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ABSTRACT

Discussed are the results of the 1988 Indiana State High School Mathematics Contest, which attracted 2,523 students. Four multiple-choice tests (each with a 90-minute time limit) based on proficiencies listed in the "Mathematics Proficiency Guide" and published by the Indiana Department of Education were used. The tests were: (1) Algebra (first course) containing 48 items; (2) Geometry, 44 items; (3) Algebra (second course), 40 items; and (4) Comprehensive, with 28 items. The paper analyzes student performance on selected items of each test, based on a 20 percent proportional stratified random sample. The analysis focused, in each test, on what the student knew, what the student did not know, and what items made a difference in student performance. Complete copies of all four tests are included in the appendix. (YP)

THE 1988 INDIANA STATE MATHEMATICS CONTEST: TESTS AND ANALYSIS OF STUDENT PERFORMANCE

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INTRODUCTION

The 1988 Indiana State High School Mathematics Contest was held on April 30 at fourteen sites across the state. Funding for the contest was obtained from the Indiana Council of Teachers of Mathematics and from student registration fees. The contest attracted 2523 students.

Major goals of the contest remain the same:

1) To stimulate interest in the study of mathematics;
2) To recognize outstanding mathematics students;
3) To foster communication among mathematics students;
4) To facilitate communication within the mathematics education community in Indiana;
5) To promote appreciation of mathematical excellence; and
6) To recognize outstanding mathematical achievement.

The Contest Committee, composed of interested college and university mathematics educators and the state mathematics consultant, has essentially remained intact since revival of the contest in 1983. All fourteen test sites have continued; however, new site coordinators have been appointed at some locations. Ball State University continues to serve as the coordinating agency for printing, test distribution, test analysis, and financial disbursements.

TEST CONSTRUCTION

Test writing responsibilites were delegated by the Contest Chairman to mathematics and mathematics education faculties at four state universities: Purdue University - North Central, Algebra (First Course); Purdue University - Calumet, Algebra (Second Course); University of Southern Indiana, Geometry; and Indiana University - Kokomo, Comprehensive. The mathematical content of each test was based on proficiencies listed in the Mathematics Proficiency Guide, published by the Indiana Department of Education. Items were formulated by the test writers, with assistance from classroom teachers throughout the state. Using standard test construction procedures, the four multiple-choice (90 minute time limit) tests were finalized with the following number of items:

Algebra (First Course)	48 items
Geometry	44 items
Algebra (Second Course)	40 items
Comprehensive	28 items

TEST ADMINISTRATION

Fourteen sites across the state served as test centers, with a total of 2523 students participating. Each site coordinator was responsible for administering and grading tests, presenting certificates to all participants and "Scholar" awards to those students who correctly answered 75% of the test questions, and arranging any local program for students.

All test results were sent to Ball State University for analysis and to determine students scoring in the top five percent on each test. These students received a special certificate designating them as "Outstanding Scholars" at a special awards ceremony in the State House. Award cut-off scores for Algebra (First Course), Geometry, Algebra (Second Course), and Comprehensive Tests were, respectively, 31, 37, 34, and 19. Top scores for the tests were, respectivley, 41, 43, 40, and 26. Table 1 provides information on student participation at each site for each test.

Table 1
1988 Indiana State Mathematics Contest
Student Participation

Site	Test: Algebra(1st)	Geometry	Algebra(2nd)	Comprehensive
Ball State	80	50	25	46
Butler	126	82	73	109
Franklin	105	50	54	56
I.S.U.	64	26	42	22
U.S.I.	55	54	48	51
I.U. Bloomington	24	12	5	1
I.U. Richmond	26	35	33	20
I.U. Kokomo	47	45	16	16
I.U. Gary	55	48	15	14
I.U.P.U. Ft. Wayne	80	41	42	40
I.U. New Albany	38	29	40	34
Purdue	44	31	34	61
Purdue Westville	76	58	34	35
Saint Mary's	66	55	35	23
Totals	886	616	496	528

TEST ANALYSIS

The remainder of this article presents an analysis of student performance on selected items of each test, based on a 20 percent proportional stratified random sample. Sample sizes for each test at each site are presented in Table 2. Complete copies of all four tests are included in the Appendix.

Table 2
Sample Sizes by Test Site and Test

Site	Test: Algebra(1st)	Geometry	Algebra(2nd)	Comprehensive
Ball State	16	10	5	9
Butler	25	16	15	22
Franklin	21	10	11	11
I.S.U.	13	5	8	4
U.S.I.	11	11	10	10
I.U. Bloomington	5	2	1	0
I.U. Richmond	5	7	7	4
I.U. Kokomo	9	9	3	3
I.U. Gary	11	10	3	3
I.U.P.U. Ft. Wayne	16	8	8	8
I.U. New Albany	8	6	8	7
Purdue	9	6	7	12
Purdue Westville	15	12	7	7
Saint Mary's	13	11	7	5
Totals	177	123	100	105

Since 1984, the analysis has focused on the following questions:

1. What do students know?
2. What don't students know?
3. What items make a difference in student performance?

Operationally, answers to each question were found, respectively, by:

1. An item difficulty of .70 or higher.
2. An item difficulty of .25 or less.
3. A discrimination of .59 or higher.

ALGEBRA (FIRST COURSE)

The Algebra (First Course) Test consisted of 44 items, with a mean of 19.67 and a standard deviation of 7.10 for the sample of 177 students. The reliab.lity of the test, using the KR-20 statistic, was .84. Scores for the sample ranged fromo 5 to 41. Outstanding Scholar Awards were presented to 53 of the 883 participants. The sample mean is unusually low compared to previous tests with a comparable number of items. Comments from teachers during past years indicate that many of them "require" Algebra I students to take the test. Since the population would then contain students with a wider range of abilities, the sample mean would be affected.

KNOWN CONTENT. Analysis of the sample data produced 6 questions with item difficulties of .70 or greater, providing an indication of what students know. Four of these items (Items 1, 6, 8, 12) were in the Comprehension level of Bloom's Taxonomy of Cognitive Objectives, while Items 5 and 12 were in the Application level. The content for these items dealt with combining like terms, evaluating an expression, solving an equation involving consecutive integers, solving a proportion, solving an equation, and solving an equation given in word form, all common topics in the Algebra (First Course) curriculum.

