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ABSTRACT

Mathematics and the use of mathematical thinking should be much more than what has been traditional school arithmetic. Much of the mathematical reasoning can be developed and experienced out of school, particularly in the home. This material is a teacher's guide designed to help parents support what is done with their children in class. Background material for parents is provided. Assessment record sheets are presented. A total of 52 activities on the following concepts and skills are included: (1) computation; (2) numeration; (3) fractions; (4) geometry; (5) arithmetic operations; (6) problem solving; (7) number relations; and (8) logic. (YP)

# MEANINGFUL MATHEMATICS

## LEVEL THREE

### TEACHER'S GUIDE TO LESSON PLANS



THIS MATERIAL WAS DEVELOPED WITH THE HELP OF NATIONAL SCIENCE FOUNDATION FUNDING - GRANT #MDR 8550460

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## LEVEL THREE

## REASONABLE OUTCOMES AT THE END OF THE YEAR

Computation:

1. Add one and two digit whole numbers to one and two digit whole numbers
2. Subtract one and two digit whole numbers from two digit whole numbers.
3. Add and subtract money decimals that have up to four digits.
4. Add and subtract proper fractions within common "families" - fourths, eighths, tenths.
5. Multiply a two digit whole number by a single digit.
6. Divide a two digit whole number by a single digit.

Numeration:

1. Read and write whole numbers through 999.
2. Count orally past 100.
3. Count objects more than 30 in numbers.
4. Count on from any starting point <100.
5. Count back from any starting point <100

Fractions:

1. Recognize fractional parts representing wholes divided into 2, 3, 4, 5, 6, 8 or 10 parts
2. Recognize equivalence of fractions within common families, e.g., 2/8 = 1/4.
3. Compute with fractions as described above.
4. Use fractions in "story problems" that involve adding and subtracting.

Geometry:

1. Distinguish and be able to classify squares, non-squares, rectangles, circles, triangles and parallelograms.
2. Distinguish and be able to classify cubes, non-cubes, rectangular solids, triangular prisms, spheres, and cylinders.
3. Recognize equality of two shapes that can be made from the same two smaller shapes, or are the result of halving a larger shape.
4. Distinguish perimeter from area and area from volume.

Arithmetic Operations:

1. Understand the origins of, and be able to symbolize, addition, subtraction, multiplication and division.

2. Use correctly the operations in situations that involve comparing, joining and separating and be able to write number sentences to show these.
3. Generate problems given three numbers so an arithmetic operation on two yields the third.

<u>Problem Solving:</u>

1. Use tables to find answers.
2. Draw pictures to represent conditions in problems.
3. Graph conditions in problems.
4. Write number sentences to represent problem situations.
5. Solve non-numeric problems.

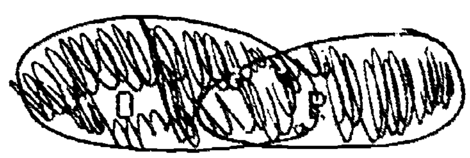
<u>Number Relations:</u>

1. Immediate recall of addition facts for numbers < 10.
2. Immediate recall of multiplication facts for numbers < 10.
3. Write equality sentences and inequality sentences in all forms, e.g. 7 = 3 + 3 + 1; 4 + 2 + 1 = 7; 6 + 3 > 6 + 2; 7 < 9; 1 + 2 + 1 < 4 + 3 + 1.

<u>Logic:</u>

1. Use AND to show multiple classification and joint requirement.


2. Use Or to show alternative possibilities.



3. Use NOT to show complementary condition

NOT

15

LEVEL THREE

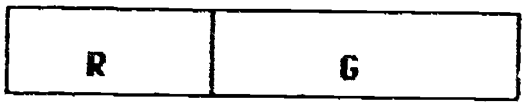
## FRACTIONS. JOINING: CUISENAIRE RODS

### LESSON ONE

Introduction: Use overhead projector version of the C-rods. The children should use rods in order to follow what you do. Establish that you are working in the SIXTHS family. PUt the D rod on the overhead to remind them this is the ONE rod.

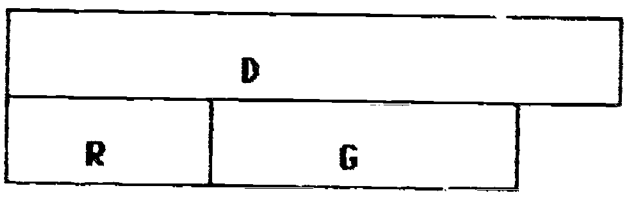
Place the R rod on the overhead projector. Ask the children which fraction this is. (1/3)

Place the G rod on the overhead projector to make a train with the R rod.



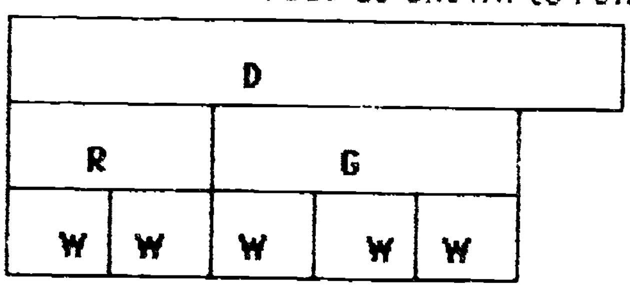

Ask the children what fraction is being added to 1/3. (1/2)

Compare this result with the D rod.



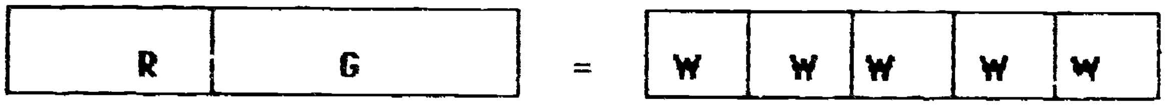

Ask the children what fraction this length is: (5/6)

Put the five W rods as shown to reinforce this.




Remind them that W = 1/6 in this family, or when D = 1, and that W MEASURES both R and G, so W will MEASURE R + G.

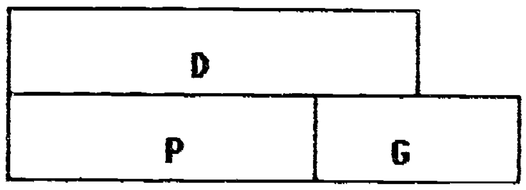
Write 1/3 + 1/2 = 5/6 to represent the results of this joining. If needed, write this in connection with the Rod equation.




1/3 + 1/2 = 5/6

Do a second joining, for example P + G.


P is 2/3, G is 1/2 so this represents 2/3 + 1/2. Again refer to the D rod and W rods.



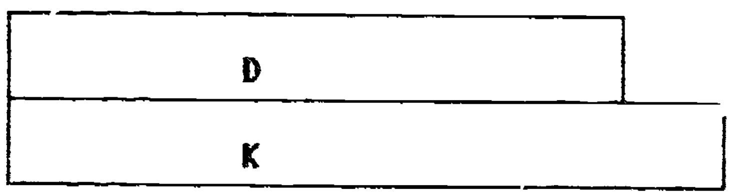

This joining is W longer than D or 1 + 1/6. Write this as:
1 + 1/6 = 1 1/6.

<u>Activity</u>: Pass out the worksheets for pairs of students to work on with rods. Emphasize referring to the D rod which is ONE and the W rod which is 1/6 when interpreting the results of the joining.

<u>LESSON TWO</u>

This lesson is to give experience with joining fractions that are greather than one, still in the SIXTHS family.

<u>Introduction</u>: Place the D rod on the overhead. Remind the children is is ONE. Place a K rod next to it.

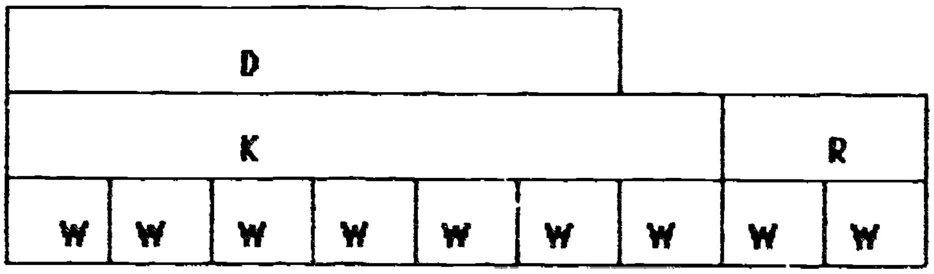



Ask the children if that is less than or more than one. HOw much more than One is it? One W rod or 1/6. The rod then represents 1 1/6

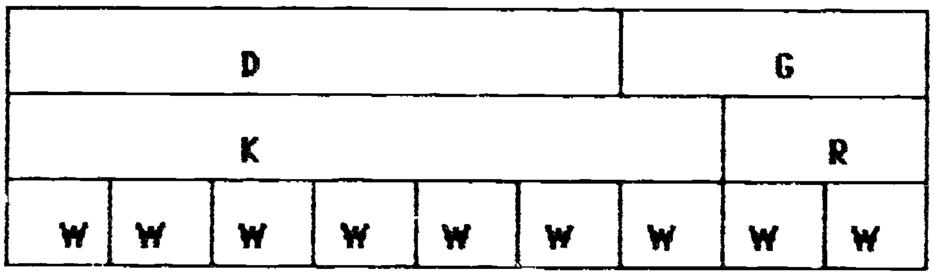
Any other rod joined to this will give something even more greater than one. Put the R rod next to K.


"Now how much more than one do we have?" (3W = G = 1/2).

Put the W rods as shown:

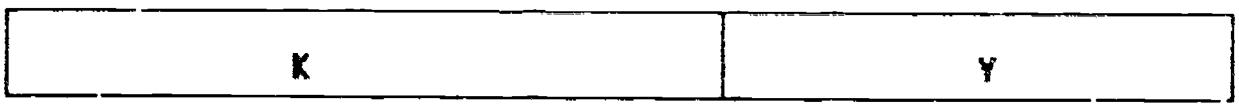



Now count them. There are 9/6. Put the G rod next to D:




"This is the same as 9W, so 9/6 = 1 1/2 = 1 3/6.

Join K with Y:




Compare this with 2D, and: 7/6 + 5/6 = 2.

"How many SIXTHS is this?"

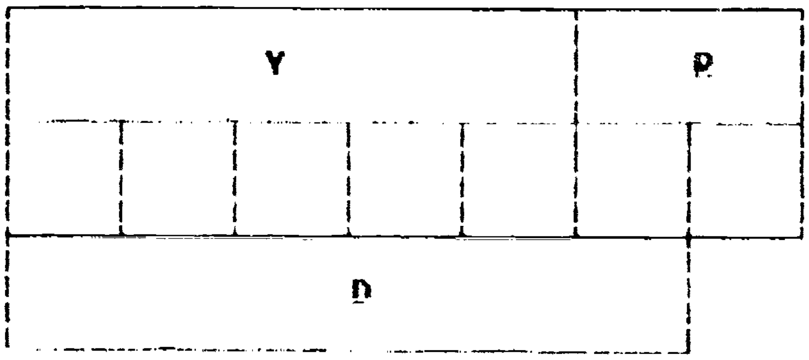
Work to get the children to see that the same fraction can be expressed many ways.

1 = 6/6 2 = 12/6 9/6 = 3/2 = 1 1/2 etc.

MATHEMATICIAN________________________________

## JOINING FRACTIONS D = ONE

"I used Cuisenaire Rods to do these"

| RODS JOINED | NUMBER SENTENCE |
| --- | --- |
| Example:  | 5/6 + 1/3 = 1 1/6 |
| W + R | |
| W + G | |
| W + P | |
| W + Y | |
| R + R | |
| R + G | |

TO: THE TEACHER

FROM: A. DEAN HENDRICKSON

Attached is something you can reproduce and send home to parents to encourage them to support what you are doing in the classroom.

# MATHEMATICS IN THE HOME

A. Dean Hendrickson, University of Minnesota-Duluth

## Introduction:

Mathematics and the use of mathematical thinking is much more than what has been traditional school arithmetic. The arithmetic of whole numbers, fractions and decimals constitutes no more than 10-15% of the mathematics we use throughout our lives. Much of the mathematical reasoning we use can be developed and experienced out of school, particularly in the home. Some of these suggestions may seem remote from the arithmetic you remember, but they will involve children in the THINKING essential to both the learning and use of mathematics in everyday life.

## Pre-Mathematical Thinking:

Before a child can understand school mathematics, certain ways of thinking and skills must be available for use. These are continuously used throughout learning of mathematics, but particularly elementary school mathematics. These include: counting, comparing, ordering, using patterns, using grouped material, using language and establishing relations and relationships. Needed experience with these can be obtained around the home. Before describing things to do with children at home to help them with their school mathematics, here are some "golden rules" based upon research and experience with learning children.

1. You must not force children since this has negative effects, such as turning them away from doing things or from you. A child learns when ready, curious, and needing to make sense of something. This goes in spades for drill on memorizing so-called "basic facts."

2. Give children positive things to do when time is available, especially those things they can do and enjoy doing. Don't ask for things beyond the child's capacity to do.

3. Give lots of praise and encouragement. If what the child does or says doesn't seem to make sense to you, don't criticize or correct. Ask questions that might lead the child to consider it in a different way.

4. Don't look for day-to-day progress or change or for immediate results. Just as with many other things, such as walking or talking, a child may seem to be making no headway and then suddenly, it's all there. Children develop in spurts and unevenly, and have long plateaus where nothing seems to be happening. That's normal and accept it. There is probably a lot going on below the surface.

5. Don't compare yours with other children. Everyone is different - thank goodness!

6. Don't worry if a particular skill, such as using language, is

coming along more slowly than you'd like or than brother John's did. Somehow most of them seem quite a lot alike by the time they are 12 or so.

Words:

A number of words commonly used in mathematics and related to teaching mathematics should be used often outside of school as well. Some examples are *some-more, a lot, more than, less than, large, small, many, few, same as, different, alike, all, some, not, left, right, ahead of, behind, above, below, front, back, long, short.*

In addition to words associated with comparing, grouping and space, the number words are important. Children must know the counting words, but even more than that, they must see the pattern in the use of counting words. The cordial words like *first, second, third,* et. are also important. Use of these words around home helps children to count objects correctly and to identify position of things in ordered arrangements.

Comparing:

Have children compare things as to size, length, are and volume whenever possible. "Which glass has more?" "Which box holds more?" "which of these is heavier? heaviest?" "Put these sticks in order of length." "Arrange the silverware so the longest is farthest from the plate and the shortest is nearest the plate." Questions like these should be frequent. They should involve different kinds of things both indoors and outdoors. Combine these with questions that make the children estimate measurements of distance and height such as "Which do you think is as high as the shed, A or B?"

Comparing of quantity leads to better understanding of number and number relationships. "Are there more chairs or lamps in this room?" "Are there more cups or teaspoons on the table?" "Have we go more red roofs or green roofs on our street?" "Put enough table knives on the table so that there are as many knives as forks." "Do you have more boys or girls in your class?" These can be asked when out walking, riding in the car, watching TV or sitting in the boat. Ask children to do things that will make one group as large as another frequently All such activity helps children build number relations into their deeper understandings, instead of as memorized associations that have no meaning - like names and dates you once memorized to pass a history test!

Ordering things that can be counted is important. Bead stringing activities are good for young children. "String some beads so the third bead is red and the fourth bead is blue." "Make a string so every other one is green," etc.

Ordering things that have lengths, areas and volumes extends comparing beyond two things. Have children place three sticks of different lengths in order from shortest to longest; place three pieces of paper of different areas into orders; place three different sized cans of jars into order. Gradually extend the number of things to more than three for these activities.

Ask frequent questions about the ordering of events as to which happens first, second...last, etc. Connect these with time estimations, "How many minutes ago do you think this happened? How many days?" etc.

Counting:

Children should keep extending their memorized sequence of counting words. This is important. But being able to say the words in right order does not mean they can count things. They need much practice at this. Have them count everything around the house that is countable - the chairs, tables, legs on chairs; the tiles on the floor, in the ceiling; the number of windows in a room; the silverware in the drawer; the cans on the shelf; the pieces of wood in the woodpile; the telephone poles going by, etc. The more they count, the better able they are to count. When they are pretty good at counting forward, have them do some counting back. For example, start with 20 clothespins. One at at time put one into a can and count aloud those that are left as each one is removed from the pile.

Patterns:

Have children look for patterns - in the carpet, in the ceiling, in wallpaper, in the drapes, on the bedspreads. Patterns of shape, or color, or sound are all important. Beads can be strung in patterns. Collections of bottle caps, old keys, buttons, screws, nuts and bolts, and similar "junk" can be put into patterns. Ask children what would come next in a pattern, or what would go where something is missing in a pattern.

Number:

Help your child learn number size by having him see the same number, such as five, in many different arrangements and materials. Playing cards can be sorted into those all having the same number. Mixed groups of say, five marbles, three buttons, three keys, six spoons, can be used. "Find me the material there are five of," etc. Put some number, seven for example, of beads or marbles into three or four different shaped glass jars, "Find a jar with seven in it." "Find another." Put the same number of one kind of thing in one jar and another kind in a second jar, etc., and do the same kind of thing. Involve the child with numbers in as many different ways, with as many different kinds of material, and as many different sizes as possible. Gradually increase the number size as the child seems able to easily handle smaller numbers.

Using Numbers:

Comparing groups with number property; combining such groups; separating larger groups into smaller groups of a given size or into equal size groups - all of these activities help children to understand when each of the four arithmetic operations are used.

Some examples of things to do in the home of this kind are:

1. Compare two different sized groups in several ways. "How many more are there in this group than in that group?" "This group has how many fewer than that group?" "How many times as many are there here as there?" These kinds of questions used with groups of all kinds

of things - knives, forks, chairs, chair legs and table legs, buttons, marbles, pieces of candy, etc., help the child with what the school is doing.

2. Join together several groups of the same size into a larger group. Rows of pennies can be arranged into an array like this and can then be looked at a different way to see 5 groups of 6 pennies:

oooooo
oooooo
oooooo
oooooo
oooooo

Both lead to a total of 30 in the array. Do this in a row at a time, having the child tell you how many are there all together each time. Separate and take apart such arrays row by row and see what is left each time. Do this with different kinds of things, different size rows and different total numbers of things. Clothes pins, ceramic tiles, beans, corn are all good for this.

3. Join together groups of different size, such as seven things with five things. Have the child describe what is happening in words. Have the child add to one group of things enough to make it the same size as another larger group. Have the child make equal two unequal size groups without adding anything more to the collection. "Here are a group of 15 clothes pins and one of 7 clothes pins. Do something so you have two equal groups."

4. Give the child large amounts - in the 20's or 30's of things to:

   a) make several groups of a given size from. Some numbers should make these smaller groups an even number of times and some should have some left that is not enough to make another of the smaller group.

   b) make a certain number of groups that will all have just as many in them.

   Examples:

   "Put these 30 beans into 6 cups, so each cup has just as many. How many are in each cu?"

   "Put these 43 beans, six at a time into cups. How many cups did you use?" "What should e done with what is left over?" "When do you have some left over?" "When don't you have anything left over?"

When you do for walks, have the children compare, add together, etc., things along the way. Do the same in the car, the supermarket, in the drugstore. "How many are there on the top shelf?" "How many re on the bottom shelf?" "How many are there on the top and bottom shelves together?"

Have the child do as much adding, subtracting, multiplying and dividing of this kind - always as related to things - as you can. DON'T try to drill your child on "addition" facts or "multiplication" facts. Let the child learn these in due time through the school activities and those you do at home as described here. DON'T have the child write number things – the school will do this. Accept verbal answers and descriptions. Get in the habit of asking your child why certain answers are given and LISTEN.

SOME FINAL HINTS:

1. Have your children count things as much as possible.

2. Ask children simple addition, subtraction questions about REAL things in the surroundings to give practice in mental arithmetic.

3. Play card games that require mathematics or related things like WAR, OLD MAID, CRIBBAGE, RUMMY (regular or gin).

4. Give thinking games for holiday gifts – CONCENTRATION, HUSKER DU, etc.

5. Get a Little Professor or some similar calculator-based program to give mental arithmetic practice.

6. Cheap mathematics games can be bought at Target, Woolworths, etc. Some examples are COVER UP, HEADS UP, SCORE FOUR, TUF, APOLLO, etc.

7. Give your child a simple four function calculator and let him or her fool around with it.

8. Encourage block play and building, sand play, making birdhouses, etc.

9. Key words are COMPARING, COUNTING, PATTERNS, COMBINING (groups), SEPARATING (large groups into smaller groups)

10. Point out mathematics wherever it is in the surroundings. Children must realize mathematics is:

    a. easy to learn
    b. useful
    c. fun

# LEVEL THREE

## USING THIS TEACHER'S GUIDE

The lessons are grouped by topic, process and/or concept. Teach each of these to mastery. Lessons may be repeated as necessary, returned to later in the year for desired review, or stretched out in time as appropriate and necessary. Some lessons review previous material, but in a more advanced context. Others are new material. These should be easily distinguished from each other.

LEVEL THREE

INTRODUCTION

Coming into this level, students will have had work in:

1. Number concept development
2. Meaning and use of the four operations
3. Place value representation
4. Numeration in base ten
5. Problem solving
6. Work with 2D and 3D shapes
7. Logical use of AND and NOT
8. Graphing
9. Measurement
10. Fractions
11. Operations in base ten

Prior to starting instruction, assess the children's knowledge in these areas, using the attached instruments.

Many of the children will complete the transition from pre-operational thinking to concrete operational thinking during this year. Activities to help them in this transition are included. These will give them experience in reversibility of thought, formal measurement of lengths and areas; and increased use of symbolism related to mathematical concepts and processes. However, it is still necessary for you to observe the practice of introducing new or unmastered concepts at the concrete level, gradually relate symbols such as numerals and pictures to these concepts, and frequently associate concrete materials, pictures and symbols in the activities.

# LEVEL THREE
## GETTING STARTED

Read the LEVEL TWO TEACHER'S GUIDE as background and for information about the previous experiences children have had. Also read THE PIAGET PRIMER, or a similar work to understand the psychological and educational theory underlying the program.

Major ideas and Processes emphasized during the year are COMPARING, COUNTING, CLASSIFYING, USING PATTERNS, USING NUMBER OPERATIONS, PROBLEM SOLVING, ESTIMATION AND REASONABLENESS OF ANSWERS, RELATIONS BETWEEN NUMBER, OPERATIONS WITH FRACTIONS, RATIO, GEOMETRY, LOGIC, MEASUREMENT, GRAPHING, INVERSE RELATIONSHIP OF OPERATIONS, USE OF CALCULATORS, FINDING AREAS, AND INTRODUCTION TO VOLUME AND TO NEGATIVE NUMBERS, and MENTAL COMPUTATION.

COMPARING will involve comparisons of weight, volumes, areas, parts of wholes, number properties of collections and the properties of shapes.

COUNTING should include oral counting for 5 minutes each day. Counting on, counting back, skip counting forward and backward, counting by tens and hundreds are all needed.

CLASSIFICATION will be related to SIMILARITIES AND DIFFERENCES and the use of AND, OR and NOT.

USING PATTERNS will emphasize patterns in number tables, and the patterns in the numeration system. Patterns of shape are emphasized more.

NUMBER OPERATIONS will be used in one-step and two-step problems that use more than one operation. The numbers will include larger whole numbers and fractions.

PROBLEM SOLVING will involve numbers and non-numeric situations that emphasize geometric properties.

ESTIMATION and REASONABLENESS of answers will be emphasized throughout. Rounding of numbers to get "ball park" bounds for answers to be found is to be emphasized.

EQUALITY, LESS THAN and GREATER THAN relations between numbers is to be continued and emphasized.

OPERATIONS WITH FRACTIONS are to be related to the operations with whole numbers, since all have their origins in comparing, joining and separating.

RATIO ideas are to be extended to the use of units and a unitary basis for what will become proportions.

GEOMETRY will involve more area measurement, comparison of, and relations between lengths, and the properties of shapes. The idea of congruence is to be emphasized.

CALCULATORS are to be used to reinforce counting, place value, patterns, and to make calculations in problem solving activity.

VOLUME is to be developed by building shapes that occupy space from units of volume.

NEGATIVE NUMBERS are introduced with emphasis on making quantities more positive or more negative.

MENTAL COMPUTATION. By the end of this year, children should have the "addition facts" and "multiplication facts" at the immediate recall level. Frequent mental computation activities in interesting contests such as games, relays, etc. should be conducted.

Problems should be related to children's experience. Frequently have children write and create problems for others to solve. Point out problem solving strategies such as making a table, searching for a pattern, drawing a picture as these are employed in whole-class activities.

MEASUREMENT is with standard units with emphasis on the process of measurement.

MATERIALS to be used during the year include:

- Base Ten blocks and Place Value mats
- Cuisenaire Rods
- Pattern Blocks and Tangrams
- Geoblocks
- Logic Blocks
- Geoboards
- Calculators
- Square Tiles
- Cubes

## LEVEL THREE - INSTRUCTIONAL PRACTICES

1. Most lessons are designed so that introduction is by demonstration of how to use the materials on the overhead. Do this and ask enough questions so children know what they are to do. When recording forms or worksheets are involved, make transparencies of these and show the children how to record in them.

2. Introduce any symbols slowly and carefully so that children know what they represent.

3. Group children into 2's, 3's and 4's as often as possible and encourage them to learn together and from each other.

4. Recording forms and worksheets should be kept in individual student folders labelled "My Work in Mathematics."

5. When children are working with materials, walk around and observe what they are doing. Ask questions of a leading or probing nature to stimulate their thinking. Avoid "playing teacher" and telling them how to do something until they have had sufficient time to work it out for themselves.

6. Children learn from their mistakes. Don't penalize them for "good" mistakes that indicate an idea that just hasn't matured yet. Children get right answers for wrong reasons and wrong answers for right reasons. You must be able to distinguish between these.

7. Plan for storage of materials in open locations where they are accessible to children. Use storage containers like plastic ice cream pails, gallon milk bottles and boxes made into drawers. Teach children to be responsible for getting materials out to work with, policing the work areas for dropped materials and for putting materials away. These are LEARNING materials and children must understand that.

8. Feel free to develop homemade "analogues" of the commercial materials suggested.

## LEVEL THREE

## ASSESSMENT PROCEDURES

<u>NUMBER</u>

<u>Hand Test</u>: Show the child the number to be tested in beans in one hand. Have the child count them. Put some of these in the other hand. Show the first hand and ask how many are in the other hand. 'How many twos?" "How many threes?", etc.

<u>Written Test</u>: Give the child a number, e.g. fourteen. Ask that this be written in part-part whole combinations in as many ways as possible, including equal part combinations.

<u>COUNTING</u>

1. Have the child count on from a given number.

2. Have the child skip count.

3. Have the child count out a collection of objects.

4. How does the child handle transitions from decade to decade? into the hundreds?

5. In rational counting, does the child touch objects as counted? Move objects as counted? Rely on eye contact with objects? Subvocalize? Count in groups? Fail to maintain a 1-1 correspondence between counting words and objects counted?

<u>FRACTIONS CONCEPT</u>

1. Have the child divide up a shape into thirds, fourths, fifths, etc.

2. Have the child color in a given number of equal parts.

<u>PLACE VALUE</u>

1. Show the child a numeral, e.g. 1 3 4.

2. What is counted by the "1"?

3. What is counted by the "3"?

4. How many tens in this number?

5. What digit is in the tens place?

6. In the hundreds place?

Geometric Shape: Show a number of shapes and have the child pick out the ________s. Give the child shapes to sort into like shape collections.

NUMBER OPERATIONS

Attached are the 14 situations that give rise to the operations of addition and subtraction. Read these to the whole class one at a time. Children should have some blocks to manipulate. On the first reading ask the children to just find the answer and write it in the space provided. Tell them to think of a number sentence to show the problem. Read it a second time. Tell them to write the number sentence and to circle the answer. Use the results to know which kinds of problems to emphasize with the class during the early part of the year.

COMPUTATION FORMS

Give the child a computation form like

$$\begin{array}{r} 24 \\ + \\ \underline{13} \end{array}$$

Ask the child to write this in horizontal form: ____ + ____ = ____.

Ask the child to show what is in computation form with base ten blocks.

EQUALITY

Give the child a split board with 8 cubes on one side and 8 cubes on the other side. Ask the child to group the objects to show, in succession 8 + 5 + 3; 6 + 2 = 8; and 1 + 7 = 4 + 4

ASSESSMENT RECORD
for

*Mathematician:*______________________________ ______

*Assessment: "Please count for me." Circle the largest number in the sequence the child uses correctly.*

| Date: | Number Sequence |
|---|---|
| | below ten 11 12 13 14 15 16 17 18 19 20 |
| | 30 40 50 60 70 80 90 |
| | 100 110 120 __________(write in) |
| Date: | below ten 11 12 13 14 15 16 17 18 19 20 |
| | 30 40 50 60 70 80 90 |
| | 100 110 120 __________(write in) |
| Date: | below ten 11 12 13 14 15 16 17 18 19 20 |
| | 30 40 50 60 70 80 90 |
| | 100 110 120 __________(write in) |

*Mathematician:* ____________________

*Assessment: "Please Count These." (rational counting of objects. Circle the largest number in the set the child correctly counts.*

| | Number Sequence | Moves Objects | Touches Objects | Mentally "Touches" |
|---|---|---|---|---|
| Date: | below ten 11 12 13 14 15 16 17 18 19 20 30 ______ (write in) | | | |
| Date: | below ten 11 12 13 14 15 16 17 18 19 20 30 ______ (write in) | | | |
| Date: | below ten 11 12 13 14 15 16 17 18 19 20 30 ______ (write in) | | | |

*Mathematician:*____________________________________

*Assessment: Numbers "Known" - all parts combinations.*
*Cirlce the largest number on that date.*

| | Hand Test | Bowl Test | Number |
|---|---|---|---|
| Date: | | | 6 7 8 9 10 11 12 13 14<br>15 16 17 18 19 20 21 22<br>23 24 ______(write in) |
| Date: | | | 6 7 8 9 10 11 12 13 14<br>15 16 17 18 19 20 21 22<br>23 24 ______(write in) |
| Date: | | | 6 7 8 9 10 11 12 13 14<br>15 16 17 18 19 20 21 22<br>23 24 ______(write in) |

*Mathematician:*______________________________

*Assessment: Recognition of Geometric Shapes. Check those recognized or differentiated from others.*

| | Triangle | Square | Circle | Rectangle | Parallellogram | Hexagon |
|---|---|---|---|---|---|---|
| Date | | | | | | |
| Date | | | | | | |
| Date | | | | | | |
| Date | | | | | | |

PROBLEMS FOR ASSESSMENT (1)

1. COMBINE 1

____Diane has 5 red blouses and 4 blue blouses. How many blouses does she have altogether?

2. CHANGE 1

____Ronnie had 5 snowballs on the windowsill. Jan gave him 3 more snowballs. How many snowballs did he have then?

3 COMPARE 1

____Fred has 9 crayons and Tim has 6 crayons Fred has how many more crayons than Tom?

4. CHANGE 2

____Debbie had 7 jelly beans. She gave 5 of them to Tom How many jelly beans did she have left?

5. CHANGE 3

____Betty had 6 paper clips. Harry gave her some more paper clips. Then Betty had 8 paper clips. How many paper clips did Harry give her?

6. COMPARE 2

____Janice has 8 sticks of gum. Tom has 2 sticks of gum. Tom has how many sticks less than Janice?

7. CHANGE 4

____Iris had 8 peanuts. She gave some to Eddie and then had 3 peanuts left. How many peanuts did she give to Eddie?

## PROBLEMS FOR ASSESSMENT (2)

1. COMPARE 3

____Barb has 6 pencils. Frank has 3 pencils more than Barb. How many pencils does Frank have?

2. CHANGE 5

____Fran had some pencils. She gave 4 pencils to Jean. Now she has 3 pencils left. How many pencils did she have to start with?

3. COMBINE 2

____Bill has 9 jelly beans. 5 are blue and the rest are brown. How many jelly beans are brown?

4. COMPARE 4

____Tony has 7 books. Janet has 4 books less than Tony. How many books does Janet have?

5. CHANGE 6

____Pete had some milk cartons. Annie gave him 3 more milk cartons. Now he has 8 milk cartons. How many milk cartons did he have to start with?

6. COMPARE 5

____Maxine has 9 sweaters. She has 5 sweaters more than Sue. How many sweaters does Sue have?

7. COMPARE 6

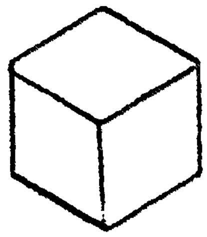
____Peter has 7 pencils. He has 3 pencils less than Tom. How many pencils does Tom have?

MATHEMATICIAN: ______________ ____________________________

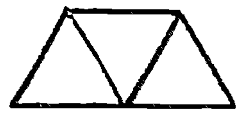
| | ANSWER | NUMBER SENTENCE |
|---|---|---|
| 1 | * | |
| 2 | * | |
| 3 | * | |
| 4 | * | |
| 5 | * | |
| 6 | * | |
| 7 | * | |

MATHEMATICIAN: ________________________

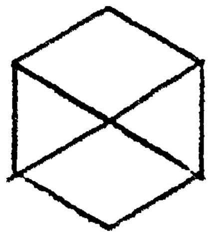
Pattern Blocks



Color 1/3 blue



Color 2/3 blue

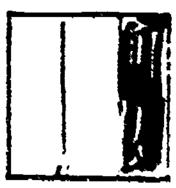


Color 3/6 red

---

Write a fraction to show what part of each shape is shaded.

 ____

 ____

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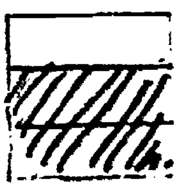
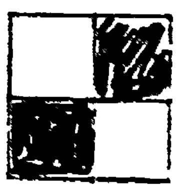
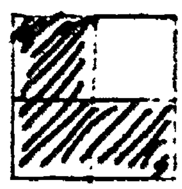
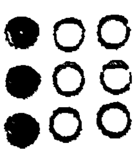
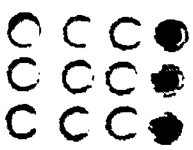
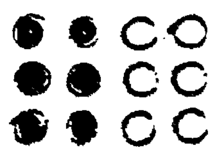
 ____

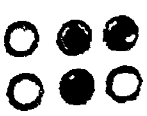
 ____

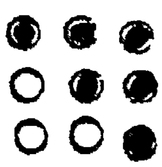
 ____

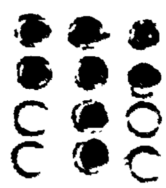
 ____

 ____

 ____

 ____

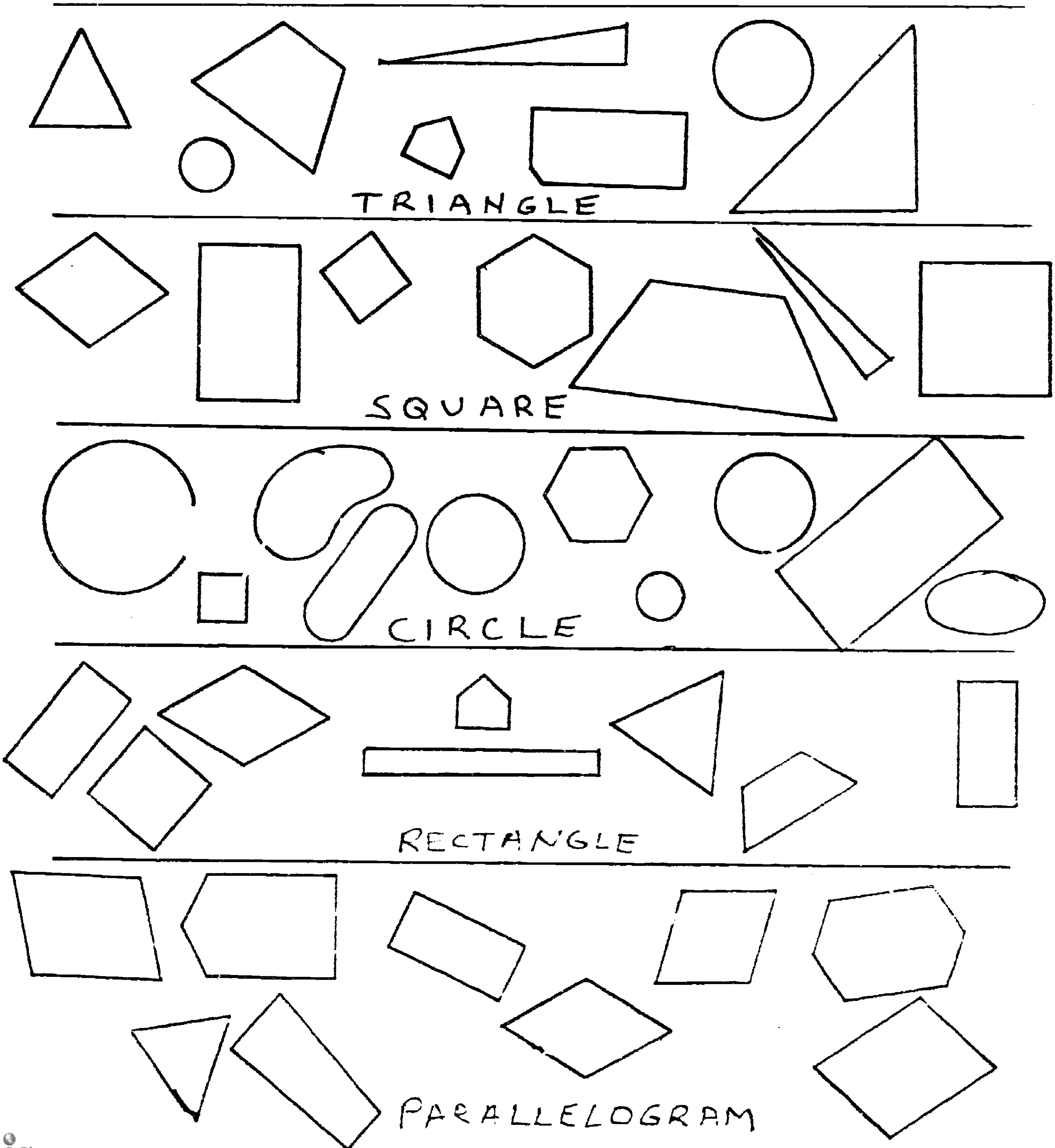
 ____

 ____

MATHEMATICIAN:________________________________________

## SHAPES

**In each section, place an "X" over the shapes for the name given.**



# LEVEL THREE ASSESSMENT RECORD

MATHEMATICIAN: ______________________________ Pg. 1

## COUNTING

| ROTE COUNTING | BEGINNING OF YEAR | END OF YEAR |
|---|---|---|
| Counting from one to? | | |
| Counting on from 6 to? | | |
| Counting on from 13 to? | | |
| Counting on from 22 to? | | |
| Counting on from 37 to? | | |
| Counting on from 41 to? | | |
| Counting on from 50 to? | | |
| Counting on from 65 to? | | |
| Counting on from 73 to? | | |
| Counting on from 82 to? | | |
| Counting on from 98 to? | | |
| Counting on from 102 to? | | |
| Counting on from 131 to? | | |
| Counting on from 206 to? | | |
| Counting on from 300 to? | | |
| Counting by tens to? | | |
| Counting by hundreds to? | | |
| Counting by tens from 12 to? | | |
| Counting by tens from 63 to? | | |
| Counting by tens from 121 to? | | |
| Counting by hundreds from 13 to? | | |
| Counting by hundreds from 59 to? | | |
| Counting by hundreds from 132 to? | | |

| RATIONAL COUNTING | BEGINNING OF YEAR | END OF YEAR |
|---|---|---|
| Number of objects correctly counted? | | |
| Moves objects as counts? | | |
| Touches objects as counts? | | |
| Visually tracks objects? | | |
| **NUMBER CONCEPT** | | |
| Largest number for which: parts are known? | | |
| multiples are known? | | |
| **FRACTION CONCEPT** | | |
| Fractional parts recognized: | | |
| 1/2 | | |
| 1/3 | | |
| 1/4 | | |
| 1/5 | | |
| 1/10 | | |
| Other fraction symbols written? | | |
| **PLACE VALUE** | | |
| Recognizes what digits represent: | | |
| in two digit numeral? | | |
| in three digit numeral? | | |
| other? (describe) | | |

| GEOMETRIC SHAPES | BEGINNING OF YEAR | END OF YEAR |
| --- | --- | --- |
| Shapes recognized: | | |
| Triangle? | | |
| Square? | | |
| Rectangle? | | |
| Parallelogram? | | |
| Circle? | | |
| Hexagon? | | |
| Cube? | | |
| Triangular Prism? | | |
| Rectanqular Prism? | | |
| Cylinder? | | |
| Sphere? | | |
| COMPUTATION FORMS | | |
| Vertical Addition | | |
| Vertical Subtraction | | |
| Horizontal Addition | | |
| Horizontal Subtraction | | |
| SYMBOL USE | | |
| Symbols Recognized: | | |
| + | | |
| – | | |
| ÷ | | |
| x | | |
| = | | |

| | BEGINNING OF YEAR | END OF YEAR |
|---|---|---|
| Symbols Used Correctly: | | |
| + | | |
| − | | |
| x | | |
| ÷ | | |
| = | | |
| **EQUALITY SENTENCES** | | |
| Whole = parts (8= 6 + 2) | | |
| Parts = Whole (1 + 2 + 3 = 6) | | |
| Parts = Parts (1 + 2 + 3 = 4 + 1 + 1) | | |
| **NUMBER OPERATIONS** | | |
| Problem Type Solved: | | |
| Combine 1 | | |
| Combine 2 | | |
| Change 1 | | |
| Change 2 | | |
| Change 3 | | |
| Change 4 | | |
| Change 5 | | |
| Change 6 | | |
| Compare 1 | | |
| Compare 2 | | |

| | BEGINNING OF YEAR | END OF YEAR |
|---|---|---|
| Compare 3 | | |
| Compare 4 | | |
| Compare 5 | | |
| Compare 6 | | |

Correct Number Sentences Written:

| | BEGINNING OF YEAR | END OF YEAR |
|---|---|---|
| Combine 1 | | |
| Combine 2 | | |
| Change 1 | | |
| Change 2 | | |
| Change 3 | | |
| Change 4 | | |
| Change 5 | | |
| Change 6 | | |
| Compare 1 | | |
| Compare 2 | | |
| Compare 3 | | |
| Compare 4 | | |
| Compare 5 | | |
| Compare 6 | | |

Table 1

Change Problems

| Problem Title | Sample Problem | Characteristics |
|---|---|---|
| Change 1. | Bill has 2 pencils. Jean gives him 3 pencils. How many pencils does Bill have then? | Increase, initial set and change set known, question about final set. |
| Change 2. | Bill has 5 pencils. He gives 3 to Jean. How many pencils does he have left? | Decrease, initial set and change set known. Question about final set. |
| Change 3. | Bill has 2 pencils. Jean gives him some more. Now he has 5. How many did Jean give him? | Increase, initial set and final set known. Question about change set. |
| Change 4. | Bill has 5 pencils. He gives some to Jean. Now he has 2. How many did he give to Jean? | Decrease, initial set and final set known. Question about change set. |
| Change 5. | Bill has some pencils. Jean gave him 2 more. Now he has 5. How many did he begin with? | Increase, change set and final set known. Question about initial set |
| Change 6. | Bill has some pencils. He gave 3 to Jean. Now he has 2. How many did he begin with? | Decrease, change set and final set known. Question about initial set. |

## Table 2

### Combine Problems

| Problem Title | Sample Problem | Characteristics |
| --- | --- | --- |
| Combine 1. | Bill has 3 red pencils and 2 green pencils. How many pencils does Bill have altogether? | Two subsets are known Question about whole set. |
| Combine 2. | Bill has 5 pencils. Three are red and the rest are green. How many are green? | Whole set and one subset are known. Question about other subset. |

Table 3

Compare Problems

| Problem Title | Sample Problem | Characteristics |
| --- | --- | --- |
| Compare 1. | Bill has 2 pencils. Jean has 5. How many more does Jean have than Bill? | Comparison stated in terms of more, referent set and compared set known. Question about difference set. |
| Compare 2. | Bill has 2 pencils. Jean has 5. How many fewer pencils does Bill have than Jean? | Comparison stated in terms of less, referent set and compared set known. Question about difference set. |
| Compare 3. | Bill has 2 pencils. Jean has 3 more than Bill. How many pencils does have? | Comparison stated in terms of more, referent set and difference set known. Question about compared set. |
| Compare 4. | Jean has 5 pencils. Bill has 3 fewer pencils than Jean. How many pencils does Bill have? | Comparison stated in terms of less, referent set and different set known. Question about compared set. |
| Compare 5. | Jean has 5 pencils. She has 3 more pencils than Bill. How many pencils does Bill have? | Comparison stated in terms of more, compared set and different set known. Question about referent set. |
| Compare 6 | Jean has 2 pencils. She has 3 fewer pencils than Bill. How many pencils does Bill have? | Comparison stated in terms of less, compared set and difference set known. Question about referent set. |

## LEVEL THREE

### COUNTING

ORAL Counting

1. SKIP Counting

| | |
|---|---|
| by twos | by twenties |
| by threes | by thirties |
| by fours | by forties |
| by fives | by fifties |
| by sixes | by sixties |
| by sevens | by seventies |
| by eights | by eighties |
| by nines | by nineties |
| by tens | by hundreds |

2. Skip counting from a given number, e.g. 3, 5, etc., as above through "by tens"

3. Counting on from any given number

4. Counting back from any given number

5. Skip counting back from a given number: "by twos...........by tens"

RATIONAL COUNTING

1. Counting of collections > 30

2. Counting of collections by twos, by threes, by fours,........ , by tens.

ORDINAL COUNTING

1. Correctly identifying any ordinal position up to 20th

## LEVEL THREE

## CLASSIFICATION: SIMILARITIES & DIFFERENCES

### LESSON ONE

Use the template provided to make red, yellow, green and blue rectangles of two sizes, circles of two sizes, squares of two sizes and triangles of two sizes - 32 in all. These can be printed on, and then cut of a heavy paper or light cardboard. Use ATTRIBUTE BLOCKS from the ESS materials if you can get them.

Introduction: Each child should have the above set of pieces of ONE color, 8 in all, of say, blue. Hold up a triangle of the same color as the students have. "Hold up a triangle that is like this one." See that all choose the SAME SIZE as yours. "Hold up a shape that is different from mine."

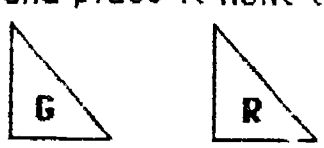
Ask a few students HOW their shape is different from yours. Pass out the remaining shapes so each pair of children have a full set of 32.

"Hold up two shapes that are different ONLY in color." "How are these ALIKE?" "Now hold up two that are different ONLY in shape." "How are these alike?" "Now hold up two that are different ONLY in size." "How are these alike?"

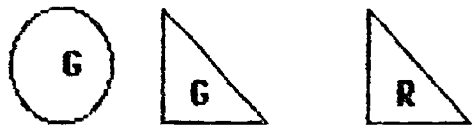
Discuss fully how things can be alike and different.

### LESSON TWO

Introduction: Place one of the over head transparency versions of the shape set on the overhead i.e., a small green triangle. "Hold up a piece that I can put next to this that is different in ONLY ONE WAY." More than one is possible, so discuss all candidates that are presented. Choose one and place it next to the first:



"Now hold up a piece I can put next to the green triangle on the OTHER SIDE that is also different in ONLY ONE WAY." Again, discuss those that are suggested. Place one of those next to the green triangle.




Activity: The children are to work in pairs and arrange all 32 pieces in a row so that each piece differs from each of its two neighbors in exactly ONE way. Challenge students to see if they arrange these in a closed loop and have each piece differ from each of its two neighbors in exactly one way.

LESSON THREE:

Introduction: Explain how difference games work.

ONE DIFFERENCE - A piece must differ on only ONE property - shape, color, size - from each of its two neighbors in the line.

TWO DIFFERENCES - A piece must differ on EXACTLY TWO properties from each of its two neighbors in a line.

THREE DIFFERENCES - A piece must differ on ALL THREE properties from each of its two neighbors in a line.

Place a transparency of Sample Logic Puzzle on the overhead. Place a starting piece in the first space. Ask the children what could go next. Remind them that one line means ONE DIFFERENCE, two lines mean TWO DIFFERENCES, and three lines mean THREE DIFFERENCES. Continue asking which pieces to put on the transparency until all are placed.

Activity: Use the logic puzzle sheets in order with pairs of children working on them with a set of all 32 attribute pieces.

LESSON FOUR:

Introduction: Make a transparency of a 4 x 4 matrix. Use the LARGE four shapes of four different colors. Place a piece in the upper left corner. Explain that all blocks in the same ROW (slide your finger along the rows) must have the same COLOR. All blocks in the same COLUMN (slide your finger down the column) must have the same SHAPE. Ask the children to tell you how to complete the matrix.

Activity: Pass out the 4 x 4 matrix worksheets for pairs of children to do. They will need all of the LARGE pieces in a set (16) or all of the SMALL pieces in a set (16) in each pair.

LESSON FIVE:

Introduction: Make LOGIC cards from the master provided tracing a different set of 25 of the 32 pieces on 10-15 cards. Each child has a card and a set of 32 pieces. Have one child draw pieces from a box with all 32 pieces in it just as bingo numbers are drawn. Each child covers the square on his/her card with the correct piece as called. If a line, column or diagonal is covered, a child calls "LOGIC."

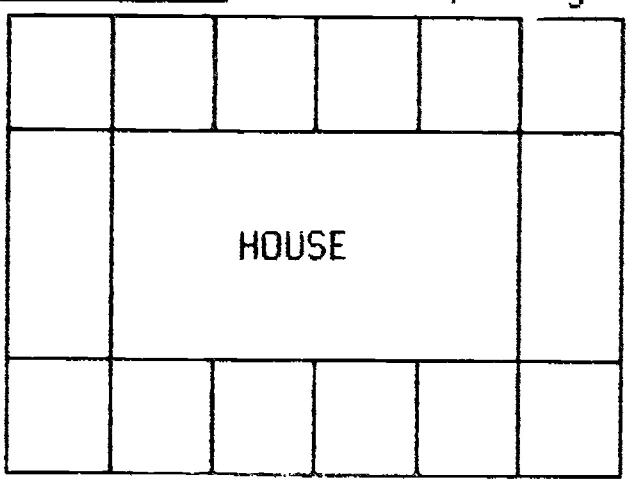
LESSON SIX

Introduction: Show a transparency of the ZOO or DISNEY WORLD. Start a piece at the entrance and show the students which gates the piece CAN or CANNOT enter. Repeat with a second piece.

Activity: Assign the ZOO and DISNEY WORLD cards to pairs of students. Each student is to record which pieces successfully entered the ZOO or DISNEY WORLD.

LESSON SEVEN: How Does Your Garden Grow?

Introduction: Use a transparency made from the House master:




Tell the children that a farmer lives in the large space called the house. The smaller spaces around the house are the fields for crops. Each space is to have a different plant. Suppose each block is a plant, ask the children to arrange them so that each plant (block) differs in 2 ways from the plant in the next field.

Ask the children: "How are neighboring plants alike?"
"How are neighboring plants different?"

The same game can be repeated with pictures of fruits and vegetables, where the children arrange these pictures so that the spaces will differ from the next space in one way.

Variations: One can try to go from the end of the path back into the center as well as going from the center to the end. You can also change the rule from different in one way to alike in one way.

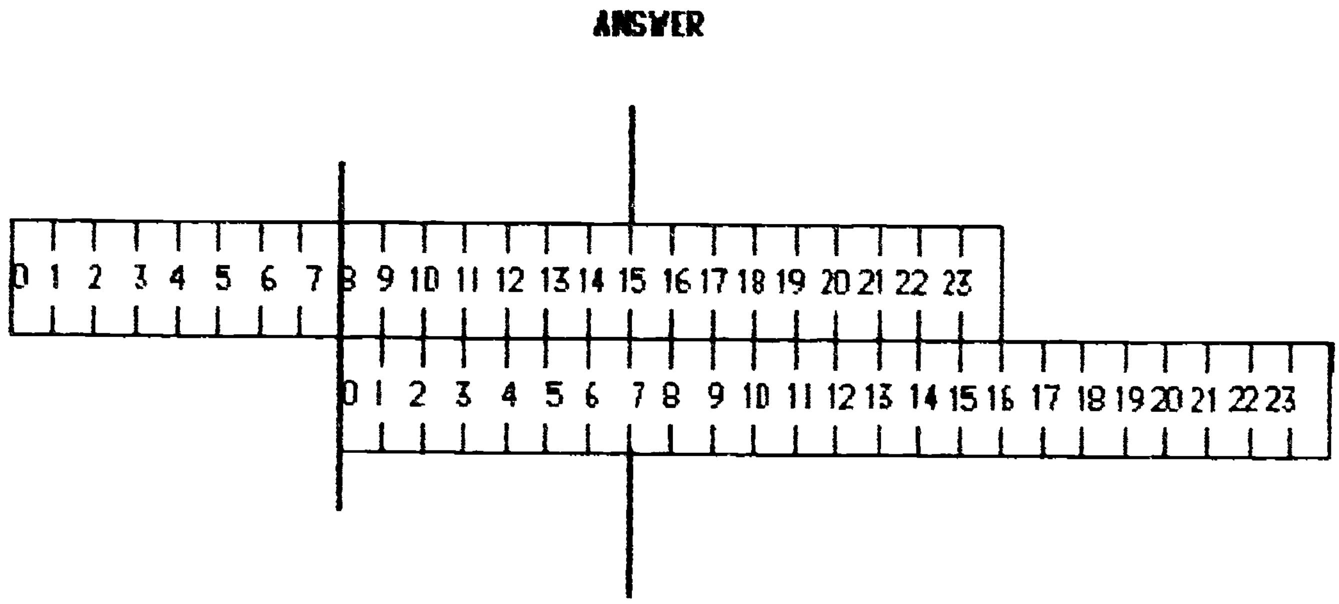
LEVEL THREE

COMPUTATION: NUMBER LINE SLIDES

LESSON ONE

Introduction: Use the black line master provided to make number line slides on a heavy paper or light cardboard. Make overhead transparency slides. On the overhead, place the slide as shown.

"This is to add 7 to 8. I set the "0" of one slide opposite "8" on the other. Opposite "7" on the first slide is the answer - "15."

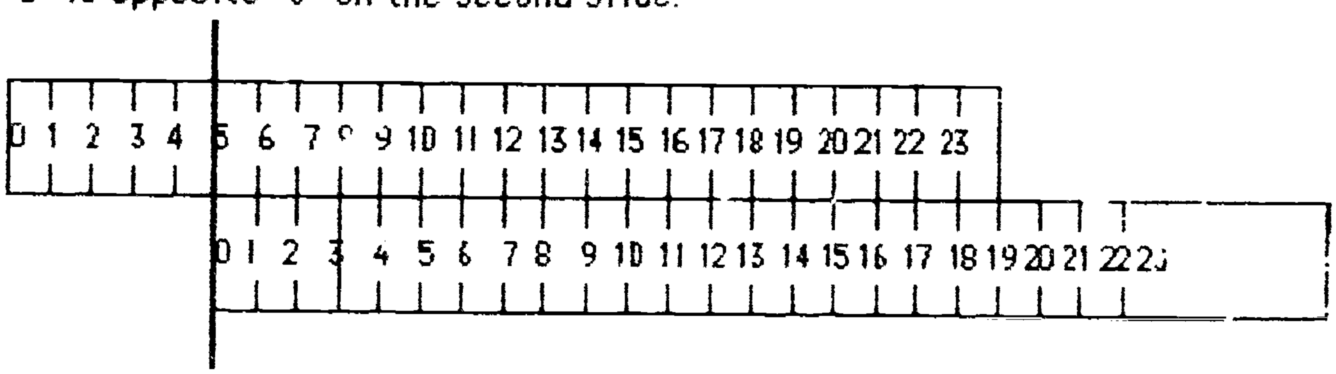



Activity: Children should use number slides to do the additions on the worksheet.

LESSON TWO

Introduction: Write the open sentence 12 - 7 = on the chalkboard. Ask the children to show you how to set the number slides to do this.

Find "12" on one slide. Set "7" on the other slide opposite this. The result "5" is opposite "0" on the second slide.



Activity: Have children use number slides to work the subtraction problems on the worksheets.

## LEVEL THREE

### FRACTIONS: PATTERN BLOCKS

### LESSON ONE

Background: This is a review lesson from Level Two. The purpose is to remind children of a whole divided into three equal parts, into two equal parts and into six equal parts. They also need to be reminded that $1/6 < 1/3 < 1/2$. At this level, work with the fractions will be with Cuisenaire rods primarily.

Introduction: On the overhead projector, cover a blue Pattern Block with two green pieces: "The green piece is what part of the blue piece?" "If the blue piece is ONE, what fraction is the green piece?"

Cover the red pattern block with green pieces: "The green piece is what part of the red piece?" "The blue piece is what part of the red piece?" (You may have to show both the blue and the red covered by green pieces so the 2 to 3 ratio can be seen.) Emphasize the idea that the green piece is a "common measuring unit" for both the red and the blue.

Tell the children the yellow piece will be the whole that all the other pieces are to be compared to. It is the ONE.

"What fraction is the red piece when the yellow is ONE?"

"What piece will be one-third?"

"What fraction is the green piece?"

When it is clear that the children see: R = 1/2; B = 1/3; G = 1/6, have them work on the activity.

Activity: The children should work in pairs. They should have a small collection of the red, blue, green and yellow Pattern Blocks. They are to complete the worksheets.

### LESSON TWO

Background: This is to give additional experience with the "Sixths" family of fractions.

Introduction: On the overhead projector, use the yellow, red, blue and green blocks to do the following:

"What fraction does the red block show?" Write 1/2.
"What fraction does the green block show?" Write 1/6.
"What fraction of the red block is NOT covered by the green block?" 1/3 or 2/6.
"The green block 'subtracts' from the red block." Write 1/2 - 2/6 = 1/3.
"What block fits on the rest of the red block?"

Cover the red block with three green blocks and write 1/2 = 3/6.

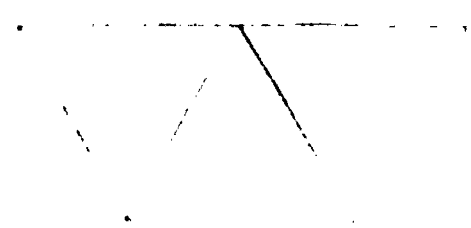
Go back to the equation 1/2 - 1/6 = 1/3 and under it write:
3/6 - 1/6 = 2/6.

Ask the children how they could show that 2/6 = 1/3 and these are two names for the same fraction like 1/2 and 3/6 are.

Activity: Group children in twos with Pattern Blocks to do the next worksheets.

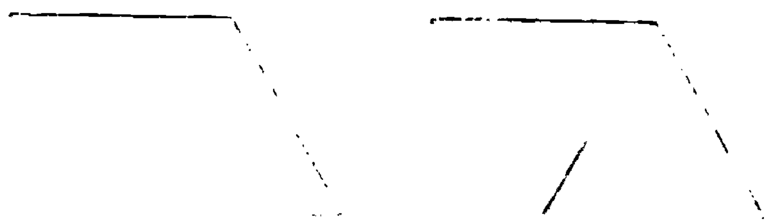
LESSON THREE

Place the red block and three green blocks as shown:



Put another green block on the red block.
"The green block is what part of the red block?"
"So one third of one half is what fraction?" Write: 1/3 x 1/2 = 1/6

Put the blue block and two green blocks as shown:

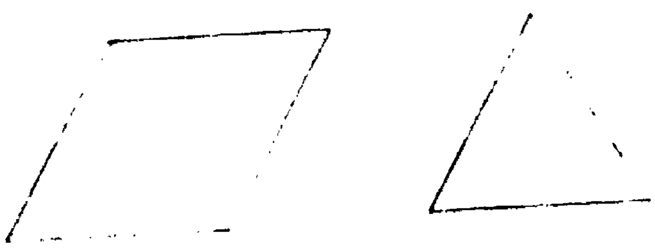


Put a green block on the blue block. "The green block is what part of the blue block?" "So one half of one third is what fraction?"
Write: 1/2 x 1/3 = 1/6.

Activity: Group children in twos with yellow, green, blue and red Pattern Blocks and have them work on the worksheets.

