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ABSTRACT

Several competencies and instructional strategies necessary to accommodate the changing role of teachers of mathematics at middle-school level are described. Also provided are teacher-generated and teacher-tested instructional activities that can be used to facilitate student success in learning mathematical concepts. After describing the competencies of a successful mathematics teacher, strategies associated with successful teaching are discussed. The strategies included are: (1) time management; (2) framing instruction in a problem-solving format; (3) teaching estimation skills and mental computation; and (4) use of technology such as computers and calculators. Errors typically made by middle-school students, a hypothetical diagnosis of each error, and an instructional strategy are presented. Forty-two activities for students and teachers are provided. Sixty-eight references are listed. A model lesson description and examples are appended. (YP)

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Successful Mathematics Teaching for Middle-School Grades

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Winter 1990

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FOREWORD

This handbook is designed for educators who must make important decisions about mathematics instruction in middle schools. These decisions and the educational impact they have on students are greatly influenced by the information available to decision makers. This handbook is intended to serve as a useful resource guide. We believe that the information provided in this resource can be of use to any practitioner working with or interested in mathematics instruction at the middle-school level. This handbook, however, has been specifically designed for middle-school mathematics teachers and supervisors.

This resource describes several competencies and instructional strategies necessary to accommodate the changing role of teachers of mathematics at the middle-school level and provides teacher-generated and teacher-tested instructional activities that can be used to facilitate student success in learning mathematical concepts.

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INTRODUCTION

Beginning with An Agenda for Action (National Council of Teachers of Mathematics [NCTM], 1980) and including the Curriculum and Evaluation STANDARDS for School Mathematics (NCTM, 1989), mathematics education policymakers have called for major changes in mathematics education. These changes are necessary because society has moved into a technological age.

Education is a mirror of society; schools serve to guide students into the changing direction of society. Citizens of a technology-oriented society require skills different from those of citizens of the industrial age (Campbell & Fey, 1988; Pollak, 1987). The mathematical expectations for future workers in the technological age are described by Henry Pollak as follows:

- The ability to set up problems with the appropriate operations.
- Knowledge of a variety of techniques to approach and work on problems.
- Understanding of the underlying mathematical features of a problem.

The ability to work with others on problems.

- The ability to see the applicability of mathematical ideas to common and complex problems.
- Preparation for open problem situations.
- Belief in the utility and value of mathematics.

To meet the challenges of the technological society effectively, teachers will need to facilitate learning, rather than dispense knowledge. Because much of the content of mathematics for middle-grade students has already been taught, teachers will need to provide classroom experiences that actively involve students in exploring, analyzing, and applying mathematics in real-world situations. They will need to increase the time spent in concept development (i.e., teaching for understanding), rather than memorizing facts. They must incorporate the use of direct instruction and multiple instructional strategies to meet the needs of all students, ranging from those at risk to the gifted. Further, they will need to maximize instructional time to accomplish these goals.

The purposes of this handbook are: 1) to describe competencies necessary to accommodate the changing role of teachers of mathematics at the middle-school level and 2) to describe instructional strategies that facilitate student success in learning. This handbook is structured around four competencies that describe characteristics of a successful middle-school mathematics teacher. These are:

1. Knowledge of the characteristics of the middle-school student.
2. Knowledge of the structure of mathematics to be taught and guidelines for teaching this structure.
3. Knowledge of teacher behaviors and strategies associated with successful teaching.
4. Knowledge of strategies for diagnosing student errors.

In addition, a section includes teacher-generated and teacher-tested activities.

COMPETENCIES OF A SUCCESSFUL MIDDLE-SCHOOL MATHEMATICS TEACHER

The challenge to mathematics teachers is to increase the number of students who are mathematically literate and who can engage in problem solving. In order to be successful, teachers will need to develop a number of competencies. These specific competencies, synthesized from current research findings, are listed below and described more fully in the succeeding chapters of this resource book.

Knowledge of Characteristics of the Middle-School Student

The successful teacher is able to identify developmental, psychological, and cultural characteristics that affect the learning of middle-school students.

Knowledge of the Structure of Mathematics

The successful teacher is able to:

1. Describe issues related to equity in the teaching of mathematics.
2. Identify strands of mathematics.
3. Describe the structure of mathematics as related to the content.
4. Describe learning theories that are applicable to teaching mathematics.

Knowledge of Strategies Associated With Successful Teaching

The successful teacher is able to:

1. Use good time-management strategies.
2. Integrate problem-solving activities in the mathematics curricula.

3. Teach problem-solving and estimation skills through direct instruction.
4. Make appropriate use of technology, such as calculators and computers.

Knowledge of Diagnosis of Student Errors

The successful teacher is able to:

1. Use a diagnostic/prescriptive approach in teaching.
2. Diagnose student errors.
3. Prescribe instructional strategies that will remediate the specific student errors.

KNOWLEDGE OF CHARACTERISTICS OF THE MIDDLE-SCHOOL STUDENT

The successful middle-school mathematics teacher knows the characteristics of the students he or she teaches and understands that these students are affected by the changes that are occurring in their bodies (changes that affect their motivation for school tasks). The successful middle-school teacher is also aware that these students are influenced by the media to which they are exposed. Television, for example, has an effect on cognitive processes. This chapter focuses on two broad sets of characteristics of middle-school students--their internal changes and the effect of media.

The middle-school child is undergoing rapid physical, sexual, psychological, cognitive, and social changes. Physiological changes, initiated by an increased output of activating hormones, include sexual maturation and growth spurts in height and weight that have an unsettling effect on the middle-school student's efforts to achieve self-confidence. Cognitive changes reflect the student's ability to think abstractly and formulate and test hypotheses. Peers play a vital role in the psychosocial development at this age by providing an opportunity to learn social skills and sharing similar problems and feelings. Because of the challenging and sometimes difficult developmental tasks that are to be mastered during this stage, it is easy to understand that middle-school students frequently display a sense of emotional liability and distractedness (Mussen, Conger, & Kagan, 1979).

During this transitional period, middle-school students begin to develop an exceptionally strong need for affiliation, which may lead to a lack of motivation in the classroom. The lack of motivation is a factor that is frequently cited by middle-school teachers as a primary cause of students' failure to learn. By the time many students enter the middle school, interest in exploring and learning, which seemed intrinsic when they entered first grade, has decreased considerably (Bruner, 1968; Goodman, 1963).

Students' lack of motivation for school tasks may result from middle-school students' focus on needs other than intellectual ones. Using Maslow's theory of human motivation as a base, Gnagely (1980) identified the motivational focus of middle-school students. In Maslow's theory, needs lower in the hierarchy must be satisfied before higher-level needs (Self-Actualization, To Know, and Aesthetic) serve to motivate behavior.

AESTHETIC
(Need for art and beauty)

TO KNOW
(Need to be well-informed)

SELF-ACTUALIZATION
(Need to develop one's talents and capabilities)

ESTEEM
(Need to be valued/respected)

LOVE AND BELONGING
(Need to be attached/accepted)

SAFETY
(Need to seek security; need to protect oneself from
threat or possible failure by refusing to try or by
lowering the level of aspiration)

PHYSIOLOGICAL
(Bodily needs, such as hunger or thirst)

Figure 1. Maslow's Hierarchy of Human Needs

Although Gnagely's findings were similar to the hierarchy proposed by Maslow, there were significant differences relevant to the middle-school student. First, the *Safety* needs, rather than the *Physiological* needs, of every age group were at the bottom of the hierarchy. One implication is that teachers need to ensure that the classroom environment is conducive to risk taking because *Safety* implies emotional as well as physical security. This is an especially important factor as the skills for successful problem solving -- exploring, discussing, guessing, and questioning -- are encouraged.

In addition, there were significant differences between the needs of males and females at the seventh- and eighth-grade levels (See Figure 2 for comparison). The differences hold implications for teachers of seventh- and eighth-graders. Because males and females show differences on self-worth dimensions, teachers need to provide differential encouragement to them. The teacher should be especially conscious of the need for males to feel valued, respected, and accepted. Perhaps male students would feel most comfortable, thus motivated, by working in groups with their friends and would respond well to teacher comments aimed at enhancing the self-concept of the male student. Female students appear to have greater concern over the changes taking place in their bodies, and teachers should display understanding and encouragement. Females at this age express more interest in developing their individual capabilities and would probably be motivated by individual challenges and competitive situations.

MASLOW	MALES	FEMALES
Aesthetic	-----	-------
To Know	To Know	Esteem
Self-Actualization	Self-Actualization	Love and Belonging
Esteem	Physiological	To Know
Love and Belonging	Love and Belonging	Self-Actualization
Safety	Esteem	Physiological
Physiological	Safety	Safety

Figure 2. Comparison of Maslow's Hierarchy of Needs with Seventh- and Eighth-Graders in Gnagely's Study.

Motivation for middle-school students is also influenced by how students determine the end product of their learning experience. Marton and Saljo (1976) describe motivational processes that begin to emerge at ages 10-12 and continue through adulthood. Some students are intrinsically motivated and complete tasks to satisfy themselves; other students are extrinsically motivated and complete tasks to satisfy an external source, such as parents and teachers.

Marton and Saljo found that students' intentions regarding a learning task were inseparable from their processing activity (i.e., some intended to reproduce the information to make good test grades [external]; others intended to understand the information being presented [internal]). They classified these two types of motivation for learning as deep or surface motivation.

Biggs (1972) and Entwistle (1975), in separate studies, indicated that findings about deep versus surface factors were true across geographical, linguistic, and cultural boundaries. Entwistle described four ways students are motivated: Meaning-Oriented, Strategic, Reproducing, and Nonacademic. These four approaches are described in the following chart:

TYPES OF MOTIVATIONS	INTENT	PROCESS
MEANING-ORIENTED	Intrinsically motivated--enjoys and values learning.	Looks beneath the surface. Tries to understand concepts rather than rote-learn disconnected facts. Tries to see relationships among ideas.
STRATEGIC	Motivated to get good grades.	Competes; often very organized and methodical in his or her study habits.
REPRODUCING	Fearful of failure.	Tries to memorize or rote-learn; not good at picking up cues as to what is expected.
NONACADEMIC	Generally uninterested and unmotivated but frequently will attend to social factors.	Very disorganized in his or her study habits.

Entwistle found that students using any of the approaches, except the Nonacademic approach, performed equally well. However, a significant difference was noted in the quality of educational experiences. Because teachers are faced with students who possess a range of motivations (those who really want to understand, those who just want to get by, and those who really do not care), their challenge is to guide students through learning experiences in a way that will provide optimum success for all students. Activities that are enjoyable and encourage students to create meaning will increase students' intrinsic motivation. The use of cooperative learning techniques is likely to be effective in motivating the "Nonacademic" student (Slavin, 1980). The use of structured approaches to learning will aid students' development of organizational skills.

Wlodkowski (1978) proposed that teachers apply motivational techniques differently at various times in a lesson. In the beginning of the lesson, when students enter and start the learning process, they should develop a positive attitude toward the teacher, the subject matter, and the general learning environment. At the same time, the teacher should ensure that the content and skills to be learned will satisfy a basic need of the student. During the lesson process, the teacher should stimulate students, maintaining attention to the instructional activity and ensuring that they are experiencing supportive affective or emotional results. At the end of the lesson, the teacher should reinforce students' increased competencies.

Research shows that there is a positive relationship between academic performance and the classroom social climate for junior-high school (middle-school) students (Chamberlain, 1981; Fry & Coe, 1980; Moen & Doyle, 1978). An awareness of the varied and changing needs of the middle-school student and a conscious effort to use strategies that foster their success should contribute to increased academic performance and a high level of motivation. This is especially important as the middle-school child moves beyond computational skills in mathematics and toward an increased understanding of concepts and applications of those concepts to solve problems. Through an understanding of the developmental characteristics of the student at the middle-school level, the teacher can have a positive influence on learning.

Because children in this country spend an average of 25 hours a week watching television (as much time as they spend in five days of classroom activities), no discussion of the characteristics of middle-school students would be complete without addressing the major influence that technology is making on their intellectual development and learning styles. In the preface to her book, Mind and Media: The Effects of Television, Video Games, and Computers

(1984), Patricia Greenfield states:

> I could only approach the subject as an anthropologist coming into a foreign culture. I wondered about the nature of the motivation and skills that children in the video/computer culture have, skills that I, as a member of a different culture, lack. I wondered why skills which were so simple and obvious for my teen-age son were difficult and sometimes impossible for me. (p. viii)

Greenfield's statement reflects a surfacing phenomenon -- students today are very different in their thought processes from students of previous years, as a direct result of exposure to television (Cullingford, 1984).

To understand the psychological changes brought about by television, it is instructive to compare its effects with those of the print media. Prior to the invention of the printing press, learning was primarily transmitted orally. Since that revolutionary invention, the printed word has dominated our approach to educational instruction. Print permits the accumulation of knowledge by creating a way to store information. Learning to read affects the process of thinking and the ways people classify, reason, and remember. There is almost a common assumption that effective reading of the printed word is the hallmark of an educated person and that a medium such as television encourages children to be passive, mindless, and unimaginative.

Several cognitive skills appear to be developed as a function of watching television. A primary one is decoding skills that are similar to, but less specialized than, those needed to read (Krier, 1982). Visual and auditory decoding skills include understanding:

Visual Decoding:

- The technique of cutting from one shot to another.
- Panning the camera from one side of a scene to another side.
- Zooming from a long shot to a close shot.
- Splitting the scene.

Auditory Decoding:

- Faceless narrators.
- Canned laughter.
- Changes in music.

Each of these techniques is a symbolic representation that stands for something in the real world. For example:

Simple cut -	change of perspective on a given scene.
Dissolve -	change of scene or change of time.

Students learn to use these cues to understand the information being presented. Although children are not taught as formally how to watch television as they are taught to read, they do acquire these skills at an early age, indicating that they are able to use inductive reasoning (Jacobs, 1988).

Research indicates two effects on students related to television. First, several studies reported that students demonstrated greater comprehension of material when television, rather than print, was the medium of instruction (Beagles-Roos & Gat, 1976; Greenfield, 1976; Runco & Pezdek, 1983). This finding presents a dilemma for mathematics teachers because video and interactive video instruction in mathematics are only beginning to be developed. One videotaped instruction program, the Challenge of the Unknown (Maddux, 1986), is an excellent example of using multimedia to set the stage for problem-solving experiences.

A second result in the literature is that students who are heavy television viewers may develop a tendency to be restless (Singer & Singer, 1977). One explanation offered is that because the pace of television presentation is not controlled by the students, they become accustomed to progressing through material by rote. As a consequence, students' reflection and persistence may decrease because they are not in control of the speed of the presentation. Teachers may need to provide a learner-paced, structured lesson to combat the restlessness. Allowing the student time to reflect and solve problems possibly will balance the effects of television.

KNOWLEDGE OF THE STRUCTURE OF MATHEMATICS

Teachers should know the structure of mathematics in order to teach for meaning. Middle-school-aged children are more apt to be motivated if the focus is on meaning than they are if instruction consists of drill and practice. Further, they will be able to apply their knowledge in real-world situations. Knowledge of the structure of mathematics involves a number of elements. Teachers need to understand the standards that mathematics educators hold for school mathematics. The Commission on Standards for School Mathematics (NCTM, 1989) proposed 13 standards as guidelines for grades 5-8 curricula. The first four of these describe approaches that need to be taken by teachers in teaching mathematics. These four standards are:

* Mathematics as problem solving.
* Mathematics as communication.
* Mathematics as reasoning.
* Mathematics as connections.

A mathematics classroom that incorporates these four standards will be different from the current norm. This new classroom will feature teachers modeling problem-solving behavior and expecting their students to be active problem solvers. Students will be encouraged to use inductive and deductive reasoning and will be able to describe their thinking processes. They will see mathematics as more than computation. The development of this new mathematics classroom includes a shift from emphasis on a product -- "finding the right answer" -- to an emphasis on process -- the process of solving the problems and discussing the solutions to problems.