UNKNOWN CONTENT. Five items (Items 3, 21, 27, 32, and 38) had item difficulties less than .25, indicating what students do not know. Item 3, at the Comprehension level, involved determining the value of 1 raised to a positive power and 1 raised to a negative power, then adding 1 plus 1 to get 2. From the numerical alternatives, 23% of the sample selected (d), the result obtained by subtracting exponents. About 47% of the sample selected alternative (e): Cannot be determined.

Item 21, at the Application level, involved finding the sum of two roots of a cubic equation, given one root. Only 21% of the sample answered correctly, while 31% selected alternative (b), the opposite value. Apparently students could divide the cubic by a linear term using long division or synthetic division to obtain a quadratic. However, they failed to recall that the obtained numerical coefficient on the x-term was the negative sum of the roots.

Focusing on the solution of an equation involving simple rational expressions and the use of a proportion or reciprocal, Item 27 was answered correctly by only 25% of the students. About 49% of the students selected alternative (e): None of these, while 13% selected alternative (a), the reciprocal of the correct answer. It is not clear why students selected (e).

Item 32, answered correctly by 20% of the sample, dealt with a typical Algebra I "work" problem." Past analyses of Algebra (First Course) tests have shown that students do well on this type of problem. The National Council of Teachers of Mathematics, in its Standards for Curriculum and Evaluation in School Mathematics, has recommended decreased emphasis on work problems. Since students performed adequately on Item 18, a "work" problem involving only two persons, perhaps the inclusion of three persons in the problem created the difficulty.

Teachers have indicated that multistep word problems cause frustration among students. Item 38 appears to fit that category. At the Application level, it involves ratios and proportions, but with three variables: men, trees, and days. Only 24% of the sample correctly answered. Alternatives (a), (b), and (d) were selected, respectively, by 15%, 12%, and 14% of the sample, whereas alternative (e): None of these was selected by 31%.

DISCRIMINATORS. There were 6 items which most strongly discriminated the highest scoring students from the lowest scoring students. Five of these items fell in the Application level (Items 15, 16, 40, 41, and 43), while Item 2 was at the Comprehension level.

Item 2 involves computation of four complex fractions, and then ordering them to find the greatest value. Over 15% of the sample chose alternative (e), a value less than 1. Past analyses have also shown students to have difficulty with complex fractions.

Only about 42% of the sample correctly answered Item 15, a multistep word problem. Incorrect responses were divided among the four alternative answers, indicating, perhaps, that many students guessed if they could not solve the problem.

Item 16 again points to a weak area for students. The item involves a simple equation involving rational expressions. Although 61% of the sample correctly responded, almost 20% selected alternative (b), the result obtained by failing to multiply both sides of the equation by the least common denominator. In whatever form they appear on the tests, items containing rational expressions apparently cause difficulties.

Item 39 merely requires students to write a series of algebraic expressions, ending with a simple linear equation of the form $ax + b = c$ to solve. Slightly over 50% of the sample correctly responded. Using trial and error would produce the correct response quickly.

Common sense leads to the correct answer for Item 41. The dog gains two feet on the rabbit for every jump they make; therefore, in 75 jumps the dog will make up the 150 feet head start of the rabbit. In equation form, we have $7x + 150 = 9x$, where the two distances for the dog and rabbit are equal after x leaps. Lack of problem solving ability appears to be the major factor in this item.

Item 43 is a "story" problem requiring students to solve a linear inequality in one variable. With the solution of the inequality resulting in a mixed numeral, students who answered incorrectly may not have known what whole number response to select from the alternatives since the question asked for the "largest possible number..."

ALGEBRA (FIRST COURSE) SUMMARY. Students in the sample mastered basic notions in algebra. The implications for instruction once again include the following:

1. Students need more help in learning to solve equations involving rational expressions.
2. Students need to have a better global view of the nature of the roots of polynomials, in particular, for quadratic polynomials.

ALGEBRA (SECOND COURSE)

On the 40-item Algebra (Second Course) Test, the range of scores for the random sample of 100 students was from 4 to 40. The sample mean was 21.81, with a standard deviation of 7.90. The reliability of the test, using the KR-20 statistic, was .88. Outstanding Scholar Awards were presented to 30 students.

KNOWN CONTENT. An item analysis identified 4 questions having item difficulties of .70 or higher. Using Bloom's Taxonomy of Cognitive Objectives, all items (Items 1, 27, 28, and 38) were found to be at the Comprehension level. Item 21 had an item difficulty of .77, but was statistically unacceptable since the discrimination index was 0.08.

Item 1 asks for the product of a binomial containing an imaginary number. Item 27 is a "word" problem requiring students to systematically work with numerical quantities; no algebra is required. Item 28 requires simple computation of a log, while the final item, Item 38, requires students to find the slope of a line given an equation of the form $ax + by = c$.

UNKNOWN CONTENT. Questions with item difficulties less than 0.25 were examined to determine what students did not know. Only Item 30, with an item difficulty of 0.13, fell in this category. At the Application level, the item involved solving a logarithmic equation using basic properties of logarithms and the definition of logarithm. This content area has been identified on previous analyses as an area of concern. In many cases, students have not covered the material by test time in April.

DISCRIMINATORS. There were eight test items which most strongly discriminated between the highest scoring and lowest scoring students in the sample. Items 2, 19, 31, and 39 were at the Comprehension level, while Items 22, 24, 26, and 34 were simple Application level problems.

Item 2 required double factoring of a simple 4-term expression in order to find a binomial factor, content typically found in the Algebra (First Course) curriculum. In Item 19, students were asked to identify the graph of $x = |y - 1|$. Although 57% of the sample correctly responded, the remainder of the sample evenly divided their responses among the other alternatives. Surprisingly, 11% chose alternative (c), the graph of $y = x + 1$. Item 31 also requires students to identify the graph of a function, in this case an exponential function $y = (1/3)^x$. As in the past, it appears that better students have a better global view of graphing and have probably studied exponential and logarithmic functions.

Item 22 involves solving an equation involving rational expressions, with a variable in the denominator. The solution is straight forward. Item 24 gives the formula for the total surface area of a rectangular prism and asks students to solve for w, the width of the prism. This is typically an Algebra (First Course) topic. Fifteen percent of the sample chose alternative (d), the result obtained by correctly factoring w from two terms, but ignoring the "+" between the terms.

