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ABSTRACT

The advent of calculators for graphing and function plotters is changing the way college algebra and calculus are taught. This paper illustrates how the machines are used for teaching the following: (1) domain and range; (2) product and quotient inequalities; and (3) the solving of equations. Instructional hints are provided for each topic with some equations and graphs. (YP)

A GRAPHICAL VIEW of DOMAIN AND RANGE INEQUALITIES and EQUATIONS

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A Graphical View of Domain and Range, Inequalities, and Equations

Function plotters are changing the way college algebra and calculus are taught. Very early in an algebra course the student is introduced to the rectangular coordinate system and how to graph a function. Even in a college algebra class many students don't know that a graph of a function is the plot of the pairs of numbers $(x,f(x))$ or (x,y) for all x in the domain of the function; so, the student needs to go through the steps of pencil and paper plotting of points for a variety of functions. This would include linear, quadratic, absolute value, radical, and rational functions. Having done this, he must know that a computer simply calculates many pairs of numbers from the rule that describes the function and then plots these pairs of numbers just as the beginning algebra student would do with 10 to 15 pairs of numbers and a piece of graph paper.

Assume that the algebra student understands that a graph of a function is nothing more that a plot of all ordered pairs of numbers that defines a function. If the instructor has access to a graphing calculator or function plotter along with an appropriate projection system for classroom use, learning can be enhanced by a visual image of a problem situation. Seeing a 'picture' of the function helps the student visualize the set of *all* x's that makes up the domain of a function or the set of all y's that makes up the range of the function. Seeing values for x that cause the graph of $(x - 2)(x + 3)(x \cdot 7)$ to be below the x-axis is an easy way to solve the inequality

$$(x - 2)(x + 3)(x - 7) < 0.$$

The student can now solve equations like

$$x^2 + 3x - 5 = 0$$

$$|x^2 - 5x + 2| - 3 = 0,$$

$$\sqrt{x - 3} = 4$$

$$\frac{x - 4}{(x - 2)(x + 3)} = 3$$

with ease and understanding.

DOMAIN and RANGE

Define the domain as the set of all ***x-coordinates*** on the complete graph of the function. Likewise, the range as the set of all y-coordinates of every point on the complete graph of the function. Graph any function. If the student thinks that the graph of a function stops with the edges of the viewing rectangle - thus implying that the values in the domain and range stop - show the student a bigger view. For example, in figure one, the three views of the function $y = x^2 - 2$ clearly show that the graph doesn't stop when $y = 10$.

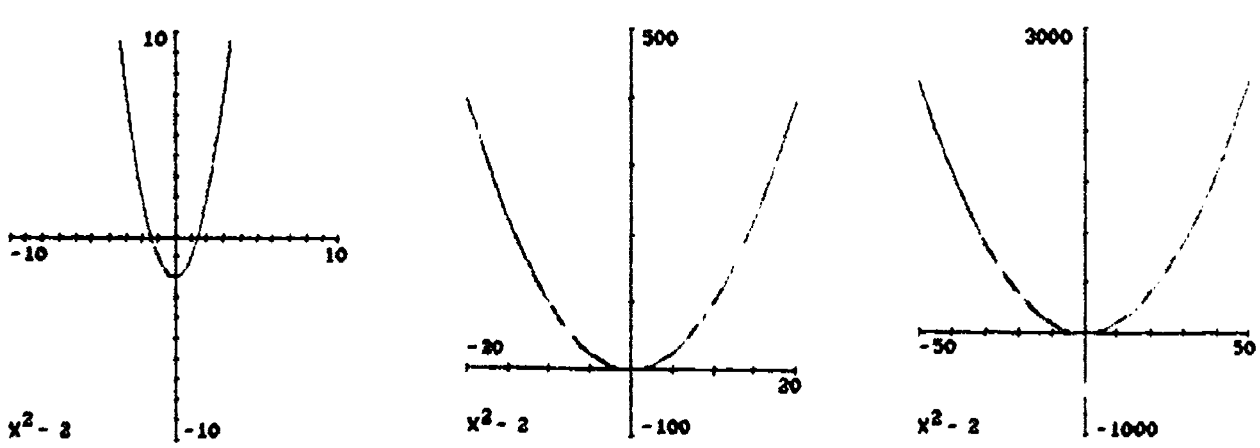

Figure 1.