Specific content that should be taught in the middle grades is described in the last nine standards proposed by the Commission, which describe the following content strands:

- Numbers and number relationships.
- Number systems and theory.
- Computation and estimation.
- Patterns and functions.
- Algebra.
- Statistics.
- Probability.
- Geometry.
- Measurement.

Although these topics are presented in traditional textbooks, there is little emphasis on them. A majority of the content for grades 6, 7, and 8 is related to computation, and there is an underemphasis on the other strands of mathematics; it is important to teach all strands.

We bellieve that mathematics content for middle-school students should be reorganized around the strands of statistics, probability, geometry, and measurement. Doing so addresses a number of problems. First, 90 percent of mathematics concepts can be taught through these strands, which are frequently underemphasized in mathematics classes. Problems introduced in these strands can provide students an opportunity to practice skills such as computing, estimating, finding patterns, and writing number sentences. Second, because much of the mathematics used in technological applications and by adults in daily life is based on these four strands, their use in teaching can help connect mathematics used in school to that to be used later in life. Materials such as those from the Quantitative Literacy Project (1987) are available for use in instruction in probability and statistics.

To teach the content of mathematics, teachers must understand the structure of mathematics. Teachers who are knowledgeable of both content and structure are more likely to be successful in "teaching for meaning" than those who are not. Many teachers realize that simple drill and practice in computational skills will not be sufficient; they must teach for understanding.

Understanding in mathematics implies knowledge of when to use one procedure rather than another to solve a proposed problem. Bruner (1964) stated, "The most important thing about memory is not storage of past experiences, but rather the retrieval of what is relevant in some usable form. This depends upon how past experience is coded and processed" (p. 2). Cognitive psychologists suggest that knowledge is stored in the learner's brain as a network of concepts or constructs. **Learning is the making of connections between a new or novel situation and the learner's existing network of knowledge.** Understanding is facilitated by the learner's organization of knowledge.

Research indicates that students who use structured knowledge to guide their encoding and retrieval will be able to recall more information than students who do not (Adelson, 1981). Teachers can aid students in the process of encoding (storing information for later retrieval) by providing them with clear and direct labels during learning experiences. For example, rather than repeatedly saying to students, "Remember this is for the test on Friday," the teacher could tie information to a concrete activity or demonstration. The teacher also could help reinforce encoding and retrieval by relating concepts to previously discussed or interrelated concepts. For example, in a lesson involving percent, a teacher could discuss the relationship between fractions, decimals, and percent (i.e., 3/4 = .75 = 75%).

In these ways, teachers can influence the students' knowledge organization (or cognitive structure), which influences what students attend to, what they recall, and, ultimately, how they solve problems. Students and teachers often see mathematics as nothing more than a collection of procedures for solving computations. However, mathematics is a unified system of concepts and operations that explains certain patterns and relationships that exist in the real world (Resnick & Ford, 1981). Understanding the structure of mathematics implies grasping: 1) the interrelations of concepts and operations and 2) the rules by which they may be manipulated and reorganized to discover new patterns and properties. For example, perimeter and area can be viewed as interrelated with geometry and measurement. Unfortunately, most students never come to an "understanding" in mathematics because of the piecemeal fashion in which such interrelated concepts are generally presented and the failure of teachers to provide the "whole picture." The use of well-organized materials that are based on appropriate instructional principles should increase meaning and understanding for students. In the end, however, it is the teacher's knowledge of mathematics that affects how concepts are presented and how students understand relationships among them.

After students have become proficient with mathematical concepts, the goal should be to help them apply the information they have stored to novel situations. Stored knowledge alone does not solve problems. Problems are solved through the transfer of knowledge to a new situation. Transfer of knowledge is a major component of problem solving and thus is an important principle for teachers to understand and facilitate.

Because novel situations often require a reorganization of existing skills, the teacher should ensure that the prerequisite skills of a particular task have been mastered. If prerequisite knowledge is lacking, the student cannot be expected to solve problems successfully. To check for necessary prerequisites, the teacher can ask: What set of actions must a student be able to take to complete a particular task?

During a learning experience, transfer of knowledge to new tasks will be facilitated if students are given an opportunity to discover relationships and apply principles within a variety of situations (Hilgard, 1956). Teachers generally recognize this need but frequently become too impatient to let the process evolve. Often a concept is explained to students, but little opportunity is provided for them to discover, through their own work, a clear insight into the principles, proofs, or operations underlying the concept. The successful teacher will use teaching procedures that stimulate students to explore various aspects of a concept and its relation to other concepts of mathematics, which likely will result in enhanced problem-solving skills. Specific suggestions for the teacher include:

1) Discuss the subject matter structure with students.
2) Find systematic ways of representing knowledge.
3) Present connections and relationships among different topics in mathematics.
4) Provide opportunities to practice new procedures and concepts in a wide variety of situations.
5) Teach as much mathematics as possible -- the more facts, procedures, and relations within a student's knowledge structure, the more likely he/she will be to invent or discover needed connections.
6) Apply and demonstrate knowledge in problem-solving situations.
7) Design direct instruction in specific problem-solving strategies.

KNOWLEDGE OF STRATEGIES ASSOCIATED WITH SUCCESSFUL TEACHING

The research literature describes specific teacher behaviors as being associated with successful mathematics teaching. These are: 1) wise use of time, including using a majority of class time for concept development; 2) framing instruction in a problem-solving format; 3) use of direct instruction in estimation skills and mental computation; and 4) the use of technology such as computers and calculators.

Time Management

Strategies that maximize class time must be implemented by teachers who desire successful student outcomes. Research findings indicate that use of a well-organized lesson plan in which "routines" are used will maximize time usage. Good, Grouws, and Ebmier (1983) describe one such plan in their book, Active Mathematics Teaching. These mathematics educators describe a model for teaching that resulted in increased learning for junior-high/middle-school students by teachers participating in the Missouri Mathematics Project. This model presents the "best" pattern or routine for lesson segments as one that: 1) reviews related concepts and vocabulary, 2) develops concepts, 3) guides practice, 4) includes seatwork, and 5) assigns homework (see Appendix A for specific detail). Good and his colleagues noted the importance of the developmental portion of the lesson; they suggest that at least 50 percent of a class period be spent in it. This type of lesson format resulted in increased student learning because students were better able to understand the material taught. Two elements are needed for this model to succeed. First, guided practice periods should be carefully monitored by the teacher; too often, the teacher engages in other activities such as checking papers during this time. Second, homework should be assigned only when students have understood the material that has been taught.

Another researcher, Leinhardt (1988), compared the teaching behaviors of experienced (defined as successful) and novice teachers and found that experienced teachers exhibit better use of classroom time than novice teachers do. Experienced teachers conduct lessons that are highly efficient in internal structure, are fluid in transition between segments of the lesson, and widely employ "routines" within the lesson segments. The lessons of novice teachers, in contrast, were characterized by fragmentation and student confusion about which procedures were to be followed in carrying out classroom activities. Interestingly, the experienced teachers in this study did not use more of the actual class time for concept development than the novice teachers did; however, the instruction provided was "direct" in nature and was clearly communicated. The experienced teachers were also able to diagnose efficiently potential learning problems in each segment of the lesson. This was not true for the novice teachers. For example, while checking homework, the experienced teachers were able to assess which students might need additional assistance with previous work, whereas novice teachers often had difficulty checking homework without distractions. Experienced teachers made use of "routines," including guided practice and checking homework, which allowed students to predict the rules for classroom activities. Novice teachers did not use well-developed routines; thus, students in these classrooms were unsure of "what comes next." Using lesson segments facilitates learning because when students can predict classroom activities to some extent, the cognitive processing required of them is reduced (Leinhardt & Putnam, 1987).

Each phase of the lesson should include a diagnostic/prescriptive approach. The teacher assesses students' understanding and performance at each step of the process. Reteaching is required if student understanding is absent. A diagnostic/prescriptive approach to instruction serves to enhance the learning process by providing immediate attention to information that has not been learned correctly.

Although the use of a structured approach to mathematics lessons may seem stifling or lacking in creativity to the teacher, there are two important advantages to using such a system. First, use of instructional time is maximized. Second, students at risk of dropping out of school benefit from direct instruction and structured classrooms. The results of Project Follow Through, a study of 1,000 low-income minority students from urban and rural settings, indicate that the most effective programs for the at risk student are "structured educational programs." Project evaluators concluded that for at risk students to succeed as adults, they need high-quality, structured instructional programs not only in elementary school, but also in the middle-school grades and beyond (Gersten & Keating, 1987). The implications of this and other studies (Brophy & Good, 1986; Steblins, St. Pierre, Proper, Anderson, & Cerva, 1977) indicate that the use of direct instruction enhances academic growth for the at risk student. Lessons can be structured so that teachers can assess the students' understanding of what has been taught. The teacher can then guide students toward better understanding, using immediate feedback and problem-solving strategies (Gersten, Carnine, Keating & Tonsic, 1987).

Integration of Problem-Solving Skills Into the Mathematics Curriculum

A critical teacher strategy is the integration of problem-solving skills into the mathematics classroom. Peterson, Fennema, and Carpenter (1988) compared teacher behavior with student achievement for two groups of teachers. The experimental group participated in problem-solving in-service activities during the school year, and the control group was not involved in those activities.

There were differences in both teachers and students. The primary differences between the two groups of teachers were that the experimental group posed questions more often, expected students to use multiple strategies for solving problems, and began lessons with story problems. Control-group teachers focused on computational and number facts.

The student performance of the two groups of teachers was also different. Students of experimental group teachers who were in the low-achievement class improved their scores significantly on a test of solving word problems. In addition, this group of students performed as well as students in the control group did in computation skills and at a higher level than the control group in recalling number facts.

These findings suggest that when teachers know how to teach problem solving, they can and do change their teaching strategies, resulting in better student skills. One factor to keep in mind, however, when generalizing the results of the study is that experimental group teachers were involved in <u>ongoing</u> in-service activities related to problem solving. It has been demonstrated that ongoing in-service activities, as opposed to "one-shot" activities, result in greater change in teacher behaviors. This programmatic aspect of the study contributed to both teacher and student changes.

What Is Problem Solving?

When many teachers see or hear the term "problem solving," they generally think of word problems that appear at the end of chapters in the textbook. Word problems are, however, only one part of problem solving. Problem solving is a process, a way of thinking or analyzing events. Engaging in problem solving requires an individual to use previously acquired knowledge in an unfamiliar situation (Krulik & Rudnick, 1980).

In order to improve their problem-solving abilities, students should be asked to solve both routine and nonroutine problems. Routine problems are those types of problems that can be solved by the application of an algorithm followed by the necessary computation to arrive at a "right" answer. Nonroutine problems, in contrast, generally involve more than one step (application of more than one algorithm) and may have several correct solutions. Nonroutine problems often involve the use of higher order thinking skills, such as inductive and deductive reasoning, to draw conclusions. Some nonroutine problems do not involve the use of numbers for successful solutions. The exercises in the backs of the most recent textbooks are often a combination of routine and nonroutine problems. Teachers need to help students differentiate between the two types of problems and learn to generate both types themselves.

Enhancing the problem-solving skills of students involves creating a classroom environment in which students are encouraged to express their ideas. Teachers should emphasize that it is "OK" to make mistakes and that it is fun as well as satisfying to try to solve a hard problem. Asking open-ended questions related to the process of solving problems will help the student express his or her ideas. There is some indication that students cooperating with one another during instruction are more likely to be good problem solvers than are those who work individually (Johnson, Skon, & Johnson, 1980). Students who learn to work together can benefit from one another's ideas and proposed solutions to problems, resulting in increased problem-solving skills. For students to feel free to discuss ideas and solutions, they must feel that they are in an accepting environment where risk-taking behavior is encouraged.

Model Lesson With Problem-Solving Focus

The critical question for teachers is: How does one integrate problem-solving skills into textbook-driven mathematics curricula? Because most of the concepts presented in middle-school curricula have been previously taught, an optimal situation is available for the integration of problem-solving activities. First, students are not motivated to practice skills that they have already learned.

However, they can be encouraged to practice these skills when real-life situations or verbal problems are presented that require their use. When the content of the lesson involves the introduction of new skills, students are likely to see a reason to learn them if they are presented through a problem-solving format. Each lesson in the middle-school mathematics classroom should involve the development of problem-solving skills by using problem-solving activities in the introduction of the skill, the practice of the skill, and the homework assignment.

We have developed a model that describes a structured lesson using a problem-solving focus. It incorporates some of the lesson segments specified by Good and his colleagues (described earlier); however, their format has been modified to fit a problem-solving approach.

I. Introduction

In this phase, the teacher spends a few minutes discussing the previously assigned homework. The teacher then gives the students a few problems, asking them to perform mental computations or provide estimates to arrive at solutions. Next, the teacher provides a brief review of the prerequisite skills or vocabulary that will be needed in the lesson for that day.

II. Verbal Practice

The teacher uses two or three verbal story problems that provide a cumulative review of past topics. The teacher then gives problems that "set the stage" for the lesson of the day. These problems do not necessarily have to include numbers. The function of these problems is to provide a context for the topic to be discussed.

III. Developmental Phase

During this phase, students are asked to apply to daily life situations knowledge that previously has been taught, or they are introduced to the new concepts in the middle-school curricula. This section will include routine and nonroutine word problems. During it, the emphasis is on applying skills to solving problems. Instruction in the developmental phase may include demonstration of the concepts underlying the skill being taught. Use of manipulatives, calculators, or computers may be beneficial. Teachers may also teach various strategies for solving problems (i.e., working backward, solving a simpler problem). The developmental section of the lesson may take only part of the class period because many of the skills taught have been encountered previously by the student. As students engage in the problem-solving process, the teacher will observe students who are having difficulty with either concepts or computation. Teacher monitoring of student performance during this phase is especially important so that he or she may provide additional assistance to students who need it.

IV. Practice

Students may practice problem-solving skills and the accompanying computational skills individually, in pairs, or in groups. Research findings suggest that both problem-solving skills and computational skills are enhanced when students engage in small-group problem-solving activities (Peterson et al.,1988). Good and his colleagues (1983) suggest a 15-minute practice period, but the practice period needed for instruction based on a problem-solving approach may be longer. Assignments may take one class period or one week.

The objective, however, is for students to practice the mathematical skill, using it to solve problems.

In the following pages, two lessons are provided to demonstrate the plan. In Appendix B, there are additional lessons.

MODEL LESSON 1

Standard 1: Mathematics as Problem Solving
Standard 4: Communication
Standard 10: Statistics
Skill: Computing Averages
Level: 6-8

Previous Lesson: Introduction of the concept of an "average" or "mean"

Introduction

A. Review: Discuss the terms average and mean. Review the concept of arranging objects, such as paper clips, into groups to find the average number.

B. Check homework. (Each student had been asked to bring a newspaper clipping that could be used to find an average. Check to ensure that the clippings were brought.)

Verbal Practice

A. Review of previous material:

1. Ask students to define average and mean.
2. Ask for situations when an average would be needed. (Example: Computing grade averages and batting averages.)

B. Lead-in for this lesson:

If you wanted to know the average number of cassette tapes that each person in the class has, how would you find this number?

Developmental

A. Teach how to compute an average. Use previous class examples of finding the average number of cassette tapes to teach this concept.

1. Have students call out the number of tapes they own (list boys' and girls' responses in different columns).
2. Demonstrate process of computing the average for three students.
3. Have students use a calculator at their desks to compute class average.
4. Compare differences between girls' averages and boys' averages.

Extension: Have girls and boys estimate the average amount spent on cassette tapes.

B. Use as a lesson with individual students:

1. Have each student estimate the average amount of time he or she spent on homework per week. Use a calculator to compute.
2. Have the students compute the average amount of time an individual spent watching television in one week. Make a chart to reflect data.
3. Have the students estimate and then compute the average number of minutes they are allowed to talk on the phone per week and month.

Practice

A. The following activity should be completed as a large group.

Use newspaper clippings to find an average. For example, if the student brought a weather map, an average temperature could be found in the same city over a given time period. Other examples for clippings might include batting averages from the sports section, stock market figures, etc. Ask students to give oral reports on their findings.