Solving the system of quadratic equations for x^2 in Item 26 immediately gives $x^2 = -1$, which leads to the correct answer of no real roots. Apparently, many students ignored or failed to obtain a "negative" and, therefore, chose alternative (b): $x = \pm 1/3$.

Item 34 was a three-step problem: 1) Find the slope of a given line; 2) Find the slope of a line perpendicular to the given line; and 3) Select an equation of a line perpendicular to the given line. Based on analysis of incorrect responses, students evidently failed to negate the reciprocal of the given line's slope.

ALGEBRA (SECOND COURSE) SUMMARY. Concerns expressed on past analyses continue to arise on the 1988 test. Implications for instruction include the following:

1. Students need a better global view of the graphs of functions. Certain classes of functions behave in the same way, and minor changes in the function create only slight variations in the graph.
2. The NCTM Standards suggest more work with exponential and logarithmic functions. In the case of the State Mathematics Contest, this would require earlier teaching of the notions so that students are more familiar with the ideas.

GEOMETRY

The Geometry Test for 1988 contained 44 items. The 95th percentile score for the entire group of students that took the test was 31, making the cut point for Outstanding Scholar somewhat lower than the arbitrary standard of 75% correct for a Scholar. This indicates that the test as a whole was more difficult for these students than might be desired. The observed range for the sample was 6 through 41 correct. There was a mean of 23.91 and standard deviation of 7.58 for the sample of 123 students.

KNOWN CONTENT. Six items meet the criteria of 75% or more of the students responding correctly. In order of ease they are Items 15, 33, 8, 25, 13, and 27. Items 20 and 28 were very close to the 75% criteria and are discussed here as well.

These easy items, or well known content, fall into four general areas. Items 8 and 13 relate to relationships among lines in a plane. This includes parallel lines, alternate angles, and transversals. Item 28 involves basic information about parallelograms, Item 15 relates to the definition of similar triangles, and Items 20, 27 and 33 all relate to basic theorems about congruent triangles. Item 33 requires that students recognize basic congruency theorems for triangles.

As might be expected, all of these items are reasonably classified at the comprehension level or lower on Bloom's taxonomy. It seems clear then, that basic relationships involving how lines behave in a plane, and what it means for triangles to be similar or congruent were well understood by these students.

UNKNOWN CONTENT. The criteria of 25% or less of the sample responding corectly, identifies five items that were not known. In order of difficulty they were Items 44, 37, 32, 17, and 22. The next most difficult item was Item 39 to which 28% of the sample responded correctly. Table G4 presents the item statistics for these items.

Three of these items, Items 17, 39 and 44, involved three dimensional figures. Item 17 involved no more than comprehension of what it means for two planes to intersect in three-space, while Items 39 and 44 involved coordinating volume relationships among two figures. These last two items would be classified as comprehension, or perhaps as high as analysis on Bloom's taxonomy. Formuli for volume were involved in both items, and perhaps the failure to recall, or re-construct volume formuli played a part in the difficulty.

Items 22, 32 and 37 all involved triangles in a plane, and all are classified at the Application or Analysis levels. While several of the "easy items" dicussed earlier also involved planar triangles, these difficult items required more than just recognition and comprehension. The most popular wrong answer for Item 22 shows that the relationship of SSA was widely accepted by students as sufficient for similarity. The wrong responses for Items 32 and 37 suggest that most students did not identify the crucial relationships and just estimated things on the basis of drawings. While they were plausable depictions of the relationships, the drawings were off just enough to deceive the careless.

Item 17 is interesting as it only requires the recognition of how two planes may intersect in space. This item can be classified no higher than comprehension. It may be that students took the language of the item to mean that the planes did in fact intersect in the usual sense of the word, and so ruled out the possibility of parallel planes. The most popular wrong answer was simply "a line". The very low discrimination factor for this item lends further support to the idea that the wording of the item was misleading.

Item 44 was the most difficult item of the test, but it had a very respectable discrimination factor. The item requires only a routine application of the knowledge of the formuli for volumes of cylinders and pyramids. The simplest interpretation is that students who did well on the test as a whole had a firmer grasp of geometric relationships in space than did other students. How much of this represents "opportunity to learn" and how much spatial perception is hard to know. As the most discriminating items are considered below, it is clear that 3-space insight characterizes the outstanding students.

DISCRIMINATORS. The items that most strongly discriminated among the participants were Items 41, 7, 16, 14, 11, 6, 5, 3, and 38. Items 15, 33 and 43 will be commented upon later.

With three exceptions, these items are at the Application or higher level. Three general content area concerns are represented by these items:

1) Ratio is an important construct in 4 of the 9 items,
2) Algebraic relationships involving supplementary angles are featured in two of them, and
3) Three of them involve solid geometric concepts.

A somewhat surprising item is Item 3. It simply requires the knowledge that pi is, by definition, the ratio of the measure of the circumference of a circle to the measure of its diameter and that it is not a rational number. Not only did most of the students fail to identify this fact (54% of them), but there were almost as many who were willing to suppose that 22/7 and 3.14 were equivalent (39%) as there were who knew what pi is (46%). It may be we still have many people who could be led to suppose, along with Dr. Goodwin, that in Indiana pi is still 3.2 or 4 (see Hallerberg, 1975).

NONDISCRIMINATORS. Items that fail to discriminate are often interesting. Items 15, 33, and 43 contributed little if anything to the discrimination. For two of them, the reason is obvious, Items 15 and 33 were so easy that they could not discriminate among subjects. Item 43, however is interesting. Aproximately half of the students responded correctly to it, but the item still failed to provide much discrimination.

Item 43 asks how many planes of symmetry a cube has. A look at the choices for wrong responses indicates preferences for 5 and 3 planes. Perhaps those who took 7 as a serious possibility, continued their thinking to see that more than 7 are present. For whatever reasons, this behavior was no more common for those who did poorly than for those who did well. The spatial visualization involved leads to speculation that the item may be measuring something quite different from just Geometric achievement.

A few generalizations are in order. Ratio and its applications seem to be important for identifying the better students. The applications of algebra to geometric ideas characterizes better students. The understanding of circles is important. And, an understanding of solid geometric relationships characterizes the better students.