<u>LESSON FOUR</u>

Introduction: Place a blue block and a green block on the overhead projector:



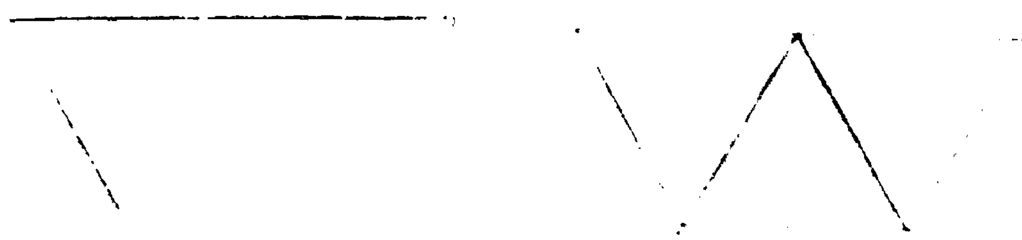
Ask the children how many green block shapes could be made from the blue block. (2) Hold up a UNIFIX cube link of four and ask how many twos could be made from it.

Write 4 ÷ 2 to show this and complete it as 4 - 2 = 2.

"What fraction is the blue piece?" Write: 1/3
"What fraction is the green piece? Write: 1/6

"How many green shapes from the blue?" Write: 1/3 - 1/6 - 2.

Talk about the similarity of these. HOW MANY OF ______ FROM ______?
Place a red piece on the overhead with three green pieces as shown:



"What fraction is the red piece?" 1/2
"What fraction would you get if you divided the red pieces into three equal parts?" 1/6
Write: $1/2 \div 3 = 1/6$

<u>Activity:</u> Group the children in twos. Each should have a few yellow, green, red and blue pieces. Have them work together on the worksheet.

LEVEL THREE

FRACTIONS: CUISENAIRE RODS 1

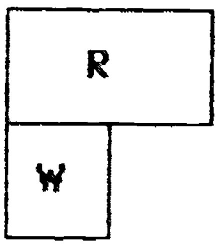
Work with fractions is based upon a recognition of equal part-whole relations and an understanding of measurement. Denominators are measuring units - the larger the unit, the fewer needed and vice versa. Hence, it takes three thirds and four fourths to measure the same ONE. When children have identified this, then the operations with fractions are just like those with whole numbers. Fractions are joined, separated into parts and compared like whole numbers are. Cuisenaire Rods or fractions strips can be used to measure and are a natural follow up to initial intuitive work with Pattern Blocks.

LESSON ONE

Background: This lesson is to focus on the simple fractions of one half, one third, one fourth, with different lengths as ONE.

Introduction: Put a red rod (R) on the overhead. Ask the children to put that rod in front of them.

"Find a rod that is one half of that." Place the rods as shown:

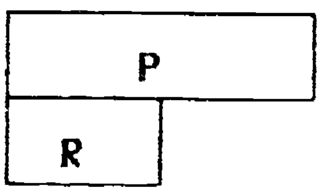



**Write W = 1/2 R**

"Find a rod that is ONE HALF of the light green rod (G)."

There is no such rod in the set. If any child thinks there is one, be sure to get the confusion about the meaning of "one half" cleared up.

"Find a rod that is ONE HALF of the purple (P) rod." Place this arrangement of rods on the overhead:



Activity: Pass out the worksheets for children to work on in pairs, using the Cuisenaire Rods. Make a chart to hang in the room with the symbols on it:

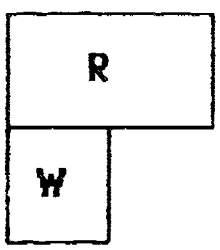
| | |
|---|---|
| White is | W |
| Red is | R |
| Light Green is | G |
| Purple is | P |
| Yellow is | Y |
| Dark Green is | D |
| Black is | K |
| Brown is | N |
| Blue is | E |
| Orange is | O |

LESSON THREE

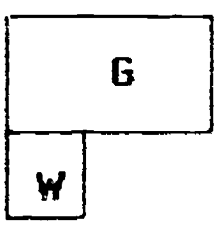
Background: In developing fraction families using Cuisenaire rods, a DIFFERENT rod or rod train will represent ONE in each family. W will always be A common measuring unit or A COMMON DENOMINATOR. Children need some experience with different ones.

Introduction: Place a red rod (R) on the overhead projector and have the children place one of this length rod in front of them.

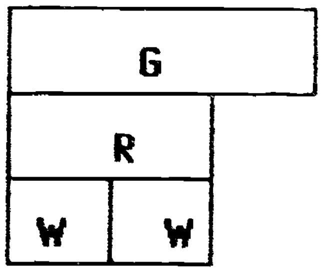
"If I let R be ONE, what fraction is W?" (1/2)




"If I let G be ONE, what fraction is W?" (1/3)

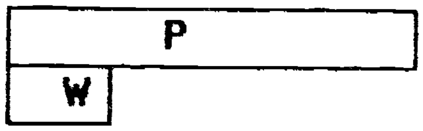



"If I let G be ONE, what fraction is R? (2/3)




Refer to the 2W to show R is two times as long as the 1/3 length, so is 2/3.

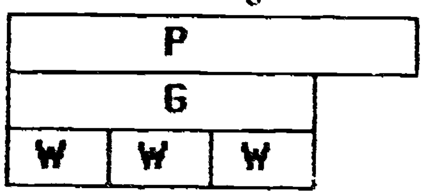
"If I let P equal ONE, what fraction is W?" (1/4)




"What fraction is R?" (1/2 or 2/4)

Emphasize the two names. R is one of two equals that make P or it is equal to two of four equals that make P.

"What rod represents three fourths?" (G). Again, show this is three of the 1/4 rod in length since G=3W.



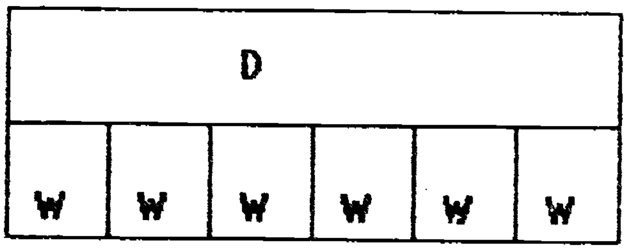

Be sure the children see how W can be used to find what part each rod is of another before asssigning work to the chilor en.

Activity: Give pairs of children a set of Cuisenaire Rods and have them work together on the worksheets.

LESSON FOUR

Introduction: As you demonstrate on the overhead projector, have the children follow what you do with Cuisenaire Rods at their desks.

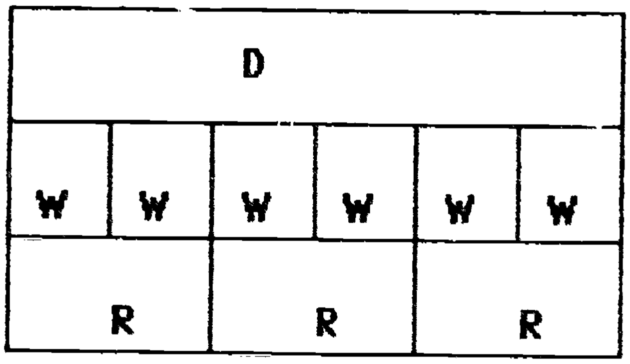
Name the dark green (D) rod as ONE. Ask the children to make a row of white (W) rods equal in length to the D rod. Place these on the overhead




"What parts of a D rod is a W rod? (1/6)

"Now MEASURE the D rod with red (R) rods."

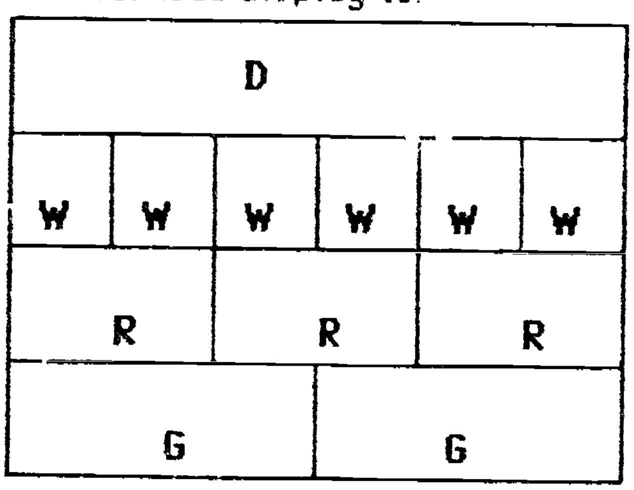
Do this on the overhead.




Have the children arrange their rods this way. Ask several questions about the relationships between the rods such as:

"Two Ws are what part of the D?"
"Three Ws are what part of the D?"

Now ask them to MEASURE the D rod with the light green (G) rods. Change the overhead display to:




Ask several questions to relate these rods such as.

"How many R rods would you need to make a G rod?"

"A G rod is how many times as long as an R rod?"

"A G rod is what part of a D rod?"

"An R rod is what part of a G rod?"

A W rod is what part of an R rod?"

Write: D = 1 = 6/6
G = 1/2 = 3/6
R = 1/3 = 2/6
W = 1/6 = (W measures EVERY rod!)

Ask the children to put these rods in order from smallest to largest.

"Which is the shortest?" Write 1/6

"Which is next?" Write 1/6 < 1/3 Point out the < symbols is used to point toward the smaller.

"Which is next?"

Write 1/6 < 1/3 < 1/2 Reinforce the use of <

Write 1/6 < 1/3 < 1/2 < 1

"Are W, R and G all LESS THAN one?"

"Is one half MORE THAN or LESS THAN one third?":

Keep pointing to the symbols as you say the words, e.g. one half - 1/2.

LEVEL THREE

FRACTIONS: EQUIVALENT FRACTIONS: CUISENAIRE RODS

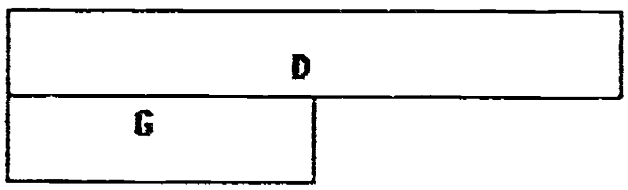
Background: Children should see that fractional parts can be expressed in more than one way. Using a common measure makes this easy to see.

Within each "family" of fractions as introduced, equivalence of fractions should be initially emphasized.

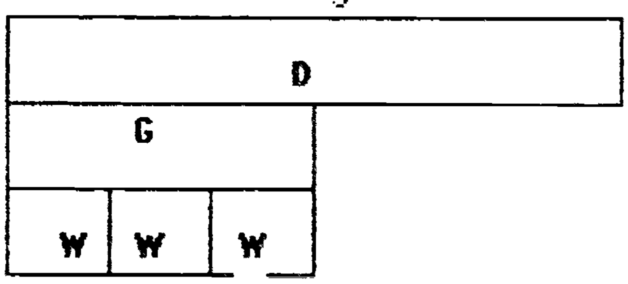
LESSON ONE

Introduction: Use overhead Cuisenaire Rods. Children should have rods to use as you demonstrate. Place the D rod on the overhead:


"Find the rod that is ONE HALF of this." (G)




"Put W rods below your G rod."



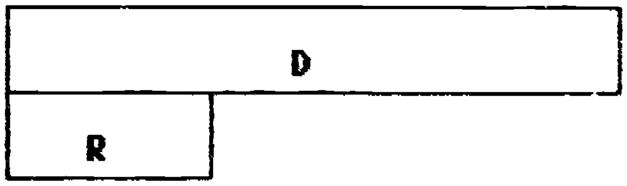

"Each W rod is what fraction?" (1/6)

"How many W rods are the same as a G rod?" (3)

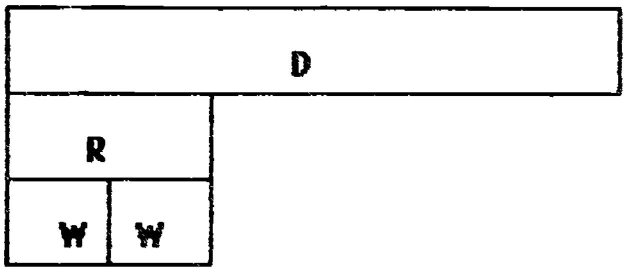
Write: 3/6 = 1/2
3 W = G

Emphasize these are two names for the same fraction

"Find the rod that is ONE THIRD of the D rod." (R)

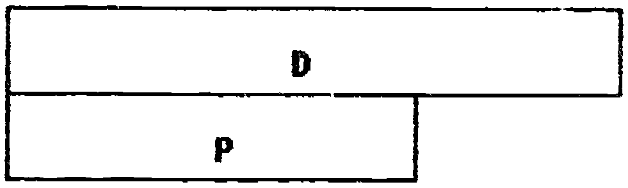



"How many W rods are the same as an R rod?" (2)

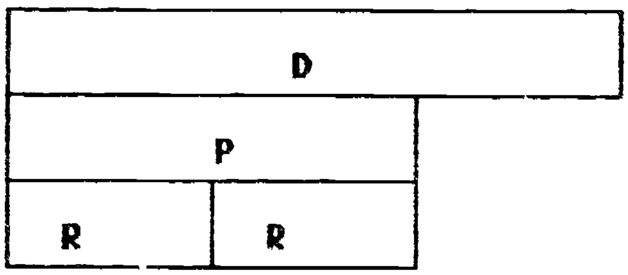



Write: 2/6 = 1/3
2 W = R

"What fraction does the P rod represent?" (2/3)

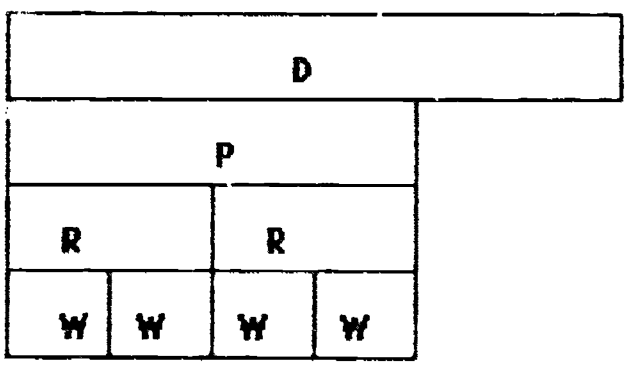



"Put two ONE THIRD (R) rods below this."




Write: 2/3 = 1/3 + 1/3
P = R + R

"Put W rods under the R rods."



Write: 2/3 = 4/6
2 R = 4 W

Emphasize: The DENOMINATOR shows how many EQUAL parts are in the ONE.

The NUMERATOR COUNTS how many of these EQUAL parts there are.

Since a whole can be divided into EQUAL parts in many ways, a fraction can have different denominators and have several names.

Summarize:

1/2 = 3/6
1/3 = 2/6
2/3 = 4/6

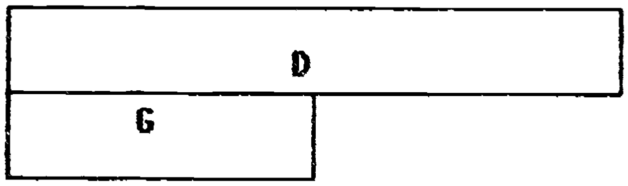
Activity: Give pairs of students C rods and the worksheets to work on. As you walk around observing what they are doing, ask question that direct their attention to the EQUIVALENCE of fractions.

## LEVEL THREE

### FRACTIONS: COMPARING: CUISENAIRE RODS

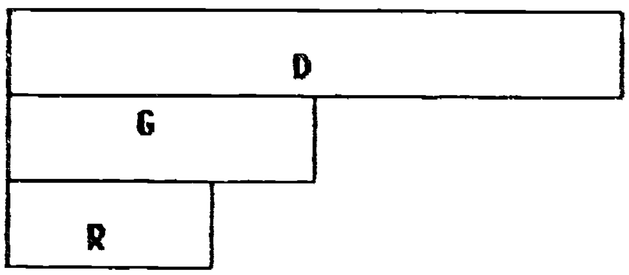
Introduction: Use the overhead to show the D rod as ONE.


Put the G rod below it:



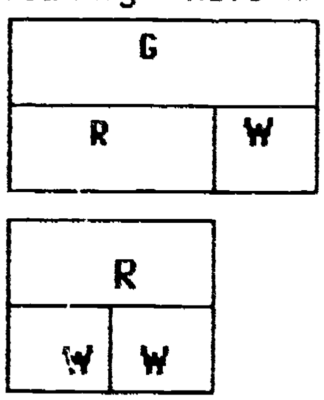

"What fraction is the G rod?" (1/2)

Put the R rod below that:




"What fraction is the R rod?" (1/3)
"Which is larger?" (1/2)
"Which rod is part of the other rod?" (R is part of G)
"Could you get more than one or less than one R rod by cutting up the G rod?" (more than one)
"What part of another R rod could you get?" (1/2)

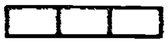
You might have to show this:




**The W, that is how much more G is than R, is 1/2 of R**

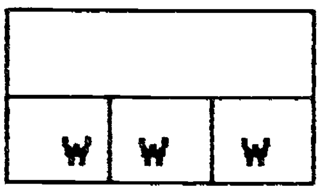
Write: G = R + 1/2R
G  R = 1 1/2

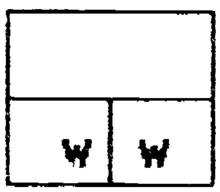
Discuss this thoroughly. Use UNIFIX cubes to relate this to whole numbers:



"Can I get a two link from this?"

"What is left is what part of another two link?" (1/2)
"So I get one + one half or One and one half when I divide the three links by two?"
"Dividing fractions is just like dividing whole numbers. I must find how many of the smaller I can get from the larger?"
"Couldn't I get a G rod from the R rod?"
"Could I get PART of a G rod from an R rod?"
"What part could I get?"
"Could I use W rods to check this?"




Write: R  G = 2/3
or   1/3  1/2 = 2/3

Emphasize that you can only get PART of a larger thing from a smaller thing, or something less than one."

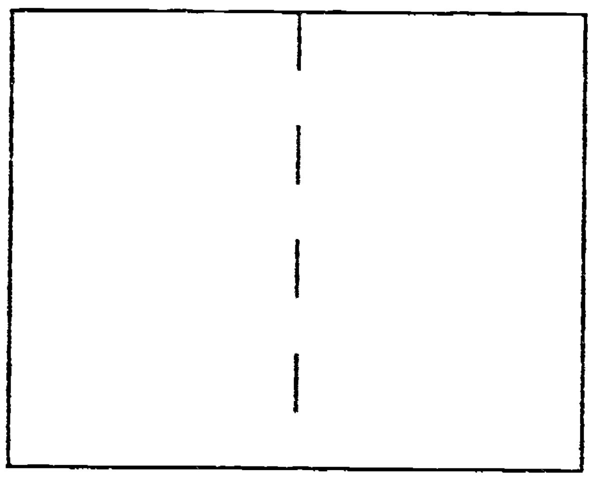
<u>Activity</u>: Give worksheets to pairs of students to work, using Cuisenaire Rods. As you monitor the work, keep asking questions about the relative sizes of the fractions. Before starting them to work on the worksheet, go over the two examples completely.

LEVEL THREE

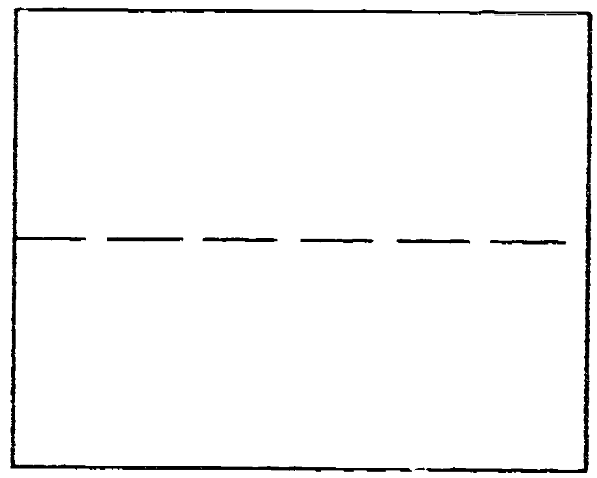
FRACTIONS: PAPER FOLDING

Introduction: Hold up a sheet of paper. Ask the children how it could be folded in half. The two most common responses will be to fold as shown by the dotted lines in A and B:

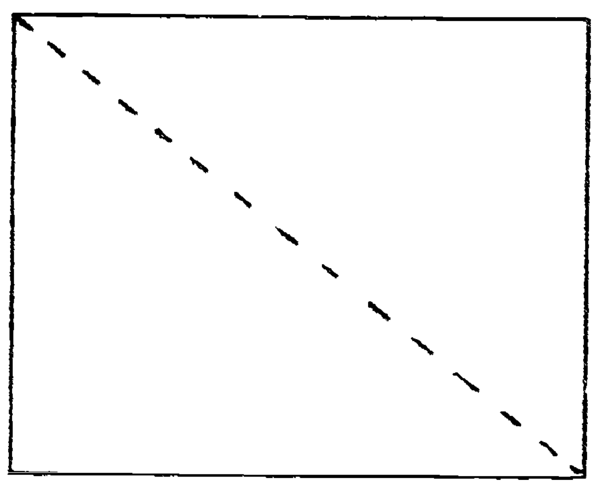
A



B



If no one suggests folding as shown in C,



C

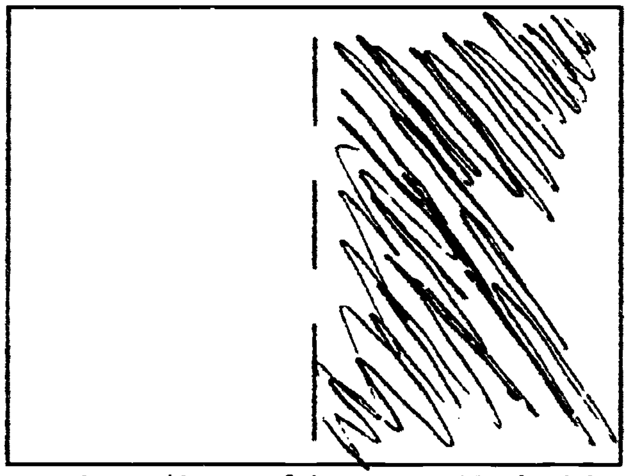
ask if folding it that way would result in it being folded in half.

"What is a half?" (The idea should come out that it is one of two equal parts!).

"How many halves are there in a whole?"

If you have to, fold as in C and tear along the fold and show that the two parts can be made to coincide. Ask the children if there are other ways to fold the sheet into two equal parts.

Pass out two plain white sheets of paper (8 1/2 x 11 works well) to each child. Ask them to take 3 very different color crayons from their desks or pass some out. Ask them to fold their sheets as in B. Have them take one color and put 5 or 6 lines on one half as shown:



Ask how it should be labelled. Write 1/2 on the board. Point out that the "2" counts how many equal parts there are, and the "1" counts which of those you ar^ working with. Have them write 1/2 on the striped half with the same color of crayon as the stripes, and refold it with the colored part inside.

"What part of the whole are you looking at now?"

"How would you find half of the ONE HALF?"

Have them fold it in half again.

"How many equal parts will there be when you open it up?"

"What will each part be called?"

Accept ONE FOURTH but NOT a quarter! Ask them to open it up.

"How many equal parts are there now?"

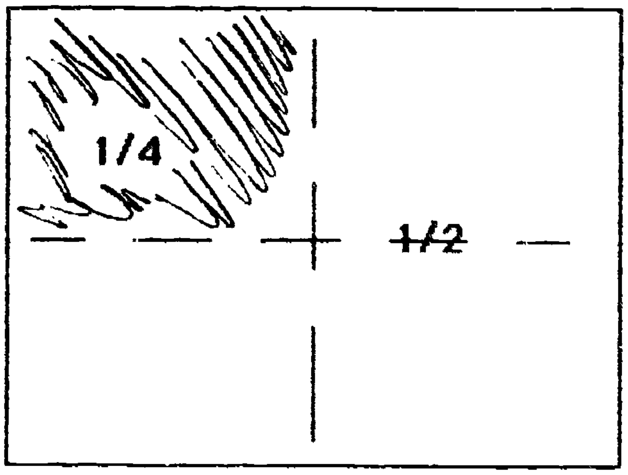
"What part of the whole is each part?"

"How many FOURTHS are in the whole?"

"Can you find a part of the paper that is two-fourths?

"Three-fourths?"

Have them take a second color and color one of the two uncolored FOURTHS as shown and label:

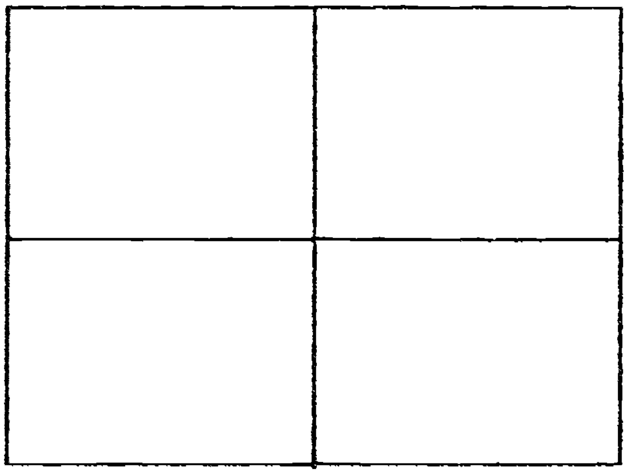



Ask how many FOURTHS are in the HALF? Write 1/2 = 2/4 on the board. Point out that the "4" shows the number of equal parts we now have and the "2" how many of these are in the HALF.

Put the following on the board and ask the children to (1) copy the problem, and (2) use the parts on their sheet to find the results:

| | | |
|---|---|---|
| 1/4 + 1/4 = | 1/4 + 3/4 = | 1/4 + 2/4 = |
| 3//4 - 1/4 = | 3/4 - 2/4 = | |
| 1/2 - 1/4 = | 1 - 1/4 = | |
| 1 - 1/2 = | 1/2 + 1/2 = | |

Ask the children to supply answers to each one, one at a time and tell how they found them. If needed draw on the board and have children shade in, for example, 1/2, then another 1/4:

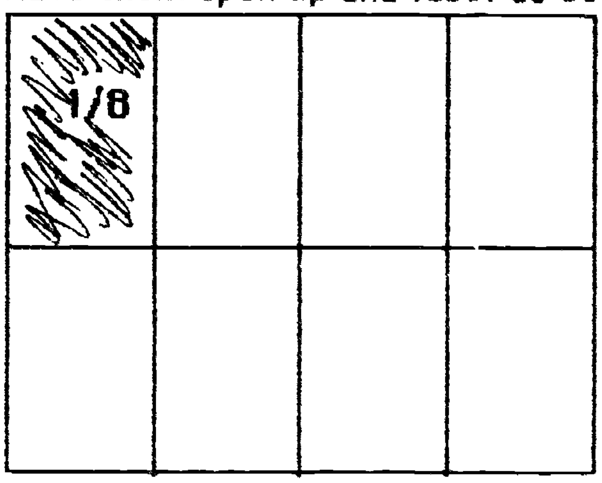


"How much is shaded?"

Reinforce with questions like, "How much is one half of one half?"

<u>Extension:</u> As before, refold the paper that has been folded in FOURTHS. Hold up and be sure the size seen is identified as ONE FOURTH. Ask children to predict how many equal parts would result if the ONE FOURTH is folded in half?

A surprising number of children will think there are 6, not 8, equal parts. Have them open up and label as before with a 3rd color of crayon:




**This will leave one unshaded piece - a ONE EIGHTH. Also, ask questions as before:**

**"How many EIGHTHS in ONE HALF?**

Write the equivalent forms as the children supply these:

1/2 = 2/4 = 4/8 1/4 = 2/8, etc.

Ask the children to find 3/8, 5/8 and 7/8 on their sheets. Then have them add 1/8 + 1/2 using the parts on the sheet. Discuss the result.

Then have them find 1/2 - 1/8 using their sheets. Be sure that they can do this using the pieces on the sheet.

Activity: Give them the worksheet and have them USE THE PIECES ON THE PAPER to find the results. Closely monitor the work, showing how the pieces are added together or one piece taken from another

FRACTIONS: SEPARATING: UNIFIX CUBES

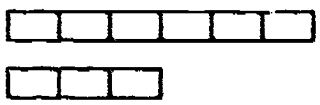
LESSON ONE

Show the students a link of six green UNIFIX cubes. Tell them this is going to be ONE like the D rod is ONE.

Using colored squares on the overhead projector, make a facsimile of the UNIFIX cube ONE with squares:

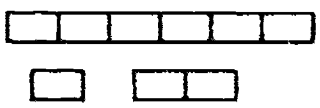
"How many equal parts are here?" (6)
"Each one represents what fraction?" (1/6)
"Two will represent what fraction?" (2/6)
"What is another name for that fraction?" (1/3)

Place 3 blue squares below this row of green squares:



"What fraction does this row of squares show?" (1/2)

Remove two of these to one side:

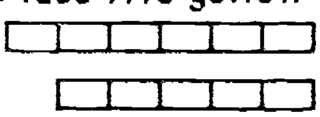


"How many sixths have I taken away from the 1/2?" (2)
"Two sixths is what fraction?" (1/3)
"What is left?" (1/6)

So write: 1/2 - 1/3 = 1/6

"Here the minus sign shows *take away*"
"I took 1/3 from the 1/2, leaving 1/6 behind."

Place five yellow squares on the overhead:



"What fraction is this?" (5/6)

Remove a square:
"What fraction did I take away?" (1/6)
"What is left?" (4/6)
"What is another name for this fraction?"(2/3)

Write: 5/6 - 1/6 = 4/6
5/6 - /6 = 2/3

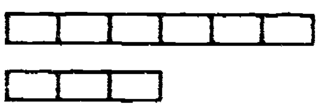
Take away another square:
"What fraction did I take away from the four sixths or two thirds?" (1/6)

Write: 4/6 - 1/6
2/3 - 1/6

"What fraction is left?" (1/2)
"How many sixths are in this fraction?' (3)

Write: 4/6 - 1/6 = 3/6
4/6 - 1/6 = 1/2
2/3 - 1/6 = 1/2

"These all describe this take away."
"What fraction is left?" (1/2 or 3/6)



Remove two squares:
"What fraction did I take away?" (2/3 or 1/3)

Write: 1/2 - 1/3
3/6 - 1/3
3/6 - 2/6

"How many sixths are left?"

Complete the number sentences:

1/2 - 1/3 = 1/6
3/6 - 1/3 = 1/6
3/6 - 2/6 = 1/6

Repeat starting with four squares (23/) if necessary.

<u>Activity</u>: Give students the worksheets and UNIFIX cubes. They are to make TWO six cube links. One is to be kept intact to show ONE. The other is to take away from. Monitor the work. Ask frequent questions about the equivalence of sixths with halves and thirds as the children work.

LEVEL THREE

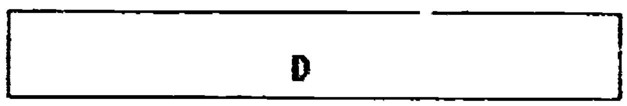
FRACTIONS — FINDING THE DIFFERENCE

It is impossible to "take away" using Cuisenaire Rods, so this is a good model to emphasize the "finding the difference" interpretation of subtraction.

LESSON ONE

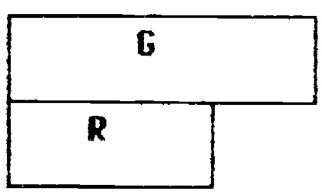
Introduction: Use overhead version of C rods. Students should have C rods to follow what you do.

Place the D rod on and remind them this is ONE:




Place the G rod on the overhead. "What fraction does this rod show?" (1/2)


Write 1/2 on the board. Put the R rod below the G rod:




"What fraction does the R rod show?" (1/3)

"What rod will fill in the space between the G rod and R rod?" (W)

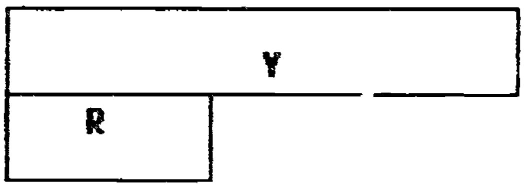
"That space is the DIFFERENCE between the G rod and the R rod. You just found that difference. What fraction is the W rod?" (1/6)

Write 1/2 - 1/3 = 1/6
G - R = W

"Here the minus sign shows the DIFFERENCE between the fractions. Give me an example of a difference between whole numbers where the minus sign is used."

Place the Y rod on the overhead:


Place the R rod below it:




"Find the rod that is the difference between 5/6 (the Y rod) and 1/3 (the R rod).)" This will be G.

"Write the number sentence to show what fraction is the difference between 5/6 and 1/3." (5/6 - 1/3 = 1/2)

"Can you write this another way." (5/6 - 2/6 = 3/6 = 1/2) (some variants of this)

<u>Activity</u>: Give pairs of students worksheets and rods to use to complete them. Monitor the work. Keep asking questions about "the DIFFERENCE" - which fraction is greater, which is smaller and if the difference joined to the smaller will give the larger.

LEVEL THREE

FRACTIONS: SQUARES

LESSON ONE

Introduction: This lesson is to give children experience in comparing fractions. Put a transparency of a square divided in half on the overhead. Label each part:

| | |
|---|---|
| 1/2 | 1/2 |

Put another square on that is in four equal parts and label each part:

| | |
|---|---|
| 1/4 | 1/4 |
| 1/4 | 1/4 |

Put another on that is divided into 8 parts and label each part:

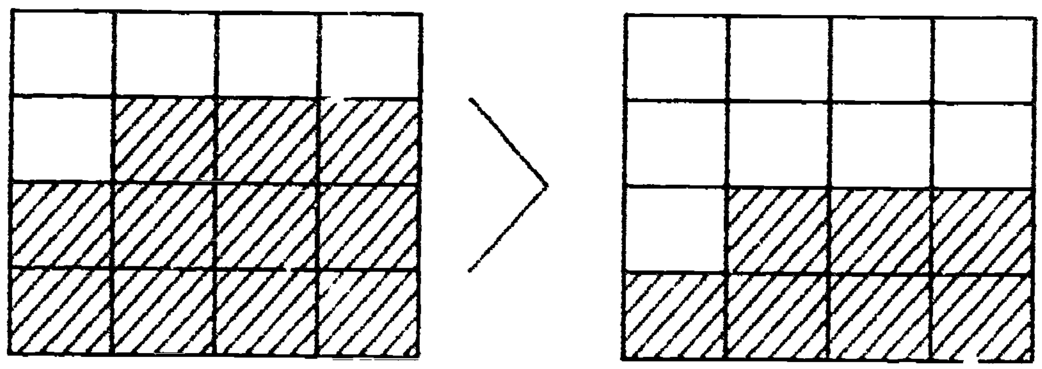
| | | | |
|---|---|---|---|
| 1/8 | 1/8 | 1/8 | 1/8 |
| 1/8 | 1/8 | 1/8 | 1/8 |

Then put the last one on - a square divided into 16 equal parts and label each part:

| | | | |
|---|---|---|---|
| 1/16 | 1/16 | 1/16 | 1/16 |
| 1/16 | 1/16 | 1/16 | 1/16 |
| 1/16 | 1/16 | 1/16 | 1/16 |
| 1/16 | 1/16 | 1/16 | 1/16 |

"How many 1/16 in 1/2?"
"How many 1/16 in 1/4?"
"How many 1/16 in 1/8?"
"How many 1/8 in 1/2?"
"How many 1/8 in 1/4?"

Put two squares that have been shaded on the overhead projector. Have the children tell you how the inequality sign should go then put it in. Emphasize that 11 is more than 7 so 11/16 > 7/16:



<u>Activity</u>: Have the children color in sixteenths as shown on the Worksheets and write the inequalities.

## LEVEL THREE

### NUMERATION

Background: This will give children an idea of HOW BIG numbers are.

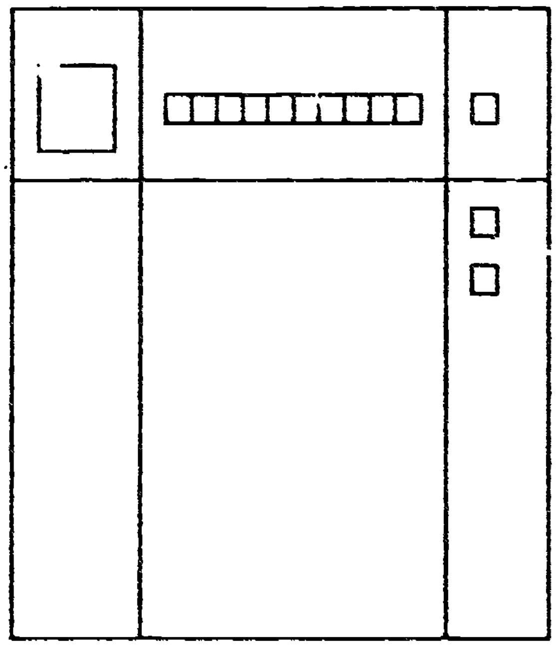
### LESSON ONE

Introduction: Group the children in fours. Give each a set of base ten blocks including the THOUSANDS block. Have them do the building worksheet first then hand out cards with different number words on them. Each group is to build the number on the care received plus the next successive ten numbers. Observe how trading is being done.

LEVEL THREE

## NUMERATION: BASE TEN BLOCKS

Introduction: On the overhead use a place value mat and two ONES. Have the children do as you do and write 2 in the Adding by Two's column of their recording forms.



Mathematician: ____________

| Adding by twos | | | Adding by threes | | | |
|---|---|---|---|---|---|---|
| H | T | O | H | T | O | etc. |
| | | 2 | | | | |
| | | 4 | | | | |
| | | | | | | |
| | | | | | | |

Add two more ONES and have them record "4" on the form.

Activity: Each child should have four TENS and twelve ONES, the place value mat and recording form.

They are to add 2 at a time in the ONES column, trading where necessary until all four TENS are used. Then they add 3 at a time in the ONES column until all four TENS are used. They should complete the recording form.

## LESSON ONE

Introduction: Children should have base ten place mats, hundreds, tens, and ones and the first recording form.

Write "35" on the chalkboard or overhead. Have the children read this in words in unison, then build it on the place mat with base ten materials. Have them draw a picture of the materials on the recording form.

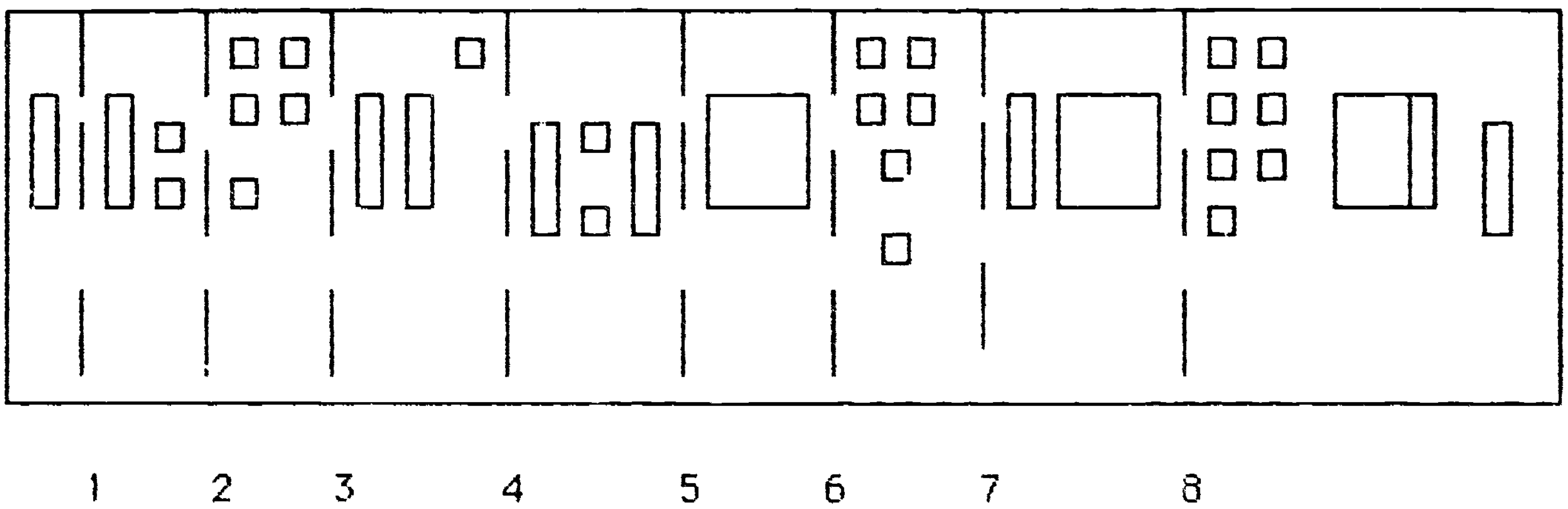
Repeat other numbers until it is clear the children know what is wanted.

Activity: Have the children do the recording forms in order, using the base ten materials. Encourage the use of a constant length heavy line for tens and heavy dots for ones in drawing the pictures.

LESSON TWO

Array a set of overhead base ten blocks in a way similar to that shown below. Make a cover to gradually slide back, exposing the base ten blocks. At each position of the cover the children should write the number visible on the recording form provided.

Reverse the direction of the cover movement to create a different situation.



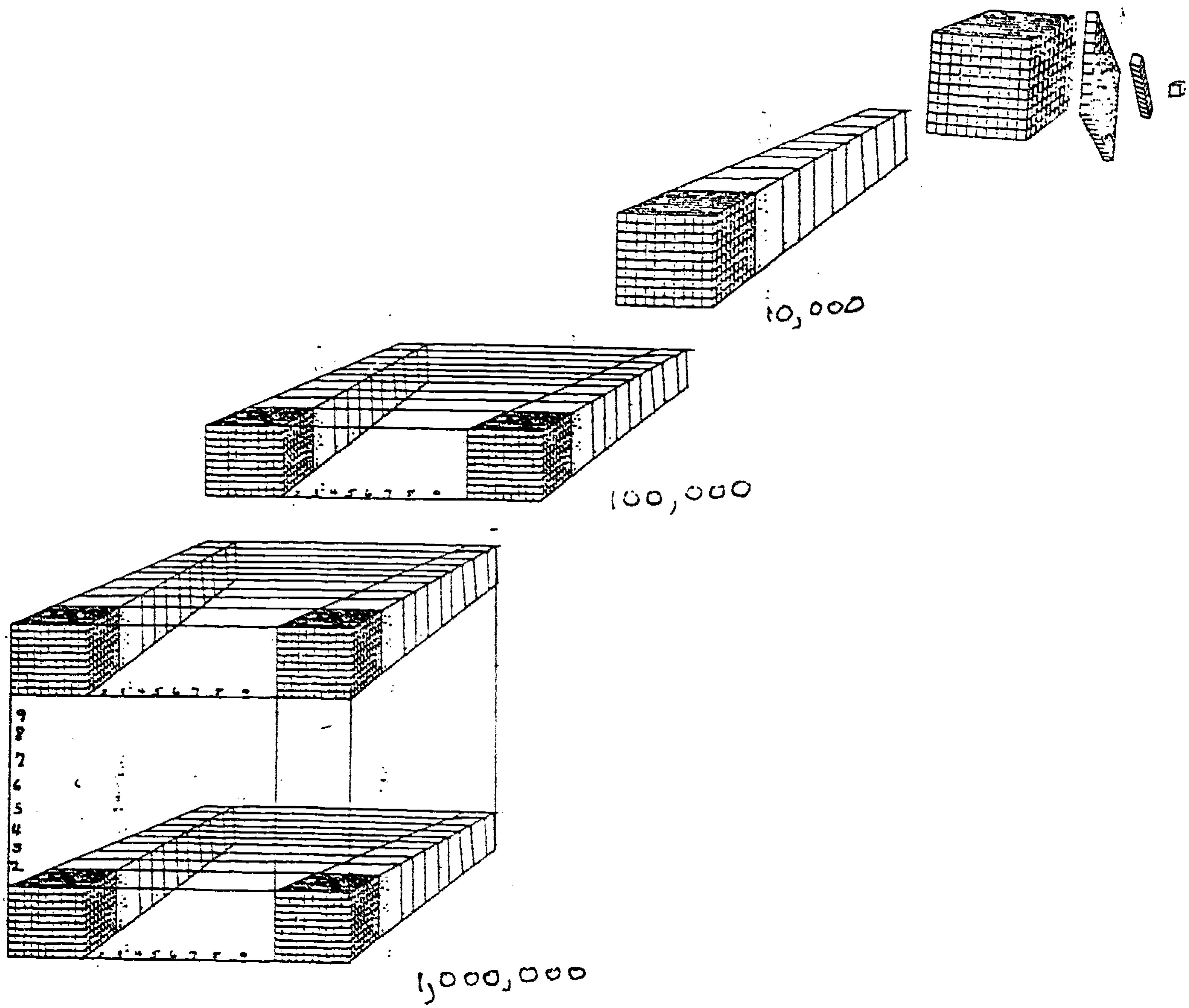

You can make a permanent "mystery box" by gluing cardboard versions of base ten blocks to a board and slide a cover over this. This is suitable for individual instruction.

## LARGE NUMBERS

Background: At this level children should extend place value to several places, though doing no paper and pencil computing with such numbers.

A natural extension is to use the thousands cube to form "longs" of ten cubes, "flats" of ten "long" and a "block" of these "flats."

You will not have thousands cubes in sufficient number to do this. Waxpaper quart milk or juice containers can be cut off to very closely approximate a thousands cube which is a cubic decimeter in volume. The places then would be:



Notice the grouping in THREE places that correspond to where commas are placed when writing the numerals (cube - long - flat - comma) These coorespond also to the multiples of 3 in the expondents:

$$b^{11}\ b^{10}\ b^{9},\ b^{8}\ b^{7}\ b^{6},\ b^{5}\ b^{4}\ b^{3},\ b^{2}\ b^{1}\ b^{0}$$

Applied to base ten, the powers are:

$$\ldots\ldots,10^{8}\ 10^{7}\ 10^{6},\ 10^{5}\ 10^{4}\ 10^{3},\ 10^{2}\ 10^{1}\ 10^{0},\ \text{where } 10^{0} = 1$$

Most calculators will display 8 digits and then go into an overflow mode or else convert to exponential notation, i.e., $1 \le d < 10 \times 10^{N}$. Some experience with seeing large numbers in symbolic form is important as a result. To repeat calculating with these without the use of a calculator makes no sense whatsoever. It is nothing more than tedious busywork!

Have the students measure a link of 10 Unifix cubes. Discuss the responses to each of the following questions:

"How long do you think a link of 100 cubes will be ?"

Reference these to lengths that are present in the room.

"How long do you think a link of 1000 cubes will be ?"

Reference these to objects or distances familiar to the children.

"How long do you think a link of 10,000 cubes will be?"

Do the same here.

"How long do you think a link of 100,000 cubes will be?"

There are approximately:

| Cubes | Distance |
|---|---|
| 10 | 7.5 inches |
| 100 | 6.25 feet |
| 1000 | 62.5 feet |
| 10,000 | 208 yards |
| 100,000 | 1.18 miles |
| 1,000,000 | 11.8 miles |

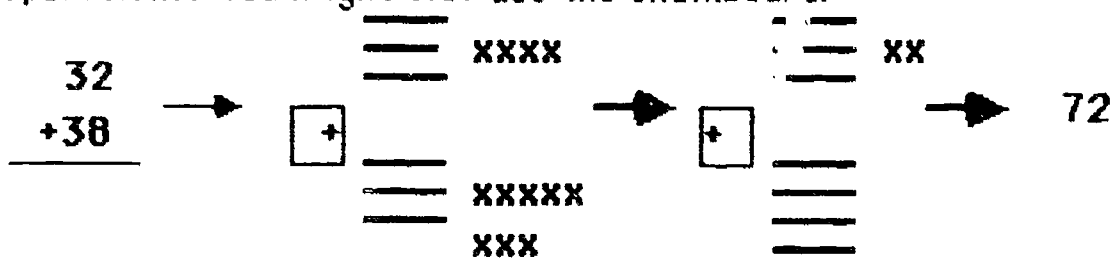
LEVEL THREE

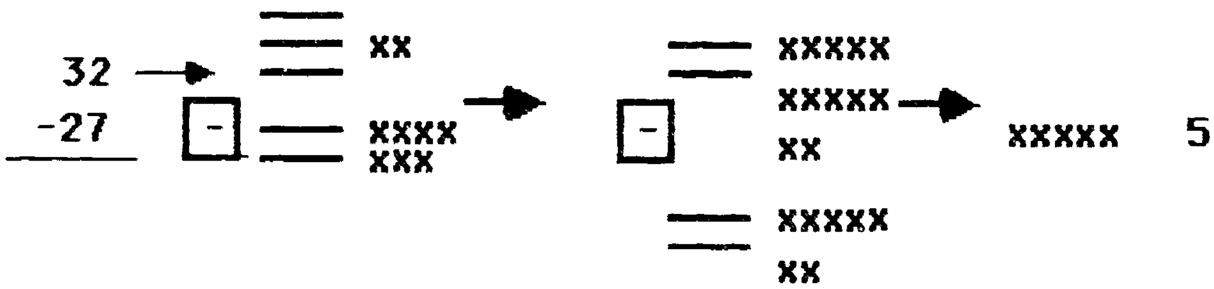
## COMPUTATION: ADDITION & SUBTRACTION

Background: Addition and subtraction in base ten should be maintenance of skill for these students. Introduce it with a brief review then periodically assign exercises, but not too often!!! Permit those who still need base ten blocks to use these, but encourage those who can to work without them.

### LESSON ONE:

Use the overhead projector with base ten blocks to review these operations. You might also use the chalkboard.



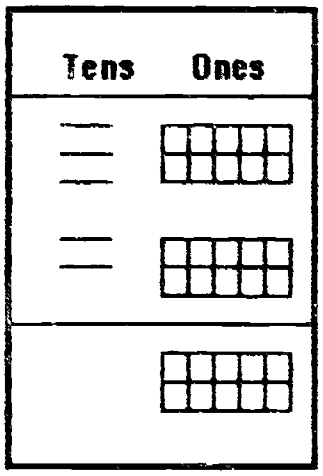

Activity: Worksheets are provided for practice where needed.

### LESSON TWO:

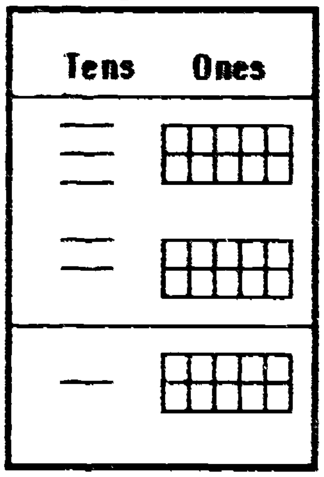
Background: Children should be encouraged to ALWAYS check these operations with the inverse operation.

Introduction: Put the following on the overhead.

$$\begin{array}{r} 34 \\ -23 \\ \hline \end{array}$$



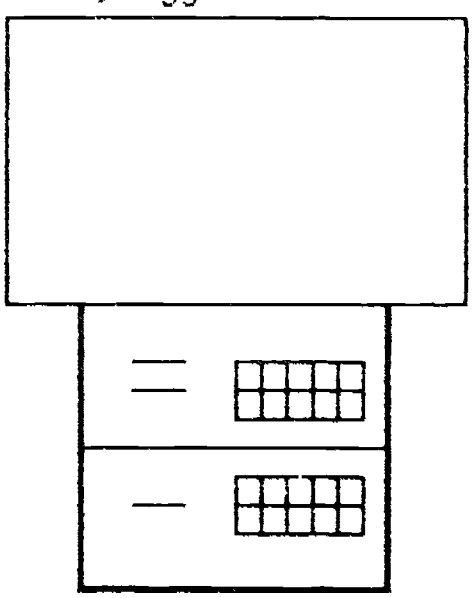

"What goes below the line? Put that in:




and write $\begin{array}{r} 34 \\ -23 \\ \hline 11 \end{array}$

"How can we be sure that is correct?"

If children suggest adding back to see if they get the larger number, fine. If not, suggest that. Cover up the top number on the place value mat:



"What is the result of adding these two numbers?"

Then show the original 34

```
Write: 34        Check: 23
     - 23             - 11
       11               34
```

"How do we check addition?"

"How do we check subtraction?"

Discuss this thoroughly. Use the master provided to prepare additional worksheets as needed.

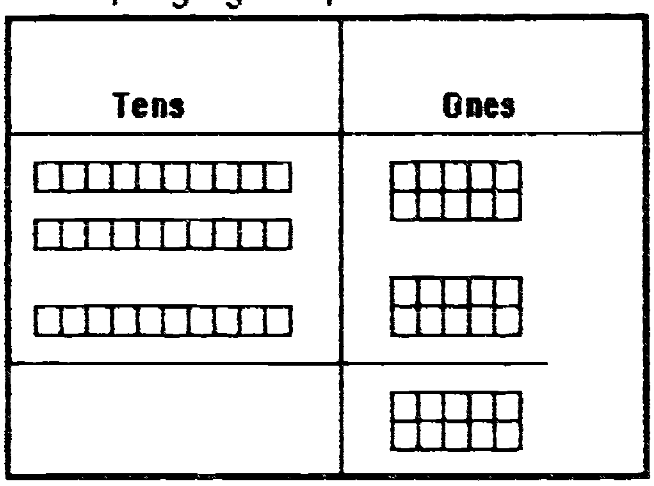
LEVEL THREE

COMPUTATION: SUBTRACTION

Background: Children can interpret the "-" sign either as "take away" or as "finding the difference." The latter involves establishing a one-to-one correspondence to see by how much the larger exceeds the smaller. The former is a separation kind of change. In this lesson you will demonstrate both interpretations and show either kind of thinking leads to the same result. Children will do whichever is most sensible and/or comfortable to them.

LESSON ONE

Introduction: Place the following on the overhead projector and write the accompanying computation form on the chalkboard.




$$\begin{array}{r} 24 \\ -13 \\ \hline \end{array}$$

Have the children arrange their place value mats and base ten blocks the same way.

"We will use the '-' sign to mean take away the number on the bottom from the number on the top."

"What number will we take away?"

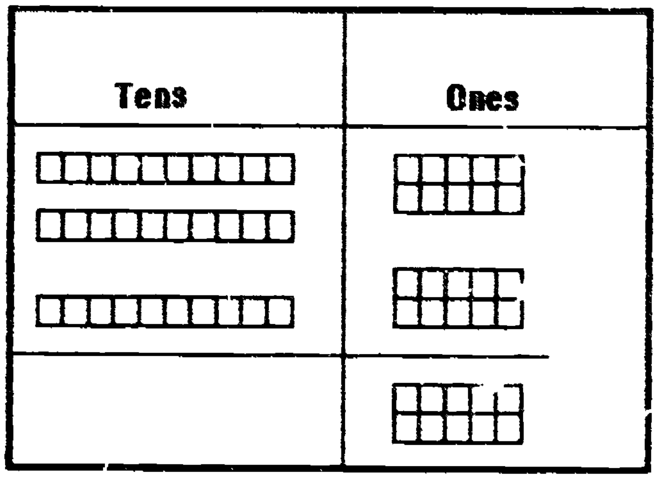
"Which chould I take away first - TENS or ONES?"

Discuss the responses. Children should see that it makes no difference.

Complete the computation form.

$$\begin{array}{r} 24 \\ -13 \\ \hline 11 \end{array}$$

Arrange the following on the overhead:




$$\begin{array}{r} 24 \\ -16 \\ \hline \end{array}$$

"Can I take away one ten?"

"Can I take away six ONES?"

Children should suggest making the trade of one ten for ones in order to take away six ones.

Complete the form:

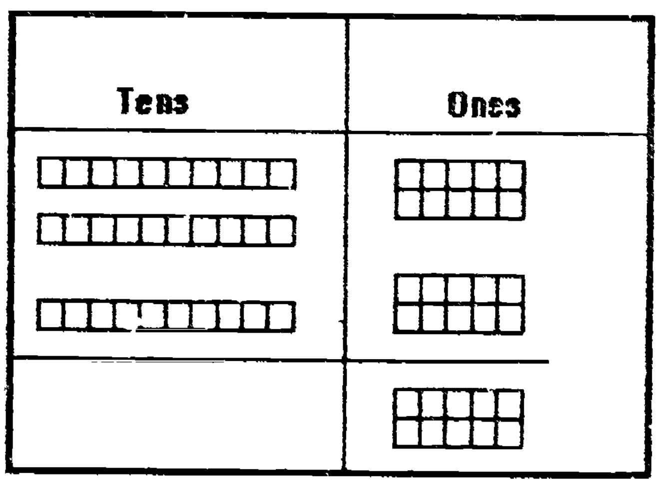
$$\begin{array}{r} 24 \\ -\ 16 \\ \hline \end{array}$$

"Would I ever have to trade more than one ten in the larger number?" Discuss.

Activity: The children should work on the activity sheets using the take away interpretation of "-".

LESSON TWO

Introduction: Set up the problem on the overhead as in Lesson One. The children should follow by arranging base ten blocks on the place value mats:




$$\begin{array}{r} 24 \\ -13 \\ \hline \end{array}$$

"This time we will think of the '-' sign as showing us to find the different between the top number and the bottom number."

"Which of these two is the larger?"

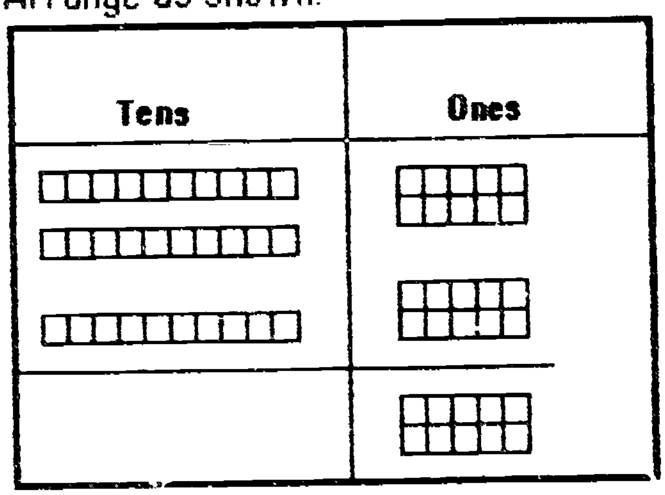
"How much of the larger can be matched by the bottom number?"

"How much of the top number is left over?"

Arrange the materials to show the one-to-one correspondence: one ten from 13 with one ten from 24, and 3 ones from 13 with 3 of the 4 ones in 24. ONE TEN and ONE ONE is the difference.

Write: 24
$$\begin{array}{r} 24 \\ -13 \\ \hline 11 \end{array}$$

Arrange as shown:




$$\begin{array}{r} 24 \\ -17 \\ \hline \end{array}$$

"Can we match a TEN from the bottom number with one from the top number?"

"Can we match all of the ONES in the bottom number with ONES in the top number?"

"What must we do?"

Make the exchange and point out the SEVEN in the top number now unmatched. Write 7 in the computation form.

