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ABSTRACT

Mathematics and the use of mathematical thinking should be more than what has been traditional school arithmetic. Much of the mathematical reasoning can be developed and experienced out of school, particularly in the home. This material contains recording forms and worksheets for activities for the Level Six experiences. Pupils are to complete these as part of their learning experiences. Forms and worksheets are included for all of the concepts and skills of the Level Six program. (YP)

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# MEANINGFUL MATHEMATICS

## LEVEL SIX

### RECORDING FORMS AND WORKSHEETS FOR PUPIL USE

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Mathematician: \_\_\_\_\_

"I first estimated the answer then worked the problem on the calculator."

MULTIPLICATION

ESTIMATE OF ANSWER

CALCULATOR COMPUTATION

Example:

$$\begin{array}{r} 4.15 \\ \times 6.23 \\ \hline \end{array}$$

25

25.8545

$$\begin{array}{r} 11.51 \\ \times 3.04 \\ \hline \end{array}$$

$$\begin{array}{r} 6.03 \\ \times 4.82 \\ \hline \end{array}$$

$$\begin{array}{r} 0.45 \\ \times 3.17 \\ \hline \end{array}$$

$$\begin{array}{r} 0.39 \\ 0.56 \\ \hline \end{array}$$

$$\begin{array}{r} 2.07 \\ \times 4.7 \\ \hline \end{array}$$

$$\begin{array}{r} 9.1 \\ \times 3.43 \\ \hline \end{array}$$

$$\begin{array}{r} 0.08 \\ \times 4.8 \\ \hline \end{array}$$

11789.2

Mathematician: \_\_\_\_\_

"I first estimated the answer then worked the problem on the calculator."

MULTIPLICATION

ESTIMATE OF ANSWER

CALCULATOR COMPUTATION

EXAMPLE:

$$\begin{array}{r} 0.43 \\ \times 1.85 \\ \hline \end{array}$$

$< 1$

0.7955

$$\begin{array}{r} 3.01 \\ \times 5.92 \\ \hline \end{array}$$

$$\begin{array}{r} 0.04 \\ \times 0.81 \\ \hline \end{array}$$

$$\begin{array}{r} 0.15 \\ \times 8.01 \\ \hline \end{array}$$

$$\begin{array}{r} 2.04 \\ \times .59 \\ \hline \end{array}$$

$$\begin{array}{r} 2.031 \\ \times 1.28 \\ \hline \end{array}$$

$$\begin{array}{r} 1.045 \\ \times 3.4 \\ \hline \end{array}$$

$$\begin{array}{r} 2.323 \\ \times 1.56 \\ \hline \end{array}$$

11789.2

Mathematician: \_\_\_\_\_

"I first decided about how big the answer would be, then did the calculation on the calculator."

Decimals

Estimates of Answer

Answer

13.41  
x 8.05

105

107.9505

15.062  
x 3.86

19.841  
x .162

By rounding  $.2 \times 20 = 4$

3.214242

431.06  
x 24.71

43.56  
x .062

.583  
x .14

198.04  
x 3.75

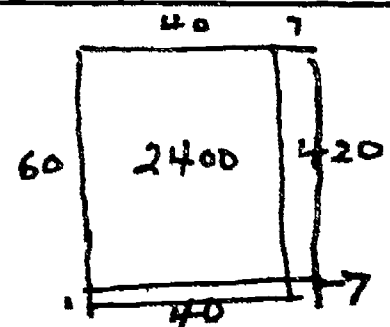
.043  
x .056

15.08  
x 3.472

9.04  
x 14

Mathematician: \_\_\_\_\_

"I did these multiplications as requested."

MULTIPLICATION	EXPANDED FORM	PICTURE OF PRODUCTS
<p>EXAMPLE:</p> $\begin{array}{r} 47 \\ \times 61 \\ \hline \end{array}$	$\begin{array}{r} 47 = 40 + 7 \\ \times 61 = 60 + 1 \\ \hline 2400 + 420 \\ \phantom{2400 + } + 40 + 7 \\ \hline 2867 \end{array}$	
$\begin{array}{r} 74 \\ \times 36 \\ \hline \end{array}$		
$\begin{array}{r} 56 \\ \times 28 \\ \hline \end{array}$		
$\begin{array}{r} 72 \\ \times 41 \\ \hline \end{array}$		

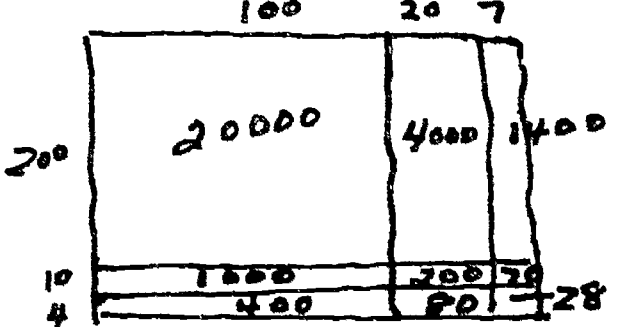
Mathematician: \_\_\_\_\_

"I did these multiplications as requested."

MULTIPLICATION	EXPANDED FORM	PICTURE OF PRODUCTS
$\begin{array}{r} 105 \\ \times 23 \\ \hline \end{array}$		
$\begin{array}{r} 216 \\ \times 38 \\ \hline \end{array}$		
$\begin{array}{r} 421 \\ \times 30 \\ \hline \end{array}$		
$\begin{array}{r} 207 \\ \times 53 \\ \hline \end{array}$		

Mathematician: \_\_\_\_\_

"I did these multiplications as requested."

MULTIPLICATION	EXPANDED FORM	PICTURE OF PRODUCTS
$\begin{array}{r} 127 \\ \times 214 \\ \hline \end{array}$	$\begin{array}{r} 100 + 20 + 7 \\ 200 + 10 + 4 \\ \hline 20000 + 4000 + 1400 \\ 1000 + 200 + 70 \\ 400 + 80 + 28 \\ \hline 27,178 \end{array}$	
$\begin{array}{r} 106 \\ \times 117 \\ \hline \end{array}$		
$\begin{array}{r} 211 \\ \times 145 \\ \hline \end{array}$		
$\begin{array}{r} 153 \\ \times 212 \\ \hline \end{array}$		

11785-1



Mathematician:

"I estimated the answer, then did the decimal division on the calculator."

DIVISION	ESTIMATE	CALCULATOR RESULT
EXAMPLE: $4.23 \div .11$	41	38.4545
$5.65 \div 1.4$		
$3.87 \div 1.01$		
$5.43 \div .08$		
$6.05 \div 1.4$		
$3.40 \div 1.7$		
$11.62 \div .15$		
$9.09 \div .11$		
$12.31 \div 1.04$		
$6.82 \div .07$		

1.25.89.2

Mathematician:

"I estimated the answer, then did the decimal division on the calculator."

DIVISION	ESTIMATE	CALCULATOR RESULT
$15.02 \div .50$		
$9 \div 1.40$		
$10.15 \div 1.01$		
$7.98 \div 2.10$		
$14.32 \div .08$		
$6.89 \div 1.41$		
$5.08 \div .08$		
$6.81 \div 1.11$		
$9.09 \div 1.1$		
$12.04 \div .06$		

1.25.89.2

Mathematician: \_\_\_\_\_

"I estimated the answer by rounding and found the answer using a calculator."

Decimals	Estimate of Answer	Answer
$14.06 \div 1.82$	$14 \div 2 = 7$ answer is more than this	7.725552747
$182.65 \div 3.04$		
$192.06 \div .158$		
$56.43 \div 13.5$		
$91.04 \div .16$	$91 \div .2 = 455$ answer is more than this	569.
$14.56 \div .24$		
$19.64 \div 15.8$		
$106.81 \div 12.07$		
$14.34 \div .23$		
$.583 \div .134$	big piece $\div$ smaller piece gives MORE THAN ONE	
$.065 \div .24$	smaller piece $\div$ by bigger piece gives much less than one	
$15.46 \div 3.81$		
$.083 \div 4.75$		

Mathematician: \_\_\_\_\_

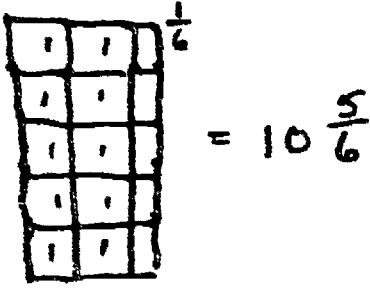
"I did these multiplications as requested."

FRACTIONS TO MULTIPLY	RESULT IN LOWEST TERMS	PICTURE
$\frac{2}{3} \times \frac{5}{8}$		
$\frac{3}{5} \times \frac{3}{8}$		
$\frac{1}{2} \times \frac{2}{3}$		
$\frac{5}{6} \times \frac{7}{12}$		
$\frac{3}{4} \times \frac{11}{12}$		

12489.1

Mathematician: \_\_\_\_\_

"i did these multiplications as requested."

FRACTIONS TO MULTIPLY	RESULT IN LOWEST TERMS	PICTURE
$\frac{1}{3} \times \frac{2}{5}$		
$\frac{1}{2} \times 1\frac{3}{4}$		
$2\frac{1}{6} \times 5$	$\frac{13}{6} \times 5 = \frac{65}{6} = 10\frac{5}{6}$ $5(2) + 5(\frac{1}{6}) = 10\frac{5}{6}$	
$3\frac{1}{2} \times 7$		
$1\frac{5}{8} \times 3\frac{1}{4}$		

12489.1

Mathematician: \_\_\_\_\_

"I rewrote these fraction multiplications to make it easier to reduce them."

FRACTION PRODUCT

CHANGED AND REDUCED

Example:  $3/5 \times 1/3$

$$1/5 \times 3/3 = 1/5 \times 1 = 1/5$$

$$\frac{4}{5} \times \frac{5}{7}$$

$$\frac{2}{3} \times \frac{3}{5}$$

$$\frac{5}{8} \times \frac{2}{5}$$

$$\frac{16}{5} \times \frac{15}{16}$$

$$\frac{11}{8} \times \frac{16}{5}$$

$$\frac{4}{9} \times \frac{9}{16}$$

$$\frac{2}{15} \times \frac{5}{8}$$

$$\frac{4}{5} \times \frac{5}{8}$$

6/13/89.2

Mathematician: \_\_\_\_\_

"I rewrote these fraction multiplications to make it easier to reduce them."

FRACTION PRODUCT

CHANGED AND REDUCED

Example:  $3/5 \times 1/3$

$1/5 \times 3/3 = 1/5 \times 1 = 1/5$

$$\frac{6}{5} \times \frac{15}{8}$$

$$\frac{6}{8} \times \frac{15}{5} = \frac{3}{4} \times 3$$

$$\frac{5}{6} \times \frac{2}{15}$$

$$\frac{5}{12} \times \frac{12}{15}$$

$$\frac{5}{8} \times \frac{8}{5}$$

$$\frac{3}{5} \times \frac{10}{3}$$

$$1\frac{4}{5} \times \frac{5}{9}$$

$$1\frac{3}{5} \times \frac{5}{8}$$

$$1\frac{3}{5} \times \frac{7}{8}$$

6/13/89.2

Mathematician: \_\_\_\_\_

"I rewrote these fraction multiplications to make it easier to reduce them."

FRACTION PRODUCT

CHANGED AND REDUCED

Example:  $3/5 \times 1/3$

$$1/5 \times 3/3 = 1/5 \times 1 = 1/5$$

$$2\frac{1}{3} \times \frac{6}{7}$$

$$3\frac{1}{4} \times \frac{4}{5}$$

$$\frac{2}{5} \times 1\frac{2}{3}$$

$$8 \times \frac{1}{8}$$

$$2\frac{1}{5} \times \frac{5}{12}$$

$$1\frac{7}{8} \times \frac{4}{5}$$

$$4\frac{1}{2} \times \frac{2}{5}$$

$$3\frac{2}{3} \times \frac{6}{15}$$

6/13/89.2



Mathematician: \_\_\_\_\_

"I did these divisions as indicated and showed the division with a picture."

DIVISION	IN EXPANDED FORM	PICTURE
<p>Example:</p> $321 \div 43$	$7 \text{ R } 20$ $40 + 3 \overline{) 300 + 20 + 1}$ $280 + 20 + 1$ <hr/> $20 \text{ REMAINDER}$	
$271 \div 53$		
$403 \div 29$		
$206 \div 18$		
$326 \div 51$		

12489.2

Mathematician: \_\_\_\_\_

"I did these divisions as indicated and showed the division with a picture."

DIVISION	IN EXPANDED FORM	PICTURE
$196 \div 33$		
$275 \div 25$		
$426 \div 40$		
$329 \div 31$		
$409 \div 39$		

12489.2

Mathematician: \_\_\_\_\_

"I did these divisions as indicated and showed the division with a picture."

DIVISION	IN EXPANDED FORM	PICTURE
$385 \div 35$		
$286 \div 34$		
$104 \div 15$		
$428 \div 62$		
$123 \div 11$		

12489.2

Mathematician: \_\_\_\_\_

DIVISION	PICTURE	SAME DENOMINATOR FORM	ANSWER
<b>EXAMPLE:</b> $\frac{1}{2} \div \frac{2}{3}$		$\frac{3}{6} \div \frac{4}{6}$	$3 \div 4 = \frac{3}{4}$
$\frac{2}{3} \div \frac{1}{2}$			
$\frac{1}{2} \div \frac{3}{8}$			
$\frac{3}{8} \div \frac{1}{2}$			
$\frac{2}{3} \div \frac{1}{4}$			
$\frac{1}{4} \div \frac{2}{3}$			
$\frac{2}{3} \div \frac{3}{4}$			

1.25.89.1

Mathematician: \_\_\_\_\_

DIVISION	PICTURE	SAME DENOMINATOR FORM	ANSWER
$\frac{3}{4} \div \frac{2}{3}$			
$\frac{5}{6} \div \frac{1}{3}$			
$\frac{7}{8} \div \frac{1}{2}$			
$\frac{1}{2} \div \frac{3}{4}$			
$\frac{3}{4} \div \frac{7}{8}$			
$\frac{7}{8} \div \frac{3}{4}$			
$\frac{5}{8} \div \frac{3}{4}$			

1.25.89.1

Mathematician: \_\_\_\_\_

DIVISION	PICTURE	SAME DENOMINATOR FORM	ANSWER
$\frac{3}{4} \div \frac{5}{8}$			
$\frac{7}{8} \div \frac{1}{4}$			
$\frac{7}{12} \div \frac{1}{2}$			
$\frac{5}{12} \div \frac{1}{2}$			
$\frac{2}{3} \div \frac{5}{12}$			
$\frac{2}{3} \div \frac{7}{12}$			
$\frac{11}{12} \div \frac{1}{2}$			

1.25.89.1

Mathematician: \_\_\_\_\_

"For each pair of fractions given, I decided which was larger, joined them, found the difference between and divided one by the other both ways."

FRACTIONS	SAME DENOMINATORS	COMPARED	JOINED	THE DIFFERENCE BETWEEN	DIVIDED
Example: $1\frac{2}{3}, 1\frac{3}{4}$	$1\frac{8}{12} = \frac{20}{12}$ $1\frac{9}{12} = \frac{21}{12}$	$\frac{21}{12} > \frac{20}{12}$ so $1\frac{3}{4} > 1\frac{2}{3}$	$\frac{20}{12} + \frac{21}{12} =$ $\frac{41}{12} = 3\frac{5}{12}$	$\frac{21}{12} - \frac{20}{12} =$ $\frac{1}{12}$ , so $1\frac{3}{4} - 1\frac{2}{3} =$ $\frac{1}{12}$	$1\frac{2}{3} \div 1\frac{3}{4}$ $\frac{20}{12} \div \frac{21}{12} = \frac{20}{21}$ $1\frac{3}{4} \div 1\frac{2}{3}$ $\frac{21}{12} \div \frac{20}{12} = \frac{21}{20}$
$\frac{8}{5}$ $\frac{3}{2}$					
$\frac{5}{6}$ $\frac{5}{9}$					
$\frac{3}{4}$ $\frac{5}{6}$					
$\frac{2}{3}$ $1\frac{1}{3}$					

Mathematician: \_\_\_\_\_

"For each pair of fractions given, I decided which was larger, joined them, found the difference between and divided one by the other both ways."

FRACTIONS	SAME DENOMINATORS	COMPARED	JOINED	THE DIFFERENCE BETWEEN	DIVIDED
$1\frac{3}{8}$ $1\frac{2}{3}$					
$2\frac{1}{8}$ $1\frac{3}{4}$					
$1\frac{3}{8}$ $\frac{7}{8}$					
$1\frac{1}{3}$ $\frac{3}{4}$					
$1\frac{2}{3}$ $\frac{4}{5}$					



Mathematician: \_\_\_\_\_

"I changed these numbers into SCIENTIFIC NOTATION (written with powers of ten)

NUMBER	IN SCIENTIFIC NOTATION	NUMBER	IN SCIENTIFIC NOTATION
Example: 143.8	$1.438 \times 10^2$	.0012	
.041	$4.1 \times 10^{-2}$	3.81	
12.44		9.05	
16.05		1,413	
1421.0		.0002	
.3141		15.41	
.0042		108.0	
158.1		904.5	
19.02		.0421	

Mathematician: \_\_\_\_\_

"I multiplied and divided these numbers by adding exponents to multiply and subtracting exponents to divide."

NUMBERS

PRODUCT OR QUOTIENT

Example:

$$14.1 \times .48$$

↓  
add exponents of 10's

$$\begin{aligned} 1.41 \times 10 \times 4.8 \times 10^{-1} &= \\ 1.41 \times 4.8 \times 10^{(1+(-1))} &= \\ 1.41 \times 4.8 \times 10^0 &= \\ 1.41 \times 4.8 &= 6.768 \end{aligned}$$

$$201.1 \div .02$$

↓  
subtract exponents of 10's

$$\begin{aligned} 2.011 \times 10^2 \div 2 \times 10^{-2} &= \\ 2.011 \div 2 \times 10^{(2-(-2))} &= \\ 2.011 \div 2 \times 10^4 &= \\ 4.022 \times 10^4 &= 40220. \end{aligned}$$

$$304.1 \times 21.6$$

$$304.1 \div 21.6$$

$$15.4 \times .004$$

$$15.4 \div .004$$

Mathematician: \_\_\_\_\_

"I multiplied and divided these numbers by adding exponents to multiply and subtracting exponents to divide."

NUMBERS

PRODUCT OR QUOTIENT

$$343 \div 21.7$$

$$486.7 \times .004$$

$$523.1 \div 3.05$$

$$.045 \times .043$$

$$.045 \div .043$$

$$6.321 \times .05$$

Mathematician: \_\_\_\_\_

"I changed these numbers to SCIENTIFIC NOTATION and multiplied or divided by adding or subtracting exponents of ten."

NUMBERS	IN SCIENTIFIC NOTATION	RESULT
$304.0 \times 201$		
$304 \div 2.01$		
$.005 \times .013$		
$.005 \div .013$		
$14.5 \times 30.2$		
$145.0 \div 30.2$		

Mathematician: \_\_\_\_\_

"I estimated the answer by rounding and found the answer using a calculator."

$452 \times 134$		
$398 \times 17$		
$436 \div 21$		
$383 \times 141$		
$693 \div 35$		
$143 \times 387$		
$126 \times 17$		
$19 \times 302$		
$21 \times 5871$		
$18 \times 4120$		
$280 \times 19$		
$280 \div 19$		
$5871 \div 21$		

Mathematician: \_\_\_\_\_

"I estimated the answer by rounding and found the answer using a calculator."

$421 \div 13$

$621.7 \div .52$

$384.1 \times 2.8$

$981 \div .03$

$14.27 \div 3.1$

$625 \div 18$

$2831 \div 14$

$98.7 \times 3.3$

$42.1 \times .06$

$973.1 \div .06$

$99.2 \div 81$

$1492 \div 12$

$386 \div .09$

Mathematician: \_\_\_\_\_

"For each UNIVERSE given, I wrote the missing term."

UNIVERSE

IS

NOT

Cars

Ford

Buildings

Not house

Trees

Birch

Sports

Not baseball

Colors

Red

Shapes

Not triangle

Subjects

Science

Patterns

Not flowered

Shells

Smooth

Pets

Not cat

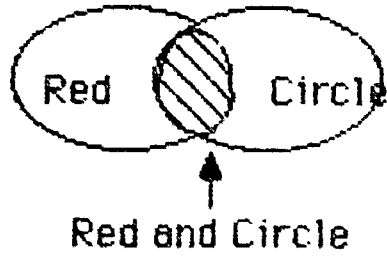
Mathematician: \_\_\_\_\_

"I drew a diagram to show each description."

COMPOUND PHRASE

DIAGRAM

Sample:  
RED AND CIRCLE



MEN AND TEACHERS

WOMEN AND TEACHERS

STRONG AND BRAVE

HIGH AND MIGHTY



Mathematician: \_\_\_\_\_

"I drew a diagram to show each description."

Pg. 2

COMPOUND PHRASE	DIAGRAM
WHITE AND FLUFFY	
RICH AND FAMOUS	
DOWN AND DIRTY	
ALIVE AND WELL	

Mathematician: \_\_\_\_\_

"I drew diagrams to show whether INCLUSIVE or EXCLUSIVE OR is shown."

OR PHRASE

DIAGRAM

BOY OR GIRL

.

MAN OR FIREMAN

HERE OR THERE

SICK OR PRESENT

RED OR SOFT

Mathematician: \_\_\_\_\_

"I drew diagrams to show whether INCLUSIVE or EXCLUSIVE OR is shown."

OR PHRASE

DIAGRAM

UP OR DOWN

BLONDE OR BLUE EYED

WOMAN OR TEACHER

MAN OR COOK

ROBIN OR BIRD

7.18.89.3

Mathematician: \_\_\_\_\_

"I changed one word to make the given true statements false."

TRUE STATEMENT	FALSE STATEMENT
<b>EXAMPLE:</b> If red, then square	If red, then triangle
If rich, then famous	
If famous, then rich	
If work, then get paid	
If get paid, then worked	

Mathematician: \_\_\_\_\_  
"I changed one word to make the given true statements false."

TRUE STATEMENT

FALSE STATEMENT

If rich, then popular

If popular, then rich

If poor, then sick

If alone, then cry

If alive, then think

If high, then mighty

If down, then out

7.18.89.4

Mathematician: \_\_\_\_\_

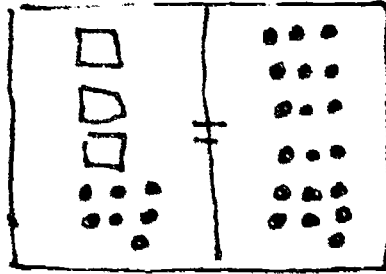
"I found what gives in the  in each case, doing one step at a time."

OPEN SENTENCE

PICTURE

STEPS TO FIND

$$3\Box + 7 = 19$$



$$3\Box + 7 = 19$$

$$3\Box = 12$$

$$\Box = 4$$

$$12 = 4 + 2\Box$$

$$17 = 2 + 3\Box$$

$$4\Box + 1 = 25$$

Mathematician:

"I found what gives in the  in each case, doing one step at a time."

OPEN SENTENCE

PICTURE

STEPS TO FIND

$$3\Box + 9 = 27$$

$$2\Box - 4 = 8$$

$$9 = 3\Box - 9$$

$$2\Box - 5 = 7$$

Mathematician:

"I found what gives in the  in each case, doing one step at a time."

OPEN SENTENCE

PICTURE

STEPS TO FIND



$$34 = 4 + 3 \square$$

$$5 \square + 3 = 18$$

$$4 \square + 1 = 13 + \square$$

$$3 \square + 7 = \square + 15$$



Mathematician: \_\_\_\_\_

"I added, and subtracted numbers or multiplied or divided by numbers  
to find ONE .

OPEN SENTENCE

WHAT I DID TO BOTH SIDES

NUMBER IN

$$7 - \square = 3$$

$$12 = 15 - \square$$

$$2\square = 9$$

$$5 = \frac{1}{3}\square$$

$$3\square = 14$$

$$2\square = 15$$

Mathematician: \_\_\_\_\_

"I added, and subtracted numbers or multiplied or divided by numbers  
to find ONE .

OPEN SENTENCE	WHAT I DID TO BOTH SIDES	NUMBER IN <input type="checkbox"/>
$\frac{1}{3} \square = 3$		
$3 \square = 18$		
$15 = 5 \square$		
$18 = 6 \square$		
$4 = \square + 1$		
$8 = \square - 3$		

Mathematician: \_\_\_\_\_

"I added, and subtracted numbers or multiplied or divided by numbers  
to find ONE  $\square$ ."

OPEN SENTENCE	WHAT I DID TO BOTH SIDES	NUMBER IN $\square$
$2\square = 16$	$\div 2$	$\square = 8$
$3\square = 27$		
$\square + 3 = 7$		
$\square - 4 = 6$		
$\frac{1}{2}\square = 4$		
$\square + 3 = 10$		

Mathematician: \_\_\_\_\_

"For each problem involving distance and time, I made a table and found the missing amount from the table or from a proportion."

PROBLEM	TABLE	PROPORTION																		
<p><b>Example:</b> A train travels 210 miles in 3 hours. How far can it travel in 5 hours?</p>	<table border="1"> <tr> <td>Miles</td> <td>70</td> <td>140</td> <td>210</td> <td>280</td> </tr> <tr> <td>Hours</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> </table> <table border="1"> <tr> <td>Miles</td> <td>350</td> <td>420</td> <td>490</td> </tr> <tr> <td>Hours</td> <td>5</td> <td>6</td> <td>7</td> </tr> </table>	Miles	70	140	210	280	Hours	1	2	3	4	Miles	350	420	490	Hours	5	6	7	$\frac{210 \text{ mi.}}{\square \text{ mi.}} = \frac{3 \text{ hr.}}{5 \text{ hr.}}$ $3 \square = 5 \times 210$ $\square = \frac{5 \times 210}{3} = 350 \text{ mi.}$
Miles	70	140	210	280																
Hours	1	2	3	4																
Miles	350	420	490																	
Hours	5	6	7																	
<p>John walks 7 miles in one hour. How far could he walk in 2 1/2 hours?</p>																				
<p>Joan can run 100 meters in 12 seconds. Assuming she can hold that speed for 300 meters, how long should a run of 300 meters take?</p>																				
<p>A car travels 300 miles in 6 hours. How far had it travelled after 4 hours?</p>																				

Mathematician: \_\_\_\_\_

"For each problem involving distance and time, I made a table and found the missing amount from the table or from a proportion."

PROBLEM	TABLE	PROPORTION
<p>Two trains start from a station at the same time. After one hour, one has gone 50 miles and the other 60 miles. How far apart will they be after 3 hours?</p>		
<p>The faster of two cars travels 400 miles while the other travels 300 miles. When the slower has gone 600 miles, how far will the faster have gone?</p>		
<p>If Diane runs 4 miles for each 3 miles run by Vivian, how far will Vivian have run when Diane has run 12 miles?</p>		
<p>Minnows swim twice as fast as crayfish. Crayfish swim three times as fast as leeches. Minnows swim how many times as fast as leeches?</p>		

Mathematician: \_\_\_\_\_

"I worked the following problems using percents."

**PERCENTS:** The hundred grid represents a whole basement. Show a good way to divide it up so that:

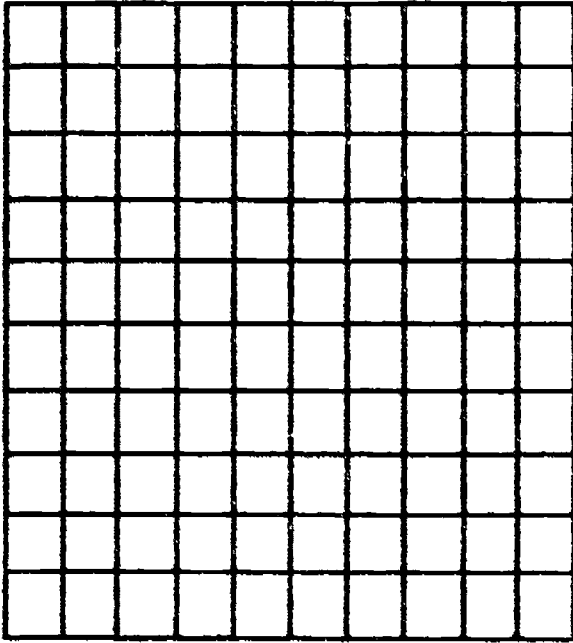
20% is a workshop

15% is a laundry area

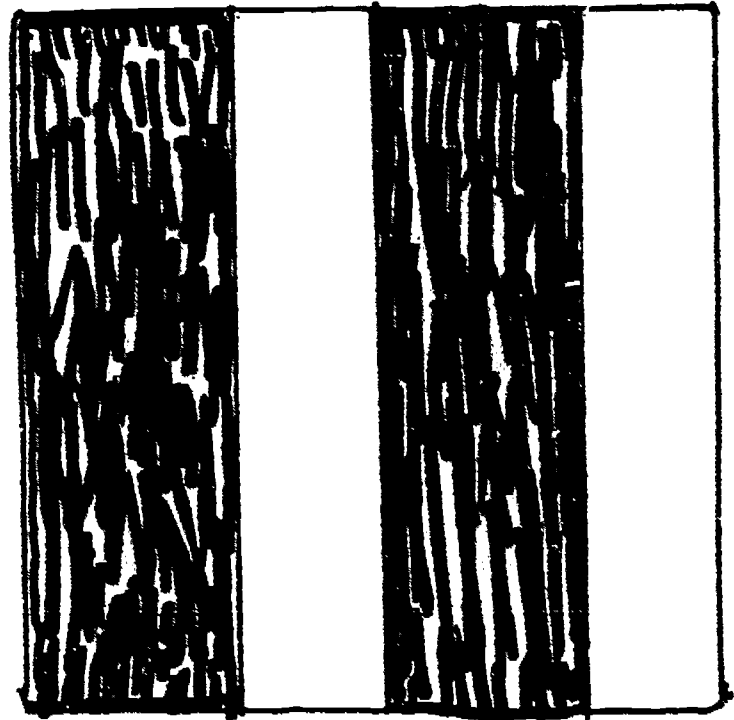
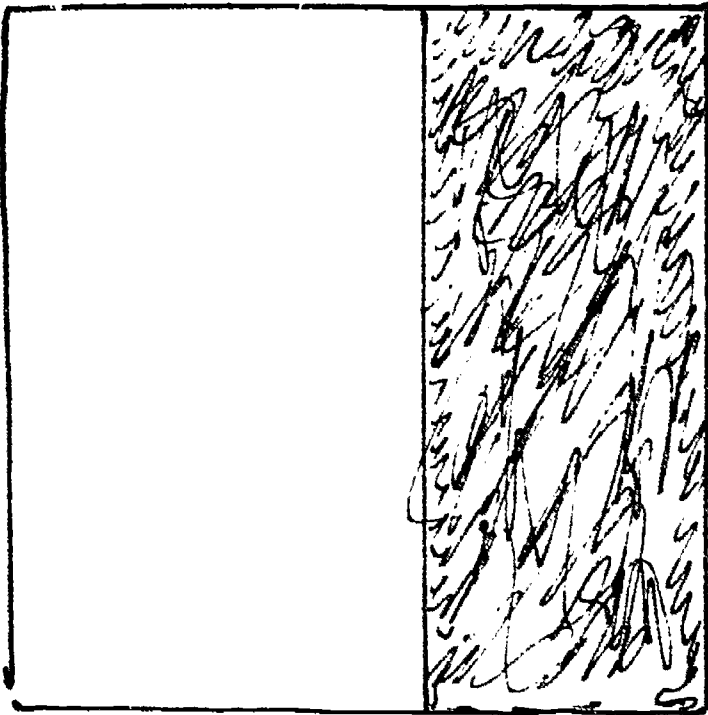
15% is sauna

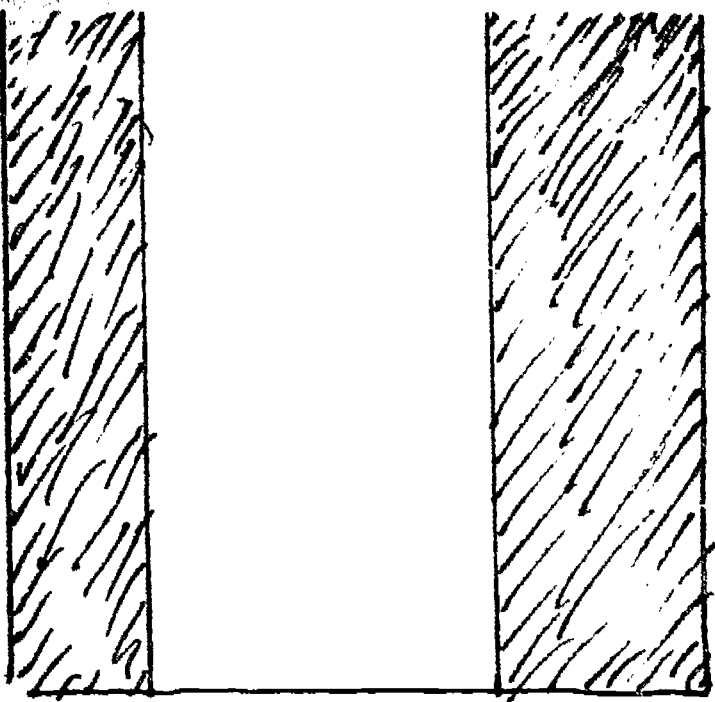
The rest is open space

"What percent is open space?"

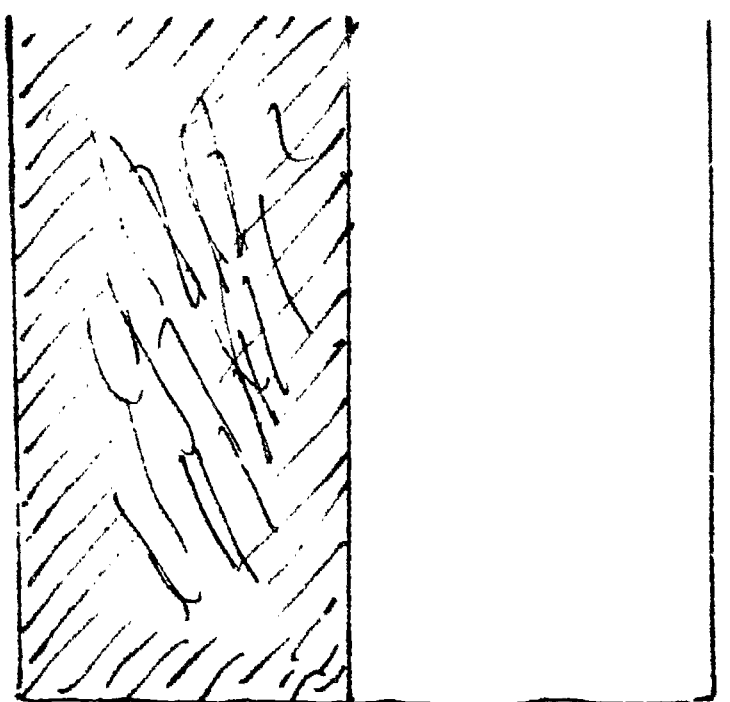


Estimate the percent of each given square that is figured or shaded. The use a HUNDRED grid the same size as the square to find the percent.