GEOMETRY SUMMARY. Students in the sample had mastery of basic ideas about lines, angles and triangles in the plane. When we move to higher cognitive levels or to three dimensions and involve spatial visualization the tasks become sharply more difficult. Students who have somehow obtained the ability to visualize these relationships are at a distinct advantage relative to their peers. Students who could relate algebraic concepts to geometric ideas did distinctly better. Finally, students who could deal well with ratio ideas likewise scored higher. Implications for instruction would include the following:

1. There is a need to help students firmly relate their geometric learning to physical objects that can be seen, handled and visualized.
2. Students need to be helped to develop relationships among the skills and techniques of algebra and geometry.
3. Simple formuli and relationships (including things as "old" in the curriculum as C = pi x d) must be more firmly attached to physical realities and experience. The ratio idea in general is difficult for students to

grasp and apply.

4. Problem solving that involves the higher cognitive levels should always be a center piece of instruction.

COMPREHENSIVE

The Comprehensive Test for 1988 contained 28 items. The 95th percentile rank for all students who took the test was at 19 of 28 correct. This was 2 items below the 75% of the test correct required for the Scholar rating. The observed range for the sample of 105 was 1 through 25 correct, with a mean of 11.09 and standard deviation of 4.77.

KNOWN CONTENT. This test was written at a relatively high cognitive level. There are no items that can be solved by knowledge alone. Most items require at least application, and many require analysis or synthesis. Not surprisingly, only one item, Item 11, met the criteria of 75% or more of the students responding correctly. Since the overall test was quite difficult for this population, the next two items in order of difficulty are included to provide some insight into areas of relative student strength. Items 1 and 14 had difficulty indices of 73 and 72 respectively.

Item # 11 is an easy comprehension item relating to factorials. Students knew what is meant by factorial notation. Item #1 was at the application level, and required the knowledge and application of slope relationships of lines when lines are stated as linear equations. Students needed to know how to determine the slope of a line from its equation, and they had to know how to determine which of the three equations represented the hypotenuse. They may have recognized and applied the theorem that two lines meet at right angles when their slopes are negative reciprocals of each other.

Item #14 requires a comprehension of what is meant by log and exponent notation. Though two or three steps are required to solve the item, no more than the basic defintion needs to be understood.

In sum, we might suppose that students who took this test had a grasp of content that included the definitions of factorials and log functions. They had substantial mastery as well of the analytic geometry of the line.

UNKNOWN CONTENT. In contrast with the Known Content, which contained no more than 3 items, 1/4 of the test items were not solved by at least 25% of the students taking this test. These unknown items are Items 23, 25, 22, 18, 7, 4, and 27.

Two of these items, Items 7 and 22, were at lower cognitive levels and so indicate topics that were not known by these students. Item 7 requires either the application of long division of polynomial functions or synthetic division. Item 22 requires the recognition of the inverse nature and meaning of the log and exponential functions. Item 18, though classified at the analysis level, also involved relationships between the log and exponential functions.

Item 27 invclved the log function as well. However it was a complex analysis item requiring identifying and relating properties of the log function, factoring, the fact that a squared is the same as (-a) squared, and the use of composition of functions. It is really a very nice item, but reasons for its relative difficulty would be most speculative. It probably has more to do with the problem solving required to see the parts and their relationship to one another than it does with just content.

The remaining three items, Items 4, 23, and 25 also are at high cognitive levels. The content is varied. It involves simultaneous circular and rational functions, conics analysis, and applications of the double angle formuli respectively; this makes content alone the unlikely cause of the relatively difficulty. The problems are simply very nice problems placed at levels of difficulty that the students were unfamiliar with. The content may well have been covered, but the reflection upon how that content may be expressed, and upon problem solving strategies involving such content were lacking.

Item 23 involving conic anlaysis was the most difficult item in the test, and its discrimination level was not acceptable. One might reasonably argue that this item represents entrapment, saying more about human nature than content or problem solving. A check of the item statistics show that 52% of the population went for the over-generalization that second degree polynomial equations in both x and y with coefficients of opposite signs are hyperboli. The fact that students fell for the over-generalization requires that they at least recognized that conics do have characteristic polynomial expressions. This suggests again that the content as such was familiar to the students. The difficulty was with problem solving.

DISCRIMINATORS. There are three items, Items 17, 12 and 19, that had discrimination factors greater than .60. Items 26, 20, 16 and 8 were close with discrimination factors of .57. Only Item 23 had a discrimination less than .10. It was also one of the "unknown items" as is discussed above.

The analysis shows that three of these items involved the geometry of the circle and/or circular functions. Two of these were classified at the application leve , and one of them at the comprehension. The last, Item 26, is the easiest to interpret. All that is required is a comprehension of how the coefficients of the function Y = a cos bX + c effect the graph of the function. Items 17 and 20 required an understanding of inscribed angles and their measures. This all suggests that students who did less well had less understanding of the geometry of the circle than others.

The content of Items 12, 16 and 18 is certainly within the range of a second year algebra student. Yet only 1/2 of the students determined the correct answers. The large discrimination factor, or course, means that it was the right half. This discrimination is probably then best accounted for on the basis of an ability to see relationships among ideas rather that upon the content of the ideas themselves. In short, general problem solving ability is being highlighted here.

COMPREHENSIVE SUMMARY. Problem solving itself emerges here as the most important aspect of the test. It identified students who were able to apply their knowledge to novel settings and see relationships among familiar ideas.

The geometry of the line is well known by these students and they were acquainted with factorials and the log function. They did not however, have a firm grasp on the geometry of the circle or upon circular functions. Neither were they proficient with inverse relationships particularly those involving the exponential and log functions.

Implications for instruction include the following:

1. There is evidence that circular, log and exponential functions are not well understood by students. Perhaps more and earlier attention can be given to these functions.
2. It is important that we review and integrate prior learning.
3. Students need many opportunities to develop problem solving orientations to their learning. The question of how what is being learned <u>can</u> be related to what has gone before is most important. This test, and other such tests, can provide nice problem sets for analysis and study.

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APPENDIX

1988

STATE HIGH SCHOOL MATHEMATICS CONTEST

SPONSORED BY THE INDIANA COUNCIL OF TEACHERS OF MATHEMATICS

ALGEBRA TEST
(FIRST COURSE)

THIS TEST WAS PREPARED BY THE MATHEMATICS
FACULTY OF PURDUE UNIVERSITY NORTH CENTRAL

DIRECTIONS FOR TEST:

DO NOT open this booklet until you are told to do so.