B. These problems could be used in a cooperative learning situation.

1. Ask students to use the clippings to generate their own problems finding averages. For example, if a grocery ad was clipped, the student might write a problem such as:

   > John was going to the store to purchase cornmeal. He wanted to find the average cost of three different brands of the same quantity. The prices were:
   >
   > | | |
   > |---|---|
   > | Martha White | $.59 per lb. |
   > | CornMeal by Betty | .78 per lb. |
   > | Pillsbury | .81 per lb. |
   >
   > What was the <u>average</u> cost?

2. Use the students' grades. Suggest that they determine their average and decide the grades needed to maintain or achieve an "A."

3. Have the students collect their grocery receipts for one month and determine the weekly average spent for food.

4. Ask students to call a movie theater to inquire about the total number of customers who enter the given theater in a given period of time. Find the average number of moviegoers per week.

MODEL LESSON 2

Standard 1:	Mathematics as Problem Solving
Standard 5:	Number and Number Relationships
Standard 7:	Computation and Estimation
Standard 10:	Statistics
Standard 12:	Geometry
Skill:	**Finding Percent of a Number**
Level:	6-8

Previous Lesson: Changing percent to decimals or fractions

Introduction

Review

1. Discuss the relationship between percent, decimals, and fractions. Review the rule that percentages must be changed to decimals or fractions before multiplying.

2. Check homework on examples of changing percent to a decimal or a fraction.

Verbal Practice

Use the following for verbal practice:

1. State this situation and ask students to convert from a fraction to a decimal and percent.

   How many students are wearing a wristwatch in our class? Find the fraction, decimal, and percent for this question.

2. Given the following problem, ask how the percent could be found (substitute applicable numbers for your classroom).

   > We have 26 students in our math class. If we have 12 boys and 14 girls, what percentage of our class is girls? What percentage is boys?

Developmental

Explain the procedure for finding the percent of a number. Use the previous example of finding percentages of boys and girls in a classroom. Example: 12/26 = N%, 14/26 = N%. Use concrete examples of dividing them according to gender. The skill of finding percent could be applied as follows:

1. Ask students to find what percentage of the class is wearing a certain color. Make a bar graph to show results.

2. Ask students to survey classmates about their favorite food. Ask students to tabulate what percentage of the class enjoys a certain food. Use a circle graph to show results.

3. Solve for percent.

   a. Estimate to solve.

      Lee wants to save 20% of $120.00 that he will earn mowing grass. How much money will he save from this summer job?

   b. Have each student determine the actual amount.

4. Ask students to think of their family's dinner bill at their favorite restaurant. Have one student report the total bill. Have the class estimate and then determine how much tip should be left for the waitperson to receive 15% of the total bill.

Practice

Additional NCTM standard addressed: Standard 4: Mathematical Connections.

Give students a map of the United States to use with the following. Conduct in a large-group setting.

1. Give the percentage of states that have names beginning with the letter "A."
2. Give the percentage of states that have North in their names.
3. Give the percentage of states that have West in their names.
4. Ask students to give the percentage of states that border Mexico.

Practice

Direct the following activity by asking your students to work cooperatively in small groups.

Activities

1. Conduct a survey of the class to find the percentage of students choosing Bon Jovi, Poison, ACDC, etc., as their favorite rock group.
2. Have them compute the percent of students preferring each rock group. Bar graphs or line graphs could be used to display results.
3. Conduct a survey of the class to find their favorite sport. Tabulate the results to find what percentage of students would choose tennis as their favorite sport.

Teaching Problem-Solving Skills
Through Direct Instruction

Teachers can model and demonstrate problem-solving skills as one method to help students become problem solvers. The strategies used in successful problem solving can be taught via direct instruction. Models for problem solving upon which instruction can be based have been developed by Polya (1980) and Krulik and Rudnick (1980). Polya proposed a four step-process that includes:

1) Understand.
2) Devise a plan.
3) Carry out the plan.
4) Look back.

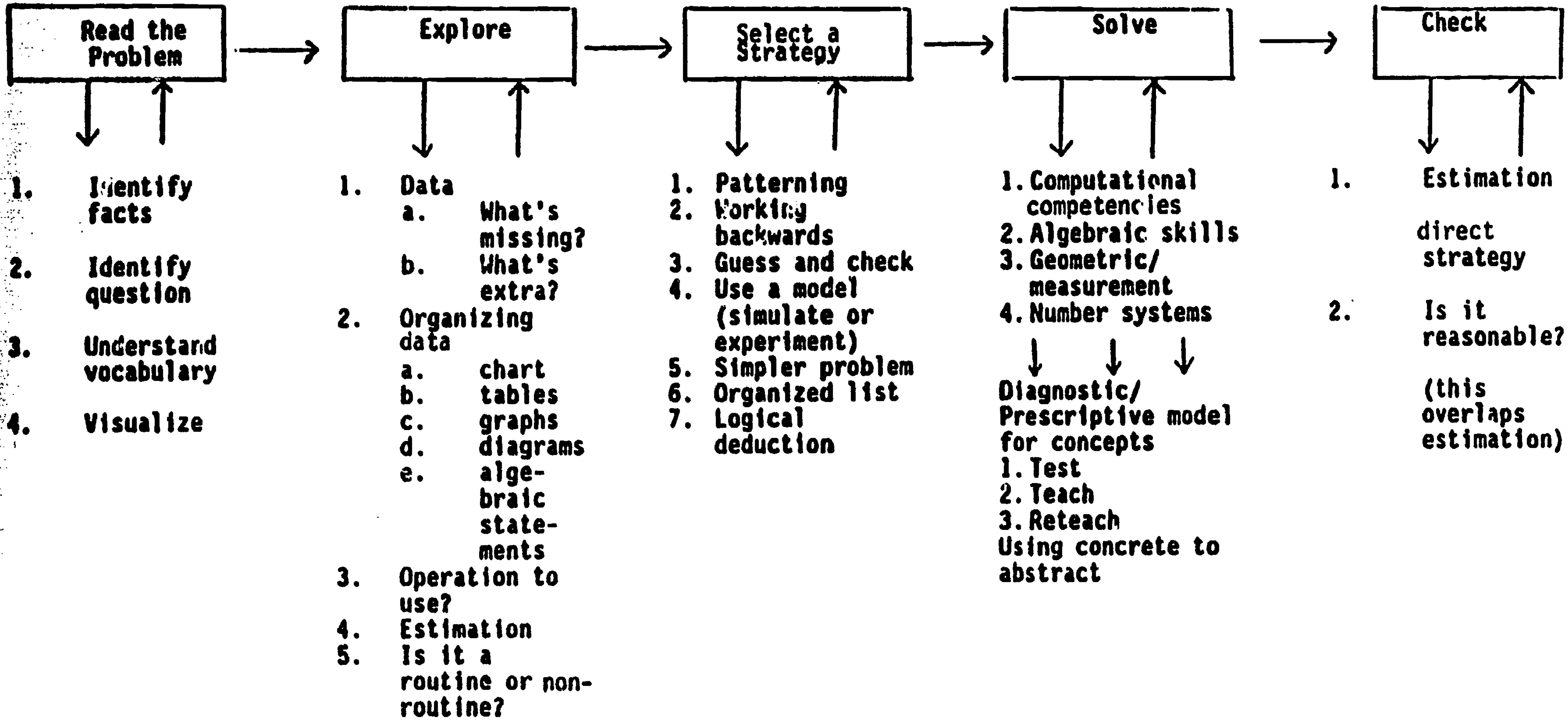
Krulik and Rudnick (p. 5) added an additional step to Polya's basic model--the Explore step. Their model is presented in Figure 3.

As described in the Krulik and Rudnick model (Read step), students first need to determine the question being asked. it is often a good idea to have students underline the question being asked and/or to write the question in their own words. In the Explore step, students identify and organize the facts presented in the problem; making a list of the facts is often helpful. Next, students determine which of the facts presented are needed to solve the problem; extraneous facts should be eliminated from the list. Needed facts that are missing also must be identified. Estimation should be encouraged.

The step in the problem-solving process in which the student must Select a Strategy is usually the most difficult. Often more than one strategy or combination of strategies may be used. Students come to know which strategy or strategies to select only through practice--and more practice--with problem-solving activities. Thus, students should be exposed frequently to situations in which they must use problem-solving skills.

Figure 3

Krulik and Rudnick Problem-Solving Model



Additional problems for practice, as well as activities involving problem-solving skills, may be found in the activities section of this handbook. Students of teachers who use this or a similar approach to teaching problem solving become confident problem solvers.

Teaching Estimation Skills and Mental Computation Through Direct Instruction

Estimation is an essential skill for both students and adults. It is probably the mathematics skill most often used by adults in daily life. Estimation skills also will be vital for workers of the future. With the increased use of technology, it will be essential to use critical thinking skills to determine reasonableness of results. Yet, these skills are taught infrequently in the mathematics curricula, and students need to improve both estimation and critical thinking skills to be effective.

Students' ability to estimate improves greatly when estimation skills are taught directly (Driscoll, 1981). One of the first issues in teaching estimation skills is to dispel the myth that mathematics and pencil and paper are inseparable. A second issue is helping students overcome anxiety about not getting the right answer. Having the students engage in mental computation activities is a good way to begin building an appropriate mental climate for estimation. It helps to establish the idea that mathematics is something to think about, but not necessarily something to write down.

Mental computation and computational estimation are both similar and different. They are similar because both are performed mentally and are used to check for correctness or reasonableness of answers. The primary difference between the two is that mental computation requires knowledge of basic facts; some types of estimation can be learned prior to learning basic facts. Students need to be able to distinguish between situations in which mental computation is appropriate and those in which estimation is appropriate. Interestingly, it is possible to be competent at mental computation, but unable to estimate; however, the converse is not true. Individuals who are competent at computational estimation are also competent at mental computation (Reys, Bestgen, Rybolt, & Wyatt, 1982).

Role of Mental Computation in the Middle Grades

Mental computation should be used beginning in the elementary grades. It can serve as a bridge between basic facts and algorithm development. In the elementary classroom, it allows both practice of basic facts and extension of these facts. In the middle grades, mental computation can be used to develop and evaluate student knowledge of basic concepts (e.g., what is 50% of 24 or 25% of 100) (Reys & Reys, 1986).

Reys and Reys (1986) have suggested that a middle-grade mathematics program emphasizing mental computation should include the following:

1. Students should appreciate the importance of mental computation.
2. Instruction should include systematic development of mental computation during the middle grades.
3. Instruction should include whole numbers, fractions, decimals, percents, and multiplying by multiples and powers of 10.

Role of Computational Estimation

Computational estimation allows students to verify reasonableness of results obtained and also provides them with an opportunity to detect computational errors in their work. Estimation involves the use of a variety of skills; it is similar to problem solving and is, in fact, a part of the problem-solving process.

It is important to remember that estimation is not a skill that should be isolated in a single unit of instruction. Although estimation skills can be taught effectively by using direct instruction, they should be integrated into the regular mathematics curriculum.

Teaching Computational Estimation

Computational estimation is a set of skills that can be taught. A comprehensive estimation curriculum in computation must address several areas:

1) Development of an awareness of and an appreciation for estimation.
2) Development of number sense.
3) Development of number concepts.
4) Development of estimation strategies.

(Reys & Reys, 1986).

Awareness and appreciation of estimation are often difficult. Many students "feel uncomfortable" about not getting the "right" answer; these students must learn to develop a tolerance for error. Some suggestions for creating an appropriate climate for estimation are: use real-world applications extensively, emphasize situations where only an estimate is required, use easy examples in the early stages, accept a variety of answers, use oral work and group discussion, and help the students determine when estimation is appropriate.

The following information on how to teach estimation skills has been taken directly from Effective Mathematics Teaching: Remediation Strategies: Grades K-5 (Clayton, Burton, Wilson, & Neil, 1986):

> The development of number sense can be associated with having the student recognize "sensible" or "reasonable" answers. Instruction in this area could begin by presenting the students with noncomputational situations in which they identify or supply a reasonable answer. For example:

Sandra carried some books to school. She carried about ___ books?

5 50 100

Another activity that can be used to build number sense is presenting examples that have been solved and having students check to find unreasonable answers. For example:

PICK OUT THE ANSWERS THAT DON'T MAKE SENSE

$$1.\ \begin{array}{r}249\\27\\\underline{416}\\685\end{array}\qquad 2.\ \begin{array}{r}98\\\underline{\text{x}24}\\23{,}502\end{array}\qquad 3.\ \begin{array}{r}\underline{22}\\6|\ 1320\end{array}\qquad 4.\ \begin{array}{r}\underline{95}\\5|\ 447\end{array}$$

(Trafton, 1978)

In developing number concepts, it has been suggested that teaching estimation skills prior to teaching the concept may enhance the student's understanding of that concept. Students' general lack of development of some number concepts can be noted by the fact that in a recent national study, it was found that 76 percent of 13-year-olds in the United States incorrectly estimated the sum of 12/13 and 7/8 (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1981). Was it because they didn't know how to estimate, or because they really didn't understand what 12/13 and 7/8 represent? The answer is probably a combination of both. It is quite probable that teaching the correct algorithm could result in greater understanding of these mathematical concepts.

Research on computational estimation has suggested that performance can improve dramatically when specific estimation strategies are taught (Driscoll, 1981). Reys and Reys (1986) suggest four strategies:

front-end estimation, rounding, compatible numbers, and clustering or averaging.

<u>Front-end</u> estimation involves using the first digits (left-most digits) in a problem to make estimates. The front-end strategy is a two-step process: estimate front-end and adjust for the other digits. This is a strategy that even young students can learn to use, and it is appropriately used in addition, subtraction, and division.

Example:

$1.26	1.	Front-end: total the front-end amounts
4.79		(1 + 4 + 1 = 6)
.99	2.	Adjust: group the cents
<u>1.37</u>		(.26 + .79 = $1.00)
		(.99 + .37 = $1.00)

Thus:	$6.00	+	$2.00	=	$8.00
	initial estimate		adjusted estimate		final estimate

The <u>rounding</u> strategy is a powerful and efficient process for estimating the product of two multidigit factors. Rounding is a three-step process: rounding, computing with the rounded numbers, and adjusting. Students should be taught that the purpose of rounding is to provide mentally manageable numbers. They need to learn to be flexible in their method of rounding--fitting it to the particular situation, numbers, and operations involved. It is important for students to realize that there is no right answer in estimation. Each rounding choice produces different, but reasonable, results. Students should also learn to adjust answers for over- and underestimation.

The <u>compatible</u> numbers strategy refers to a set of numbers that is easy to manipulate mentally. The choice of a particular set of compatible numbers involves a flexible rounding process. This strategy is particularly helpful when estimating division problems. For example:

Estimate: $7\overline{)3388}$	compatible sets	not compatible
	$7\overline{)3500}$	$7\overline{)3000}$
	$8\overline{)3200}$	$7\overline{)3300}$
	$8\overline{)4000}$	$8\overline{)3400}$

The <u>clustering or averaging</u> strategy is well-suited for problems when a group of numbers cluster around a common value. For example, estimate the total attendance:

Olympic Games (Sept. 1-7)

Monday	105,000
Tuesday	109,975
Wednesday	95,492
Thursday	103,823
Friday	100,224
Saturday	97,795

(1) Since all the numbers are relatively close in value, the use of clustering is appropriate.
Estimate an "average" . . . . about 100,000.

(2) Multiply the "average" by the number of values.
6 x 100,000 = 600,000 (pp. 149-152)

Use of Technology

Technology is changing the way we obtain mathematical solutions to problems. Students' use of two tools, the calculator and the computer, is increasing. Mathematics educators are urging teachers to use these tools in their classrooms, implying that teachers and students will need to know not only how to use these tools, but when their use is appropriate.