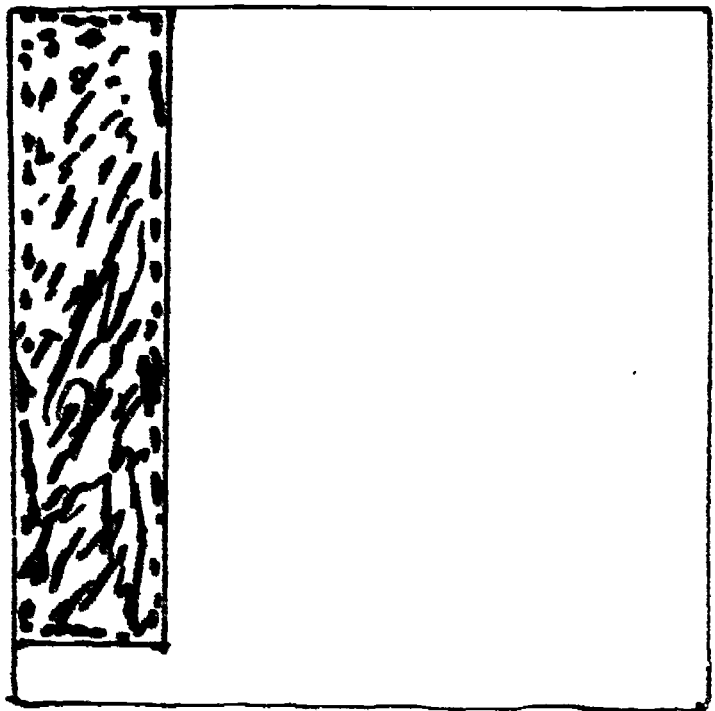




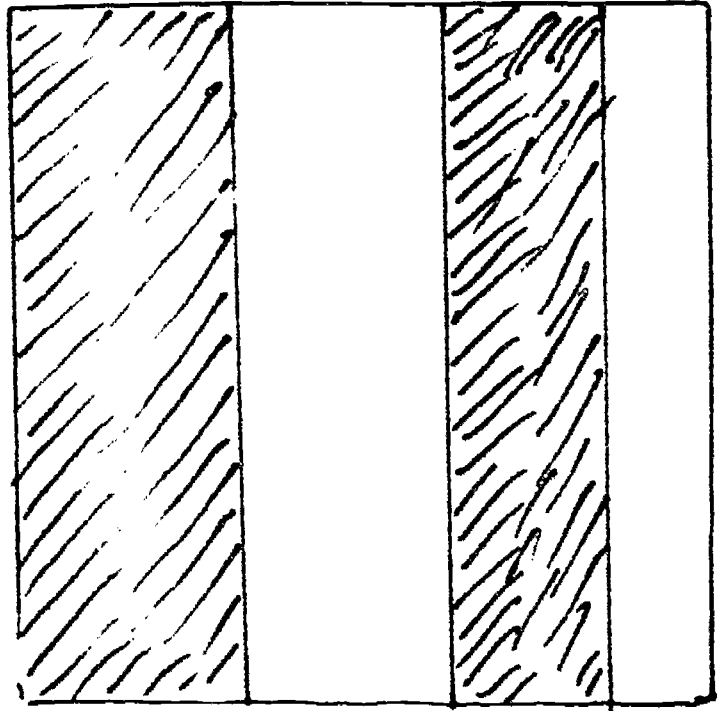
Estimate \_\_\_\_\_ Answer \_\_\_\_\_



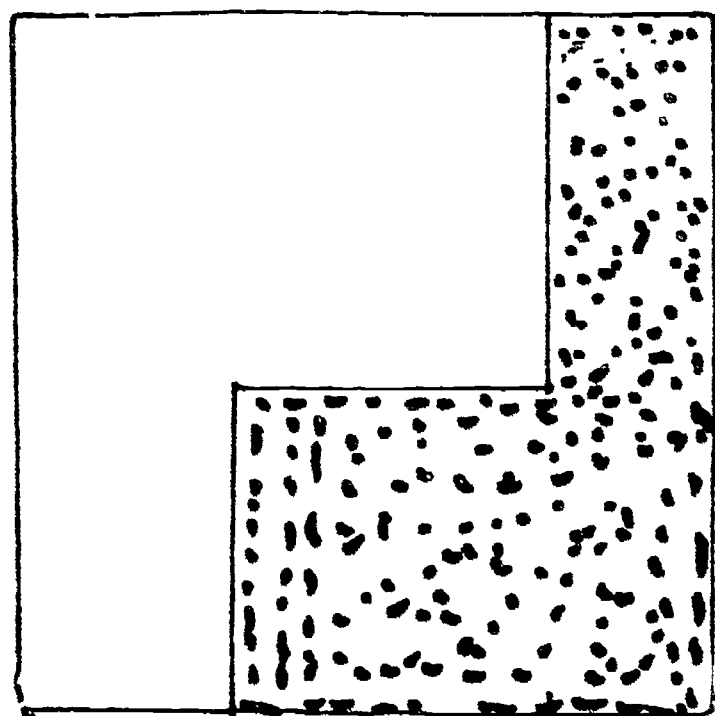
Estimate \_\_\_\_\_ Answer \_\_\_\_\_



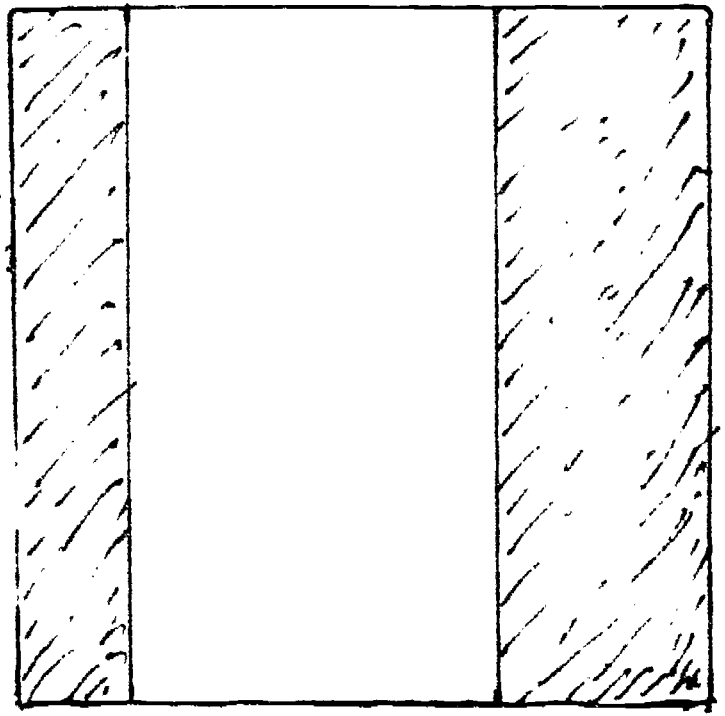
Estimate \_\_\_\_\_ Answer \_\_\_\_\_



Estimate \_\_\_\_\_ Answer \_\_\_\_\_

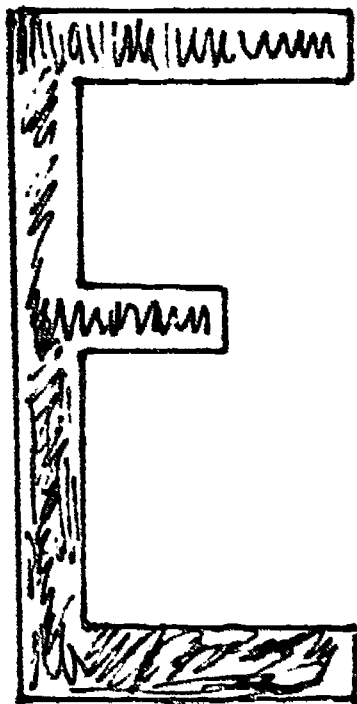


Estimate \_\_\_\_\_ Answer \_\_\_\_\_

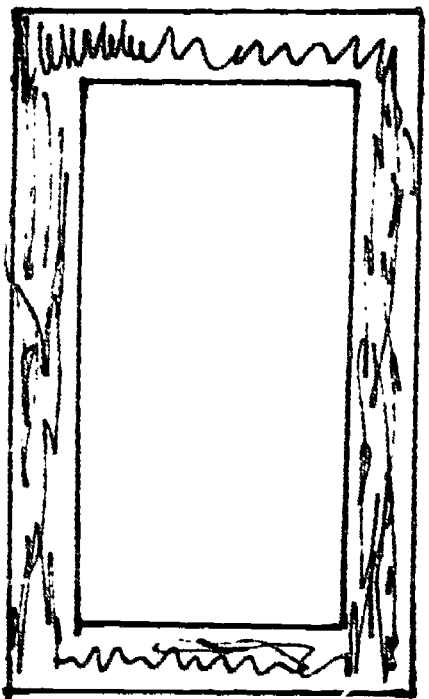


Estimate \_\_\_\_\_ Answer \_\_\_\_\_

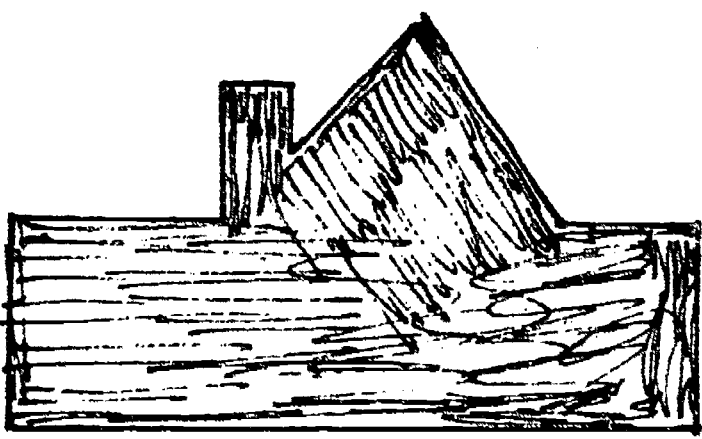
Estimate the percent of a 100 grid shown by each colored figure given. Then use a transparent 100 grid to find the answer.



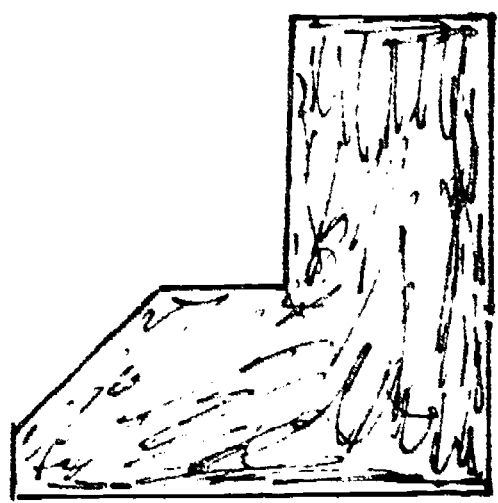
Estimate \_\_\_\_\_  
Answer \_\_\_\_\_



Estimate \_\_\_\_\_  
Answer \_\_\_\_\_



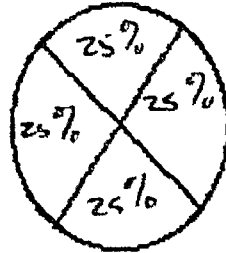
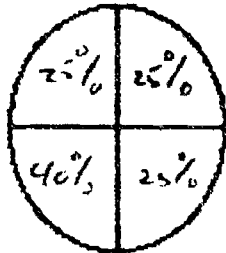
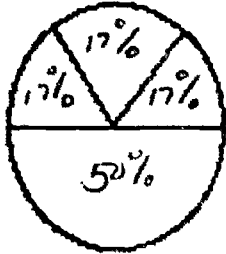
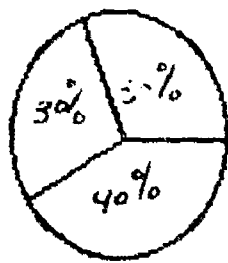
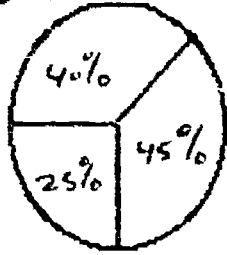
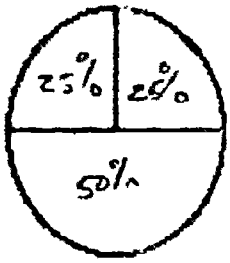
Estimate \_\_\_\_\_  
Answer \_\_\_\_\_



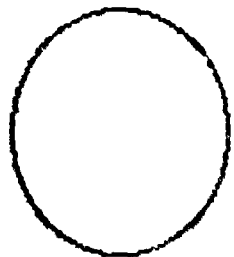
Estimate \_\_\_\_\_  
Answer \_\_\_\_\_



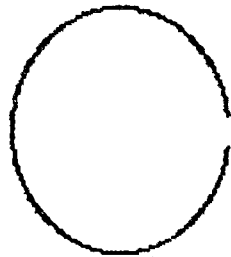
Circle the circle graphs that are reasonable:



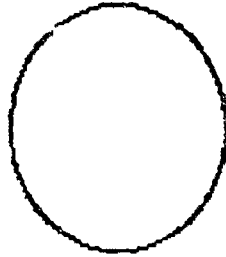
Draw lines in to divide each circle according to percents given:



60%, 20%, 20%



25%  
25%  
50%



50%  
25%  
15%  
10%

For each problem, make a table if that helps to see the proportion how the answer is found.

**Problem**

**Solution**

If Frank pays back the \$900 he borrowed in a month, 12% interest will be charged. How much interest does he pay?

Bill made a 10% cash down payment on a stereo set that cost \$289. How much did he pay as a down payment?

Tess's father leaves a 15% tip for waitresses. The check when the family ate out at The Eatery was \$50. How much tip did Tess's father leave?

During December, Joyce made 28 of 32 free throws. What percent did she make?

At the end of the day, Bob said the painting job was 95% complete. What percent was left to do?

Minnesota had a 7% sales tax. How much should be paid in sales tax on a \$14,500 car?

When Bob filled 28 boxes, his employer told him his job was 80% completed. How many boxes did Bob have to fill to start with?

What percent of the students are girls in a school where there are 62 girls and 78 boys?

Tom went to borrow \$100 at the bank. He said he would pay it back after a year. The bank gave him \$96. What rate of "discount interest" did the bank charge Tom?

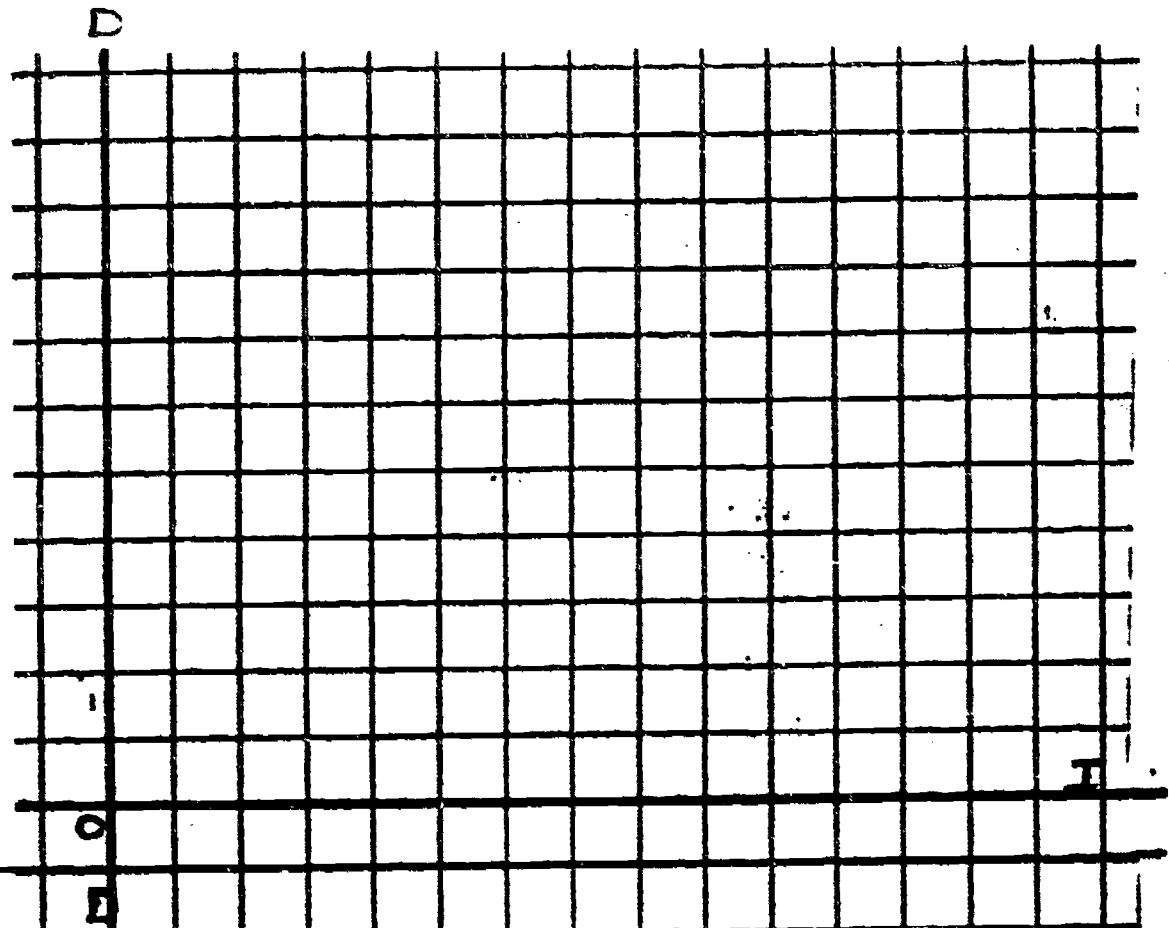
Mathematician: \_\_\_\_\_

"I completed each table of values for D and I, found the constant change in D and the rule and graphed this."

TABLE

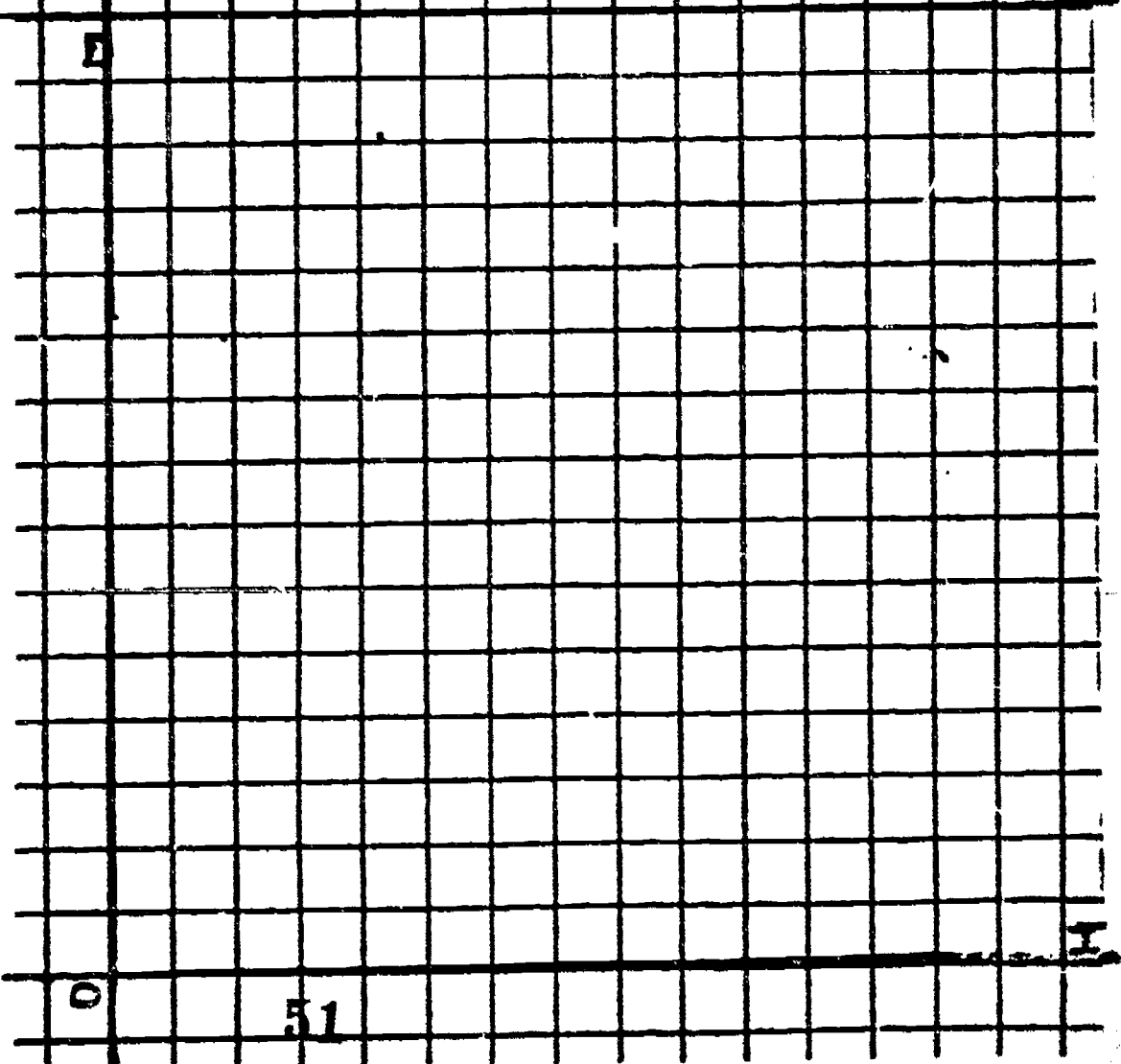
I	D	Change in D
.		
1	2	2
2	4	2
		2
		2
		2
		2

GRAPH



RULE:

I	D	Change in D
0	5	
		1
		1
		1
		1
		1
		1



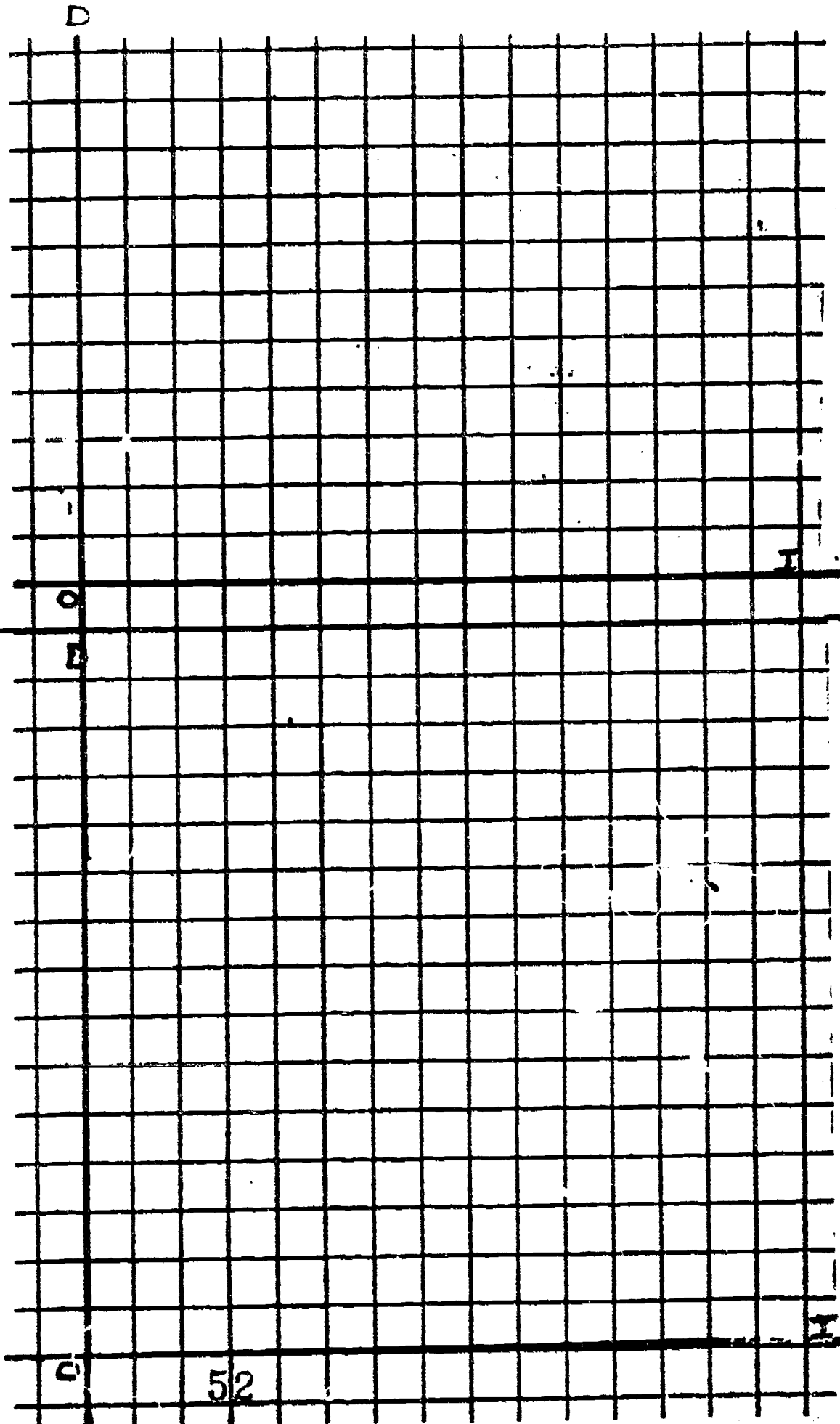
Mathematician: \_\_\_\_\_

"I completed each table of values for D and I, found the constant change in D and the rule and graphed this."

TABLE

I	D	Change in D
0		
1		
2		
3		
4	5	
5	$5\frac{1}{2}$	
6	6	

GRAPH



RULE:

I	D	Change in D
0	1	$\frac{1}{3}$
1	$1\frac{1}{3}$	$\frac{1}{3}$
2	$1\frac{2}{3}$	

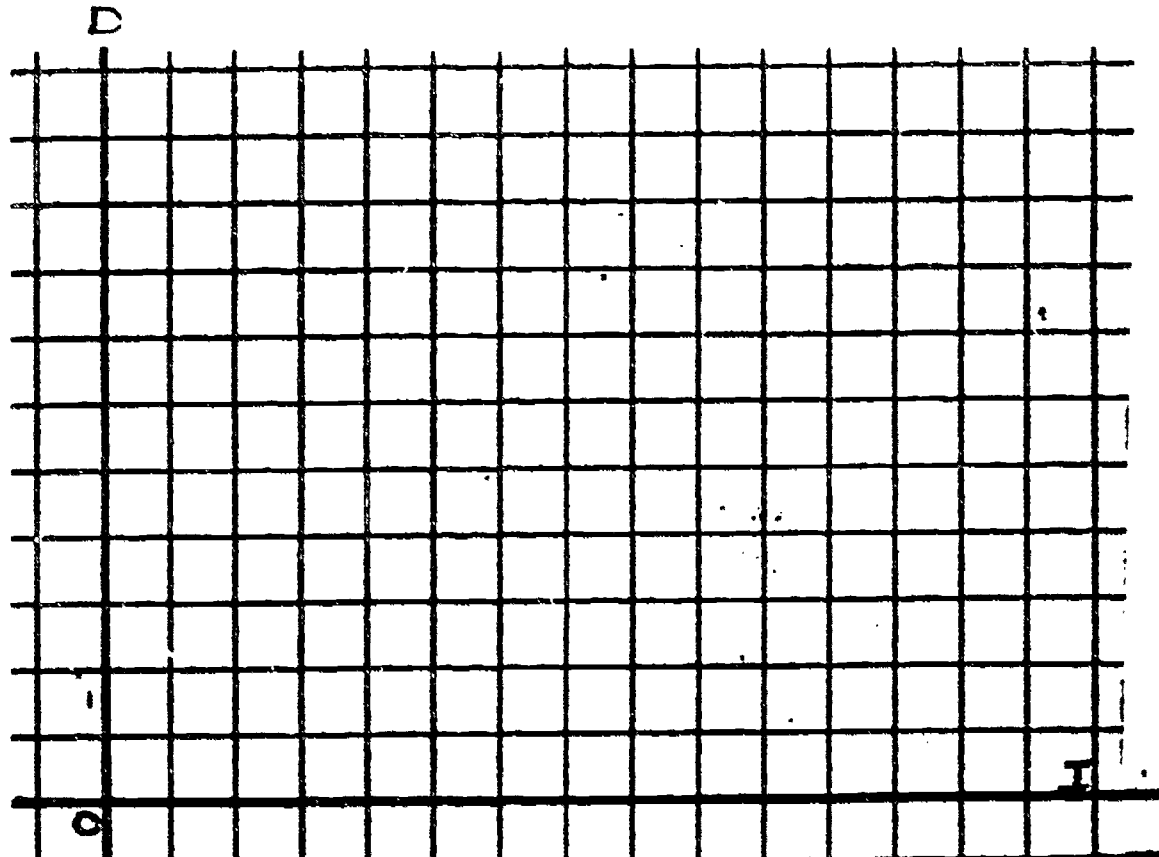
Mathematician: \_\_\_\_\_

"I completed each table of values for D and I, found the constant change in D and the rule and graphed this."

TABLE

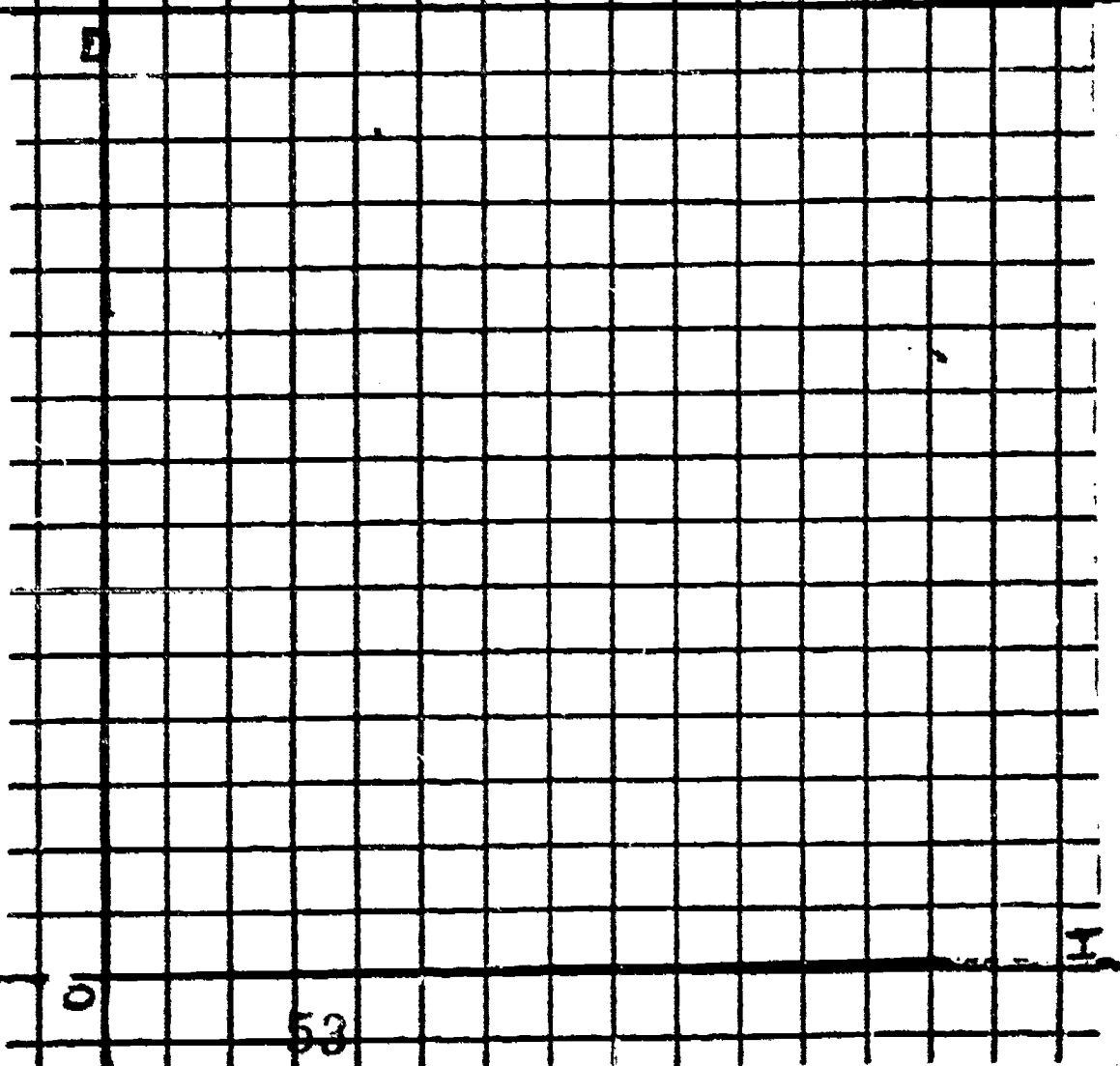
I	D	Change in D
0	2	
		5
		5
		5
		5
		5

GRAPH



RULE:

I	D	Change in D
1	3	
2	5	
3	7	



Mathematician: \_\_\_\_\_

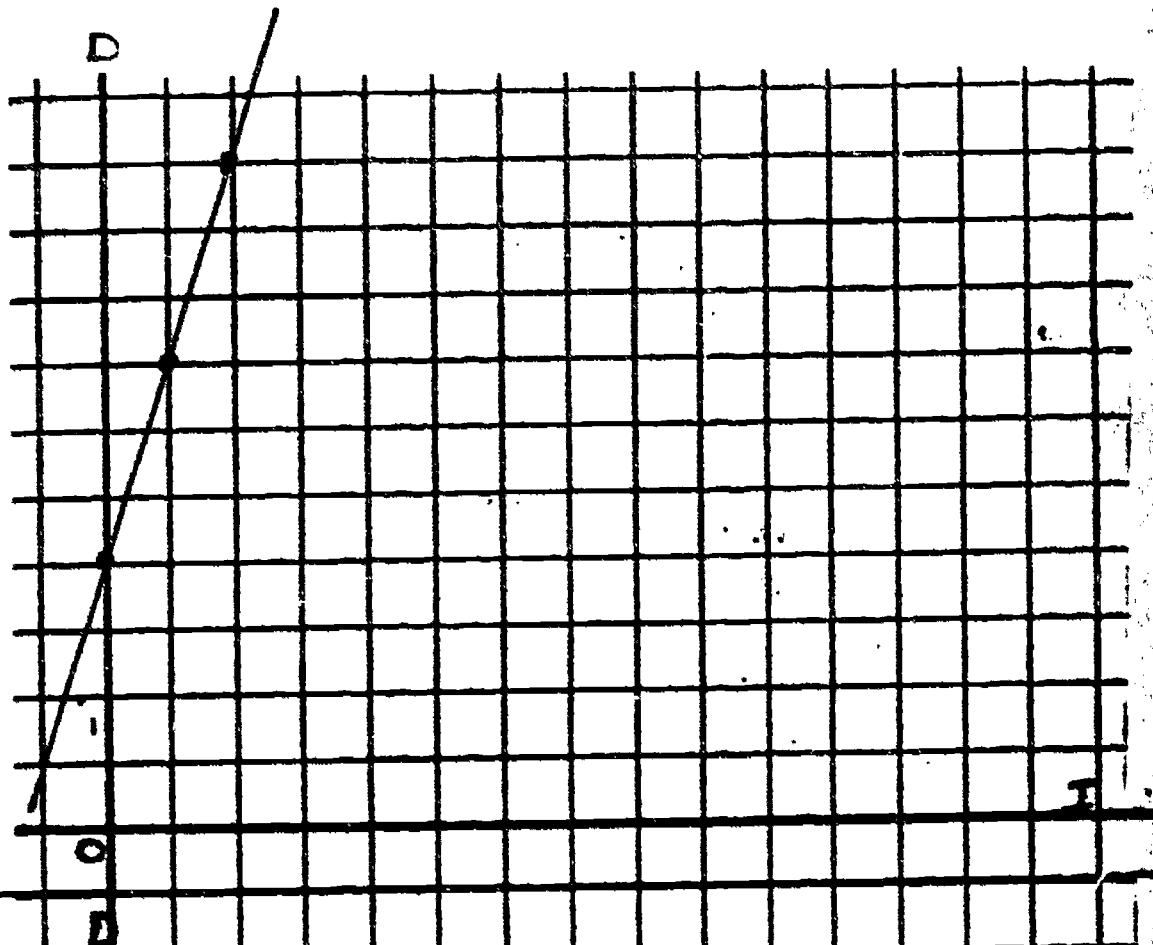
"I completed each table of values for D and I, found the constant change in D and the rule and graphed this."