This is a test of your competence in high school algebra, first course. For each of the 44 problems there are listed 5 possible answers. You are to work each problem and determine which is the correct answer and indicate your choice by filling in the circle in the correct place on the separate answer sheet provided. A sample follows:

1. If $x + 2 = 6$, then x equals

   A. 8

   B. 3

   C. $\frac{1}{3}$

   D. 4

   E. none of these

	A	B	C	D	E
1.	①	②	③	●	⑤

The correct answer for the sample is "4", which is answer D; therefore, you should answer this question by filling in the circle D as indicated above.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any question. If you are unable to work any particular problem, it is to your advantage to guess at the answer rather than leave it blank. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet and begin work. When you have finished one page, go on to the next page. The working time for the entire test is 90 minutes.

DIRECTIONS FOR ANSWER SHEET:

Fill in your name in the blanks provided. Above your name write the name of your school and the city where it is located, including zip code.

Along the side of your name indicate your sex and grade by filling in the circle provided. A sample follows:

SAMPLE: Mary A. Brown, who goes to Western High School in Muncie and is in the ninth grade, would write across the top and fill in along the side.

WESTERN HIGH SCHOOL - MUNCIE, INDIANA 47306

NAME (Last, First, M.I.)																	
B	R	O	W	N		M	A	R	Y		A						

SEX	GRADE
Ⓜ	●
●	⑩
	⑪
	⑫

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.

ALGEBRA I
1988 SPRING
STATE CONTEST

(1) $-(a - 2) - [2a - (a-3)] =$

a) $2a - 1$ b) $-2a + 1$ c) $2a + 1$ d) $-2a - 1$ e) $-2a + 3$

(2) Which of the following has the greatest value?

a) $\frac{1 - \frac{1}{3}}{3}$ b) $\frac{-3}{-\frac{1}{3}}$ c) $\frac{-3}{\frac{1}{3}}$ d) 0 e) $\frac{3 - \frac{1}{3}}{3}$

(3) Find the value of $\left(1^{5a} + 1^{-2a}\right)$.

a) 0 b) 1 c) 2 d) 3a e) cannot be determined

(4) Which of the following is the simplest form of $\dfrac{x - \frac{1}{y}}{y - \frac{1}{x}}$?

a) $\frac{x}{y}$ b) $\frac{y}{x}$ c) $\frac{x - 1}{y - 1}$ d) $\frac{y - 1}{x - 1}$ e) $\frac{xy - 1}{xy}$

(5) Six consecutive integers are given. The sum of the first three is 51. What is the sum of the last three?

a) 53 b) 54 c) 56 d) 57 e) 60

(6) Find the value of $\dfrac{x}{x - a} + \dfrac{a}{a - x}$ when $x = 50$ and $a = 10$.

a) 0 b) 1 c) 10 d) 5 e) 50

(7) If x is greater than 4, which of the following has the least value?

a) $\frac{4}{x + 1}$ b) $\frac{4}{x - 1}$ c) $\frac{4}{x}$ d) $\frac{x}{4}$ e) $\frac{x + 1}{4}$

(8) For what value of x is $\frac{1}{x} = \frac{1}{x - 2}$ a true statement?

a) 0 b) 2 c) -2 d) 1 e) None of these

(9) A bag contains 28 pounds of sugar which is to be separated into packages containing 14 ounces each. How many such packages can be made?

a) 2 b) 4 c) 8 d) 16 e) 32

(10) Which of the following is the square of $\sqrt{1 + \sqrt{1}}$?

a) 2 b) 1 c) 4 d) 3 e) $1 + \sqrt{2}$

(11) If the length of a rectangle is increased by 30% and the width is decreased by 30%, what percent change occurs in the area?

a) 0% b) increases 6% c) increases 9%
d) decreases 6% e) decreases 9%

(12) If $\frac{p \cdot p \cdot p}{p + p + p} = 3$ and p is positive, then p is

a) $\frac{1}{3}$ b) $\frac{1}{9}$ c) 27 d) 3 e) 9

(13) Three times a number less seven is thirty-two. What is twice the number?

a) 13 b) 17 c) 26 d) 32 e) 39

(14) If p erasers cost c cents, how many erasers can be purchased for D dollars?

a) $\frac{Dp}{c}$ b) $\frac{100c}{Dp}$ c) $\frac{100cp}{D}$ d) $\frac{100cD}{p}$ e) $\frac{100Dp}{c}$

(15) Two-thirds of the faculty of a high school are women. Twelve of the men of the faculty are unmarried while $\frac{3}{5}$ of the male teachers are married. The total number of faculty members in this school is

a) 30 b) 50 c) 60 d) 72 e) 90

16) $\frac{4m + 3}{5} - \frac{m + 4}{2} = 1$ Solve for m.

a) 6 b) 5 c) 10 d) 8 e) 4

(17) Find the remainder when $(x^3 + 3x^2 - 2x - 7)$ is divided by $(x - 2)$.

a) 1 b) -1 c) 9 d) -9 e) -7

(1(Sam can do a job in 1 hour, and Joe can do the same job in 40 minutes. How long will it take them to do the job together?

a) 32 minutes b) 24 minutes c) 16 minutes
d) 22 minutes e) 36 minutes

(19) The numerator of a fraction is 6 less than the denominator. If the numerator is increased by 5 and the denominator is increased by 8, the value of the fraction is unchanged. What is the sum of the numera r and denominator of the fraction?

a) 20 b) 22 c) 24 d) 26 e) 28

(20) A student has an average of 87 for ten tests. How many grades of 100 must he receive to bring his average up to 90?

a) 3 b) 4 c) 5 d) 2 e) 6

(21) If x = 2 is one solution of the equation $x^3 - 4x^2 + x + 6 = 0$ What is the sum of the other two?

a) -3 b) -2 c) -1 d) 2 e) 3

(22) $(x^{1/3}x^{1/6})^{12}x^{-3} =$

a) 1 b) x c) x^2 d) x^3 e) x^4

(23) The difference of two numbers is 12. If one-half their sum is substracted from four-fifths of the larger, the remainder is zero. Find the smaller of the two numbers.

a) 10 b) 12 c) 14 d) 16 e) 18

(24) If (x, y, z) is the solution to the system

$$\begin{aligned} x + y &= 8 \\ y + z &= 4 \\ z + x &= 8 \end{aligned}$$

Then $x + y + z =$ ___.