LEVEL THREE

COMPUTATION: CHECKING ADDITION AND SUBTRACTION

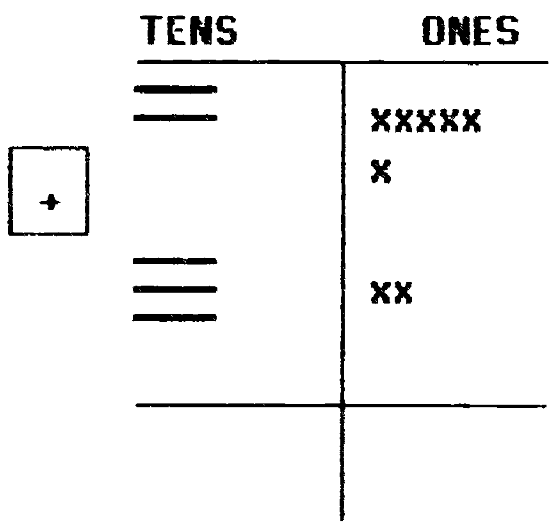
LESSON ONE

Introduction: Children should have base ten pieces and place value mats. Use the overhead projector versions of base ten blocks. Write:

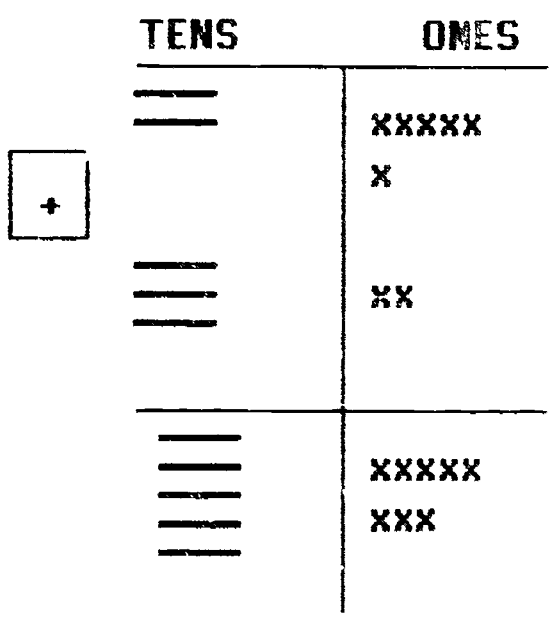
$$\begin{array}{r} 26 \\ +\ 32 \\ \hline \end{array}$$

**on the chalkboard**

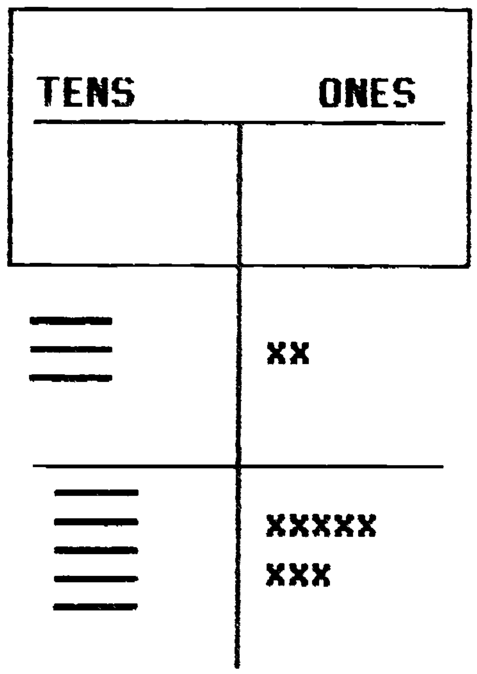
Place the base ten blocks on the overhead version of a place value mat:




Ask the children to set this up on the workmat. Ask for responses and set up as:




Ask the children how we could tell if the answer is correct. Accept subtracting either 32 or 26 from 58. If this is not suggested, ask if this would be reasonable to do. Cover up the top row and remove the plus sign as shown:




Point to the top number and tell the children to set up their place value mats to subtract this from the number below the line. When they have done that, uncover the top row of material and ask the children if this is the same answer. Remind them of 4 + 5 = 9 and 9 - 5 = 4 and that subtraction is used to check addition.

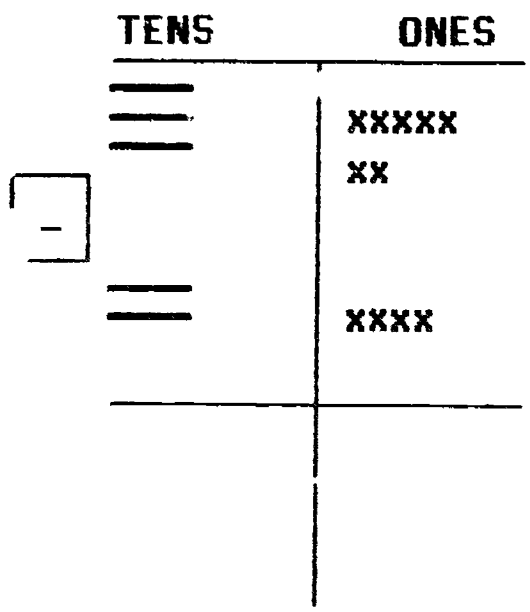
Activity: Pass out the first worksheet. Have the children check each addition done with base ten blocks by a subtraction.

LESSON TWO

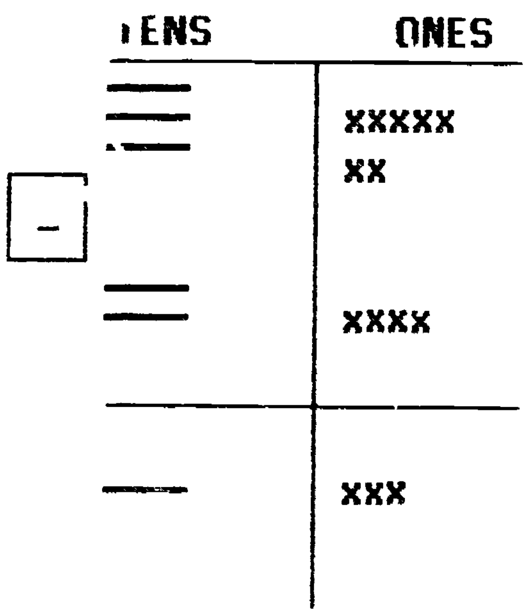
Introduction: As in LESSON ONE, children should have base ten blocks and work mats and you should demonstrate on the overhead projector. Place a subtraction problem on the blackboard:

$$\begin{array}{r} 37 \\ -\ 24 \\ \hline \end{array}$$

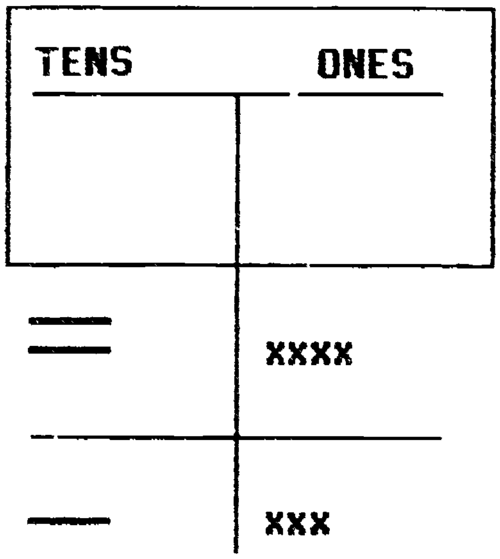
Show this on the overhead with base ten blocks and a transparency minus sign:




Have the children set this up and work it. Then do it on the overhead:




"Did you get the same answer?" "How can we check our answers?" You should elicit if you can and reinforce the idea of adding the result to the smaller number to see if you get the larger number. Cover up the latter as before and have the children set the other two numbers up to add.




When they are finished, uncover your top number and see if that is what the children found. Remind them that you can always check adding by subtracting and check subtraction by adding.

Activity: Have the children work on the second worksheet, using base ten blocks and place value mats. As you circulate, keep reminding them of the inverse nature of these - subtraction "undoes" addition and addition gives back what you started with.

LESSON THREE

Use the worksheets provided and the master copy to make additional worksheets as you see needed. Encourage:

1. Adding the subtraction "answer" to the smaller number in the check.

2. Subtracting either original number from the addition "answer" to get the other addend.

3. ALWAYS checking answers to these two computations by using the inverse operation.

LEVEL THREE

## MULTIPLICATION 1

Background: The children will have had developmental work in the meaning of multiplication and division, using tiles, beans and cups and UNIFIX cubes. This should be repeated at this level using larger numbers.

### LESSON ONE: UNIFIX CUBES

Introduction: Children should have 20-30 UNIFIX cubes. Make several links with 2 UNIFIX cubes in them ahead of time.

"Make links with two cubes in them of your UNIFIX cubes."

When this is done, hold up a link. Join a second to it.

"How many 2's are in this new link?"

Write 2 x 2 = 4 on the chalkboard. Add another link to this.

"How many 2's are in this link now?"

Write 2 x 3 = 6 on the chalkboard.

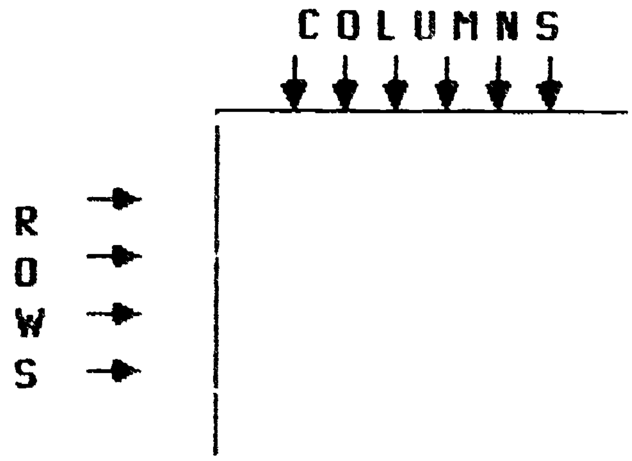
Activity: Pass out the recording forms and have the children complete these using their UNIFIX cubes. Check to be sure the number sentences are being written correctly as you circulate among the students.

Prepare additional recording forms as needed from the master provided and repeat the lesson with ever increasing numbers.

### LESSON TWO: TILES

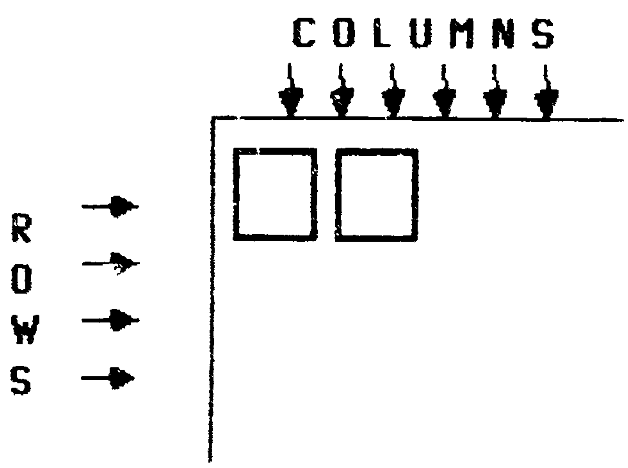
Background: You should have a large tagboard multiplication table that can be filled in by the children as they develop the number combinations. This is the same as the table that they are completing when working with tiles This activity gives an area model of multiplication that (1) reinforces understanding of area, and (2) leads into the model for multiplication in a place value system.

Introduction: Have an overhead transparency model of the tile workmat, the recording table and some square tiles to use on the overhead.




| x | 1 | 2 | 3 | 4 | 5 | etc. |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |
| etc. | | | | | | |

Place two tiles as shown:

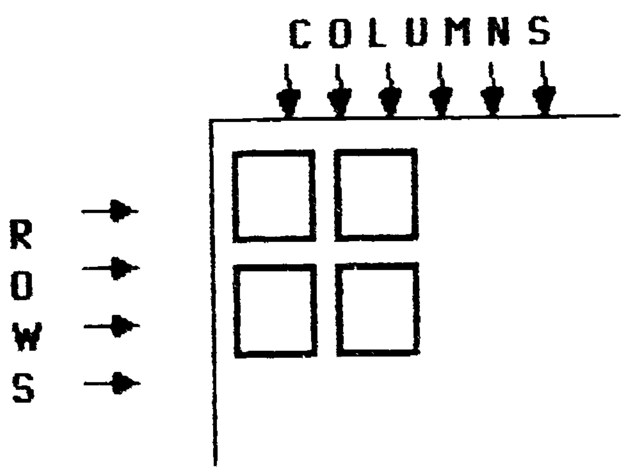


"These tiles are all in the same row. How many columns are there?"

"How many tiles are there?" Record:

| x | 1 | 2 | 3 | 4 | 5 | etc. |
|---|---|---|---|---|---|---|
| 1 | | 2 | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |
| etc. | | | | | | |

Put another row of 2 tiles:




"How many rows are there?"

"How many columns are there?"

"How many tiles in each row?"

"How many tiles in each column?"

"How many tiles altogether?"

Record:

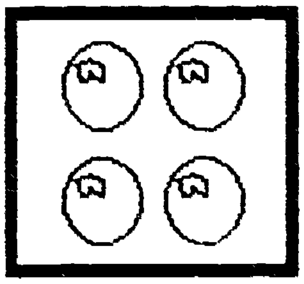
| x | 1 | 2 | 3 | 4 | 5 | etc. |
|---|---|---|---|---|---|---|
| 1 | | 2 | | | | |
| 2 | | 4 | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |
| etc. | | | | | | |

Activity: The children should have work mats and several square tiles of wood or carboard or ceramic material, etc. They should have recording forms. As they enter in these, the number facts will begin to go into memory. Several repetitions of creating the table along with class activity using the tables will assure mastery of these facts. As you circulate, see that the table entries are being made correctly.

LESSON THREE: BEANS AND CUPS

Introduction: Use an overhead transparency mat with circles on it to simulate cups, and some beans. The children should have beans and cups to use.

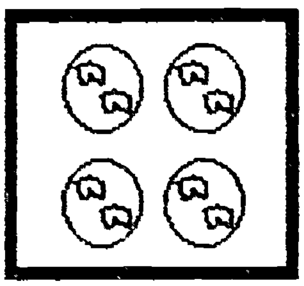
"We'll put one bean in each of these ______ cups."



"How many beans did we use?"

Write 4 x 1 = 4.

"Put one more bean into each cup."



"How many beans did we use altogether?"

Write: 4 x 2 = 8.

Show an overhead transparency of the recording form and how to write into it.

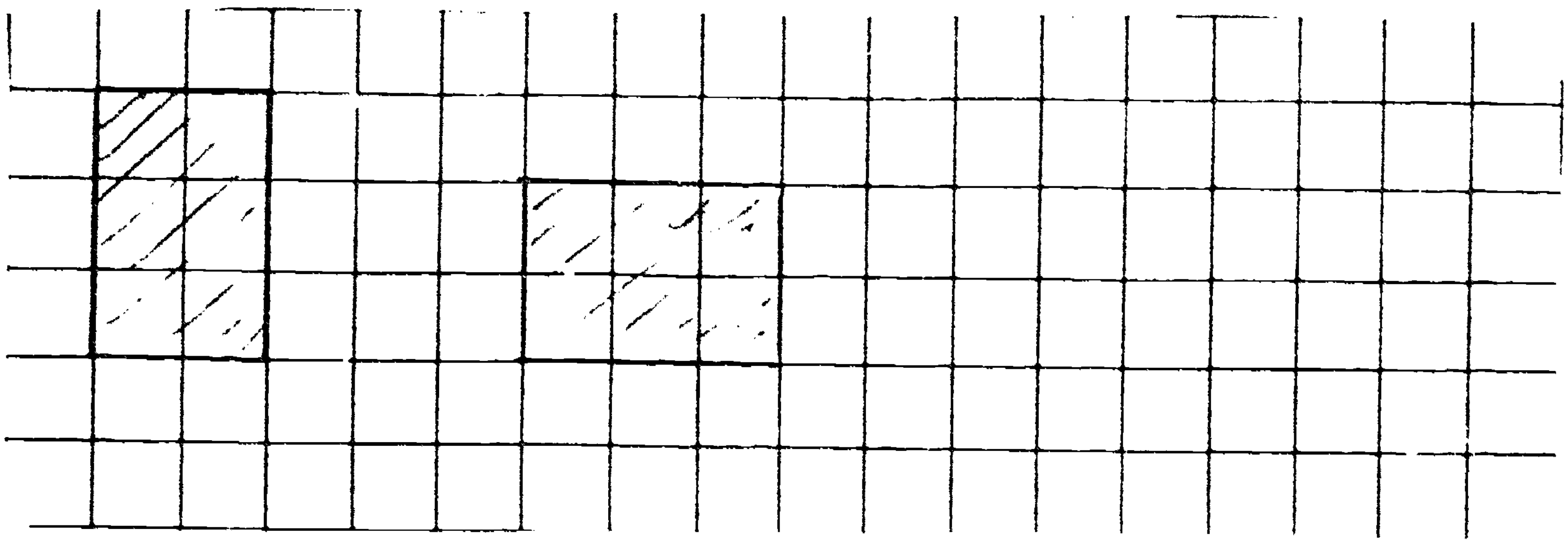
Activity: Give the children beans recording forms and blank multiplication tables and have them put beans into cups to complete both.

LESSON FOUR: BASE TEN ONES

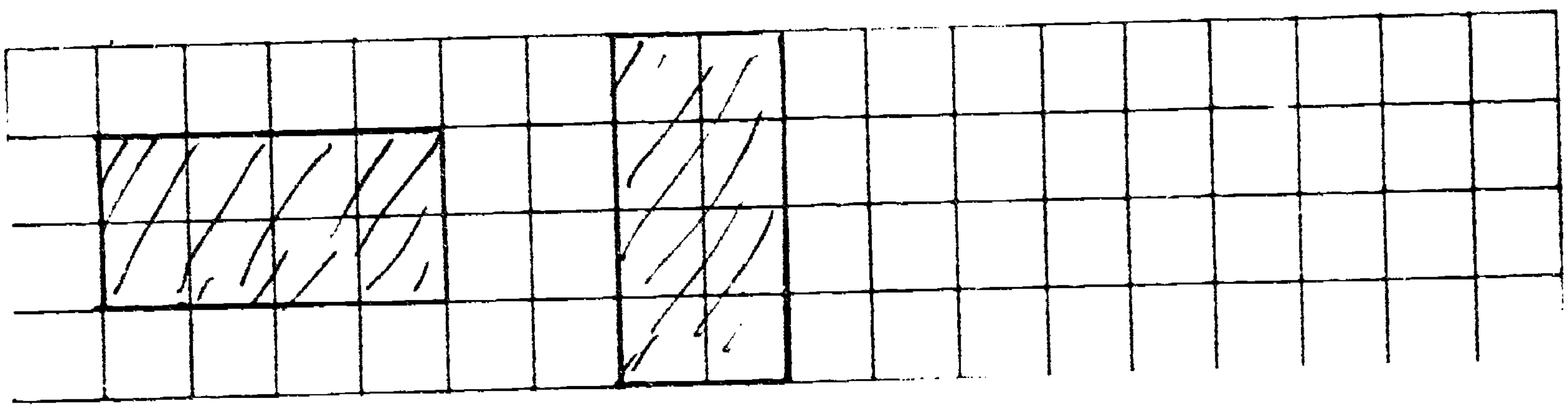
Background: This lesson is to directly lead into multiplication using Base Ten blocks.

Introduction: Children should have several base ten ones and graph paper to use. You should have an overhead transparency of the graph paper and some base ten ones.

On the graph paper build a rectangle of base ten ones.



Write 2  3 = 3 x 2 = 6 on the chalkboard. Build another.



Write the number sentences for this: 4 x2 = 2 x 4 = 8.

Activity: Set the children to work building rectangles of Base Ten ones to complete the worksheets.

LEVEL THREE

MULTIPLICATION FACTS: Tiles

LESSON ONE:

Introduction: Have an overhead transparency multiplication table and tiles. Arrange a 4 x 6 tile rectangle. Have the children tell you how many tiles are there. Go to the transparency table and circle as shown:

| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | |
| 2 | | | | | | | | | |
| 3 | | | | | | | | | |
| 4 | | | | | | △ | | | |
| 5 | | | | | | | | | |
| 6 | | | | ○ | | | | | |
| 7 | | | | | | | | | |
| 8 | | | | | | | | | |
| 9 | | | | | | | | | |

"Is there another place that shows 4 times 6?"

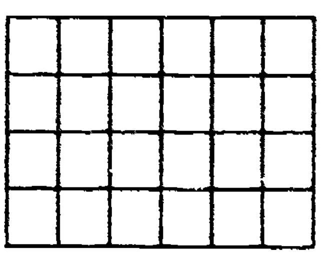
Allow for time so children can think about this. Then put the △ in the table and 24 inside each of these.

Activity: Children should work in pairs to use tile rectangles to complete the table.

LESSON TWO

Introduction: In this lesson Base Ten ones and tens are to be used to generate entries in the multiplication table.

On the overhead projector make a rectangle of overhead transparency base ten ONES. Have the children tell you how many ONES are in this.

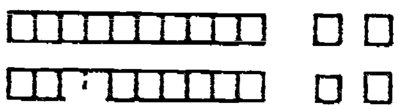


Record this in an overhead version of a blank multiplication table.

| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | |
| 2 | | | | | | | | | |
| 3 | | | | | | | | | |
| 4 | | | | | | ○ | | | |
| 5 | | | | | | | | | |
| 6 | | | | ○ | | | | | |
| 7 | | | | | | | | | |
| 8 | | | | | | | | | |
| 9 | | | | | | | | | |

Ask the children which of these two places should be used. "Does it make any difference which way the rectangle is looked at?"

Trade ONES for TENS on the overhead to get:



Point out that this is the same as TWENTY FOUR ONES.

Activity: Have students use base ten blocks to complete the worksheets supplied and to make entries into a blank multiplication table.

LEVEL THREE

## DIVISION

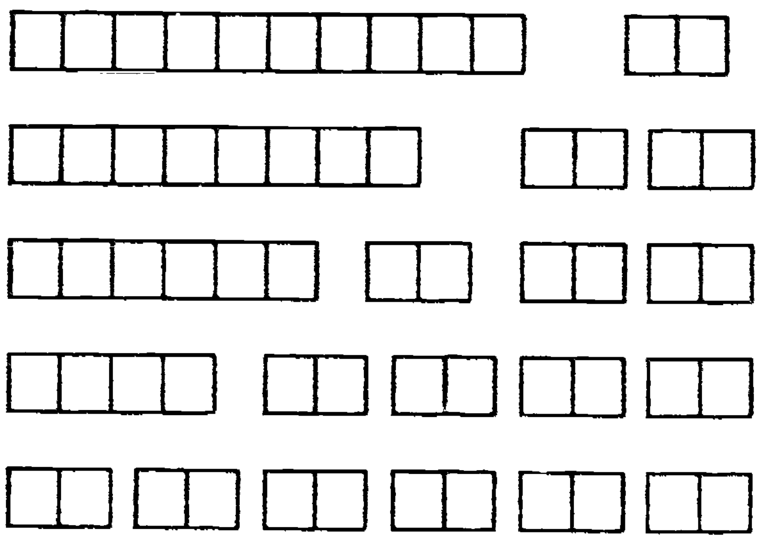
<u>Introduction:</u> Division is the inverse of multiplication. By using the same materials to develop understanding of both, children see that these operations are related to each other.

### LESSON ONE: UNIFIX CUBES

<u>Introduction:</u> Children should have 20-25 UNIFIX cubes of the same color each. You demonstrate the separation of a longer link into several shorter links and have them follow you.

"I have a UNIFIX cube link of 12 cubes. I will see how many 2's are in it."

Break off 2 links and count these as you do this:



**"one"**

**"two"**

**"three"**

**"four"**

**"five, and the last one left makes six."**

"So there are SIX 2's in twelve or twelve divided into twos is SIX."

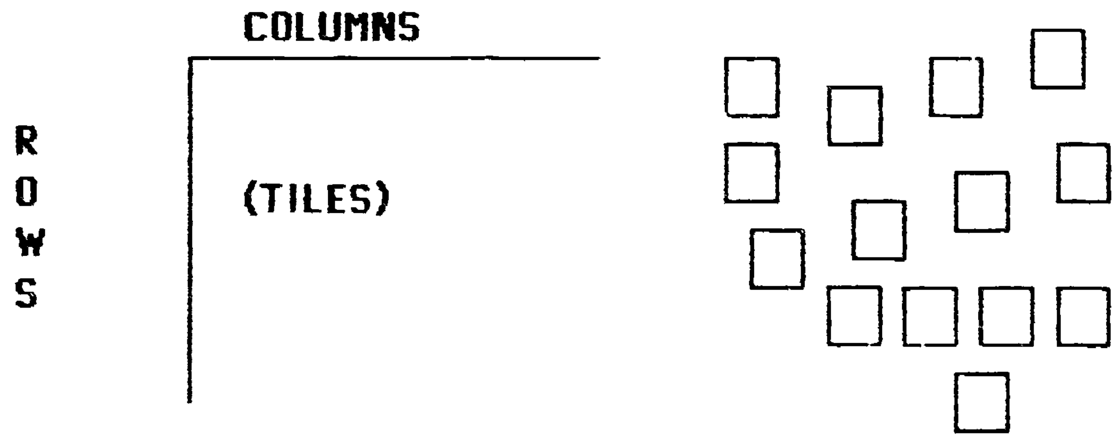
Repeat by breaking off threes to show twelve divided by three is four.

<u>Activity:</u> Pass out the recording forms and have children complete these by breaking off smaller links from a long UNIFIX cube link. Modify this later by having children count a group of UNIFIX cubes and making links of a given size from these.

Repeat as needed so children all begin writing correct number sentences.

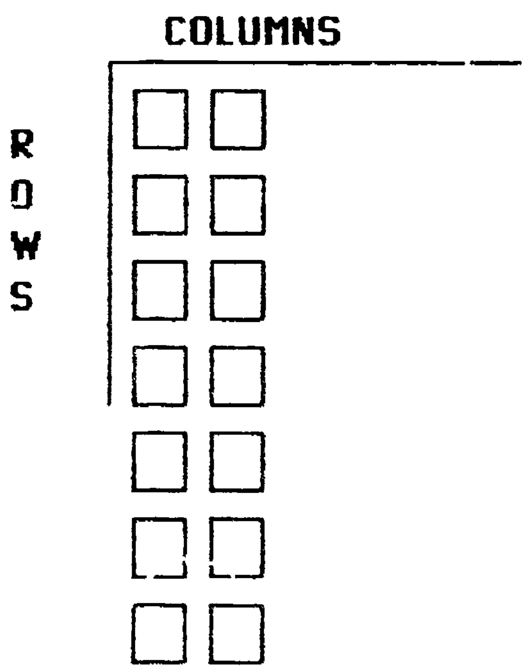
## LESSON TWO: TILES

Introduction: Use a tiles work mat transparency and inch square pieces of some kind on the overhead.




"I want to arrange these fourteen tiles into 2 columns. How many rows will I get?"

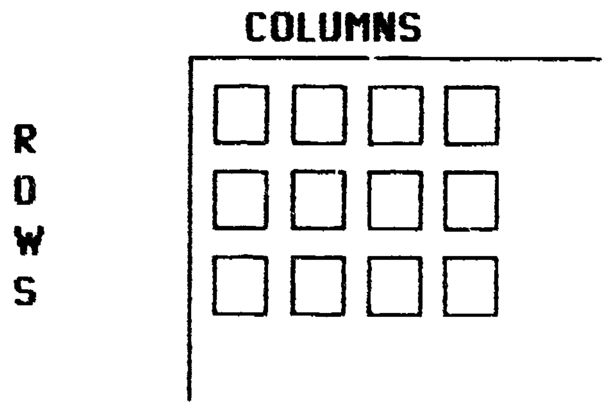
Do it:




"Fourteen divided into two's is seven."

Write: $14 \div 2 = 7$.

Arrange twelve squares into three ROWS:



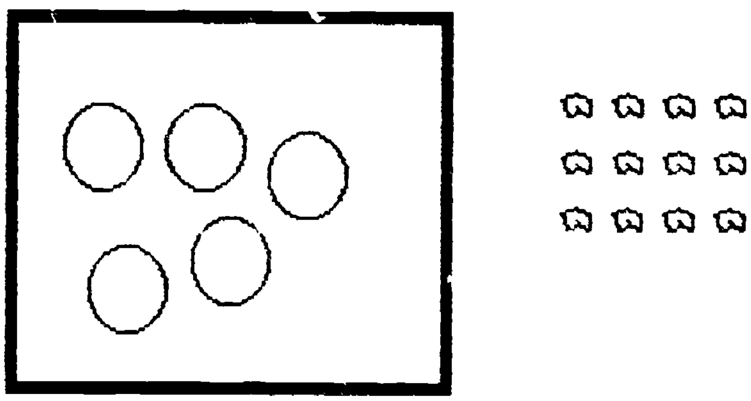

"Twelve divided by three is four." Write: $12 \div 3 = 4$.

Activity: Children should have 25-30 tiles each, the workmat and the recording forms. As you monitor the work, watch for correct use of the sign in particular.

LESSON THREE: BEANS AND CUPS

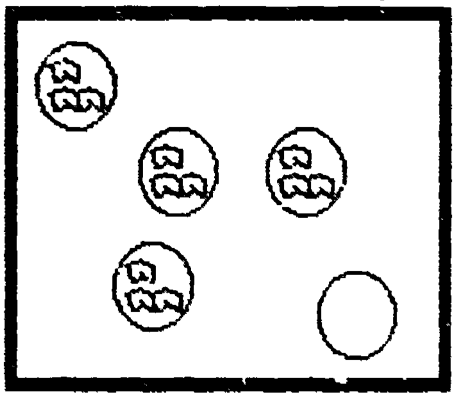
Background: This lesson involves the repeated subtraction version of division, or the measurement interpretation. The size of the groups being formed is constant, and one counts how many groups can be made.

Introduction: Use an overhead transparency version of a "cups" mat and beans:



"I have twelve beans and will put three beans in each cup. How many cups will be used?"

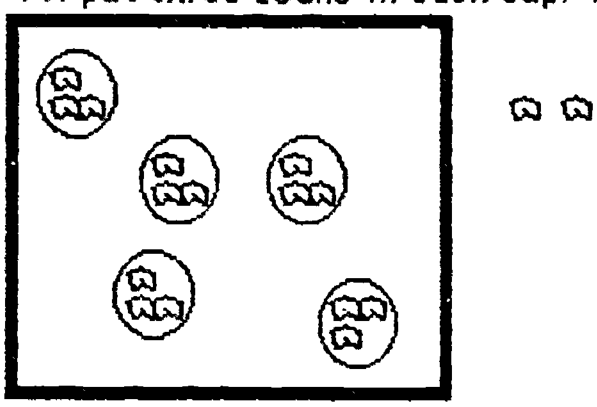
Measure out three/cup until the beans are used:



"Since twelve beans will go three at a time into FOUR cups, we write: 12÷ 3 =4."

Add more beans to the group to number seventeen.

"I'll put three beans in each cup. How many cups will I need?"



"I have two left. That's not enough to make another three, so I can't use another cup. The two is a remainder and we write:
17÷ 3 = 5 R2."

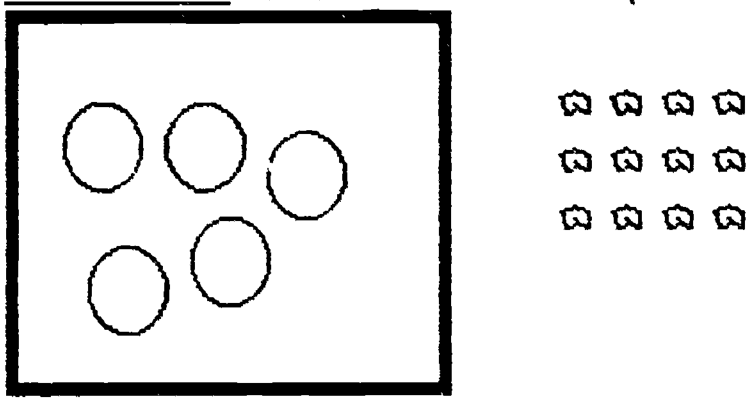
Discuss the two examples so children can do the following activity.

Activity: Children should have 40-50 beans and 10 cups and recording forms. As the children work, walk around to check on their use of symbols.

LESSON FOUR:

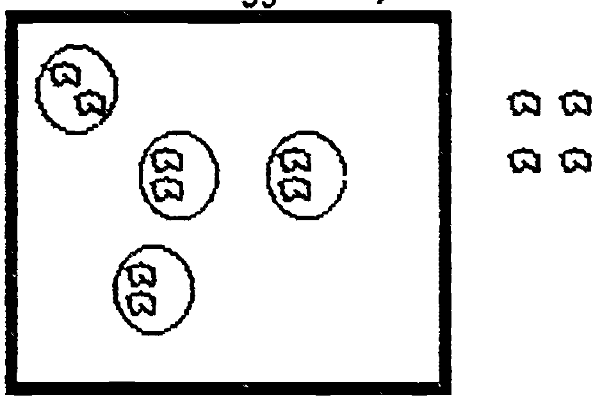
Background: This partitioning interpretation of division, whereby some scheme must be developed to assure equality is more difficult for children than the measurement or repeated subtraction in Lesson Three. In partitioning, the number of groups formed is constant, the same number must be in each group, and one counts how many are in the groups.

Introduction: Use the overhead "cup" workmat and some beans.



"I want to sort these twelve beans into four of these cups so that each cup has just as many beans. How can I do this?"

If someone suggests putting three into each cup, press to find out why they would be sure there would be just the right number of beans. If one or two are suggested, do this:



When completed, discuss how to be sure each cup as JUST AS MANY beans.

Write: $12 \div 4 = 3$ on the board.

So a second problem, but incorporate a remainder and have the children tell you what to do.

"I have fourteen beans to divide equally among four cups. How should I do this?"

Discuss the remainder and be sure you write the sentence as:
$14 \div 4 = 3$ R2.

Talk about how many more you must start with to avoid a remainder when dividing by four (or other numbers).

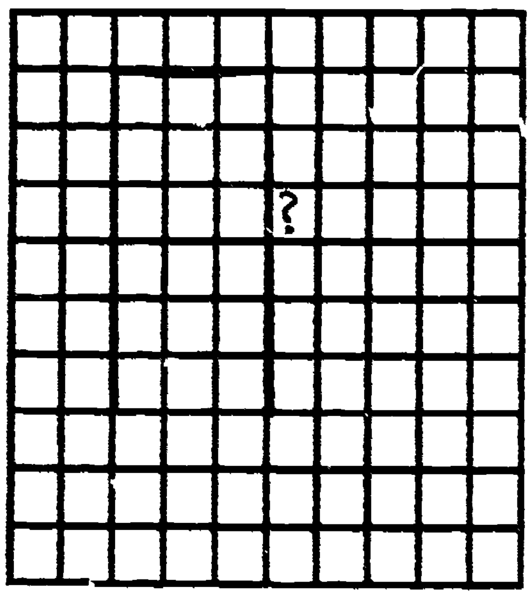
"Will you get a remainder when dividing twelve by two? by three? by four? by five?", etc.

LESSON FIVE: BASE TEN ONES

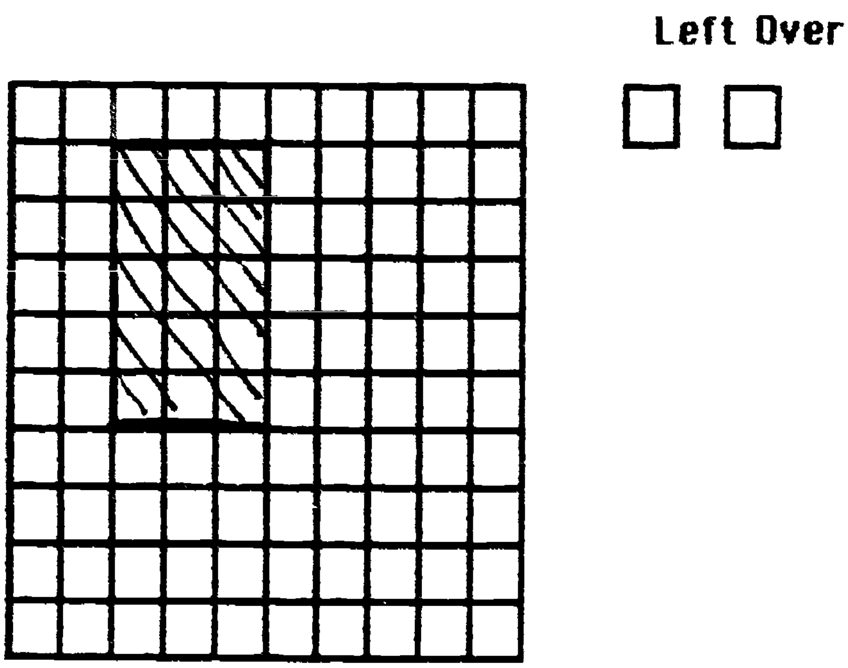
Background: This forming of rectangles with a given dimension will give the background needed to introduce the children to division in base ten.

Introduction: Use an overhead transparency graph paper and base ten ones on the overhead.

"I have seventeen ones. What is the other side of the biggest rectangle I can make with side three?"



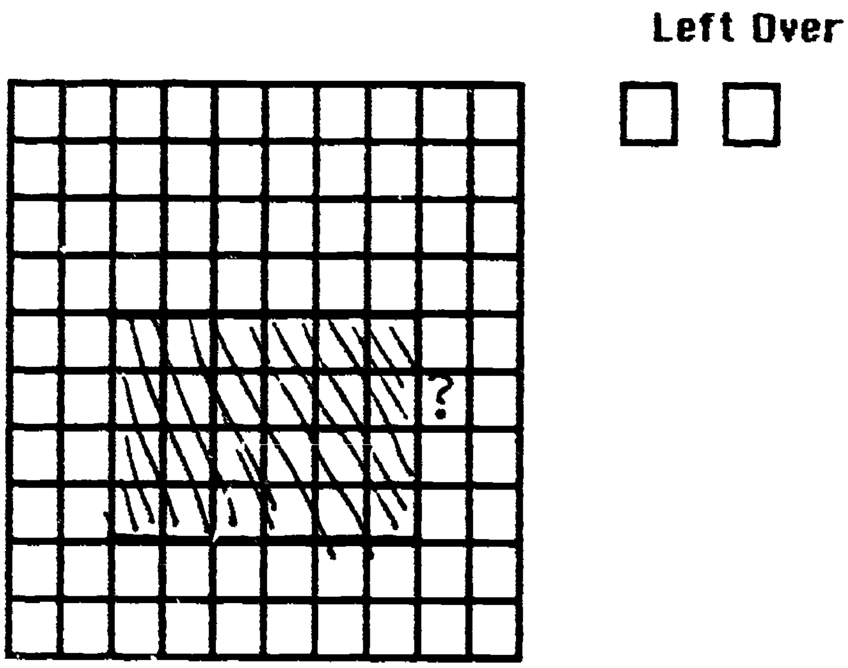

Make it.



"The biggest one is FIVE long and there are TWO left over. I write: 17    3 = 5 R2."

Take 25 base ten ones and place on the overhead.

"I want to make a rectangle 6 long. How wide will it be?"




"It will be FOUR wide with ONE left over. I write: 25    6 = 4 R1."

Discuss these to prepare children for the activity.

Activity: Children should have 30-40 base ten ones, graph paper and the recording forms. Monitor the work to see that the number sentences are written properly.

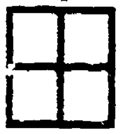
All of the multiplication and division lessons should be repeated using increasing numbers during the year until base ten multiplication is introduced.

## LEVEL THREE

### SQUARES

### LESSON ONE

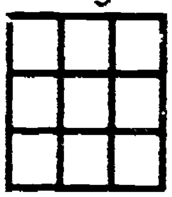
Introduction: Children should have tiles and graph paper. On the overhead projector, build a square two on a side from tiles:



Write: $4 = 2 \times 2$

Have the children outline this square in the upper left hand corner of a piece of graph paper.

Enlarge the square to one three on a side:



Write: $9 = 3 \times 3$

"How many tiles did I use to make this square one unit longer on each side?"

"How many tiles were added to the first square to make this one?"

Activity: Have the children use tiles to make squares as specified on the worksheet. Each square made should be outlined on graph paper.

### LESSON TWO

Activity: Have children build squares on geoboards like those they have recorded on the graph paper in LESSON ONE.

## LEVEL THREE

### MULTIPLICATION: BASE TEN BLOCKS

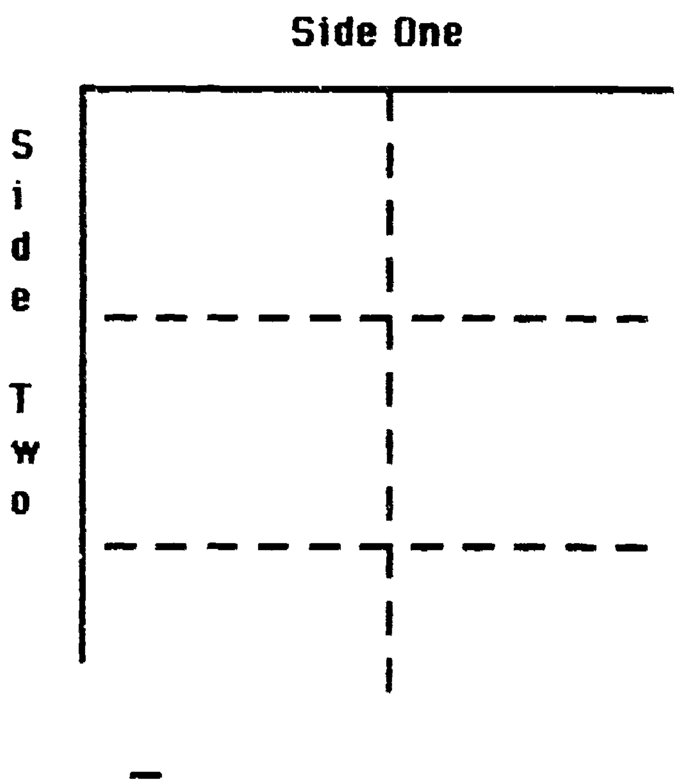
Background: Because of the existence of calculators it is unnecessary for children to become highly proficient in the use of the computation algorithms in base ten. They should understand the relationship between multiplication and division, place value representation and the properties of multiplication and division in base ten so as to be able to interpret outputs of calculators and computers. This means ability to estimate quickly the results of these operations in terms of size of number and ability to use scientific notation for base ten numerals.

The model used to perform multiplication and division in base ten is the rectangle model. This SHOWS how multiplication and division are related and the standard algorithms do not. These algorithms are carried out with no more than 3 digits in one number and the long division algorithm is introduced with no more than 2 digit divisions. This is done only because division of polynomials is still needed for some procedures in algebra later.

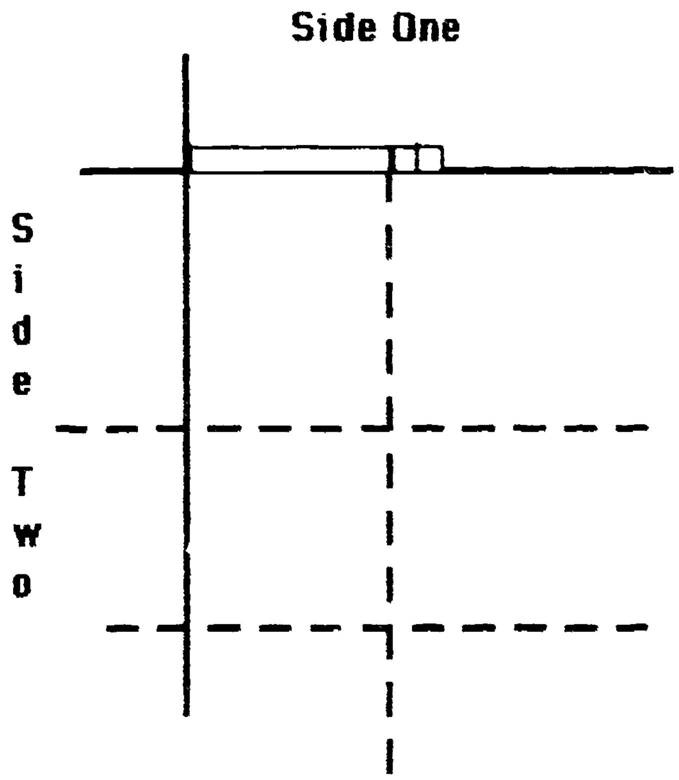
### LESSON ONE: COUNTING NUMBERS

Background: Students must understand numeration and place value before this is done.

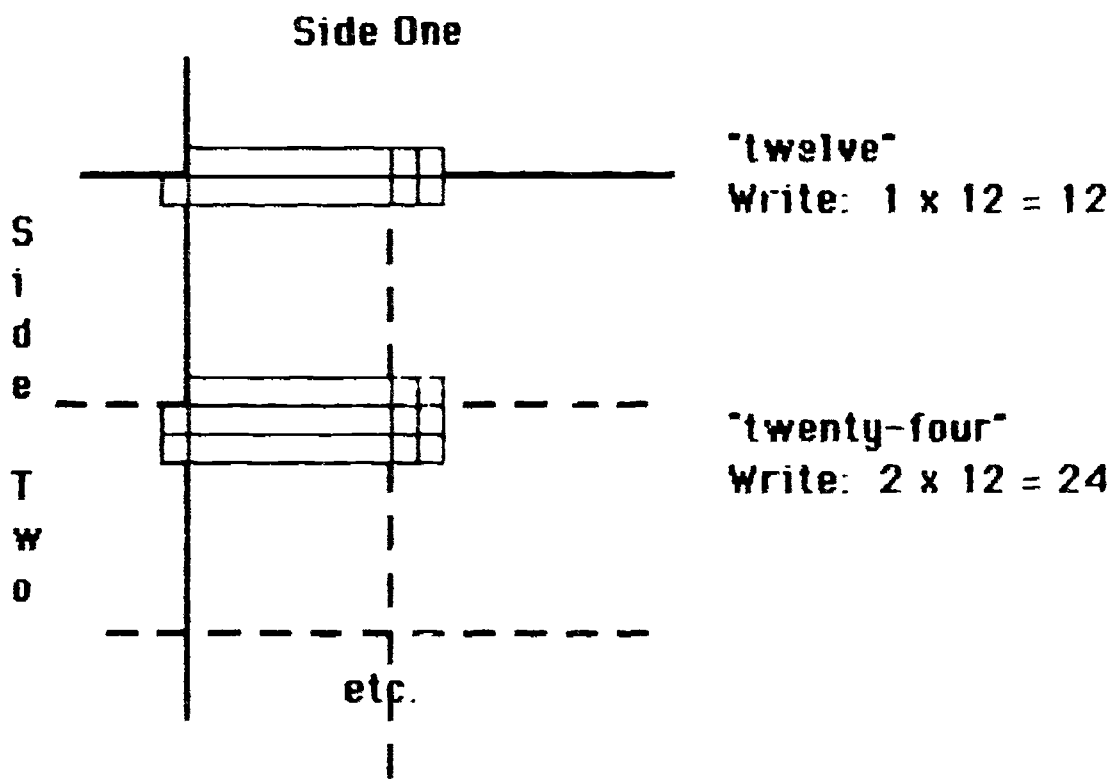
Introduction: Use the base ten mat in transparency form and overhead base ten blocks:



Put twelve in base ten blocks on Side One:




Use base ten ones along side two to show multiplication successively by one, two, three.......nine. Ask students for the result each time:



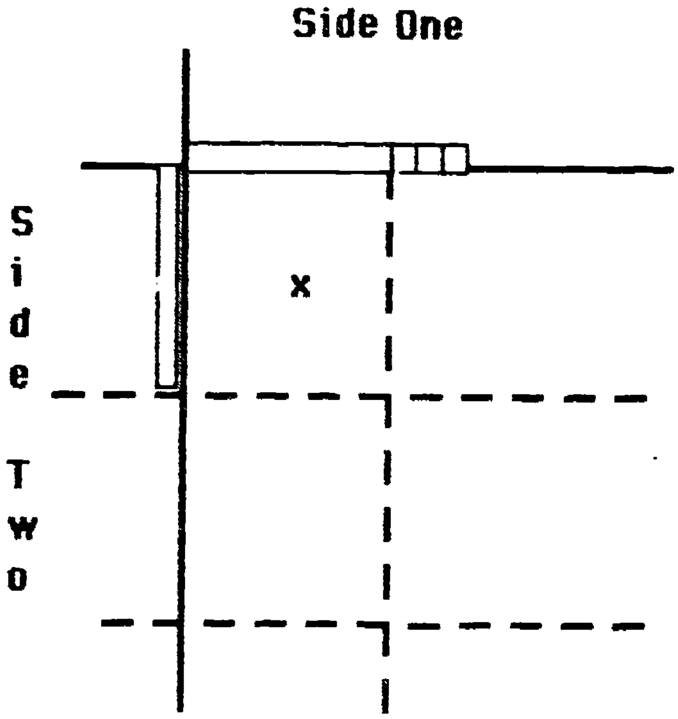

<u>Activity</u>: Give students the multiplication-division corner mats, base ten blocks, and the worksheets and have them do this with the base ten blocks. Encourage the students to keep the base ten materials packed into a rectangle each time.

## LESSON TWO: TENS

Background: Children must realize that multiplying by the base (ten) changes each place to the next larger place. Thus, when multiplying by ten, 3 tens become 3 hundreds, 3 ones become 3 tens, etc. If multiplying by 2 tens, 3 tens become 6 hundreds, 3 ones become 6 tens, etc. This lesson should help them see that.

Introduction: Use base ten blocks and the transparency mat.

Put up thirteen and a ten on the other side:

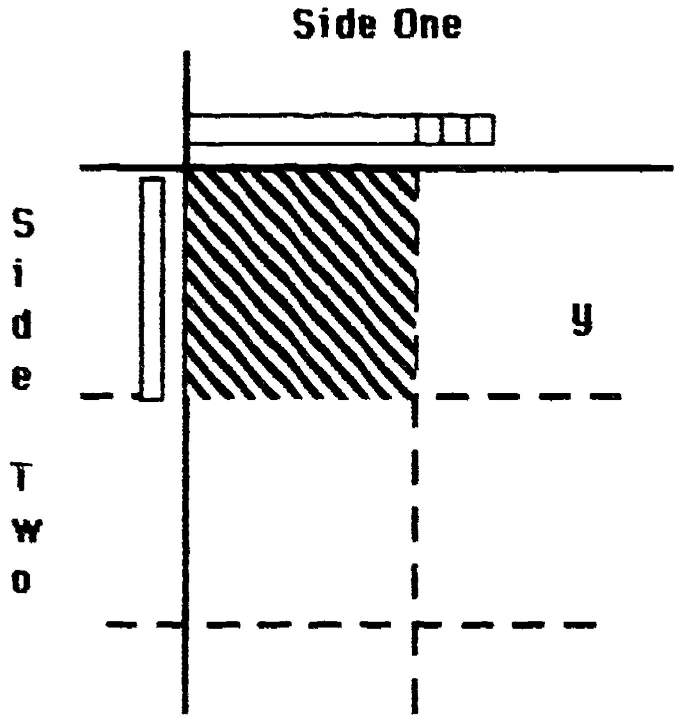



Point to the area "x". "What base ten piece fits in here?"

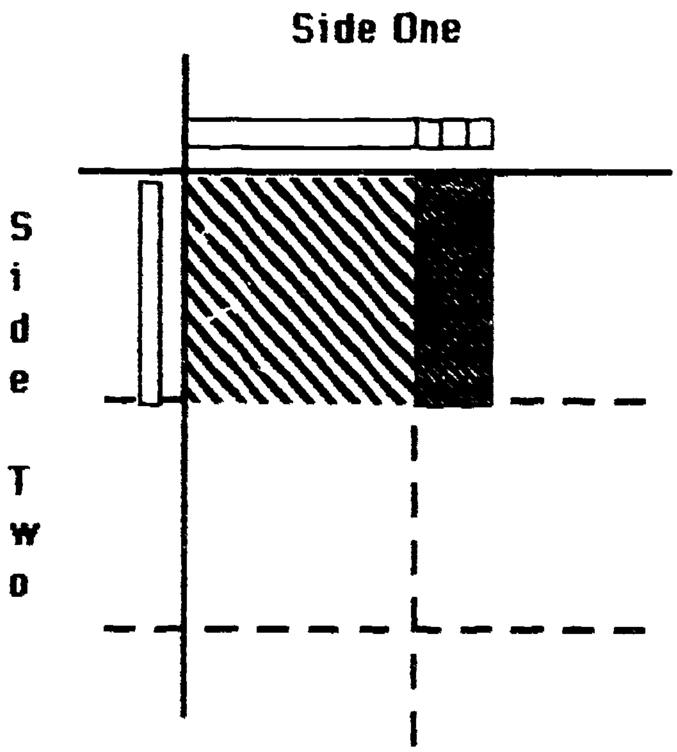
When the hundreds piece is suggested, put that piece there.

"What is the result of multiplying this ten (point to the one on Side Two) times this ten (point to the one on Side One)?" "What pieces will go in here?"

Point to the "y" area:




"Three tens fit in here."




"What do you get when you multiply this ten (point to the one on Side Two) times the three ones?"

Again emphasize that multiplying by a ten raises the ten to a hundred and the three ones to three tens.

Activity: Children should use base ten blocks to do the worksheets. This lesson should be coordinated with oral decode multiplying activity.

"Ten times three is?" "Ten times four is?" "Ten times twenty is?", etc.

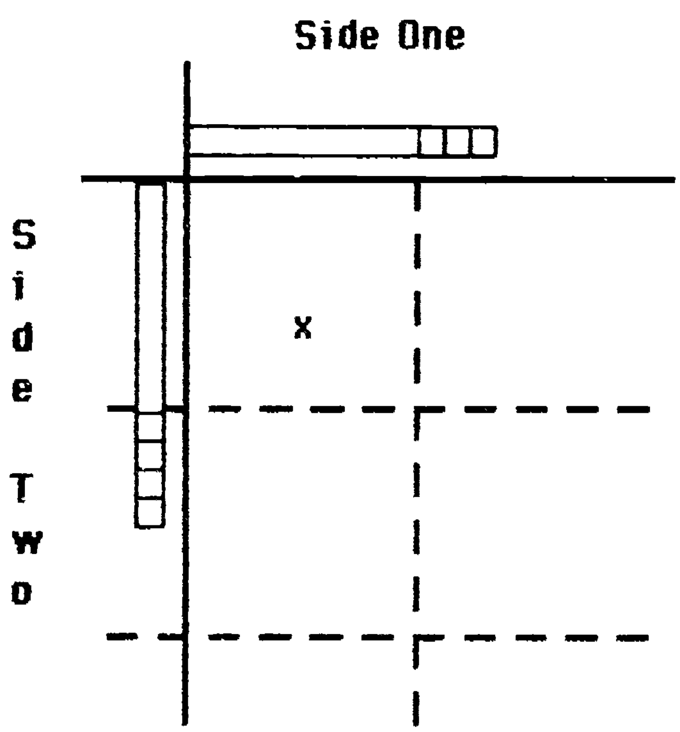
Extend the lesson to the rise of two tens. Vary the factor on Side One to 13, 14, etc. Include several tens in the oral activity.

"Twenty times one is _____?" "Twenty times 6 is _____?" "Twenty times eleven is?", etc.

LESSON THREE: TENS AND ONES

Background: This puts both tens and ones together and completes the development of the rectangle model for multiplication.

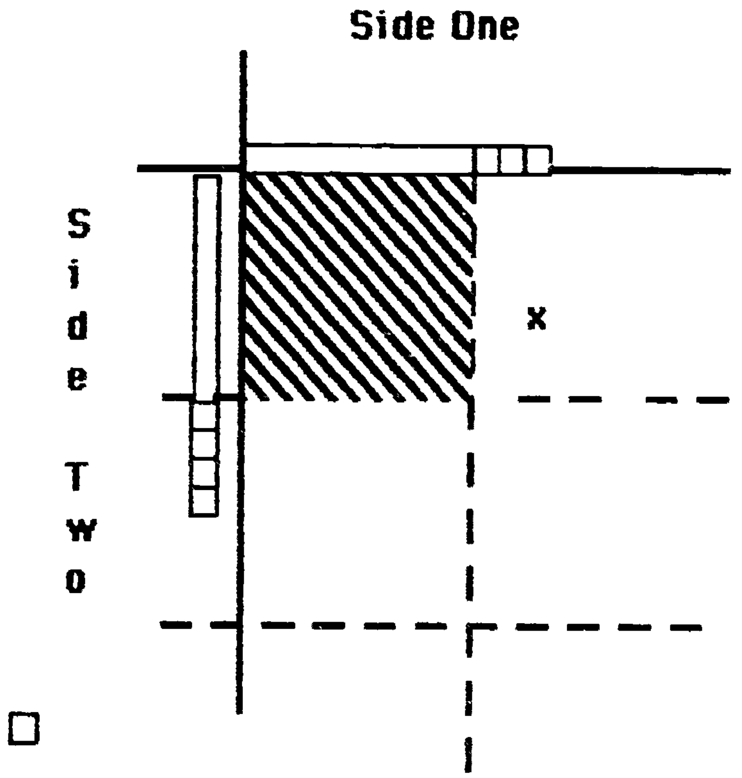
Introduction: Put thirteen on Side One and fourteen on Side Two on the overhead projector forms:




Point to the "x" area

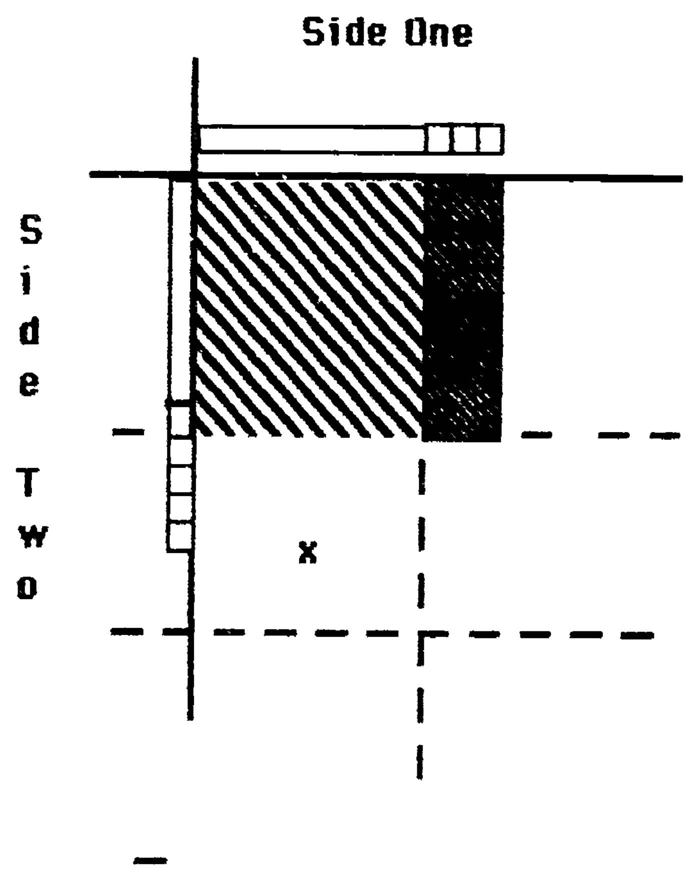
"What fits in here?" (If students can't uniformly respond with "a hundred," they have not had enough experience with Lesson Two activities.

Put the hundreds piece in:



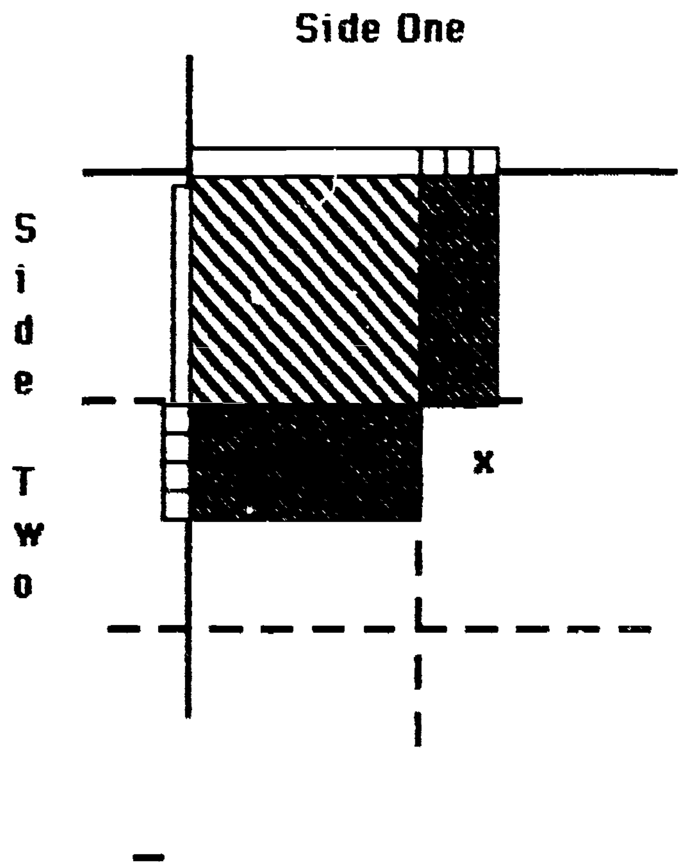

Point to the next "x" area. "What fits in here?" Work to get the idea of three tens - the base raises the 3 ones to 3 tens.