Example:

TABLE

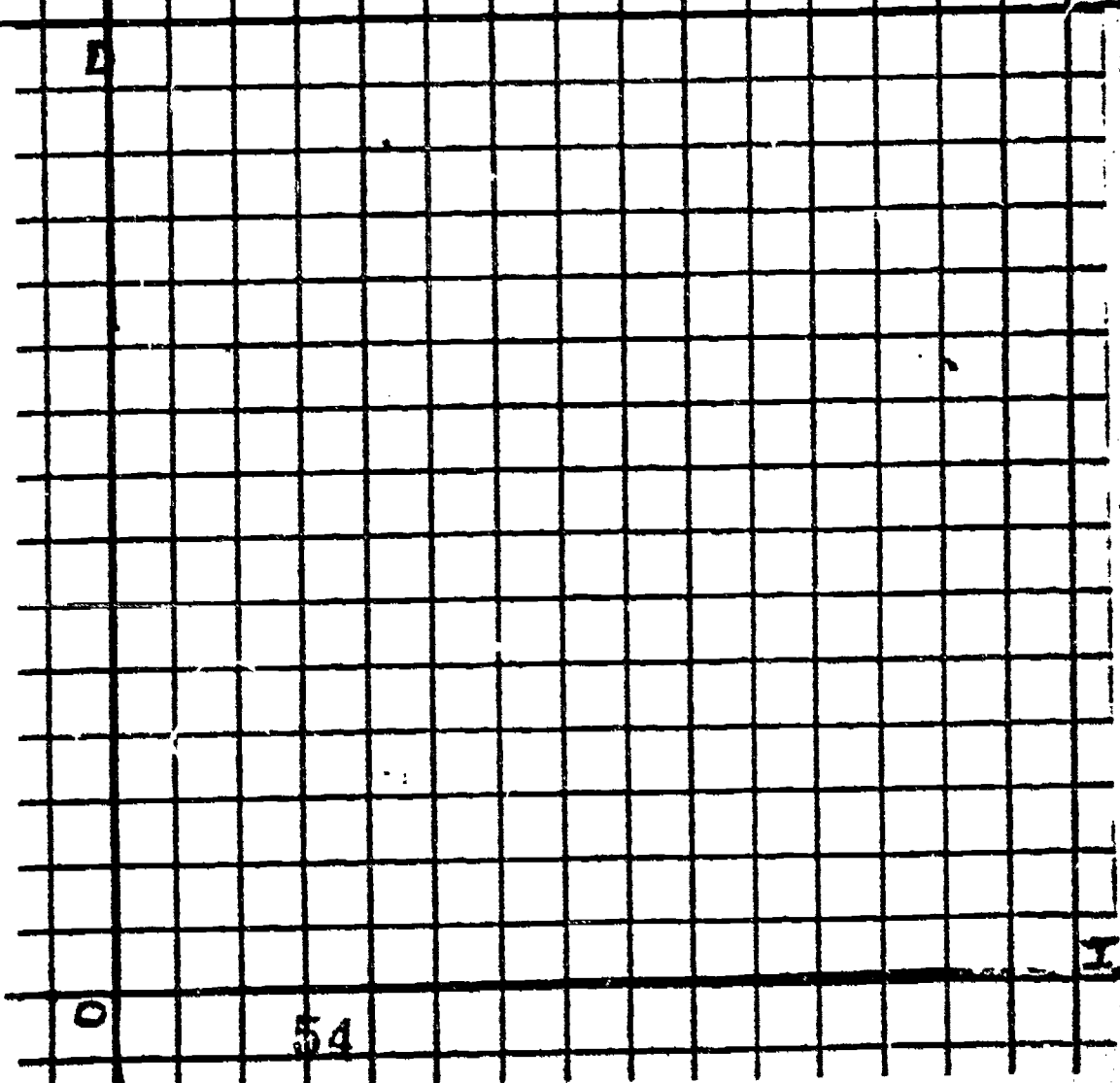
I	D	Change in D
0	4	
1	7	3
2	10	3
3	13	3
4	16	3

GRAPH



RULE:

I	D	Change in D
0	2	
1	3	
2	4	
3	5	



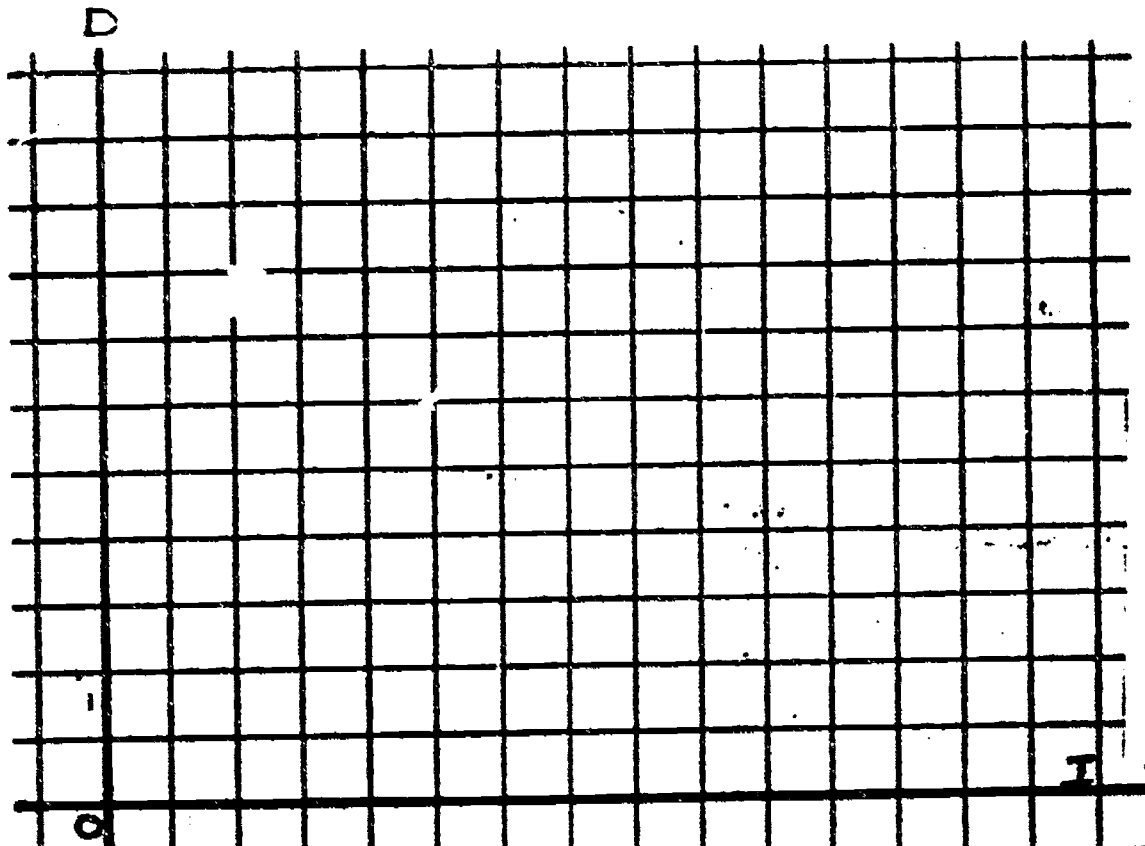
Mathematician: \_\_\_\_\_

"I completed each table of values for D and I, found the constant change in D and the rule and graphed this."

TABLE

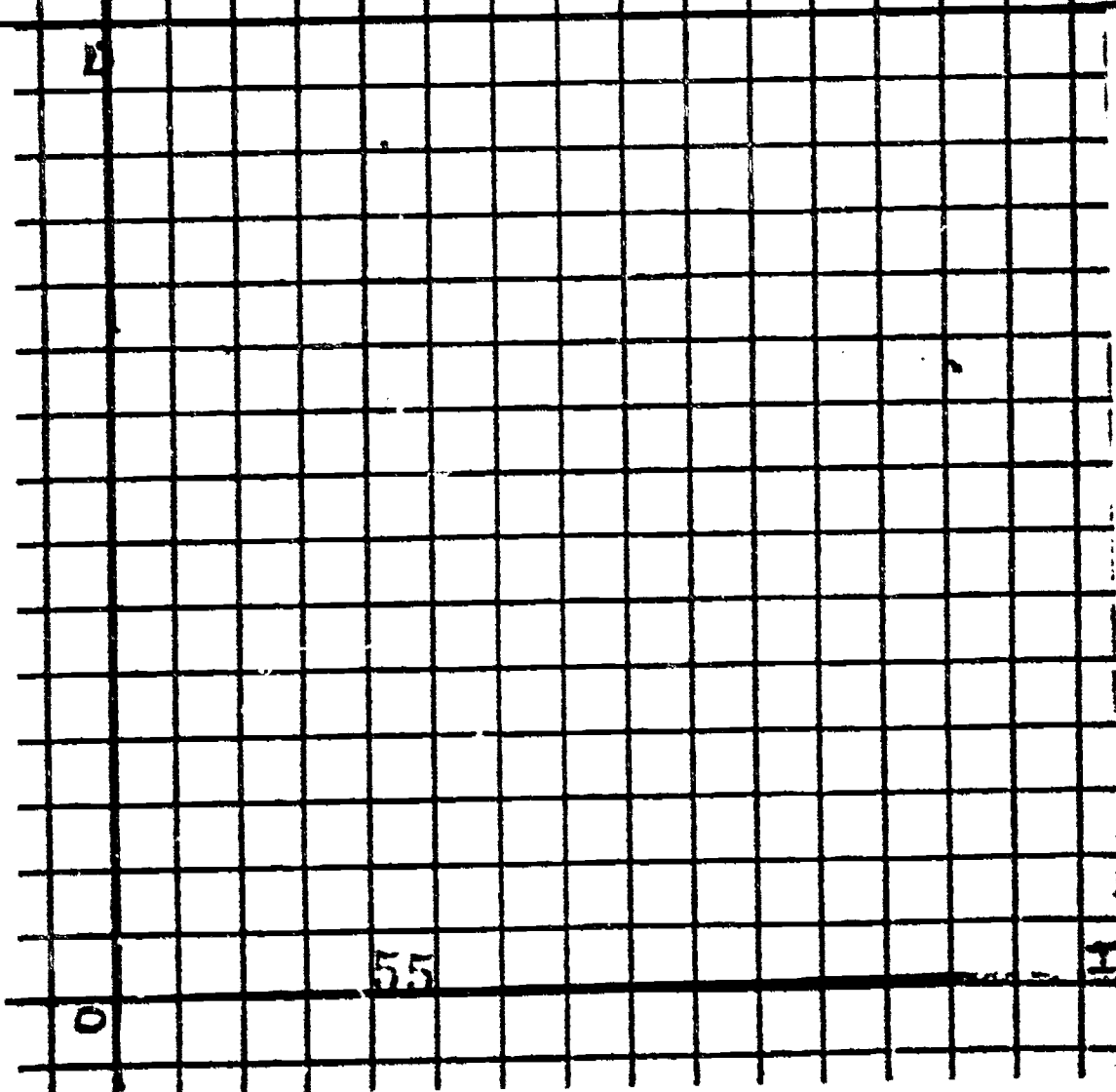
I	D	Change in D
0	3	
		2
		2
		2
		2
		2

GRAPH



**RULE:**

I	D	Change in D
1	5	4
2	9	4
		4



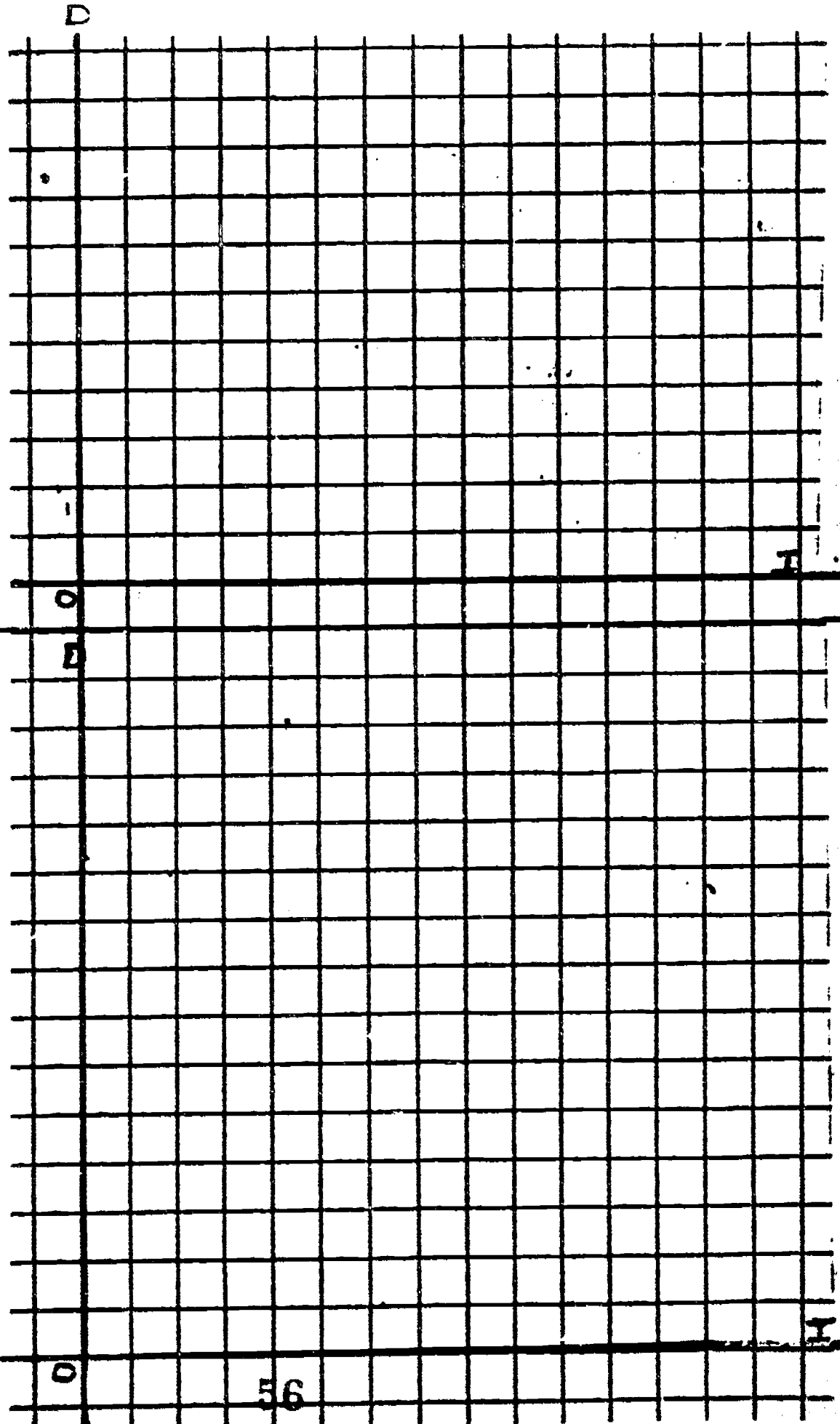
Mathematician: \_\_\_\_\_

"I completed each table of values for D and I, found the constant change in D and the rule and graphed this."

TABLE

I	D	Change in D
0	0	
1	3	
2	6	
3	9	

GRAPH



RULE:

I	D	Change in D
0	-2	
1	1	3



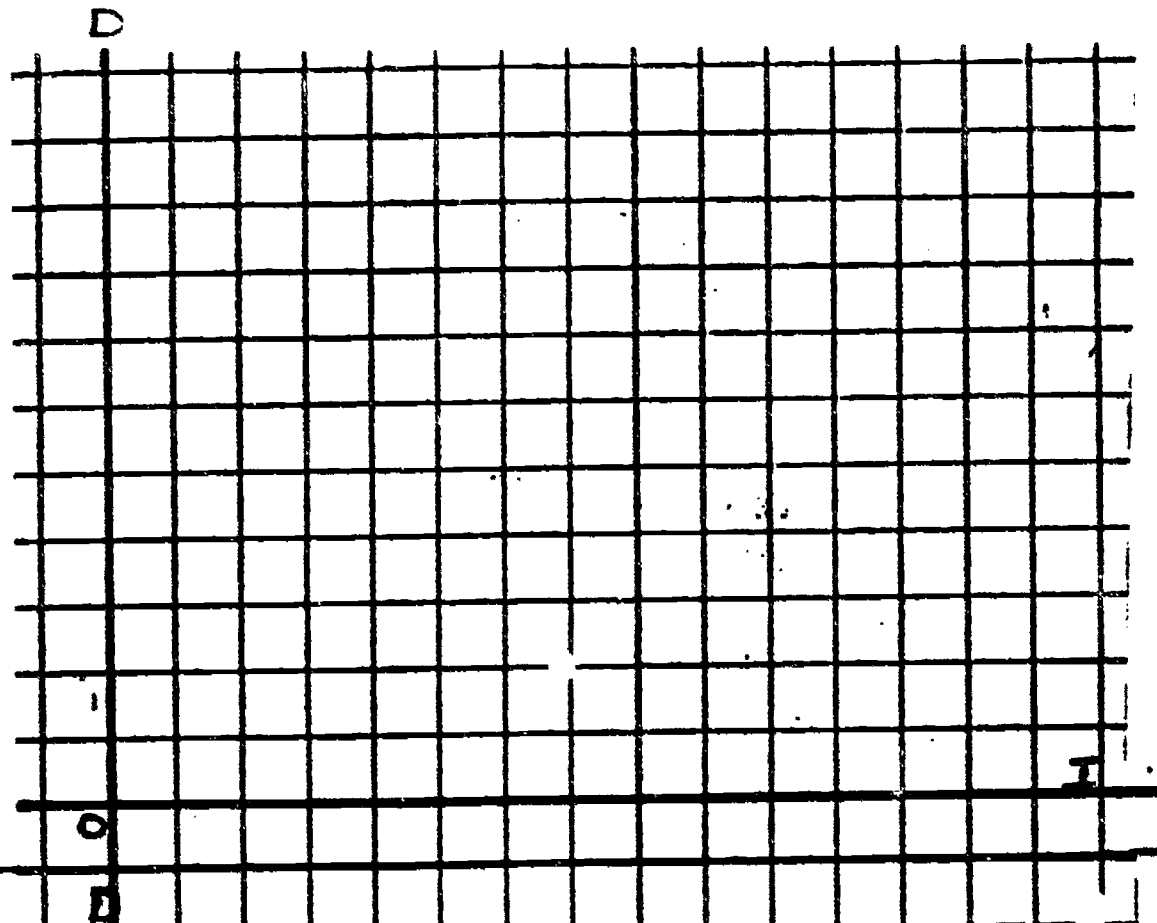
Mathematician: \_\_\_\_\_

"I completed each table of values for D and I, found the constant change in D and the rule and graphed this."

TABLE

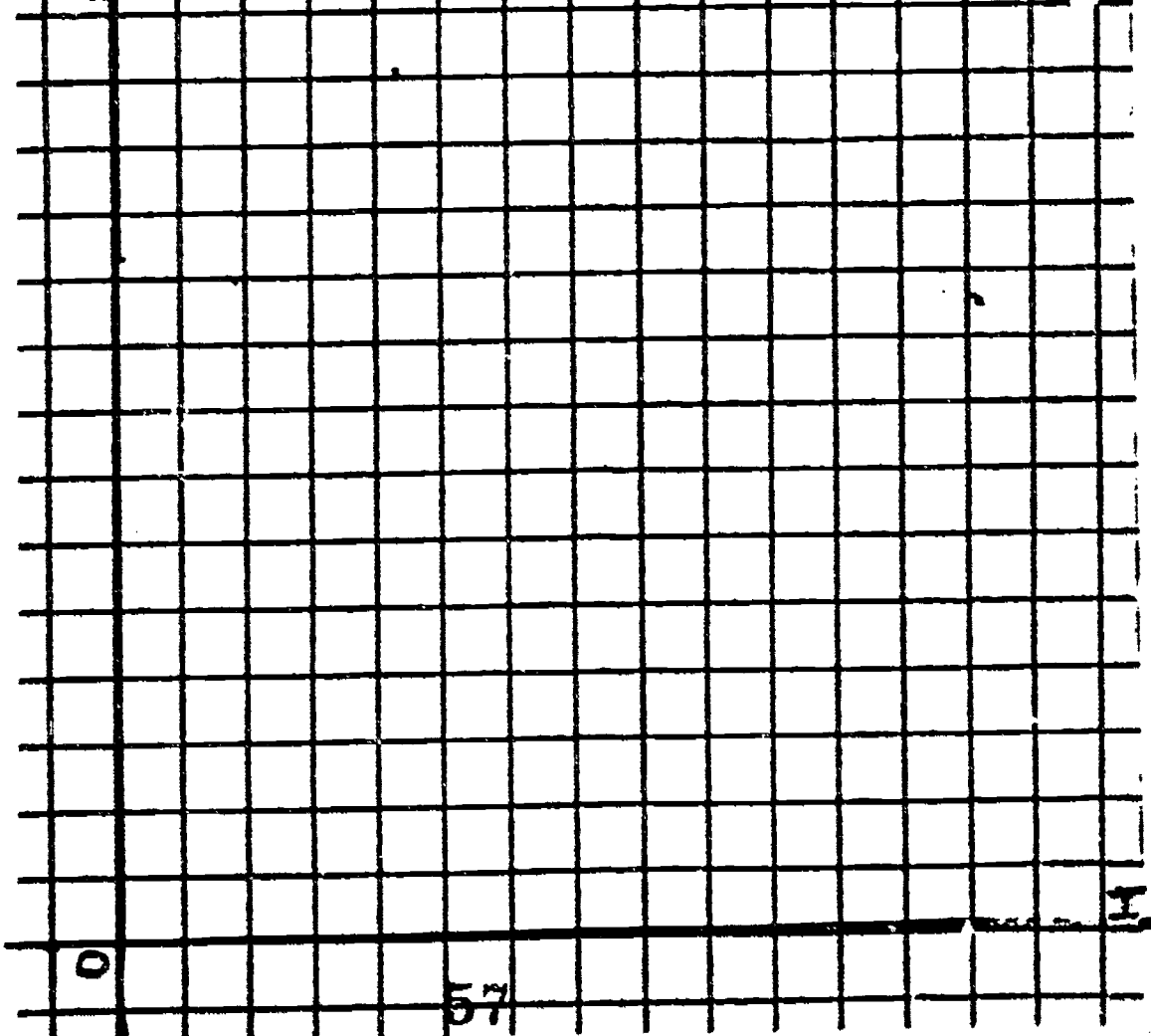
I	D	Change in D
4	5	
5	$5\frac{1}{2}$	
6	6	

GRAPH



RULE:

I	D	Change in D
0	0	
1	.1	
2	.2	
3	.3	
4	.4	
5	.5	



Mathematician: \_\_\_\_\_

"I completed the table for these ratios and found the missing term in the proportions."

"There are 3 blue buttons for every 5 brown buttons in a box."

Blue buttons	3										
Brown	5										
Total	8										

6:10 = 15 :

20:32 = 25 :

15:  = 21:56

:40 = 30:50

"For every 2 students with brown hair in Northern Elementary school, one can find 3 students with blonde hair."

Brown Hair	2										
Blonde Hair	3										
Total	5										

10:30 =  :21

8:24 =  : 30

6:15 = 10:

15:25 =  : 50

Mathematician: \_\_\_\_\_

"A grocer stocks 6 loaves of white bread for every loaf of rye bread."

White bread											
Rye Bread											
Total											

$$3: \boxed{\phantom{00}} = 5: 30$$

$$30 : 5 = \boxed{\phantom{00}} : 6$$

$$\boxed{\phantom{00}} : 14 = 24: 28$$

$$4 : 28 = 5 : \boxed{\phantom{00}}$$

"Tom has 4 red marbles for every 7 green marbles."

Red marbles											
Green marbles											
Total											

$$8: 14 = 12: \boxed{\phantom{00}}$$

$$7 : 4 = 21 : \boxed{\phantom{00}}$$

$$4 : 11 = \boxed{\phantom{00}} : 66$$

$$11 : 7 = \boxed{\phantom{00}} : 28$$

Mathematician:

"I found totals using the rates given. I cancelled units to get the correct units for the totals."

RATE

TABLE OF TOTALS

50¢ per box

Units EXAMPLE:

boxes	number of things	1	2	3	4	5	6	7
¢	Total	50	100	150	200	250	300	350

12 in. per foot

Units

	number of things							
	Total							

3 ft. per yard

Units

	number of things							
	Total							

5 boys per room

Units

	number of things							
	Total							

18 mi per gal.

Units

	number of things							
	Total							

3 tsp per cup

Units

	number of things							
	Total							

Mathematician:

"I found totals using the rates given. I cancelled units to get the correct units for the totals."

RATE

TABLE OF TOTALS

Units

8 strings  
per  
guitar

	number of things							
	Total							

Units

60 mi  
per  
hour

	number of things							
	Total							

Units

\$5 per  
lb.

	number of things							
	Total							

Units

6 lbs  
per  
day

	number of things							
	Total							

Units

7 grams  
per  
liter

	number of things							
	Total							

Units

3 oz.  
per  
cup

	number of things							
	Total							

Mathematician: \_\_\_\_\_

"For the situations given I completed the table, wrote the equation and the ratio."

Example:

"Tuna costs 60¢ per can."

¢	60	120	180	240	300	360	420	480	540	600
cans	1	2	3	4	5	6	7	8	9	10

$$\text{¢} : \text{cans} = 60 : 1$$

$$\text{Total ¢} = \frac{60\text{¢}}{\text{can}} \times \boxed{\phantom{00}} \text{ cans}$$

"Tom's sister runs a steady 8 meters/sec."

Meters										
Seconds										

$$\text{meters} : \text{seconds} = 8 : 1$$

$$\text{Total meters} = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} \times \boxed{\phantom{00}} \text{ —————}$$

"Jerry bought apples that cost 25¢ each."

¢									
Apples									

Ratio

Equation

"A paint formula calls for 1/2 oz. of pigment "A" for each quart of paint.

oz. pigment A		1/2							
paint	1 pt.	1 qt.	2 qts.	3 qts.	1 gal.	5 qts.	6 qts.	7 qts.	2 gal.

ratio oz. pigment : qts. paint = 1 : 2

equation

"An outboard engine calls for 4 oz. of oil for each gallon of gasoline."

oz. oil									
gal. gas.									

Ratio oz. oil " : gal. gasoline =  :

"Joyce gets \$2 per hour for babysitting."

hours							
Total pmt.							

Ratio \$ : hours =

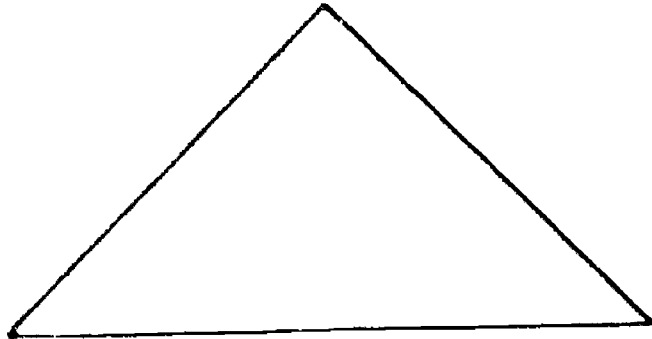
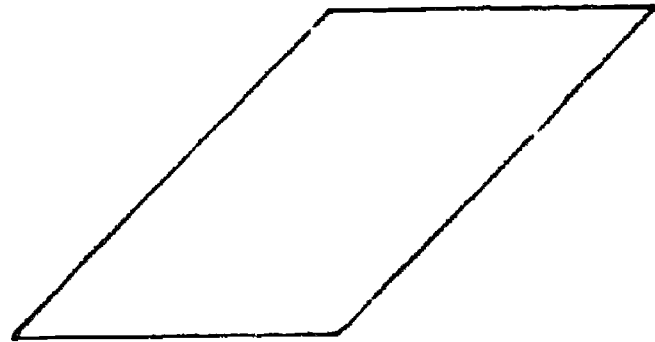
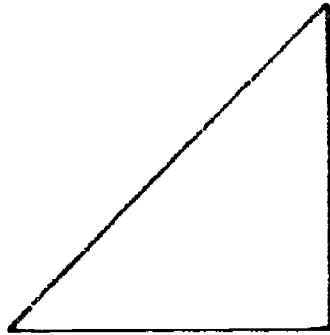
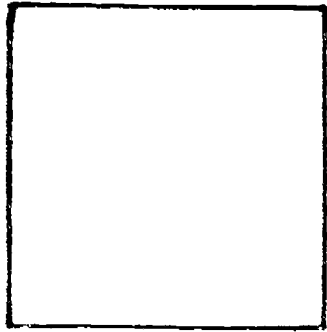
Equation

Mathematician: \_\_\_\_\_

"I wrote the rule for this set of shapes."

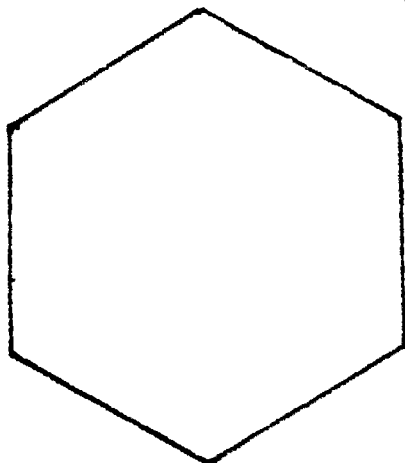
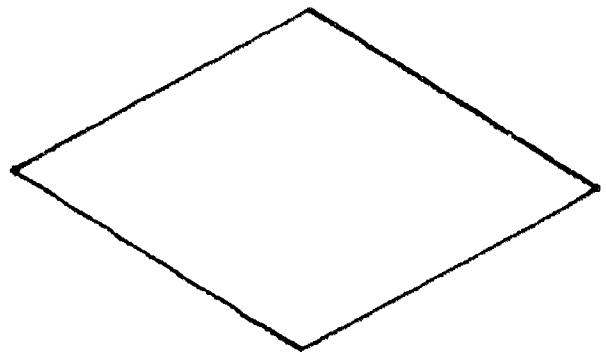
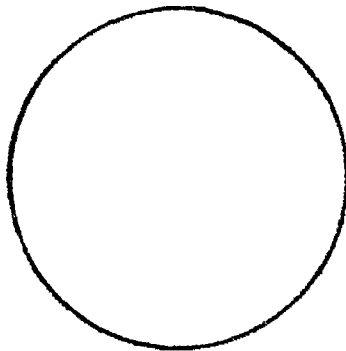
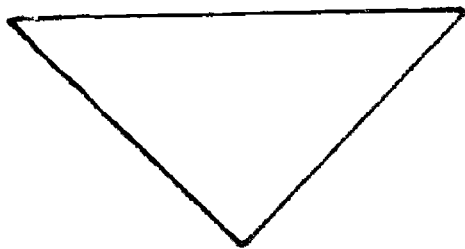
**THESE SHAPES FOLLOW THE RULE**

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**THESE SHAPES DO NOT FOLLOW THE RULE**



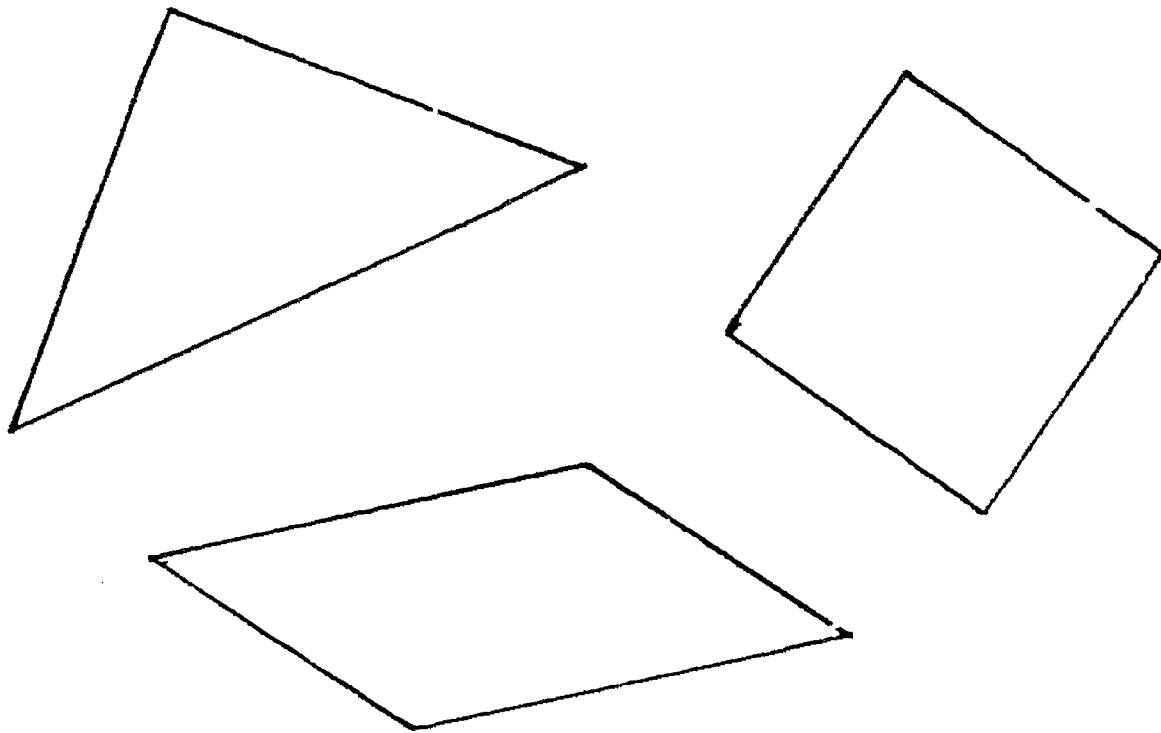


Mathematician: \_\_\_\_\_

"I wrote the rule for this set of shapes."

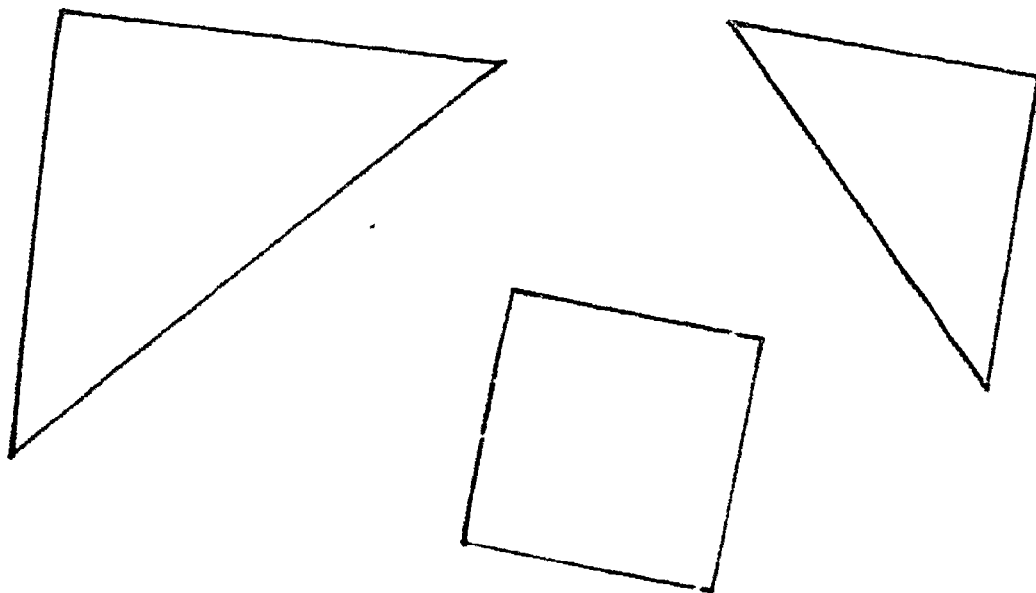
**THESE SHAPES FOLLOW THE RULE**

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**THESE SHAPES DO NOT FOLLOW THE RULE**

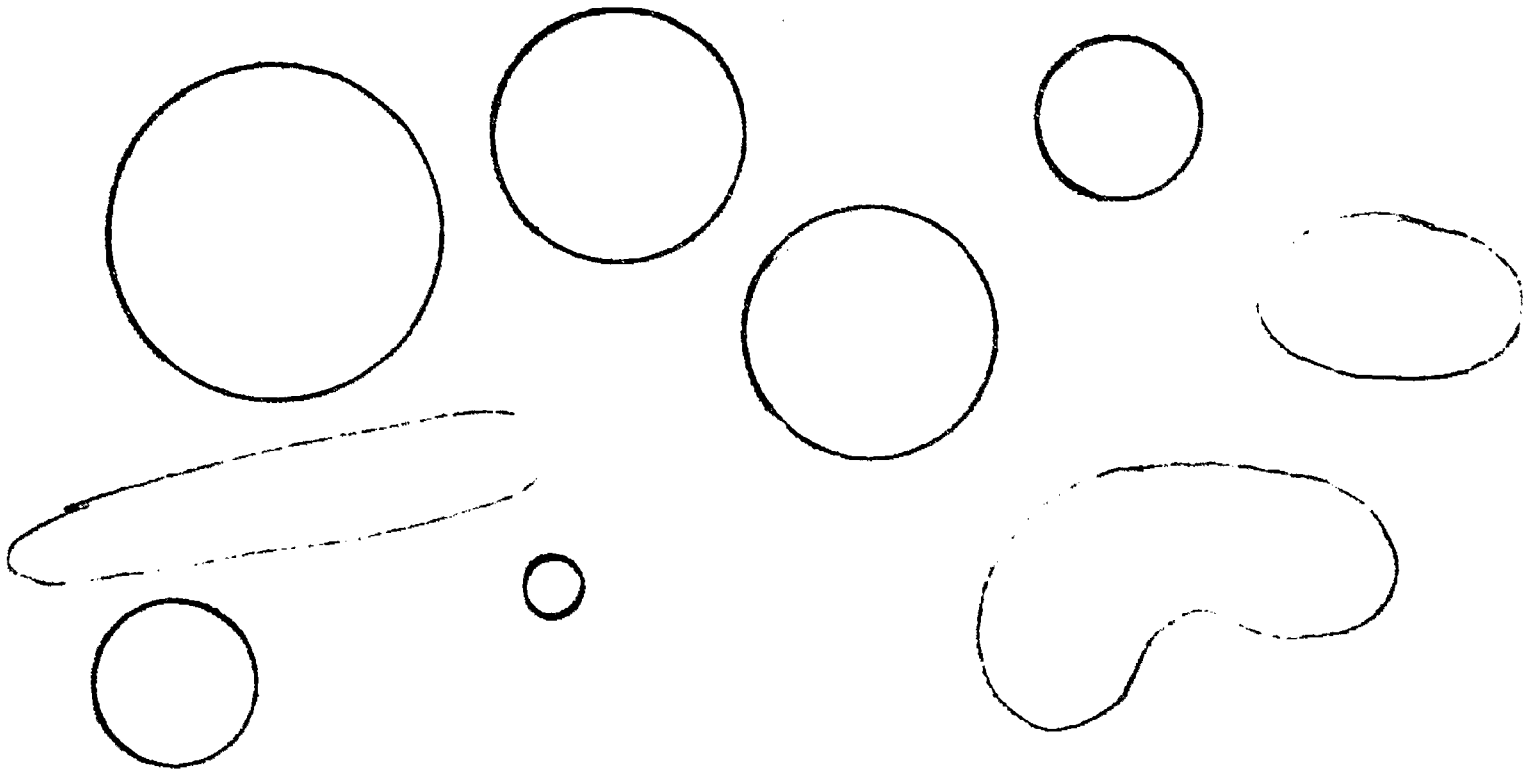


Mathematician: \_\_\_\_\_

"I wrote the rule for this set of shapes."

