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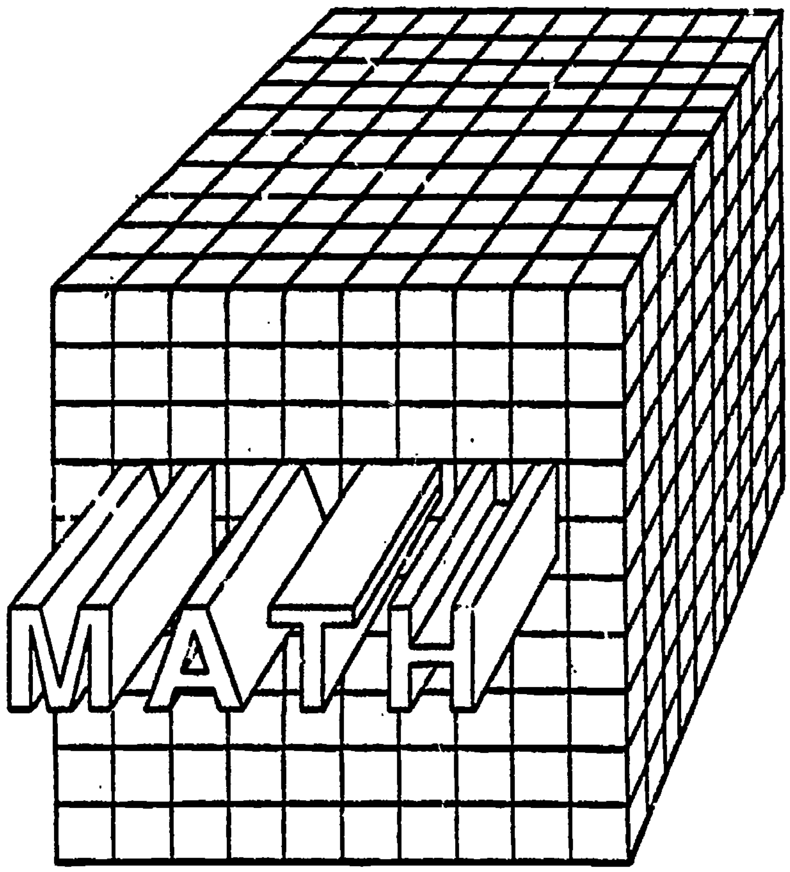
ABSTRACT

This document aims to assist teachers, curriculum specialists, and other educators in their endeavors to improve their K-12 mathematics education programs. This is a companion document to the "Michigan Essential Goals and Objectives for Mathematics Education." The document illustrates the integration of mathematical content with process skills. The s :: process skills are: conceptualization; mental arithmetic; estimation; computation; applications and problem solving; and calculators and computers. The major purpose of this interpretive document is to provide examples and specific information on the content strands, the mathematical processes, and specific objectives. Vocabulary, comments, and examples for each objective are presented at different grade levels, such as K-3, 4-6, or 7-9. The mathematical content strands are: (1) whole numbers and numeration; (2) fractions, decimals, ratio, and percent; (3) measurement; (4) geometry; (5) statistics and probability; (6) algebraic ideas; (7) problem solving and logical reasoning; and (8) calculators. (YP)

AN INTERPRETATION OF :

THE MICHIGAN ESSENTIAL GOALS AND OBJECTIVES FOR MATHEMATICS EDUCATION



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MICHIGAN STATE BOARD OF EDUCATION

MICHIGAN STATE BOARD OF EDUCATION

AN INTERPRETATION OF:

THE MICHIGAN ESSENTIAL GOALS AND OBJECTIVES

FOR MATHEMATICS EDUCATION

The Michigan Department of Education is pleased to present this document entitled "An Interpretation Of: The Michigan Essential Goals and Objectives For Mathematics Education." The purpose of this document is to assist Michigan teachers, curriculum specialists, and other educators in their endeavors to improve their K-12 mathematics education programs. This document is a companion document to the Michigan Essential Goals and Objectives for Mathematics Education, published in 1988.

The Michigan Department of Education functions as a partner with local and intermediate school districts to help them meet the future educational needs of our children and our society. This document was developed in cooperation with The Michigan Council of Teachers of Mathematics through the input of many educators from throughout the state.

The Michigan Essential Goals and Objectives for Mathematics Education and this companion document provide program guidelines for kindergarten through grade nine. It is hoped that they will help elementary and middle school staff shape an active mathematics curriculum that prepares students for the challenges of secondary school and beyond. The building blocks of an academic foundation are in place in many of our elementary schools. Yet, so important is the foundation that we must make every effort to ensure that, in every school and for every child, the elementary and middle school curriculum is the best that we can provide.

The content of this document is to provide a better mathematics curriculum for all students in grades K-9. I know that all school districts are committed to this goal and hope this document serves as a valuable asset to districts as they examine the relevance and adequacy of their mathematics curriculum.

Donald L. Bemis

Donald L. Bemis

March, 1989

ACKNOWLEDGMENTS

This document was prepared by the Michigan Council of Teachers of Mathematics through a grant funded by the Michigan Department of Education. An Interpretation Of: The Michigan Essential Goals and Objectives for Mathematics Education was written to assist educators in the establishment, implementation and evaluation of mathematics programs. The document is organized by the eight content strands of the Michigan Mathematics Framework. Each of the eight chapters includes an overview and a complete listing of the objectives, with examples and instructional comments. The objectives are identified with the codes used in annual reports of the Michigan Educatinal Assessment Program. The sections in this document were prepared by varied groups of people rich with mathematics education expertise. Different styles and examples in the content sections illustrate different writing styles of the various authors and the special needs of the content itself.

Many people, dedicated to our students' understanding and ability to use mathematics, have contributed to this effort. The Executive Board of the Michigan Council of Teachers of Mathematics and its president, Christian Hirsch, initiated the project. The Michigan Department of Education's Assessment Unit and its supervisor, Edward Roeber, funded the development of this work. Charles Allan, Mathematics Specialist, Michigan Department of Education, took responsibility for the final form of the document. The following people were responsible for writing this Interpretive Booklet.

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March, 1989

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PREFACE

The Michigan Essential Goals and Objectives For Mathematics Education* is designed to assist administrators and teachers in planning, developing and implementing grades K-9 mathematics programs and to provide some guidelines for grades 10-12 instruction. They provide a philosophical foundation and curricular framework from which educators may construct a comprehensive local program to meet the instructional needs of their students.

That document illustrates the integration of mathematical content with process skills. In addition, it upgrades the bench mark expectations for achievement to be commensurate with the demands of a technological society.

Introduction

Quantitative thought and understanding form the keystone for the new mathematics objectives for Michigan. Support for this comes from a National Science Foundation report which stated that "...quantitative thought and understanding continue to become more important for more people..."

This keystone is used in setting a new direction for curriculum and teaching that will help students learn to think, reason, solve problems and apply mathematics in real life situations. The objectives reflect mathematical knowledge that is essential for all students' educational development and employment. Rote skills learned in a meaningless way do not prepare students for the future.

The view of computation is broadened to include basic facts, mental arithmetic, estimation and calculators. With the availability of calculators and computers in the workplace, the emphasis must be on conceptual development and problem solving. The use of calculators and computers to achieve the important mathematics objectives is suggested in Educating Americans for the 21st Century.

In its Agenda for Action: Recommendations for School Mathematics of the 1980s, the National Council of Teachers of Mathematics recommended that problem solving be the focus of school mathematics, that basic skills be defined to encompass more than computational facility and that mathematics programs take full advantage of the power of calculators and computers at all grade levels.

A later report from the National Science Foundation, The Mathematical Sciences Curriculum K-12: What is Still Fundamental and What is Not, in recommendations for elementary and middle school mathematics, stated "A principal theme of K-8 mathematics should be the development of number sense, including the effective use and understanding of numbers in applications as well in other mathematical contexts."

* Pages iv-vi and viii are taken from the March, 1988 publication of the Michigan State Board of Education which contained the objectives themselves.

The changes suggested in the NSF report would "replace excess drill in formal paper-and-pencil computations with various procedures to develop better number sense on the part of the student."

Substantially less emphasis was to be placed on paper and pencil execution of the arithmetic operations and de-emphasis was suggested for drill with larger numbers and fractions with larger denominators.

The following content recommendations were included in the NSF report:

1. Mastery of basic number facts
2. Selective use of calculators and computers
3. Mental arithmetic, estimation and approximation
4. Problem solving, including the use of calculators and computers as tools
5. Elementary data analysis, statistics and probability
6. Place value, decimals, percent and scientific notation
7. Intuitive geometry, including formulas for perimeter, area and volume
8. Algebraic symbolism and techniques, particularly in grades seven and eight.

Over the past two decades there has been an explosion of research on mathematics learning and teaching. Syntheses of some of the research can be found in companion publications, Research within Reach: Elementary School Mathematics and Research within Reach: Secondary School Mathematics. Among the major national data sources on mathematics learning and mathematics teaching are those reported in the National Assessment of Education Progress and the Second International Mathematics Study.

There is unanimity from all reports and research results that conceptualization of mathematics and understanding of problems should be valued more highly than just correct solutions to routine exercises. Yet there is extensive documentation that students are now lacking both conceptual understanding and problem solving skill. For example, only half of the students finishing eighth grade could give the meaning of the decimal 0.52 and only 20% of them could estimate the answer to 3.04 x 5.3 as 16 given the choices of 1.6, 16, 160 and 1600. For too long, mathematics instruction has been thought of as presenting rules and providing enough drill for students to master the skill. As Simon pointed out, to survive in mathematics courses, many students attempt to compensate for their lack of understanding by memorizing mathematical procedures and formulas.

The mathematics objectives have been written to a framework of eight content strands and six process strands. The content strands are: l) whole numbers and numeration, 2) fractions, decimals, ratio and percent, 3) measurement, 4) geometry, 5) statistics and probability, 6) algebraic ideas, 7) problem solving and logical reasoning, and 8) calculators. The process strands are: l) conceptualization, 2) mental arithmetic, 3) estimation, 4) computation, 5) applications and problem solving, and 6) calculators and computers. Many of the objectives from previous documents have been retained but enhanced with a strong focus on mathematical thinking and higher order skills. The chart that follows illustrates the relational nature of the content strands to the process strands and constitutes the framework of the mathematics objectives.

Framework:
Michigan Mathematics Objectives

Mathematical Processes

Mathematical Content Strands

	Conceptualization	Mental Arithmetic	Estimation	Computation	Applications and Problem Solving	Calculators and Computers
Whole Numbers and Numeration						
Fractions, Decimals Ratio and Percent						
Measurement						
Geometry						
Statistics and Probability						
Algebraic Ideas						
Problem Solving & Logical Reasoning						
Calculators						

Objective Coding

Each of the objectives in this interpretive document is coded to identify the content strand, major objective, and process strand with which it is associated. A four component code is used.

1. The first component is a letter to identify the content strand of the objective. The following abbreviations are used:

Whole Numbers and Numeration	W
Fractions, Decimals, Ratio and Percent	F
Measurement	M
Geometry	G
Statistics and Probability	S
Algebraic Ideas	A
Problem Solving and Logical Reasoning	P
Calculators	C

   Example: In the code F2Cn1--F refers to the Fractions, Decimals, Ratio and Percent strand.

2. The second component is the major objective under which a particular objective can be found. The major objectives are simply referred to in numerical order.

   Example: In the code F2Cn1--2 refers to the second major objective in the Fractions, Decimals, Ratio and Percent strand.

3. The third component identifies the process of the objective. The following abbreviations are used:

Conceptualization	Cn
Mental Arithmetic	MA
Estimation	Es
Computation	Cm
Applications and Problem Solving	PS
Calculators and Computers	Ca

   Example: In the code F2Cn1--Cn identifies the objective as a conceptual item.

4. The fourth component indicates the number of the objective.

   Example: In the code F2Cn1--the 1 indicates that this is the first objective in the conceptual section of the second major objective of the Fractions, Decimals, Ratio and Percent strand.

New Directions

The following seven topics illustrate the thrust established by the broadened framework of content strands and mathematical processes.

1. Conceptualization of mathematical topics receives much greater emphasis. Conceptual understandings are essential for all content strands, are the basis for the teaching of computational topics and are essential for success in problem solving. The increased emphasis on quantitative thinking and conceptual knowledge requires a concomitant increase in the use of manipulative materials at all grade levels, K-12.

2. Problem solving permeates all content strands. Because of their central purpose in mathematics and in practical situations, problem solving and logical through are viewed as threads that run through all content areas. Beyond the use of problem solving in the other content areas, there are problem solving strategies in a separate strand that can help students be better problem solvers.

3. Graphical representation and interpretation are strengthened. The topics are important in mathematics and just as important in social science and science. The importance of the topic for the ordinary citizen is further underscored by the extensive use of graphs to present quantitative information in newspapers and magazines.

4. The needs of students in a technological society are reflected. The objectives take into account the major shift in the use of calculators and computers to do routine computations by the ordinary citizen and by people in business and industry. Paper-pencil computation remains as an expectation but with limits on the expectations. The objectives reflect the value of the calculator as a teaching tool and as a tool for students in solving problems beginning in the early grades. Capstone objectives for grades 4, 7, and 10 specify objectives which should be met without a calculator and those for which a student may use a calculator.

5. Mental arithmetic and estimation receive more attention and are given greater importance. One major reason for this is at least three-fourths of the everyday use of mathematics is without paper and pencil. Success depends upon sound concepts of numbers and continuing emphasis on thinking skills.

6. Spatial visualization and geometry are broadened. This shift in emphasis in the content reflects the needs of the ordinary citizen as she/he lives in a three-dimensional world, as well as the needs in later mathematics courses. The increased attention to conceptualization requires more concrete models with more emphasis on spatial and quantitative visualization.

7. Algebraic concepts and symbolism are introduced earlier. A major reason for this is the economy of algebraic symbolism to express concisely a generalization from arithmetic. For example, algebraic symbolism is essential in expressing the formulas for perimeter, area and volume. Further, with content from the first year of algebra becoming increasingly essential for future career choices of all students, a better transition from arithmetic to algebra is essential. Earlier use of algebraic symbolism in a meaningful way will help much in the transition from arithmetic to the more formal work in first year algebra.

Framework:
Michigan Mathematics Objectives

		Mathematical Processes					
		Conceptualization	Mental Arithmetic	Estimation	Computation	Applications and Problem Solving	Calculators and Computers
Mathematical Content Strands	Whole Numbers and Numeration						
	Fractions, Decimals Ratio and Percent						
	Measurement						
	Geometry						
	Statistics and Probability						
	Algebraic Ideas						
	Problem Solving & Logical Reasoning						
	Calculators						

OVERVIEW

The 1988 publication of The Michigan Department of Education, Michigan Essential Goals and Objectives for Mathematics Education, contained a general overview of changes and new directions for mathematics education in Michigan, as well as statements of objectives for each content strand. That document did not contain an overview of each content strand nor specific examples to illustrate the meaning of the objectives.

The major purpose of this interpretive document is to provide the needed examples and specific information on the content strands, the mathematical processes, and specific objectives. The hope is that teachers, other school personnel, college instructors and the general public will understand better the nature of the objectives and the rationale for choosing them.

THE OBJECTIVES—CONTENT AND PROCESS

The mathematics objectives were written using eight content strands:

1. Whole Numbers and Numeration
2. Fractions, Decimals, Ratio and Percent
3. Measurement
4. Geometry
5. Statistics and Probability
6. Algebraic Ideas
7. Problem Solving and Logical Reasoning
8. Calculators

For each content strand, major objectives are given to provide a general overview of the content and these are followed by more specific objectives.

The specific objectives, identified by grade level K-3, 4-6, or 7-9 are organized by process, giving added information about the type of knowledge reflected in the objectives. The six processes are:

1. Conceptualization
2. Mental Arithmetic
3. Estimation
4. Computation
5. Applications and Problem Solving
6. Calculators and Computers

This chart illustrates both the content and processes that constitute the framework of the mathematics objectives.

Framework:
Michigan Mathematics Objectives

Mathematical Processes

Mathematical Content Strands	Conceptualization	Mental Arithmetic	Estimation	Computation	Applications and Problem Solving	Calculators and Computers
Whole Numbers and Numeration						
Fractions, Decimals Ratio and Percent						
Measurement						
Geometry						
Statistics and Probability						
Algebraic Ideas						
Problem Solving & Logical Reasoning						
Calculators						

CONTENT STRANDS AND MATHEMATICAL PROCESSES

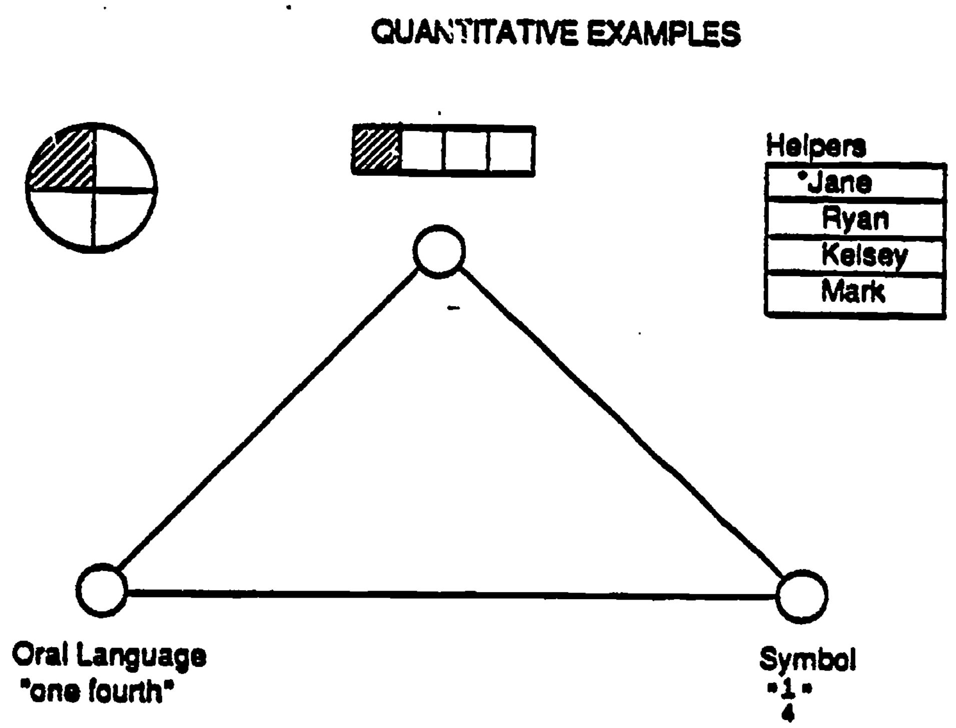
This booklet has been organized by eight content strands. For each content strand, an overview presents the content of the strand, the way the content is developed, reasons for teaching the objectives, and suggestions for teaching. Comments and specific examples are included to provide further clarification of the intent of the objectives. Each objective in the first six strands is classified according to the six mathematical processes. The objectives for the content strands, Problem Solving and Logical Reasoning, and Calculators do not have a mathematical process classification.

The meaning of each of the six mathematical processes can be inferred from examination of the comments and examples. What follows is a general description of the major features of each.

CONCEPTUALIZATION

Conceptual knowledge is rich in relationships with new knowledge connected in many ways to prior knowledge. It is knowledge that draws upon many experiences in school and out of school. It includes intuitions, perceptual information and invented strategies. It is knowledge that the learner understands. It is knowledge acquired by knowing both the how and the why. It is knowledge that is not overly compartmentalized but connected in various ways.

As an example, consider the concept of the fraction one-fourth. The concept contains "part of a cookie, not the whole cookie" acquired by a two-year-old. It contains "equal parts", essential knowledge acquired by four friends of age six who know that it is "only fair to share the cookie equally." It contains examples in the varied experiences of children involving pieces of fruit, other foods such as pizza and sheets of paper marked in four equal parts for a poster. It contains within it the knowledge of the whole number 4, showing the number of equal parts. Finally, the knowledge is communicated by the oral language "one fourth" and the written symbol "$\frac{1}{4}$" at age 7 or 8. The major connections among the various quantitative ideas, the written symbol and the oral name can be represented in a diagram such as this:



The concept of one-fourth is limited initially by a continuous object. Many years later, examples extend the concept to include discrete sets such as $\frac{1}{4}$ of a dozen eggs, distance such as $\frac{1}{4}$ inch and the result of dividing 1 by 4.

In summary, conceptualization of the fraction is a rich quantitative background with many connections to the experiences of the learner developed over a relatively long period of time.

Conceptual Objectives in the Content Strands: Each of the content strands has major objectives that are conceptual with expectations that the concept is to be connected with appropriate oral language and written symbols.

Whole Numbers and Numeration objectives include concept of number, the meaning of the four operations, addition, subtraction, multiplication and division and relations between place value models and algorithms.

The *Fraction, Decimal, Ratio and Percent* objectives include a quantitative understanding of the numbers, of equivalent fractions and of equivalent decimals. Also included as conceptual objectives are the meanings of the operations on fractions, decimals and percent and the relations among fractions, decimals and percent.

The *Measurement* strand is heavily conceptual with emphasis on the meaning of length, area, volume, perimeter, surface area, mass, liquid capacity, time, temperature and money. The concept of the measurement process is used to develop the conceptual objectives for the metric system.

The objectives in the *Geometry* strand are primarily conceptual with almost all other objectives classified as problem solving. Conceptual objectives include shapes and properties of geometric figures, relations among geometric objects, specifying positions using locations and distance, geometric transformations, and visualizing-sketching-constructing. Special attention is needed in relating geometric vocabulary to these spatial concepts.

Conceptual objectives for *Statistics and Probability* include reading and interpreting tables and graphs, the meaning of descriptive statistics such as mean, median, outlier and quartile, and the meaning of probability, likelihood, certainty, and impossibility.

The objectives for *Algebraic Ideas* are primarily conceptual with emphasis on the meaning of variables, models for relations and open sentences as well as manipulative and pictorial models for the distributive property. Number concepts listed in the algebraic ideas strand include integers, exponents and graphs of powers and roots. Function concept includes tables of values, graphs and expressing relationships between two sets.

Conceptual knowledge is implicit in the *Problem Solving and Logical Reasoning* content strand and in all its sub-sections: Patterns, Understanding Problems, Problem Solving Strategies, Evaluating Solutions and Logical Reasoning. Understanding the mathematical ideas used in the problems always is prerequisite knowledge for solving the problems.

The *Calculators* strand includes the recognition of keys and special features, the use of the calculator to compute and limitations. Conceptual knowledge from the other content strands is essential in deciding how to use the calculator and which buttons to press. For example, to find the decimal for $\frac{4}{17}$, students must understand that $\frac{4}{17}$ can be interpreted as 4 ÷ 17 to know how to keystroke the calculator, $\boxed{4}$ $\boxed{\div}$ $\boxed{1}$ $\boxed{7}$ $\boxed{=}$. Understanding of decimals is needed to interpret the display of 0.2352941 and to round the result to the desired degree of precision.

Effective teaching of conceptual knowledge can be characterized as follows:

EFFECTIVE TEACHING OF CONCEPTS

DOES	DOES NOT
.. include real life experiences	.. use only textbook examples
.. use many types of quantitative examples	.. use a single example
.. use manipulative materials, concrete models and diagrams	.. rely solely on oral descriptions or symbolic explanations
.. give attention to oral names and symbols concepts, oral language and	.. assume students relate symbols
.. allow sufficient time for careful development	.. hurry development
.. ask many questions	.. over-explain
.. provide time for thinking	.. hurry the thinking process
.. relate concepts to prior knowledge	.. use isolated instances
.. value understanding	.. stress rote procedures
.. assess understanding as an integral part of evaluation	.. assess only computation skill and vocabulary

PROCEDURAL KNOWLEDGE

Somewhat in contrast to conceptual knowledge is procedural knowledge. In school mathematics, most procedural knowledge consists of a linear set of steps on how to manipulate written symbols. It is knowledge characterized by rules or algorithms. Procedural knowledge characterizes the computation objectives in the Michigan Essential Goals and Objectives for Mathematics Education. Included are computational objectives for operations on whole numbers, fractions, decimals and integers. Computation objectives for percent are reserved for calculators. In the computation objectives, limitations are placed on the size and type of numbers so that the computation procedures can be limited, allowing the needed time for teaching concepts and a broader range of mathematical content.

Both conceptual knowledge and procedural knowledge are important in mathematics. Further, the two types of knowledge need to be related in a more substantive way. Conceptual objectives that relate models and algorithms are intended to show this relationship between the two types of knowledge. One of the difficulties with procedures is that they can be learned in isolation from other things students know. To minimize such isolated learning, conceptual objectives are included that relate algorithms to concrete models.

If procedures for computation are understood, students are more likely to be able to perform related tasks that have not been taught. Research many years ago showed that if students understand two-digit subtraction, they are more likely to be able to do three-digit subtraction. Recent research has shown that teaching kindergarten and first grade children to count by tens meaningfully using bundles of sticks makes them much more successful with practical word problems with two-digit numbers. An abundance of research on fractions, decimals and percent shows the value of understanding the quantitative nature of the numbers as the basis for computational understanding, success with computation, and retention over a longer period of time.

Beyond the ability the knowledge provides for new tasks, understanding helps in deciding which algorithm goes with which problem. It is not uncommon to see second or third grade students regrouping in addition or subtraction when it is not required, sixth through eighth grade students using reciprocals when multiplying fractions and cross-multiplying at inappropriate times. While understanding regrouping and fraction algorithms will not remedy all errors, students are much more likely to detect their own, to remedy faulty procedures themselves and to apply algorithms to given exercises correctly.

Effective teaching of procedures can be characterized as follows:

EFFECTIVE TEACHING OF PROCEDURES FOR COMPUTATION

DOES	DOES NOT
.. relate ways to think about basic facts (fact strategies) to concepts of operations	.. approach fact learning purely as memorization
.. value thinking about facts and algorithms	.. rely solely on teacher or textbook procedures
.. connect algorithms to concepts of numbers and operations	.. begin with step-by-step procedures
.. take the necessary time to relate concepts and algorithms	.. give a single example and then move to the algorithm
.. help students see why each major step is taken	.. give students only a rule
.. spend time building conceptual understanding followed by short but frequent amounts of computational drills	.. spend as much time practicing memorized procedures for algorithms

The overall goals are to build sound quantitative concepts and use them to build procedures for computation that are based in conceptual knowledge. Conceptual and procedural knowledge interact with each other and each makes a distinctive contribution to a person's total mathematical knowledge.

MENTAL ARITHMETIC

Mental arithmetic is the process of finding exact answers mentally without the aid of paper and pencil or calculator. Problems can be presented orally or in writing with pencils used only to write the answer. Answers may also be given orally.

Concepts of numbers and operations are essential components of mental arithmetic. For example, to find 27 + 45 mentally, one needs to see 2 and 4 as tens. To find $3 - \frac{1}{4}$ mentally instead of with a cumbersome written algorithm, one needs concepts of fractions, wholes and subtraction. With these prerequisites, it is easy to see $\frac{1}{4}$ removed from one whole leaving $2\frac{3}{4}$.

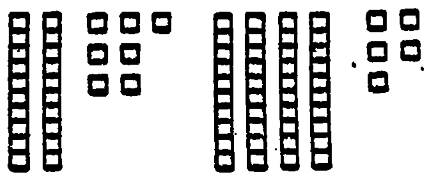
Mental arithmetic also requires procedures, but usually relatively simple ones. To find the answer for 27 + 45, basic facts and the concept of place value are prerequisites. To find 3 X 98 mentally as 3 X 100 - 3 X 2, three computation procedures are needed as well as the concept of place value.

Many mental arithmetic examples can be illustrated with models to help students see a variety of procedures for finding the answer. Using base ten blocks or a sketch as shown, 27 + 45 could be found by thinking:

a. 20 + 40 = 60; 60 + 7 = 67; 67 + 5 = 72

b. 27 + 40 = 67; 67 + 5 = 72

c. 20 + 40 = 60; 7 + 5 = 12; 60 + 12 = 72



In teaching mental arithmetic, the following suggestions should be kept in mind:

1. Encourage alternate ways to think. While the teacher should provide examples of the way he/she thinks, students will suggest other correct and very usable procedures.

2. There is no one way to find a correct answer. In a given class, it is expected that more than one method will be used by students to find correct answers.

3. To avoid having students reach for pencils to do the mental computation, use a time limit. Examples can be given orally or displayed on the overhead projector or chalkboard for a limited period of time.

Michigan objectives include mental arithmetic for operations on whole numbers, fractions, decimals and percent. There are mental arithmetic objectives for selected fractions related to equivalent fractions and for fraction-decimal-percent equivalents. In all cases for whole numbers and for fractions, the size and type of numbers are limited in recognition that mental arithmetic should be kept relatively simple and easy for all students to learn.

Mental arithmetic is important to teach because it is very practical, it facilitates students' subsequent learning, and it is an essential prerequisite to estimating computation.

ESTIMATION

Estimation is closely related to mental arithmetic yet it does have distinct characteristics. Estimation is similar to mental arithmetic in its requirement for sound concepts of numbers, place value, and concepts of the operations. Both also require skill with powers of ten. Estimation goes further by including not only estimates related to computation but estimates of number and measures. Estimation can be done orally or with written examples. For example, estimating the product of 235 X 62 as about 200 X 60 could be done orally or in written form but in either case a time constraint is essential. Estimating the part of a region that is shaded can be done in written form because there is no computation procedure to follow.

In estimation, students and teachers need to develop a tolerance for error because the acceptance or rejection of an estimate depends on its practical use. Different estimates could all be correct and there is not likely to be a single correct answer. A range of acceptable answers suggests a flexible set of procedures and decision making rules.

Mental arithmetic and estimation comprise over $\frac{3}{4}$ of the practical uses of arithmetic. The increased attention to these objectives reflects the resurgence in their importance in everyday consumer affairs, money management and problem solving.

Mental arithmetic, estimation, paper/pencil computation and calculators are interrelated. With proper balance among these, less time is needed for page-by-page textbook practice on computation. Much of the whole number computation that still dominates the middle and upper grades can be deleted and supplanted with more important and more interesting mathematics topics.

COMPUTATION

The computation objectives are considered to be paper/pencil objectives primarily because of stating clearly what is to be done by hand. Viewed more broadly, computation could be considered as mental arithmetic, computation done with calculators, estimation as well as paper/pencil algorithms.

Paper/pencil computation objectives are limited severely. For example, multiplication of whole numbers is limited to multiplying by two-digit numbers. Division is limited to two-digit divisors to 30 or multiples of ten. *There are no whole number computation objectives for grades 7-9.* Fraction and decimal computation is restricted to examples that are easy to illustrate with concrete models. In percent, all computation is reserved for calculators with major attention given to concepts, mental arithmetic and estimation.

The decreased requirements for paper/pencil computation in today's world allows the instructional time needed for teaching conceptualization and a broader range of mathematical topics.

APPLICATIONS AND PROBLEM SOLVING

Problem solving is listed both as a content strand and as a process strand. As a content strand, *Problem Solving and Logical Reasoning* contains objectives more closely related to problem solving strategies. *Applications and Problem Solving* process objectives are written for the other content strands. For example, there are *Applications and Problem Solving* objectives on when to use addition, subtraction, multiplication or division with whole numbers, fractions or decimals and in solving problems with percent. There are *Applications and Problem Solving* objectives for measurement, geometry, statistics and probability, and algebraic ideas. The use of *Applications and Problem Solving* in all these content strands and the content strand on *Problem Solving and Logical Reasoning*, illustrate the central role of problem solving in the mathematics classroom.

CALCULATORS/COMPUTERS

The major attention in this process is given to calculators because a separate document, (Michigan) Essential Goals and Objectives in Computer Education has been prepared for computers. There is also a content strand for calculators containing objectives about the functions and use of a calculator. The objectives in the content strand are useful in seeing how a calculator can be used in instruction, for example how to use a calculator to skip count for multiplication.

The process objectives include computations that are too complex to be done accurately and quickly with paper and pencil. Problem Solving objectives that require computation assume that calculators are available for use if desired by the student.

In teaching computation, mental arithmetic, estimation and the use of the calculator, it is essential that students be helped to see when each is appropriate to be used.

SUMMARY

The new directions set in the Michigan Essential Goals and Objectives For Mathematics Education are exciting and invigorating. Substantially less time will be needed for paper/pencil whole number computation, complex fraction and decimal computation, and percent computation with the integral use of calculators in instruction and in testing. Excessive drill for skill that often kills interest can give way to smaller amounts of practice over a longer period of time. Instructional time will consequently be gained for other, more important and more interesting objectives.

Mental arithmetic and estimation are elevated to their rightful place in the heirarchy of objectives. Problem solving, logical reasoning and applications become a major focus for mathematics in school. More careful attention and greater emphasis on conceptualization provides a sounder foundation for all of mathematics. A broader scope of topics to include major amounts of geometry, measurement, probability, statistics and algebraic ideas will provide our students with an expanded view of mathematics and its use.

The overriding goal is to provide a mathematics program truly fitting for our citizens of the 21st century. Our industry and our practical needs demand it. Insightful, stimulating, and effective teaching is required. A new kind of instructional material is essential with increased use of manipulatives, more hands-on activities and greater use of everyday examples. Nothing less than the best effort from all of us on this exciting venture can be accepted. It will be to the benefit of us all.

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Framework:
Michigan Mathematics Objectives

Mathematical Processes

Mathematical Content Strands	Conceptualization	Mental Arithmetic	Estimation	Computation	Applications and Problem Solving	Calculators and Computers
Whole Numbers and Numeration						
Fractions, Decimals Ratio and Percent						
Measurement						
Geometry						
Statistics and Probability						
Algebraic Ideas						
Problem Solving & Logical Reasoning						
Calculators						

W

WHOLE NUMBERS: AN OVERVIEW

What is included in this strand?

The whole number strand contains six subdivisions:

Numeration
Addition
Subtraction
Multiplication
Division
Number Properties

Numeration receives more attention since the whole number operations depend on place value concepts. Whole number paper/pencil computation receives less attention. There are no whole number computation objectives for grades 7-9.

Number properties is a new category. Some of the objectives in this subdivision have not previously been a part of the state objectives and others have, in the past, been included elsewhere. Scientific notation and prime factorization have not previously been included and multiples were found in the fraction strand.

How does the development flow?

Not only do the objectives listed in this strand represent a child's first experiences with mathematics, it forms the foundation for much of other mathematics the child is to learn. For this reason, special concern has been placed on the development of understanding as opposed to rote memorization of discrete mathematical facts and algorithmic processes.

Approximately one-fourth of the objectives are concerned with conceptual development. This emphasizes the importance of understanding with meaning of numbers and operations, and avoids memorizing senseless rules. Most of the conceptual objectives require "hands-on" use of manipulatives by students.

Integrated within this development are mental arithmetic, estimation and use of the calculator.

Mental arithmetic needs to be done both orally and visually and in a timed situation.

Estimation is extremely important as a way to check computation and calculator results, as well as for its practical value. Adults spend more time with mental arithmetic and estimation than they do with paper/pencil computation. Estimation processes need to be taught and practiced and become an integral part of instruction.

Adults use calculators. Children should also use calculators for some whole number computation and especially so for all more-complex computation. It is extremely important to learn the appropriate time to use a calculator. There should also be a limit to the degree of difficulty of any paper/pencil computation given to children. The "rule of thumb" used by the objectives writers was, "If an adult would get up and walk to another room for a calculator, a child should not be expected to do it with paper and pencil." Calculators should be available for many test situations as well.

Why teach these objectives?

Much of practical mathematics rests on understanding and use of whole numbers. Furthermore, whole numbers are prerequisite for much, but not all, of later mathematics. Computational proficiency with the multiplication and division algorithms, however, is not prerequisite knowledge for other mathematics such as fractions, geometry, statistics, probability, measurement and algebraic ideas. Whole number work does not have to be completed prior to studying other mathematics. Conceptual knowledge is essential for problem solving with whole numbers as is certain computation proficiency.

Yes, basic facts must be learned! They are included in the objectives as is paper/pencil computation. Included also are thinking strategies helpful in learning the basic facts. However, an attempt has been made to limit whole number computation. In all too many instances, drill on whole number computation dominates the elementary mathematics curriculum. As we prepare for the technological world, students must learn more than routines and formulas for solutions. Conceptualization must be given added importance.

What are the implications for instruction?

Throughout this strand, emphasis has been placed on conceptual development. Base ten blocks are used as the manipulative model for this strand. Instruction in place value concepts, addition, subtraction, multiplication and division can be greatly enhanced with the use of this model. Children should also be familiar with both comparison and take away subtraction; partition and measurement division. Basic fact strategies make fact learning easier and more efficient.

Suggestions have been made to limit the size of the numbers in paper/pencil computation problems. Limitations are as follows:

Addition:	Two three-digit numbers and three two-digit numbers in grades K-3; four three-digit numbers in grades 4-6
Subtraction:	Three-digit, one regrouping in grades K-3; three-digit, two regroupings in grades 4-6 (This objective should be mastered by the end of fourth grade.)
Multiplication:	One-digit times three-digits, no regrouping, with fact restrictions, grades K-3; two-digit times three-digit in grades 4-6
Division:	Divisors of thirty or less or multiples of 10 to 100; and dividends of no more than four digits; grades 4-6.

Mental arithmetic has been extended to expect third grade students to add two-digit numbers, no regrouping, and subtraction of tens and hundreds. Addition and subtraction of any two-digit numbers mentally are expected for grades 4-6.

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Vocabulary

K-3

- addend
- base ten blocks
- difference
- division (using both measurement and partition models)
- odd and even
- place value (ones, tens, hundreds, thousands)
- product
- rounding
- subtraction (including both comparison and take away models)
- sum

4-6

- common factor
- common multiple
- divisor
- factor
- multiple
- place value (ten thousands, thousands, hundred thousands, millions)
- prime
- prime factorization
- quotient

7-9

- scientific notation

Resources

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WHOLE NUMBERS AND NUMERATION: THE OBJECTIVES

NUMERATION: To read, write, compare, order and round numbers

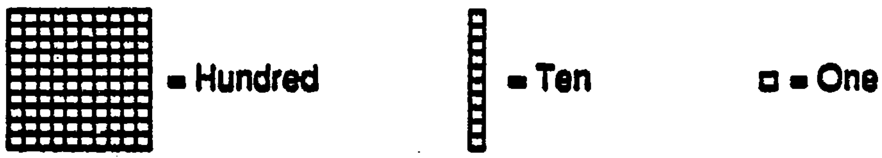
CONCEPTUALIZATION: To translate among models, word names and symbols [W1Cn1]

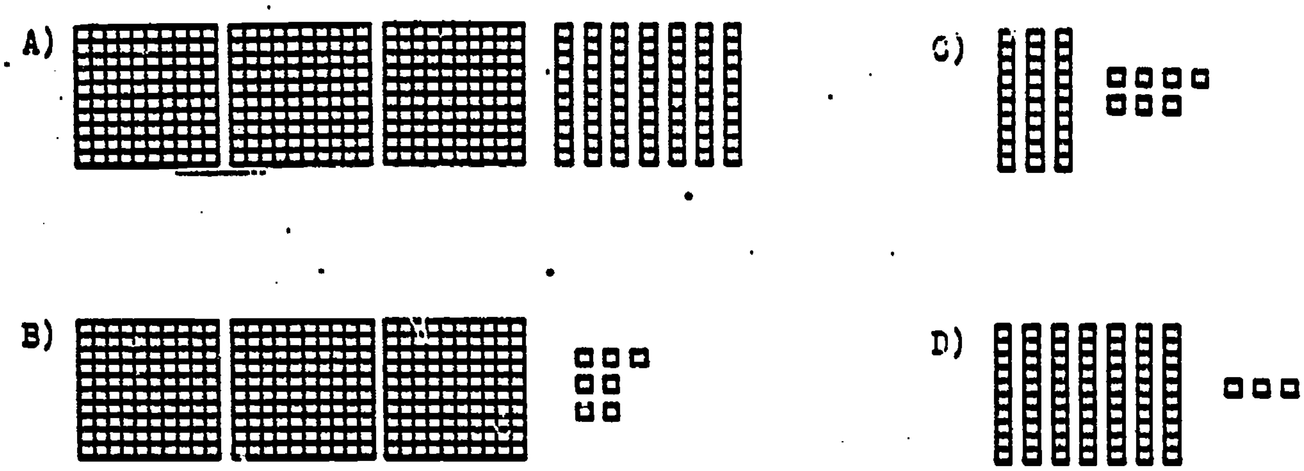
K-3 Comment:

Use base ten blocks as the model. Use ones, tens, and hundreds to show place value.

K-3 Example:

Which shows 307?





(Answer: B)

CONCEPTUALIZATION: To read numbers and recognize place value [W1Cn2]

K-3 Comment:

Use up to four-digit numbers

K-3 Example:

Which numeral represents 4 tens, 9 ones, 7 hundreds?

(Answer: 749)

4-6 Comment:

Use up to six-digit numbers

4-6 Example:

How is the number ⌣8,407 read?

(Answer: Sixty-eight thousand, four hundred seven)

COMPUTATION: To compare and order numbers [W1Cm1]

K-3 Comment:

Use up to four digit-numbers

K-3 Example:

In which numeral is the value of four the greatest?

A. 4,732
B. 400
C. 42,000
D. 429

(Answer: C)

4-6 Comment:

Use up to five-digit numbers

4-6 Example:

How many times as large is the 4 in 4,732 than the 4 in 1,740?

(Answer: One hundred times as large)

Arrange ine digits 2, 7, 3, 4, 1 to write the greatest number.

(Answer: 74,321)

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COMPUTATION: **To regroup numbers using place value as needed for computation algorithm** [W1Cm2]

K-3 Comment:

Use up to four-digit numbers

K-3 Example:

Which of the following is another name for 463?

A. 4 hundreds, 3 ones
B. 4 hundreds, 6 tens
C. 46 tens, 3 ones
D. 40 tens, 3 ones

(Answer: C)

4-6 Comment:

Use up to six-digit numbers.

4-6 Example:

How should 3004 be regrouped in the subtraction example?

```
 3004
-1467
```

(Answer: 299 14)

ESTIMATION: To round numbers [W1Es1]

K-3 Comment:

Use up to four-digit numbers rounded to the largest place.

K-3 Example:

What is 548 rounded to the nearest hundred?