Put these in:



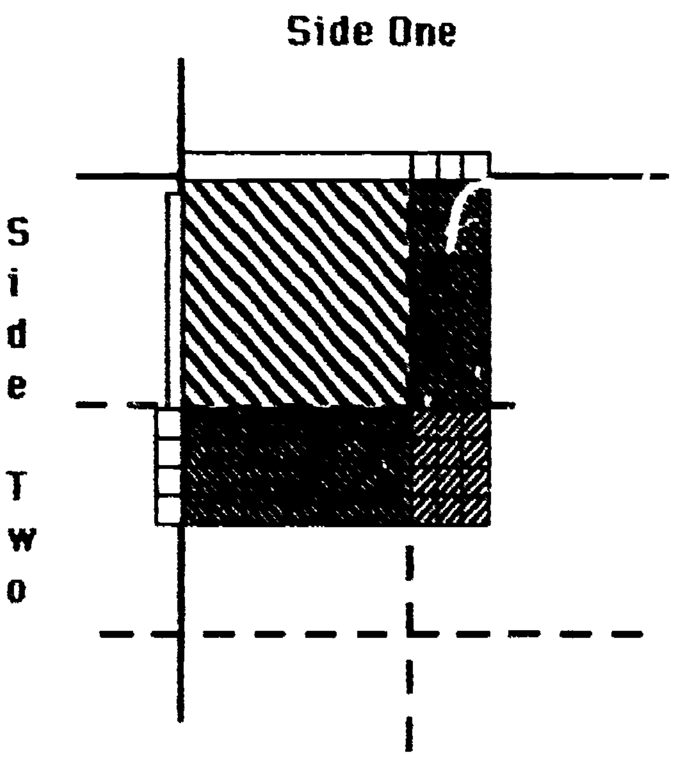
"We have multiplied by this (point to it) ten. Now we will multiply by the four ONES." "What goes here?" (Point to the next "x" area). "That's right - four ones count out four tens when multiplying the ten."

Put these in:




"We now have three parts of the product." "What goes in here?" Point to the last "x" area.

"Since ones multiplied by ones give more ones, and 4 x 3 = 12, we have twelve ones. Notice they make a 4 x 3 rectangle like the tiles did, and the ones did before."




"Now we can count up ONE HUNDRED + THREE TENS + FOUR TENS + TWELVE ONES." Point to each as you describe it. "How many tens altogether?"

"ONE HUNDRED + SEVEN TENS + TWELVE ONES."

"What must we do with ten of the ones?" "How many tens doe     at make?"

So we have (describe orally as you write) 1 8 2

"The number sentence is: $14 \times 13 = 100 + 80 + 2$ or 182"

"This came from: $10 + 4 \times 10 + 3 = 100 + 40 + 30 + 12$

$= 100 + 70 + 12$

$= 100 + 80 + 2$

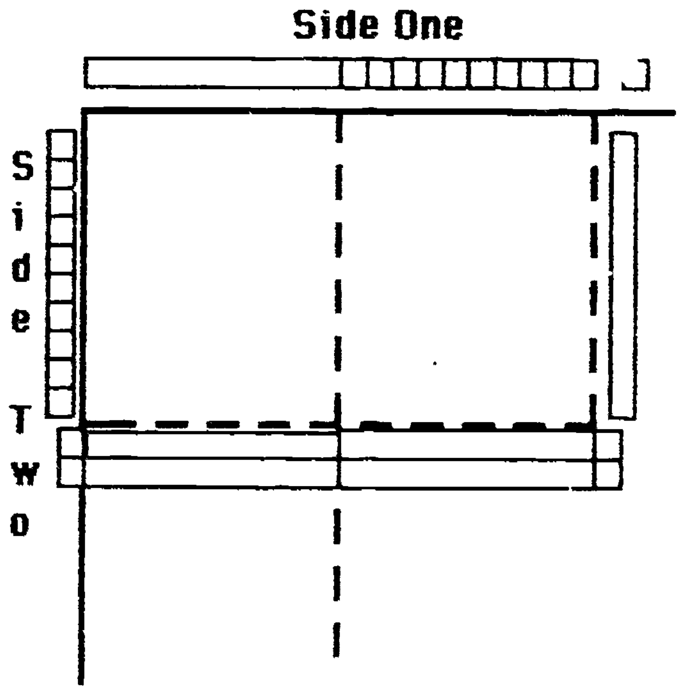
$= 182$

## COMPUTATION: STANDARD FORM-MULTIPLICATION

### LESSON ONE

Work the following multiplication using the rectangle model on the overhead. Have the children follow what you do at their seats with the recording forms. Have a copy of the recording form on a transparency.

Do 21 x 12 by the rectangle method:




**Record as:**

| Hundreds | Tens | Ones |
|---|---|---|
| 2 | 0 | 0 |
| | 4 | 0 |
| | 1 | 2 |
| 2 | 5 | 2 |

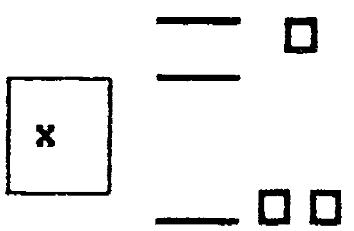
"TENS multiplied by TENS gave what?"

"TENS multiplied by ONES gave what?"

"ONES multiplied by ONES gave what?"

Point to the partial products as you discuss these.

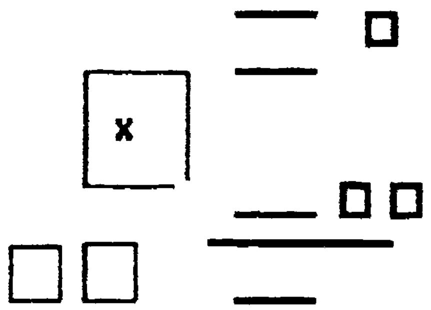
Take the "square corner" mat off, move the materials in the rectangle over to the side, and arrange the following on the overhead projector:




```
  2 1
x 1 2
-----
```

"What is the result of multiplying ONE TEN by TWO TENS?"

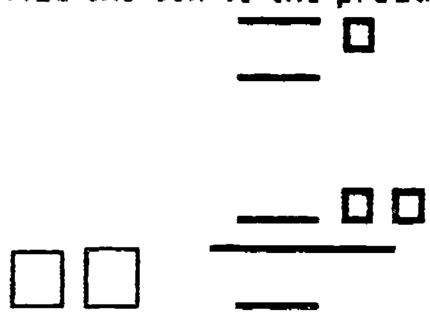
Arrange as shown:




Point to the 200 in the recording form.

"What is the result of multiplying ONE TEN times ONE ONE?"

Add the ten to the product:

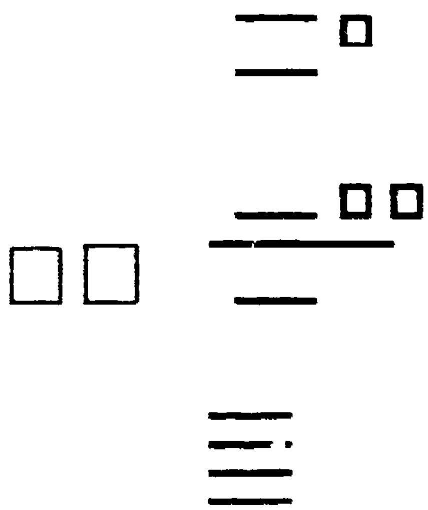


Point to the 10 in the recordin., form.

"What do you get when you multiply TWO ONES time TWO TENS?"

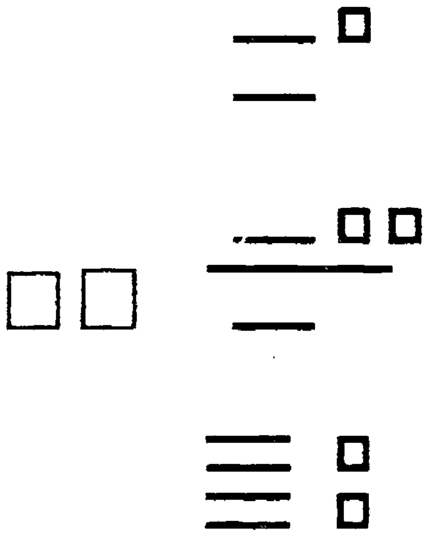
You may have to discuss this. Each ONE counts TWO TENS - TWO ONES count FOUR TENS

Point to the 4. ›e recording form and arrange four tens:



"What is two times one?

Point to the 2 in the recording form and put the base ten ones in as shown:



Compare the base ten pieces found this way with those put aside from the rectangle. Emphasize that the results are the same.

<u>Activity:</u> Pass out the activity sheets for the children to work on with base ten blocks. Check to see the partial products are recorded correctly and that any required trades are made in obtaining the final result.

LEVEL THREE

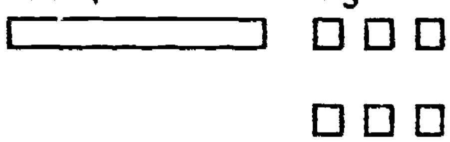
DIVISION: BASE TEN BLOCKS

Background: We have considered multiplication in terms of finding the area of a rectangle, given the lengths of the two sides. We now consider division as being the finding of one side of the largest rectangle to be made from a potential area and the length of the other side.

LESSON ONE:

Introduction: Use the overhead transparency base ten blocks. Have the children follow using these materials in standard form.

Put up the following:



**and**

□ □ □

"How many of these

**(point to the** □ □ □ **)**

can we get from the ten and three ones?"

Most students will see the single group of three ones. Push them to find those in the ten.

"To make the groups of 3, what should be done with the ten?"

The suggestion to EXCHANGE the Ten for ten ones must be elicited and discussed:

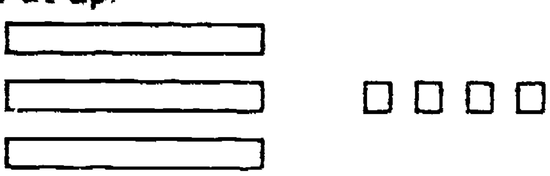
□ □ □ □ □ □ □ □ □ □    □ □ □

**becomes**

□    □ □ □    □ □ □    □ □ □    □ □ □

"So we get four "threes" with one left over.
Write: 13 ÷ 3 = 4 R 1

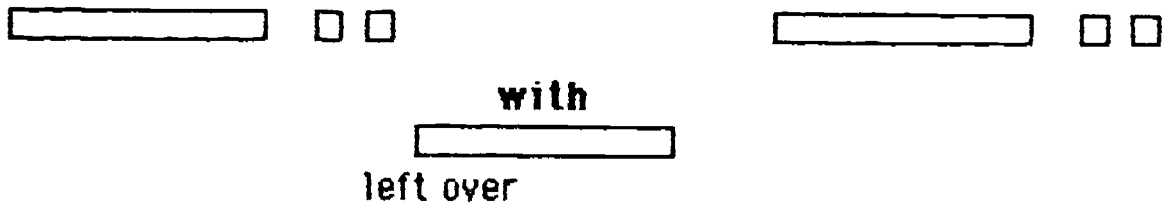
Put up:



"We want to divide this up into two equal groups."

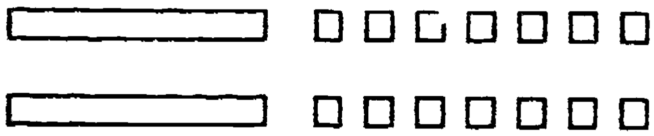
"How can we do this?"

Follow student suggestions and see what is right or wrong with each. Eventually separate as:




"What must we do with the Ten to get something to put into each group?"

Decompose the Ten and put five ones into each group.

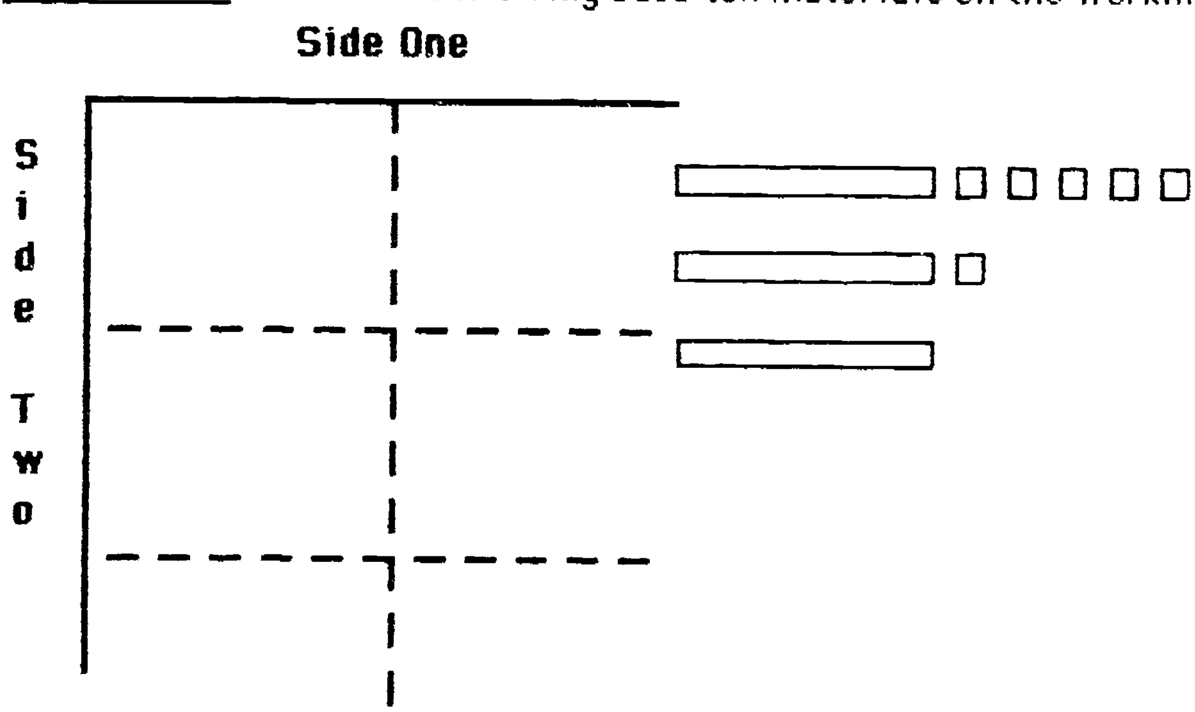


Write: 34 ÷ 2 = 17

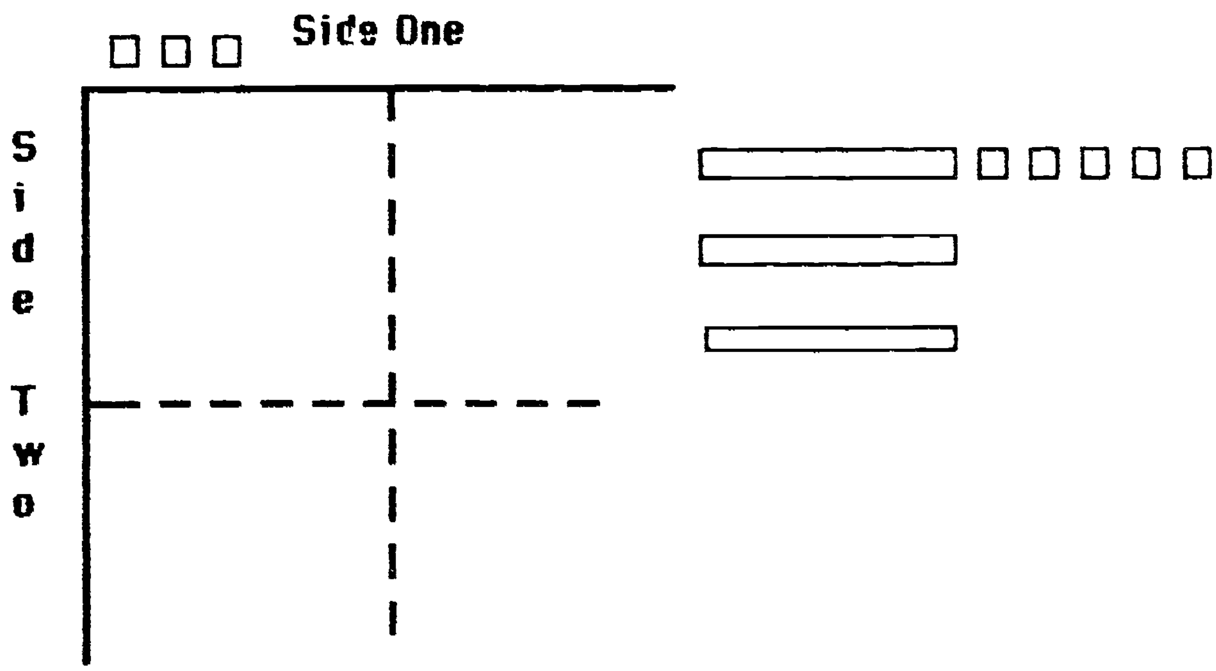
<u>Activity</u>: Pass out the worksheets and have the children use the base ten blocks to work the exercises.

<u>LESSON TWO:</u>

<u>Introduction:</u> Place the following base ten materials on the workmat on the overhead:

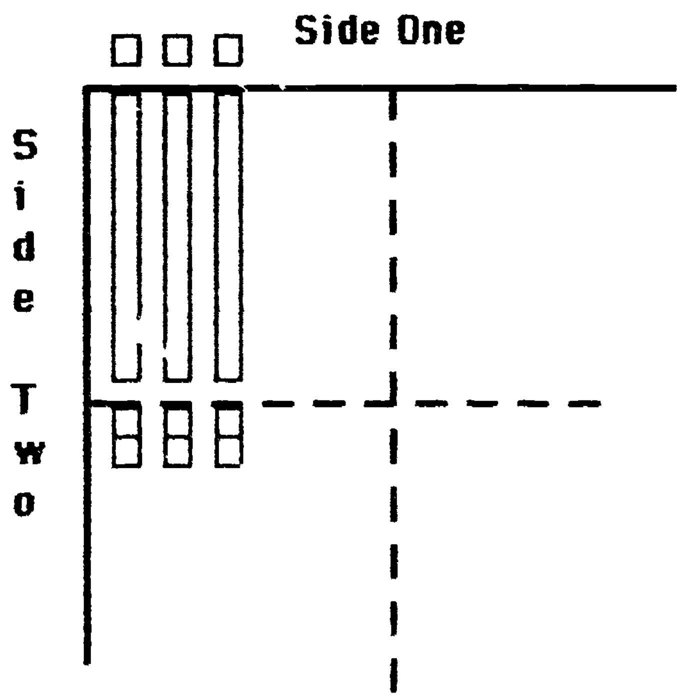



Make Side One with 3 additional ONES.




"How do we arrange the tens and ones we have into a rectangle having this side?"

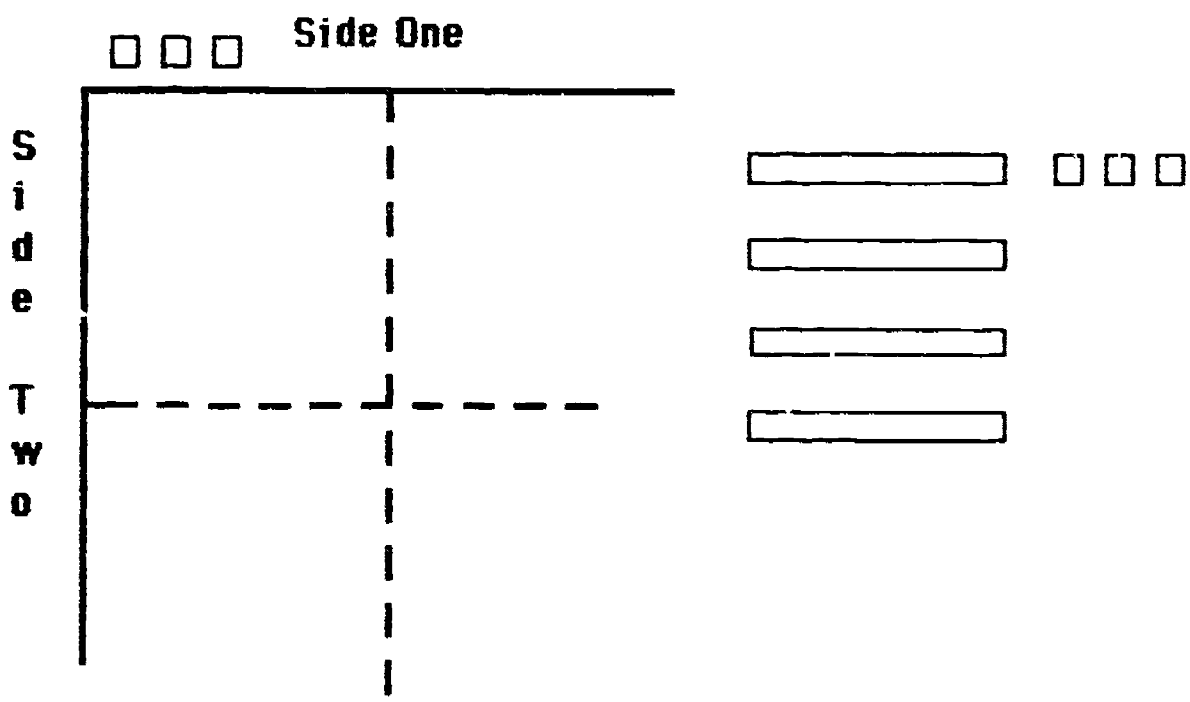
Put the materials into the rectangle:




"How long is Side Two?"

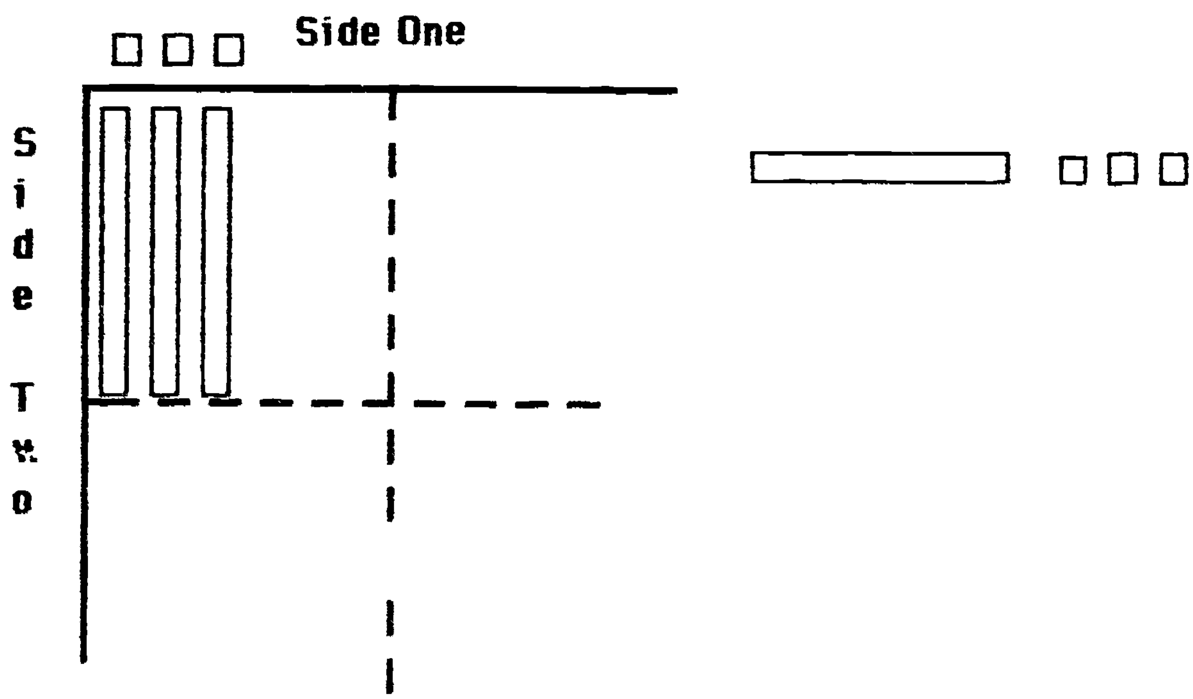
Write: 36÷ 3 =12

Place the following on the overhead:



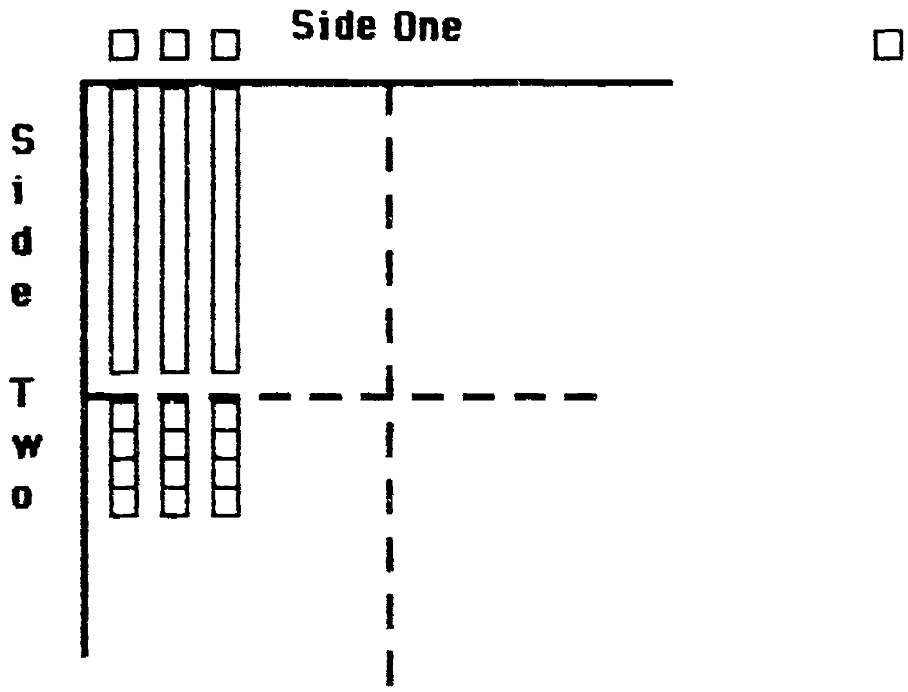

"How do I make a rectangle of these materials having a side of three?"

You probably will get the suggestion to do this much first:




"What must we do now to extend Side Two?"

Exchange the Ten for ones and extend, using twelve of the ones.




Write: 43÷3 =14 R 1

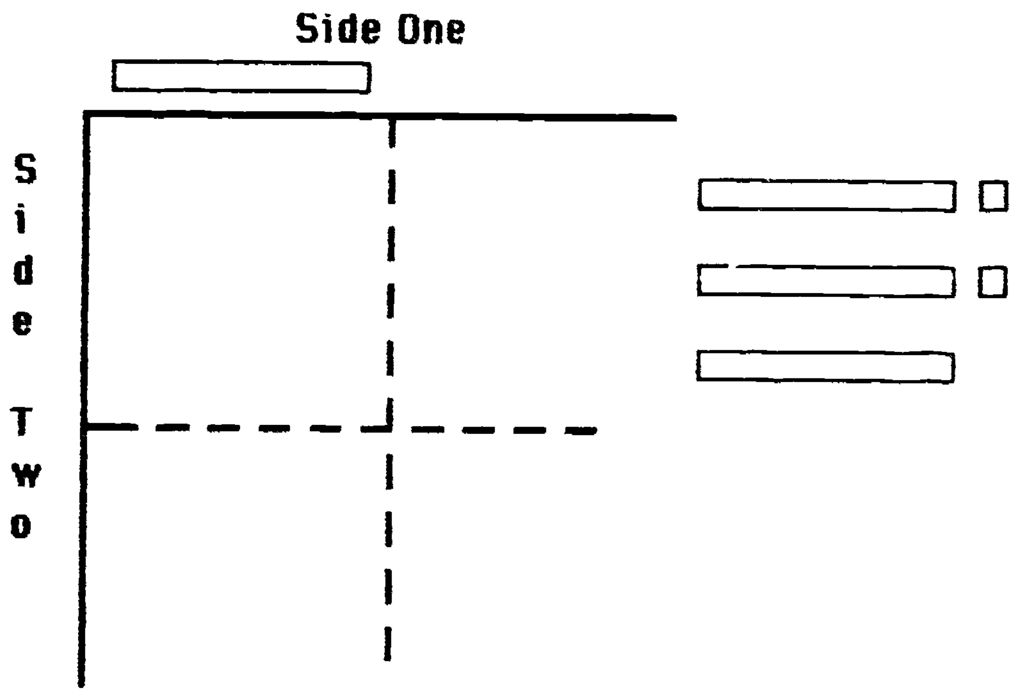
Activity: Pass out worksheets and have children work on these with Base Ten blocks.

LESSON THREE

Background: This is to emphasize dividing by ten. Just as multiplying by ten causes each numeral to count the next larger place, so dividing by ten causes each numeral to count the next smaller place. This should be emphasized as the lesson in multiplication and this one are developed.

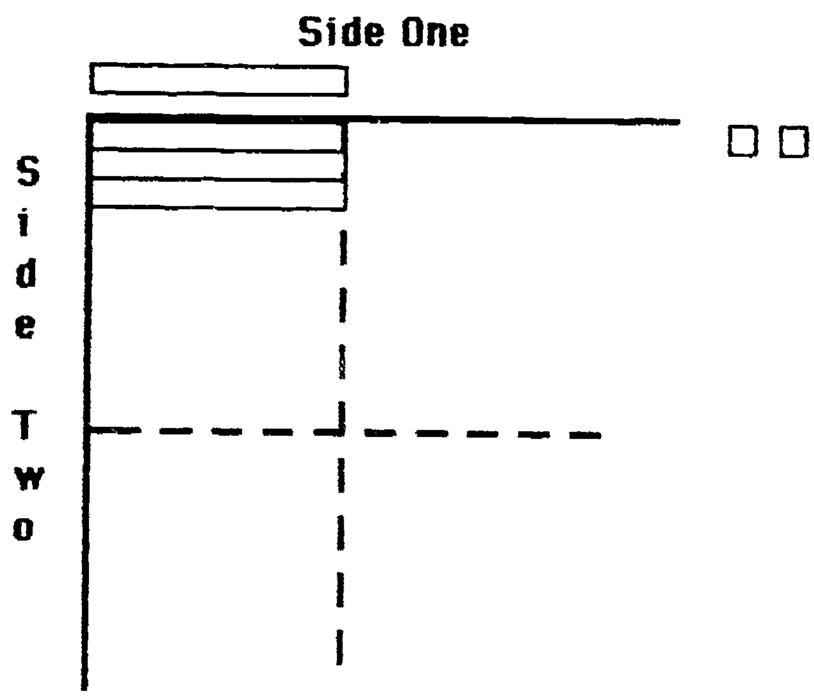
Introduction: The children should have base ten blocks and the "corner" mat to follow what you do on the overhead.

Place the following material on the overhead:




"How much of this will go into a rectangle having this length of Side One?"

On the overhead arrange:

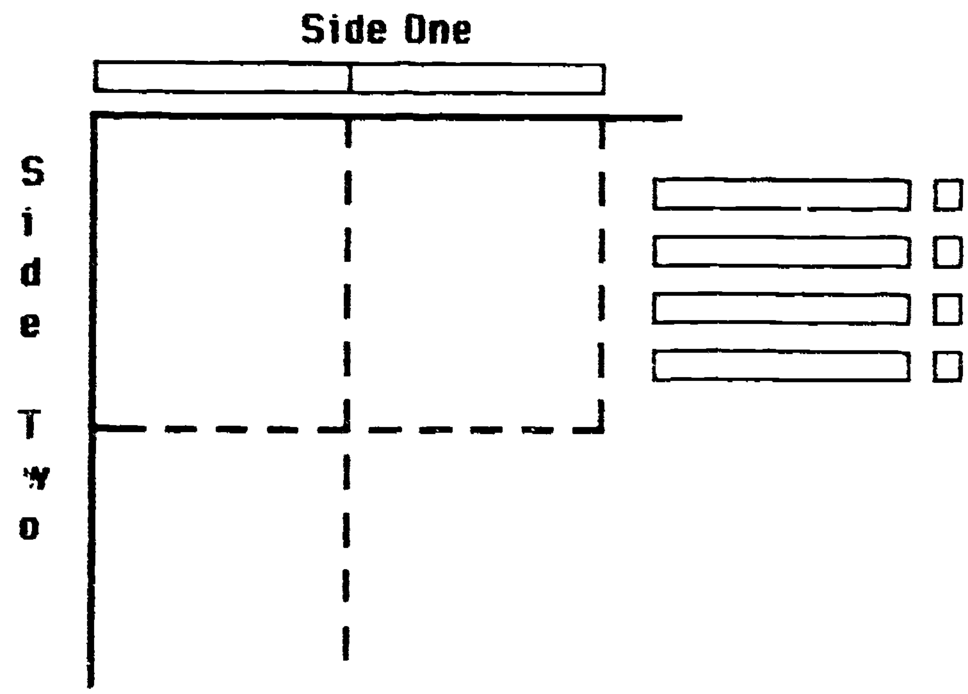



"How long is Side Two?"

"Are the two ones enough to make another Side One?"

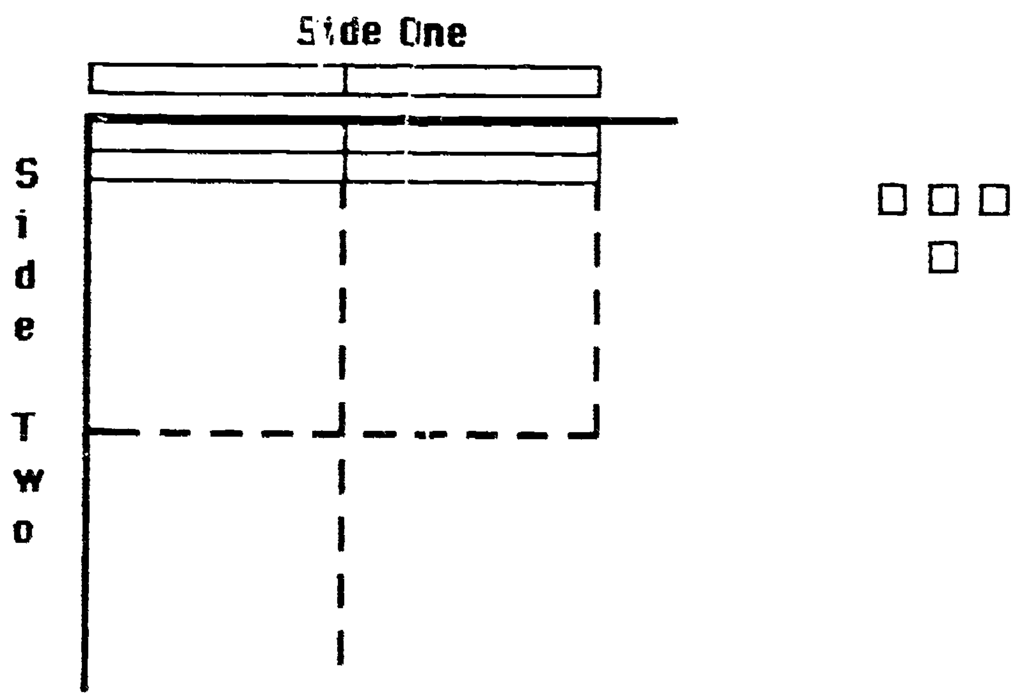
Write: $32 \div 10 = 3$ R 2

Point out the 3 has moved from the tens place to the ones place. Put the following on the overhead:




"What should I use in the rectangle?"

Arrange:




Do I have enough ones to extend the rectangle more?"

"What would I need to extend the rectangle more?"

"How long is Side Two?"

Write: 4÷20 = S R 4

Point out that 4 TENS   2 TENS is just like 4 UNIFIX   2 UNIFIX. The result is 2.

Activity: The children should work on the worksheets using base ten blocks and the "corner" mat. Observe the correct placement of the tens in the rectangle and the correct identification of the side made and the remainder, if there is one.

*Note: This lesson should be accompanied by decade mental division as suggested:*

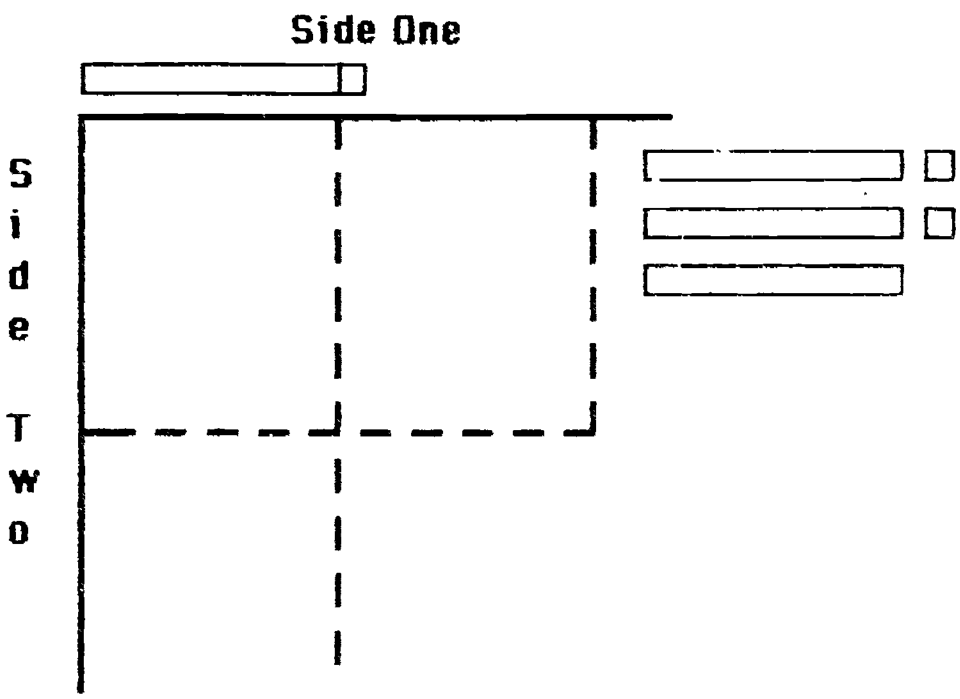
*"What is twenty divided by ten?"*

*"What is forty divided by twenty?" etc.*

LESSON FOUR

Background: This lesson puts together dividing by tens and dividing by ones into the use of a "two digit" division. It should not be used until the first three are thoroughly understood. With this lesson children will need to recognize when to exchange tens for ones and will have to try different possibilities for Side Two, so expect some errors to begin with. Allow plenty of time for the base ten block activities.

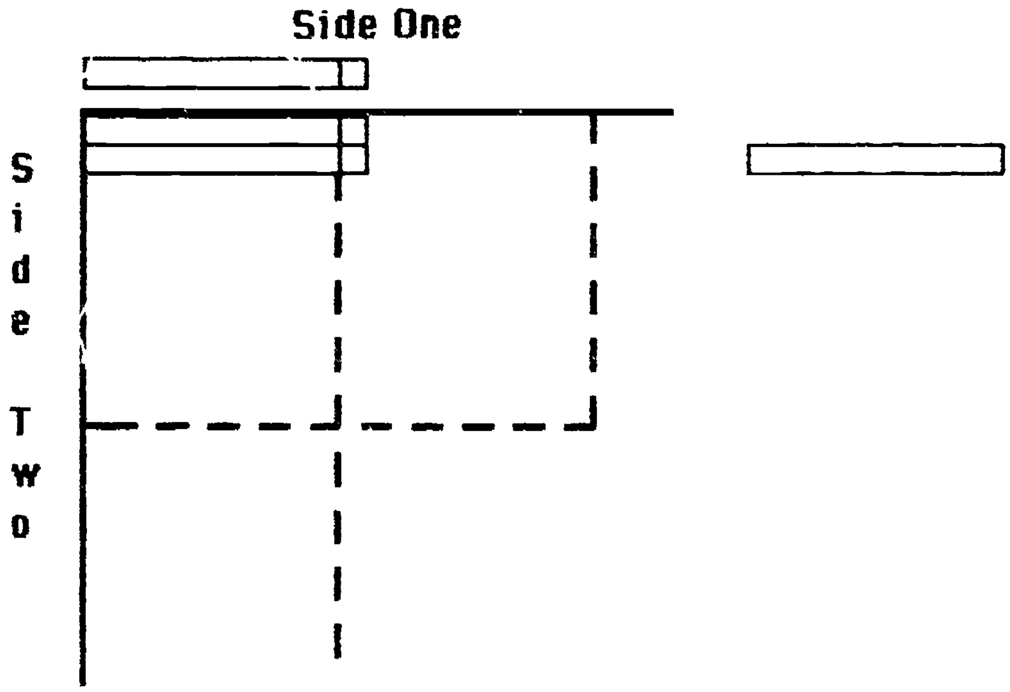
Introduction. Children should have base ten blocks to follow what you do on the overhead. On the overhead arrange:




"I want to make the largest rectangle with this side (point to Side One) from these base ten blocks (point to the materials available.)"

"How many tens and ones can I use?"

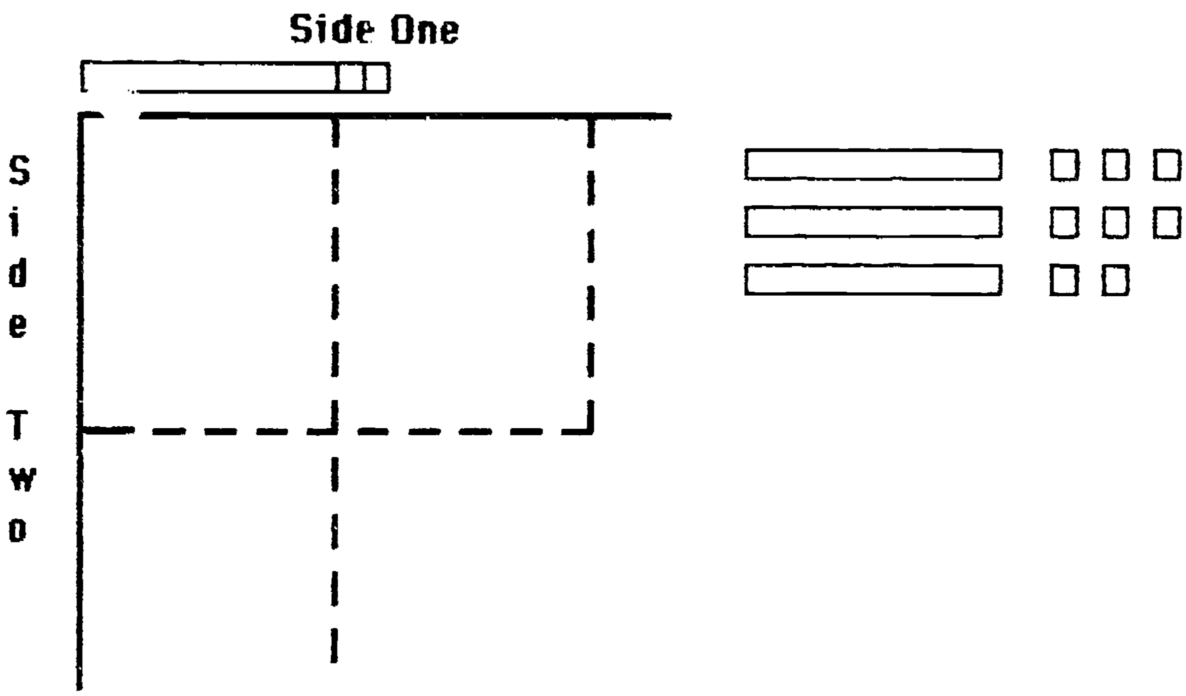
Arrange as




"The ten left won't make another row across to comple' ·ectangle so it must stay as a remainder."

Write: 32÷1 = 2 R 10

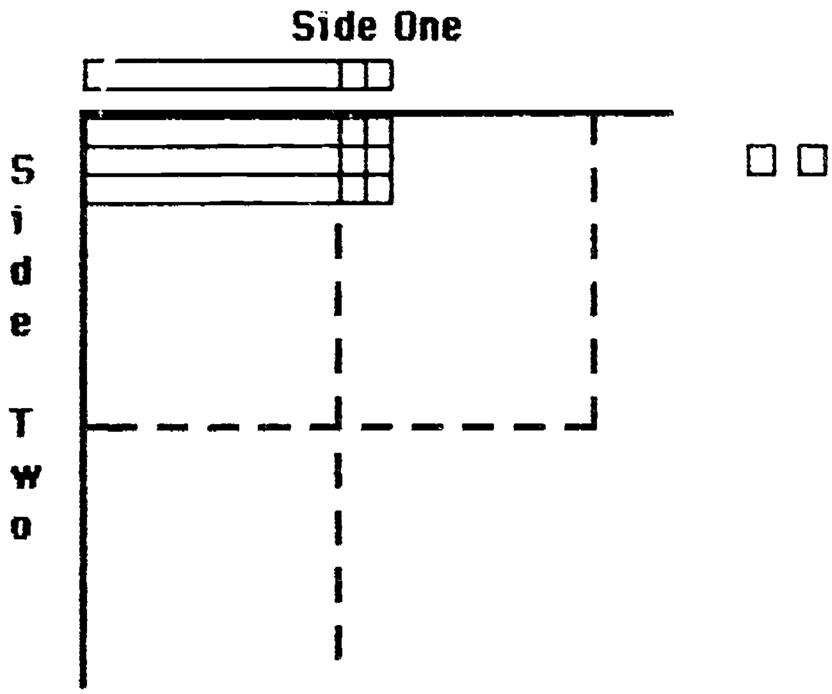
Do another:




"How many tens can I use in the rectangle?"

"How many ones can I use?"

Arrange as:




"The two is a remainder."

Write: 38 ÷ 12 = 3 R 2

Activity: When it is clear the children realize they must make the LARGEST rectangle possible with the given side from the given materials, put them to work to do the worksheets with base ten blocks.

LEVEL THREE

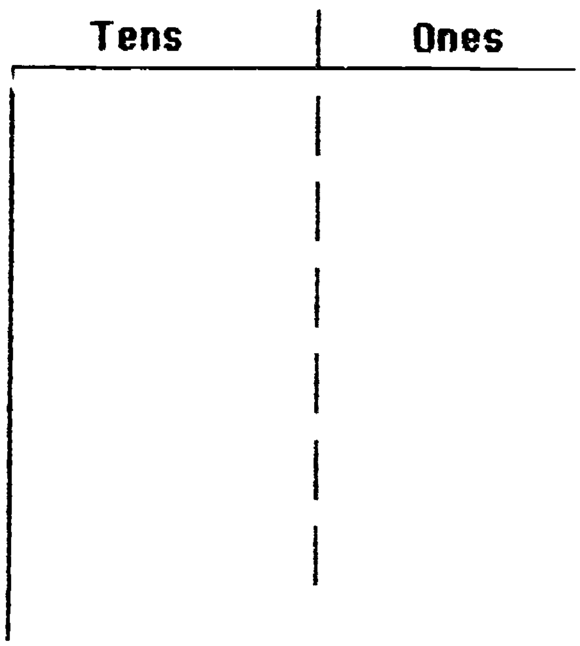
COMPUTATION: STANDARD FORM FOR DIVISION

Background: After children have worked with the rectangle model they can be introduced to the standard form using a single digit division or a divisor showing tens.

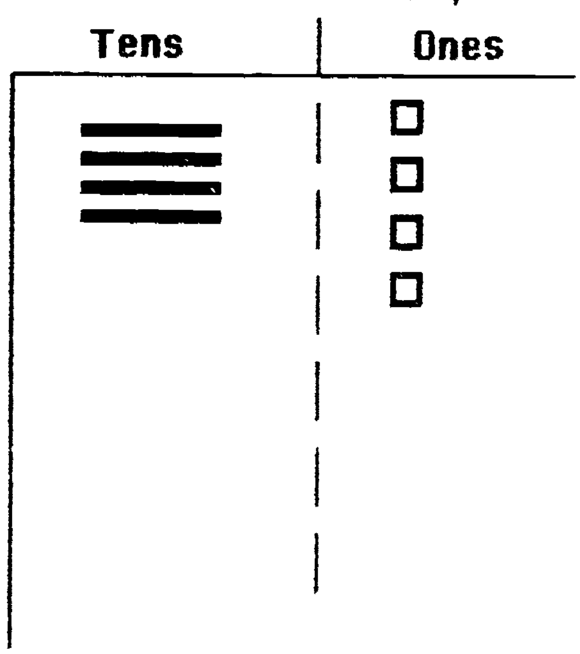
LESSON ONE

Introduction: Write $4\overline{)44}$ on the chalkboard. Explain that this shows division of 44 by 4.

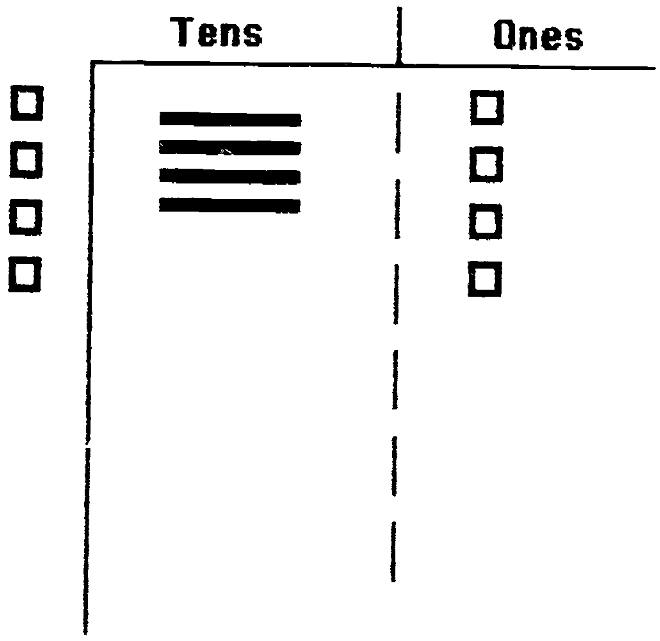
On the overhead place a division workmat in overhead transparency form:




Put the 44 with base ten pieces:



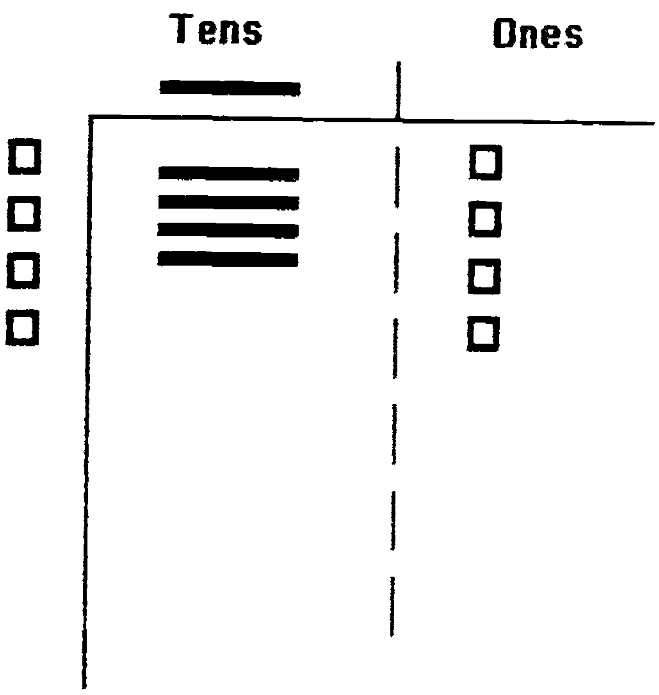
Put the 4 ONES as a divisor as shown:




"What is four TENS divided by four ONES?"

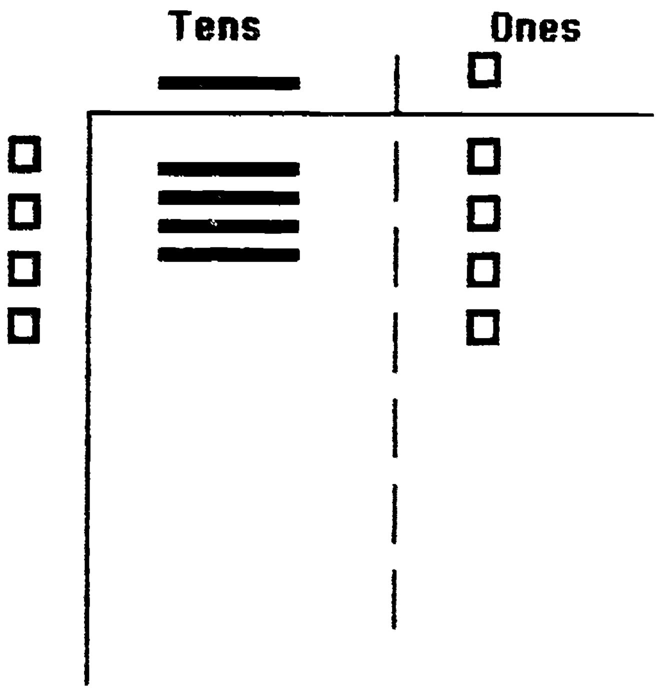
"Notice that four divided by four is one - but one WHAT?" ONE TEN

Put the TEN in the "Quotient"




"What is four ONES divided by four ONES?"

"How many groups of four ones are in four ones?" ONE
Put that in:




so $4\overline{)44}$ with quotient 11 results. Emphasize the 1st "1" counts TENS and the 2nd "1" counts ones!!

Activity: Children should use base ten blocks to show the divisions on the worksheets provided.

## LEVEL THREE

### SYMBOLS: + AND X USED TOGETHER

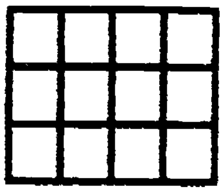
Background: Children should begin to recognize that addition and subtraction are basic operations and multiplication and division are basically renaming operations. This 12   4 is another name for 3, and 5 x 2 is another name for 10. Hence, in 3 x 3 + 2, 3 x 3 is done first to get the number to be ADDED to 3. In general, "x" and "   " are always done first to rename numbers. "+" and "-" are done last in any order.

### LESSON ONE

Introduction: Use tiles on the overhead with children following what you do with tiles at their desks. Place 12 tiles on the overhead.

"How might I arrange these in a rectangle?"

Accept the first suggestion, i.e., 3 by 4 and arrange these:

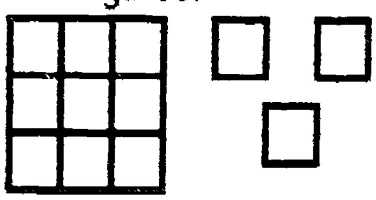


Write 12 = 3 x 4.

"Can I make a different rectangle?" Arrange this.

You should get the children to see that there are three rectangles, - 3 x 4, 6 x 2, 12 x 1. "What is the largest square I can make from these twelve tiles?"

Arrange as:



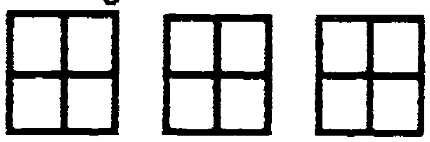
Point out the three left over. Write: 12 = 3 x 3 + 3.

Emphasize the 3 x 3 is 9. Three added to this gives the 12 tiles.

"Is there another square we can make?"

"Can we make more than one?"

Arrange:



We get THREE 2 x 2 squares. Write: $12 = 3 \times (2 \times 2)$

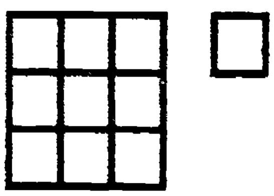
Point out the parentheses around the 2 x 2 to show how the 3 squares are made.

<u>Activity:</u> Give students tiles and the recording forms to work on. Check to be sure the number sentences are being written correctly.

<u>LESSON TWO</u>

<u>Introduction:</u> This lesson is to have children interpret sentences having operation signs into arrangements of materials. You lead with tiles on the overhead and follow by arranging tiles at their desks.

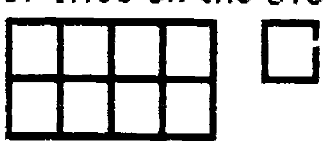
Write $10 = 3 \times 3 + 1$ on the chalkboard or on the overhead projector. Have the children suggest how the tiles can be arranged to show this. Arrange the tiles and have the children arrange theirs:



Discuss this, if needed.

Write a second number sentence: $9 = 2 \times 4 + 1$.

Ask the children to arrange their tiles to show this. Put the arrangement of tiles on the overhead:



<u>Activity:</u> Have the children work on the worksheets provided, using square tiles.

Monitor carefully to see that the number sentences are interpreted so the "x" is used to rename numbers BEFORE terms are combined using "+" or "-".

LESSON THREE

Pair the children. Within each pair, they are to take turns generating number sentences for the other to construct using tiles.

LEVEL THREE

PROBLEM SOLVING 1

Background: Study the material provided on the fourteen kinds of adding and subtracting problems arising from comparison, joining and separating, and part-part-whole relations in the LEVEL TWO Guide. These are summarized and exemplified in the beginning of this Guide.

Involve the children with problem solving of this kind at least once each week. The kinds of problem solving to use are:

1. The 14 cases of addition and subtraction with increasingly larger numbers. Give more practice with those that are presenting more difficulty - those other than combine 1, change 1, change 2, and compare 1.

2. Multiplication and division based upon repeated addition and subtraction and distributive division.

3. Problems that are multi-step in that more than one operation is used. (See problem solving material in Level Two Guide for sample problems. Put larger numbers into these.)

4. Non-numeric problems that involve Pattern Blocks and Tangrams. Some examples are provided along with sources of commercial materials that are good for this.

CHANGE PROBLEMS (PROBLEM SOLVING)

This is a procedure for giving oral problems to children to get them to understand the different kinds of problems, and to learn to write number sentences that represent these.

Children should have blocks, cubes or something to represent the objects in the problems. They should have recording forms.

"I will read the problem slowly. Listen for the question. Think of the number to answer the question. Write on the recording form."

Read the problem. Display a printed form of it on the overhead while reading slowly. Give children time to think and to manipulate objects if they need it.

"This time think of the number sentence that shows what is in the problem. Write this. Circle the number in the number sentence that answers the question in the problem."

Again give time.

Discuss each problem and its number sentence after the children have done it.

Below are the 6 change problems with the number sentences as samples. You supply the numbers appropriate for the time of the year.

Change 1: (easy)

Tom had ______ boxes. Jane gave him ______ boxes. How many boxes did he have?

$$N_1 + N_2 = \boxed{N_3}$$

Change 2:

______ boys were sitting in the gym. ______ boys got up and left the gym. How many boys were ~~still~~ in the gym?

$$N_1 - N_2 = \boxed{N_3}$$

Change 3: (hard)

Paul has _____ UNIFIX cubes. Paula gave him some more UNIFIX cubes. Paul then had _____ UNIFIX cubes. How many did Paula give him?

$$N_1 + N_2 = N_3$$

Change 4:

_____ girls were waiting for the bus. A few girls got tired and left so there were _____ girls waiting for the bus. How many girls left the bus stop?

$$N_1 - N_2 = N_3$$

Change 5:

Fran had some Pattern blocks in a box. Marge put _____ more Pattern Blocks in the box. Fran counted _____ Pattern Blocks in the box. How many Pattern blocks did Fran have to start with?

$$N_1 + N_2 = N_3$$

Change 6:

Tom had some tennis balls. He gave _____ of them to Peter, and had _____ left in his bag. How may tennis balls did he have to start with?

$$N_1 - N_2 = N_3$$

LEVEL THREE

PROBLEM SOLVING: GENERATING PROBLEMS

Background: Children should interpret open sentences into (a) arrangements of materials and (b) words involving those operations.

LESSON ONE

Introduction: Write an open sentence on the chalkboard: 4 + 7 = □ "What does the " □ " show?" "What operation is to be used?" "Someone suggest a story to go along with the number sentence."

Discuss these stories, particularly terms like "in all," "altogether," "How many more", etc. Try to elicit different problem types that could be represented by this - combine, join, etc. Using the same numbers, write the following open sentences and ask the children to write a story for each. Remind them, the story must have a question to be answered - as shown by the " □ ".

□ = 7 + 4; 4 + □ = 7; □ + 4 = 7.

LESSON TWO

Introduction: Write the open sentence. 7 - □ = 4 on the board. Ask the same set of questions about " □ ", "-" and the numerals. Ask for stories to go along with this and discuss. Then put the following on the chalkboard and have the children write stories to go with them: 7 - 4 = □ ; □ - 4 = 7; □ - 7 = 4

LESSON THREE

Introduction: Put 7, 8 and + on the chalkboard. Ask the children to think of a story with a QUESTION that involves 7 and 8 and +. Discuss 3 or 4 of these suggested stories. Write one of them on the board and have the children write the number sentence for it, solve it. Follow up with others such as: 21, 15, -; 5, 4, +; 21, 7, -; 5, 9, +; 9, 4, +, 20, 16, -; 16, 9, -; 9, 4, -; 17, 8, +; etc. Collect the problem samples and sentences written. Some of these can be used for other problem solving.

LESSON FOUR

Give the children only numeral pairs and let them write story problems using these. Have them write number sentences for each problem written.

## PROBLEM SOLVING: MULTIPLICATION AND DIVISION

Background: Most problems children have with verbal problems requiring multiplication and division arise from a lack of understanding of what these operations mean. Their experience with the situations that give rise to these operations is too limited. Multiplication too often is taught only as "repeated addition" and division as "repeated subtraction." Although these operations arise from consideration of part-part whole situations, joining, separating, and comparing like addition and subtraction, other more complex situations also require multiplication and division.

