, is drawn, intersecting $\overrightarrow{XZ}$ at Q. *m* is the ⊥ bisector of $\overline{C'Z}$, so Q is equidistant from C' and Z, making QC' = QZ.





Now focus on $\Delta A'C'Q$ (in blue). From the Triangle Inequality,

$$A'C' < A'Q + QC'$$

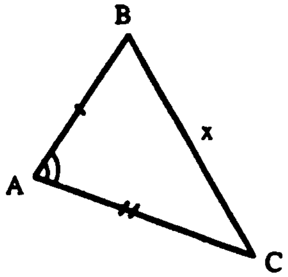
But A'C' = AC, A'Q is XQ and QC' = QZ. Substituting,

$$AC < XQ + QZ.$$

So $AC < XZ$ by the Betweenness Theorem.

Let us summarize what has been deduced about triangles given SAS. From the lengths AB and AC of two sides of a triangle, you can compute a range of possible lengths for the third side BC using the

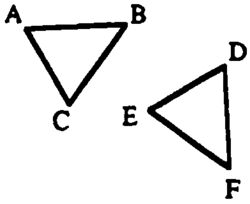
Triangle Inequality. The larger m∠A is, the larger BC is. If you know m∠A, the length of the third side is uniquely determined. This length can be found using trigonometry, a branch of mathematics that we introduce later in this course.



Questions

Covering the Reading

1. If AB = DF, AC = DE, and m∠A > m∠D in the figure below, then BC is (greater than, less than, equal to) EF.



2. State an application of the SAS Inequality Theorem.

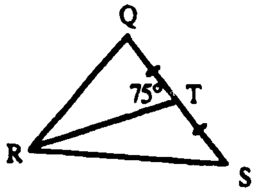
3. Multiple choice. Which of the following is *not* used in the proof of the SAS Inequality?
- **a.** Betweenness Theorem
- **b.** Isosceles Triangle Symmetry Theorem
- **c.** Isosceles Triangle Theorem

4. The Triangle Inequality is applied to which triangle in the proof of the SAS Inequality?

5. Suppose in ΔABC that AB = 6", BC = 3" and m∠B = 62.
- **a.** Is AC uniquely determined?
- **b.** What branch of mathematics studies the calculation of AC from this given information?

Applying the Mathematics

6. Use the figure below as marked. Explain why RS > QR.



7. a. Construct a circle with radius x. $\overline{\quad x \quad}$

   b. Construct a circle with radius y. $\overline{\qquad y \qquad}$

   c. What theorem justifies why you should widen your compass from part (a) to part (b)?

8. a. Draw ΔABC with $AB = 9$ cm, $BC = 6$ cm and $m\angle B = 40$.

   b. Draw ΔDEF with $DE = 9$ cm, $EF = 6$ cm and $m\angle E = 80$.

   c. Measure AC and DF.

   d. Which is longer?

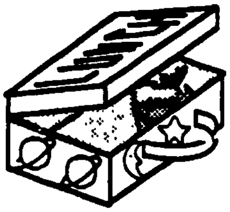
   e. Why?

In 9 and 10, suppose Percy's jaws BA and BC are of the same length, 8 cm.

9. What are the largest and smallest possible lengths for AC, the opening of his mouth?

10. What will be the length of $\overline{AC}$ when $m\angle ABC = 60$?

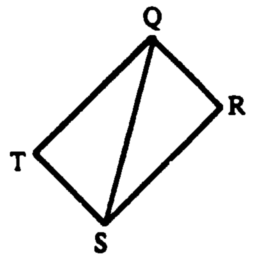
11. What theorem explains the fact that as a lunchbox is opened, the distance between the front of the top and the handle increases?



Review 12. Prove or produce a counterexample: If one angle of a quadrilateral is bisected by a diagonal and the angles not cut by the diagonal are congruent, then the quadrilateral is a kite. *(Lesson 7-7)*

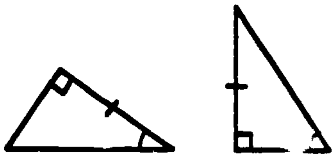
13. Given: $QT = RS$
$TS = QR$

Prove: a. $\Delta QTS \cong \Delta SRQ$
b. $\overline{QT} \,//\, \overline{RS}$ *(Lesson 7-6)*

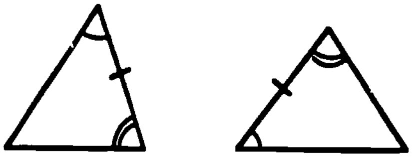


In 14 and 15, tell whether the triangles are congruent. If so, what triangle congruence theorem justifies the congruence. *(Lesson 7-2)*

14.



15.

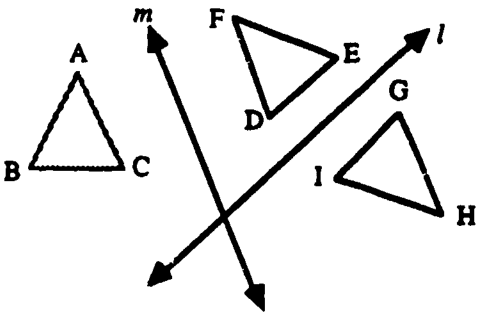


16. a. Draw a triangle with sides of length 3, 7, and 8 units.
b. Draw a triangle with sides of length 3, 5, and 7 units.
c. Measure the angles of these triangles to verify that two of the angles are congruent and two are supplementary.
(Lesson 7-1)

In 17 and 18, ΔGHI has been reflected over line *l*, and then its image has been reflected over line *m*.

17. ΔGHI ≅ _____ ≅ _____. *(Lesson 6-5)*

18. ΔABC is a (reflection, rotation, translation) image of ΔGHI.
(Lesson 6-3)



19. One side of a triangle is double the length of a second side. The third side is triple the length of the second side. Explain why this is impossible. *(Lesson 1-9)*

Exploration 20. The SAS Inequality is sometimes called the Hinge Theorem. Explain the reasoning behind this nickname.