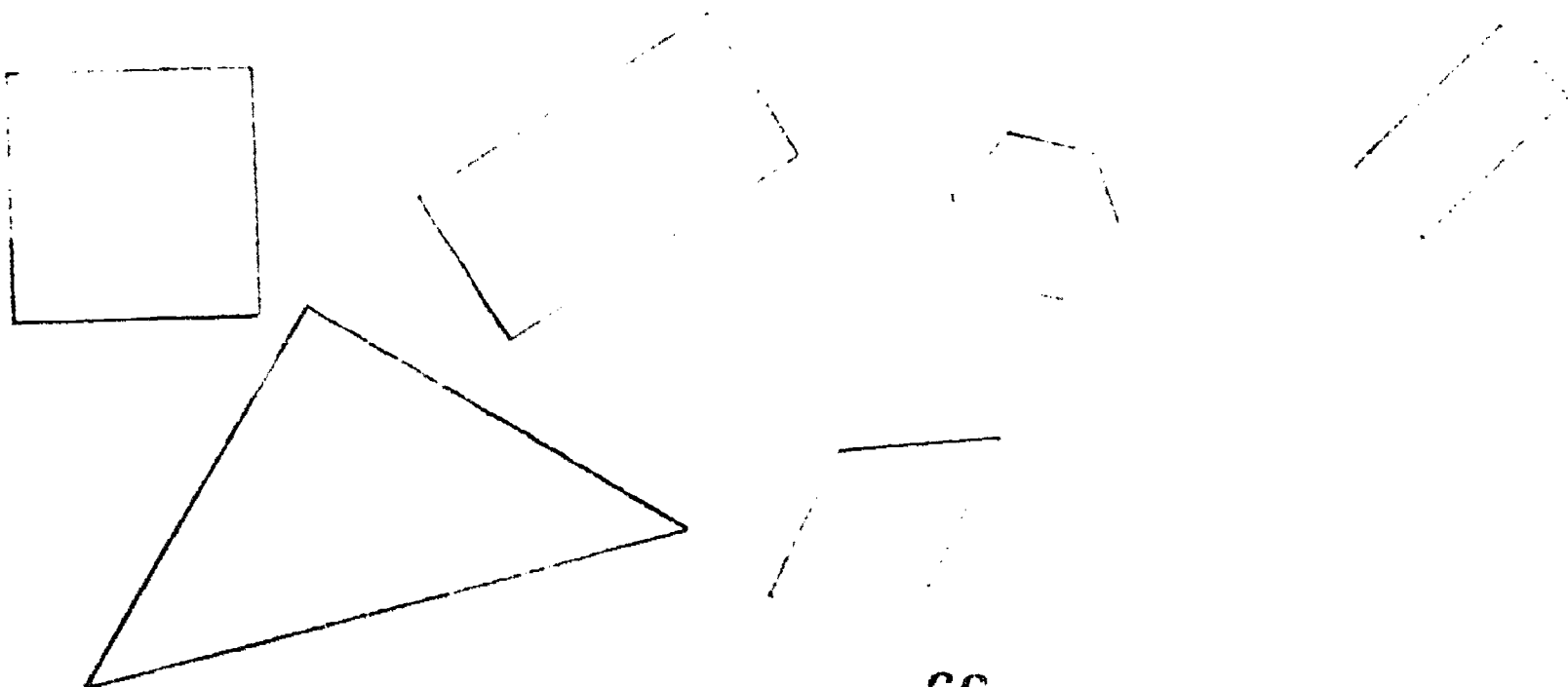
THESE SHAPES FOLLOW THE RULE

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THESE SHAPES DO NOT FOLLOW THE RULE



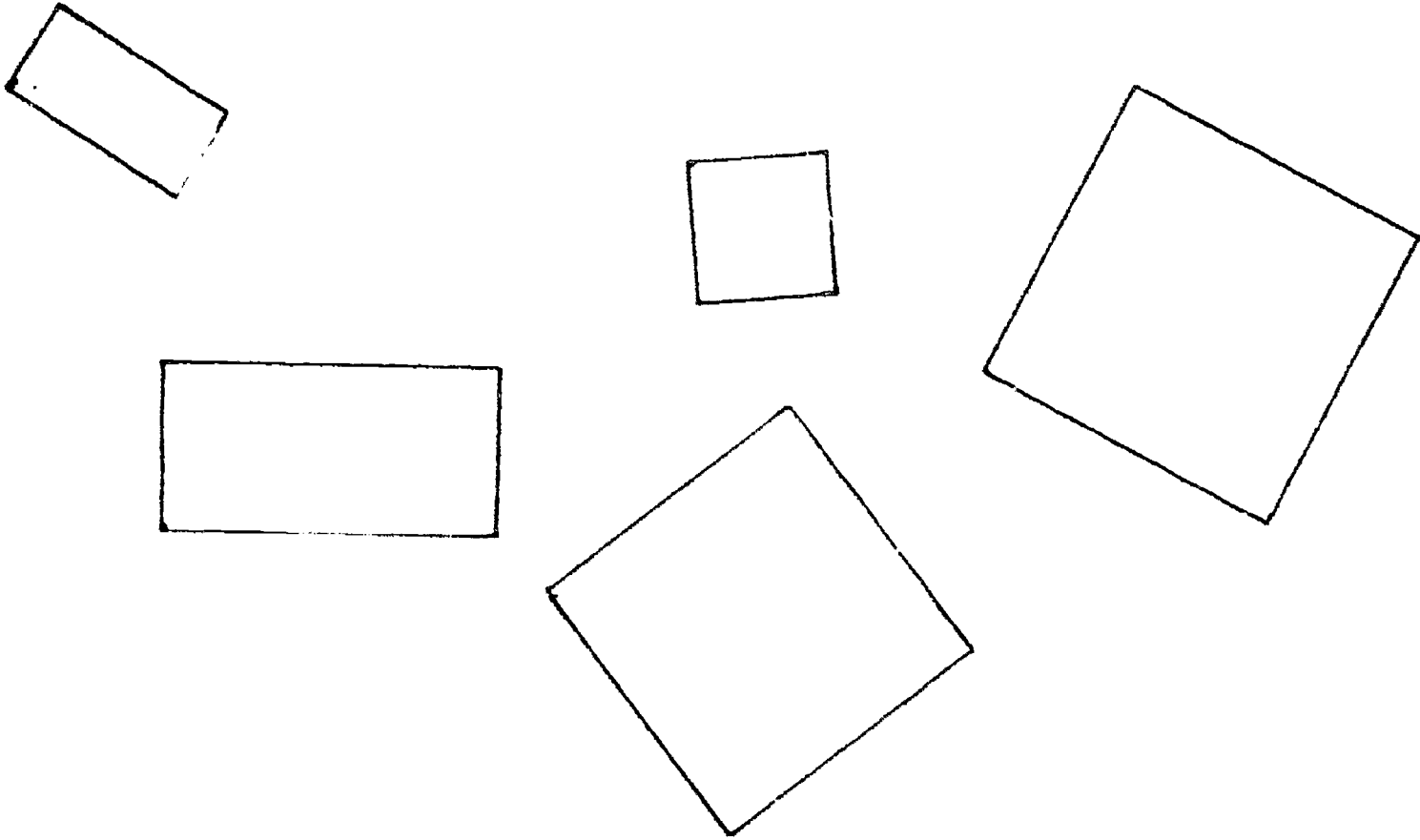
2-18-77

Mathematician: \_\_\_\_\_

"I wrote the rule for this set of shapes."

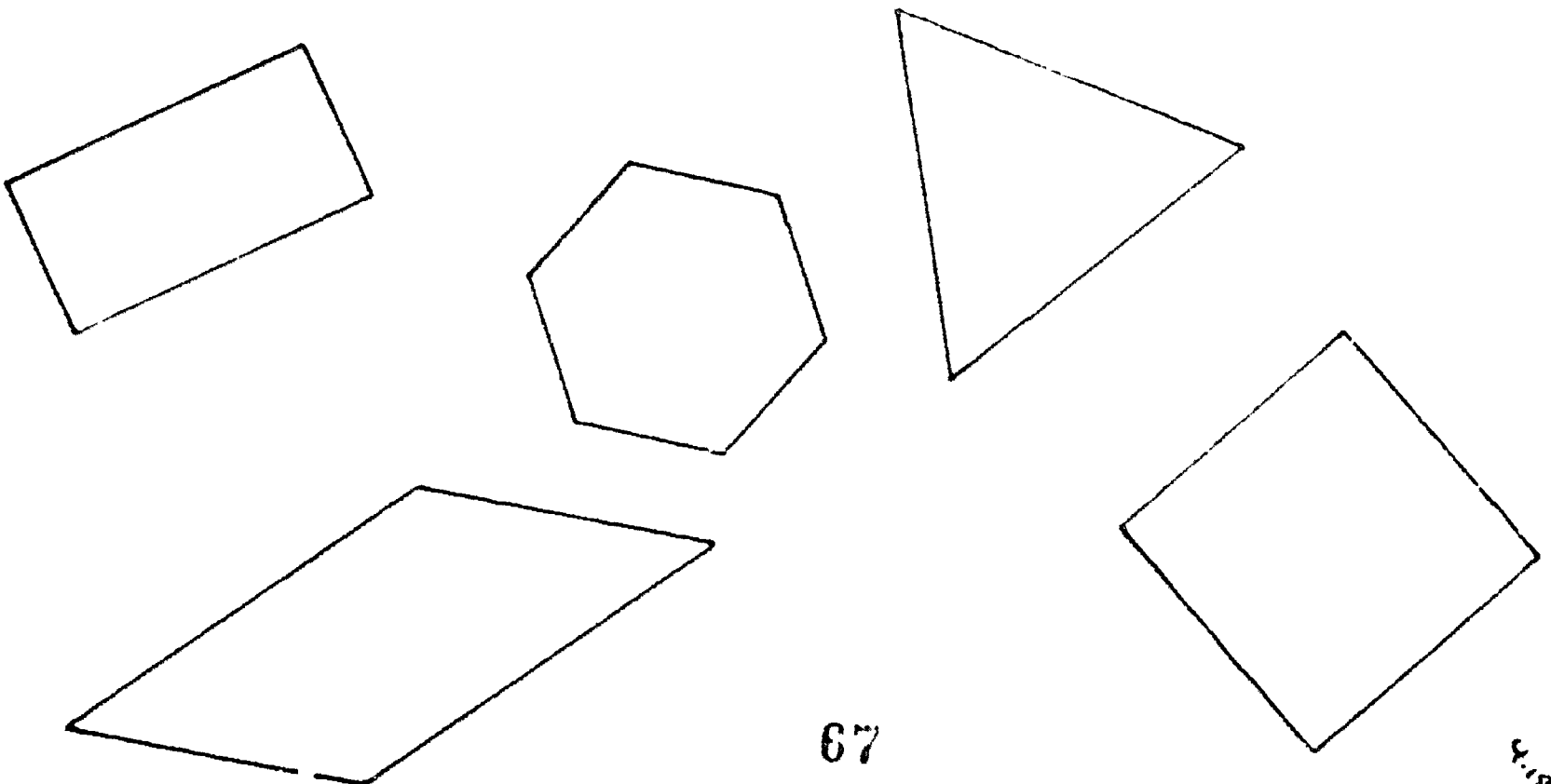
**THESE SHAPES FOLLOW THE RULE**

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**THESE SHAPES DO NOT FOLLOW THE RULE**

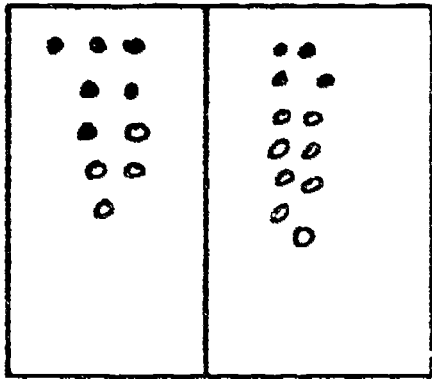


Mathematician: \_\_\_\_\_

"I wrote the equality or inequality shown by the split board with chips."

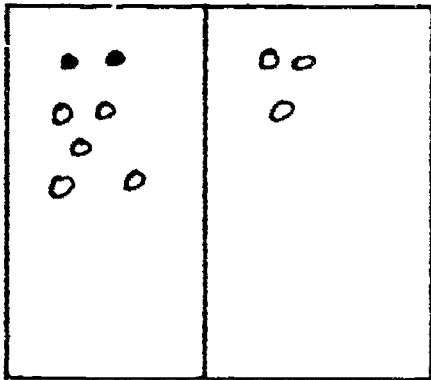
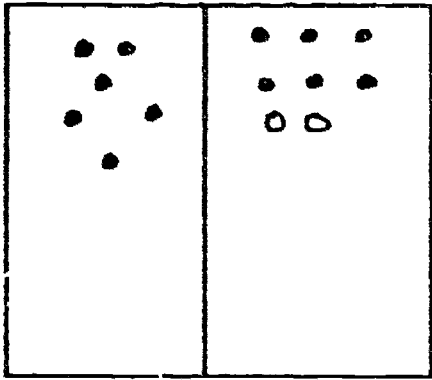
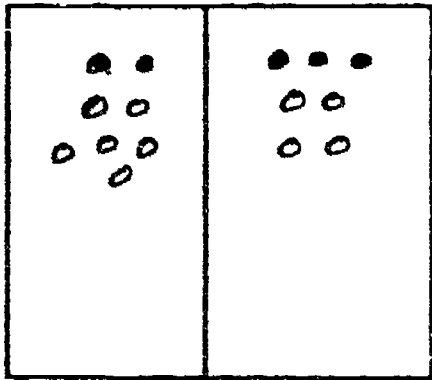
SPLIT BOARD

NUMBER STATION



Example

$$2 > -4$$

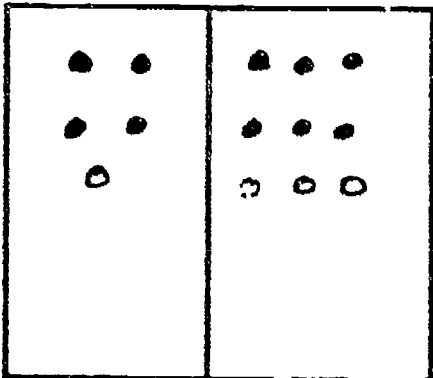
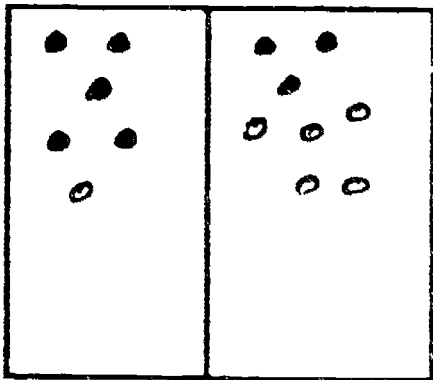
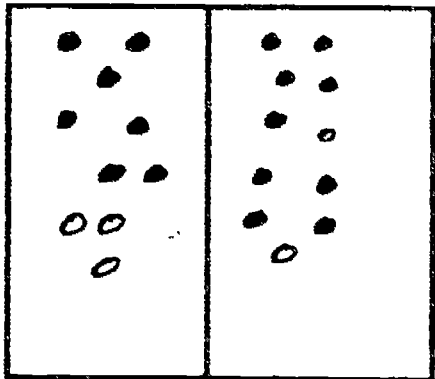
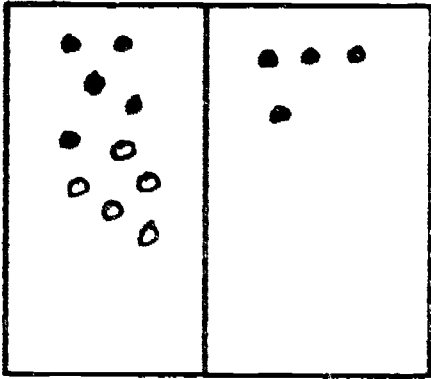


Mathematician: \_\_\_\_\_

"I wrote the equality or inequality shown by the split board with chips."

SPLIT BOARD

NUMBER STATION

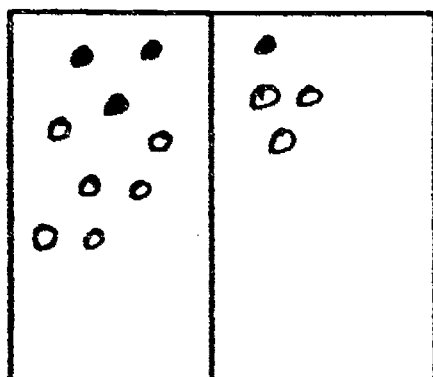
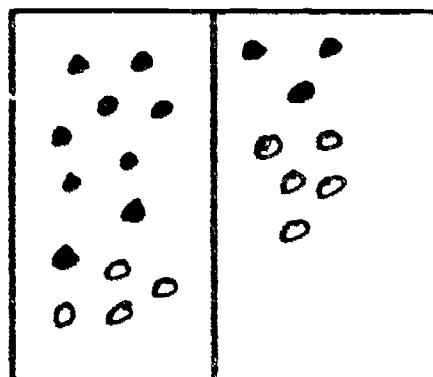
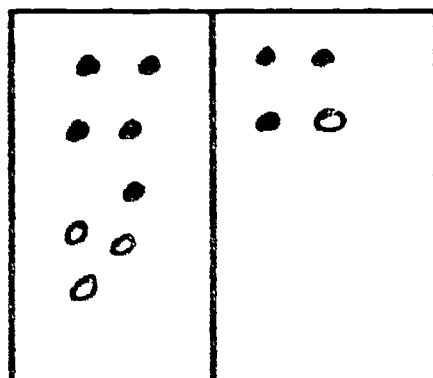
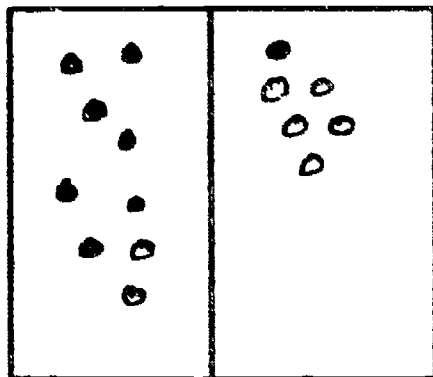


Mathematician: \_\_\_\_\_

"I wrote the equality or inequality shown by the split board with chips."

SPLIT BOARD

NUMBER STATION



Mathematician: \_\_\_\_\_

"I added or subtracted the number as shown to get a new equation or inequality."

NUMBER SENTENCE	HOW TO CHANGE	NEW SENTENCES
Example: $4 < 7$	$-2$	$2 < 5$
$12 = \square + 3$	$-3$	$9 = \square$
$6 > -1$	$+2$	
$2\square + 5 = 15$	$-5$	
$9 > 2$	$+6$	
$-2 > -6$	$+4$	
$3\square - 4 = 18$	$+4$	
$0 > -2$	$-3$	
$5 > 3$	$-3$	

Mathematician: \_\_\_\_\_

"I added or subtracted the number as shown to get a new equation or inequality."

NUMBER SENTENCE	HOW TO CHANGE	NEW SENTENCES
$9 = 2\square + 1$	$-1$	
$8 > 2$		$6 > 0$
$\square + 5 = 13$		$\square = 8$
$9 > 1$	$-5$	
$2 < 6$	$-2$	
$-1 < 4$	$+3$	
$-3 < 2$	$-4$	
$5 < 12$	$-4$	
$8 < 17$	$-7$	



Mathematician: \_\_\_\_\_

"I completed these number sentences with signed numbers."

$$2\Box + (-4) = 10$$

$$\Box + 7 < 16$$

five numbers that fill the box are:

$$\Box - (-1) = 4$$

$$\Box - 3 > 9$$

five numbers that fill the box are:

$$8 = 2\Box - 6$$

$$7 > \Box + 3$$

five numbers that fill the box are:

$$15 > 12 - \Box$$

five numbers that fill the box are!

BE CAREFUL!

$$8 - \Box = 5$$

$$16 - 2\Box = 9$$

$$8 - \Box < 16$$

What is one number that CANNOT go in the box?

$$2\Box + 1 > 9$$

$$\Box - (-4) < 6$$

five numbers that fill the box are:

Mathematician: \_\_\_\_\_

"I completed these number sentences with signed numbers."

$$3\Box + (-1) = 14$$

$$6 + \Box < -7$$

Name five numbers that can go into the box!

$$18 = 4\Box + 2$$

$$5 < 2\Box - 1$$

Name five numbers that can go in the box!

$$8 > \Box + (-2)$$

Name five numbers that can go into the box!

$$21 = 2\Box + 17$$

$$3\Box + (-4) = 11$$

$$3\Box + \Box < 16$$

Name five numbers that can go in the box!

$$\Box + (-4) < 2$$

Name five numbers that can go in the box!

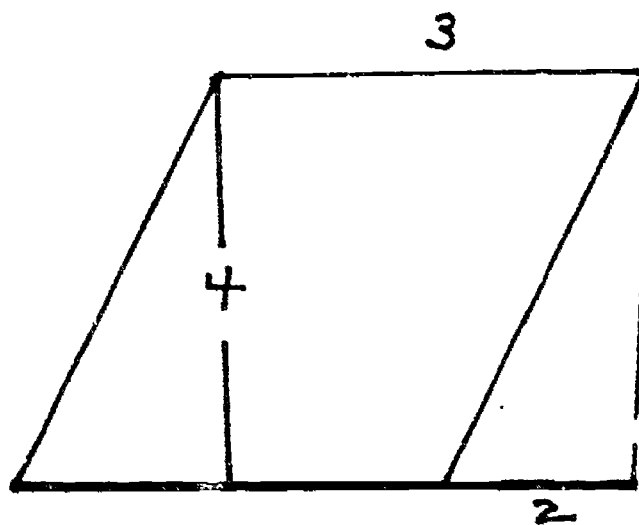
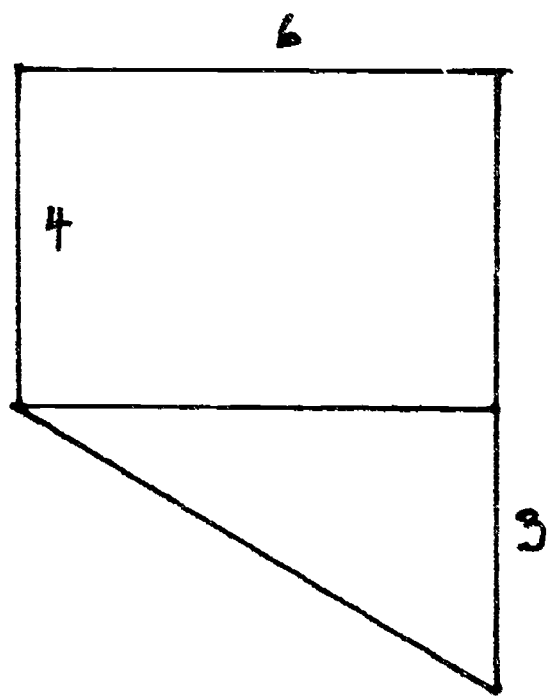
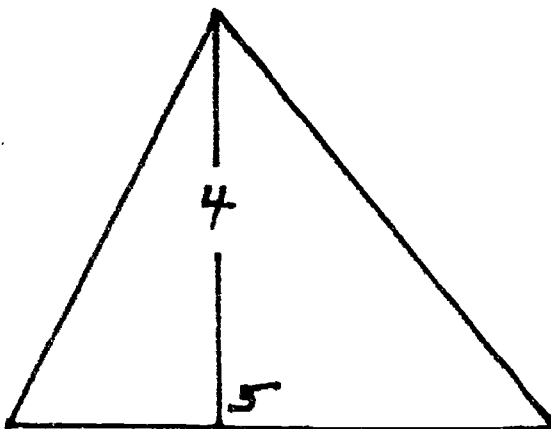
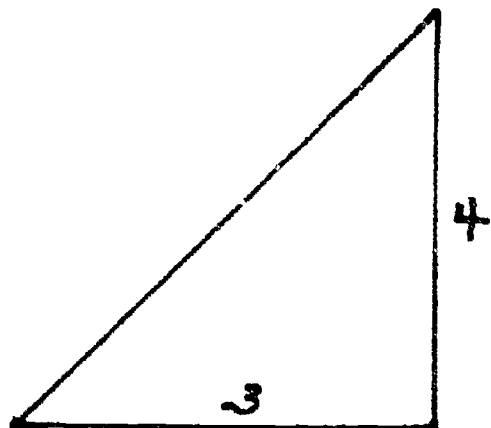
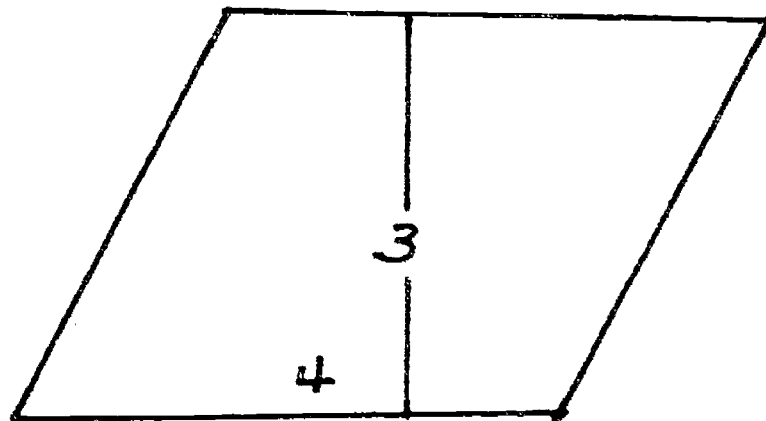
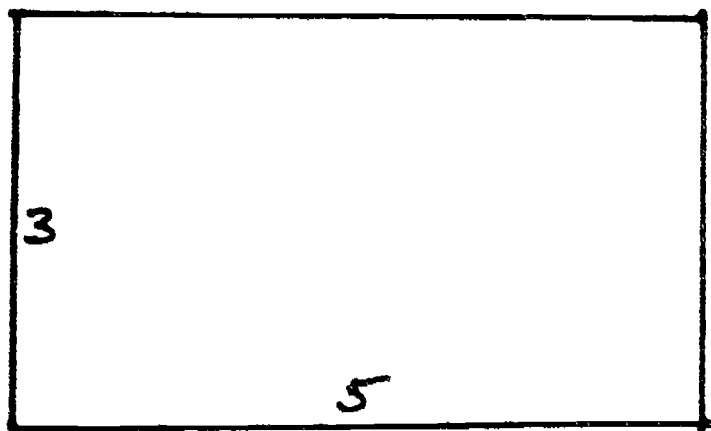
$$3\Box + 1 = 16$$

$$2\Box + (-4) = 14$$

$$0 > \Box + (-2)$$

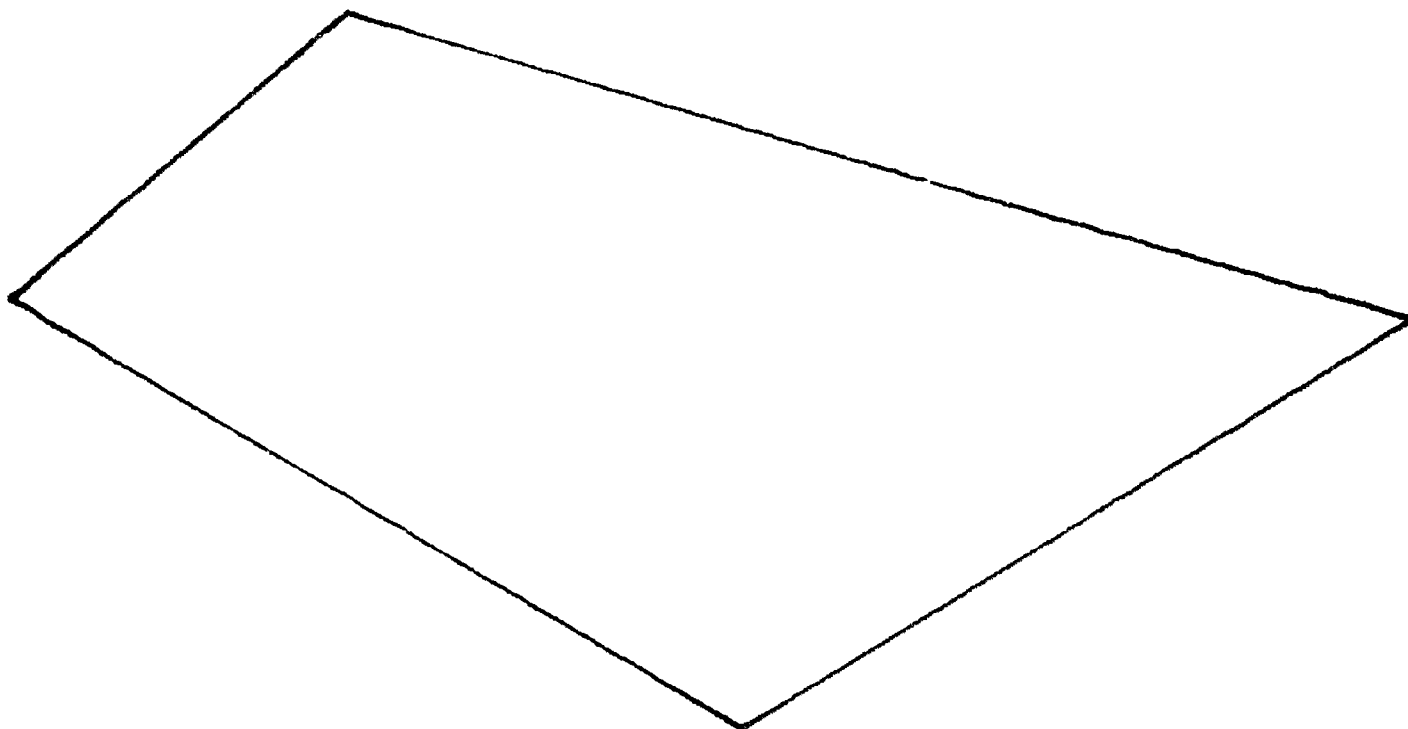
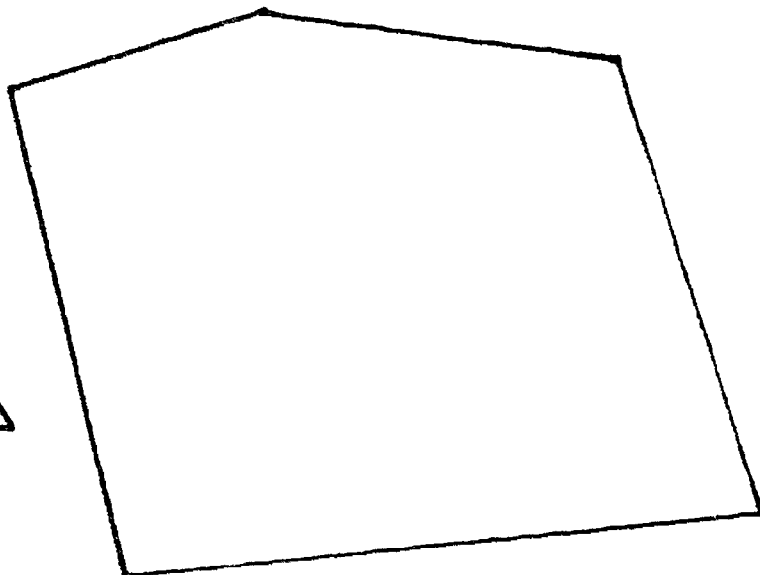
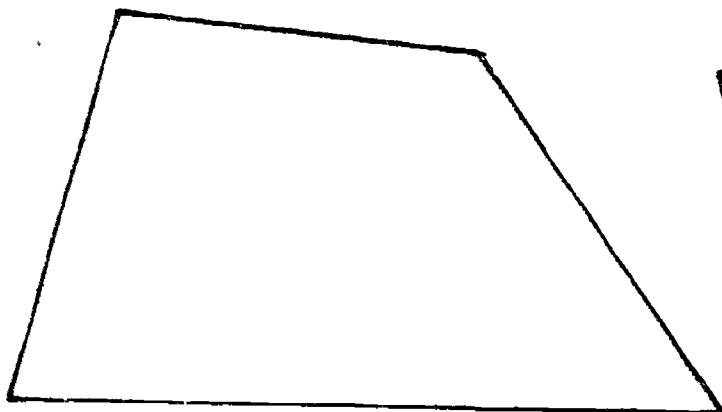
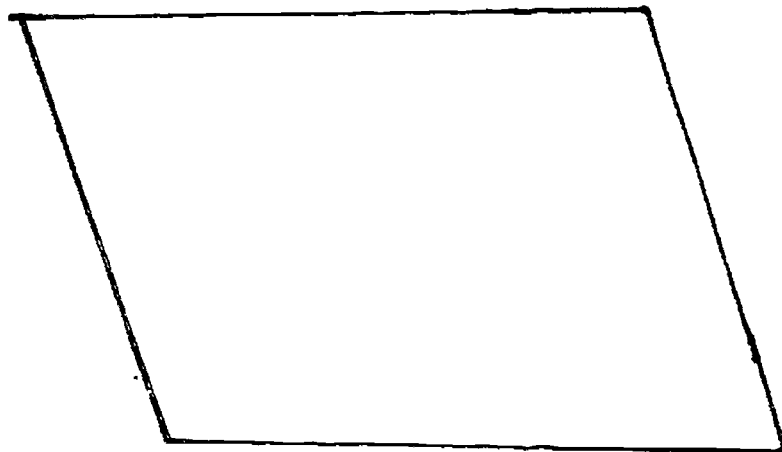
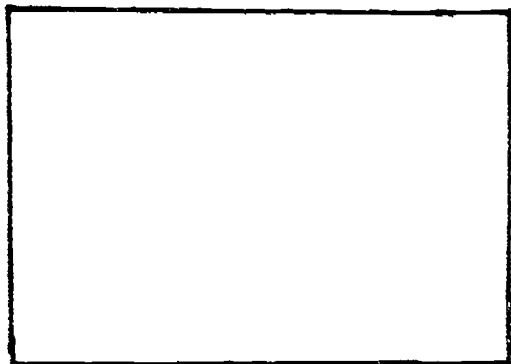
MATHEMATICIAN: \_\_\_\_\_

"I found the areas for the shapes given."