Use of Calculators

Although the use of calculators has been recommended by the National Council of Teachers of Mathematics since 1980, they are not widely used in classrooms. There appear to be three main barriers to calculator use:

1) Teachers are uncertain about when calculator use is appropriate and are unsure about how to use the calculator (Clayton, et al., 1986).
2) There is concern that use of calculators will result in declining emphasis on computational skills, although multiple research studies have indicated that this concern is not valid (Hembree & Dessart, 1986; Suydam, 1981).
3) Because standardized testing assesses students' paper-and-pencil computational skills, teachers are hesitant to spend instructional time in ways that may not be reflected in testing.

The first two of these concerns can be dealt with effectively by demonstrating to teachers appropriate calculator use in the classroom and explaining the research findings to critics. The third concern, the testing issue, is, however, valid. Policymakers in mathematics education are demanding that companies producing standardized tests change evaluation procedures. Such changes would enhance teachers' perceptions regarding calculator use since there would be a direct relationship between instruction and assessment in an attempt to ensure curricular validity.

To encourage use of calculators in mathematics classrooms, teachers should ensure the following:

- Every student is taught how and when to use a calculator.
- Concentration is on the problem-solving process, rather than calculations associated with problems.
- Students gain access to mathematics beyond their level of computational skills.
- Students explore, develop, and reinforce concepts, including estimation, computation, and properties.
- Students experiment with mathematical ideas and discover patterns.
- Students perform tedious calculations that arise when working with real data in problem-solving situations (NCTM, 1987).

Calculator uses range from computation and concept development to solving problems that require a higher level of computational skills than possessed by the student. First, calculators may be used to perform tedious computations. Reys and Reys (1986) suggested that if a computation takes longer than one minute, it should be done by calculator. An Agenda for Action (NCTM, 1980) recommends that computations of more than 3- to 4-digit numbers be completed with a calculator.

A second use is in concept development. Comstock and Demana (1987) describe the use of a calculator to develop and reinforce the concept of percent. They say that one reason that middle-school students have difficulty figuring percentages is that the computational aspect is so cumbersome that there is not enough practice time for mastering the concept. These researchers suggest that use of a calculator makes it possible for students to do many more problems in the same period of time. The use of the calculator in this situation is enhanced when teachers emphasize the process used by questions such as, "What did you do to get your answer?"

Calculators are also used in concept building with area and perimeter. These are concepts that are frequently confused by students; they can be clearly differentiated by activities such as building a table of areas and perimeters using a calculator. Comstock and Demana (1987) presented the following tables as examples of such an activity:

Table 1. Distinguishing Between Area and Perimeter

Width (m)	Length (m)	Perimeter (m)	Area (m^2)
2	5	14	10
3.5	____	____	____
5	____	____	____

Table 2. Extending Table 1 to See More Patterns

Width	Length	Perimeter	Area
2	5	14	10
3.5	6.5	20	22.75
5	8	26	40
____	10	____	____
____	____	38	____
____	____	____	54

Calculators are also used in problem-solving activities. Using a calculator allows the student to focus on the process of solving the problem, rather than the computation. When using calculators, students can attempt to solve problems more complex than those solved with paper-and-pencil computation. For example, the following problem would be considered impossible using paper-and-pencil calculations:

> What is the number of hamburgers sold by McDonald's since it opened and the average number of these hamburgers purchased by each American?

Yvon (1987) suggests two problems that might interest middle-school students.

They include:

1) How many minutes or hours does your class watch television each day? Each week? Each year? Do boys and girls differ in the amount of television viewed?
2) What is the total distance the class could walk in one hour figured on the basis of a five-minute sample?

Thus, calculators should be used: 1) after basic skills have been initially taught and mastered, 2) for tedious computations, 3) for development and reinforcement of skills and concepts, and 4) as an aid in problem solving. Students who are encouraged to use calculators describe themselves as having more positive attitudes toward mathematics and as being better problem solvers than do students who are not encouraged to use calculators. Teachers are encouraged to use calculators in their classrooms. Many activities that incorporate calculators in classroom activities are presented in the Activities section of this handbook.

Use of Computers

Research on the effectiveness of computers in instruction is limited because most studies investigate the effects of drill-and-practice or tutorial programs (Clark, 1985; Krulik, Bangert, & Williams, 1983; Krulik, Krulik, & Cohen, 1980). The results of these studies are mixed -- some indicate increased achievement, and some indicate little or no gain in achievement. A problem in generalizing these results is that variables such as the amount of use, type of software, and method of measuring gains differ from study to study. Selection of appropriate software, adequate use, and reliable measuring instruments may produce gains in learning.

However, using computers in instruction has other advantages. Computers:

- Enhance motivation.
- Permit students to work at their own pace.
- Provide immediate feedback and reinforcement.
- Allow teachers to specify student activities.
- Allow for active learning by the student. (Clayton et al., 1986)

Until recently, students have used computers primarily for drill-and-practice activities or for computer literacy (Becker, 1985). There is little or no research on the effectiveness of using computers for problem solving, the present focus of mathematics education (Kansky, 1986). Thus, appropriate use of computers in problem-solving situations cannot be determined at this time. It may, however, be helpful to teachers to consider using a computer to solve some problems.

When using a computer for problem solving, the student must determine what type of software is appropriate. The type of problem will dictate the type of software needed. Some examples of available software packages are: spreadsheets, statistical packages, data bases, graph-generation packages, and packages that assist in use of geometric shapes.

Use of Paper and Pencil, Calculators, or Computers for Solving Problems

Concepts should be developed in the mathematics curriculum to allow students to learn to select and use an appropriate strategy for computing, choosing from mental arithmetic (computation or estimation), paper and pencil, or the use of a calculator or computer.

In the following example, use of a calculator would be appropriate:

> A set of cards is prepared, each one bearing the price of an object and a particular discount in percentages (e.g., $10.95, 15%). Each of the two players has a calculator. One player turns over a card to reveal a price and a discount. Then both players estimate the final, discounted price. They use the calculators to find the discounted price, and the player who comes closest to the actual discounted price earns one point. A game played to 10 points takes 10 minutes or less. (NCTM, 1989, p. 97)

When using the calculator, it is important for students to use the results appropriately. The student should be able to decide if the answer is reasonable.

> A tray can hold 12 salads. How many trays are required for 244 persons?

In this example, the calculator reflects a repeating decimal, and the answer would be rounded to the largest whole number.

Using mental computation and estimation helps one check for the reasonableness of results when using a calculator.

> Marsha will earn $9.40 per day for babysitting. She needs to decide how many days she will need to work to purchase a bicycle costing $85.00.

When using estimation, she multiplied 9 x 9 = 81. However, when she entered the problem into the calculator, she entered 850 ÷ 9.4. Her estimate suggested that this answer is not reasonable.

Mental computation could be employed in the following situation:

> Mary's family spent $69.25 at their favorite restaurant for dinner. What would be 15% of this total for the tip?

Finally, a computer would be appropriate when information needs to be stored for later retrieval. An example is storing information necessary to determine how long it would take to pay off a credit card balance at a given interest rate with a revolving balance.

In summary, it is very important that the student learns to choose the appropriate method for performing a computation. Appropriate strategies that incorporate estimation, mental computation, calculator, and computer activities can aid in the development of the student's mathematical skills and sense of numbers.

KNOWLEDGE OF DIAGNOSIS OF STUDENT ERRORS

The ability to diagnose errors made by students is one of the competencies possessed by a successful teacher. The successful teacher uses a diagnostic/prescriptive approach to instruction during each lesson segment. Thus, by constantly monitoring student performance, the teacher is able to identify and remediate patterns of errors in students' work. Diagnosis of students' errors prior to attempting to remediate the skill is critical. It is only after the type of error has been diagnosed that appropriate instruction to remediate the error can be prescribed. Leaders in the field of mathematics education have suggested that diagnosis should not be limited to slow learners or underachievers but should be used with all students as a preventive measure. In general, symptoms of faulty learning include: the inability to work three or four examples of a type of problem correctly, failure to improve with practice, confusion of mathematics processes, and inability to apply what has been learned in practical situations (Clayton et al., 1986).

Procedures for Diagnosing Errors

Two procedures generally used for diagnosis of error are:

1) Analysis of written work.
2) Oral interview.

Analysis of written work can include homework, test papers, and class work. In examining a student's written work, teachers can generate tentative hypotheses about the errors. However, prior to providing remediation instruction, the teacher should discuss the errors with the student. An appropriate statement would be: "Tell me how you got this answer." As the teacher listens to the student's explanation, the teacher can determine the type(s) of error being made. Three general types of errors have been identified by mathematics researchers (Englehardt, Ashlock, & Wiebe, 1984). These include:

1. Careless: incorrect responses resulting from responding without fully engaging in the task. $8X(4+3)=27$
2. Procedural: incorrect responses resulting from misordered or inappropriate procedures, such as subtracting the minuend from the subtrahend.
   $$\begin{array}{r} 7.5 \\ \underline{+3.8} \\ 10.13 \end{array}$$
3. Conceptual: absent or misunderstood concept/principles, such as inappropriate concepts of zero or regrouping in subtraction.
   $$\begin{array}{r} \underline{10.1} \\ 4.5/40.5 \end{array}$$

Teacher strategies for remediating each of these error types will be quite different. Thus, remediation will not be efficient or effective unless the type of error being made can be identified. For example, if a student consistently makes careless errors, the teacher's strategy is to slow the student's work down and encourage checking the work. The last two types of errors present the greatest problem for the teacher, both in diagnosis and in remediation. Learning to discriminate between procedural and conceptual errors is very important because the remediation strategies for these errors are quite different.

If a student has made a procedural error, the teacher's major goal is to reteach the rule or concept that has not been learned. After the teacher demonstrates the rule, the student should practice the rule with the teacher observing to be sure the procedure has been learned correctly.

Finally, if a student has made a conceptual error, the teacher must assess where the student's breakdown in learning occurred; e.g., what concepts have not been learned. The teacher then will need to reteach these concepts at a lower functional level (with semiconcrete or concrete materials). Conceptual errors will not be successfully remediated if only numbers (abstract symbols) are used.

Thus, as can be seen, the types of remediation strategies that should be used are dependent on the types of errors that have been made. The successful teacher continuously diagnoses errors made by students and assesses the performance of the entire class to ensure that appropriate remediation strategies are employed.

In the following pages, errors typically made by middle-school students in each of the strands of mathematics are presented. A hypothetical diagnosis of each error, as well as an instructional strategy that could be used for remediation, also is presented.

ERRORS, DIAGNOSES, AND STRATEGIES

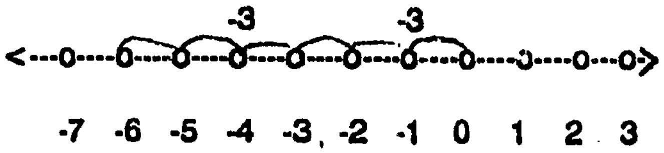
NUMERATION

Error	Diagnosis	Strategy
45,062 Student writes above numeral for 4,506,002.	Student has difficulty writing larger than six digits containing 0s.	Use manila folder with appropriate place value periods designated. Student places numeral cards on blanks.
180.149 rounded to the nearest tenth would be 180.2	Student determines answer by rounding all digits rather than only hundredths.	Use colored markers to emphasize the number to the immediate right of the digit to be rounded.
Student writes 3.4 < 3.18795	Student does not understand decimal place value.	Use 0s as place holders to emphasize comparison. 3.40000 3.18795

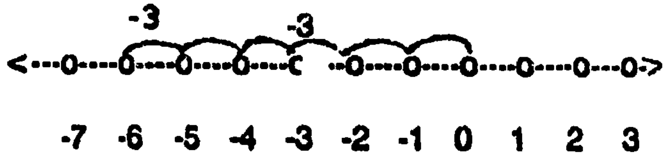
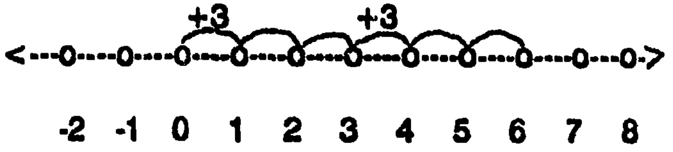
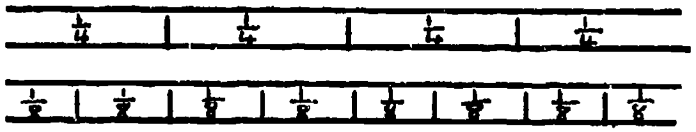
COMPUTATION

Error	Diagnosis	Strategy
21 7/1407	Student ignores 0 when using division algorithm.	Have student check for reasonableness of answer. Use mnemonic device to show algorithm. Does Divide McDonald's Multiply Sell Subtract Canned Compare Biscuits Bring down
0.89 x 0.001 .89000	Student uses 0s improperly as place holders.	Write the rule for multiplying decimals: 1. Multiply the factors as if they were whole numbers. 2. Count the number of digits that are to the right of the decimal points. Use this number to position the decimal point in the product so that the product has this number of digits to the right of the decimal point.
133.2 3/401 -3 10 - 9 11 - 9 2	Student writes whole number remainder as decimal in quotient.	Use place value model to illustrate the 2 as 2/3 not 2/10.

COMPUTATION (Cont.)

Error	Diagnosis	Strategy
4.24 0.2/8.48	Student does not transfer decimal point in dividend when dividing by decimal divisor.	Explain that when the divisor is a decimal, the divisor must be changed to a whole number. Whenever the divisor is changed, the dividend must also be changed. 0.35 /1.75 35 /175 The divisor and dividend were multiplied by 100. Now demonstrate the short cut for multiplying by moving the decimal point. Have students state the following rule: 1. Move the decimal point in the divisor to make the divisor a whole number. Move the decimal point in the dividend the same number of places. 2. Divide as with whole numbers. 3. Write the decimal point in the quotient directly above the decimal point in the dividend.
(+5) x (-4)= +20 (-5) x (+4)= -20	When the factors have different signs, the student lets the product have the same sign as the factor with the greater absolute value.	Have the pupil use a number line to show multiplication as repeated addition. A positive first factor tells how many times to use the second factor as an addend. Example: +2 x - 3 

COMPUTATION (Cont.)

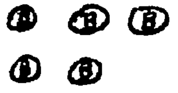
Error	Diagnosis	Strategy
		A negative first factor tells how many times to use the opposite of the second factor as an addend. Example: -2 x 3  Example: -2 x -3 
3/4 = 6/7	Difficulty finding equivalent fractions -- student does not understand the meaning of equivalent fractions.	Use fractions bars cut from equal wholes. Ask students to manipulate to prove that 3/4 = 6/8. 
-2 + 3 = 5 or -5 -4 + -3 = -7 or +7	Student adds the absolute values of the addends and the sum is positive or negative in an apparently random pattern.	Use different colored dots or markers to represent positive and negative numbers. Review adding two positive numbers and adding two negative numbers. Example: +2 +7 = +9 

COMPUTATION (Cont.)

Error | **Diagnosis** | **Strategy**

Example Two

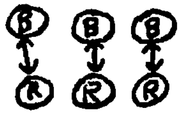
-3 + -2 = -5



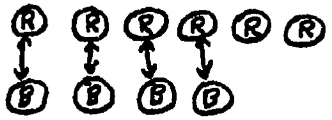
Example Three

When we add numbers with different signs, two different colored dots cancel each other. Have the students use this idea when adding positive and negative integers.

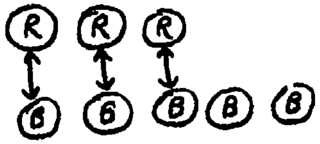
-3 + 3 = 0



+6 - 4 = +2



-5 + 3 = -2

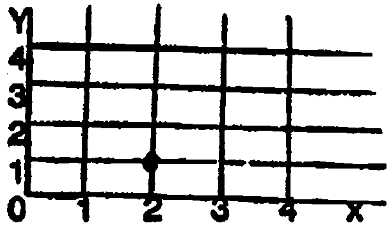


COMPUTATION (Cont.)