Similar to CHANGE 1 addition problems are "repeated addition" CHANGE 1 multiplication problems. An initial set of some size is given, a number indicating how many sets of this size are joined is given and the larger resulting set must be found. An example is: *"John put 6 handfuls of 3 nickels each into a bank. How many nickels did John put into the bank?"*

An inverse of this, CHANGE 2, results when a large set is given, the size of a smaller set is given, and one needs to find the change number describing HOW MANY of that size can be made. This is the "measurement" or "repeated subtraction" division. An example is: *"Susie has 24 cookies. She gives 3 cookies to each of the children on the playground. How many children get cookies?"*

This is essentially a counting process. The child must count how many 3's can be obtained from 24.

CHANGE 3 problems involve a large initial set, a known change number and the SIZE of the final EQUAL sets that can be made from this must be found. This is the PARTITION interpretation of division. An example is: *"Susie has 24 cookies. She gives the same number of cookies to each of 4 friends. How many cookies does each friend get?"*

Here the child must develop some strategy for assuring EQUALITY of the resulting sets. Comparison of two sets involves a comparison set and referent set, along with either a differences set or a correspondence between elements of the set. If questions involve "more than" or "less than", addition and subtraction result. If questions are "how many TIMES AS MANY?" or WHAT PART OF ?" are asked, multiplication or division may result.

COMPARE 1 problems result when one is given the referent set and a correspondence and needs to find the comparison set. An example is: *"Lis has 3 TIMES AS MANY nickels as dimes. She has 4 dimes. How many nickels does she have?"*

Multiplication is needed to answer the question.

COMPARE 2 problems result when one is given the comparison set and the correspondence and the referent set must be found. An example is: "*Iris has 15 nickels. She has 3 times AS MANY nickels as dimes. How many dimes does Iris have?*"

Division is used to answer this question.

COMPARE 3 problems result when the two sets are given and the correspondence must be found. An example is: "*Frank has 24 nickels and 8 dimes. He has HOW MANY TIMES as many nickels as dimes?*"

Division gives the answer.

COMPARE 4 problems involve finding the correspondence in the other direction. An example is: "*Frank has 24 nickels and 8 dimes. Frank's dimes are what fractional PART OF his nickels?*"

This is really the finding of a ratio of dimes to nickels:
8:24 or 1:3. However, the division - 8/24 - usually is expressed as a fraction - 1/3. The result is real conflict between a child's concept of fraction in terms of equal parts of a whole and the concept of ratio as a correspondence.

COMPARE 5 problems result when the correspondence sought is MANY-TO-MANY. An example is: "*There are 12 girls and 16 boys in the room. HOW MANY times as many boys are there as girls?*"

Again, division to find the answer results in a conflict between fraction and ratio.

COMPARE 6 problems ask for the many-to-many correspondence, but in the opposite direction. An example is: "*There are 12 girls and 16 boys in a room. The girls represent WHAT PART OF the boys?*"

Division again results in a fraction - this time <1. Note the difficulty in finding language to avoid creating the ratio-fraction conflict.

COMPARE 7 problems result when one set and a many-to-many correspondence is given and the other set must be found. An example is: "*There are 16 boys in the class. There are 4/3 AS MANY boys as girls. How many girls are there?*"

Dividing by the fraction is needed. If the larger set is sought, COMPARE 8 problems result. An example is: "*There are 12 girls in the room. The number of boys is 4/3 the number of girls. How many boys are in the room?*"

Multiplication gives the answer.

RATE problems involve a constant rate of change of one variable relative to another, or a constant comparison of one variable relative to another. Rate problems involve a total of one variable found by multiplying a rate x units of the second variable. The rates are usually expressed in units such as mi./hr., $/lb.; lb./cu. ft.; %/(parts/100) etc. These problems should not be introduced to students until they have a good understanding of the rate function in general. That is the purpose of introducing *"Guess My Rule"* early and gradually working it into consideration of constant rate, or linear functions.

RATE 1 problems result when one is given the rate and the value of the INDEPENDENT variable and must find the value of the related DEPENDENT variable. An example is: *"Fred pays $12.00/square yard (the rate) for carpet. How much will 16 square yards (value of the dependent variable) cost him?"*

Notice that you must find a total (of the dependent variable), multiplying $12.00/sq. yd. x 16 sq. yds. gives $192.00. The units of the independent variable cancel to give an answer expressed in units of the independent variable.

RATE 2 problems result when one is given the rate and value of the dependent variable and must find the value of the corresponding independent variable. An example is: *"John paid $162.00 for carpeting that cost $9/square yard. How many square yards did he buy?"*

Division gives the answer: $162.00 = $9/sq. yd x _☐_ sq. yds. _☐_ sq. yds. = $162 divided by $9/sq. yds.

Here the $ units cancel and the answer is in the units of the other variable.

RATE 3 problems result when a pair of values of the two variables is given and one must find the rate. An example is: *Peter paid $342 for 200 eight foot '2 x 4's." What was the cost in dollars of each '2 x 4'?"*

$342 = \_\_$☐_/'2x4' x 200 '2x4's' so

_$☐_/'2x4' = $342 divided by $200 '2x4's' = $1.71 for each '2x4'.

This is the rate: $1.71/'2x4'.

Several kinds of problems are based upon the MULTIPICATION PRINCIPLE. Among these are the selection problems.

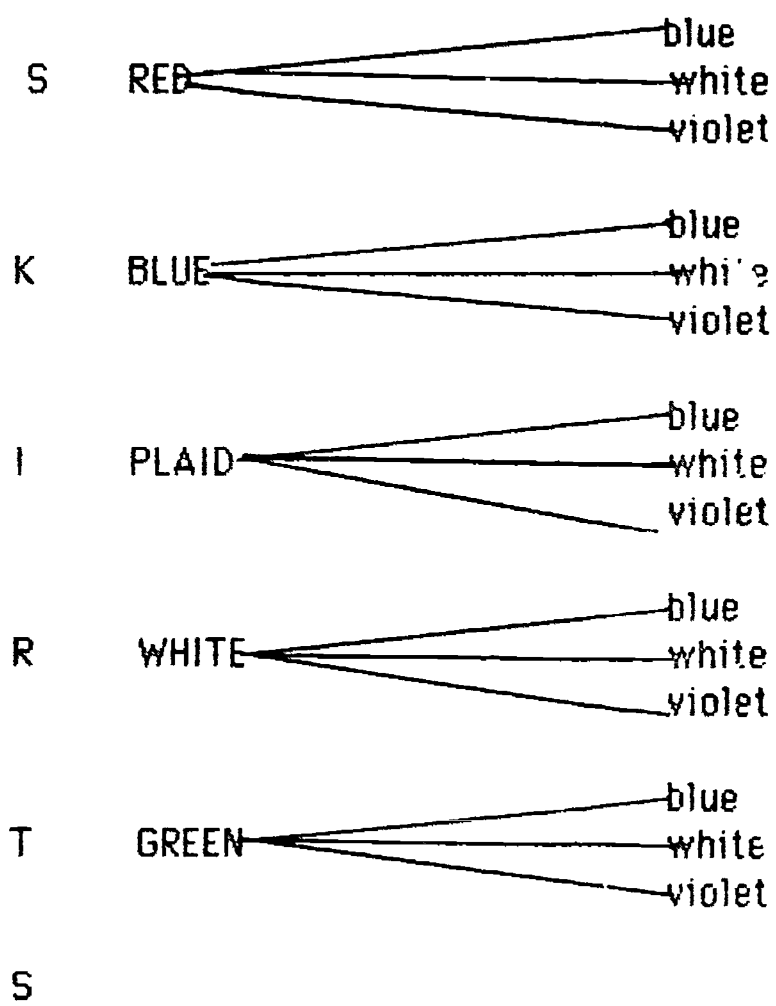
SELECTION 1 problems involve simple ordered pairs where the choice set for each element of the ordered pair as given the number of possible ordered pairs is sought. An example is: *Jane as 3 sweaters with different patterns. She also has 5 different color skirts. How many outfits consisting of a sweater AND skirt can she select to wear?"* The ordered pairs can be seen at the intersections of rows and columns in a matrix.

SKIRTS

| SWEATERS | Red (R) | Blue (B) | Plaid (P) | White (W) | Green (G) |
|---|---|---|---|---|---|
| Blue (b) | b,R | b,B | b,P | b,W | b,G |
| White (w) | w,R | w,B | w,P | w,W | w,G |
| Violet (v) | v,R | v,B | v,P | v,W | v,G |

They also can be seen from a factor tree.

SWEATERS



In either case we have 3 x 5 or 5 x 3 = 15 pairs.

SELECTION 2 problems result when one is given one choice set and the total number of pairs and the second choice set is sought. An example is: *"Jane has 24 sweater-skirt outfits. She has 4 sweaters. How many skirts does she have?"*

Division is needed to find the answer: 24 4 = 6 skirts.

SELECTION 3 and SELECTION 4 problems involve triples made from 3 choice sets, quadruples made from 4 choice sets, or the extended *-tuples, >2.*

Examples are:

*"Frank has 5 sport coats, 3 vests and 5 pairs of trousers. How many outfits consisting of a sport coat, vest and pair of trousers can he choose from?"*

Multiplication is required to obtain 5 x 3 x 5 = 75 outfits.

*"Frank can make 24 different outfits consisting of a sport coat, vest and trousers. He has 3 sport coats and 4 pairs of trousers and some vests. How many vests does he have?"*

The problem requires multiplication AND division: Multiply 3 x 4 and then divide 24 by this product.

Children should have experience with these situations using concrete objects. Symbols probably should be used with just the CHANGE problems at this level.

PROBLEM SOLVING: An Emphasis on Strategies

LESSON ONE:

1. Make up a problem.

2. Have children put information in a table.

3. Give questions to answer.

Example: Last week on Monday; 40 boys and 32 girls ate hot lunch; on Tuesday, 28 boys and 23 girls ate hot lunch; on Wednesday 34 boys and 41 girls ate hot lunch; on Thursday 10 boys and 8 girls ate hot lunch; on Friday 34 boys and 39 girls ate hot lunch:

**Number Eating Hot Lunch**

| | Monday | Tuesday | Wednesday | Thursday | Friday |
|---|---|---|---|---|---|
| | | | | | |
| Boys | | | | | |
| Girls | | | | | |

1. How many people ate hot lunch on Wednesday?

2. How many more boys ate hot lunch on Monday?

3. How many boys ate hot lunch during the week?

4. How many people ate hot lunch during the week

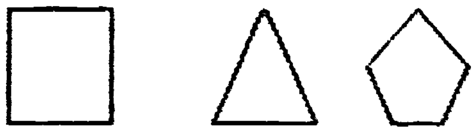
5. How many more girls ate hot lunch on Wednesday than on Monday?

LESSON TWO:

1. Give problems that require the children to try out different possibilities or to guess and check.

Example: Tom has 4 coins that total 50 cents. What coins does he have?

Each shape has sides:



How many squares would you need to get 16 sides?

How many sides would 5 squares and 3 triangles have?

How many triangles and how many pentagons would you need to have 29 sides?

LESSON THREE:

Use multi-step problems: Twenty-four children were taking Suzuki violin. Last week, 3 children quit and 7 children started the class. How many children are now taking Suzuki violin?

These kinds are made easier by having a "number" day with UNIFIX cubes. For example: Start with a twelve UNIFIX cube link. Have children join to and take away UNIFIX cubes from this link, writing the number sentences each time, i.e.:

12, join two, take away three, $12 + 2 - 3 = 11$.

LESSON FOUR:

Give students problems that have more information than needed so they must use the RELEVANT information.

Example: John has 4 horses, 6 sheep, 14 chickens and 10 rabbits. How many 4-legged pets does he have?

LESSON FIVE:

Children should also recognize when there is not enough information to solve a problem.

Example: Fred and Tony bought 4 bags of marbles How many marbles did they get?

Jean and Fran shared their cookies with three other girls. How many cookies did each girl get?

LESSON SIX:

Children should be able to round to the nearest decade, i.e. 37 to 40, or to the nearest hundred, i.e. 195-200 in order to estimate an answer. When they find the answer they then know if it makes sense.

Example: The third grade has 28 children and the fourth grade has 31 children. Is the number of children in both grades closer to 6 or 60?

There are 6 stickers on a page. Janet used 8 1/2 pages. Is the number of stickers she used closer to 40 or 50?

LESSON SEVEN:

Children should use tables to put several known examples into to answer questions.

John's father sold 3 cottages for every 10 houses he sold. If in a year, he sold 12 cottages, how many houses did he sell?

| | | | | |
|---|---|---|---|---|
| Houses | 10 | 2 x 10 = 20 | 3 x 10 = 30 | 4 x 10 = 40 |
| Cottages | 3 | 2 x 3 = 6 | 3 x 3 = 9 | 4 x 3 = 12 |

If you have done the UNIFIX ratio activities with some frequency, children should be able to generate the table.

Sources of problems: Problem Solving Experiences in Mathematics, Charles, Mason & Martin, Addison Wesley Publishing Company (Grades 3 and 4).
Mathematics Problem Solving Activities, Dale Seymour Publications (Grades 3 and 4.)

LEVEL THREE

Examples of Problems Appropriate for this Level:

1. John had ______ crayons. Donald had _____ more than John. Together they had 40 crayons. Find all pairs of numbers showing their crayons so the answer will be 40 crayons.

2. Jean has _____ UNIFIX cubes. Patsy had _____ fewer UNIFIX cubes than Jean. Together they had 28 UNIFIX cubes. Find all pairs of numbers for their UNIFIX cubes so the answer will be 28 UNIFIX cubes.

3. Terri brought 3 dozen cookies for the class party. She took 4 back home. How many cookies were eaten at the party?

4. Peter put in 6 rows of lettuce with 12 seeds in each row. Half of the seeds grew. How many lettuce plants came up? Insects destroyed 10 plants. How many lettuce plants were left?

5. Johnson's bean plants grew 1/3 inch daily for 2 weeks. How much taller were they after 2 weeks?

6. At the school store, three oranges plus two apples cost $1.55. Two oranges plus three apples cost $.05 more. What is the cost of one orange? Of one apple?

7. Joan's mother lost 45 pounds on a diet. She now weighs 126 pounds. How much did she weigh before dieting?

8. In the basektball tournament, Tammy scored 32 points, Iris scored 19 points and Lil scored 24 points. Tammy scored how many more points than Lil? Iris scored how many fewer points than Lil? How many more points would Iris have to score to have as many as Tammy?

9. John and his sister get the same allowance - $1.00 each week. One week John spent $.38 on a candy bar and $.25 for a pencil. His sister spent $.49 for a scarf. Which had more money left? How much more?

10. The classes at Franklin School are always bigger than those at the Merritt School. Jan's third grade at Franklin has 28 students. There are 50 third graders at the two schools. How many are in third grade at Merritt?

11. Tim is 9 years old and his brother is 13 years old. How old will Tim be when his brother is 19 years old?

12. Mrs. Erickson had four boxes of UNIFIX cubes with two links of ten in each box. She spilled them all on the floor. How many UNIFIX cubes were on the floor?

13. Patsy bought 4 candy bars at \$.40 each and 5 packages of gum. The bill was \$2.85. How much is each package of gum?

14. Kathy found some colored links in the closet. Each link was 3/4 inch long. She made a chain of 20 links. How many inches long was the chain?

15. Tom's mother gave him some money to buy groceries. He bought two tomatoes at \$.40 each and 3 packages of carrots for \$.09 each. He received \$.03 change. How much money did his mother give him?

16. Bill and Carol decided to pick 100 pussy willows for the classroom. After one hour, Bill had picked 32 and Carol 38. How many more pussy willows did they need to pick?

17. Judy and Bob sold tickets to the class carnival. Bill sold 12 and Judy sold 4 more than Bill. How many tickets did they sell together?

18. Ann noticed that each row at the Arena had one more seat than the row below it. The first row has 13 seats and there are 9 rows. How many seats are in the ninth row?

19. Tom made ______ snowballs. He gave ______ snowballs to each of 4 friends. Find as many pairs of numbers for the snowballs as you can so that they would be equally distributed to 4 friends.

20. Bob has ______ bags of marbles. Each bag has ______ marbles. Bob has 60 marbles. Find as many pairs of numbers for bags and marbles as you can so that Bob will have 60 marbles.

21. Bernice has ______ books. On her birthday she gets 11 new books. She then has______ books. Find as many number pairs for the books as you can so she will have received11 new books each time.

22. Tom has ______ toy cars. Nine are red and ______ are blue. Find as many number pairs for toy cars and blue ones so there are always 9 red cars.

23. Jean has ______ pencils. She has 6 pencils more than Bill. Bill has ______ pencils. Find as many number pairs as you can so Jean always has 6 pencils more than Bill.

24. Sissy has ______ pencils. She has 9 pencils fewer than Joyce. Joyce has ______ pencils. Find as many number pairs for the pencils as you can so Sissy will always have 9 fewer than Joyce.

25. Alice has ______ skirts. She gets ______ more skirts on her birthday. She now has 14 skirts altogether. Find as many number pairs as you can so she always has 14 skirts.

26. Frank has ______ records. His mother gives him 12 more records. He then has ______ records. Find as many number pairs as you can so Frank's mother will have given him12 more records.

27. Sarah has ______ puppies. She gives ______ puppies to Jill. Sarah has 5 puppies left. Find as many number pairs as you can so that Sarah always has 5 puppies left.

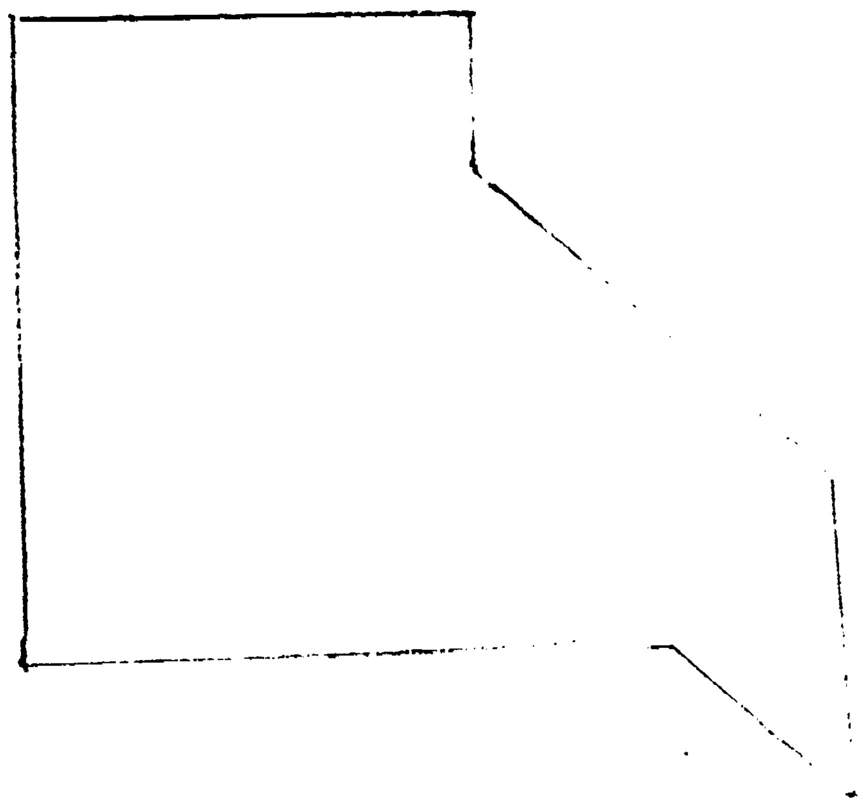
28. Felix has 13 cats. ______ of his cats ran away. He has ______ cats left.

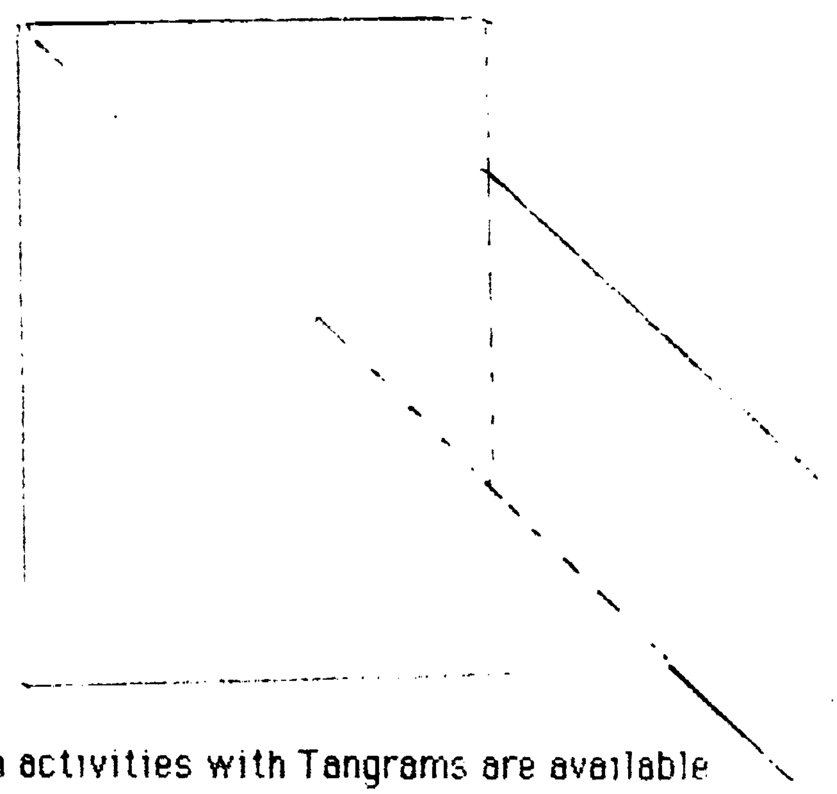
29. Archie has ______ pencils and Reggie has ______ pencils. Reggie has 6 more pencils than Archie. Find as many number pairs as you can so Reggie has 6 more pencils than Archie.

30. Betty has 12 books. Veronica has ______ fewer books than Betty. Veronica has ______ books. Find as many number pairs as you can so Betty will always have 12 books.

PROBLEM SOLVING: TANGRAMS

Background: You may want to demonstrate on the overhead with problem transparencies and tangrams first. Then assign worksheets to the children to complete with tangrams on those that do not have outlines of tangram pieces, have children trace the tangrams to show where they were placed in solving the problem.

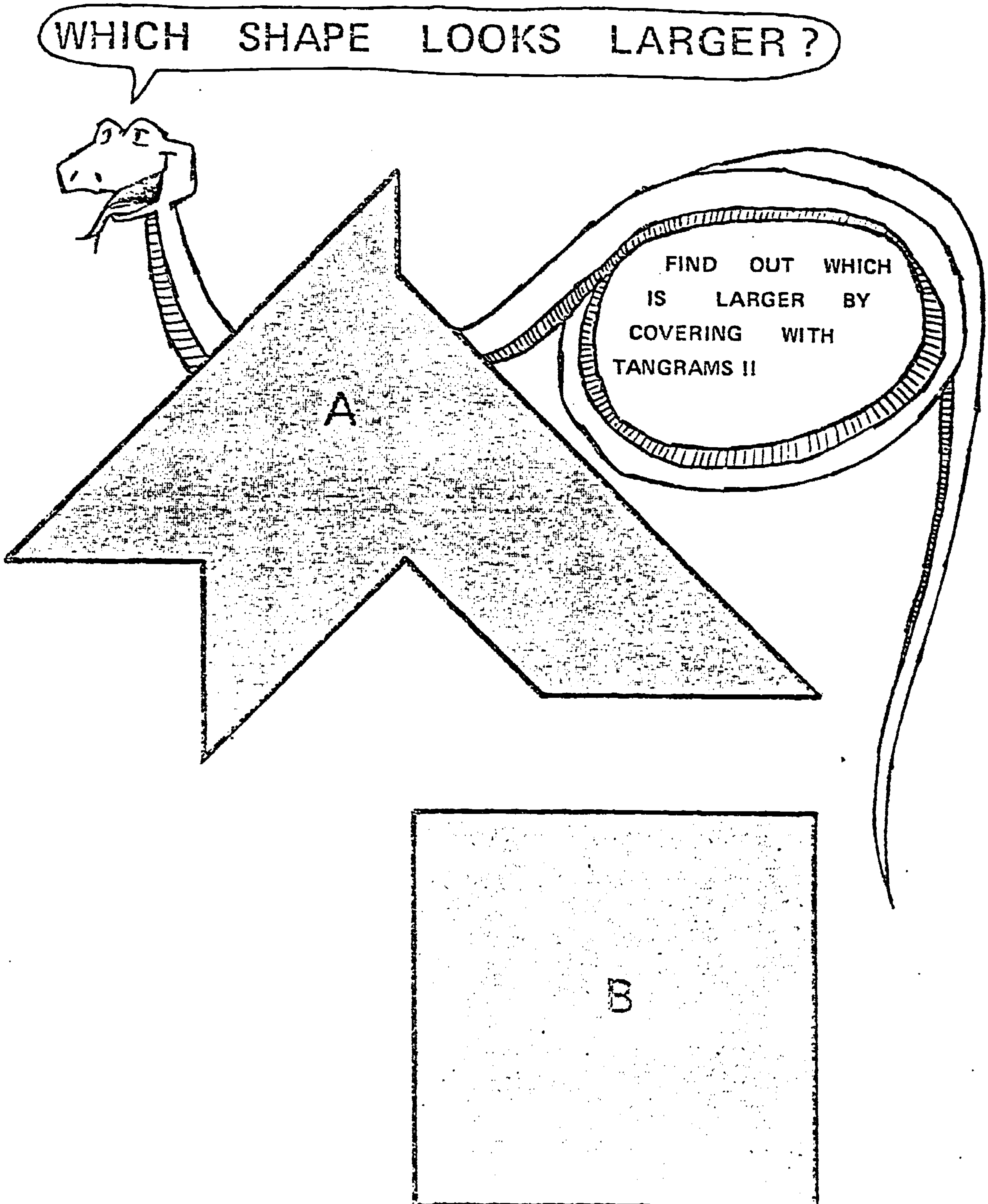


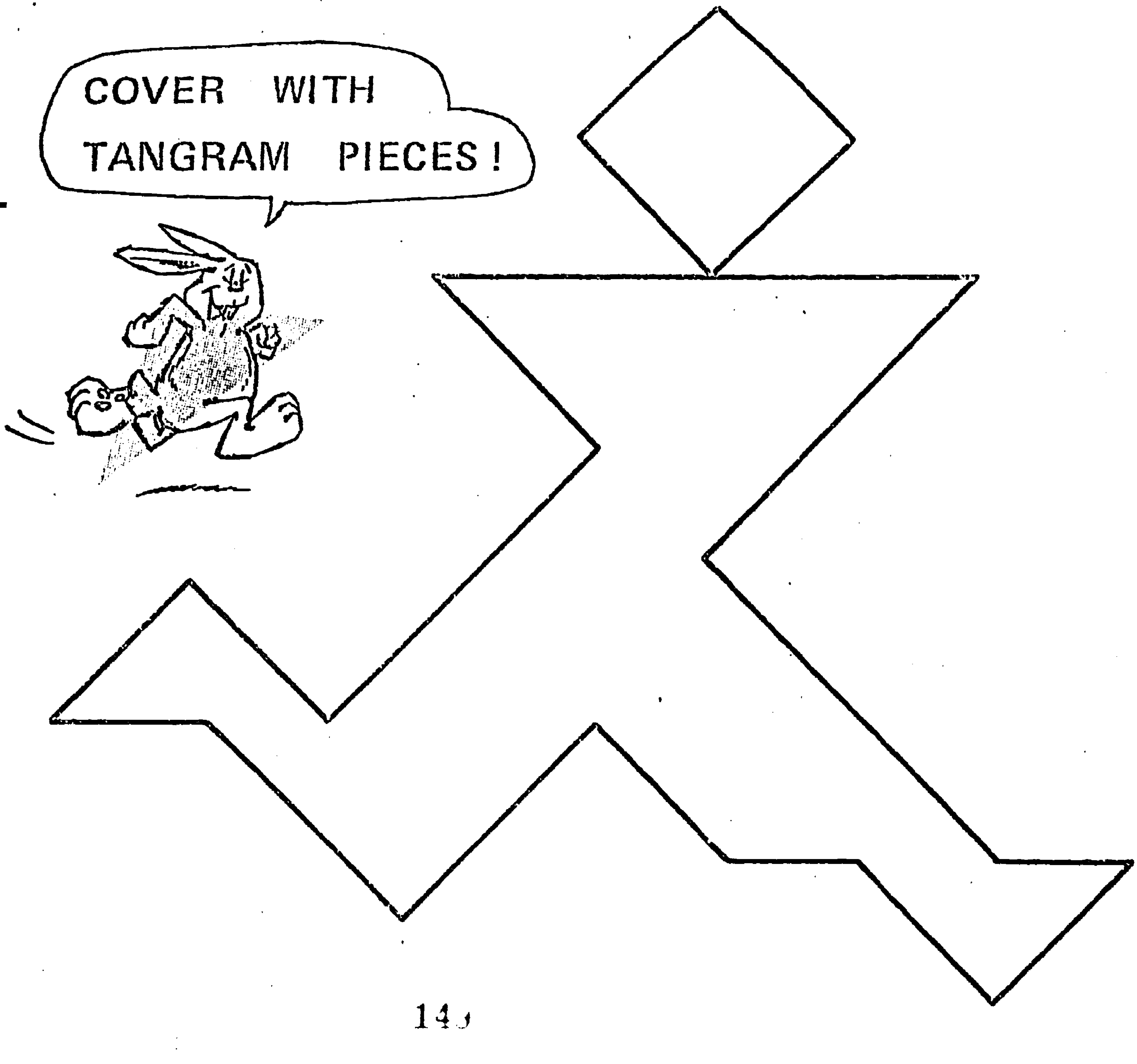


Additional problem activities with Tangrams are available

1. Tangram Patterns - Creative Publications
2. Hands on Tangrams 3 - Creative Publications
3. Tangram Cards - Elementary Science Study Unit

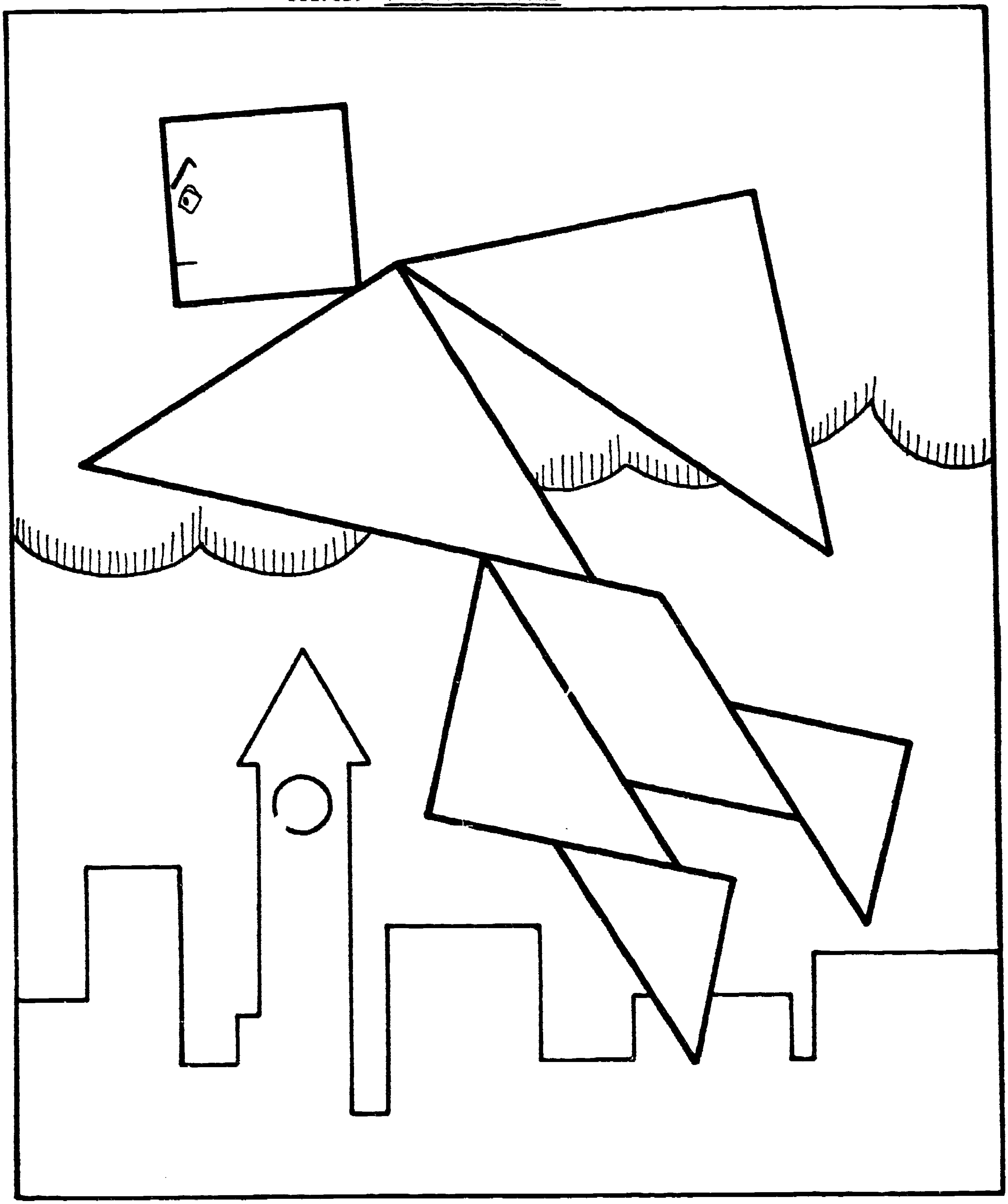
## MORE COMMERCIAL TANGRAM ACTIVITIES
Source: Tangramath




COVER WITH TANGRAM PIECES!

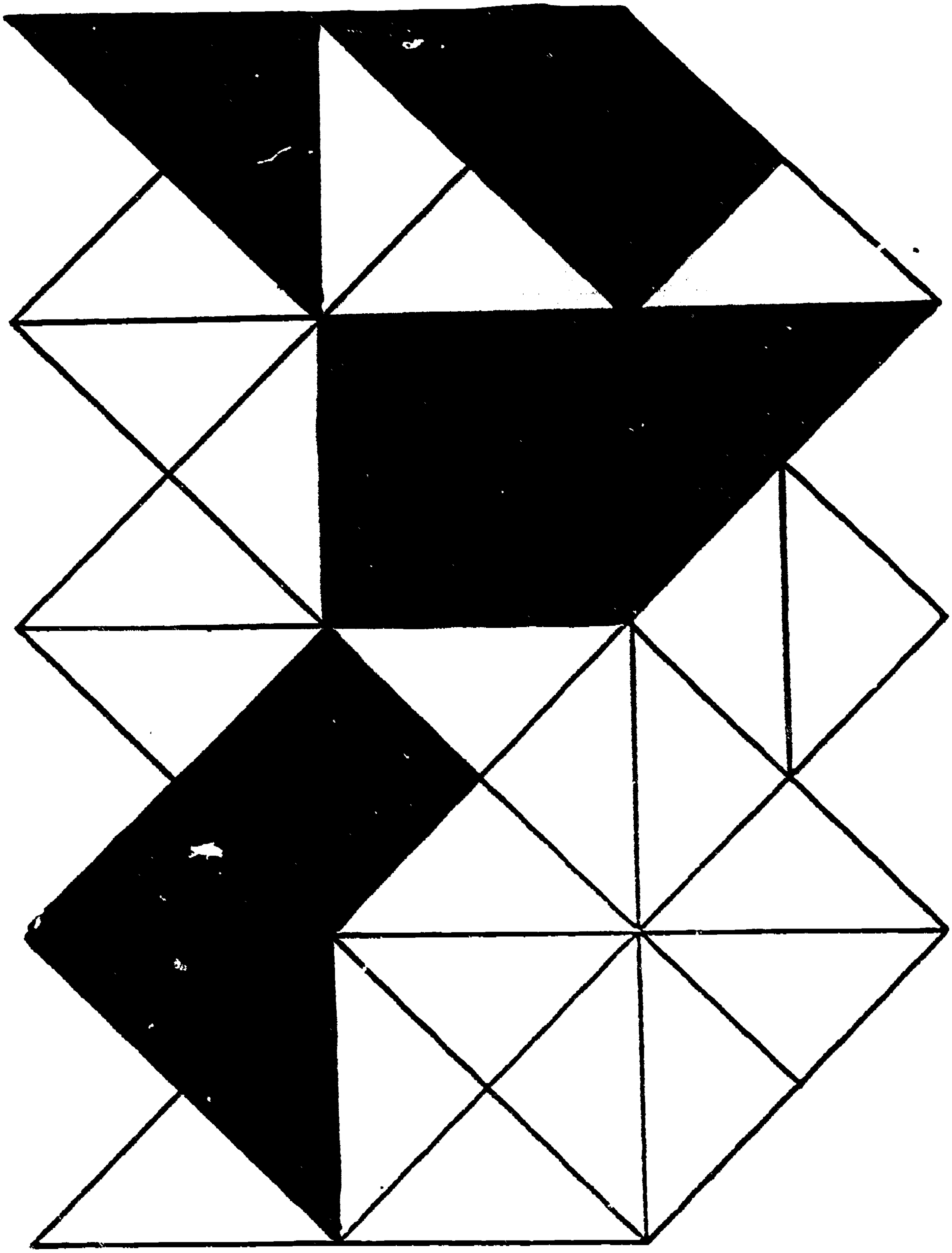
EXAMPLE OF A COMMERCIAL TANGRAM ACTIVITY

Source: TANGRAM PATTERNS



Superman flying

# A SAMPLE 'TANGRAM PROBLEM GENERATOR' PROBLEM



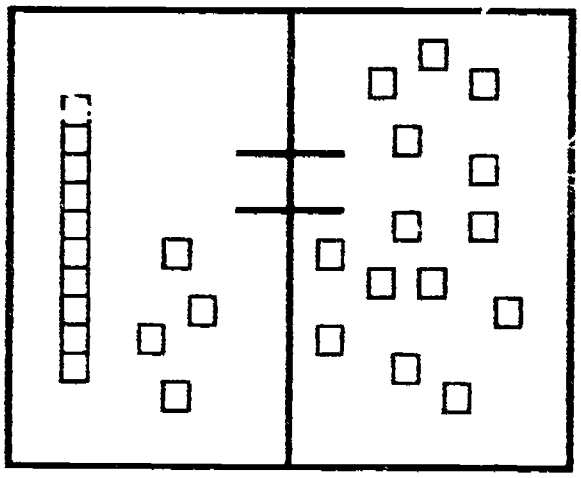
EQUALITY

Background: At this level equality activities should involve place value with numbers of 2 digits used.

Challenge children to write the numbers in as many different ways as possible.

LESSON ONE

Introduction: On the overhead projector place an equality mat with base ten blocks as shown. Tell children to arrange theirs the same way as you do.

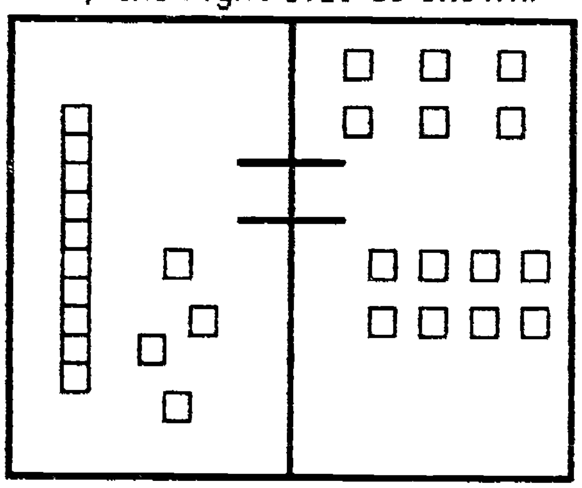


"How much is shown on the left side?"

"How much is shown on the right side?"

"Is the same amount shown on both sides?

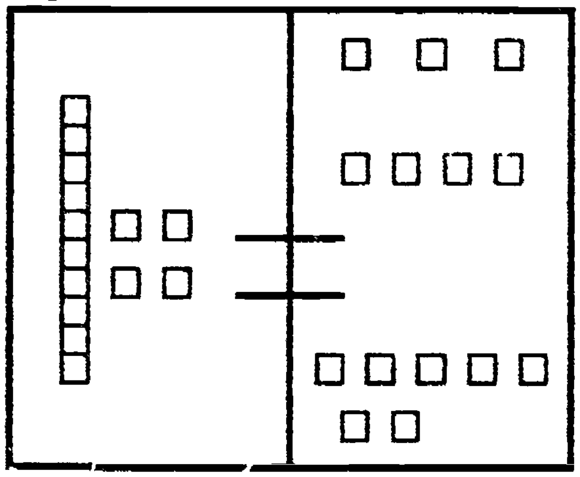
Group the right side as shown:



"What are the parts of the fourteen on the right side?"

Write: 14 = 6 + 8

Regroup into:



"What are the parts on the right side now?"

Write: 14 = 3 + 4 + 7 below the other. Write several starts of number sentences below these: 14 =

14 =

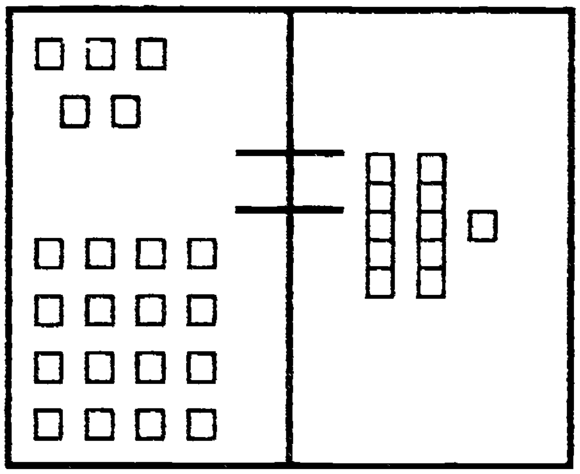
14 =, etc.

Have the children group the ONES on the right side so as to get a number sentence DIFFERENT from any on the board. Write these down as generated.

Activity: Pass out recording sheets and have the children generate as many number sentences for each given number as they can. Repeat this lesson at least once a month.

LESSON TWO

Introduction: Arrange the following on the overhead. Children should have equality boards and base ten blocks:



"Arrange yours to be like mine."

"Are both sides equal?"

Write: 5 + 16 = 21

"Can this be written another way? - look at the ONES on the bottom."

Write: 5 + 4 x 4 = 21

"Four times four is another name for sixteen."

"Arrange the left side in 3 groups." Ask for number sentences from several children and write these down.

Examples:

6 + 5 + 5 = 21
4 + 3 + 14 = 21, etc.

Discuss each case presented.

<u>Activity</u>: Have the children work on the prepared activity sheets. Check on each child to determine whether or not number sentences are being written correctly. Repeat parts of the lesson as needed to remediate number sentence writing problems.

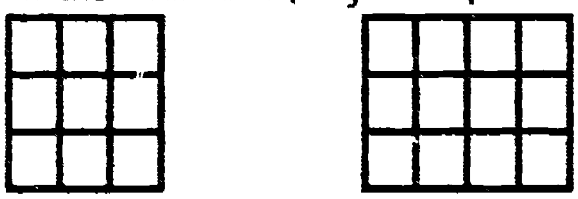
LEVEL THREE

INEQUALITY

LESSON ONE

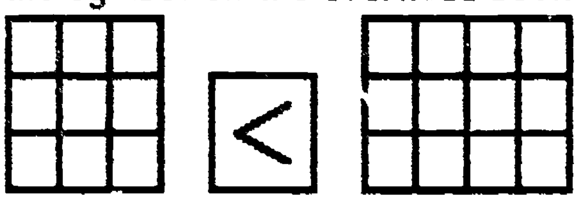
Introduction: Prepare about 25 number cards with number words from nine to forty on them. Have 2 or 3 of each kind available. Children should have tiles and the recording forms.

On the overhead projector place the following tile arrays:



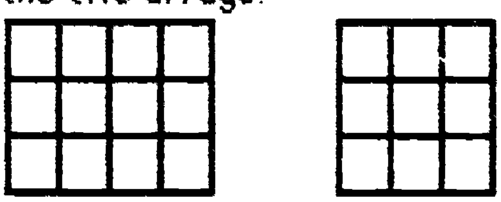
"Which rectangle has more tiles?"

"We use this sign to show that the open side points to larger number." Put the symbol on the overhead between them:



Write: $9 < 12$

"The sign always points to the smaller number. Switch the positions of the tile arrays:



"How should the sign point now?" Have one of the children arrange it. Repeat with two different numbers of tiles.

Activity: Pass out two different number cards to each child. They are to build a tile arrangement to show the two numbers and write the number sentence to show the comparison of them.
All remaining cards should be placed in a deck accessible to the children. When they finish with two they are to remove two more from the deck and place their two on the deck. Monitor to see the children are not always putting the larger number to the left or right.

LEVEL THREE

USING CALCULATORS: CALCULATORS

Background: You should have a classroom set of four function calculators with large keys that are solar powered or have long life batteries with automatic power shift off after a short period of disuse. First find out how the calculator operates. Most use arithmetic logic: 5 + 3 = yields 8. These are the kind of use since they parallel the number sentence structure of canonical form - a + b = c.

First make sure ALL students know which keys to depress in which order. Also determine whether the first ADDEND entered or the second ADDEND entered is repeated when = is depressed, and whether the first FACTOR or the last FACTOR is repeated when = is depressed.

LESSON ONE: FREE EXPLORATION

Give each child a calculator. Let them push buttons at will - they can't hurt the calculator. Be prepared to answer questions about the unusual displays they might obtain - exponential notation resulting from large number overflows of the display.

LESSON TWO: COUNTING

Have the children use the repeat function of the calculator to keep pressing the = to add the desired number repeatedly.

1. Count from 1 -

| | | | | |
|---|---|---|---|---|
| 1 | + | 1 | = | = |

.........

2. Count from any N -

| | | | | | |
|---|---|---|---|---|---|
| 4 | 8 | + | 1 | = | = |

.......

3. Skip counting

**Example:**

| | | | | | |
|---|---|---|---|---|---|
| 2 | + | 2 | = | = | = |

| | | | | | |
|---|---|---|---|---|---|
| 3 | + | 3 | = | = | = |

In skip counting, use a hundreds chart and have the children circle each number that appears on the display. Encourage them to look for patterns, to predict which number comes next in the skip counting, etc.

Develop worksheets similar to that provided to help children recognize the patterns in number sequences or on a table.

LESSON THREE: PLACE VALUE

Introduction: Have the children push [2] on the calculator.

"In what place is the 2?"

Push [3] on the calculator.

"In what place is 3?"

In what place is 2?"
"What number is shown?"

Push [4]

"In what place is 4?"

"In what place is 3?"

"In what place is 2?"

"In displaying a number, the numeral for WHICH PLACE is pushed first?" second?" last?"

Activity: The children should have base ten blocks and calculators. Have them build numbers out of the base ten blocks and then push buttons on the calculator to display these. The worksheets should be completed

An additional source of sheets to use is "KEYSTROKES - Counting and Place Value" - available from several instructional materials catalogs.

## LEVEL THREE

### GUESS MY RULE

Background: You as teacher will be a rule machine. Children will give you small whole numbers. You will use the rule on these to generate a response. Children are to "guess" your rule. Establish the ground rule that a child is not to blurt out the rule when discovered. This spoils it for other children. When a child thinks he or she knows the rule, the correct test is, "If I gave you ________(number), will you give me ________(number) back?" You answer by "yes" or "no".

Introduction: Tell the children you have a rule. Ask one child for a number. Use the rule to return a number. Then ask for another, etc.

Example: Rule: IN number + 7 = OUT number

Response:

| IN | OUT |
|---|---|
| "3" | "10" |
| "6" | "13" |
| "2" | " 9" |

If no child suggests it, you suggest putting numbers IN ORDER in the IN column to see the rule more easily.

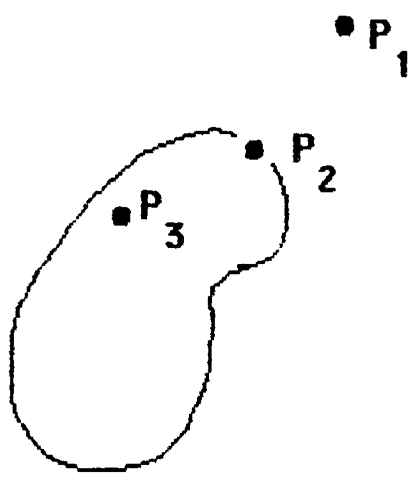
These rules can involve any of the basic arithmetic operations.

LEVEL THREE

## GEOMETRY: BASIC PROPERTIES

Background: In their development of geometric ideas, children first recognize TOPOLOGICAL properties. These include closed v. open shapes:

Inside-on-outside point location relative to a bounded area:




**$P_1$ is OUTSIDE**

**$P_2$ is ON**

**$P_3$ is INSIDE**

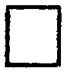
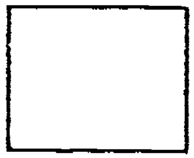
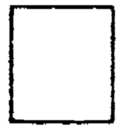
And whether or not shapes have one "hole" or two "holes". The latter is more closely related to their recognition of printed English letters than geometric shapes encountered.

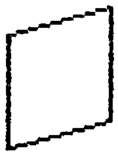
They also recognize TRANSFORMATIONAL properties. A shape does not change by rotating it about a point or "flipping" it over a line or moving it in space. They can recognize symmetry about a line or point, for example.

EUCLIDEAN properties associated with measurement and assigning of numeric values to length, angle, area, etc. develop more slowly since number is involved.

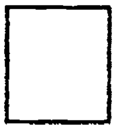
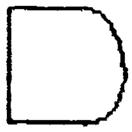
Recently a model of development of geometric ideas was proposed by the Van Hieles. Very briefly it is as follows:

Level 0 (the student considers visually identifable properties as a whole, without regard to components). Students at this level think orientation in space, proportionality of shape (similarity) differentiate shapes:

**Hence**  **is different from**  **and** 

**is different from** 

They also ignore some properties such as straight v. curved lines:

**Thus**  **and**  **are the same.**

Students at this level will shift from one property to another when sorting or classifying sha es.

Level 0 (the student can compare component properties of shapes but does not use class inclusion consistently, e.g. a square is a parallelogram). Students will often focus on a single property to classify instead of multiple properties, e.g. "equal sides AND one right angle."

Students will often identify shapes by several properties rather than the minimum set of sufficient properties.

Students at Level Three are likely to be at one of these levels or transitional to the next level where they use definitions, can modify these, or recognize equivalent definitions (OR) can recognize class inclusion and sort shapes using the minimum set of sufficient conditions. They also make correct use of "if, then" statements. Students at Level 1 also can make multiple classifications: "A square is a rectangle". "A square is an equiangular shape."

The lessons in geometry are fomulated to help children consolidate thinking at the level they are at and to help them move to the next evel.

## LEVEL THREE

## GEOMETRY: PROPERTIES OF SHAPES

### LESSON ONE

Introduction: Put an overhead transparency of different shapes that has been prepared from the supplied master on the overhead projector. Ask the children to look at all of the different geometric shapes

"Which of these can you name?"

"What properties do they have that make them different?"

"For example, they don't have the same number of sides."

List these properties as they are generated. Some likely to come forth that you might solicit through probing questions, are:

The number of sides
Whether sides have the same lengths
The kinds of angles
Whether all straight lines, all curved lines or a mixture
Whether open or closed shape

Point to two, i.e., the square and the rectangle:

"How are these alike?" "How are these different?" "Which of these could be grouped together?"

Write these on the blackboard.

Continue in this fashion, having the children compare two figures at a time to identify how they are ALIKE and how they are DIFFERENT from each other.

"Which of these are special cases of others?" "Is a square a special kind of rectangle?" "How is it different from other rectangles?" The purpose is to get children to focus on:

side relationships
angle relationships
opposite side relationships

LEVEL THREE

GEOMETRY: SIMILARITIES AND DIFFERENCES

Introduction: Place some shape on the overhead projector, i.e., a rectangle. "What shape that you know the name of differs from this in just one way - sides, angles, etc.?"

Discuss the first shape suggested thoroughly. How is it like the given shape? How is it different from the given shape?" "Suggest another shape different in just one way from the given shape, but not in the same way as before."

Discuss this thoroughly. Contrast it with the first shape suggested.

Repeat the procedure with a second shape on the overhead.

Activity: Assign the worksheet to pairs of students. Monitor the work. Ask questions to get them to look at as many different properties of shapes as you can.

## LEVEL THREE

### GEOMETRY: GEOBLOCKS 1

Background: Children should have experience in comparing volumes of frequently found shapes, identify how they are alike and how they are different and see how their linear dimensions are related to volume.

Introduction: Show the children two different geoblocks. Ask them how they are ALIKE. You may have to prompt them to look at the number of edges, corners, or faces, or the shapes of the faces. Explore this as fully as you can.

Then ask them how the blocks are DIFFERENT. Try to get at the idea of how much space they occupy. Ask the children how they might determine if they occupied the SAME amount of space, or which occupied MORE space.

Activity: Put masking tape labels on 5 different geoblocks to label them A,B, C, D and E. Two of these should have different shapes, but occupy the same amount of space. Put these at a work station along with the recording forms. Have children go to the work station in pairs over a period of days to complete the worksheets. When all have done this, discuss the results with the whole group.

## LEVEL THREE

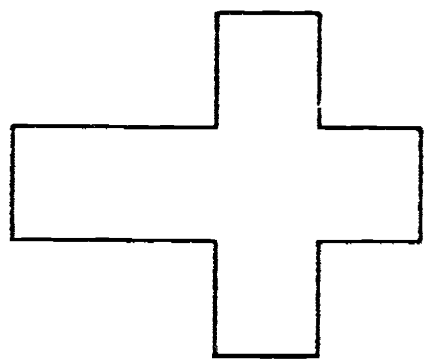
### GEOBLOCKS: SURFACE AREA

Introduction: Children should work in pairs. Each pair is to have 3 blocks not too radically different from each other.

"Hold up a block that has FIVE faces."

Discuss FACES and the kinds of geometric shapes found on them.

On the overhead projector, show how to make a "geojacket" by tracing the faces of a block to make such an outline. The geojacket for a CUBE is shown as an example:



Activity: Ask the children to make a geojacket like this for each of their geoblocks by tracing faces on graph paper. Upon completion:

"Which of your geojackets have the same area?"

"Which of your geoblocks have the same volume, but different area geojackets?"

"If you cut out your geojackets, could you fold it up to make a 'paper' geoblock?"

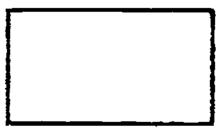
Follow-up Activity: The geojackets should be cut out, folded, taped together, and colored. They could be hung around the room as mobiles.

# LEVEL THREE

## GEOMETRY: GEOBLOCKS 2

### LESSON ONE: Rectangles

Children see rectangles in textbooks that (1) have the long side in a horizontal position and (2) are a close approximation to the Golden Rectangle.



They rarely see rectangles that are oriented differently or that have long sides and very narrow widths.

Introduction: Take one of the rectangular solid geoblocks. Trace around each face on the chalkboard or on the overhead transparency. Point out how the rectangles are different and alike.

Activity: Groups of children should have a set of geoblocks each. They should also have a large sheet of wrapping paper on which to trace. Have each group find and trace as many different rectangles as they can. The wrapping paper should NOT have two tracings of the same rectangle. Children can check on this by placing a face of a block or two rectangles thought to be alike to see if they are. When all groups are complete, hang the wrapping paper up where all can see. Ask questions about the results.

"How many rectangles are there?"

"How many of these are squares?"

"How many non-square rectangles are there?"

"Which rectangles look like they have one side of the same length?"

If children discover that a group has made two or more tracings of the same rectangle, have this checked by placing the appropriate face of a block on each.

LESSON TWO: Triangles

Choose a triangular base solid (ramp). Trace all faces on the chalkboard or overhead. Point out the triangles.

"How are they alike?"

"What do a rectangle and a triangle have in common?"

Activity: Children should be organized and given materials as in Lesson One. They are to trace all DIFFERENT triangles that they find on the faces of the geoblocks. When this is complete discuss as with the rectangles. Try to get children to identify the different RIGHT triangles and the non-right triangles they have found.

LESSON THREE

Hold up two dissimilar geoblocks:

"How are these alike?" DISCUSS

"How are these different?" DISCUSS

Clarify terminology. Solid Shapes like the geoblocks have FACES, EDGES and VERTICES.

Look of assumptions of EQUAL edge length, EQUAL areas of faces, etc.

LEVEL THREE

GEOMETRY: GEOBOARDS

Background: Students sometimes tend to think of area as a number resulting from substituting numbers into a formula. Work with Tangrams, Pattern Blocks, tiles, graph paper and geoboards give them concrete experiences that lead to thinking of area as that part of a plane within a boundary.

LESSON ONE: Square

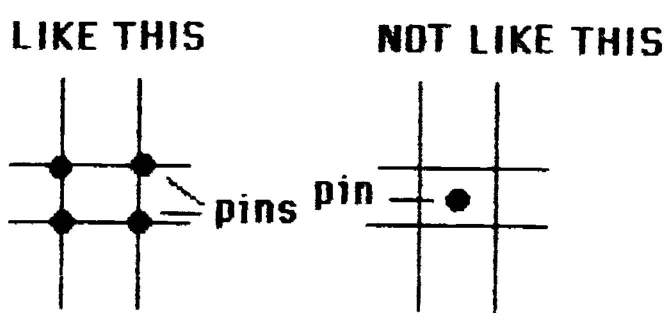
Introduction: Use either an overhead geoboard, a large demonstration geoboard or the dotpaper template provided.

Make a square on it. Ask the children how many of the smaller squares of the geoboard are inside this square. Tell them that the number of these squares is the AREA.

"What is the area of the square I made?" "If I cut a square out of paper just like the one I made on the geoboard, how many of the 'geoboard squares' would I have?"

"Could I use the small geoboard squares to measure how much paper is in that square?" "Would the area of the square I made on the geoboard be the same as the area of the paper square?" "Could I measure the desk top with little squares all the same size?"

Activity: Give each child a geoboard. These should be made with a coordinate background so the pins are placed:

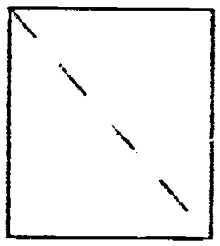



Have each child make a square on the geoboard.

"Hold up your geoboards." "Who has the square with the least area?" "What is the area of the square you made?" Have the children make squares and record their areas on the form with a picture of the square.

<u>LESSON TWO: Triangles</u>

<u>Introduction:</u> Fold a piece of paper as shown:



Tear it in half and place the two triangles together.

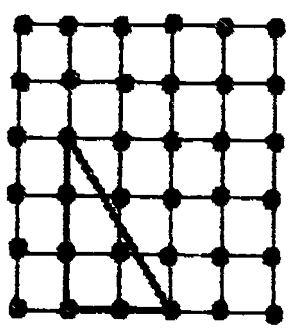
"Are the two triangles the same?" "What do we know when 2 equal things make a larger one?" "Each triangle has area half of the rectangle!" "If we can find the areas of rectangles we can find the areas of RIGHT triangles."

Be sure children can identify:

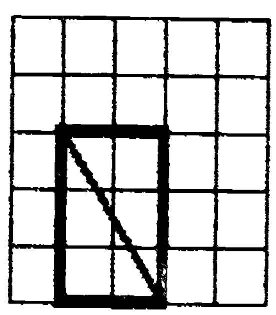
1. a RIGHT triangle
2. the sides that meet in the right angle as sides of a rectangle.

<u>LESSON THREE: Triangles</u>

<u>Introduction:</u> On the overhead geoboard build a triangle.



**Make a rectangle around it**

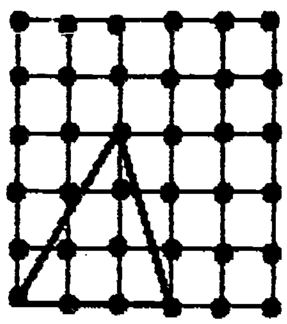


Remind the children the triangle is half of the rectangle, so to find the triangle area, they should make a rectangle that includes it and take half of it.