(Answer: 500)

4-6 Comment:

Use up to six-digit numbers

4-6 Example:

When rounding to the hundreds place, you think of 7,645 as being between which two numbers?

A. 7,500 and 7,600
B. 7,600 and 7,700
C. 7,640 and 7,650
D. 76 and 77

(Answer: B)

ADDITION: To add whole numbers using manipulative models and computational algorithms

CONCEPTUALIZATION: To identify and use models and thinking strategies for basic facts [W2Cn1]

K-3 Comment:

This objective is very important in the addition sequence, however, the students should have learned the basic facts *before* fourth grade. With this in mind, this objective is not appropriately tested at grade four or beyond.

Thinking strategies include COUNT ON (8 + 3 is 8 ... 9, 10, 11), DOUBLES (7 + 7 = 14), NEAR DOUBLES (7 + 8 ... 7 + 7 and 1 more), and ADDING 9 (7 + 9 ... 10 + 7 = 17 so 1 less or 16; or 9 + 1 = 10 and 6 more = 16).

K-3 Example:

Since 10 + 8 = 18, what is 9 + 8?

(Answer: 17)

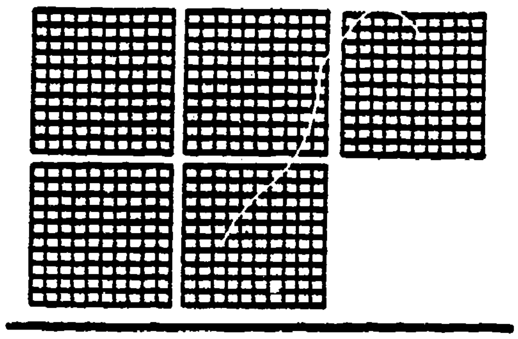
Since 8 + 8 = 16, what is 8 + 9?

(Answer: 17)

CONCEPTUALIZATION: **To identify models to show addition of multiples of 10 and 100 [W2Cn2]**

K-3 Example:

This picture shows which addition?



A. 3
<u>+2</u>

B. 300
<u>+200</u>

C. 30
<u>+20</u>

D. 300
<u>+20</u>

(Answer: B)

CONCEPTUALIZATION: **To use models to show the addition algorithm, identifying results of regrouping [W2Cn3]**

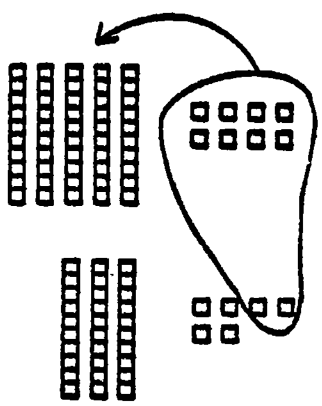
K-3 Comment:

Use no more than two two-digit numbers with one regrouping. When illustrating with concrete base ten blocks, the final step of regrouping ten ones as a ten is shown with blocks. In pictures, the regouping is shown with an arrow.

K-3 Example:

Which picture would be used to find 58
<u>+36</u>

(Answer:



COMPUTATION: To recall basic facts from memory [W2Cm1]

K-3 Comment:

Problems are done orally, using addition facts where both digits are greater than or equal to five.

K-3 Example:

"I will read the problem, repeat it once and go on. Write the answer. Then show the answer by shading the circles."

Then, in your normal voice:

1. **Read the problem;**
2. **Repeat the problem;**
3. **Wait three seconds;**
4. **Repeat this process for the next problem.**

7 + 8 = ?

Write the answer	__	__
Show the answer	⓪	⓪
	①	①
	②	②
	③	③
	④	④
	⑤	⑤
	⑥	⑥
	⑦	⑦
	⑧	⑧
	⑨	⑨

COMPUTATION: To add two numbers [W2Cm2]

K-3 Comment:

Use no more than two three-digit numbers involving double regrouping, written vertically or horizontally.

K-3 Example:

356 + 448 = ?

(Answer: 804)

COMPUTATION: To add three or more numbers [W2Cm3]

K-3 Comment:

Use no more than three one- or two-digit numbers in horizontal or vertical form.

4-6 Comments:

Use no more than four two- or three-digit numbers in horizontal or vertical form.

MENTAL ARITHMETIC: To add multiples of ten or multiples of 100 mentally [W2MA1]

K-3 Example:

Use the timed format described in addition facts.

80 + 60 = ?

(Answer: 140)

MENTAL ARITHMETIC: To add two two-digit numbers mentally [W2MA2]

4-6 Comment:

Use two two-digit numbers involving regrouping.

4-6 Example:

Use the timed format described in addition facts.

56 + 78 = ?

(Answer: 134)

ESTIMATION: To estimate the sum of two, three, or more numbers [W2Es1]

4-6 Comment:

Use two three-digit numbers which easily round (within 20 of a multiple of 100).
Items need to be administered under timed conditions. See mental arithmetic for examples.

4-6 Example:

Estimate the sum, 398 + 203.

A. 500
B. 600
C. 6000
D. 5000

(Answer: B)

7-9 Comment:

Use more than two numbers which round easily, administered in a timed condition.

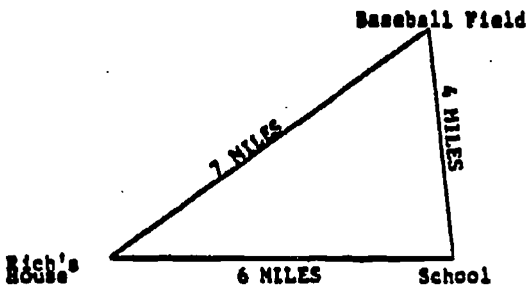
APPLICATIONS AND PROBLEM SOLVING: **To solve problems involving addition [W2PS1]**

K-3 Comment:

The solution of the problem must include addition. Use two three-digit numbers or up to three two-digit numbers.

K-3 Example:

Rich left home and went to baseball practice. The he went home to change his clothes and went to school. How far did he travel?



(Answer: 20 miles)

4-6 Comment:

The solution of the problem must include addition. Use up to three three-digit numbers or four two-digit numbers.

CALCULATORS: To add any numbers in column or horizontal form [W2Ca1]

K-3 and 4-6 Comment:

Calculators are allowed. Paper and pencil formats will not be used. Problems will use verbal directions or story problem formats

The restriction on number of addends does not hold when using calculators.

K-3 Example:

Find this sum.

87 + 139 + 462 + 62

(Answer: 750)

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SUBTRACTION: To subtract whole numbers using manipulative models and computational algorithms

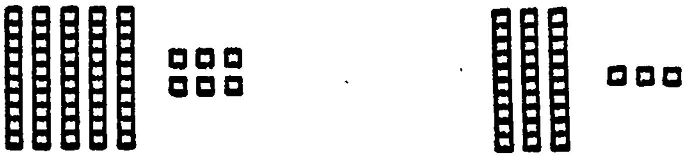
CONCEPTUALIZATION: To identify and use take away and comparison models for subtraction [W3Cn1]

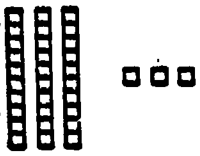
K-3 Comment:

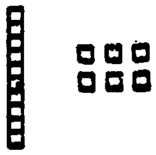
Both numbers are two-digits and do not involve regrouping. Use pictures of base ten blocks to show possible solutions to a problem.

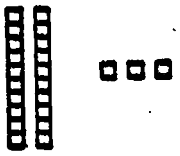
K-3 Example:

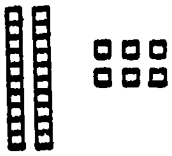
Which picture shows how much larger the first group is than the second?



A 

B 

C 

D 

(Answer: C)

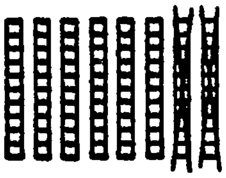
CONCEPTUALIZATION: To identify models to show subtraction of multiples of 10 and 100 [W3Cn2]

K-3 Comment:

Use base ten blocks as the manipulative.

K-3 Example:

Which picture would be used to find $\begin{array}{r} 80? \\ \underline{-20} \end{array}$

(Answer: )

CONCEPTUALIZATION: To relate subtraction to addition [W3Cn3]

K-3 Comment:

All basic fact triples (number families) are used.

K-3 Example:

Which number sentence completes the group?

8 + 7 = 15
7 + 8 = 15
15 - 8 = 7

A) 7 + 1 = 8
B) 8 - 7 = 1
C) 15 - 7 = 8
D) 15 + 7 = 22

(Answer: C)

4-6 Comment:

If a subtraction example is given, then a corresponding addition example must be chosen.

4-6 Example:

If 27 - n = y, then which is true?

A. n + y = 27
B. n - y = 27
C. y - n = 27
D. 27 + y = n

(Answer: A)

CONCEPTUALIZATION: To use thinking strategies for basic facts [W3Cn4]

K-3 Comment:

The major strategy is counting back to subtract 1, 2 or 3. For other facts, the main strategy is using addition.

K-3 Example:

Which addition fact do you use to find 15 - 7?

(Answer: 7 + 8 = 15)

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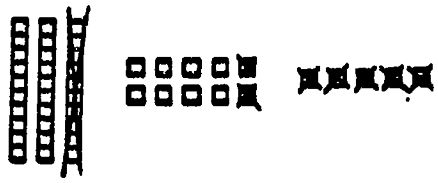
CONCEPTUALIZATION: To use models to show the subtraction algorithm, identifying results of regrouping [W3Cn5]

K-3 Example:

This is a picture for which problem?



A. 45 - 17
B. 45 + 17
C. 28 - 17
D. 28 + 17

(Answer: A)

4-6 Comment:

This objective is not appropriately tested at grade 7.

COMPUTATION: To recall basic facts [W3Cm1]

K-3 Comment:

Problems are done orally, using basic facts in a timed situation. See ADDITION FACTS for format.

COMPUTATION: To find differences of two- and three-digit numbers involving regrouping [W3Cm2]

K-3 Comment:

Only one regrouping is used.

K-3 Example:

$$\begin{array}{r} 358 \\ \underline{-173} \end{array}$$

(Answer: 185)

4-6 Comment:

Use two regroupings, including zeros in the middle.

This objective should be mastered by the end of fourth grade.

4-6 Example:

702
-195

(Answer: 507)

MENTAL ARITHMETIC: To find differences of multiples of ten and 100 mentally [W3MA1]

K-3 Example:

Use the timed format as described in ADDITION FACTS.

130 - 60 = ?

(Answer: 70)

MENTAL ARITHMETIC: To find differences of two-digit numbers mentally [W3MA2]

4-6 Example:

Use the timed format as described in ADDITION FACTS.

94 - 47 = ?

(Answer: 47)

43

ESTIMATION: To estimate to find approximate differences [W3Es1]

K-3 Comment:

Use two two-digit numbers which are within three of a multiple of ten. The problem can be placed in a story setting or administered orally in a timed setting.

K-3 Example:

Jean went to a flower shop. She ordered 71 roses, the store had only 18. They would have to order about how many more to fill Jean's order?

A. 30
B. 40
C. 50
D. 60

(Answer: C)

Use a timed format as described in ADDITION FACTS.

Is 82 - 39 about 30, about 40, about 50 or about 80?

(Answer: About 40)

4-6 Comment:

Use two three-digit numbers which are within 20 of a multiple of 100. The problem can be placed in a story setting or administered in a timed setting.

4-6 Example:

The cafeteria needs 432 lunches. They have 281 made. About how many lunches do they still need to make?

A. 250
B. 700
C. 50
D. 150

(Answer: D)

Use a timed format as ADDITION FACTS.

Is 591 - 115 about 100, about 400, about 500 or about 600?

(Answer: About 500)

APPLICATIONS AND PROBLEM SOLVING: **To solve problems involving subtraction [W3PS1]**

K-3 Comment:

The solution of the problem must include subtraction. Use no more than three-digit numbers with one regrouping.

K-3 Example:

What operation should be used to solve this problem?

John and Sam were each given a jar of jelly beans. John's jar contained 156 jelly beans. Sam's jar contained 275 jelly beans. How many more jelly beans does Sam have than John?

A. addition
B. subtraction
C. multiplication
D. division

(Answer: B)

4-6 Comment:

The solution of the problem must include subtraction. More than one step is allowed.

4-6 Example:

For an evening program, 565 chairs were needed in the gym. The senior class set up 142 chairs. The custodian set up 275 chairs. How many more chairs are still needed?

(Answer: 148)

CALCULATORS: To subtract any whole numbers [W3Ca1]

K-3 Comment:

The restriction of three-digit numbers on subtraction does not apply to calculator problems. Story problem format is preferred.

MULTIPLICATION: **To multiply numbers using manipulative models and computational algorithms**

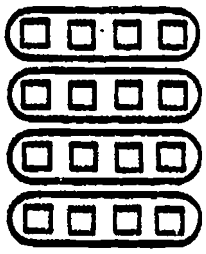
CONCEPTUALIZATION: **To identify and use models and thinking strategies for basic facts [W4Cn1]**

K-3 Comment:

The major concrete model uses sets with the same number in each.

Thinking strategies include SKIP COUNTING, (6 x 5 ... 5, 10, 15, 20, 25, 30), USING 9 (9 x 6 ... 10 sixes = 60, one less 6 is 54) and DOUBLES (4 x 9 ... 2 nines = 18; 18 + 18 = 36).

K-3 Example:



Which of the following can be answered by the picture above?

A. How many sets of squares are there?
B. How many squares are there in each set?
C. How many squares are there altogether?
D. All of the above

(Answer: D)

If 2 x 9 = 18, what is 4 x 9? (Answer: 36)

If 10 x 7 is 70, what is 9 x 7? (Answer: 63)

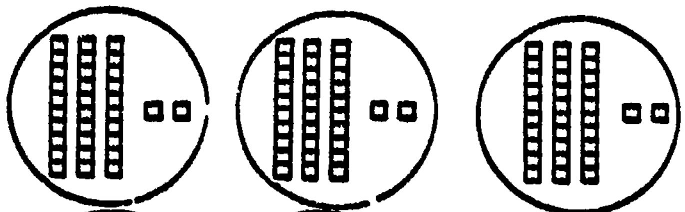
CONCEPTUALIZATION: **To use models to show the multiplication algorithm [W4Cn2]**

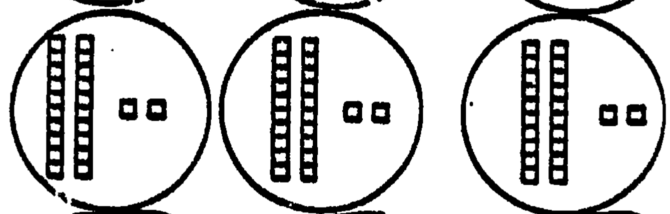
K-3 Comment:

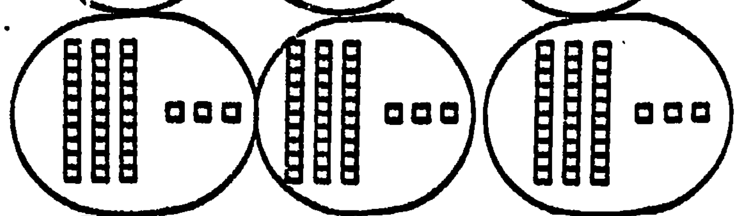
Use pictures of sets of objects or base ten blocks. Use no more than one-digit times two-digit numbers, not involving regrouping.

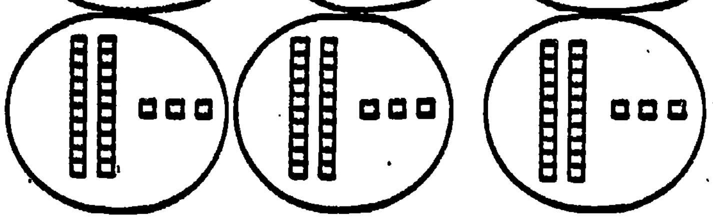
K-3 Example:

Which picture shows three sets of 32?

A. 

B. 

C. 

D. 

(Answer: A)

MENTAL ARITHMETIC: **To recall selected basic facts from memory [W4MA1]**

K-3 Comment:

Problems are done orally using only multiplication facts of 1, 2, 5, and 9.

K-3 Example:

9 x 9 = ?

(Answer: 81)

4-6 Comment:

The other facts are needed in grade 4 but are not appropriate for testing in grade 7.

MENTAL ARITHMETIC: **To find products of multiples of 10 and 100 [W4MA2]**

K-3 Comment:

Problems are done orally using multiples of 10 and 100 times 1, 2, 5, or 9.

A timed format is used as described in ADDITION FACTS.

K-3 Example:

20 x 8 = ?

(Answer: 160)

MENTAL ARITHMETIC: **To multiply one-digit and two-digit numbers and find other appropriate special products mentally [W4MA3]**

4-6 Comment:

Problems are given orally in a timed situation with no regrouping.

4-6 Example:

72 x 6 = ?

(Answer: 432)

7-9 Comment:

Problems are given orally in a timed situation with regrouping allowed. Special products INCLUDE FACTORS OF 15, 25, and 50.

7-9 Example:

86 x 7 = ? (Answer: 602)

98 x 50 = ? (Answer: 4900)

ESTIMATION: **To use multiples of 10 and 100 to estimate products [W4Es1]**

4-6 Comment:

Use two two-digit numbers which are within three of a multiple of 10 in a timed situation.

4-6 Example:

The product of 38 x 23 is between

A. 60 and 120
B. 120 and 600
C. 600 and 1,200
D. 6,000 and 12,000

(Answer: C)

COMPUTATION: **To multiply two numbers up to a two-digit by a three-digit number [W4Cm1]**

K-3 Comment:

Use one-digit times two- or three-digit numbers, no regrouping and limited to restrictions on basic facts.

K-3 Example:

841
x 2

(Answer: 1,682)

4-6 Comment:

Use any three-digit number times a two-digit number.

4-6 Example:

277
x89

(Answer: 24,653)

49

APPLICATIONS AND PROBLEM SOLVING: **To solve problems involving multiplication [W4PS1]**

K-3 Comment:

Use only multiplication facts of 1, 2, 5, and 9.

K-3 Example:

There are 9 girls on each softball team. How many girls are on three softball teams?

(Answer: 27)

4-6 Comment:

Use up to two-digit by three-digit numbers.

4-6 Example:

Ernest packed 144 party favors in each box. He packed 8 boxes. How many party favors did he pack?

(Answer: 1,152 favors)

CALCULATORS: To multiply any numbers [W4Ca1]

4-6 Comment:

Calculators are allowed and paper and pencil not used. Problems will use verbal directions or story problem formats. Problems may involve more than one step.

4-6 Example:

Use the calculator to solve:

> Tracy earns $3.65 an hour.
> She works six hours each day.
> How much does she earn if she works five days?
>
> (Answer: $109.50)

DIVISION: To divide whole numbers using manipulative models and computational algorithms

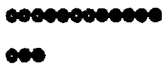
CONCEPTUALIZATION: To identify and use measurement and partition models for division [W5Cn1]

K-3 Comment:

Problems are limited to small two-digit numbers divided by a one-digit number, with limits on restrictions to basic facts.

K-3 Example:

Which shows the number of threes in 12?

A. 

B. 

C. 

D. 

(Answer: C)

CONCEPTUALIZATION: To relate division to multiplication [W5Cn3]

K-3 Comment:

Problems are restricted to single digit multiples of 1, 2, 5, and 9.

K-3 Example:

Which of the following numbers go together using multiplication or division?

A. 2, 9, 18
B. 3, 6, 9
C. 2, 7, 9
D. 20, 10, 5

(Answer: A)

51

4-6 Comment:

Problems use products of numbers no larger than two-digits times three-digits.

4-6 Example:

Which of the following sets of numbers go together using multiplication or division?

I. 7, 8, 56
II. 9, 9, 81
III. 23, 56, 1288

A. I and II
B. I and III
C. II and III
D. I, II and III

(Answer: D)

If $28 \div x = y$, then which is true?

A. $x \cdot y = 28$
B. $x \div y = 28$
C. $28 \cdot y = x$
D. $28 = y \cdot x$

(Answer: A)

CONCEPTUALIZATION: To use thinking strategies for finding basic facts [W5Cn3]

K-3 Comment:

The basic fact triples (number families) which include 1, 2, 5, and 9 as factors are used.

The main thinking strategy for division facts is to use multiplication.

K-3 Example:

If 5 nines = 45, what is $45 \div 5$?

(Answer: 9)

4-6 Comment:

The other facts are needed in grade 4 but are not apropriate for testing in grade 7.

CONCEPTUALIZATION: To interpret the remainder [W5Cn4]

4-6 Comment:

Divisors are less than 30 and dividends no more than four digits.

4-6 Example:

Which sentence is related to $17\overline{)5772}$?

A. 17 x 9 + 339 = 5772
B. 17 + 9 x 339 = 5772
C. 17 x 339 + 9 = 5772
D. 17 x 339 - 9 = 5772

(Answer: C)

What is the least number of buses needed to take 716 students to the zoo, if each bus can take 24 students?

A. 29
B. 29 5/6
C. 30
D. 30 5/6

(Answer: C)

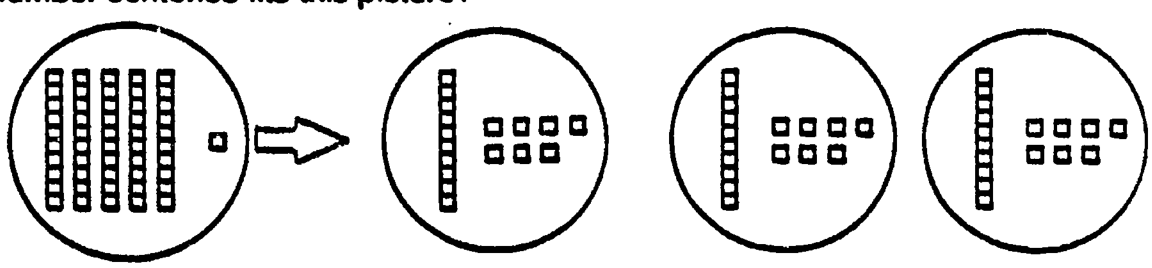
CONCEPTUALIZATION: To relate models to the division algorithm [W5Cn5]

4-6 Comment:

Divisors are 30 or less or multiples of 10.

4-6 Example:

Which number sentence fits this picture?



A. 51 ÷ 3
B. 51 x 17
C. 51 ÷ 17
D. 51 x 3

(Answer: A)

MENTAL ARITHMETIC: **To divide multiples of 10, 100, and 1000 getting quotients that are multiples of 10, 100, or 1000 [W5MA1]**

4-6 Example:

Use a timed format as described in ADDITION FACTS.

$5600 \div 70 = \underline{?}$

(Answer: 80)

ESTIMATION: **To use multiples of 10, 100 and 1000 to determine the number of places in the quotient [W5Es1]**

4-6 Comment:

Testing of estimation is difficult. If students have the opportunity, they will use paper and pencil to compute when asked to estimate answers. Estimation could be tested as shown in the example.

4-6 Example:

"I will show you a problem for which you are to select the best estimate. After reading the problem, you will have only a short time (four seconds) before the next problem is read, so do not attempt to find the exact answer. Select the response which fits the problem."

Show a card with the problem $35\overline{)4856}$.

"What is the estimate of the quotient?"

A. Between 0 and 10
B. Between 10 and 100
C. Between 100 and 1000
D. Between 1000 and 10000

(Answer: C)

How many digits are in the quotient for $6\overline{)6054}$?

(Answer: Four)

ESTIMATION: **To determine the first digit and its place value in the quotient [W5Es2]**

4-6 Comment:

Divisors will be 30 or less or multiples of 10 and dividends no more than four digits.

4-6 Example:

$38\overline{)4687}$

What does the first number in the quotient indicate?

A. The number of hundreds
B. The number of thousands
C. The number of tens
D. The number of ones

(Answer: A)

COMPUTATION: **To find the quotient and remainder for one and two-digit divisors (up to 30, multiples of 10, 40 through 90) with up to four-digit dividends [W5Cm1]**

4-6 Example:

Find $17\overline{)5679}$

A. 334 R1
B. 334
C. 423 R5
D. 423

(Answer: 334 R1)

55

APPLICATIONS AND PROBLEM SOLVING: **To solve problems involving division [W5PS1]**

K-3 Example:

Kelsey used 10 stamps to mail letters. She put 2 stamps on each letter. How many letters did she mail?

(Answer: 5 letters)

4-6 Example:

One of the wells on a farm pumps 300 gallons of water every hour. How many hours does it take to pump enough water to fill a 750 gallon tank?

Which operation is used to solve the problem?

A. Addition
B. Subtraction
C. Multiplication
D. Division

(Answer: D)

CALCULATORS: **To divide any numbers [W5Ca1]**

4-6 Example:

Use a calculator to find $386\overline{)57932}$

(Answer: 150.0829)

NUMBER PROPERTIES: To recognize and use properties of whole numbers

CONCEPTUALIZATION: To demonstrate and use the meaning of odd and even [W6Cn1]

K-3 Comment:

Use numbers less than 100.

K-3 Example:

Which of the following sets contain only even numbers?

A. 14, 28, 56
B. 15, 30, 45
C. 2, 4, 9
D. 5, 10, 20

(Answer: A)

4-6 Example:

Which sets have an odd number of numbers?

X: 3, 5, 9, 13
Y: 15, 17, 23
Z: 2, 4, 6
W: 6, 10, 12, 15

A. X and Y
B. Y and Z
C. Z and W
D. X, Y, Z and W

(Answer: B)

CONCEPTUALIZATION: To demonstrate and use the meaning of multiple and common multiple [W6Cn2]

4-6 Comment:

Limit to multiples of numbers less than 20.

4-6 Example:

Which set contains a number which is not a multiple of three?

A. 0, 12, 36
B. 3, 9, 27
C. 6, 51, 81
D. 6, 12, 17

(Answer: D)

Which two numbers are common multiples of both 3 and 4?

A. 12, 24
B. 3, 4
C. 9, 16
D. 6, 8

(Answer: A)

CONCEPTUALIZATION: To demonstrate and use the meaning of factor and common factor [W6Cn3]

4-6 Comment:

Use numbers to 200 for factor and numbers to 50 for common factor.

4-6 Example:

How many factors of 8 are there?

(Answer: 4)

Which numbers have only one as a common factor?

A. 3, 6, 15
B. 10, 30, 55
C. 28, 34, 56
D. 24, 25, 49

(Answer: D)

CONCEPTUALIZATION: To demonstrate and use the meaning of prime number and prime factorization [W6Cn4]

4-6 Comment:

Use primes less than 100 for the meaning of primes.

4-6 Example:

Which of the following is not prime?

A. 31
B. 43
C. 91
D. 97

(Answer: C)

The number 81 has how many prime factors?

A. 1
B. 2
C. 3
D. 4

(Answer: A)

7-9 Comment:

Prime factorization is appropriate for 7-9.

7-9 Example:

How many primes are there between 36 and 56?

A. Fewer than 5
B. Exactly 5
C. More than 10
D. Exactly 10

(Answer: B)

Which is the prime factorization of 36?

A. $6 \cdot 6$
B. $4 \cdot 9$
C. $2^2 \cdot 3^3$
D. $2^2 \cdot 3^2$

(Answer: D)

CONCEPTUALIZATION: **To demonstrate and use the meaning of scientific notation [W6Cn5]**

7-9 Comment:

Operations using scientific notation should be subject to the same limitations as whole number computation except that each will have a power of ten.

7-9 Example:

Three million five hundred thousand can be written in scientific notation as:

A. 35×10^6
B. 3.5×10^5
C. 35×10^5
D. 3.5×10^6

(Answer: D)

COMPUTATION: **To determine when sums, differences, products and quotients are even or odd [W6Cm1]**

4-6 Example:

Indicate whether the following are even or odd:

A. the sum of two even numbers (even)
B. the product of two even numbers (even)
C. the sum of an even and an odd number (odd)
D. the product of an even and an odd number (odd)
E. the difference of two odd numbers (even)

COMPUTATION: **To find multiples of numbers less than 20 [W6Cm2]**

4-6 Example:

The numbers 36 and 16 are both multiples of 4. Write two multiples of 4 which are greater than 41.

(Answers will vary)

COMPUTATION: **To find factors of numbers less than 200 [W6Cm3]**

4-6 Example:

Two factors of 198 are 2 and 3. Find four other factors of 198.

(Answer: Possibilities - 1, 6, 9, 11, 18, 22, 33, 66, 99, 198)

COMPUTATION: **To find common factors of two numbers, each less than 50 [W6Cm4]**

4-6 Example:

The numbers 2 and 3 are factors of both 24 and 30. Write another factor common to both 24 and 30.

(Answer: 1 and 6)

COMPUTATION: **To find common multiples of two numbers, each less than 16 [W6Cm5]**

4-6 Example:

Which of the following contains a common multiple of both 3 and 8?

A. Numbers 10 to 20
B. Numbers 20 to 30
C. Numbers 30 to 40
D. Numbers 50 to 60

(Answer: B)

COMPUTATION: **To determine prime numbers less than 100 [W6Cm6]**

4-6 Example:

Write the prime numbers between 50 and 75.

(Answer: 53, 59, 61, 67, 71, 73)

COMPUTATION: **To find prime factorization of numbers 100 or less [W6Cm7]**

4-6 Example:

Which is the prime factorization of 48?

A. $2^2 \cdot 4 \cdot 3$
B. $2 \cdot 8 \cdot 3$
C. $16 \cdot 3$
D. $2^4 \cdot 3$

(Answer: D)

COMPUTATION: **To express whole numbers in scientific notation and conversely [W6Cm8]**

7-9 Example:

Which of the following are in scientific notation?

A. 30
B. 3×5
C. 30×10^8
D. 3.5×10^{-15}
E. $.35 \times 10^{-16}$

(Answer: D)

Framework: Michigan Mathematics Objectives

	Mathematical Processes					
Mathematical Content Strands	Conceptualization	Mental Arithmetic	Estimation	Computation	Applications and Problem Solving	Calculators and Computers
Whole Numbers and Numeration						
Fractions, Decimals Ratio and Percent						
Measurement						
Geometry						
Statistics and Probability						
Algebraic Ideas						
Problem Solving & Logical Reasoning						
Calculators						

F

FRACTIONS, DECIMALS, RATIO AND PERCENT
AN OVERVIEW

What is included in this strand?

This strand contains the concepts and skills involved in using fractions, decimals, ratio, proportion and percent to compute and to solve problems. Mental computation and computational estimation objectives are included under each of these four major topics. The calculator is included primarily under the decimal, ratio-proportion and percent topics.

How does the development flow?

The objectives in this strand underscore the importance of a strong conceptual foundation for the learning of subsequent computation and problem solving skills. Fraction concepts begin at the K-3 level. Denominators of tenths will help the introduction of decimals. Objectives which involve conceptualization place emphasis on concrete models and illustrations. Rectangles and circles are used initially and are followed by number lines and sets of objects in higher grades. Students are to comprehend the process and understand the meaning of the numbers and operations they use. This comprehension should be further strengthened in a context of real-life applications. We seek to promote the student's ability to reason and to apply the mathematics they learn.

The initial fraction objectives, i.e., meaning, equivalence and comparison should precede the initial work with decimals. However, not all of the fraction material needs to be covered prior to teaching decimals. The major interrelationship of decimals and fractions occurs at the beginning of the percent topic. Here, a student is to see a ratio expressed as either a fraction, a decimal or a percent. The hundred's grid is an excellent model to help establish this relationship. It is important that adequate time be given to helping a student see how fractions, decimals and percents are interrelated. Concrete models should be utilized at all grade levels to continuously reinforce the meaning of the numbers and operations. Estimation and mental computation strengthen this conceptualization. Written procedures for computing with fractions, decimals and percents come after understanding.

The underlying theme of percent is ratio and proportion. Students must also be able to interpret various uses of percent in equation form. Estimation and mental computation help build a strong intuitive feeling for percents. Paper and pencil computation is restricted because calculators are used for much of the computation with decimals and percents.

The interrelatedness of the strands is obvious but needs emphasis. The study of fractions, decimals, ratio-proportion and percent is strongly connected to measurement, similarity, probability, algebra and problem solving. Also, earlier learning in the areas of whole numbers, numeration and geometry promote learning in this strand.

Why teach these objectives?

Computation is one of the major areas of mathematics instruction in the elementary and middle grades. As we consider the education which will best serve our students in the twenty-first century, we seek to balance the importance of computation with that of reasoning, problem solving and application. The objectives in this strand represent a broadened definition of computation which includes more than just written computation skills. Computational estimation, mental computation and the use of calculators are important skills in everyday uses of computation.

Computational estimation and mental computation are not only practical skills on their own merit, they also support subsequent understandings of the quantitative relationships which undergird the student's "number sense". We believe that a student's ability to compute, to estimate mentally and to perform other estimation tasks at one grade level helps him/her to acquire concepts at a later level.

What are the implications for instruction?

The objectives are written with the view that the teacher will be actively involved in questioning, clarifying, presenting, and demonstrating mathematics with models and real-life applications. Fractions, decimals, ratio-proportion and percent cannot be adequately taught solely from the textbook. Estimation and mental computation require active classroom participation. Also, the teacher must provide experiences with manipulatives and pictorial representations (at all grade levels) to help provide the required understandings.

It is crucial that adequate time be given to instruction and evaluation using the typical models such as the rectangular and circular regions, the hundreds square and the number line. The teacher should instruct and assess the student's ability to make connections between concrete examples, pictures, vocabulary and symbols.

Problem solving is a major reason for studying fractions, decimals, ratio and percent. Thus, both teaching and testing should reflect this priority. Paper-and-pencil procedures for computation with fractions, decimals, ratios and percents are still somewhat important. This written computation must, however, use less complex examples and stress understanding over rote manipulation of symbols.

Simple fractions are recommended for developing concepts, practicing procedures or testing. By simple fractions we mean those fractions whose denominators are 2, 3, 5, or 7 and some easy multiples of these prime numbers. Simple fractions are said to be related if their denominators are multiples of the same number.

Suggested Fractions

Halves	Thirds	Fifths	Sevenths
4ths	6ths	10ths	14ths
8ths	9ths	20ths	
16ths	12ths	50ths	
	18ths	100ths	

This set of fractions is not meant to be restrictive, but should serve as a guideline so that instruction does not make excessive use of complicated fractions.

Vocabulary

K-3 Level	4-6 Level	7-9 Level
Fraction	Common Denominator	Proportion
Unit	Decimal	Reciprocal
	Denominator	Rate
	Equivalent Decimals	
	Equivalent Fractions	
	Higher Term Fraction	
	Lowest Term Fraction	
	Numerator	
	Percent	
	Ratio	

Resources

Allinger, Glen D. and Joseph N. Payne. Estimation and Mental Arithmetic with Percent. In Estimation and Mental Computation, 1986 Yearbook. (H. L. Schoen and M. J. Zweng, Editors). Reston Virginia: National Council of Teachers of Mathematics, 1986, pp. 141-155.

Bennett, Albert J., Jr., Decimal Squares. (Grades 4-8). Workbook and manipulatives, Creative Publications or Cuisenaire.

Bennett, Albert and Davidson, Patricia, Fraction Bars. (Grades 4-8). Workbook and manipulatives. Dale Seymour Publication or Creative Publications.

Ellerbruch, Larry W. and Joseph N. Payne. A Teaching Sequence from Initial Fraction Concepts through the Addition of Unlike Fractions. In Developing Computational Skills, 1978 Yearbook. (M. N. Suydam and R. E. Reys, Editors). Reston, Virginia: National Council of Teachers of Mathematics, 1978, pp. 129-147.

Holden, Linda, The Fraction Factory Program. (Grades 3-6). Activity books and manipulatives, Creative Publications or Cuisenaire.

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Reys, Robert, Trafton, Paul, Reys, Barbara, and Zawojewski, Judy. Computation Estimation. (Grades 6-8). Transparency masters and worksheets, Dale Seymour.

The Mathematics Resource Project, University of Oregon, Ratio, Proportion and Scaling. (Grades 5-9). Teacher notes, activities, games, and worksheets, Creative Publications.

FRACTIONS AND MIXED NUMBERS: THE OBJECTIVES

MEANING: To demonstrate and use the meaning of fractions

CONCEPTUALIZATION: To relate fractions to concrete models [F1Cn1]

K-3 Comment:

Use rectangular, square and circular regions. Use word names and fraction symbols for halves, thirds, fourths, fifths, sixths, sevenths, eighths, ninths, and tenths.

Vocabulary:

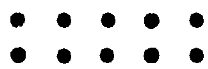
Fraction: A number $\frac{a}{b}$ that shows a of b equal parts in a unit.
Unit: That which is being considered as one or the whole.

4-6 Example:

Make a mark with an arrow on the number line where you think $1\frac{2}{3}$ is located.

7-9 Example:

This set of dots is $\frac{5}{6}$ of the total set. Draw a picture of the total set.



CONCEPTUALIZATION: To relate fractions to division using the necessary vocabulary [F1Cn2]

4-6 Example:

Three pies are to be shared equally by 5 people. How much pie will each person get?

Draw a picture to represent your solution.

4-6 Comment:

Draw pictures to show $\frac{a}{b}$ as $a \div b$.
Master the vocabulary: Numerator, denominator, and unit fraction.

Vocabulary:
Denominator: The number in a fraction that tells the number of equal parts in a unit.
Frequently used denominators: halves, thirds, fourths, fifths, sixths, eighths, ninths, tenths.

4-6 and 7-9 Comment:

Emphasize the meaning of unit fraction (numerator of 1) in the context of a set of objects, e.g., one-sixth of a set of 24 objects.

ESTIMATION: **To estimate fractions and sizes of regions using easily recognized fractions [F1Es1]**

4-6 Example:

Circle the picture below which is closest to one-half shaded?

7-9 Comment:

This shape is $\frac{1}{4}$ 

Which shape is closest to $\frac{3}{7}$? Estimate.

Comment: Estimation activities are usually characterized by oral/mental processes carried out in a reasonable time limit.

APPLICATIONS AND PROBLEM SOLVING: **To solve problems involving the meaning of fractions [F2PS1]**

K-3 Example:

Jane cut her pizza into 8 equal pieces. She ate 3 pieces. What fraction of the pizza did she eat?

Draw a picture to show your solution.

7-9 Example:

There are 3 boys and 5 girls on the Equations team. What fraction of the team is boys?

(Answer: $\frac{3}{8}$)

EQUIVALENT FRACTIONS: To find equivalent fractions using concrete models and generalizations for equivalent fractions

CONCEPTUALIZATION: **To relate concrete models and equivalent fractions [F2Cn1]**

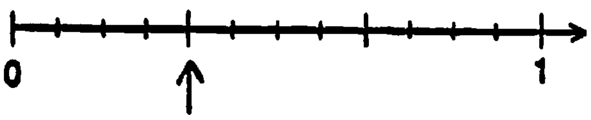
4-6 Example:

Use your fraction strips (or other manipulatives) to show several fractions which are equivalent to $\frac{2}{6}$.

Vocabulary:
Equivalent Fractions:
Fractions which name the same amount or same number.

7-9 Example:

Name two equivalent fractions located at the arrow.



(Answer: $\frac{4}{12}$, $\frac{1}{3}$)

MENTAL ARITHMETIC: **To find equivalent fractions for easily recognized fractions [F2MA1]**

4-6 and 7-9 Comment:

Emphasize higher term fractions at grades 4-6 and lowest term fractions at grades 7-9.

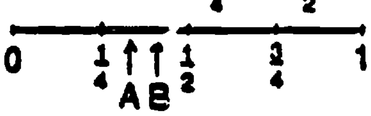
Vocabulary: *Higher Terms Fraction:* A fraction with a numerator and denominator having a common factor. *Lowest Terms Fraction:* A fraction with a numerator and denominator that do not have a common factor greater than one.

When a fraction is simplified, it is expressed in lowest terms.

ESTIMATION: **To estimate fractions using easily recognized fractions [F2Es1]**

4-6 Example:

Use equivalent fractions to estimate and find two fractions located between $\frac{1}{4}$ and $\frac{1}{2}$.



Acceptable answers for A and B:
$\frac{3}{8}$ and $\frac{2}{5}$, $\frac{5}{16}$ and $\frac{7}{16}$, etc.

7-9 Comment:

Include mixed numbers and whole numbers.

COMPUTATION: **To find equivalent fractions and mixed number/fraction equivalents [F2Cm1]**

4-6 Example:

Write a fraction that is equivalent to $\frac{3}{4}$ with a denominator of 20.

$$\frac{3}{4} = \frac{\square}{20}$$

Draw a picture to show that the two fractions are equivalent.

7-9 Comment:

Include the use of mixed whole numbers.

Example:
23 fifths is equivalent to 4 and ______ tenths.

(Answer: 6)

APPLICATIONS AND PROBLEM SOLVING: **To solve problems with equivalent fractions [F2PS1]**

4-6 Example:

Jody has $\frac{1}{4}$ of the cake left from her party.
If the whole cake had been sliced into 32 equal pieces, how many pieces does Jody have?

7-9 Example:

Write all of the fractions equivalent to $\frac{3}{4}$ that can be made using these numbers:

6,7,8,10,15,20,25

(Answers: $\frac{6}{8}$, $\frac{15}{20}$)

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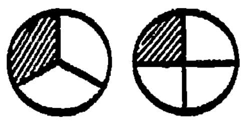
COMPARE/ORDER: To compare and order fractions

CONCEPTUALIZATION: To compare and order using models and appropriate fractions [F3Cn1]

K-3 Example:

Which is the smaller fraction?



one-third one-fourth

4-6 Comment:

Use a number line to show the order of fractions from least to greatest.

7-9 Example:

Write these fractions in order from least to greatest.

$\frac{4}{5}$ $\frac{1}{2}$ $\frac{2}{3}$ $\frac{1}{6}$

ESTIMATION: To estimate fractions using easily recognized fractions [F3Es1]

4-6 Example:

Which fraction below is greater than $\frac{1}{2}$?

Estimate.

$\frac{17}{50}$ $\frac{29}{50}$ $\frac{23}{50}$

4-6 Example:

Which fraction below lies between $\frac{1}{4}$ and $\frac{1}{3}$?

Estimate.