Order of operations $(3+1) \times 3^2 = 144$	Failure to follow order of operations rules. Student multiplies by 3 then squares the product.	Use ACRONYM Please (Parenthesis) Excuse (Exponent) My (Multi) or Dear (Div) Aunt (Add) or Sally (Subtract)

FUNCTIONS AND RELATIONS

Error



A = (1,2)

Diagnosis

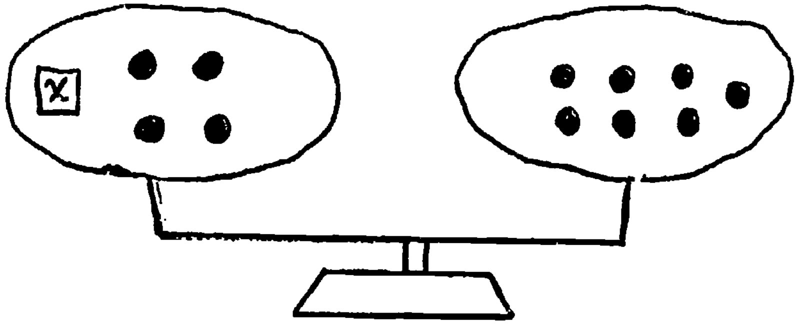
Student confuses the first number of the ordered pair with the Y axis and the second number with the X axis

Strategy

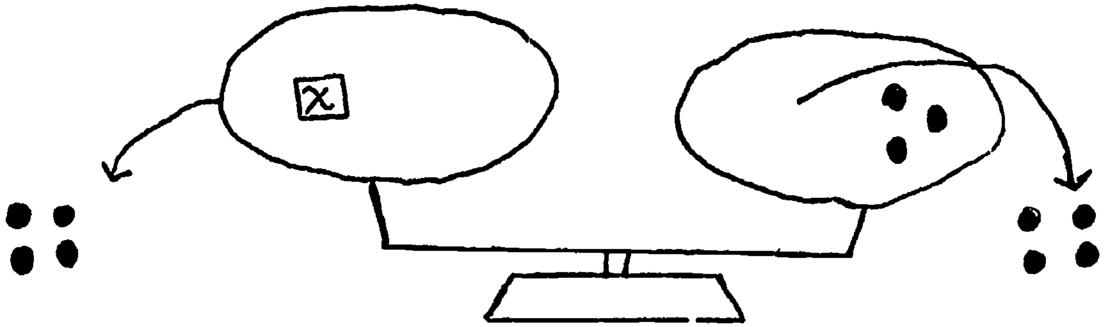
Use grid paper to demonstrate the intersecting number lines. Plot a message using ordered pairs for your friend to solve.

PRE-ALGEBRA AND ALGEBRA

Error	Diagnosis	Strategy
$X + 4 = 7$ $X = 7 + 4$ $X = 11$	Student fails to apply the inverse operation to both sides of this equation.	To aid in remembering the inverse operation rule for solving equations, use a balance scale:



To find X or to get X by itself, 4 weights must be removed from the left side. To keep the balance, however, what is done to one side must also be done to the other side. 4 weights must also be removed from the right side.



This shows that X+4-4 = 7-4, or X=3. Encourage students to check all results.

$3 + 4 = 7$

$7 = 7$

PRE-ALGEBRA AND ALGEBRA (Cont.)

Error	Diagnosis	Strategy
$b - 5 + 3 \times b$ If $b = 8$ as $8 - 5 \times 3$ or $8 - 5 + 3 \times b$	Fails to substitute a value for all variables.	Have the pupil code, either with shapes or colors, each variable and its corresponding replacement value. For example, $\triangle - 5 + 3 \times \triangle$ if $\triangle = 8$ Emphasize that <u>each</u> $\triangle$ must be replaced with the number 8. This works particularly well on an overhead projector. Then have the pupil substitute the replacement value (12) for each variable and evaluate the expression. $\triangle - 5 + 3 \times \triangle$ if $\triangle = 12$

FUNCTIONS AND RELATIONS

Error

$\frac{2}{3} = \frac{n}{12}$

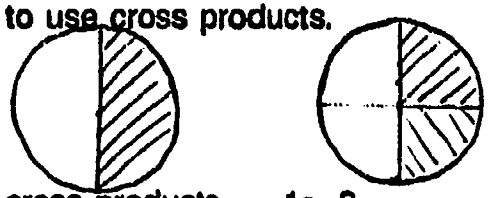
$2n = 36$
$n = 18$

Diagnosis

The student doesn't understand the idea of proportion as equivalent fractions and has multiplied numerators and then denominators as in multiplying fractions.

Strategy

First, use models to show that 1/2 and 2/4 are equivalent fractions. Then show that the cross products 1x4 and 2x2 are equal. Next show this relationship as a proportion 1/2 = n/4. Work through the equation to show how to use cross products.



cross products are equal $\frac{1}{2} \times \frac{2}{4}$

$1 \times 4 = 4$
$2 \times 2 = 4$

numerator products $\neq$ denominator products

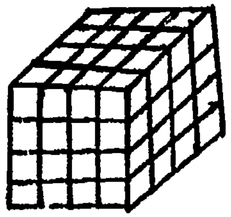
$1 \times 2 = 2$ $\quad$ $2 \times 4 = 8$

To solve proportion $\frac{1}{2} = \frac{n}{4}$

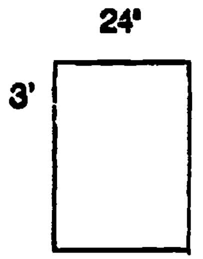
use cross products $\frac{1}{2} \times \frac{n}{4}$

$2 \times n = 1 \times 4$
$2n = 4$
$n = 2$

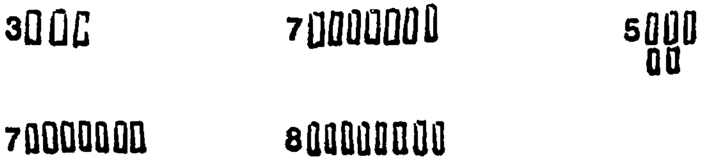
GEOMETRY

Error	Diagnosis	Strategy
Find the volume  V = 16 cubic units	Student counts the visible surface area.	Develop the concept of volume, using 3-dimensional models composed of cubic units. Define volume as the number of cubic units contained in a solid figure. Have the students compare a cubic unit with a  linear unit —— and a square unit. □ Illustrate how the bottom layer of a rectangular solid corresponds to the length and width of the figure and how the number of layers corresponds to the height of the figure. After the pupil can determine the volume of 3-dimensional models, use drawings of rectangular solids. Finally guide them in discovering the formula as follows. 1. Have the student determine the length, width, and height of the solid. 2. Draw units on the visible faces. 3. Help the student to see that the volume of any rectangular solid is given by V = lwh 4. Finally, substitute the value for each dimension in the formula.

MEASUREMENT

Error	Diagnosis	Strategy
 P = 54" or P = 54'	Student does not convert to equal measurements before finding perimeter.	Use 12 rulers to construct a rectangle on the floor 2' x 4'. Ask student to count number of "feet" in perimeter. Convert to inches if necessary.

PROBABILITY AND STATISTICS

Error	Diagnosis	Strategy
What are the chances, or probability, of drawing a red marble from a bag if a black, a red, and two yellow marbles are put in the bag?	A pupil may not be able to find a probability because he or she cannot determine what the possible outcomes are.	Explain that any possible result is an outcome. Flipping a coin has two possible outcomes -- heads or tails. Rolling a die has six possible outcomes - 1, 2, 3, 4, 5, and 6. If each outcome is just as likely to occur as any other outcome, then all the outcomes are <u>equally likely</u>. Use a coin and a die to illustrate this point. Draw a diagram showing all the outcomes possible in each example.
Compute the median. 5,695 7,869 +10,851 3,589	Student has difficulty computing median and recalling the definition of median.	Arrange 30 items, such as paper clips, into 5 groups as follows:  Explain that the mean is the number of objects in the groups that would be in each group if each group had the same number of objects. Have the student arrange the objects so that there is the same number in each group.  So, the mean is 6. Use the calculator to compute the mean.

PROBLEM SOLVING

Error	Diagnosis	Strategy
In January, 1.03 cm of rain fell. In February, rainfall amounted to 2.85 cm. The snowfall for January was 32.12 cm. How much more rain fell in January than February? Answer: 1.03 2.85 36.00 32.12 -32.12 36.00 3.88 3.88 cm more rain fell in January than in February.	Student has difficulty with irrelevant information. The student 'uses' all numbers given rather than analyzing the question.	1. Have the students omit the question from the problem. Discuss possible questions that could be asked for the problem and discuss the relevant facts pertaining to the question. 2. Have students generate their own problems. Stress relevant information for each question. 3. Make a list of the needed facts and a list of the extraneous facts.
If a telephone call from Illinois to California costs $1.25 for each minute, how many minutes can you talk for $5.00? Answer: $5.00 x1.25 6.25 You can talk for 6 hours and 25 minutes.	The student is not able to analyze the data since he/she chose the wrong operation when translating the problem to mathematics.	Have the student illustrate the problem (or simulate). Emphasize that the diagram or drawing need not be elaborate. This visual aid will help pupils to understand the question. Give example of how students would simulate.

PROBLEM SOLVING (Cont.)

Error	Diagnosis	Strategy
A bottle of juice holds 67.6 ounces. If this makes 13 servings, how many ounces are in one serving? Answer: 67.6 -13. 54.6 54.6 ounces per serving	Some pupils may be unable to determine what operation or equation to employ because they are concerned about the size of the numbers.	1. Encourage students to substitute small, simpler numbers when reading the problem until they decide on which operation to use. 2. Use the calculator, once operation has been chosen, to alleviate fear of large numbers.

ACTIVITIES

NOTE: Many of the activities included in this handbook have been drawn directly or modified from Tom Layne's teacher-developed activities for problem solving. These are identified by an asterisk at the beginning of the activity and have been used with Layne's permission. For further information, Layne may be contacted at Shenandoah College in Winchester, VA. Two types of activities are provided:

1. Those designed to be used directly with the students.
2. Those designed specifically for teachers to use in organizing instructional activities for the students.

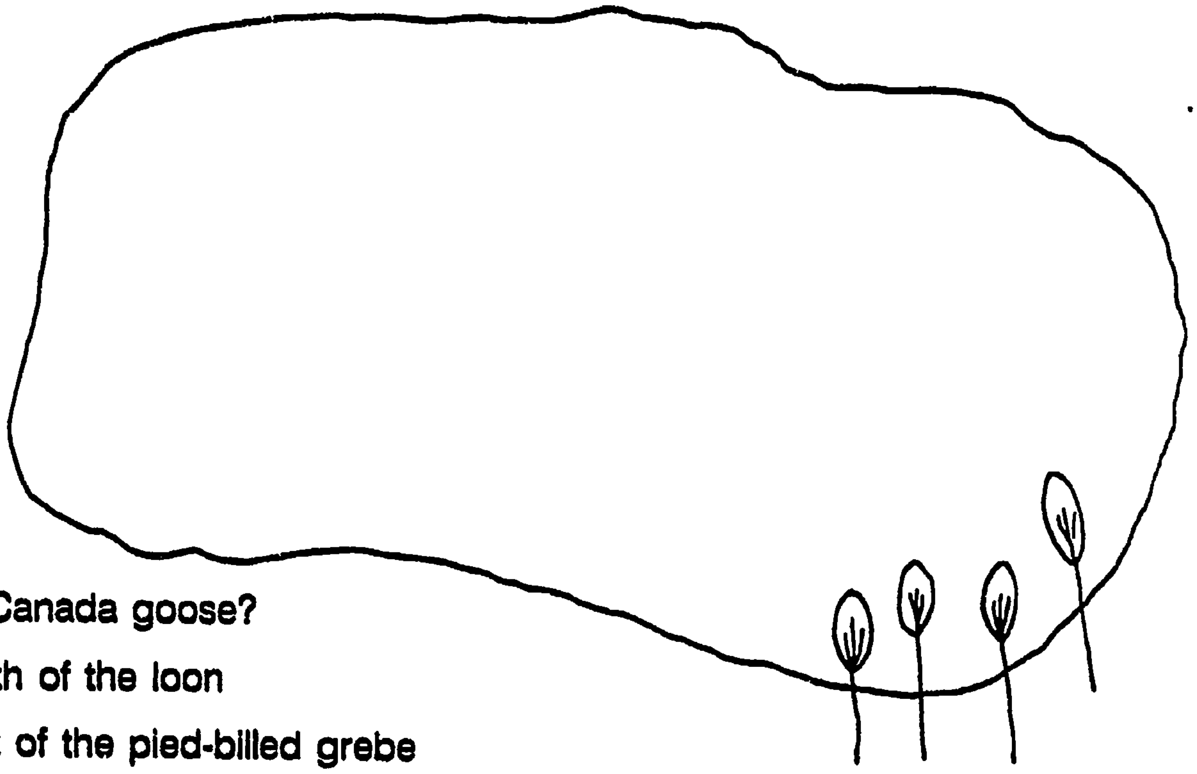
Standard 1: Mathematics as Problem Solving
Skill: Visualizing in Problem Solving
Level: 6-8

*<u>Solve the following by visualizing</u>:

A coot, a loon, a Canada goose, a black duck, a pintail, and a pied-billed grebe were swimming on a pond.

Following the directions below, place the ducks on the pond.

The black duck is in front of the Canada goose. The pied-billed grebe is south of the black duck while the pintail is northeast of the pied-billed grebe. The coot and the loon are south of the pintail near the cattails along the bank.



Where is the Canada goose?

- A. south of the loon
- B. east of the pied-billed grebe
- C. west of the black duck
- D. north of the pintail

Standard 1: Mathematics as Problem Solving
Skill: **Identify the Question**
Level: 6-8

* Use a TV Guide or a newspaper listing for television schedules to tell what the question could be for each answer given. These answers can be given by the teacher or by a classmate.

Local Cable Companies

Atlanta	IND	WTBS	17
Baltimore	ABC	WJG	13
Hagerstown	PBS	WWPB	25
Pittsburgh	CBS	KDKA	2
Weston	CBS	WDTV	5

Answers

1. ABC
2. Channel 25
3. PBS
4. KDKA
5. Weston

Standard 1: Mathematics as Problem Solving
Skill: Identifying Facts
Level: 6-8

* Ask students to group into pairs. Have one partner generate a problem, similar to the one below, asking for facts to be identified.

The 7th and 8th grades have a hockey match. Each team can have 10 players. There are 15 7th-graders and 17 8th-graders. The game started at 1:30 p.m. and lasted until 2:45 p.m. Five 7th-graders scored 3 goals each. Six 8th graders scored 2 goals each.

1. Who played in the hockey game? ____________
2. How long did the game last? ____________
3. How many players had to sit on the bench for each grade?
   7th grade _______ 8th grade _______
4. What was the final score? ______
   Who won? ____________

Standard 1: Mathematics as Problem Solving
Skill: Problem Solving
Level: 6-8

To set up a resource center, use three problem-solving file boxes labeled as shown below.

Problems

Hints

Solutions and Extensions

The "Problems" box contains a collection of problems. One problem should be on each numbered card. The cards could be grouped by levels of difficulty or by chapter to correspond with strategies developed in the student book. The "Hints" box contains cards numbered to correspond to the problem cards. One side of each card lists questions that, if answered correctly, would help students find a solution. On the other side of the card are answers to the questions.

In the "Solutions and Extensions" box, one side of each card shows one or two solutions to the problem. The other side has a problem that is an extension of the original problem. These cards can also be numbered to correspond to the numbers on the "Problems" and "Hints" cards.

Standard 1: Mathematics as Problem Solving
Skill: Using the Calculator With Fractions
Level: 6-8

1. Find out how many hours of your life you have slept. (Multiply the average number of hours per night times 365¼ times your age.)
2. How much air do you breathe each day? (Human beings take in about ½ liter at a time. Count the number of breaths you take each minute with a watch and then multiply the number of breaths/minute times ½ times 60 times 24.)
3. Your heart pumps about 1/3 of a quart of blood with each contraction. How much blood does your heart pump each day? (Find your pulse rate per minute. Multiply your pulse rates times 60 times 24 times 1/3 to find the number of quarts.)

