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ABSTRACT

Mathematics and the use of mathematical thinking should be much more than what has been traditional school arithmetic. Much of the mathematical reasoning can be developed and experienced out of school, particularly in the home. This material is a teacher's guide designed to help parents support what is done with their children in class. End-of-the-year assessment material is presented. A total of 35 activities on the following concepts and skills are included: (1) computation; (2) scientific notations; (3) logic; (4) word problem; (5) ratio; (6) inequality; (7) area; (8) graphing; (9) geometry; (10) problem; (11) estimation; (12) using data; and (13) use of LOGO. (YP)

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MEANINGFUL MATHEMATICS

LEVEL SIX

TEACHER'S GUIDE TO LESSON PLANS

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SE 051 198

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LEVEL SIX ASSESSMENT RECORD

Mathematician:______________________________

	LEVEL OF MASTERY					
	Date:		Date:		Date:	
TOPIC	Progress Made	Mastery Attained	Progress Made	Mastery Attained	Progress Made	Mastery Attained
Computation						
whole numbers						
fractions						
decimals						
calculator						
Thinking						
Exponents						
Scientific Notation						
Money						
Logic						
AND						
OR						
NOT						
IF-THEN						
OPEN SENTENCES						
w/positive numbers						
w/negative numbers						
RATE APPLICATIONS						
distance/time						
parts of wholes						
percents						

LEVEL SIX ASSESSMENT RECORD

LEVEL OF MASTERY

	Date:		Date:		Date:	
	Progress Made	Mastery Attained	Progress Made	Mastery Attained	Progress Made	Mastery Attained
Guess My Rule						
Ratio						
Inequalities						
Area						
rectangles and squares						
triangles						
parallelograms						
circles						
Graphing						
Squares						
enlarging						
shrinking						
differences between						
Triangles						
congruence						
similarity						
corresponding parts						

LEVEL SIX ASSESSMENT RECORD

LEVEL OF MASTERY

	Date:		Date:		Date:	
	Progress Made	Mastery Attained	Progress Made	Mastery Attained	Progress Made	Mastery Attained
Magic Triangles						
Magic Squares						
Probability						
Use of Number Line						
Problem Solving						
Estimation						
Using Data						
mean						
range						
graphing						
Using LOGO						
use of variables						
use of recursion						

ERIC

LEVEL SIX

INTRODUCTION:

All but a very few of students should be concrete operational and capable of conserving number, length, area and volume. Some may be able to use formal reasoning processes within certain contexts. Hence, first order symbol use will be expected of all. Students should understand the use of =, +, −, ×, ÷, >, <, : and %. Computation with whole numbers, fractions and decimals should be maintained through use in problem solving activities. Students should know WHEN to use WHICH of the four arithmetic operations. Review lessons are provided for these topics emphasized in LEVELS Four and Five, but be prepared to re-use those lessons for students who have not yet achieved mastery.

Increased emphasis is placed on the following topics at this level:

............algebra
............geometry, including congruence and similarity
............use of exponential and scientific notation
............use of proportions
............calculator use
............graphing in the coordinate plane
............use of percent and other constant rate applications
............more complete programming LOGO.

Lessons are arranged by topics. Most topics should be mastered before moving to another. Make every attempt to relate old material to new material and to integrate several different processes and skills into problem solving activities: Little paper and pencil computation should be expected. This is to be done by calculator when it cannot be done quickly mentally.

Repeat lessons as required for mastery. Alter examples given to meet special needs. The same problems can be worked using in successions whole numbers, fractions and decimals as data, for example.

Most lessons give ONE or TWO examples for explanation. Use more when needed, before assigning group seat work.

Great emphasis is placed on the use of number sentences at this level.

Commercial materials to supplement the Teacher's Guide and Worksheets provided are referred to in the lessons.

Spend five minutes or so during each class period on orally presented problems that require mental solutions and/or computations.

LEVEL SIX
ASSESSMENT TASKS

COMPUTATION: Give the student the accompanying test. The test is in two parts. Part One permits no writing material - answers should be found mentally. The second part requires students to first estimate, then calculate with a hand held calculator.

EXPONENTS AND SCIENTIFIC NOTATION: Give the student the accompanying test.

LOGIC: Give the student the accompanying test.

OPEN SENTENCES: Give the student the accompanying short test.

RATE APPLICATIONS AND RATIO: The problems furnished should give you a good idea of how these are being handled. A percent and ratio test is supplied for you to use.

AREA: A test is supplied to give students.

SQUARE: A test is supplied for enlarging, shrinking and finding the difference between squares.

TRIANGLES: A test is given for this.

PROBABILITY AND USING DATA: A test is supplied.

USING LOGO: You should be able to observe the child's progress here relative to use of variables and recursion.

Mathematician:____

PROBABILITY

GIVEN	**PROBABILITY OF**
4 red balls, 3 green balls	**selecting a green ball is** ☐
3 white balls, 4 red balls, 2 black balls	**selecting a green ball is** ☐

USING DATA

For this set of data find the information asked for and make a beam balance on the range. Mark the center to balance it.

53, 71, 64, 67, 60

Median (midscore) __________

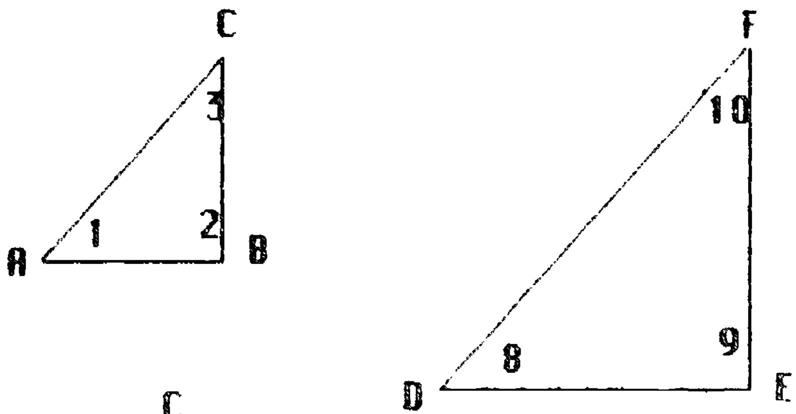
Mean __________

Range __________

7 17 89 1

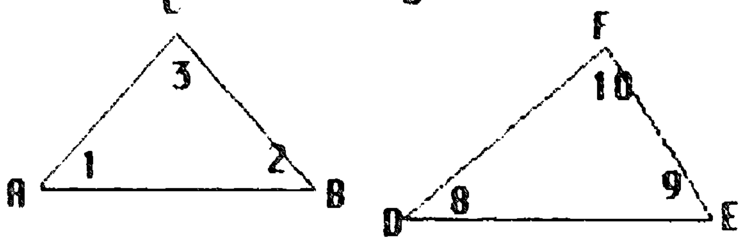
MATHEMATICIAN:______________________

TRIANGLES



☐ congruent?

☐ similar only?



☐ congruent?

☐ similar only?

For those triangle pairs that are congruent list the pairs of corresponding sides and angle.

1
2
3.
4
5.
6.

7 17 89 1

Mathematician:____

SQUARES

$(5 + 1)^2 =$

$(T + 3)^2 =$

$(S - 2)^2 =$

$(2B + 1)^2 =$

$(3A - 1)^2 =$

$7^2 - 4^2 = (\quad + \quad)(\quad - \quad)$

$A^2 - T^2 = (\quad + \quad)(\quad - \quad)$

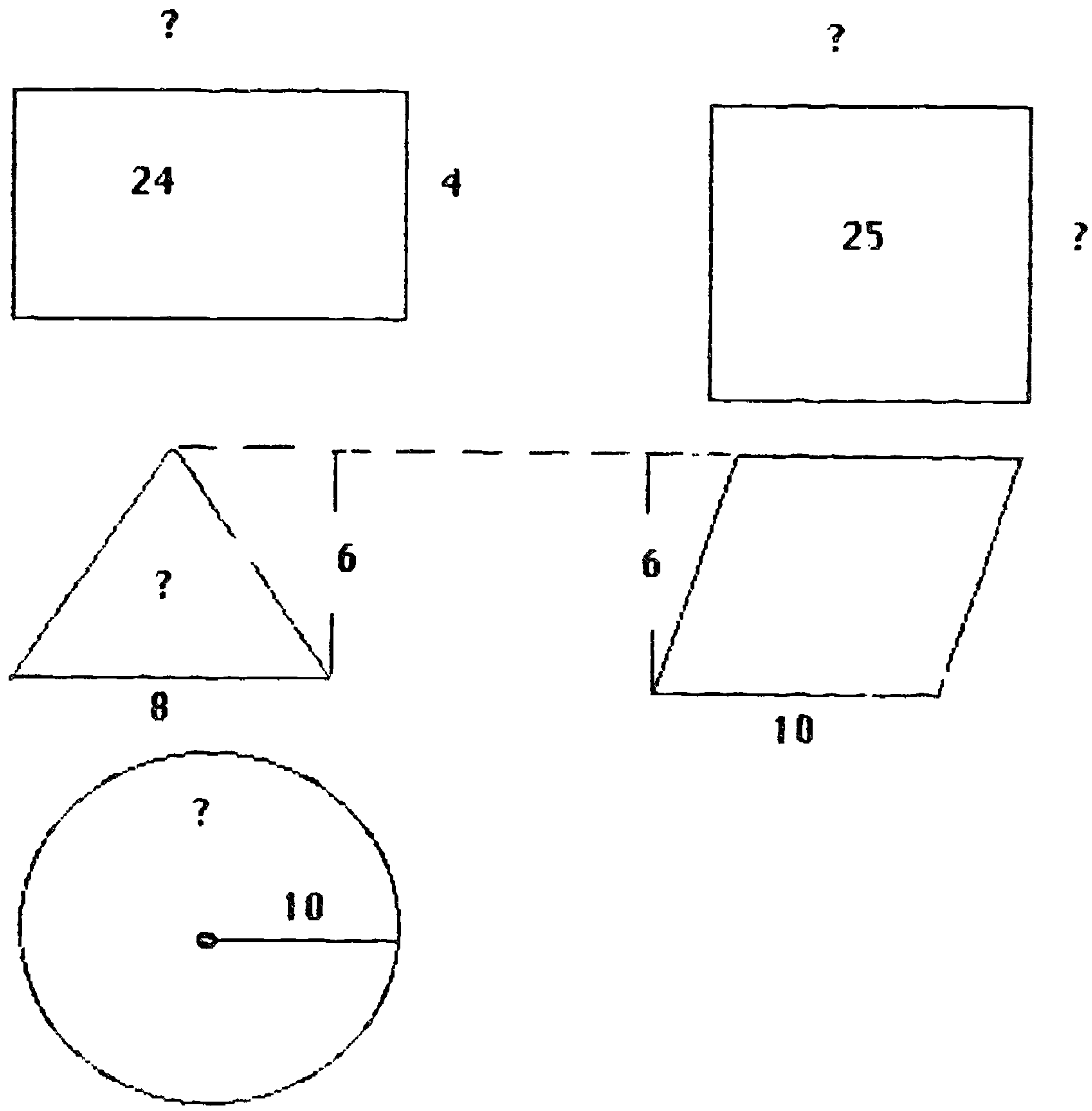
$10^2 - 3^2 = (\quad + \quad)(\quad - \quad)$

7 14 89 1

Mathematician:________________________________

AREA

The area is given inside the shape. Supply what is missing for each shape.



7 17 89 3

Mathematician: ______________________________

OPEN SENTENCES

$3\square + 4 = 13$

$18 = 3\square - 6$

$6 - \square = 1$

$^{+}2 - \square = ^{-}3$

$\square + ^{-}3 = ^{-}2$

$\square - ^{-}2 = 4$

$7 - \square = ^{-}3$

$5 = \square + ^{-}2$

$2\square < 5$

$\square$ can be ______________________________

$3\square + 1 > 16$

$\square$ can be ______________________________

$2\square - 3 < 11$

$\square$ can be ______________________________

Matematician:________________________________

PERCENT

40% of $75 is ____________

$24 is __________% of $48

A discount is $10. This is 25% of what price? [ ]

A store advertises 33 1/3% off on all items. What would you pay for an item that is priced at $90?

[ ]

If a bank pays 5% interest at the end of each month, how much interest would your deposit of $200 earn the first month?

[ ]

2 for $5 is the same as [ ] for $15.

A trail mix that has twice as many peanuts as raisins, has how many peanuts and raisins if there are 72 pieces total

Peanuts [ ] Raisins [ ]

7.17.89.5

ERIC

Mathematician:____

COMPUTATION

I. Find these answers mentally.

45 + 71 = ☐   ☐ = 63 - 29

☐ = 2 x 26   ☐ = 154 ÷ 7

2/3 + 3/4 = ☐   4/5 - 1/4 = ☐

☐ = 1 1/3 x 2   ☐ = 3/4 ÷ 2/3

17.1 + 23.8 = ☐   ☐ = 34.0 ÷ 16.2

☐ 4 x 3.03   80.4 ÷ 4 = ☐

II. First estimate, then calculate.

434 + 385 = ☐
Estimate: __________

983 - 347 = ☐
Estimate: __________

83.4 x 16.02 = ☐
Estimate: __________

304.5 ÷ 21.3 = ☐
Estimate: __________

18.45 x 103.6 = ☐   .29 x 16 = ☐

42.39 ÷ 1.39 = ☐   ☐ = .39 + .19 + .49

☐ = 1.39 + 62.3   ☐ = 10.00 = 1.99 - 4.99

7 17 89 6

Mathematician: ____________________

EXPONENTS

I. Simplify these by adding or subtracting exponents.

$2^2 \times 2^5 = \square$ $3^3 \times 3^4 = \square$

$T^2 \times T^3 = \square$ $\square = T^4 \times T^2$

$\square = 2^7 \div 2^3$ $3^5 \div 3^2 = \square$

$T^7 \div T^2 = \square$

$T^5 \div T^3 = \square$

II. Express each in scientific notation.

3475 = ____________ $\times 10^{\square}$

487.2 = ____________ $\times 10^{\square}$

98.43 = ____________ $\times 10^{\square\square}$

.043 = ____________ $\times 10^{\square}$

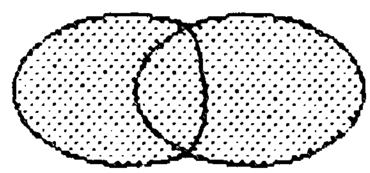
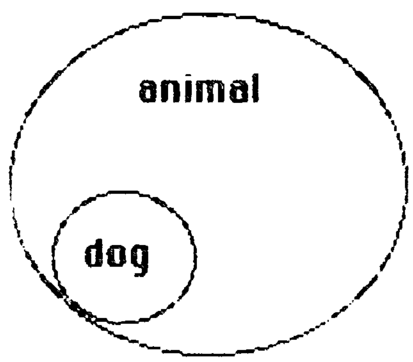
.00032 = ____________ $\times 10^{\square}$

7.17.89.7

Mathematician:______________________________

LOGIC

1. Draw a diagram to show the given phrase
 or
 Write the phrase the diagram shows

Phrase	Diagram
Soft and Wet	
	
Up or down	
	

7.18.89.1

LEVEL SIX

DECIMALS

Background: Decimals are a special form of fractions with place value representation. The same ideas need to be emphasized.

...Multiplication of a quantity by something GREATER THAN ONE gives as LARGER quantity

...Multiplication of a quantity by something LESS THAN ONE gives a SMALLER quantity

...Division of a quantity by something GREATER THAN ONE gives a SMALLER quantity

...Division of a quantity by something LESS THAN ONE gives a LARGER quantity

Students should always estimate the results of operations with decimals before doing the computation

LESSON ONE: Multiplication

Introduction: Write a number on the chalkboard or overhead: 45.713

"What is the whole number part of this number?" (45)

"What is the decimal part?" (.713)

Write a second number: 45.713
x 3.04

"What is the whole number part of the multiplier?" (3)

"What is the decimal part?" (.04)

"Will the whole number part of the product be in the 10's or 100's? (100's) (close to 135)

"Then the decimal point is placed after the THIRD digit to show hundreds."

"Do this on the calculator to find the exact result." Write: 45.713
x .15

"Will the product be more or less than 45?" (LESS THAN)

"Why?" "What is .1 x 45? .2 x 45?"

"The product should be between what numbers? (4 and 9)

"Use the calculator to find the exact product."

Do two or three examples of each kind, with the same kinds of questions emphasizing (1) where the result should be, and (2) estimating the result.

<u>Activity:</u> Have pairs of students work on the worksheets together, using calculators to verify the results.

<u>LESSON TWO: Division</u>

<u>Introduction:</u> Write: 45.713 ÷ 3.04

"Is the answer more or less than 45?"

"Why?"

"The answer should be near what number?"

"Do the division on the calculator."

Write: 45.713 ÷ .15 "Should this answer be more or less than 45?" "Why?"

"The answer should be between what numbers? (230 - 450)

"Do the division on the calculator to find the answer."

"Do two or three examples of each kind emphasizing approximating the answer and judging the relative magnitude of the answer.

<u>Activity:</u> Have students work in pairs to do the worksheet using calculators to find the answers after estimating.

ERIC

COMPUTATION REVIEW

LESSON ONE: Multiplication with Whole Numbers

Introduction: Put this example on the overhead or chalkboard:

```
 46
x53
```

"How many digits do you expect in the answer? (4) Why?"

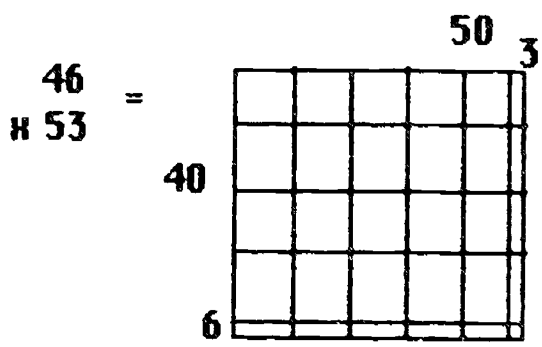
Emphasize the estimating by mentally multiplying 50 x 50 which is in the thousands or 4 digits to represent.

Point to the top multiplier. "What digit is in the tens place?" in the ones place?" Do the same with the bottom multiplier.

Write:

```
 46   =   40 + 6
x53   x   50 + 3
```

Draw:



"How many hundreds to you see?(20) Write: 2000

"How many 60's do you see?(5) Write:

```
  2000
+  300
+  120
```

"What is 6 x 3" Write:

```
  2000
+  300
+  120
+  180
-------
```

and add to get 2438.

ERIC

Relate these partial products to the expanded form. 40 + 6
50 + 3

50 x 40 = 2000
50 x 6 = 300
3 x 40 = 120
3 x 6 = 18

Write a second example and repeat the process. Emphasis should be on multiplication by places and recognition of what ALL of the digits - in multipliers and in the product - are counting.

Have students use the worksheet provided to review the multiplication.

LESSON TWO: Review of Multiplication of Decimals

Introduction: Put the following example on the overhead projector or chalkboard.

4.82
x3.01

"How is this written in expanded form?" Write in response to student suggestions. It should finally be: 4.82 = 4 + 80/100 + 2/100
x3.01 = 3 + 1/100

"What is the result of multiplying the whole number parts?" (12)

Write: 4.82 = 4 + 80/100 + 2/100

x3.01 = 3 + 1/100

12.

"What is the result of multiplying 3 x .82?" Write:

4.82 = 4 + 80/100 + 2/100

x 3.01 = 3 + 1/100

12 12 + 240/100 + 6/100

+2.46

"What is the result of multiplying .01 (1/100) x 4?" Write:

4.82 = 4 + 80/100 + 2/100

x3.01 = 3 + 1/100

12. = 12 + 246/100 + 4/100

2.46

.04

"What is the result of multiplying .01 (1/100) x .82 (82/100)

Write: 4.82 = 4 + 80/100 + 2/100

x3.01 = 3 + 1/100

12. 12 + 246/100 + 4/100 + 82/1000

2.46

.04

.0082

Total these:

4.82 = 4 + 82/100

3.01 = 3 + 1/100

14.5082 = 12 + 2 + 46/100 + 4/100 + 82/1000

Do a second example, pointing out the parts. Have the students do both examples on the calculator and read the results.

Emphasize how the multiplication is done in places, just like whole numbers. The decimal point is placed (1) by identifying the whole number range of the product or (2) the largest decimal in the product.

Consider this example to emphasize the latter: .43
x.27

The largest decimal is .2 of .4 or .08 so the decimal point is placed so the answer is in the range of .08 - .1 or .2. Do the example on the calculator to verify this .1161.

Give several examples for students to do with the calculator, after first estimating where the decimal point should go.

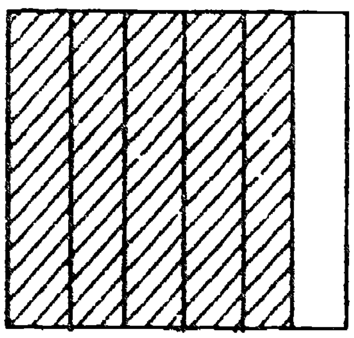
Have students do the worksheet provided.

LESSON THREE: Multiplication of fractions - Review

Introduction: Place the following example on the board: 2/3 x 5/6

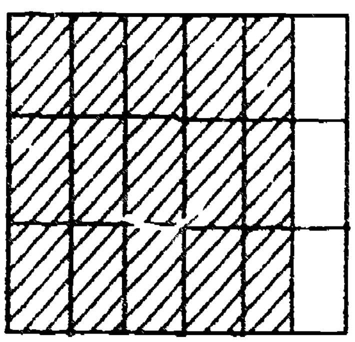
"Do you expect a fraction more than or less than 5/6? why?"

Emphasize the idea of taking only a part of 5/6 so the answer is smaller than that. "Let's look at the picture of this:

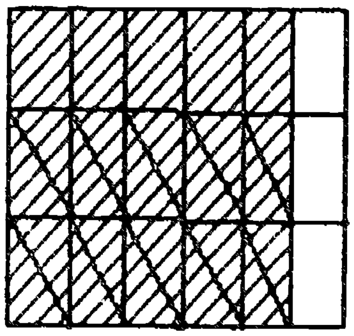


5/6

"How can we take 2/3 of this?" Emphasize that it must be divided into 3 parts first:



"Now we can take 2 of the 3 parts of 5/6."



"What size are these parts?"
(eighteenths)
"How many are there?" (5 x 2 = 10)

"So the result of the multiplication of 2/3 x 5/6 = 10/18 (10: the number of new smaller parts and 18: the number smaller parts in a whole)."

"This can be reduced by seeing that 10 and 18 both have a factor of 2 so 10x18 - 2x5/2x9   2/2 = 1 so this is 1 x 5/9 - 5/9.

Do a second: 3/4 x 1 2/5

"We must find the total number of fifths. The problem then is....."

1 2/5 = 5/5 + 2/5 = 7/5. Write: 3/4 x 7/5

"As before 3 x 7 gives the number of new parts. 4 x 5 gives the number of these parts in a whole, or one."

3/4 x 7/5 = 21/20 or 1 1/20, since 20/20 = 1.

Consider: 1 1/3 x 2 1/5

"This must be more than 2 1/5" "What is the number of thirds in 1 1/3?" Write: 4/3

"What is the number of fifths in 2 1/5?" Write: 4/3 x 11/5 = 44/15, or just under 3. This is greater than 2 1/5 as expected.

Write: 4/3 x 11/5 = 44/15 = 2 14/15

Do a few more examples. Emphasize:

1. Finding the new number of smaller parts that are in a whole by finding the product of denominators.

2. Finding the number of these parts in the answer by finding the product of the denominators.

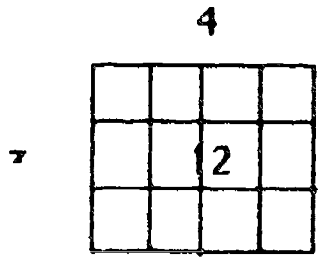
3. Changing mixed numbers to fractions, and

4. Reducing fractions by finding factors common to numerals and denominator.

LESSON FOUR: Division of Whole Numbers

Introduction: Put the following example on the chalkboard: $3 \times 4 = 12$

"We can show this by":



(draw on the board)

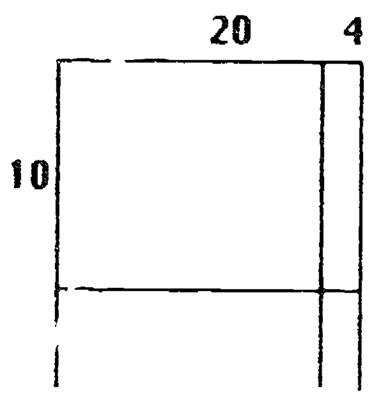
"The related divisions are: $12 \div 3 = 4$ and $12 \div 4 = 3$."

"These are also shown on the rectangle where we know one side and the area and must find the missing side:

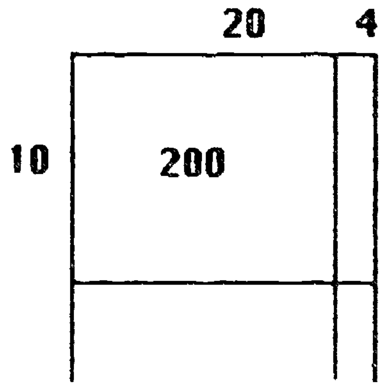
$12 \div 3 = \square$ and $12 \div 4 = \square$

"Consider 287 ÷24" (write on the board)

We can think of 287 as possible area and 24 as the given side

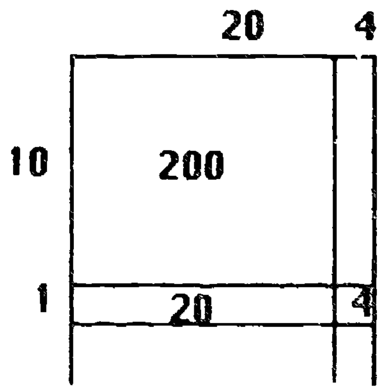


"A side of 10 would use up area as shown."



$$\begin{array}{r} 287 \\ -\,240 \\ \hline 47 \end{array}$$

"Only one 24 can be obtained from 47."



$$\begin{array}{r} 47 \\ -24 \\ \hline 23 \end{array}$$

"So the result is 287 ÷ 24 = 11 R 23. The representation of the remainder, 23, divided by 24 as a decimal can be done on the calculator. In expanded form this division would be:

```
          10 + 2
         ______________
20 + 4  | 200 + 80 + 7
          200   40
         ______________
                40 + 7
                20 + 4
                ______
                20 + 3
```

1) 200 ÷ 20 is 10
2) 40 ÷ 20 is 2, but that would require 8, so use 1
3) 23 is the remainder

<u>LESSON FIVE: Division of Decimals</u>

<u>Introduction</u>: Students must realize that

1. Division of a decimal - a fraction - by a whole number results in still smaller fractional parts, and

Division of a decimal by a decimal less than one requires finding how many SMALLER parts are in a number. This results in a number MORE THAN the original.

Write this on the board: $5 \div 1/10$

"How do we get 1/10ths in these ones?"

"Each is divided into 10 equal parts so the number of tenths is 50."

"When we divide by a fraction less than one we are looking to see how many SMALLER pieces can be made."

"Will we have more than we started with, or less?"

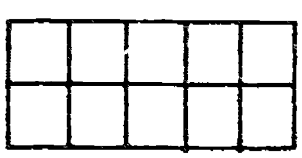
"Notice that dividing by 1/10 or .1 is the same as multiplying by 10."

"What would $5 \div 1/100$ be?"

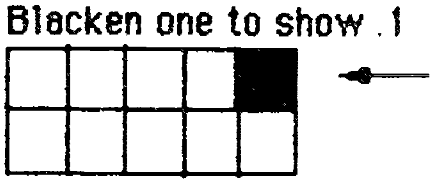
"Here we have 100 from each of the 5 or 500 altogether."

Use overhead transparency graph paper for the pictorial model.

Put the graphic on the board to show a one divided into tenths.

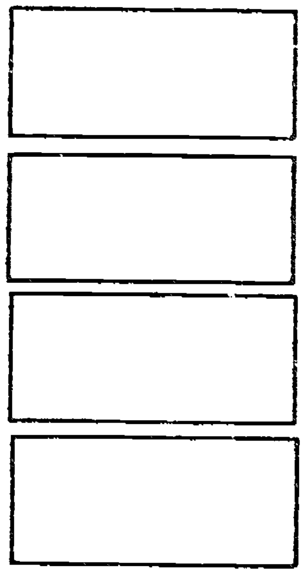


Blacken one to show .1

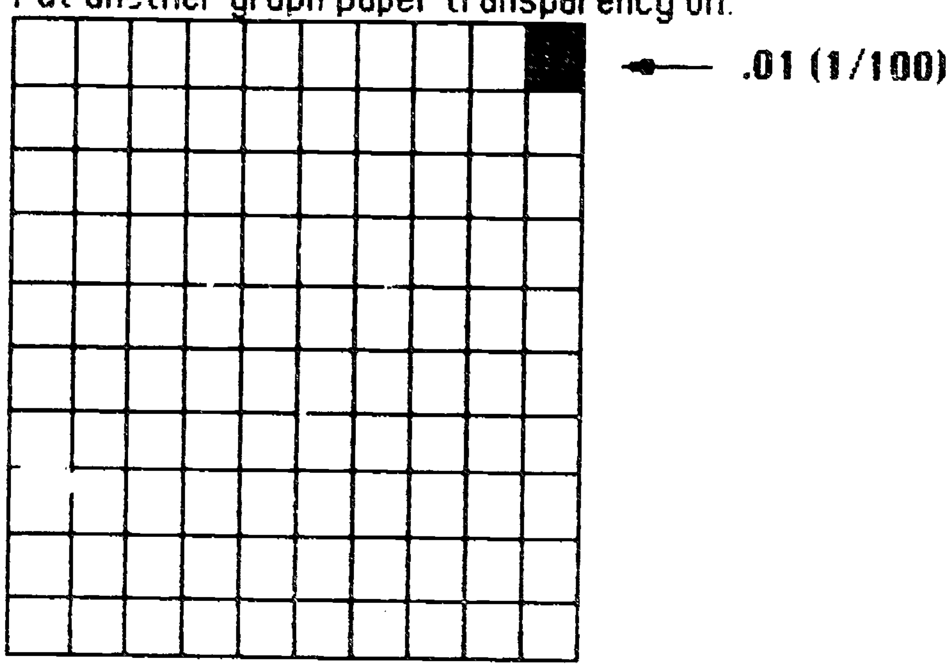


.1 (1/10)

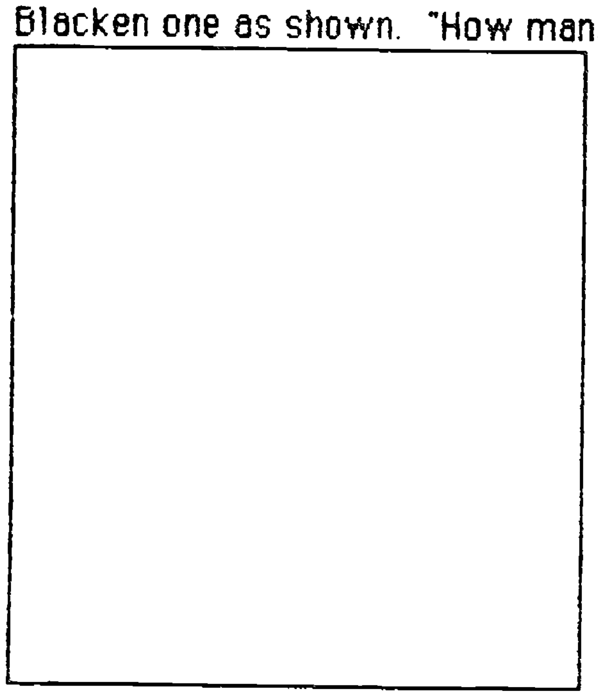
"Here are 4 ones. How many of the .1 are in these?"

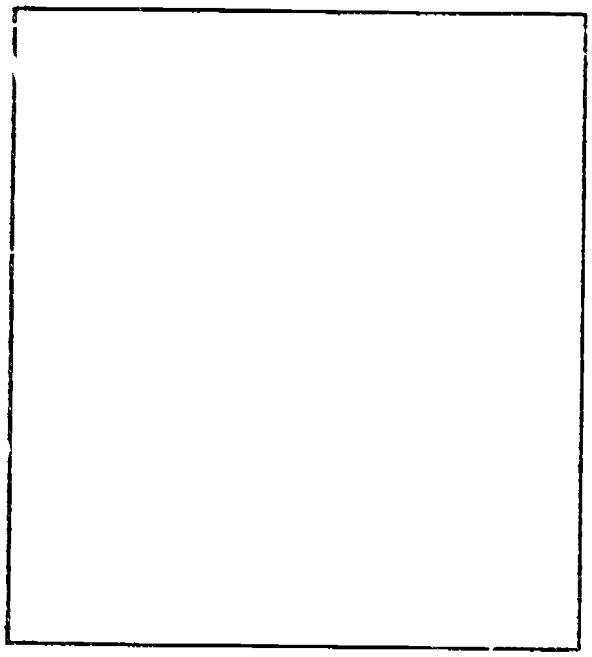


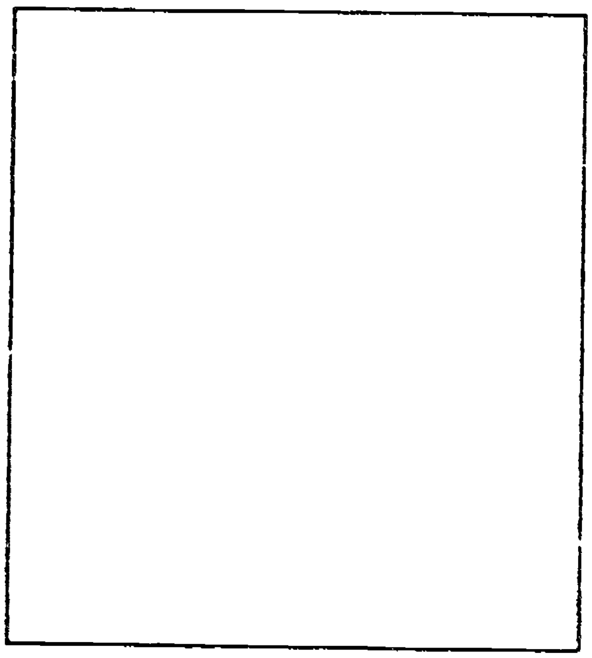
Put another graph paper transparency on:



Blacken one as shown. "How many of the .01 are in these 3 ones?"







Estimate this quotient: 4.10 ÷ .2

(An estimate between 20 and 21 should be given.) "There are 5 of the .2 in each one, so 4 has 20. There is half of another one, so 10.5 is the answer."

<u>Activity:</u> Assign the worksheets to pairs of students who have calculators.

<u>LESSON SIX: Division of Fractions</u>

Introduction: Division of fractions is just like division with whole numbers ONCE BOTH ARE EXPRESSED WITH THE SAME DENOMINATOR. Emphasize the fact that a denominator NAMES the size of the part.

"6 apples ÷ 3 apples" = 3 (no name, just a number)

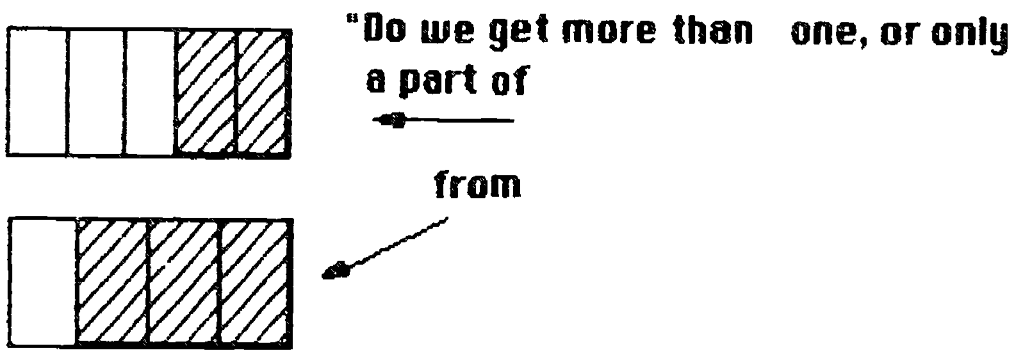
"15 kites ÷ 3 kites" = 5 (no label, just the number of 3 kites groups)

"8 fifths ÷ 2 fifths" = 4 (no name, just a number)

Write: 8/5 ÷ 2/5 = 8 ÷ 2 = 4

"To divide fractions, express each with the same denominator and divide the whole number numerators." Consider: 3/4 ÷ 2/5

"What kind of answer do we expect?" Draw the picture:



"We expect an answer of more than one."