$\frac{3}{19}$ $\frac{8}{19}$ $\frac{13}{19}$ $\frac{5}{19}$

COMPUTATION: To compare and order fractions [F3Cm1]

4-6 Example:

Which is least?

$\frac{3}{4}$ $\frac{7}{8}$ $\frac{9}{16}$ $\frac{9}{10}$

4-6 Comment:

Emphasize special cases during instruction: Unit fractions, e.g., $\frac{1}{3}$ is greater than $\frac{1}{5}$.

Same denominator, e.g., $\frac{7}{10}$ is less than $\frac{8}{10}$.

Use models to reinforce the example.

7-9 Comment:

Students may either rename with a common denominator or use the cross-products rule to compare fractions.

Emphasize writing 3 or more fractions in order from least to greatest.

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CALCULATORS: **To compare and order fractions using decimal equivalents [F3Ca1]**

7-9 Example:

Order from least to greatest
You may use a calculator. $\frac{11}{24}$ $\frac{4}{9}$ $\frac{5}{8}$ $\frac{7}{15}$

APPLICATIONS AND PROBLEM SOLVING: **To solve problems involving comparing or ordering fractions [F3PS1]**

4-6 Example:

Tom ate $\frac{1}{4}$ of the pizza; Sam ate $\frac{1}{3}$ and Jack ate $\frac{1}{6}$. Tilly ate $\frac{3}{12}$ of the pizza. Who ate the most?

4-6 and 7-9 Comment:

Instruction and testing should include examples where notation involves whole numbers, mixed numbers and common fractions all in the same exercise.

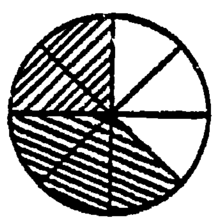
ADD/SUBTRACT: To add and subtract fractions including combinations with whole numbers

CONCEPTUALIZATION: **To relate the addition and subtraction operations to models and to each other [F4Cn1]**

4-6 Comment:

Like denominators using simple fractions, e.g., $\frac{1}{2}, 3\frac{1}{2}, \frac{1}{4}, \frac{5}{4}, \frac{2}{3}$

Use illustrations



2 eighths
<u>+ 3 eighths</u>

Use word names for denominators.

Have students draw a picture to illustrate their procedure and answer.

7-9 Comment:

Unlike denominators. Illustrate the need for a common denominator. Illustrate the need for renaming in subtraction.

$$\begin{array}{r} 2 \\ -1\frac{1}{3} \\ \hline \frac{2}{3} \end{array}$$

start with 2
show $1\frac{1}{3}$

Remove $1\frac{1}{3}$

Vocabulary:
Common Denominator:
When two or more fractions have the same denominator they are called like fractions and are said to have a common denominator.

Stress the relationship of addition to subtraction, e.g., $\frac{11}{8} - \frac{6}{8} = \frac{5}{8}$ and $\frac{5}{8} + \frac{6}{8} = \frac{11}{8}$.

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MENTAL ARITHMETIC: **To find sums or differences of like fractions mentally [F4MA1]**

4-6 Comment:

Limited to halves, fourths, eighths.

Limited to like denominators. Use oral exercises.

7-9 Example:

Use a pencil only to write answers. Give exercises orally, e.g., "Add $3\frac{1}{4}$ and $2\frac{3}{4}$."

ESTIMATION: To estimate sums and differences [F4Es1]

4-6 Example:

Estimate. Is this sum greater than 2? Explain your thinking.

$$\frac{2}{3} + \frac{7}{8}$$

(Answer: No. Each addend is less than 1.)

7-9 Example:

Estimate. Is this sum less than 12? Explain your thinking.

$$5\frac{3}{8} + 2\frac{2}{5} + 4\frac{11}{13}$$

(Answer: No. The sum of the 3 fractions is greater than 1 and the sum of the whole numbers is 11.)

COMPUTATION: To find sums or differences [F4Cm1]

4-6 Comment:

Addition involves 2 addends with like or related fractions.

Subtraction is limited to related fractions with no renaming.

Example: Yes / Not used

$$5\tfrac{3}{4} - 2\tfrac{1}{8} \qquad\qquad 5\tfrac{1}{4} - 2\tfrac{7}{8}$$

7-9 Comment:

Limited to 2 or 3 addends. Subtraction includes renaming. Common denominators are restricted to simple cases (denominators are easy multiples).

APPLICATIONS AND PROBLEM SOLVING: To solve problems involving addition and subtraction with fractions [F4PS1]

4-6 Example:

The school record for the running long jump was $21\frac{3}{4}$ feet. Sarah jumped $19\frac{1}{2}$ feet. How far was she from the school record?

7-9 Example:

Ali drove a $2\frac{1}{2}$ inch nail through two boards. The board was $1\frac{1}{8}$ inches thick and the second was $1\frac{3}{4}$ inches thick. How close to the other side of the second board is the point of the nail?

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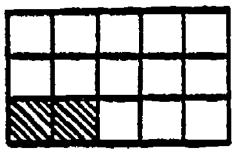
MULTIPLY/DIVIDE: To multiply and divide fractions including combinations with whole numbers

CONCEPTUALIZATION: To relate the multiplication and division operations to models and to each other [F5Cn1]

4-6 Example:

What multiplication is represented by the shaded area?



(Answer: $\frac{1}{3} \times \frac{2}{5}$)

7-9 Example:

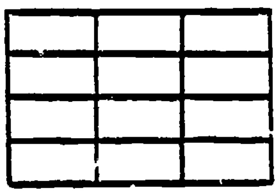
Which multiplication equation is related to this division question?

Division	Multiplication
How many $\frac{3}{4}$'s are in $1\frac{1}{2}$?	A. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ B. $2 \times \frac{3}{4} = 1\frac{1}{2}$ C. $1\frac{1}{2} \times \frac{3}{4} = \frac{9}{8}$

(Answer: B)

Use the model below to find the product of $\frac{3}{4}$ and $\frac{2}{3}$. Shade in the area which represents the product.

$\frac{3}{4}$



$\frac{2}{3}$

7-9 Comment:

Vocabulary:
Reciprocal: Two fractions whose product is one are called reciprocals. The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.

4-6 and 7-9 Comment:

Encourage students to interpret the operation.
$1\frac{1}{2}$ divided by $\frac{1}{4}$ can be interpreted as, "how many fourths are there in $1\frac{1}{2}$?"

$2\frac{3}{8}$ divided by 4 can be interpreted as showing $2\frac{3}{8}$ units in 4 equal pieces. Illustrate each with a diagram.

MENTAL ARITHMETIC: **To find a fractional part of appropriate whole numbers mentally [F5MA1]**

4-6 and 7-9 Comment:

Finding a fractional part of a whole number is limited to denominators that are factors of the whole number.

Oral activities are stressed, e.g., "What is one-third of twelve?"

The 4-6 level is restricted to unit fractions, i.e., fractions with one as the numerator.

ESTIMATION: To estimate products and quotients [F5Es1]

4-6 Example:

Estimate the product of $\frac{3}{8}$ and $\frac{15}{16}$. Will the product be closer to zero, one-half or one.

7-9 Example:

Estimate the answer to this division. Choose the best answer.

$2\frac{3}{8} \div 4$ is about...

A) 8 B) 12 C) 0 D) $\frac{1}{2}$

(Answer: D)

COMPUTATION: To find products and quotients [F5Cm1]

4-6 and 7-9 Comment:

The general rule for paper and pencil division of fractions is limited to the 7-9 level. Manipulatives and illustrations with simple fractions are used at the 4-6 level.

APPLICATIONS AND PROBLEM SOLVING: **To solve problems involving multiplication and division with fractions [F5PS1]**

4-6 Example:

A recipe that serves two people, calls for $\frac{2}{3}$ cup of flour. How much flour would be needed if you adjusted the recipe to serve eight people?

7-9 Example:

How many $\frac{1}{4}$ foot boards can be cut from $2\frac{1}{2}$ feet of board?

Draw a picture.

DECIMALS

MEANING: To demonstrate and use the meaning of decimals

CONCEPTUALIZATION: To relate decimals to models [F6Cn1]

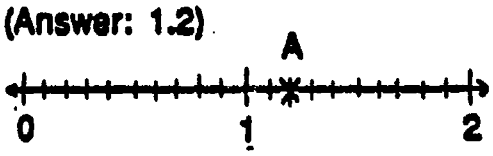
K-3 Comment:

A strip of paper folded into 10 equal parts is an effective model.

4-6 Example:

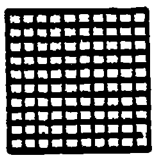
Which decimal names point A on the number line?

(Answer: 1.2)



7-9 Example:

Shade the figure to represent this decimal

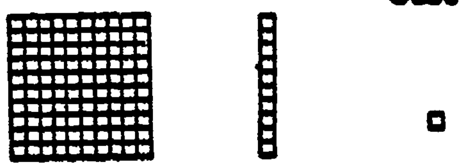


0.345

4-6 Comment:

Use base-ten blocks.

Flat	Long	Small Cube



Unit	Tenth	Hundredth

Vocabulary: *Decimal*: A numeral which uses place value and a decimal point to express a fraction. The denominator is a power of ten.

Limit grades 4-6 to thousandths.

4-6 and 7-9 Comment:

Use square regions and number lines.

CONCEPTUALIZATION: **To use place value and to read and write decimals to thousandths [F6Cn2]**

4-6 Example:

Use oral dictation exercises to help students write decimals. e.g., Write these decimals: "2 tenths 6 thousandths" (0.206); "fifty-three hundredths"(0.53); "fourteen tenths" (1.4).

7-9 Example:

What is the place value of the position indicated by the arrow and what does the digit mean?

↓
7.583

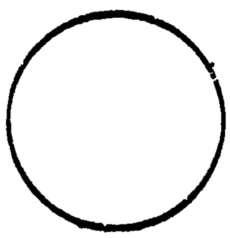
(Answer: The 8 is in the hundredths place and means 0.08, eight hundredths).

ESTIMATION: To estimate decimals using whole numbers and models [F6Es1]

4-6 Example:

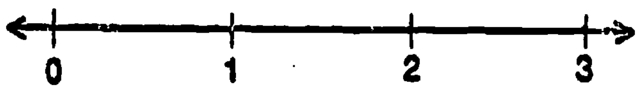
Shade this amount of the circle. Estimate.

0.23



7-9 Example:

Place these decimals on the number line at their approximate locations.



0.3 2.5 3.09 .75 .903 1.5112

ESTIMATION: To round decimals to a given place [F6Es2]

4-6 Example:

Round these numbers to the nearest whole number.

4.06 0.83

Round these numbers to the nearest tenth.

0.62 0.98

7-9 Example:

Round these numbers to the nearest hundredth.

0.037 0.296 0.423

Round these numbers to the nearest thousandth.

34.0124 0.8097

APPLICATIONS AND PROBLEM SOLVING: **To solve problems involving the meaning of decimals [F6PS1]**

4-6 and 7-9 Comment:

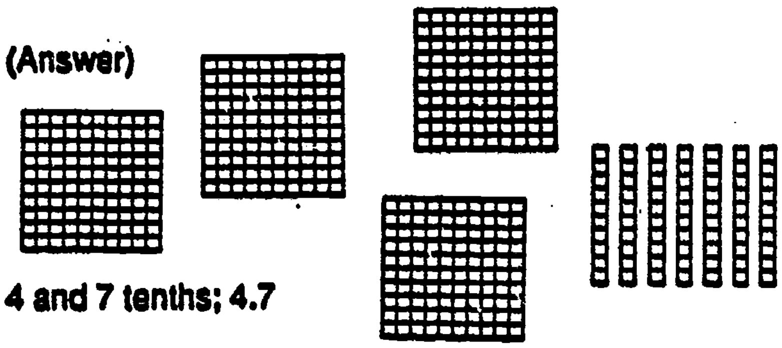
Interpret money and metric measurements as decimals.

4-6 Example:

Have students represent the decimal using an appropriate model or drawing a diagram, say the name of the decimal, and write the symbol.

David has 4 dollars and 7 dimes.

(Answer)



4 and 7 tenths; 4.7

4 flats and 7 longs

7-9 Example:

Bob saw a sign advertising gasoline for 87.9 cents per gallon. What decimal names this number in dollars?

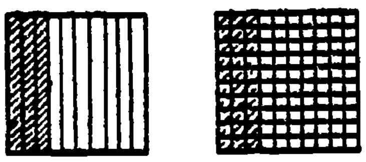
EQUIVALENT DECIMALS: To find equivalent decimals using models and generalizations for equivalent decimals

CONCEPTUALIZATION: **To identify equivalent decimals using models and generalizations for equivalent decimals [F7Cn1]**

4-6 Example:

Show 0.3 and 0.30 using models. "What can be said about the comparison of these two numbers?"

(Answer: They both show the same amount.)



7-9 Example:

Which pair of decimals is equivalent?

a. 0.07 and 0.007
b. 0.7 and 0.700

a. Five tenths and 0.50
b. Fifty hundredths and 5 thousandths.

4-6 Comment:

Limited to thousandths.

Vocabulary: *Equivalent Decimals:* Decimals which name the same number.

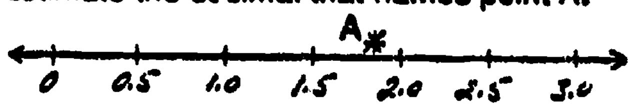
4-6 and 7-9 Comment:

Emphasize interpreting a decimal in more than one way using place value, e.g., 0.35 first as 3 tenths 5 hundredths and then as 35 hundredths.

ESTIMATION: **To use equivalent decimals to make estimates using models or using decimals** [F7Es1]

4-6 and 7-9 Example:

Estimate the decimal that names point A.



(Answer: About 1.8)

APPLICATIONS AND PROBLEM SOLVING: **To solve problems with equivalent decimals [F7PS1]**

4-6 Example:

The stopwatch showed that Laura's time in the 100 meter race was 12.030 seconds. Which time below shows the same as 12.030?

12.3 seconds or 12.03 seconds

7-9 Example:

Amber walked 0.7 kilometers.
Trevor walked 0.07 kilometers.
Marie walked 0.700 kilometers.
Who walked the same distance?

CALCULATORS: **To interpret calculator displays for decimal equivalents [F7Ca1]**

4-6 Example:

Multiply 6 x $1.25 on a calculator. What does the display show?

(It shows 7.5 instead of 7.50. Help students interpret this as $7.50 and to realize that most calculators do not show ending zeros to the right of the decimal point.)

7-9 Example:

Suppose that a calculator display does not show zeros to the right of the decimal point. How would such a calculator show 18 dollars and thirty cents? (Answer: 18.3)

COMPARE/ORDER: To compare and order decimals

CONCEPTUALIZATION: **To compare or order decimals using concrete models, word names or decimal symbols [F8Cn1]**

K-3 Example:

Use a paper strip divided into tenths as a model.

Show one and six tenths; eight tenths; one and three tenths Arrange these in order from smallest to largest.

4-6 Example:

Using base-ten blocks, let the flat represent one unit, the long represent one tenth, and the small cube represent one hundredth. Show 3 tenths and and 28 hundredths. Which is larger?

Show 8 hundredths, 25 hundredths; one and 4 hundredths; 2 tenths. Arrange these decimals from smallest to largest.

ESTIMATION: **To estimate decimals using easily recognized fractions [F8Es1]**

4-6 Example:

Circle the decimals that are close to zero. 0.13 0.62 0.7 1.003 .002
Circle the decimals that are close to one-half. 0.49 0.8 0.05 0.54 0.23
Circle the decimals that are close to one. 1.93 1.06 0.97 0.4 1.2

7-9 Example:

Which fraction could be used to estimate this decimal, 0.3174?

Choices: $\frac{1}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{3}$

(Answer: $\frac{1}{3}$)

APPLICATIONS AND PROBLEM SOLVING: **To solve problems involving comparing or ordering of decimals [F8PS1]**

4-6 Example:

Use the digits 4, 7, and 3 to write the greatest number that is possible. Now write the smallest number that is possible.
(Hint: You are allowed to use a decimal point.)

(Answer: greatest 743;
smallest 0.347)

7-9 Example:

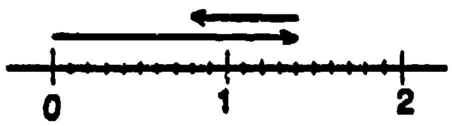
It snowed 9.3 cm on Monday, 9.08 cm on Tuesday, and 8.764 cm on Wednesday. On which day did it snow the most?

ADD/SUBTRACT: To add and subtract decimals

CONCEPTUALIZATION: **To relate the addition and subtraction operations to models and to each other [F9Cn1]**

4-6 Example:

Which operation is shown on the number line?

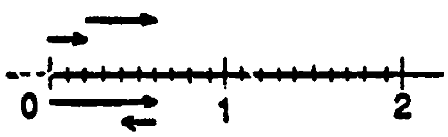


(Answer: 1.4 - 0.6)

Use base-ten blocks to show 2 and 6 tenths, then show 3 tenths. Now put these two numbers together to get 2 and 9 tenths.

7-9 Example:

Use models to explain the inverse relationship between addition and subtraction.



0.2 + 0.4 = 0.6
0.6 - 0.4 = 0.2

MENTAL ARITHMETIC: To add and subtract selected decimals mentally [F9MA1]

4-6 Example:

Use oral dictation exercises.
Ask questions such as:
What is four plus 6 tenths?

(Answer: 4.6)

What is seven minus three tenths?

(Answer: 6.7)

7-9 Example:

Use oral dictation exercises.
Ask questions such as:
What is 7 and 7 tenths minus 4 and 3 tenths?

(Answer: 3.4)

ESTIMATION: To estimate sums and differences [F9Es1]

4-6 Example:

Encourage students to use front-end estimation or rounding. Estimate the answer to 8 + 2.73 + 0.12 = ?

(Answer: About 10 or 11)

7-9 Example:

Place the decimal point in each answer to make it correct (the answers have been rounded).

6.8338261 + 96.6801624 = 10351

(Answer: 103.51)

4.43114908 - 2.21268214 = 22185

(Answer: 2.2185)

COMPUTATION: To add and subtract decimals [F9Cm1]

4-6 and 7-9 Comment:

Paper and pencil computation parallels the limits used with whole numbers. Calculators are used for larger multi-digit computations. Limit grades 4-6 to whole numbers, tenths, and hundredths.

8C

APPLICATIONS AND PROBLEM SOLVING: **To solve problems involving addition and subtraction of decimals [F9PS1]**

4-6 Example:

Pat is 1.7 meters tall. Daniel is 1.4 meters tall. How much taller is Pat than Daniel?

Amber rode her bike 1.3 km to the park, then 3 km to the store, and then 2.4 km home. How far did she ride her bike?

7-9 Example:

Alan hiked 7.58 km in the morning. Then he hiked 12.6 km in the afternoon. How far did he hike?

4-6 and 7-9 Comment:

Calculators may be used to perform the computations involved in these problems.

CALCULATORS: To add and subtract decimals [F9Ca1]

4-6 and 7-9 Comment:

Use numbers which contain more digits than those used for paper/pencil or mental computations. Do not present the exercises in place-value column alignment.

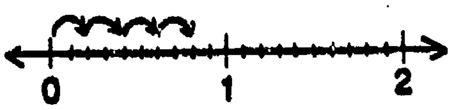
Example: Find the difference: 15 minus 0.98721

MULTIPLY/DIVIDE: To mulitply and divide decimals

CONCEPTUALIZATION: **To relate the multiplication and division operations to models and to each other [F10Cn1]**

4-6 Example:

What multiplication does the number line show?

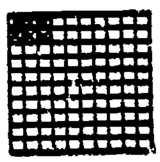


(Answer: $4 \times 0.2 = 0.8$
or
Using base-ten blocks, show 4 groups of 2 tenths each. How much is this altogether?)

7-9 Example:

$0.2 \times 0.3 = ?$

Build a rectangle that is 2 tenths wide by 3 tenths long on this 10 x 10 square that represents one unit.



0.3 divided by 0.06 = ?

Ask "How many groups of 6 hundredths are there in 3 tenths?" Use three longs to represent 3 tenths and measure out groups of 6 hundredths.

CONCEPTUALIZATION: **To relate equivalent expressions for the operations, including multiplication of a whole number and a decimal [F10Cn2]**

4-6 Example:

What expression means the same thing as:

$0.2 + 0.2 + 0.2 + 0.2$

(Answer: 4 times 0.2)

7-9 Example:

Which expression is equivalent to this equation?

0.6 divided by 0.02 = N

(Answer: 0.02 times some number, N, is equal to 0.6.)

MENTAL ARITHMETIC: To multiply and divide with decimals and powers of ten [F10MA1]

4-6 Example:

What is 1.2 x 10? What is 46.8 x 100? What is 27 ÷ 10? What is 68.9 ÷ 100?

7-9 Example:

What is 6 x .1?
What is 4 x .01?
What is 9 x .5?
What is 2 x .07?

4-6 Comment:

Limit to multiplying and dividing by 10, by 100, and by 1000, and by multiples of these numbers.

7-9 Comment:

Limit to multiplying by 0.1, by 0.01, by 0.001, and by multiples of these numbers.

ESTIMATION: To estimate products and quotients [F10Es1]

4-6 Example:

15.1 x 3.02 is about 4.5 or 45 or 450?

1.63 ÷ 3.8 is about 0.4 or 4.0 or 40?

7-9 Example:

Place the decimal point in each answer to make it correct (the answers have been rounded).

2.12491781 x 90.6528926
= 192629

(Answer: 192.629)

23.6053759 ÷ 3.52169911
= 670283

(Answer: 6.70283)

COMPUTATION: To multiply and divide decimals up to thousandths [F10Cm1]

4-6 Comment:

Use paper and pencil methods to solve problems such as;

4.3 x 0.7 = ?
0.15 x 0.3 = ?
18.16 ÷ 2 = ?
0.63 ÷ 0.7 = ?

CALCULATORS: To find products and quotients [F10Ca1]

4-6 and 7-9 Comment:

Calculators are used for multiplication and division with multi-digit decimals. Paper and pencil computation will be used for exercises in which the numbers are somewhat more difficult than can be handled by mental computation.

APPLICATIONS AND PROBLEM SOLVING: To solve problems involving multiplication and division of decimals [F10PS1]

4-6 Example:

One kilogram of hamburger costs $1.19. How much will 2.7 kilograms cost?

7-9 Example:

Suppose you need 78.75 meters of wire. The wire comes 12.6 meters per spool. How many spools of wire do you need? (Include part of a spool if needed.)

(Answer: 6.25 spools)

4-6 and 7-9 Comment:

Calculators may be used to perform the computations involved in these problems.

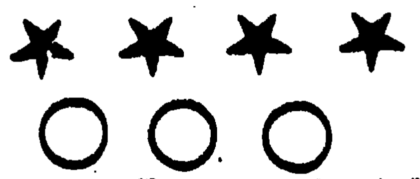
RATIO AND PROPORTION

RATIO: To use ratio in practical situations

CONCEPTUALIZATION: To determine ratios from models that are part-to-part, part-to-whole, or rates and recognize verbal expressions for ratio [F11un1]

4-6 Example:

What is the ratio of stars to circles?



(Answer: 4 to 3 or $\frac{4}{3}$).

Vocabulary: *Ratio*: A relationship between two numbers which can be expressed as a fraction.

7-9 Comment:

Use a variety of verbal expressions for ratios, e.g.,

5 to 8 5 out of 8 5 per 8
5 for every 8

Use a variety of rates: e.g., Mile per hour, words per minute, cents per pound, revolutions per minute, kilograms per liter, beats per minute.

APPLICATIONS AND PROBLEM SOLVING: **To solve problems involving ratios [F11PS1]**

4-6 Example:

Team	Won	Lost	Games Back
Pistons	40	20	
Hawks	37	24	$3\frac{1}{2}$
Bulls	32	28	8

What is the ratio of wins to total games played for the Pistons?

(Answer: 40 to 60. Lowest terms fractions not required.)

7-9 Example:

Suppose you typed 5 pages of an essay. There were 750 words and it took you 30 minutes. What rate of words to minutes did you type?

(Answer: $\frac{\text{25 words}}{\text{1 minute}}$)

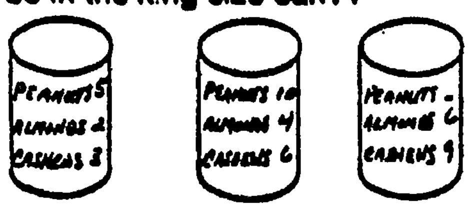
Vocabulary: *Rate:* A ratio which compares two quantities which are expressed in different units.

EQUIVALENT RATIOS/PROPORTIONS: To identify and find equivalent ratios

CONCEPTUALIZATION: **To demonstrate the meaning of equivalent ratios using models or practical situations [F12Cn1]**

7-9 Example:

Each size container of mixed nuts is to have the same ratio of peanuts to cashews. How much peanuts should be in the king size can?1



Regular Large King

(Answer: 15)

7-9 Comment:

Use tables.

Example:

Tires	5	10	15	20
Cars	1	2	3	?

COMPUTATION: To find equivalent ratios and solve proportions [F12Cm1]

7-9 Example:

Solve this proportion

$\frac{N}{3} = \frac{100}{2}$

7-9 Comment:

This is similar to finding equivalent fractions where solutions to equivalent ratios are whole numbers. Use cross-products to work with ratios where the second and fourth terms are not multiples or factors of each other.

Vocabulary: *Proportion*: A statement that two ratios are equal.

APPLICATIONS AND PROBLEM SOLVING: To solve proportion problems [F12PS1]

7-9 Example:

An 8 ounce cup of yogurt costs \$1.28. What is the cost per ounce?

(Answer: \$0.16)

7-9 Comment:

Practical situations involving rates or scale drawings are amphasized.

Calculators may be used to perform the computations involved in these problems.

CALCULATORS: To solve proportions with larger numbers or proportion problems with more difficult computation [F12Ca1]

7-9 Example:

Use your calculator to solve this proportion:

$\frac{N}{21} = \frac{30,422}{34,440}$

(Answer: N = 18.55)

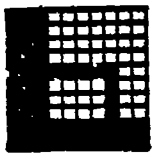
PERCENT

MEANING: To demonstrate the meaning of percent as a ratio whose second term is 100

CONCEPTUALIZATION: To use models to represent percents [F13Cn1]

4-6 Example:

State the indicated percent for the square.



area shaded _______%

area not shaded _______%

7-9 Example:

Your reference set is 100 unit cubes. Build a model and state what percent of the 100 cubes you use in each case.

A) 4 cubes long, 4 cubes wide, 4 cubes high.
B) 10 cubes long, 5 cubes wide, 2 cubes high.

Another example: Let a meter (100 cm) be your reference set. What percent of a meter is each of the following:

A) The length of a piece of chalk which is 5 cm.
B) Your height which is 120 cm.

Vocabulary: *Percent*: A ratio of one number compared to 100. The symbol for percent is %.

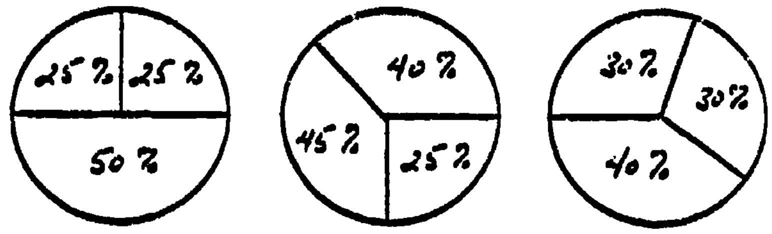
4-6 and 7-9 Comment:

A formal introduction to percents usually occurs in grade 7 or 8. However students are exposed to and encounter percents in many everyday situations such as free-throw averages in basketball and percent of students who are boys, etc. The experience at this level should rely on the 100 grid as the basic model. Percent is defined as a special fraction whose denominator is 100. The concept of percent can then be transferred to other models such as 100 cents, 100 days, or 100 cm for the later grades

APPLICATIONS AND PROBLEM SOLVING: **To use the meaning of percent in solving practical problems [F13PS1]**

4-6 Example:

Two of these circle graphs have sensible percents. Find the circle with percents that are wrong. Explain why these percents are impossible.



Suppose you pay a tax of 9 cents on the dollar. What percent of tax are you paying?

7-9 Example:

What percent of a dollar is 5 dimes? 150 cents? 3 quarters and 4 pennies? 6 nickels and two half-dollars?

What are the fewest number of coins needed to make 20% of a dollar? 137% of a dollar?

PERCENT, FRACTION, DECIMAL EQUIVALENTS:
To express ratios as percents, fractions, or decimals and to relate each form to the other two

CONCEPTUALIZATION: **To recognize equivalent expressions involving selected fractions, decimals and percents using models or easily recognized fractions [F14Cn1]**

4-6 and 7-9 Comment:

For level 4-6 only simple fractions, decimals and percents (denominators are factors of 100) should be used. The emphasis should be that there are three equivalent ways to express a ratio. The 100 grid is an excellent model for establishing this relationship. At level 7-9 more complicated ratios can be used as well as other models.

4-6 Example:

In which of the following sets does each of the numbers represent the same number?

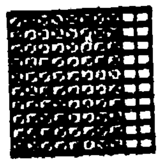
$\frac{1}{2}$ 50% .05

25% $\frac{1}{4}$ .25

.10 $\frac{1}{10}$ 1%

7-9 Example:

Circle all of the numbers below which represent this shaded area.



80% 0.80 0.08

$\frac{4}{5}$ $\frac{8}{10}$ 8%

MENTAL ARITHMETIC: **To use easily recognized fractions and give fraction, decimal and percent equivalents [F14MA1]**

7-9 Comment:

For both levels the appropriate evaluation is oral examination or questions dictated orally with students given a limited amount of time to write the answers.

ESTIMATION: **To estimate equivalents for fractions, decimals and percent using easily recognized fractions [F14Es1]**

7-9 Example:

Which of the following numbers is closest to $\frac{1}{2}$?

$\frac{245}{500}$ 59% 0.501

(Answer: 0.501)

CALCULATORS: To express any ratio as a percent or decimal [F14Ca1]

4-6 Comment:

Use ratios with decimals that terminate, as tenths or hundredths.

Examples: $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{5}$

7-9 Example:

Use a calculator to help fill in the chart. Round your answers to the nearest tenth.

Fraction	Decimal	Percent
$\frac{21}{32}$	____	____
$\frac{512}{115}$	____	____
$\frac{7}{215}$	____	____

APPLICATIONS AND PROBLEM SOLVING: To solve problems using fraction, percent and decimal equivalents [F14PS1]

4-6 Comment:

Use verbal situations of a practical nature.

7-9 Example:

Circle the answers that have the same meaning as 75%.

5 no's for each 6 votes;
15 out of 50;

75 cents out of a dollar; 3 out of 4; 150 compared to 200

75 hits for 100 times at bat; three quarters for each dollar.

Who has the better free throw percentage?

Mike: 49 out of 79 tries
Sue: 53 out of 70 tries
Terry: 62 out of 84 tries

USING PERCENT: To find a percent of a number

CONCEPTUALIZATION: **To recognize and use the meaning of percent in finding either the part (percentage) or the whole (base) when the percent (rate) is given [F15Cn1]**

4-6 Comment:

All percent problems involve a proportion of two equivalent ratios. One ratio always has a denominator of 100. The two equivalent ratios can be thought of as $\frac{A}{100} = \frac{B}{C}$, and we are always looking for one of the missing parts.

For the 4-6 level only simple percents such as 25%, 50% will be used. The two equal ratios should be easy to compute.

7-9 Comment:

In the case of finding a percent of a number such as 20% of 25, this can be thought of as:

$$\frac{20}{100} = \frac{B}{25}$$

so B = 5 and it can be found by using equivalent fractions. This problem, 20% of 25, can also be related to multiplication of fractions or decimals:

20% of 25 is equivalent to $\frac{20}{100} \times 25$

20% of 25 is equivalent to .20 x 25.

MENTAL ARITHMETIC: To find selected percents of a number mentally [F15MA1-3]

7-9 Comment:

Evaluation is best done orally with the teacher asking students to calculate 50% of 200 or 25 out of 50 is what percent. Students will be given a limited amount of time to respond. Selected percents used include the following:

a. 1%, 10%, 50%, 100%
b. 200%, 300%, and other multiples of 100%
c. 5%, 15%, 20%, 25%

ESTIMATION: **To estimate the percent of a number using easily recognized fractions [F15Es1]**

7-9 Example:

Which is closest to 27% of 840?

$\frac{1}{2}$ of 840

$\frac{1}{4}$ of 840

$\frac{1}{7}$ of 840

$\frac{1}{10}$ of 840

Estimate your total bill for purchasing a video if the cost is $216.95 plus a 4% sales tax?

CALCULATORS: To find a percent of a number [F15Ca1]

7-9 Example:

Use a calculator. Find 17 $\frac{3}{4}$ percent of $16,724.

Round your answer to the nearest whole number of dollars.

(Answer: $2969)

APPLICATIONS AND PROBLEM SOLVING: **To solve percent problems, including percent of increase or decrease [F15PS1]**

7-9 Example:

An appliance store bought a refrigerator for $475 and marked it up 40% to sell. What was the selling price?

(Answer: $665.00)

7-9 Comment:

A calculator may be used to perform the computations involved in these problems.

Framework:
Michigan Mathematics Objectives

Mathematical Processes

Mathematical Content Strands	Conceptualization	Mental Arithmetic	Estimation	Computation	Applications and Problem Solving	Calculators and Computers
Whole Numbers and Numeration						
Fractions, Decimals Ratio and Percent						
Measurement						
Geometry						
Statistics and Probability						
Algebraic Ideas						
Problem Solving & Logical Reasoning						
Calculators						

M

MEASUREMENT: AN OVERVIEW

What is included in this strand?

The measurement objectives are in two major groups. One is Length, Area, Volume, Angles. The other is Capacity, Mass, Time, Temperature, Money.

Man's experience with measurement has developed from use of non-standard units to comprehensive systems of measures. Non-standard units may come from nature (stones, twigs, ropes with knots, etc.) or from parts of a person's body (hands, feet, strides, arm spans, etc.) In the United States there are two systems in common use, the customary system (feet, gallons, pounds, degrees Fahrenheit) and the metric system (meters, liters, grams, and degrees Celsius). Our schools must provide students with instruction on the metric system and selected customary units. Since our industry, commerce, and society are moving to join the world by being metricated, only knowledge of the metric system, not the customary system, will be evaluated on the Michigan Educational Assessment Program.

How does the development flow?

The students first will learn the basic concepts of each of the various types of measurement. For example, what do we mean by length? by liquid capacity? At the same time they are learning the basic concepts, they are acquiring names of many different units of measure and their symbols. After this basic introduction, the students learn to use and read a number of different scales which give them the measures. Then there are objectives which involve applications and problem solving.

There is another flow within a number of the objectives. The precision of measurements increases as the student goes through the grades from K to 9. For example, in K-3 measurements of length are made with a centimeter scale marked only with whole centimeters, while in 4-6 the scale has millimeter markings. As students' skill with numbers progresses from whole numbers to decimals, they are expected to express measures with decimals.

Why teach these objectives?

Measurement is a very important tool in our lives. Measurements are made as we work, as we play, as we grow, as we feed ourselves, as we clothe ourselves, as we house ourselves, and as we enter into trade or commerce with others.

As our students progress in school with studies in science, technical subjects, art, and music as well as higher mathematics, they will need to understand and use measurement. Then as they become adults, they will use these skills in many careers - engineering, designing clothes, transportation, construction, landscaping, meteorology, and finance to name just a few. In addition, these skills are needed for personal reasons - cooking, handling finances, and recreation.

What are the implications for instruction?

Instruction should be provided to each student so that the concepts of measurement can grow from non-standard units to the standard units.

This calls for many hands-on experiences even before a measuring device is used. There are many sets of manipulatives that can be procured for this purpose, but items found in the classroom (pencils, sticks, paper squares, paper clips, etc.) can also be used. The main thing is that students at any grade level must be actively involved in the process so that they can grasp the concepts. As the students learn, instruction can progress from manipulating concrete items to viewing and drawing pictorial representations and finally to using models and formulas to apply the concepts to problem solving.

As students are learning about the metric system, the teacher should help them identify parts of their bodies that are an approximation of the units. The small finger width may be used for a centimeter, the hand width or span from tip of thumb to tip of first finger for decimeter, and distance from floor to hips for a meter.

Teacher and student should use the terminology appropriate for measurement and the proper names and symbols for the units. There should be ample opportunities so that students can say the vocabulary of measurement aloud, not just read it on paper. It helps in the learning of measurement concepts (as in all of mathematics) to verbalize the process and give proper names to the result. It should be understood that the measurement objectives do involve more than an arithmetic computation.

After the basic concepts of whole units are learned, the students must be helped to apply concepts of powers of ten and decimals to measurement. Fortunately, the same ideas that apply to the numeration system do apply to the metric system. As they learn to multiply or divide by 10, 100, or 1000, they can change from meters to centimeters or from grams to kilograms. As they learn about rounding off numbers, they can be helped to transfer that idea to precision of measurement. Knowledge and use of measurement concepts can strengthen learning of decimals, and knowledge and use of decimals can strengthen the learning of measurement.

As the student masters measurement and the metric system, the interrelationship of the cubic units, capacity units, and the mass units must be presented. The cubic unit - one cubic decimeter - is the same as the capacity unit - one liter. The mass (weight) of one cubic centimeter of water at sea level is 1 gram.

Teachers and students should 'think' within the metric system. Conversion between the customary and metric units will not be done.

Vocabulary

The metric system uses prefixes to apply to the basic units of gram, liter, and meter to get other units equal to 0.001, 0.01, 0.1, 10, 100 and 1000 times the basic unit. However we do not use all of the prefixes. The ones that are used most frequently are listed below. Students should have knowledge of them for testing on MEAP.

Length	Capacity	Mass (Weight)
meter - m	liter - L	gram - g
millimeter - mm	milliliter - mL	milligram - mg
centimeter - cm	kiloliter - kL	kilogram - kg
decimeter - dm		
kilometer - km		

Measurement Vocabulary Words

K-3

a.m.
area
Celsius
centimeter - cm

cubic unit
degree
dimension
distance
gram - g
kilometer - km
kilogram - kg
length
linear unit
liter - L
measurement
meter - m
metric
monetary value
money
p.m.
rectangular box
ruler
scale
thermometer
unit
volume
weight
width

New in 4-6

decimeter - dm
elapsed time
estimate
kiloliter - kL
milliliter - mL
millimeter - mm
milligram - mg
perimeter
protractor

New in 7-9

circumference
pi
precision
surface area

Resources

NCTM 34th Yearbook - Instructional Aids in Mathematics (pp. 286-7 list measurement devices with procurement sources.)

NCTM 37th Yearbook - Mathematics Learning in Early Childhood - Chapter 10. This chapter is worthwhile for teachers at all grade levels K-9

Creative Publications, Activity Resources, Dale Seymour, and other similar sources can provide measurement devices for demonstration and student manipulation; such as rulers, tapes, tiles, protractors, clocks, play money, 2 cm cubes (for volume), balance and metric weights, trundle wheel, beakers (or other metric liquid containers, bathroom scales.)

MEASUREMENT: THE OBJECTIVES

LENGTH, AREA, VOLUME, ANGLES: To measure length, area, volume and angles

CONCEPTUALIZATION: **To identify and describe the concept of length and the relative sizes of the standard units [M1Cn1]**

K-3 Comment:

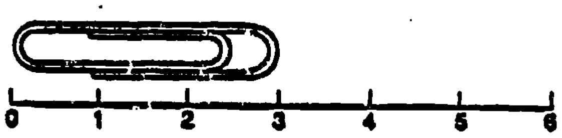
Non-standard units, centimeters, meters, and kilometers will be used to state measures of familiar items to whole units. Although customary units (inches and feet) will still be discussed in the classroom, only metric measures will be used on the test. Centimeter rulers used will be in whole units (no millimeters.) For this objective the student will not be required to place a centimeter ruler on a figure to find a measurement. Instead the ruler will be used by the teacher or shown on a figure.

By comparing lengths students should select the one that is the longest, shortest, or if they are the same length.

The teacher should help each student relate the approximate size of a centimeter by using a body part, possibly the small finger.

K-3 Example:

On the centimeter scale below, find the length of the paper clip to the nearest whole unit.



(Answer: 3 cm)

4-6 Comment:

To measure familiar items only mm, cm, dm, m, and km should be used. Centimeter scales should be marked with millimeters, etc. Any scale over 10 cm should have markings so that decimeters are easily recognized.

The student should be able to answer questions about the relative sizes of the linear units and the relative sizes of lengths of a set of items (less than, greater than, same length, smaller, largest.) The students should relate dm and m to body parts.

The students should be able to select appropriate metric units to measure lengths of items familiar to students.

CONCEPTUALIZATION: To identify and describe concepts of area, perimeter, volume, and angle measure [M1Cn2]

4-6 Comment:

A centimeter or non-standard grid will be used to find the area of a rectangle. A centimeter or non-standard scale will be used to find the perimeter of a rectangle.

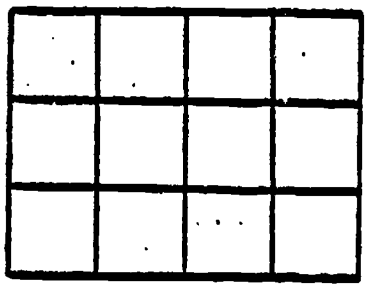
Classroom demonstration involving student participation filling boxes with blocks should be used to learn the concept of volume. Then pictures of rectangular solids marked to suggest centimeter cubes may be used. Discuss how many blocks in a layer, how many layers, and how many blocks in all. Then ask for volume with correct units.

Teach the use of "square unit", "cubic unit", and superscripts for area and volume. (cm^2, cm^3)

For testing of this objective, the centimeter scale, grid, or blocks will be shown on the figure.

4-6 Example:

Find the perimeter of the rectangle to the nearest centimeter.



(Answer: 14 cm)

7-9 Comment:

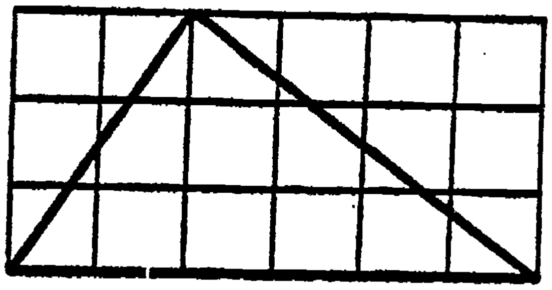
See discussion for 4-6 for developing concept of volume.