To calculate the domain of a function like

$$y = \frac{2}{x^2 - 9}$$

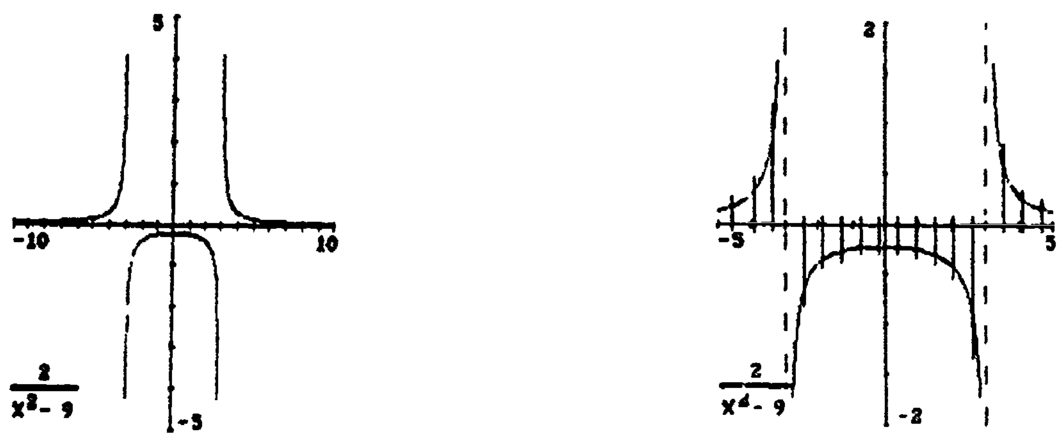

Figure 2. {...-5...-3.9...~~-3~~...-2...0...2.1...~~3~~...3.5...4...}

draw a series of vertical lines(see figure two) starting on the left and proceeding right. When the line crosses the x-axis and the graph simultaneously, that x-coordinate is an element of the domain. As the vertical line moves to the right, the student sees that the domain does not include -3 or 3 because the line doesn't cross the graph for those values of x. If all the values for x are listed in set notation, the student will see a pattern developing that describes the domain of the function.

To find the range of this function, draw a series of horizontal lines starting at the bottom and proceeding upward(see figure three). Again, there are no y-coordinates on the graph in (-.22,0] but every where else. Thus the range must be all real numbers except (-0.22,0]. If the student wants more accuracy on the range, zoom in on the region near -0.22. At the algebra level, it is not crucial that the student know how to prove that (0,-2/9) is the local maximum. But with the function plotter, this value can be found with ease.

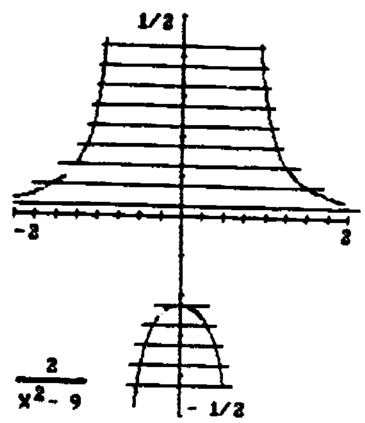

Figure 3. {...-1/2...-3/10...-.22 ¦????¦ 0...(.2)...(.4)...}

PRODUCT and QUOTIENT INEQUALITIES

The graph of $y = (x - 3)(x + 2)(x - 1)$ in figure four is the set of all points where the y-coordinate equals the *product* of the *x-coordinate minus 3, the x-coordinate plus 2 and the x-coordinate minus 1.* This means that every where else the y-coordinate ***does not*** equal the product. In fact, above the graph of $y = (x - 3)(x + 2)(x - 1)$, the y-coordinate is ***larger than*** the product. Below the graph of $y = (x - 3)(x + 2)(x - 1)$, the y-coordinate is ***less than*** the product.