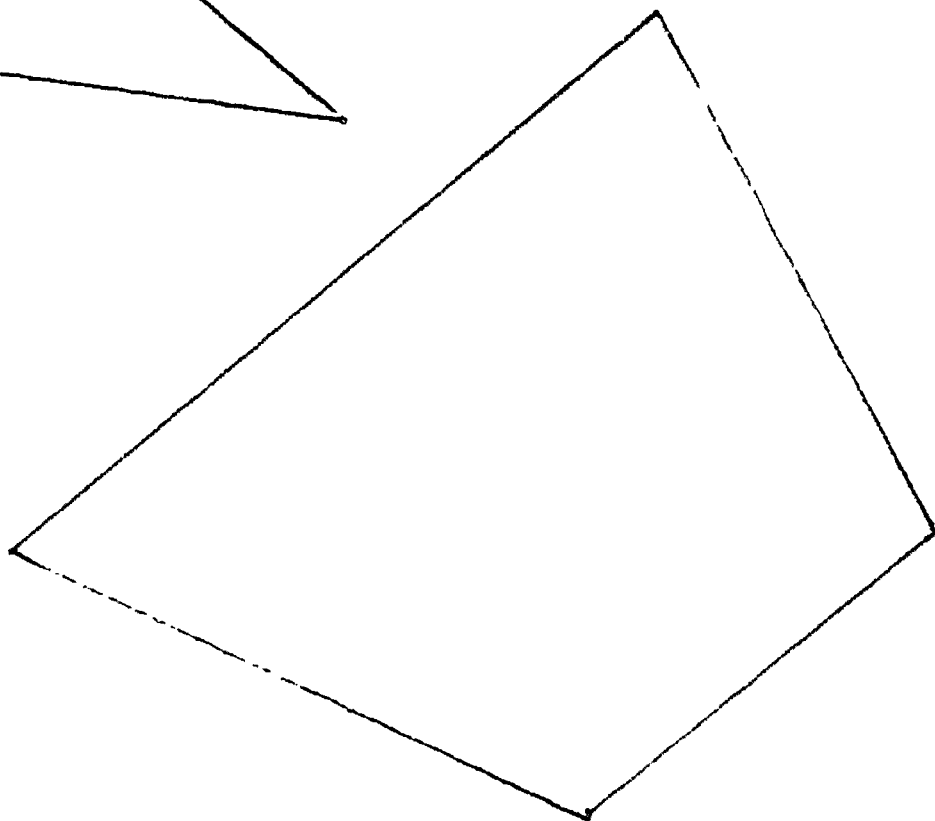
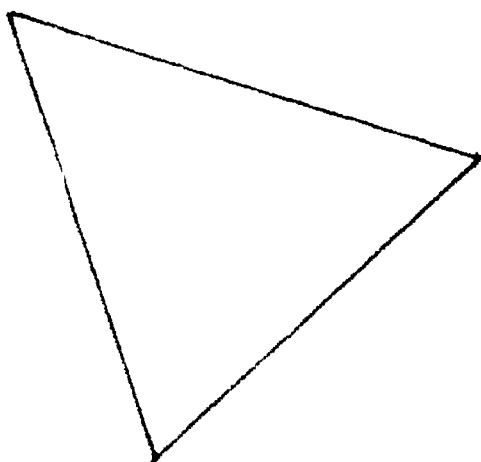
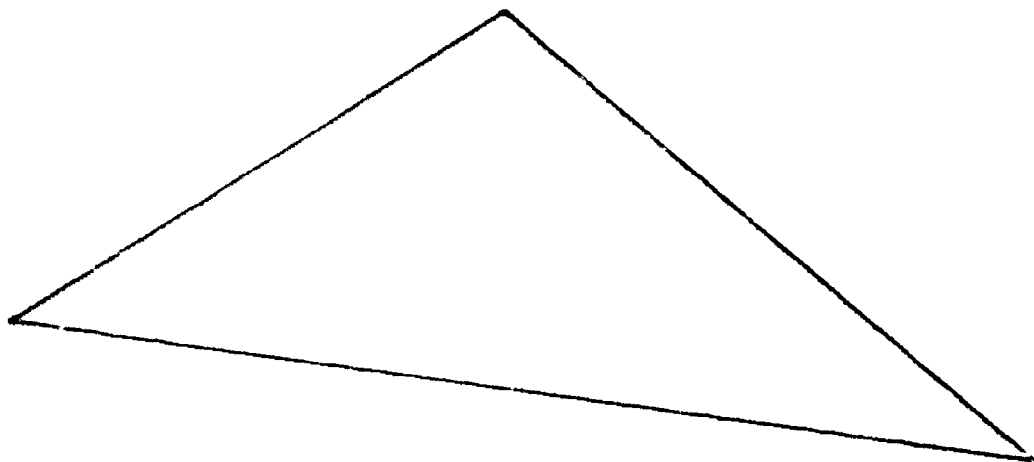
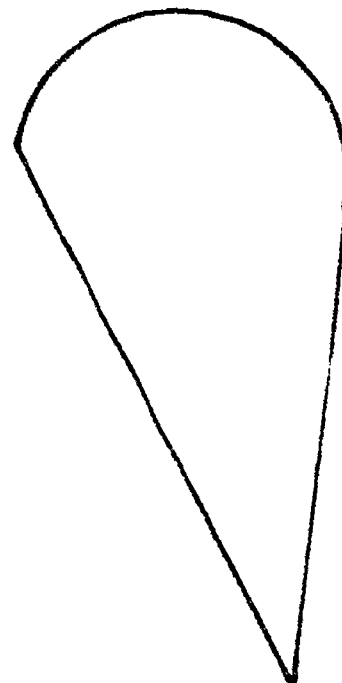
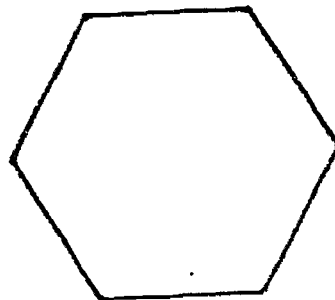
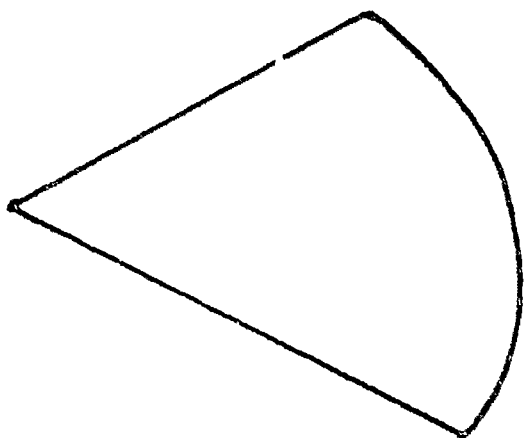
Mathematician: \_\_\_\_\_

"I used the graph paper transparency to find the areas of these rectangles and other parallelograms. I wrote the area on the shape."



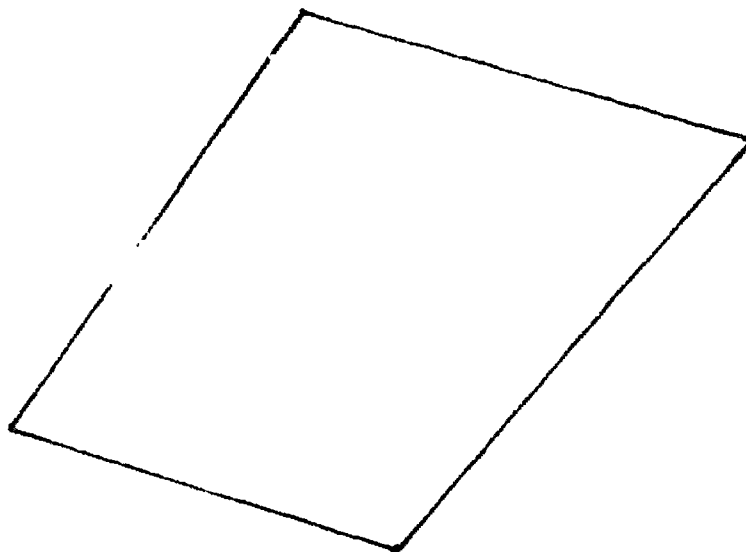
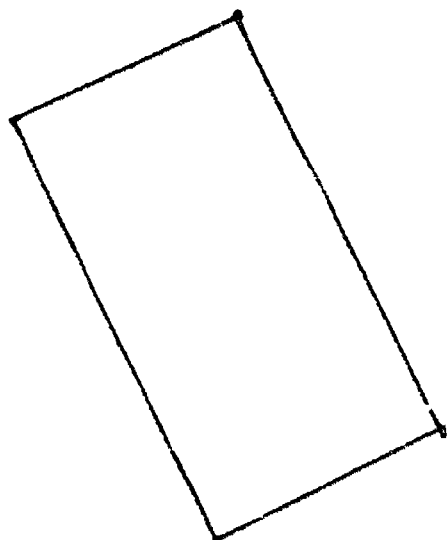
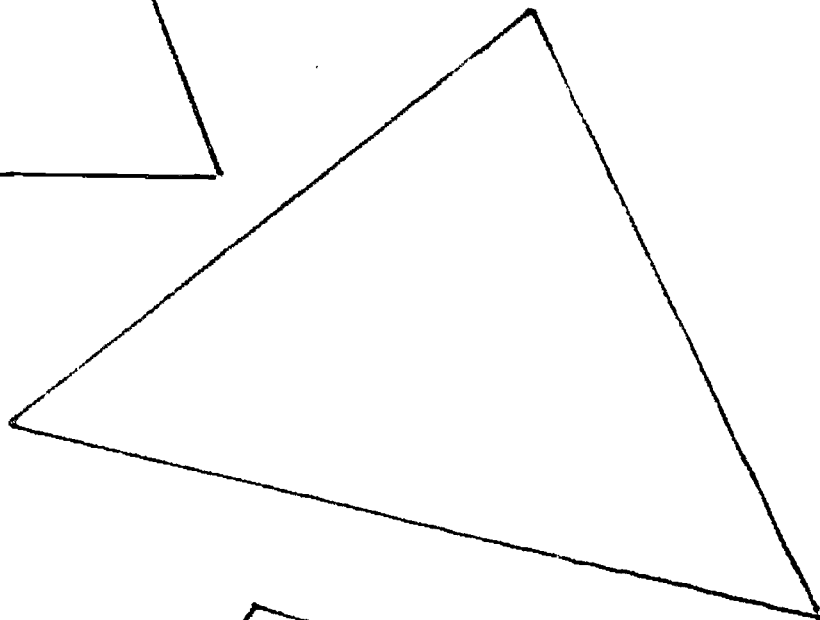
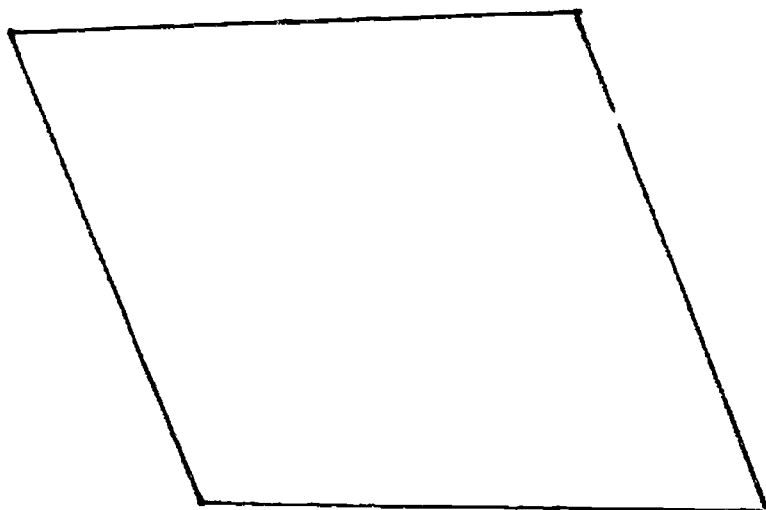
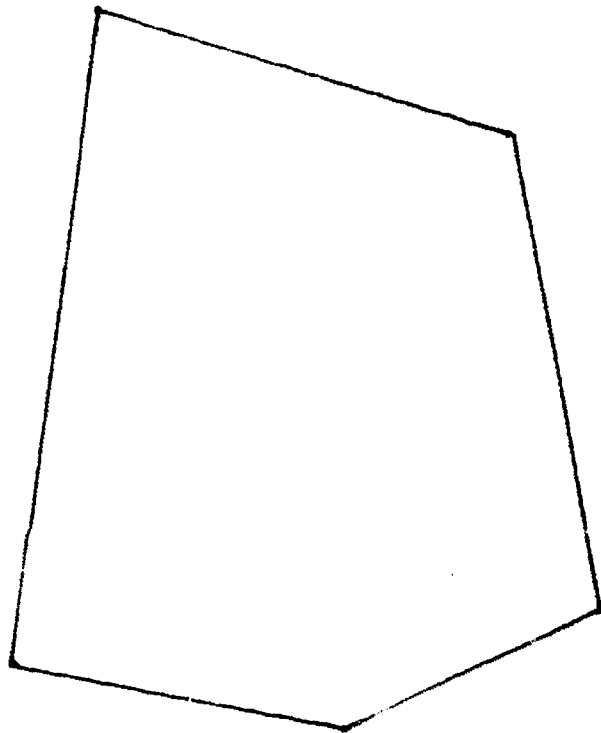
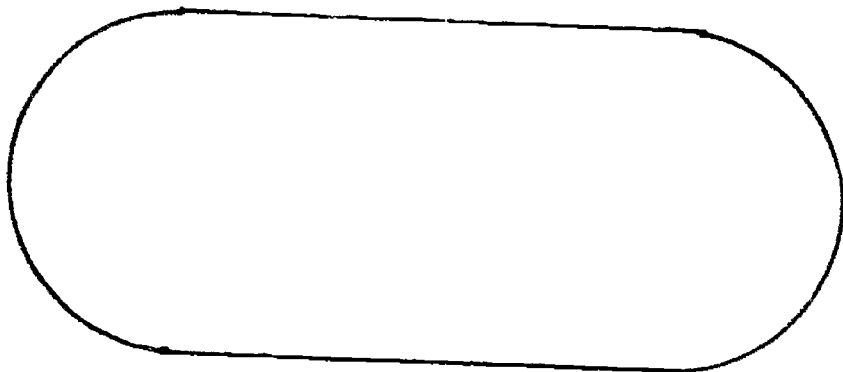
Mathematician: \_\_\_\_\_

"I used the graph paper transparency to find the areas of these rectangles and other polygons. I wrote the area on the shape."



**Mathematician:** \_\_\_\_\_

**"I used the graph paper transparency to find the areas of these rectangles and other parallelograms. I wrote the area on the shape."**



Mathematician: \_\_\_\_\_

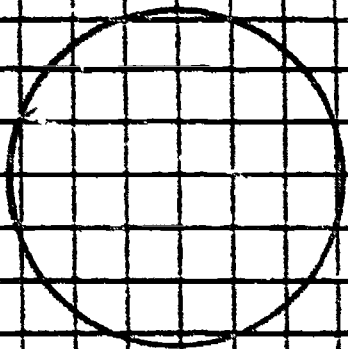
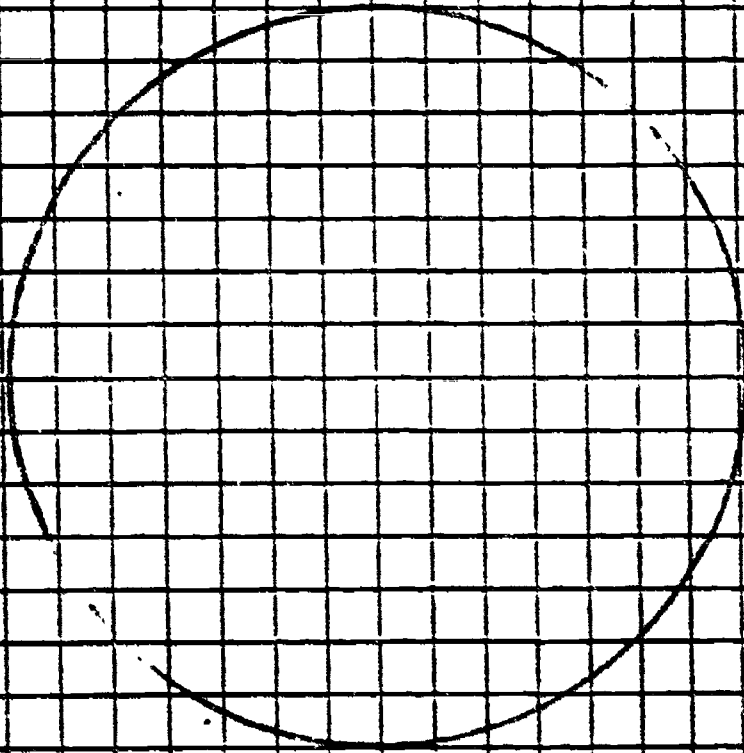
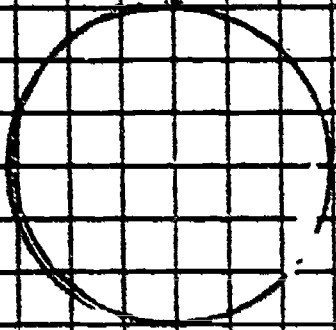
"For the circles given I found the area as best I could and calculated the radius. I put these all in the table."

CIRCLES

AREA

RADIUS

RADIUS<sup>2</sup>



Mathematician: \_\_\_\_\_

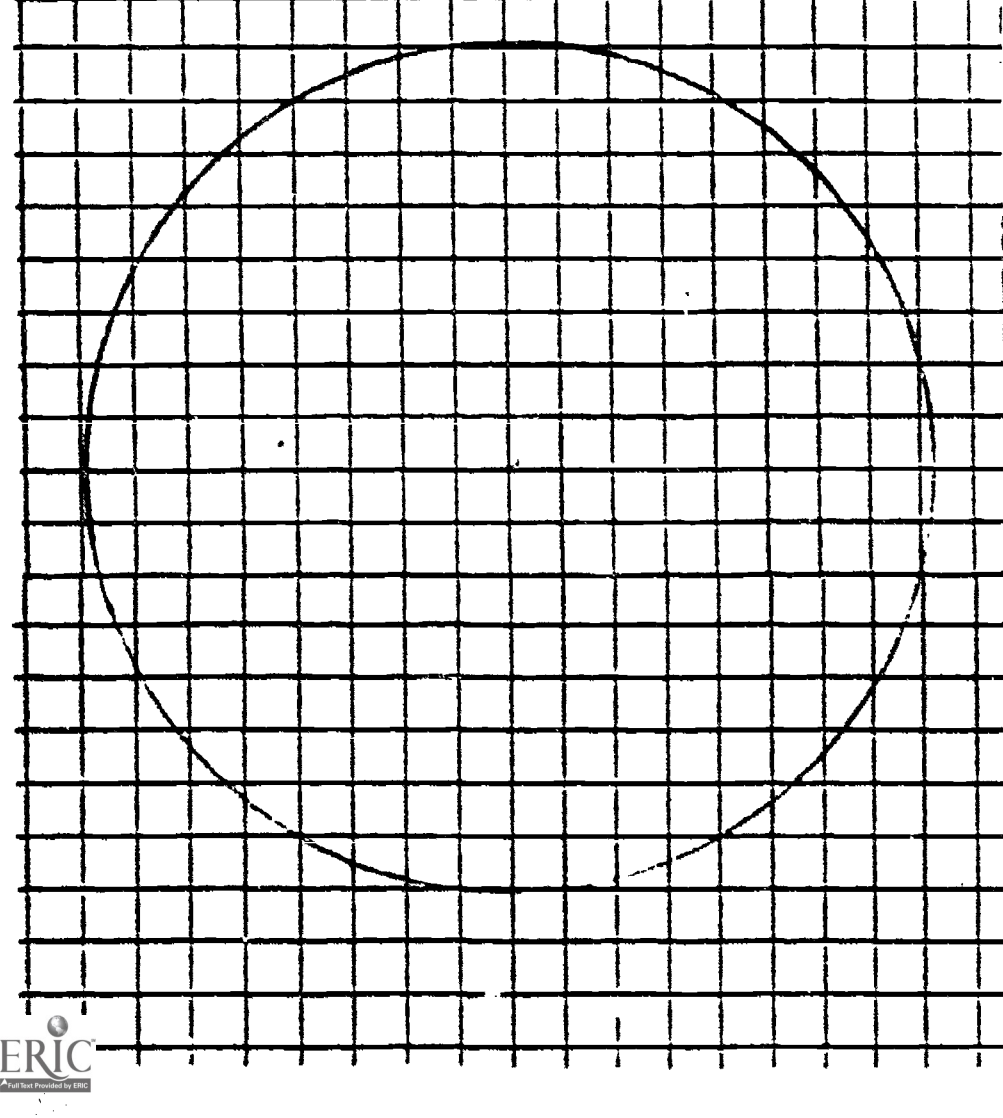
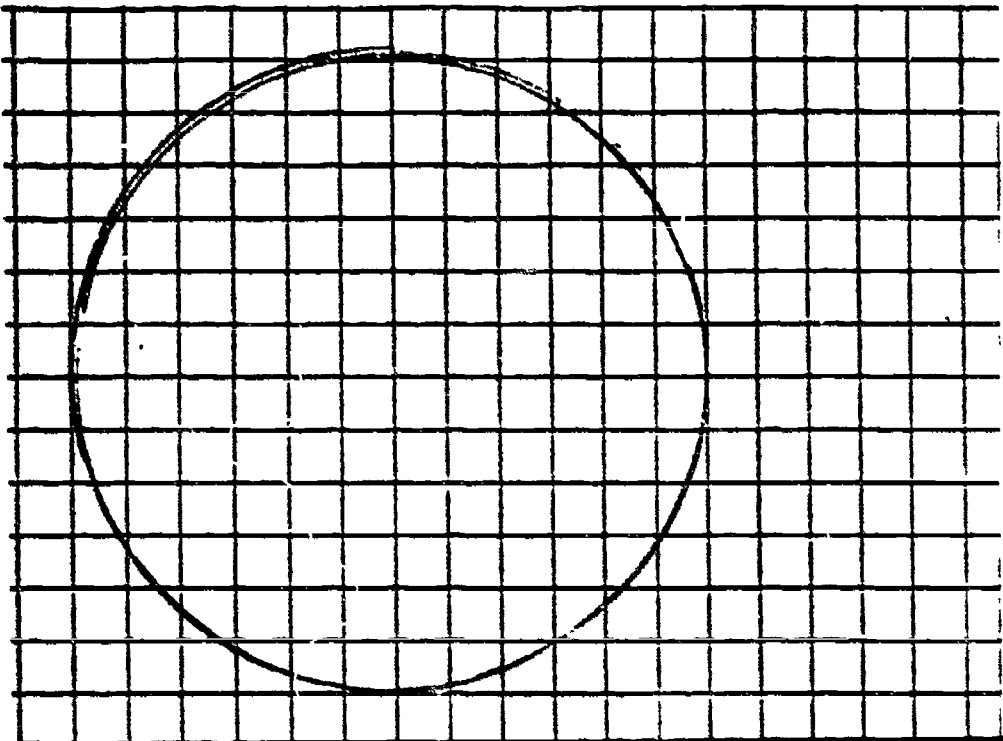
"For the circles given I found the area as best I could and calculated the radius. I put these all in the table."

CIRCLES

AREA

RADIUS

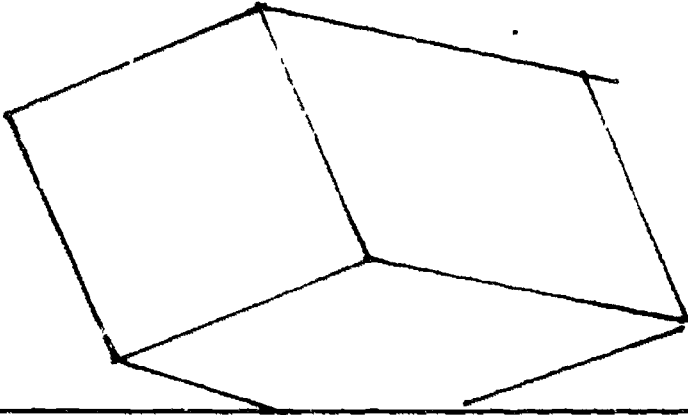
RADIUS<sup>2</sup>





Mathematician: \_\_\_\_\_

"Given the dimensions of the base and the height of solids, I found the volumes."

GIVEN DIMENSION	PICTURE OF SOLID	VOLUME
Base: 9 sq. in. height: 6 in.		$9 \times 6 = 54 \text{ cu in.}$
Base: Rectangle $4 \times 6$ Height: 3		
Base: Triangle base = 2 height = 3 height = 4		
Base: square $3 \times 3$ Height: 5		

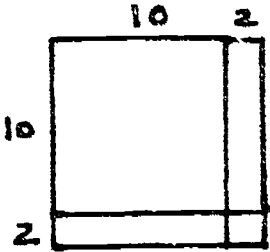
Mathematician: \_\_\_\_\_

"Given the dimensions of the base and the height of solids, I found the volumes."

GIVEN DIMENSION	PICTURE OF SOLID	VOLUME
Height: 6 Base: parallelogram base = 4 height = 3		
Height: = 12 Base: = rectangle 3 x 2		
Height: 9 Base: square 5 x 5		
Base: Circle area = 14.6 Height: 5		

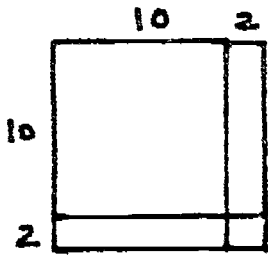
Mathematician: \_\_\_\_\_

"I built the squares from base ten pieces and made pictures of these."

SQUARE SIDE	PICTURE	COMPUTATION
EXAMPLE:		$12^2 = (10+2)^2 = 100 + 2(20) + 4$ $= 144$
12		
13		
14		
15		

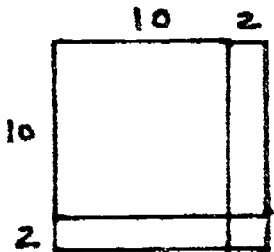
Mathematician: \_\_\_\_\_

"I built the squares from base ten pieces and made pictures of these."

SQUARE SIDE	PICTURE	COMPUTATION
EXAMPLE:  12		$12^2 = (10+2)^2 = 100 + 2(20) + 4$ $= 144$
$T + 3$		
$T + 4$		
$T + 5$		

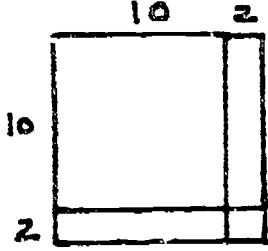
Mathematician: \_\_\_\_\_

"I built the squares from base ten pieces and made pictures of these."

SQUARE SIDE	PICTURE	COMPUTATION
EXAMPLE:		$12^2 = (10+2)^2 = 100 + 2(20) + 4$ $= 144$
16		
17		
18		

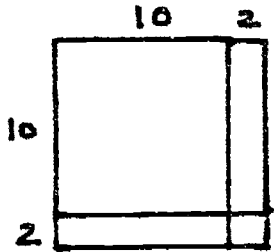
Mathematician: \_\_\_\_\_

"I built the squares from base ten pieces and made pictures of these."

SQUARE SIDE	PICTURE	COMPUTATION
EXAMPLE:		$12^2 = (10+2)^2 = 100 + 2(20) + 4$ $= 144$
19		
T+1		
T+2		

Mathematician: \_\_\_\_\_

"I built the squares from base ten pieces and made pictures of these."

SQUARE SIDE	PICTURE	COMPUTATION
EXAMPLE:		$12^2 = (10+2)^2 = 100 + 2(20) + 4$ $= 144$
$T + W$		
$T + 2W$		
$T + 3W$		

Mathematician: \_\_\_\_\_

"I drew pictures of these squares and wrote out all products."

SQUARE SIDE	PICTURE	PRODUCTS
Example:		$(10+4)^2 = 100 + 2(40) + 4^2$ $= 100 + 80 + 16$ $= 196$
13		
12		
11		



Mathematician: \_\_\_\_\_

"I drew pictures of these squares and wrote out all products."

SQUARE SIDE	PICTURE	PRODUCTS
15		
16		
17		
18		

2-6-88

Mathematician: \_\_\_\_\_

"I drew pictures of these squares and wrote out all products."

SQUARE SIDE	PICTURE	PRODUCTS
19		
20		
21		
22		

2.4.8.

Mathematician: \_\_\_\_\_

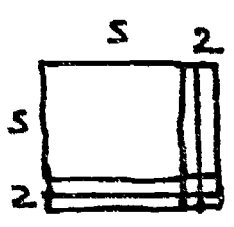
"I drew pictures of these squares and wrote out all products"

SQUARE SIDE	PICTURE	PRODUCTS
31		
41		
51		
61		

2-1-87

Mathematician: \_\_\_\_\_

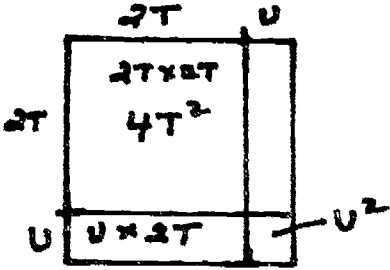
"I drew pictures of these squares and wrote out all products."

SQUARE SIDE	PICTURE	PRODUCTS
$S+2$		$(S+2)^2 = S^2 + 2(S) + 2^2$ $= S^2 + 2S + 4$
$S+3$		
$S+4$		
$S+n$		

2.4.5.

Mathematician: \_\_\_\_\_

"I drew pictures of these squares and wrote out all products."

SQUARE SIDE	PICTURE	PRODUCTS
<p>EXAMPLE: <math>2T + U</math></p>		$(2T)^2 + 2(2TU) + U^2$ $4T^2 + 4TU + U^2 = (2T+U)^2$
<p><math>A + 2B</math></p>		
<p><math>C + D</math></p>		
<p><math>3M + N</math></p>		

10371

Mathematician: \_\_\_\_\_

"I drew pictures of these squares and wrote out all products."

SQUARE SIDE	PICTURE	PRODUCTS
$4T + 1$		
$2T + 5$		
$3M + 2$		
$B + 6$		

C. C. S.

Mathematician: \_\_\_\_\_

"I drew pictures of these squares and wrote out all products."

SQUARE SIDE	PICTURE	PRODUCTS
$z + 3w$		
$x + 2y$		
$y + \frac{3}{4}$		
$z + 4$		

2.4.0

Mathematician: \_\_\_\_\_

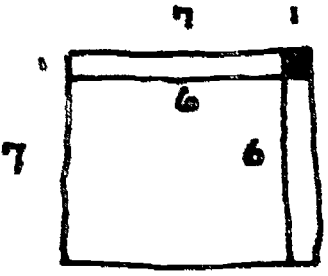
"I used the pattern for squaring numbers ending in 5."

NUMBER	COMPUTATION
<b>EXAMPLE:</b> 15	$15^2 = (\underline{1 \times 2})25 = 225$
25	$25^2 =$
35	$35^2 =$
45	$45^2 =$
55	$55^2 =$
65	$65^2 =$
75	$75^2 =$
85	$85^2 =$
95	$95^2 =$



Mathematician: \_\_\_\_\_

"I shrank each square by the amount given to make a new square, drew a picture and wrote out all of the parts in a sentence."

SIDE OF THE GIVEN SQUARE	SHRINK THE SIDE BY	PICTURE	SENTENCE
<p>EXAMPLE:</p> <p>7</p>	<p>1</p>		$(7-1)^2 = 7^2 - 2 \times 7 + 1$ $= 49 - 14 + 1$ $= 36$ $6^2 = 36$
<p>8</p>	<p>3</p>		
<p>12</p>	<p>1</p>		
<p>11</p>	<p>3</p>		

Mathematician: \_\_\_\_\_

"I found the differences between the given squares and drew the picture."

SQUARE SIDES	PICTURE	DIFFERENCES
<p>Example: 10 and 6</p>		$10^2 - 6^2 = (10-6)(10+6) = 4 \times 16 = 64$
<p>12 and 10</p>		
<p>15 and 10</p>		
<p>S and T</p>		

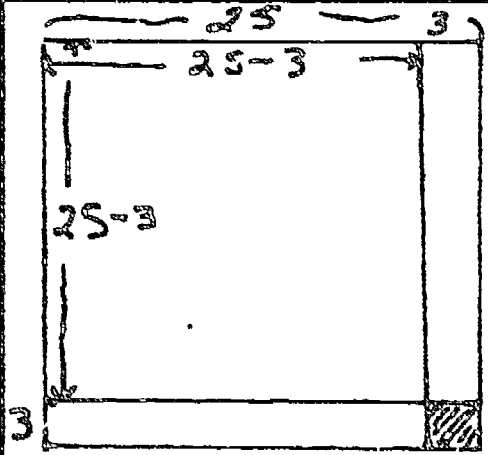
Mathematician: \_\_\_\_\_

"I found the differences between the given squares and drew the picture."

SQUARE SIDES	PICTURE	DIFFERENCES
A and B		
2T and S		
2M and 2N		
X and Y		

Mathematician: \_\_\_\_\_

"I shrank each square by the amount given to make a new square, drew a picture and wrote out all of the parts in a sentence."

SIDE OF THE GIVEN SQUARE	SHRINK THE SIDE BY	PICTURE	SENTENCE
T	1		
25	3		$(25-3)^2 = (25)^2 - 3 \times 25 + 3^2$ $= 45^2 - 65 + 9$
X	2		
T	$\frac{2}{3}$		

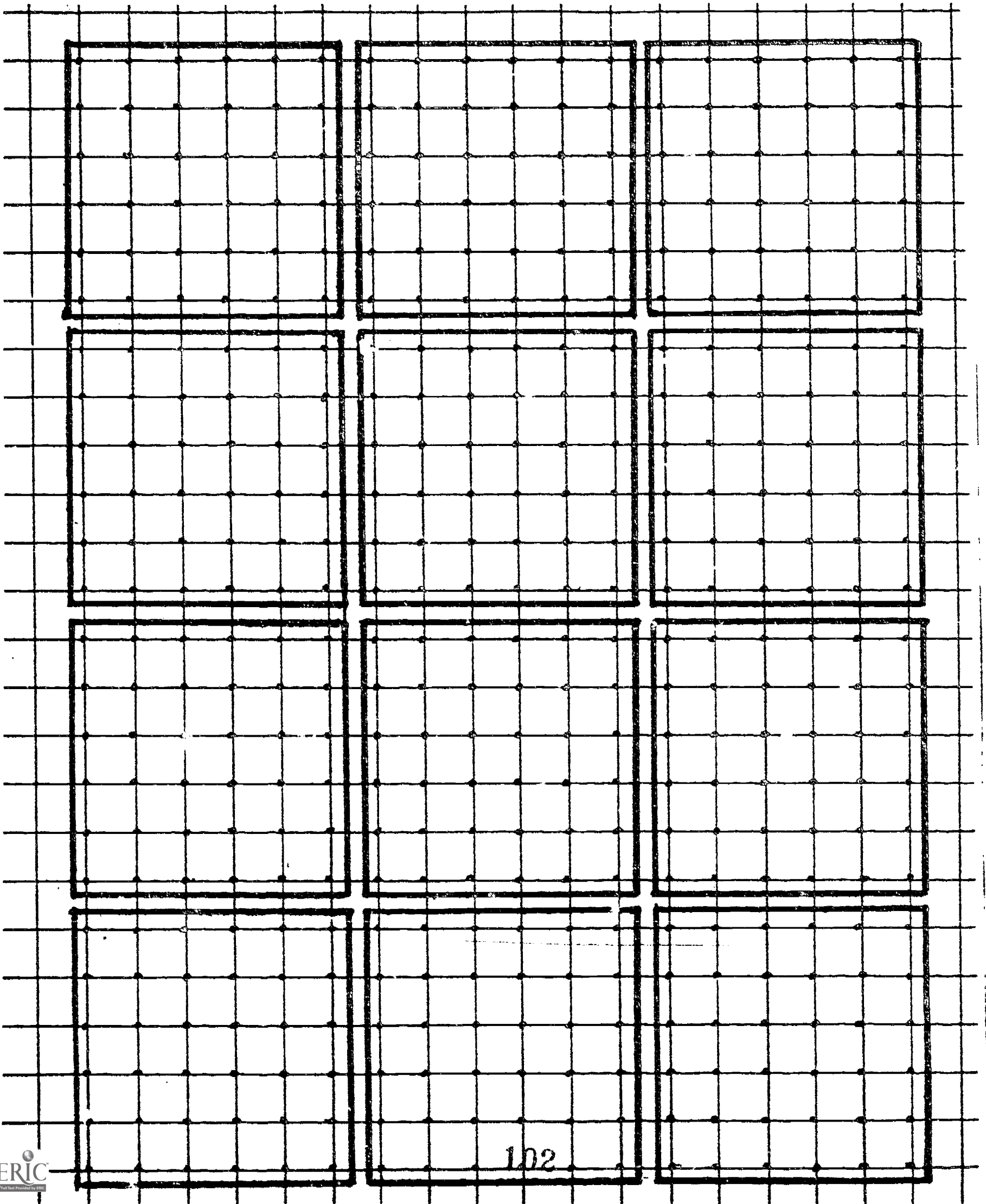
Mathematician: \_\_\_\_\_

"I shrank each square by the amount given to make a new square, drew a picture and wrote out all of the parts in a sentence."