3/4 ÷ 2/5 = 15/20 ÷ 8/20 = 15 ÷ 8 = 15/8

"15/8 is more than one - almost two (1 7/8)

Do a second in the same way. The easy way to express both with the sane denominator is to cross multiply to get numerators and multiply denominators, e.g. in the above example: $\frac{5 \times 3}{4 \times 5} \div \frac{4 \times 2}{4 \times 5}$

Assign worksheets to pairs of students to work.

LEVEL SIX

REVIEWING FRACTIONS

Background: Students should understand how to perform ALL of the operations with fractions as a consequence of previous work. They now should apply their knowledge of fractions and operations with fractions in problem solving.

LESSON ONE

Introduction: Place 2 fractions on the overhead projector: 3/5 2/3

"How can we determine which of these is greater?"

Get to the idea that they cannot be compared unless expressed using the same measuring unit (denominator).

"What is a denominator that both can be expressed with?" Rewrite (with students' help)

3/5 (x 3/3) = 9/15 2/3 (x5/5) = 10/15

"Now we can do what we choose with them - compare, add, subtract divide."

3/5 = 9/15 2/3 = 10/15

"Since 10 > 9, 2/3 > 3/5."

"Joining them gives 9/15 + 10/15 = 19/15."

"2/3 is 1/15 larger than 3/5."

"If we divide the larger fraction by the smaller should we expect something less than one or more than one?"

"If we divide the smaller by the larger, should we expect something more than one or less than one?"

Discuss fully. Then write:

LARGER	SMALLER
2/3 = 10/15	4/5 = 9/15

LARGER ÷ SMALLER = 10/15 ÷ 9/15 = 10/9, more than one

SMALLER ÷ LARGER = 9/15 ÷ 10/15 = 9/10, less than one

Do a second example.

1. Multiply denominators to get common denominator
2. Compare
3. Join
4. Find the difference
5. Divide both ways
6. Draw pictures if this becomes necessary

For Example:

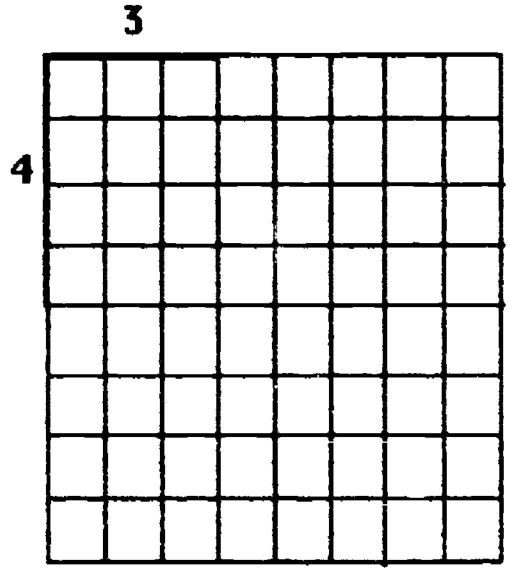
 = 2/3

 =3/5

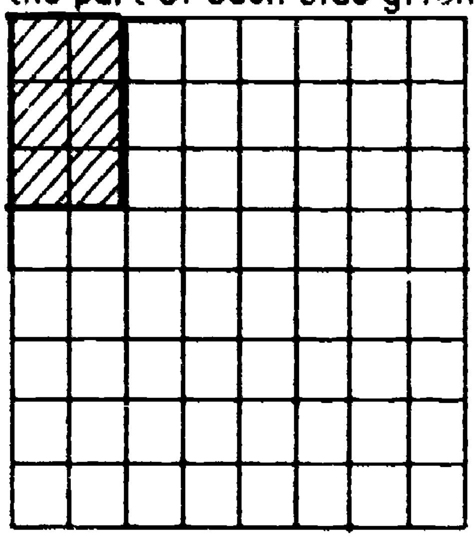
<u>Activity:</u> Pass out worksheets and have pairs of students work on these.

FRACTION REVIEW: Rectangles Inside Rectangles

Introduction: Write the multiplication: 2/3 x 3/4 on the board. On the overhead projector use a transparency graph paper and mark off as shown:



"First we make a rectangle with the denominators as sides. Then we show the part of each side given by the numerator and make that rectangle."



"This rectangle has area 6 and is part of the rectangle with area 12." It is half of it, so: 2/3 x 3/4 = 1/2

"Multiplying fractions, then, is comparing the rectangle made by using the numerators with that made by using the denominator.

"Remember to use the fact that anything except 0 divided by itself is 1." Write:

3/3 = 1   4/4 = 1   5/5 = 1

-Use equivalent fraction forms to replace one with large numbers by an equivalent with small numbers. In the problem we just did, 2/3 x 3/4, this could be written as 3/3 x 2/4 since the order of numbers multiplied doesn't matter. This is:

1 (3/2 = 1) x 1/2 (2/4 = 1/2) or 1/2

Pass out worksheets for pairs of students to work on.

ERIC

THINKING

Background: The following activities should be done periodically throughout the year.

LESSON ONE: Points In Argument

Introduction: In any argument there are usually points upon which both sides can agree, points on which the sides clearly disagree and some point which are irrelevant to the case in point. This lesson will give students experience in classifying points brought out in arguments as one of these three kinds.

Use the following as an example: "Some schools require students to follow a dress code by specifying a kind of 'uniform' such as white shirts or blouses and dark pants or skirts." How many of you are in favor of such a code?"

Identify these students and reseat them on one side of the room.

"How many of you would be opposed to such a code?"

Seat these on the other side. Any students not committing to either side should serve as a jury. Ask each side to give reasons for their choice and write these down:

FOR	AGAINST

When both lists are complete, go down each item by item. Ask the opposing group if they could agree with the item. If so, label is "A".

"Are any items in either list irrelevant in that they have nothing to do with the case in point?" Label these "I".

All others are then labelled "D".

Other argument points to analyze in the same way to use are:

"There is much violence on TV in the form of fights, killings, bomb explosions, etc. This has a negative effect on young viewers."

"A person who is starving has a right to steal food in order to survive."

LESSON TWO: Dependence of evidence

Background: Points given in an argument often depend on other evidence or assumptions made, either state or unstated. For example:

"Schools should be established for gifted children." An argument in favor of this might be that they would learn more in such a school. Hidden in this argument are the assumptions that:

1. they could proceed more rapidly;
2. they would have to compete with others of similar ability;
3. they would probably have better teachers

The latter, in turn, assumes good teachers would be attracted by the prospect of having brighter students. Students should be able to distinguish hard or independent evidence from dependent points based upon assumptions or other things being true first.

Present the following situation to students:

"Some jobs, like cleaning sewers or collecting garbage, are unpleasant, but necessary. Other jobs require more skill and are more pleasant - teaching, librarians, etc. A politician introduces a bill to make the pay for "unpleasant jobs" the same as that for "more pleasant" jobs. What are the arguments for and against his bill?"

When these are generated, go over them one at a time to see which are independent and which are dependent upon other assumptions or facts.

LESSON THREE: Value of Evidence

Background: Arguments depend upon a small set of key points. If these can be refuted in some way, the whole argument collapses like a house of cards. Other points give support to the argument to varying degrees. Children should have experience in judging worth of points made in support of a case.

Introduction: "Some people have advocated the discontinuance of standardized testing." Once standardized tests are defined by example, elicit arguments for and against this proposition. As a group, evaluate each as a KEY point, as STRONG or WEAK support. Other propositions to use for experiences like this:

1. "A higher tax rate should be placed on rich people."

2. "Some people oppose the use of animals in medical and other scientific research."

3. "Films and TV shows that show excessive violence should be censored or restricted in showing."

LESSON FOUR: Opinion or fact?

Arguments based only upon opposing opinions cannot be resolved. Some opinions have more validity that others if they are based upon a set of facts. Children should be able to:

1. distinguish opinion from fact; and
2. judge validity of opinions.

Introduction: Present the following situation to children:

"A movement in the United States is toward offering more choice to students and parents as to which school children should attend. Should the taxpayers pay to send children to private schools if parents and children choose these?"

As opinions are expressed, list these. When they are listed, go over these with the class and identify each as OPINION based on feelings, prejudices, beliefs or guesses and FACTS based upon personal experience or documentable evidence. Other propositions to similarly handle are:

1. "A freeway is proposed which will go through a neighborhood and cause removal of several homes and a school. What are arguments for and against this?"

Evaluate the arguments as a group as to the degree of OPINION or FACT contained in each.

2. "The state taxes heavier cars more than lighter cars. Is this a justifiable practice?"

3. "Many sporting events are held on Sunday. Some people feel Sundays should be TOTALLY devoted to church or family activities. What are your opinions on this?"

LESSON FIVE: The Other Side

Background: In many arguments, opposing sides never really see what the other side's argument looks like. It is helpful for children to have to define the opposing side in terms of listing the points made in the position.

"There is considerable evidence that smoking contributes to lung cancer, heart disease and emphysema. Some legislators favor more restrictive laws limiting smoking, while others say current laws are good enough. List the arguments in favor of each side."

Discuss these arguments. Have children then vote on each side of the issue, giving reasons why they ignored points made by the opposing side.

Other propositions to handle in this way include:

1. "Although most of our parents, grandparents, great grandparents, etc. came to the United States from another country, mostly Europe, some people want to restrict the immigration of people from the near East, far East, Africa, South and Central America and Mexico. Should such immigration be restricted?"

2. "In some countries TV is available only from 8 to 11 PM. Is this a good policy?"

LESSON SIX: Any Resolution?

Background: What is the result of an argument? Are any conclusions drawn? Has anyone's opinion been changed? Has one side been demonstrated to be superior in its case? Here are 7 possible results, ranging from most satisfactory to least:

1. An agreement was reached;
2. A compromise was made;
3. A set of alternatives was established;
4. A list of priorities for future discussion was agreed upon;
5. Agreement was reached on points of agreement and difference in positions;
6. Each side identified the other's position;

7. The argument was a waste of time.

Introduction: "Organ transplants are expensive. Side 1 argues in favor of federal funding of these while side 2 argues this money should be spent on basic research or prevention of disease. What are some possible outcomes of this debate?"

Have children list these and then try to order them as to the level of resolution that each outcome represents. Here are some others to handle in this way:

Issue	Side 1	Side 2
1. Capital punishment	opposed	favors
2. Sexually segregated schools for children 12-18	opposed	favors
3. Public employees (police, firemen, snowplow operators, teachers) have a right to strike	opposed	favors
4. Creating jobs at the expense of the environment	opposed	favors

LESSON SEVEN: Supporting Arguments

Background: There are several ways to try to show that "you are right" in an argument. You can

A) describe probable results of actions;
B) refer to evidence or authorities,
C) label or name events or situations;
D) use judgmental language to try to sway

Children should be able to identify which tactics are being employed.

Introduction: Emphasis should be placed on what is useful rather than what is interesting. Have children give arguments for and against this.

Go over the list of arguments and categorize these as primarily type A,B,C or D, and give reasons why judging them as such. Other propositions to handle this way are:

1. "Service is better in neighborhood stores than in supermarkets."

2. "Scientists share responsibility for the misuse of their discoveries by engineers and politicians."

3 "A large number of families now have 2 wage earners. Raising children properly is more important than paying bills."

4. "People choose clothing and jewelry in accordance with their values and beliefs, so it is proper to judge people by their appearance and the clothes they wear.":

LESSON EIGHT: Dismissing Arguments

Background: Some arguments can be attacked by pointing out the use of such practices as

A. exag[illegible]on
B. taking examples to extremes
C. generalizing from too few cases
D. misstating facts
E. mistaken identification; and
F. misinterpretation of statements.

Introduction: "If workers get the wages they desired, there would be no strikes." Have students generate arguments in favor of this conclusion and arguments in opposition. Analyze these arguments as to the presence of factors A,B,C,D,E & F. Point out how these weaken arguments or inhibit resolving differences.

Other similar questions to handle this way include

1. "Money spent on expensive military equipment would be better spent on hospitals, medical research, schools, etc."

2. "Children should be given cash allowances by their parents instead of earning their own money."

ERIC

3. "Money spent that is raised by taxes is different from money spent that is raised by charging consumers for goods or services."

4. "Wars sometimes become necessary."

ERIC

LEVEL SIX

EXPONENTS AND SCIENTIFIC NOTATION

Background: Prior to using exponential notation, which arises in the interpretation of calculator displays, students must understand the use of exponents, especially as related to decimals and scientific notation.

LESSON ONE: Exponents

Introduction: Write: 5 x 5 x 5 on the overhead or chalkboard. "How many times is 5 used as a multiplier?" Write: $5 \times 5 \times 5 = 5^3$

"The raised 3 shows how many times 5 is used as a multiplier."

Write: 10 x 10 x 10 x 10 on the overhead or chalkboard.

"What number should I write with the 10 to show it is used as a multiplier FOUR times?" Write: $10^4 = 10 \times 10 \times 10 \times 10 = 1000$

"How many 0's follow the 1 in the numeral? How do we write this as a POWER OF TEN?"

Write: $100 = 10^?$ (10^2)
Then: $1000 = 10^?$(10^3)
Then: $100000 = 10^?$ (10^5)

Write: 2x2x2 X 2x2

"How many times is 2 a multiplier in the first number? in the second number? in the number that is the product?" (3, 2, 5)

"What is this result?" $8 \times 4 = 32 = 2^5$
Consider: 10 x 100 = 1000

"Each number written with an exponent is $10^1 \times 10^2 = 10^3$

"What do you do with the exponents when multiplying numbers made up of the same multipliers?"

$A = 10^2$

$B = 10^4$

"What is the result of multiplying A by B?" (10^6)

Pass out the worksheets for pairs of students to work on.

<u>LESSON TWO: Separating Numbers into Powers of Ten</u>

<u>Introduction:</u> Write: 600 on the overhead or chalkboard.

"Let's see how many ways we can write this as a product of two numbers."

600 = 2 x 300
600 = 3 x 200
600 = 4 x 150
600 = 5 x 120
600 = 6 x 100

"Notice we have a small number less than 10 multiplied times a power of 10." Write: $600 = 6 \times 10^2$

Write: 625 on the board or overhead. "How do we divide this by 10?"

$$
\begin{array}{r|l}
10 & \overline{625} \\
 & \underline{600} \\
 & 25 \\
 & \underline{20} \\
 & 5
\end{array}
$$

"Written as a decimal this is 62.5

Write 625 = 62.5 x 10

"The most useful form is to have a small number less than 10 times a power of ten. How can we divide 62.5 by 10?"

```
      6.25
  10 | 62.5
       60
       --
        25
        20
        --
         5
```

"So 62.5 = 6.25 x 10"

"Now we can write 625 is 6.25 x 10 x 10 or 6.25 x 10^2."

"Remember that dividing by 10 changes each place to the next smaller, so 625 ÷ 10 = 62.5

6 in the hundreds place goes to the tens place
2 in the tens place goes to the ones place
5 in the ones place goes to the TENTH place."

"Dividing by 100 changes each place to the second smaller, so
625 ÷ 100 = 6.25 x 10^2

6 goes hundreds to ones
2 goes tens to tenths
6 goes ones to hundredths."

"How would we write 387 as a number between and 10 times a power of ten?"

We get the number between one and ten (one digit) by dividing by 100 so 387 = 3.87 x 10^2."

"Notice that multiplying on the right side gets back to the 387."

Write: 1.23 x 10^3

"How do we write this as a single numeral? One way is by 10 at a time:
1.23 x 10 = 12.3 x 10 = 123 x 10 = 1230
(1) (2) (3)
so 1.23 x 10^3= 1230
"To Summarize: To multiply by powers of ten, move the decimal point the exponent number of places to the RIGHT. To divide by powers of ten, move the decimal point the exponent number of places to the LEFT."

This is why having numbers in the form of a small number multiplied by powers of ten is so important to know."

Consider: 2×10^3 = 2000

$\underline{3 \times 10^5}$ x $\underline{300000}$

6×10^8 600000000

Pass out worksheets for students to work on in pairs.

ERIC

LEVEL SIX

MENTAL SHORTCUTS IN COMPUTING WITH MONEY

Background: In working with money, students should learn to use estimation and rounding techniques to obtain mental answers. Some examples follow:

In adding or subtracting:

Consider 42 +59 adding and subtracting 1

gives 41 +60 a much easier mental computation.

Another is 42 -27 adding 3 to both gives

45 -30 a much easier mental computation

In multiplying or dividing using 25¢ or 50¢, students should use the facts 25 = 100/4 and 50 = 100/2 since multiplication by 100 is so easy mentally

$.25 x 482 then becomes 482 ÷ 4 = $120.40

$.50 x 261 becomes 261 ÷ 2 = 130.50

Multiplying and dividing factors gives easier computations that can be done mentally.

40 x 36 = 20 x 72 = 10 z 144 = 1440
65 ÷ 5 = 130 ÷ 10 = 13
118 ÷ 25 = 11800 ÷ 4 = 2950

In working with money, so many prices end in xxx9 that rounding and subtracting works. Consider:

.39 x 7 = .40 x 7 - 7 =$2.73

or 1.99 x 6 = 2.00 x 6 - 6 = 11.94

or .29 x 9 = 2.70 - 9 = 2.61

If adding several dollar amounts, round to dollars, add, then mentally subtract differences.

3.89 + 6.79 + 9.99

4 + 7 + 10 - .11 - .21 - .01

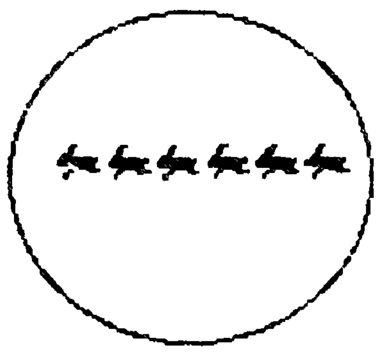
21.00 - .33 = $20.67

LOGIC

Background: Students at this level have had several years of work with the use of AND, OR, NOT, and with If-Then reasoning. These lessons build upon that understanding.

LESSON ONE: Review of AND, OR, NOT

Introduction: Place a diagram on the board:



"If BUNNIES are isolated in the circle, what is the best description for what is outside the circle?" (NOT BUNNIES)

"Remember NOT is always with reference to a defined UNIVERSE. Some universes BUNNIES might belong to are (1) ANIMALS, (2) MAMMALS, (3) 4 LEGGED CREATURES, etc. NOT bunnies would refer to (1) all other animals, (2) all other mammals and (3) all other 4 legged creatures."

Help me complete this table:

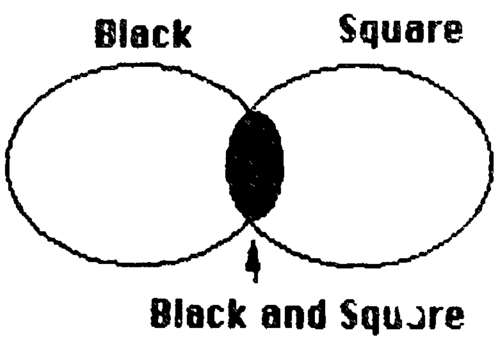
UNIVERSE	HAVES	NOTS
hair colors	blondes	not blondes
colors	reds	
dogs	collies	
cars		not Fords

Activity: Have pairs of students complete the worksheet provided.

LESSON TWO

Place this phrase on the board: "Black and square."

"How do I show this with a diagram?" Draw out and finally illustrate on the board or on the overhead.



"Remember this indicates both BLACK AND SQUARE AT THE SAME TIME."

"What is another way to say black and square?" (Black squares)

Activity: Have pairs of students complete the worksheet provided.

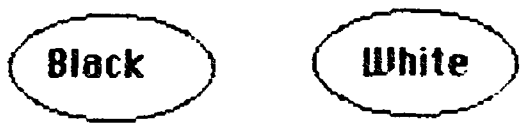
LESSON THREE

"What is the difference between INCLUSIVE and EXCLUSIVE OR?"

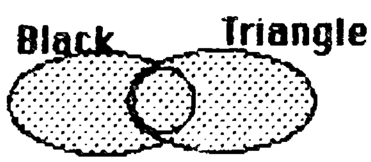
Discuss. The big idea is that INCLUSIVE OR includes the possibility of AND, while EXCLUSIVE OR separates into two distinct groups.

Examples to use:

Black or white



Black or triangle



0There can be black triangles.)

Activity: Pass out worksheets for pairs of students to work on.

LESSON FOUR

"This is a true statement."

"If well, then alive."

"What one word would I change to make it false?" Discuss.

If well, then dead (or not alive) is the only way to make it false." Give two or more similar examples. The part following "then" must be denied to make it false.

Activity: Have pairs of studnts complete the worksheet.

ERIC

OPEN SENTENCES

Background: Students use concrete models to work with open sentences with both positive and negative numbers in them. These lessons will give some experience with those.

LESSON ONE: Review

"Recall what we can do with open sentences." Write:

1. Add the same number to both sides of the equality
2. Subtract the same number from both sides of the equality
3. Multiply both sides of the equality by the same number
4. Divide both sides of the equality by the same non-zero number

"This is true for BOTH POSITIVE and NEGATIVE numbers."

Write: □ + 3 = 9
"Subtract 3" − 3 −3
□ = 6

Write: □ −4 = 7
"Add 4" +4 +4
□ = 11

Write: 3□ = 12
"Divide by 3" $\frac{3□}{3} = \frac{12}{3}$
□ = 4

"Remember, we want to see what ONE □ is so do what must be done to get that."

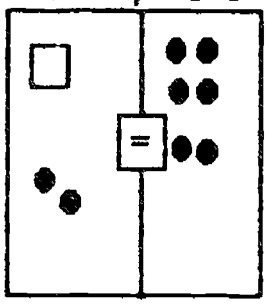
Write:

1/4□ = 3
"Multiply by 4" 4(1/4□) = 4(3)
□ = 12

"To get ONE □ , we add or subtract other things. We also divide several □ by a number and multiply a fractional part of a □ by a number to get ONE □."

Do several more examples of each type.

<u>Activity:</u> Have pairs of students work on the worksheet. Have split boards and chips available for use if that is needed.



□ by subtracting ●● from both sides, so

□ = 4

<u>LESSON TWO: Multi-step Solutions</u>

<u>Introduction:</u> Write on the board or on the overhead projector:

2□ + 3 = 15

"What should we do first?" Discuss the idea of getting the □ alone first.

$$\begin{array}{rcl} 2\square + 3 &=& 15 \\ -3 && -3 \\ \hline 2\square &=& 12 \end{array}$$

"Now what do we do to see ONE □ ?" Discuss the need to divide by the multiplier of □

$$\frac{2\square}{2} = \frac{12}{2}$$

$$\square = 6$$

"I'm sure everybody knew that 2 x 6 = 12 so 6 had to go in the □."

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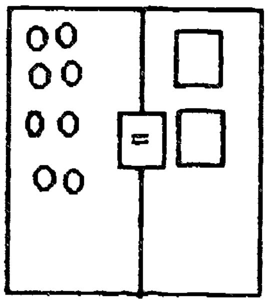
"Here is another one."

$7 = 15 + 2\square$

"What should we do first?"

$$\begin{array}{rcl} 7 & = & 15 + 2\square \\ \underline{-15} & \underline{=} & \underline{-15} \\ 8 & = & 2\square \end{array}$$

"Let's see what this looks like."



"What must be under each box so the board balances?"

"So we write: $-4 = \square$

Write: $7 = 15 - 2\square$

"When the $\square$ are subtracted, it is easier to ADD that number of $\square$ to both sides first.

$$\begin{array}{rcl} 7 & = & 15 - 2\square \\ \underline{+2\square} & & \underline{+2\square} \\ 7 + 2\square & = & 15 \end{array}$$

"Now we should SUBRACT what?"

$$\begin{array}{rcl} 7 + 2\square & = & 15 \\ \underline{-7\quad} & & \underline{-7} \\ 2\square & = & 8 \end{array}$$

"How much must be in each □?"

Activity: Have pairs of students work on the worksheets. Have split boards. Cardboard squares to represent □ and chips for the numbers.

LEVEL SIX

RATE APPLICATION: Distance/Time

Background: Another common example of the use of linear rate is in distance/time/rate problems. These lessons deal with that.

LESSON ONE: Introduction

"A train is travelling at a steady 60 miles per hour. Let's see how far it travels during different periods of time."

distance	60	120	180	240	300	360	D=rxt
time	1	2	3	4	5	6	+
rate	60	60	60	60	60	60	r

"How far will the train go in 2 1/2 hours? (150 mi.) in 4 1/2 hours?" (270 miles)

"A jogger runs 1 mile in 8 minutes. Let's look at how far he runs in different numbers of minutes."

distance	1	2	3	4	5	6	7
time	8	16	24	32	40	48	56

"How long would it take the jogger to go 3 1/2 miles?" (28 minutes) 5 1/2 miles?" (44 min)

"I could use a proportion instead of the table."

$$\frac{\square}{2\ 1/2} = \frac{60}{1} \qquad \text{so} \quad \square = 2\ 1/2\ (60) = 150 \text{ miles}$$

$\longleftrightarrow$
2 1/2

$$\frac{\square}{3\ 1/2} = \frac{8}{1} \qquad \text{so} \quad \square = 3\ 1/2\ (8) = 28 \text{ minutes}$$

$\longleftrightarrow$
3 1/2

Activity: Have students do the worksheet in pairs.

RATE APPLICATIONS: Parts of Wholes

Background: Several of the applications of rates have to do with parts of wholes such as mixtures, etc. These lessons deal with several of these.

LESSON ONE: "At the hamburger shops, seven Big Macs are sold for every twelve cheeseburgers. Let's look at different numbers of Big Macs and cheeseburgers sold."

Big Macs	**7**	**14**	**21**	**28**	**56**	**63**	
Cheeseburgers	**12**	**24**	**36**	**48**	**96**	**108**	

"On a day where 100 cheeseburgers are sold about how many Big Macs were sold?"

"If 24 Karat gold is pure gold, how much gold is in a 14 Karat gold ring weighing 2 oz.?"

The proportion is: $\frac{14 \text{ karats}}{24 \text{ karats}} = \frac{\quad \text{oz.}}{2 \text{ oz.}}$

$24 \square = 14 \times 2$ so $\square = \frac{28}{24} = \frac{7}{6}$ oz.

"Oranges are sold at 1.69 for 4 lbs. What does this rate table look like?"

oranges	**4**	**8**	**12**	
cost	**1.69**	**3.38**	**5.17**	

"What should 2 lbs. cost?"

"What is the cost of 1 lb. of these oranges?"

"Is it easier to find the cost of any number of pounds if you know the cost of one pound?"

"This is called the UNITARY rate. In many problems involving the cost of things, the UNITARY rate is easiest to use."

Find UNITARY rates for these:

3.65 g. of potassium in each 10 grams of compound.
A recipe calls for 1 cup of sugar and 2 1/2 cups flours.
10 oz. of paint pigment for each 32 oz. of linseed oil.
Working 2 1/2 hours of $12. $5.00 for a 20 lbs. box of cherries.

Discuss these and apply the unitary rates found to several different amounts.

<u>Activity:</u> Give pairs of students worksheets to complete.

RATE APPLICATIONS: Percent

LESSON ONE: introduction:

"Mr. Jones says he is right 100% of the time. Do you believe him? Why or why not?"

Give the class this question and discuss what 100% means.

"The high school basketball team shot 21% from the field in the last game. Do you think they wr · the game? Why or why not?"

This discussion should get around to looking at this as being about 1 in 5.

"The sportscaster described a player as giving 150% effort. What do you think of this?"

This discussion should get at the idea that you can't use more than the WHOLE (or 100%) of anything.

LESSON TWO

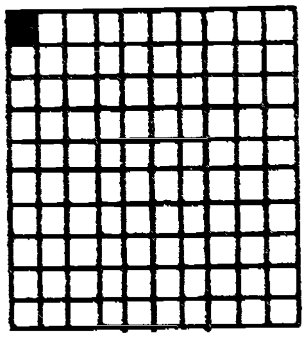
Introduction: As an application of the rate relationship, percent was looked at in terms of a proportion:

$$\frac{\text{percentage}}{\text{base}} = \frac{\text{rate}}{100}$$

Many applications of percent involve small whole number rates. In this it is easier for some to look at first finding 1%. Finding 1% is dividing by 100, which is a shift to the left of TWO digits in the base. This lesson gives experience with that.

.How many parts in a hundred does 1% show?"

Show this 1 part in 100 on a hundreds square:



"What is the easiest way to find 1/100, or divide something by 100?"

Give several quantities for students to find 1% of 600, 870, 1500.

"Once you find 1%, then multiply this by the total percent to get the required percentage. Example:

7% of $900
1% is 9(x7 = $63)

LESSON THRE: Percent Problems

Introduction: In this lesson, show the students they have a choice when finding percentage - proportion or the % method.

"The Johnsons spent 8% of their $2,000 income one month on food. How much was spent on food?"

% method

1% of $2,000 = $20
8% of 2000 = 8 x 20 = $160

Proportion method

$\frac{\square}{2000} = \frac{8}{100}$ so $\square$ = 20(x 8 = $160)

← 20

"A TV is on sale at 40% off. How much is saved on a TV costing $500?"

% method

1% of 500 = $5 x 40 = $200

Proportion method

$$\frac{\square}{\$500} \quad \frac{40}{100}$$

← 5

$\square$ = 5 x 40 = $200

Activity: Assign problem sets. Allow students to use either method.

ERIC

GUESS MY RULE:

Children should have been introduced to this previously. If you are not familiar with the rules for playing "Guess My Rule," see this section in LEVEL FIVE Teacher's Guide for rules and some sample activities. Review with the children:

1. The concept of variable
2. Deriving the rules using tables of values
3. Graphing the rules
4. The difference between dependent and independent variables; and
5. The difference between open and closed sentences

Here are some rules to use with children to help in the review of these ideas:

3I + 2 = D	2I 5 + D
4I - 1 = D	5I + 2 = D
I - 1 = D	3I + 5 = D

For each rule, have children

1. complete the table of values
2. order the "I" values
3. find the common difference in D values
4. graph the rule

Using the Linear Relation

Background: When the general linear rule - D = rI + k, where D is the dependent variable, r is the constant rate of change in D compared to change in I and I is the independent variable and k is a constant, is simplified by setting k = o, direct variation or proportions result. Some examples are D = 2/3I or D/I = 2/3. This results in the following equivalence class:

D	2	4	6	8	10	12	14	etc.	
I	3	6	9	12	15	18	21		
multiple	1	2	3	4	5	6	7		

In every case the comparison can be reduced to 2/3. Picking any two from the class results in a proportion, i.e., 4/6 = 8/12 or 2/3 = 10/15, or 6/9 = 14/21. As proportions these would be written:

2:3 :: 10:15, 4:6 :: 8:12, etc.

Any measurable quantities can be used as variables. Hence, 2 apples : 3 peaches as 4 apples : 6 peaches. When ratios and proportion are reviewed, this should be pointed out.

In D = 4I, the units of measurement will agree. For example: D = r x t is an example of this. D is Distance in miles, r is rate in miles/hour and t is time in hours:

$$\text{miles} = \frac{\text{miles}}{\text{hour}} \times \underline{\text{hours}} \quad \text{reduced to miles = miles}$$

Other examples are:

Total cost in \$ = \$/can x cans

Total cost in ¢ = ¢/piece x pieces

Total cost in \$ = \$/tire x tires

Total quarts - quarts/gallon x gallons

Total inches = inches/foot x feet

Total cost in \$ = \$/shirt x shirts

Total eggs = eggs/carton x cartons

Total boxes = boxes/case x cases

These are some of the many special cases of the D=rI relationship. To develop this general idea, several lessons will integrate "Guess My Rule" with ratio and proportion.

<u>LESSON ONE: Review of Ratio and Proportion</u>

<u>Introduction:</u> "In a school there are 2 boys for every 3 girls. How many of each should we expect to see in classes of different size?"

Place the following table on the overhead in transparency form:

Boys									
Girls									
Class size									

"We shall complete this table by multiplying the 2:3 ratio by whole numbers in order."

Do this step by step and complete the table. Take time to show how the whole number distributes over the ratio, i.e. 2 (2:3) = 4:6, 5 (2:3) = 10:15, etc. The result will be:

Boys	2	4	6	8	10	12	14	16	18	20
Girls	3	6	9	12	15	18	21	24	27	30
Class size	5	10	15	20	25	30	35	40	45	50

"Notice that any of these can be compared."

Give several examples and discuss these:

8:12 = 16:24	Boys : Girls
6:15 = 16:40	Boys: Class Size
9:15 = 21:35	Girls: Class Size

<u>Activity:</u> Pass out the worksheets and have pairs of students complete the tables and the proportions.

<u>LESSON TWO: Using Units and "Per"</u>

<u>Background:</u> The idea of per as quantity of something for each unit of measurement needs to be reinforced with students. This lesson reviews the concep and provides practice in working with units.

<u>Introduction:</u> "Think of as many rates as you can. One example is miles per hour."

Write down as many as students generate. If necessary provide an additional example to stimulate their thought. Show how totals are arrived at by using the rate. Example:

6 bottles per carton can be written as 6 bottles/carton. The total bottles in several cartons is total bottles = $\frac{\text{6 bottles}}{\text{cartons}}$ x no of cartons

Point out how the cartons "cancel" (cartons/cartons - 1) so the total is described using bottles.

Use the rates students provided to generate cases where totals must be found using the rates. Some likely examples include:

Total miles = $\frac{\text{60 miles}}{\text{hours}}$ x hours driven

Total cost = $\frac{\text{50¢}}{\text{candy bar}}$ x candy bars bought

Total inches = $\frac{\text{12 inches}}{\text{feet}}$ x no. of feet

Activity: Have students work in pairs on the worksheets provided.

LESSON THREE: Review of "Guess My Rule"

Introduction: Review the rules with the class.

1) DO NOT BLURT OUT THE RULE WHEN YOU FIND IT. TEST AS INDICATED
2) TEST BY "If I give you (______), will you give me (______) back?"

"I have a rule (3I + 1 = 0)." As children input small numbers, develop the table on the board or overhead projector:

I	O
0	1
1	4
2	7
3	10
4	13
5	16
etc.	

Find entries in the "Change in O" column:

I	O	Change in O
0	1	3
1	4	3
2	7	3
3	10	3
4	13	3
5	16	3
etc.		

Relate the "change in O" constant to the coefficient of I in the rule.

Relate the value of O when I = 0 to the constant in the rule.

Rename I & O as I and D because:

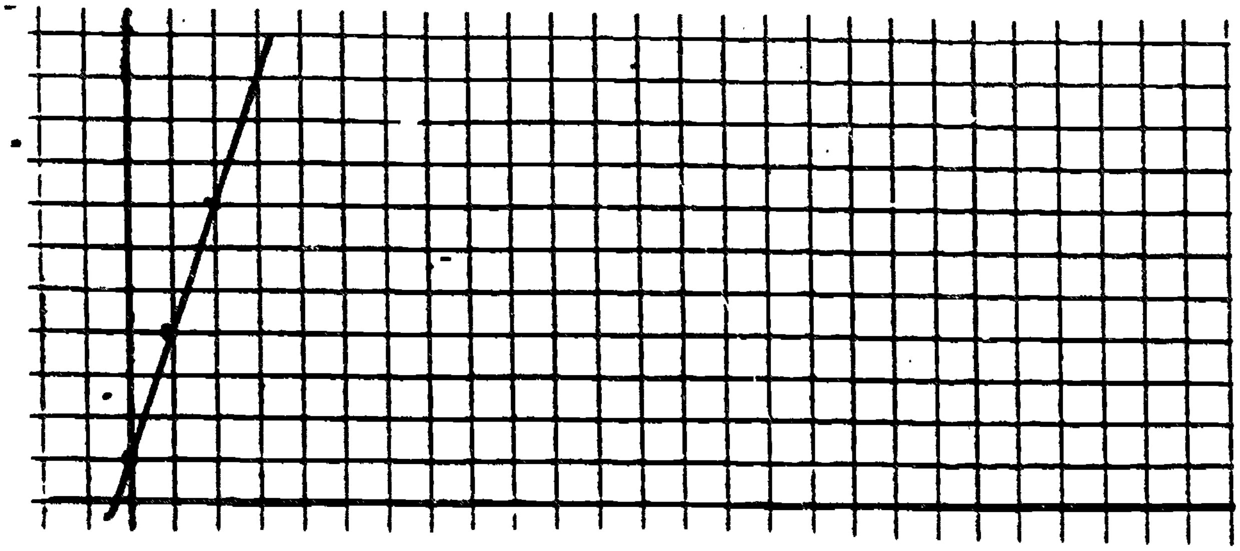
1) I is the INDEPENDENT VARIABLE since there is freedom of choice of a value for it
2) D is the DEPENDENT VARIABLE because its value for a given I is determined by the rule.

Point out the Change in D of 3 occurs for EACH change in I of 1.