A centimeter grid will be shown on the figures in finding areas of rectangles or triangles. Centimeter marks on a triangle or rectangle will be shown to find perimeter.

From figures of angles, the student should be able to determine the largest or smallest and be able to put them in order by size. No actual measuring is needed for this objective.

7-9 Example:

Determine the area of the triangle by counting the squares which cover it.



(Answer: 9 cm^2)

CONCEPTUALIZATION: To distinguish among situations which call for measuring length, area or volume [M1Cn3]

4-6 Comment:

For this objective, the students will not actually find length, area or volume. Students will consider actual familiar situations and determine whether length, area or volume is required.

The student should select from units, such as cm, cm^2, cm^3, or mm, cm, m, km the appropriate one for a given situation.

4-6 Example:

Give the metric unit used to state the distance from Grand Rapids to Lansing.

(Answer: km)

7-9 Example:

Mr. Jones is going to fence his rectangular backyard which extends 20 meters behind his house and is 30 meters wide. His house, which is 15 meters long, will serve as part of the fence. How much fence does Mr. Jones need?

What do you need to find to solve this problem, area, volume, or perimeter?

(Answer: Perimeter)

CONCEPTUALIZATION: To identify and describe concepts of circumference and surface area [M1Cn4]

7-9 Comment:

Students should participate in discussions about circles. Running a finger around the 'rim' of the circles will help with understanding circumference while moving a finger in the interior of the circle suggests area. The terms diameter and radius need to be known.

This objective does not require any computations for circumference or area of circles. Students need to be able to determine whether a practical situation calls for finding area or circumference.

In discussing pi (π) be sure that students understand that it is the ratio of the circumference to the diameter and that pi is a fixed number, the same for circles of all sizes.

Models of solids, such as cans or cereal boxes, may be used for discussion so that students may learn about surface area. Practical uses of surface area, such as painting a room or house, applying wall paper, etc., should be introduced.

7-9 Example:

Jan is going to paint the bottom of her circular pool. Before she can find how much paint to buy, what does she need to compute area or circumference?

(Answer: Area)

CONCEPTUALIZATION: To determine the length of an object or a line segment with an appropriate unit, and a standard measuring instrument using hands-on activities [M1Cn5]

K-3 Comment:

Actual items found in the classroom or pictures of familiar items that suggest line segments should be used for students to practice measuring length.

The centimeter ruler used should be marked with whole centimeters, not millimeters. Students need help with the placement of the ruler next to the item to be measured so that the starting point of the ruler and the starting point of the item line up.

K-3 Example:

Find the width of the mathematics book.

(Answers will vary)

4-6 Comment:

The units for 4-6 are mm, cm, dm, and m. Scales to be used are centimeter (marked with millimeters) ruler, meter stick, or tape.

Items to be measured should be familiar to students. Hands-on experience is essential so that students are comfortable placing the measuring device and reading the measurement to the nearest unit.

4-6 Example:

Use your centimeter ruler to measure this line segment to the nearest centimeter.

(Answer: 9 cm)

CONCEPTUALIZATION: To measure area (square units) and volume (cubic units) by the process of covering, filling, and counting, and to recognize the relative size of standard units [M1Cn6]

4-6 Comment:

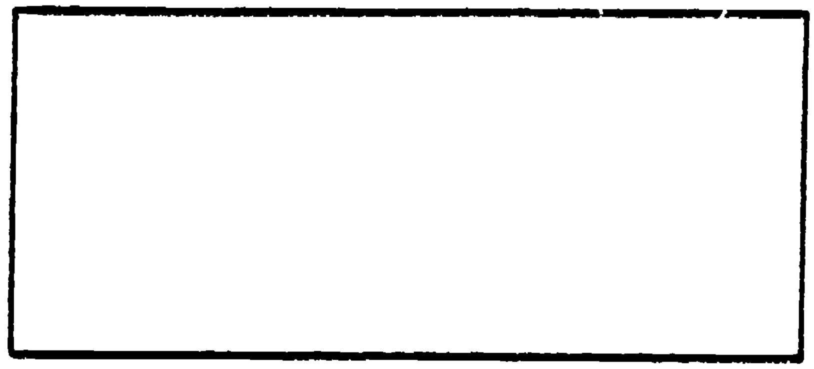
There should be hands on experience with laying square unit pieces of paper on a figure and counting to find the area. In the testing situation there will be a clear plastic centimeter grid to lay over a figure to assist in counting to find the area. Experience in the classroom with such a device will help to prepare the student.

There should be hands on experience filling a box with cubes and counting to find the volume. Limit volume problems to boxes that require up to 50 cubes.

Student activities with "covering" and "filling" should lead to development of the formulas for area of a rectangle and volume of a rectangular solid.

4-6 Example:

Find the area of the rectangle by using your centimeter grid.



(Answer: 36 cm^2)

7-9 Comment:

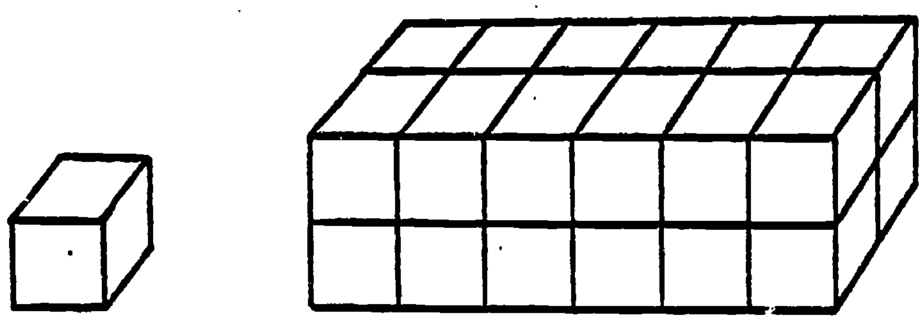
Each student should actually cover a rectangle with squares of paper or cardboard, count to find the area, and then state the area with the correct units.

Volume of a rectangular solid is determined by filling the box with cubes, counting. The volume should be stated with correct units.

There should be classroom discussions with hands-on activities so that every student will be familiar with the names and relative sizes of linear, square and cubic standard units.

7-9 Example:

Find the volume of this box.



(Answer: 24 cm^3)

CONCEPTUALIZATION: To measure a given angle and to draw an angle of a given size [M1Cn7]

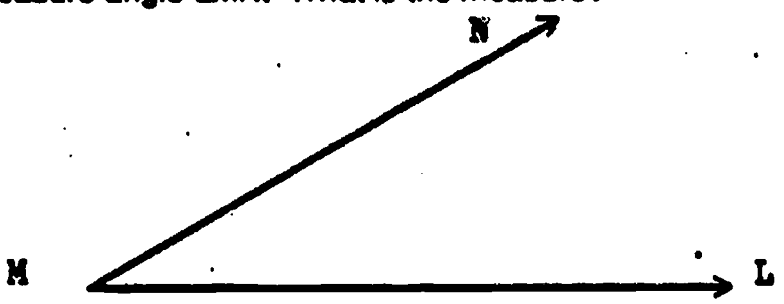
7-9 Comment:

It is essential to have hands-on experience with a protractor. Proper placement of the device on an angle and the reading of the measure of angle is important. Every student should measure angles of various sizes and placement on the page including those which are part of a larger geometric figure. Students should know what is meant by various names for angles (for example, one letter for vertex and three letters for side, vertex, side.) Students should have experience drawing angles of given measures. In the testing situation the student will use a protractor to measure an angle and to draw an angle of specific size.

Discuss the terms obtuse angle, acute angle, right angle, straight angle, complementary angles, supplementary angles. Although these are geometry objectives, wording of test problems may involve these vocabulary words. The student should understand what is meant by the sum of the angles of a polygon.

7-9 Example:

Use a protractor to measure angle LMN. What is the measure?



(Answer: 30°)

CONCEPTUALIZATION: To read various scales such as rulers and protractors [M1Cn8]

K-3 Comment:

There should be hands-on experience using a centimeter ruler to read the length of a segment to the nearest cm. As classroom activity, using a meter stick, the student should measure a length less than 1 meter. In the testing situation the student will use a centimeter scale to measure pictures of items to the nearest centimeter.

4-6 Comment:

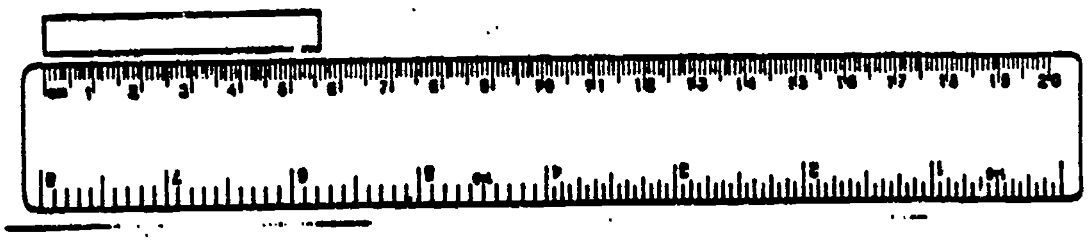
Students should use a centimeter ruler marked with millimeters to measure to the nearest millimeter. Instruction should be provided on the proper placement of the ruler on the item to be measured, and what it means to read the measure to the nearest millimeter. Measurements to the nearest decimeter may be made using a meter stick or centimeter ruler. A meter stick or metric tape should be used to measure lengths over 5 meters and less than 40 meters.

A protractor is used to measure an angle to the nearest ten degrees. Students will need help with the proper placement of the protractor on the angle and reading from the proper scale.

A centimeter ruler is used to measure the sides of polygons.

4-6 Example:

Use the centimeter ruler to find the length of the ribbon to the nearest millimeter.



(Answer: 56 mm)

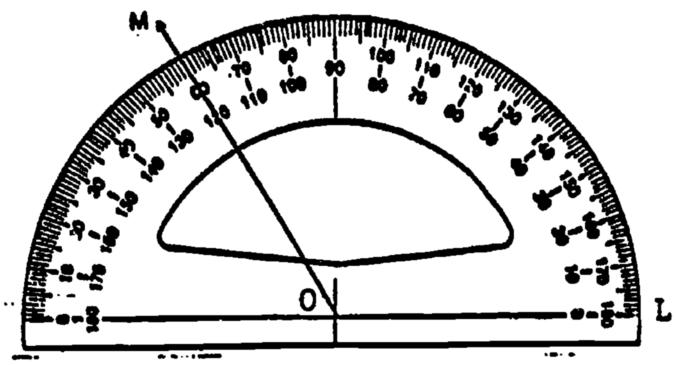
7-9 Comment:

The students should read protractors to the nearest degree.

The students should read a centimeter scale to give ___ cm ___ mm. Lengths up to 50 m can be measured using a metric tape or meter stick.

7-9 Example:

Use the protractor to measure angle LOM.



(Answer: 120°)

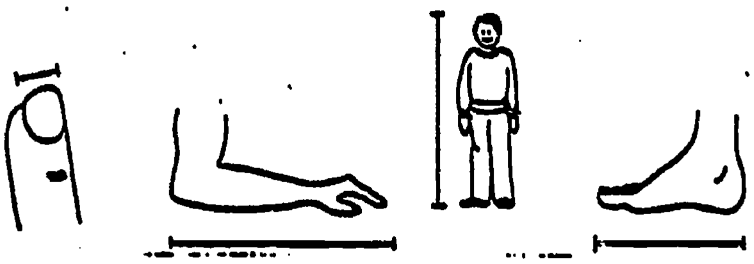
ESTIMATION: **To estimate the length of a familiar object or drawing [M1Es1]**

K-3 Comment:

Estimations of lengths should be made primarily in centimeters. Some discussion could involve meters. Classroom activities should have the students considering items for which they know the length in making estimates of other similar items. For example, if they have measured the length of a crayon or a pencil, have them consider other classroom items such as a book, a window sill, a poster, etc., making an estimate of the length of that object visually, not by using the crayon or the pencil as a scale. Everyone in the class may make estimates, then see how good their estimates were by making an actual measurement. The process can be turned around. A length can be given, for example, 25 cm. The students may then be asked to select from a set of familiar items the one that is that length.

K-3 Example:

Which of the following is a picture of an object which has a measurement of one centimeter?



A B C D

(Answer: A)

4-6 Comment:

The activities for K-3 should be extended to estimating in mm and m as well as cm.

4-6 Example:

Estimate the height of the door of your classroom.

(Answers will vary)

ESTIMATION: **To estimate the area or volume of a familiar object or drawing [M1Es2]**

4-6 Comment:

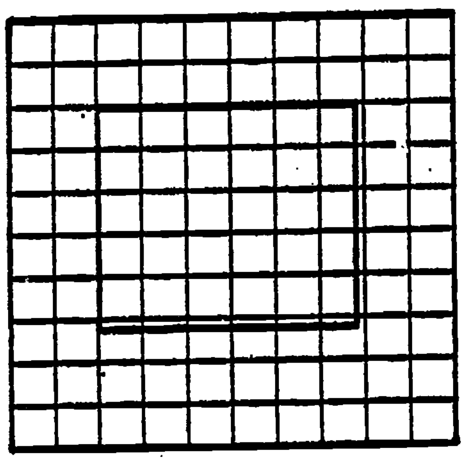
In estimating volume of a rectangular box, dimensions should not be greater than 10 units.

Figures in a drawing for area may have straight or curved sides and have either straight sides such as a rectangle or curved or jagged sides such as a drawing of a hand. A grid with squares from 1 cm to 1 inch may be shown on the figure to assist in the estimation.

Another type of estimation that students should experience is that of rounding off dimensions given for a rectangle or rectangular box and then stating the computation that would be necessary. For example, if the dimensions of a rectangle are 98 cm by 73 cm, an estimate would be given as 100 x 70 cm^2.

4-6 Example:

What is a good approximation of the area of the rectangle below?



(Answer: 30 cm^2)

7-9 Example:

Show how you would find a good approximation of the volume of a box which is 2.6 cm x 1.9 cm x 9.2 cm.

(Answer: 3 x 2 x 10 cm^3 or 60 cm^3)

ESTIMATION: **To estimate length, area, and volume using all appropriate units of measure [M1Es3]**

7-9 Comment:

Estimations of lengths should be made in mm, cm, dm, or m, as appropriate. Classroom discussions should consider estimations of the volume of a part of a house or school (airspace), volume of concrete needed for a driveway, or volume of an aquarium. The students should consider the appropriate units to use for practical estimates. For example, if you were estimating the height of a tree, would you use mm, cm, dm or m?

7-9 Example:

Mrs. Smith needs to estimate the volume of her kitchen. What would be the appropriate unit of measure for her to use?

(Answer: m^3)

APPLICATIONS AND PROBLEM SOLVING: **To determine the perimeter of an object or of a polygon [M1PS1]**

4-6 Comment:

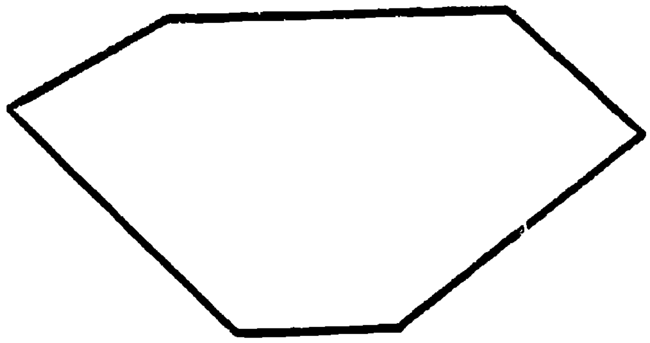
Students should use a metric scale (millimeters, centimeters and meters) to measure sides of a polygon to find perimeter. Polygons should not have more than six sides.

Perimeter of a rectangle can be found if length and width are given. This can be extended to a practical application, such as the perimeter of a room, plot of ground, or picture frame.

Students must be able to distinguish between a problem calling for perimeter and one calling for area.

4-6 Example:

Measure the sides and find the perimeter of this hexagon.



(Answer: 17 cm)

APPLICATIONS AND PROBLEM SOLVING: **To use the formula, A = L x W, to find the area of a rectangular object or drawing [M1PS2]**

4-6 Comment:

Dimensions of a rectangle may be provided by labeling or by specifying length and width. Students should understand how the formula A = L x W is developed. Dimensions should be limited to more than 4, less than 40.

Be sure that the students understand the difference between area and perimeter. Also be sure that answers are always given with correct units. In the testing situation for an area problem, one of the choices may be perimeter or the correct number with incorrect units.

4-6 Example:

Find the area of a rectangle if its length is 15 m and its width is 7 m.

(Answer: 105 m^2)

APPLICATIONS AND PROBLEM SOLVING: **To determine the circumference of a circle, the area of a geometric shape, and the volume of a cylinder or rectangular prism [M1PS3]**

7-9 Comment:

Formulas will be used to find areas of triangles, parallelograms, and circles. Formulas will also be used to find the volume of a cylinder or rectangular prism.

Students should select proper data to use for computations. For example, a triangle may have three sides and an altitude labeled; a parallelogram may have two sides and an altitude labeled; a circle or cylinder may have either a radius or a diameter labeled.

For computation in circle or cylinder problems $\pi = 3.14$ will be used. Be sure that the student gives an answer complete with appropriate units.

7-9 Example:

Mrs. Brown bought a large pail. The pail is 50 cm high and it has a diameter of 20 cm. Using $\pi = 3.14$, find the volume of her pail to the nearest whole unit.

(Answer: 15,700 cm^3)

APPLICATIONS AND PROBLEM SOLVING: **To use a formula to relate lengths, areas, and volumes. (For example, to find the effect on the area or volume of an object by changing one dimension.) [M1PS4]**

7-9 Comment:

Most problems for this objective will be stated without units. However, if units are used, they should be only metric ones (cm, dm, m). Geometric figures should include rectangles, traingles, circles, rectangular solids, and cylinders.

Classroom discussions should consider the effect on area or volume when one or more dimensions are doubled, tripled, or halved. The multiplier of all dimensions does not need to be the same. For example, one dimension of a rectangle may be multiplied by two while the other dimension is cut in half.

7-9 Example:

If you multiply the length of a side of a cube by 3, what happens to the volume?

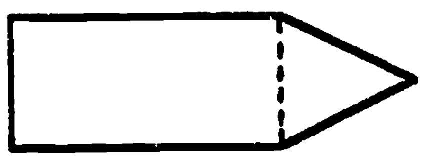
(Answer: The volume is 27 times as large)

APPLICATIONS AND PROBLEM SOLVING: **To find the area and volume of figures resulting from combining or separating common geometric figures [M1PS5]**

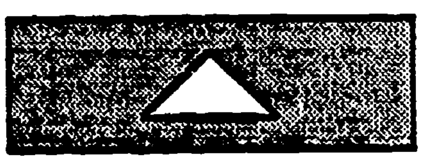
7-9 Comment:

Problems presented to the students may involve area or volume of figures which are the combination of two simple figures as well as a separation (subtraction) of one figure from another. Shapes may include triangles, rectangles, circles, half or quarter-circles, rectangular solids, or cylinders.

combination:

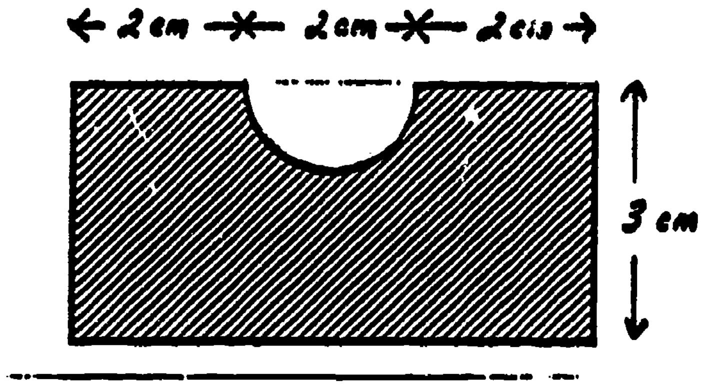


separation:



7-9 Example:

Find the area of the shaded figure to the nearest whole unit. ($\pi = 3.14$)



(Answer: 16.43 cm^2)

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CAPACITY, MASS, TIME, TEMPERATURE:

To measure and use liquid capacity, mass (weight), time, temperature, monetary value and the relationships of the basic metric units

CONCEPTUALIZATION: **To recognize and use the concepts of mass, liquid capacity, time and temperature, including standard units, relative sizes, comparisons, and their abbreviations and symbols [M2Cn1]**

K-3 Comment:

Classroom activities should involve the students in filling and pouring liquids together with a standard liter (L) container.

Each student should participate in finding the mass (weighing) of familiar items with a metric scale or a balance scale using metric weights. They should find their own mass in kilograms. The terms mass, gram (g), and kilogram (kg) should be discussed. Sometimes non-standard units may be used in representing weight.

Discussions of temperature should involve the Celsius scale, and students should know what the measures of 'room temperature' and 'comfortable outdoor temperature' are. The relationship of temperature to proper outdoor attire during the various seasons helps students think about the Celsius measures.

Units and terms for time measure at this level are minutes, hours, days, months, years, a.m., p.m.

K-3 Example:

A new pencil weighs about the same as how many paper clips?

4-6 Comment:

Similar classroom activities as in K-3 should be used, but extend units to mL, L, kL, g, mg, kg.

Freezing and boiling points for water on the Celsius scale should be discussed. Students can participate in measuring and recording outdoor temperature daily. Students should be able to subtract starting and ending times to find elapsed time, and to add to find what an ending time will be when starting and elapsed times are known.

The students should learn the relationship of the cubic centimeter and cubic decimeter to the liter, and the relationship of the cubic centimeter to the gram.

4-6 Example:

The temperature of boiling water is ? degrees C.

(Answer: 100° C)

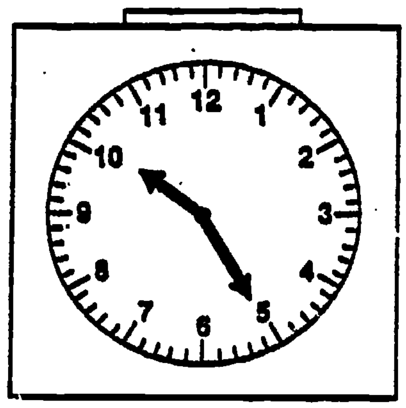
CONCEPTUALIZATION: To tell time to the nearest five minutes [M2Cn2]

K-3 Comment:

Looking at the face of a clock, students should be able to read time to the nearest five minutes. Some questions may be phrased such as "What time is it an hour earlier, an hour later, a half hour earlier?"

K-3 Example:

What time will it be one hour later?



(Answer: 11:25)

CONCEPTUALIZATION: To measure liquid capacity and mass (weight) using appropriate standard units and measuring instruments [M2Cn3]

K-3 Comment:

Classroom activities should involve students in measuring liquid capacity and mass (weight). Students should discuss the names of the standard metric units which will be attached to these measurements. Actual measurement will not be done in the testing situation. Rather students will demonstrate knowledge of the names of the metric units for liquid capacity and mass, properly selecting the unit to be used for a familiar situation, and also being able to select the most appropriate measure.

K-3 Example:

What is the correct unit to use to tell how much milk a cup will hold?

(Answer: mL)

4-6 Comment:

Metric units for liquid capacity are extended to mL, L, and kL and for mass (weight) to mg, g, and kg.

As students learn to multiply or divide by 10, 100 or 1000, practice converting measures to larger or smaller units, e.g., mL to L, and kg to g. Some problems should begin or end with decimals.

4-6 Example:

A jar that holds 2,500 mL holds how many L?

(Answer: 2.5 L)

CONCEPTUALIZATION: To recognize and use U.S. coins and bills, $5 or less [M2Cn4]

K-3 Comment:

There should be hands-on experience with coins and paper money (real or play). There should be practice giving the name and value of coin or bill shown or pictured. The total amount of money shown or pictured can be found by adding, but students should be able to 'count' the sum mentally while looking at the coins and bills or touching them. Students should be able to make exchanges of equal values of money, for example, exchange a half dollar for 4 dimes and 2 nickels.

K-3 Example:

Show a group of other coins that are equal in value to the following coin.

(Answer: 10 pennies, 1 nickel and 5 pennies, 2 nickels)

CONCEPTUALIZATION: To read various scales, such as a thermometer [M2Cn5]

K-3 Comment:

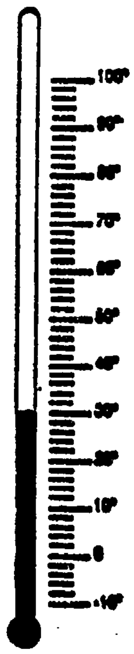
For this objective, the student must learn to read several scales. Students will need practice in reading a thermometer marked with ten degrees. There should be practice in reading the amount of liquid in a container scaled with mL or L. The amount should be read to the nearest 10 mL or whole L. Mass (weight) should be read from a scale marked in kg.

4-6 Comment:

The same comments for K-3 apply to grades 4-6, except that measurements are read to the nearest unit. Practice using thermometers scaled in 2 degrees or 5 degrees before expecting skill with 'nearest degree'.

4-6 Example:

Read the thermometer.



(Answer: 30°)

CONCEPTUALIZATION: To recognize and use the characteristics of the measurement process, including selection of appropriate units, derived units, the role of approximation, and the conversion-of-units process [M2Cn6]

4-6 Comment:

Eventually a student must become completely familiar with the process of measurement and, for the MEAP test, metric measurement. The names of the units and their symbols must be known. The whole structure of the metric system may be discussed, but only certain units need to be mastered here: mg, g, kg for mass and mL, L and kL for capacity.

The student should know the relationship among the units and be able to convert from metric units, such as mg to g or kg to g. At no time will there be conversion from metric to customary units, or vice versa. The student should think within the metric system.

Selection of the most appropriate unit for mass or for liquid capacity is important. While a person's mass could be given in grams, kilograms would be a more appropriate unit.

Students at this level should be able to convert among different time measures: days to weeks, hours to minutes, etc.

4-6 Example:

5 kg is equal to how many grams?

(Answer: 5,000 g)

7-9 Comment:

In addition to the topics listed for grades 4-6, the students learn to convert linear, square, and cubic measures to larger or smaller units. Decimals should be required in most of the problems. Multiplication or division by a power of ten is a required skill.

7-9 Example:

A rectangle has an area of 3230 cm^2. Express this area in dm^2.

(Answer: 32.30 dm^2)

CONCEPTUALIZATION: **To recognize and use the metric system, including the decimal relationship among the various units and the relationships among cubic units, capacity units and mass units [M2Cn7]**

7-9 Comment:

With this objective the students learn how to use the interrelationships that exist within the metric system. Not only is it necessary to develop a good understanding of the relationship within the liquid capacity units and within the mass units, but also the relationship between the cubic units, mass units, and liquid capacity units. Help the students to see that all these relationships are not complicated. By definition, a cubic decimeter (or 1000 cubic centimeters) is the same as one liter. Also, by definition, the mass of one cubic centimeter basically has a mass of one gram. A lot of discussion and demonstration with appropriate containers with student interaction will help.

In addition to relationships such as mL to L or kg to g, additional relationships involve cm^3 to g, mL to cm^3, L to kg, or L to dm^3. Problems should be fairly simple and involve only one of these three relationships. Problems requiring conversions such as mL to dm^3 should be reserved for enrichment.

Problems presented for this objective will most frequently involve decimals. Therefore, the basic operations of multiplying and dividing by powers of ten will need to be mastered.

7-9 Example:

The volume of a container is 12.78 dm^3. How many liters will it hold?

(Answer: 12.78 L)

CONCEPTUALIZATION: To recognize and use the concept of precision of measurement [M2Cn8]

7-9 Comment:

When we count the number of pages in a book, we can state the answer as a precise number. However, when we make a measurement of length, area, mass, liquid capacity, etc., the measurement is given as a number, but that number does not exactly reflect the length, area, mass, etc. We may state that the length of a room is 5 meters to the nearest meter. This means that the actual length of the room could be anywhere from 4.5 m to 5.5 m. If the length is stated as 5.0 meters, then the precision of measurement is greater and the range of the measurement is from 4.95 m to 5.05 m.

Students should learn that measures are not exact, that a degree of precision is involved, and that a stated measure has a related range within which the true measure lies.

If a measure is given, as 12.3 g, it is possible to state the range as 12.25 g to 12.35 g, obtained by subtracting and adding one-half the unit of measure, 0.1 g; 12.25 is the least measure and 12.35 is the greatest measure that could be stated as 12.3 g. The measurement is given to the nearest tenth of a gram.

This objective should not be confused with estimation.

Skill with decimals is a prerequisite for this objective. Practice with this objective will strengthen the students ability to work with decimals.

7-9 Examples:

If a measure of a line segment is given as 2.8 cm, what is the range of the true measure of the line segment?

(Answer: 2.75 cm to 2.85 cm)

If a measurement is given as 3.80 L, we may infer that the measurement was made to the nearest ?.

(Answer: 0.01 L).

ESTIMATION: To make estimations involving temperature, time and money [M2Es1]

K-3 Comment:

All students should have an opportunity to participate in making estimates. They can learn much from making estimates and then seeing how their estimates relate to the actual measure. Success will improve with guided practice. Use classroom activities to estimate current temperature (Celsius) or the temperature appropriate for certain activities, the amount of money needed for activities and items known by the students and time needed to complete things understood by the students.

4-6 Comment:

Extend the comments made for K-3 to situations which would be appropriate for middle grade students.

ESTIMATION: **To make estimations of the capacity of various common containers in terms of metric units [M2Es2]**

4-6 Comment:

As classroom activities, have students estimate the capacity of various containers in liters. Since many items are now being sold in liter containers, estimations can be made by comparison to one of these. Do not expect great accuracy for the estimates. Also, have students estimate the capacity of various containers in milliliters, recognizing that this may prove more difficult than liters. Containers used for estimates should be ones that are familiar to the middle grade students. One liter and 250 milliliter containers should be available for the students to study visually as estimates are being made.

7-9 Comment:

See 4-6 Comments, but extend difficulty to those reaching into high school.

ESTIMATION: **To make estimations of weight in terms of metric units [M2Es3]**

7-9 Comment:

This is probably the most difficult of the estimation objectives, even for adults. Items familiar to the students should be used in making comparisons and estimates. For example, let students know the mass (weight) of their mathematics textbook and of a new pencil. Have another book (either lighter or heavier) and an object weighing a little more than the pencil passed around the class and have all students write down their estimates. The actual mass can be found after everyone has written down an estimate. A discussion will follow about actual measures.

Units to be used are g and kg. Students should have an opportunity to estimate weight of a familiar object based on the "feel" of a brass kg or g weight. You may ask if a loaf of bread has the mass of the kilogram weight. Another activity would be for students to select from a group of items displayed on a table at the front of the room one that would weigh about the same as the kg weight. After the students have made their selections, put the kilogram weight on one side of a balance scale and each of the items, one at a time, on the other side to see how close the selections were.

APPLICATIONS AND PROBLEM SOLVING: **To solve one-step verbal arithmetic problems posed within a measurement context, including elapsed time and money [M2PS1]**

K-3 Comment:

Opportunities should be provided for students to solve measurement problems along with their basic computational skills. This objective does not call for a separate unit to be taught. As measures are taught, students may be provided verbal problems which require them to perform addition, subtraction or simple multiplication on the measures. As money is discussed, the verbal problems may involve total cost of several different items, total cost of several of the same items, how much change will be received, or how much money is left. When time is discussed, problem solving may deal with total time spent or elapsed time between two stated times. For elapsed time, times stated may involve both a.m. and p.m. and quarter and half hours but subtractions should not require regrouping.

It is important that each student has the basic reading and vocabulary skills to read and understand the verbal problems.

K-3 Example:

Find the total cost of these purchases.

Soap	$ .79
Toothpast	$1.23
Cereal	$2.47
Corn	$ .44

(Answer: $4.93)

4-6 Comment:

For grades 4 to 6, the comments are the same as for K to 3. The measurement skills and computational skills will make it possible to give more difficult verbal problems involving different measurement units. Elapsed time problems should involve no regrouping, the same as K-3. At this level students will be expected to compute change of temperature also.

In the mathematics class period, it is important that the teacher provide special instruction so that the student learns how to read a mathematics problem posed for solution.

4-6 Example:

Damon arrived at the movies at 1:15 p.m. and left at 3:45 p.m. How long was he at the movies?

(Answer: 2 1/2 hours)

APPLICATIONS AND PROBLEM SOLVING: **To use a table of equivalents to solve simple problems involving the conversion of units within a system of measurement [M2PS2]**

4-6 Comment:

Using a table converting U.S. money to a foreign currency, the students should find the value (in foreign currency) of a given amount of U.S. money. Using a table for mass (weight) or capacity the student should select the proper data and solve a problem. Notice that the student is not required to 'know' the basic conversion equivalents, but rather know how to read from a table what is necessary and then perform a computation.

4-6 Example:

Use the following table of information for the problem below.

U. S. A.	Canada
$100.00	$134.00
$200.00	$268.00
$300.00	$402.00

Jim has $50.00 in U.S. funds. What will that be in Canadian funds?

(Answer: $67.00)

7-9 Comment:

The comments for 7-9 are the same as for 4-6. The computations required will be in keeping with the computational skills of 7-9 students. Much of the computation will probably involve decimals.

7-9 Example:

Use the following table of information for the problem below.

Mass (weight) 1000 milligrams (mg) = 1 gram (g)
1000 grams (g) = 1 kilogram (kg)
1000 kilograms (kg) = 1 metric ton

The weight 1 dm^3 of silver is 10.5 kg. How many grams is that?

(Answer: 10,500 g)

APPLICATIONS AND PROBLEM SOLVING: **To solve multi-step verbal problems posed within a measurement context [M2PS3]**

4-6 Comment:

All problems solved for this objective will require two or more steps and involve any of the measurement units included in the objectives. Money and elapsed time may also be expected. Any of the problems solved under Length, Area, Volume, and Angles may be involved just so they are expanded to be two or more steps.

4-6 Example:

Find the cost of carpet for a living room which measures 5 m by 6 m. The unit cost for the carpet is $12.98 per square meter.

(Answer: $389.40)

7-9 Comment:

The multi-step problems may involve the relationship of volume or capacity to mass (weight). More complex formulas should be provided if needed, e. g., the volume of a cylinder.

7-9 Example:

You have an aquarium which measures 3 dm x 5 dm x 4 dm. You must treat the water with a chemical which calls for 1 g to 5 liters. How much of the chemical should you use?

(Answer: 12 g)

Framework: Michigan Mathematics Objectives

Mathematical Processes

Mathematical Content Strands	Conceptualization	Mental Arithmetic	Estimation	Computation	Applications and Problem Solving	Calculators and Computers
Whole Numbers and Numeration						
Fractions, Decimals Ratio and Percent						
Measurement						
Geometry						
Statistics and Probability						
Algebraic Ideas						
Problem Solving & Logical Reasoning						
Calculators						

GEOMETRY: AN OVERVIEW

The world about us is viewed in a variety of ways. One way employs the concept of number in response to questions of "how many" or "which one." A second way the world is viewed uses the ideas of geometry in response to questions of "what is the shape" or "how does one shape differ from another." A third view of the world combines the previous two by applying numbers to geometric shapes using the process of measuring. Considering these three ways of viewing the world, number and geometry are relatively equal in importance.

What is included in this strand?

The geometry strand has six major objectives that deal with the following ideas.

1. Shape--What is this shape?

2. Shape properties--What are the characteristics of this shape?

3. Relations among shapes--How are these shapes related?

4. Position--On a coordinate system what are the coordinates of a vertex of this shape?

5. Transformations--Would the shape look the same if it were rotated, reflected, or flipped? Does the shape have symmetry?

6. Visualizing-Sketching-Constructing--What would this shape look like from another viewpoint? Can you draw the shape? Can you construct it?

The fundamental idea of the geometry strand is that of shape or geometric figure. Examples of shapes include triangle, line, pyramid, square, angle, cube, etcetera.

The objectives ask that students be able to identify shapes and their properties, to identify and use relations between shapes, to locate point using coordinates, and to transform, sketch, construct and visualize shapes. These are capabilities useful in life and in further mathematics.

How does the development flow?

Instruction should begin with shape identification. The shapes should include common three-dimensional shapes such as cubes, cones, cylinders and spheres as well as plane figures such as circles, triangles and rectangles. Additional shapes are added over time.

As familiarity with shapes increases, their properties, such as equal sides, containing right angles or having symmetry, are introduced. Later, relations are introduced between two shapes such as two triangles being congruent or similar or two lines being perpendicular or parallel.

Early in the instructional sequence students should be taught to sketch shapes. As the shapes get more specialized, the demand for accuracy of the sketches should be greater. Students should be taught to sketch three dimensional shapes also. Early attempts will be clumsy, but will improve with experience and instruction.

Once familiarity with a shape is achieved, shapes may be reflected, rotated, slid or transformed in other ways. Students should be taught to carry out these transformations and the resulting image figures should be compared with the originals in regard to lengths of sides, measures of angles, tilt, position in the plane or space, and other properties.

Why teach these objectives?

Geometry is a tool for describing the world. To describe the world it is vital that students have a good intuitive working knowledge of the important shapes, relationships and modes of representing geometric ideas. Further, geometry helps students comprehend other mathematics. The introduction of coordinates to locate points allows students to relate geometry and algebra. The knowledge of shapes permits the concepts of area, volume and linear measure to be adequately conceptualized. Geometric models of number concepts, such as fraction, relate geometry to number. Thus knowledge of geometric content is central to understanding and using much of the other mathematics included in the K-9 curriculum. Other content strands in these objectives depend upon geometry to aid in their representation and comprehension.

What are the implications for instruction?

Geometry should be taught with real objects. Pictures and words are not sufficient. Both solids and plane figures need to be stressed. The faces of solids can be used to introduce plane shapes. When introducing the cylinder, for example, soup cans, tuna cans, and other models should be central to instruction. Initially, the focus is on identification and discrimination. Later, attention turns to characteristics such as its circular bases, its curved surface, its "rollability"; then to its symmetry, its similarity and dissimilarity to other solids such as cones and prisms, its representation via a freehand sketch, its representation using equations in a three dimensional coordinate system. The base of a cylinder is a good model of a circle. When evaluating student learning, real objects should be used as well as drawings and verbal descriptions.

Practical applications of geometric knowledge abound in everyday life and should be used in the instructional and evaluation process as appropriate.

Such applications range from a question like "Why is it safer to have roads intersect at right angles?" to "Can bathroom tiles have the shape of a trapezoid?" to "How can you find the center of a rectangular ceiling?" to "How much aluminum siding is needed to complete a job?" Whenever geometric concepts can be used in an application, it needs to be pointed out.

Vocabulary

K-3

- angle
- box
- circle
- closed
- cone
- corner
- cylinder
- cube
- edge
- face
- folding symmetry
- inside
- on
- open
- outside
- rectangle
- same shape
- same size and shape
- segment
- side
- solid
- sphere
- square
- tile
- triangle

4-6

- angle
  - acute
  - obtuse
  - right
- circle
  - center
  - diameter
  - radius
  - semicircle
- congruent
- coordinates
- diagonal
- image
- intersect
- line
- origin
- parallel
- perpendicular
- polygon
  - 5-gon(pentagon)
  - 6-gon-(hexagon)
  - 8-gon (octagon)
- prism
  - square
  - triangular
- pyramid
  - square
  - triangular
- quadrilateral
  - kite
  - parallelogram
  - rhombus
- ray
- reflection
- rotations
  - half-turn
  - quarter-turn
- similar
- translation
- triangle
  - equilateral
  - isosceles
  - right
  - scalene

7-9

- angle
  - complimentary
  - corresponding
  - interior
  - supplementary
  - vertical
- circle
  - chord
  - sectors
  - secant
  - tangent
- coordinate system
- cross section
- midpoint
- polygon
  - altitude
  - median
  - n-gon
  - regular n-gon
- prism with any base
- pyramid with any base
- Pythagorean relation
- size transform
  - enlargement
  - reduction
- trapezoid

4-6 Continued -

- vertex
- view
  - front
  - side
  - top

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University of Oregon Mathematics Resource Project. Geometry and Visualization. Palo Alto, CA: Creative Publications. 1977 (4-9)

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GEOMETRY: THE OBJECTIVES

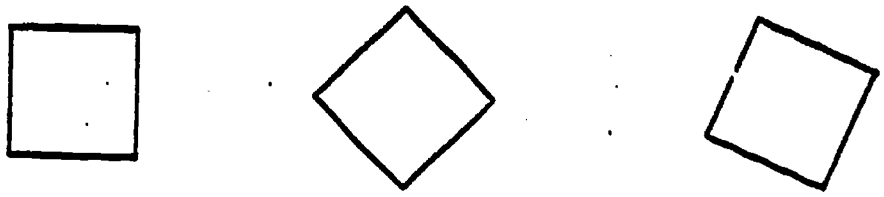
SHAPE: To recognize and use shapes in one, two and three dimensions

CONCEPTUALIZATION: To identify and illustrate appropriate geometric shapes [G1Cn1]

K-3 Comment:

Shapes include circles, triangles, rectangles (including squares), line segments, angles, spheres, cubes, cones, cylinders and boxes.

Activities used to introduce a shape should include a variety of examples of the shape, i.e., equilateral and right triangles as well as scalene, and displaying the shape in a variety of positions, for example, a square in horizontal and non-horizontal orientation.

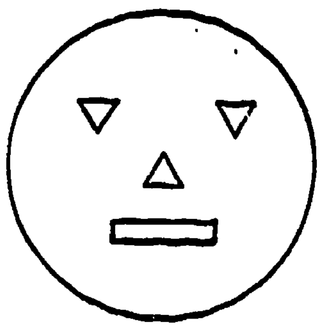


Triangles should not be shown always with a side parallel to the edge of the chalkboard or paper.

Definition: A box is a solid whose faces are rectangles. The technical term is rectangular parallelepiped (so we chose box instead).

K-3 Example:

How many triangles are in the picture?



(Answer: 3)

4-6 Comment:

Shapes include isosceles, right, scalene and equilateral triangles; angles, rays, and lines; parallelograms, kites, and rhombi as quadrilaterals; polygons with 5, 6, and 8 sides; square and triangular prisms or pyramids.