a) 8 b) 9 c) 10 d) 11 e) None of these

(25) $\frac{1}{y} + \frac{2}{3 + y} =$

a) $\frac{3}{3 + 2y}$ b) $\frac{3y + 3}{+ 27}$ c) $\frac{y + 1}{y + y^2}$ d) $\frac{y + 3}{3y + y^2}$ e) $\frac{3y + 3}{3y + y^2}$

(26) If $\frac{3}{3k + x} = \frac{1}{k + 1}$ and $k \neq -1$, then $x = ?$

a) $\frac{1}{3}$ b) 1 c) 3 d) 3k e) 6k + 3

(27) If $\frac{1}{x} - \frac{1}{a} = \frac{1}{b}$, then $x =$

a) $\frac{a + b}{ab}$ b) $\frac{ab}{a + b}$ c) $\frac{a - b}{a + b}$ d) $\frac{a + b}{a - b}$ e) none of these

(28) If $2^{2x} = 64$, then x is

a) 3 b) 5 c) 4 d) 6 e) 7

(29) Determine the value of k such that (-1, -1) is on the graph of $4kx - (1 - y)k = 3(2k - 1)$

a) $\frac{3}{8}$ b) $\frac{1}{4}$ c) $\frac{1}{3}$ d) $\frac{1}{6}$ e) None of these

(30) If x_1 and x_2 are solutions of the equation

$$10 - \frac{x^2}{6} = 6 - \frac{x}{3}$$

Then the sum of x_1 and x_2 is

a) 2 b) -2 c) -10 d) 10 e) -6

(31) Traveling downstream, a man goes 12 miles in 2 hours. Going upstream, he makes $\frac{2}{3}$ the distance in twice the time. The rate of the current is

a) 6 mph. b) 2 mph. c) 4 mph. d) 0 mph. e) 3 mph

(32) John, Joe and Jeff, working together, can do a job in 4 hours. If John can do the same job in 10 hours, and Joe in 12 hours, how many hours would it take Jeff to do the work?

a) 14 b) 16 c) 12 d) 15 e) 13

(33) On an algebra test, 39 more students passed than failed. On the next test, 7 who had passed the first test failed and one-third of those who failed on the first test passed the second test. As a result, 31 more passed the second test than failed it. What was the number of students who passed the first test?

a) 46 b) 48 c) 38 d) 42 e) 44

(34) What must $(x-y)$ be subtracted from in order to obtain $(x + y)$?

a) $2x + 2y$ b) $2x - 2y$ c) $2x$ d) $2y$ e) $x + 2y$

(35) One factor of $6x^2 - 7xy^2 - 20y^4$ is

a) $3x + 4y$ b) $2x - 5y^2$ c) $6x - y^2$ d) $3x + 5y^2$ e) None of these

(36) An airplane travels $\frac{a}{8}$ feet in r seconds. At this constant rate how many feet does it travel in 4 minutes?

a) 10ar b) $\frac{ar}{2}$ c) $\frac{30r}{a}$ d) $\frac{30a}{r}$ e) $\frac{10a}{r}$

(37)

$$\left[\frac{1}{2+\frac{3}{4+\frac{5}{6+\frac{7}{8}}}}\right]^{-1}$$

a) $\frac{152}{33}$ b) $\frac{137}{52}$ c) $\frac{52}{137}$ d) $\frac{33}{152}$ e) None of these

(38) Leonardo Fibonacci (1170-1250) was one of the most talented mathematicians of the Middle Ages. The following problem was taken from one of his works called the Liber Abaci. "A certain king sent 30 men into his orchard to plant trees. If they could set out 1,000 trees in 9 days, in how many days would 36 men set out 4,400 trees?"

a) 35 b) 31 c) 33 d) 32 e) None of these

(39) The following problem was adapted from Chuquet's Triparty en la science des numbres (1484). Nicolas Chuquet was one of France's finest fifteenth century mathematicians. "A merchant visited three fairs. At the first, he doubled his money and then spent \$30; at the second, he tripled his money and then spent \$54; at the third, he quadrupled his money and then spent \$72 at which time he had \$48 left. How much money did he have at the start?"

a) \$25 b) \$29 c) \$33 d) \$37 e) \$39

(40) The equation $|3x - 2| = 4$ has two solutions. The product of the solutions is ___.

a) $-\frac{4}{3}$ b) $\frac{2}{3}$ c) $\frac{3}{2}$ d) $\frac{3}{4}$ e) $-\frac{3}{2}$

(41) Alcuin of York (775 AD) compiled a collection of brain teasers entitled <u>Problems for Quickening of the Mind</u>. Solve the following problem from the collection. "A dog chasing a rabbit, which has a headstart of 150 feet, jumps 9 feet every time the rabbit jumps 7 feet. In how many leaps does the dog catch up with the rabbit?"

a) 60 b) 75 c) 70 d) 55 e) 79

(42) The following problem was found in a manuscript unearthed in 1881 at Bakhshali, in northwest India. The manuscript may have been written in the third century. "A merchant pays duty on certain goods at three different places. At the first he gives $\frac{1}{3}$ of the goods, at the second $\frac{1}{4}$ of the remainder, and at the third $\frac{1}{5}$ of the remainder. The total duty is 24. What was the original value of the goods?

a) 34 b) 39 c) 36 d) 40 e) 44

(43) Ida said: "I sold 3 more tickets than twice the number of tickets Fred sold." Penelope replied: "I sold 32 tickets, and that is more than you sold." What is the largest possible number of tickets Fred could have sold?

a) 14 b) 15 c) 13 d) 11 e) 12

(44) How many ounces of distilled water should be added to 16 ounces of a 25% solution of antiseptic to prepare a 10% solution of antiseptic?

a) 32 b) 28 c) 26 d) 30 e) 24

1 9 8 8

STATE HIGH SCHOOL MATHEMATICS CONTEST

SPONSORED BY THE INDIANA COUNCIL OF TEACHERS OF MATHEMATICS

A L G E B R A   T E S T

(SECOND COURSE)

THIS TEST WAS PREPARED BY THE MEMBERS OF THE DEPARTMENT OF MATHEMATICAL SCIENCES OF PURDUE UNIVERSITY CALUMET.

DIRECTIONS FOR TEST:

DO NOT open this booklet until you are told to do so.