Define $r = \frac{\text{change in D}}{\text{change in I}}$, so

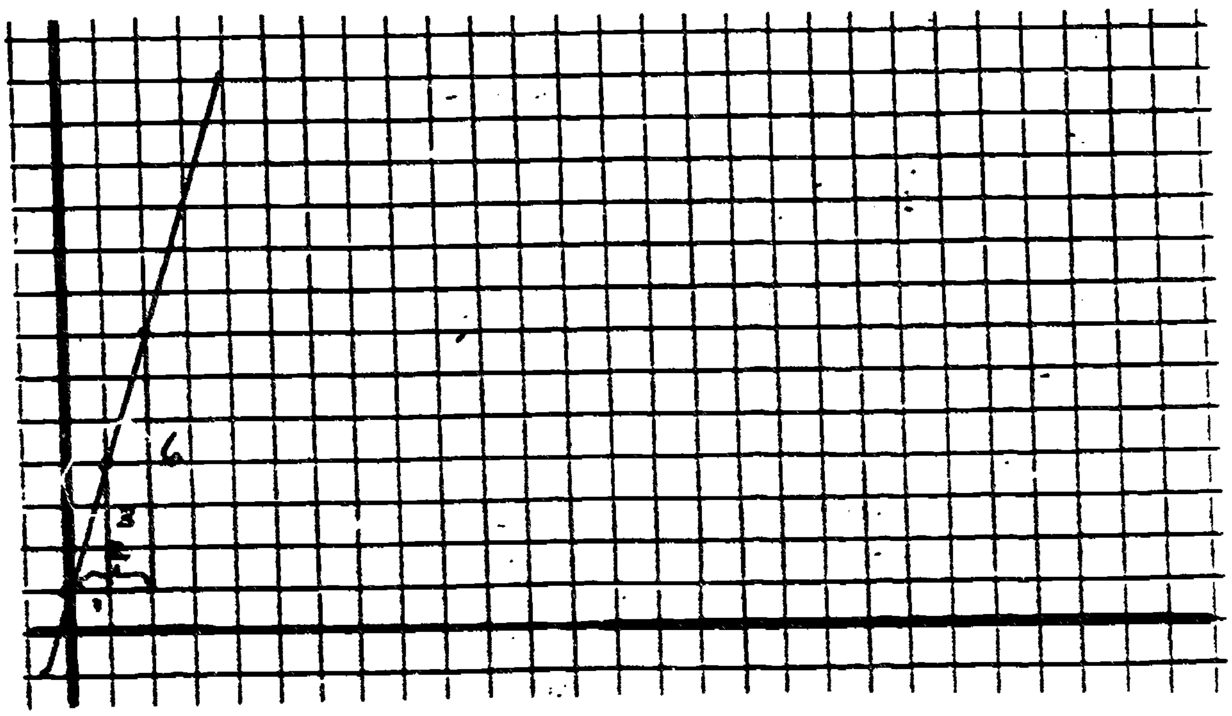
r in this case = $\frac{3}{1} = 3$

Graph this rule on a coordinate scale on a transparency on the overhead:



Draw the line connecting the three points. Identify the change in D to the change in I and point out using the graph that:

(1 to 4) <u>3</u> = <u>6</u> (1 to 7)
(0 to 1) 1   2 (0 to 2)



No matter where the total change in D is compared to the corresponding total change in I, the result will be 3.

<u>Activity</u>: Have pairs of students complete the worksheets given

LESSON FOUR: Constant Change and Ratio

Background: In this lesson rules of the form D = r.I where r is a constant and represents the change in D comparison are related to ratio and proportion.
change in

Introduction: "Consider this rule - D = 2 I. The table looks like this."

Put in the successive values for I and have students give the corresponding D values.

I	0	1	2	3	4	5	6	7	8	9	10	
D	0	2	4	6	8	10	12	14	16	18	20	

"Notice that in each case the ratio of D to 1 is 2 : 1."

$\frac{4 : 2}{2} = 2 : 1$ $\frac{6 : 3}{3} = 2 : 1$

$\frac{8 : 4}{4} = 2 : 1$

"D = 2 I tells us D is always twice I." "The ratio 2: 1 for D : I tells us the same thing.

"Mr. Jones sells 3 Fords for every 2 Toyotas." "How do we show this in a table?"

Fords	3	6	9	12	15	18	21	24	27	30		
Toyotas	2	4	6	8	10	12	14	16	18	29		

"We could show .his as Fords = 3 Toyotas."

Be sure children see that this is correct using several cases from the table. "Fords = $\frac{3}{2}$ Toyotas and Fords : Toyotas = 3 : 2 gives us the same information.

Activity: Have pairs of students work on the worksheets together. Those who need colored chips to support their thinking should have them.

LESSON FOUR:

Background: Probably the most frequently used example of D=rl and the resulting direct proportions is in the use of percents. This lesson will develop that use.

Introduction: "Does anyone know why a one cent piece is called a CENT?"

Develop the idea of 100 being related to CENT.

"What other words that make you think of 100 have CENT in the word?"

Some possibilities are: century; centenarian

"In mathematics, the term PER CENT means parts PER 100. So 07% means 20 parts per hundred." "Let's look at several of these in table form."

10% %

1	2	3	4	5	6	....	10
10	20	30	40	50	60	....	100

Work on 2 per 20 being the same as 10%

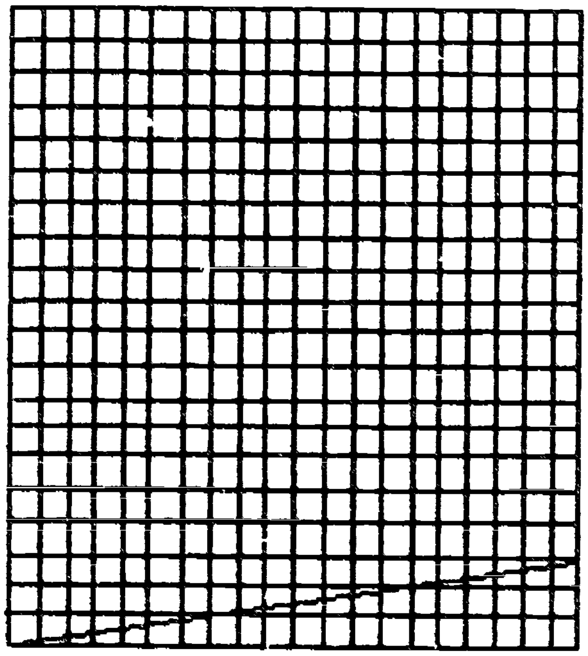
Work on 5 per 50 being the same as 10%

Write some proportions.

3/10 = 10/100   6/60 = 10/100   etc.

"These all show the same ratio comparison."

Put the following graph on the overhead projector.



"BASE is what base the rate is applied to. Percentage is the part of that BASE we get by applying the rate."

In the table above, label the parts at this line.

Rate

Percentage	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Base	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**

"What is a common percent you have heard of?" "Where did you hear of it or see it?"

Use that percent to generate a table and a graph as above.

"Some per cents commonly used in discounting the prices of goods are 20%, 25%, 33 1/3%, 50%." These look like:

							Rate
Percentage	1	2	3	4	5	6	 20
Base	5	10	15	20	25	30	 100

								Rate
Percentage	1	2	3	4	5	6	....	35
Base	4	8	12	16	20	24	....	100

									Rate
Percentage	1	2	3	4	5	6	7	....	33 1/3
Base	3	6	9	12	15	18	21	....	100

"Why do you suppose 33 1/3% is used more often than 30%?"

"How do we apply percents?" "As seen from the tables, the following proportion holds:

$$\frac{\text{percentage}}{\text{base}} = \frac{\text{rate}}{100}$$

Using this, the rate can be applied to ANY base."

For example: "25% of the 60 students at Bray Elementary School bought tickets to the school play. How many students bought tickets?"

percentage (this is what we're looking for - the part c´ the base the rate gives)

base (this is the quantity the rate is applied to)

rate (this is the per cent or parts in a hundred)

In this problem: $\frac{\text{percentage}}{60} = \frac{25}{100}$

This simplifies to $\frac{\text{percentage}}{60} = 1/4$

ERIC

so percentage - 15 students bought tickets. Show how the 15 can be obtained by the rate table:

1	5	15								
4	20	60								

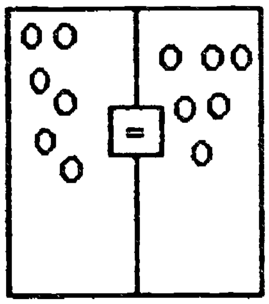
or by dividing 60 by 4.

WORKING WITH INEQUALITIES AND EQUALITIES

Background: Students will have had previous experience with these topics. Those Lessons (RELATIONSHIPS and OPEN SENTENCES) at Level Five could be reviewed.

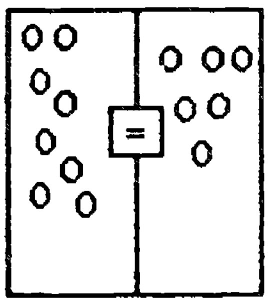
LESSON ONE: Adding and Subtracting Positive Numbers in Relationships

Introduction: Place a split board transparency on the overhead with colored chips arranged as shown.

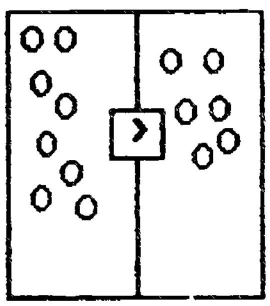


Have some transparency [>] symbols available.

Add 2 chips to one side:

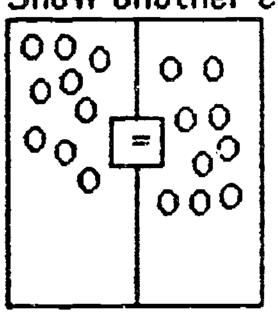


"Is this still an equality?" "How should a symbol be placed to show the relationship?"
Replace the = with > as shown:

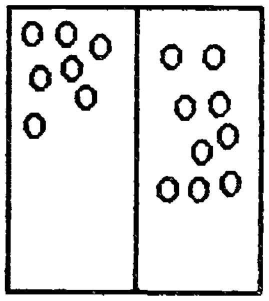


Write: 6 = 6 but 8 > 6

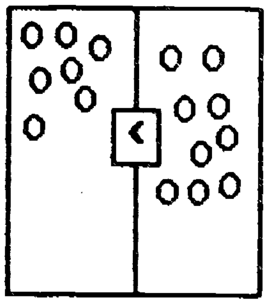
Show another equality



"This time I'll subtract 2 chips from one side."

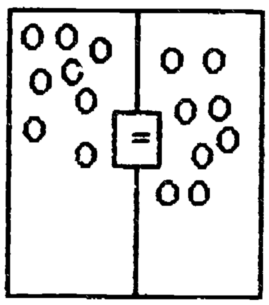


:How should I place the sign?"



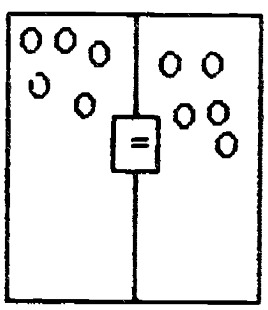
Write: 9 = 9 but 7 < 9

"Adding to or subtracting from one side of an equality changes it to an inequality." Put another equality on the overhead:



"If I add 2 to BOTH sides, what is the result?" (10 = 10)

"If I now subtract 5 from each side, what is the result?" (5 = 5) Show:

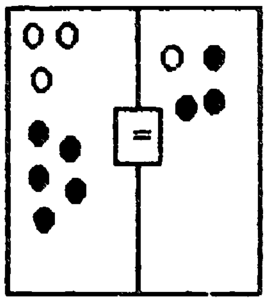


"Adding to or subtracting from BOTH asides doesn't change the equality. The result is still an equality."

Activity: Have pairs of students work on the worksheets provided. Have split boards and counters available for those who might need them.

LESSON TWO: Adding or Subtracting Negative Numbers in Relationships

Introduction: Place the following array on the overhead. A split board with TWO colors of chips to show positive and negative:

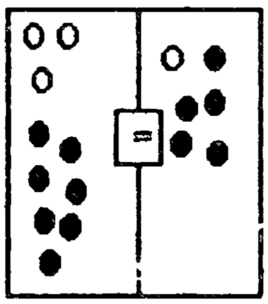


● **positive**

o **negative**

"Are these sides equal?" "What is the number on each side?"

Add 2 ● to each side.

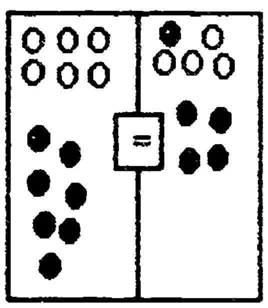


"Do we still have the same number on both sides?"

"What is the number on both sides?"

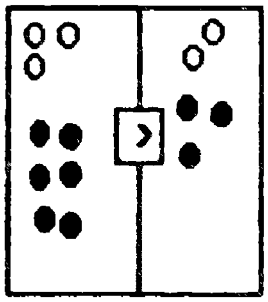
"Does adding the SAME positive to both sides change the relationship of equality?"

Add 3 O to both sides



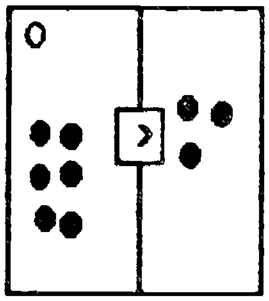
"Are the two sides equal?" "What number is on each side?" "Does adding negative to BOTH sides change the relationship of equality?"

Put this on the overhead projector.



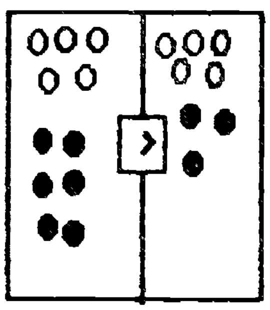
"What is the number on each side of the inequality?" Write: 3 > 1

"I shall subtract two negatives from each side."



"How is this inequality written with numerals?" 5 > 3

"I'll now add 5 negatives to each side."



"How is this inequality written with numerals?" 0 > -2

" Is this true?"

"Notice that in each of these inequalities the left side is 2 more than the right. Does adding or subtracting negative from both sides change n inequality?"

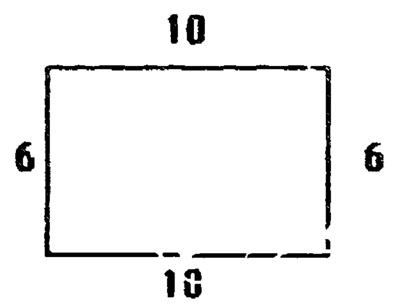
<u>Activity</u>: Give pairs of students worksheets. Have split boards, symbol cards and 2 colors of chips available for those who need them.

LEVEL SIX

LESSON ONE: Area

Students will have had previous experience in finding ares of rectangles and triangles, so this should be a review.

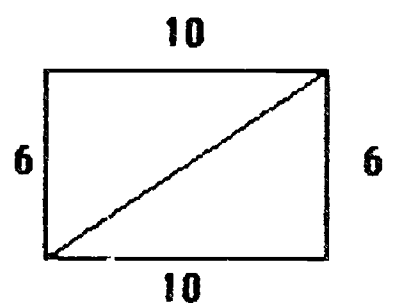
Introduction: Place a rectangle on the overhead projector and label as shown:



"What is the area of this rectangle?" (60)

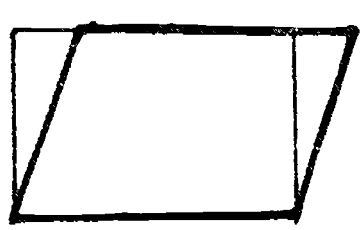
"What is the distance around it?" (32)

Draw a line as shown:



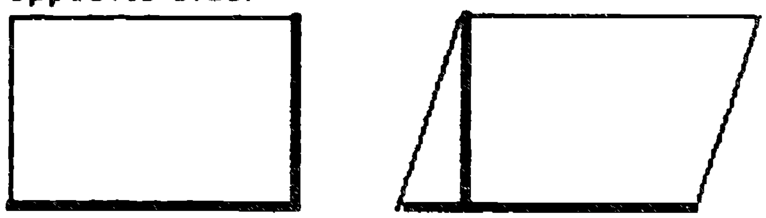
"I divided the rectangle into 2 equal parts." "What is the area of each triangle?"

Put this on the overhead:



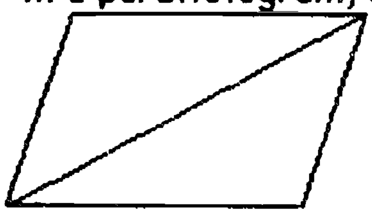
"Notice the same triangle is added on as is taken off, so the parallelogram has the same area as the rectangle."

"In both cases, the area is found by multiplying a side by the distance to the opposite side."



Clearly point these out in the figures shown.

"In a parallelogram, a line can be drawn to divide it into 2 equal triangles."

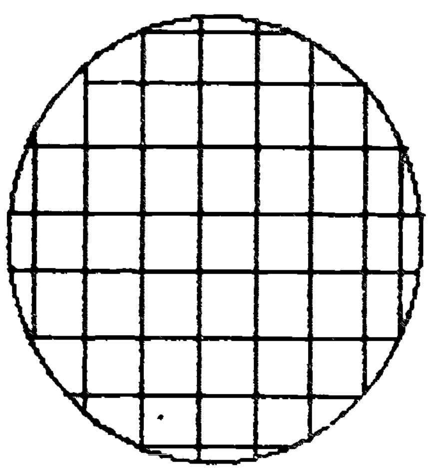


"A triangle has half the area of a rectangle or parallelogram with the same dimensions."

Activity: Have pairs of students work on the worksheets provided. Give them transparent graph paper to check their work.

LESSON TWO: Areas of Circles

Introduction: Place a circle with a grid background on the overhead projector.



"First count all of the squares inside the circle, then try to put together the pieces of squares to get a good ESTIMATE of the area.

"A measurement of the DIAMETER (distance across) the circle is: ________."

"One half of that is the RADIUS of the circle or the distance from the center of the circle to any point on the circle.

"See if you can find a relationship between the radius of the circle and the area.

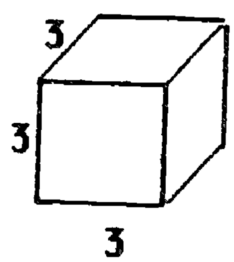
Do a second circle with a larger radius and repeat the question.

"In your assignment you'll find the area and the radius for several circles to help you discover a relationship."

LESSON THREE: Volume

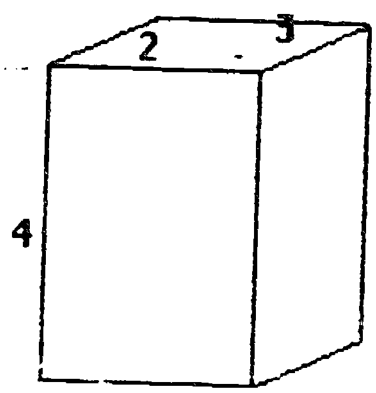
Students have had experience calculating volumes of geoblocks. This lesson will review what they have had.

Introduction: "This is a cube with edges as shown."



"What is the area of each 'layer'?" "How many layers are there?" Write:
Volume = 3 x 3 x 3 = 27

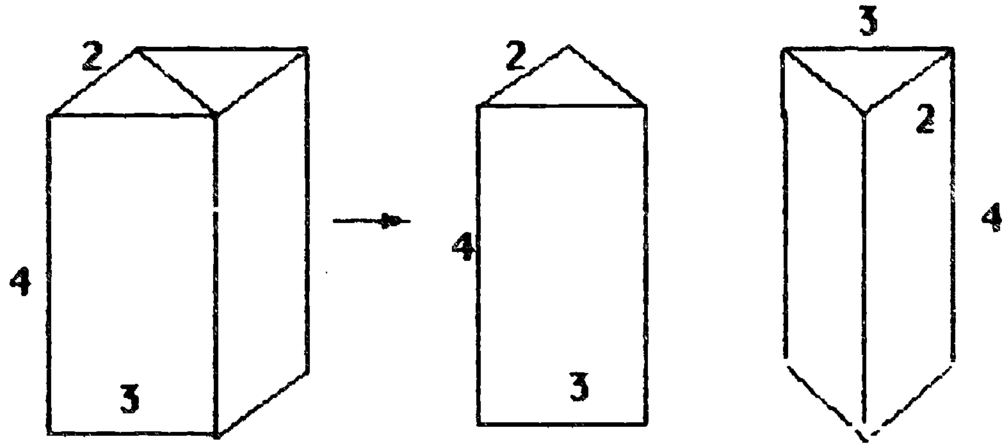
"This is a rectangular solid with edges of different lengths from each other."



"What is the volume of this solid?" Draw lines in to show unit cubes if needed.

Write: Volume = 4 x 2 x 3 = 24

"Just as triangles can be half of rectangles, rectangular solids can be cut in half to give triangular prisms.



"These have triangular bases and each is half of same rectangular solid. The volume is found by taking HALF of the product of the 3 dimensions."

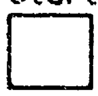
Activity: Have pairs of students work on the worksheets togethe

LEVEL SIX

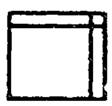
BUILDING SQUARES: Base Ten Squares

LESSON ONE

Introduction: Use transparent base ten pieces on the overhead projector to start with. Place a hundreds piece on the overhead projector:



"I shall enlarge this square by making each side one unit longer." Place pieces as shown:

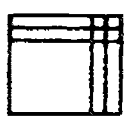


"It took TWO tens - one on each side - and a one to fill in the corner."
"How long is each side of this NEW SQUARE?" Write:

$11 = 10 + 1 \qquad 11^2 = (10 + 1)^2 = 100 + 2(10) + \quad = 121$

"$100 = 10^2$, $1 = 1^2$ and the other product is TWO times the crossproduct of 1 and 10 since we must add to TWO sides of the square."

"We shall now add one more to each side so the ORIGINAL SQUARE sides will be 2 more." Place pieces as shown



"What are the total number of pieces here?" In response to student responses, write:

1 HUNDRED
$2 \times 2 = 4$ TENS
$2^2 = 4$ ONES

"What length is each side of the square?" ($12 = 10 + 2$)

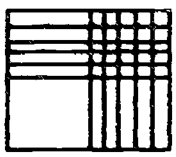
"How many tens are on each side?"

ERIC

"What is the size of the little square in the corner the ONES are filling?"
Write:

$$
\begin{aligned}
(10 + 2)^2 &= 100 + 2\,(2 \times 10) + (2 \times 2) \\
&= 100 + 40 + 4 \\
&= 144\text{, so} \\
12^2 &= 144
\end{aligned}
$$

"We shall now really enlarge the square to a new one that is fifteen on each side." "How many TENS are needed on each side?" (5) "How many ONES will fill the small square in the corner?" Summarize this model:



$$
\begin{aligned}
15^2 = (10 + 5)^2 &= 100 + 2\,(50) + 5^2 \\
&= 100 + 100 + 24 \\
&= 225
\end{aligned}
$$

<u>Activity:</u> Pass out the base ten pieces and the worksheets and have pairs of students complete these.

<u>LESSON TWO: Patterns in Making Squares</u>

<u>Introduction:</u> At this point, the patterns in squaring numbers with 5 in the ones place should be analyzed. Put the following summary on the board:

$$
\begin{aligned}
15^2 = (10 + 5)^2 &= 100 + 2(50) + 25 \\
&= 225
\end{aligned}
$$

$$
\begin{aligned}
25^2 = (20 + 5)^2 &= 400 + 2(100) + 25 \\
&= 625
\end{aligned}
$$

$$
\begin{aligned}
35^2 = (30 + 5)^2 &= 900 + 2(150) + 25 \\
&= 1225
\end{aligned}
$$

$$
\begin{aligned}
45^2 = (40 + 5)^2 &= 1600 + 2(200) + 25 \\
&= 2025
\end{aligned}
$$

"What pattern do you see in the products?" (get to the last two digits of 25)

"How are the digits ahead of 25 related to the first digit of the number being squared?" (get to (digit x (digit t1))

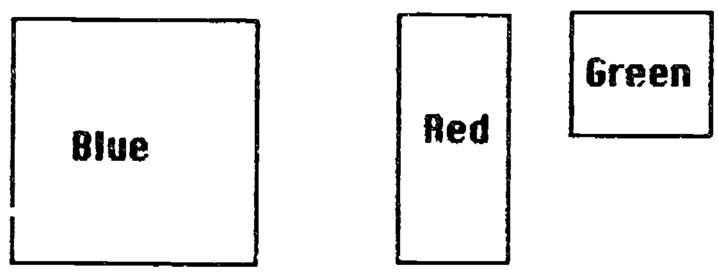
"Use the pattern seen to find 65^2."

Reinforce by writing: 65^2 (6 x 7) 25 = 4225
Discuss as needed

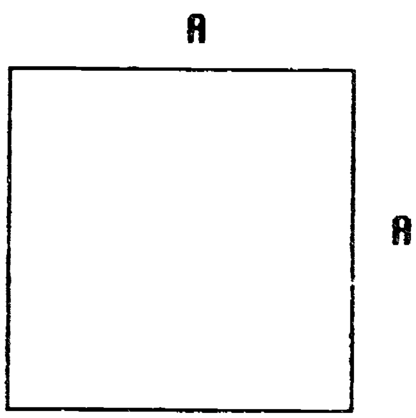
Activity: Have pairs of students work on the worksheets.

LESSON THREE: Generalized Square Building

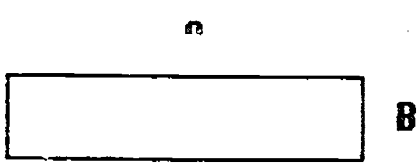
Introduction: This lesson involves building squares with non-numerically described sides. Use the template provided to make squares and rectangles for transparency use. You can use different colors for the squares and rectangles. Example:



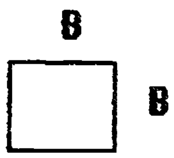
Put a large square on the overhead and label the sides "A".



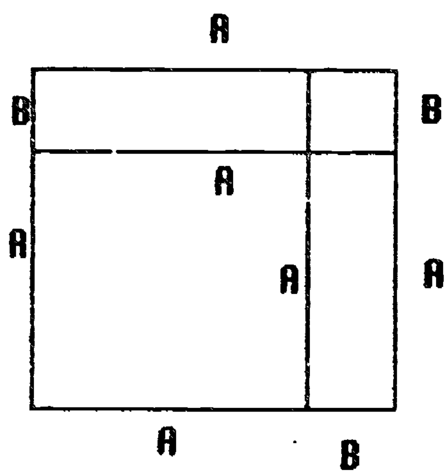
Put an associated rectangle on the overhead and label as shown:



Put a small square on the overhead projector and label the sides B.



Build a square of an A square, 2 A X B rectangles and a B square.

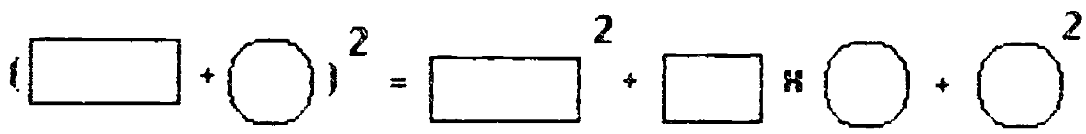


"What is the length of the side of the square I made?" (A + B)

"Of what parts is this square made?" (A^2, 2 AB and B^2)

Write: $(A + B)^2 = A^2 + 2\ A \times B + B^2$

We can rewrite this as:



"Now anything can be put into the shapes."

"Putting numerals in – a different one in each shape – gives something like this:"

ERIC

$$\left(\boxed{2} + \textcircled{1}\right)^2 = \boxed{2}^2 + \left(\boxed{2} \times \textcircled{1}\right) + \textcircled{1}^2$$

$$3^2 \qquad = 4 + 4 + 1 = 9$$

or

$$\left(\boxed{30} + \textcircled{2}\right)^2 = \boxed{30}^2 + \left(\boxed{30} \times \textcircled{2}\right) + \textcircled{2}^2$$

$$= 900 + 120 + 4$$

$$= 1024, \text{ so}$$

$$32^2 = 1024$$

"We could put other letters into the shapes."

$$\left(\boxed{T} + \textcircled{W}\right)^2 = \boxed{T}^2 + 2\left(\boxed{T} \times \textcircled{W}\right) + \textcircled{W}^2$$

$$= T^2 + 2\,TW + W^2$$

"We could even put more complicated things in."

$$\left(\boxed{D+4} + \textcircled{S}\right)^2 = \boxed{D+4}^2 + 2\left(\boxed{D+4} \times \textcircled{S}\right) + \textcircled{S}^2$$

$$= D^2 + 8D + 16 + 2DS + 8S + S^2$$

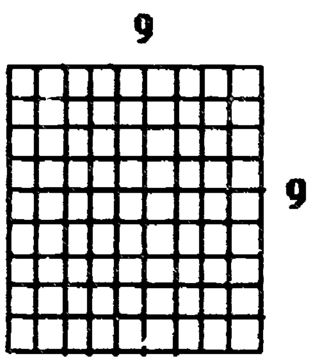
Activity: Assign pairs of students to work on the worksheets provided.

SHRINKING SQUARES

Background: Just as squares can be enlarged, they can be shrunk by shortening the sides. These lessons systematically shrink squares

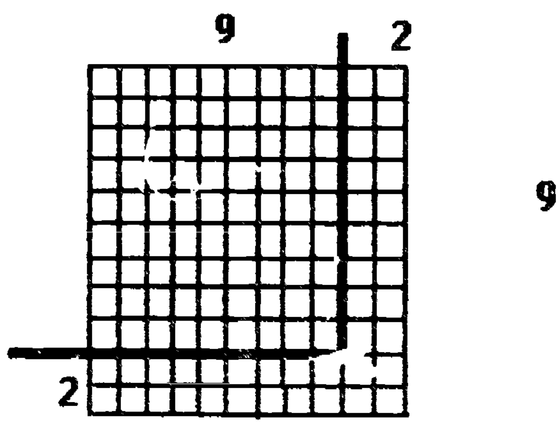
LESSON ONE: Shrinking by Units

Introduction: Put an overhead graph paper square like that shown on the projector:

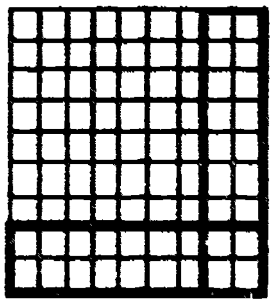


"We are going to shorten each side of the square by the same amount."

Put into the diagram as shown:



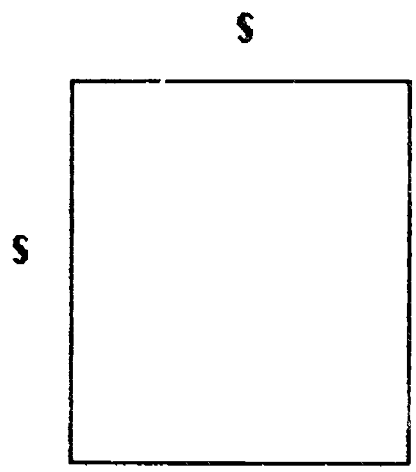
"To find the area of the new square, we can subtract some of the area of the original square from that area. Highlight the rectangle:



Point out the square in the lower right hand corner is subtracted TWICE if the rectangles are subtracted, so ONE square must be added back. Write:

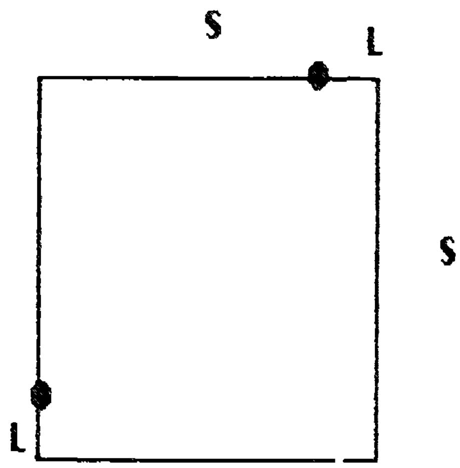
$$7^2 = 9^2 - 2(2 \times 9) + 2^2$$
$$= 81 - 36 + 4$$
$$= 49$$

Now put a square without any units on it on the overhead. Label as shown:

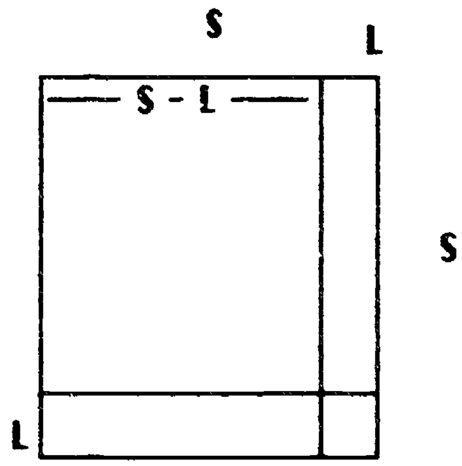


"S is the side of the square which we haven't measured to describe in units."

"We'll take the same length away from each side of the square."



Complete the diagram.



"The square S - L on a side is the square S on a side with some area subtracted. When the TWO L x S rectangles are subtracted, the square with side L is subtracted TWICE. Add ONE L2 back in." Write:

$$(S - L)^2 = S^2 - 2 L X S + L^2$$

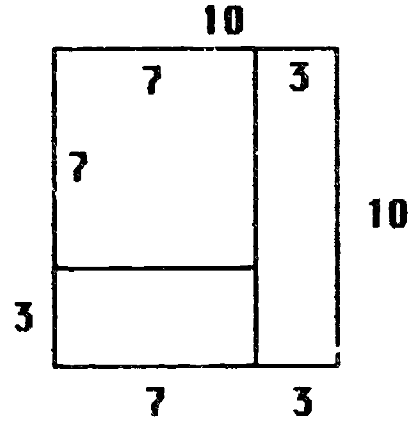
Point out the parts on the diagram often as needed until the reason for adding the smaller square back in is seen.

Activity: Pass out the worksheets for pairs of students to work on.

DIFFERENCE OF TWO SQUARES

Background: Using the difference of two squares for mental computation is useful just as squaring sums or differences is.

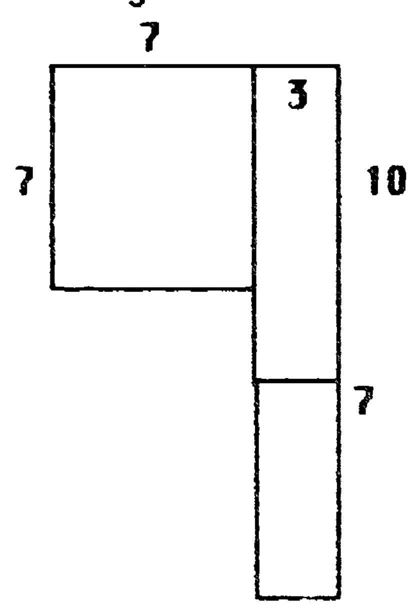
Introduction: Put the following square array on the overhead projector. The pieces can be cut from overhead transparency:



"The two rectangles shown are the DIFFERENCE BETWEEN the 10 inch square and the 7 inch square. This should equal 100-49 = 51.

"Since both rectangles have a width of 3 we can join them together in one having a longer length. This 3 is the difference between square sides, 10-7."

Rearrange into:



"This rectangle has length 17 and width 3, so has area 3 x 17 = 51, as expected." "We can write this as:

$$10^2 - 7^2 = (10 + 7)(10 - 7)$$

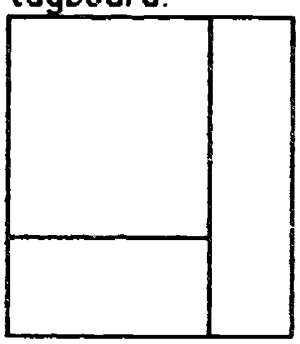
Show the following hold by calculat.ng each side: $12^2 - 5^2 = (12 + 5)(12-5)$

$144 - 25 = 17 \times 7$
$119 = 119$
$15^2 - 5^2 = (15 + 5)(15 - 5)$

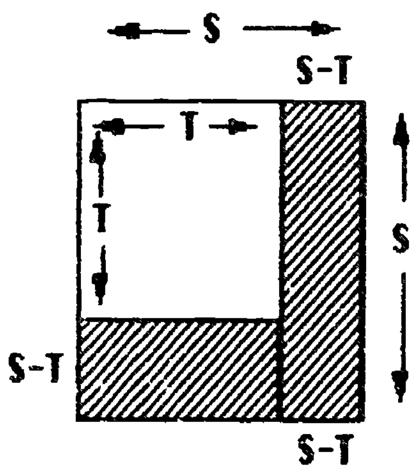
$225 - 25 = 20 \times 10$
$200 = 200$
$20^2 - 6^2 = (20 + 6)(20 - 6)$

$400 - 36 = 26 \times 14$
$364 = 364$

Make the following configuration from heavy colored transparency or from tagboard.