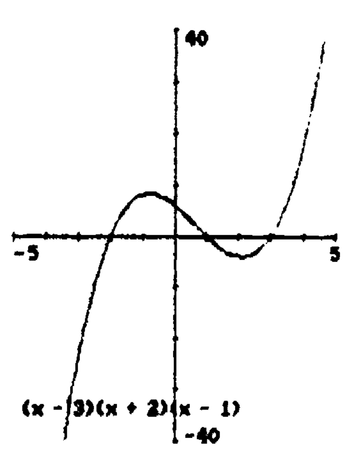

Figure 4.

Therefore, to solve the inequality, $(x - 3)(x + 2)(x - 1) < 0$, graph the corresponding function (in figure five)

$$y = (x - 3)(x + 2)(x - 1) \text{ and } y = 0$$

and find values for x where the graph - y-coordinate or

$(x - 3)(x + 2)(x - 1)$ - is below the x-axis ($y = 0$).

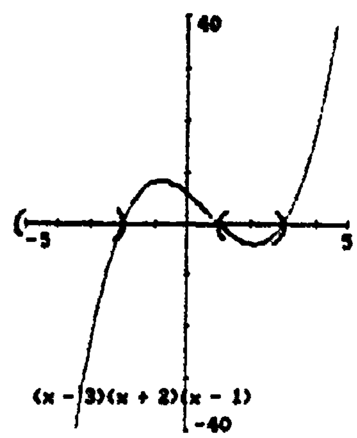

Figure 5.

In this example, the values for x where the graph changes from greater than zero to less than zero or conversely are relatively easy to find, -2, 1, and 3; thus, the solution is $(-\infty, -2) \cup (1,3)$.

To solve the problem $x^2 - 4x + 2 > 0$, graph the corresponding function(figure six.) $x^2 - 4x + 2$ and zoom in on the x-intercepts with the function plotter, or use the quadratic formula to calculate exact values for the x-intercepts. The solution to the inequality will be those values of x where the graph is above $y = 0$ (the x-axis), or $(-\infty, .59) \cup (3.41, \infty)$. If desirable, this approach to solving product inequalities can then be used to develop the cut-point algebraic method for solving the inequality.

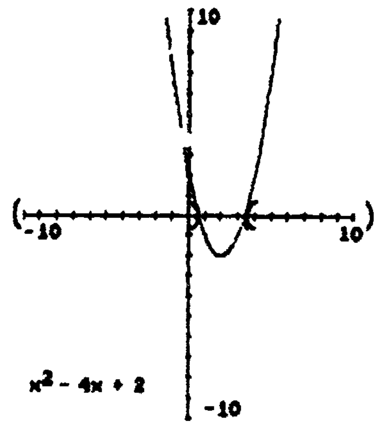

Figure 6.

The solution to rational inequalities can be done in the same fashion. To solve

$$\frac{x - 4}{(x - 3)(x + 2)} > 0$$

graph(figure seven)

$$y = \frac{x - 4}{(x - 3)(x + 2)}$$

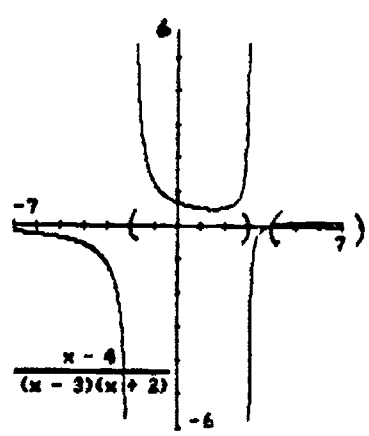

Figure 7.

and find values of x where the graph is above the x-axis - using the zoom feature of the calculator or software. Depending on the instructor's preference, the x-intercept and asymptotes could also be found easily by algebraic methods. However, from the graph, it is clear that the function is above the x-axis when $x \in (-2,3) \cup (4,\infty)$.