SIDE OF THE GIVEN SQUARE	SHRINK THE SIDE BY	PICTURE	SENTENCE
12	4		
27	1		
9	5		
10	3		

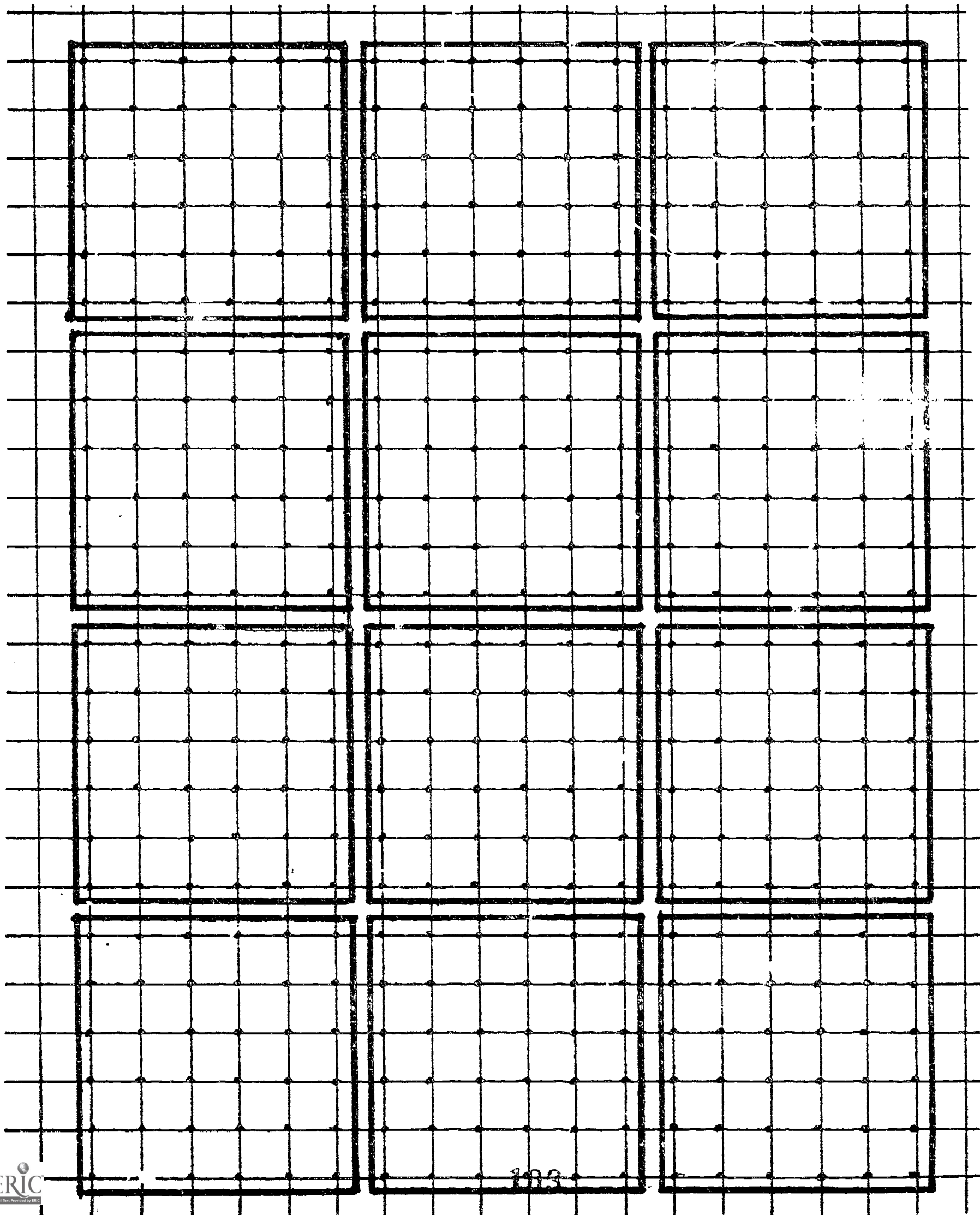
Mathematician: \_\_\_\_\_

"This is my record of tasks done on the geoboard."



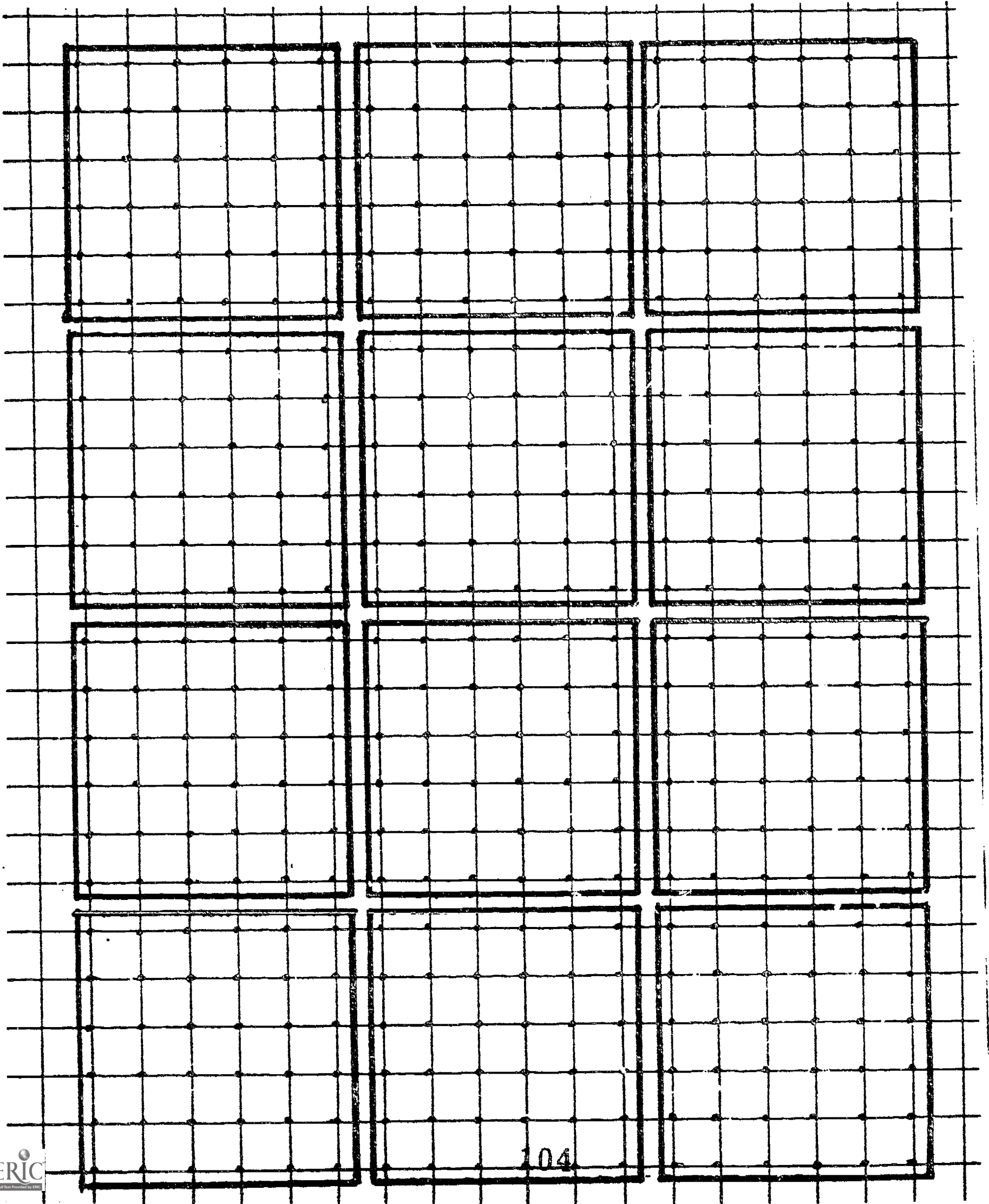
Mathematician: \_\_\_\_\_

"This is my record of tasks done on the geoboard."



Mathematician: \_\_\_\_\_

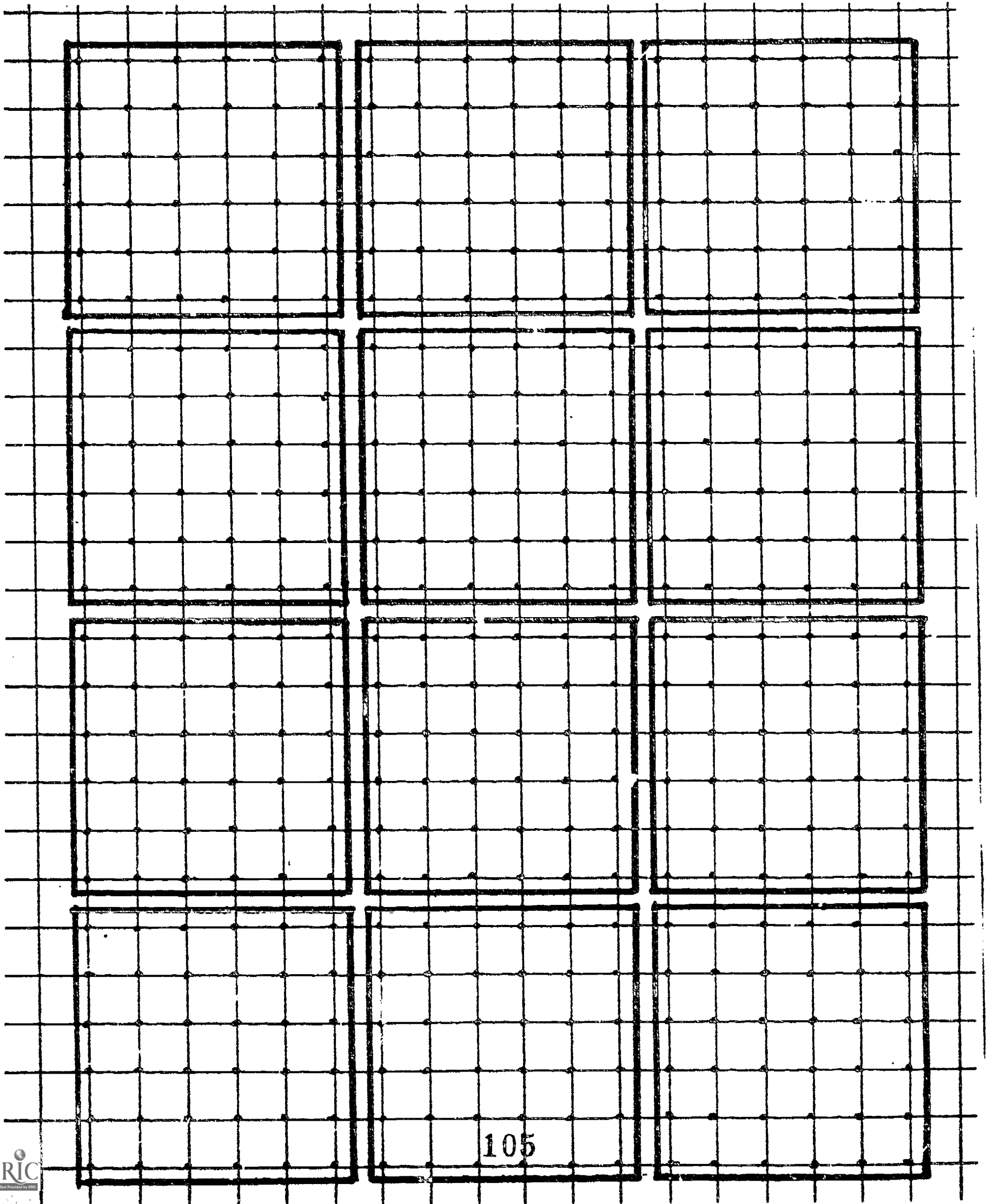
"This is my record of tasks done on the geoboard."





Mathematician: \_\_\_\_\_

"This is my record of tasks done on the geoboard."



Mathematician: \_\_\_\_\_

"I made the shapes with the areas given and recorded this on the geoboard picture."

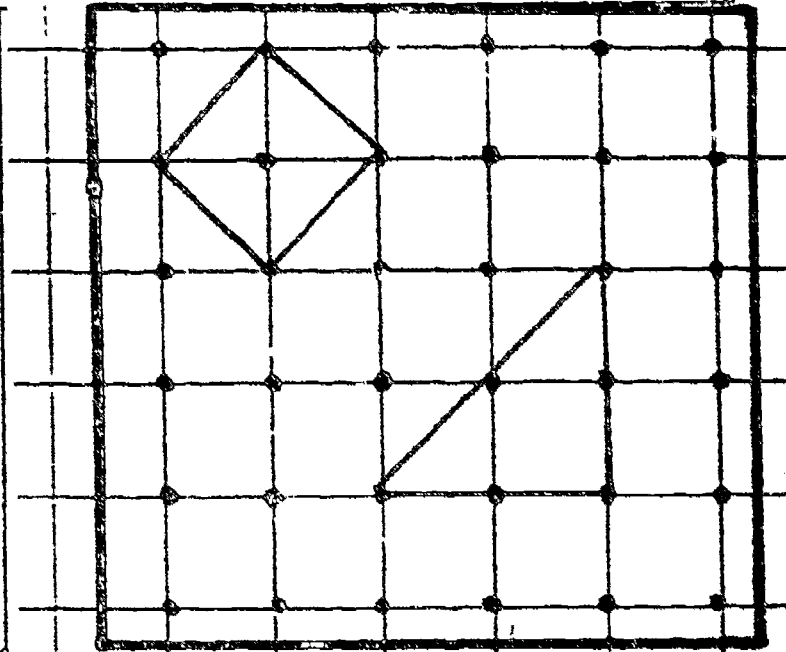
SHAPES

AREA

GEOBOARD

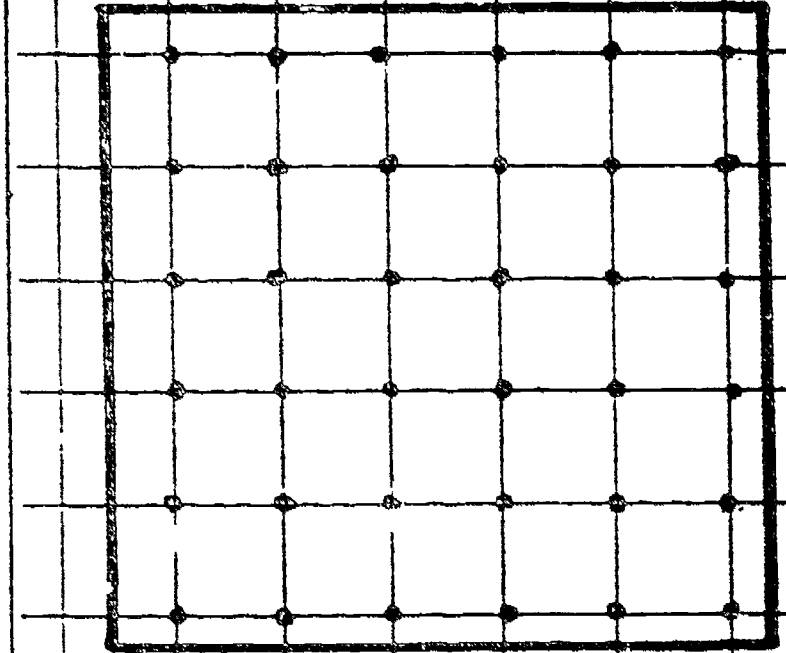
a  
Triangle  
and  
a Square

2



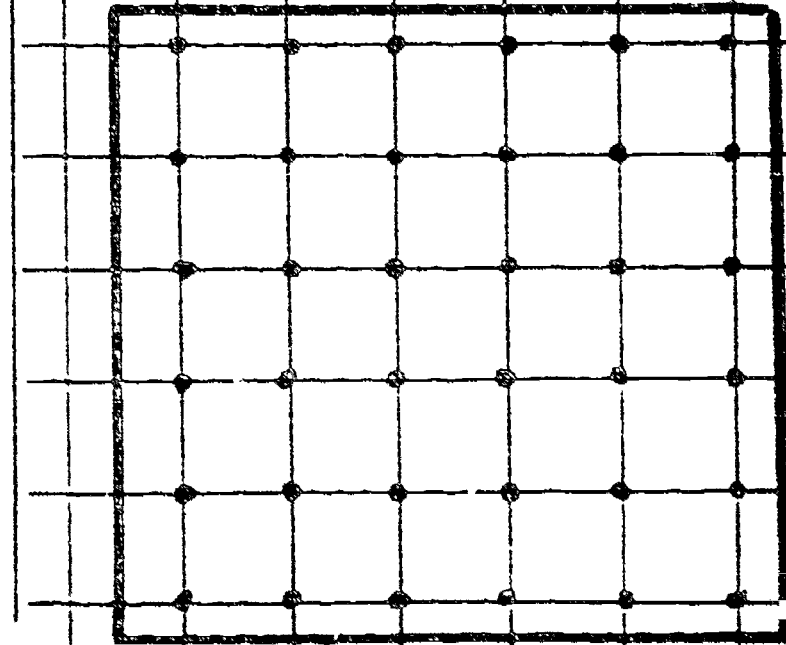
Two  
Squares

2 and 5



Two  
Triangles

2 and 3



Mathematician: \_\_\_\_\_

"I made the shapes with the areas given and recorded this on the geoboard picture."

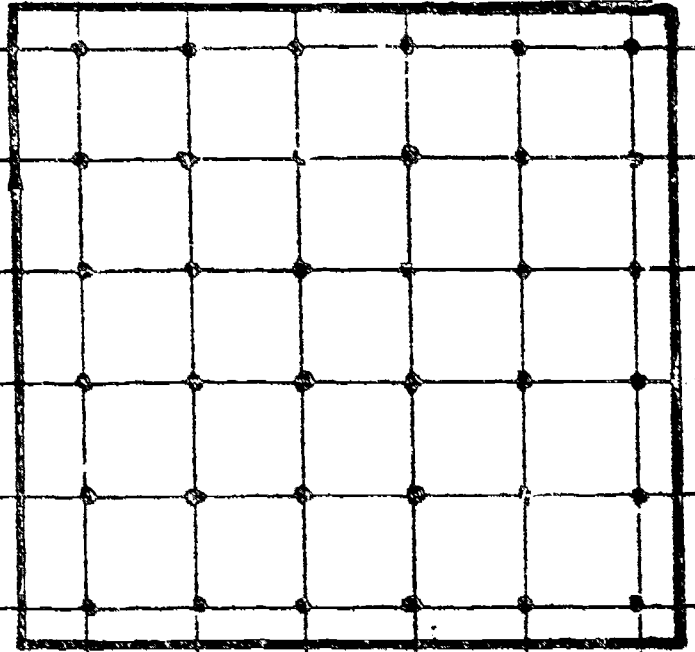
SHAPES

AREA

GEOBOARD

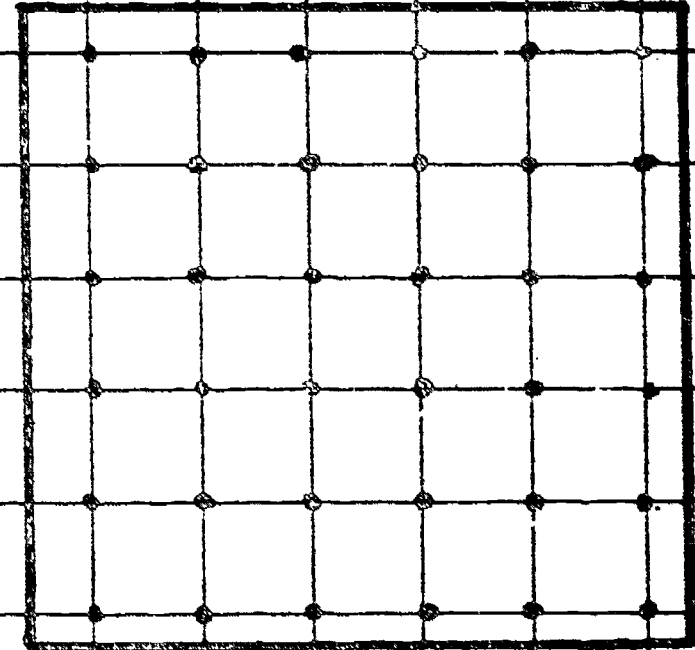
Triangle

5



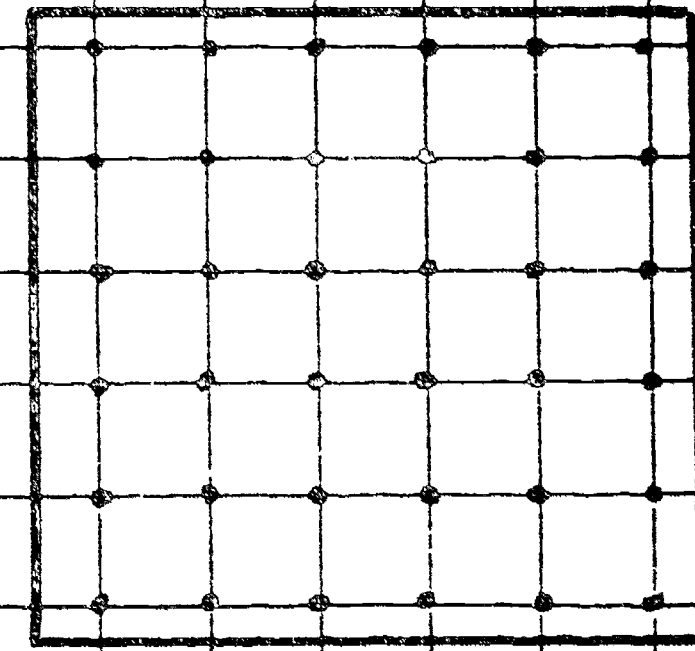
4 sides

7



rectangle

6



Mathematician: \_\_\_\_\_

"I made the shapes with the areas given and recorded this on the geoboard picture."

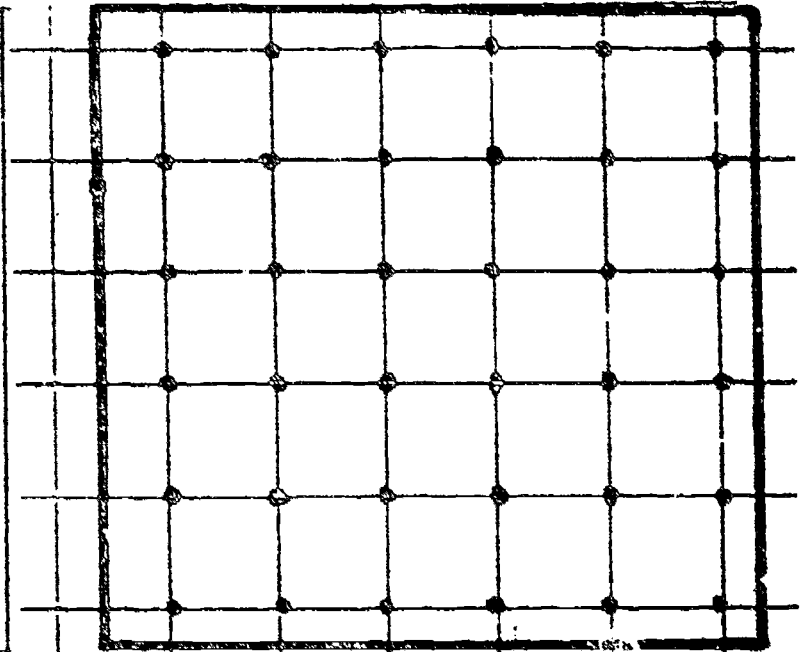
SHAPES

AREA

GEOBOARD

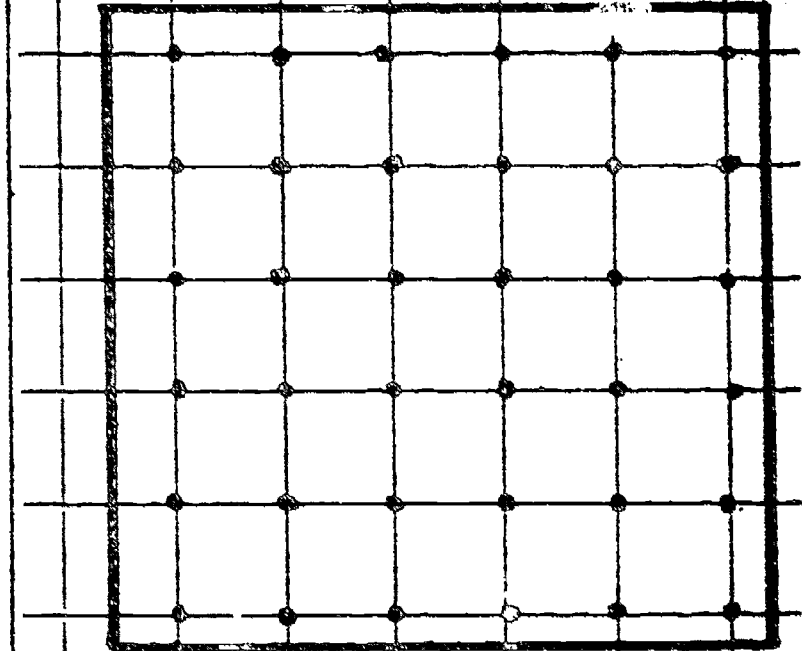
Parallelogram

6



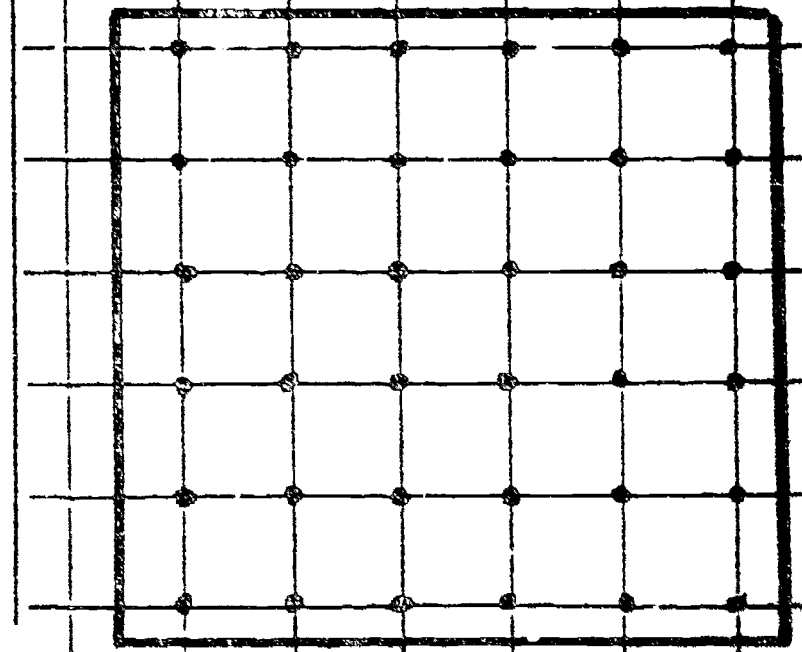
A triangle  
and  
a square

equal



Two  
Congruent  
triangles

4



Mathematician: \_\_\_\_\_

"I made the shapes with the areas given and recorded this on the geoboard picture."

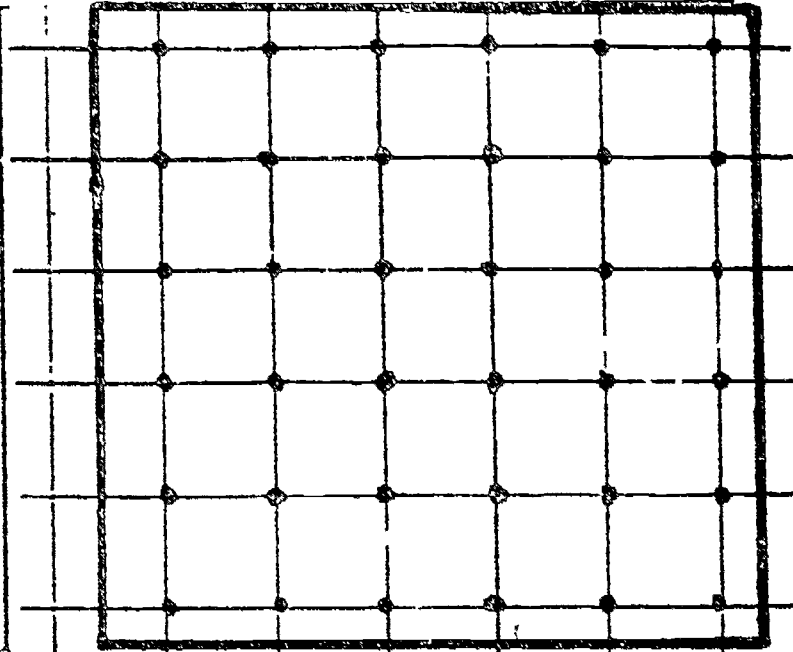
SHAPES

AREA

GEOBOARD

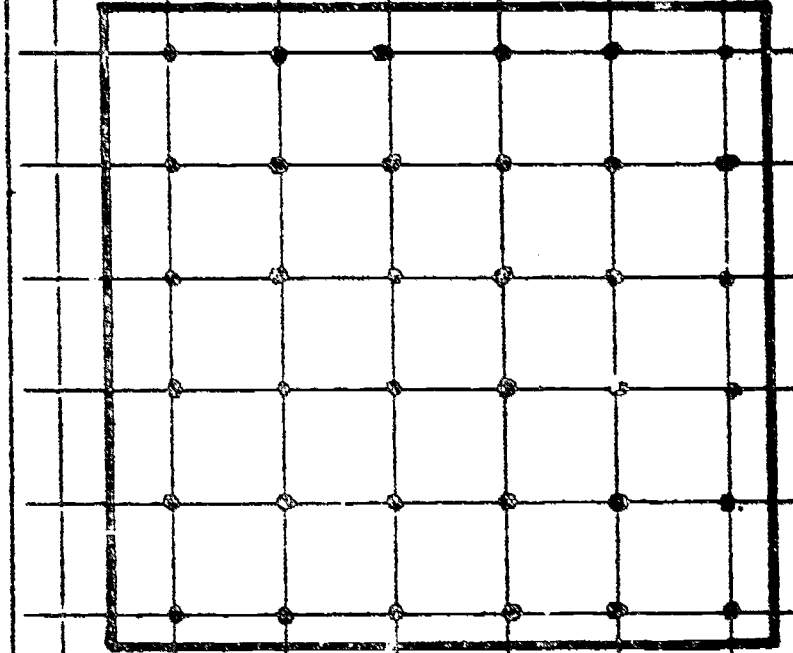
SQUARE

10



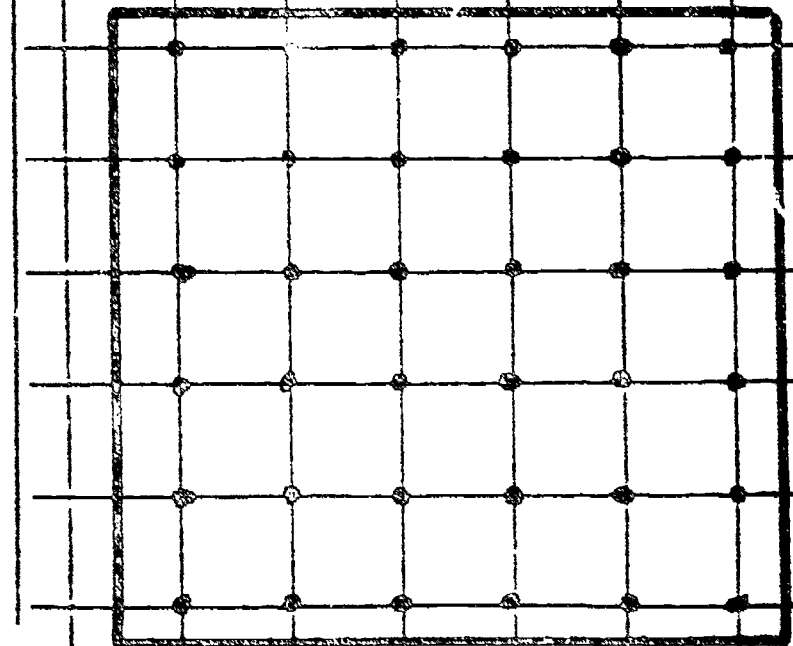
2  
different  
rectangles

4



A  
right triangle

4



Mathematician: \_\_\_\_\_

"I made the shapes with the areas given and recorded this on the geoboard picture."

SHAPES

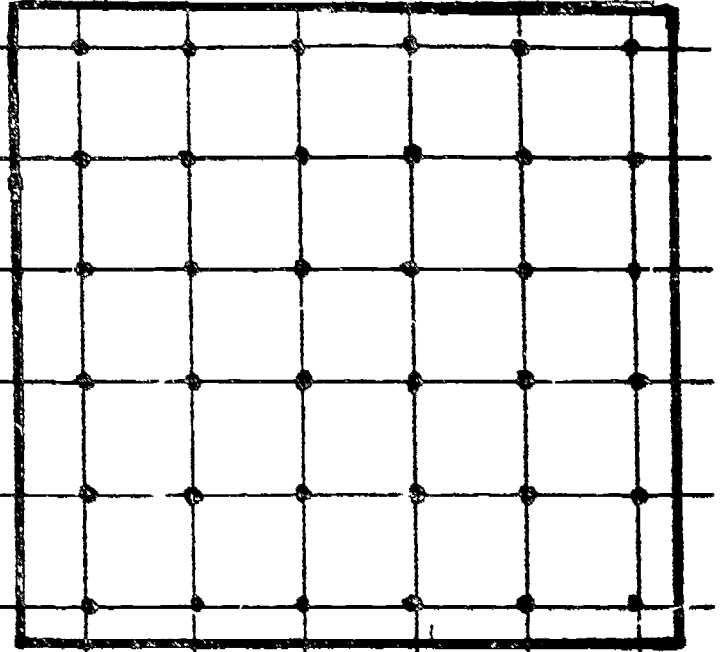
AREA

GEOBOARD

2

different  
triangles

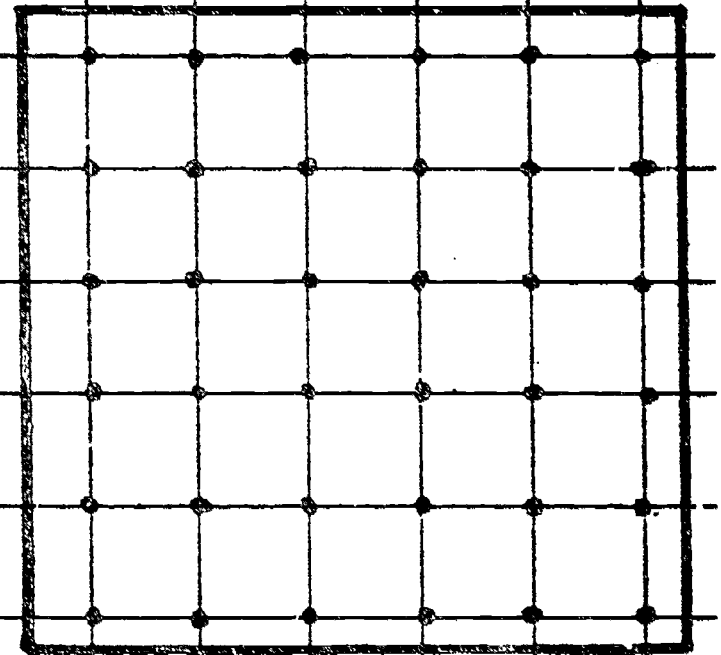
4



5

sided

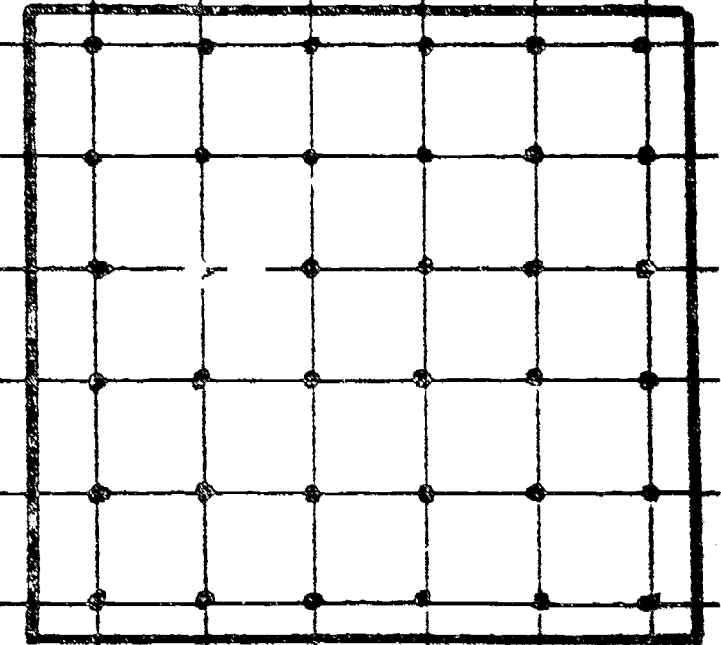
5



6

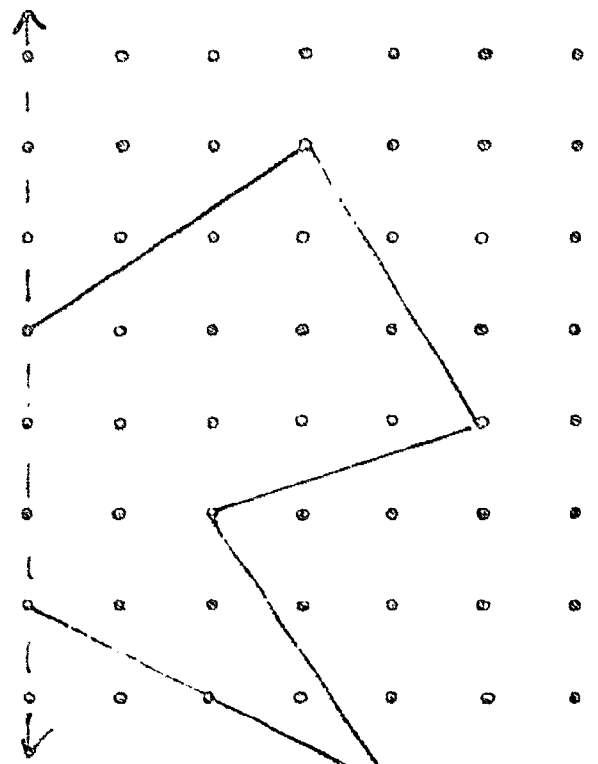
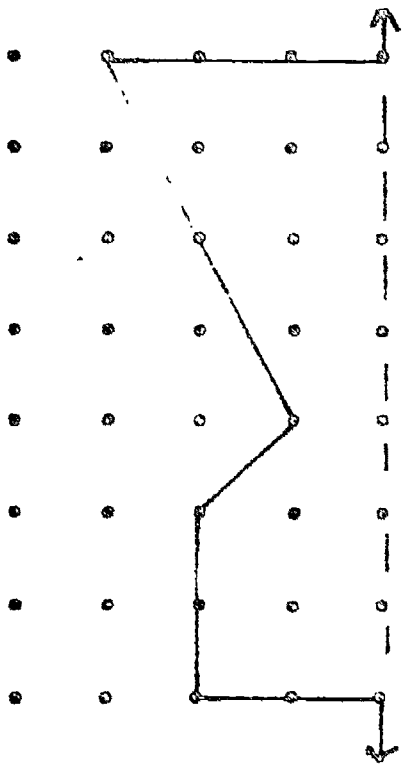
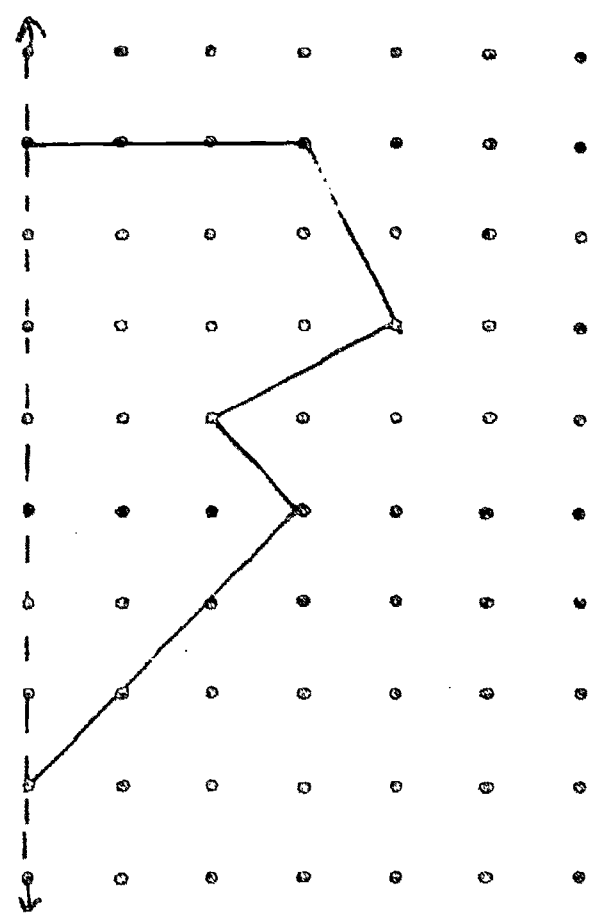
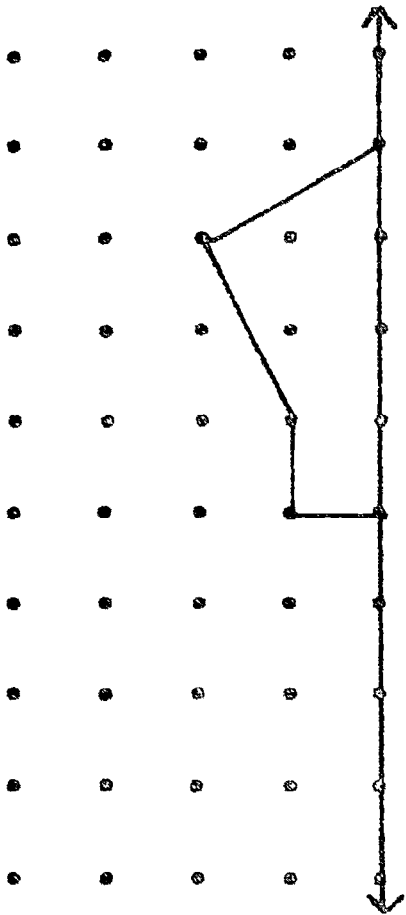
SIDED