Place this on the overhead and label as shown.

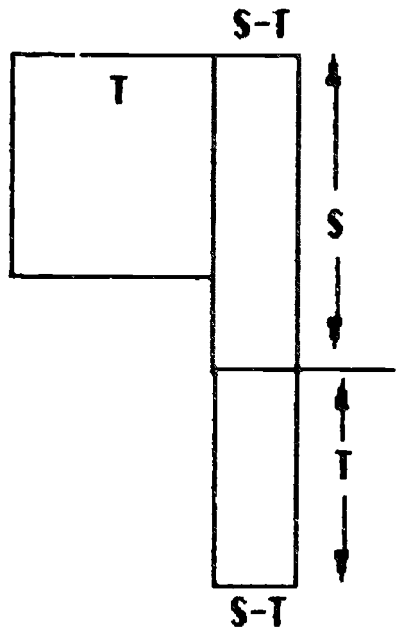


"The shaded area is the DIFFERENCE BETWEEN the square with side S and the smaller square with side T."

"The larger rectangle has length S and width S-7."

"The smaller rectangle has length T and width S-7."

"Since they have a width in common we'll join the two rectangles like this." Arrange the pieces as shown:



"This new rectangle has length S + T and width S - T.

"The difference of the 2 squares having areas S^2 and T^2, or $S^2 - T^2$, is the rectangle having length S + T and width S - 7. We write:

$$S^2 - T^2 = (S + T)(S - T)$$

"Any numbers can be put in for S and T as we saw in the first example."

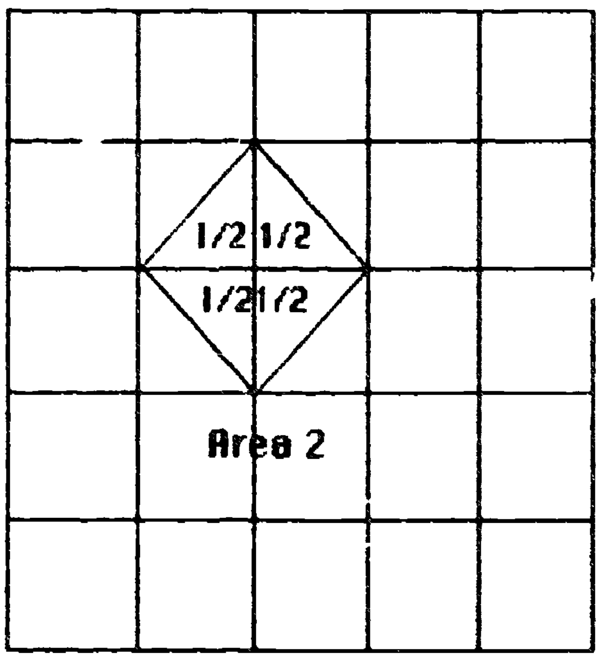
$(X + 1)^2 - X^2 = (X + 1 + X)(X + 1 - X)$
$= 2X + 1$
$(a + 2b)^2 - b^2 = (a + 2b + b)\ a + 2b - b)$
$= (a + 3b)(a + b)$

<u>Activity</u>: Give pairs of students the worksheets to complete.

GEOMETRY: Geoboards

LESSON ONE: More About Squares

Introduction: Place the numerals from 1 to 25 on the chalkboard.
"We are going to find squares having area equal to as many of these as we can on the geoboard. Use a single band and make a square having area '1'. It has ONE of the geoboard squares inside it. Show me your squares with area 1. Now make a square with area 2." (This may present some difficulty until students realize bands do not have to lie on the rows and columns of pins.)



"Now experiment to find squares with different areas. When you find one no one else has found before, let me know." Circle the numbers as children show you squares that have given areas. When completed the following should be circled:

1	11	21
2	12	22
3	**13**	23
4	14	24
5	15	**25**
6	**16**	
7	**17**	
8	18	
9	19	
10	20	

When all have been found by someone, so the list is as shown, have students make s ,ares having all areas circled on the list.

LESSON TWO: Triangles with Different Areas

Have students make triangles that have the following areas in succession
1, 2, 3, 4, 5, 6, 7, 8

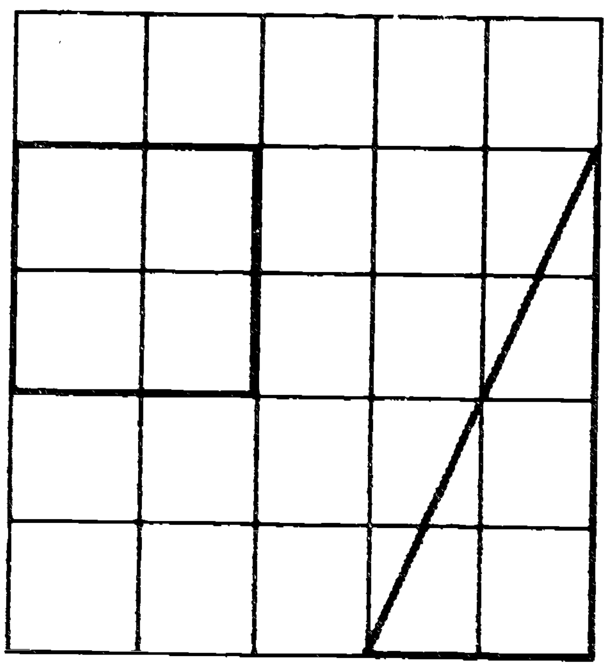
"What is the area of the larges triangle you can make on the geoboard?"

"Can you make more than one with this area?"

"Construct a square and a triangle with the same area."

LESSON THREE: Shapes with Equal Areas

Make a triangle and a square with the same area with two bands. Use an overhead transparency geoboard or hold up a regular geoboard.



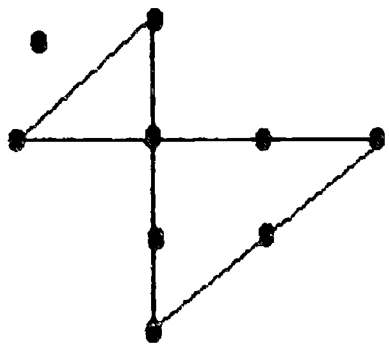
"Make these two shapes and make sure the areas inside the shapes are the same."

"Make the following shape pairs with the areas given. Pass out the activity sheets and recording forms. Monitor the work. Keep reminding students of the relationships between rectangles and parallelograms and that triangles are always half of a rectangle or a parallelogram.

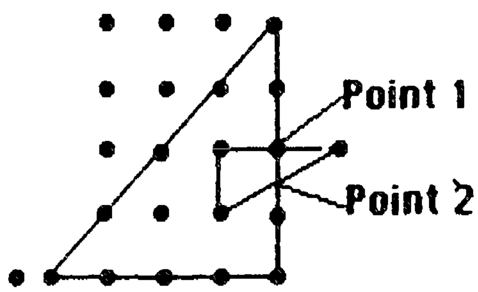
LESSON FOUR

Have student use geoboards and record the results on geoboard dot paper forms.

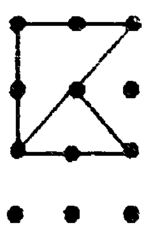
"Make two triangles that have just one point in common." Be sure they have this correct. An example is:



"Now make two triangles that have just two points in common." Here you will have to distinguish between points ON the triangle and points in the INSIDE of a triangle. A triangle is a closed curve, or a line! An example is:



Some students might think the following is all right.

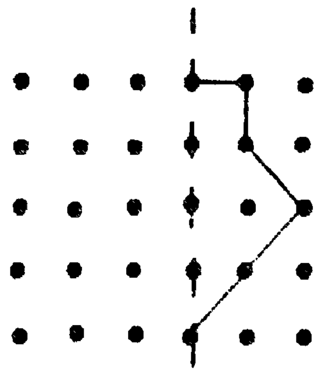


The triangles have not just two points in common, but the line segment determined by them as well.

Activity: Assign the worksheets to pairs of students to work. They are to make and record pairs of triangles that have 3, 4, 5 and 6 points in common.

LESSON FIVE: Dot Paper

Introduction: Make the following on a piece of dot paper transparency and place on the overhead projector.



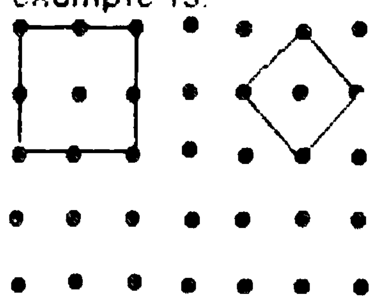
"This is half of a shape and the line is a line of symmetry. How should I complete the shape?"

Follow the suggestions that students give that are correct. Discuss those that aren't after putting in the lines as suggested and it is seen that this doesn't work.

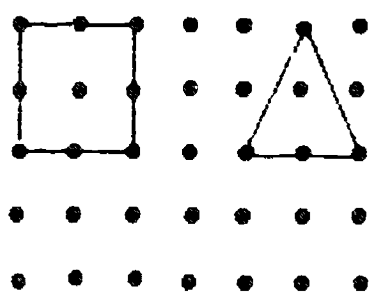
Activity: Have pairs of students complete the dot paper sheets to complete shapes given lines of symmetry.

LESSON SIX

Prepare dot paper transparency shapes and show on the overhead. An example is:



"How are these alike?" Discuss responses.
"How are these different?" They are both squares! Another example is:

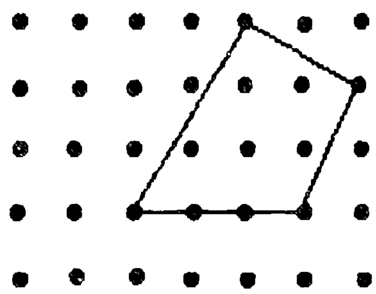


Same questions and use of them.

Activity: Have students work on the dot paper activity sheets.

LESSON SEVEN:

Use an overhead geoboard or one large enough to be seen by the entire class. Make a general quadrilateral on it:



"How should I make this into a trapezoid?" Discuss the properties of a trapezoid (one pair sides 11). "How should I transform the trapezoid into a parallelogram?" Discuss the differences between these two shapes.

"How should I transform a parallelogram into a rectangle?" Discuss the differences between these shapes. "How should I transform the rectangle into a square?" Discuss the difference between these shapes." "How should I transform the square into a rhombus?" Discuss the difference.

QUESTIONS FOR DISCUSSION

1. Is every square a rectangle? Why?
2. Is every rectangle a square?
3. Is every square a parallelogram?

Discuss

4. Is every parallelogram a square?
5. Is every rectangle a parallelogram?

Discuss

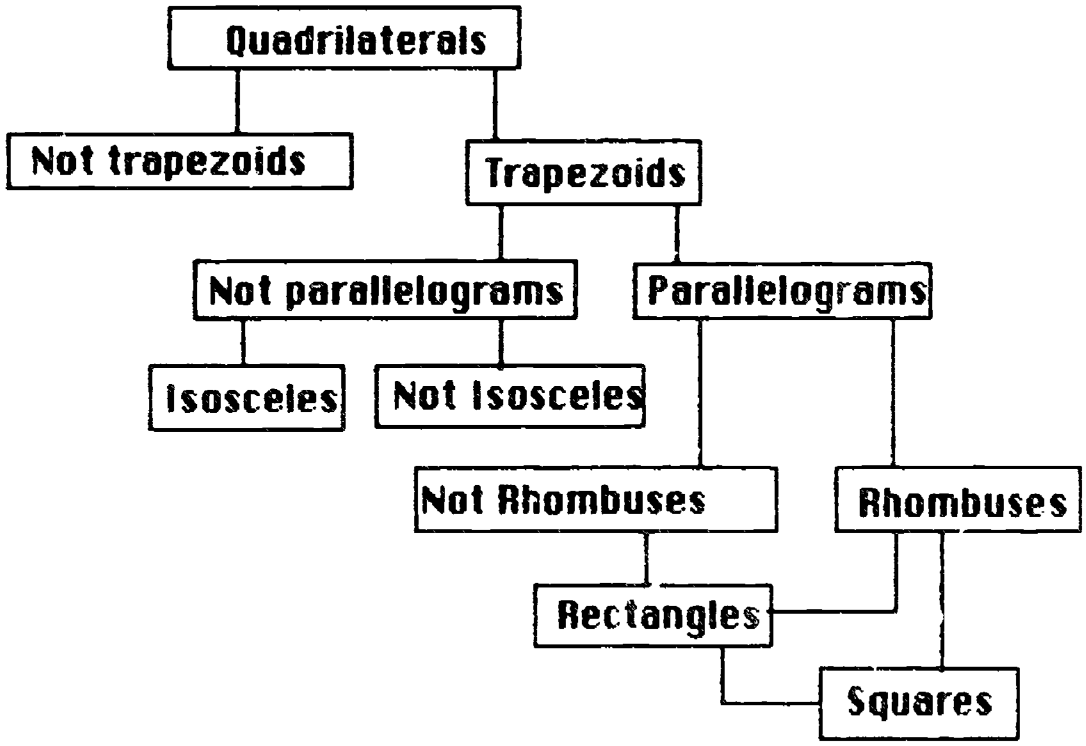
6. Is every parallelogram a rectangle?
7. Is every square a trapezoid?"

Discuss

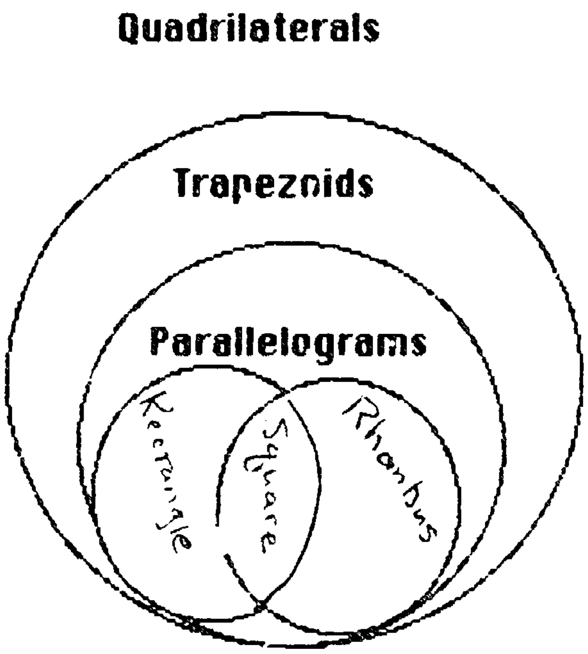
8. Is every trapezoid a square?'
9. Is every rhombus a parallelogram?
10. What is the most general four-sided shape?"
11. What four-sided shape must satisfy the most conditions?

11. What four-sided shape must satisfy the most conditions?

The conclusion of this lesson should be the following hierarchy shown.



or



LESSON EIGHT

Give pairs of students the worksheet to transform a rhombus through stages into a general quadrilateral.

ERIC

GEOMETRY:

LESSON ONE: Congruence and Similarity

Use the template provided to make an envelope of triangles for each student. Have the students sort these into groups of triangles that:

(1) match exactly (one fits on top of the other)
(2) look alike, but some are "photo enlargements" of another
(3) do not fit into either of these groups

Discuss completely what characteristics determine how triangles fit into each of the three groups.

In what group are all of the angles of one equal to all of the angles of the other?"

"In those that don't fit into Group 1 or Group 2, do any have 2 sides the same as triangles in Group 1 or Group 2?"

"What is alike about those in Group 1 or Group 2?"

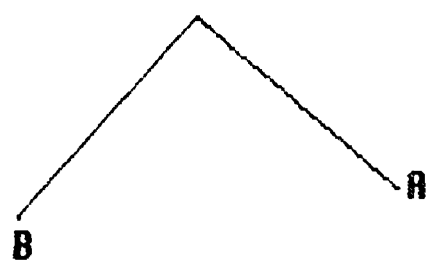
"Make a list of which angles are equal, i.e. A3 = B2."

"Make a list of triangles that have 2 sides equal."

"Make a list of triangles that have one side and one angle equal."

LESSON TWO: Congruence

Introduction: Place the following on the overhead projector. Use Transparency Angle #1:

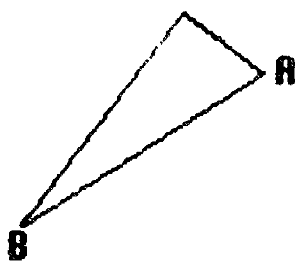


"How many different triangles can result from connecting A to B?"

"When two sides and the angle they make are fixed, the triangle is DETERMINED.

1. Notice that if i change either side, a different triangle results.

Use the Transparency Triangle #2:

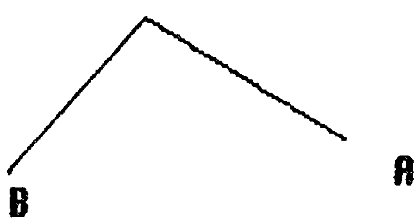


Place this transparency o the other so the students can see ONE side and the ANGLE are the same, but the SECOND side is different.

Use Transparency #3:

Do as above, but then compare with both previous triangles.

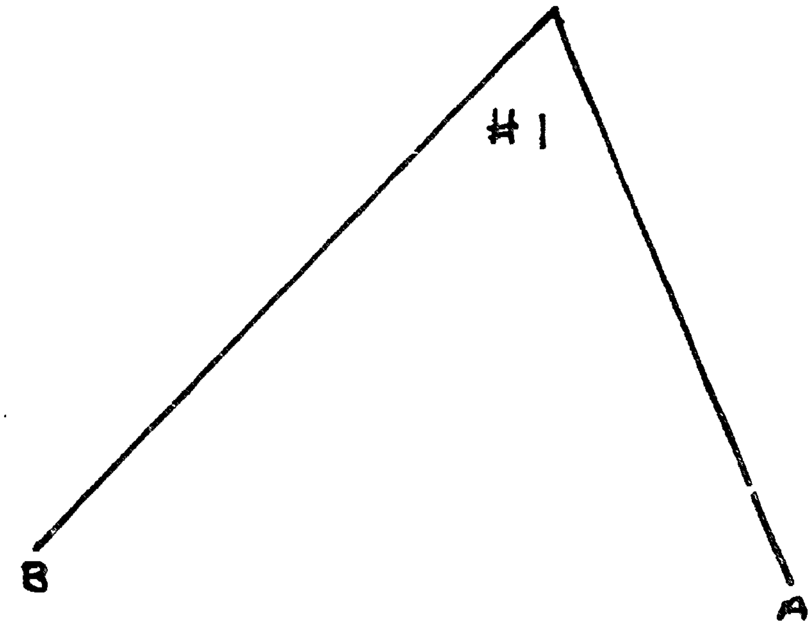
Use Transparency #4:

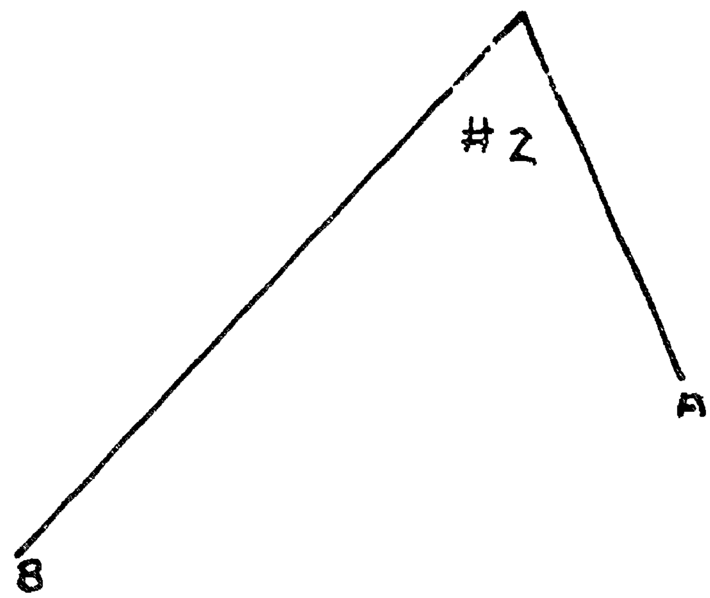


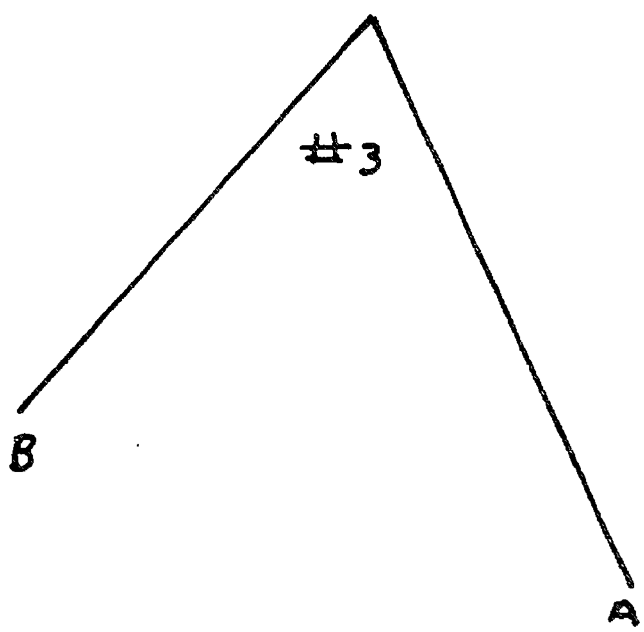
Make this triangle and compare w'th Transparency #1.

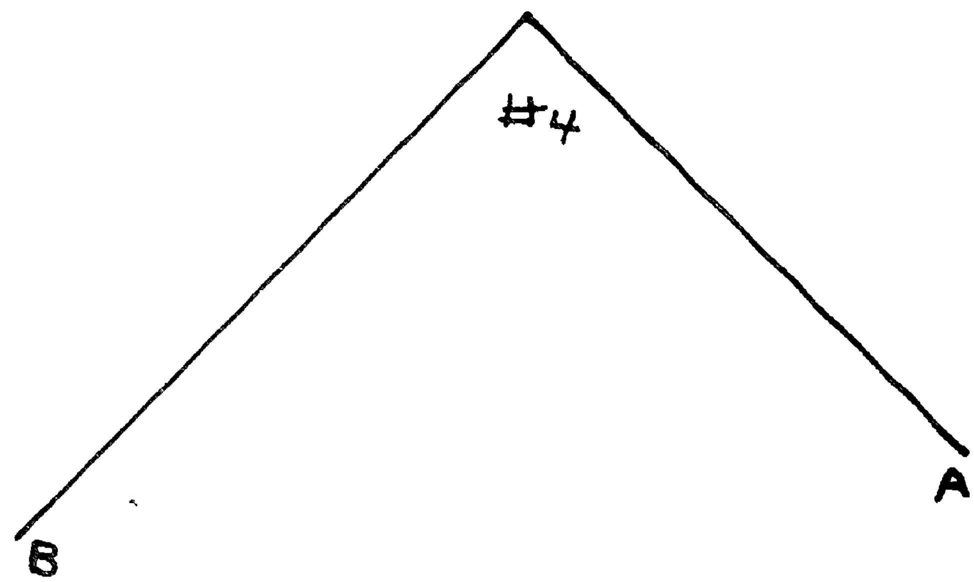
"3 parts - an angle and the two sides making the angle - must be the same in 2 triangles if they are to match."

<u>Activity</u>: Pass out the workshee's for pairs of students to work on.


#1
B
A

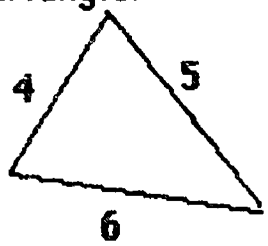

#2
B
A


#3
B
A

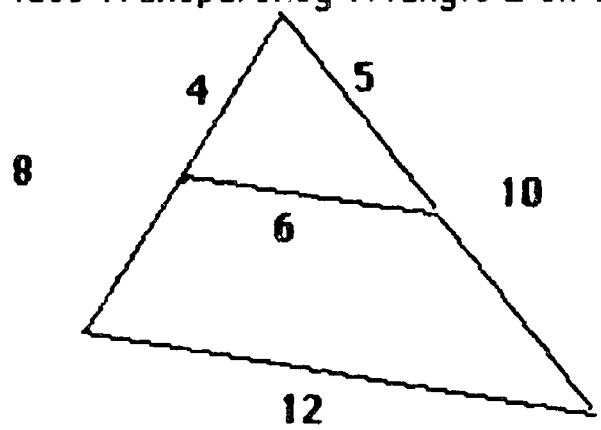

#4
B
A

LESSON THREE: Similarity

Introduction: Use Transparency Triangle 1. Point out the sides of the triangle.



Place Transparency Triangle 2 on top of the first:



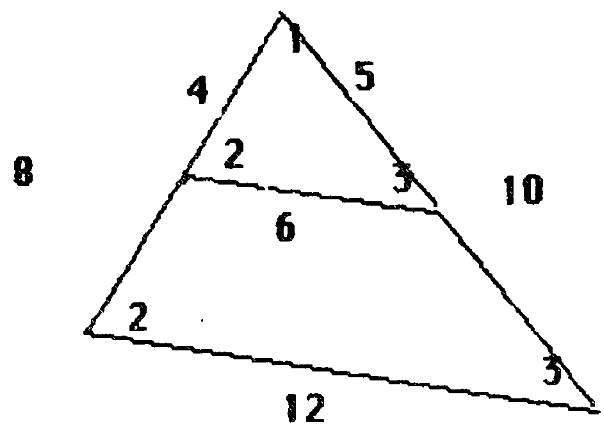
Point out how the sides of the smaller compared to those of the larger are in the ratio 4:8, 5:10, 6:12. These all belong to the 1:2 family of ratio so are the same.

"The sides of these triangles are PROPORTIONAL since they are all in the same ratio."

Place the corresponding angles to match one pair at a time so the students can see the angles are equal in pairs.

"The angles of these two triangles are equal in pairs."

"How are the proportional sides related to the angles?"



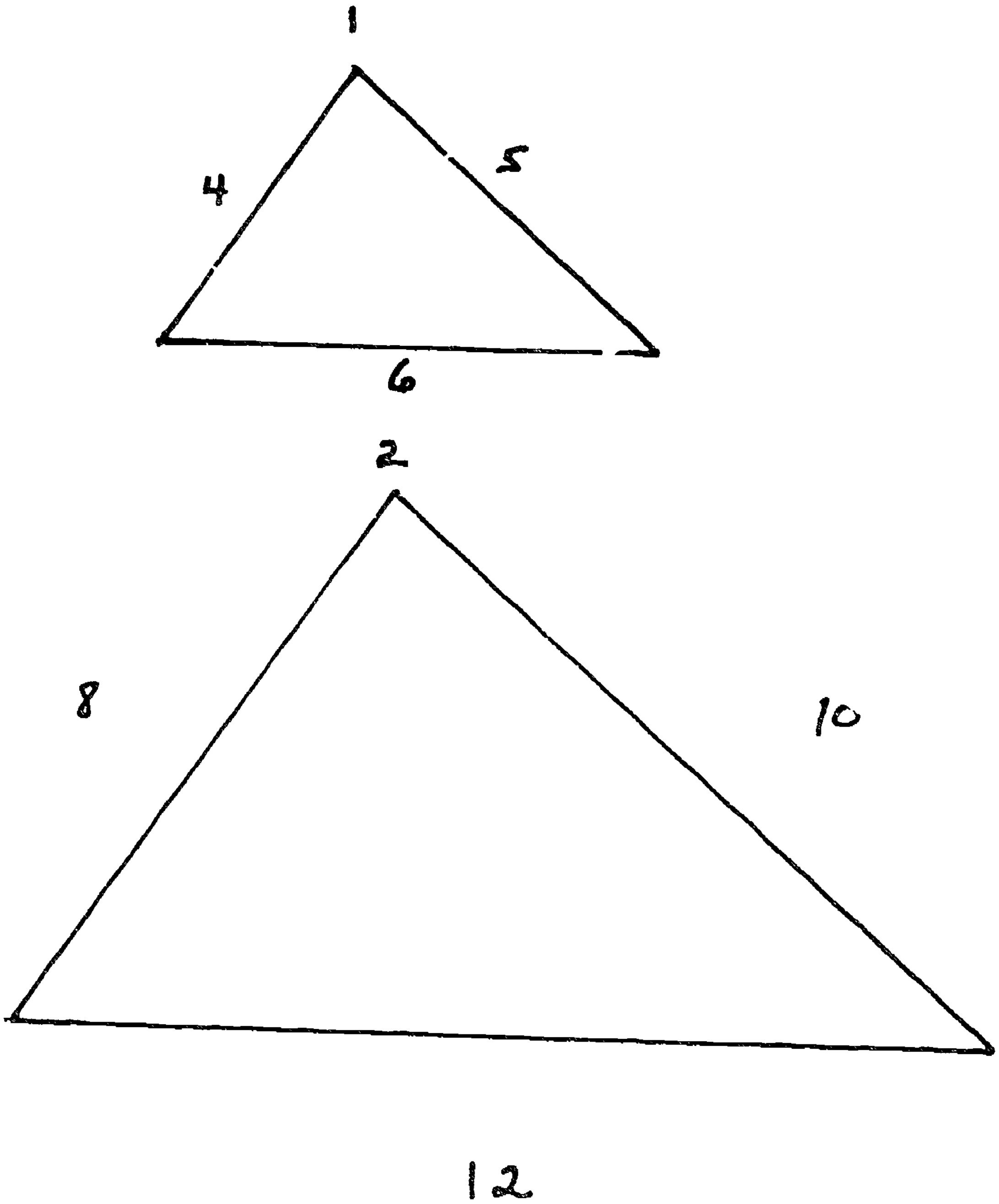
"Are the angles labelled with the same number equal?"

"Proportional sides lie opposite the equal angles."

"What else is true of the longest sides of the two triangles?" (parallel)

<u>Activity</u>: Have students do the worksheets working in pairs.

TRANSPARENCY
TRIANGLES

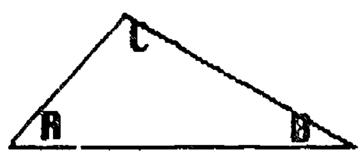


ERIC

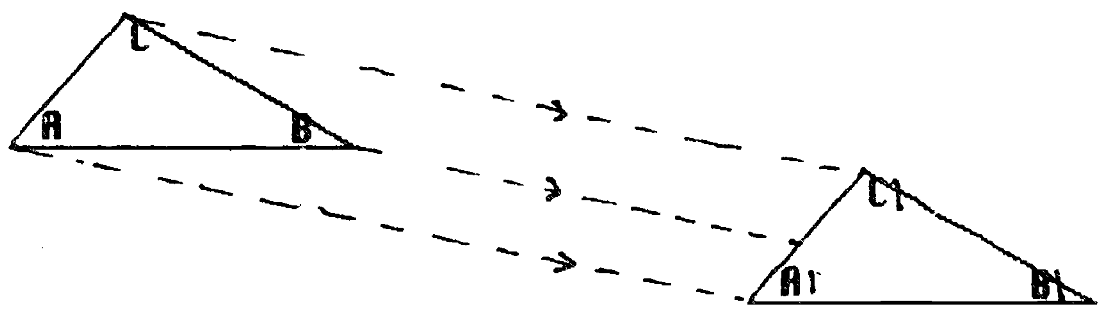
LEVEL SIX

CONGRUENCE: Corresponding Parts

Introduction: Place a cut out transparency triangle with the vertices labelled on the overhead projector.



Move it to a new location on the overhead and trace the shape.



Label the vertices as shown.

"Which angles in the new triangle are the same as which angles in the original triangle?" (A & A1, B & B1, C & C1)

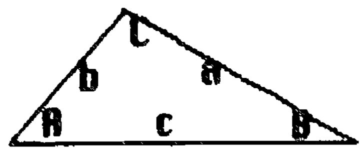
"Which angles APPEAR to be the largest?"

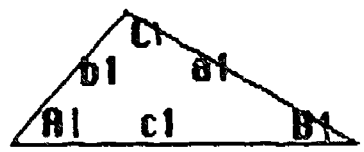
"Which sides APPEAR to be the longest?" (opposite C & C1)

"Which angles APPEAR to be the smallest?" (B & B1)

"Which sides are the shortest?" (opposite B & B1)

"CORRESPONDING parts are those that are in the same place in the two triangles. Corresponding sides lie opposite the angles that are equal."





Labels the sides as shown.

"Side a lies opposite angle A. Side b lies opposite angle B. Side c lies opposite angle C."

"Which sides lie opposite angles A1, B1 & C1?"

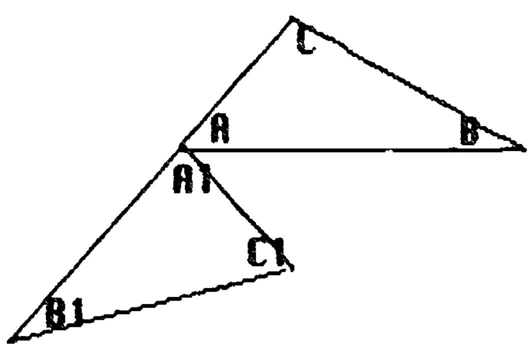
"How does side a compare with side a1?"

"How does side b compare with side b1?"

"How does side c compare with side c1?"

"These two triangles are CONGRUENT - same size and shape. Their CORRESPONDING PARTS are all equal - angles and sides."

Rotate the original triangle as shown.



Do this slowly so the students can see C move into C1 and B into B1.

"Is this new triangle CONGRUENT to the original?"

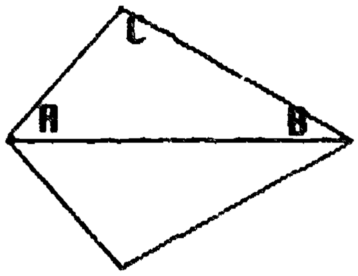
"What angle corresponds to angle A?"

"What angle corresponds to angle B?"

"What angle corresponds to angle C?"

"Does moving a triangle to a new position change its size or shape?"

Flip the triangle over as shown.



"How should I label the angles in this triangle?"

Discuss these cases thoroughly. Emphasize:

1. Corresponding sides lie opposite equal angles
2. Corresponding angles lie opposite equal sides
3. Corresponding angles and sides of CONGRUENT triangles are equal.

Activity: Assign pairs of students to work on the practice sheets.

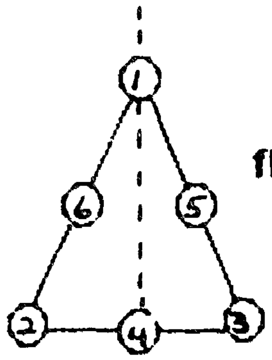
LEVEL SIX

PROBLEM SOLVING
LESSON ONE

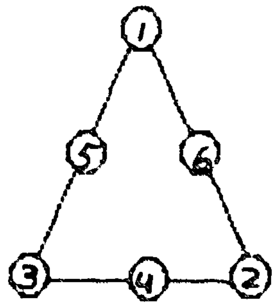
Introduction: Make an overhead transparency of the "magic triangle" provided. Place this on the projector.

"We will use the numbers 1 through 6 and put them in the circles so the numbers along each side add to 9."

Look at all solutions suggested:

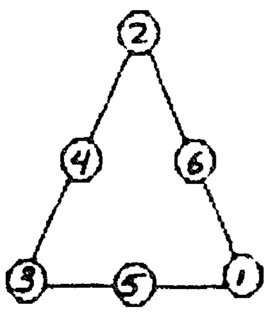


flips over line l to give



"How many lines are there to flip over?"

and rotate through to give



"How many ways are there to rotate?"

Discuss the relationships. The sum of 1 + 2 + 3 + 4 +5 + 6 = 21

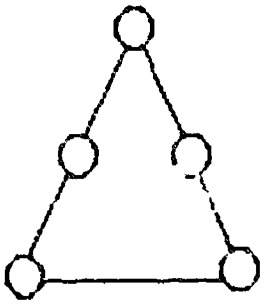
The sum of the 3 sides = 9 + 9 + 9 = 27

"Which numbers are used twice in the 27?"

"What must their sum be?"

"What three numbers, 1 through 6, added give 6?"

"Is there any other way to do this other than putting 1, 2 and 3 in the 'corners'?"



"This time we want the sides to add to 10."

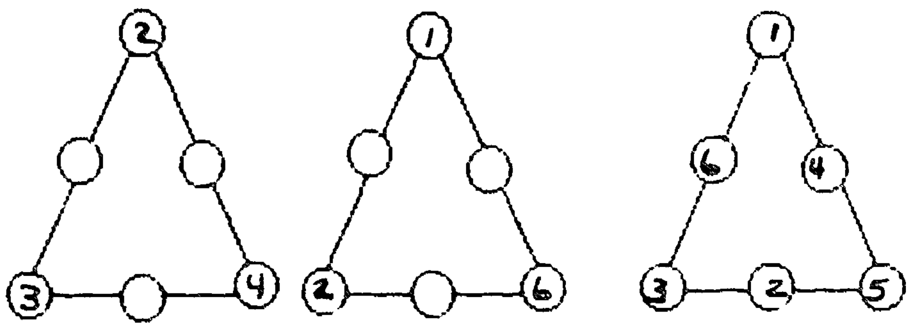
"What will be the sum of the 3 sides?" (30)

"The sum of 1 + 2 + 3 + 4 + 5 + 6 is still 21."