SOLVING EQUATIONS

Using a function plotter, the student has one more tool to use when solving equations. For example, the student may know how to solve the quadratic equation: $x^2 + 3x - 5 = 0$ by the completing-the-square method or the quadratic formula. In addition to these methods, if the graph is drawn with the aid of a calculator or computer, the student can see that the graph is zero at approximately 1.2 and -4.2(See figure eight.) With zoom-in ability, the solutions can be obtained to several place accuracy.

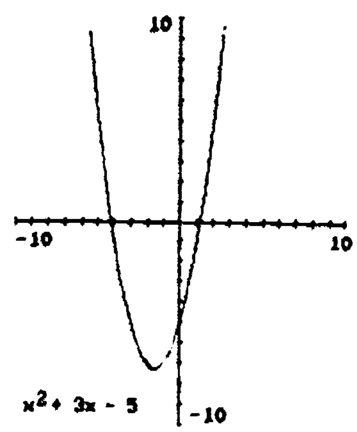

Figure 8.

How many traditionally taught college algebra students are asked to solve:

$$|x^2 - 5x + 2| - 3 = 0 \; ?$$

Maybe a few, but if the graphical method is known, the graphing is made easy by the computer. Correct interpretation of the graph by the student is likely because graphing software or graphing calculators are used on a daily basis in teaching college algebra using a graphical approach. In figure nine, can't most students see that the function is zero at -.2, 1.4, 3.6, and 5.2?

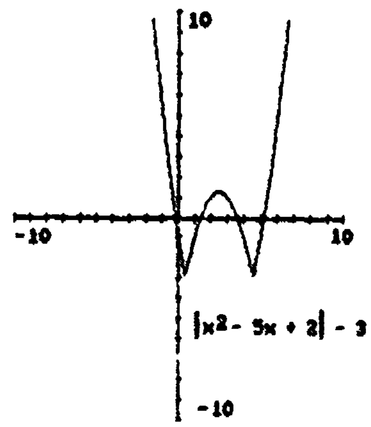

Figure 9.

Solve:

$$\frac{x - 4}{(x - 2)(x + 3)} = 3$$

Subtract 3 from both sides and graph the function(in figure ten):

$$\frac{x - 4}{(x - 2)(x + 3)} - 3$$

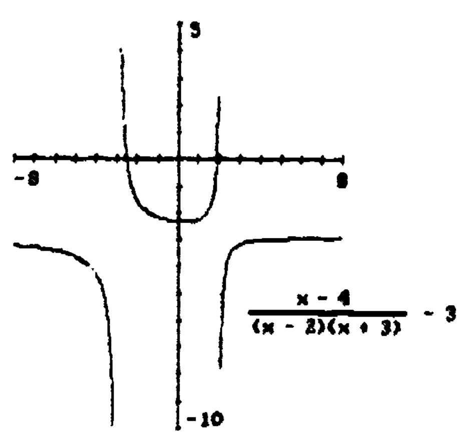

Figure 10.

While this equation can be solved algebraically, most of the students in a college algebra class will probably not remember how to simplify the difference of the fractions. They may miss the problem because of lack of skills in subtraction of fractions. How important is this when the topic is to solve the equation containing fractions? Again, knowing the behavior of functions tells the student the graph is correct and the solution to the corresponding equation is where the function crosses the x-axis: 1.9 and -2.5.

Students can solve equations by the graphical method when the equations are impossible to solve with algebraic methods or when the basic algebraic techniques have been forgotten or never learned.