5



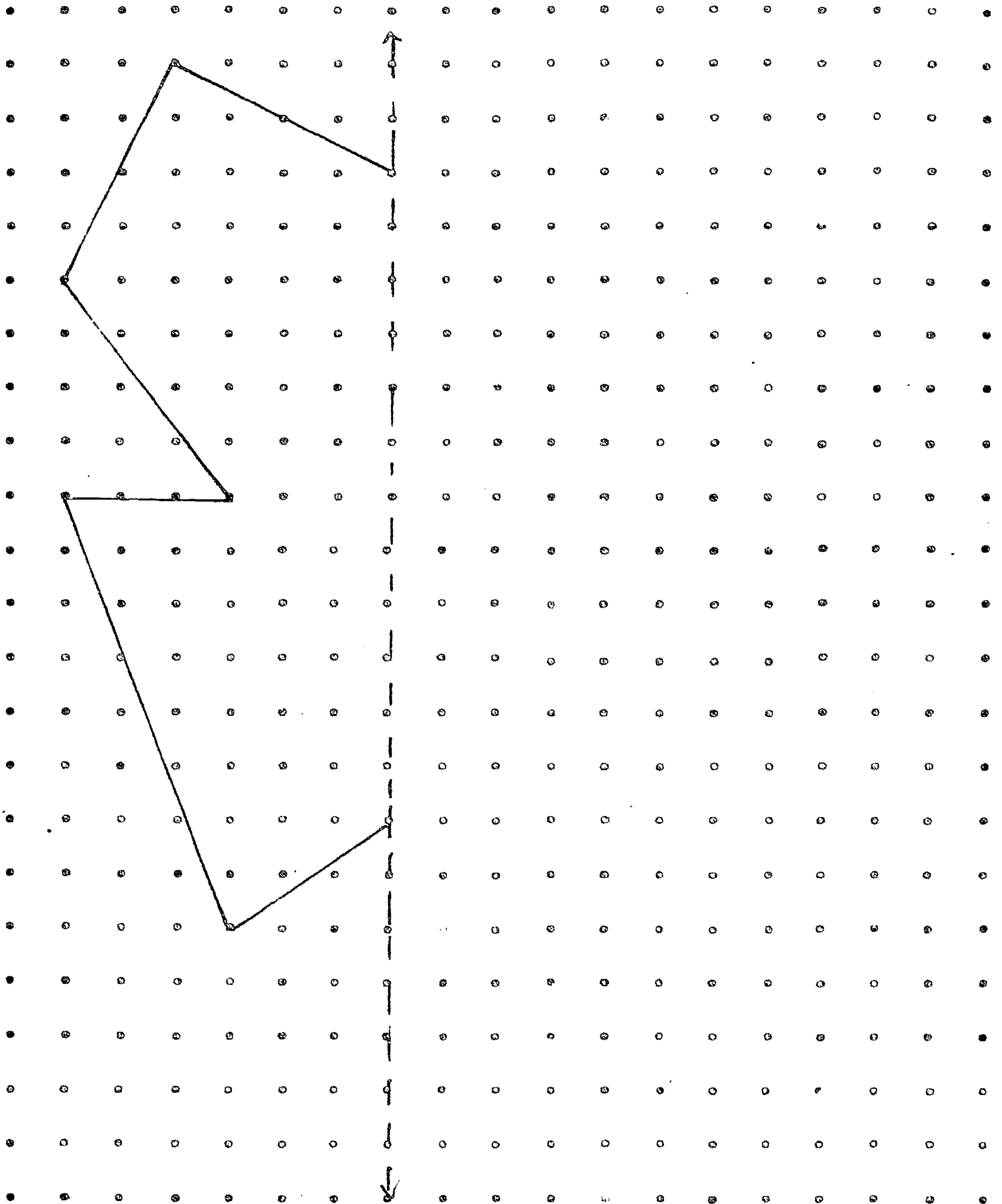
Mathematician: \_\_\_\_\_

"I completed these figures so the new shape has the indicated line of symmetry."



Mathematician: \_\_\_\_\_

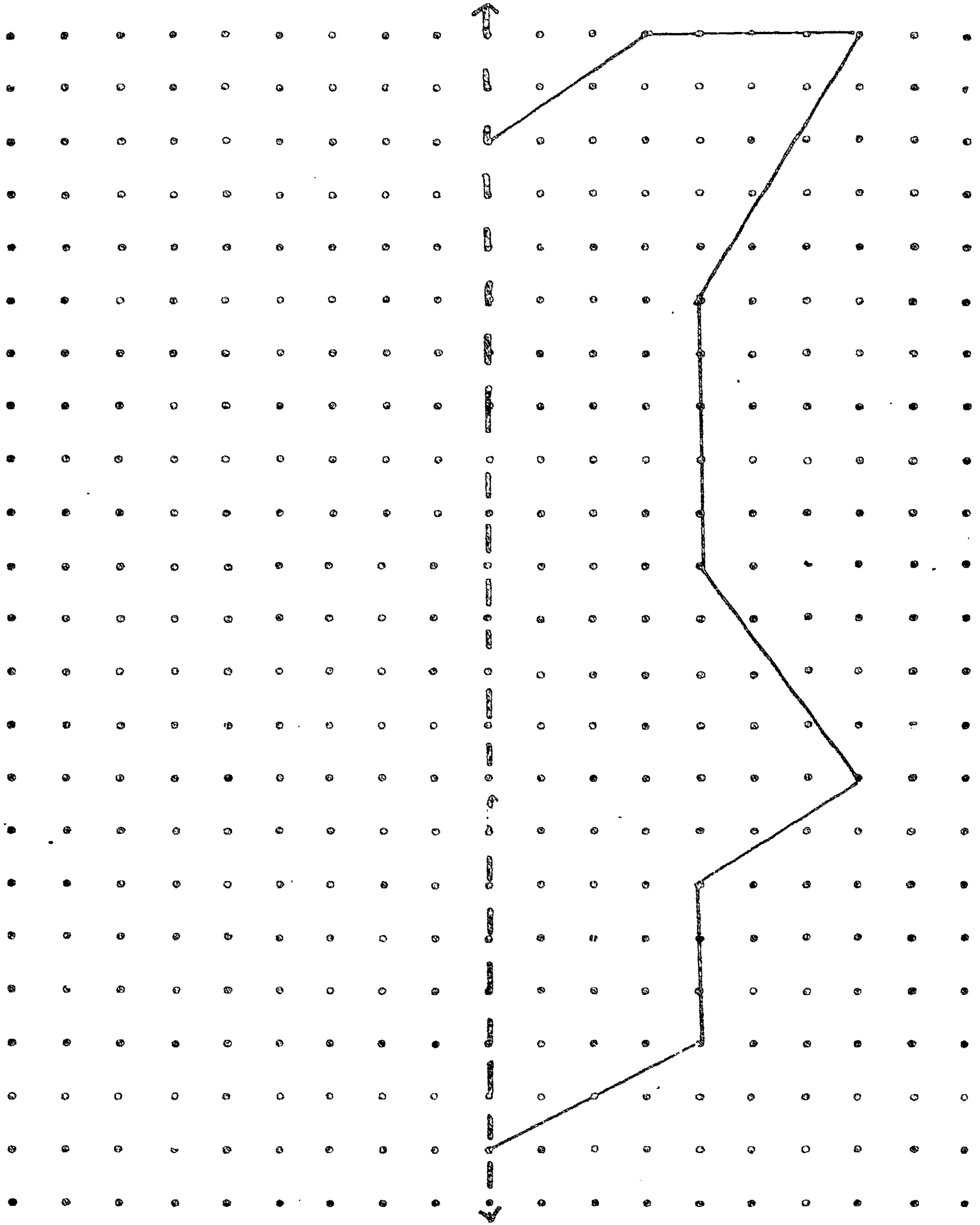
"I completed these figures so the new shape has the indicated line of symmetry."





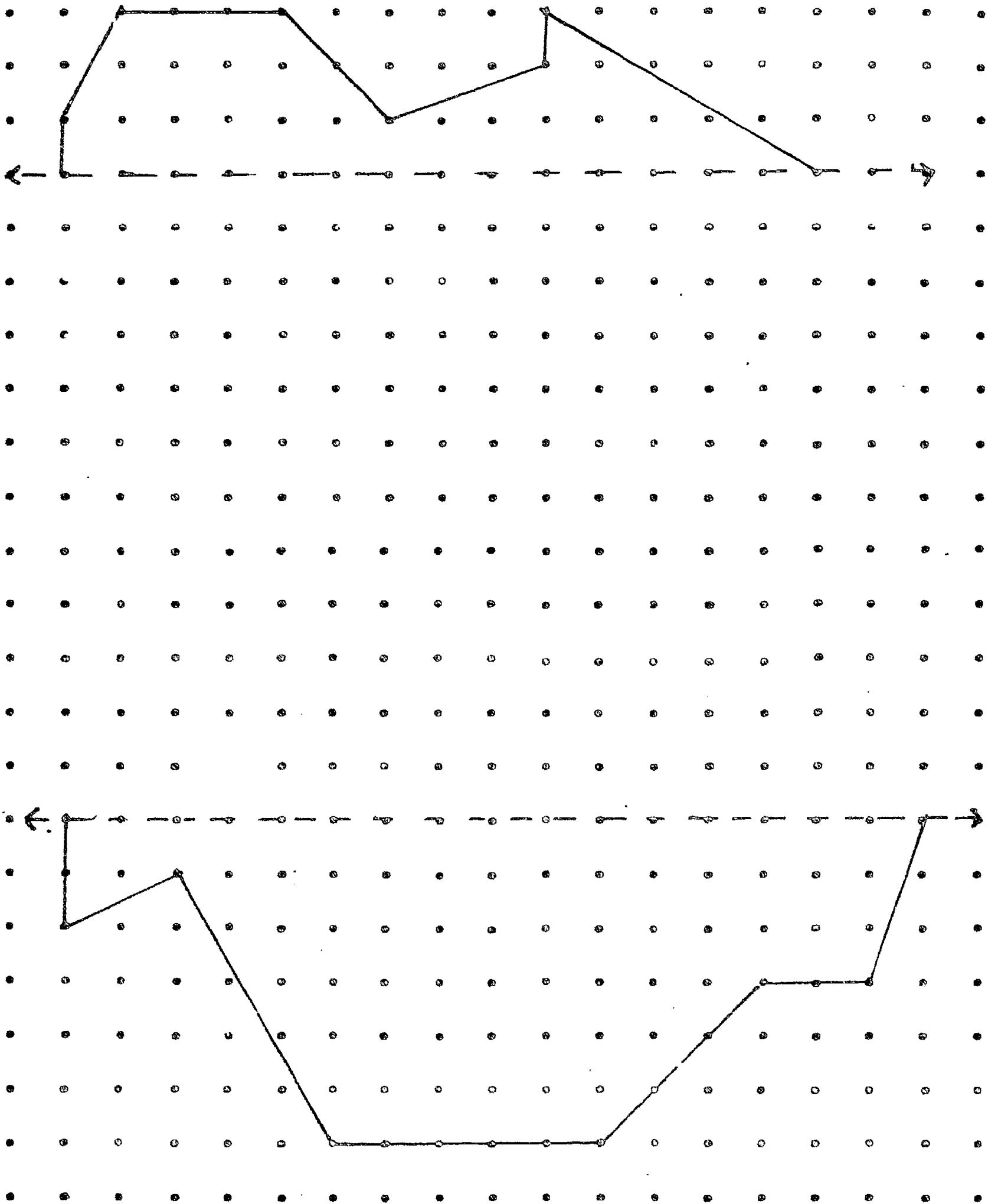
Mathematician: \_\_\_\_\_

"I completed these figures so the new shape has the indicated line of symmetry."



Mathematician: \_\_\_\_\_

"I completed these figures so the new shape has the indicated line of symmetry."



Mathematician: \_\_\_\_\_

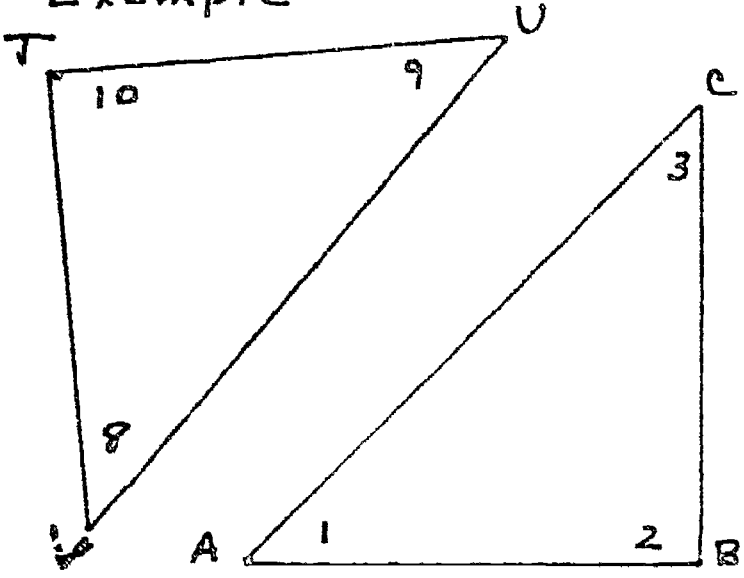
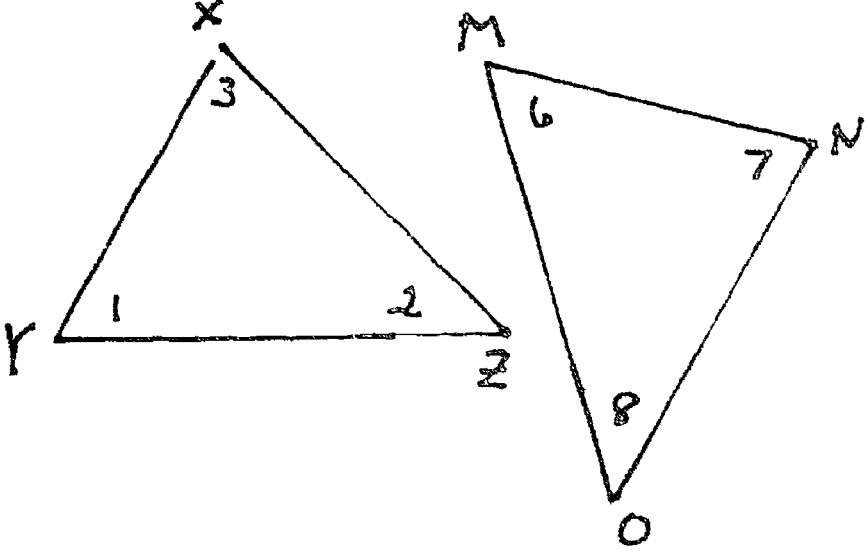
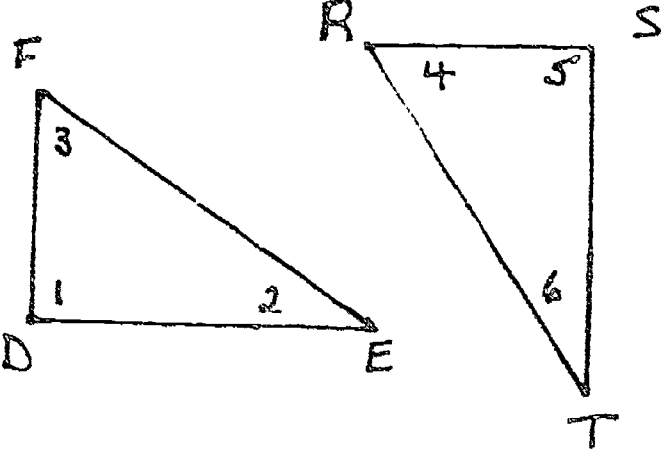
"I found all pairs of congruent triangles and all pairs of similar triangles on sheets I & II and wrote the letters of these."

CONGRUENT TRIANGLES

SIMILAR TRIANGLES


Mathematician: \_\_\_\_\_

"I found the corresponding parts for the congruent triangles given and wrote them in the columns."

TRIANGLES	CORRESPONDING SIDES	CORRESPONDING ANGLES
<p>Example</p> 	<p>TU and BA TV and BC AC and UV</p>	<p><math>\angle 1</math> and <math>\angle 9</math> <math>\angle 10</math> and <math>\angle 2</math> <math>\angle 3</math> and <math>\angle 8</math></p>
		
		

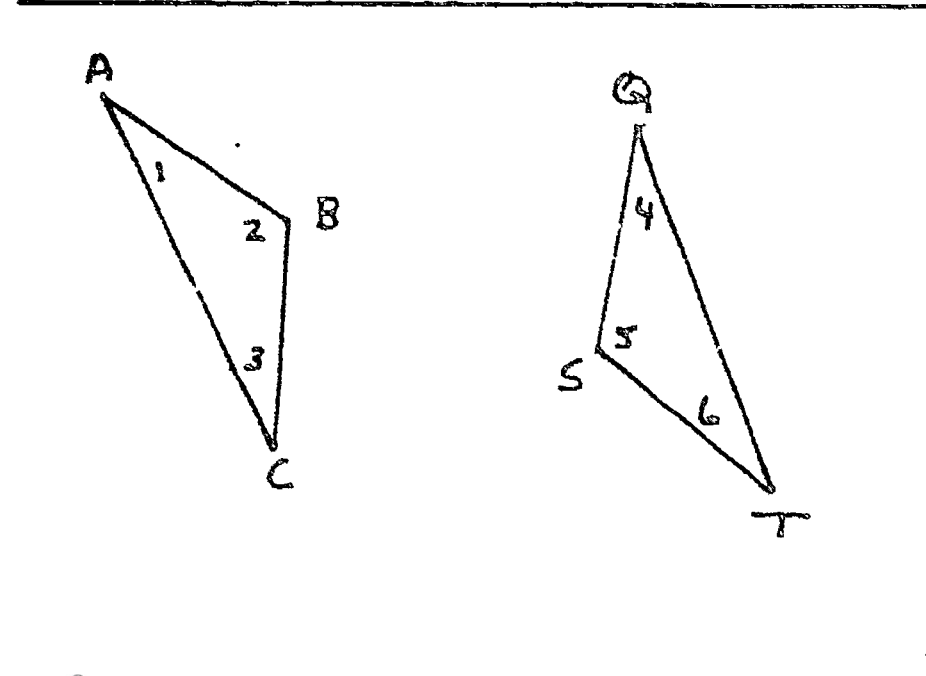
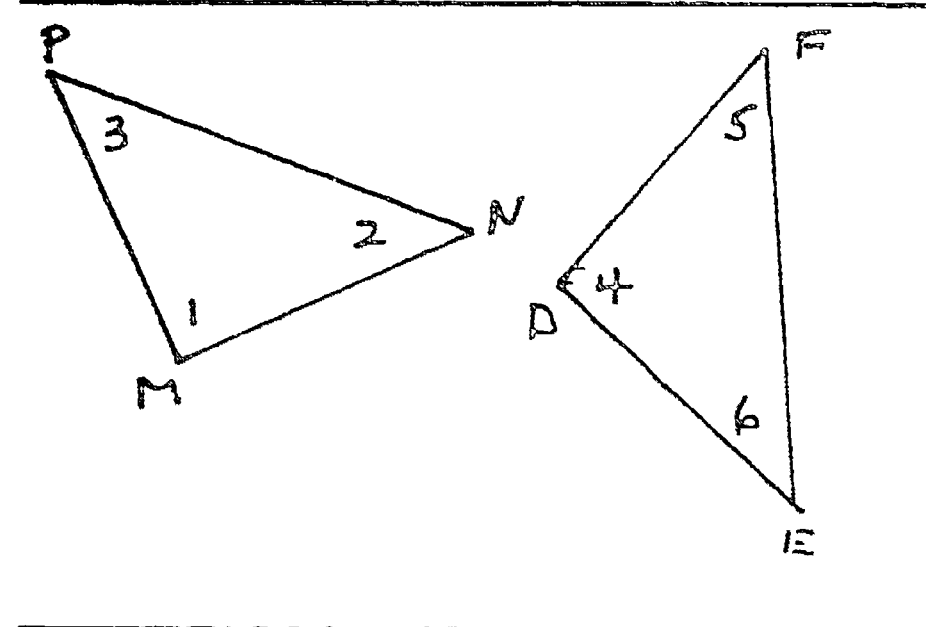
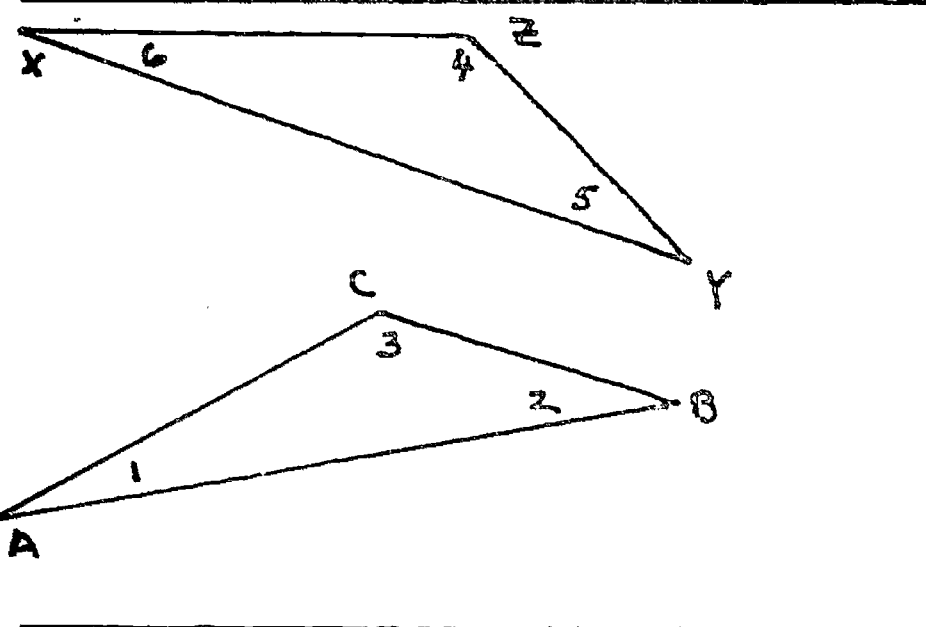
Mathematician: \_\_\_\_\_

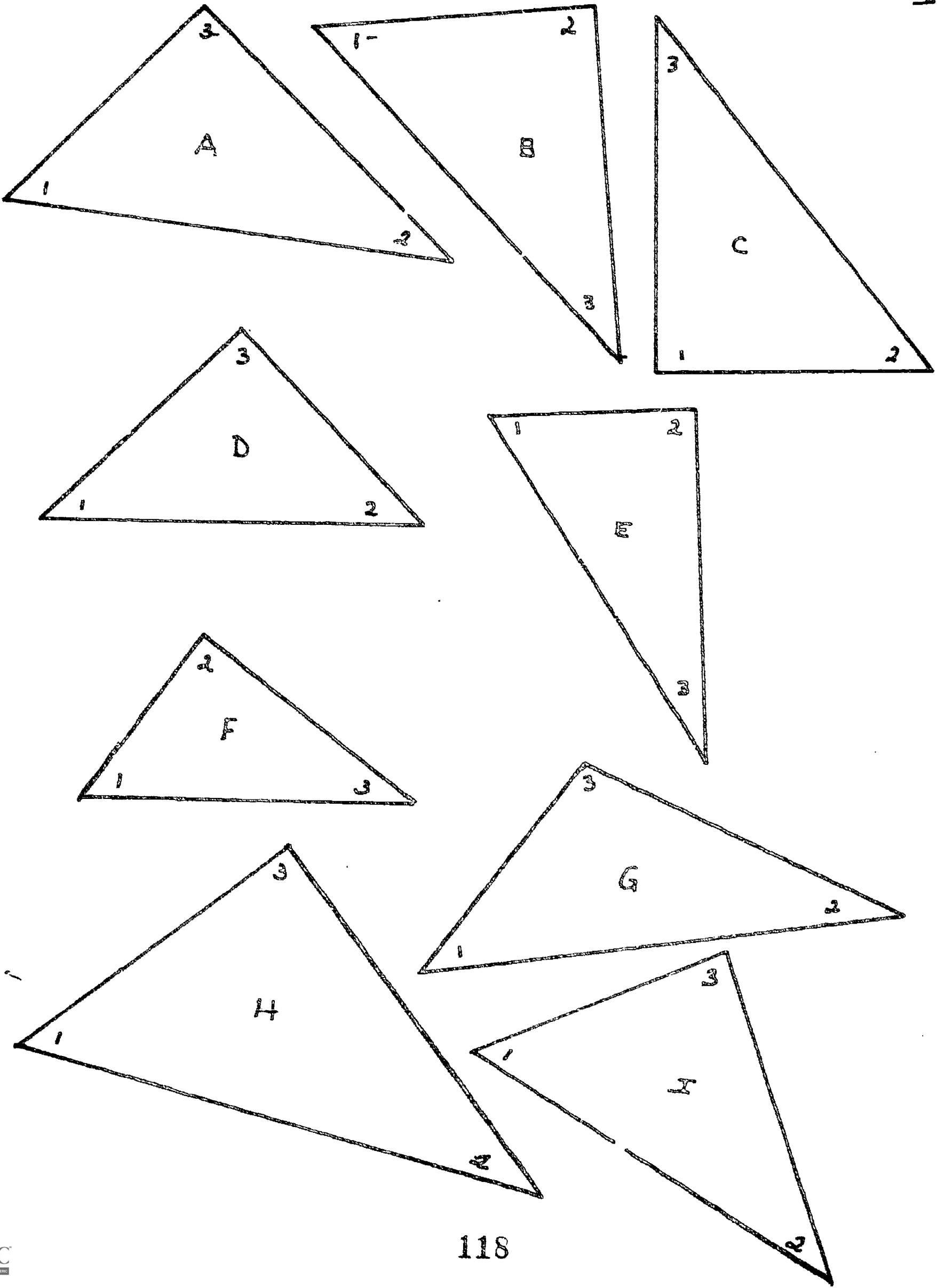
"I found the corresponding parts for the congruent triangles given and wrote them in the columns."

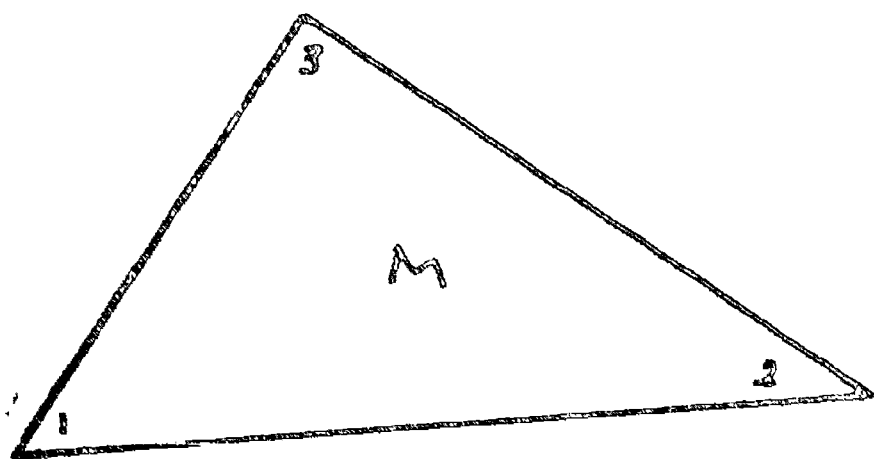
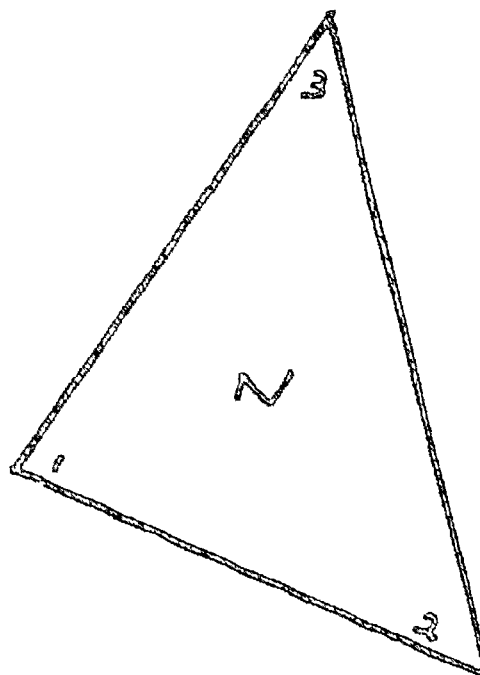
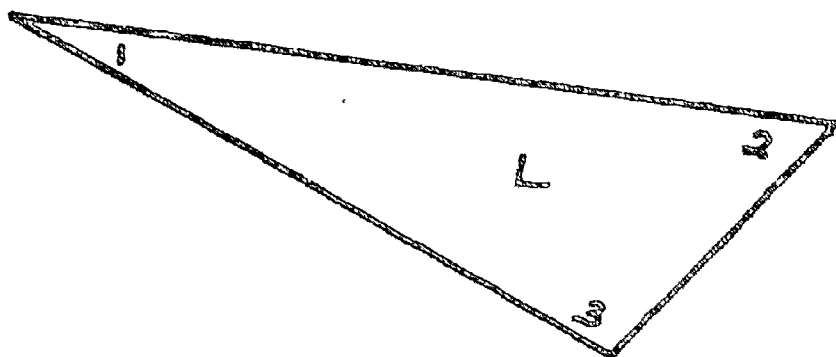
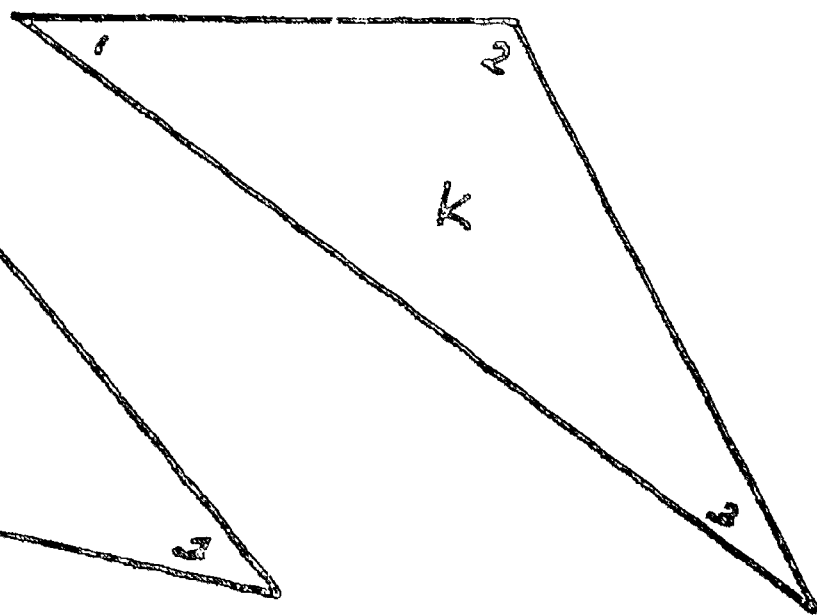
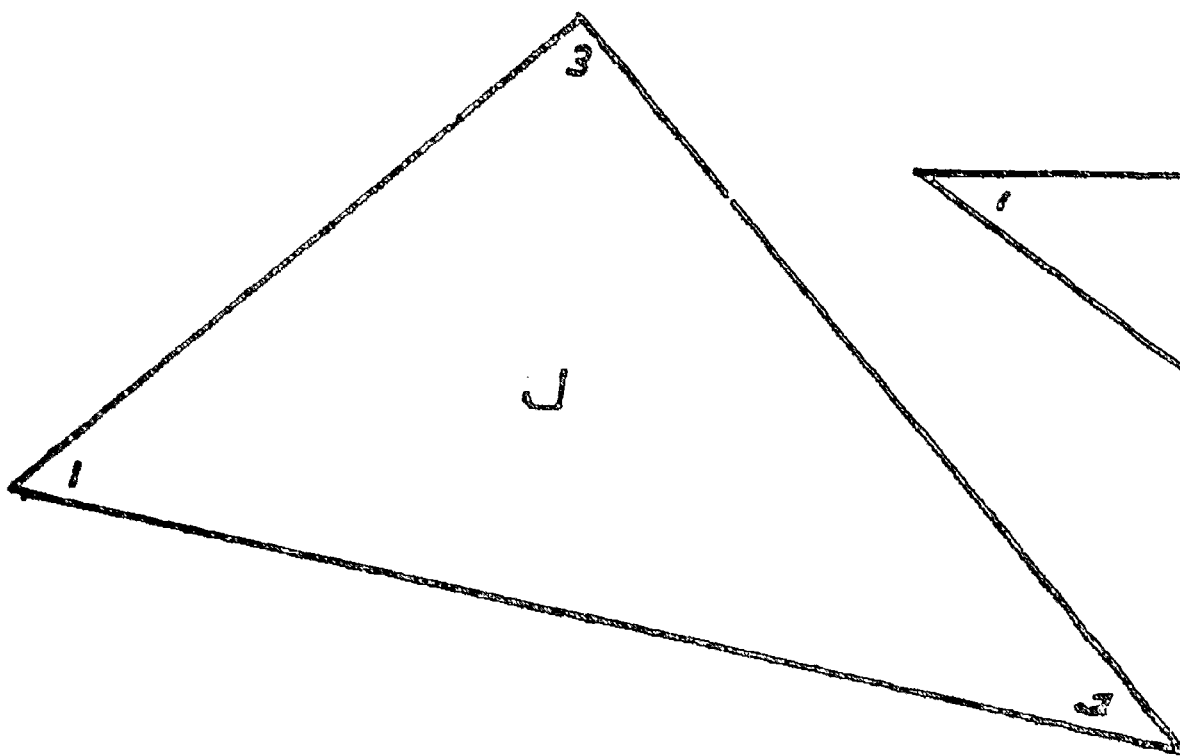
TRIANGLES

CORRESPONDING  
SIDES

CORRESPONDING  
ANGLES





Mathematician: \_\_\_\_\_

"We recorded the letters of every pair of similar triangles on sheets I & II and wrote the reason - all angles equal or sides proportional."