"The numbers used twice must add to what number?"

"What numbers 1-6 are candidates for the 3 corners?" [3 that add to 9 are (2,3,4) (1,2,6) (1,3,5)]

Show all of these:



Won't work **Won't work** **Works**

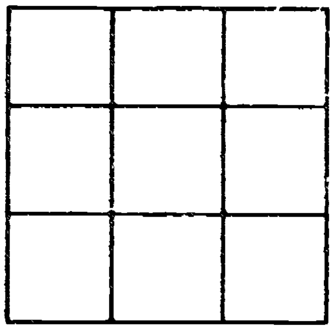
"What is there about the numbers in (2,3,4), 1,2,6)and (1,3,5) that should lead you to suspect the first two won't work?"

"Remember that this triangle can be flipped over three lines of symmetry and rotated through two different rotations."

Assign the activity sheet to pairs of students to work.

LESSON TWO

Introduction: Make an overhead transparency of a "magic square". Place this on the projector.



"This is a magic square. It has nine places for numbers so we'll will use 1, 2, 3, 4, 5, 6, 7, 8, & 9."

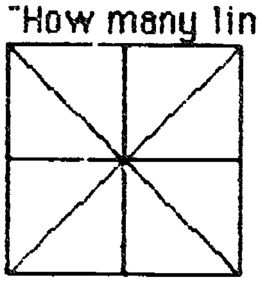
"We want each row to add to 15, each column to add to 15 and each diagonal to add to 15."

"How should I put the numbers in?"

Let the students explore the possibilities and find solutions. Write all of these on the board. One solution is:

2	9	4
7	5	3
6	1	8

"How many lines of symmetry does this square have? (4)



"Let's flip over each line of symmetry to get other solutions and see if we can find them in those you found."

2	9	4
7	5	3
6	1	8

gives

6	1	8
7	5	3
2	9	4

2	9	4
7	5	3
6	1	8

gives

4	9	2
3	5	7
8	1	6

2	9	4
7	5	3
6	1	8

gives

8	3	4
1	5	9
6	7	2

2	9	4
7	5	3
6	1	8

gives

2	7	6
9	5	1
4	3	8

"What number stays in the same place in all of these different solutions?"
(5)

"How is this number related to the required sum?"

"What is this number's position in the sequence of digits?"

"Given these numbers: 4, 5, 6, 7, 8, 9, 10, 11, 12 - what number should go in the middle of a magic square?" (8)

"What would be the sum of the rows, columns and diagonals of this magic square?"

7	**12**	**5**
6	**8**	**10**
11	**4**	**9**

is a solution

8 is center

3x8 = 24 is the sum used.

"How many additional solutions can we get from this?" (4)

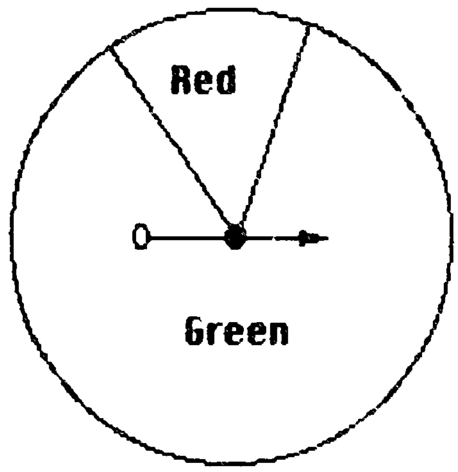
Have pairs of students work on the activity sheet.

LEVEL SIX

PROBABILITY

LESSON ONE

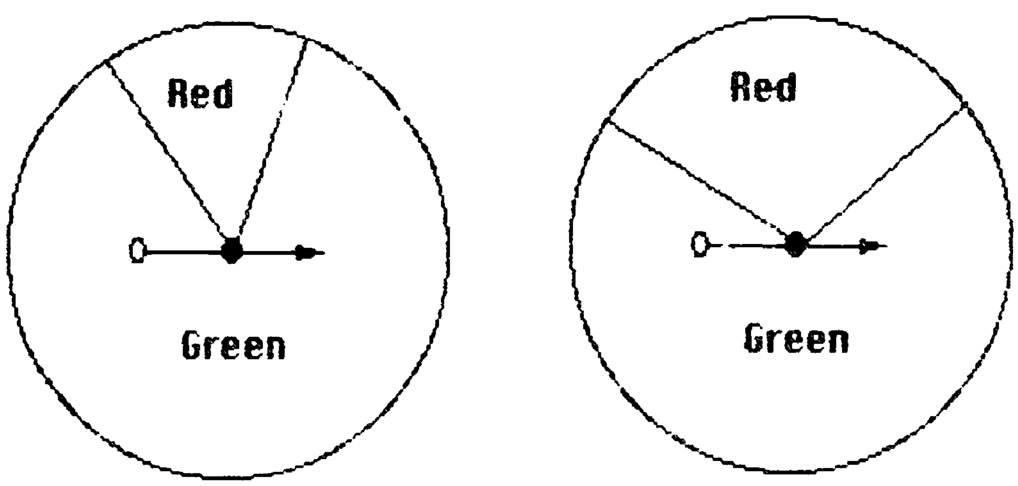
Introduction: Put an overhead transparency spinner on the overhead projector:



"If I spin this, is it more likely to land on red or green? Why?" Discuss this thoroughly.

"If I spin it 100 times, how many times do you think it will land on red? Why?" discuss this in terms of relative amount of green versus relative amount of red.

"Do you think I could spin it 100 times and never get red?" Discuss the idea of independent events. Each new spin is unrelated to the results of any previous spin. Put a second spinner on next to the first.

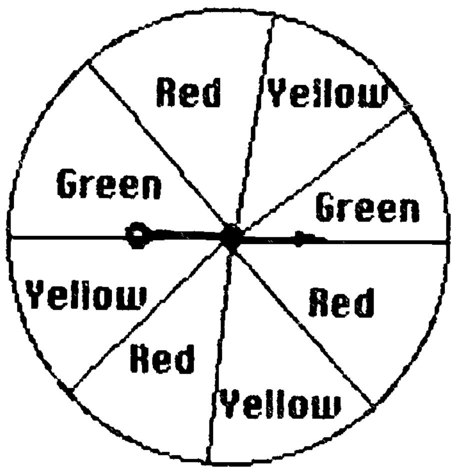


"Am I more likely to get a red in 100 spins now? Why?" "Do you think I would get twice as many red with this spinner as with the first one?"

Activity: Have students use spinners of the first kind and spin 100 times and record red or green each time, and fill in the sheet. Give a spinner of the second kind and have students fill a second sheet. Students should work in pairs.

Post Activity Exercise: Tally red and green totals from each group and graph the red totals. Compare the graphs from the two spinners.

LESSON TWO



Introduction: Put spinner on the overhead and ask these questions:

"Which color is most likely to be spun to? Why?"

"Which is least likely to come up?" Discuss these.

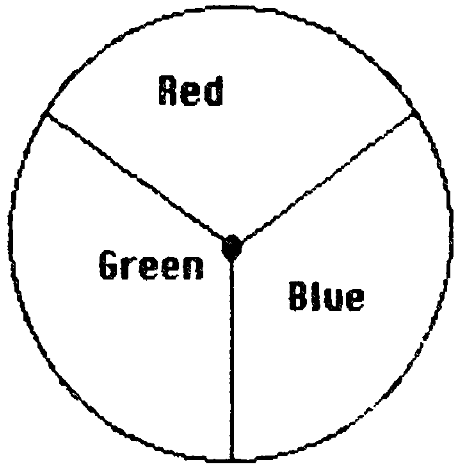
"If I spin 100 times, how many times do you think it will rest on red? on green? on yellow?"

Activity: Have pairs of students work with the spinner and a recording form. They should spin 100 times.

Post Activity Exercise: Combine totals from all groups and show the results. Find the fractional part that is red; that is green, that is yellow.

LESSON THREE

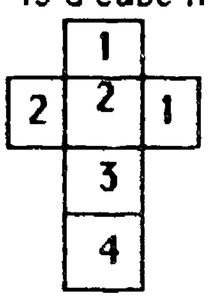
Introduction: The PROBABILITY of an event happening is the fraction
Happens
Total events Thus, the probability of "red" for this spinner is 1/3



It is 1/3 for green and 1/3 for blue.

"What is the probability of a "4" showing on a cube that is rolled, if the cube has numerals 1, 2, 3, 4, 5, 6 on the faces?" Discuss. Each face is equally likely, so a 4 should appear 1/6 of the time.

"Is a cube made like this a "fair" cube?"



"Why or why not?" "Which numeral is most likely to appear? Why?"

"We have a 'fair's '^*ion if all outcomes are EQUALLY LIKELY. On a 'fair' cube each numera ' d come up 1/6 of the time."

<u>Activity:</u> Assign the class into groups of 3 - one is to tally outcomes, one to roll the die and the other to check. The die is to be rolled 6 times. Record results in the form.

<u>Post Activity Exercise:</u> Use group results to complete the attached table.

<u>LESSON FOUR</u>

<u>Introduction:</u> This activity seems to violate the equally likely idea. That is because the human value element enters in. On the face of it, it seems that if told to write a numeral 1-4, 1/4 would write "1", 1/1 "2", 1/4 "3", and 1/4

"4". However experience shows "3" is chosen most frequently. Here is the chance to check this with your class.

"Get your pencil ready to write a number down on a piece of paper." "Ready? Write down a number from 1 to 4 quickly."

Have a tally form ready to tally the results.

1	2	3	4

"How many wrote "1"? "2"? "3"? "4"? Discuss the results. "Get your pencil ready to write down a number. Ready? Write down 7, 8 or 9!"

7	8	9

Compare with the earlier results. Each here should be close to 1/3 while each earlier deviated to some degree from 1/4.

Discuss with students the idea that "all conditions must be the same" for probability predictions to hold. When human values are factored in, the "laws of chance" are altered.

LEVEL SIX

PROBABILITY AND LOGIC

LESSON ONE: A Game to Use

Introduction: Students are to have the recording sheet provided. On the overhead projector show the students a set of colored chips and ask them to record how many of each kind there are. A good starting collection is 4 Red, 3 Green, and 1 Yellow.

Individual students are to blindly select a given number of the chips from a bag as a choice in the game. A good beginning number is three. Each student is to choose a given number of statements from the set of statements given. A good beginning number is one.

A rule is established to determine whether a move can be allowed or not. A good beginning choice is to have the statement TRUE of the chosen subset of chips.

Example of The Game:	Colored chips available:	4B, 3G, 1Y
	Subset to be chosen:	3 chips
	Statement set:	(see attached)
	Statements to be chosen:	1
	Move rule:	Statement is TRUE about the chosen set.
	Number of Moves to End:	7

Each child has a sheet with the given statements and the recording forms. All information about the conditions should be entered into the form. Be sure this has been done.

"Each of you choose one of the eight statements. Record this in your form."

Check that this has been done.

"_______________, please come and choose 3 chips from the bag."

Show these on the overhead projector.

"If your statement is true of this set, write YES, otherwise No."

"How many of you could answer yes? A yes is one move toward the end. You need seven yes's to win."

Example:

(R) (R) (Y)

Statements True: 3, 4, 5
Statements False: 1, 2, 6, 7, 8

Replace the chips in the bag. "______________, please come up and choose three chips." Show these on the overhead and go through the same series of questions and statements.

Children many have questions about the language in the statements. Take time to discuss these. Some examples are:

"Does 'there is a' mean exactly one or only one. '

"Does 'there are' mean exactly two, or only two?"

Example of second choice:

(G) (G) (R)

Statements true: 3, 4, 6, 7, 8
Statements false: 1, 2, 5

The students might question the use of different - different from each other? or different from the red?

Discuss this with them with regard to the need for precision in the use of language.

Continue having children choose sets until there is a winner, or winners, i.e, 7 yes's.

"Which choice of a statement gives the best chance of winning the game? Why?

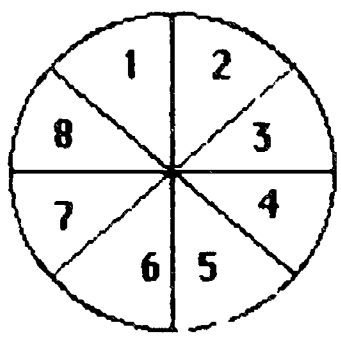
This question is likely to elicit the observation that the number of Reds, Greens, and Yellows affects the validity of the set of statements.

LESSON TWO: Give the children the option of writing their own statement to use in the game and play it that way.

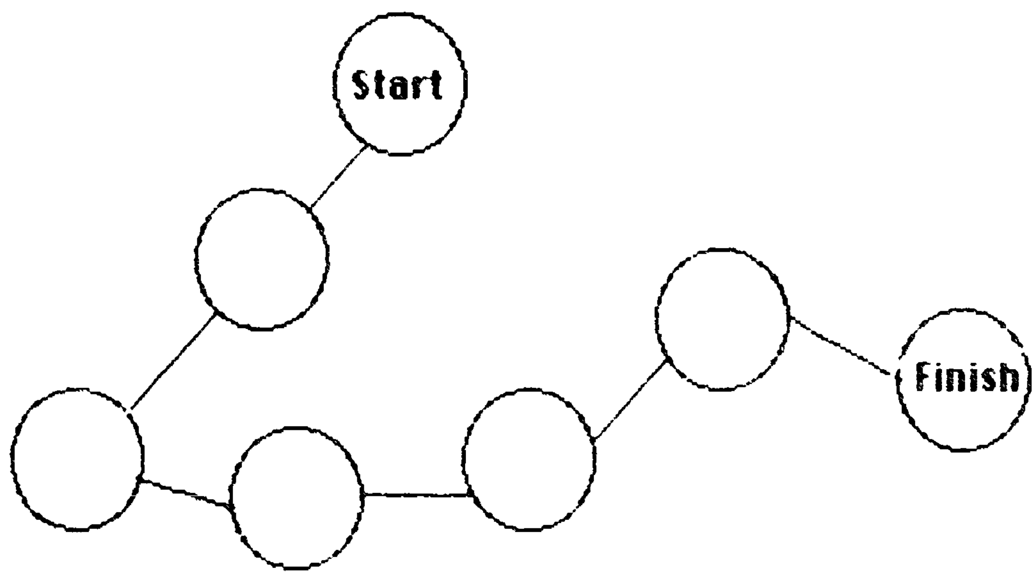
LESSON THREE: Play the game as in Lesson One but change the rule so that a move can be made if the chosen statement is FALSE.

LESSON FOUR: Have students choose TWO of the statements, or choose one and write one. Then the Rule can be: BOTH TRUE
BOTH FALSE
ONLY ONE TRUE (OR FALSE)

LESSON FIVE: Have students work in pairs. A fixed subcollection of chips is used. A spinner is used to select a statement to be judged TRUE or FALSE:



A game board is used for each child with some marker to move on it if the Rule is satisfied.



LESSON SIX: A deck of cards is made from the statements and these are turned over one at a time, instead of a spinner.

VARIATIONS ON THE GAME FOR OTHER LESSONS:

1. Change the material to numerals. The statements would then relate to properties of numbers: add even, multiples of, a factor of, prime or not, etc.
2. Change the materials to shapes. The statements would then relate to oroperties of shapes: angle sizes, number of sides, numbers of right angles, etc.
3. Change the set that is available.
   Change set size of chosen set.
   Change statements given.
   Change number of statements chosen
   Change the rule to be able to move.
   Change the number of moves to get to Finish line.

SOME QUESTIONS TO RAISE, OR THAT STUDENTS MIGHT RAISE:

1. Are any statement choices or pairs or triples of statement choices "sure bets"?
2. Which statement choices give a better chance of winning?
3. Can colors, for example, other than those in the "available set" be mentioned in statements?
4. What is the difference between:
   "NOT ALL ARE RED" and
   "ALL ARE NOT RED", or
   "NOT ONE IS GREEN", and
   'ONE IS NOT GREEN"?

(The placement of words like some, all, none, and not are important to the meaning!)

ERIC

MORE ABOUT PROBABILITY

LESSON ONE: Independent/Dependent Events

Introduction: Events are DEPENDENT if one affects the other. They are INDEPENDENT if one event does not affect the other. A good example is drawing a card from the deck.

"Draw one, replace, draw another." These are independent events since in both cases the possible outcomes are the same.

"Draw one, draw another." These are dependent. The first event has affected the possible outcomes for the second event.

Use the following examples. Have the students write down whether the events are dependent or independent. Discuss each one as to "why" the events should be considered dependent or independent.

A.

1. John plays Nintendo one hour each day.
2. John scores more points in the next Nintendo tournament.

B.

1. The Chicago Bears will win the Super Bowl next year.
2. Mt. St. Helens will erupt again next year.

C.

1. Tom will get an A on his next math test.
2. Tom got an A on his last math test.

D.

1. It will snow tonight.
2. Bill will be late to school tomorrow.

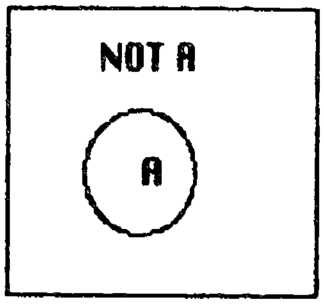
E.

1. The next child born in the local hospital will be a girl.
2. The last child born in the local hospital was a girl.

ERIC

LESSON TWO: Complementary Events

Introduction: Complementary events involve the probability of the NOT event. For example, in tossing a fair die, the probability of a FIVE is 1/6; the probability of a NOT FIVE is 5/6. Note that for complementary events, the sum of the probabilities is always 1. That is because the sum of A and the Complement of A is the entire universe being considered.



The term "odds" is used when the numerators of probability fractions are compared by a ratio. For example:

Tossing a die:	
Get a FOUR	1/6
NOT GET a four	5/6
ODDS for getting a FOUR	1 : 5
ODDS AGAINST getting a FOUR	5 : 1

Give the class this problem and ask these questions. Discuss the responses. Emphasize the difference between ODDS FOR and ODDS AGAINST.

"You have nine keys in your key case. One opens the front door and one opens the back door. The other seven cannot help you into the house. In the dark, you pick a key:"

"What are the odds against your key opening the front door?" (8:1)
"What are your odds against your key opening the back door?" (8:1)
"What are your odds against your key opening either door?" (7:2)

"If your keys all look alike except for one small key, how do the odds change in these three cases?"

LESSON THREE: Compound Events

Introduction: If an event consists of two or more simple events occurring together, it is a COMPOUND EVENT. For example, the outcome of the toss of two coins is a compound event since the individual outcomes are independent.

Ask the students to bring two different coins to class. Have them identify the head and tail for each.

"How many different outcomes could we have if we flipped BOTH of the coins?"

Discuss this, then make a table on the chalkboard or the overhead:

Two Heads	**One Head One Tail**	**Two Tails**	**Total**

Show the students how to flip and catch the coins and place on desk top without seeing the result until on the desk top.

"Flip BOTH of your coins."

"How many have TWO HEADS?"

"How many have ONE HEAD and ONE TAIL?"

"How many have TWO TAILS?"

Tally the results in the table.

Have each student toss the two coins 10 times and record the results on the form supplied.

Accumulate the results in a table like that above. Then the fraction is formed in each case:

$$\frac{\text{TWO HEADS}}{\text{TOTAL}} \qquad \frac{\text{ONE HEAD, ONE TAIL}}{\text{TOTAL}} \qquad \frac{\text{TWO TAILS}}{\text{TOTAL}}$$

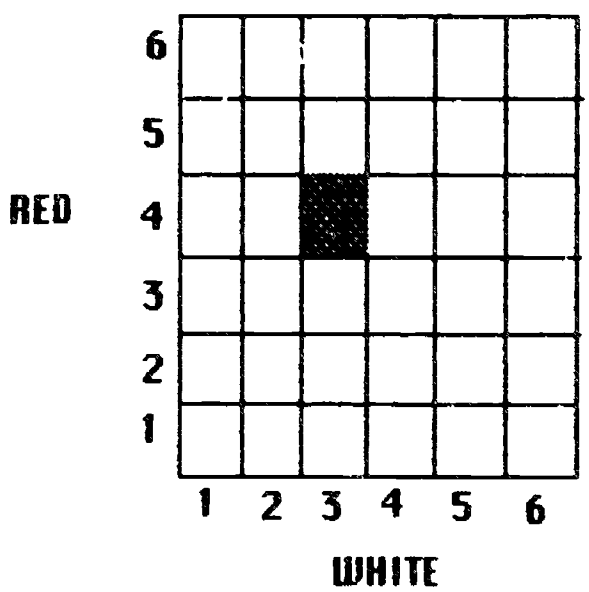
should be very close to 1/4.

"Why do you think each fraction is close to 1/4?"

Discuss. Point out that the events are INDEPENDENT so one doesn't affect the other.

P (2H) = 1/2 x 1/2 = 1/4
P (H,T) = 1/2 x 1/2 = 1/4
P (T,T) = 1/2 x 1/2 = 1/4

All of the outcomes in a 2 event compound event can be shown in a matrix. For example, in rolling 2 dice - one red, one green:



The ▩ shows the outcome on a "4" on the Red and a "3" on the white.

Make a matrix like this and pose the following questions for the students:

"What is the probability of getting "doubles"? Point out there are 6 possible ways of getting this result, so the probability is: 6/36 = 1/6

"What is the probability of getting a sum of 8?"
"What is the probability of getting a sum of 7?"
"What is the probability of getting a sum of 11?"

"What is the probability of getting an 8 AND doubles?"

"If both dice show the same number, what is the probability their sum is 8?"

RELATIONSHIPS

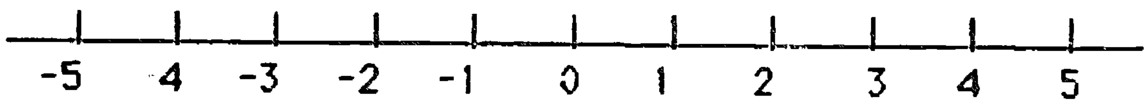
<u>Background:</u> Students have had experience with inequalities, equalities and signed numbers. These lessons carry this work further.

LESSON ONE

<u>Introduction:</u> "Which of these is true?" $3 < 5$, $3 = 5$, $3 > 5$

"Which of these is true?" $-3 < -5$, $-3 = -5$, $-3 > -5$

Discuss the difference between these cases. Show the consistency on the number line where the larger quantity (more positive)) is ALWAYS to the RIGHT, or "higher", on the number line.



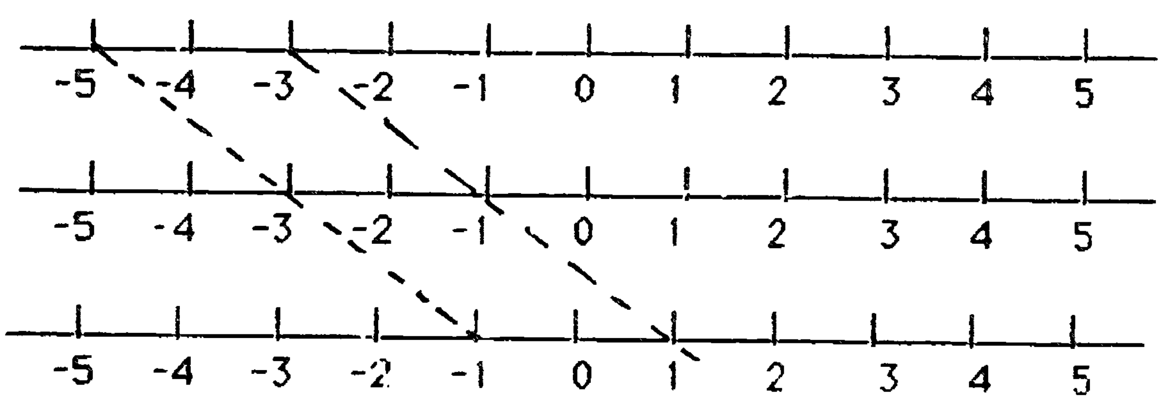
"What can we do with inequalities?" $3 < 5$

"Add 2 to both sides. Is it still the same direction?" $3 + 2 < 5 + 2 = 5 < 7$

"Is the difference between them still the same? Can you add the same number to this inequality?" $-5 < -3$?

Let's see: $-5 + 2 < -3 + 2 \longrightarrow -3 < -1$ "add 2 again: $-1 < 1$
"add 2 again: $1 < 3$

"We can add as much as we like and not change it. We just keep moving farther to the RIGHT on the number line."



"What do you think would happen if we subtracted 2 from each side of these inequalities?"

$3 < 5$
$-5 < -3$

"Subtracting 2 pushes the numbers toward more negative, while adding 2 pushes them toward more positive."

Activity: Assign pairs of students the worksheets to complete.

LESSON TWO

Introduction: "Remember how we make numbers more negative or positive." Place on the chalkboard or overhead:

MORE +	MORE -
ADD +	ADD -
SUBTRACT -	SUBTRACT +

"How do we interpret 4 x -5? This is ADDING negative 4 times so is even more negative."

+ x - = -

"How do we interpret -4 x -5? This is SUBTRACTING negative 4 times or making more positive."

- x - = +

Use overhead transparency number cards and give combinations to add, subtract and multiply. Examples:

+4 -3	Add +1 Subtract +4 - -3 = +7 -3 - +4 = -7 Multiply -12
-5 -2	Add -7 Subtract -5 - -2 = -3 -2 - -5 = +3 Multiply +10
-1 -1	Add -2 Subtract 0 Multiply +1

Activity: Have pairs of students complete the worksheets.

LESSON THREE: Number Sentences with Inequality

Introduction: Write the following on the chalkboard: □ + 2 < 7

"How do we isolate the □?" Write below the other: □ <+5

"What counting numbers can be used in the □?" (+1, +2, +3, +4)

"Can 0 be put in the □?" "How many negative numbers could be put in the□?"

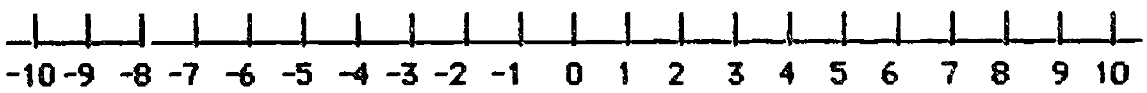
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

"The open circle at 5 shows 5 CANNOT GO into the $\square$!"

Write: $\square - 3 > {}^{+}4$

"How do we isolate the $\square$?" Write below the other: $\square > {}^{+}7$

Have a student come to mark on the number line all numbers that can go into the $\square$.



SUMMING THINGS UP

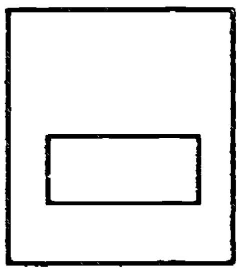
LESSON ONE

Introduction: Each child should have a paper square about 4" x 4". Lead the class in a paper folding and tearing activity as described. Use the overhead projector to display the paper parts as illustrated. Hold up the paper square.

"This is one. If I fold it in half and tear along the fold into two parts, what fraction will each part be?"

"Carefully fold yours in half as I do, crease it and tear it into two parts."

Demonstrate by folding and tearing the square. Place one of the pieces on the overhead projector and keep the other half in your hand.



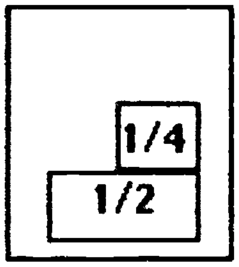
"Place one half on the desk as I have done on the overhead." Write on the board as shown: 1/2
1st fold

"This shows the part that is on the desk now." Hold up the half you kept.

"If I fold this into half and tear it into two parts, what fraction will each part be?"

Discuss "half of half - 1/2 x 1/2 = 1/4" if need be.

"Carefully fold, crease and tear your half in half as I do." Place one fourth on the overhead projector as shown. Keep the other fourth in your hand.

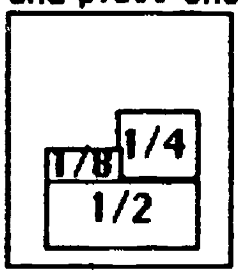


"Place one fourth next to the half as shown." Write: 1/2 + 1/4
1st fold 2nd fold

"This sum now shows how much of the paper square is on your desk and on the overhead." Hold up the fourth you kept.

"If I add half of this to the parts on the overhead, what fraction will I add?"

Discuss the idea of 1/8 being half of 1/4. Fold, crease and tear the fourth and place one part on the overhead as shown:



Write: 1/2 + 1/4 + 1/8
1st fold 2nd fold 3rd fold

At this point, stop and discuss 1/4 as 1/2 x 1/2. Show how this is written as $(1/2)^2$ where the exponent shows how many times 1/2 is used as a multiplier.

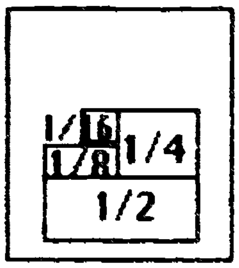
"How would I write 1/8 using 1/2 as a multiplier?" Write under the other material:

1/2 + 1/4 + 1/8
1st fold 2nd fold 3rd fold

$1/2 + (1/2)^2 + (1/2)^3$

Hold up the eighth you have: "What fractions result from folding this in half?"

"Carefully fold, crease and tear your eighth to get the 1/16th." Place on the overhead as shown:



Continue the written material as shown:

	$1/2$ +	$1/4$	+ $1/8$ +	$1/16$
Folds	1	2	3	4

$$1/2 + (1/2)^2 + (1/2)^3 + (1/2)^4$$

"Half of the sixteenth would give what fraction?" "The fraction pieces that are added each time, what is left is halved are recreating the paper fraction I started with, so each new fraction added gets me closer to ONE."

Write:

$1/2 + (1/2)^2 + (1/2)^3 + (1/2)^4 + (1/2)^5$ is close to one
$1/2 + (1/2)^2 + (1/2)^3 + (1/2)^4 + (1/2)^5 + (1/2)^6$ is even closer

"How close to ONE do you think folding and tearing in half will get me after 20 folds?" "Will there still be a fraction part left to halve again?" "Will I ever get ALL of the original paper into the square I am building?"

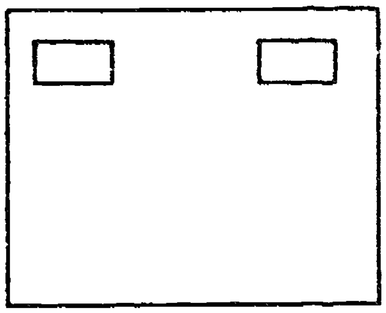
<u>LESSON TWO</u>

<u>Introduction:</u> As in Lesson One, each child should have a paper square. You lead the class activity as in that lesson, using the overhead projector.

"This is again to be ONE. This time I shall divide it into THREE equal parts by folding it and tearing out the three pieces."

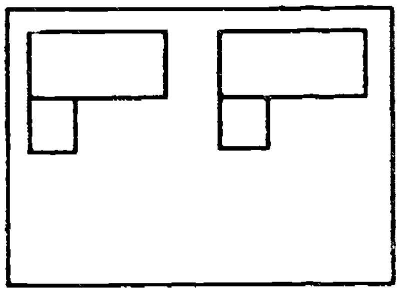
Carefully fold it, crease it and tear into three pieces. Show these to the class.

"Do the same with your square. If you keep one piece to fold and tear again, how many piles can you start with the other two pieces?" Place two of your pieces on the overhead projector as shown.



"What fraction of the original square is in each pile." (Third)

"Now I'll fold this third into three equal parts. What fraction will each part be?" (Ninth) Carefully fold, crease and tear these pieces and put two of them as shown:

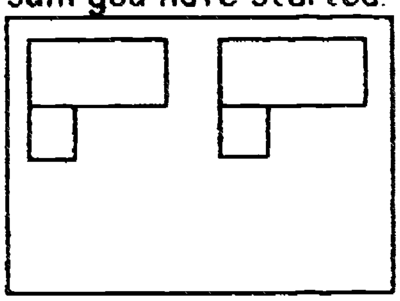


"Do the same with your paper." Write: 1/3 + 1/9

"This is the amount of paper in each of the two piles 'now'."

"Hold up the ninth you have - what will be the size of each piece when I fold this into three equal part?" (Twenty-seventh)

Fold, crease and tear it. Put two of the three as shown and add 1/27 to the sum you have started.



1/3 + 1/9 + 1/27

"If we continue taking one third of each piece of paper left and putting one third into each of these two arrangements, what fraction of the original square will be in each place?" (One Half)

"How much closer to HALF in each pile will I get if I fold and tear 10 times?" "No matter how many times I do this, will I always have a piece to tear into 3 parts?" "Can I ever get exactly HALF in each pile?"

LESSON THREE

The procedure is the same as for the earlier lessons.

"If I divide this square, which is ONE, into four equal parts, how many different pil.s can I start if I keep one fourth to divide further?" (Three)

"As I continue to keep one part to divide into four parts, what part of the square will begin to accumulate in each of these three piles?" (One Third)

"Let's look for a pattern in what we have done."

Write:

$1/2 + (1/2)^2 + (1/2)^3 + \dots \longrightarrow 1$

$1/3 + (1/3)^2 + (1/3)^3 + \dots \longrightarrow 1/2$

$1/4 + (1/4)^2 + (1/4)^3 + \dots \longrightarrow 1/3$

"How is the fraction these su..s are getting closer to related to the number of parts into which we divide the square?"

"Consider dividing the square into TEN equal parts. How many piles will we get with the pieces if we keep one to divide into ten again?"

"What fraction of the square is each pile getting closer to?"

"Let's add up the parts in each of these nine piles."

1) $1/10 + (1/10)^2 + (1/10)^3 + \dots$

2) $1/10 + 1/100 + 1/1000 + \dots$

3) .1 + .01 + .001 +

4) .1
.01
<u>.001</u>
.111 with 1's continuing

Discuss the sequence in going from 1 through 4 above. Point out the two ways to write one ninth:

1/9 .11111111...

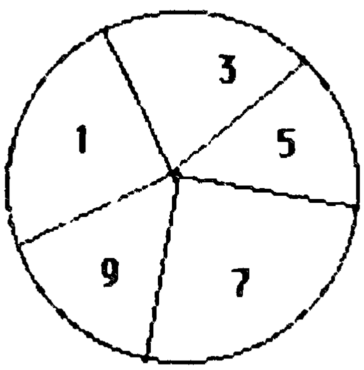
ERIC

The following parts of stamps have the costs shown. HOw much should a pair of green stamps cost?

G	W	G	R	W	R	G	G
Cost 20¢		Cost 11¢		Cost 15¢		Cost	

If you bought 6 rabbits for $9, and sold them for $2.00 each, how much money did you make?

John's change purse had a quarter, five nickels and some dimes. If he used the quarter, the nickels and some dimes to buy a book for $1.20, how many dimes did he use?



If you add numbers from the wheel three at a time, how many sums are possible?

Soda pop costs $2.49 for a six pack + a 10¢ deposit on each can. What is the total cost of 2 six packs?

What 5 odd numbers added together give 13? How many different ways can this be done?

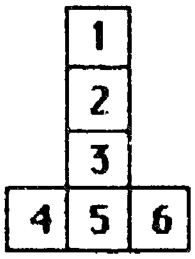
The school parking lot has 7 rows. 20 cars can park in each row. 19 spaces are empty. How many cars are in the lot?

9 students took a make up test. The average score was 85. 8 of the students' scores are given. What was the 9th student's score?
80,92,95,87,81,77,93,85

If pizzas are cut up into 8 equal parts, how many pizzas are needed to give 2 pieces to each of 32 people at a party?

Jane earned $24 babysitting. She put half in the bank and spent 1/3 of the rest on a lunch at the circus. How much money did she have left?

The track team went to the meet in cars and a bus. 4 people rode in each of 5 cars and 15 people rode the bus. They decided to return with 3 people in each car. How many rode the bus?



This was folded up to make a cube. What numeral is on the face opposite the "2"?

In decorating for the school bazaar, Bill worked 2 hours more than Tim. Tim worked 1 hour less than Joan, who worked 3 hours. Tess worked half as long as Bill. How long did Tess work?

John had one line 133 inches long, another 79 inches long. How much did he have to erase off the long line and add to the short line to make two equal lines?

What three digit number is twice the product of its digits?

Joyce bought 7 items at the store for $23. Cassettes cost $2, videotapes cost $3 and compact discs were $5. How many compact discs did she buy?

Frank spent $29.60 for a shirt and tie for his father for Christmas. The shirt cost $12 more than the tie. The tales tax was $2.40. What was the price of the tie?