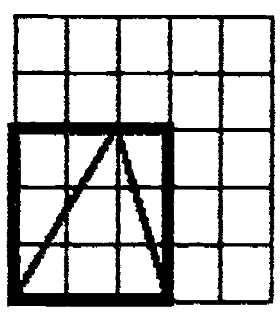
"How long are the sides of the triangle that make a right angle?" (3 and 2)
"How long are the sides of the rectangle?" (3 and 2)
"How many squares inside the rectangle?"
"How many squares inside the triangle?"
"Some of the squares are made up of several small pieces."

Discuss this so the children see they can ALWAYS find a rectangle associated with a right triangle and hence find the area of the right triangle.

"What if this triangle has no right angle?"



**Make a rectangle around it**



"How many right triangles do you see?" "Can you find the area of these?" "If you subtract the areas of the right triangles from the area of the rectangle, what do you get?"

Discuss this thoroughly. Children can ALWAYS find the area of ANY triangle on the geoboard by such a subtractive process of areas of right triangles from enclosing (or circumscribing!) rectangles.

## LEVEL THREE

### GEOMETRY: TANGRAMS

### LESSON ONE

There are several shape outlines in the student book. Give each child a set of Tangrams and have them work those and outline the Tangram pieces as in place when the problem is solved.

### LESSON TWO

The children have a Tangram Problem Generator in their books. Have each child make a shape by coloring 16 small triangles that all connect together. A sample is provided to make a transparency to show how this is done.

Each child should use Tangrams to try to solve it. Collect all shapes made. Copy them. Put these into a box called "Challenges." Children can take one and try to convert it successfully with Tangrams. If done, it is placed in a box "Can Be Solved." If not, it is placed in a box, "NOT YET SOLVED." The idea is to have remaining in the NOT YET SOLVED box only those NO ONE has been able to do.

LEVEL THREE

USING LOGO

Background: Read this section on Using LOGO in the LEVEL TWO Teacher's Guide. If children have had LOGO experience as suggested there, they should be familiar with the use of ST, BK, FD, LT, RT, HOME, CLEARSCREEN.

After children are adept at using these commands to make shapes, to hit targets in the screen field, etc. - they should learn to control the Turtle's leavings to leave a trail or to not leave a trail.

PENUP PU This command does not affect the turtle's movements - it lifts the pen up so the turtle leaves no trail.

Closely related is: PENERASE PE

This command will remove a line previously made by having the turtle retrace that trail.

PENDOWN PD This command puts the pen back down so the turtle again leaves a visible trail.

HIDETURTLE HT This command hides the tu. When children KNOW what they are doing in moving the turtle, this command can be used. It removes the turtle so only the turtle's TRAILS are shown. The commands are executed more quickly.

Children like color. In LOGO both the background color and the pen color can be set. The colors available are:

| | | |
|---|---|---|
| 0 | BLACK | BK |
| 1 | WHITE | W |
| 2 | GREEN | G |
| 3 | VIOLET | V |
| 4 | ORANGE | O |
| 5 | BLUE | Bl |
| 6 | REVERSE | RV |

Setting background and/or pen color is done by giving the command followed by the color number.

SETPC This command sets the pen color. For example SETPC 2 sets the pen color to GREEN.

SETBG This command sets the background color. For example SETBG 5 sets the background color as BLUE

These two commands followed by turtle movement commands will leave a GREEN turtle trail on a BLUE background.

A good experience is to have children recreate on the screen using color the concrete materials they are working with. They can make the Pattern Block shapes, the Cuisenaire Rods, tiles, geoboards, etc. To do some of these will require use of the REPEAT command.

Children will soon notice that certain sequences of commands are repeated to accomplish things. An example is making a square:

Using FD and RT commands alone a square can be made by:

FD 50
RT 90 }side one, turn for side two

FD 50
RT 90 } side two, turn for side three

FD 50
RT 90 } side three, turn for side four

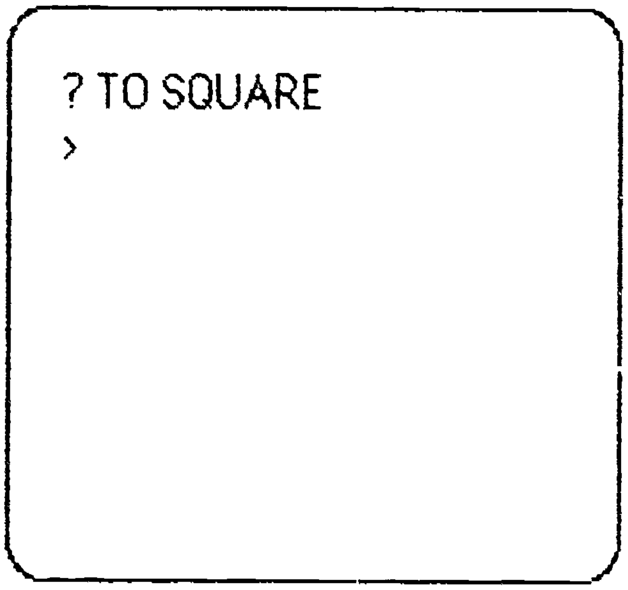
FD 50 } side four

Notice the turtle is not HOME - it is pointing to the left.

REPEAT [commands] will cause the sequence of commands within the brackets to be repeated " " number of times. So to make the square:

REPEAT 3[FD 50 RT 90]
FD 50

Once children can use this new set of commands, they are ready to create procedures. The primitive TO followed by a word that names the procedure changes LOGO from the graphic or turtle mode to a PROGRAMMING mode. For example: TO SQUARE followed by the Return Key will give the following:



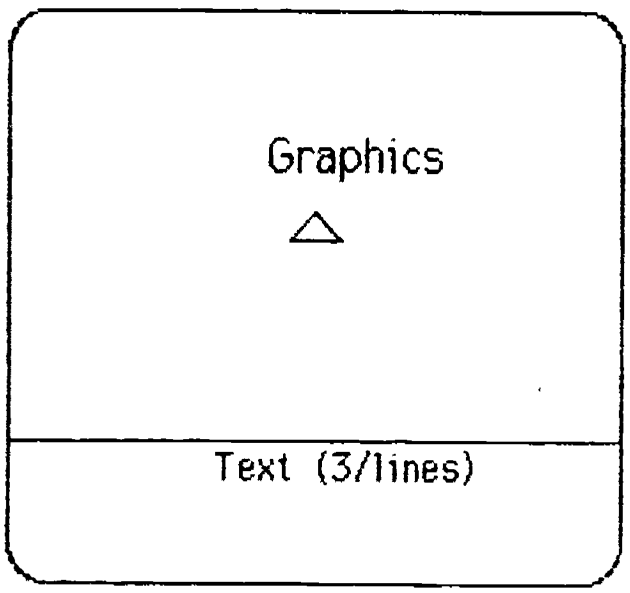

The ">" indicates a command is to be typed in. Doing this followed by a RETURN will bring up another ">". You can continue to write commands to define the procedure named by the word after TO when this is to your satisfaction. END will complete the definition and house that procedure. It is called up and executed by typing in the name.

Example:

```
? TO SQUARE
> REPEAT 3 [ FD 50  RT 90 ]
> FD 50
> END
```

defines SQUARE

Typing in SQUARE will result in the turtle's drawing of the square. Normally the screen first appears as a SPLITSCREEN:



To get a full screen for graphics, one must type in FULLSCREEN. Then no text will appear on the screen if one goes into the procedure defining mode.

To get a full screen for text, as for example when defining a procedure, type in TEXTSCREEN. Now all of the text, such as commands that have been typed in, are seen. Use the references given and introduce as much in the way of cleaning up the memory, printing out procedures in the memory, saving procedures on a disk, etc. as the children can learn to handle.

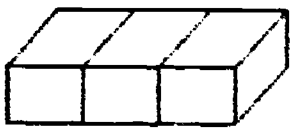
It is not appropriate at this level to introduce VARIABLES or RECURSION to these children. Most of them are just becoming concrete operational in their thinking rather than functioning as pre-operational thinkers. This year is when a majority of them will make the transition from being primarily pre-operational to being somewhat concrete operational in their thinking. The concepts of VARIABLE and the process of RECURSION are too abstract for all but one or two of them.

VOLUME: MULTILINKS

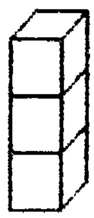
Background: Children have exposure to volume through work with geoblocks. Building rectangular sol
ids out of multilinks and then inch CUBES keeps visible the units used to measure volume

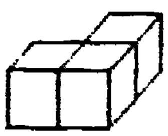
LESSON ONE

Introduction: This lesson involves having children building rectangular solid houses with specified numbers of rooms (as represented by multilinks.) Show the children THREE multilinks. Manipulate these to show the only way to build houses with square or rectangular floors is:



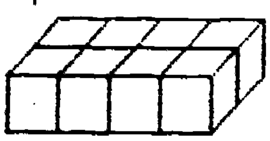
or



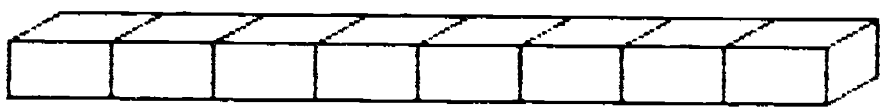


does not have a rectangular floor

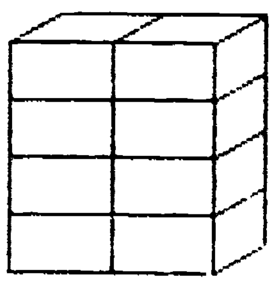
Repeat this with EIGHT multilinks, showing how houses can be made with rectangular or square floors:



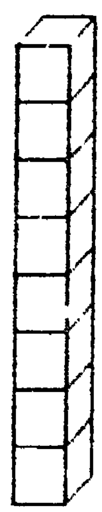
or



or



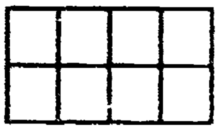
or



Activity: Children are to use multilinks to build "rectangular solid" houses on given floors.

LESSON TWO

Place an overhead transparency on that has a rectangle as shown. The units are the same size as multilinks:



"How many multilinks does it take to cover this rectangle?"
"How many 'rooms' are in this ONE STORY house?"
"How many multilinks do I need to put a second STORY on this house?"
"How many rooms would be in that two story house?"
"How do you find how many rooms are in a many story house when you know the size of the floor?"

Activity: Have the children work in pairs to use MULTILINKS to build houses on given floors, writing the number sentences to show how many rooms are in it.

LESSON THREE

Introduction: Make these multilink "Houses" 2 x 2 x 3 and 2 x 2 x 1. Hold them up one at a time and ask children to build one of each just like them.

"What is the volume of (hold up the 2 x 2 x 3)?"
"What is the volume of (hold up the 2 x 2 x 1)?"

Link them together into a 2 x 2 x 4.

"What is the voume of this one made of the other two?"

Point out that 2 x 2 x 3 = 12   2 x 2 x 1 = 4   2 x 2 x 4 = 16

"Would the volume change if the two original houses were joined in a different way?"
"Join them in a different way and count the cubes in the new house." Discuss.

Activity: Have children use multilinks with the worksheet provided.

## MEASUREMENT: NON-STANDARD UNITS (Lengths)

### LESSON ONE

Background: Measurement is based upon the acceptance of a standard against which properties are compared - a length used to compare other lengths, a weight used to compare other weights, a volume used to compare other volumes. These standards are often defined by societies to provide uniformity of measurement. In this lesson, a variety of different units are used to measure lengths.

Introduction: Make a line on an overhead transparency. Use matchsticks of 2 different lengths to measure the line. Point out to the students that MORE of the SHORTER unit are needed and FEWER of the LONGER unit.

Activity: Select a set of shorter objects in the room to be measured such as the windowsill, a table width, height of the chalkrail from the floor, etc. Organize children into groups of four. Some groups should have UNIFIX cubes, other groups paper clips or plastic links, other groups drinking straws and cellophane tape. Have each group measure the selected objects with their material by forming UNIFIX links, chains of paper clips and taped together lengths of drinking straws. These measurements should be recorded on the recording form. The links, chains and drinking straw lengths for each measurement should be retained. When the question comes up of what to do if the length is too long for "x" units but not long enough for "x + 1" units, inform the children to use the smaller number. Have one UNIFIX group put their links up in front of the room. The rest of the class is to determine which links measured which objects.

Select one of the objects measured. Have one of each material group tell what number of those units were needed to measure that length. Discuss why more paper clips were needed than drinking straws and related questions.

Select a different unmeasured object in the room. Ask each group to estimate how many of their units would be needed to measure that.

"For lengths that you couldn't quite measure, what would be wrong with using drinking straws as much as possible, then using paper clips for the rest of the length?"

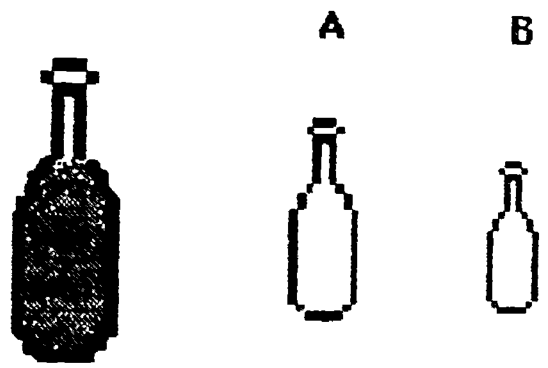
LESSON TWO: (Weight)

Introduce this by weighing an object with two different "units" such as washers and UNIFIX cubes. Discuss the results. Use OHAUS balances or home-made balances for this (See Level Two Guide.)

Activity: Select objects to be weighed and units to be used. Organize groups as in Lesson One and basically repeat the process used there.

LESSON THREE: (Volume)

Introduction: Show children a larger jar or clear bottle filled with colored water and 2 different smaller jars:




"Will the water in the jar fill more of this jar (point to A) or this jar (point to B)?"

Discuss the responses. Then take several of A and of B. First fill the A's and have the children count these. Pour the water back in the large jar. Now fill B's and count these Discuss.

"How is this like measuring lengths and weights?"

Reinforce the idea that the SMALLER the unit, the MORE needed, and the LARGER the unit, the FEWER needed.

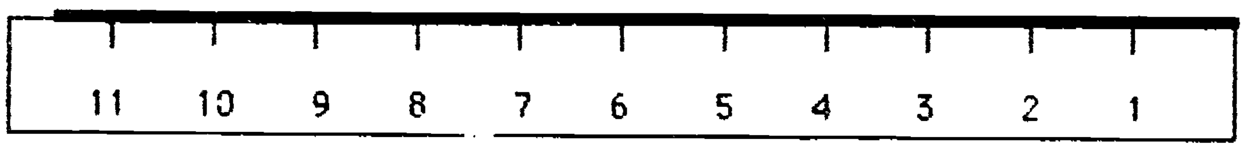
LEVEL THREE

MEASUREMENT: STANDARD LENGTH UNITS

Background: After considerable experience using non-standard units to measure objects and emphasis on the idea more of smaller units are needed and fewer of larger units are needed for a particular length, children can be introduced to standard units - both English and metric units.

LESSON ONE

Introduction: Place a transparency on the overhead that has a straight line on it. Using a clear plastic foot ruler, show where the end of the line comes on the foot ruler.




"Which is the end of the line closest to - 10 or 11?"

Discuss responses. Point out you have measured the line to the NEAREST INCH.

Put up a second line, place one end of the ruler on an end and have the children tell you how long it is to the NEAREST INCH.

Activity: Have children work in pairs to measure selected objects in the room to the nearest INCH. A recording form has been supplied for this purpose.

LESSON TWO

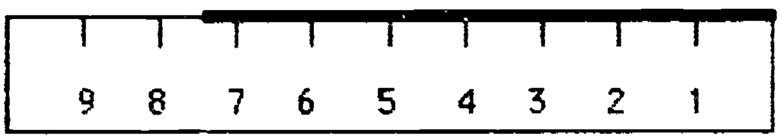
Lesson ONE with measurements made to the nearest HALF-INCH.

LESSON THREE

Lesson ONE with measurements made to the nearest FOURTH of an inch.

LESSON FOUR:

Introduction: Draw a line on the overhead projector. Place a decimeter metric ruler on the line. Have the children tell you how long the line is to the nearest CENTIMETER.




Measure a second line the same way.

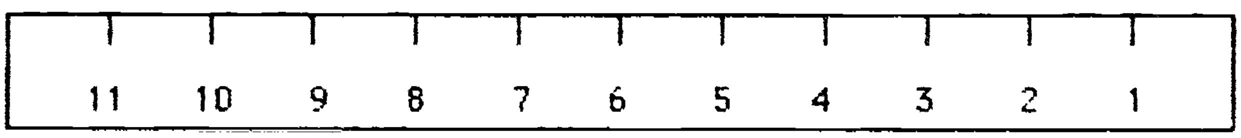
Activity: Have the children measure selected objects and record the results as with English units.

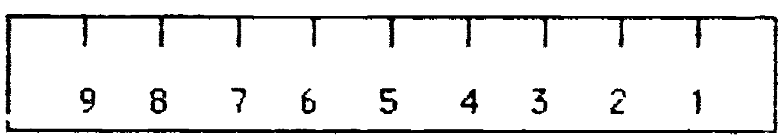
LESSON FIVE:

Same as Lesson FOUR, but to the nearest HALF CENTIMETER.

LESSON SIX:

Introduction: Place two transparent rulers - one in English measure and one in metric measure on the overhead projector.




"Are inches longer than centimeters?"

"A centimeter is about what part of an inch?"

"How many centimeters would I need to measure a line 2 inches long?" Remind the children of the need to use MORE of a shorter unit and FEWER of a longer unit.

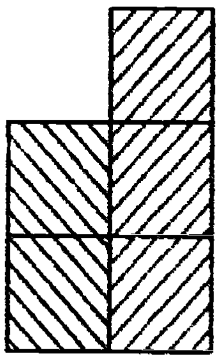
## LEVEL THREE

### CORRESPONDENCES

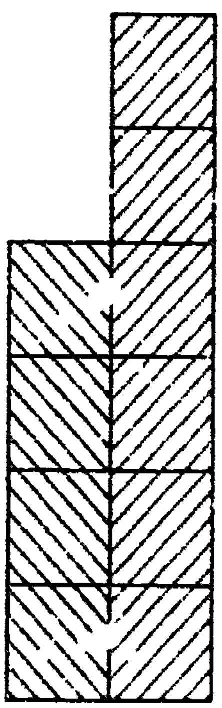
Background: Children need to have experiences with correspondences other than 1 to 1 to develop concept of ratio. These lessons will give that kind of experience.

#### LESSON ONE

Use colored squares on the overhead projector. Children should have UNIFIX cubes of two different colors, about 20 of each color. If possible they should have crayons of the same color as the UNIFIX cubes and graph paper. Place a column of two of one color on the overhead. Beside it place a column of three of the second color:



Tell the children to make a link of TWO of one color of UNIFIX and another link of THREE of the other color. They should color in as shown on the graph paper (see A below). Then have them make a second link of TWO of the first color and another link of THREE of the second color as before. These should be attached to the first links to get:

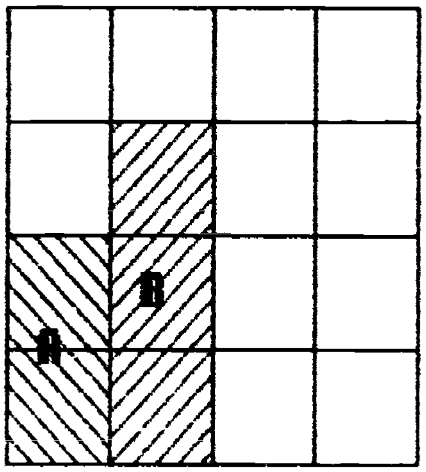


Have them color these in next to the first pair of links on the graph paper (see B below).

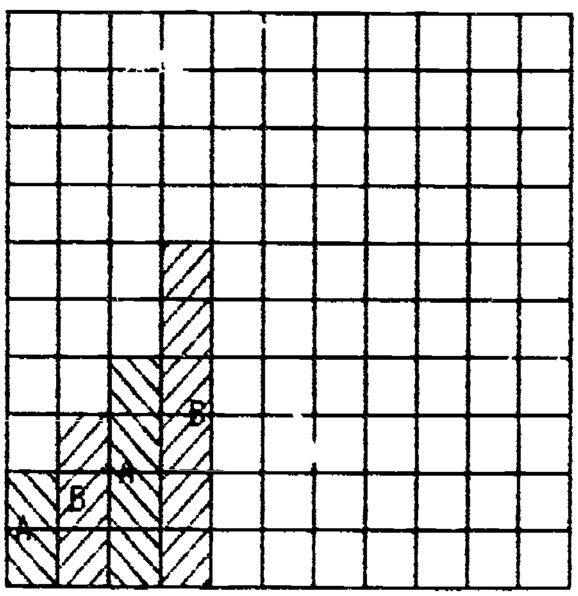
The children should continue to make links of the two colors, join to already existing links and color this new pair on the graph paper. When they have exhausted the capability of the graph paper, have them use a straight edge to draw lines as shown on the graph paper. (See C below).

"Which line climbs faster?" "Why does it climb faster?" "Could you get the TWO links and THREE links back from the longer links?"

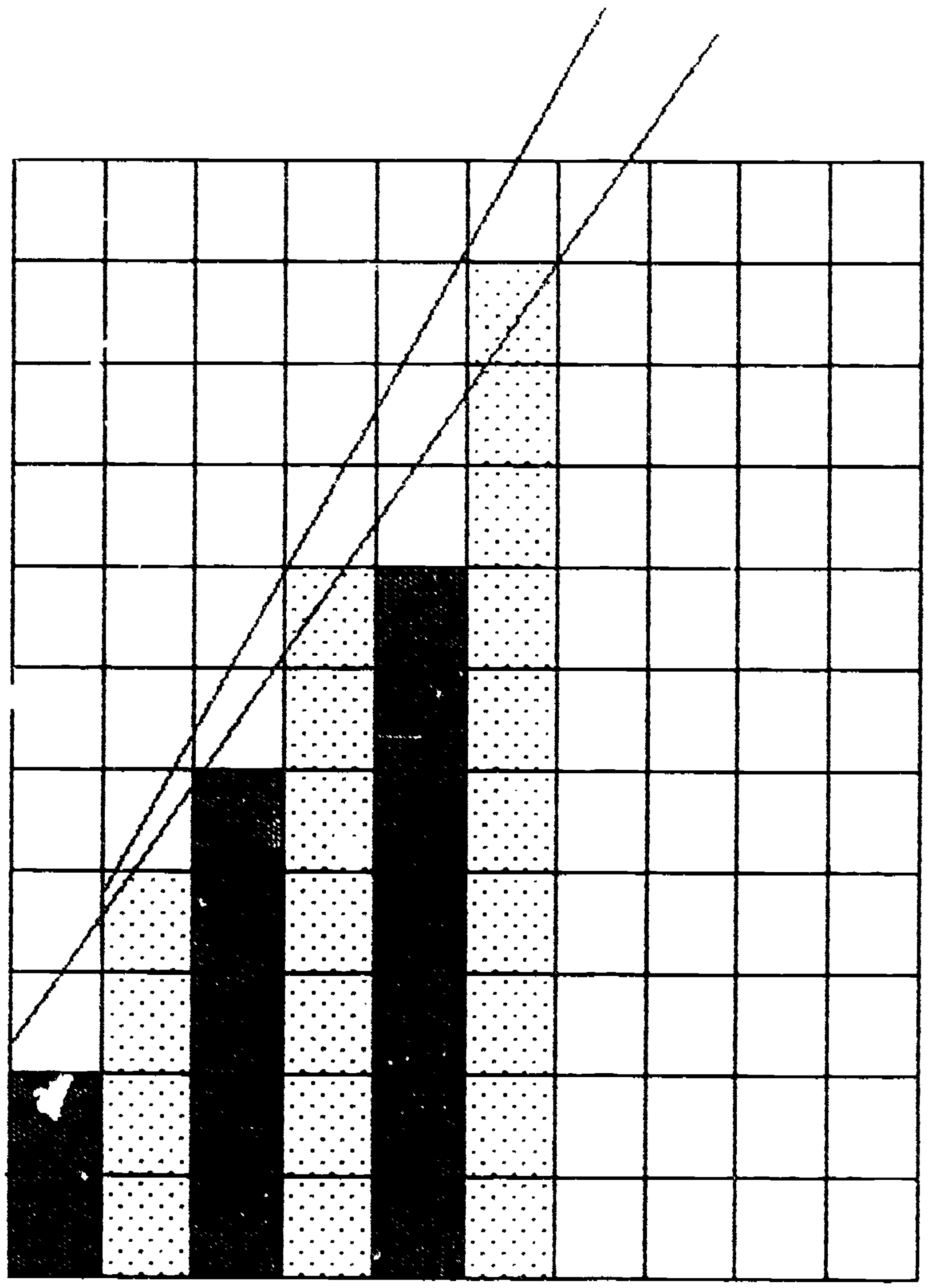
A




B



C



Use UNIFIX cubes to have children look at other correspondences such as 3:4; 1:3; 4:1; 3:2; 2:5, etc.

LESSON TWO

Introduction: Children are to use 2 DIFFERENT shapes from Pattern Blocks, or 2 DIFFERENT COLORS of chips, or animal shapes, etc., to explore correspondence. Always emphasize that the groups can be recovered and that e.g. in 2:3, multiplying by 2, 3, 4, etc., in turn gives a new comparison, but the original 2's and 3's can be obtained if desired. Hence we have EQUIVALENCE CLASSES for each basic pair. An example is 3:4 = {3:4, 6:8; 9:12, 12:16; 15:20, etc.}

## LEVEL THREE

COMPUTATION WITH MONEY

LESSON ONE: This lesson is to have children identify bill and coin arrangements. Have the children do the worksheets provided.

LESSON TWO:

Introduction: Make a transparency of the Menu From Harry's. Ask the children to identify the costs of items on the menu. Have them orally determine some simple computations such as two bagels, etc., using the calculators. Review the reading of the output of the calculators.

Activity: Give a copy of Harry's Menu to each child and the Order worksheet to complete, using calculators.

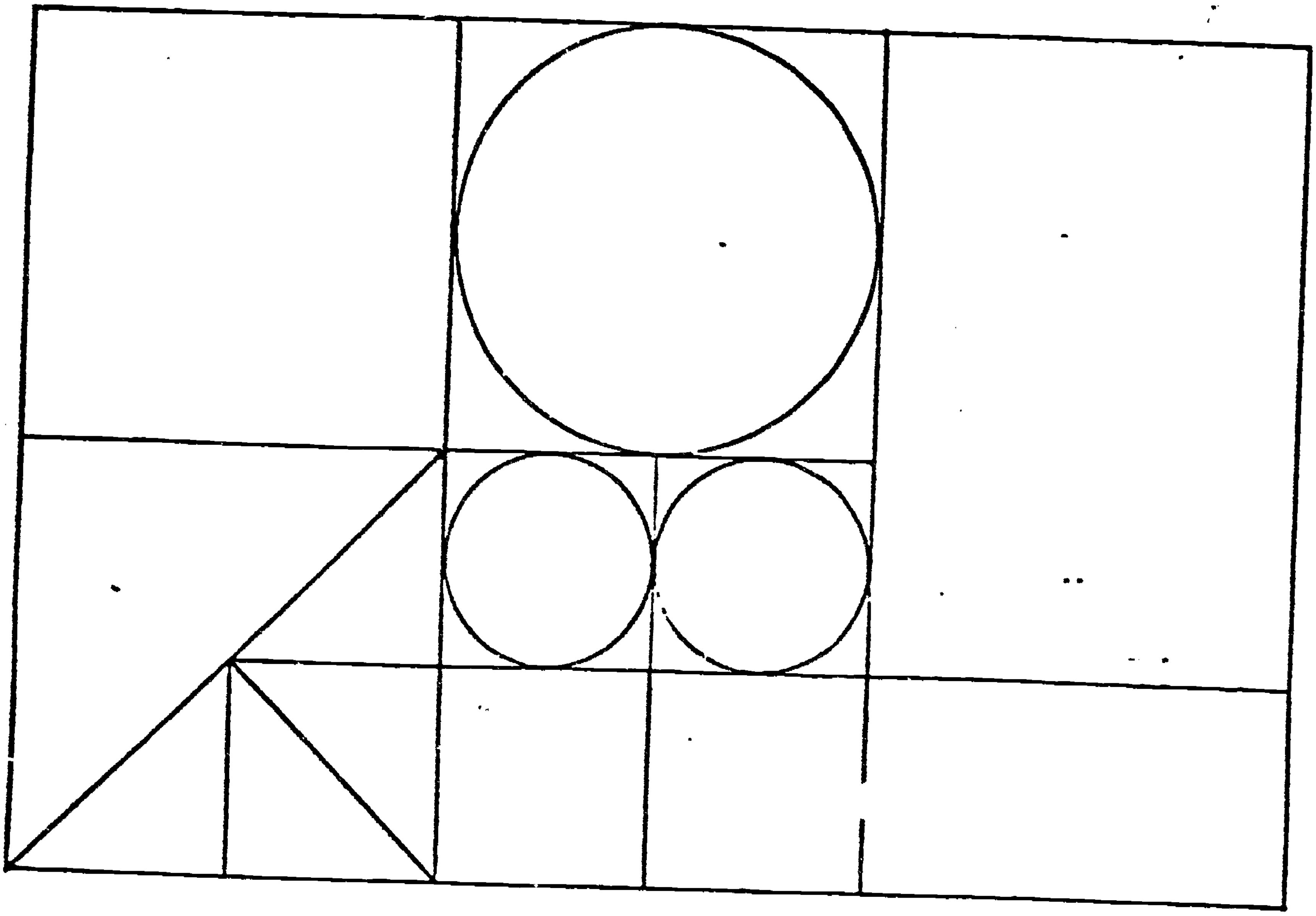
# BLACK LINE MASTERS

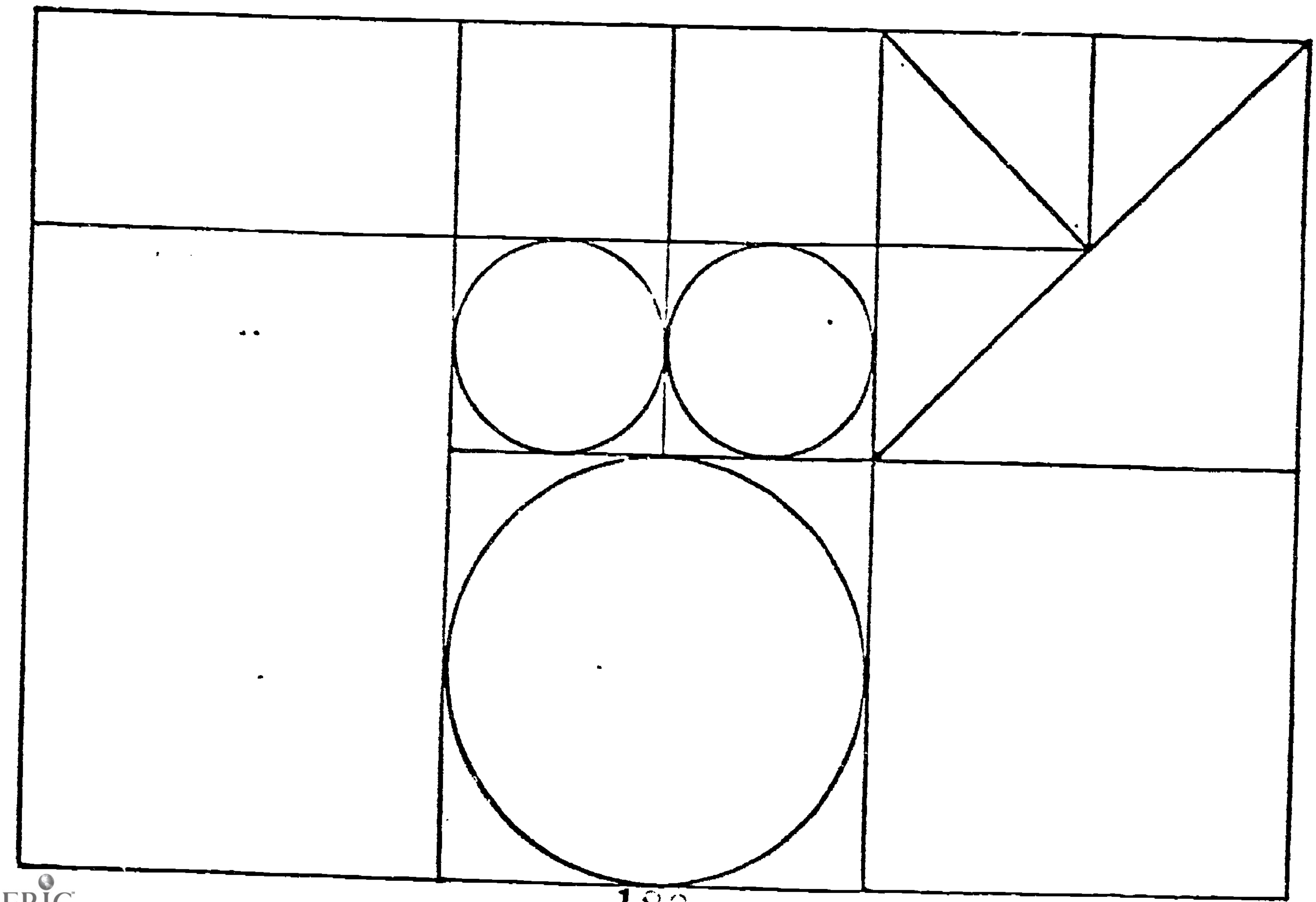
## for many of the

## WORKSHEETS

## and

## RECORDING FORMS

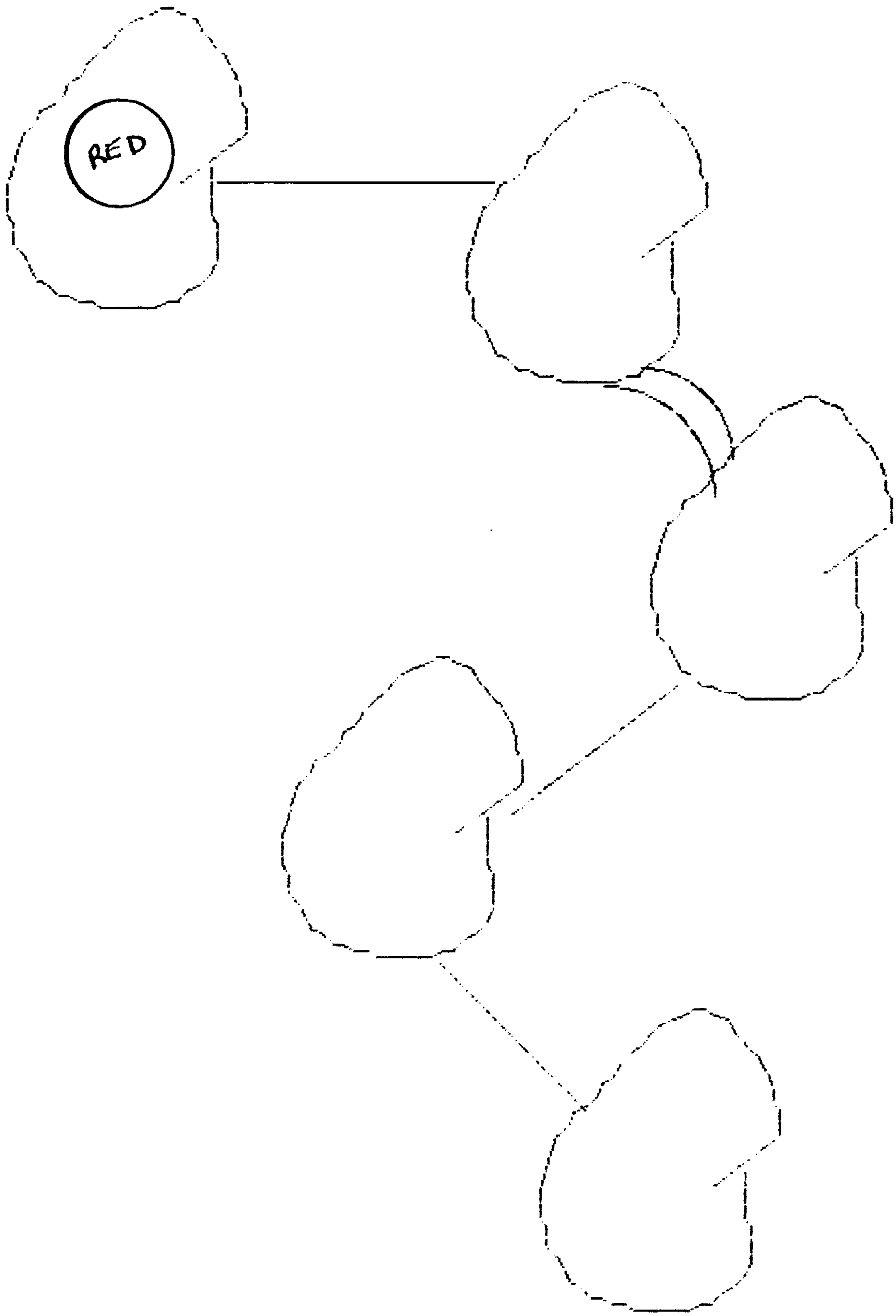




## IDENTIFICATION CARDS I

| | |
|---|---|
| LARGE | SMALL |
| TRIANGLE | NOT GREEN |
| CIRCLE | BLUE |
| GREEN | NOT RECTANGLE |
| RED | SQUARE |
| RECTANGLE | YELLOW |
| NOT SQUARE | NOT CIRCLE |
| NOT BLUE | NOT TRIANGLE |
| NOT YELLOW | NOT RED |

SAMPLE LOGIC PUZZLE



A 4 x 4 MATRIX

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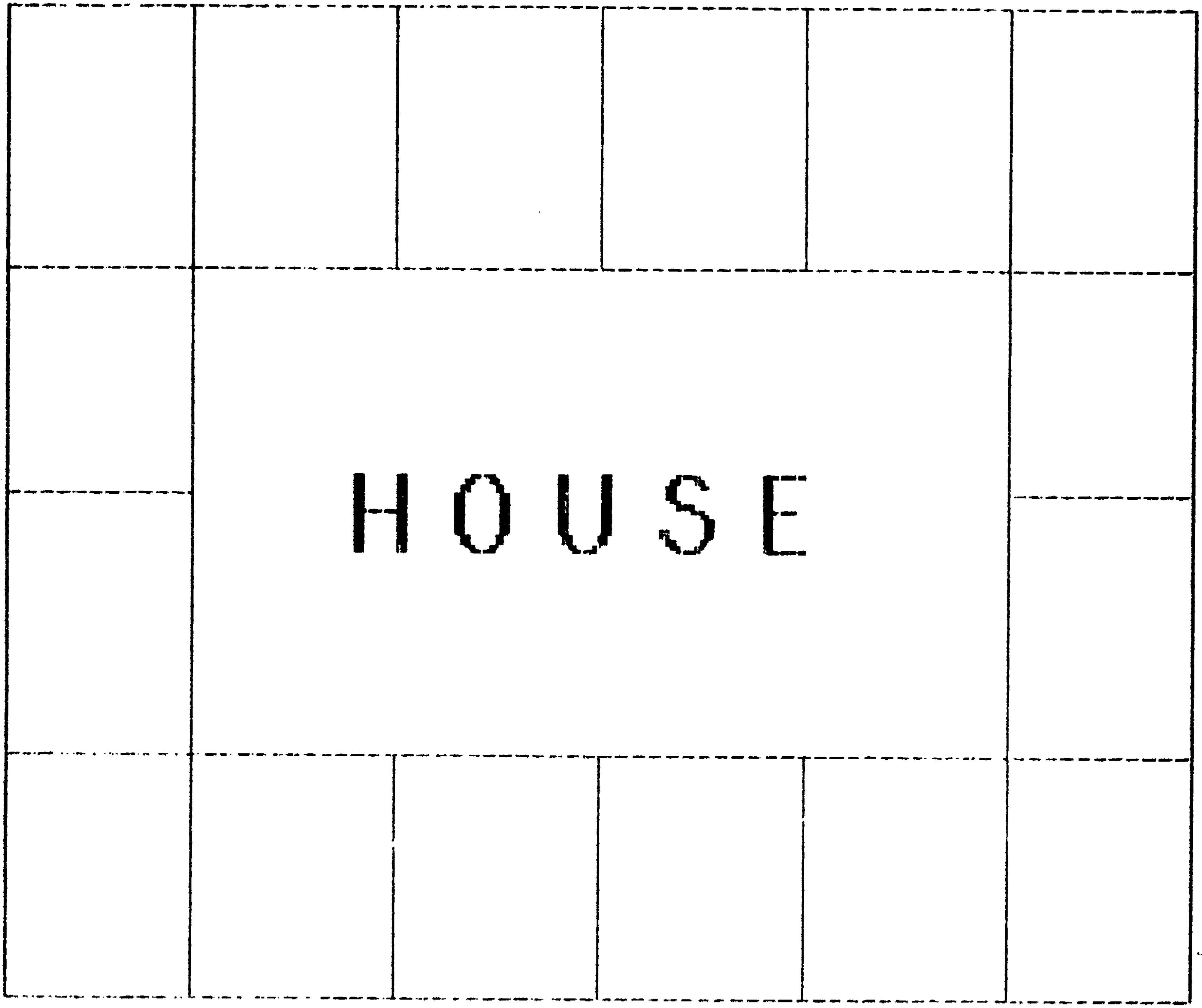
HOUSE

# L O G I C

PURPOSE: Logic/Bingo can be used to test and familiarize the children with the names of the logic blocks. Instead of Bingo we will use LOGIC as the name of our game.

MATERIALS: Using The Master, make several different cards to look like Bingo (Logic) cards.

| L | O | G | I | C |
|---|---|---|---|---|
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| | | FREE | | |
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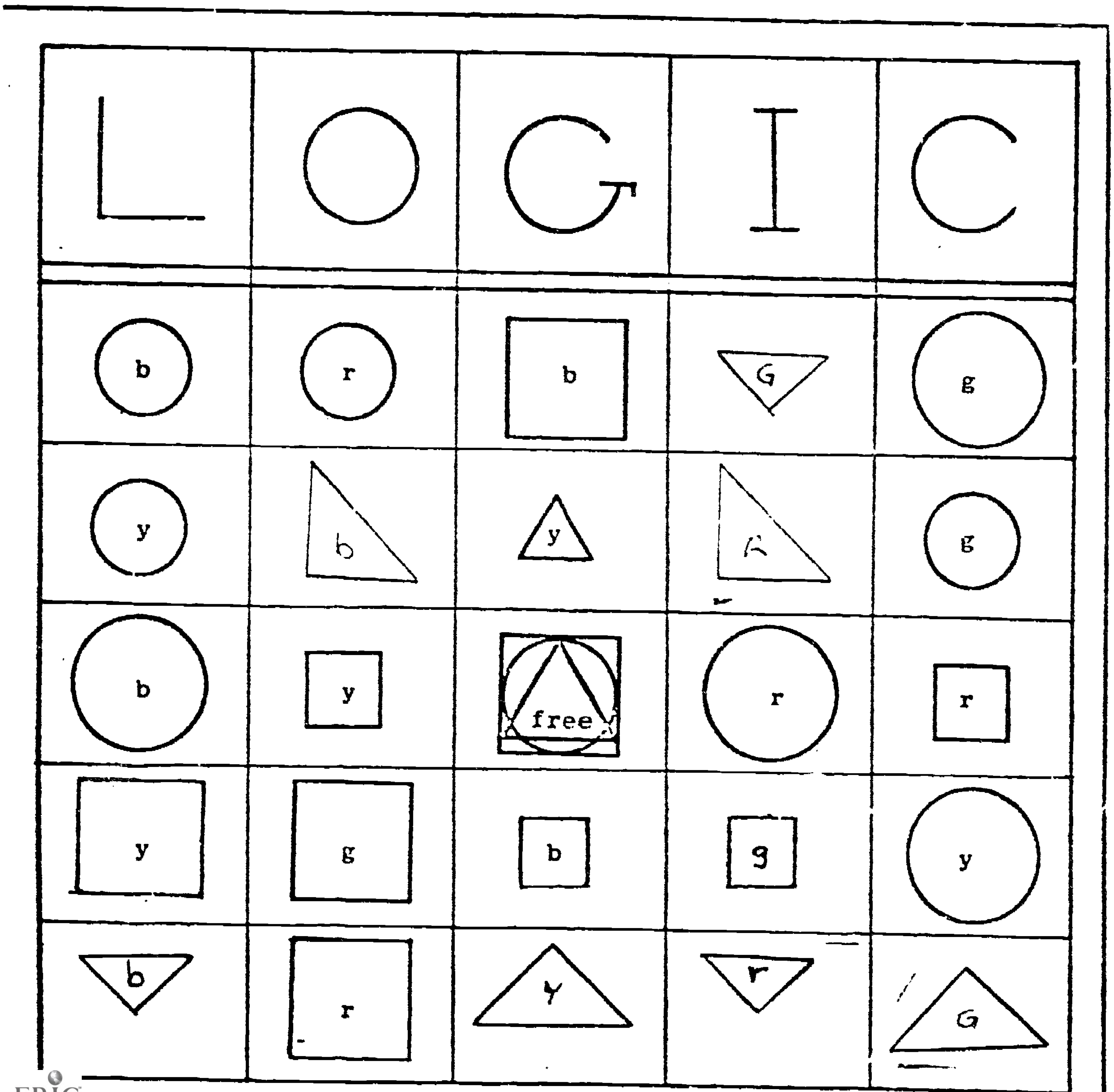
# LOGIC PLAYING CARD SAMPLE



Mathematician:________________________________

"I used number slides to solve the number sentences."

| | | |
|---|---|---|
| | | |
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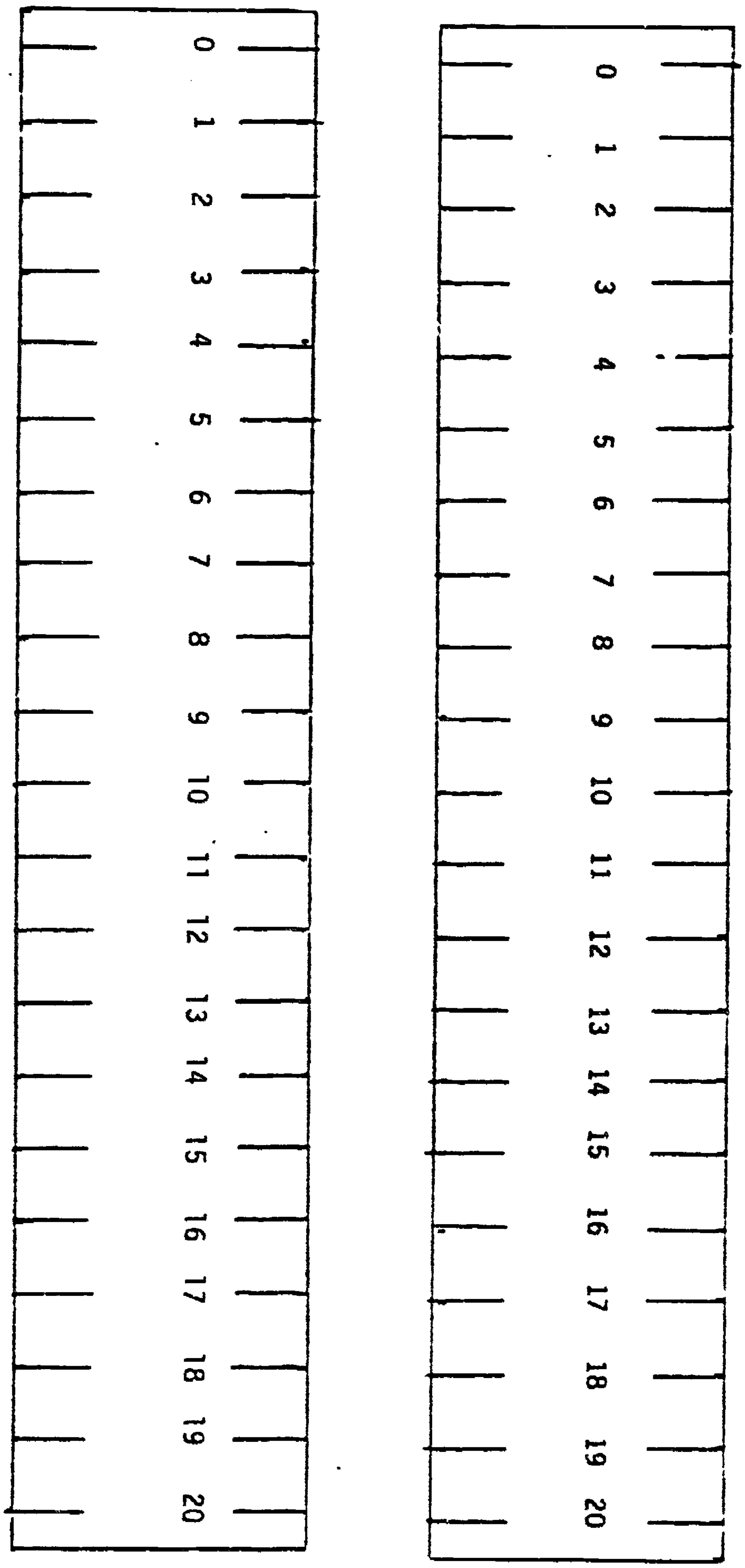
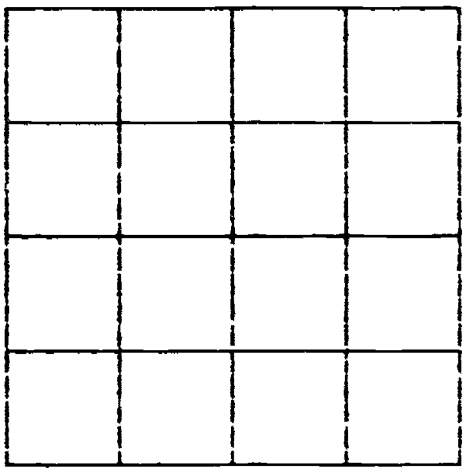
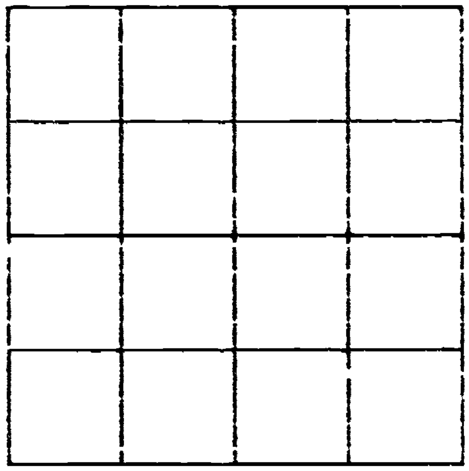
# NUMBER SLIDES MASTER

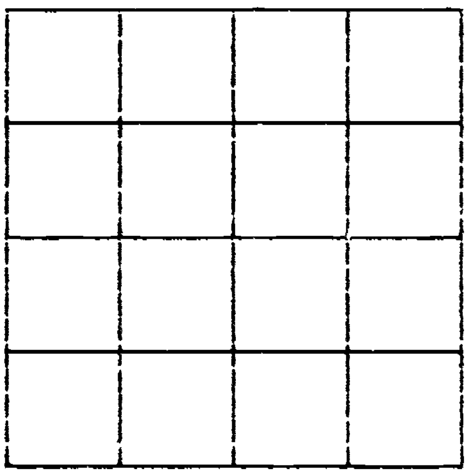


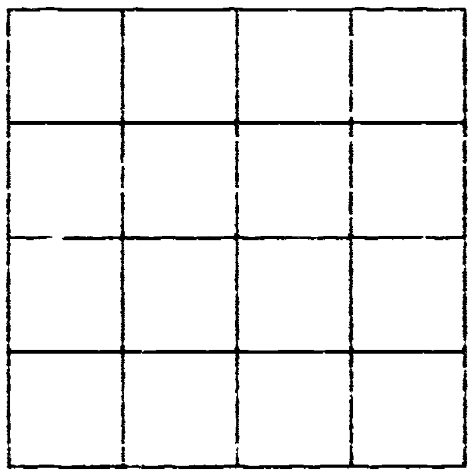
Mathematician:________________________________________

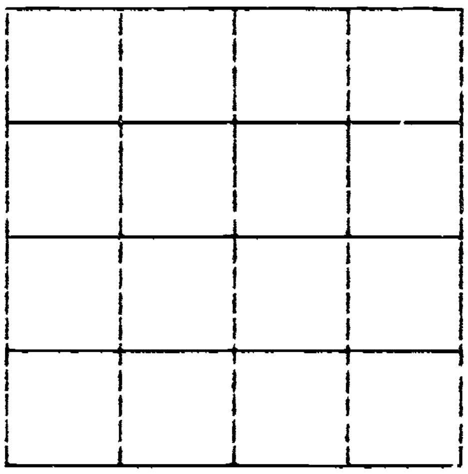
"I colored in the fractions as shown and used "<" or ">" to show which fraction was larger."

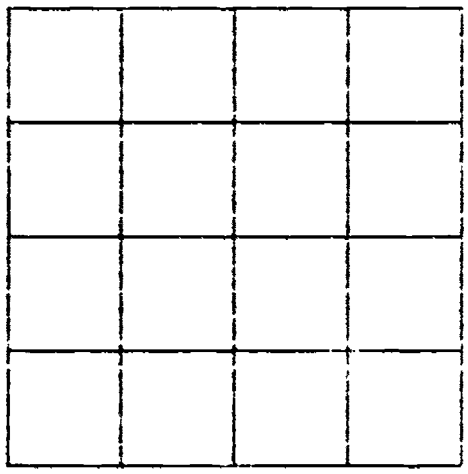












MATHEMATICIAN: ______________________________

## My Hundreds Computer

| Number | Hundreds | Tens | Ones | | □ | ▬ | ● |
|---|---|---|---|---|---|---|---|
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Mathematician:______________________________

"I wrote the numeral for each base ten number."