Students may wish to conduct additional research.

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Standard 1: Mathematics as Problem Solving
Standard 5: Number and Number Relationships
Standard 7: Computation and Estimation
Skill: Using Fractions
Level: 6-8

Pam is going to make chocolate pudding using the following recipe, but she only wants to make half as much as the recipe makes. How much of each ingredient does she need?

3 T. cream corn starch
1/3 c. sugar
1/2 t. salt
2 c. milk
1 t. vanilla
1 1/2 squares unsweetened chocolate

Standard 1: Mathematics as Problem Solving
Standard 5: Number and Number Relationships
Standard 7: Computation and Estimation
Standard 9: Algebra
Skill: Finding Interest
Level: 6-8

* Use the formula I=prt to solve each problem. (I=interest, p=principle, r=rate, t=time)

1. Doug needs $16.00. He borrows the money from a bank for 1½ years. If the yearly interest rate is 14%, how much interest will he owe at the end of 1 1/2 years?
2. Jerry paid $36.00 interest when he borrowed $600.00 at 1% per month. How many months did Jerry borrow the money?

Employ the calculator to solve.

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Standard 1: Mathematics as Problem Solving
Standard 7: Computation and Estimation
Skill: Working Backwards in Problem Solving
Level: 6-8

Problem

Benny, Amy, and Mark are taking a ski trip to Colorado with Benny's parents. Their plane leaves Dulles Airport at 10 a.m. It takes 2½ to 3 hours to drive to Dulles from Petersburg. They want to allow time to stop for breakfast, park the car at the airport, and check in with the airlines. What time should they leave Petersburg to allow sufficient time to make their flight? We don't want them to have to wait at the airport too long before the flight.

Strategy

Use the working backwards strategy to solve this problem. Use estimation to find the solution.

Standard 1: Mathematics as Problem Solving
Standard 7: Computation and Estimation
Skill: Computing
Level: 6-8

A teacher will need to direct students in playing "GCF" (Greatest Common Factor) Bingo. On a 3x3 grid drawn on their papers, students write these numbers, one to a box, in any order: 9, 1, 5, 2, 12, 15, 6, 10, 8. Then the teacher calls out pairs of numbers from below in random order. Students find the GCF and circle that number on their papers. The first to circle three in a row or diagonal is the winner.

18,27
35,75
22,26
36,60
45,75
56,40
72,78
20,90
15,17

Standard 1: Mathematics as Problem Solving
Standard 7: Computation and Estimation
Skill: Finding Estimates
Level: 6-8

* Several stores in the malls are putting up signs indicating an additional percentage off the sale price instead of marking the new price on the tickets.

Suppose you look at a $69.99 coat that has a sale price of $49.99. A sign in the store allows an additional 20% discount. Use estimation and your calculator to find out how much you will save and how much you will pay for the coat.

1. How much will you save on the coat? (Estimate with your partner.)
2. How much will you pay for the coat? (Estimate using rounding or front-end estimation.)

Standard 1: Mathematics as Proolem Solving
Standard 7: Computation and Estimation
Skill: Compute by Estimating
Level: 6-8

* Cecile sees a $49.99 dress that is on sale for $34.99 plus an additional 20% off. She decides the dress will cost her $28.99.

Estimate to see if Cecile is correct.

Standard 1: Mathematics as Problem Solving
Standard 7: Computation and Estimation
Skill: Computing Using Consumer Skills
Level: 6-8

Use a "Pizza Hut" menu or newspaper ad to write 5 questions of your own. Exchange with your partner.

Examples:

1. How much would two large cheese pizzas cost?
2. What would be your change if you purchased a large supreme pizza and gave the cashier a $20 bill? Use a calculator to solve.

Jessica and Lori went to Pizza Hut. They spent $10.66 together. How much of this did they leave for a tip? Here is their bill:

1 Big Topper Pan Pizza	$2.69
1 Pepperoni Pan Pizza	$1.79
2 Salad Bars	$1.49 each
1 Medium Mountain Dew	.65
1 Large Pepsi	.70
5% Sales Tax	

Standard 1:	Mathematics as Problem Solving
Standard 7:	Computation and Estimation
Skill:	**Computation**
Level:	6-8

Directions: Use the calculator to solve.

16,396 people watched the state high school football championship game; 853 students from the two schools each paid \$3.50 to attend; 762 people under age 13 each paid \$1.75; and 12,480 people over 13 each paid \$5.25 to attend. The rest were admitted for free. How much money was collected at the game?

Write down the order in which you entered this problem into the calculator.

Standard 1: Mathematics as Problem Solving
Standard 7: Computation and Estimation
Skill: Computation/Estimation
Level: 6-8

* The Smith family (two parents and two children) decided that they want to visit Washington, DC, for four days during their next summer vacation. They live in Los Angeles and want to travel as cheaply as possible. Should they take the train, fly, or drive their own car? If they drive their car, they will eat in restaurants and sleep in motels. If they travel by train, they will eat in the dining car and rent a sleeper. If they fly or take a train, they will rent a car when they get to Washington. (Note: Students must be provided with sources stating airline costs, train fares, gasoline cost per gallon, hotel average, etc. As a variation, the students could use newspaper, magazine, and telephone calls to find the information needed.)

Standard 1: Mathematics as Problem Solving
Standard 7: Computation and Estimation
Skill: Estimating
Level: 6-8

Use your estimation skills to first estimate the answer. Then use your calculator to compute. Use a chart to show your estimated and calculated answers.

1. Find out the ratio of program time to commercial time during a long television movie.
2. An estimation (only) of the number of blades of grass on a lawn figured on the basis of a small sample.
3. Find out the amount of paint needed to cover the walls of your classroom.
4. Find out the average number of hours spent completing homework with all students in your grade level.

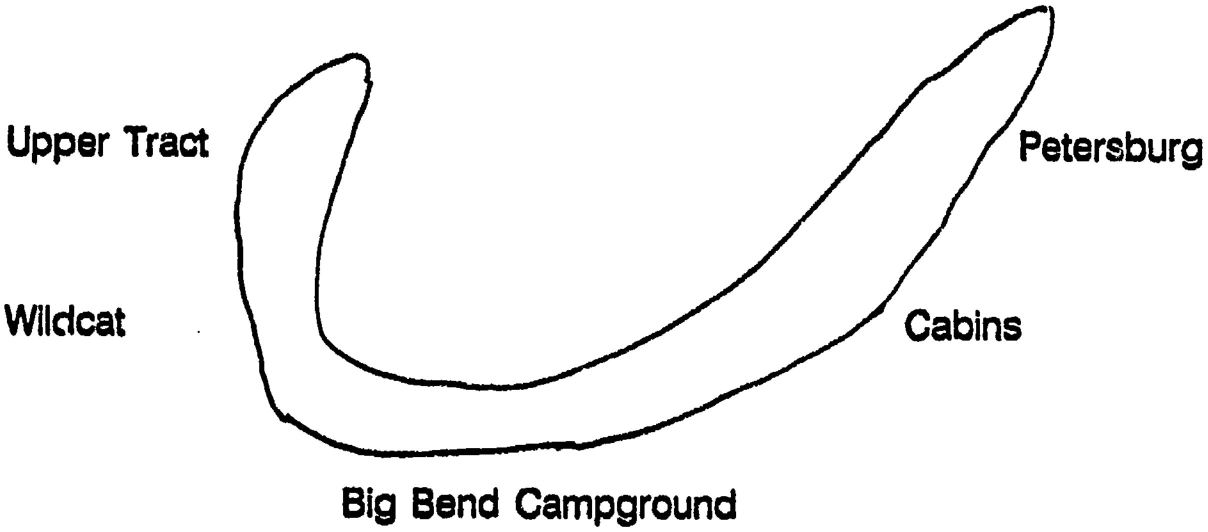
Standard 1: Mathematics as Problem Solving
Standard 7: Computation and Estimation
Skill: Using Calculator to Compute
Level: 6-8

*__Directions__: Work with a partner to calculate the following. Use your calculator.

CAR A	CAR B
27 m.p.g.	20 m.p.g.
Uses super unleaded	Uses regular unleaded
@ $.99 p.g.	@ $.82 p.g.

__Question__: On a trip of 200 miles, determine the gas consumption (number of gallons) for each car.

Standard 1: Mathematics as Problem Solving
Standard 2: Mathematics as Communication
Standard 3: Mathematics as Reasoning
Standard 4: Mathematics Corrections
Standard 7: Computation and Estimation
Skill: Problem Solving, Estimation, Reasoning
Level: 6-8



* If I paddled south from Upper Tract to my friends' camp at Cabins, how long will it take if traveling to Big Bend takes 4 hours? Big Bend is the halfway mark. Can I make the trip in daylight? If so, how?

Standard 1: Mathematics as Problem Solving
Standard 5: Number and Number Relationships
Standard 7: Computation and Estimation
Skills: Changing Fractions to Decimals
Level: 6-8

* Solve the problem by changing the fractions below to decimals using a calculator. Decide if they would be terminating or repeating. Note: Some calculators may round off, and an answer such as 0.4666666 may appear as 0.4666667.

In the United States, 2/3 of the population is under 75 years old. Find the decimal to represent the population over 75 and under 75.

Standard 1: Mathematics as Problem Solving
Standard 7: Computation and Estimation
Standard 10: Statistics
Skill: Identifying Facts, Reading Tables, Estimation, Finding Percentages, Graphing
Level: 6-8

Provide each small group with a basketball record sheet from a recent game. Answer these questions, using the record sheet.

1. What teams played on what date?
2. Who were leading scorers for each team?
3. State the halftime score.
4. What percentage of field goals was successful for each team?
5. Were any three point goals scored? If so, by whom and what team?
6. What percentage did foul shots account for in each team's total score?
7. Using the individual scores as a guide, construct a bar graph showing the highest to lowest scorer.

Standard 1:	Mathematics as Problem Solving
Standard 7:	Computation and Estimation
Standard 10:	Statistics
Skills:	**Finding Averages, Estimating**
Level:	6-8

Solve using a calculator.

1. Record the odometer reading on your car when the tank is filled with gas, and record the reading again the next time you fill up the tank. Record the amount of gas pumped. Estimate the miles per gallon for your car by subtracting the odometer readings and dividing the difference by the amount of gas.
2. Find the mean amount of money spent on grocery items per week. Save all grocery receipts for two months. Add the totals, and divide the sum by the number of weeks in the two-month period.

Standard 1: Mathematics as Problem Solving
Standard 2: Mathematics as Communication
Standard 7: Computation and Estimation
Standard 10: Statistics
Skill: Writing Mathematical Sentences
Level: 6-8

Write a mathematical sentence for the following situations. Use a calculator to solve.

1. You have the opportunity to spend $1000.00 from your favorite mail-order catalog. Determine how this money will be spent, with accuracy to the penny. Include sales tax, shipping charges, etc.
2. How many gallons of water are used at your home per month, week, day, hour? Find an average over six months. (Use your water bill.)
3. How many names would you find in your local telephone directory? Find an average on a one-page sample to determine the total.
4. Find the number of hamburgers McDonald's has sold since it opened, and find the average number consumed for every person in the country.
5. Fenway Park, in Boston, sells about 22,000 hot dogs at each home game. The hot dogs are each 15 cm long. What would the length be if all the hot dogs sold in a season were placed end to end?
6. How many minutes or hours does your class watch television each day? week? year?

Standard 1: Mathematics as Problem Solving
Standard 7: Computation and Estimation
Standard 10: Statistics
Standard 13: Measurement
Skill: Using the Calculator in Problem Solving
Level: 6-8

<u>Use the calculator to</u>:

1. Compare your height with the <u>average</u> height of all your classmates. Compare different grade levels. Make a chart to show your results.
2. How many cubic centimeters of air are available to each student in the room? How does this compare with other classrooms, the gym, the cafeteria, etc.?
3. Find out the number of beef cattle it takes to supply a fast-food restaurant for a day, a month, a year.
4. Find out the total distance the class could walk in one hour figured on the basis of a five-minute sample.

Standard 1:	Mathematics as Problem Solving
Standard 8:	Patterns and Functions
Skill:	**Using Formulas to Solve Problems**
Level:	6-8

* Use your calculator to solve these problems. Work cooperatively in pairs. (Remember: distance = rate x time (d = rt).)

1. A car was driven at a speed of 65 miles per hour. How far did the car travel in 5 1/4 hours?
2. A racer finished a 210-mile race in 3 hours. How fast was he going?

Standard 1: Mathematics as Problem Solving
Standard 2: Mathematics as Communication
Standard 3: Mathematics as Reasoning
Standard 8: Patterns and Functions
Skill: Pattern Recognition
Level: 6-8

A. State which doesn't belong, and give reason.

1. Volleyball, Basketball, Soccer, Softball
2. Bounce, Chest, Forward, Baseball, Punt
3. Chevrolet, Buick, Cadillac, Subaru, Ford

B. State how these are alike and explain your answer.

1. Honky Tonk, Macho Man, Steamboat, The Animal, The Giant
2. Redskins, Giants, Bears, 49ers, Raiders
3. Speaking, Writing, Reading, Nonverbal, Listening

Standard 1: Mathematics as Problem Solving
Standard 9: Algebra
Skill: Using Variables and Equations
Level: 6-8

*Problem:

Sparks Department Store is having a 30 percent off sale. If x is the original price of an item, write an equation that says the sale price of an item is $50.

The students should use calculators to compute. Variation: Ask for a table showing this variable, as well as other original prices, in tabular form.

Standard 1: Mathematics as Problem Solving
Standard 10: Statistics
Skill: Finding Averages
Level: 6-8

* Use the following information to place the students in order from the highest average to the lowest average.

1. Paul's average is lower than Charlotte's, but higher than Lisa's.
2. Chad's average is lower than Shana's.
3. Jeff's average is lower than Michelle's, but higher than Charlotte's.
4. Velvet's average is lower than Lisa's, but higher than Shana's.
5. Lori's average is lower than Kim's, but higher than Michelle's.

Standard 1: Mathematics as Problem Solving
Standard 10: Statistics
Skill: Finding Averages
Level: 6-8

The Johnsons are planning a vacation. They estimate they will travel 2,250 miles. Their car averages 18 miles per gallon of gas.

1. How many miles can they travel on a tank with 10 gallons of fuel?
2. If gas is $.98 per gallon, how much should they set aside for gasoline?
3. If they only want to spend about $100 on gasoline, how many miles can they travel?

Standard 1: Mathematics as Problem Solving
Standard 10: Statistics
Standard 11: Probability
Skill: Using Data to Generate Graphs, Charts
Level: 6-8

*Use your calculator to answer the following:

1. During the reading contest, Mary read four books, Sue read two, and Amy read five. Joan read twice as many as Sue and Jerry read half as many as Mary.

   Draw a bar graph showing how many books each student read.

2. Using a batting mean chart or average chart from a newspaper, answer the following.

   A. Determine the mean batter.

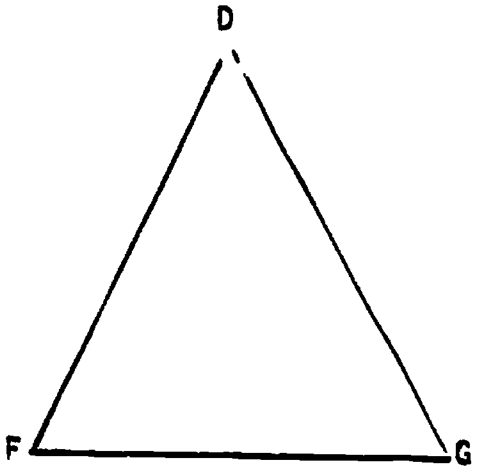
   B. Name the one farthest from the mean.