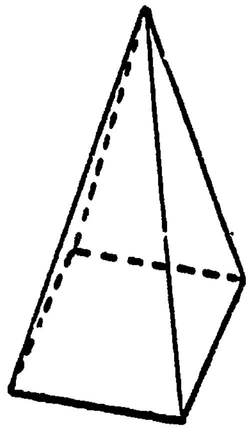
Prisms and pyramids are named by the shape of the base(s). Thus, a prism or pyramid with a pentagon for a base is a pentagonal prism or pyramid.

Definitions: A kite is a quadrilateral with two distinct pairs of congruent adjacent sides.
A parallelogram is a quadrilateral with two pairs of opposite sides parallel.

A polygon is a closed figure made up of line segments attached end to end. An n-gon is a polygon with n sides.

4-6 Example:

What kind of pyramid is shown?



(Answer: Square pyramid)

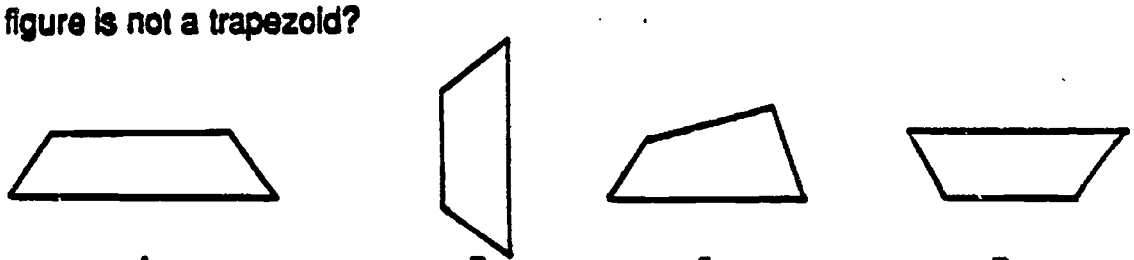
7-9 Comment:

The shapes are expanded to include trapezoids: all polygons named by the number of sides, both regular and otherwise; all prisms, pyramids and cones. Draw attention to how names of geometric ideas convey their meaning, e.g., hexagon (six angles), diagonal (joining two angles).

Definition: A trapezoid is a quadrilateral with only one pair of parallel sides. There are two definitions for trapezoid commonly used by teachers. One specifies that only one pair of sides is parallel; the other specifies that at least one pair of sides is parallel. If the latter is used, all parallelograms are trapezoids, but this is not true when the first definition is used. The definition chosen for the test is the first. Thus, parallelograms are not trapezoids.

7-9 Example:

Which figure is not a trapezoid?



(Answer: C)

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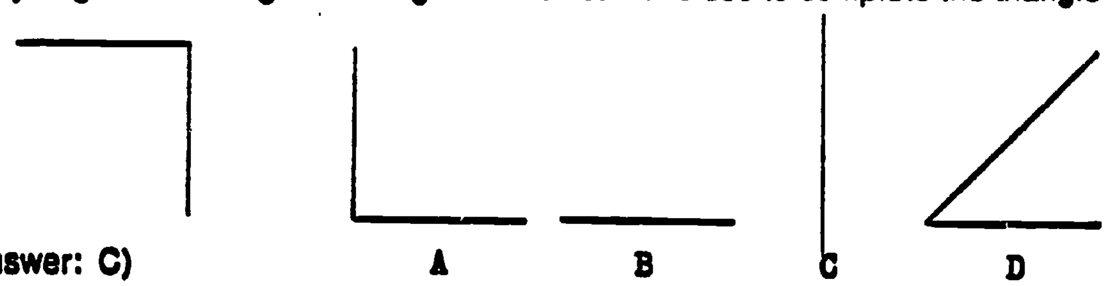
APPLICATIONS AND PROBLEM SOLVING: **To solve problems involving appropriate geometric shapes [G1PS1]**

K-3 Comment:

The shapes are those listed for the K-3 conceptualization objective.

K-3 Example:

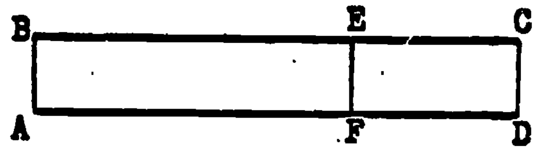
Mary began a drawing of a triangle. Which can she use to complete the triangle?



(Answer: C)

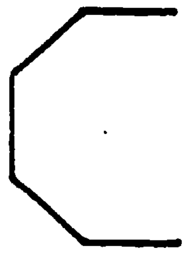
4-6 Example:

What is the smallest number of line segments needed to draw three different size rectangles. You may use a segment in more than one rectangle.



(Answer: 5, Segments $\overline{AB}$, $\overline{FE}$, $\overline{DC}$, $\overline{AD}$ and $\overline{BC}$)

How many sides are needed to complete this octagon?



(Answer: 3)

7-9 Comment:

Counting faces and sides or looking for hidden shapes are good problem solving activities.

7-9 Example:

Exactly four of the faces of a solid are triangular. Choose true or false for each of these statements.

T F The figure could be a triangular pyramid. (True)

T F The figure could be a hexagonal pyramid. (False; There are six triangular faces.)

T F The figure could be a triangular prism. (False; There are only two.)

SHAPE PROPERTIES: To recognize and use properties of one, two, and three dimensional shapes such as equal sides, equal angles, and symmetry

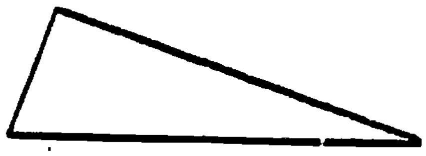
CONCEPTUALIZATION: To identify or illustrate properties of appropriate geometric shapes [G2Cn1]

K-3 Comment:

The shapes are those noted under the K-3 Comment in the shape category. Properties include characteristics of the faces, sides, and corners (angles); open or closed shapes; folding symmetry and tiling quality; solids that roll or slide. A shape is said to "tile a plane region" if you can cover the region with the shape leaving no gaps and having no overlaps. Gaps at the edges of the region do not count, you simply cut off the excess as you would in tiling a bathroom floor. A shape has folding symmetry if there is a way to fold it so that two identical halves are made.

K-3 Example:

How many square corners does this triangle have? Check using a folded paper model of a square corner.



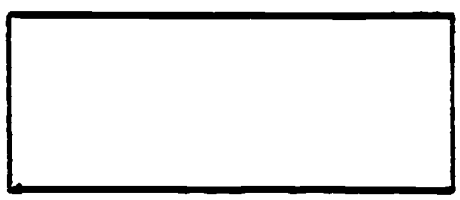
(Answer: one)

4-6 Comment:

The parts of a circle (diameter, radius, center, semicircle) are appropriate content as are diagonals of polygons, types of angles (acute, obtuse, right), equality, parallelism or perpendicularity of sides or faces of shapes, symmetry of plane and solid shapes, and the ability to tile a region with several copies of a smaller region.

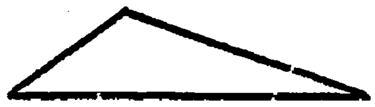
4-6 Example:

Draw all lines of symmetry in this rectangle. How many did you find?



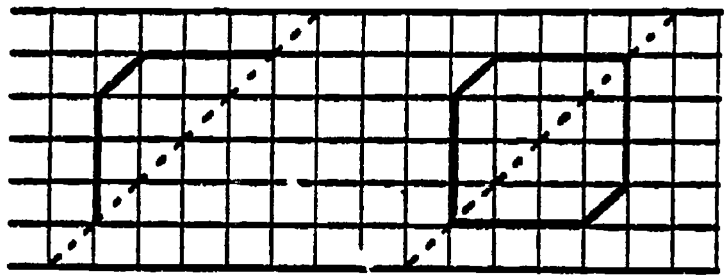
(Answer: 2)

Can this figure shown be used to tile a plane region?



(Answer: Yes, every triangle will tile a plane region.)

Complete the drawing so that the dotted line becomes a line of symmetry.



7-9 Comment:

Tangents, secants, chords and sectors of circles are included. Supplementary, complementary and vertical angles can be introduced as well as concepts such as altitude, median, interior angles of a polygon and the Pythagorean Relation.

Definitions: For a circle:

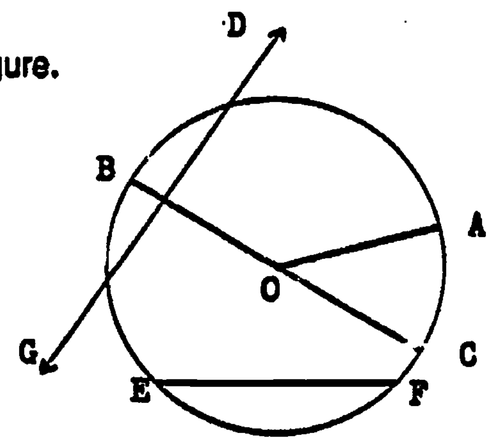
A tangent is a line intersecting the circle in one point.
A secant is a line intersecting the circle in two points.
A chord is a segment with its end points on the circle.
A sector is a region bounded by two radii and an arc.

For a polygon:

An altitude is a perpendicular from a vertex to an opposite side.
A median of a triangle is a segment joining a vertex to a midpoint of a non-adjacent side.

7-9 Example:

Name the secant shown in the figure.



(Answer: $\overleftrightarrow{GD}$)

ESTIMATION: **To compare visually the measures (sizes) of segments, angles and plane regions [G2Es1]**

K-3 Comment:

Gross estimates are appropriate here, and students should be given opportunity to check their estimates against a model. For example, if you have several angles and seek the one closest to a square corner, let students check their choices against a square corner.

K-3 Example:

Which drawing is the best square corner?

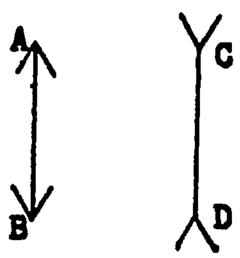
(Answer: A.)

4-6 Comment:

Some optical illusions can be useful to motivate the need for measurement

4-6 Example:

Which is longer, line segment AB or line segment CD?



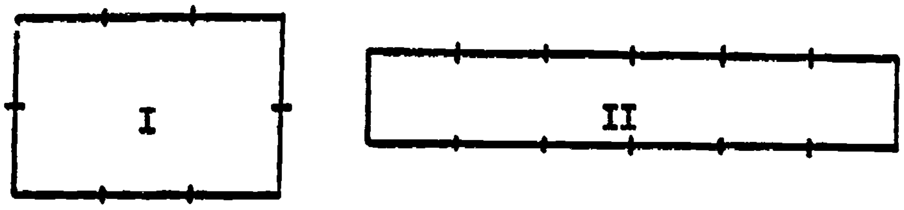
(Answer: $\overline{AB} = \overline{CD}$)

7-9 Comment:

The area of a region is often judged by the length of its perimeter even though there is no relation. Help kids learn to make the discrimination.

7-9 Example:

Which shape has the greater perimeter?



(Answer: II)

The greater area?

(Answer: I and II are the same.)

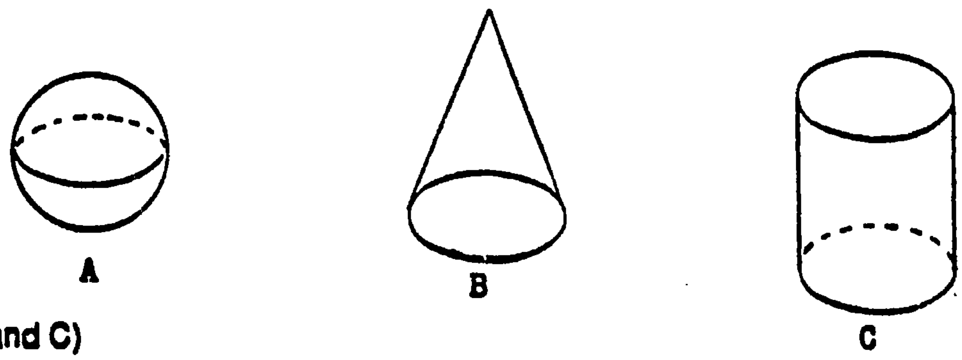
APPLICATIONS AND PROBLEM SOLVING: **To solve problems using properties of appropriate geometric shapes [G2PS1]**

K-3 Comment:

Experimentation and hands-on experience is vital in solving problems here. The appropriate shapes are those identified in the K-3 comment under shape.

K-3 Example:

Which of the solids can "roll in a straight line"?



(Answer: A and C)

4-6 Comment:

Drawing shapes, cutting them out and testing are important activities in geometric problem solving. Geoboards or dot paper provide many activities such as "draw a pentagon with two right angles."

4-6 Example:

Who am I? I have 4 sides. I have exactly one line of symmetry which goes through two vertices.

(Answer: Kite)

I have 4 sides, and 2 lines of symmetry that do not go through the vertices.

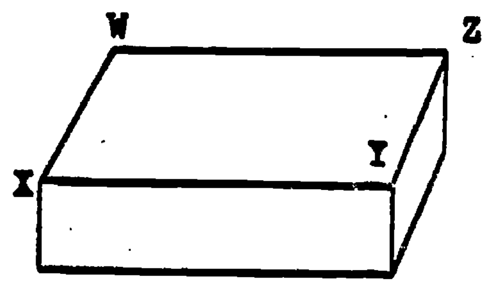
(Answer: Rectangle)

7-9 Comment:

Problems can be "problems within geometry" as well as applications which may involve measurement.

7-9 Example:

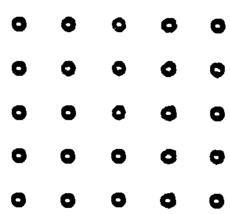
In the box shown, WXYZ is a rectangle as are the other faces. Sketch the planes of symmetry for the box. How many are there? If WXYZ were a square, how many symmetry planes for the box would there be?



(Answer: 3, 5 --The diagonal planes through line segment $\overline{XZ}$ and $\overline{WY}$ are symmetry planes also.)

7-9 Example:

How many line segments, 5 units in length, can be drawn on the dot paper below?



(Answer: 8 --Think - How many 3 x 4 right triangles can be drawn and use the Pythagorean Relationship.)

RELATIONS AMONG GEOMETRIC OBJECTS:

To recognize and use the relations of congruence, similarity, intersection, parallelism, and perpendicularity for appropriate figures in one, two and three dimensions

CONCEPTUALIZATION: **To identify and illustrate appropriate relations among figures** [G3Cn1]

K-3 Comment:

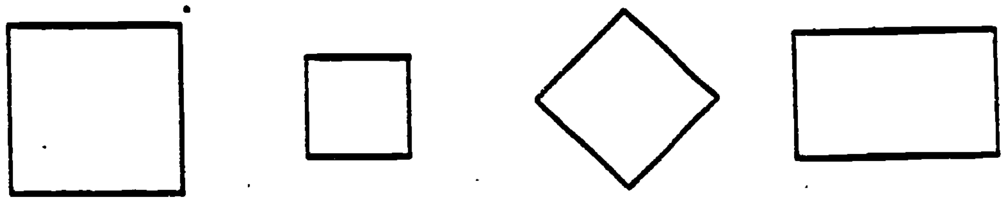
Figures with the same shape, or same shape and size should be stressed, but the technical vocabulary of similar (same shape) and congruent (same shape and size) are not to be used at this level. Real objects can be used to illustrate these ideas. For example, two ignition keys for the family car are the same shape and size, while wallet size school pictures and a 5 x 7 enlargement are the same shape.

"Same shape," as used in this objective, means "similar." Note that "same shape" is also often used to designate a class of shapes such as all triangles or all rectangles. We say a long and thin rectangle is the same shape as a thick one--they are both rectangles and thus, the same shape even though they are not similar. Care needs to be taken.

The approximate shapes are those included in the vocabulary list for K-3.

K-3 Example:

Which figures have the same shape as  ?



(Answer: All but the last one.)

4-6 Comment:

All the relations listed in the global objective above are appropriate here and their technical names should be used, i.e., "congruence" for "same size and shape."

4-6 Example:

True or False: Perpendicular lines do not intersect.

(Answer: False)

True or False: Parallel lines cannot be perpendicular.

(Answer: True)

7-9 Comment:

No new geometric relations are introduced at this grade level. Attempts should be made to ensure all students understand these relations and learn to apply them to applications in mathematics and the real world. Stress drawing models to illustrate congruence, similarity, perpendicularity, and parallelism since these relations are vitally important in "proof" geometry.

7-9 Example:

Completion: Triangle ABC is similar to triangle XYZ, so $\frac{AB}{XY} = \frac{?}{ZY}$

(Answer: BC)

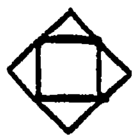
APPLICATIONS AND PROBLEM SOLVING: **To solve problems using the appropriate relations among shapes [G3PS1]**

K-3 Comment:

Recognition of same size and/or shape in complex and non-routine situation is appropriate.

K-3 Example:

If △ = 5 and □ = 20, what is the value of the figure?



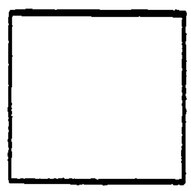
(Answer: 40)

4-6 Comment:

As geometric shapes and their properties become familiar, those properties and shapes can be the bases of problem solving activities.

4-6 Example:

A square is folded along a symmetry line forming two shapes. Name the shapes you could get. What can you say about each pair?



(Answer: Rectangle or right triangle. In each case the two shapes formed are congruent.)

7-9 Comment:

The relationships between the areas, the perimeters and the volumes of similar figures are important. Students should be helped to master the relations. If the ratio of similarity is K, then perimeter is multiplied by K, the area by K^2 and the volume by K^3.

7-9 Example:

Find the area of a square, if its side is four times the side of a square with area 2.

(Answer: $32 = 2 \times 4^2$)

A box holds 4 liters of wood chips. How many liters of wood chips will a similar box with dimensions doubled hold?

Answer: $32 = 4 \times 2^3$)

POSITION: To recognize and use informal and formal coordinate systems on lines and planes to specify locations and distances

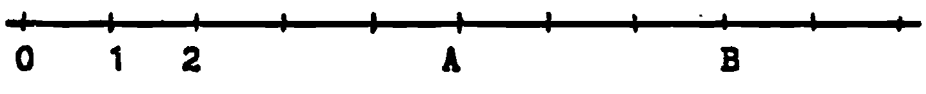
CONCEPTUALIZATION: To identify and produce points satisfying given conditions [G4Cn1]

K-3 Comment:

A natural coordinate system is the number line. It can be used to determine distances and is useful in representing number properties. Concrete examples include rows of books on a shelf and rows and columns of seats in a classroom. Inside, on and outside a figure are basic positional ideas and easily made into game-like learning situations.

K-3 Example:

A partially numbered number line is shown. What number is at A? How far is it from A to B?



(Answer: 5,3)

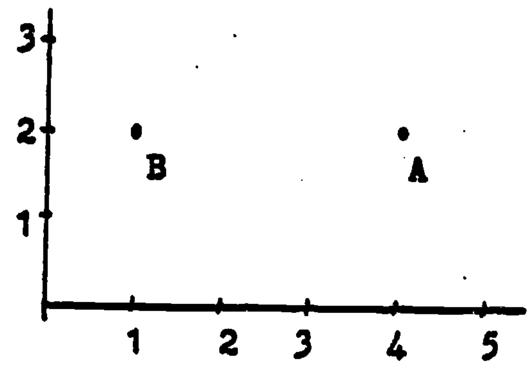
4-6 Comment:

Maps provide a handy introduction to coordinate systems as do classrooms and auditoria arranged in rows and columns. Choosing the lower left hand corner for the origin, the mixed number-letter scheme used in a map can be changed to a number-pair organization. Horizontal and vertical distances are easily computed in such a system.

Definitions: A coordinate system is a correspondence of numbers to points. In the plane, the usual coordinate system is defined by two number lines intersecting perpendicularly at the zero point of the number lines.

4-6 Example:

What are the coordinates of A? How long is BA?



(Answers: (4,2); 3)

7-9 Comment:

The coordinate system should be completed by this grade level, i.e., all four quadrants. Use coordinate methods to calculate distances between points and midpoints of segments.

7-9 Example:

X has coordinates (-3,4), Y has coordinates (5,-2). Find the distance between X and Y. Find the midpoint of line segment XY.

(Answers: 10; (1,2))

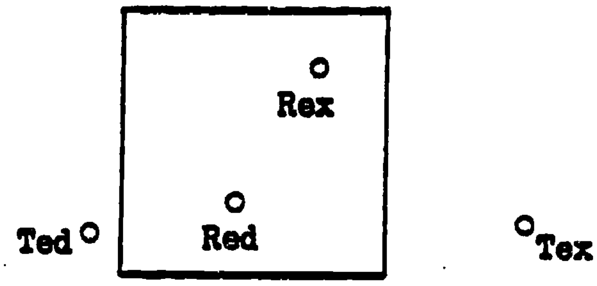
ESTIMATION: To estimate distances and positions in the coordinate plane [G4Es1]

K-3 Comment:

Use the concept of "near" with those of "on", "inside" and "outside" to practice comparing distances.

K-3 Example:

The square is the corral fence. Name the horse that is nearest to the corral fence.



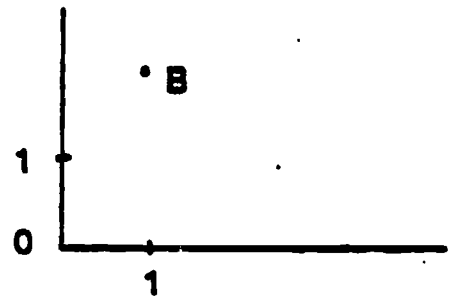
(Answer: Ted)

4-6 Comment:

Use whole number coordinates to estimate positions and distances. Check the accuracy of estimates by completing coordinate system or using a clear grid.

4-6 Example:

In whole numbers, estimate the coordinates of B.



(Answers will vary.)

7-9 Comment:

The distance formula can be used to check the accuracy of estimates. Use calculators to do the computations.

7-9 Example:

Using whole numbers, estimate the distance between A (8,4) and B(-3,-1).

(Answers will vary.)

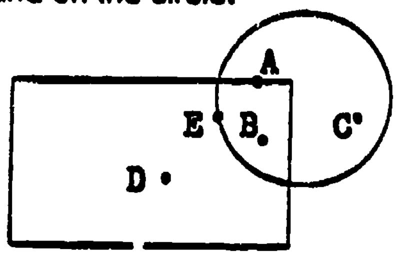
APPLICATIONS AND PROBLEM SOLVING: To solve problems using position, concepts and notation [G5PS1]

K-3 Comment:

Drawing shapes on the floor and asking students to stand at various positions such as inside, on, outside or a combination of these is a good problem solving activity.

K-3 Example:

Name the point that is inside the rectangle and on the circle.



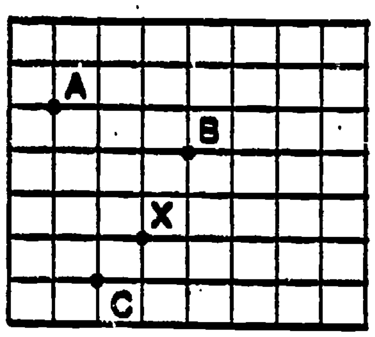
(Answer: E)

4-6 Comment:

Traveling along city streets gives a different distance between points. Explore "taxi cab" distance. "Taxi cab" distance is the distance along the streets in a city or the roads in the country rather than "as the crow flies."

4-6 Example:

Emergency care is located at A, B and C at the Fair Grounds. Mary falls and cuts her hand at X. To which care unit should she go if she must follow streets?



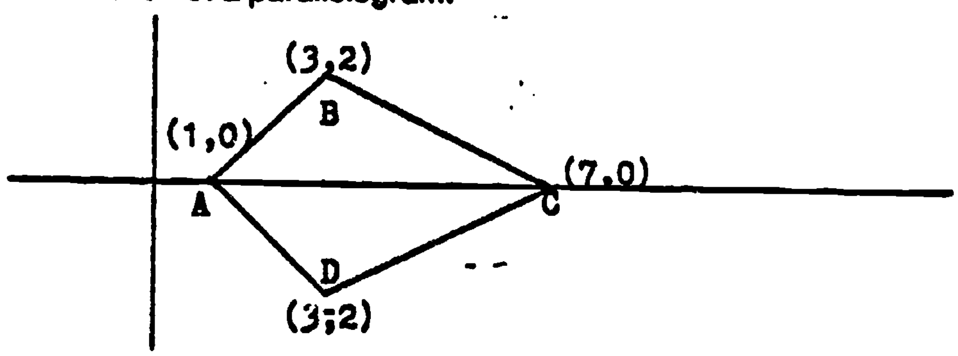
(Answer: C)

7-9 Comment:

Distance ideas can be used to determine the nature of geometric shapes by using knowledge of the shapes.

7-9 Example:

Show that quadrilateral ABCD is not a parallelogram.



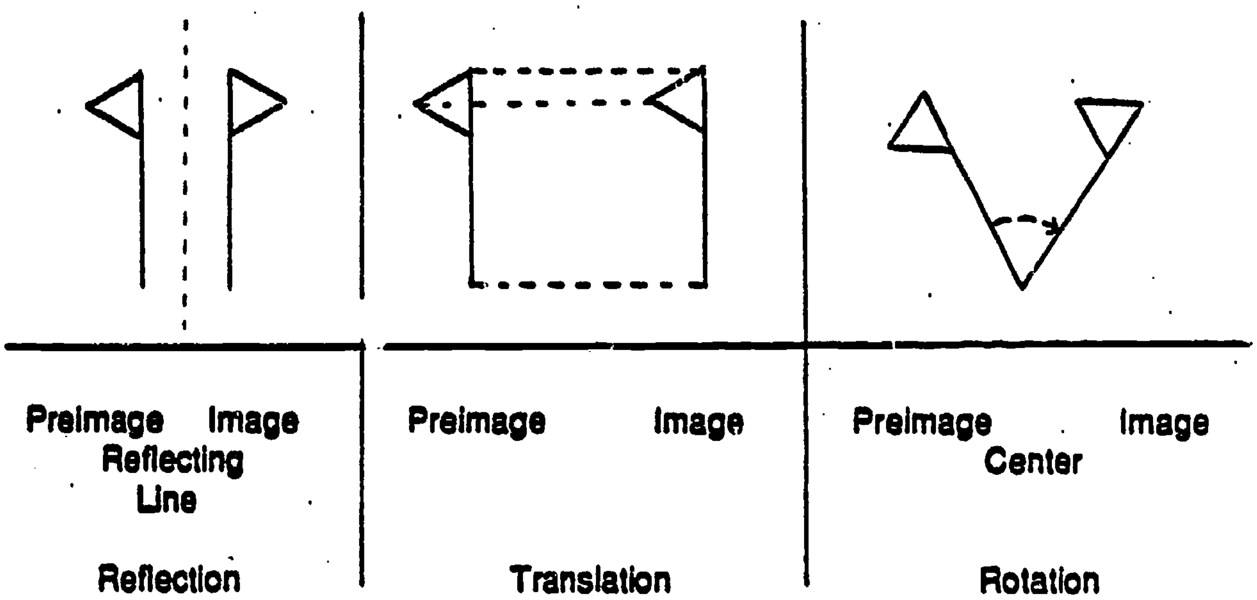
(Answer: Show that opposite sides are not equal.)

TRANSFORMATIONS: **To recognize and use the transformations of reflection in a line (flip), translation (slide), rotation about a point (turn), and size change (enlargement or reduction).**

CONCEPTUALIZATION: **To recognize and produce appropriate transformations [G5Cn1]**

4-6 Comment:

The terms reflection, translation and rotation are fancy terms for easy ideas. Any object that is turned over without turning is reflected. A good example is flipping a page of a book--the page is reflected in the crease of the bending. Similarly when an object is slid from one place to another without turning or flipping, it is translated. Driving an automobile along a straight road illustrates a translation. We turn doorknobs, dials, hands of a clock. Each exemplifies the idea of a rotation, namely, turning about a fixed point as center.

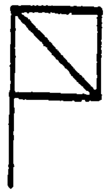


Activities illustrating these notions using real objects should be used. After such activity, the students should be taught to draw figures and their images under each transformation. One way to draw a reflection image is 1) to draw the figure on tracing paper 2) fold along the reflecting line, and 3) trace the shape on the other half of the tracing paper. Rotations can be done similarly: 1) draw the figure on one sheet of paper 2) trace that figure 3) with the pencil at the center, turn the tracing 4) trace the tracing. Translations are simply slides. First draw a figure and make a copy of it; 2) slide the copy; 3) trace the copy.

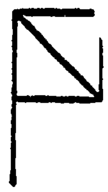
Reflections over any line, half and quarter turns, slides any distance and folding symmetry form the content emphasis.

4-6 Example:

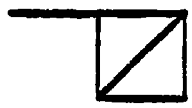
Draw the image of the flag after doing each of the following transformations.



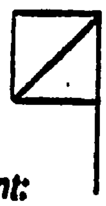
1. Slide 7cm to the right.



2. Rotated $\frac{1}{4}$ turn clockwise about the bottom of the staff.



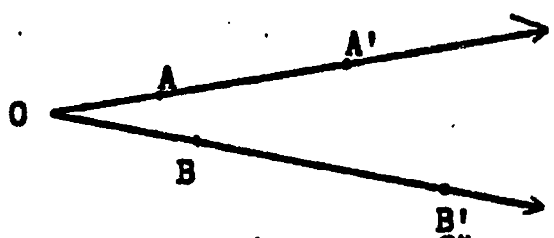
3. Reflected in its staff.



7-9 Comment:

Drawing the results of performing a series of transformations helps learners internalize the transformation idea. Note that objects transformed are congruent to the original objects whenever the transformation is a reflection, translation or rotation. If the transformation is a size transformation, the image is similar to the original object.

Enlargement/Reduction (size transformation) is new here. The idea is the same as a photo enlarger or slide projector. There is a point O, called the center, and a positive number K, called the ratio or magnitude of the size transformation.



A' is the image of A if and only if $OA' = k \cdot OA$ (or $\frac{OA'}{OA} = k$) and A, A' are on the same line. For each point in the plane, the same relatinship holds, i.e., $OB' = k \cdot OB$ and B' is on $\overrightarrow{OB}$.

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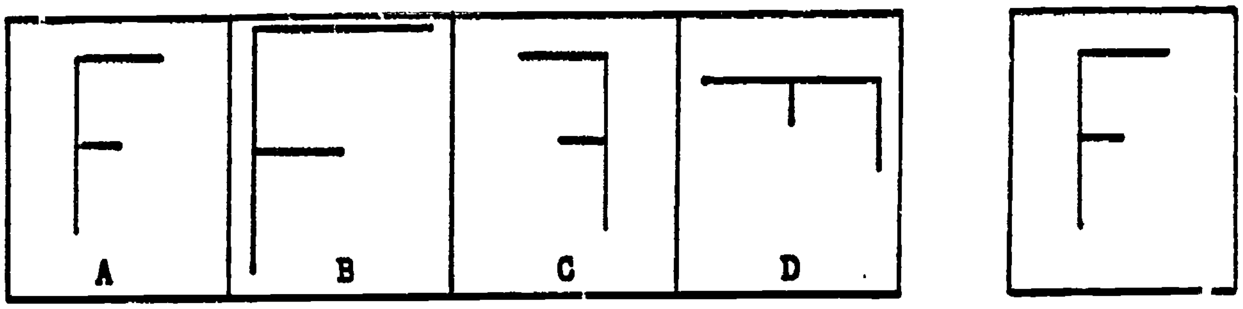
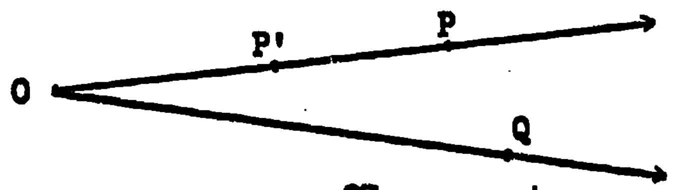
7-9 Example:

Which "F" could not be the result of reflecting, rotating and/or sliding the "F" at the right?



(Answer: B)

The size transformation image of P is P'. Find the image of Q.



(Answer: Draw OQ. Compute $\frac{OP'}{OP}$. Locate Q' using a ruler so that
OQ' = k · OQ.)

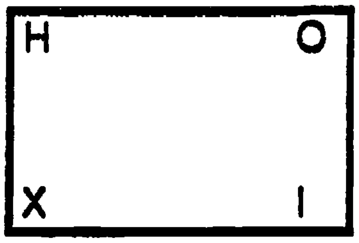
APPLICATIONS AND PROBLEM SOLVING: **To solve problems using appropriate transformations [G5PS1]**

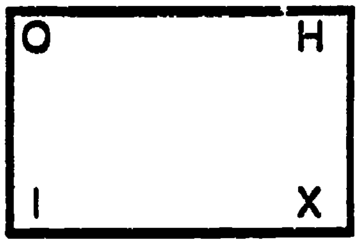
4-6 Comment:

Using paper or cardboard models which can be turned, folded, slid, marked on, cut out and otherwise distinguished is useful in learning to analyze problems asking students to discover transformations.

4-6 Example:

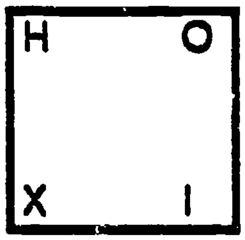
What transformation of the rectangle at the right produces the rectangle below?

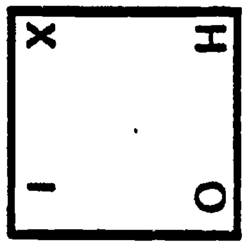




(Answer: Reflection in the perpendicular bisector of line segment $\overline{HO}$ or reflection in line segment $\overline{HX}$ or in line segment $\overline{OI}$.)

What transformation of the square at right produces the square below?





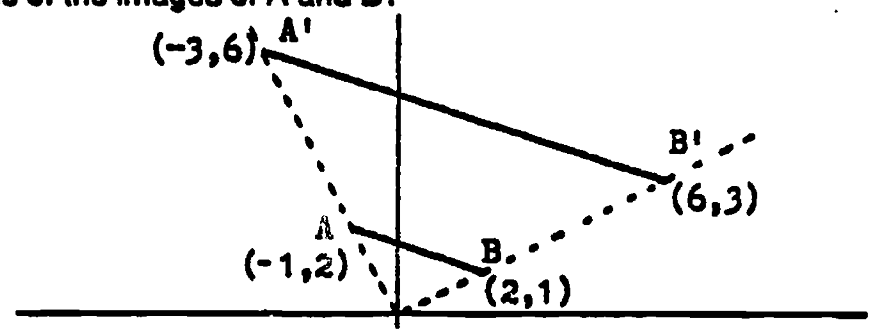
(Answer: Rotation 90 degrees --Quarter turn--clockwise)

7-9 Comment:

Coordinate representations of size changes are especially instructive to illustrate the multiplicative property of the transformation.

7-9 Example:

Line segment $\overline{AB}$ is subjected to a size change with the origin as center and magnitude 3. What are the coordinates of the images of A and B?



(Answer: (-3,6) and (6,3)--The answer is shown in the drawing as well.)

VISUALIZING-SKETCHING-CONSTRUCTING:
To visualize, sketch and construct geometric objects

CONCEPTUALIZATION: **To visualize, sketch and construct geometric shapes or relationships [G6Cn1]**

K-3 Comment:

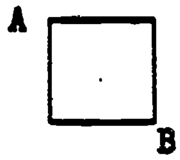
All the plane shapes introduced at this level should be the subjects of drawing or sketching. Plane figures can be cut apart and put together again so that students can recognize and discuss various shapes contained in a given shape. (Tangrams are good for this). Solids should be viewed from a particular point of view such as the front or side and a sketch made of what is seen. Sketching of solids with "dotted lines for the hidden edges" is not expected, but sketches for solids as they appear to the student should be emphasized.

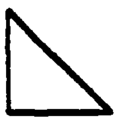
K-3 Example:

What figure do you see by looking at the bottom of a soup can? Draw it.

(Answer: Circle -- )

Cutting a sandwich from A to B produces what shapes? Draw one.



(Answer: Right triangle -- )

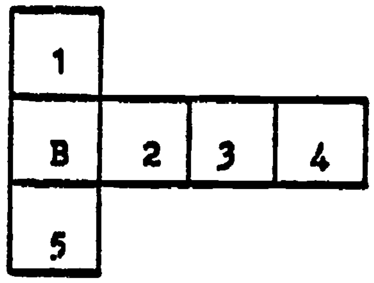
4-6 Comment:

All the shapes of this level should be material for drawing. In addition, sketches of solids familiar to the students should be done--especially cubes, boxes, cylinders and cones. Drawing front, top and side views of objects (like buildings) is also a good exercise. Circles should be constructed using a compass. Cross sections of solids can also be investigated. Use the art teacher to help students begin to make a solid look right in a sketch on the plane.

4-6 Example:

The square marked "B" is to be the bottom of a cube folded from the pattern shown. Which square will be the top?

Draw a cube.



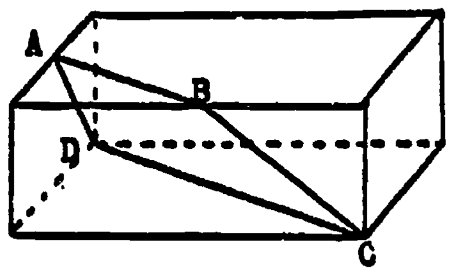
(Answer: 3)

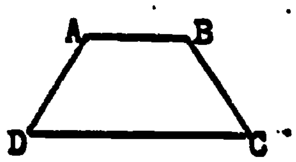
7-9 Comment:

Hidden lines in drawings of solids should be shown as dotted lines. Constructions include copying an angle, copying a segment, constructing an angle bisector and perpendicular bisector. Sketches of all cross sections of a solid should be done accurately.

7-9 Example:

A cross section of a cube is made by cutting at ABCD. Sketch the shape of the face of the cut.



(Answer: 

)

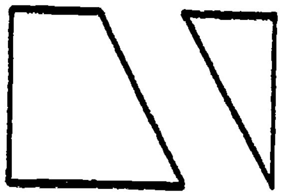
APPLICATIONS AND PROBLEM SOLVING: To solve problems requiring visualizing, sketching or constructing geometric shapes or relationships [G6PS1]

K-3 Comment:

Puzzles--dissected shapes--make good visualization activities. Provide cutouts so that students can experiment.

K-3 Example:

Given the two pieces at the right, draw (sketch) three familiar shapes you can form with them.



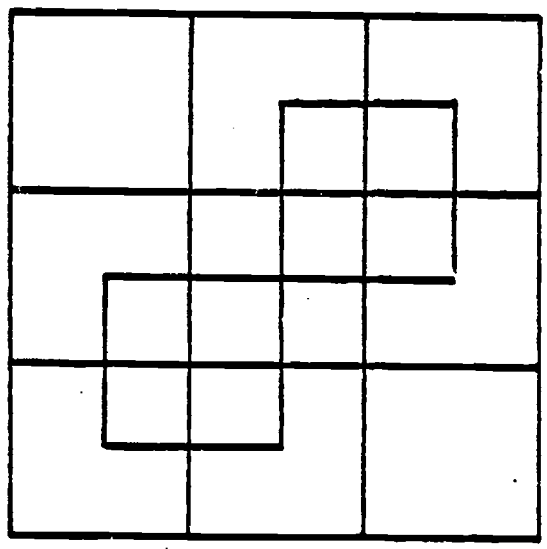
(Answer: Triangle, Rectangle, Parallelogram)

4-6 Comment:

Counting activities when figures overlap provide good experience in analyzing figures. The skill is useful in later mathematics and in life.

4-6 Example:

How many squares?



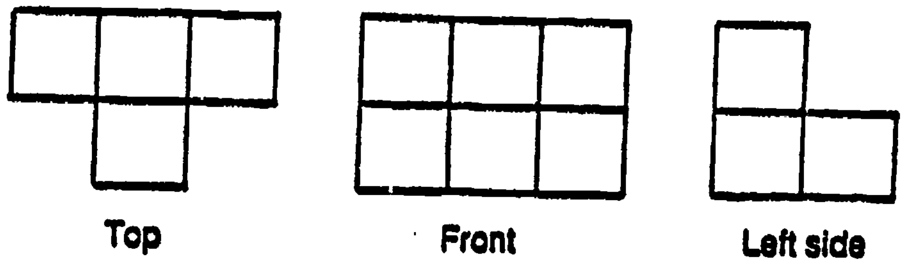
(Answer: 26)

7-9 Comment:

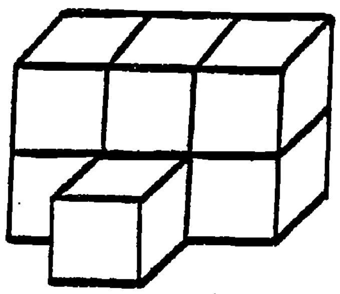
Cubes can be used to make shapes which provide practice in visualizing and in sketching.

7-9 Example:

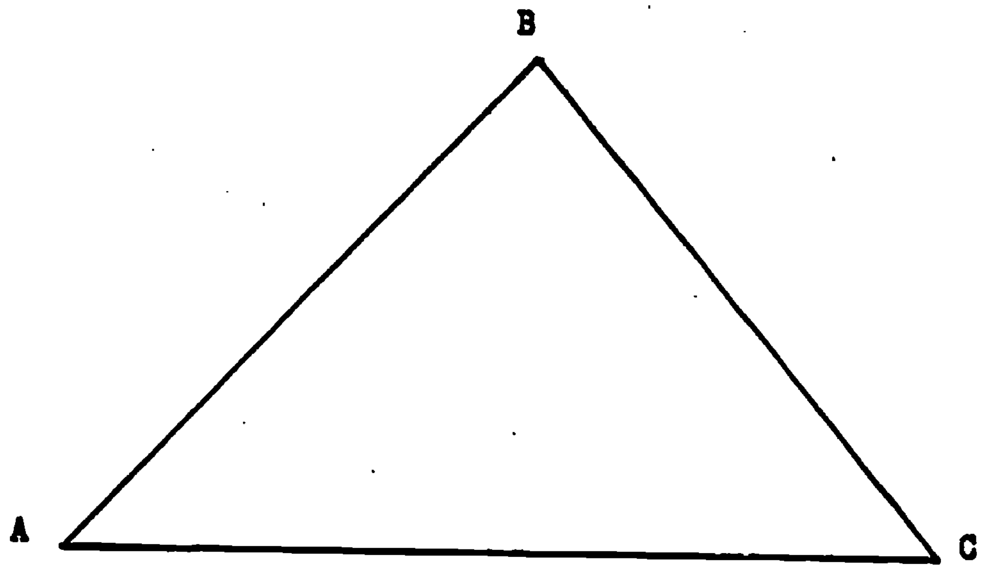
Sketch a block house if its top, front and left side are as shown.