Mathematician:____________________ Another Numeral Record

| Hundreds | Tens | Ones | Hundreds | Tens | Ones |
|---|---|---|---|---|---|
| | | | | | |
| Hundreds | Tens | Ones | Hundreds | Tens | Ones |
| | | | | | |
| Hundreds | Tens | Ones | Hundreds | Tens | Ones |
| | | | | | |
| Hundreds | Tens | Ones | Hundreds | Tens | Ones |
| | | | | | |

Mathematician:______________________________

"I built the numbers with base ten blocks and wrote the numerals."

| Number Words | Picture | Numeral |
|---|---|---|
| Starting Number | | |
| | | |
| | | |
| | | |
| | | |
| | | |

**MATHEMATICIAN:**______________________________

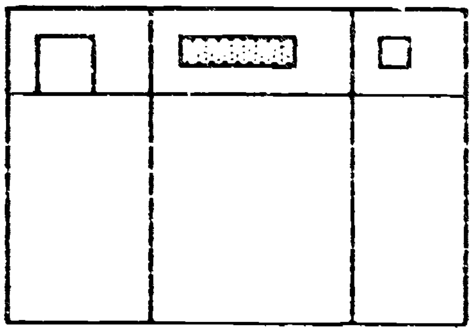
**"My pictures, numerals, and words for numbers I built with base ten blocks."**

| Words | Pictures | Numerals |
|---|---|---|
| | Hundreds \| Tens \| Ones | |
| | Hundreds \| Tens \| Ones | |
| | Hundreds \| Tens \| Ones | |
| | Hundreds \| Tens \| Ones | |

MATHEMATICIAN:______________________________

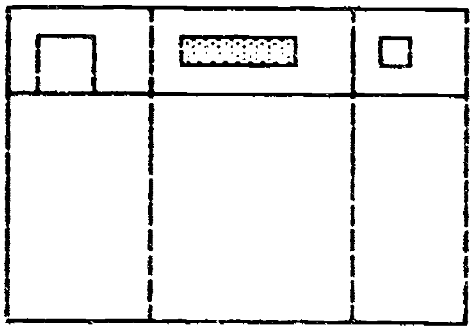
**"My pictures, numerals, and words for numbers I built with base ten blocks."**

| Pictures | Words | Numerals |
|---|---|---|



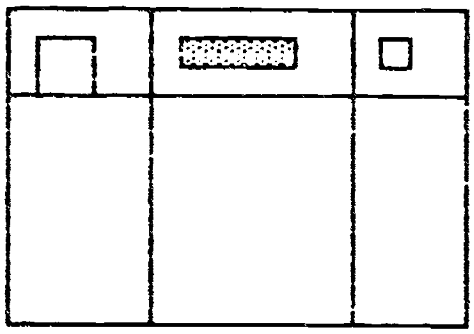
___Hundreds ___Tens ___Ones

| Hundreds | Tens | Ones |
|---|---|---|
| | | |



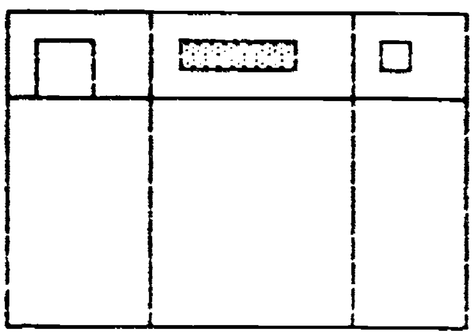
___Hundreds ___Tens ___Ones

| Hundreds | Tens | Ones |
|---|---|---|
| | | |



___Hundreds ___Tens ___Ones

| Hundreds | Tens | Ones |
|---|---|---|
| | | |



___Hundreds ___Tens ___Ones

| Hundreds | Tens | Ones |
|---|---|---|
| | | |

MATHEMATICIAN:______________________________

"I added to the ONES column, traded as needed until I reached FORTY."

| Adding Twos | | Adding Threes | | Adding Fours | | Adding fives | |
|---|---|---|---|---|---|---|---|
| T | O | T | O | T | O | T | O |
| | | | | | | | |
| | | | | | | | |
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MATHEMATICIAN: ____________________________________________

## Picture/Numerals Worksheet

| Picture | | Numerals | |
|---|---|---|---|
| | | TENS | ONES |
| | | | |
| | | | |

| | | | |
|---|---|---|---|
| | | TENS | ONES |
| | | | |
| | | | |

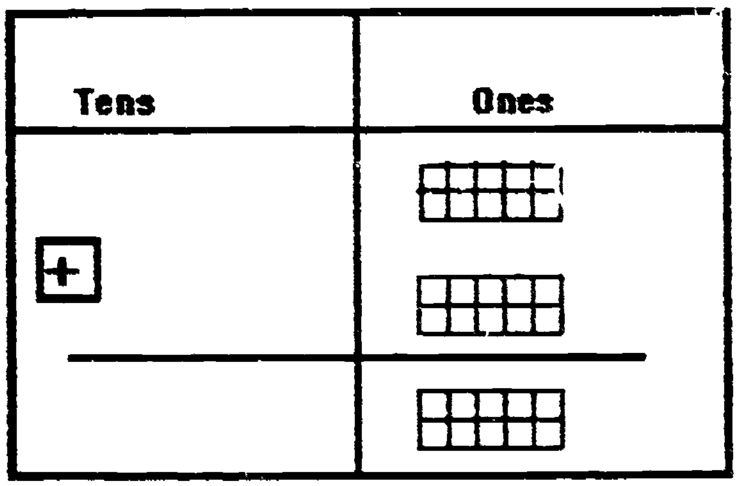
*Mathematician:* ______________________________

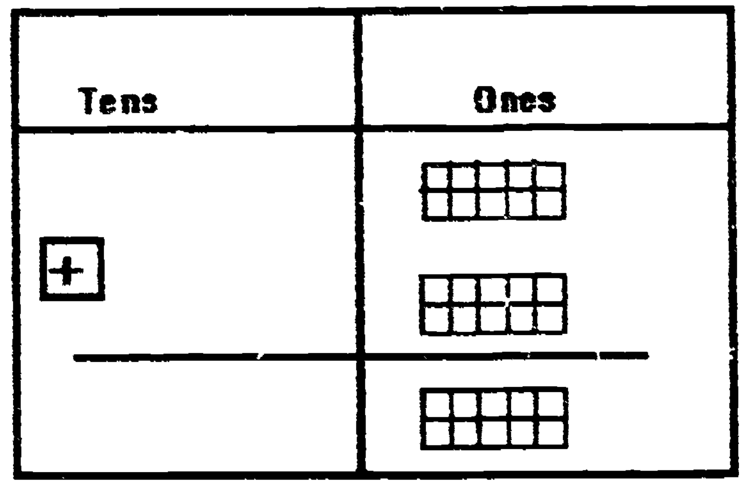
## My Base Ten Block Record

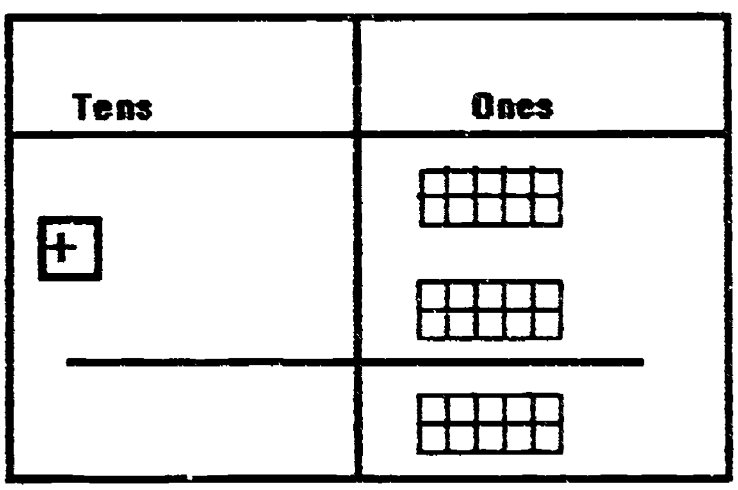
| Picture | Numerals | | Picture | Numerals | |
|---|---|---|---|---|---|
| Example: | | | | | |
| | TENS | ONES | | TENS | ONES |
| | | | | | |
| | | | | | |
| | TENS | ONES | | TENS | ONES |
| | | | | | |
| | | | | | |
| | TENS | ONES | | TENS | ONES |
| | | | | | |
| | | | | | |
| | TENS | ONES | | TENS | ONES |
| | | | | | |
| | | | | | |

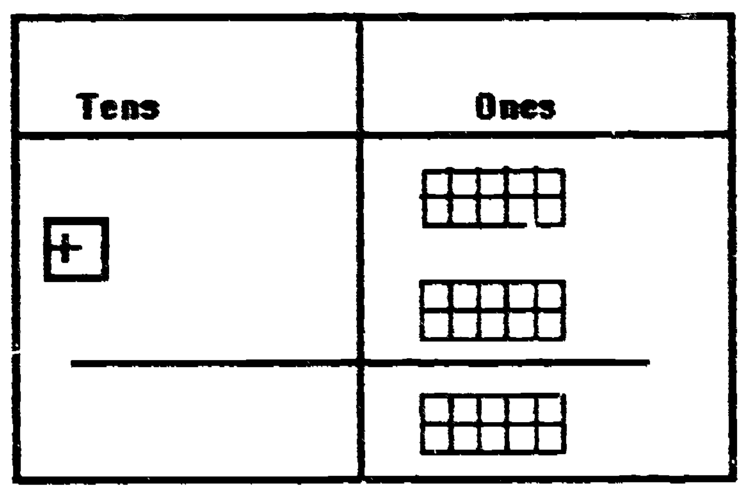
Mathematician:________________________________

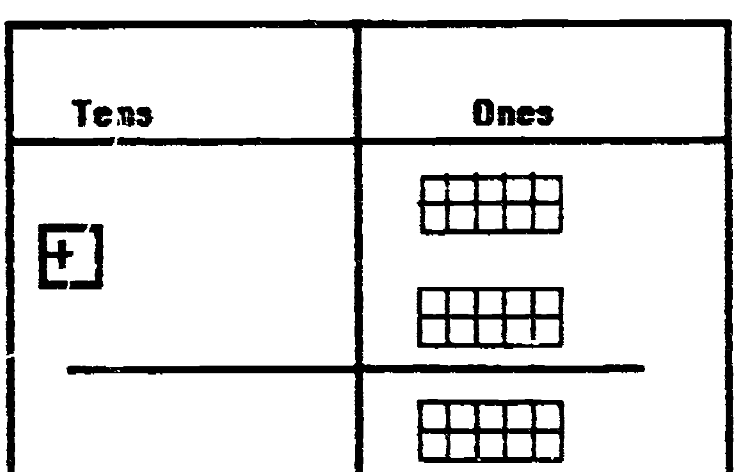
"I added the bottom number to the top number to find the answer

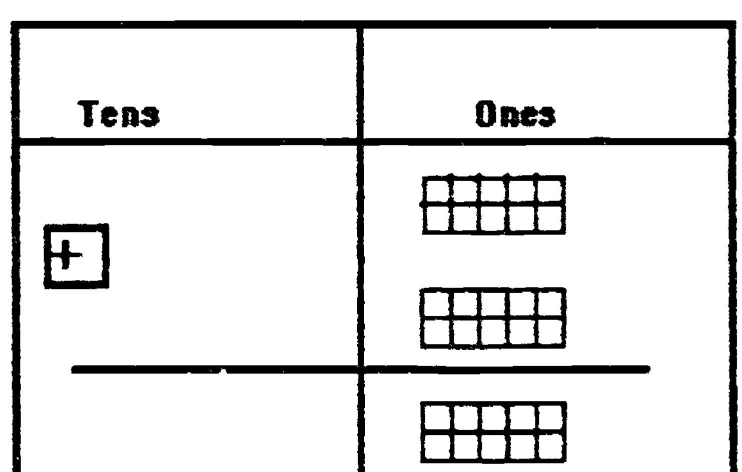


Mathematician:______________________________

"I took the bottom number from the top number to find the answer."

| Tens | Ones |
|---|---|
| − | |
| | |
| | |

| Tens | Ones |
|---|---|
| − | |
| | |
| | |

| Tens | Ones |
|---|---|
| − | |
| | |
| | |

| Tens | Ones |
|---|---|
| − | |
| | |
| | |

| Tens | Ones |
|---|---|
| − | |
| | |
| | |

| Tens | Ones |
|---|---|
| − | |
| | |
| | |

Matematician: ______________________________

"I did these additions and checked the answer by subtraction."

| Hundreds | Tens | Ones |
|---|---|---|
| | | |
| | | |

| Hundreds | Tens | Ones |
|---|---|---|
| | | |
| | | |

| Hundreds | Tens | Ones |
|---|---|---|
| | | |
| | | |

Check:

| Hundreds | Tens | Ones |
|---|---|---|
| | | |
| | | |

| Hundreds | Tens | Ones |
|---|---|---|
| | | |
| | | |

| Hundreds | Tens | Ones |
|---|---|---|
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| Hundreds | Tens | Ones |
|---|---|---|
| | | |
| | | |

| Hundreds | Tens | Ones |
|---|---|---|
| | | |
| | | |

| Hundreds | Tens | Ones |
|---|---|---|
| | | |
| | | |

Check:

| Hundreds | Tens | Ones |
|---|---|---|
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| Hundreds | Tens | Ones |
|---|---|---|
| | | |
| | | |

| Hundreds | Tens | Ones |
|---|---|---|
| | | |
| | | |

MATHEMATICIAN: ____________________

"I did the adding or subtracting and checked with the INVERSE OPERATION."

| TENS | ONES |
|---|---|
| | |

| TENS | ONES |
|---|---|
| | |

| TENS | ONES |
|---|---|
| | |

CHECKS:

| TENS | ONES |
|---|---|
| | |

| TENS | ONES |
|---|---|
| | |

| TENS | ONES |
|---|---|
| | |

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| TENS | ONES |
|---|---|
| | |

| TENS | ONES |
|---|---|
| | |

| TENS | ONES |
|---|---|
| | |

CHECKS:

| TENS | ONES |
|---|---|
| | |

| TENS | ONES |
|---|---|
| | |

| TENS | ONES |
|---|---|
| | |

Mathematician________________________________

"I did these additions and checked the answer by subtraction."

| Tens | Ones | Tens | Ones | Tens | Ones | Tens | Ones |
|---|---|---|---|---|---|---|---|

Check:

| Tens | Ones | Tens | Ones | Tens | Ones | Tens | Ones |
|---|---|---|---|---|---|---|---|

| Tens | Ones | Tens | Ones | Tens | Ones | Tens | Ones |
|---|---|---|---|---|---|---|---|

Check:

| Tens | Ones | Tens | Ones | Tens | Ones | Tens | Ones |
|---|---|---|---|---|---|---|---|

Mathematician: ______________________________

"I did these subtractions and checked the answer by addition."

| Hundreds | Tens | Ones |
| --- | --- | --- |
| | | |
| | | |

| Hundreds | Tens | Ones |
| --- | --- | --- |
| | | |
| | | |

| Hundreds | Tens | Ones |
| --- | --- | --- |
| | | |
| | | |

Check:

| Hundreds | Tens | Ones |
| --- | --- | --- |
| | | |
| | | |

| Hundreds | Tens | Ones |
| --- | --- | --- |
| | | |
| | | |

| Hundreds | Tens | Ones |
| --- | --- | --- |
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| Hundreds | Tens | Ones |
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| Hundreds | Tens | Ones |
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| Hundreds | Tens | Ones |
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| | | |

Check:

| Hundreds | Tens | Ones |
| --- | --- | --- |
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| Hundreds | Tens | Ones |
| --- | --- | --- |
| | | |
| | | |

| Hundreds | Tens | Ones |
| --- | --- | --- |
| | | |
| | | |

Mathematician: ______________________________

"I did these subtractions and checked the answer by addition."

Tens Ones | Tens Ones | Tens Ones | Tens Ones

Check:

Tens Ones | Tens Ones | Tens Ones | Tens Ones

Tens Ones | Tens Ones | Tens Ones | Tens Ones

Check:

Tens Ones | Tens Ones | Tens Ones | Tens Ones

MATHEMATICIAN:________________________________________

"MY RECORD OF MAKING RECTANGLES OUT OF TILES."

COLUMNS

ROWS

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | | | | | | | | | | | |
| 1 | | | | | | | | | | | |
| 2 | | | | | | | | | | | |
| 3 | | | | | | | | | | | |
| 4 | | | | | | | | | | | |
| 5 | | | | | | | | | | | |
| 6 | | | | | | | | | | | |
| 7 | | | | | | | | | | | |
| 8 | | | | | | | | | | | |
| 9 | | | | | | | | | | | |
| 10 | | | | | | | | | | | |

Mathematician:________________________________

"I made squares from tiles. I recorded these on graph paper."

| Side of Square | Tiles Used | Number Sentence | Area of the Square |
|---|---|---|---|
| | | | |
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Mathematician: ______________________ "These are graph paper records of my work."

MATHEMATICIAN: ____________________________________________

"I put beans into cups to complete this form. I wrote Number Sentences to show this."

| CUPS USED X | BEANS IN EACH CUP = | TOTAL BEANS USED | NUMBER SENTENCES |
| --- | --- | --- | --- |
| | | | |
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Mathematician:____________________________________

"I made these rectangles from base ten ones, entered the number in the table, and made trades to show this as tens and ones."

| Rectangle Size Made | TENS | ONES |
| --- | --- | --- |
| | | |
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Mathematician: ____________________

"I made smaller UNIFIX cube links from long UNIFIX cube lengths."

| UNIFIX CUBE LENGTH | ÷ UNIFIX LINK LENGTH | = NUMBER OF LINKS | + REMAINDER | NUMBER SENTENCE |
|---|---|---|---|---|
| | | | | |
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Mathematician: ____________________________________________

"I made rectangles if I could from the tiles."

| Tiles ÷ | Rows = | Columns | + Remainder | Number Sentence |
|---|---|---|---|---|
| | | | | |
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Mathematician: ______________________________

"I used tiles to complete the open sentence. I then wrote sentences that were related to that one."

| OPEN SENTENCE | SAME FACT ANOTHER WAY | 2 RELATED SENTENCES | |
|---|---|---|---|
| | | | |
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MATHEMATICIAN: ______________________________

"I made rectangles from tiles in as many ways as I could."

| Number of Tiles | Largest Rectangles made | Other Rectangles made | Number Sentences |
|---|---|---|---|
| | | | |
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Mathematician:________________________________

"I made tiles arrangements to show two numbers and compared them. I wrote a number sentence using "<" to show this."

| Numerals | Tile Pictures | Number Words |
|---|---|---|
| | | |
| | | |
| | | |
| | | |

MATHEMATICIAN: ____________________

"My Beans and Cups Multiplication Table."

↓ C U P S ↓

→ BEANS IN EACH CUP →

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | | | | | | | | | | | |
| 1 | | | | | | | | | | | |
| 2 | | | | | | | | | | | |
| 3 | | | | | | | | | | | |
| 4 | | | | | | | | | | | |
| 5 | | | | | | | | | | | |
| 6 | | | | | | | | | | | |
| 7 | | | | | | | | | | | |
| 8 | | | | | | | | | | | |
| 9 | | | | | | | | | | | |
| 10 | | | | | | | | | | | |

MATHEMATICIAN ______________________

" I used tiles to make rows and columns."

| Tiles Used | Rows | Columns | Remainder | Number Sentence |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |
| | | | | |
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MATHEMATICIAN: ____________________

"These are my rectangles made of Base Ten Ones and the Number Sentences for them."

MATHEMATICIAN: ______________________________

"Here are pictures of my multiplication showing all partial products."

| Side One | Side Two | Picture | Partial Products |
|---|---|---|---|
| | | | |
| | | | |
| | | | |
| | | | |

MATHEMATICIAN: ______________________________

"Here are multiplications showing tens and ones."

| Side One | Side Two | Number Sentences |
| --- | --- | --- |
| | | |
| | | |
| | | |
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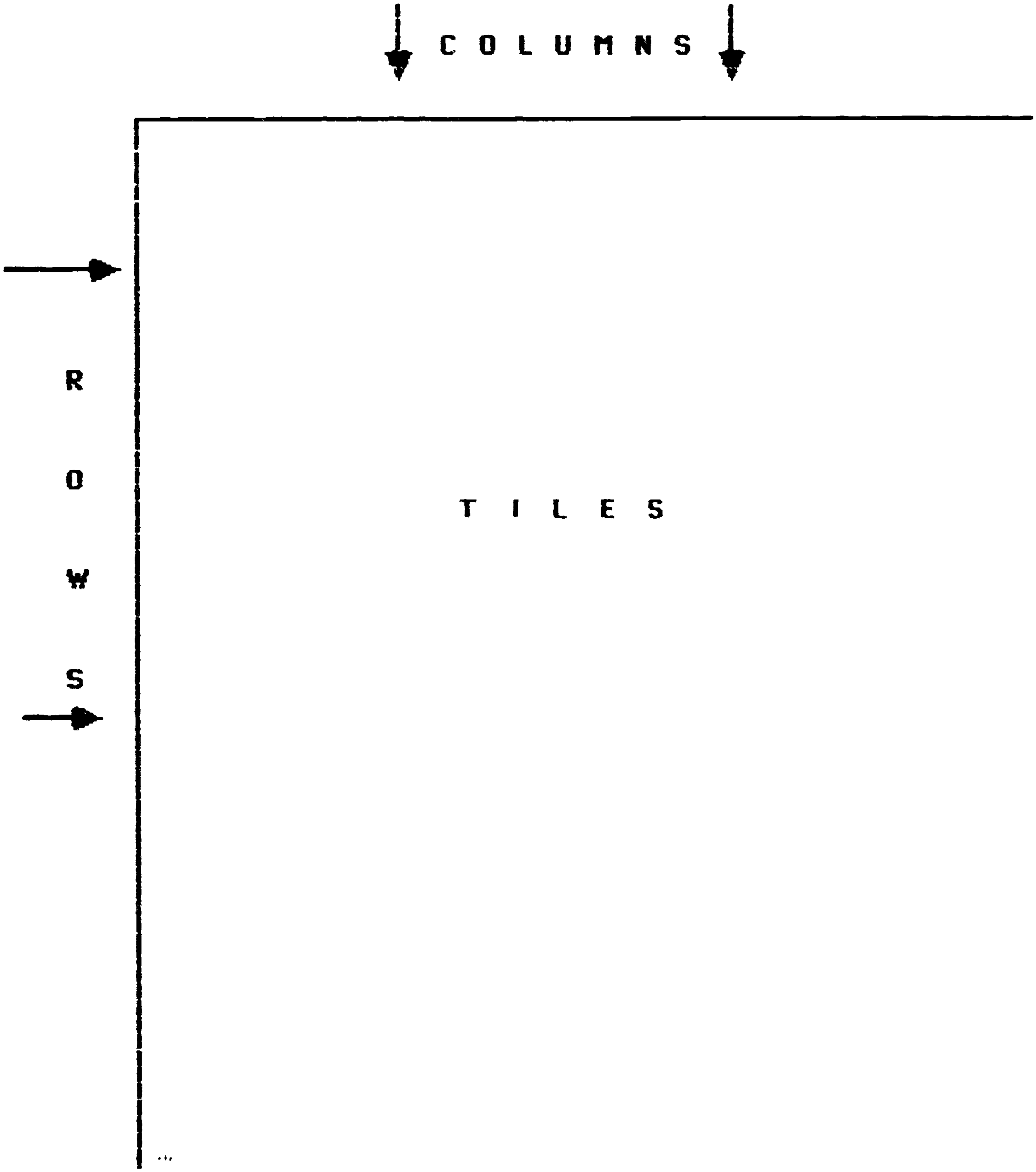
| | | |
| | | |

Mathematician: ______________________________

"I worked these multiplications by building rectangles with base ten blocks."

| Side One | Side Two | Rectangle | Number Sentence |
|---|---|---|---|
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | |
| 2 | | | | | | | | | | |
| 3 | | | | | | | | | | |
| 4 | | | | | | | | | | |
| 5 | | | | | | | | | | |
| 6 | | | | | | | | | | |
| 7 | | | | | | | | | | |
| 8 | | | | | | | | | | |
| 9 | | | | | | | | | | |
| 10 | | | | | | | | | | |


COLUMNS
ROWS
TILES

Mathematician: ____________________

"I made rectangles from base ten blocks to do these multiplications."

| Multiplication | Rectangles Made | Products Recorded |
| --- | --- | --- |
| Tens \| Ones | x | Hundreds \| Tens \| Ones |
| Multiplication | Rectangles Made | Products Recorded |
| Tens \| Ones | x | Hundreds \| Tens \| Ones |
| Multiplication | Rectangles Made | Products Recorded |
| Tens \| Ones | x | Hundreds \| Tens \| Ones |

SIDE ONE

SIDE TWO

Mathematician: ______________________________

"Here are multiplications done by making rectangles with base ten blocks."

| Picture | Computation Form | Length x width = area |
| --- | --- | --- |
| | Tens / Ones<br>+ | |
| | Tens / Ones<br>+ | |
| | Tens / Ones<br>+ | |

Mathematician:______________________________________

"I worked the multiplications by using base ten blocks, drew pictures of the base ten materials used."

| Computation Given | Picture of Multiplication | Partial Products and Answers |
| --- | --- | --- |

| Hundreds | Tens | Ones |
| --- | --- | --- |
| | | |
| | | |
| | | |
| + | | |
| | | |

| Hundreds | Tens | Ones |
| --- | --- | --- |
| | | |
| | | |
| | | |
| + | | |
| | | |

Mathematician: ______________________________

"I made up multiplication problems and worked them by making rectangles from base ten blocks."

| Rectangle Picture | Computation Form | Rectangle Picture | Computation Form |
|---|---|---|---|
| x | Hundreds / Tens / Ones | x | Hundreds / Tens / Ones |
| x | Hundreds / Tens / Ones | x | Hundreds / Tens / Ones |
| x | Hundreds / Tens / Ones | x | Hundreds / Tens / Ones |
| x | Hundreds / Tens / Ones | x | Hundreds / Tens / Ones |

MATHEMATICIAN:______________________________

"I used Base Ten blocks and I traded tens for ones when I had to in order to divide."

| Base Ten Materials | Side One | Side Two | Number Sentences |
|---|---|---|---|
| | | | |
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| | | | |

Mathematician: ________________________________

"Here are multiplications done by making rectangles with base ten blocks."

| Picture | Computation Form | Length x width = area |
| --- | --- | --- |
| | Tens Ones<br>+ | |
| | Tens Ones<br>+ | |
| | Tens Ones<br>+ | |

MATHEMATICIAN: ______________________________

"I used ______________________________
to work these."

Mathematician: ______________________

"From the base ten blocks given, I built the largest rectangle I could inside the outline."

| Base Ten Blocks | | | Outline |
|---|---|---|---|
| Hundreds | Tens | Ones | |
| | | | |
| | | | |

MATHEMATICIAN: ____________________

"I worked divisions by making rectangles with base ten blocks."

| Rectangle Pictures | Computation Form | Rectangle Picture | Computation Form |
|---|---|---|---|
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

MATHEMATICIAN:__________________________________

"I made rectangles from tiles in as many ways as I could."

| Number of Tiles | Largest Rectangles made | Other Rectangles made | Number Sentences |
|---|---|---|---|
| | | | |
| | | | |
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Mathematician: ______________________________

"This is my record of playing GUESS MY RULE."

| In | Out | | In | Out | |
|---|---|---|---|---|---|
| | | Rule: | | | Rule: |
| In | Out | | In | Out | |
| | | Rule: | | | Rule: |
| In | Out | | In | Out | |
| | | Rule: | | | Rule: |
| In | Out | | In | Out | |
| | | Rule: | | | Rule: |
| | | | | | |

# MENU

# HARRY'S DESERT INN

| | |
|---|---|
| Quarter Pounder | $ 1.39 |
| with cheese | $ 1.69 |
| Harry's Special | $ 2.24 |
| Hot Ham 'n' Cheese | $ 1.83 |
| Roast Beef | $ 1.89 |
| Bagels | $ .49 |
| Cheese | $ .69 |
| Tomatoes | $ .15 |
| Lettuce | $ .10 |
| Bacon Burger | $ 1.79 |
| Apple Turnover | $ .69 |

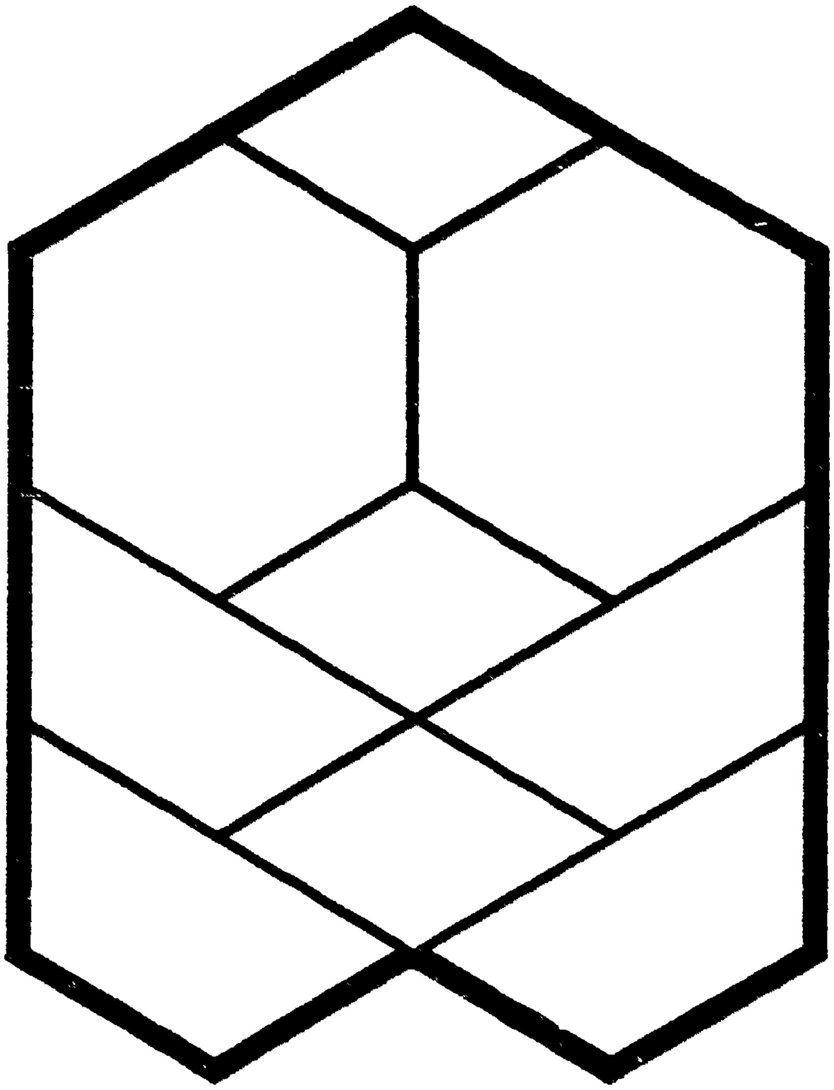
**Soft Drinks:**

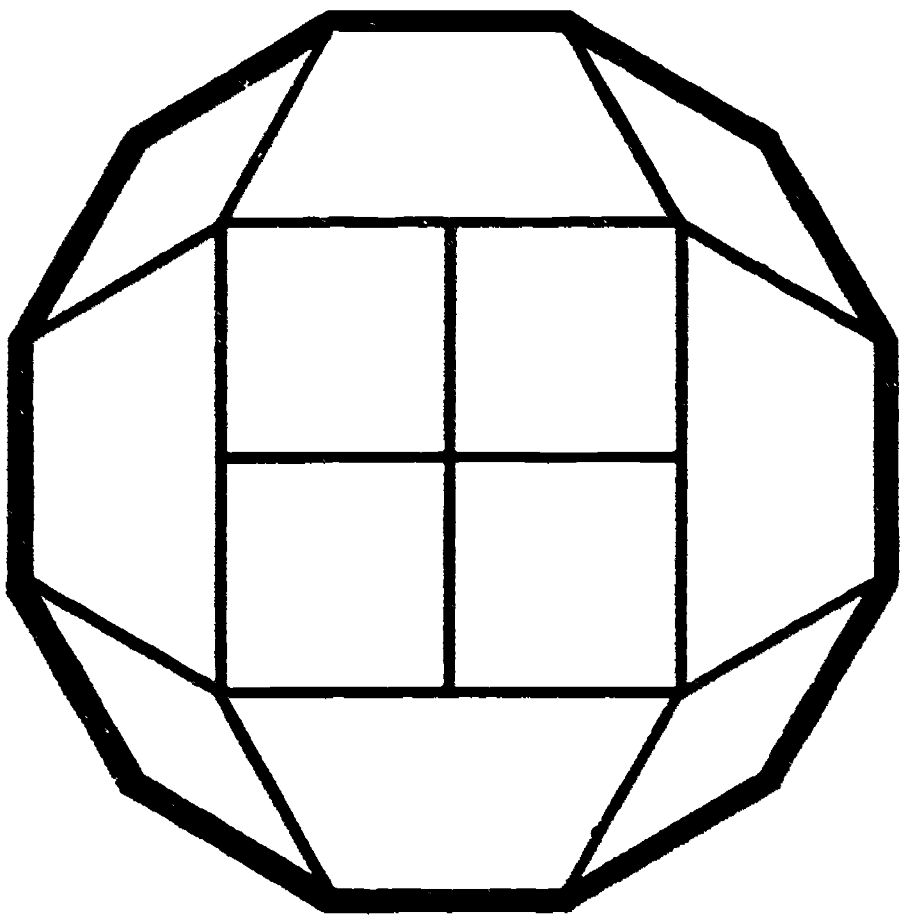
| | |
|---|---|
| Large | $ .75 |
| Small | $ .49 |
| Milk: Large | $ .60 |
| Small | $ .45 |
| Milk Shakes | $ 1.29 |
| Malted Milks | $ 1.39 |
| Ice Cream Bars | $ .39 |

**Ice Cream Cones:**

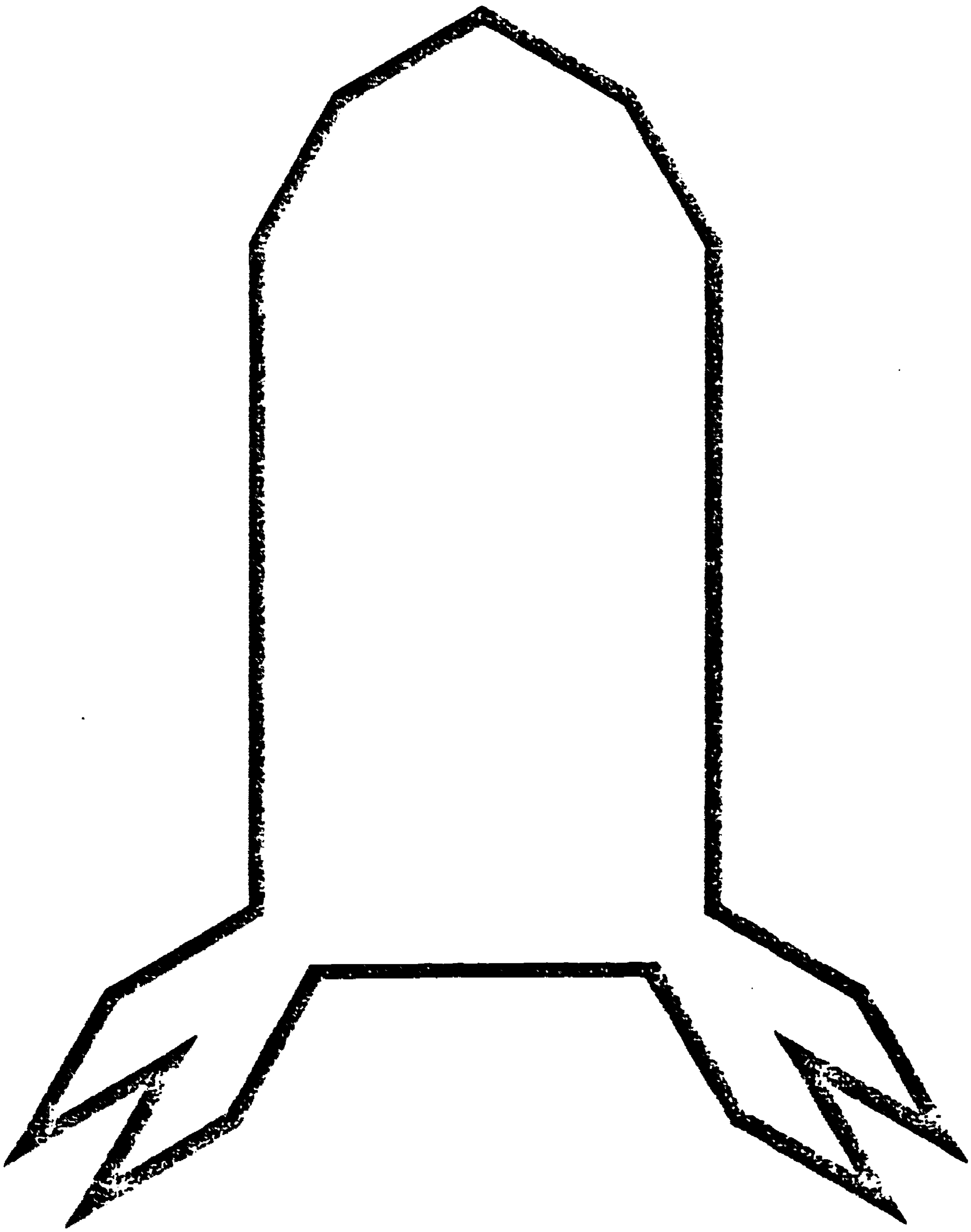
| | |
|---|---|
| One scoop | $ .60 |
| Two Scoops | $ .89 |
| Super Cones | $ 1.19 |

Copy.



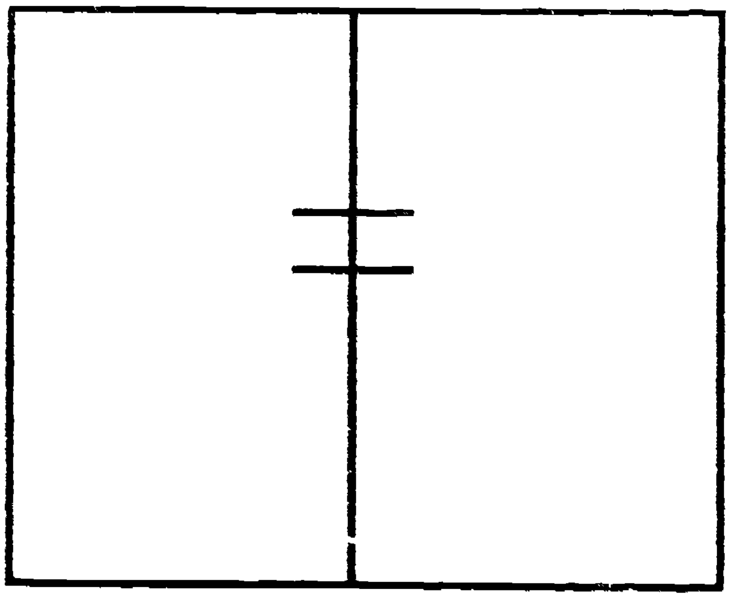


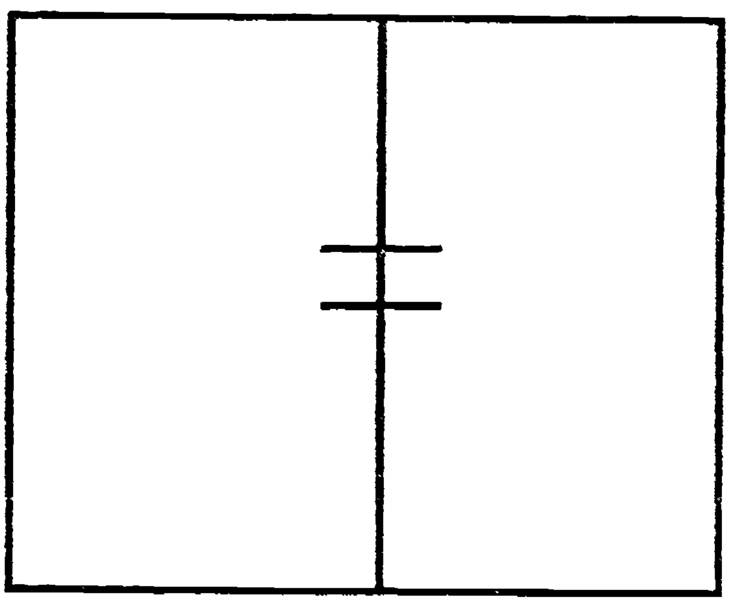
Use between 18 and 24 blocks.



Mathematician: ______________________________

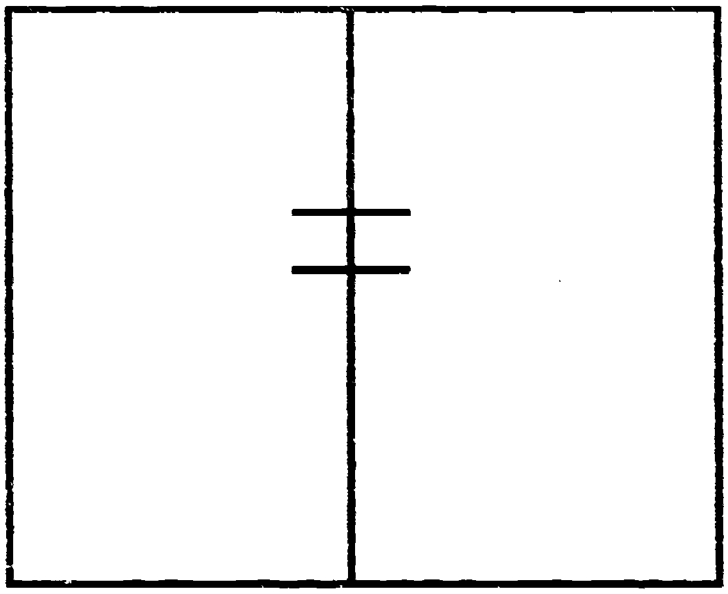
"I made an equality board as shown, grouped the ONES in different ways and wrote as many DIFFERENT number sentences as I had room to write."

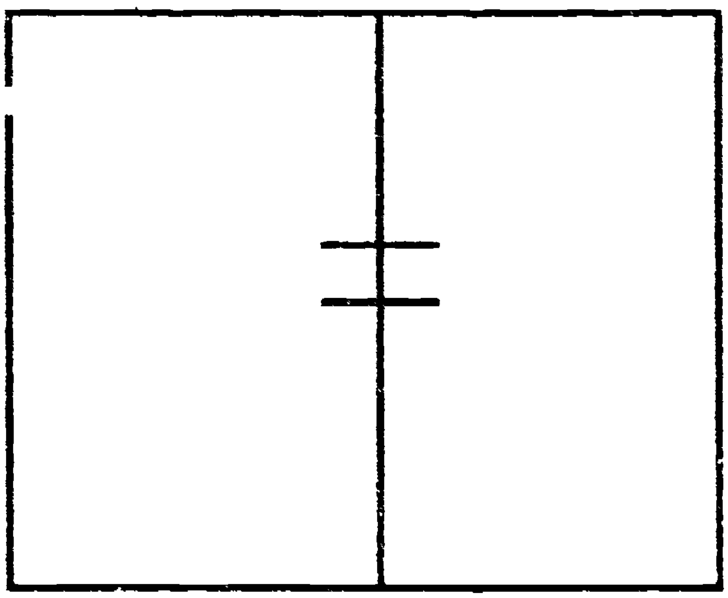




Mathematician:________________________________________

"I arranged the base ten pieces as shown, grouped the LEFT side and wrote the number sentences.





**Mathematician:**________________________________________

**"I put the '<' and '>' signs between the base ten numbers to show their sizes."**

Mathematician: ______________________________ _____

"I made tiles arrangements to show two numbers and compared them. I wrote a number sentence using "<" to show this."

| Numerals | Tile Pictures | Number Words |
| --- | --- | --- |
| | | |
| | | |
| | | |
| | | |

MATHEMATICIAN: ______________________________

"I built these tables by using a number key, [+] and [=] many times."

| Number Entered | Number of Times [=] used | Display | Number Sentence |
|---|---|---|---|
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

Mathematician: ____________________

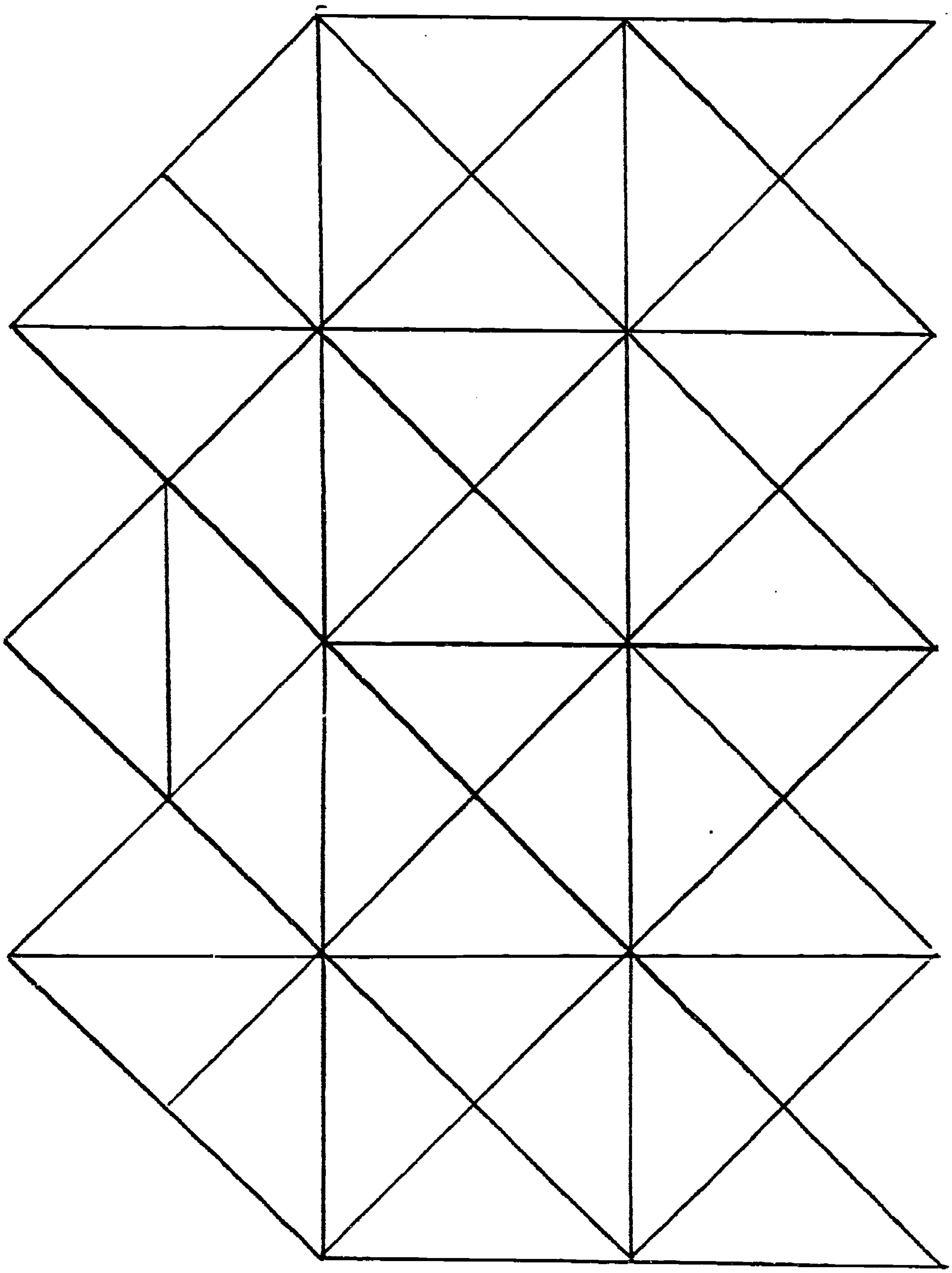
"I made base ten numbers and displayed them on the calculator."

| Base Ten Blocks Used | | | Buttons Pushed | | | |
|---|---|---|---|---|---|---|
| Hundreds | Tens | Ones | First | Next | Next | |
| | | | ◯ | ◯ | ◯ | |
| | | | ◯ | ◯ | ◯ | |
| | | | ◯ | ◯ | ◯ | |
| | | | ◯ | ◯ | ◯ | |
| | | | ◯ | ◯ | ◯ | |
| | | | ◯ | ◯ | ◯ | |
| | | | ◯ | ◯ | ◯ | |
| | | | ◯ | ◯ | ◯ | |
| | | | ◯ | ◯ | ◯ | |

MATHEMATICIAN: ______________________________

"I worked the Tangram problems and traced to show where the pieces fit."

# TANGRAM PROBLEM GENERATOR



Mathematician: ____________________

"I used the given number of multilinks to build houses with rectangle or square floors."

| Multilinks Used | Floor | Height | Number Sentence |
| --- | --- | --- | --- |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

Mathematician:________________________________________

"I built houses with the given floors and numbers of stories of Multilinks. I found the total number of rooms in these houses."

| Floors | Rooms/Story<br>Area | Stories<br>Height | Total Rooms<br>Volume |
|---|---|---|---|
| | | | |
| | | | |

Mathematician:________________________________

| What was Measured | Number of __________s used |
|---|---|
| | |
| | |
| | |
| | |
| | |

Mathematician: ______________________________

"Our group measured objects in the room."

Units Used: ______________________________

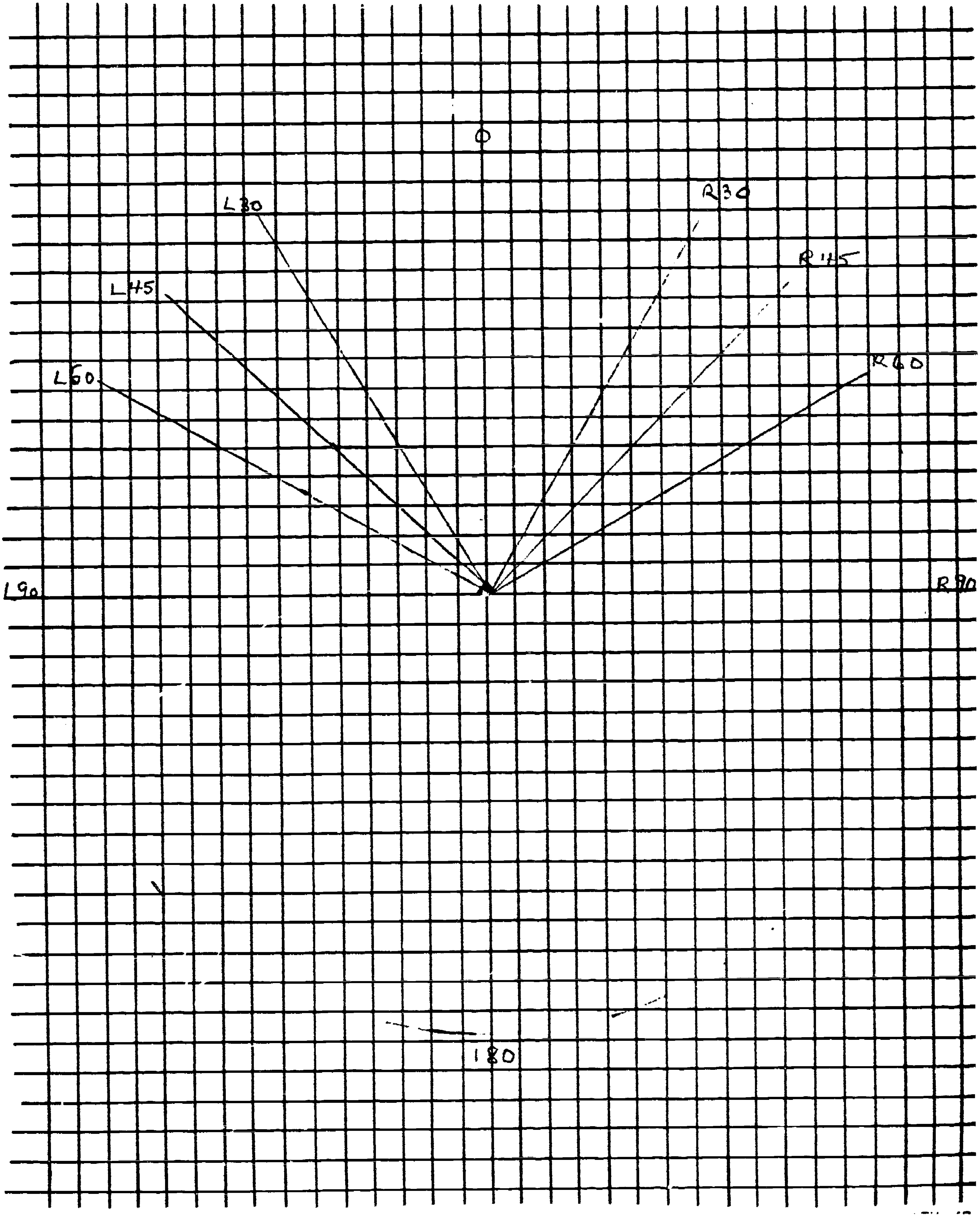
Nearest Unit: ______________________________

| Length Measured | Measurement |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |

Mathematician:________________________________________

"I found the value of each set of coins and bills given."

| COINS | VALUE |
|---|---|
| | |


0
L30
R30
L45
R45
L60
R60
L90
R90
180

# Verbal Addition and Subtraction Problems: Some Difficulties and Some Solutions

*By* Charles S. Thompson *and* A. Dean Hendrickson

Many of the difficulties that children have in solving verbal (story) problems involving addition and subtraction arise because of their limited understanding of the arithmetic operations that are involved. They don't know when to use addition or subtraction because they lack specific knowledge regarding the various situations that give rise to these operations. Often, children are taught addition only as "putting together" and subtraction only as "taking away," but many other settings involve addition and subtraction operations. Children need to receive specific instruction in different contexts if they are to become good solvers of verbal addition and subtraction problems. This article describes the contexts and then explains a successful sequence of activities that teach verbal problems.

## Categories of Verbal Problems

In elementary school mathematics, three categories of verbal problems suggest addition and subtraction operations. These categories—Change, Combine, and Compare—are described by Nesher (1981). Various types of problem situations exist within each category.

Table 1
**Change Problems**

| Problem title | Sample problem | Characteristics |
|---|---|---|
| Change 1 | Bill has two pencils. Jean gives him three pencils. How many pencils does Bill have then? | Increase, initial set and change set known, question about final set |
| Change 2 | Bill has five pencils. He gives three to Jean. How many pencils does he have left? | Decrease, initial set and change set known, question about final set |
| Change 3 | Bill has two pencils. Jean gives him some more. Now he has five. How many did Jean give him? | Increase, initial set and final set known, question about change set |
| Change 4 | Bill has five pencils. He gives some to Jean. Now he has two. How many did he give to Jean? | Decrease, initial set and final set known, question about change set |
| Change 5 | Bill has some pencils. Jean gave him two more. Now he has five. How many did he begin with? | Increase, change set and final set known, question about initial set |
| Change 6 | Bill has some pencils. He gave three to Jean. Now he has two. How many did he begin with? | Decrease, change set and final set known, question about initial set |

Let's look first at the Change category. Change problems involve increasing or decreasing an initial set to create a final set. One sample Change problem is a familiar "putting together" situation (fig. 1).

> Bert has two books. On his birthday he gets three new books. How many books does Bert have then?

All Change problems have three quantities: an initial set, a change set, and a final set. In the problem given, the *initial set* is two books, the *change set* is three books, and the *final set* is unknown. The unknown quantity in Change problems can be any one of the three sets, yielding three kinds of problems. Furthermore, the change can be either an increase or a decrease, thus yielding two problems for each of the three kinds, for a total of six types of Change problems. These problems are described and characterized in table 1.

The second category of problems is called Combine, or part-part-whole. Combine problems describe an existing, static condition involving a set and its several component subsets. A major difference between Change and Combine problems is that no action is involved in Combine problems. A sample problem is as follows:

> Consuelo has five buttons. Three are round and the rest are square. How many are square?

See figure 2.

A typical Combine problem has three related quantities—one subset, the other subset, and the whole set. These yield only two types of problems. In our example, the whole set and one subset are known. In the

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Fig. 1 Change problems involve increasing or decreasing an initial set to create a final set.

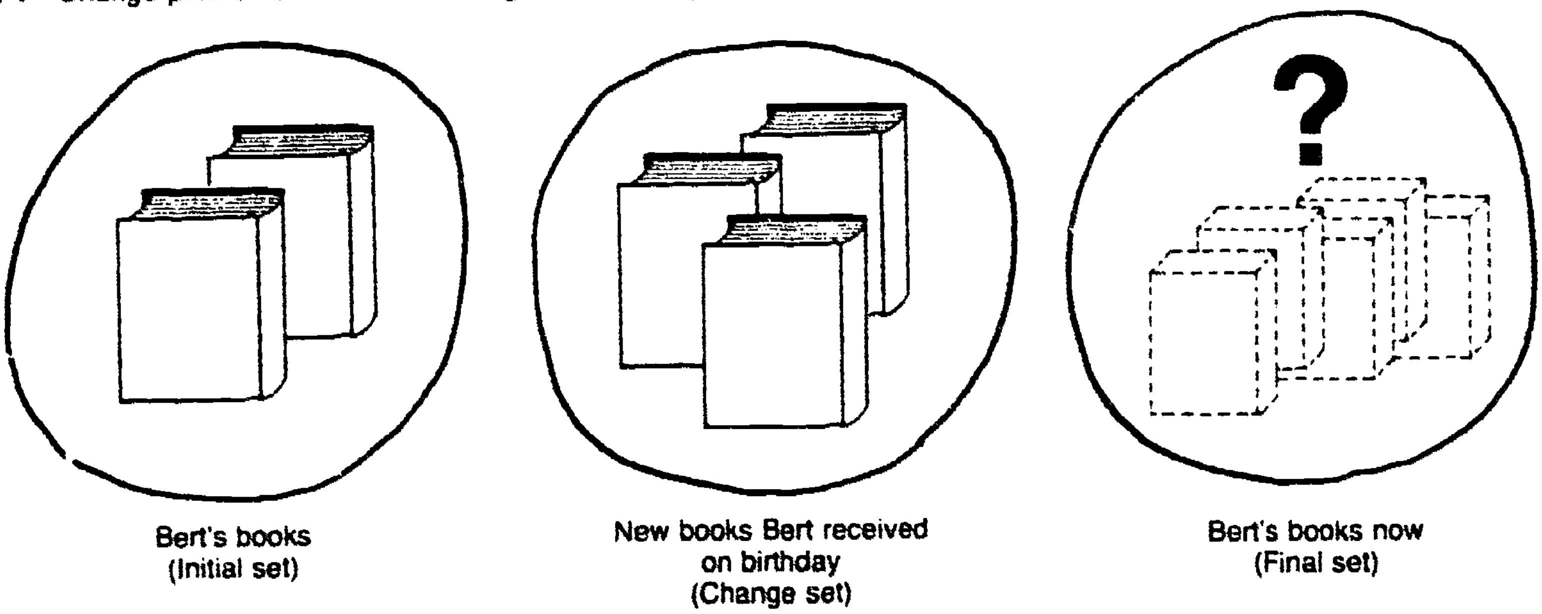



Fig. 2 "Combine" problems describe an existing condition involving a set and its several component subsets.

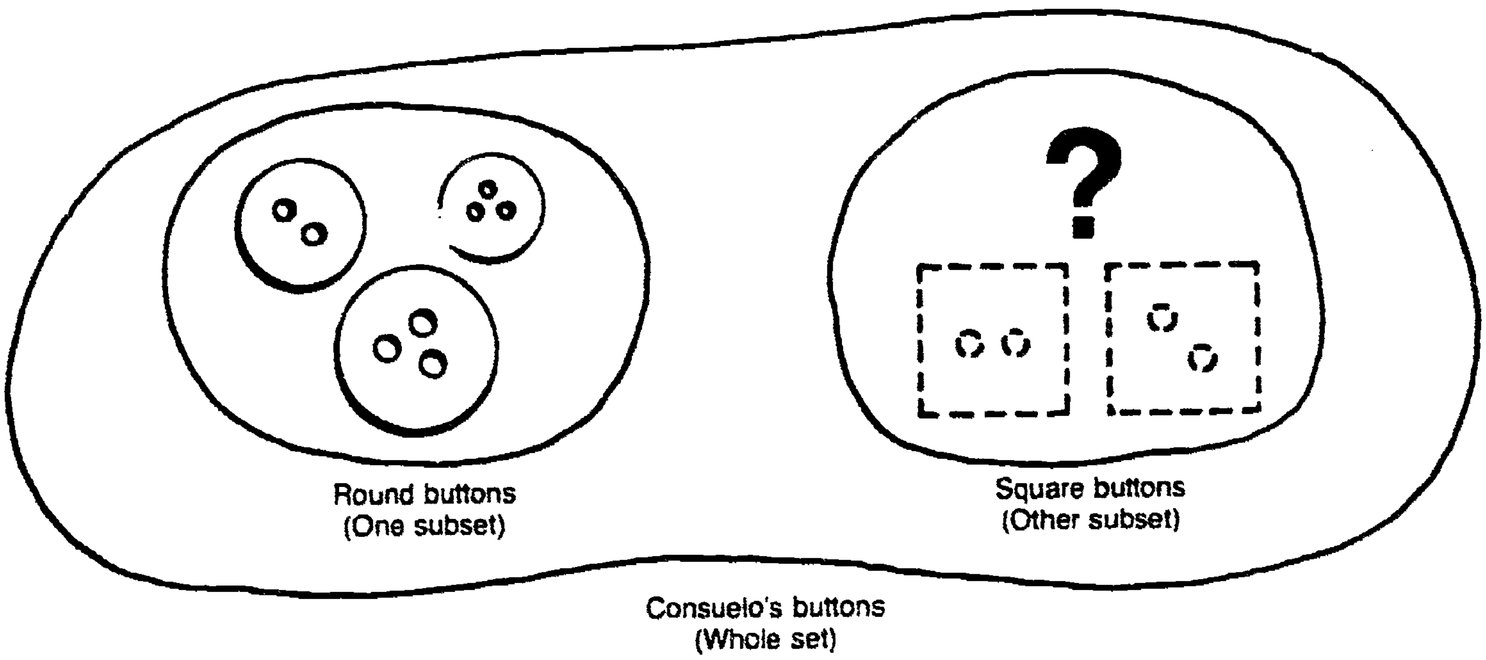
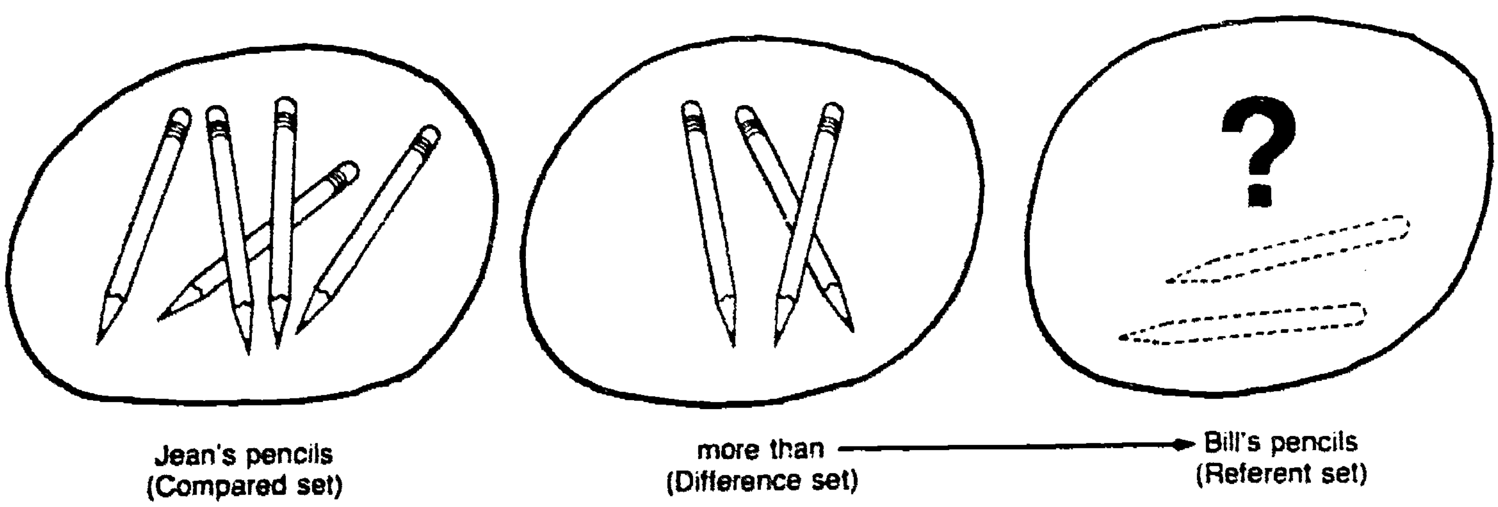



Fig. 3 Compare problems involve a comparison of two existing sets.



other type of problem, both subsets are known and the whole set is unknown. Table 2 summarizes these Combine problems.

Other Combine problems involve more than two subsets and the whole set. These problems typically involve a two-step process and are not discussed here.

The third category of problems is called Compare. Compare problems, which involve a comparison of two existing sets, are probably the most ignored type of problem in school curricula. Yet many children's experiences involve comparisons. Here is a sample problem:

> Jean has five pencils. She has three more pencils than Bill. How many pencils does Bill have?

See figure 3.

Each Compare problem has three expressed quantities—a referent set, a compared set, and a difference set. The referent set is the set to which the comparative description refers. In the sample problem, Bill's pencils compose the referent set, since Jean "has three *more* pencils *than Bill*." The compared set is the set being compared to the referent set. In the sample problem, Jean's set of five pencils (the compared set) is compared to Bill's set (the referent set). The difference set is the difference between the referent set and the compared set.

There are six types of Compare problems. The unknown quantity can be the referent set, the compared set, or the difference set. For each of these three possibilities, the comparison can be stated in two ways: (1) the (larger) compared set is *more than* the (smaller) referent set, or (2) the (smaller) compared set is *less than* or *fewer than* the (larger) referent set. Table 3 summarizes and gives examples of the six types of Compare problems.