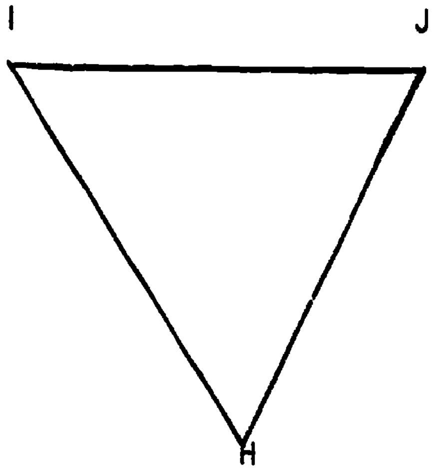
   C. Name the one closest to the mean.

Standard 1: Mathematics as Problem Solving
Standard 9: Algebra
Standard 11: Probability
Skill: Using Positive and Negative Numbers
Level: 6-8

* Use the stock market report to find examples of stocks that have increased in price and examples of stocks that have decreased in price. Each stock listed in the stock market report found in the daily newspaper shows as net change. This net change can be a gain, such as + (1/8) to show that this price went up 1/8 dollar, or a loss, such as - (1/8) to show that the price went down 1/8 dollar.

Standard 1:	Mathematics as Problem Solving
Standard 12:	Geometry
Skill:	**Congruent Angles**
Level:	**6-8**





* Triangle DFG is congruent to triangle HIJ, with the length of DG equal to 2 inches; the sum of angle I is equal to 62 degrees.

1. Name the three pairs of congruent angles.
2. Name the three pairs of congruent sides.
3. What is the length of HJ?
4. What is the measure of angles H and F?

Standard 1: Mathematics as Problem-Solving
Standard 13: Measurement
Skill: Using Metric Measures
Level: 6-8

Use a meter stick or metric rules to measure the following items in the classroom. Use the appropriate measure (m, cm, or mm). Be sure to make a reasonable estimate.

1. Length of the room.
2. Width of the room.
3. Width and length of bulletin board.
4. Width and length of chalkboard.
5. Length and width of desk top.
6. Length of this sheet of paper.
7. Length of a pencil.
8. Length and width of your thumb.
9. Length of a paper clip.
10. Width of a window.

Standard 1: Mathematics as Problem Solving
Standard 13: Measurement
Skill: Finding Volume, Area
Level: 6-8

At Home Math

Use the calculator to solve.

1. Find the inside volume of the refrigerator (or freezer or microwave oven) in cubic inches. Measure the inside dimensions of the refrigerator to the nearest inch. Since there are 1,728 cubic inches in one cubic foot, divide the volume (in cubic inches) by 1,728 cubic inches in one cubic foot. The result is the capacity of the refrigerator in cubic feet, the usual way in which such appliances are compared and advertised.
2. Determine how many boxes of tile are needed to cover the floor of a room. Measure the length and width of the room and find the square feet of the floor area that is to be covered. Assume a box holds 12 tiles, each 1 square foot.

Standard 1: Mathematics as Problem Solving
Standard 13: Measurement
Skill: Finding Area
Level: 6-8

* What is the area of a floor that measures 14 1/2 feet by 20 1/4 feet?

Use your calculator to solve. The following solution applies if you are using only a four-function calculator with one memory:

1 ÷ 2 + 14 = M+ 1 ÷ 4 + 20 = x MR = .

Explain how the calculator could be used to solve this problem from left to right if the calculator had parentheses or multiple memories.

Standard 1: Mathematics as Problem Solving
Standard 7: Computation and Estimation
Standard 13: Measurement
Skill: Finding Area, Perimeter, Fractions
Level: 6-8

1. Lisa wants to paint her living room. The living room is 16 feet long by 12 feet wide and 8 feet high. Find the total area to be painted.
2. Using a mail-order catalog, find a tablecloth to fit a table measuring 36 x 60. Determine the surface area and perimeter of the table.
3. Using your favorite recipe, rewrite the recipe so that you could double the yield. Plan a party to include this recipe.

A calculator can help you solve these problems quickly and accurately.

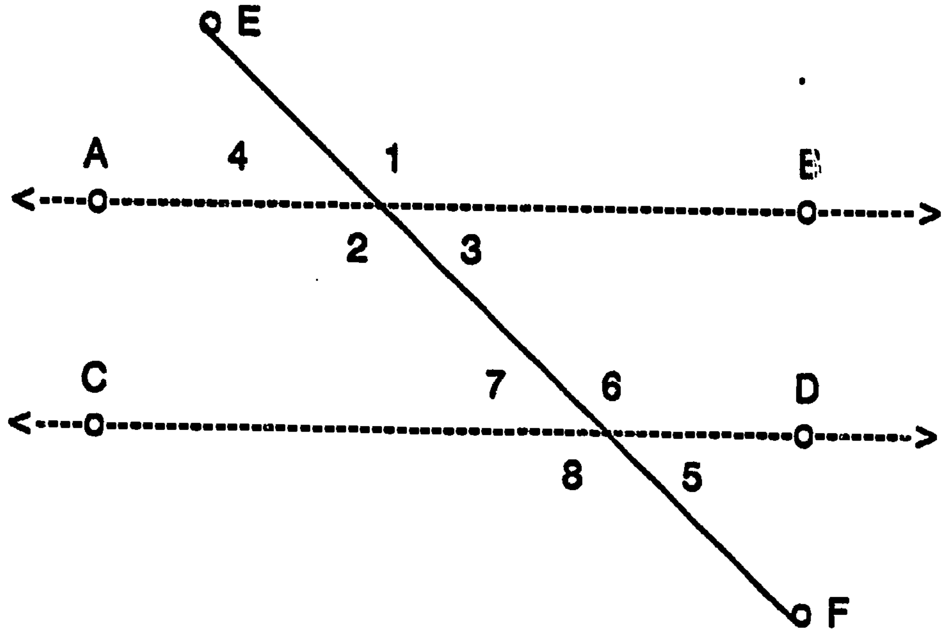
Standard 2:	Mathematics as Reasoning
Standard 3:	Reasoning
Standard 10:	Statistics
Skill:	**Collecting Data & Recording Data**
Level:	6-8

Use a calculator to solve the following problems:

1. Make a bar graph showing the costs of one pound of various food items such as hamburgers, tomatoes, butter, etc.
2. Keep a record of the daily high temperatures. Find the average for the week. Make a line graph using the data collected.
3. Determine the amount of time spent at each of the following in a typical day: sleep, school, play, and other. Express the results as percents (to the nearest whole percent). Make a circle graph of this data.

Standard 4: Mathematics as Reasoning
Standard 12: Geometry
Skill: **Vertical Angles, Corresponding Angles, Supplementary Angles**
Level: 6-8

AB is parallel to CD and EF is a transversal; the sum of angle 3 is 40 degrees.



Looking at the above geometric drawing, work in your assigned small group to answer the following.

1. Name 4 pairs of vertical angles.
2. Name 4 pairs of corresponding angles.
3. Name 4 pairs of supplementary angles.

Standard 5: Number and Number Relationships
Skill: **Finding Percents**
Level: 6-8

Students find examples of the use of percent in the advertisements or articles in the newspaper and then use that data to generate several problems involving percent. Set aside a bulletin board area so that the students can read and solve each other's problems.

Standard 5: Number and Number Relationships
Skill: **Inverse Operations**
Level: 6-8

In each space on a tic-tac-toe grid for the transparency, write a number sentence similar to those listed below.

Directions: Write the inverse statement.

1\. 40÷2=20 2. 15-3=12
3\. 9+6=15 4. 10-6=4
5\. 2x8=16 6. 8+2=10

Place the transparency on the overhead projector, but before turning it on, cover each problem with a slip of paper. Divide the students into two teams. Each team decides where they would like to place an X or an O. A team must give two related sentences correctly for the problem in that space to win it. Teams take turns until one of the teams has tic-tac-toe.

Standard 5: Number and Number Relationships
Standard 8: Patterns and Functions
Skill: Finding Percents
Level: 6-8

A 30 Percent-Off Sale
at Sparks Department Store

Original Price ($)	Discount ($)	Sale Price ($)
20	0.30 (20)=6	20-6=14
30		
40		

* Students have a good deal of trouble mastering the topic of percents in the middle school. One reason may be that students are not able to do a sufficient number of problems to gain experiences necessary to master the concept. Calculators make it possible for students to work more problems in the same amount of time. We could begin by having the students compute several cases of a problem in tabular form. By noting the form in which entries are recorded in the above table, the student can focus on the process used to arrive at an answer, rather than on the answer itself. Students will see that the entry in the second column is 0.30 times the entry in the first column, and the last entry is the first minus the second. The use of a number pattern like 20, 30, 40 in the first column helps students discover that the pattern for the last column is 0.70 times the entry in the first column or that the sales price is 70 percent of the original price.

Standard 7: Computation and Estimation
Skill: Adding Decimals
Level: 6-8

PALINDROMES are words, numerals, etc., that say the identical thing when read backwards as they do when read normally. Words such as: dad, mom, radar, bib, etc., are considered palindromes, as are numbers such as: 2332, 46864, etc. Below are some exercises whose answers are palindromes. See if you can form others.

23.85 + 1.67 = __________
5.9 + 6.31 = __________
98.55 +24.771 = __________
189.925 +400.06 = __________

Standard 3:	Mathematics as Reasoning
Standard 7:	Computation and Estimation
Skill:	**Mental Computation Using the Calculator**
Level:	6-8

Mental computation includes rounding or changing numbers so that they are mentally manageable. Give students examples to be mentally rounded and then computed by the calculator.

1. In a problem such as 2267.9, changing 2267 to 2270 allows for estimation before the calculation on the calculator is completed.
2. $35.00 - 2 x $1.75 should be solved on a calculator by first multiplying $1.75 x 2, then subtracting the answer from $35.00.
3. Assign problems that can be solved using the calculator, enabling students to determine if a displayed number is reasonable. They should know if the most significant digits of the answer are in the correct place. (Example: Should my answer to the following problem be 2300 or 23000?)

Jon moved 75,285 bars on his job in a week. He moved 52,285 in the first 3 days. How many did he move the remaining four days?

Standard 5: Numbers and Number Relationsnips
Standard 7: Computation and Estimating
Skill: Rounding/Estimation/Proportion
Level: 6-8

Use individual road maps and allow students to choose a destination, plan a route, estimate the number of miles, and (given an approximate miles per gallon) figure the cost of gasoline for the trip.

Activity: Dice Operation (All Operations, Exponents, Order of Operations)

Use 3 number cubes on dice, allowing each student to roll 3 times and record each set of numbers. Students may then arrange each set of numbers in any order, using any operation to acquire the largest (or smallest) possible number. The three numbers can then be arranged to find the total score.

Activity Equation Baseball (Simple ----> Complex Equations)

Make at least 48 equations ranging from simple to complex on 3x5 index cards. Divide and label them according to difficulty as SINGLES, DOUBLES, TRIPLES, and HOME RUNS. Also, include 12 cards, each labeled with one of the following: strike out, flied out, grounded out, double play, etc. Divide groups into two teams; draw a facsimile of a baseball diamond on the board labeling 1st, 2nd, 3rd, and HOME. Each "batter" must draw a card and answer the equation correctly in order to get on base. "Runs" are scored as teammates continue to get on base. Each inning ends when three OUT cards are drawn.

Standard 5: Number and Number Relationships
Standard 7: Computation and Estimation
Standard 10: Statistics
Standard 11: Probability
Skill: Using Consumer Skills
Level: 6-8

Check No.	Date	Check Issued To	Amount of Check	Amount of Deposit	Balance
					$250.00
546	4/12	Monarch Mills	$25.00	$ 225.00	
547	4/13	Ben Franklin's	10.98		215.02
	4/15	Deposit		150.00	365.02
548	4/20	Grant Bank	174.90		290.12
549	4/21	Fort Hill Exxon	8.25		281.87

* Is your balance reasonable? Use your calculator to decide.

Standard 6: Number Systems and Number Theory
Standard 7: Computation and Estimation
Standard 10: Statistics
Standard 11: Probability
Skill: Using Consumer Skills
Level: 6-8

Provide small groups with various mail-order catalogs. Assign them an amount of spending money and see what items they can purchase. Provide an order form that designates a blank for shipping charges and sales tax.

This activity is also easily adapted by assuming that certain items in the catalog are to be discounted in general or if more than one is purchased.

A good enrichment variation on this activity would be to have students who use discount catalogs (Service Merchandise, etc.) to actually compare the retail price stated in the catalog with the price charged in a local department store. Does the discount catalog list a reasonable retail price or an inflated one?

Standard 7: Computation and Estimation
Standard 10: Statistics
Skill: Finding Averages and Rounding
Level: 6-8

1. Have students average the number of people who live in the houses of 11 class members. Round to the nearest tenth.
2. Students could average the number of cousins, etc., also.
3. Class members could average their height in both standard and metric measurement.
4. A record of daily high/low temperature using Celsius and Fahrenheit scales (C&F) could be recorded for one week after which an average C&F temperature could be obtained for the week.

Standard 7: Computation and Estimation
Standard 10: Statistics
Standard 11: Probability
Skill: Estimating
Level: 6-8

Have students record the mileage shown on the odometer of their parents' car. They should record the mileage at weekly intervals for 3 to 4 weeks, calculating the mileage driven during each week. After the allotted time, have students estimate the yearly mileage for their family car. This activity should show a specific example where exact figures are not any more helpful than estimates.

Standard 8: Patterns and Functions
Skill: Developing Skills for Interpreting Graphs
Level: 6-8

After students have developed skills for interpreting the titles, labels, and scales for a graph, they could be given simple graphs and asked to write stories that might accompany the graph in a magazine or newspaper. Have students share their stories with the class. Ask them to check their interpretations with the authors, if available.

Standard 10: Statistics
Standard 11: Probability
Skill: Finding and Using Data From a Weathermap
Level: 6-8

Provide students with weather maps and the following topics written on index cards. Students study the weather maps using the topics as guides; they then generate word problems about their observations.

1. Average temperature in state capitals.
2. Rainfall in inches.
3. Snowfall in inches.
4. Barometric readings.
5. Cities with temperatures above 90°.

Standard 12: Geometry
Skill: Distinguishing Between Polygons
Level: 6-8

Give students a collection of cardboard strips and brass fasteners and have them make different types of polygons using the various strips. This activity may be extended to include the skills of congruency and symmetry, using their shapes.

Standard 12: Geometry
Skill: **Geometric Concept**
Level: 6-8

Use index cards that contain geometric concepts, such as the ones shown below. Students can then play a game in which they match a shape card with a card that contains the corresponding symbol. To vary this game, have the students place the cards in rows and play a game of geometry concentration.

Rhombus	Perpendicular Lines	Transversal
Equilateral Triangle	Scalene Triangle	Trapezoid

Standard 12:
Geometry
Standard 13: Measurement
Skill: **Finding Surface Area**
Level: 6-8

Prepare the following task cards, allowing students to use six cubes to determine answers:

1. How many different ways can you arrange the cubes so that at least one face of each cube joins one face of another cube?
2. Find the surface area of each arrangement.
3. Make a chart to show your results.
4. What is the smallest surface area?
5. What is the greatest surface area?

Standard 13: Measurement
Skill: Identifying Perimeter and Volume
Level: 6-8

Cut one-inch squares from graph paper. On about 15 index cards, write measurements such as those shown below:

A=3 P=8 A=5 P=12 A=7 P=16
A=8 P=14 A=8 P=16 A=6 P=12
A=7 P=12 A=10 P=17 A=10 P=20

The cards are shuffled and placed facedown in a pile, and the inch squares are divided among the players. One player in the group draws a card and reads the measurements aloud. Each player then takes the number of squares to form a region with the area and perimeter called for on the card.

Ex: A=6 P=14

The first player to form the designated region wins the squares. The winner is the person who has the most squares at the end of playing time.