(Answer:



Construct the perpendicular bisectors of the sides of this triangle. Do they intersect in a point? Is the point inside, on, or outside the triangle?



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Framework:
Michigan Mathematics Objectives

Mathematical Processes

Mathematical Content Strands	Conceptualization	Mental Arithmetic	Estimation	Computation	Applications and Problem Solving	Calculators and Computers
Whole Numbers and Numeration						
Fractions, Decimals Ratio and Percent						
Measurement						
Geometry						
Statistics and Probability						
Algebraic Ideas						
Problem Solving & Logical Reasoning						
Calculators						

S

STATISTICS AND PROBABILITY: AN OVERVIEW

The overview for this strand contains a summary description of the content included, the developmental flow of the content with further interpretations and grade level emphasis, reasons that the objectives are important for inclusion in the curriculum and some suggested implications for instruction. The Resources section contains references with further suggestions for curriculum and for teaching.

The second part on Interpretation of the Statistics and Probability strand contains objectives identified by process for the objective --Conceptualization, Mental Arithmetic, Estimation, Computation, Problem Solving and Applications, or Calculators and Computers. Following each objective are comments and examples organized by grade levels, K-3, 4-6 and 7-9 to help in interpreting the objective and providing specifications to assist in the assessment of the objective.

What is included in this strand?

In the statistics and probability strand, students should be able to:

1. construct, read, and interpret tables;
2. construct, read, and interpret graphs;
3. read, interpret, detemine and use distributions of data (i.e., descriptive statistics) in regard to:
   a. the center of the distribution;
   b. the way in which a distribution is spread out; and
   c. the existence of extreme values
4. read, interpret, determine, and use probability in regard to chance and uncertainty.

How does the development flow?

Tables:

Tables are often used as the first step in organizing data. Students begin with tables containing a small number of rows and columns, using pictures and tallies and progress to more complex tables. They also progress from simply answering questions about entries in the table to interpretive or comparative questions based on data. To construct tables, students need to learn organizational skills and recording skills.

Graphs:

Graphs provide students with a means to communicate observations they have made about data. In the lower elementary grades students make *object* graphs, using actual objects, for example, shoes, mittens, toy cars. They progress to *pictorial* graphs, for example, pictures or cutouts of animals, teeth, cars, etc. They then move to *symbolic* graphs, for example, a square standing for a pet. As they begin to use more abstract graphs, they need to learn appropriate techniques of scaling to make a graph.

Besides conventional graphing forms, picture graphs, bar graphs, line graphs and circle graphs, students need to read, construct and interpret newer types of graphs such as Line Plots, Stem-and Leaf Plots, Box Plots, and Scatter Plots in Grades 7-9.

Students need to learn to use graphs for prediction. This includes recognizing trends and patterns, interpolating and extrapolating.

Descriptive Statistics:

Three types of descriptive statistics are presented: (a) Measures of Central Tendency showing the center of distribution - mean and median; (b) Measures of spread - the range and quartiles; and (c) Extreme values - outliers. These are defined and described in the discussion of objectives.

Students should not just be able to identify and determine descriptive statistics. They should be able to use these measures in predicting outcomes, and in choosing wise courses of action.

Students should learn that a single number summary, for example the mean or range, is not sufficient to describe a data set. Students should learn to use descriptive statistics along with graphical methods as ways to deal with data.

Probability:

Knowledge of probability allows students to make reasonable predictions in situations involving uncertainty. Students should progress from equally likely to non-equally likely events, to determining probabilities experimentally, and to investigating compound events. Ultimately, students should use their knowledge of probability to simulate real world events and situations.

Why teach these objectives?

Everyone is confronted daily with data presented in tables and graphs. The ability to deal with information in graphs and tables is an essential skill in today's world. In addition to the more standard types of graphs, several new forms, for example Line Plots, Stem-and-Leaf Plots, Box Plots, and Scatter Plots are easy to construct, convey a lot of information simply, and are easy to interpret. These graphs are widely used in business and industry, and students need to be familiar with them.

Collecting and analyzing data are crucial skills. Learning to organize data in tables and to display data in graphs will be of great assistance in practical situations and in the study of other content areas such as science and social science.

Descriptive statistics help people to describe distributions quickly. Ability to recognize and use measures of central tendency and measures of spread has been considered part of the mathematics curriculum for many years. With the familiar mean, median, and range, the equally useful ideas of quartile and outlier are included. Mode is not included because it has a limited utility in statistical work.

Probability has long appeared in texts. It deserves greater attention, since it helps us to make predictions in the face of uncertainty, and it serves as the basis for simulating real-world events in the classroom.

Life is a series of choices. Statistics and probability provide an opportunity to develop the critical thinking skills necessary to make the most beneficial choices. The skills of asking appropriate questions, collecting relevant data, gaining information from the data, and answering the questions will serve students at all stages in their life.

What are the implications for instruction?

Statistics and probability should be taught actively. Students should collect data of interest to themselves by searching through sources, by conducting surveys, and by performing experiments. Rulers and graph paper should be readily available for constructing graphs and tables. Probability devices, such as dice (at least 6-sided and 20-sided), spinners, coins, playing cards, and flat counters with two distinct sides should be part of the equipment in each classroom.

Students should:

Be taught to make predictions before investigating a topic, conducting a survey, or carrying out an experiment, and then use the results to verify or disprove their predictions;

be trained to make inferences based on results obtained or displayed;

be encouraged to use calculators routinely for computation during the study of statistics and probability, and also learn to use computers for long-run simulations; and

learn to write summary sentences or paragraphs describing their conclusions based on data available.

Statistics and probability relate to all areas of the curriculum. Studies, surveys, and experiments can and should be carried out in social science, science, and other areas, as well as in mathematics. The more that examples from other areas are encountered, the more students will see the usefulness of statistics and probability in their world.

Vocabulary

K-3	4-6	7-9
Bar graph	Descriptive statistics	Box plot
Data	Equally likely events	Compound event
Picture graph	Frequency	Circle graph
	Interpolate	Mutually exclusive
	Line graph	Extrapolate
	Line plot	Outlier
	Mean	Quartile
	Median	Scatter plot
	Probability	Stem-and-Leaf Plot
	Random	Whisker
	Sample	
	Simple event	
	Statistics	

Resources

Freeman, Maeji. Creative Graphing. New Rochelle, NY: Cuisenaire Company of America, Inc., 1986. (K-3, 4-6, 7-9).

Garcia, Adela & Palinary, Agis. Math/Primary Fun With Graphs. Huntington Beach CA: Creative Teaching Press, Inc., 1986. (K-3).

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Murphy, Elaine C., Developing Skills With Tables and Graphs. Palo Alto, CA: DaleSeymour Publications, 1981. (K-3, 4-6).

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2. Newman, Claire M., Obremski, Thomas E., and Scheaffer, Richard L., Exploring Probability, 1987, (4-6, 7-9).
3. Gnanadeskan, Mrudulla, Scheaffer, Richard L., and Swift, Jim. The Art and Techniques of Simulation, 1987, (7-9).

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Teaching With Student Math Notes. Reston, VA: NCTM, 1987. pp. 51-56, 83-94, 101-106,113-118. (4-6, 7-9).

Yager, Tom and Kepchar, Georgeann, Tables, Charts, & Graphs. St. Louis, MO: Milliken Company, 1985. (K-3, 4-6).

STATISTICS AND PROBABILITY: THE OBJECTIVES

TABLES: To construct, read and interpret tables

CONCEPTUALIZATION: To read tables and identify existing patterns in tables [S1Cn1]

K - 3 Comment:

Tables should be no larger than 4 rows by 2 columns, or the reverse. Entries in the table may be numbers, words, pictures, or tallies.

K - 3 Example:

How many fish are in the tank?

IN THE FISH TANK	
	~~////~~
	//
	///
	/

(Answer: 5)

4 - 6 Comment:

Tables may be any size.

4 - 6 Example:

What can you say about the average attendance, beginning in 1980-81?

Pistons Attendance at the Silverdome

	GAMES	TOTAL	AVG.
1978-79	41	389,936	9,510
1979-80	41	333,233	8,128
1980-81	41	228,349	5,569
1981-82	41	406,317	9,910
1982-83	41	522,063	12,733
1983-84 (Led NBA)	41	652,865	15,923
1984-85 (Led NBA)	41	691.540	16,867
1985-86 (Led NBA)	41	695,239	16,957
1986-87 (Led NBA)	41	908,240	*22,152

*NBA All-Time Record

(Answer: The average attendance increased each year, 1980-81 through 1986-87.)

COMPUTATION: To construct tables from data [S1Cm1]

K-3, 4-6 Comment:

Data should be interesting to the student, and, if possible, collected by the student or the class. Once the data are available, discussion should center on setting up the categories in the table.

COMPUTATION: To record data in existing tables [S1Cm2]

K-3 Comment:

Recording should be by tallying, keeping the total number of entries small.

4-6 Comment:

Recording should be done by tallying and then the frequencies represented by numbers.

APPLICATIONS AND PROBLEM SOLVING: To use tables for comparison [S1PS1]

4-6 Comment:

Some comparison should be within a table, for example, mileage between pairs of cities, some between tables, for example, most 10-year-olds prefer the Disney Channel, while most 13-year-olds prefer MTV.

CALCULATORS AND COMPUTERS: To generate tables using calculators and computers [S1Ca1]

4-6, 7-9 Comment:

Spreadsheets are good examples of computer-generated tables.

GRAPHS: To construct, read and interpret graphs

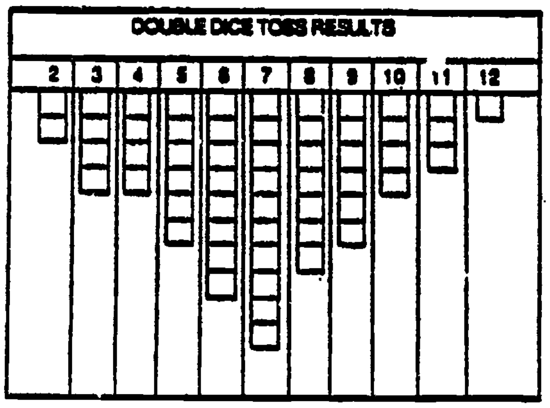
CONCEPTUALIZATION: **To read picture graphs and bar graphs [S2Cn1]**

K-3 Comment:

Graphs should be displayed that read in various directions: Left to right, top to bottom,
bottom to top, right to left.

K-3 Example:

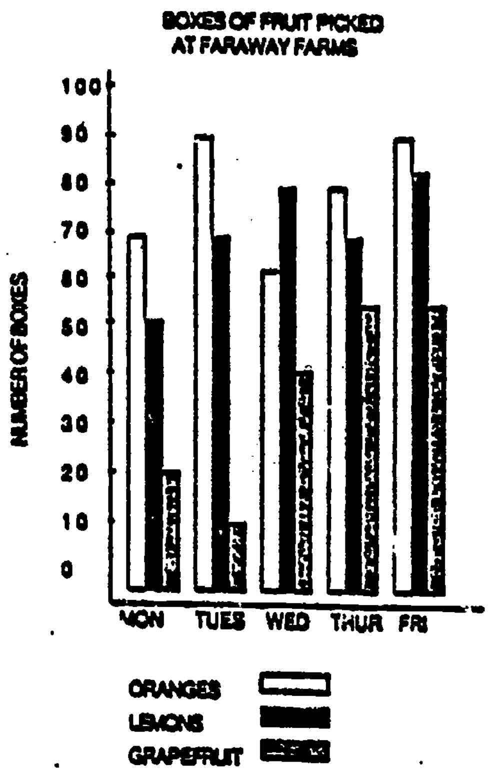
Which two numbers are hardest to get?



(Answer: 2 & 12)

4-6 Example:

Which fruit sold the most boxes?



(Answer: Oranges)

CONCEPTUALIZATION: To read line graphs and line plots [S2Cn2]

4-6, 7-9 Comment:

A *Line Plot* uses a portion of a number line, with each value marked at the appropriate spot. Repeated values are marked vertically. A line plot is useful for recording data directly. Students must learn that the number line must have a constant scale.

7-9 Example:

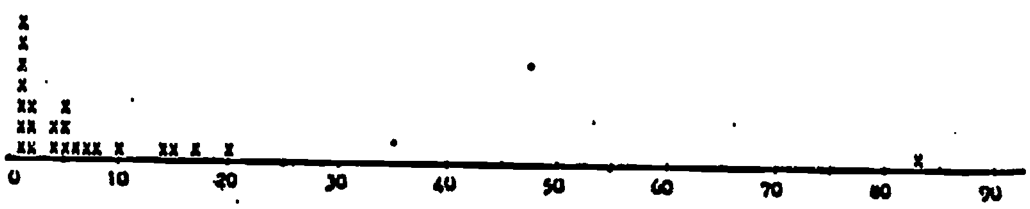
1984 Summer Olympics

Number of Gold Medals

Australia	4	South Korea	6
Austria	1	Mexico	2
Belgium	1	Morocco	2
Brazil	1	Netherlands	5
China	15	New Zealand	8
Finland	4	Pakistan	1
France	5	Portugal	1
West Germany	17	Romania	20
Great Britain	5	Spain	1
Italy	14	Sweden	2
Japan	10	United States	83
Kenya	1	Yugoslavia	7

How many nations won 15 or more Gold Medals?

Line Plot of Gold Medals



Number of Gold Medals

(Answer: 4 nations)

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CONCEPTUALIZATION: To read circle graphs [S2Cn3]

7-9 Example:

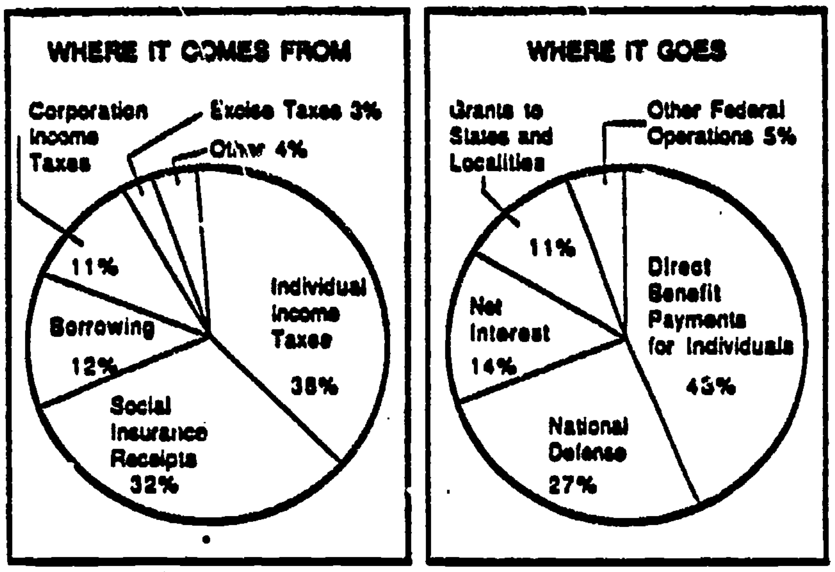
What percent of the 1969 Reagan Budget income is due to borrowing?

What percent of the 1989 Reagan Budget will be paid out in interest?

THE REAGAN BUDGET

1989 BREAKDOWN

Fiscal Year Outlay Estimate: $1,094,200,000,000



(Answers: 12%, 14%)

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CONCEPTUALIZATION: **To read stem-and-leaf plots, box plots and scatter plots [S2Cn4]**

7-9 Examples:

The table below is used to provide examples of stem-and-leaf plots, box plots, and scatter plots

Item	Calories	Fat (gm)	Carbohydrates (gm)	Sodium (mg)
HAMBURGERS				
Burger King Whopper	660	41	49	1083
Jack-in-the-Box Jumbo Jack	538	28	44	1007
McDonald's Big Mac	591	33	46	963
Wendy's Old Fashioned	413	22	29	708
SANDWICHES				
Roy Rogers Roast Beef	356	12	34	610
Burger King Chopped-Beef Steak	445	13	50	966
Hardee's Roast Beef	351	17	32	765
Arby's Roast Beef	370	15	36	869
FISH				
Long John Silver's	483	27	27	1333
Arthur Treacher's Original	439	27	27	421
McDonald's Filet-O-Fish	383	18	38	613
Burger King Whaler	584	34	50	968
CHICKEN				
Kentucky Fried Chicken Snack Box	405	21	16	728
Arthur Treacher's Orig. Chicken	409	23	25	580
SPECIALTY ENTREES				
Wendy's Chili	266	9	29	1190
Pizza Hut Pizza Supreme	506	15	64	1281
Jack-in-the-Box Taco	429	26	34	926

Source: Consumer Reports, September 1979.

Stem-And-Leaf Plot:
A stem-and-leaf plot presents numerical information succinctly, using a *stem*, for example, tens or hundreds, and *leaves*, the final significant digit in each number.

Grams of Carbohydrates in Fast Food

Stem	Leaf
1	6
2	57799
3	24468
4	469
5	00
6	4

Legend

4|6 represents 46 grams of carbohydrates

In this case, the stems are the tens digits, while the leaves are the ones digits. The legend is important because it tells how to read the particular Stem-and-Leaf Plot.

Usually data are recorded on one Stem-and-Leaf Plot, and then the leaves are arranged in numerical order on a second plot.

Back-To-Back Stem And Leaf Plot
Stem-and-Leaf Plots can be displayed back to back for comparison purposes.

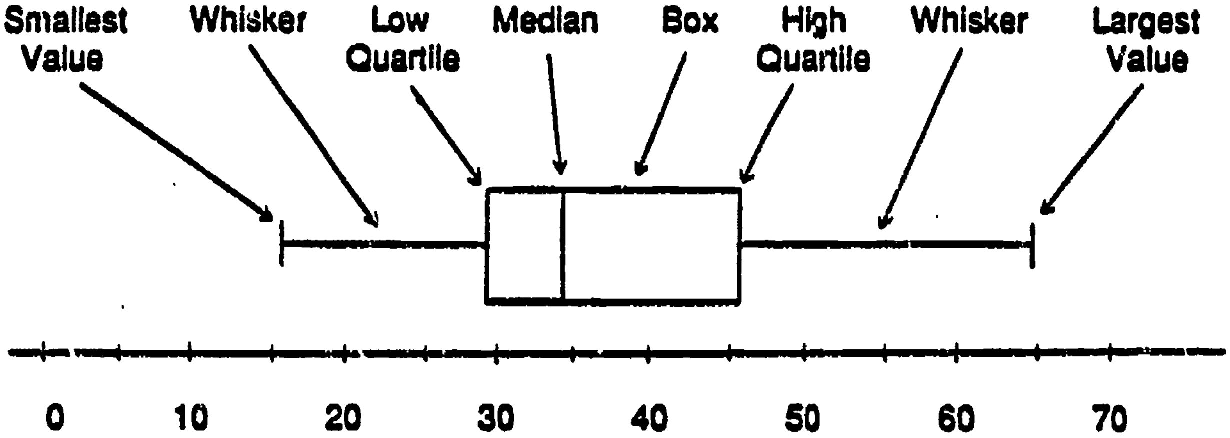
Fast Foods

Grams of Fat		Grams of Carbohydrates
9	0	
875532	1	6
877632	2	57799
43	3	24468
1	4	469
	5	00
	6	4

Legend

1|4| represents 41 grams of fat

Box Plot

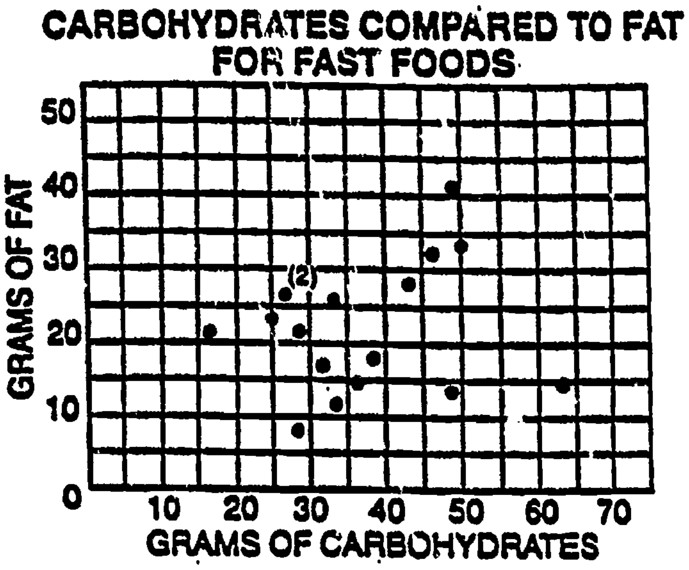


Grams of Carbohydrates in Fast Foods

The Box Plot is prepared using five numbers: The smallest value, the low quartile, the median, the high quartile, and the largest value. The box contains the middle 50% of the values, 25% on each side of the median. Each whisker contains 25% of the values.

Box Plots are frequently used side-by-side for comparison.

Scatter Plot



A Scatter Plot is used to graph data with pairs of values (*Bivariate Data*). The Scatter Plot can be studied for examples of *positive* relationships, when one variable increases, the other variable tends to increase, or *negative* relationships, when one variable increases, the other variable tends to decrease.

ESTIMATION: To make comparisons among graphs [S2Es1]

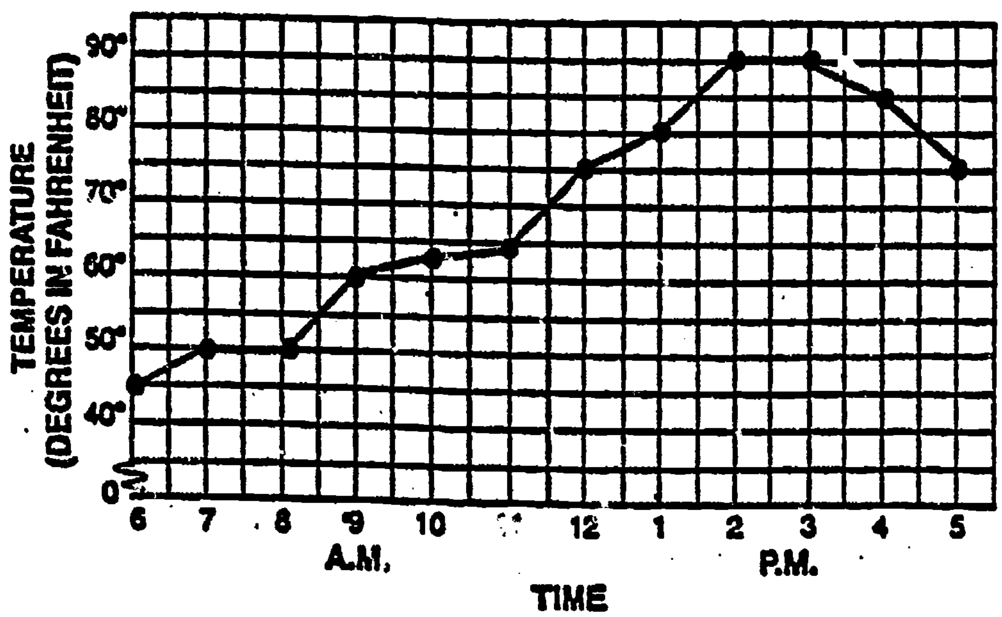
4-6 Example:

Use the graph of fruit from Faraway Farms for the first Conceptualization example on graphs.

How many more boxes of lemons were sold than grapefruits on Thursday?

(Answer: About 14)

ESTIMATION: To interpolate on graphs [S2Es2]



4-6 Example:

What was the approximate temperature at 1:30 p.m.?

(Answer: About 84 degrees)

7-9 Example:

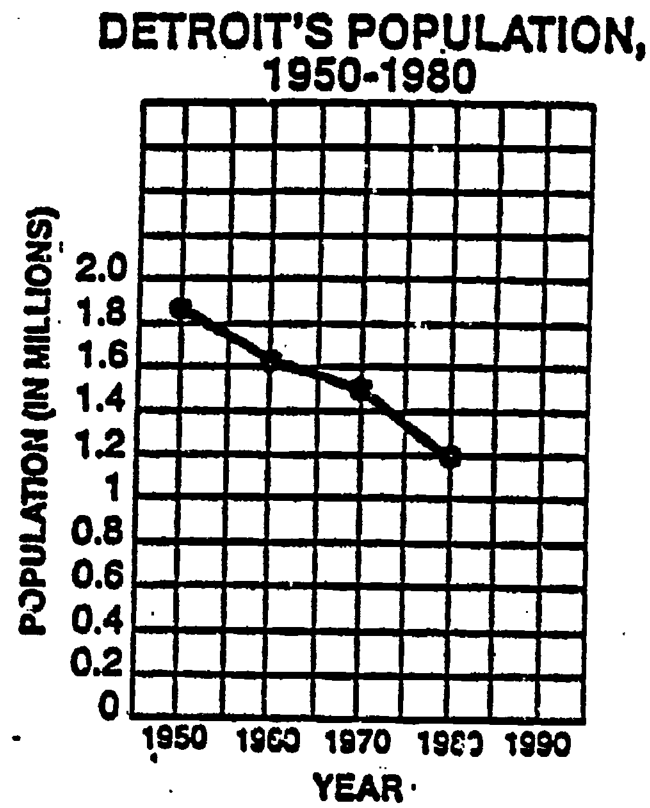
About when did the temperature first reach 70 degrees?

(Answer: About 11:30 a.m.)

ESTIMATION: To extrapolate on graphs [S2Es3]

7-9 Example:

Predict Detroit's population in 1990, assuming that the trends continue.



(Answer: About 0.9 million or 900,000)

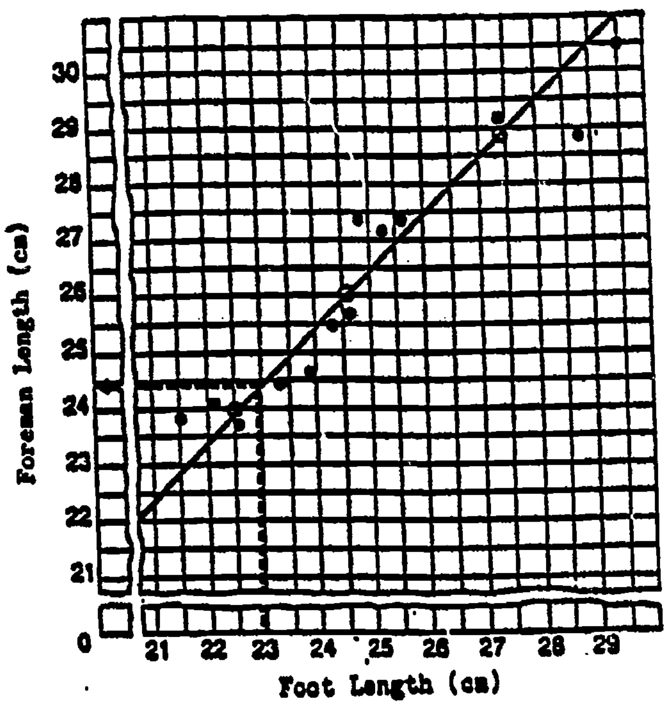
ESTIMATION: To use a fitted line on a scatter plot for prediction [S2Es4]

7-9 Comment:

A line may be fitted to a Scatter Plot by any of several techniques, which vary in precision. The purpose of fitting a line is to allow prediction. The skill of fitting a line to a Scatter Plot is not included in these objectives. However, predicting from a line already fitted is a useful skill.

7-9 Example:

Predict the forearm length of a person whose foot is 23 cm long.



(Answer: 24.5 cm)

Note that the axes are "broken" with only the relevant portion of the axes displayed.

COMPUTATION: To determine appropriate scales for graphs [S2Cm1]

4-6, 7-9 Comment:

Students need to be reminded that numbers are equally spaced along an axis.

COMPUTATION: To construct graphs [S2Cm2]

K-3, 4-6, 7-9 Comment:

The types of graphs appropriate for construction are the same ones that students are to read.

APPLICATIONS AND PROBLEM SOLVING: To select a graph that fits given information [S2PS1]

APPLICATIONS AND PROBLEM SOLVING: To determine patterns, see trends, predict outcomes, and make wise choices using graphs [S2PS2]

4-6, 7-9 Comment:

This "omnibus" objective is intended to allow any type of "higher level" activities involving graphs. The difficulty level of exercises, problems or test items should be appropriate to the developmental level of the students.

DESCRIPTIVE STATISTICS: **To read, interpret, determine, and use descriptive statistics**

CONCEPTUALIZATION: **To define terms - mean, median, range, and frequency [S3Cn1]**

4-6 Comments:

Note: *Mode* is not included because it is not a measure of Central Tendency and also because it has limited utility in statistical work.

Mean: The "average", the sum of the values divided by the number of values. For data sets of any size, calculation of the mean using a calculator is appropriate.

Median: The "middle" value in a distribution *when the distribution is ordered according to size.* When there is an odd number of values, the median is one of those values (in the set 1, 1, 3, 4, 5, 8, 9, the median is 4). When there is an even number of values, the median is the average (mean) of the two "middle" values (in the set 1, 1, 3, 4, 5, 5, 8, 9, the median is 4.5 - the average of 4 and 5).

Range: The difference between the highest and lowest values in distribution. The range can be expressed as a difference (40 - 18) or as a single number (22).

Frequency: The number in a category. Notice how frequency is used in the following table.

Results of 60 Dice Rolls

Number Showing	Tallies	Frequency
1	𝍸 ///	8
2	𝍸 𝍸 //	12
3	𝍸 ////	9
4	𝍸 𝍸 ///	13
5	𝍸 𝍸	10
6	𝍸 ///	8
TOTAL		60

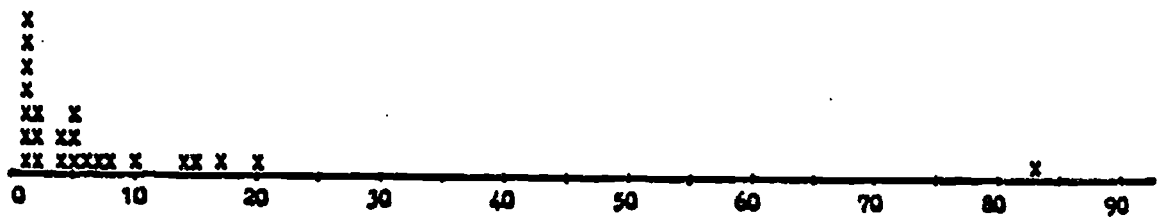
CONCEPTUALIZATION: To define outlier and quartile [S3Cn1]

7-9 Comment:

An outlier is an especially extreme value. When using Box Plots, an outlier is 1.5 times the length of the box above the high quartile or below the low quartile. If outliers exist in a Box Plot, the whisker may stop at the last value that is not an outlier, with the outlier(s) shown as dots.

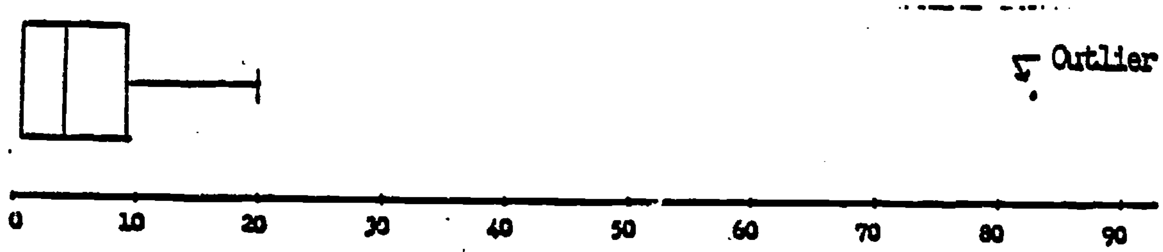
7-9 Example:

Number of gold medals won by countries, 1984 Summer Olympics



83 is an outlier

If the Line Plot data is shown as a Box Plot, it would look like this:



No lower whisker - lowest value & low quartile are both 1.

Outliers are always of special interest. In this case, the absence of Russian athletes may have resulted in the U.S. dominance.

7-9 Comment:

The quartiles, with the median, divide a distribution into four equal parts. Once the median is found, the quartiles are found by finding the median of each half of the distribution. In a Box Plot, the quartiles are the edges of the box. In the Box Plot above, the low quartile is 1 and the high quartile is 9.

CONCEPTUALIZATION: To explain how extreme values affect the median and mean [S3Cn2]

7-9 Comment:

In most distributions, adding one or two extreme values to a set will have little effect on the median, but will cause the mean to change dramatically. If the value 50 is included in the set 1, 1, 3, 4, 5, 8, 9, the median changes only from 4 to 4.5, while the mean changes from about 4.4 to about 10.1.

COMPUTATION: To order data in ascending or descending order [S3Cm1]

K-3 Comment:

This is an important pre-skill for determining the median and the quartiles.

COMPUTATION: To determine mean, median, and range [S3Cm2]

4-6, 7-9 Comment:

See the comments under conceptualization.

COMPUTATION: To determine outlier and quartile [S3Cm3]

7-9 Comment:

See the comments under conceptualization.

APPLICATIONS AND PROBLEM SOLVING: To determine patterns, see trends, predict outcomes, and make wise choices using descriptive statistics [S3PS1]

4-6, 7-9 Comment:

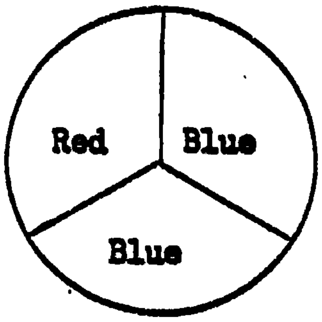
This "omnibus" objective is intended to allow any type of "higher-level" activities involving descriptive statistics. The difficulty level of exercises, problems or test items should be appropriate to the developmental level of the students.

PROBABILITY: To read, interpret, determine and use probabilities

CONCEPTUALIZATION: To compare the likelihood of simple events [S4Cn1]

K-3 Example:

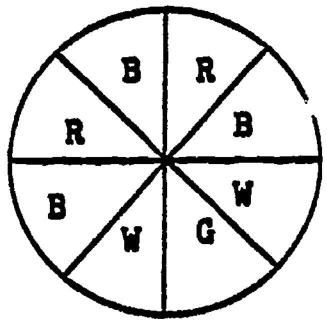
Which is more likely, blue or red?



(Answer: Blue)

4-6 Example:

Which is more likely, blue or red?



(Answer: Blue)

CONCEPTUALIZATION: To designate events as certain or impossible [S4Cn2]

7-9 Comment:

A *certain* event has probability 1.
An *impossible* event has probability 0.
All other events have probabilities between these extremes.

7-9 Examples:

(a) Is the event "meeting William Shakespeare, the well known author, on a trip to England" certain or impossible?

(Answer: Impossible)

(b) When a standard die is rolled, what is the probability of getting a number less than seven? What do we call this kind of event?

(Answer: 1; certain)

CONCEPTUALIZATION: To give one as the sum of the probabilities of all possible outcomes [S4Cn3]

7-9 Comment:

In any situation, the sum of the probabilities of all the mutually exclusive events is one. If a die is thrown, the outcomes 1, 2, 3, 4, 5, 6 are mutually exclusive. An alternative set of mutually exclusive events would be "Odd number thrown" and "Even number thrown". However, a "6" and "an even number" are *not* mutually exclusive.

MENTAL ARITHMETIC: **To determine probabilities of simple events** [S4MA1]

7-9 Example:

When two dice are rolled, what is the probability of getting a sum of 9 on the top faces?

(Answer: $\frac{4}{36}$ or $\frac{1}{9}$)

MENTAL ARITHMETIC: **To determine the probability an event will not occur, given the probability the event will occur** [S4MA2]

7-9 Comment:

If the probability of an event occurring is p, the probability of the event not occurring is 1-p. Either an event will occur or it will not; these are mutually exclusive and exhaustive; there are no other possibilities.

7-9 Example:

The probability of throwing a 7 with two dice is 1/6. What is the probability of not throwing a 7?

(Answer: $\frac{5}{6}$)

COMPUTATION: **To determine probabilities of compound events** [S4Cm1]

7-9 Example:

What is the probability of getting exactly 2 heads when 3 coins are tossed?

(Answer: $\frac{3}{8}$ - There are eight ways for three coins to fall and three of these ways contain two heads)

APPLICATIONS AND PROBLEM SOLVING: **To use probability devices to simulate real world events** [S4PS1]

4-6, 7-9 Comment:

The ability to simulate real world events is possibly the most powerful use of probability. Standard probability devices include coins, discs with two distinct sides, spinners, dice (6-sided and 20-sided), playing cards, colored marbles, and random digits.

4-6 Example:

Suppose a family has four children. Use a coin toss for each birth. Head = boy; tail = girl. How many boys and how many girls does the family have as shown by the coins?

7-9 Example:

A batter with an average of .350 bats five times in a game. Simulate 50 games to answer the question, "How likely is it that he will get exactly two hits?" (Use random digits: 01-35 is a hit; 36-99 & 00 are not hits).

(Answer: Depends on the outcomes of the actual simulations.)

CALCULATORS AND COMPUTERS: **To use calculators to determine probabilities [S4Ca1]**

CALCULATORS AND COMPUTERS: **To use computers to simulate compound events [S4Ca2]**

7-9 Comments:

Once simulations have been carried out a few times in the classroom using probability devices, computer simulations may be run to provide a large number of trials quickly. This will allow a fairly accurate approximation of the probabilities.

Framework: Michigan Mathematics Objectives

Mathematical Processes

Mathematical Content Strands	Conceptualization	Mental Arithmetic	Estimation	Computation	Applications and Problem Solving	Calculators and Computers
Whole Numbers and Numeration						
Fractions, Decimals Ratio and Percent						
Measurement						
Geometry						
Statistics and Probability						
Algebraic Ideas						
Problem Solving & Logical Reasoning						
Calculators						

A

ALGEBRAIC IDEAS: AN OVERVIEW

What is included in this strand?

The Algebraic Ideas strand is broken into three broad categories:

Variables
Real Numbers & Their Properties
Functions and Graphs

Each of these major sections includes subsections with multiple objectives.

How does the development flow?

The major emphasis of this strand is on conceptual knowledge beginning early in a child's school experience. The objectives do not stress the manipulation of the more complex algebraic symbols traditionally done in a secondary school algebra course. Traditional algebraic symbols are not neglected, however, and begin with the K-3 objectives. Symbols are useful because of their efficiency, simplicity and clarity in expressing mathematical ideas. The focus is not on just mastering the symbols but on using the symbols naturally as a way to communicate.

Problem solving as a central focus of mathematics can be seen in the algebra strand as in the other strands. Problem solving constitutes the second greatest number of objectives in the algebraic ideas strand.

The mental arithmetic, estimation and calculator objectives illustrate further the practicality of algebraic ideas. As an example, the two mental arithmetic objectives, one at grades 4-6 and the other at grades 7-9, involve using the distributive property as a short cut for mental computation of products. This is a process which has been demonstrated to be extremely effective with children who have had the opportunity to use such techniques.

The strand includes 55 objectives, 9 for grades K-3, 21 for grades 4-6 and 25 for grades 7-9. All of the Mathematical Processes have objectives written for them, however, mental arithmetic, estimation and calculator objectives are not included until grades '-6. Of these 55 objectives, 19 deal with conceptual learning, 14 with computation, 13 with problem solving, 4 with estimation, 3 with calculators and computers and 2 with mental arithmetic.

Why teach these objectives?

Algebraic ideas provide a gateway from arithmetic to mathematics but they do not come only after arithmetic is completed. Algebraic ideas are introduced concomitantly with arithmetic and other mathematical topics. Algebraic ideas are essential in expressing important generalizations from arithmetic, measurement, statistics and other branches of mathematics. The distributive property, $a(b + c) = ab + ac$ expresses an important generalization in its own right but one that is essential in understanding the algorithm for multiplying whole numbers. In measurement, $A = lw$ is a compact way to show the relationship between the length, width and area of a rectangle. In statistics, the mean (average) of three numbers, a, b and c can be expressed as Mean $= (a + b + c) \div 3$. Thus one major goal in teaching algebraic ideas is to teach generalizations and the symbols needed to express important generalizations. Equations using variables, including formulas, are the most common means to express such generalizations.

Variables are useful also in representing unknowns. Missing factors expressed in equations such as $3 \times \square = 12$ are useful in introducing division. Unknowns are useful also in solving problems where a variable is used to represent some quantity that you wish to determine and then the variable is used to write an equation or inequality. Evaluating algebraic expressions using unknowns not only teaches the meaning of the expressions but also provides alternate ways to practice arithmetic and to use the calculator in more interesting contexts.

All every day situations cannot be described using positive whole numbers and fractions. Therefore, the extension of these systems to integers and real numbers is necessary. Although this extension is implicit in the whole number strand it is dealt with in greater detail in the algebraic ideas strand.

What are the implications for instruction?

The concept of variable is introduced early. Missing addends and missing factors can be considered as variables. K-3 objectives have variables represented by both open figures, ⬡ and closed figures, ⬢ but not by letters. During grades 4-6 the transition is made to letters as variables.

Symbolization for multiplication also develops throughtout the strand. In the K-3 objectives multiplication is identified by the x sign ($3 \times \square = 6$). In the 4-6 objectives multiplication is depicted with a dot ($3 \cdot N = 6$) and the 7-9 objectives use juxtaposition ($3n = 6$).

The use of variables for unknowns and to express generalizations should be done naturally in the course of teaching many mathematics topics. As skill in addition of fractions is being taught, for example, present an example such as $\frac{3}{8} + n = \frac{7}{8}$, requiring almost the same skill but in a slightly different and more interesting way. As areas of rectangles are being found, use 5 for the width and let the variable l stand for the length, asking students to see how they could express the area (5 times the length or 5xl).