In writing all of the odd numbers 1-49, how many times is the numeral (2) written?

In one day, Americans eat 8 million pounds of butter and 7 million pounds of margarine. How much of these spreads do Americans eat in a year?

In one day, cows produce 47 million gallons of milk. If Americans drink one pint each day on the average, this is enough milk for how many Americans?

One nice thing about the metric system is the fact that 1 litre of water weighs 1 kilogram. A 15 litre pail weighs .5 kilogram empty. What does it weight with 10 litres of water in it?

ERIC

The price of a gallon changes so fast, the station was using a pump that showed only half the price. What was the price charged when the pump showed 59.9¢?

The students in Mrs. Moore's class decided to see what they weighed in pairs. John and Bill together weighed 129 lbs. Bill and Tony together weighed 134 lbs. John and Tony together weighed 153 lbs. How much did each boy weigh?

Terry's father is paid 23¢/mile by the University for using his own car. He travels 1043 miles. How much must the University pay him?

Jim's mother planted 1/3 of the garden with marigolds. The remaining 16 square feet were in flowering shrubs. How many square feet are in the garden?

Jorge received * in Christmas gift. He planned to save the same amount each week for . weeks to buy a pair of $60 skates. How much did he save each week?

Kris and Lisa were looking at a field.
Kris: "One third of the field is 24 square feet."
Lisa: "No, one fourth of the first is 3/4 of that."
Kris: "We are both right."

How many square feet are in the field?

A motorcycle and a car pass each other going in opposite directions at 60 mph. How far apart will they be in 10 minutes?

Tow fractions are each one third of the way between 2/3 and 1/12. What are they?

Of the perfect square less than or equal to 100, some are o 'd and some even. What is the ratio of odd perfect squares to even perfect squares?

Four fifths of what number is the same as two thirds of 24?

Grace's father noted the tank showed "3/4FULL". The car's tank holds 14 gallons and the car averages 28 miles/gallon. Does he have enough gasoline to complete a 300 mile trip?

Peter bought pints of berries at 6 for $10, and sold them at $2.50 each. He made a total of $100 in profit. How many pints of berries did he sell?

On a particular day, the sun rose at 5:00 AM and set at 9:00 PM. John looked at the clock and said, "The number of hours of daylight left is one third the number of hours of daylight that have gone by." What time is it?

Janene decided to raise hens that lay eggs. In three weeks the red hen laid 6 more eggs than the white hen. She gathered a total of 36 eggs. How many did the white hen lay?

At O'Hare field a plane lands every 30 seconds. How many planes land in a 6 hour period?

The price tage on a suit that Chuck wanted to buy was $150. The salesman told him he could have it for 30% less. What would Chuck pay for this suit?

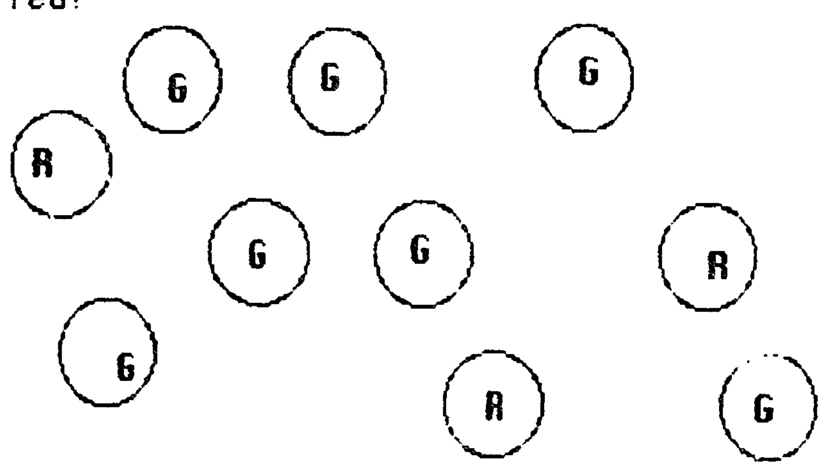
In Helen's row the test socres were:

Helen	75%
Jean	50%
Willie	80%
George	85%
Tom	90%

There were 20 questions on the test. Which student missed 5 questions?

Tom had some sticks measuring 10 inches, 12 inches and 16 inches. How could he use these to draw a 5 inch line? an 18 inch line?

Becky had some chips that were red on one side and green on the other. They were lying on the table as shown. How many did she turn over so half were red?



Janine has several one cent pieces, nickels and dimes. Show how many different ways she can offer coins to pay for a 29¢ purchase. Which way uses the fewest coins? The greatest number of coins?

1¢	5¢	10¢	Total
			29¢
			29¢
			29¢
			29¢
			29¢
			29¢
			29¢
			29¢

Jean and Tom each have some dimes and some nickels. Each has 12 coins. Jean has 5 dimes and Tom has 20¢ more than Jean. Show what coins each has:

	Dimes	Nickels
Tom		
Jean	5	

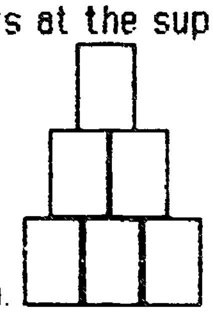
Bill drops a rock from the Split Rock Lighthouse. The table shows how far it falls:

Time	1 sec.	2 sec.	3 sec.	4 sec.	5 sec.
Distance in each second	32 feet	32 feet	32 feet		
Total distance at end of second	32 feet	64 feet	96 feet		

How far will it have fallen after 5 seconds? If the lighthouse is 192 feet high, how long will it take to hit the ground?

Put 4 tiles together in five different shapes. If each tile is 1 inch on an edge, which shapes has the least perimeter? Draw that shape.

Bob works at the supermarket. His boss told him to stack some coffee cans as shown.



He was given 45 cans to stack. How many rows were in his stack?

When the bantam hockey team stopped at MacDonalds, all twenty ordered fries, hamburgers, or both. Twelve ordered hamburgers. Eight ordered fries and hamburgers. How many ordered ONLY fries?

A book of stamps has twenty 2 ¸ stamps. How much does it cost?

Sam's father sent him to buy the family tickets for the barbershop concert. He paid $3.00 for adult tickets and $2.00 for children's tickets. He gave the ticket cashier $16. What are the possible arrangements of adults and children in Sam's family?

Denise's mother played 18 holes of golf with a score of 72. She took 4 fewer strokes of the back 9 holes than on the front 9 holes. What was her score on each of the 9 holes?

What length and width does a rectangle have that has area 48 and perimeter 28?

During the season, the Colts and the Blackhawks scored a total of 62 goals. The Blackhawks scored 8 more than the Colts. How many goals did the Colts score?

Susan bought 36 inches of frame to make a photograph frame. The picture is twice as long as it is wide. What are the dimensions of the picture frame?

Betsy worked the first 10 problems on a 25 problem test in 15 minutes. If she works the rest of the problems at the same rate, how long will it take her to finish the test?

Long distance companies charge the following rates:

A .40 for the 1st minute
.25 for each additional minute

B .35 for the first minute
.30 for each additional minute

The two companies would charge the same amount for a call of how many minutes?

John's brother received the following subscription proposal in the mail for a monthly magazine:

SAVE MONEY FOR A LONGER SUBSCRIPTION!
No money now - we'll bill you later!

☐ 9 mos. $12 ☐ 12 mos. $15 ☐ 18 mos. $23 ☐ 24 mos $29

Name:______________________________

Address:____________________________

City:_______________**State**________ **Zip**_______

Is this a new ☐ or renewal ☐ subscription?

A family uses 3 gallons of milk each week. How much is saved by buying half gallon containers at $1.99 instead of quarter containers at $1.09?

Finding Information Needed to Solve Problems

How long would it take a salmon to swim from Chicago to St. Sault Marie if it swims at 3 miles/hour? How long would it take a motorboat to travel this distance at 35 miles/hour?

How much closer to Minn apolis is Duluth than Chicago?

How many cats would weight the same as a Bengal Tiger?

What city in Minnesota is farthest from St. Paul?

How many telephone numbers are in the residential section of your telephone director?

How long is the flagpole on the school grounds?

What fraction of the people with telephones in your city have last names beginning with "H"?

On a recent bicycle trip, Frances left at 8:00 AM. She rode 60 miles at 20 miles/hour and rested for an hour. She completed the 140 mile trip at the same rate. At what time did she arrive at her destination?

A baseball player's "slugging" rating is based on Total Bases hit for. A single is one base, a double two, a triple three, and a home run four. Willie Brown hit 75 singles, 48 doubles, 20 triples and 52 home runs. How many total bases did Willie hit for?

Gloria gets an average of 4 of every 5 math problems correct on tests. This is what percent? How many should she get right on a 30 item test? on a 100 item test?

Pete has found that 2 large pizzas serve 10 people and 5 small pizzas will serve 10 people. How many large pizzas would Pete need for 50 people? How many small pizzas for 50 people?

In a recent survey, 6 out of 10 people said they liked Westerns. What percent of the people surveyed liked Westerns?

How many words are on the front page of (most popular local paper).

What is your favorite TV show? For how many hours is it on the air during a year?

Barbara's father estimated the total area of grass in the yard to be 3000 square yards. A bag of Wee-N-Feed will treat 800 square yards. How many bags should Barbara's father buy?

A can of cola holds 12 ounces. If you drink one can each day, how many gallons will you have drunk after a year?

On a trip to Yellowstone Park, Susan's father filled the tank four times. These tool 12.8 gallons, 13,3 gallons, 11.9 gallons, and 12.7 gallons. At an average price of 109.9¢ per gallon, how much did Susan's father pay for gasoline on the trip?

On the same trip, he paid the following restaurant meal charges - $12.95; $26.00; $31.25; and, $26.85. Did he spend more for food or gasoline? How much more?

When Roberta has read 240 pages, what percent of a 400 page book has she read?

What is the amount of sales tax on a $240 CR if the tax is 6%?

Sears recently marked down a set of prices. A washing machine that originally cost $429 was sold for $319. What was the percent of discount.

The label on a peanut butter jar showed it contained 10% oil. How many ounces of oil in a one pound jar of peanut butter?

The bank advertised one year "Certificates of Deposit" as paying 8.7%. How much interest should be paid at the end of the year for a $7500 "CD"?

The weatherman said the total snowfall was 140% of last year's snowfall, which was 84 inches. How many inches is this year's snowfall?

Pauline's test was scored at 80%. The test had 25 items. How many did Pauline have correct?

The sales price on a coat was $119. This was after a 30% discount. What was the original price of the coat?

On an 8 x 8 checkerboard, the largest rectangle is the whole board - 64 square units. The smallest is 1 x 1 - one square unit. How many rectangles that have different areas and dimensions are on the checkerboard? (A rectangle 8 x 2 and a square 4 x 4 are different because they have different dimensions, even though they have the same area.)

Don's brother had a hole dug for the basement of his house. The hole was in the shape of a rectangular solid with dimensions 6 ft. x 32 ft. x 48 ft. How many cubic YARDS of dirt were removed?

PROBLEM SOLVING - 2

Students at this level should have had considerable experience with the sixteen situations that give rise to addition and subtraction and the sixteen situations that give rise to multiplication and division. At this level they should successfully combine these operations in multistep problems and apply these operations in problems involving rates, interest, area, volume and in handling data. They should use the calculator to do the computation in real problems that involve real, not contrived, numbers.

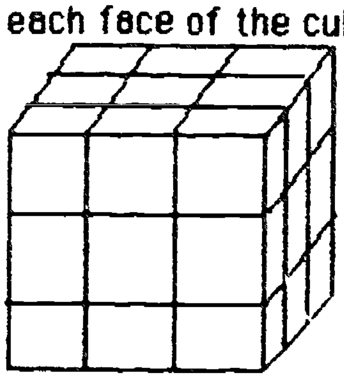
1. The population of Indianapolis is
On the weekend of the Indy 500, the population was
How many people who did not live in Indianapolis attend the Indy 500?

2. If you can buy 14 grapefruit for $2.00, how much should 35 grapefruit cost?

3. On a recent trip of 1250 miles, the family car averaged 24 mpg. The tank was full when they started and had to be refilled when it showed EMPTY after 336 miles. If they filled the tank each time it showed EMPTY, how many gallons were in the tank when they finished the trip> Assume their speed averaged over an hour was constant.

4. Two-thirds of Mr. Jones class are boys. If 4 boys transfer to the other 6th grade and 4 girls come into Mr. Jones's class, one half of the class will be boys. How many students are in Mr. Jones class?

5. Joyce has several picture cubes with her boy friend's picture on each face of the cube. She stacks these as shown:



How many more pictures are hidden than are visible on the outside of this larger cube?

6. A clock runs slow. It shows only 55 minutes have passed when an hour has really passed. If the clock is set at 12 noon, what is the real time when the clock shows 2:45 PM?

7. "On the first day of Christmas, my true love game to me - A partridge in a pear tree.

"On the third day of Christmas, my true love game to me -
3 French hens
2 turtle doves
A partridge in a pear tree

"On the fifth day of Christmas, my true love gave to me

5 golden rings
4 calling birds
3 French hens
2 turtle doves
A partridge in a pear tree

How many gifts did this person receive on the twelfth day of Christmas?"

8. A fence must be placed around a playground that is 60 feet by 90 feet. Posts are to be placed 10 feet apart. What will be the cost of fencing this playground on all but one short side if fencing costs $2.79/ft. and fence posts cost $3.49 each?

9. Piotr had $108 in his wallet. He had an equal number of ones, fives, tens and twenty dollar bills. How many tens did he have?

10. When she inquired about the cost of a phone call overseas, Joanne was told the cost was 50¢ for the first minute and 22¢ for each additional minute or part of a minute. She called her aunt and the bill was $2.92 How long did she talk?

11. A year ago, Denise's father planted 30 arbor vitae, 18 white spruce and 44 white pine trees on their lake property. The drought killed 8 of each kind. How many trees are still living?

12. For the school science fair, Patricia made a large insect collection. She had five beetles for every butterfly and three butterflies for every moth. She also had 6 other insects she could not identify. Her collection had 158 insects. How many of these were beetles?

13. Mr. Smith hired a boy to mow his lawn every six days over the 96 days of summer. he paid him $6 for each mowing. How much did having his lawn moved cost Mr. Smith?

14. When Paul went to the lake for the month of June, his grass was 1 1/2 inches long. The average growth was 1/4 inches each day. He left instructions to cut it when it got to be 2 1/2 inches long. How many times was it cut during the month of June?

15. Bob and Tim were playing marbles.
Tim: "If you give me 5 of yours, I'll have as many as you."
Bob: "Yes, but if you give me 5 of yours, I'll have twice as many as you." How many marbles does each boy have?

16. A square 10 inches on a side is cut into pieces and reformed into 2 unequal squares whose sides are in the ratio 4 : 3. How much greater is the total perimeter of the two squares than the perimeter of the original square?

17. What month of the year is 1989 months from May?

18. If wallpaper must be overlapped 1 inch, how many pieces of 12 inch wallpaper are needed to paper a wall that is 21 feet, 9 inches long?

19. Duluth is 56 miles from Eveleth. Tom starts from Duluth, goes halfway and realizes he doesn't have enough gas to get there. He goes back 16 miles to a gas station. How far is the gas station from Eveleth?

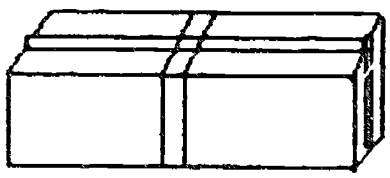
20. In a marathon which is 26 miles, water stations are placed every two miles. What part of the race has a runner run, when she gets water for the 5th time?

21. When the Minnesota lottery started, one store sold 756 tickets at $10.00. 79 persons won two dollars, 4 people won 5 dollars, 2 people won $20, and 1 person won $100. What profit did the state make on this sale of lottery tickets? this store?

22. Tom working alone can mow a lawn in 6 hours. If Marty helps him, it will take only 4 hours. How many hours would it take Marty to mow this lawn?

23. Ruth bought three books at the same price and received $15.53 change from a $20 bill. How much did each book cost?

24. A package is decorated as shown:



The package is 12 inches longer than it is wide. It is also 6 inches wider than it is high. If the ribbon needed to go around both ways as shown is 96 inches, what are the dimensions of the package?

25. 2 hamburgers and a Coke cost $4.27. 2 hamburgers and an order of fries cost $4.07. 2 hamburgers and 3 Cokes cost $6.56? What can you tell about the prices of these items?

26. Christmas trees come priced by the foot. If a 6 foot tree costs $18.60, how much should be charged for an 8 foot tree?

27. Holding the baby, Paul weighs 46 pounds more than Judy. When Judy holds the baby, she weighs as much as Paul. How much does the baby weigh?

28. Two-thirds of Mrs. Hill's class were boys. When twin girls joined the class, only three-fifths of the class were boys. How many boys are in Mrs. Hill's class? How many students are in Mrs. Hill's class?

29. On Jack's father's farm 8 hands can cut and bale 4 acres of hay in one day. Jack has 126 acres of hay. After working two weeks, 3 of the hands had to leave. How long do those left work to finish the job?

30. If you started with 1 penny and doubled your money each day, how long would it take you to get your first million? ($1,000,000)

31. Last winter, during January it snowed 3 inches on a Monday, 5 inches on Tuesday and a foot on Wednesday. It snowed again on Thursday. If the average snowfall/day was 9 inches, how many inches of snow fell on Thursday.

32. Paula and Francine stepped on the scale together It read 234 pounds. Paula knew that if Francine lost 30 pounds she would still weigh 10 pounds more than Paula. How much does each weigh?

33. Bernice can shovel the drive in 40 minutes. Betty can shovel it in 30 minutes. How long should it take them shovelling together?

34. Mrs. Thomas and her class and five of the other teachers went to a movie. Adult tickets cost $3.50 and the children paid $1.50 The total cost was $69.00. How many children are in Mrs. Thomas' class?

35. The small bag of popcorn holds 8 ounces and cost $.50. The larger box holds 20 ounces and costs $1.25. How much money do you save by buying the box if you expect to eat 20 ounces of popcorn?

36. The President of the United States is elected by the Electoral College. It has 538 votes. In 1976, Carter received 56 more electoral votes than Ford. How many electoral votes did each receive?

37. If a dozen eggs cost 90¢, how much would you pay for one egg?

38. After Tricks or Treat, Len had too much candy, so his mother made him give some away. He gave 1/4 to his neighbor friend, 1/2 of what was left to his brother and he ate the remaining 9 pieces. How many pieces of candy did he receive on his Tricks or Treat?

39. Mary was printing out the numerals from 1 to 1000. She tired after writing 630 digits. What is the last number she wrote?

40. At the end of a recent baseball season, Nolan Ryan had been pitching for 20 years and averaged 250 strikeouts per season. How many strikeouts did he have at the end of 20 years?

41. In the Arena, sections have as close to 22 rows of seats with 14 seats in each row as possible. The sections are numbered with letters starting with A. If the Arena holds 5680 seats, what is the letter assigned to the last section?

42. Last year, the Center City Titons played 162 games. They never lost more than 2 in a row and never won more than 5 games in a row. What is the most games they could have won in that season? Could have lost in that season?

LEVEL SIX

ESTIMATING AND CALCULATING

LESSON ONE

Introduction: Have the students make estimates of the following amounts:

1. pounds of food eaten in the past month
2. books read so far in your life
3. hours spent watching TV in a month
4. number of hamburgers eaten in the past year
5. number of cans of soda pop drunk in the last year
6. number times looked at a clock in the last month

Discuss how the estimates were made. Compile the results and use the data for the following assignments:

1. graph each set of data
2. find the mean and range of each set of data

LESSON TWO

Give the students the attached True-False test. Discuss the results. Have the students find the correct answers for those that are false. The answer key for the test is:

1. F	2. T	3. F	4. T	5. T
6. T	7. F	8. F	9. F	10. F
11. T	12. F	13. T	14. T	15. T

LESSON THREE

Have the class plan a picnic. Have them first estimate the cost/person and the resulting total cost. Have them plan the menu, estimate the total cost of each item for the entire class. Then have them find prices for the item quantities planned and find the real cost for the picnic.

Mathematician:______________________ ______________________

"I circled T for TRUE statements and F for FALSE statements. I GUESSED for those I wasn't sure of."

1. T F The average adult woman weighs 155 lbs.

2. T F Most 6th graders can jump at least 1 meter in the standing long jump.

3. T F Most people can lift 3 times their weight.

4. T F A motorcycle can travel faster than a motor boat.

5. T F Detroit is east of Chicago.

6. T F New York City is closer to Texas than to Oregon.

7. T F The average person can walk a mile in about 10 minutes.

8. T F The Soviet Union has a greater population than India.

9. T F The family car should be able to travel 1000 miles in 8 hours.

10. T F The average person will sleep about one million hours in a lifetime.

11. T F A million seconds is longer than a week.

12. T F If a triangle and a square have the same perimeter, the triangle has the greater area.

13. T F It costs ONE PERSON less to fly from New York City to Los Angeles than to drive from one to the other.

14. T F Montana is larger than Pennsylvania.

15. T F A liter contains more than a quart.

USING DATA

LESSON ONE

"Several students in Mrs. Hanson's class volunteered to count the cars that went past the school. They each watched for 15 minutes and recorded the results. The table shows what 30 children counted."

Students	Cars Counted
Tom	4
June	9
Willie	6
Frank	8
Ruth	11
Joanne	13
Bob	7
Jackie	5
Gloria	10
Tammy	9
Sammy	8
George	4
Joyce	11
Barbara	3
Grace	5

"Who counted the most cars?"

"Who counted the fewest?"

"What was the difference between the fewest counted and the most counted?"

"This is called the RANGE of the data."

"What was the number of cars counted most frequently?"

"This is called the MODE of the data."

"What do you think the average number of cars counted was?"

"Let's put the data into an arrangement where we can see it better. Remember how we arranged data in 'Guess My Rule'?"

"What was the greatest number of cars counted?"

Write: Joanne 13

"What was the next largest number?"

As students continue supplying these in order, write them:

Joanne	13
Ruth	11
Joyce	11
Gloria	10
Jane	9
Tammy	9
Frank	8
Sammy	8
Bob	7
Willie	6
Jackie	5
Grace	5
George	4
Tom	4
Barbara	3

"Now what do you think the average is?"

"What is the middle number?"

"The ARITHMETIC AVERAGE is the sum of the numbers divided by the number of numbers."

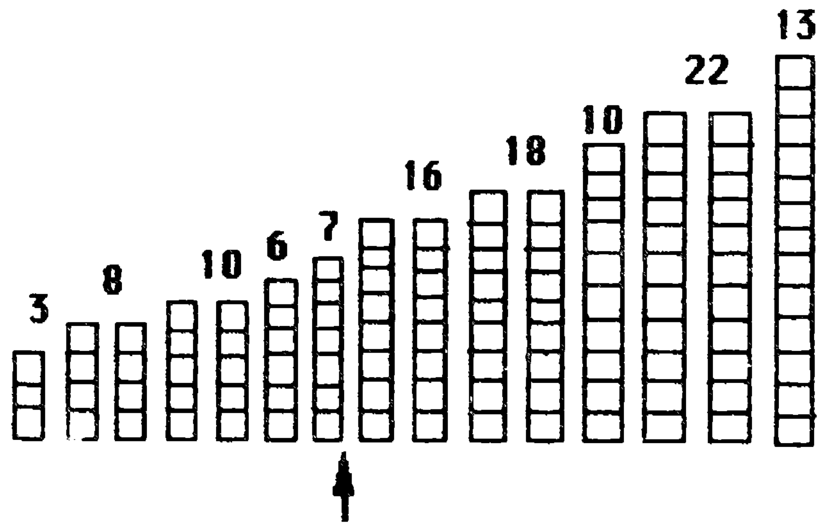
"Use your calculator to find the average of these 15 numbers." (7.533)

"Is this the number of cars anyone counted?"

"Could you count this number of cars?"

"Does this number or the middle number give you a better idea of the 'average' number of cars in a 15 minute period?"

Graph the data using UNIFIX cubes.



"The arrow points to the average you computed."

"Do you think there are more Unifix cubes to the left or right of this place?"

"Now look at HOW MANY numbers are to the right and left?"

"Obviously AVERAGE is an artificial number of some kind. It is the definition of the sum of numbers ÷ the number of numbers."

"The average of 1 & 3 is 2; 4 ÷ 2

The average of 1, 3 and 5 is 3; 9÷3."

"Notice it is two away from 3 as is 5."

"The average of 1, 3, 5, 7 and 9 is 5."

"Notice 3 & 7 are just as far from 5 and 1 and 9 are just as far from 5."

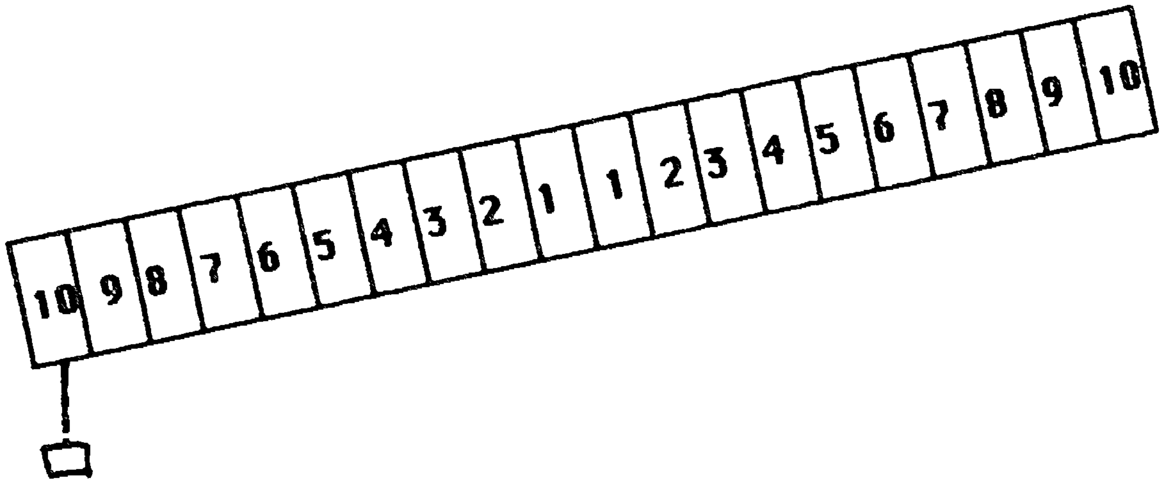
"Perhaps the distance that numbers are from the average is important."

"We'll see more about that later."

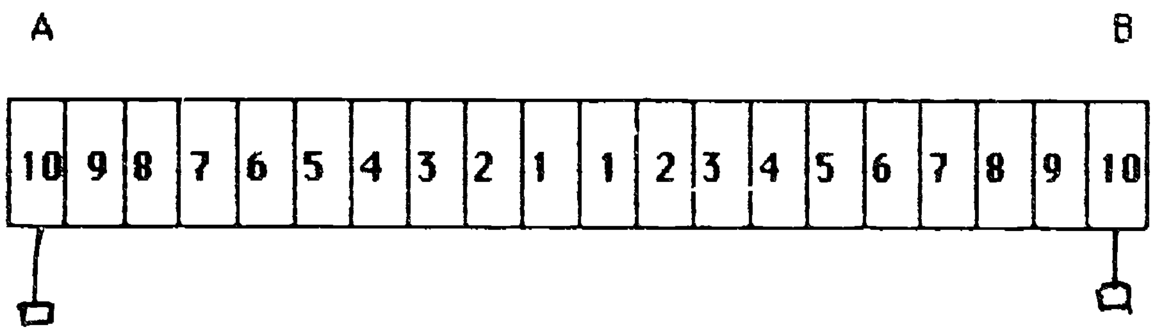
<u>Activity</u>: Assign pairs of students with calculators to complete the recording forms.

LESSON TWO

Introduction: Use beam balance that has places to hang weights or to insert weights.

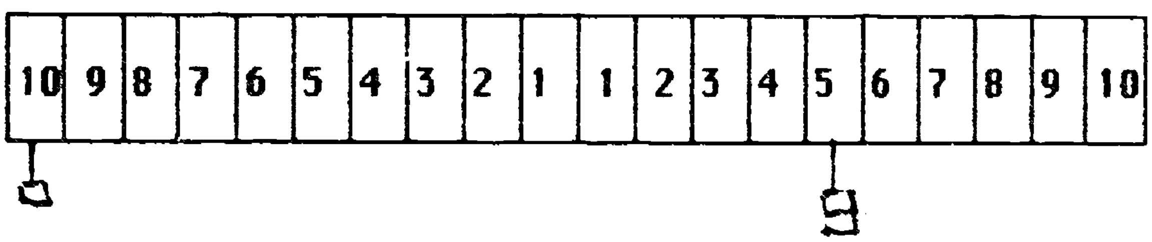


Place a weight as shown. "Where do I put a weight on the other side so this will balance?"



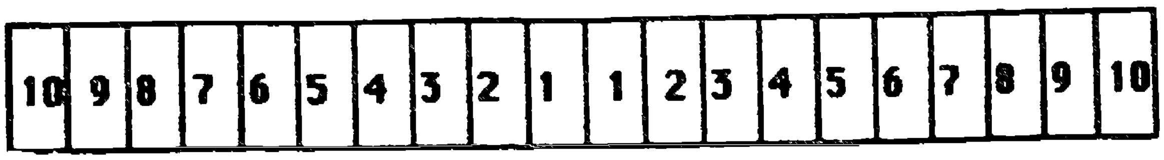
Take the weight from B. "I want to put TWO weights on that side. Where do I put them?"

Place as children suggest. If not at 5, follow up by asking why they think it doesn't balance? Eventually get to:



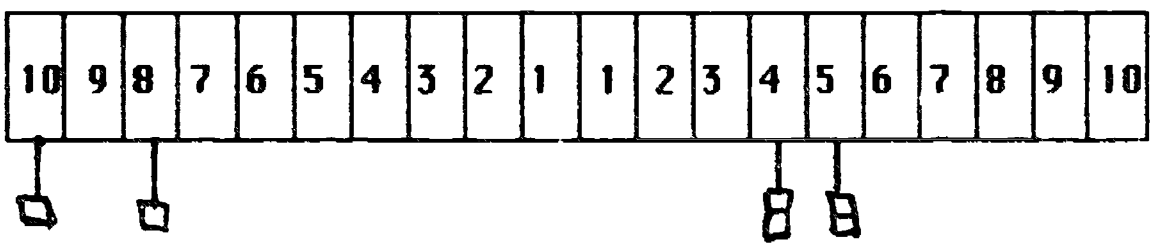
"How can TWO weights on one side balance ONE weight on the other?"

Discuss importance of DISTANCE the weight is from the center. Place a weight as shown:



"Why does the beam go down when there are TWO weights on each side?" Discuss.

"Where should I put TWO weights on the other side so the beam will balance again?" The eventual outcome of this interaction should be:



"Notice that we have on one side 10 x 1 + 8 x 1 = 18. On the other side we have 4 x 2 + 5 x 2 = 8 + 10 = 18."

"The turning caused by the weights depends on the distance they are from the center."

<u>Activity</u>: Have students work in pairs on the worksheet

LESSON THREE

Introduction: "Here is a set of test scores." Write the following on the chalk board or overhead:

Ronald	75
Sally	80
Bill	60
George	65
Grace	75

"What should we do with these first?" Put them in order:

Sally	80
Grace	75
Ronald	75
George	65
Bill	60

"What is the average of these scores?" (80 + 2(75) + 65 + 60 ÷ 5 = 71

"Consider a beam with 71 as center and the scores as weights on the beam. What is the range of scores?" (20 points)

"We'll make the beam 21 units long ranging from 60 to 80. It is 21 units so 60 and 80 can be included."

60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80

"Put the center at the average - 71."

60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80

"Each score is a weight."
"How far is 65 from 71?" (6)
"How far is 60 from 71?" (11)
"How far is 75 from 71?" (4)
"There are how many weights at 75?" (2)

"How far is 80 from 71?" (9)

"Now we check the weights x distance from center on each side."
(6 x 1 + 11 x 1 = 17   (4x2) + 9 = 8 + 9 = 17

"The turning effect of the weights is the same on one side as on the other side so it is a balanced beam."

"The average you find of the scores is the center of a beam of the length of the range of scores that is in balance."

Here's another set of numbers:

Alice	36 points
Joyce	18 points
Agnes	17 points
Terri	13 points
Janine	11 points

"What is the middle number?" (17)

"What is the average number of points?" (19)

"What is the range of points?" (25)

"We'll make a beam 25 units long."

11	12	13	14	15	16	17	18	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36

"Where does the center go?" (19)

"Where are the weights placed?"

11	12	13	14	15	16	17	18	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	[illegible]	36

ERIC

"Why do you think that is so?"

"Why is the average away from the middle number?"

Point out the turning effect of the 36 is 17 x 1 since there is just one 36. The turning effect in the opposite direction is:

$1 \times 1 + 2 \times 1 + 6 \times 1 + 8 \times 1 = 1 + 2 + 6 + 8 = 17$, so the beam balances.

"You have to be careful using the average because very big values or very small values affect it. Sometimes the middle number better describes the data.

Activity: Have pairs of students complete the worksheet.

LEVEL SIX

USING DATA 2

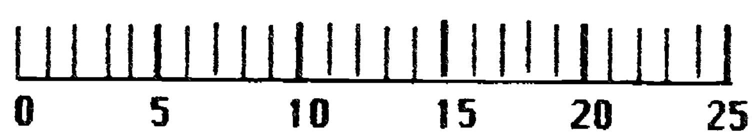
LESSON ONE: The Origin of the Mean

Introduction: Place the following data on the board:

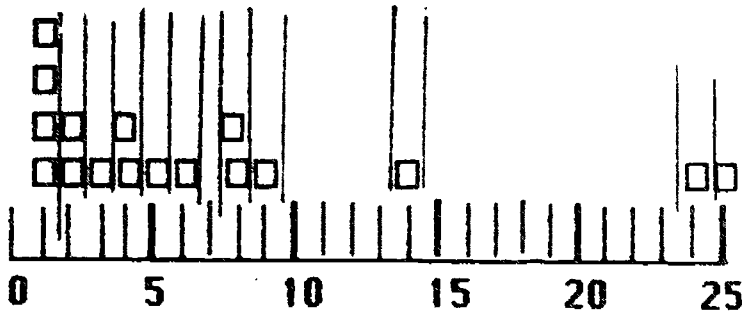
Austria	1
Canada	4
Czechoslovakia	6
Finland	13
France	3
E. Germany	24
W. Germany	4
Great Britain	1
Italy	2
Japan	1
Liechtenstein	2
Norway	9
Sweden	8
Switzerland	5
USSR	25
United States	8
Yugoslavia	1

"In a recent Winter Olympics, the following total medals were won by competitors from these countries."

"We make a line plot of these." Place the RANGE line on the overhead.



"We'll put each number in its proper lace on this line by going through the list. Austria won 1 medal so we put a □ on the line." Use cardboard squares on the overhead line. Continue on until the list is completed.



"Which numbers of medals are farthest away from the others?" (24 or 25)

"Where are numbers of medals grouped together?"

"Which number of medals did most country representatives win?" (1)

"The athletes from how many countries had ten or fewer medals?" (14)

"Where is the United States relative to the others?" (about in the middle or just above)

"What do you think is the 'average' number of medals won?"

"What do you think is the 'middle' number of medals won?"

Have the class made line plots for data collected from the class. Some suggested sources are:

grades on the last test
ages of fathers in the class
numbers born in each of the 12 months of the year.

LESSON TWO: Splitting Up the Range
Introduction: Consider this set of data:

Planet	Number of Moons
Mercury	0
Venus	0
Earth	1
Mars	2
Jupiter	16
Saturn	23
Uranus	15
Neptune	2
Pluto	1

"One way of looking at this data is to use the MEAN or average." Write

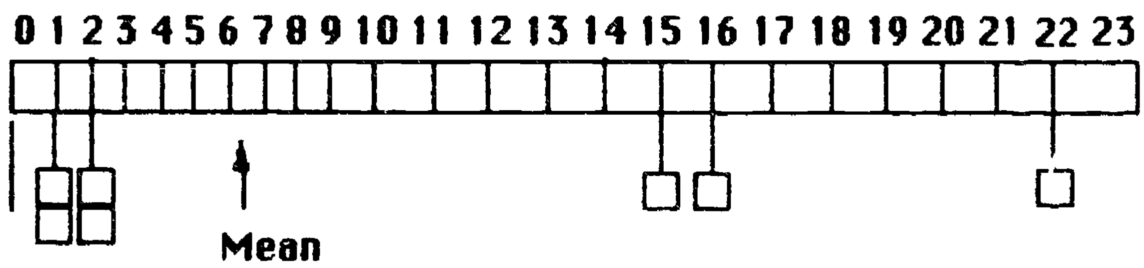
$$\text{average} = \frac{0+0+1+2+16+23+15+2+1}{9}$$

$$= 6.666 \text{ or } 6\ 2/3$$

"Does any planet have 6 2/3 moons?"