PAIRS OF SIMILAR TRIANGLES

REASONS

PAIRS OF SIMILAR TRIANGLES	REASONS

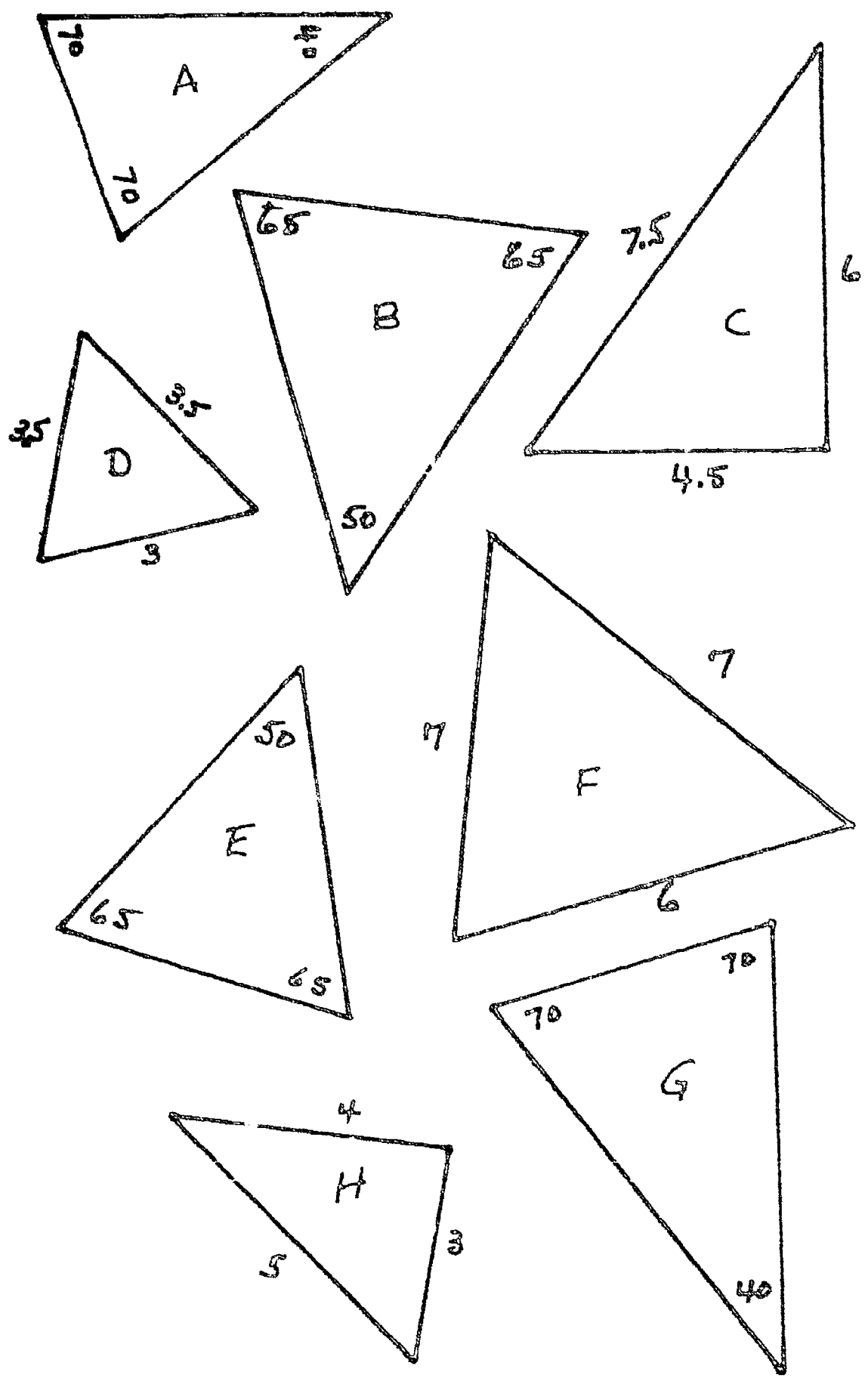


Mathematician: \_\_\_\_\_

"I picked out the triangles that are similar and wrote why they are."

GIVEN TRIANGLES

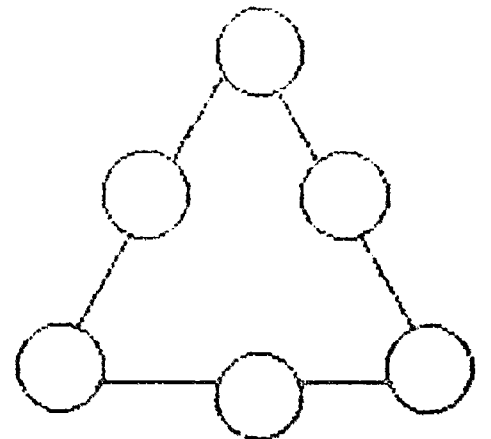
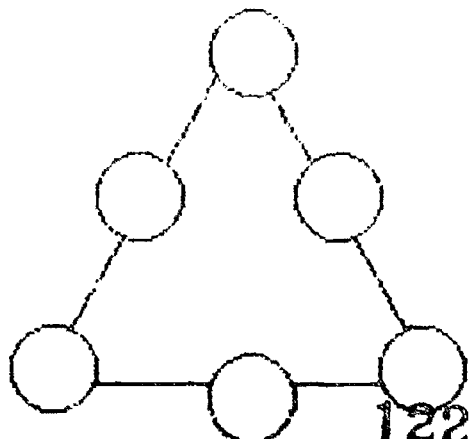
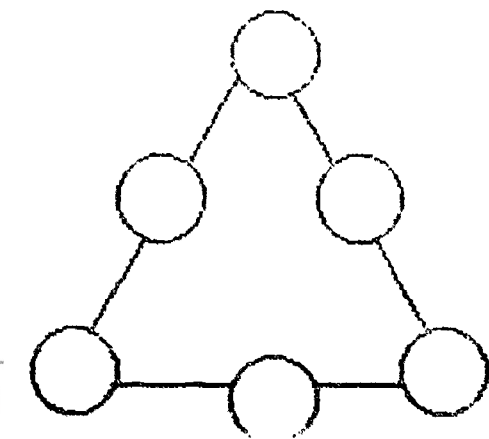
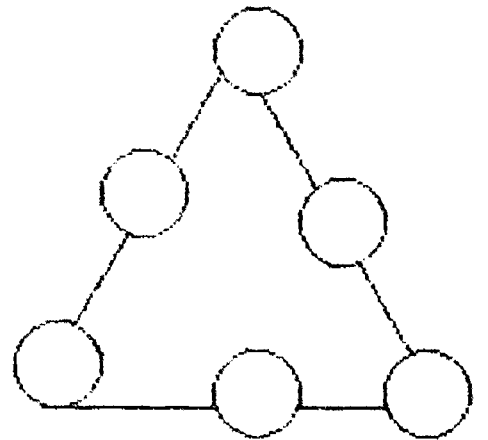
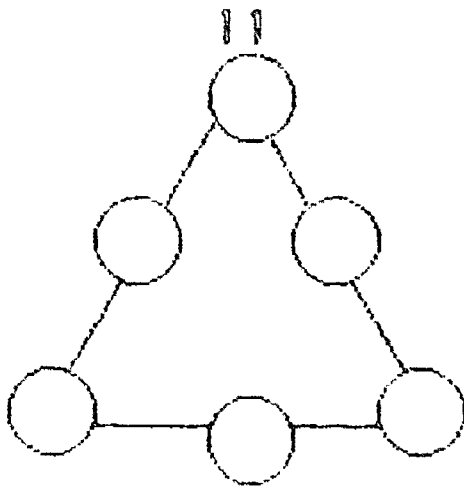
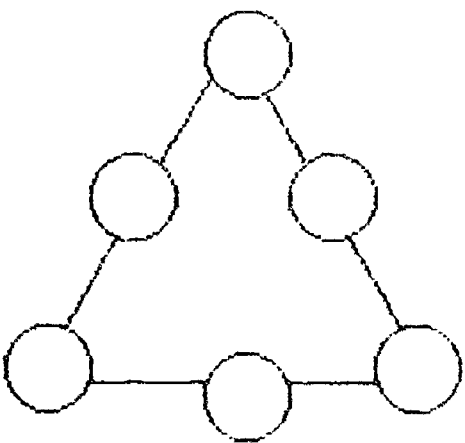
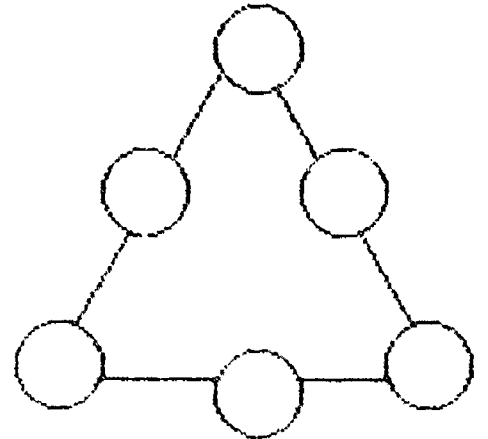
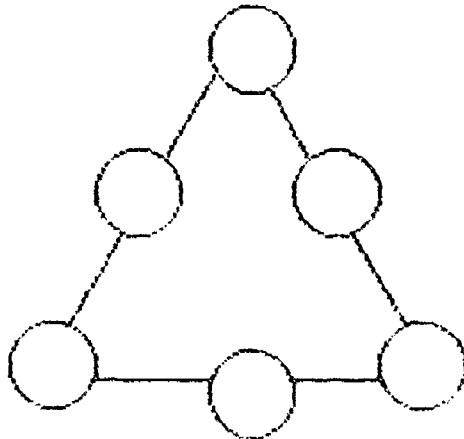
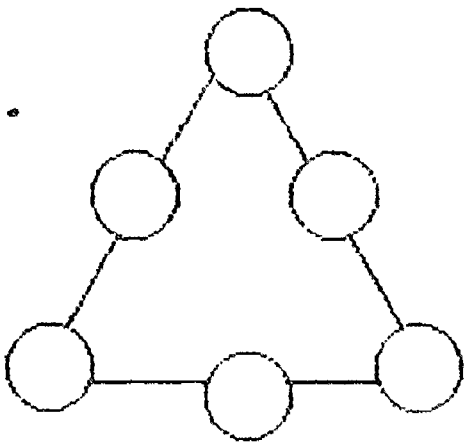
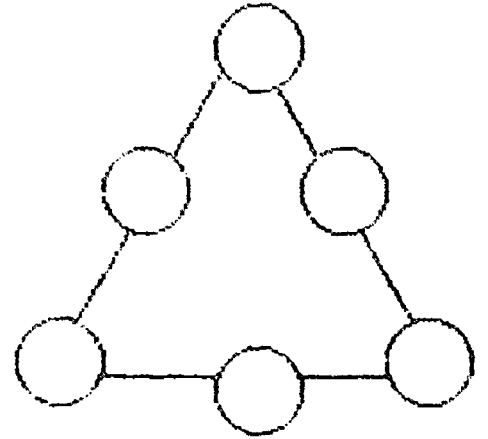
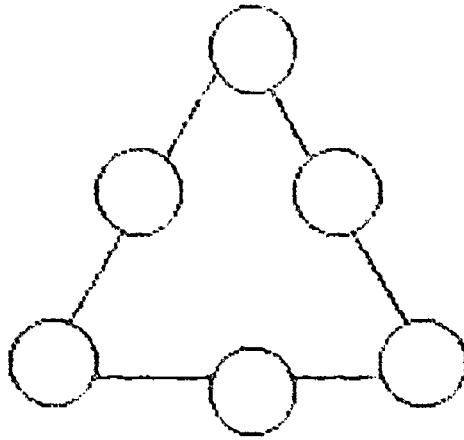
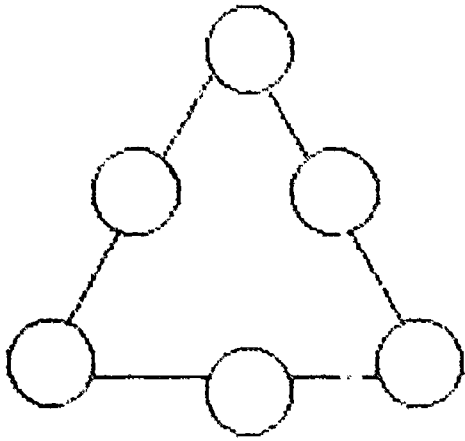
SIMILAR TRIANGLE P



Mathematician: \_\_\_\_\_

"I used the numbers 1, 2, 3, 4, 5 and 6 in the circles so the sides added to the number given."

10



122

305571

Mathematician: \_\_\_\_\_

"For each series of 9 numbers given, I found the number to go into the middle square and the sum for the rows, columns, and diagonals of the MAGIC SQUARE."

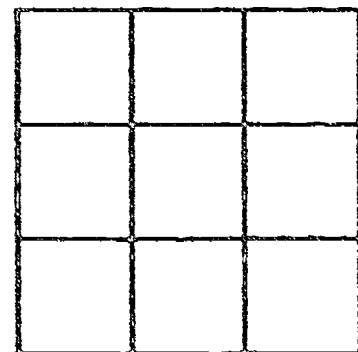
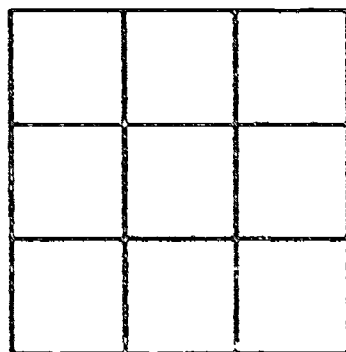
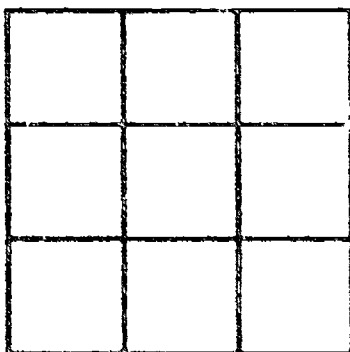
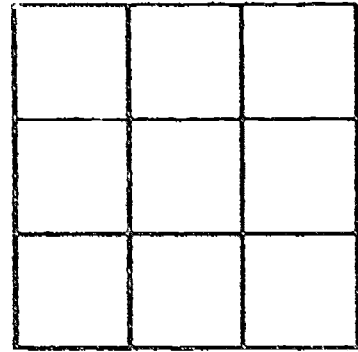
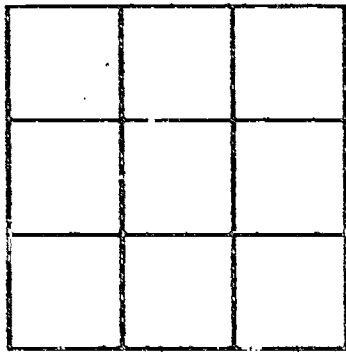
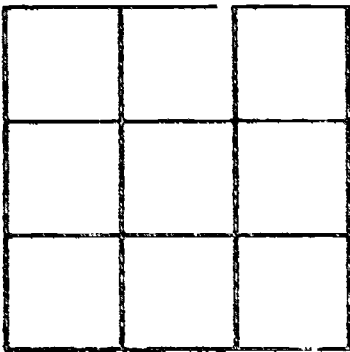
Number series  $\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \frac{11}{4}, \frac{13}{4}, \frac{15}{4}, \frac{17}{4}$

"Middle" number

Row, Column, Diagonal

Sum for the Square

### THE MAGIC SQUARES



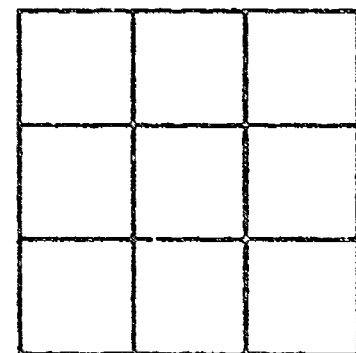
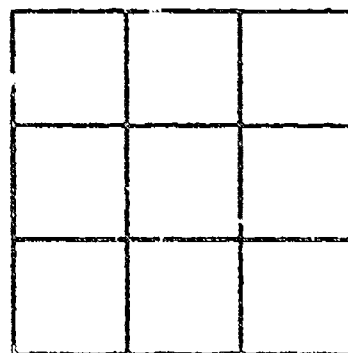
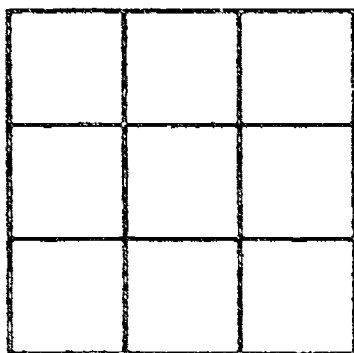
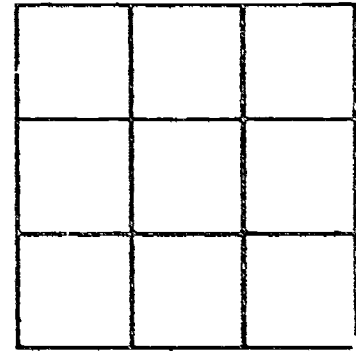
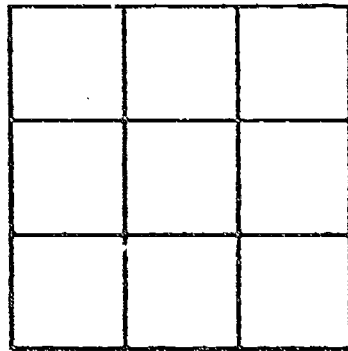
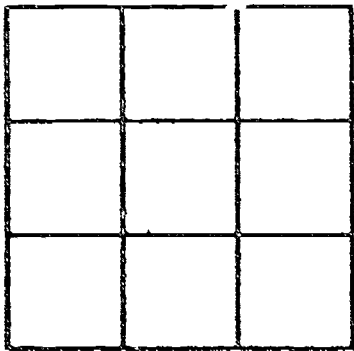
Mathematician: \_\_\_\_\_

"For each series of 9 numbers given, I found the number to go into the middle square and the sum for the rows, columns, and diagonals of the MAGIC SQUARE."

Number series  $-3, -1, 1, 3, 5, 7, 9, 11, 13$

"Middle" number  
Row, Column, Diagonal  
Sum for the Square

### THE MAGIC SQUARES



Mathematician: \_\_\_\_\_

"For each series of 9 numbers given, I found the number to go into the middle square and the sum for the rows, columns, and diagonals of the MAGIC SQUARE."

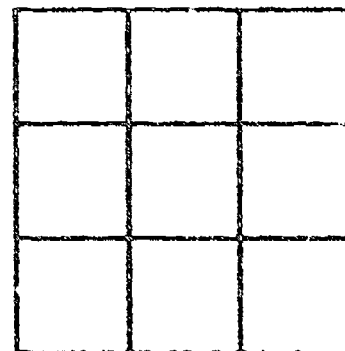
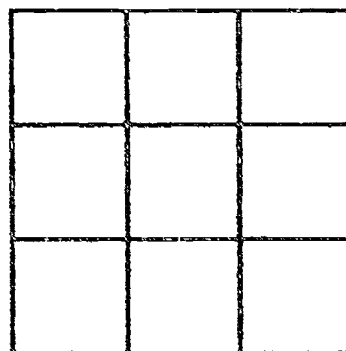
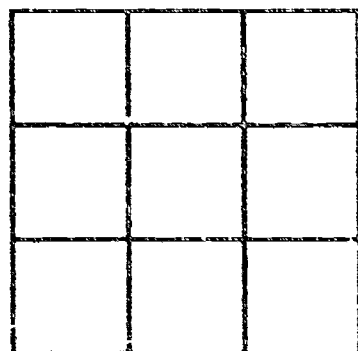
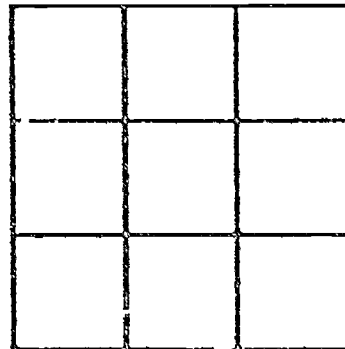
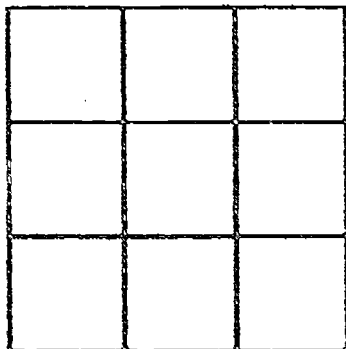
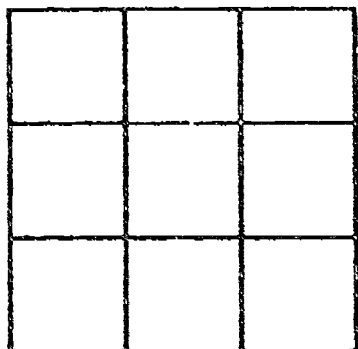
Number series 0, 2, 4, 6, 8, 10, 12, 14, 16

"Middle" number

Row, Column, Diagonal

Sum for the Square

### THE MAGIC SQUARES



Mathematician: \_\_\_\_\_

"For each series of 9 numbers given, I found the number to go into the middle square and the sum for the rows, columns, and diagonals of the MAGIC SQUARE."

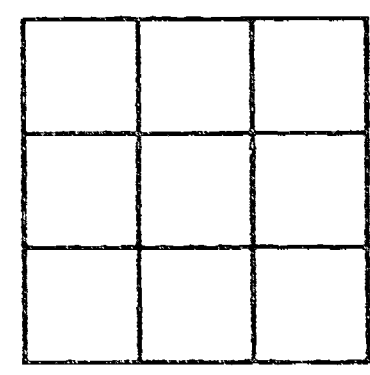
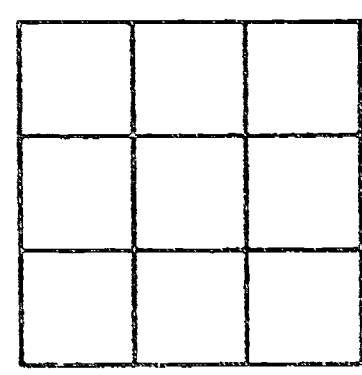
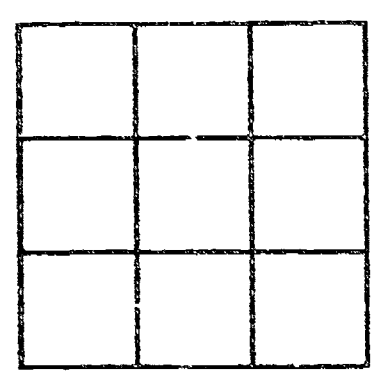
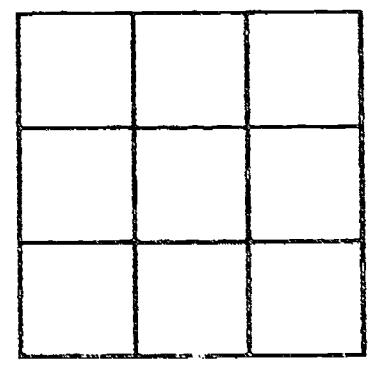
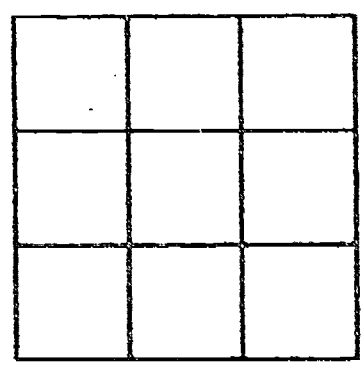
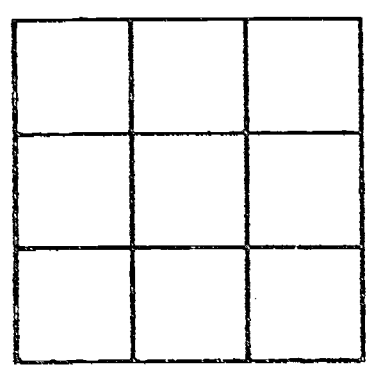
Example:

Number series 1, 3, 5, 7, 9, 11, 13, 15, 17

"Middle" number 9

Row, Column, Diagonal Sum for the Square 27

### THE MAGIC SQUARES









In a family with one child, there are two possibilities - a boy or a girl. In a family of two children the possibilities are shown in the table. Complete the table for a family with 4 children and a family with 5 children.

Boys	Girls	Total
1	0	1
0	1	1
2	0	2
0	2	2
1	1	2
		4
		5

DIGITS: The digits 1 and 2 make 21 and 12. Show all numbers possible using the given digits. Do not use any digit more than once.

DIGITS	NUMERALS
1,2,3	
1,2,3,4	
0,4,6	

Mathematician: \_\_\_\_\_

"I chose the given number of statements, recorded the allowed number of moves and the results of the game."

CHIPS AVAILABLE

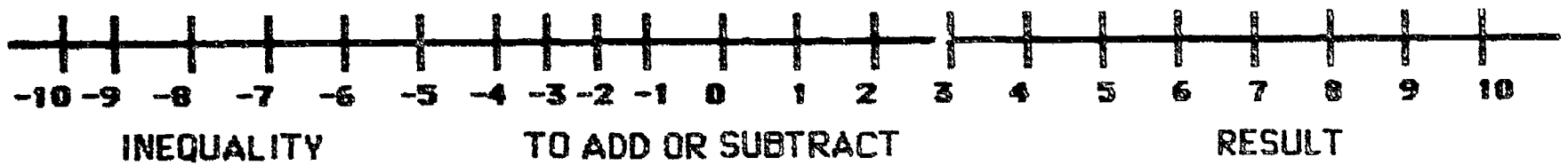
RED	
GREEN	
YELLOW	

RULE: \_\_\_\_\_  
 \_\_\_\_\_

	Moves to win			Statement(s) Chosen	Allowed to Move?	
	Number of chips chosen				YES	NO
	RED	GREEN	YELLOW			
1st choice						
2nd choice						
3rd choice						
4th choice						
5th choice						
6th choice						
7th choice						
8th choice						
9th choice						
10th choice						

Mathematician: \_\_\_\_\_

"I used the number line to add to and subtract from the given inequalities."



$8 > 6$	$+ 4$	
$-3 < -1$	$- 2$	
$0 < 4$	$- 6$	
$2 < 7$	$+ 2$	
$3 < 11$	$- 5$	
$4 > -1$	$- 6$	
$4 > 3$	$+ 6$	
$-3 < 0$	$- 2$	
$6 > 1$	$- 4$	
$2 < 8$	$+ 3$	

Mathematician: \_\_\_\_\_

"I used the number line to add to and subtract from the given inequalities."



INEQUALITY

TO ADD OR SUBTRACT

RESULT

EXAMPLE: $\square + 2 > 7$	$- 2$	$\square > 5$ can you list 4 of these?
$\square + 1 < 5$		
$\square - 3 < 4$		
$2\square + 1 > 9$		
$\square + 5 < 11$		
$2\square - 1 < 7$		
$\square - 3 < 4$		
$2\square - 3 < 7$		
$2\square - 3 > 3$		
$\square + 6 < 13$		

Mathematician: \_\_\_\_\_

For the following sets of data, I

1. found the mean
2. found the range
3. graphed the data given on the balance beam
4. found the MOMENTS on each side

TEST SCORES

BALANCE BEAM

20, 25, 25, 30  
42, 45

Mean \_\_\_\_\_ Range \_\_\_\_\_

Moments clockwise \_\_\_\_\_

Moments counterclockwise \_\_\_\_\_

15, 18, 20, 20, 20,  
25, 29

Mean \_\_\_\_\_ Range \_\_\_\_\_

Moments clockwise \_\_\_\_\_

Moments counterclockwise \_\_\_\_\_

85, 92, 94, 96, 99,  
100

Mean \_\_\_\_\_ Range \_\_\_\_\_

Moments clockwise \_\_\_\_\_

Moments counterclockwise \_\_\_\_\_

Mathematician: \_\_\_\_\_

"For the following sets of data, I

1. found the mean
2. found the range
3. graphed the data given on the balance beam
4. found the MOMENTS on each side

TEST SCORES

BALANCE BEAM

39, 43, 47, 51,  
55, 57, 61

Mean \_\_\_\_\_ Range \_\_\_\_\_

Moments clockwise \_\_\_\_\_

Moments counterclockwise \_\_\_\_\_

5, 6, 8, 10, 12,  
15, 16, 18, 19

Mean \_\_\_\_\_ Range \_\_\_\_\_

Moments clockwise \_\_\_\_\_

Moments counterclockwise \_\_\_\_\_

9, 9, 12, 15, 16,  
17, 18

Mean \_\_\_\_\_ Range \_\_\_\_\_

Moments clockwise \_\_\_\_\_

Moments counterclockwise \_\_\_\_\_

Mathematician: \_\_\_\_\_

"For each data set I found the range and the average. Then I showed how the data would balance a beam with the center at the average."

DATA SET	BEAM BALANCE
	<div data-bbox="505 618 1767 682" style="border: 1px solid black; height: 26px;"></div>
	<div data-bbox="505 1041 1767 1106" style="border: 1px solid black; height: 26px;"></div>
	<div data-bbox="505 1555 1767 1619" style="border: 1px solid black; height: 26px;"></div>
	<div data-bbox="505 2008 1767 2073" style="border: 1px solid black; height: 26px;"></div>



Mathematician: \_\_\_\_\_

"For each data set I found the range and the average. Then I showed how the data would balance a beam with the center at the average."

DATA SET	BEA11 BALANCE
	<div data-bbox="498 632 1754 700" style="border: 1px solid black; height: 27px; width: 650px;"></div>
	<div data-bbox="498 1060 1754 1128" style="border: 1px solid black; height: 27px; width: 650px;"></div>
	<div data-bbox="498 1577 1754 1645" style="border: 1px solid black; height: 27px; width: 650px;"></div>
	<div data-bbox="498 2043 1754 2111" style="border: 1px solid black; height: 27px; width: 650px;"></div>

## Problems That Involve Data Analysis

1. These tables show the births and deaths in the United States in 5 years.

	Units of Measure	1970	1975	1980	1982	1983
Births	1000	3732	3144	3612	3631	3614
Deaths	1000	1921	1893	1990	1986	2010
Difference between births & deaths	1000					
Population millions		203.3	215.5	226.5	231.8	234

- Complete the table
- Find the part of the increase in population for each period that is due to the difference between births and deaths
- Graph the data in the table using a line graph.

2. This table shows the school enrollment and school expenditures for several years for public elementary and secondary schools:

	Unit of Measure	1970	1975	1980	1982	1983
Enrollment	millions	60.4	60.1	57.3	57.9	57.7
Expenditures	\$billions	46.0	70.9	102.5	119.1	127.0

- What is the expenditure in dollars/unit of enrollment for each year? \_\_\_\_\_
  - Why do you think expenditures increased while enrollments decreased?
- c. Does the graph showing births have any relationships to the decline in enrollment over this period?

3. This table shows deaths from various causes for several years:

Causes of Death	1970	1980	1981
		in 1,000's	
All causes	1921	1989.8	1978
Cardiovascular diseases	1008	988.5	973
Malignancies	330.7	416.5	422.1
Accidents	114.6	105.7	100.7
Obstructive Conditions	30.9	56.1	58.8
Pneumonia & Flu	62.7	54.6	53.7
Diabetes	38.3	34.9	34.6
Other			

- a. Fill in the "other" line of the table.
- b. In which year was the percent of deaths that were due to accidents the highest? \_\_\_\_\_
- c. The deaths due to cardiovascular disease were how many 1000's more than all other causes? \_\_\_\_\_

4. This table shows incidence of tooth decay in children for 2 periods.

Age	1971-1974		1979-1980	
	No. of Children (1000)	Average No. of Fillings	No. of Children (1000)	Average No. of Fillings
10	4378	4.14	3515	2.60
11	4088	4.58	3479	3.00
12	4116	6.36	3601	4.18
13	4153	8.67	3822	5.41
14	4046	9.60	4063	5.53

- a. Why do you think the average number of fillings increased with age?
- b. Why is the average number of fillings lower for 1979-80 than for 1971-74?
- c. If the average number of fillings declines at the same rate, what should the average number of fillings be for a 12 year old in 1985-86?

"For each frequency table given, I made a horizontal bar graph, a vertical bar graph and a circle graph."

**FAVORITE TELEVISION SHOWS OF STUDENTS:**

Detective	60
Comedy	45
Variety	38
Western	50

**FAVORITE COLORS OF STUDENTS**

Blue	50
Pink	45
Red	54
Orange	49
Yellow	52

**STEREO TAPE SALES**

Monday	150
Tuesday	130
Wednesday	140
Thursday	115
Friday	155

**CHOICES OF A FAVORITE SPORT**

Tennis	40
Hockey	36
Soccer	34
Baseball	44
Football	46

**FAVORITE SUBJECTS OF STUDENTS**

Math	60
English	56
Spanish	44
Science	50
History	46
French	44

**MATHEMATICS: TEST SCORES OF A CLASS**

<u>Score</u>	<u>Number Receiving</u>
95	15
85	25
80	50
75	30
70	20
60	15

# LEVEL SIX LOGO WORKSHEET

Mathematician: \_\_\_\_\_

"I ran the procedures given using 10 as input. I answered the questions."

(INPUT) MYSTERY1 10

```
TO MYSTERY1 :NUMBER  
IF :NUMBER = 0 [STOP]  
PRINT :NUMBER  
MYSTERY1 :NUMBER - 1  
END
```

What was the first number printed? \_\_\_\_\_

What was the last number printed? \_\_\_\_\_

(INPUT) MYSTERY2 10

```
TO MYSTERY2 :NUMBER  
IF :NUMBER = 0 [STOP]  
MYSTERY2 :NUMBER - 1  
PRINT :NUMBER  
END
```

What was the first number printed? \_\_\_\_\_

What was the last number printed? \_\_\_\_\_

Explain the difference in the two printed lists. All statements in the two procedures are the same; they are just ordered differently!

This is my procedure to print:

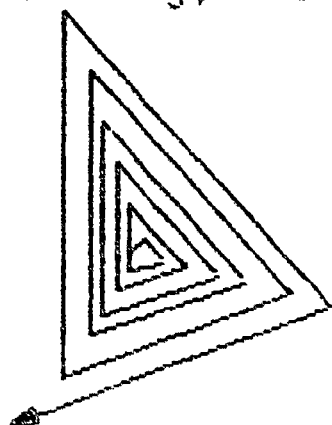
- 1
- 2
- 3
- 4
- 5
- 6
- 7
- 8

This is my procedure to print:

1 2 3 4 5 6 7 8



This is my procedure to give:



# LEVEL SIX LOGO WORKSHEET

Mathematician: \_\_\_\_\_

"I ran the procedures given using 10 as input. I answered the questions."

(INPUT) MYSTERY1 10

```
TO MYSTERY1 :NUMBER  
IF :NUMBER = 0 [STOP]  
PRINT :NUMBER  
MYSTERY1 :NUMBER - 1  
END
```

What was the first number printed? \_\_\_\_\_

What was the last number printed? \_\_\_\_\_

(INPUT) MYSTERY2 10

```
TO MYSTERY2 :NUMBER  
IF :NUMBER = 0 [STOP]  
MYSTERY2 :NUMBER - 1  
PRINT :NUMBER  
END
```

What was the first number printed? \_\_\_\_\_

What was the last number printed? \_\_\_\_\_

Explain the difference in the two printed lists. All statements in the two procedures are the same; they are just ordered differently!

This is my procedure to print:

- 1
- 2
- 3
- 4
- 5
- 6
- 7
- 8

This is my procedure to print:

1 2 3 4 5 6 7 8