Transition from ordinary language to the language and symbolism of algebra should be a continuing goal of mathematics instruction. For example, an explanation of 3x27 might progress over a period of time from "You have 3 sevens and you also have 3 twenties" to "3 sevens and 3 twenties" to "3x(20 + 7) = 3x20 + 3x7" and finally to the algebraic statement of the generalization, a(b + c) = ab + ac. The goal is to see the increasing levels of generality expressed first informally with oral language using supporting concrete models, and resulting finally with compact algebraic symbolism.

Vocabulary

K-3	4-6	7-9
Variable	Positive Integer	Function
Expressions	Negative Integer	Cube Root
Mathematical Sentences	Exponent	
Equation	Square Root	
	Parentheses	
	Formula	
	Inequality	
	Distributive Property	

Resources

Coxford, Arthur F., The Ideas of Algebra, K-12, Reston, VA: National Council of Teachers of Mathematics, 1988.

ALGEBRAIC IDEAS: THE OBJECTIVES

VARIABLES

EXPRESSIONS: **To understand and use expressions containing variables**

CONCEPTUALIZATION: **To recognize and use the concept of variable in expressions [A1Cn1]**

K-3 Comment:

At this level variables will be represented with a △ or □ and similar symbols. Multiplication will be represented with X.

K-3 Example:

If △ =15 and □ = 9
then △ + □ means:

A. 15 x 9
B. 159
C. 15 + 9
D. 15 - 9

Answer: C.

4-6 Comment:

At this level variables may be represented with letters. Multiplication will be denoted with a raised dot between two numbers or variables. For example, 3 • 4 = 12 means "3 times 4 equals 12." This is not normally found in text books at this level.

4-6 Example:

3 • a + 2 means

A. a + a + a + 2
B. 3 + a + 2
C. a • a • a + 2
D. a + a + a + a + a

Answer: A.

7-9 Comment:

At this level multiplication is denoted by juxtaposition of a number and a variable, or two or more variables.

7-9 Example:

3 (p + q) - 5 is NOT the same as

A. (p + q) + (p + q) + (p + q) - 5
B. 3p + q - 5
C. 3p + 3q - 5
D. p + p + p + q + q + q - 5

Answer: B.

COMPUTATION: To evaluate expressions [A1Cm1]

Comment: The order of operations is as follows: 1. Do what is inside the parentheses; 2. Do multiplications and divisions in order from left to right; 3. Do addition and subtraction in order from left to right. If these are exponents, perform that operation after step 1.

4-6 Example:

□ + 2 • △ = 24

Which set of numbers does not make a true sentence?

A. □ = 0, △ = 12
B. □ = 4, △ = 10
C. □ = 1, △ = 12
D. □ = 24, △ = 0

Answer: C

7-9 Example:

If n=3 and p=5 then n + 2(p + 3)=

A. 40
B. 19
C. 16
D. 13

Answer: B

ESTIMATION: **To estimate values of expressions [A1Es1]**

Comment: Often to assess the ability to estimate, a timed situation is essential. Otherwise, students will first compute to find an exact answer.

4-6 Comment:

Only one operation will be used in the expression to be evaluated.

7-9 Comment:

Expressions will include two operations. The example below should be done in a timed situation.

4-6 Example:

If A = b• h, the most reasonable estimate of a when b = 7 3/4 and h = 3 1/4 is...

A. 7 x 3
B. 7 x 4
C. 8 x 3
D. 8 x 4

Answer: C.

7-9 Example:

If A = p + p r t
p = 500
r = .1
t = 2.5

Then A is in what range?

A. 0-100
B. 100-1000
C. 1000-2000
D. Over 2000

Answer: B

CALCULATORS: **To use calculators to evaluate expressions [A1Ca1]**

7-9 Example:

Use a calculator to evaluate. $\frac{Z - xy}{3}$ when x = 91, y = 2.7, Z = 24

A. - 60.3
B. - 73.9
C. - 224.1
D. - 19.29

Answer: B.

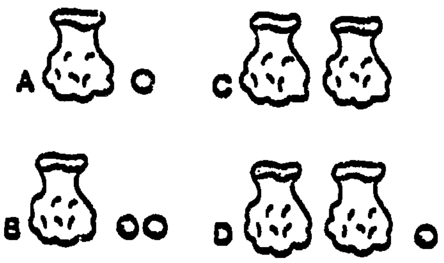
VERBAL, SYMBOL, MODEL RELATIONS:

To use variables in translating among verbal expressions, symbols and situations that are pictorial or practical

CONCEPTUALIZATION: To recognize physical or pictorial models for relations and operations [A2Cn1]

K-3 Example:

Ina has a bag of marbles. John gave her 2 more marbles. Which picture shows what she has now?



Answer: B

4-6 Example:

Which picture shows 3 more than twice 4?

A. ####

B. ###
####
###

C. #########

D. ####
###
####

Answer: D

4-6 Comment:

May use phrases such as increased by, decreased by, and half of.

4-6 Example:

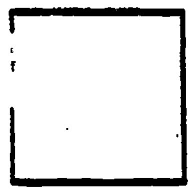
Carmen has $20 to spend on party favors. She needs favors for 15 people, including herself. Which expression represents how much she may spend for each favor?

A. $20 + 15$
B. $20 - 15$
C. $20 \cdot 15$
D. $20 \div 15$

Answer: D.

7-9 Example:

The square below has sides of length s. If the sides are increased by 2 what will the new area be?



A. s^2
B. $2s^2$
C. $4s^2$
D. $(s + 2)^2$

Answer: D.

OPEN SENTENCES: To use variables to write and solve open sentences.

CONCEPTUALIZATION: To recognize and use the concept of variable in open sentences [A3Cn1]

K-3 Example:

Which describes:
"I am a number less than 5?"

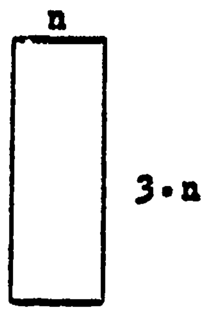
A. △ -5
B. △ <5
C. △ >5
D. △ =5

Answer: B.

4-6 Example:

Which equation could NOT be used to solve the following problem?

The figure is a race track for ants. If an ant travels 24 cm while going around the track three times, what are the dimensions of the track?



A. $3 \cdot (6n + 2n) = 24$
B. $3 \cdot 2 \cdot 4 \cdot n = 24$
C. $3 \cdot (n + 3n + n + 3n) = 24$
D. $3 \cdot (n + 3 \cdot n) = 24$

Answer: D.

7-9 Example:

The dimensions of a cube are increased by 2. Its volume is increased by 152. Which equation could be used to determine its original size?

A. $(s + 2)^2 - s^2 = 152$
B. $2s^2 - 3^2 = 152$
C. $2s^3 - s^3 = 152$
D. $(s + 2)^3 - s^3 = 152$

Answer: D.

COMPUTATION: To find solutions to open sentences [A3Cm1]

K-3 Comment:

Use only one variable. Use only one operation. Multiplication is indicated by x. Use only linear equations or inequalities. No letters for variables and no open frames are used.

4-6 Comment:

Use Linear equations. Two operations may be involved. Multiplication is indicated by a dot. e.g. $3 \cdot 4 = 12$ Letters may be used for variables.

K-3 Example:

$\triangle + 3 = 8$ What is $\triangle$?

A. 5
B. 8
C. 3
D. 11

Answer: A

4-6 Example:

$2 \cdot (b + 3) = 16$

A. 8
B. $6\frac{1}{2}$
C. 5
D. 13

Answer: C

7-9 Comment:

Equations may be of degree 2. Situations may include proportions.

7-9 Example:

If x may only be 2, 3, -2, -3, or -5 solve $x^2 + x = 6$.

A. 2, -2
B. -2, 3
C. 3, -3
D. 2, -3

Answer: D.

APPLICATIONS AND PROBLEM SOLVING: **To find solutions to problems stated verbally [A3PS1]**

K-3 Example:

I am a number. Nine is two more than I am. Who am I?

A. 11
B. 9
C. 8
D. 7

Answer: D.

4-6 Example:

Kim has a collection of stamps. After being given 10 stamps and selling 7 she had 20. How many stamps did she have originally?

A. 13
B. 30
C. 23
D. 17

Answer: D.

7-9 Example:

Lynn has a scale model of a chair. The model is $\frac{2}{5}$ the size of the real chair. If the real chair is 60 cm tall, how tall is her scale model?

A. 12 cm　C. 30 cm

B. 24 cm　D. 150 cm

Answer: B

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REAL NUMBERS AND PROPERTIES

DISTRIBUTIVE PROPERTY: To recognize and apply the distributive property

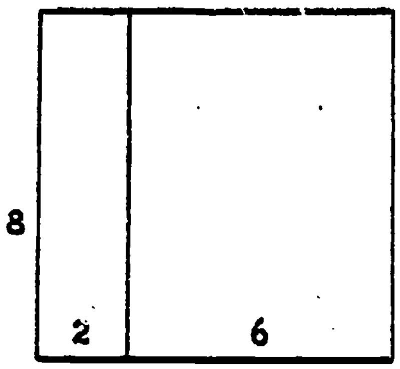
CONCEPTUALIZATION: To recognize equivalent manipulative or pictorial representations of the distributive property [A4Cn1]

4-6 Comment:

Use whole numbers only. Use multiplication and addition only.

4-6 Example:

Which expression represents the area of the square?



A. $8 \cdot 2 \cdot 6$
B. $8 \cdot 2 + 8 \cdot 6$
C. $8 \cdot 2 + 6$
D. $8 + 2 \cdot 6$

Answer: B.

7-9 Comment:

Include the use of division and subtraction. Include fractions, decimals and percent.

7-9 Example:

$21 \cdot 3\frac{1}{7}$ is equal to:

A. $21 \cdot 3 \cdot \frac{1}{7}$

B. $21 + 3\frac{1}{7}$

C. $21 \cdot 3 + 21 \cdot \frac{1}{7}$

D. $21 \cdot 3 + \frac{1}{7}$

Answer: C

MENTAL ARITHMETIC: To use the distributive property for mental arithmetic short cuts [A4MA1]

Comment: Mental arithmetic and estimation are difficult to test with a paper-pencil test. Unless something is done to force students to work problems mentally they will use paper and pencil and then select the appropriate response. Placing students in a timed situation is one way of forcing them to solve problems mentally.

4-6 Comment:

Items may be administered in a timed situation with the numbers not shown. No regrouping in the sum allowed if timed.

7-9 Comment:

Regrouping is allowed in the sum with timed situations.

4-6 Example:

Timed situation, numbers not shown. Find the product of 71 and 6.

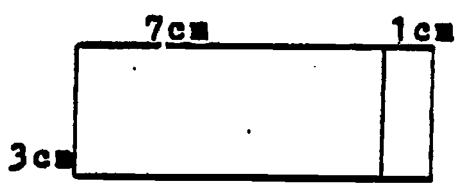
7-9 Example:

Timed situation, numbers not shown. Find 78 x 8.

APPLICATIONS AND PROBLEM SOLVING: To apply the distributive property to problem solving situations [A4PS1]

4-6 Example:

A rectangle 7 cm by 3 cm has a 1 cm by 3 cm strip added to it. Which expression represents the area of the new rectangle?

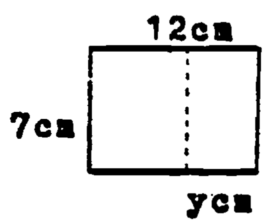


A. 3 • 7
B. 3 • 7 + 1
C. 3 • 7 + 3 • 1
D. 3 + 7 + 1

Answer: C.

7-9 Example:

A rectangle is 7 cm by 12 cm. A strip of width y is cut from the rectangle. What is the area of the remaining rectangle?



A. 7 • 12
B. 7 • 12 + 7y
C. 7y
D. 7 (12 - y)

Answer: D.

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INTEGERS: To recognize, use and compute with integers

CONCEPTUALIZATION: To interpret and compare integers in familiar situations [A5Cn1]

4-6 Comment:

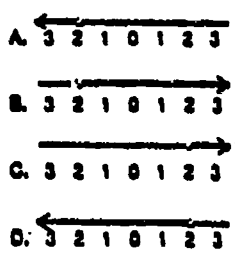
Use a thermometer or a number line. No two-step problems used.

7-9 Comment:

Use business, sports or science applications. Multiple step applications.

4-6 Example:

Which graph shows the numbers less than -2?

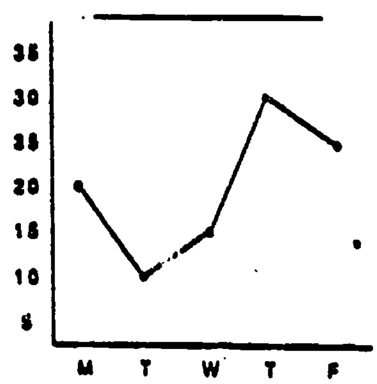


Answer: A

7-9 Example:

Paul graphs daily gains/losses of his stock. For the week indicated, he had

A. a net loss of 1.
B. a net loss of 5.
C. a net gain of 5.
D. a net gain of 2.



Answer: C.

COMPUTATION: To determine the sign of the answer for integer computation [A5Cm1]

7-9 Example:

If a and b are negative integers and a > b, what can be said about a - b?

A. a - b is negative
B. a - b is zero
C. a - b is positive
D. It is impossible to tell

Answer: C.

COMPUTATION: To compute with integers [A5Cm2]

7-9 Example:

Find $\frac{-12\,(-6)}{-2}$

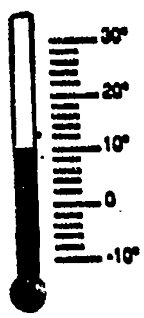
A. 36
B. -36
C. 18
D. -18

Answer: B

APPLICATIONS AND PROBLEM SOLVING: To use integers in everyday situations [A5PS1]

4-6 Example:

It is 10° C and the temperature drops 6° C per hour, what is the temperature after 2 hours?



A. -2° C
B. 4° C
C. 22° C
D. -22° C

Answer: A.

7-9 Example:

Jack was 3 under par on Saturday and 2 over par on Sunday. If par for the course is 65, how many strokes had he taken in two days?

A. 130
B. 131
C. 125
D. 129

Answer: D.

EXPONENTS, POWERS AND ROOTS: To recognize and use concepts of exponents, powers and roots

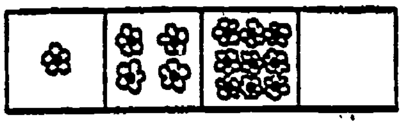
CONCEPTUALIZATION: **To recognize and use patterns of squares and cubes [A6Cn1]**

K-3 Comment:

Use only squares in pattern situations.

K-3 Example:

How many flowers belong in the 4th space?



1st 2nd 3rd 4th

A. 9
B. 10
C. 15
D. 16

Answer: D.

4-6 Comment:

Use squares or cubes.

4-6 Example:

Sixty-four 1 cm cubes are used to build a larger cube. How long is the edge of the cube which has been built?

A. 4 cm
B. 8 cm
C. 12 cm.
D. 16 cm

Answer: A

CONCEPTUALIZATION: **To recognize and use exponents and power notation [A6Cn2]**

4-6 Example:

y^3 means

A. $y + y + y$
B. $3 \cdot y$
C. $y \cdot y \cdot y$
D. $y \cdot 3$

Answer: C.

7-9 Example:

If $3^x = 9^2$, what whole number does x represent?

A. 2
B. 3
C. 4
D. 5

Answer: C.

CONCEPTUALIZATION: To read graphs of powers and roots [A6Cn3]

7-9 Comment:

No fractional exponents used.

7-9 Example:

The square root of a number n is written as $\sqrt{n}$. Which statement is true of $\sqrt{n}$?

A. Twice $\sqrt{n}$ equals n
B. $\sqrt{n}$ times $\sqrt{n}$ equals n
C. $\sqrt{n}$ equals n
D. n times n equals $\sqrt{n}$

Answer: B.

ESTIMATION: To estimate square roots [A6Es1]

4-6 Example:

Square	Side	Area
X	8	64
Y	9	81
Z	?	140

About how long is a side of square Z?

A. between 8 & 9
B. between 9 & 10
C. between 10 & 11
D. between 11 & 12

Answer: D.

7-9 Example:

$\sqrt{145}$ is

A. a little larger than 12
B. a little smaller than 12
C. a little larger than 13
D. exactly 13

Answer: A.

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CALCULATORS: To use calculators to find or approximate solutions to exponential equations [A6Ca1]

4-6 Comment:

Instruction should begin at this level by using the calculator and the constant multiplier to solve exponential equations of the form $X^y = Z$. However, only equations where X & Y are known, non negative integers will be used in a testing situation.

4-6 Example:

$3^{15} =$

A. 45
B. 4782969
C. 14348907
D. 315

Answer: C.

7-9 Example:

Use your calculator to find the smallest whole number n, so that $1.7^n > 800,000$.

A. 8
B. 20
C. 25
D. 26

Answer: D

APPLICATIONS AND PROBLEM SOLVING: To solve problems involving powers and roots [A6PS1]

7-9 Example:

Look at the pattern below.

$1 + 3 = 4$
$1 + 3 + 5 = 9$
$1 + 3 + 5 + 7 = 16$
$1 + 3 + 5 + 7 + 9 = 25$
$1 + 3 + ... + n = 169$

What is the value of n?

A. 25
B. 13
C. 31
D. 9

Answer: A.

FUNCTIONS AND GRAPHS

FUNCTIONS: To recognize and use function concepts

COMPUTATION:To represent a function with a table of values or a graph [A7Cm1]

4-6 Comment:

One operation used.

4-6 Example:

Below is a partial table for the rule y + 4 = d

y	8	10	15
d			

Find the numbers for d

A. 12 14 19
B. 4 6 11
C. 12 16 20
D. 9 11 16

Answer: A.

7-9 Example:

Use the table below.

First number	1	3	5	8	?
Second number	7	11	15	21	29

If 29 is the Second number, what is the First number?

A. 12
B. 15
C. 21
D. 63

Answer: A.

COMPUTATION: To recognize, describe and express in symbols a relationship between two sets [A7Cm2]

4-6 Example:

Which rule goes with this table?

y	d
0	0
1	3
2	6
3	9

Answer: y•3 = d

7-9 Example:

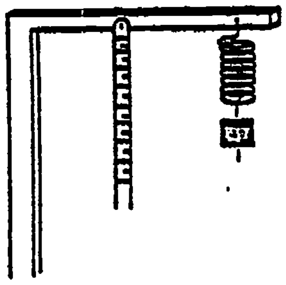
The following ordered pairs describe a rule:
(2,4), ($\frac{2}{5}$, $\frac{4}{25}$), (1.2, 1.44).
Determine the rule.

A. (x, x^2)
B. $(x, \sqrt{x})$
C. $(x, 2x)$
D. $(x, x/2)$

Answer: A

4-6 Example:

Mrs. Smith places weights on a spring and records the following data:



Weight in kg	Length of Spring in cm
2	6
3	9
4	12
5	?

How long is the spring with 5 kg on it?

A. 5 cm
B. 10 cm
C. 8 cm
D. 15 cm

Answer: D.

7-9 Comment:

Answers may involve variables.

7-9 Example:

Mr. Smith places weights on another spring and records the following data.

Weight in kg	Length of Spring in cm
1	5
2	7
3	9
4	11
x	?

If x is the weight, how long will the spring be?

A. $2x - 3$
B. $x + 2$
C. $x + 4$
D. $2x + 3$

Answer: D.

GRAPHS: To identify and interpret graphs representing situations, tables of values or sentences

CONCEPTUALIZATION: To identify an appropriate graph given a table of values or an equation, and conversely [A8Cn1]

4-6 Comment:

Bar graphs and discrete point graphs are used. Coordinates are non-negative integers.

4-6 Example:

Determine which table can NOT be associated with the graph.

A.

x	1	4	5
y	1	4	5

B.

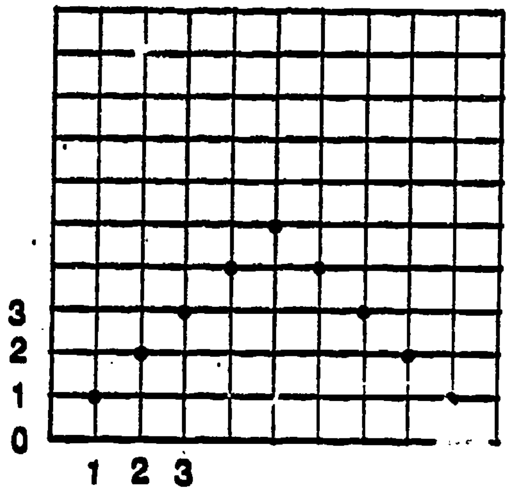
x	1	2	3	4
y	1	2	3	4

C.

x	3	4	5	6
y	3	4	5	6

D.

x	1	2	5	7
y	1	3	5	3



Answer: C.

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7-9 Comment:

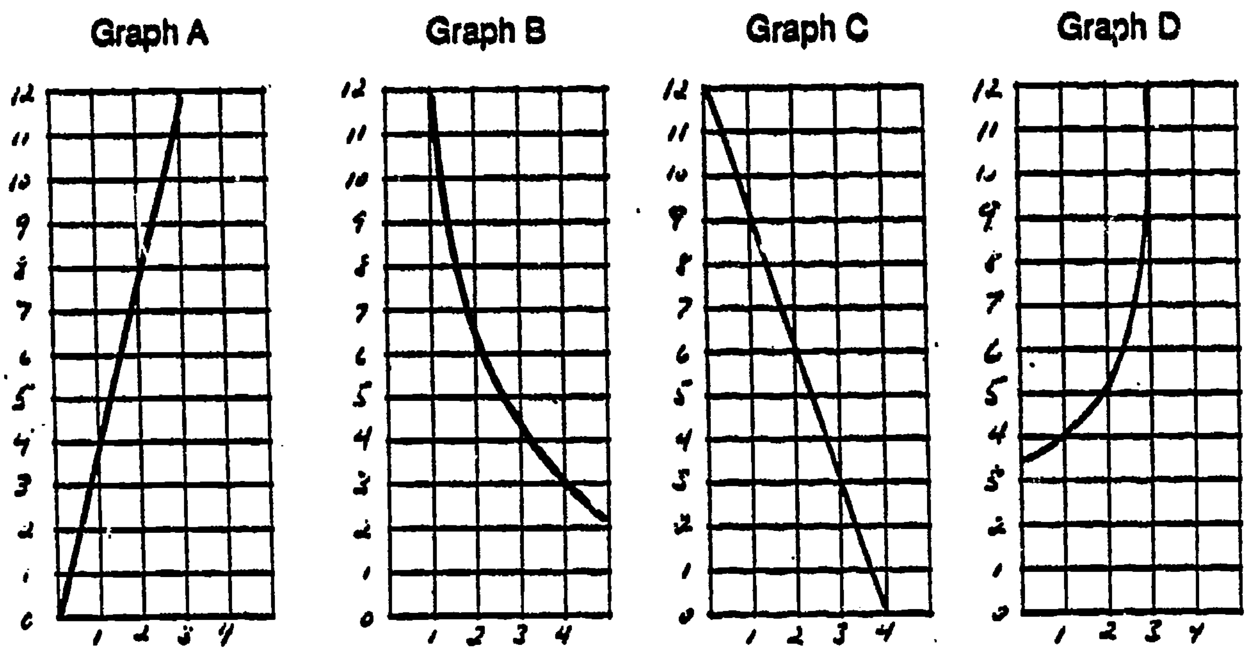
Include continuous graphs

7-9 Example:

Three values on a graph are shown in the table:

x	1	2	3
y	4	5	12

Which graph below includes all of these points?



A) Graph A
B) Graph B
C) Graph C
D) Graph D

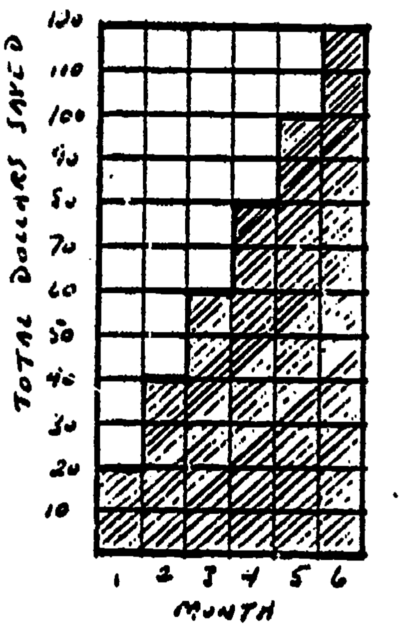
Answer: D.

APPLICATIONS AND PROBLEM SOLVING: **To use graphs to solve problems [A8PS1]**

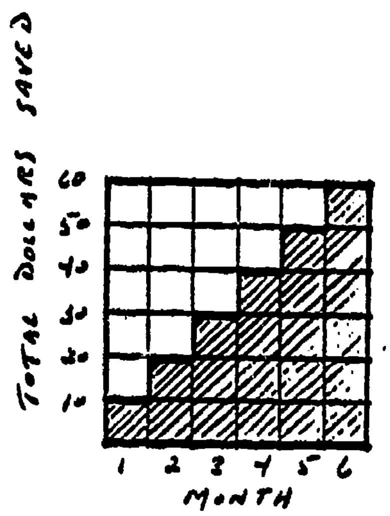
7-9 Example:

Susan Deposits $20 into her savings each month, starting in January. At the end of January she has $20. At the end of February she has $40, and so on. Which graphs shows the <u>total amount</u> of money she has saved by the end of <u>each</u> of the first 6 months of the year?

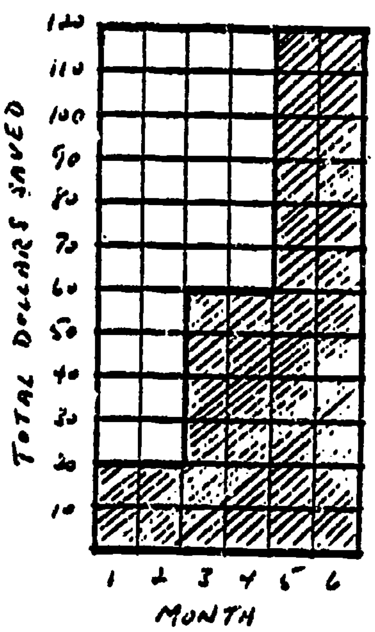
GRAPH A



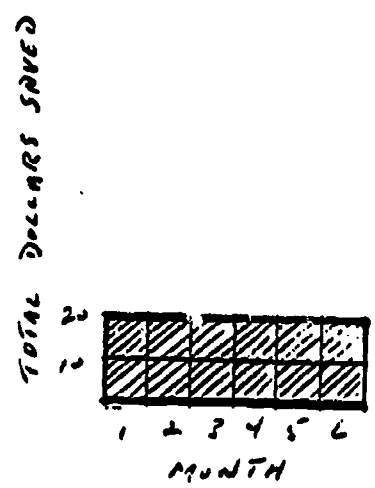
GRAPH B



GRAPH C



GRAPH D



A) Graph A
B) Graph B
C) Graph C
D) Graph D

Answer: A

Framework: Michigan Mathematics Objectives

Mathematical Content Strands	Mathematical Processes					
	Conceptualization	Mental Arithmetic	Estimation	Computation	Applications and Problem Solving	Calculators and Computers
Whole Numbers and Numeration						
Fractions, Decimals Ratio and Percent						
Measurement						
Geometry						
Statistics and Probability						
Algebraic Ideas						
Problem Solving Logical Reasoning						
Calculators						

P

PROBLEM SOLVING: AN OVERVIEW

What is included in this strand?

Problem solving involves coordinating previous experience, knowledge and intuition in an attempt to determine a method for resolving a situation whose solution and outcome are not known. Problem solving and logical reasoning is both a process and a content strand. Process is interpreted as the methods, procedures, strategies and heuristics that students use in solving problems. In order to develop the process, the skills or content of problem solving must be taught.

The major objectives of the Problem Solving strand include the following:

1. *Patterns* - Patterns permeate all of mathematics, therefore, students should be able to identify, use and construct patterns in all areas of mathematics.

2. *Understanding Problems* - The objectives here involve the first step in the formulation of a plan for solving any given problem (heuristics). Sufficient time must be devoted to gathering data, determining what is known, unknown, insufficient, contradictory or redundant.

3. *Problem Solving Strategies* - The following list represents the strategies included in this strand:

   - Identify and use a pattern
   - Use and construct a table
   - Make an organized list
   - Guess and test
   - Work backwards
   - Make or use a drawing, a graph or physical model
   - Write an open sentence
   - Solve a simpler problem
   - Eliminate possibilities
   - Select the appropriate operation(s)

4. *Evaluating Solutions* - This is the last and most neglected step in the problem solving process. It incorporates checking the result in the argument, deriving the result differently and using the result or method for solving some other problems.

5. *Logical Reasoning* - Formal and informal reasoning encompasses the ability to identify likenesses and differences, to classify and categorize objects and numbers by their attributes and to draw valid conclusions.

A pattern of improvement concentrated in the area of basic skills implies severe consequences for our nation's continued technological leadership in the global marketplace. The need for a mathematically literate citizenry is more important today than ever before. Given the pace of change in our society there is no sufficient way to learn everything that we will need to know about mathematics in the future. Consequently, the school mathematics curriculum must focus on ways to equip students with the ability to continue to learn. There will be an ever increasing demand for people who can analyze a problem and devise a means of solving it. Thus, any mathematics curriculum is no longer satisfactory that does not give *direct*, serious attention to developing problem solving ability in students.

What are the implications for instruction?

Regardless of the amount of time available for teaching mathematics, we must make a commitment to establishing problem solving as a significant part of the curriculum. Daily problem-solving experiences communicate to students the importance of problem solving and promote the improvement of problem-solving performance.

An important step in incorporating problem solving into the curriculum is the development of a plan for sequencing problem-solving experiences. Attention must be given to factors that affect problem difficulty, the age level of students, and the mathematical content requirements of particular problem-solving experiences.

When teaching problem solving, the emphasis should NOT be on the development or practice of skills. When the purpose of instruction is to develop problem-solving ability, the mathematical content of the problem-solving experiences should be kept behind the content of the regular program.

It is absolutely essential that the classroom climate be conducive to problem solving. The content and sequence of instruction, evaluation practices, grouping patterns, and the teacher's attitude and actions all interact to form the classroom climate. A classroom atmosphere conducive to problem solving is one in which students are aware of the freedom and desirability of exploring any idea for solving a problem. Some suggested teacher actions that contribute to this climate are:

1. Allow sufficient time to solve problems.
2. Use questions to focus student's attention on the pertinent information given in the problem.
3. Encourage students to consider different strategies that might be used to solve the problem.
4. Have students use, or make available for them to use, manipulative materials.
5. Avoid "censorship" of students' ideas.
6. Organize for small group "cooperative" experiences.
7. Expose students to error and encourage them to detect and to demonstrate what is wrong and why.

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For all problem-solving experiences, and particularly for those at the beginning of the year, the emphasis of instruction should be on behaviors such as willingness and perseverance in problem solving and selecting and using problem-solving strategies - NOT on getting correct answers. Good problem-solving experiences will ultimately result in higher student achievement.

An integral part of any instruction is evaluation. It is essential to make careful observations, question students, and analyze students' solutions to assess problem-solving performance. A "point system" can be used to facilitate this process. The most important thing in this system is that the answer is only part of the scoring scheme. Two plans for such a system are given:

PLAN 1

Score	Solution Stage
0	*Noncommencement:* The student is unable to begin the problem or hands in work that is meaningless.
1	*Approach:* The student approaches the problem with meaningful work, indicating some understanding of the problem, but an early impasse is reached.
2	*Substance:* Sufficient detail demonstrates that the student has proceeded toward a rational solution, but major errors or misinterpretations obstruct the correct solution process.
3	*Result:* The problem is very nearly solved; minor errors produce an invalid final solution.
4	*Completion:* An appropriate method is applied to yield a valid solution.

PLAN 2

Understanding the problem	0-Completely misinterprets the problem 1-Misinterprets part of the problem 2-Complete understanding of the problem
Solving the problem.	0-No attempt or a totally inappropriate plan 1-Partly correct procedure based on part of the problem interpreted correctly 2-A plan that could lead to a correct solution with no arithmetic errors
Answering the problem	0-No answer or wrong answer based on an inappropriate plan 1-Copying error; computational error; partial answer for problem with multiple answers; answer labeled incorrectly 2-Correct solution

Regardless of the scoring scheme, it is important that evaluation be used constructively to motivate, not discourage, students. By assigning partial credit for evidence of some understanding and for all attempts to solve problems, one can motivate students to initiate and carry out attempts to solve problems. Furthermore, with a de-emphasis of answers and rewards to partial successes, students are more likely to attempt to understand and solve problems.

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Vocabulary

K-3	4-6	7-9
attribute	arithmetic pattern	generalization
classify	at least	
element	at most	
extraneous information	extrapolate	
given information	geometric pattern	
pattern	if...then	
set	insufficient information	
strategy	rule	
	statement	
	Venn diagram	

PROBLEM SOLVING STRATEGIES

K-3

Identify and use a pattern
Use and construct a table
Make an organized list
Guess and test
Make or use a graph, drawing or physical model
Eliminate possibilities
Write an open sentence

4-6

Work backwards
Solve a simpler problem

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PROBLEM SOLVING: THE OBJECTIVES

PATTERNS: To identify, use and construct patterns

To identify a pattern and determine a missing element [P1Cn1]

K-3 Comment:

The problem can be expressed with a picture, shape, or number pattern that is to be duplicated and/or missing element supplied. Number patterns can be ascending (additive) or descending (subtractive), whole numbers only (not to exceed three-digits), and involving only one operation. Shape patterns may change in shape, size, color or position with no more than two attribute changes in a given pattern.

4-6 Comment:

The problem can be expressed with pictures, tables, shapes (two or three-dimensional) or numbers including whole numbers, decimals (through hundredths), and fractions (denominators twenty or less). Number patterns can be additive, multiplicative, ordered pairs and involve two operations in the sequence. Shape patterns can include up to three attribute changes.

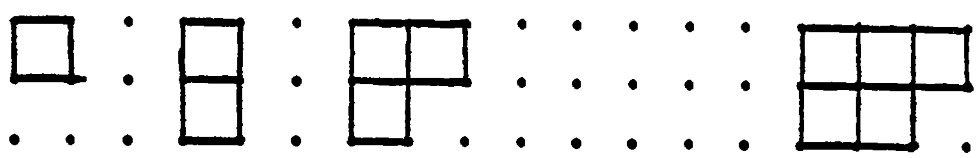
7-9 Comment:

Number patterns can involve the whole set of real numbers.

Comment: When asking students to determine a missing element, it is important always to supply a final element; otherwise, more than one pattern rule might apply to the sequence.

K-3 Example:

Draw the missing shape.



4-6 Example:

One number in the sequence below is incorrect. What should be the correct number?

3 4 6 9 13 18 24 33

A) 7 B) 24 C) 31 D) 34

(Answer: C, because 33 should be 31)

To create a pattern, given a formal rule [P1Cn2]

Comment: A formal rule can be stated in a variety of ways, in a table, as a formula, or with verbal statements.

4-6 Example:

Use the pattern rule to complete the table.

Rule: $\square = 3 \cdot \triangle + 2$

$\triangle$	0	1	2	3	4	5	6
$\square$					14		

(Answer: 2, 5, 8, 11, 14, 17, 20)

7-9 Example:

Write a pattern that fits this pattern description.

Pattern Description: The pattern has two alternating, independent sequences, one made up of numbers and the other of letters. Each number is 3 times the previous one. The letters are in reverse alphabetical order, each time skipping 2 letters.

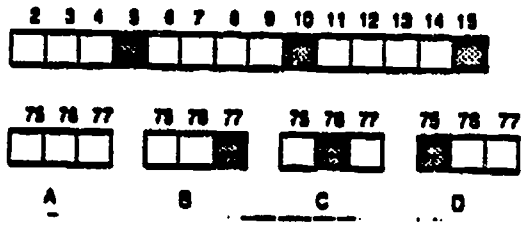
(Possible answer: 1 Z 3 W 9 T 27 Q)

To extrapolate by developing a rule for a pattern [P1Cn3]

Comment: Extrapolation here means the process of making an inference or conjecture from observed data in order to predict any other element that may be part of the pattern.

4-6 Example:

Which of the following shows the way 75th, 76th, and 77th squares will be if the pattern below is extended?



7-9 Example:

Under which letter will the number 999 appear if this pattern is continued?

A	B	C	D	E	F	G
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16					

(Answer: E)

UNDERSTANDING PROBLEMS: To demonstrate an understanding of a problem

Comment: Some useful techniques for helping students understand the facts and conditions associated with a problem include:

1. Restating the problem in their own words
2. Listing given information
3. Listing given conditions
4. Writing down the stated goal in their own words
5. Listing relevant facts
6. Listing implicit conditions
7. Describing related known problems

To determine what is to be found [P2Cn1]

4-6 Example:

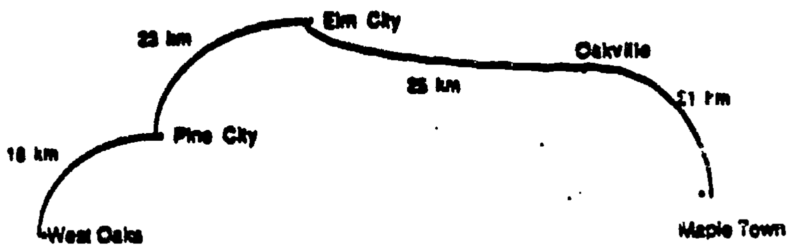
Rephrase the question in this problem in your own words.

> Lou weighed 153 pounds. How much did he weigh after he had lifted weights, eaten more, and gained 19 pounds?
> (Possible response: How much did Lou weigh after he gained?)

To identify necessary information to solve a problem [P2Cn2]

K-3 Example:

Circle the numbers on the map that you must use to solve the problem. How many kilometers is it from Pine City to Oakville?



(Answer: 23 km, 25 km)

7-9 Example:

Underline the numbers NOT needed to solve the problem.

Out of a total of 60 school days, Harold was present 57 days. His sister Nancy was absent 6 days. What is the ratio of Nancy's days present to her days absent?

(Answer 57)

To determine insufficient information [P2Cn3]

4-6 Example:

The problem below has data missing. Make up appropriate data, then find the answer using your data.

> Jane and Pete collected baseball cards. Jane collected 214 cards and Pete collected the rest. How many cards did Jane and Pete collect altogether?
>
> (Possible Answer: Pete collected 356 baseball cards.
> 214 + 356 = 570
> They collected 570 cards altogether.)

To formulate appropriate questions [P2Cn4]

K-3 Example:

Write a subtraction question for this picture.

(Possible Answer: How many more dogs are playing than are sleeping?)

4-6 Example:

Lake	Length (miles)	Greatest Depth (ft)	Area (sq mi)	Volume (cu mi)
Superior	350	1333	31,800	2930
Michigan	307	923	22,400	1180
Huron	206	750	23,000	849
Erie	241	210	9,910	116
Ontario	193	802	7,600	393

Use the table and write a question that could be answered by solving this number sentence.

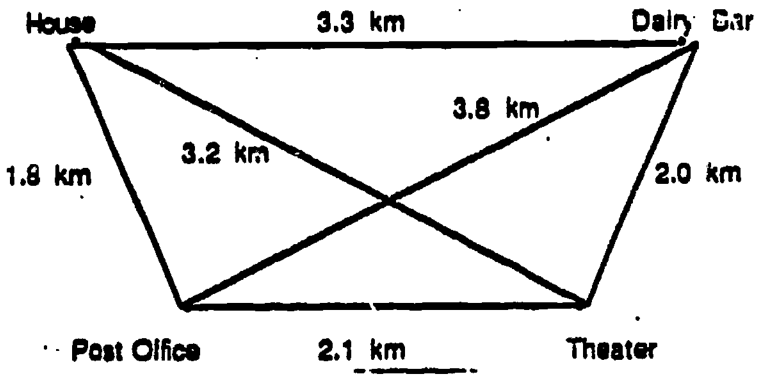
(350 + 307 + 206 + 241 + 193) ÷ 5 = _____

(Possible Answer: What is the average length of the Great Lakes

To formulate a problem for mathematical expressions or number sentences [P2Cn5]

4-6 Example:

Write a story problem that this picture will help you solve.



(Possible Answer: Linda and Debbie are sisters. They had tickets to the theater and left the house at the same time. Debbie went directly to the theater but Linda had to stop at the Post Office. How much farther did Linda walk than Debbie?)

7-9 Example:

Write a word problem for this equation: 2x + 10 = 40

(Possible Answer: Mary's dad is 40 years old. He is 10 years older than twice her age. How old is Mary?)

PROBLEM SOLVING STRATEGIES: To select and apply appropriate problem solving strategies

To identify and use a pattern to solve a problem [P3PS1]

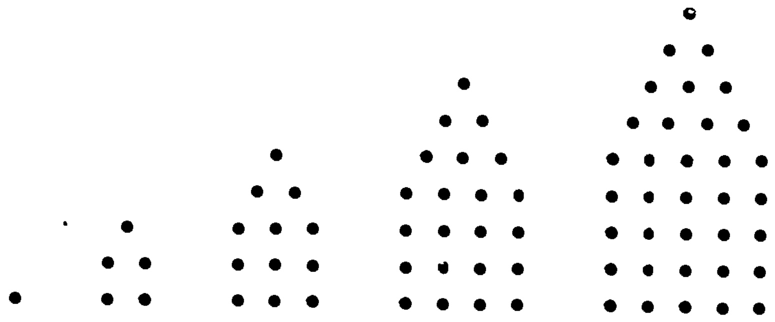
Comment: If students are expected to use a strategy they must have focused instruction and practice on the strategy. We must teach each strategy with the same degree of seriousness that we devote to any other mathematical technique or procedure. Then, when a student encounters a novel problem, s/he will have a repertoire of strategies from which to choose. It is also important to note that in many cases more than one strategy can and may be used to solve a problem.