"Could a planet have 2/3 of a moon?"

"Remember the mean is an artificial kind of number. It need not be any of the numbers in the data set." On the board place:



"Remember that the mean is the center of turning and the turning effects of the weights balance each other.

Counter clockwise	Clockwise
(16 2/3 x 2)+(5 2/3 x 2) (4 2/3 x 2)	(8 1/3 x 1)+(9 1/2 x 1)+16 1/3 x 1)
13 1/3 + 11 1/3 + 9 1/3	8 1/3 + 9 1/3 + 16 1/3

"Let's look at this data a different way." Arrange as:

23
16
15
2
2
1
1
0
0

"What is the middle score?"

"In this case it is the 5th from the top and 5th from the bottom."

Write: $9 = 4 \times 2 + 1$

"An odd number of scores always has a middle score."

"Do you think 6 2/3 or 2 gives a better idea of the 'average' number of moons?"

"Notice on the table a kind of symmetry. The number declines closer toward the sun from Saturn and farther from the sun out from Saturn."

"If there were two planets beyond Pluto with no moons, the symmetry would be almost perfect."

<u>LESSON THREE: Stem and Leaf Plots</u>

Make a copy of the attached list of ages of Presidents at their deaths for each student.

"We're going to show this data a different way."

Did any Presidents die in their 30's?" (No)

"How many Presidents died in their 40's?" (2)

"What President was the oldest?" (90 - Adams)

Make a line as shown:

```
4|
5|
6|
7|
8|
9|
```

"These numbers are STEMS since they are the first digits of the ages. All ages are between the 40's and 90's."

"Now we put in the second digits on the other side of the line."

"What are the two ages in the 40's?"

ERIC

Ages of U. S. Presidents at Their Death

The table below lists the Presidents of the United States and the ages at which they died.

Washington	67	Filmore	74	Roosevelt	60
Adams	90	Pierce	64	Taft	72
Jefferson	83	Buchanan	77	Wilson	67
Madison	85	Lincoln	56	Harding	57
Monroe	73	Johnson	66	Coolidge	60
Adams	80	Grant	63	Hoover	90
Jackson	78	Hayes	70	Roosevelt	63
Van Buren	79	Garfield	49	Truman	88
Harrison	68	Arthur	57	Eisenhower	78
Tyler	71	Cleveland	71	Kennedy	46
Polk	53	Harrison	67	Johnson	64
Taylor	65	McKinley	58		

4	6 9
5	
6	
7	
8	
9	

Put in the 6 + 9.

"This shows two pieces of data - 46 and 49."

"What numbers should we put in for the digit 5?"

4	6 9
5	3 7 7 8
6	
7	
8	
9	

Have the students copy and complete this plot.

"Who was the youngest at death?"

"Who was the oldest at death?"

"Which four Presidents were assassinated?"

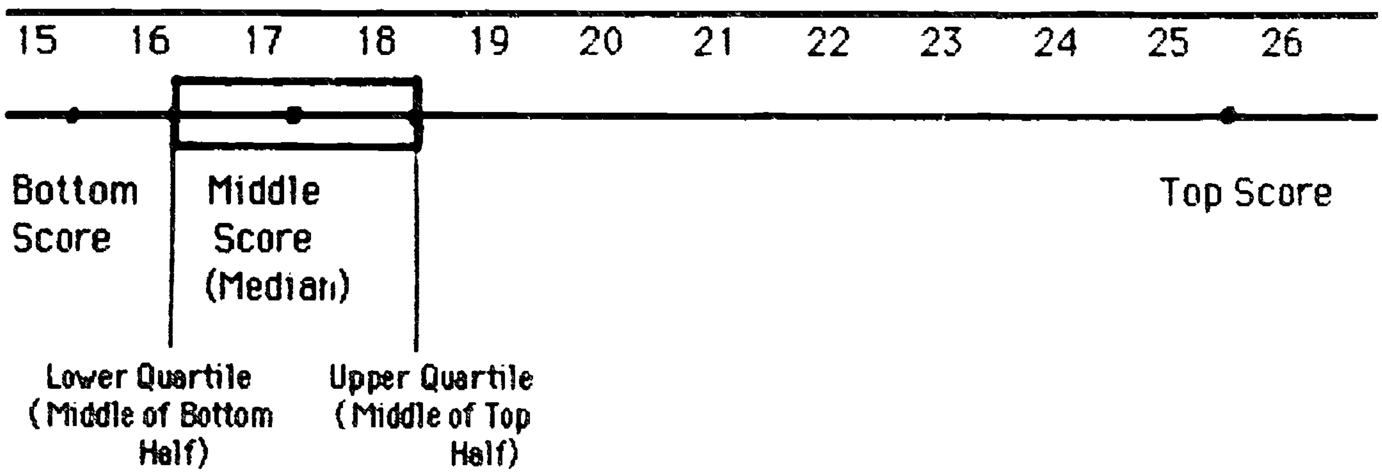
"Were any of these in their 60's or older?"

"Which of these died while in office?"

"If we had TWO sets of related data, how could we add to this plot to show both sets of data?"

<u>LESSON FOUR: Box Plots</u>

<u>Introduction:</u> Make an overhead transparency of the attached data and show it to the students.



"What percent of ratings are below median?"

"What percent of ratings are above median?"

"What percent of ratings are below lower quartile?"

What percent of ratings are above top quartile?"

What percent of ratings are in the box?"

(Go to next page)

TELEVISION RATINGS		
	Program	Rating
1.	The Cosby Show	25.5
2.	Family Ties	21.9
3.	Dallas	21.4
4.	Cheers	19.7
5.	Newhart	18.4
6.	Falcon Crest	18.3
7.	"Alfred Hitchcock Presents"	18.0
8.	60 Minutes	17.9
9.	Knots Landing	17.8
10.	A-Team	17.6
11.	Murder, She Wrote	17.6
12.	Night Court	17.6
13.	Highway to Heaven	17.0
14.	Facts of Life	16.8
15.	"Missing, Have you Seen This Person?"	16.5
16.	Kate & Allie	16.3
17.	Sara	16.3
18.	Who's the Boss?	15.9
19.	Trapper John, M>D>	15.7
20.	Love Boat	15.5
21.	Scarecrow & Mrs. King	15.4

Source: A. C. Nielson Company.

"Is there a middle in this list of data?" (17.6)

"What is the middle of the top half of the data?" (between 18.3 and 18.4)

"Can you estimate this?"

"What is the middle of the bottom half of the data?" (16.3)

"We'll show this on the RANGE LINE. Use the overhead to show:

TELEVISION RATINGS

	Program	Network	Ratings
1.	The Cosby Show	NBC	25.5
2.	Family Ties	NBC	21.9
3.	Dallas	CBS	21.4
4.	Cheers	NBC	19.7
5.	Newhart	CBS	18.4
6.	Falcon Crest	CBS	18.3
7.	"Alfred Hitchcock Presents"	NBC	18.0
8.	60 Minutes	CBS	17.9
9.	Knots Landing	CBS	17.8
10.	A-Team	NBC	17.6
11.	Murder, She Wrote	CBS	17.6
12.	Night Court	NBC	17.6
13.	Highway to Heaven	NBC	17.0
14.	Facts of Life	NBC	16.8
15.	"Missing, Have You Seen This Person?"	NBC	16.5
16.	Kate & Allie	CBS	16.3
17.	Sara	NBC	16.3
18.	Who's the Boss?	ABC	15.9
19.	Trapper John, M.D.	CBS	15.7
20.	Love Boat	ABC	15.5
21.	Scarecrow & Mrs. King	CBS	15.4
22.	"Miss Hollywood '85"	ABC	15.4
23.	"Lace II," Part 1	ABC	15.3
24.	Miami Vice	NBC	15.2
25.	Simon & Simon	CBS	15.2
26.	Riptide	NBC	15.2
27.	Cagney & Lacey	CBS	15.0
28.	"Adam"	NBC	14.9
29.	Crazy Like a Fox	CBS	14.6
30.	MacGruder and Loud	ABC	14.3
31.	20/20	ABC	14.3
32.	"Life's Embarassing Moments"	ABC	14.2
33.	Hill Street Blues	NBC	14.0

Source. A. C. Nielsen Company.

Have the students put the box plots for ABC, NBC and CBS on the same number line as you put the total data for all three networks.

Activity: Copy the attached table of Basic Interest Scales and give to each pair of students. Have them answer this set of questions about the table. The top box plot is for girls. The bottom one for boys. The * shows the score of a particular sixth grade girl.

"For which subjects is this girl's interest score in the top 25% of all girls?"

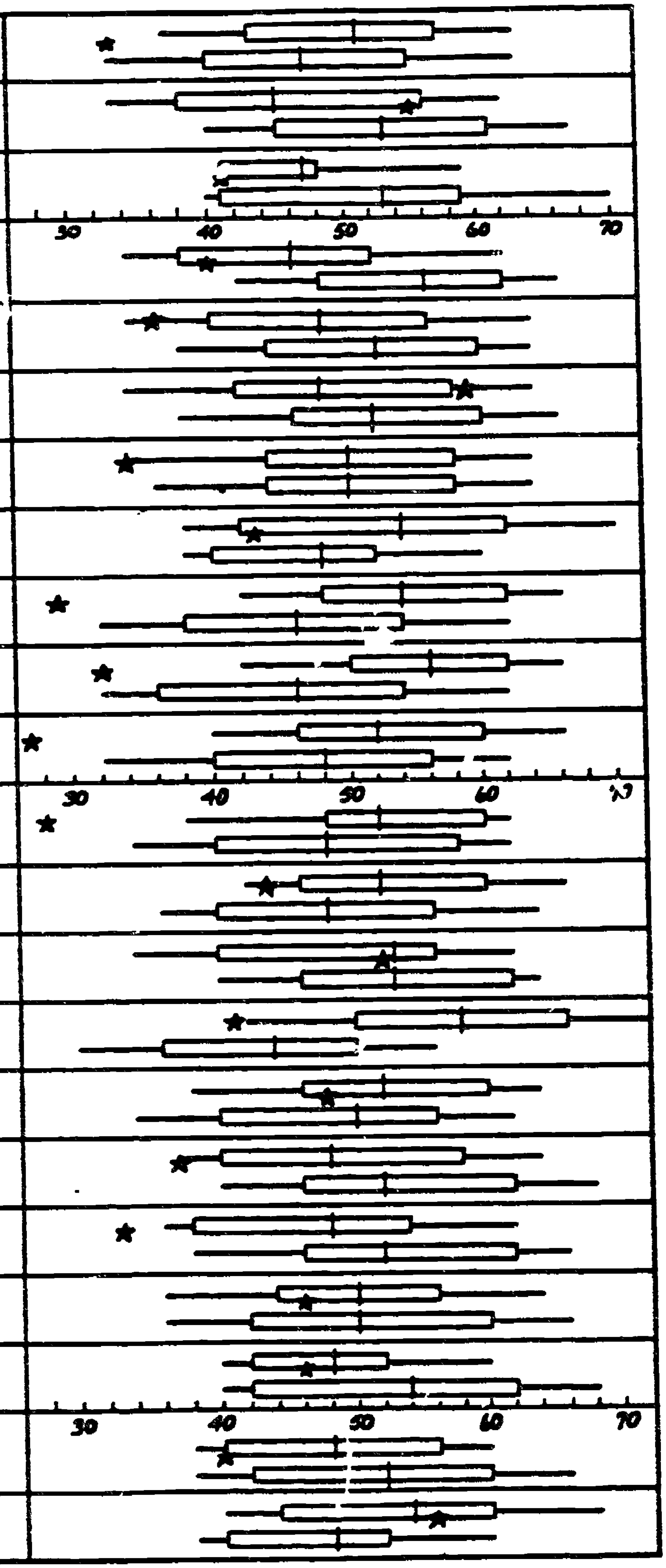
"For which subject is this girl's interest lowest?"

"For which subjects is there much more interest by girls than boys?"

"For which subjects is there much more interest by boys than girls?"

BASIC INTEREST SCALES

NATURE	33	V-LOW
ADVENTURE	55	MOD-H
MILITARY ACTIVITIES	41	V-LOW
MECHANICAL ACTIVITIES	40	AVER.
SCIENCE	36	LOW
MATHEMATICS	59	HIGH
MEDICAL SCIENCE	34	LOW
MEDICAL SERVICE	43	MOD-L
MUSIC/DRAMATICS	29	V-LOW
ART	32	V-LOW
WRITING	26	V-LOW
TEACHING	28	V-LOW
SOCIAL SERVICE	43	MOD-L
ATHLETICS	52	AVER.
DOMESTIC ARTS	41	LOW
RELIGIOUS ACTIVITIES	48	AVER.
PUBLIC SPEAKING	37	MOD-L
LAW/POLITICS	33	LOW
MERCHANDISING	45	AVER.
SALES	46	AVER.
BUSINESS MANAGEMENT	40	MOD-L
OFFICE PRACTICES	55	AVER.



LEVEL SIX

GRAPHING

Students were introduced to the construction of horizontal and vertical bar graphs and to the construction of circle graphs. At this level, briefly review these and assign the enclosed data sets for students to graph. Have them work in pairs.

ERIC

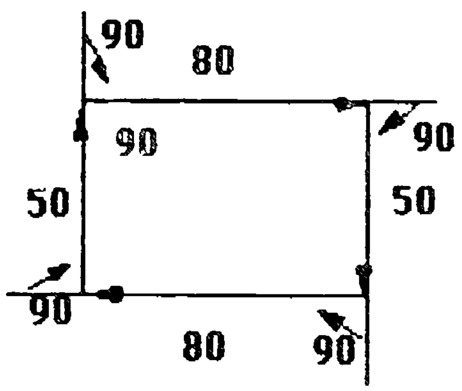
USING LOGO

Background: Students at this level should have had experience in creating programs, and packaging smaller programs into larger ones using the simplest commands. At this level they should learn to use RECURSION. RECURSION is a process in which the output of a program is fed back into that same program as an input. The result is the possibility of expanding shapes, repeating shapes, shrinking shapes, etc. However, when an output is fed back into a process as an input, this will continue indefinitely unless the process is stopped. The process can be stopped by setting the number of times it is run, or by setting a test so that when some variable value meets the test, the process stops. A few programs including a simple recursion will illustrate this process as well as the use o variables critical to using recursion.

Consider the procedure To Rectangle.

```
REPEAT 2[FD 50 RT 90 FD 80 RT 9]
END
```

This creates the rectangle below:



There are two dimensions to the rectangle that can be varied - the length and the width. Thus we can write a general procedure to create ANY rectangle with length and width we specify at the outset. We use variables to do this.

```
TO RECTANGLE : WIDTH . LENGTH
  REPEAT 2 [FD : WIDTH RT 90 FD : LENGTH FD 90]
    END
```

To make a rectangle 50 x 80 now we input: TO RECTANGLE 50 80

We can generalize further by using diffci ent variable names

```
TO RECTAN      :SIDE 1  :SIDE 2
 REPEAT [F      DE 1 RT 90  :SIDE 2 RT 90]
     END
```

Now we can make the rectangle longer horizontally or vertically.

To Rectangle 50 80 makes it horizontal.

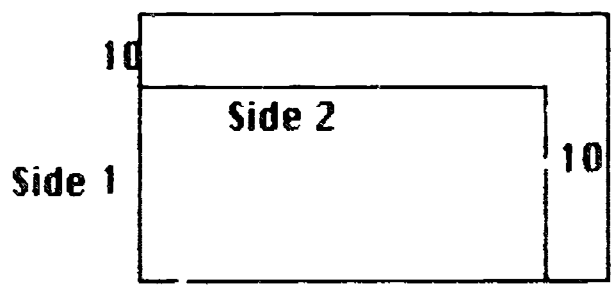
To rectangle 80 50 makes it vertical.

We can make a series of larger rectangles by increasing the length, width or both. We do this by repeating the creation of rectangles but with increased dimension(s) each time until we get as many as we want or as large as we want. Consider this procedure:

```
TO RECTANGLE :SIDE 1  :SIDE 2
     REPEAT 2[FD :SIDE 1 RT 90  :SIDE 2  :90]

     RECTANGLE  :SIDE 1 + 10  :SIDE 2 + 20
          END
```

The first run through the procedure gives a rectangle with inputed Side values - the next run through increased e.ch side by 10 and makes a rectangle.



This would go on until the capacity of the machine to handle numerical values is reached, or until you mechanically stop the procedure. You can control this recursion by putting in a test value.

There are several conditional (if-then) statements in LOGO. Usually the then is left out of the command. Consider the last procedure. We want to make expanding rectangles until one side reaches 90. The procedure is added to as follows:

```
TO RECTANGLE :SIDE 1  :SIDE 2
IF  :SIDE 1 = 90 [STOP]
REPEAT 2 [FD  :SIDE 1 RT 90  FD :SIDE 2 RT 90]
```

```
RECTANGLE :SIDE 1 + 10 :SIDE 2 + 20
     END
```

To use this conditional :SIDE 1 must be entered as a multiple of 10. Look at how this would work with input of 20 30

No. of Rectangles	:SIDE 1	:SIDE 2
1	20	30
2	30	50
3	40	70
4	50	90
5	60	110
6	70	130
7	80	150
8	90	170

:SIDE 1 is 90 after 8th rectangle made so when the 1st command of program is returned to, the conditional IF is satisfied and the STOP is executed.

Another way to stop a recursive process is after it has been done a given number of times. Say we wanted to make just five rectangles. We could add a 3rd variable to serve as a counter.

```
TO RECTANGLE :SIDE 1 :SIDE 2 :COUNTER
     IF:COUNTER = 0[STOP]
REPEAT 2[:SIDE 1 RT 90 :SIDE 2 RT 90]
RECTANGLE :SIDE 1 + 10 :SIDE 2 + 20 :COUNTER - 1
     END
```

Now let's work through input of

	20	30	5	
No. Rectangles		SIDE 1	SIDE 2	COUNTER
1		20	30	5
2		30	50	4
3		40	70	3
4		50	90	2
5		60	110	1
6		70	130	0

Now when RECTANGLE is attempted, the conditional is satisfied and the process stops. But we have 6, not 5, rectangles! Why? Look at the digits

0 1 2 3 4 5 - 6 of them. Since we set the counter to change on the 1st recursion, not the first time a rectangle is made by the REPEAT command, we get 6 rectangles, including 5 by recursion.

We could recognize this and input 4, or put the change in the :COUNTER variable elsewhere in the program. Where would it go?

The value of programming in LOGO is that students make mistakes, figure out how to correct them (debug) and improve their logical thinking.

A worksheet is provided to start students on RECURSION and Conditional stopping or counter stopping of the recursion. You should have LOGO books with problems in them so these assignments can be made. By this level students should pursue their own bent and interests in using LOGO.

TO: THE TEACHER

FROM: A. DEAN HENDRICKSON

Attached is something you can reproduce and send home to parents to encourage them to support what you are doing in the classroom.

MATHEMATICS IN THE HOME

A. Dean Hendrickson, University of Minnesota-Duluth

Introduction:

Mathematics and the use of mathematical thinking is much more than what has been traditional school arithmetic. The arithmetic of whole numbers, fractions and decimals constitutes no more than 10-15% of the mathematics we use throughout our lives. Much of the mathematical reasoning we use can be developed and experienced out of school, particularly in the home. Some of these suggestions may seem remote from the arithmetic you remember, but they will involve children in the THINKING essential to both the learning and use of mathematics in everyday life.

Pre-Mathematical Thinking:

Before a child can understand school mathematics, certain ways of thinking and skills must be available for use. These are continuously used throughout learning of mathematics, but particularly elementary school mathematics. These include: counting, comparing, ordering, using patterns, using grouped material, using language and establishing relations and relationships. Needed experience with these can be obtained around the home. Before describing things to do with children at home to help them with their school mathematics, here are some "golden rules" based upon research and experience with learning children.

1. You must not force children since this has negative effects, such as turning them away from doing things or from you. A child learns when ready, curious, and needing to make sense of something. This goes in spades for drill on memorizing so-called "basic facts."

2. Give children positive things to do when time is available, especially those things they can do and enjoy doing. Don't ask for things beyond the child's capacity to do.

3. Give lots of praise and encouragement. If what the child does or says doesn't seem to make sense to you, don't criticize or correct. Ask questions that might lead the child to consider it in a different way.

4. Don't look for day-to-day progress or change or for immediate results. Just as with many other things, such as walking or talking, a child may seem to be making no headway and then suddenly, it's all there. Children develop in spurts and unevenly, and have long plateaus where nothing seems to be happening. That's normal and accept it. There is probably a lot going on below the surface.

5. Don't compare yours with other children. Everyone is different - thank goodness!

6. Don't worry if a particular skill, such as using language, is

coming along more slowly than you'd like or than brother John's did. Somehow most of them seem quite a lot alike by the time they are 12 or so.

Words:

A number of words commonly used in mathematics and related to teaching mathematics should be used often outside of school as well. Some examples are *some-more, a lot, more than, less than, large, small, many, few, same as, different, alike, all, some, not, left, right, ahead of, behind, above, below, front, back, long, short.*

In addition to words associated with comparing, grouping and space, the number words are important. Children must know the counting words, but even more than that, they must see the pattern in the use of counting words. The cordial words like *first, second, third,* et. are also important. Use of these words around home helps children to count objects correctly and to identify position of things in ordered arrangements.

Comparing:

Have children compare things as to size, length, are and volume whenever possible. "Which glass has more?" "Which box holds more?" "which of these is heavier? heaviest?" "Put these sticks in order of length." "Arrange the silverware so the longest is farthest from the plate and the shortest is nearest the plate." Questions like these should be frequent. They should involve different kinds of things both indoors and outdoors. Combine these with questions that make the children estimate measurements of distance and height such as "Which do you think is as high as the shed, A or B?"

Comparing of quantity leads to better understanding of number and number relationships. "Are there more chairs or lamps in this room?" "Are there more cups or teaspoons on the table?" "Have we go more red roofs or green roofs on our street?" "Put enough table knives on the table so that there are as many knives as forks." "Do you have more boys or girls in your class?" These can be asked when out walking, riding in the car, watching TV or sitting in the boat. Ask children to do things that will make one group as large as another frequently All such activity helps children build number relations into their deeper understandings, instead of as memorized associations that have no meaning - like names and dates you once memorized to pass a history test!

Ordering things that can be counted is important. Bead stringing activities are good for young children. "String some beads so the third bead is red and the fourth bead is blue." "Make a string so every other one is green," etc.

Ordering things that have lengths, areas and volumes extends comparing beyond two things. Have children place three sticks of different lengths in order from shortest to longest; place three pieces of paper of different areas into orders; place three different sized cans of jars into order. Gradually extend the number of things to more than three for these activities.

Ask frequent questions about the ordering of events as to which happens first, second...last, etc. Connect these with time estimations, "How many minutes ago do you think this happened? How many days?" etc.

Counting:

Children should keep extending their memorized sequence of counting words. This is important. But being able to say the words in right order does not mean they can count things. They need much practice at this. Have them count everything around the house that is countable - the chairs, tables, legs on chairs; the tiles on the floor, in the ceiling; the number of windows in a room; the silverware in the drawer; the cans on the shelf; the pieces of wood in the woodpile; the telephone poles going by, etc. The more they count, the better able they are to count. When they are pretty good at counting forward, have them do some counting back. For example, start with 20 clothespins. One at at time put one into a can and count aloud those that are left as each one is removed from the pile.

Patterns:

Have children look for patterns - in the carpet, in the ceiling, in wallpaper, in the drapes, on the bedspreads. Patterns of shape, or color, or sound are all important. Beads can be strung in patterns. Collections of bottle caps, old keys, buttons, screws, nuts and bolts, and similar "junk" can be put into patterns. Ask children what would come next in a pattern, or what would go where something is missing in a pattern.

Number:

Help your child learn number size by having him see the same number, such as five, in many different arrangements and materials. Playing cards can be sorted into those all having the same number. Mixed groups of say, five marbles, three buttons, three keys, six spoons, can be used. "Find me the material there are five of," etc. Put some number, seven for example, of beads or marbles into three or four different shaped glass jars, "Find a jar with seven in it." "Find another." Put the same number of one kind of thing in one jar and another kind in a second jar, etc., and do the same kind of thing. Involve the child with numbers in as many different ways, with as many different kinds of material, and as many different sizes as possible. Gradually increase the number size as the child seems able to easily handle smaller numbers.

Using Numbers:

Comparing groups with number property; combining such groups; separating larger groups into smaller groups of a given size or into equal size groups - all of these activities help children to understand when each of the four arithmetic operations are used.

Some examples of things to do in the home of this kind are:

1. Compare two different sized groups in several ways. "How many more are there in this group than in that group?" "This group has how many fewer than that group?" "How many times as many are there here as there?" These kinds of questions used with groups of all kinds

of things - knives, forks, chairs, chair legs and table legs, buttons, marbles, pieces of candy, etc., help the child with what the school is doing.

2. Join together several groups of the same size into a larger group. Rows of pennies can be arranged into an array like this and can then be looked at a different way to see 5 groups of 6 pennies:

```
oooooo
oooooo
oooooo
oooooo
oooooo
```

Both lead to a total of 30 in the array. Do this in a row at a time, having the child tell you how many are there all together each time. Separate and take apart such arrays row by row and see what is left each time. Do this with different kinds of things, different size rows and different total numbers of things. Clothes pins, ceramic tiles, beans, corn are all good for this.

3. Join together groups of different size, such as seven things with five things. Have the child describe what is happening in words. Have the child add to one group of things enough to make it the same size as another larger group. Have the child make equal two unequal size groups without adding anything more to the collection. "Here are a group of 15 clothes pins and one of 7 clothes pins. Do something so you have two equal groups."

4. Give the child large amounts - in the 20's or 30's of things to:

   a) make several groups of a given size from. Some numbers should make these smaller groups an even number of times and some should have some left that is not enough to make another of the smaller group.

   b) make a certain number of groups that will all have just as many in them.

   Examples:

   "Put these 30 beans into 6 cups, so each cup has just as many. How many are in each cu?"

   "Put these 43 beans, six at a time into cups. How many cups did you use?" "What should e done with what is left over?" "When do you have some left over?" "When don't you have anything left over?"

When you go for walks, have the children compare, add together, etc., things along the way. Do the same in the car, the supermarket, in the drugstore. "How many are there on the top shelf?" "How many re on the bottom shelf?" "How many are there on the top and bottom shelves together?"

Have the child do as much adding, subtracting, multiplying and dividing of this kind - always as related to things - as you can. <u>DON'T</u> try to drill your child on "addition" facts or "multiplication" facts. Let the child learn these in due time through the school activities and those you do at home as described here. DON'T have the child write number things - the school will do this. Accept verbal answers and descriptions. Get in the habit of asking your child why certain answers are given and LISTEN.

SOME FINAL HINTS:

1. Have your children count things as much as possible.

2. Ask children simple addition, subtraction questions about REAL things in the surroundings to give practice in mental arithmetic.

3. Play card games that require mathematics or related things like WAR, OLD MAID, CRIBBAGE, RUMMY (regular or gin).

4. Give thinking games for holiday gifts - CONCENTRATION, HUSKER DU, etc.

5. Get a Little Professor or some similar calculator-based program to give mental arithmetic practice.

6. Cheap mathematics games can be bought at Target, Woolworths, etc. Some examples are COVER UP, HEADS UP, SCORE FOUR, TUF, APOLLO, etc.

7. Give your child a simple four function calculator and let him or her feel around with it.

8. Encourage block play and building, sand play, making birdhouses, etc.

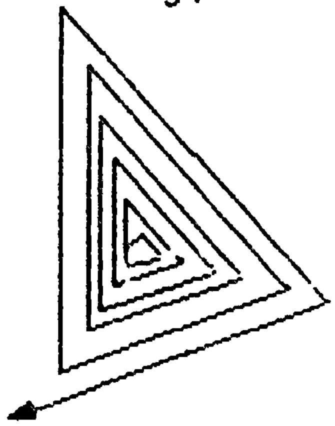
9. Key words are COMPARING, COUNTING, PATTERNS, COMBINING (groups), SEPARATING (large groups into smaller groups)

10. Point out mathematics wherever it is in the surroundings. Children must realize mathematics is:

    a. easy to learn
    b. useful
    c. fun

ERIC

This is my procedure to give:



BLACK LINE MASTERS

for many of the

WORKSHEETS

and

RECORDING FORMS

ERIC

Mathematician:____

"I first estimated the answer then worked the problem on the calculator."

MULTIPLICATION	ESTIMATE OF ANSWER	CALCULATOR COMPUTATION

11789.2

ERIC

Mathematician:____

"I did these multiplications as requested."

MULTIPLICATION	EXPANDED FORM	PICTURE OF PRODUCTS

11789.1

Mathematician:________________________________

"I estimated the answer, then did the decimal division on the calculator."

DIVISION	ESTIMATE	CALCULATOR RESULT

1.25.89.2

Mathematician: ______________________________

"I did these multiplications as requested."

FRACTIONS TO MULTIPLY	RESULT IN LOWEST TERMS	PICTURE

12489 1

Mathematician: ______________________________

"I rewrote these fraction multiplications to make it easier to reduce them."

FRACTION PRODUCT	CHANGED AND REDUCED
Example: 3/5 x 1/3	1/5 x 3/3 = 1/5 x 1 = 1/5

6/13/89.2

ERIC

Mathematician: ______________________________

"I did these divisions as indicated and showed the division with a picture."

DIVISION	IN EXPANDED FORM	PICTURE

12489 2

ERIC

Mathematician:_______________________________

DIVISION	PICTURE	SAME DENOMINATOR FORM	ANSWER

1.25.89.1

ERIC

Mathematician:________________________________

"For each pair of fractions given, I decided which was larger, joined them, found the difference between and divided one by the other both ways."

FRACTIONS	SAME DENOMINATORS	COMPARED	JOINED	THE DIFFERENCE BETWEEN	DIVIDED

Mathematician: ______________________________

"I changed these numbers into SCIENTIFIC NOTATION (written with powers of ten)

NUMBER	IN SCIENTIFIC NOTATION	NUMBER	IN SCIENTIFIC NOTATION

6.12.89.7

ERIC

Mathematician: ____________________

"I multiplied and divided these numbers by adding exponents to multiply and subtracting exponents to divide."

NUMBERS	PRODUCT OR QUOTIENT

ERIC

Mathematician:____

"I changed these numbers to SCIENTIFIC NOTATION and multiplied or divided by adding or subtracting exponents of ten."

NUMBERS	IN SCIENTIFIC NOTATION	RESULT

6.12.99.2

ERIC

Mathematician:______________________________

"I estimated the answer by rounding and found the answer using a calculator."

Mathematician: ______________________________

"I found what gives in the ☐ in each case, doing one step at a time."

OPEN SENTENCE	PICTURE	STEPS TO FIND ☐

2.22.99.3

ERIC

Mathematician: ________________________________

"I added, and subtracted numbers or multiplied or divided by numbers to find ONE □."

OPEN SENTENCE	WHAT I DID TO BOTH SIDES	NUMBER IN □

ERIC

Mathematician:______________________________________

"For each problem involving distance and time, I made a table and found the missing amount from the table or from a proportion."

PROBLEM	TABLE	PROPORTION

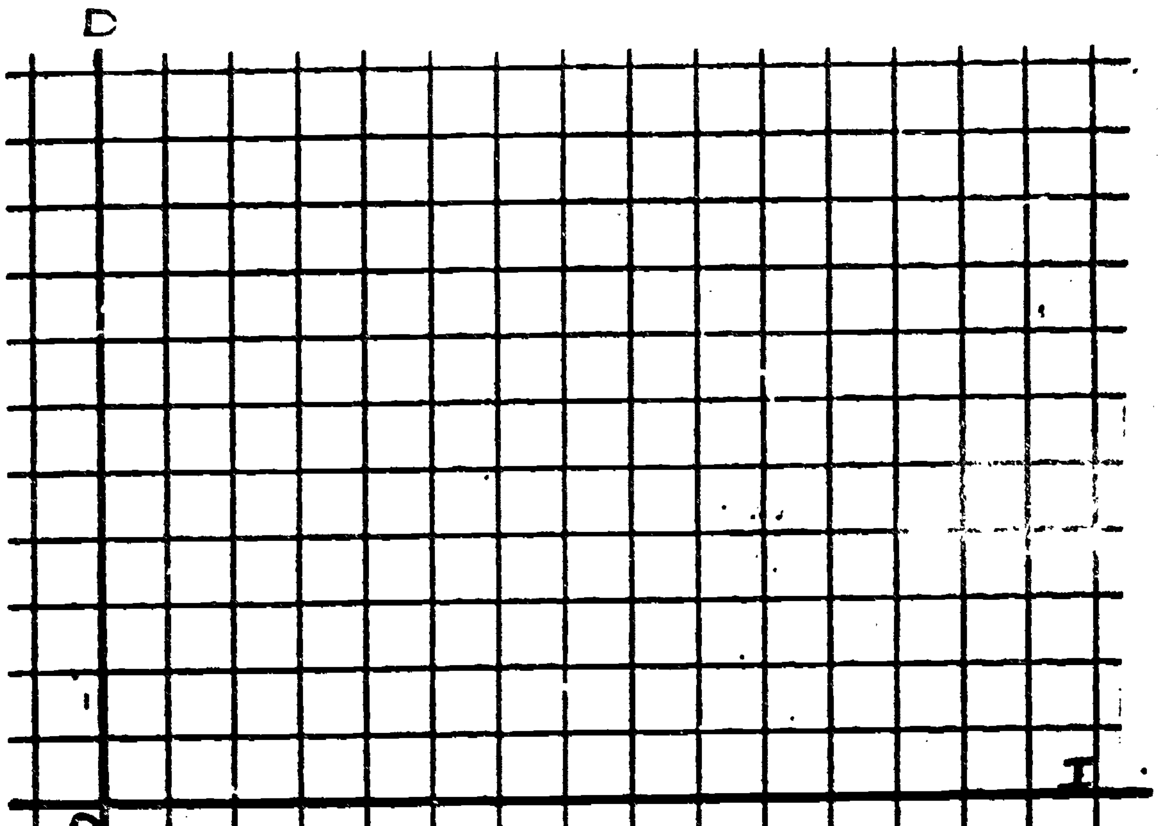
Mathematician:______________________________

"I completed each table of values for D and I, found the constant change in D and the rule and graphed this."

TABLE

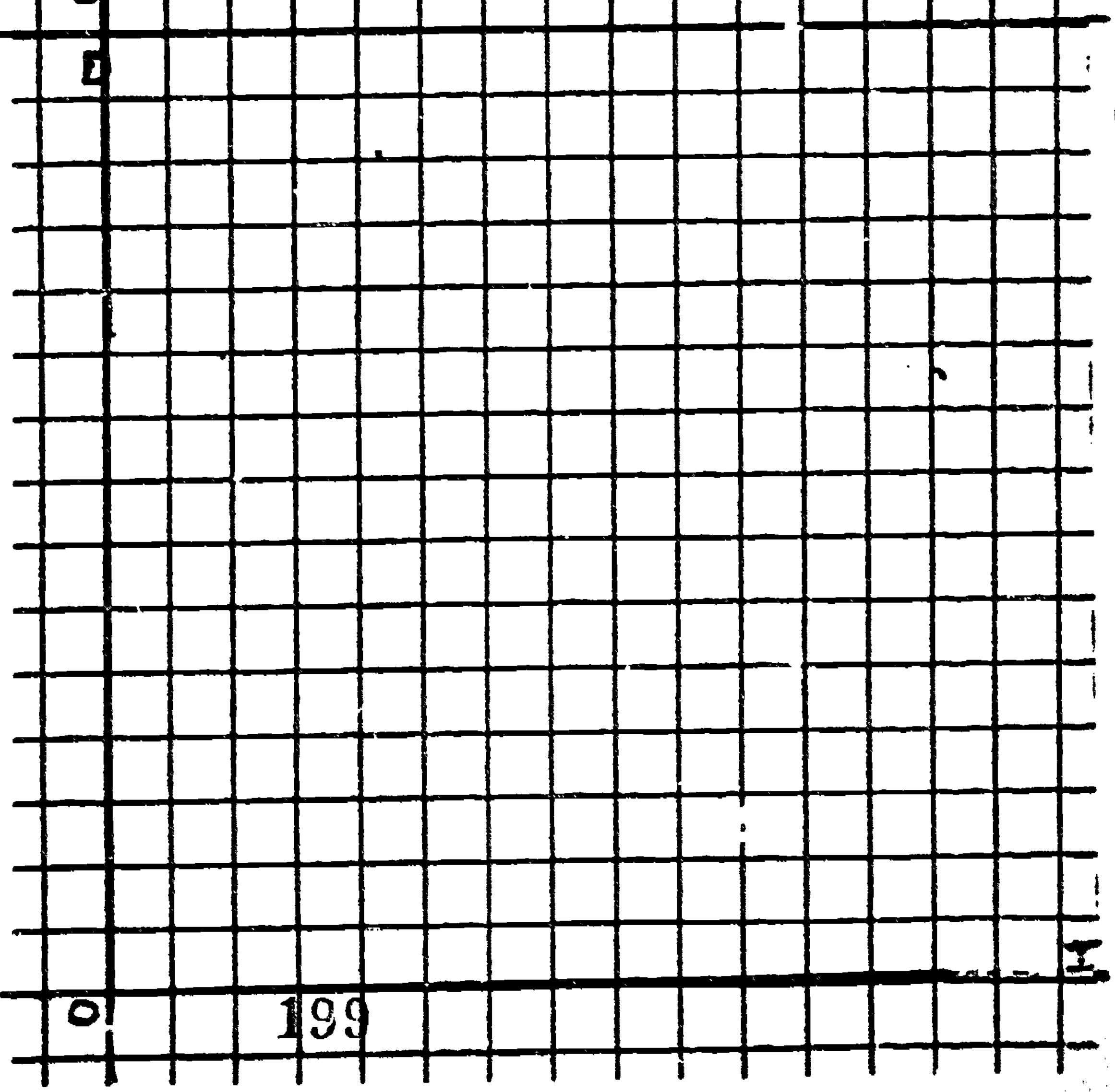
I	D	Change in D

GRAPH



Rule:

I	D	Change in D



Rule:

Mathematician: ______________________________

"I found totals using the rates given. I cancelled units to get the correct units for the totals."