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APPENDIX A

GOOD, GROUWS, AND EBMIER MODEL LESSON DESCRIPTION

MODEL LESSON DESCRIPTION

An example of the model lesson proposed by Good and his colleagues (1983) is as follows:

I. The Introductory Phase (12 minutes)
  A. A brief review associated with homework (1-2 minutes)
  B. Checking of homework (3-5 minutes)
  C. Mental computation (2-5 minutes)
II. Developmental Phase (28 minutes)
  A. Verbal problem solving
  B. Comprehension
  C. Teacher questions to assess comprehension
  D. Guided practice
III. Seatwork (15 minutes)
  A. Student completing work
  B. Student accountability for work
IV. Assignment of homework (p. 32)

Each phase of the model lesson is comprised of specific goals and employs the use of routines to accomplish these goals. Establishment of classroom routines decreases the amount of time that must be spent in giving directions for the lesson, thus increasing the amount of time available for instruction (McKinney, 1986).

Introductory Phase. Because several activities must be accomplished in the introductory phase (review, homework check, and mental computation), teachers must move rapidly through this phase. There is a tendency for many teachers to spend too much time going over homework; this should not be necessary if sufficient time has been spent in the developmental phase of the lesson during the previous day. One of the teacher goals during this phase of the lesson is to informally assess student learning so that remedial instruction can be provided if necessary. A second goal is to make a fluid transition from the work of the previous day to the work of the present day. One way to make this transition is to use verbal problem solving.

Developmental Phase. The developmental phase of the lesson is considered to be critical. The developmental part of a lesson has been defined by Good (1986) as the part of the lesson devoted to increasing the comprehension of skills and concepts, as well as allowing for meaningful practice in a controlled situation and extending these skills through application. It has been suggested that too little time is spent on the developmental activities. In one study, Good et al., (1983), found that only 14 percent of class time was used for the development of concepts and ideas. Chambers (1987) stated that good teachers devoted at least 50 percent of the class time to instruction in new material. Thus, teachers should be provided instruction in the elements of a good developmental lesson and should be provided opportunities to create these segments. In the development section of a lesson, when a new skill is being taught, teachers should focus on why an algorithm works, how certain skills are related, the association of labels with a given concept (to assist in the retrieval of information), and the extension or application of ideas to facilitate transfer of ideas (Good, 1986).

Some criteria that have been reported as being necessary for the developmental aspect of a lesson are attending to:

* Prerequisite skills.
* Interrelatedness of ideas.
* Representation of ideas as related to real-world situations.
* Generality of the concept or skill (how it is used in other situations) (Good et al., 1983).

The teacher's role during the initial part of the developmental section is to use instructional strategies that facilitate student learning. Activities often include teacher explanations and demonstrations, use of manipulative materials by both teachers and students, and use of concrete examples to determine common features. During the middle portion of the lesson, the teacher poses both process and product questions. Process and product questions differ because students respond to product questions with a "right answer," whereas process questions evoke thinking responses. For example, given the problem, .75 x 4, a product question would be: "What is the answer to this problem?" Process questions would be: "How did you get this answer?" "How did you go about solving this problem?" Student responses to these questions allow the teacher to assess the comprehension of the concept or skill. Additional explanation may be necessary, or guided practice may be appropriate because students seem to understand what they are doing.

Guided Practice. When a majority of students appear to have understood the lesson topic, the teacher will move the students to the guided practice section of the lesson. The purpose of this section of the lesson is for students to increase their proficiency at solving problems. Only a few problems (no more than five) are assigned at a time. These problems are checked verbally to further allow the teacher to ensure that the students have learned the material presented.

When students appear to understand how to work the problems independently, reinforcement of the skill or concept is undertaken. Students have an opportunity to increase their proficiency and consolidate their learning. This involves practice. In the model described by Good and his colleagues (1983), the method for practice may include seatwork problems. However, the authors of this handbook suggest that other types of practice may be used as well. For example, the practice section of the lesson could involve cooperative problem solving. In still other cases, microcomputers or calculators might be used to enhance practice. Because the purpose of this segment of the lesson is practice, the method of practice used will depend on the type of content being taught.

A potential hazard associated with the practice section of the lesson is that teachers often stop active supervision after the assignment has been made. They may become involved with recording homework grades or working extensively with one student. Often students do not use practice time productively because they know the teacher does not "check" seatwork. In order to maximize use of practice time, the teacher must monitor practice time. It is crucial that students be held accountable for this work. It has been suggested that collecting the practice work helps the students adjust to the expectation that practice work must be completed.

Homework. During the final phase of the lesson, homework is assigned. Homework should be assigned only when students understand the material developed during the class. A devastating misuse of homework occurs when students are asked to complete problems for which an inadequate background has been developed in class (Good et al., 1983). Thus, at the end of a lesson, if the students do not understand the material being practiced, no homework should be assigned. Good and his colleagues suggest that the teacher score the homework papers on Thursday. This allows the teacher to note if recurrent errors are being made and assist in the development of the instructional plan for the next week.

APPENDIX B

MODEL LESSONS

DEVELOPED BY

CLAYTON AND ASSOCIATES

MODEL LESSON I

Standard 1:	Mathematics as Problem Solving
Standard 3:	Mathematics as Reasoning
Standard 11:	Probability
Skill:	**Determine Random Probability of an Event**
Level:	6-8

Previous Lesson: Tree Diagrams showing possible outcomes.

Introduction

A. Review: Discuss how tree diagrams can be used to determine possible outcomes for a given event.

B. Check homework to determine students' understanding of how tree diagrams can be used as predictors.

Verbal Practice

A. Tell students that today one of them will be the winner. You have a prize (bonus points, pencils, candy, etc.) for the winner of today's game. Ask if anyone can guess what the possibility is that he/she will be the winner.

B. Ask students to determine the probability that their football team will win the toss prior to kickoff.

Developmental

A. Ask each student to write his or her name on a 3x5 index card that you provide, and place it in a container. Tell the students that today's lesson will show a new method for determining the probability of an individual's name being drawn to receive a" no-homework" coupon.

B. Ask one student to lay the cards facedown on a table. Can he/she make a guess about the probability that he/she could select his/her own name card on the first try to receive the coupon?

C. Students should be able to discover the probability formula from this exercise.

$$P = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

D. Ask students to determine the probability that the name card drawn would begin with the letter "S," that it would contain more than eight letters, etc.

E.
1. Use a brown paper bag and fill it with colored marbles, chips, or tiles of the same size. Be sure to allow students to examine the items before you place them in a bag.
2. Let the class determine the probability of selecting each color from the bag.
3. After recording probabilities, have five students draw an item from the bag.
4. Have the class rank these students' selections, assuming that the most valuable outcome is that which was least likely to occur.

F. Use an overhead spinner to demonstrate this activity for the large group. If a spinner has the numbers 1-8 evenly divided, what is the probability you will spin a #7? An even number? A number whose name begins with a vowel? A number whose name begins with a consonant? A number whose name has three or fewer letters, etc.?

Practice

A. Use an actual multiple-choice test with four possible answers. Ask the students to find the probability that a guess will be correct. What about a test with five possible answers? Six? What can we conclude?

B. If your book bag contains three green pencils, two blue pencils, and one Snoopy pencil, what is the probability you will reach in and draw the Snoopy pencil?

C. Have students examine the Wheel of Fortune wheel on TV or use a Wheel of Fortune spinner from the commercial game to determine the probability of spinning a bankruptcy.

D. Ask students to use a standard deck of playing cards and determine the following probabilities:

1. Drawing a queen.
2. Drawing a black jack.
3. Drawing a red 3.
4. Drawing an ace of spades.
5. Drawing a 2 of hearts.

E. Ask students to record all the different probabilities that exist concerning a random selection from their sock drawer, etc.

F. Some students may want to research states that have lotteries and report on the odds of winning one.

MODEL LESSON II

Standard 1: Mathematics as Problem Solving
Standard 12: Geometry
Standard 13: Measurement
Skill: Finding Area of Rectangle
Level: 6-8

Previous Lesson: Finding Perimeter

Introduction

A. Review: Discuss the term perimeter. Review this concept by asking a student to "walk" the perimeter of the classroom.

B. Check homework on examples of finding the perimeter of different polygons.

Verbal Practice

A. Examples of verbal problems to solve:

1. Ask students: How could we use perimeter to find how much carpet we need to carpet our classroom?

2. Ask for other situations when one would need to determine the area.

Developmental

A. Use the following problem situation to teach the concept of area:

> We want to find out how much carpet we will need to replace the old carpet in our classroom. How will we find the area of our classroom?

1. Ask students to make a scale drawing of the room (teacher should have a demonstration on display). Mark off square units to derive the formula for finding area.

2. A variation could include placing square units on the scale drawings to foster an understanding of this concept.

3. Another strategy to teach this concept would be to use 12x12 carpet tiles to place in the actual room to determine area.

B. Using a bulletin board in the classroom, measure and discuss how a decision could be reached about how much cloth to purchase to cover the bulletin board.

C. Have pupils use grid paper to draw as many different figures as they can that have an area of 12 square centimeters. Each square centimeter must have at least one side coinciding with a side of another square centimeter. Label the dimensions of each figure.

Practice

A. As a large group, discuss the following:

1. Give each student a drawing of a section of a floor plan. Discuss finding the dimensions of each room by measuring the length and width. Use the formula for finding area to label the area of each room.

2. Giving specific dimensions, ask students to draw the rectangle and find the area.

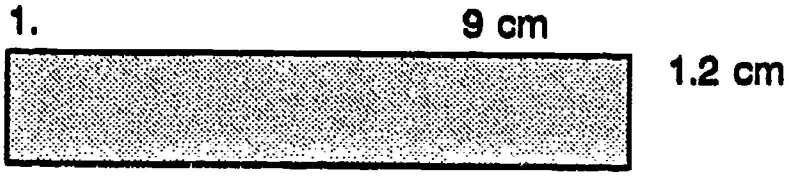
B. Ask students to determine the area by solving the following problem:

> You have decided to landscape your lawn. Measure your yard and determine how much sod will be needed to cover the area. Also, measure and determine the perimeter for fencing around your lawn.

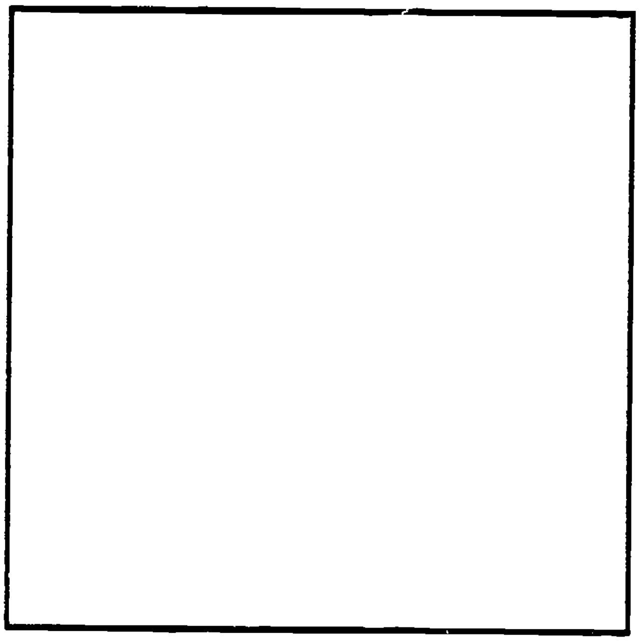
C. Given various rectangles, find the area.

EXAMPLES OF FIGURES TO FIND AREA:

1.



2. If the measure of each side equals 9 cm, find the area.



3. This activity could be used with pairs or small groups. Design a floor plan for your own mall. Label the dimensions, and find the area for each store.

4. Determine how much wallpaper is needed to repaper a wall in the classroom. Ask for a scale drawing to be developed by a small group to explain the area.

MODEL LESSON III

Standard 1: Mathematics as Problem Solving
Standard 3: Mathematics as Reasoning
Standard 8: Patterns and Functions
Skill: Graphing Ordered Pairs
Level: 6-8

Previous Lesson: Identifying Ordered Pairs
Definition of Ordinate and Coordinate Axes

Introduction

A. To review previous concepts of identifying ordered pairs, use the example of the order in which a date is written, e.g., the month and then the day. This example will show the importance of the order of the numbers. For example: 5/3 means May 3 and 3/5 means March 5. Point out that both dates involve the same two numbers, 3 and 5, but the order is very important.

B. Check homework of identifying ordered pairs according to x and y axes.

Verbal Practice

A. Use the example of a city map having an index, such as C2. Ask students to explain verbally how to use the index to locate their school. (Point out that the cities on a map are often located by a pair of coordinates, usually a letter and a number.)

B. Ask students to identify x and y axes by definition.

C. Ask for a sentence to define coordinates or ordered pairs.

Developmental

A. Use an overhead projector to show how to locate points in a plane by ordered pairs of numbers. Distinguish between x and y axes. Use the example of an airplane taking off. The airplane must first taxi (just as ordered pairs first follow x), and then the airplane starts to gain altitude. This is the same principle as moving across the x axis and then up the y axis.

B. Have students graph each of the following sets of ordered pairs on a sheet of grid paper. As each pair is graphed, that point should be joined to the previous point by a line segment. Then, the last point should be joined to the first point to complete a graph picture. Demonstrate on the overhead as they work on graph paper.

(-2,0), (2,3), (-1,4), (-1,11), (0,14), (1,11), (1,4), (2,3), (2,0), (1,1), (1,0), (-1,0), (-1,1)

Practice

A. 1. Give the students 21 identical slips of paper; write one of each of the following numbers: -10, -9, . . ., +9, +10. Each number should occur only once.

2. Place the numbers in a bag.

3. Have a pupil draw two slips of paper.

4. The first number drawn will be the x-coordinate. The second number drawn will be the y-coordinate.

5. The pupil then graphs the point on the overhead projector.

B. 1. Have each pupil make a graph picture and then write a list of ordered pairs for that picture.

2. The pupils can exchange lists and plot each other's graph pictures.

C.

1. Place the students in rows to plot their seating.
2. Assign each person a coordinate and ordinate pair, according to the position of his or her desk in the classroom. For example, the student sitting on the first row, second seat, would be assigned (1,2).
3. Ask students to plot their ordered pair on a grid.

MODEL LESSON IV

Standard 12: Geometry
Skill: **Pythagorean Theorem**
Level: 6-8

Previous Lesson: Square Roots (Using Tables/Calculators)

Introduction

A. Review: Discuss the differences in squaring a number and finding its square root.

B. Check homework to ensure that students can find squares and square roots via a table and calculator.

Verbal Practice

A. Ask students the following question: If there are 90 feet between each base on a baseball diamond, how far is it from 2nd base to home plate?

B. 1. Ask students how they could form two triangles from the baseball diamond and discuss the best answer.

2. Have students square the distances from home to first and first to second on calculators and record their answers for future use.

Developmental

A. 1. Use tiles (commercially produced) to construct a small-scale field, allowing each tile to represent 10 feet.

2. Lay squares in a diagonal pattern from second base to home plate.

3. Define the legs and hypotenuse of the right triangle formed in this exercise, and record each measurement.

B. 1. Ask students to square each leg and then find the sum of those squares.

2. Follow by asking them to square the hypotenuse.

3. Ask if they can draw an inference from this information.

C. 1. Give each student one tile and ask each to construct a right triangle using the tile to measure the two legs.

2. Now instruct them to use the tile to draw squares on the sides of the triangle.

Thus illustrate the theorem
$a^2 + b^2 = c^2$

Practice

A. Tell students that a bicycle ramp must rise 4 feet over a horizontal distance of 10 feet. Ask them how long the ramp board needs to be.

B. If a speed boat leaves the dock on the local lake and travels 4 miles east, then turns and travels 3 miles south, how far is the ship from port?

C. If a telephone pole is braced by a wire 74 feet long that is anchored 24 feet from the pole, how far up the pole should the wire be fastened?

D. Have students compute and check the diagonal measurement of your classroom door (cabinet, etc.) by measuring only the base and height.

E. Ask students to measure the base and height of a TV or computer screen and compute the diagonal measurement. (This is the advertised screen size.)

Advanced Problems for Practice

A. Find the length of the diagonal of a 5x5x5 cube.

B. A square has diagonals that are 13 cm long. How long are the sides?

C. Use the Pythagorean theorem and decide if the following are right triangles, given these lengths:

1. 6, 8, 10
2. 8, 15, 17
3. 8, 37, 39
4. 20, 22, 29