Table 2
**Combine Problems**

| Problem title | Sample problem | Characteristics |
|---|---|---|
| Combine 1 | Bill has three red pencils and two green pencils. How many pencils does Bill have all together? | Two subsets are known, question about whole set |
| Combine 2 | Bill has five pencils. Three are red and the rest are green. How many are green? | Whole set and one subset are known, question about other subset |

Table 3
**Compare Problems**

| Problem title | Sample problem | Characteristics |
|---|---|---|
| Compare 1 | Bill has two pencils. Jean has five. How many more does Jean have than Bill? | Comparison stated in terms of "*more*," referent set and compared set known, question about difference set |
| Compare 2 | Bill has two pencils. Jean has five. How many fewer pencils does Bill have than Jean? | Comparison stated in terms of *less (fewer)*, referent set and compared set known, question about difference set |
| Compare 3 | Bill has two pencils. Jean has three more than Bill. How many pencils does Jean have? | Comparison stated in terms of *more*, referent set and difference set known, question about compared set |
| Compare 4 | Jean has five pencils. Bill has three fewer pencils than Jean. How many pencils does Bill have? | Comparison stated in terms of *less (fewer)*, referent set and difference set known, question about compared set |
| Compare 5 | Jean has five pencils. She has three more pencils than Bill. How many pencils does Bill have? | Comparison stated in terms of *more*, compared set and difference set known, question about referent set |
| Compare 6 | Jean has two pencils. She has three fewer pencils than Bill. How many pencils does Bill have? | Comparison stated in terms of *less (fewer)*, compared set and difference set known, question about referent set |

## Relative Difficulties of Verbal Problems

Examination of the various types of problems and observations of children solving these problems lead to the conclusion that some types of problems are more difficult to solve than others. In general, it appears that the inherent structure of the problem is the crucial factor in determining its difficulty. For example, Combine-1 problems are structurally straightforward (table 2).

> Combine 1. Bill has three red pencils and two green pencils. How many pencils does Bill have all together?

The two subsets are given. Children can count those subsets separately. Then, they must simply recount the entire collection of objects to determine the solution to the problem. Or, depending on instruction they have received, they might use "all" or "all together" to transform it to a Change problem.

Combine-2 problems, by comparison, are not straightforward. The sets to be considered are not separate from one another.

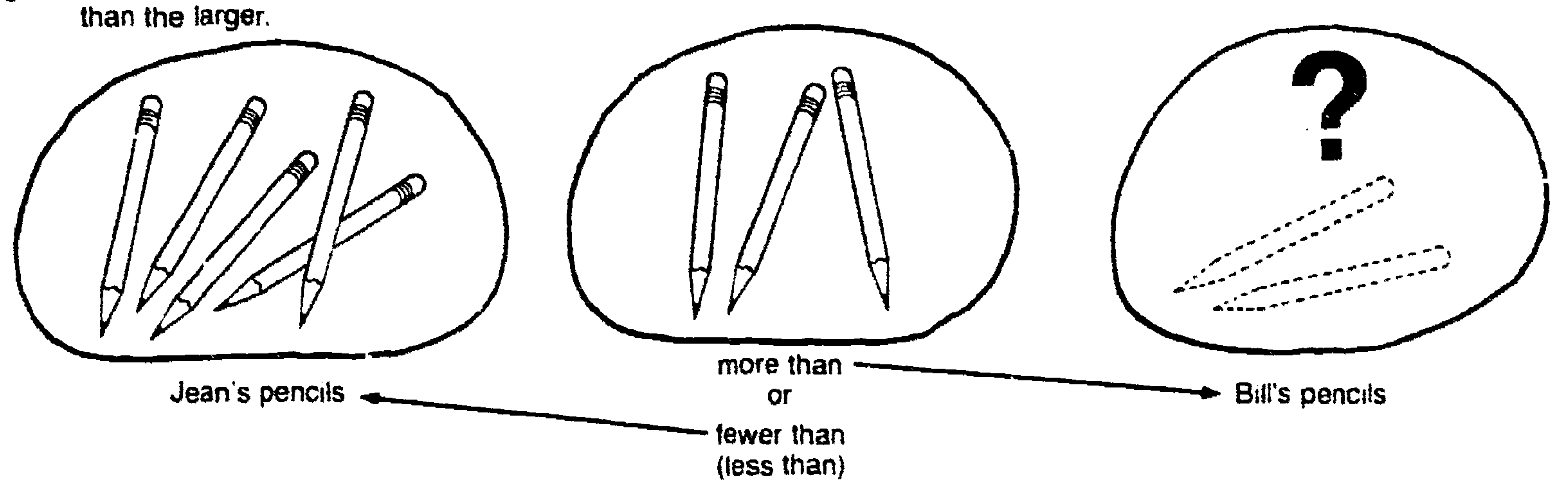
> Combine 2. Bill has five pencils. Three are red and the rest are green. How many are green?

The children must have a well-developed part-whole understanding. The whole set and one subset are given. To solve this kind of problem, children must know that the given subset is contained within the whole set mentally or physically to separate that subset from the whole set and then count the other subset. This problem can be transformed correctly into a Change-2 problem by many children. Other children transform it incorrectly into a comparison of the two subsets.

Another major factor affecting the difficulty of a problem is its semantics. How the relationships between the sets are expressed determines, to some extent, which cognitive structures must be used by the child to solve the problem. For example, study the following Compare-4 and Compare-5 problems:

Compare 4. Jean has five pencils. Bill

Fig. 4 The child must understand that the larger set is three more than the smaller set and the smaller set is three less than the larger.




has three pencils *fewer than* Jean. How many pe^cils does Bill have?

Compare 5. Jean has five pencils. She has three *more* pencils *than* Bill. How many pencils does Bill have?

See figure 4.

In each problem the larger set, of the two being compared, and the difference set are given. The child is to determine the smaller set. In the Compare-4 problem the expression used to relate the larger and smaller sets is "The smaller set is three pencils *fewer than* the larger (known) set." To solve this problem, the child might simply create what is described, by removing three pencils from the larger set to create the smaller set of two objects. This behavior transforms the problem into a Change-2 problem. In the Compare-5 problem, however, the statement used to relate the larger and smaller sets is, in effect, "The larger set is three pencils *more than* the smaller (but unknown) set." In this problem the child must use a different cognitive structure to determine what to do. Three pencils cannot be added to the smaller set, since its quantity is not known. The child must understand that if the larger set is three *more than* the smaller set, then the smaller set is three *fewer than* the larger. The child must have a well-developed cognitive structure called *reversibility*. The child must understand that the statement "$x$ is $a$ more than $y$" is equivalent to "$y$ is $a$ less than $x$." Only then will the child know that removing objects from the larger set will create the "more than" relationship expressed in the verbal problem. This same reversibility enables some children to transform Combine-2 problems into Change-2 problems.

Another factor affecting the difficulty of Compare problems is that in Compare-3, 4, 5, and 6 problems, the difference set must be mentally constructed by the child. It is not actually part of the compared set or the referent set. Furthermore, after the difference set is mentally constructed, the child must mentally add it to, or subtract it from, one given set to determine the unknown set.

Another difficulty is the varying use of the expressions *more than*, *less than*, and *fewer than*. The phrase *fewer than* is common in these fourteen types of problems, since discrete, countable sets are involved. *Fewer than* suggests counting strategies more readily than does *less than*. However, *more than* is used to express relationships between either countable or noncountable quantities. Further, the word *more* is often used in Change problems in another way, as in "John gave Frank four more."

The relative difficulties of all fourteen types of verbal problems have not yet been fully determined. But informal observations of children solving these problems, careful analysis of the problems structures and semantics (Nesher et al. 1982), and analysis of research results (Carpenter and Moser 1981; Nesher 1981; Riley 1981; Steffe 1971; Tamburino 1981) provide preliminary information about the difficulty of problems. Currently available information indicates four levels of difficulty:

Easiest: 1. Change 1 & 2, Combine 1
2. Change 3 & 4, Compare 1 & 2
3. Combine 2, Change 5 & 6, Compare 3 & 4
Hardest: 4. Compare 5 & 6

## Instructional Procedures

We have been working in a conceptually oriented, materials-based elementary mathematics program. The children in first, second, and third grades have received instruction in solving verbal problems of the fourteen types that have been described. The following general instructional sequence has been followed over a period of weeks:

1. Problem situations are presented orally to children. The children use countable materials that can be grouped, linked, and separated to aid them in solving problems. Their answers are expressed *orally*.

2. Children use countable objects to explore combinations of numbers that make larger numbers. For example, they separate five counters into two subsets in different ways and describe the results orally, such as "three and two" or "one and four."

3. Children use prepared numeral cards (0-9), and cards with the "+," "−," "=," and "□," in conjunction with activities similar to those previously described in step 2. They con-

struct number phrases and sentences with the prepared sign cards to represent the objects being used. This task helps them to connect the signs to the concepts involved. For example, if a child uses five counters and covers two of them, then a partner can create the open sentence 3 + ☐ = 5 then insert a "2 card" to complete the open sentence.

4. The problem situations are presented orally to children as in step 1. They use countable objects to solve the problems and now use the *prepared cards to construct number sentences* to represent the objects used and the conditions of the problem. For example, consider the following problem:

Change 1. Bill has two pencils. Jean gives him three pencils. How many pencils does Bill have now?

To solve this problem, children frequently make separate links of cubes to represent the two sets, join the two links, and arrange cards as shown:

[2] [+] [3] [=] [5]

5. Children use countable materials to solve orally presented problems and then *write number sentences* to indicate how they interpreted the problems. In particular, children circle their answers in the number sentences. In many problem situations several possible number sentences can be written. Consider this problem:

Compare 1. Jean has five pencils. Bill has two pencils. How many more pencils does Jean have than Bill?

Some children will interpret this as an addition problem and write 2 + ③ = 5. Others will interpret it as subtraction and write 5 − 2 = ③. Both interpretations are correct.

6. Open sentences in written form are given to children, who use countable materials to solve them.

7. Materials are not used, and children solve written verbal problems mentally while writing the corresponding number sentences.

8. Children solve open sentences (not directly tied to verbal problems) in written form without the use of countable materials.

From a broad perspective, the sequence has used the following steps: (1) develop concepts using materials, (2) connect signs to the concepts, (3) construct symbolic forms (number sentences) using prepared symbols, (4) write symbolic forms, and (5) interpret prepared symbolic forms. This sequence has resulted in students being able to interpret these problems and translate them into number-sentence models.

In conjunction with these activities, children participate in numerous counting exercises. They learn to count on from any given number and to count back from any given number. Counting on is useful in many problems, particularly in part-whole situations, in which one subset and the whole set are known, and in compare situations, where equalizing of the two sets is the strategy to be used. Counting back is also used frequently, especially in Change problems. For example, in Change-2 problems the children often count back from the larger (initial) set to create the smaller (final) set.

## Instructional Results So Far

The instructional sequence described seems to be effective in enabling children in the primary grades to solve verbal problems. Of crucial importance seem to be the use of countable materials, the use of the prepared numeral and sign cards, and the practice of circling answers when writing number sentences.

Using the countable materials enables the children to create or model the conditions presented in the problems. The children can then determine which sets to count, compare, separate, or join to solve the problems. The use of the prepared cards allows the children quickly to attach numerals to the quantities represented and to construct the corresponding number sentences. We have found that children who have not used numeral cards experience greater difficulty in writing number sentences corresponding to a verbal problem. The practice of having children circle answers when writing number sentences helps teachers understand how the children are thinking about the verbal problems. Indeed, for many of the types of problems, either an addition or a subtraction number sentence is appropriate. These practices also help teachers to recognize when children are successfully using the class-inclusion relation, reversibility of both actions and relations, and equalization of two sets.

In summary, we have learned that children can become good solvers of verbal problems. What they need is an instructional program that proceeds from the concrete to the symbolic *and* the opportunity to encounter the various problem situations that occur in real life.

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# Verbal Multiplication and Division Problems: Some Difficulties and Some Solutions

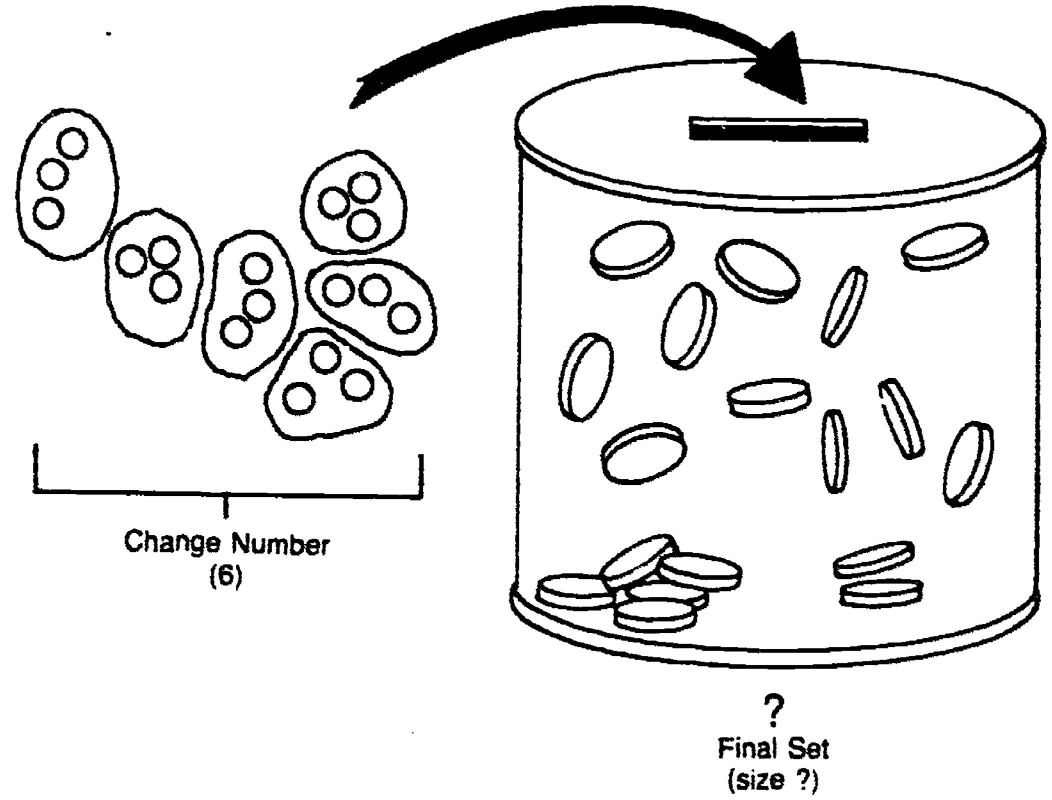
*By* **A. Dean Hendrickson**

Verbal problems that involve multiplication and division are difficult for children to solve. Many of these difficulties arise because of their limited understanding of these arithmetic operations. Their experience with the different kinds of situations that call for these operations is also limited. At the same time, these problems cannot be categorized easily because the situations that require these operations are varied. Nonetheless, multiplication is often taught only as "repeated addition" and division only as "repeated subtraction." Children must have specific instruction in *all* the situations that require multiplication and division as arithmetic operations if they are to apply them successfully to verbal problems.

## Change Problems

Extensions of the "change problems" for addition and subtraction can lead to multiplication and division. In this particular kind of problem we have an initial set, a change number, and a final set. Given an initial set of small size and a change number that describes how many of this size set are joined, we find the size of the larger final set by multiplication. These problems are *change 1*, or repeated addition, problems. Here is an example (fig. 1):

Fig. 1 An example of a *change 1* or "repeated addition" problem: John put 6 handfuls of 3 tokens each into a bank. How many tokens did John put in the bank?




John put 6 handfuls of 3 tokens each into a bank. How many tokens did John put in the bank?

*Change 2* problems result when a large initial set is given along with the size of a smaller final set, and a change number needs to be found that describes *how many* sets of that size can be made from the initial set. This problem represents the *measurement*, or repeated-subtraction, interpretation of division. Here is an example (fig. 2):

Susie has 24 cookies. She gives 3 cookies to each of the children on the playground. How many children are on the playground?

A child who can reverse the "putting together" transformation can relate a measurement interpretation of the division of countable materials to the repeated-addition kind of multiplication. In some ways the division is easier, since the child must retain only the final set size and count the number of sets that can be made. The count is constructed in the process and the size of the initial set is not important, since the count stops whenever the process runs out of objects. In repeated addition, both the count num-

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ber and the size of the initial set must be retained mentally along with the result at the end of each successive joining.

*Change 3* problems involve a large initial set and a known change number; the size of the final, *equal* sets that can be made from the initial set must be found. This is the *partition* interpretation of division. An example follows (fig. 3):

> Susie has 24 cookies. She gives an equal number to each of her 4 friends. How many cookies does each friend get?

Change 2, or measurement division, is easier, since only the size of the set being formed repeatedly must be retained and a count of these sets kept as they are made. Change 3, or partition division, requires a strategy to assure the equality of the sets being made and hence is more difficult.

## Comparison Problems

Questions involving "less than" or "more than" lead to addition and subtraction problems. These problems involve a comparison set, a difference set, and a referent set. When we compare two sets and the comparison involves questions of "how many times as many" or "what part of," we use multiplication and division. Such problems involve a comparison set, a referent set, and a correspondence other than a one-to-one correspondence between these sets. In figure 4, if the question is asked, "*A* has how many times as many as *B*?" then *A* is the comparison set, *B* is the referent set, and the correspondence of *A* to *B* is sought.

*Compare 1* problems result when the referent set and a many-to-one correspondence are given and students are asked to find the comparison set. The following is an example (fig. 5):

> Iris has 3 times as many nickels as dimes. She has 4 dimes. How many nickels does she have?

Multiplication is used to find the answer: $3 \times 4 = 12$.

*Compare 2* problems occur when the comparison and a many-to-one

Fig. 2 An example of a *change 2* problem, measurement or repeated-subtraction interpretation of division: Susie has 24 cookies. She gives 3 cookies to each of the children on the playground. How many children are on the playground?

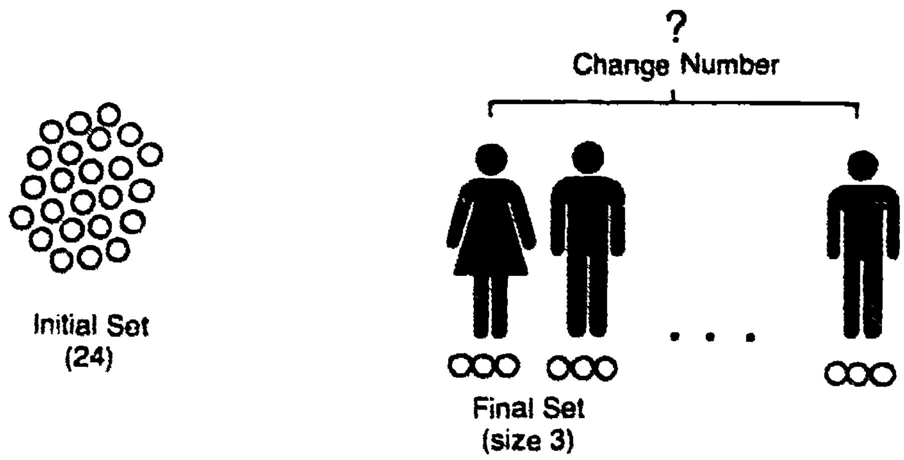



Fig. 3 An example of a *change 3* problem, a partition interpretation of division: Susie has 24 cookies. She gives them in equal numbers to her four friends. How many cookies does each friend get?

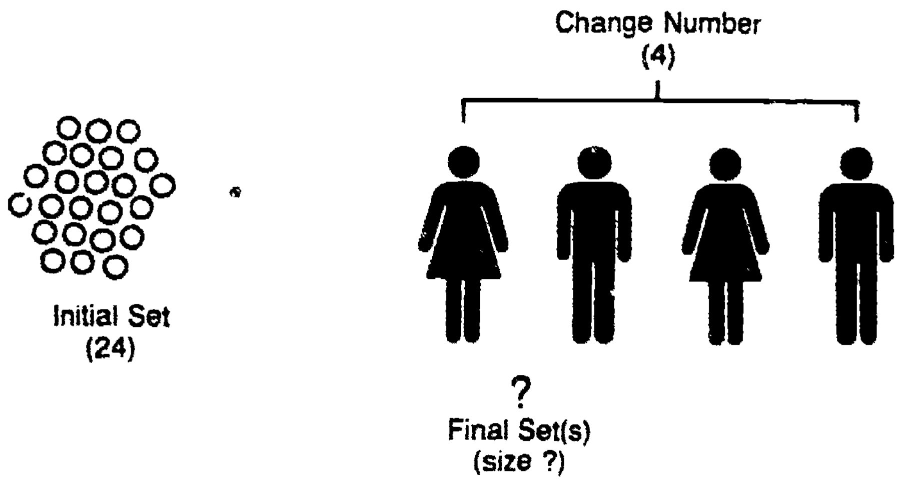



Fig. 4 A comparison problem: Find the correspondence of *A* to *B*. *A* has how many times as many as *B*?

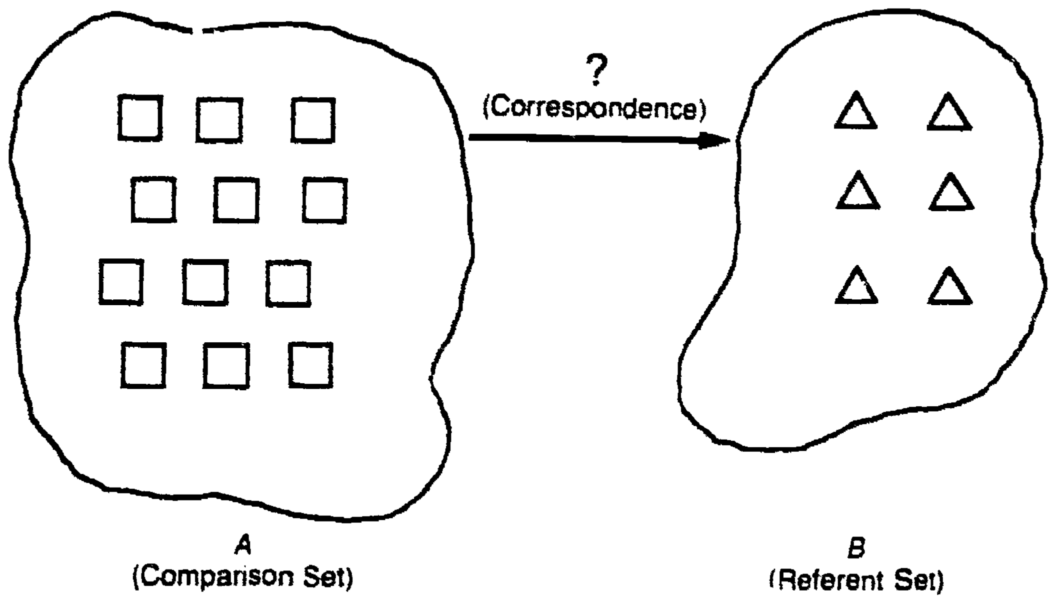


Fig. 5 A *compare 1* problem: Iris has 3 times as many nickels as dimes. She has 4 dimes. How many nickels does she have?

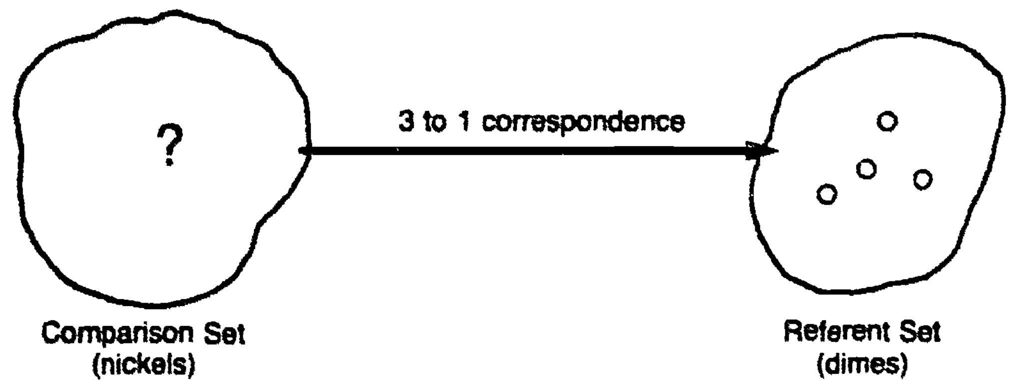



Fig. 6 A *compare 2* problem: Iris has 15 nickels. She has 3 times as many nickels as dimes. How many dimes does Iris have?

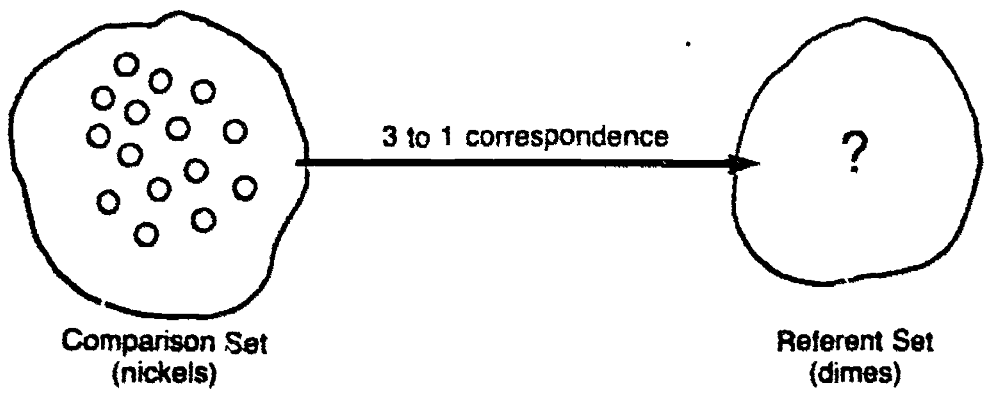



Fig. 7 A *compare 3* problem: Frank has 24 nickels and 8 dimes. He has how many times as many nickels as dimes?

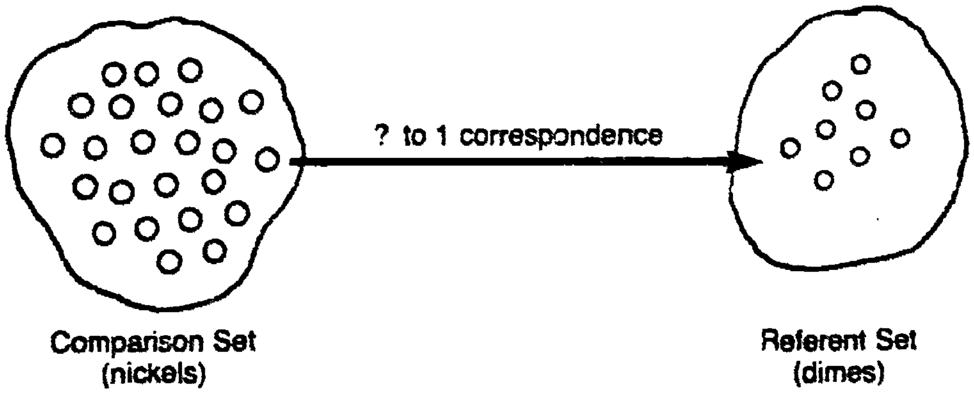
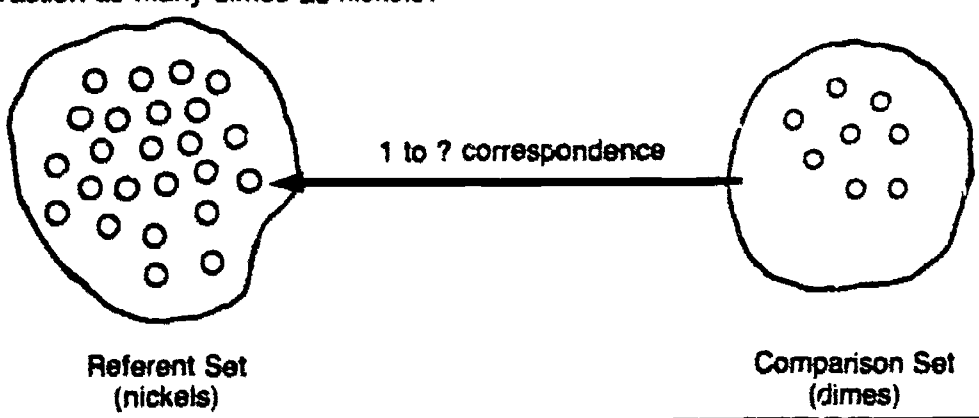



Fig. 8 A *compare 4* problem: Frank has 24 nickels and 8 dimes. He has what fraction as many dimes as nickels?




correspondence are given and the referent set must be found. Here is an example (fig. 6):

> Iris has 15 nickels. She has 3 times as many nickels as dimes. How many dimes does Iris have?

Division is used to find the answer: $15 \div 3 = 5$.

*Compare 3* problems result when the comparison set and referent set are known and a many-to-one correspondence must be found (fig. 7):

> Frank has 24 nickels and 8 dimes. He has how many times as many nickels as dimes?

Division is used to find the answer: $24 \div 8 = 3$.

*Compare 4* problems occur when a comparison set and a referent set are given and a one-to-many correspondence is sought. In this case, the comparison set is the smaller of the two. Here is an example (fig. 8):

> Frank has 24 nickels and 8 dimes. He has what fraction as many dimes as nickels? (or, Frank's dimes are what fractional part of his nickels?)

The result is division of a smaller by a larger number or formation of a rational number, usually expressed as a fraction: $8 \div 24 = 1/3$.

This kind of question puts a child's concept of *fraction* being equal parts of a whole into conflict with this ratio situation. What other language can be used to ask for this correspondence? Because of the difficulty of finding suitable language, questions related to finding this correspondence are seldom found in textbooks.

*Compare 5* problems arise when the comparison set and the referent set are given and a many-to-many correspondence is sought (fig. 9):

> There are 12 girls and 16 boys in the room. How many times as many boys are there as girls?

One divides to find the answer ($16 \div 12 = 4/3$). Here again a fraction tells how many times as much, although a ratio correspondence is made in the thinking.

*Compare 6* problems occur when the comparison set is smaller than the referent set and the correspondence is sought (fig. 10):

> There are 12 girls and 16 boys in a room. The number of girls is what part of the number of boys?

The result is found by division again, $12 \div 16 = 3/4$, and the same conflict between ratio and fraction results.

*Compare 7* problems result when the larger comparison set and the many-to-many correspondence are given and the size of the smaller referent set is sought (fig. 11):

> There are 16 boys in a class. There are 4/3 as many boys as girls. How many girls are there?

The answer is found by dividing: $16 \div 4/3 = 12$.

*Compare 8* problems arise when the smaller referent set is given along with a many-to-many correspondence. The size of the larger comparison set is sought (fig. 12):

> There are 12 girls in the room. The number of boys is 4/3 the number of girls. How many boys are in the room?

The answer is found by multiplying: $4/3 \times 12 = 16$.

The compare problems that involve many-to-many correspondences are difficult, since they bring into conflict the child's recognition of a fraction as comparing a given number of equal parts to the whole and the idea of ratio as a correspondence. The use of the same symbolism for both fractions and rational numbers compounds this difficulty.

Thinking in ratios, equating ratios, and applying ratios to situations involve formal operational thought. Very few elementary children are capable of this kind of reasoning. In fact, few eighth and ninth graders can think through the Mr. Tall–Mr. Short problem:

| | Mr. Tall | Mr. Short |
|---|---|---|
| Measured in match sticks | 9 | 6 |
| Measured in paper clips | 12 | ? |

Fig. 9 A *compare 5* problem: There are 12 girls and 16 boys in the room. How many times as many boys are there as girls?

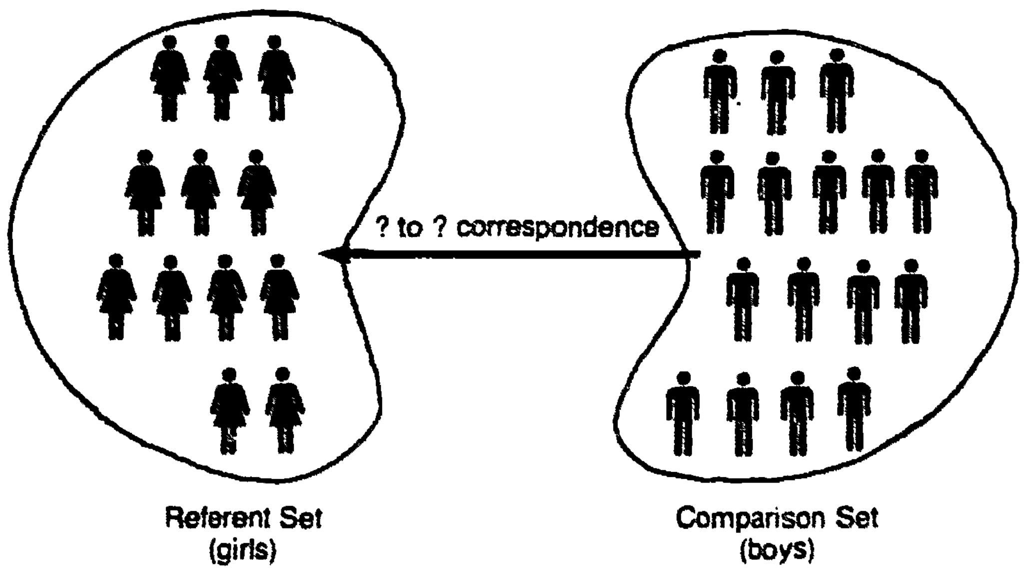



Fig. 10 A *compare 6* problem: There are 12 girls and 16 boys in a room. The girls are what part of the boys?

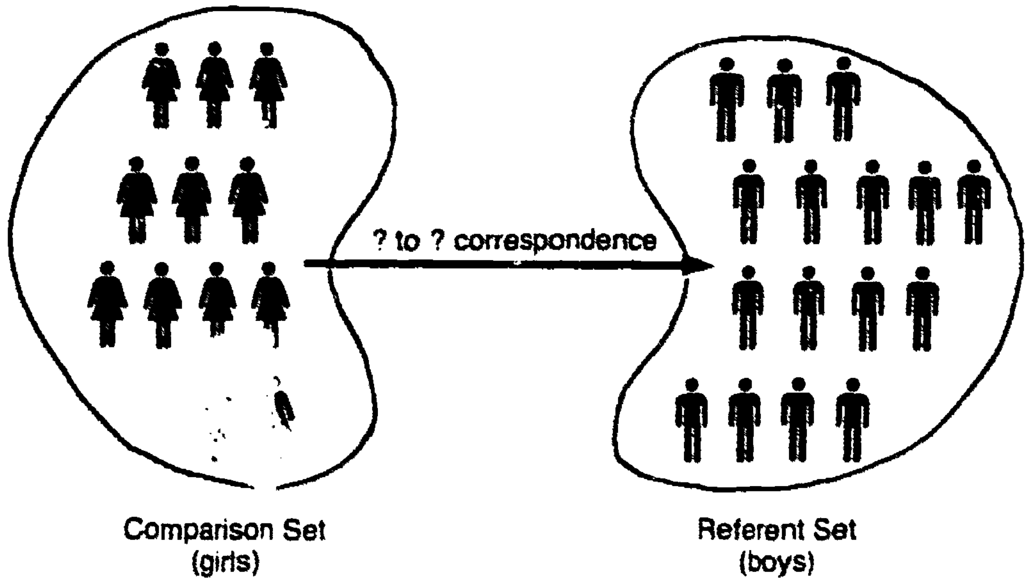



Fig. 11 A *compare 7* problem: There are 16 boys in a class. There are 4/3 as many boys as girls. How many girls are there?

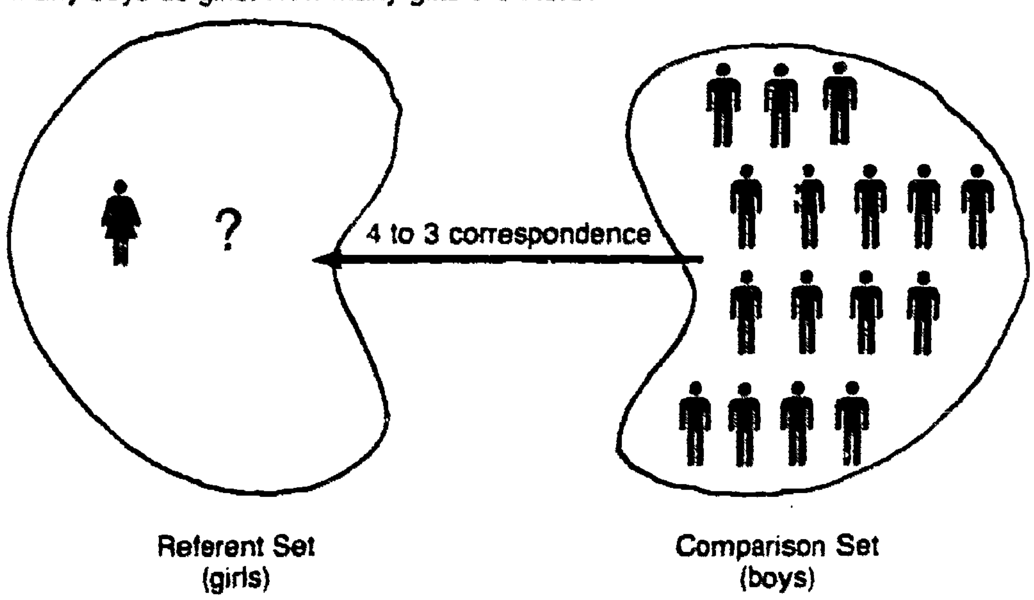
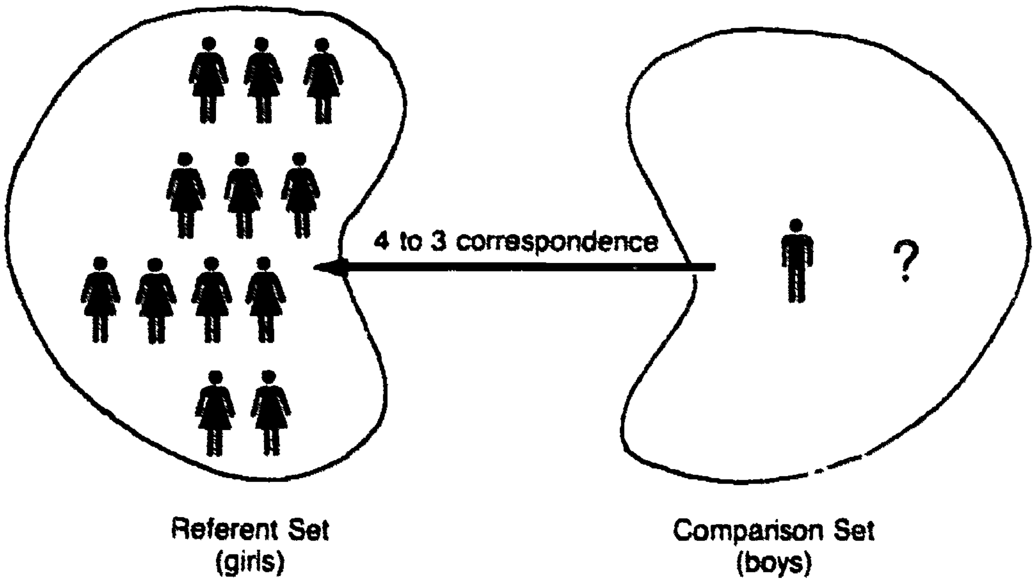


Fig. 12 A *compare 8* problem: There are 12 girls in the room. The number of boys is 4/3 the number of girls. How many boys are in the room?




## Rate Problems

The kind of proportional reasoning used in equating ratios is also involved in thinking about rate problems. These are commonly found in intermediate textbooks. A rate problem involves two variables—one independent and one dependent—and a rate of comparison between them. An example is distance (miles) = rate (miles per hour) × time (hours). Here the number of hours is the independent variable, the distance in miles (a total) is the dependent variable, and the ratio of miles to hours is the rate.

Some common rate examples are these:

Fig. 13 A *rate 1* problem: Fred pays $12.00 a square yard for outdoor carpeting. How much will 16 square yards cost?

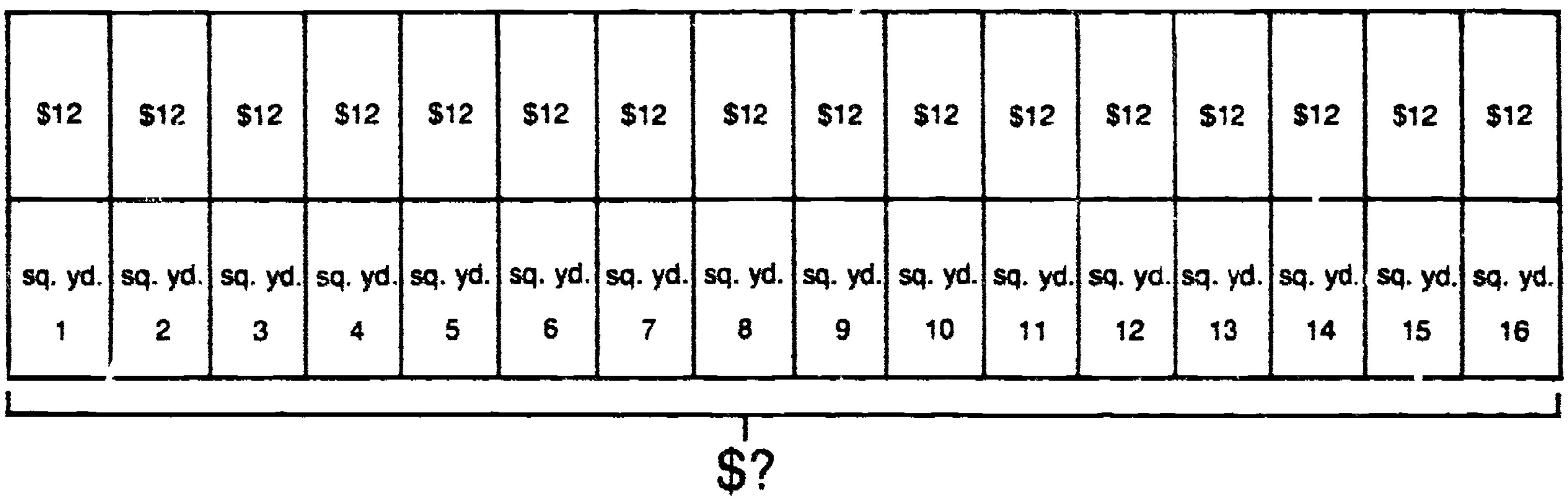
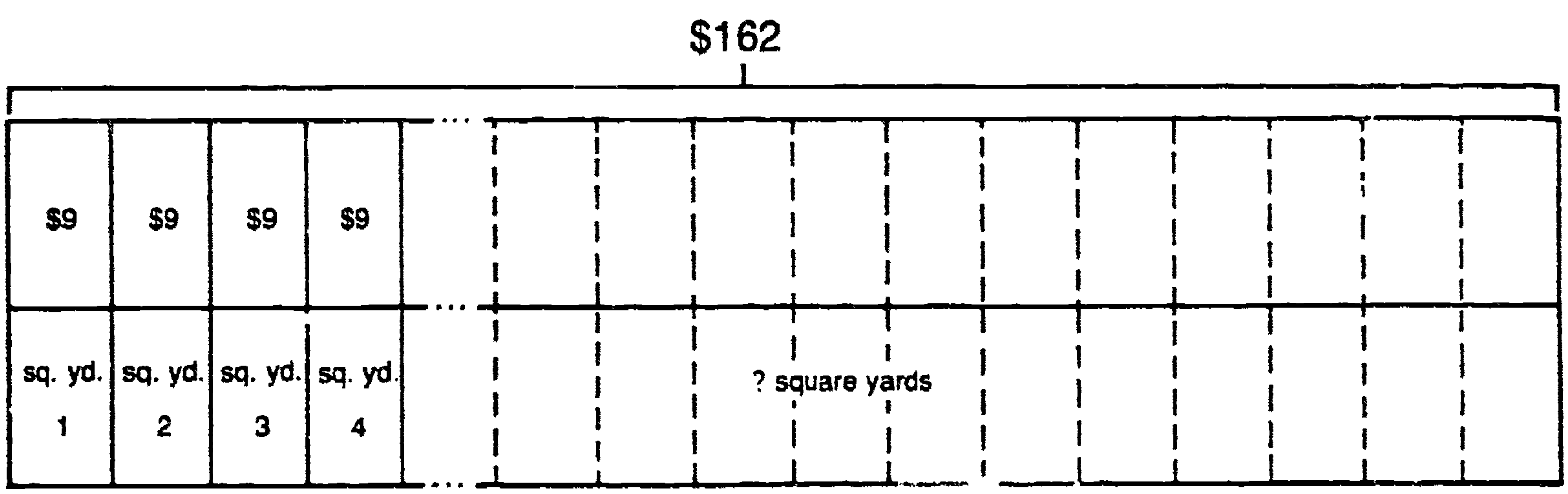



Fig. 14 A *rate 2* problem: Jane pays $162 for carpeting at $9 a square yard. How many square yards did she get?



- feet per second
- dollars per pound
- pounds per cubic foot
- gallons per minute
- cents per kilowatt hour
- parts per hundred

Children who are unable to think about rates and ratios will have difficulty doing these problems in any way other than substituting numbers into memorized formulas. Problems dealing with percentages are probably the best example of this difficulty.

*Rate 1* problems result when the rate and the value of independent variable quantity are given (usually in units of measurement) and the value of the dependent variable, usually a total, must be found (fig. 13):

> Fred pays $12 a square yard for outdoor carpeting. How much will 16 square yards cost him?

The resulting application,

total cost
= cost/sq. yd. × number of sq. yd.
= $12/sq. yd. × 16 sq. yd. = $192,

is the easiest of the rate situations to use.

*Rate 2* problems result when the rate and the value of the dependent variable are given and the value of the independent variable is sought (fig. 14):

> Jane pays $162 for carpeting at $9 a square yard. How many square yards does she get?

We have

$162 = $9/sq. yd. × ☐ sq. yd.

or

$$\text{sq. yd.} = \frac{\$162}{\$9/\text{sq. yd.}} = \boxed{18}.$$

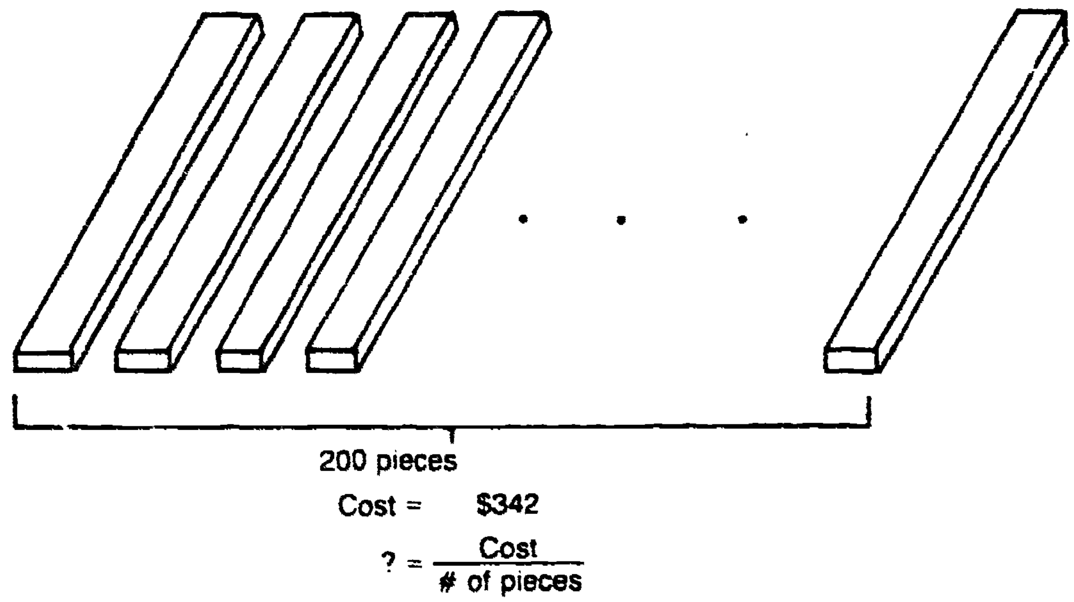
*Rate 3* problems result when the values of the dependent and independent variables are given and the ratio or comparison rate is sought (fig. 15):

> Peter paid $342 for 200 eight-foot two-by-fours. What was the cost in dollars of each two-by-four?

We have

$342 = [$ ]/board × 200 boards

Fig. 15 A *rate 3* problem: Peter paid $342 for 200 eight-foot 2 x 4's. What was the cost in dollars of each 2 x 4?




or

$$\$\ \text{cost/board} = \frac{\$342}{200\ \text{boards}} = \$1.71/\text{board}$$

## Selection Problems

Among the most difficult problems are those that require multiplication. These belong to a more general group of selection problems.

*Selection 1* problems involve simple ordered pairs where the choice sets for each element of the ordered pair are given and the number of ordered pairs possible is sought. The pairs are ordered in the sense that one choice set is associated with one element and a second choice set with the other. No ordering occurs in the writing or selection. In the following example, (skirt, sweater) is not different from (sweater, skirt). See figure 16.

> Amy has 3 sweaters with different patterns. She also has 5 different skirts. How many outfits consisting of a sweater and a skirt are possible?

The pairs can be determined from a matrix (table 1) or from a "factor tree." Either way, multiplication is used: 3 × 5 = 15 outfits.

*Selection 2* problems result when one choice set and the number of pairs are given and the other choice set is sought. These problems are similar to selection 1 problems.

Table 1
**A Matrix to Record the Pairs in Figure 16**

| | Skirts | | | | |
|---|---|---|---|---|---|
| Sweaters | 1 | 2 | 3 | 4 | 5 |
| A | A, 1 | A, 2 | A, 3 | A, 4 | A, 5 |
| B | B, 1 | B, 2 | B, 3 | B, 4 | B, 5 |
| C | C, 1 | C, 2 | C, 3 | C, 4 | C, 5 |

*Selection 3* problems involve triples, quadruples, or other extended $n$-tuples ($n > 2$) and the choice sets for each place in the $n$-tuple.

> Frank has 5 sport coats, 3 vests, and 5 pairs of trousers, all of which are color compatible. How many different outfits consisting of a sport coat, vest, and pair of trousers are in his wardrobe?

Here a 3-tuple must be formed (sport coat, vest, trousers) where ordering is not important. Finding the total number of 3-tuples uses the multiplication principle: 5 × 3 × 5 = 75.

*Selection 4* problems give the number of $n$-tuples and the sizes of all but one choice set, which is sought. An example follows:

> Frank can make 24 different outfits consisting of a sport coat, vest, and trousers. He has 3 sport coats and 4 pairs of trousers. How many vests does Frank have?

Fig. 16 A *selection 1* problem: Amy has 3 sweaters with different patterns. She also has 5 skirts of different colors. How many outfits, consisting of a sweater and a skirt, are possible?

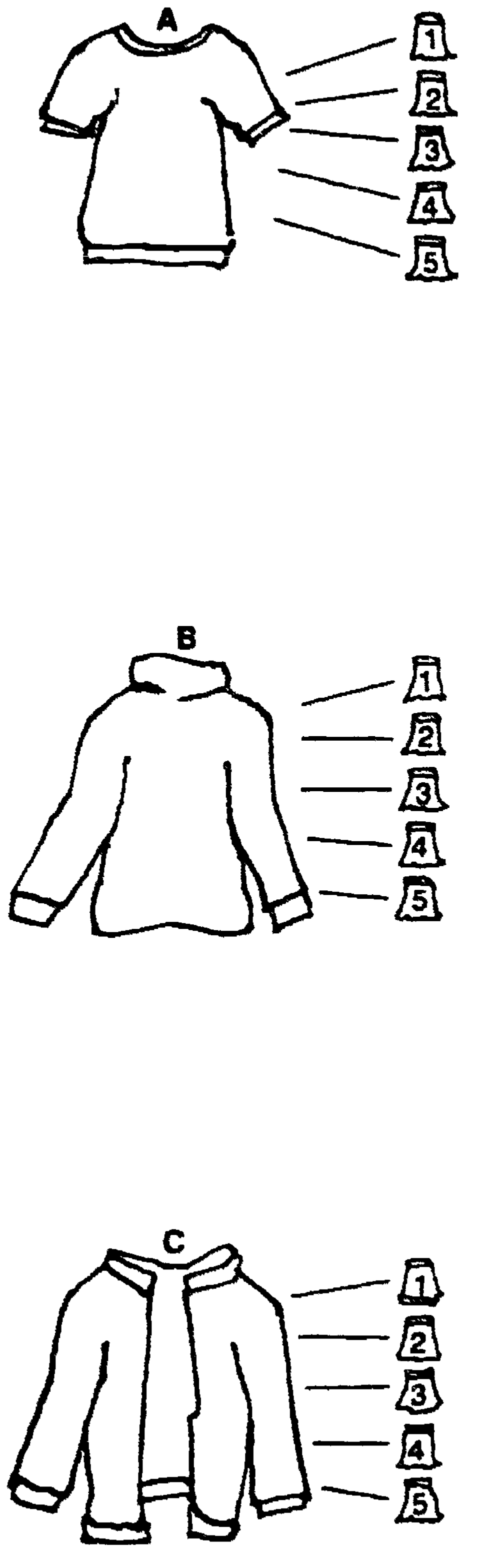



Fig. 17 Ceramic tiles can be used to link the repeated-addition idea of multiplication to area: Make 4 rows of 6 tiles each. How many tiles are used?

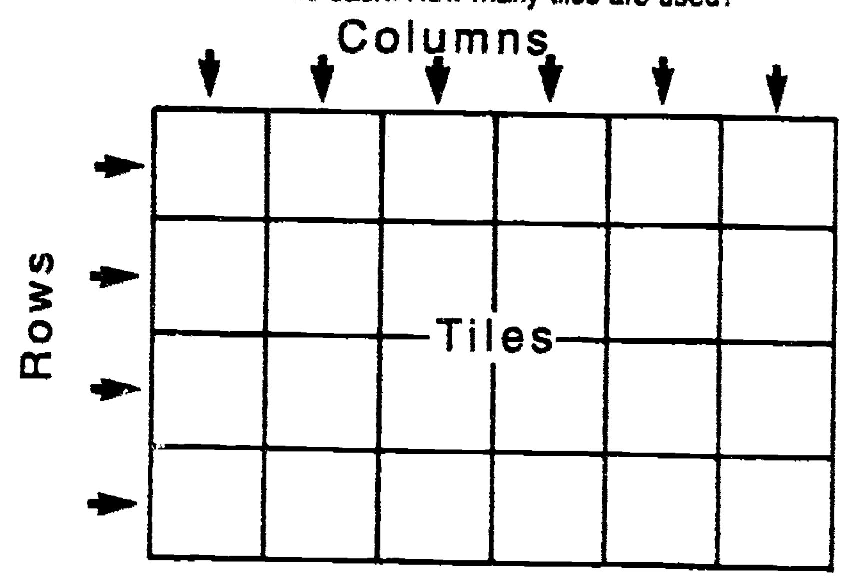



Table 2
**Change Problems**

| Problem title | Sample problem | Characteristics |
|---|---|---|
| (Change 1) Repeated addition | Fred has 3 boxes with 4 cars in each box. How many cars does Fred have? | Initial (smaller) set sizes and change number known; question about final (larger) set. |
| (Change 2) Repeated subtraction (measurement) | Jean had 12 cookies. She gave 3 cookies to each of her friends. How many of her friends got cookies? | Initial (larger) set and final (smaller) set sizes known; question about change number. |
| (Change 3) Partitioning into equal sets | Paul had 24 marbles that he gave away to 4 friends. Each friend received the same number of marbles. How many marbles did each friend get? | Initial (larger) set and change numbers known; question about the size of final (smaller) sets. |

This is a two-step problem: first multiply and then divide, or successively divide.

The selection group of problems involves the multiplication principle or one aspect of what Piaget calls combinatorial reasoning—the ability to consider the effect of several variables simultaneously. Selection 1 problems can be thought of as cells in a matrix. The thinking needed to solve them is similar to that used to solve area problems, such as being given two dimensions and finding the area and being given the area and one dimension and finding the other dimension.

## Overview

If students are going to apply multiplication and division to everyday situations, they must have experience with materials that represent these different situations.

The change situations that involve joining and separating can be introduced with materials that can be joined, separated, and arranged. Unifix cubes can be used to illustrate repeated additions and repeated subtractions as well as measurements. Ceramic tiles can also be used to link the idea of repeated addition to area (fig. 17). The measurement concept of division can also be introduced with tiles. The following kinds of questions can be asked:

- Given 24 tiles, how many rows can be made with 4 tiles in each row?
- Make 4 rows of 6 tiles each. How many tiles are used?

Beans and paper cups can be used to give experience with the partition interpretation of division as well as to the repeated-addition and repeated-subtraction interpretations of multiplication and division. Some examples

### Table 3
### Compare Problems

| Problem title | Sample problem | Characteristics |
|---|---|---|
| Compare 1 | Joellen has 3 pairs of sandals. She has 4 times as many pairs of shoes. How many pairs of shoes does she have? | Referent set and many-to-one correspondence known; question about the comparison set. |
| Compare 2 | Irene has 30 pennies. She has 5 times as many pennies as Pat has. How many pennies does Pat have? | Comparison set and many-to-one correspondence known; question about the referent set. |
| Compare 3 | Donald has 5 marbles. Peter has 15 marbles. Peter has how many times as many marbles as Donald? | Comparison set and referent set given; question about kind of (many-to-one) correspondence. |
| Compare 4 | Bonnie has 16 white blouses and 4 colored blouses. Her colored blouses are what (fractional) part of her white blouses? | Comparison set and referent set given; question about kind of (one-to-many) correspondence. |
| Compare 5 | Our class has 16 boys and 12 girls. There are how many times as many boys as girls? | Comparison set and referent set given; question about the (many-to-many) correspondence. |
| Compare 6 | Our class has 16 boys and 12 girls. The girls are what (fractional) part of the boys? | Comparison set and referent set given; question about many-to-many correspondence. |
| Compare 7 | Fred has 25 baseball cards. He has 5/4 as many cards as Jim has. How many baseball cards does Jim have? | Comparison set and many-to-many correspondence given; question about referent set. |
| Compare 8 | Erica has 25 stickers. Peggy has 4/5 as many stickers as Erica. How many stickers does Peggy have? | Referent set and many-to-many correspondence given; question about comparison set. |

### Table 4
### Selection Problems

| Problem title | Sample problem | Characteristics |
|---|---|---|
| Selection 1 | Paula has 3 kinds of cheese and 2 kinds of sausage. How many different cheese-and-sausage pizzas can she make? | Number given from which to select for each pair element; question about number of pairs possible. |
| Selection 2 | Frank makes 18 different cheese-and-sausage pizzas. He has 6 kinds of cheese. How many kinds of sausage does he have? | Number in one choice set and number of pairs given; question about number in other choice set. |
| Selection 3: extended *n*-tuple | Dave has 3 different-sized sets of wheels, 4 kinds of bodies, and 3 different motors. How many different cars with wheels, a body, and a motor can he put together? | Number given from which to choose for each portion in *n*-tuple; question about number of *n*-tuples possible. |
| Selection 4: extended *n*-tuple | Dave has 3 different-sized sets of wheels and 4 kinds of bodies; he can make 96 different cars with wheels, bodies, and motors. How many different kinds of motors does he have? | Number given from which to choose for all but one position in *n*-tuple and also number of *n*-tuples; question about remaining position. |

### Table 5
### Rate Problems

| Problem title | Sample problem | Characteristics |
|---|---|---|
| Rate 1 | Lisa buys 18 cans of polish at $0.72 per can. What is the total cost? | Given the rate and the independent variable value; question is about the dependent variable. |
| Rate 2 | Peter buys a suit on sale. The price, after a 25% discount, is $90. What was the original price? | Given the rate and the dependent variable value; question is about the independent variable. |
| Rate 3 | Corrine runs 200 meters in 72 seconds. What is her average speed in meters per second? | Given the values of the dependent and independent variables; question is about the rate. |

are the following:

- Given 21 beans, put 3 beans in cups until the beans are gone. How many cups did you use?
- Given 35 beans, put an equal number of beans into each of 5 cups. How many beans are in each cup?
- Given 4 cups, put 5 beans in each cup. How many beans were needed?

The *ratio comparison* situations can be introduced with two different shapes, two different colors of chips or cubes, or any other materials that can be put into sets and compared using the multiplication- and division-related questions in the examples.

The *selection* ideas can be introduced best with colored cubes or several geometric shapes in different colors, forming pairs and triples of these materials. Subsequently using situations that involve items from the students' experience, such as stickers, pizza toppings, clothing, and record labels, can help children apply these basic ideas of multiplication to the real world.

*Rate* problems should probably be introduced after establishing the idea of a constant rate of change in two related variables. This introduction must be done slowly and carefully and timed to the stage of cognitive development of the students. The demands are primarily on the proportional-reasoning capability of the students.

Introducing problems involving such relationships as *distance = time × rate*, *cost = cost/unit × units*, and *percentage = percent × base* should be within the more general context of rate of change. Otherwise students may substitute values into formulas without understanding the processes involved.

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