RATE **TABLE OF TOTALS**

Units

	number of things							
	Total							

Units

	number of things							
	Total							

Units

	number of things							
	Total							

Units

	number of things							
	Total							

Units

	number of things							
	Total							

Units

	number of things							
	Total							

Mathematician:______________________________

"I wrote the rule for this set of shapes."

THESE SHAPES FOLLOW THE RULE

THESE SHAPES DO NOT FOLLOW THE RULE

ERIC

Mathematician:______________________________

"I wrote the equality or inequality shown by the split board with chips."

SPLIT BOARD	NUMBER STATION

6.12.89.11

Mathematician: ______________________________

"I added or subtracted the number as shown to get a new equation or inequality."

NUMBER SENTENCE	HOW TO CHANGE	NEW SENTENCES

ERIC

Mathematician:____

"I completed these number sentences with signed numbers."

ERIC

4.5.89.2

Mathematician:____

"I used the graph paper transparency to find the areas of these rectangles and other parallelograms. I wrote the area on the shape."

Mathematician:____

"For the circles given I found the area as best I could and calculated the radius. put these all in the table."

CIRCLES	AREA	RADIUS	RADIUS^2

Mathematician: ______________________________

"Given the dimensions of the base and the height of solids, I found the volumes."

GIVEN DIMENSION	PICTURE OF SOLID	VOLUME

ERIC

Mathematician:________________________________

"I drew pictures of these squares and wrote out all products."

SQUARE SIDE	PICTURE	PRODUCTS

Mathematician:____

"I found the differences between the given squares and drew the picture."

SQUARE SIDES	PICTURE	DIFFERENCES

ERIC

Mathematician:________________________________

"I shrank each square by the amount given to make a new square, drew a picture and wrote out all of the parts in a sentence."

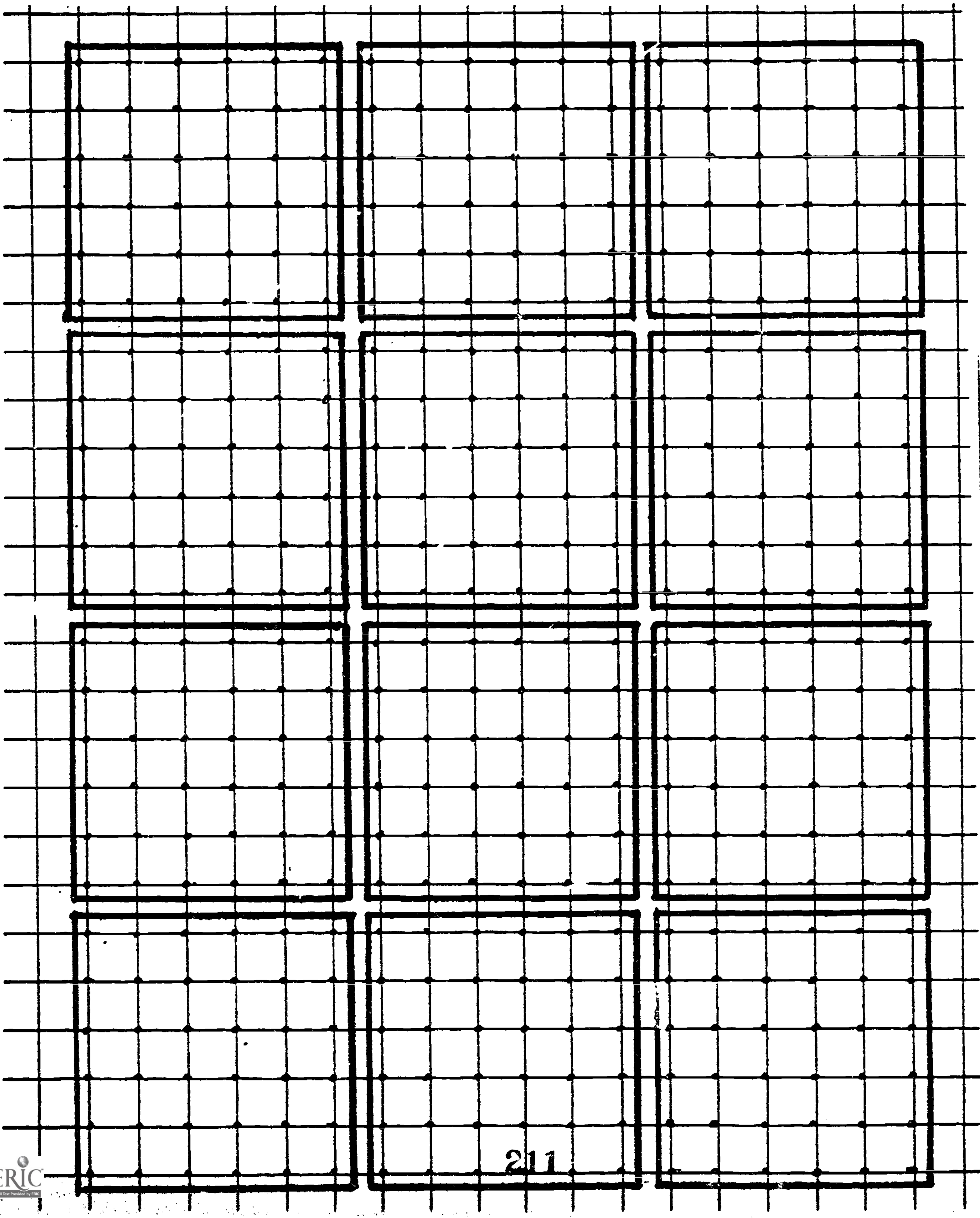
SIDE OF THE GIVEN SQUARE	SHRINK THE SIDE BY	PICTURE	SENTENCE

6.12.89.14

ERIC

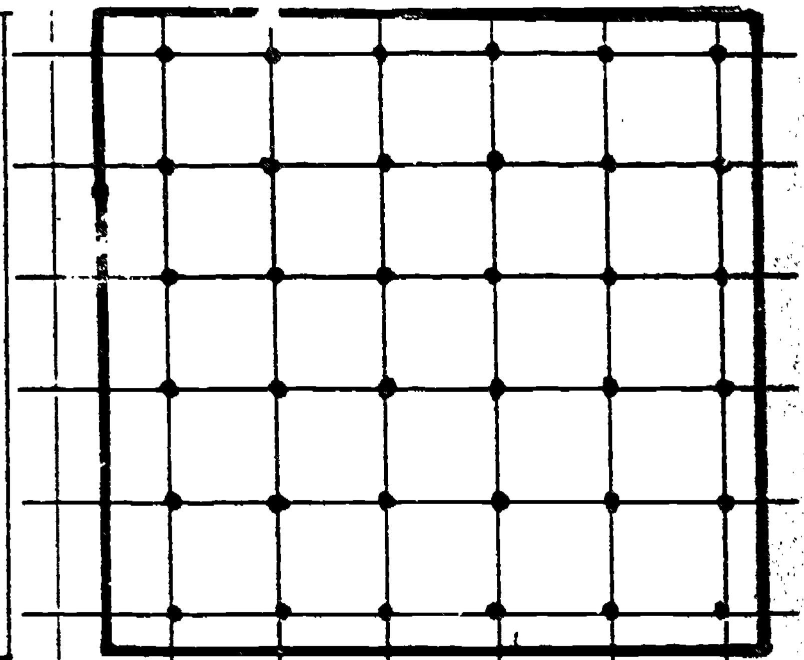
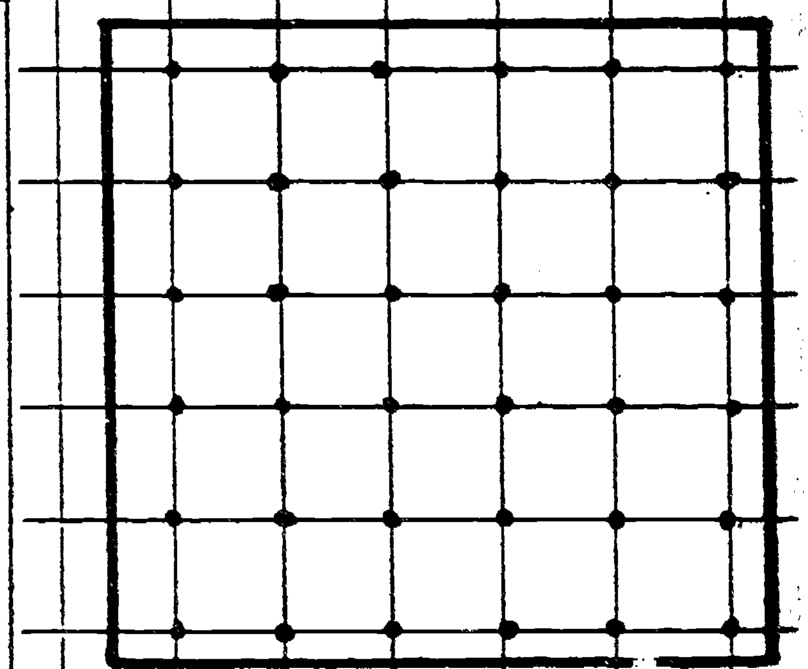
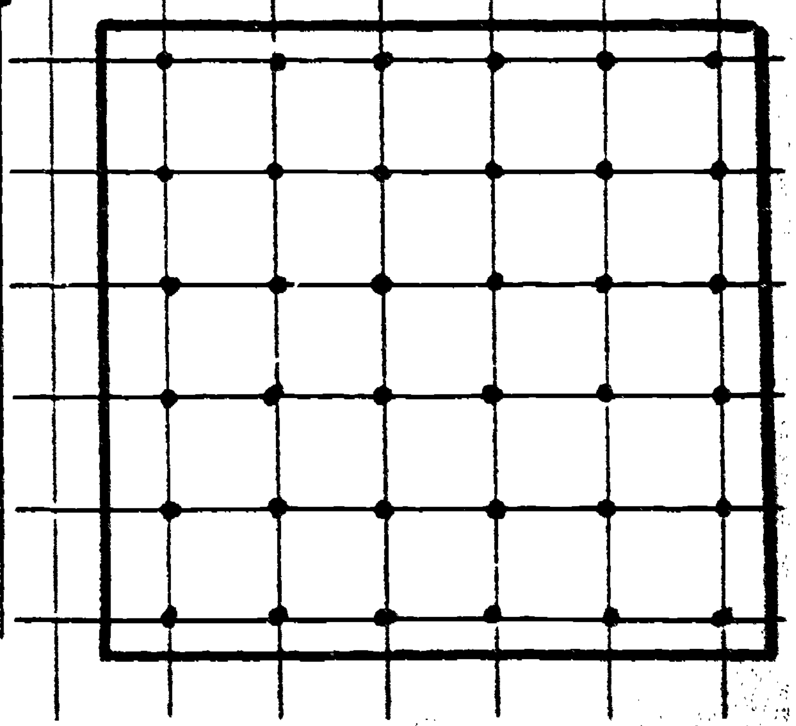
Mathematician:______________________________

"This is my record of tasks done on the geoboard."



Mathematician:________________________________

"I made the shapes with the areas given and recorded this on the geoboard picture."

SHAPES	AREA	GEOBOARD
		
		
		

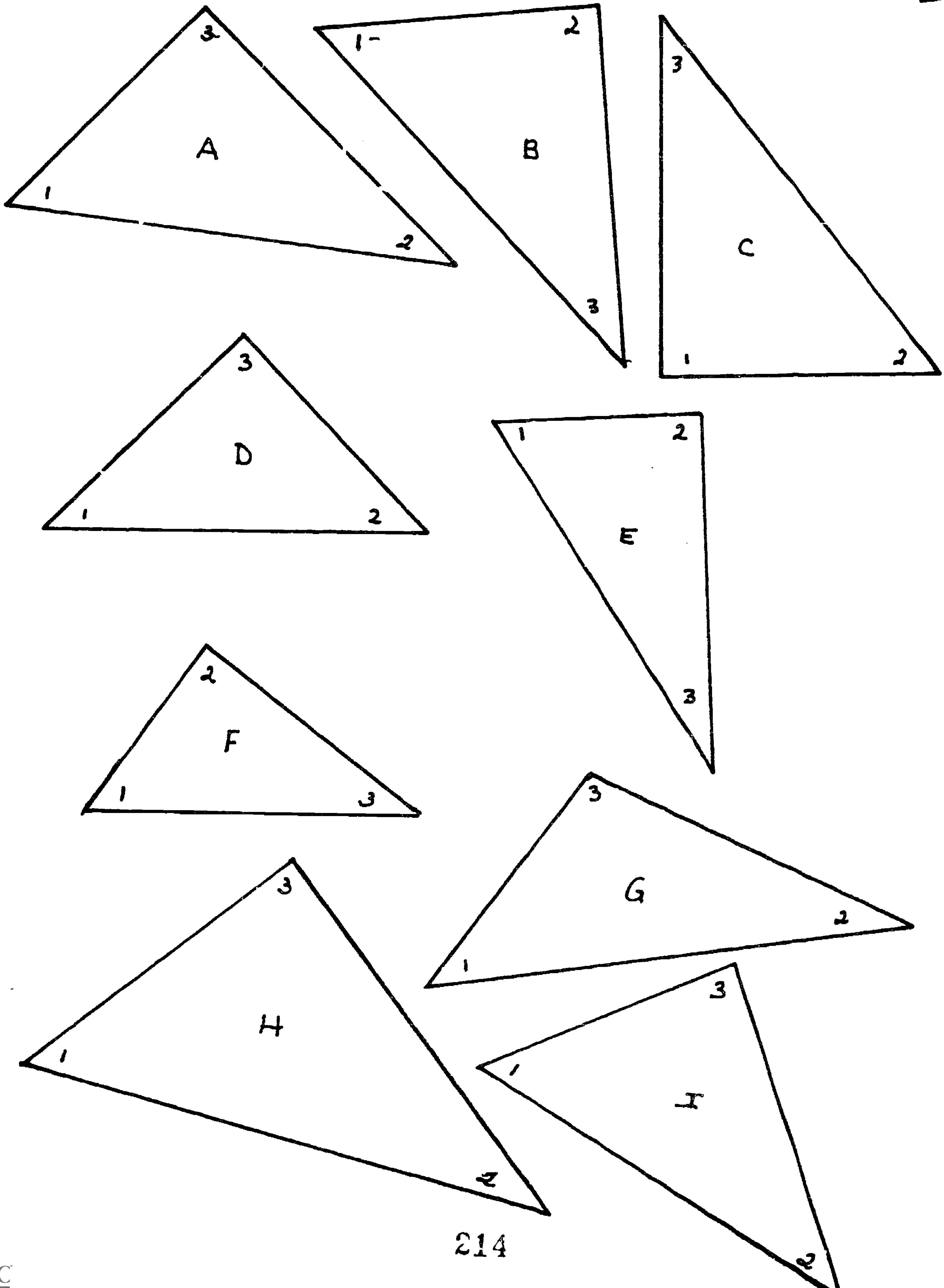
ERIC

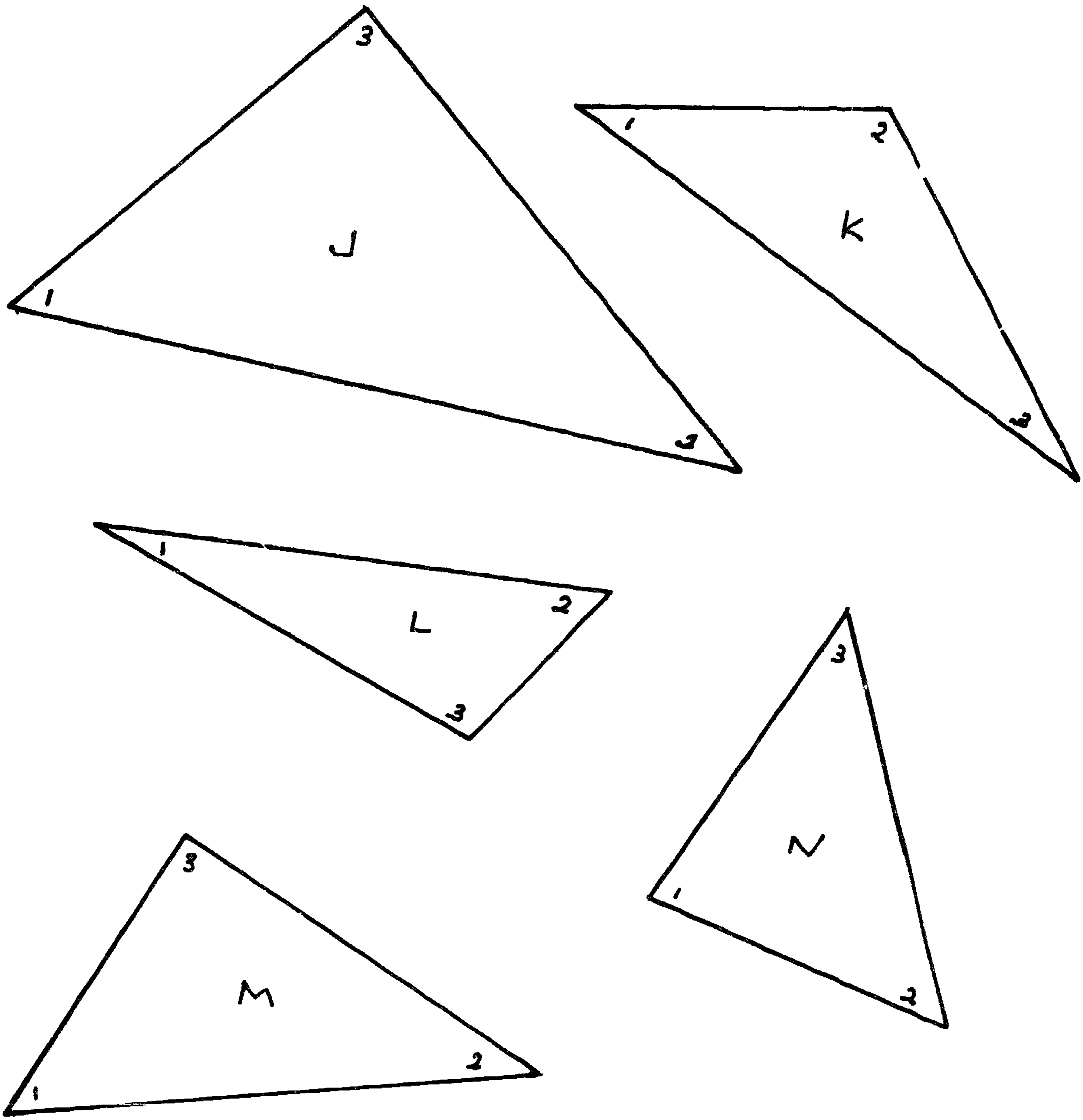
Mathematician: ____________________

"I completed these figures so the new shape has the indicated line of symmetry."

ERIC

I




J
K
L
M
N

Mathematician: ______________________________

"We recorded the letters of every pair of similar triangles on sheets I & II and wrote the reason - all angles equal or sides proportional."

PAIRS OF SIMILAR TRIANGLES	REASONS

Mathematician:____

"I found the corresponding parts for the congruent triangles given and wrote them in the columns."

TRIANGLES	CORRESPONDING SIDES	CORRESPONDING ANGLES

Mathematician:____

"For each series of 9 numbers given, I found the number to go into the middle square and the sum for the rows, columns, and diagonals of the MAGIC SQUARE."

Number series

"Middle" number

Row, Column, Diagonal

Sum for the Square

THE MAGIC SQUARES

continued

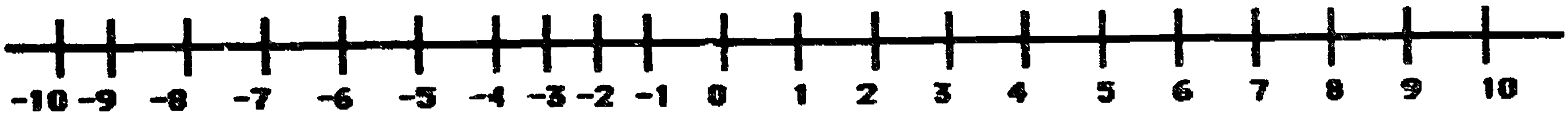
Predicted Color				Actual Color		
Red	Green	Yellow		Red	Green	Yellow
			TOTALS			

ALL CLASS TABLE

GROUP	⚀	⚁	⚂	⚃	⚄	⚅	TOSSES
1							60
2							60
3							60
4							60
5							60
6							60
7							60
8							60
9							60
10							60
TOTALS							

Mathematician:____

"I used the number line to add to and subtract from the given inequalities."



INEQUALITY	TO ADD OR SUBTRACT	RESULT

4.5.89.1

ERIC

Mathematician: ______________________ ______

"I chose the given number of statements, recorded the allowed number of moves and the results of the game."

CHIPS AVAILABLE

RED	
GREEN	
YELLOW	

RULE: ______________________

____

______ Moves to win

______ Number of chips chosen

	RED	GREEN	YELLOW	Statement(s) Chosen	Allowed to Move? YES	NO
1st choice						
2nd choice						
3rd choice						
4th choice						
5th choice						
6th choice						
7th choice						
8th choice						
9th choice						
10th choice						

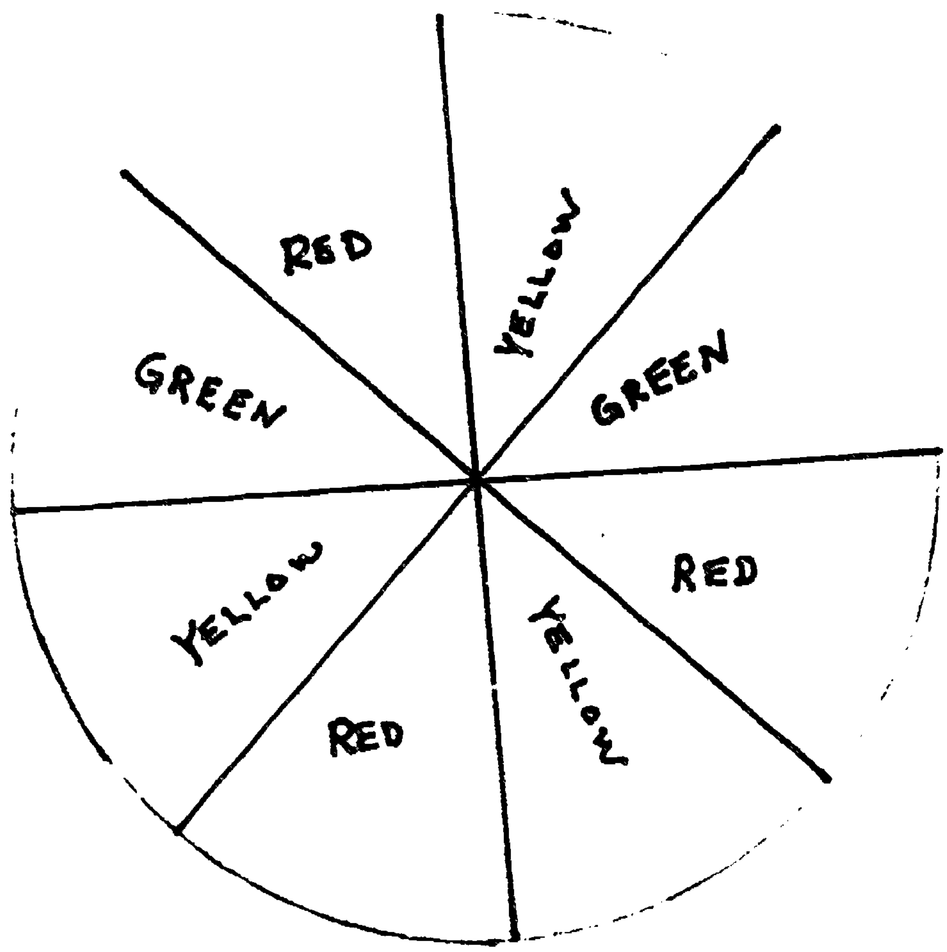
4.18.89

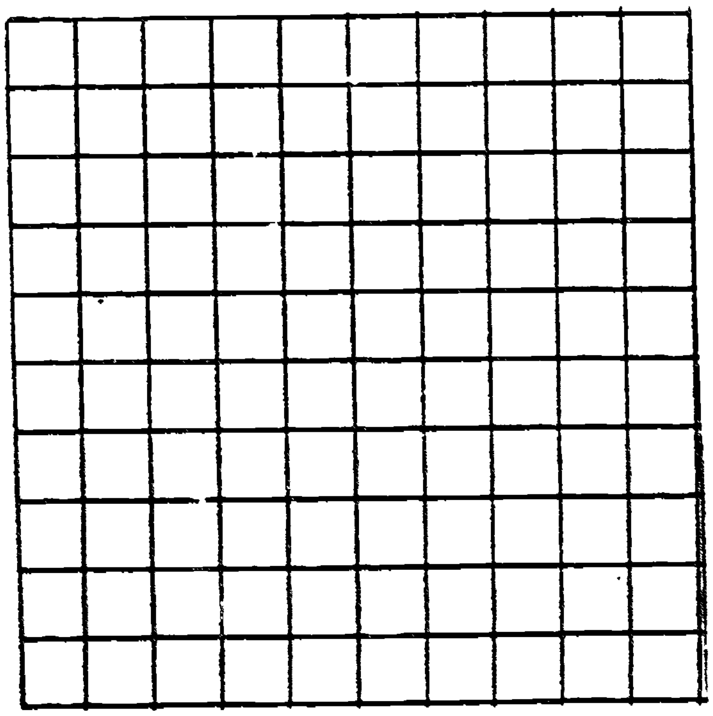
Mathematician:____

"For each data set I found the range and the average. Then I showed how the data would balance a beam with the center at the average."

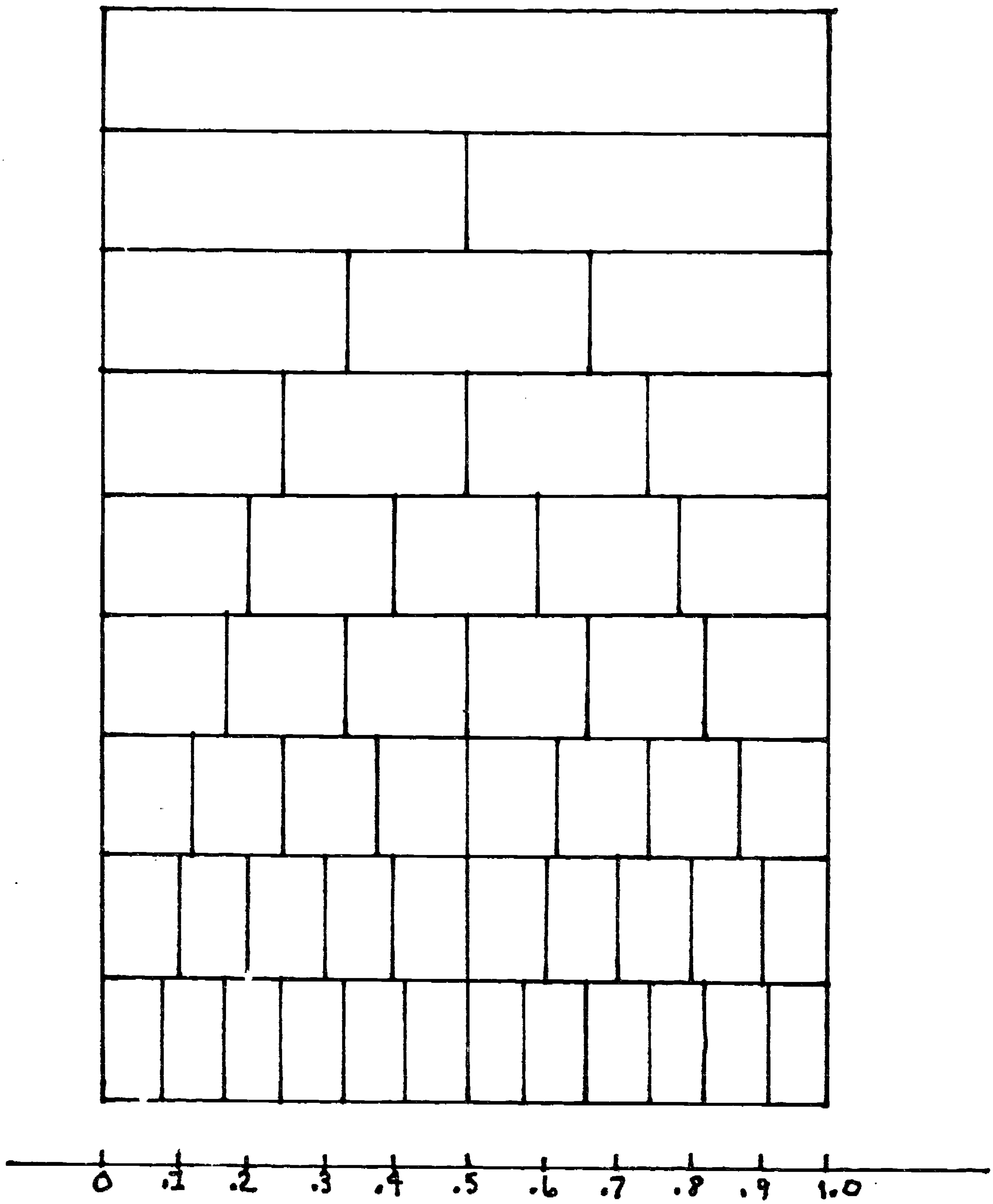
DATA SET	BEAM BALANCE

TEMPLATE FOR LESSON TWO SPINNER

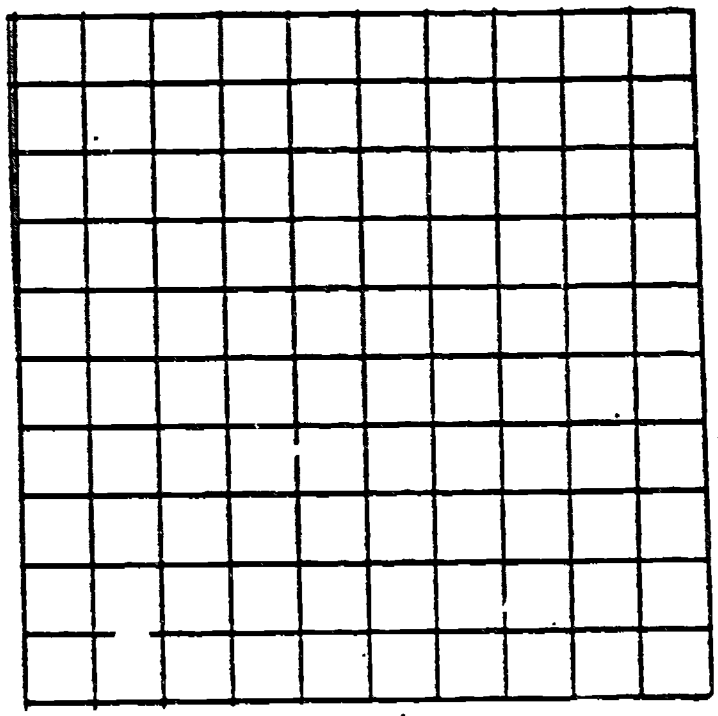




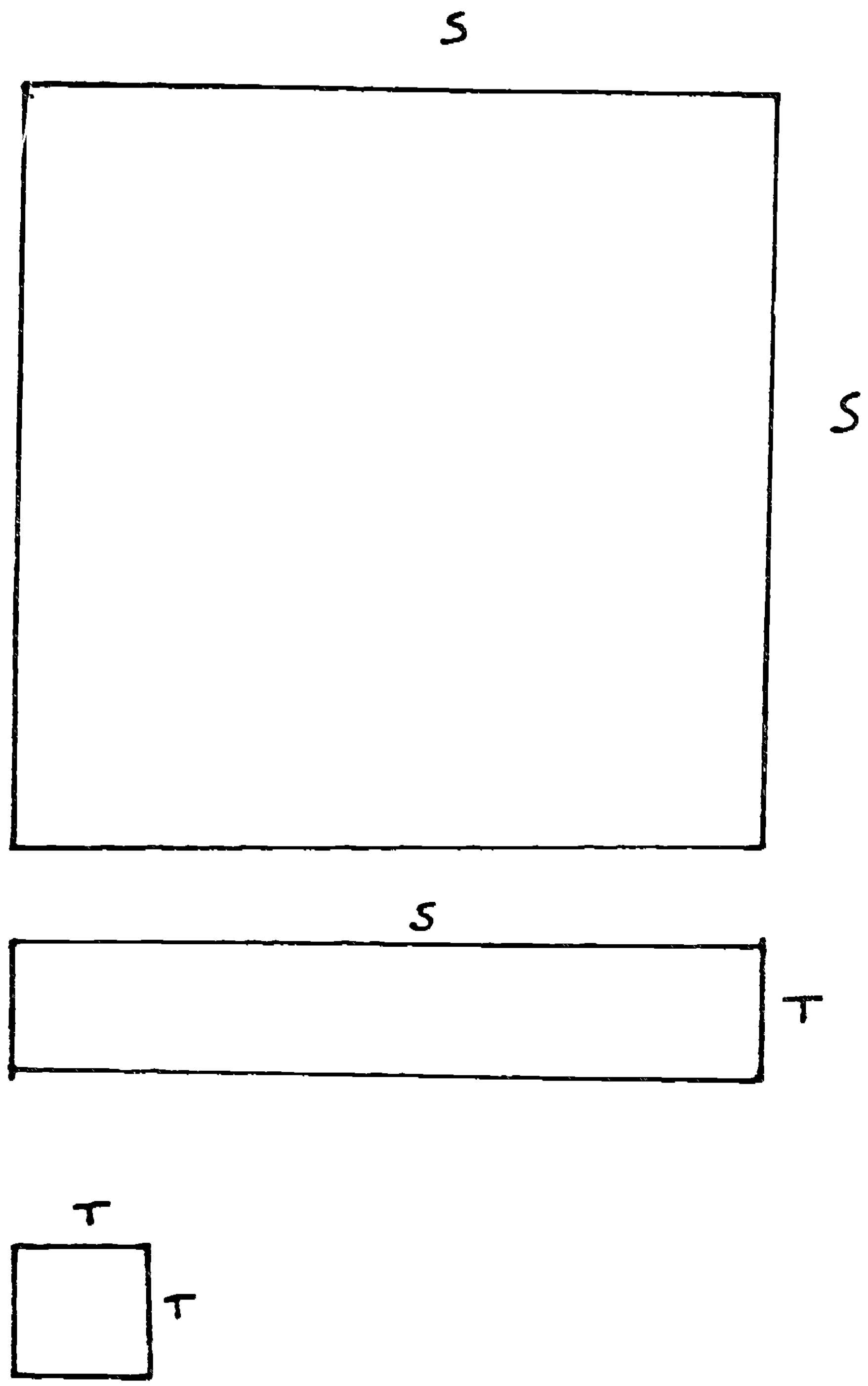
Overhead Template for Fraction Bars



TRANSPARENCY MASTER FOR A PERCENT
HUNDREDS SQUARE



TEMPLATE FOR BUILDING SQUARES



ALSO USE AS A TEMPLATE FOR CONGRUENCE/SIMILARITY