K-3 Example:

A group of children took 5 toboggans down a steep hill. 1 child was on the first toboggan. Three children were on the second toboggan and 5 children were on the third toboggan. If the pattern continues, how many children rode the fifth toboggan? (Answer: 9 children)

7-9 Example:

The dots in the figures below are arranged to form the shape of a pentagon. Thus, each figure below represents a pentagonal number. How many dots are in the tenth pentagonal number of this sequence?
(Answer: 145)



To make an organized list or table to solve a problem [P3PS2]

Comment: Students should know how to read and draw conclusions from simple tables and charts as well as be able to condense numerical information into more manageable and meaningful terms by setting up simple tables and charts. Once data have been organized in a table, the problem's solution often becomes obvious.

K-3 Example:

How many more picture books were read by Mrs. Fonda's class than by Mr. White's class?

Books	Mrs. Fonda's class	Mr. White's class
Picture Books	23	17
Storybooks	26	24

(Answer: 6)

4-6 Example:

Complete the table below to find the answer for this problem.

Suppose that you roll two number cubes, each of which has faces numbered from 1 through 6. How many times does 8 appear in the table? (Answer: 5)

+	1	2	3	4	5	6
1	2	3	4			
2	3	4	5			
3	4	5	6			
	7	8	9	10		

4-6 Example:

Each of the 13 players on the Ridewood girls' basketball team is to have a two-digit number on the back of her shirt. The coach said they each could pick their own number. She also said no girl is to use the same digit twice. She gave them 4 digits to use: [2], [3], [4], [5]. Can each girl pick a different two-digit number? Make an organized list to find your answer.

Answer:				
	23	32	42	52
	24	34	43	53
	25	35	45	54

Since there are only 12 possible combinations, one girl would not have a number.

7-9 Example:

You have $10.00. Which three different items can you order to spend as much of the $10.00 as possible?

<u>The Seafood Shack</u>
Please order by letter

A.	Bowl of Clam Chowder	$2.45
B.	Shrimp Cocktail	$3.75
C.	Baked Clams	$4.50
D.	Baked Potato	$1.25
E.	Lobster Salad	$3.50

(Prices include tax and tip.)

Make an organized list of all possibilities first, then solve the problem.

(Answers: ABC ACD BCD CDE
ABD ACE BCE
ABE ADE BDE

ABE is closest, with a total of $9.70.)

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To guess and test to solve a problem [P3PS3]

Comment: This is probably the most natural of all problem-solving strategies. Unfortunately, solving problems through the application of the guess and test strategy instead of the direct application of a particular algorithm is often discouraged. As a result, many students are very reluctant to guess at the solution of a problem. Thus, to teach this strategy it is first necessary to encourage students to make guesses. After they are comfortable making guesses, the following essential features of the guess and test strategy can be taught.

1. Make an "educated" guess at the solution.
2. Check the guess against the conditions of the problem.
3. Use the information obtained in checking to make a "better" guess.
4. Continue this procedure until the correct answer is obtained.

K-3 Example:

Problem: I counted 9 cycle riders and 24 cycle wheels going past my house. How many bicycles and how many tricycles passed my house?

Concrete materials (such as bingo chips) can be used to develop an understanding of this problem using the guess and test strategy. Some questions that would assist understanding are:

> If we use bingo chips to stand for wheels, how many should we count out for this problem? (24) Why?
> How many chips represent the wheels of a bicycle? (2) tricycle? (3)
> Can we use all 24 chips and put them in groups of 2 or 3?
> How many groups do we want altogether? (9) Why?

Students should be encouraged to move the chips around until they get a total of 9 groups of 2 or 3 chips.

(Answer: 6 tricycles 3 bicycles
$6 \times 3 = 18$ $3 \times 2 = 6$
$6 + 3 = 9$ -- cycle riders $18 + 6 = 24$ -- wheels)

4-6 Example:

Pam has some coins. Her friends try to guess what coins she has. Here are the clues she gave them:

> She has nickels and quarters.
> She has 2 more nickels than quarters.
> The coins total $3.40.

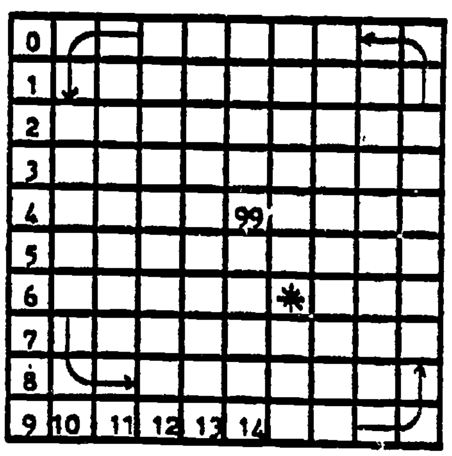
Pete guessed that she had 8 quarters and 10 nickels.
Pat's guess is incorrect. How do you know? (The value is less than $3.40).
Gail guessed that she had 12 quarters and 14 nickels. Why is Gail's guess wrong? (The value is more than $3.40).
Based on the information you got from these incorrect guesses, what would be your next guess for the number of quarters?
(Anything less than 12 but more than 8).

To work backwards to solve a problem [P3PS4]

Comment: The strategy of working backwards is useful with fewer problems than other strategies but still is a worthwhile strategy to develop. Certain problems with several steps resulting in a known value can be solved by working backwards one step at a time. The strategy of working backwards from the end is used because the end is where the concrete information is found.

7-9 Example:

If you were to continue the number pattern until you get to the star, what number would you put in the star's square? Can you determine this number without filling in or counting all the numbers between?
(Answer: 90)



7-9 Example:

Work backwards to figure out what the digits are.

```
     YY
8 ) X65
    XZ
     X5
     XZ
      1
```

(Answer: X = 2, Y = 3, Z = 4)

To make or use a drawing, a graph or a physical model to solve a problem [P3PS5]

Comment: This strategy is a useful technique not only for students to solve a problem but to understand it as well. Frequently, just changing the setting of a problem from words to a picture is enough to help students solve the problem. If the language is confusing, or if it is clear that geometry is involved, a drawing, graph, or physical model frequently is helpful. Occasionally students are reluctant to draw a picture either because they feel it is beneath them or because they simply don't want to spend the time. Remind them that research has found that students who draw pictures solve problems better and faster than those who do not.

K-3 Example:

The glee club was performing for the spring concert. Their teacher put them in rows. One person was in the first row, 2 in the second row, 3 in the third row, and so on. There were 8 rows of students. Make a drawing and find how many students were in the glee club.

```
       x    1
      xx    2
     xxx    3
    xxxx    4
   xxxxx    5
  xxxxxx    6
 xxxxxxx    7
xxxxxxxx    8
           36
```

(Answer: There were 36 in the glee club)

4-6 Example:

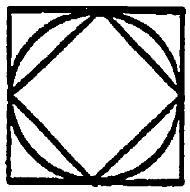
Make a drawing and find how many people are at the show. There are 6 rows, with 10 chairs in each row. The first 2 rows are filled. The third row is filled except for 4 seats on the end. The fourth row is empty and the last 2 rows each contain 8 people.

(Answer: 42 people)

7-9 Example:

One square is constructed so that each side touches a circle that is inside the square. Another square is constructed so that it is inside the circle, and each vertex of the square is on the circle. Draw a diagram and find the greatest number of points that the circle and the two squares can have in common.

(Answer: 4)



To write an open sentence to solve a problem [P3PS6]

Comment: Students should first be expected to select correct equations for specific problems (from choices provided) before being asked to write equations without assistance.

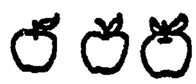
K-3 Example:

Write a multiplication number sentence to find the number of apples.

3 apples

3 apples

3 apples



3 apples

(Answer: 4 X 3 = 12)

4-6 Example:

Bob travels 156 meters in jogging around the edge of a rectangular field. If the length of the rectangle is twice as long as the width, how long is each side? Letting L denote the length, write a number sentence which could be used to solve the problem.

(Answer: $(\frac{1}{2} L + L) \bullet 2 = 156$)

$(2 \bullet L + 2 \bullet \frac{1}{2} L = 156)$

7-9 Example:

Together Jenny, Mack and Kasia sold 91 boxes of candy for their school fund raiser. Jenny sold the most candy. Mack sold one box less than Jenny. Kasia sold half as many boxes as Jenny.

Write an equation you could use to find the number of boxes sold by Jenny if Jenny sold J boxes.

(Answer: $J + (J - 1) + \frac{1}{2} J = 91$)

To solve a simpler problem to solve a problem [P3PS7]

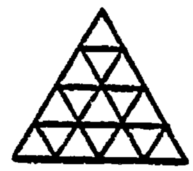
Comment: Realistic problems often contain very large numbers or a great many cases. Simplifications employs breaking the problem into its component parts, working with the parts to achieve sub-solutions, then fitting these sub-solutions together to develop the overall solution. Alternatively, reword the problem using smaller or compatible numbers or a more familiar setting to help discover the operation which should be used.

4-6 Example:

How many triangles are in this figure?
Make the problem simpler by counting all the triangles made up of 1 triangle first.

Answer:

1	16
4	7
9	3
16	1
TOTAL	27

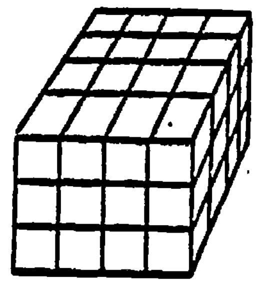


7-9 Example:

How many cubes are in this figure?
Make the problem simpler by finding all the 1 by 1 cubes first.

Answer:

1x1x1:	48
2x2x2:	18
3x3x3:	4
TOTAL	70

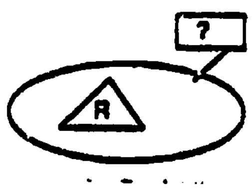


To eliminate possibilities to solve a problem [P3PS8]

Comment: Three important aspects of the elimination process are:

1. The careful selection of the first clue to be used. (Consideration must be given to the one that is easiest to use, and/or that will eliminate the most possibilities.)
2. The use of direct reasoning in the process of elimination.
3. The use of the indirect method, whereby a possible solution is tested and a contradiction is obtained.

K-3 Example:



Red	Green	Blue
Yellow	Big	Little
○	△	□

Cross out the labels that cannot describe this picture.
(Answer: All but Red, Little, and the triangle.)

4-6 Example:

Guess the number.

Clue #1 The number is greater than 2 and less than 20.
Clue #2 The number is prime.
Clue #3 The number divided by 4 leaves a remainder of 3.
Clue #4 The number plus 2 is a perfect square.

Find the number by writing all the possible numbers given by Clue #1; cross-out all the numbers eliminated by Clue #2; draw a circle around the numbers eliminated by Clue #3 and a box around the numbers eliminated by Clue #4. The number is ______.

(Answer: Number is 7)

7-9 Example:

Determine the number from the given clues.

It is a three-digit number.
It is equal to the sum of the cubes of its digits.
It is between 100 and 200.
Its digits are odd numbers.

(Answer: 153)

To select the appropriate operations(s) to solve a one-step or multi-step problem [P3PS9]

K-3 Example:

There are 32 students in Mrs. Thornberry's class. On a Monday morning, she counted 24 students in all. How many students were absent?

To solve this problem you:

A) add 32 and 24
B) subtract 24 from 32
C) multiply 32 and 24
D) divide 32 by 24

(Answer: B)

4-6 Example:

Seats in the bleachers cost $7 and box seats cost $10 each. Thomas bought 6 bleacher seats and 4 box seats. What was his total cost for the tickets?

Which is the appropriate *first step* in solving the above?

A) Find the total number of seats.
B) Find the total cost for the bleacher seats and the total cost for the box seats.
C) Find the total cost for the tickets.
D) Find the total number of tickets and the total cost of the tickets.

(Answer: B)

7-9 Example:

A tennis racket costs $30 plus 4% sales tax. If you buy the racket, how much change will you get back from $50?

A) 50 - (.04 x 30) B) 50 - (1.04 x 30)
C) .04 x (50 - 30) D) 50 - (.96 x 30)

(Answer: B)

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EVALUATING SOLUTIONS: To interpret and evaluate the solution to a problem

Comment: The problem solver should be able to determine whether or not the answer makes sense. This process might involve rereading the problem and checking the answer against the relevant information (conditions and variables) and the question. Students might also use various estimation techniques to determine if an answer is reasonable. Following is a list of ideas to consider:

1. Have students check to be sure they used all important information in the problem.
2. Have them check their arithmetic.
3. Have them answer problems in complete sentences.
4. Use activities that develop students' abilities to give the answer to a problem and check their work.
   a. Have them estimate to find answers.
   b. Have them estimate to check answers.
   c. Give them the numerical part of an answer and have them state the answer in a complete sentence.
   d. Give them a problem and an answer. Have them decide whether the answer is reasonable.

To check the solution(s) with the conditions of the problem [P4PS1]

4-6 Example:

Determine whether an answer of $7\frac{2}{5}$ cars given for this problem is reasonable. If the answer is not reasonable, explain why.

> A class of 37 students was going on a picnic. How many cars are needed if each car holds five students and one driver?
>
> (Answer: $7\frac{2}{5}$ cars is not reasonable, because it is not possible to have $\frac{2}{5}$ of a car.)

7-9 Example:

Which statement best describes why the answer given for this problem is reasonable?

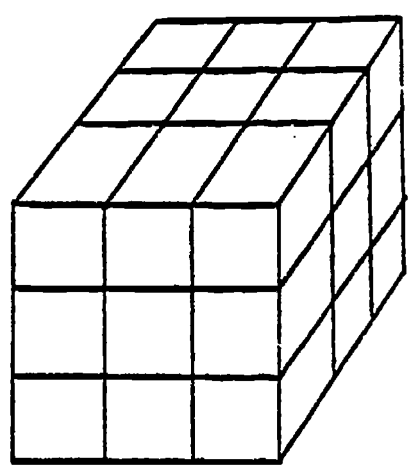
> Helen put \$98 in her savings account in one year. At the end of the year she had earned $8\frac{1}{2}$% interest. How much money did she have in her savings account at the end of the year?
>
> Answer: \$106.33
>
> A) The answer is greater than \$98.
> B) \$98 + 8% is \$106.
> C) \$106.33 rounds to \$106; 106 - 98 = 8.
> D) $8\frac{1}{2}$% of \$100 is \$8.50; \$98 + \$8.50 = \$106.50
>
> (Answer: D)

To find and evaluate alternative processes for solving the problem [P4PS2]

Comment: Early in the solution process it is important to maintain an open mind when evaluating possible strategies. Many times we fall into the trap of following a particular solution path just because it looks familiar. The better you become at suspending initial judgments and actively seeking novel alternatives, the greater the likelihood of uncovering solutions to unfamiliar problems.

4-6 Example:

A large cube was dropped into red paint. Then the cube was cut into 27 smaller cubes of equal size. How many of the smaller cubes have exactly 2 faces painted red?



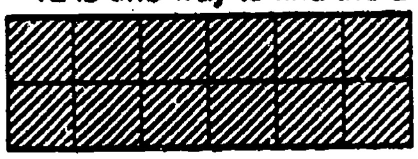
One way to solve this problem is to look at the three rows of the large cube and count the number of small cubes in each row with exactly two painted faces.
Which of the following is another way to solve the problem?

A) Counting the total number of faces shown in the picture.
B) Finding the number of corners of the large cube would give the number of small faces with exactly two faces painted red.
C) Subtract the number of corner cubes from the total number of cubes.
D) None is correct.

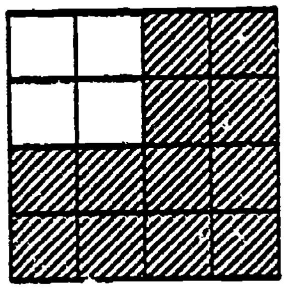
(Answer: B)

7-9 Example:

$2 \times 6 = 12$ is one way to find the area of this rectangle.



This same area can be represented as



Write an equation that represents this drawing of the area.

(Answer: $4^2 - 2^2 = 12$)

To formulate an extension of the problem [P4PS3]

Comment: Students must be encouraged strongly to reexamine their successes, refine their results and *imagine problems which have been or could be solved with similar techniques.* If each solution is carefully reconsidered in light of past experiences and coordinated with prior results, the ability to solve novel problems in the future may be enhanced.

4-6 Example:

Tom is 10 centimeters taller than Jack, who is 150 centimeters tall. To find Tom's height the boys added 150 + 10 and got 160 centimeters.

Which of the following would represent an extension of the above problem?

A) My sister is 1 year younger than I am. I am 15 years old. How old is my sister?
B) I have 15 times as much money as Tanya. Tanya has $1. How much money do I have?
C) My score in darts was 15. I made 1 more point. What is my score now?
D) Joey's and Gregg's ages add to 15 years. Gregg is 1 year old. How old is Joey?

(Answer: C)

7-9 Example:

A person mows a lawn which is a square 10 meters on a side in 20 minutes. How long will it take that person, working at the same rate, to mow a lawn which is a square 5 meters on a side?

Which of the following is NOT an extension of the above problem?

A) How long will it také the person to mow a square 15 meters on a side?
B) How much could the person mow in one hour?
C) A person mows a square 10 meters on a side in 5 minutes. How long will it take that person to mow a square 5 meters on a side?
D) Two persons mow the lawn. It takes on 20 minutes to mow a square 10 meters on a side and it takes the other person 15 minutes to mow this same size square. How long will it take them together to mow a square 5 meters on a side?

(Answer: D)

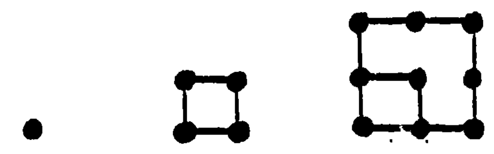
To formulate a generalization of a given problem [P4PS4]

Comment: Trying to detect a sense of some underlying pattern is called *generalizing*. It means noticing certain features common to several particular examples and ignoring other features. Once articulated, the generalization turns into a conjecture which must then be investigated to see if it is accurate.

7-9 Example:

Square Numbers

1st 2nd 3rd

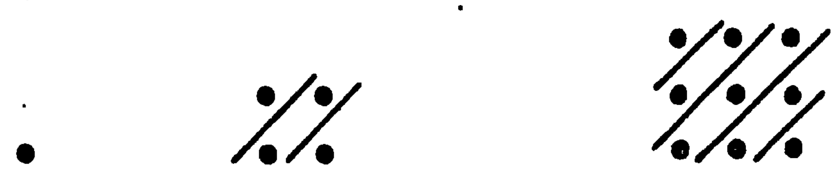


1 4 9

Write the rule for finding the nth square number.

(Answer: $n \times n$ or n^2)

7-9 Example:



$1 = 1^2$ $1 + 2 + 1 = 2^2$ $1 + 2 + 3 + 2 + 1 = 3^2$

If you start with the square arrays of dots and draw lines like those above, it seems natural to write these equations:

Which of the following is a generalization about the above relationship?

A) $1 + 2 + 3 + 4 + 3 + 2 + 1 = 4$
B) $1 + 2 + 3 + ... + n + ... + 3 + 2 + 1 = n^2$
C) $1 + 2 + 3 + ... + n = n$
D) $\frac{n(n+1)}{2}$ is an expression for the nth term.

(Answer: B)

LOGICAL REASONING: To use logical reasoning

To determine the attributes used to classify a set and vice versa [P5PS1]

Comment: Classifying is the process of putting things into sets. Use of Venn diagrams to show sets gives students a precise (and nonverbal) way of recording, organizing and communicating their thoughts about information to be classified.

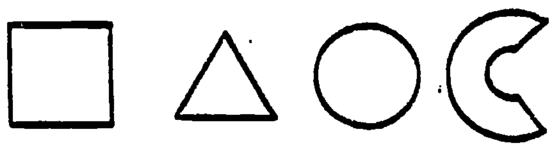
The objects we ask students to classify must have defined attributes (characteristics); otherwise, attention will be distracted by the problem of deciding whether a certain object does or does not have a certain attribute.

Classification problems include negation and:

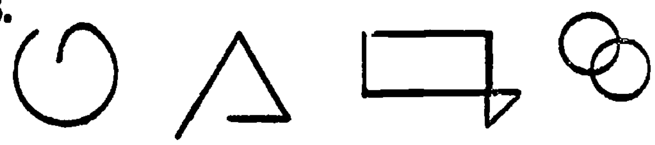
No more than two attributes (shape, not number) K-3
No more than two attributes (shape & number) 4-6
No more than three attributes (shape & number) 7-9

K-3 Example:

These shapes are ZIMS.



These shapes are not ZIMS.



Draw a shape that could be a ZIM.

(Answer: Any closed curve that does not cross itself)

4-6 Example:

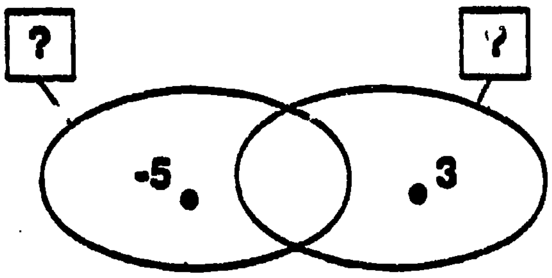
Write three numbers that meet all the conditions listed below.

All odd, but not prime
The largest number is less than 40
When divided by 4, have a remainder of 1

(Possible answers: 9, 21, 25, 33)

7-9 Example:

Select from the list to label each loop correctly.



Multiples of 2	Multiples of 3	Multiples of 4	Multiples of 5
Multiples of 10	Positive Divisors of 12	Positive Divisors of 18	Positive Divisors of 20
Positive Divisors of 24	Positive Divisors of 27	Larger Than 50	Larger Than -10
Smaller Than 50	Smaller Than -10	Odd Numbers	Positive Prime Numbers

(Answer: Left is multiples of 5 larger than -10. Right is multiples of 3; positive prime numbers; positive divisors of 24 and 27)

To interpret and use statements involving logical operations and quantifiers (and, or, not, if...then, every, all, some, no, at least, at most, each, exactly) [P5PS2]

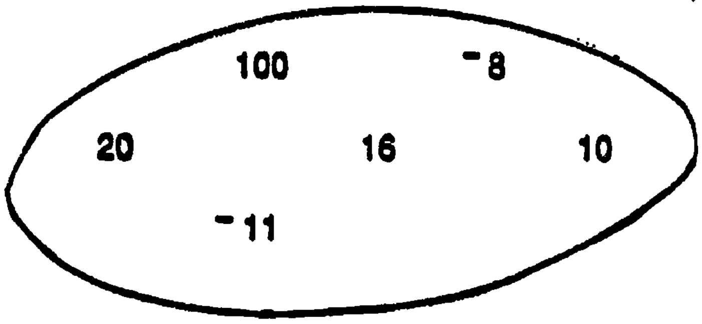
Comment: In logic, a *statement* is a sentence that is either true or false but not both.

Logical operations are: and, or, not, if...then
Logical quantifiers are: every, all, some, no, at least, at most, each, exactly

In logic, "or" is used as in the legal sense of "and/or". "All" means every single one. "Some" means at least one.

4-6 Example:

Below is a set of exactly six numbers. Below the set are some statements. Circle T if the statement is true and F if it is false.



T F Each member is a multiple of 4.
T F Some member is a multiple of 4.
T F At most one member is a multiple of 4.
T F Exactly four members are multiples of 4.
T F At least three members are negative.
T F At most three members are negative.
T F Every even number is a multiple of 4.

7-9 Example:

Which figure is described by the following clues?

It is not a square and it is not green.
It is red or big.
It is not a triangle and it is not small.

A) big red square B) small yellow square
C) big blue circle D) big green circle

(Answer: C)

To recognize and draw valid conclusions from given information [P5PS3]

4-6 Example:

My number is greater than Jill's but less than yours.

Which is a true statement?

A) Your number is greater than Jill's.
B) Your number is greater than mine.
C) Your number and Jill's are the same.
D) Your number is less than Jill's.

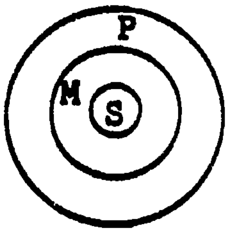
(Answer: B)

7-9 Example:

Draw a diagram that proves that the following is a valid conclusion.

All S is M.
All M is P.
Therefore, all S is P.

Answer:



Framework:
Michigan Mathematics Objectives

Mathematical Content Strands	Mathematical Processes: Conceptualization	Mental Arithmetic	Estimation	Computation	Applications and Problem Solving	Calculators and Computers
Whole Numbers and Numeration						
Fractions, Decimals Ratio and Percent						
Measurement						
Geometry						
Statistics and Probability						
Algebraic Ideas						
Problem Solving & Logical Reasoning						
Calculators						

CALCULATORS: AN OVERVIEW

What is included in this strand?

Included in this strand are three major objectives, somewhat sequential in nature, related to calculator keys, computation and limitations of calculators.

The first major objective is: *To recognize specific calcultor keys.* The skills which are included in this objective relate to the mechanics of using a calculator. At some future date when all students have learned to use calculators on a routine basis, it may not be necessary to emphasize this objective. It is necessary, however, for students to show facility with this objective before they proceed with the next two objectives.

The second major objective is: *To perform appropriate computations with a calculator.* This objective deals with using calculators in the mathematics program.

The third major objective is: *To recognize certain common limitations of calculators and be able to interpret selected calculator display symbols such as E for "error".* Students must be thoroughly acquainted with the operation of the calculator that they are using. Otherwise, the calculator's potential will be limited by the knowledge of the user.

What is the developmental flow?

The calculator strand is unique in that it is not only a strand in and of itself but is included as an objective within other strands. Problem solving is the other strand with dual purposes. The calculator strand addresses the use of the calculator, whereas, the other strands use the calculator to explore mathematical topics and to attain certain content objectives. As a result, development of the objectives within the calculator strand directly relate to the mathematical content introduced at each grade level.

A scientific calculator can be profitably used at the middle school level. The MEAP Objectives, however, can all be accomplished with a four-function calculator. There is no need to limit the use of any of the keys to specific grade levels. The mathematics curriculum should dictate which keys are appropriate for a given grade. For example, just because the calculator has a square root key or percent key does not mean that these topics should be taught at earlier grades or that these keys should be covered up in the primary grades.

What are the implications for instruction?

Calculator use is encouraged at all grade levels, from kindergarten on, for students of all abilities. As calculator usage is incorporated into the mathematics program new instructional implications must be considered.

Students must be able to decide if calculations should be done mentally, with paper and pencil, or with a calculator. The calculator can decrease a student's computational load in problem solving.

As students begin to use calculators for computations, mental computation must be emphasized. Mental computations can be used whenever possible thereby reducing the number of calculator keystrokes that need to be made. An emphasis on estimation skills will be necessary so that students can determine if the results displayed by the calculator are reasonable.

The calculator may be used to enhance a student's ability in problem solving and to develop critical thinking skills. By removing the drudgery associated with tedious computations and by helping students gain access to computations which exceed the student's paper and pencil ability, students will be able to explore and apply mathematics to interesting every day situations. It is assumed that calculators are available to students for instruction on problem solving and for MEAP test items for problem solving.

Calculator usage raises new issues not only pertaining to instruction but also evaluation. As calculators are more widely used, evaluation will move away from a focus on calculator literacy and move towards assessing a student's ability to solve more complex mathematical problems aided by technology.

Why teach these objectives?

It has been estimated that 80% of the jobs available to people in the year 2000 do not even exist today. Use of the calculator will allow teachers to devote more instructional time to the development of higher order thinking skills and problem solving skills, because these skills are necessary for success in this technological age. Furthermore, equipped with the knowledge of how to effectively use the calculator, all students, not just the more able ones, will be motivated to do more mathematics while exploring real world problems.

Procedures for using the calculator

Percent Key - %

The percent key [%] acts as an equals key for percent operations. The decimal point appears in the correct position when the percent [%] key is used. The effect of the percent [%] key is to multiply by 0.01.

1. The percent (%) key must be pressed last. For example, the keystrokes for 6% of $375 are 6 [X] 375 [%] or 375 [X] 6 [%]. In either case, the display shows 22.5.

2. The [%] key may be used in conjunction with addition or subtraction keys. For example, to find the cost of a new $59.99 coat with a 4 percent sales tax, the keystrokes are: 59.99 [X] 4 [%] [+] [=]. This will result in the tax being added for a total cost of $62.3896 ($62.39 rounded to the nearest cent). To find the sale price of a $59.99 coat with a 15 % discount, keystroke 59.99 [X] 15 [%] [-] [=] and see a display of 50.9915, or $50.99.

There are some variations in the way the percent [%] key functions on some calculators. The MEAP Objectives assume the kind of results suggested here.

Error Sign

If an E sign appears in the display it is because the calculator cannot do what it is being asked to do. When this happens no further entries may be made until the calculator cleared. The problem has to be worked again from the beginning. An error message may appear for the following reasons:

> **Overflow - the number entry or answer is outside the range of the calculator.**
> **Dividing a number by zero.**
> **Square root of a negative number.**

Square Root

When the square root sign is pressed after entering a number the square root of the number is immediately shown, e.g., 16 [√] is keystroked the display shows 4.

Plus/Minus

When the [+/-] is pressed after a number, the display immediately shows the opposite of that number, for example, 7 [+/-] changes the display from 7 to 7⁻, 7⁻ [+/-] changes 7⁻ to 7.

Memory Keys

The memory is a special place in the calculator where a number is stored. The [M+] key adds an amount to the memory and [M-] subtracts a given amount from memory. The [CM] key clears the memory. It cannot be cleared by just using the [C] or [CE] key. On other calculators there are different ways to clear memory. [RM] brings the amount in memory back to the display so that it can be used
Note the keystroking for this problem. "You have $5.00. You buy 2 half gallons of milk at $1.69 each and a loaf of bread at $1.09. How much change will you get?" The keystrokes are: 5.00 [M+] 2 [X] 1.69 [M-] 1.09 [M-] [RM] and the answer is 0.53.

Suppressed Zeros

In decimal notation unnecessary zeros to the right of the decimal point are not displayed. For example, for keystrokes .67 [+] .23 [=], the answer will be displayed as 0.9.

MEAP Calculator Specifications

The calculator literacy test questions for objectives 1 and 3 will not require the use of a calculator. However, a facsimile of a calculator face will be provided. MEAP test questions which allow the use of a calculator will be separated from those which do not allow it.

Many kinds of calculators are available on the market today. The following characteristics are necessary in order to effectively use a calculator for the MEAP objectives.

1. A display of 8 digits.

2. Four functions - addition, subtraction, multiplication and division.

3. Floating decimal point.

4. Special function keys: $\boxed{\%}$, $\boxed{+/-}$, $\boxed{\sqrt{\ }}$, $\boxed{C}$, $\boxed{CE}$, $\boxed{M+}$, $\boxed{M-}$, $\boxed{RM}$, $\boxed{C}$.

5. Addition Constant - the last addend entered is a constant upon repeated consecutive use of the equal key. For example, 3 $\boxed{+}$4 $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ will show 7, 11, 15, in effect counting on by 4 from 3.

6. Subtraction Constant - the subtrahend is a constant upon repeated consecutive use of the equal key. For example, 42 $\boxed{-}$ 7 $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ will show 35, 28, 21 in effect counting backwards by 7 from 42.

7. Multiplication Constant - the first factor entered is a constant upon repeated consecutive use of the equal key. For example,

   3 $\boxed{\times}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ shows 9, 27, 81 and

   3 $\boxed{\times}$ 6 $\boxed{=}$ 8 $\boxed{=}$ 4 $\boxed{=}$ shows 18, 24, 12.

8. Division Constant - The first factor entered is the constant upon repeated consecutive use of the equal key. For example, 56 $\boxed{\div}$ 2 $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ shows 28, 14, 7.

9. Keystroke Logic--Operations are performed in order from left to right. For example, 2 $\boxed{\times}$ 6 $\boxed{+}$ 3 $\boxed{=}$ will display 15.

   Note: The $\boxed{C}$ and $\boxed{CE}$ keys may be combined into one key. Also $\boxed{CM}$ and $\boxed{RM}$ can be combined.

Vocabulary

An understanding of the meaning of the following terminology is necessary:

K-3

calculator display	equals key
suppressed zero	plus key
keystrokes	minus key
constant feature	multiplication key
clear key	

NEW AT 4-6

clear entry key

decimal key

division key

Memory and the functions of the keys: Memory Plus [M+] Memory Minus [M-] Clear Memory [CM] Recall Memory [RM]

sequence

E (error sign)

order of operations

NEW AT 7-9

square root key [√]

"opposite of" key [+/-]

overflow

underflow

percent key [%]

percent of increase

percent of decrease

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CALCULATORS: THE OBJECTIVES

CALCULATOR KEYS AND FEATURES: To recognize specific calculator keys and selected calculator features

For this major objective, the following are suggested:

1. All students should have actual calculators at their desks at all times during instruction.
2. When evaluating this objective in the classroom children may or may not have actual calculators, depending on the teacher's preference.
3. On the MEAP Test actual calculators will not be used, but a facsimilie of a calculator face may be used.

To recognize specific calculator keys [C1Cn1]

K-3 Comment:

1. Specific keys to be used are: [C] [+] [-] [=] [X]
2. Use single digit numbers to minimize keystroke errors, thereby concentrating on calculator applications.

K-3 Example:

What key on the calculator would be used to find a sum?

(Answer: [+]

4-6 Comment:

Additional keys to be used are: [CE] [÷] [.].

7-9 Comment:

Additional keys to be used are: [√] [%] [+/-].

7-9 Example:

What would the calculator display after these keys have been pressed?

7 [+/-] [+] 6 [=]

(Answer: 1- or -1)

To recognize apropriate key sequences for automatic constant features [C1Cn2]

K-3 Comment:

Use the addition and subtraction constants only for skip counting.

[+] 5 [=] [=] [=] displays 5, 10, 15.

18 [-] 5 [=] [=] [=] displays 13, 8, 3.

K-3 Example:

What answer does the calculator display after these keys have been pressed?

[C] [+] 2 [=] [=] [=] [=] [=]

(Answer: 10)

4-6 Comment:

1. There is another use of the addition and subtraction constants.

   0 [+] 5 [=] 8 [=] 7 [=] displays 5, 13, 12.

   18 [-] 5 [=] 9 [=] 6 [=] displays 13, 4, 1.

2. In multiplication the constant is the first number entered. The multiplication constant feature may be used three different ways:

   6 [X] [=] [=] [=] displays 36, 216, 1296.

   6 [X] 2 [=] [=] [=] displays 12, 72, 432.

   6 [X] 3 [=] 5 [=] 7 [=] displays 18, 30, 42

3. In division, the constant is the first factor entered. The division constant feature may be used two different ways:

   56 [÷] by 8 [=] [=] [=] [=] displays 7, .875, .109375, .0136718 because 8 is the repeated divisor.

   56 [÷] by 8 [=] 2 [=] 4 [=] displays 7, .25, .5 because 8 is the divisor.

4. Use basic facts when first teaching the use of the constants so children can readily see how the constants operate.

4-6 Example:

What does the calculator display after each [=] in: 0 [+] 2 [=] 7 [=] 9 [=] 3 [=]

(Answer: 2, 9, 11, 5)

7-9 Comment:

The [%] key may be used in place of [=] while taking advantage of the multiplication constant feature in determining a given percent of various amounts.

7-9 Example:

What does the calculator display after each [%] in this problem?

[C] 10 [X] 720 [%] 460 [%] 180 [%]

(Answer: 72, 46, 18, because 10 remained as a factor each time)

To recognize appropriate calculator keys related to selected terms associated with mathematical operations [C1Cn3]

K-3 Example:

Which key makes this number sentence true?

7 [ ] 8 [=] 15

(Answer: [+])

4-6 Comment:

1. Additional keys to be used are: [M+] [M-] [RM] [CM].
2. The memory keys on some calculators are: [MR] or [MRC] or [MC].
3. Practical problems using purchasing can effectively make use of the memory keys. For example, explain the keystroking to find the total cost of buying two pair of shoes at $26.95 each and two volleyballs at $8.47 each.

4-6 Example:

What number appears in the display after this sequence is completed?

[CM] [C] 2 [X] 26.95 [M+] 2 [X] 8.47 [M+] [RM]

(Answer: 70.84)

7-9 Comment:

The Commutative Property may be applied in a percent problem such as: "What keystroking will show 7% of 14?" The keystroking may be done either of two ways and the answer will be the same.

7 [X] 14 [%] or 14 [X] 7 [%].

In either case, the answer is 0.98.

7-9 Example:

What keystroking, using [%], will find 6% of $200?

(Answer: Either [C] 6 [X] 200 [%] or [C] 200 [X] 6 [%] will show 12 in the display.)

COMPUTATION: To perform appropriate computations with a calculator

For this objective, the following are suggested:

1. All students should have actual calculators at their desks at all times during instruction.

2. When testing this objective all students should have actual calculators at their desks.

3. On the MEAP Test actual calculators will be used.

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To use a calculator to compute sums and differences using whole numbers [C2Cn1]

K-3 Comment:

1. Instead of using only paper and pencil, problems should use verbal directions or story problem formats.

2. Use larger numbers than normally met at a given grade level.

3. More than one operation may be used in a given problem. As an example: "Fifty-six boys and eighty-nine girls rode to the zoo on the school bus. Sixteen students were picked up by parents at the zoo, so they didn't ride the bus back to school. How many rode home on the school bus?"

4. Some problems should use the same operation with more than two addends (Associative Property).

5. If students use large numbers it may be necessary to explain the meaning of E in the display.

K-3 Example:

Find the total cost if you buy a hamburger for one dollar and fifty-five cents, fries for seventy-nine cents, a milkshake for sixty-five cents, and a large orange drink for seventy-five cents.

(Answer: $3.74)

To use a calculator to compute appropriate sums, differences, products, and quotients with whole numbers, decimals, and fractions [C2Cn2]

4-6 Comment:

1. Instead of using paper and pencil formats, problems should use verbal directions or story problem formats.

2. Use larger numbers or a greater difficulty level than normally met at a given grade level.

3. In fraction problems eliminate easily converted common fractions such as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$ because these can be readily memorized.

4. Improper fractions are allowed.

5. Use common fractions not easily converted to decimals, e.g. $\frac{183}{375}$, $\frac{725}{250}$, so that the emphasis is on the conversion process.

6. Use only terminating decimals of five digits or less.

4-6 Example:

During the school year our class will use twelve reams of looseleaf paper. If there are thirty-four students in our class, and if a ream is five hundred sheets, about how many sheets of paper will be used by each student? Round to the nearest ten.

(Answer: 180)

7-9 Comment:

1. Instead of using only paper and pencil formats, problems should use verbal directions or story problem formats.

2. Use greater numbers or a greater difficulty level than normally met at a given grade level.

3. Avoid non-terminating decimals.

7-9 Example:

Find the product of 0.38 and $\frac{5}{16}$.

(Answer: 0.11875)

To use a calculator to compute answers to percent problems including percent of increase or percent of decrease [C2Cn3]

7-9 Comment:

1. Fractions may be converted to percents.

2. Keystroke sequences which convert fractions to percents should be known. For example, to convert $\frac{14}{16}$ to a percent the keystrokes are 14 $\boxed{\div}$ 16 $\boxed{\%}$ and the answer is 87.5.

3. Use large numbers when measuring the amount of increase or decrease, after using small numbers to teach the meaning of percent increase and percent decrease.

7-9 Example:

A television set costing $450 is now on sale for fifteen percent off. Find the sale price.

(Answer: 450 $\boxed{\times}$ 15 $\boxed{\%}$ $\boxed{-}$ $\boxed{=}$; display shows 67.5, the amount of discount and then 382.5 for the sale price of $382.50.)

LIMITATIONS AND CALCULATOR DISPLAY:
To recognize certain common limitations to calculators and be able to interpret selected calculator-displayed symbols

For all the objectives for this major objective, the following are suggested:

1. All students should have actual calculators at their desks when teaching this objective.

2. When evaluating this objective in the classroom children may or may not have actual calculators, depending on the teacher's preference.

3. On the MEAP Test actual calculators will not be used, but a facsimiile of a calculator face may be used.

To recognize and interpret the calculator display [C3Cn1] .

K-3 Comment:

1. Suppressed zeros may appear when working with money problems. e.g. $0.45 + $0.55 + 1.

2. It will be necessary to introduce the $\boxed{.}$ to work with money problems.

K-3 Example:

How much does it cost to buy a candy bar for $0.35 and a can of pop for $0.45? (The calculator displays .8, so the answer must be converted to $0.80.)

4-6 Comment:

The calculator's display limits of eight digits must be recognized.

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4-6 Example:

When adding 22,222,222 and 99,999,999 the display shows $1.2222222_{E.}$ What does this display mean? What is the correct sum?"

(Answer: E means "error". The sum can be found in two steps, adding the last six digits first.)

7-9 Comment:

Underflow will result when dividing a non-zero number by a very large number. The display will show "0", zero.

7-9 Example:

In the division problem 2 [÷] by 99900000 [=] the calculator display shows 0. What does this mean?

(Answer: The quotient is closer to 0, zero, than the smallest decimal the calculator can show, 0.0000001.)

To recognize the limitations of the calculator regarding decimal numbers display and order of operations [C3Cn2]

4-6 Comment:

1. Whole number remainders may be determined from a quotient that is a decimal numeral. Use single digit divisors.

2. Use dollar amounts between $1.00 and $10.00 to show suppressed zeros in the sums.

3. Use single digit numbers with the operation of addition first followed by the operation of multiplication. e.g. 1 [+] 6 [X] 3 [=] (Answer: 21)

4. When dividing by zero the calculator may show E O or E.EEE.

5. Introduce the meaning of a negative (-) symbol. Some calculators show -2 and others show 2-.

4-6 Example:

Find the whole number remainder if the calculator shows that 3978 [÷] by 8 [=] 497.25.

(Answer: One keystroking is .25 [X] 8 [=] showing 2 as the remainder.)

7-9 Comment:

1. Use multi-digit numbers in order to discourage paper and pencil computation.

2. Whole number remainders may be determined from a quotient that is a decimal numeral. Use two digit divisors. For example, 189 $\boxed{\div}$ 14 = 13.5; as a remainder, the quotient is 13R7.

3. On some calculators the square root of a negative number results in O E in the display. Such a display might result from 1 $\boxed{-}$ $\boxed{\sqrt{}}$ $\boxed{=}$.

7-9 Example:

Find the whole number remainder if the calculator shows that 1615 $\boxed{\div}$ 38 = 42.5.

(Answer: One keystroking is .5 $\boxed{\times}$ 1615 $\boxed{=}$ showing 19 as the remainder.)

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