This is a test of your competence in the second course of high school algebra. For each of the 40 problems there are listed 5 possible answers. You are to work each problem and determine which is the correct answer, and indicate your choice by filling in the circle in the correct place on the separate answer sheet provided. A sample follows:

1. If $x + 2 = 6$, then x equals

   A. 8

   B. 3

   C. $\frac{1}{3}$

   D. 4

   E. none of these

	A	B	C	D	E
1.	①	②	③	●	⑤

The correct answer for the sample is "4", which is answer D; therefore, you should answer this question by filling in the circle D as indicated above.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any question. If you are unable to work any particular problem, it is to your advantage to guess at the answer rather than leave it blank. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet and begin work. When you have finished one page, go on to the next page. The working time for the entire test is 90 minutes.

DIRECTIONS FOR ANSWER SHEET:

Fill in your name in the blanks provided. Above your name write the name of your school and the city where it is located, including zip code.

Along the side of your name indicate your sex and grade by filling in the circle provided. A sample follows:

SAMPLE: Mary A. Brown, who goes to Western High School in Muncie and is in the eleventh grade, would write across the top and fill in along the side.

WESTERN HIGH SCHOOL - MUNCIE, INDIANA 47306

NAME (Last, First, M.I.)																
B	R	O	W	N		M	A	R	Y		A					

SEX
Ⓜ
●

GRADE
⑨
⑩
●
⑫

DO NOT TURN PAGE UNTIL YOU ARE TOLD TO DO SO.

1. Which of the following is the product of $(x + 7i)^2$?

a. $x^2 - 49$ b. $x^2 - 14x - 49$ c. $x^2 + 14ix - 49$

d. $x^2 + 14i^2$ e. $2x + 14i$

2. One of the binomial factors of $3ac - 6bc + ad - 2bd$ is

a. $3a - b$ b. $3c + d$ c. $a + 3c$

d. $a + 2b$ e. $2b - d$

3. Express $\frac{2 + 3i}{8 - 6i}$ in the form $a + bi$.

a. $-\frac{1}{50} + \frac{9}{25}i$ b. $-\frac{1}{5} + \frac{9}{5}i$ c. $\frac{1}{4} - \frac{1}{2}i$

d. $\frac{17}{50} + \frac{18}{50}i$ e. $\frac{4}{25} + \frac{9}{25}i + \frac{9}{50}i^2$

4. Choose the <u>equation</u> needed to solve the given word problem. How many units of 20% acid solution and 45% acid solution must be mixed together to obtain 100 units of a 30% acid solution? Let x represent the number of units of 20% solution.

a. $65x = 3000$ b. $20x + 4500 - 45x = 3000$

c. $-25x = 300$ d. $65x = 300$

e. none of the above

5. Which of the following represents the complete factorization over the integers of the given polynomial? (-2 is one zero)

$$p(x) = x^3 - x^2 - 10x - 8$$

a. $(x - 2)(x - 4)(x + 1)$ b. $(x + 2)(x^2 - 3x + 4)$

c. $(x + 2)(x + 4)(x - 1)$ d. $(x + 2)(x - 4)(x + 1)$

e. $(x + 2)(x^2 - 8)$

6. The remainder when $3x^4 - 2x^3 + 2$ is divided by $x^2 - 1$ is

a. $2x + 5$ b. $-2x + 5$ c. $2x - 1$

d. $x + 2$ e. 3

7. Solve the following equation for x: $x^2 - \sqrt{5}x - 11 = 0$

a. $\frac{\sqrt{5} \pm 7}{2}$ b. $\frac{7 \pm \sqrt{5}}{2}$

c. $\frac{-5 \pm i\sqrt{39}}{2}$ d. $\frac{\sqrt{5} \pm \sqrt{69}}{2}$

e. none of the above

8. If $f(x) = 2x^2 - x + 5$, what is $f(x + h) - f(x)$

a. $2x^2 + 4xh + 2h^2 - x - h + 5$

b. $2x^2 + 2xh + 2h^2 - x + h + 5$

c. $2h^2 - h + 5$

d. $2h^2 + 4xh + h$

e. $4xh + 2h^2 - h$

9. Simplify: $\dfrac{\frac{1}{x} + \frac{1}{y}}{\frac{2x^2 + 3xy + y^2}{x^2y^2}}$.

a. $\frac{(x + y)^2(2x + y)}{x^3y^3}$ b. $\frac{x^2y^3 + x^3y^2}{(x + y)(2x + y)}$

c. $\frac{2x^2y^2}{(x + y)^2(2x + y)}$ d. $\frac{xy}{2x + y}$

e. none of the above

10. $\dfrac{\sqrt{x} + \frac{6}{\sqrt{x}}}{\sqrt{x}}$ can be written as:

a. $\frac{x + 6}{x}$ b. $1 + \frac{6\sqrt{x}}{x}$

c. $\frac{\sqrt{x} + 6}{\sqrt{x}}$ d. $x + 6$ e. 7

11. The dimensions of a rectangle are measured in meters. The length is one less than twice the width. The diagonal is one more than twice the width. What is the length of the rectangle?

a. 4 m or 8 m b. 17 m

c. 15 m d. 8 m

e. 24 m

12. Perform the indicated operations on $\frac{3}{y - 2} - \frac{y - 3}{y^3 - 8}$.

a. $\frac{3y^2 + 5y + 9}{(y - 2)(y^2 + 2y + 4)}$ b. $\frac{3y^2 + 5y + 15}{(y - 2)(y^2 + 2y + 4)}$

c. $\frac{y^2 + y + 7}{(y - 2)(y^2 + 2y + 4)}$ d. $\frac{2y^2 + 5y - 30}{(y - 2)(y^2 + 2y + 4)}$

e. $\frac{y}{y^3 - y - 10}$

13. Which of the following is equal to $\left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)^2$?

a. $x^{\frac{1}{4}} + 2 + x^{-\frac{1}{4}}$ b. $x^2 + 2x + 1$

c. $\frac{x^2 + 1}{x}$ d. $x + 2 + x^{-1}$

e. 2

14. Al has a handful of dimes and quarters. The number of quarters is three more than twice the number of dimes. If d represents the number of dimes then the value of all the coins in cents is:

a. 1.35d b. 0.45d + 0.30 c. 3d + 3

d. 0.60d + 0.75 e. 60d + 75

15. Which of the following is the same number as $\frac{-\sqrt{2} + 2}{1 - \sqrt{2}}$?

a. 2

b. $\frac{1}{\sqrt{2}}$

c. $-\sqrt{2}$

d. $\frac{2}{\sqrt{2}}$

e. $\frac{4 - 3\sqrt{2}}{3}$

16. Which of the following describes the roots of $4x^2 - 1 = 2x$?

a. 2 irrational roots

b. 1 real, rational root

c. 2 rational roots

d. 2 complex roots

e. 1 real root and 1 complex root

17. Solve for x: $x^2 - 2x + 4k = 0$

a. $1 \pm i\sqrt{3k}$

b. $1 \pm i\sqrt{4k + 1}$

c. $1 \pm 2i\sqrt{4k - 1}$

d. $-1 \pm 2\sqrt{1 - 4k}$

e. $1 \pm \sqrt{1 - 4k}$

18. The solution of the inequality $x^2 - 7x - 8 \geq 0$ is

a. $-1 \geq x \geq 8$

b. $x \leq 8$ and $x \geq -1$

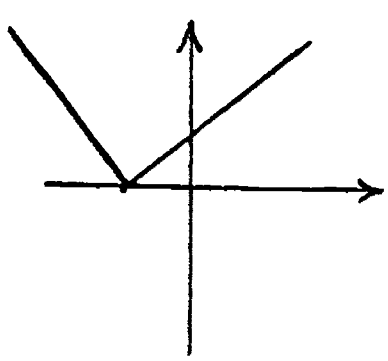
c. $x \leq -1$ or $x \geq 8$

d. $x \leq -1$ and $x \leq 8$

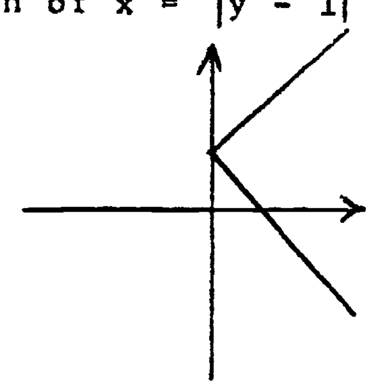
e. x is any real number

19. Which of the following is the graph of $x = |y - 1|$?

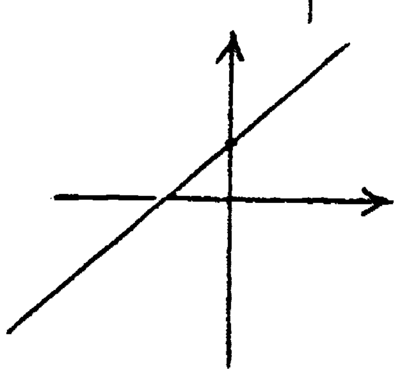
a.



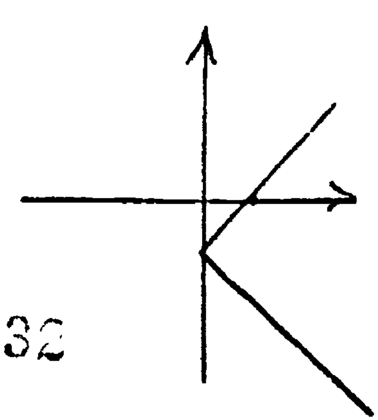
b.



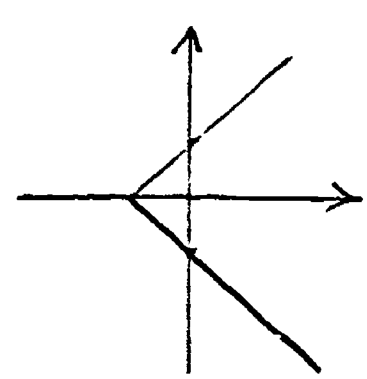
c.



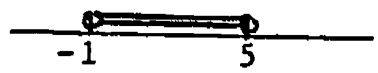
d.

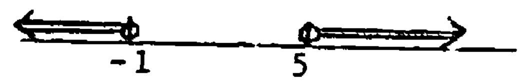


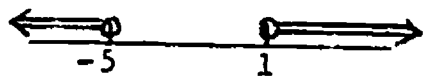
e.

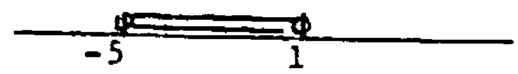


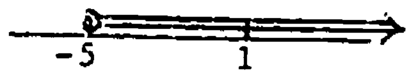
20. Which of the following represents the solution set of $\frac{x + 5}{x - 1} > 0$?

a. 

b. 

c. 

d. 

e. 

21. Which statement is true about the graph of $y = 3 + 5x - 2x^2$?

a. The graph opens downward.

b. The graph has 2 x-intercepts.

c. The graph has a maximum point.

d. The graph has 1 y-intercept.

e. All of the above statements are true.

22. Solve the following equation for h: $\frac{4}{h - 3} + \frac{2h}{3 - h} = 5$.

a. $h = \frac{19}{7}$ b. $h = \frac{11}{3}$ c. $h = \frac{1}{2}$

d. $h = -3$ or $h = \frac{-19}{7}$ e. $h = 3$ or $h = \frac{19}{7}$

23. How many solutions does the following equation have?

$$\frac{x}{\sqrt{x + 1}} - \frac{2x}{\sqrt{x + 3}} = 0$$

a. 3 distinct real solutions

b. 1 distinct real solution

c. 2 distinct real solutions

d. 1 real solution and 1 extraneous solution

e. 0 real solution

24. Solve the following formula for w: $S = 2Lw + 2Lh + 2wh$

a. $w = \frac{S - Lh}{L + h}$

b. $w = S$

c. $w = S - 1$

d. $w = \frac{S - 2Lh}{2Lh}$

e. $w = \frac{S - 2Lh}{2L + 2h}$

25. In how many points do the graphs of these two equations intersect?

$$\begin{cases} x^2 + y^2 = 25 \\ x = y^2 - 5 \end{cases}$$

a. 0 points

b. 1 point

c. 2 points

d. 3 points

e. 4 points

26. Which of the following is true regarding the solution of the system of equations:

$$\begin{cases} 6x^2 - 3y^2 = -13 \\ x^2 + y^2 = 4 \end{cases} ?$$

a. $x = \frac{1}{3}$

b. $x = \pm\frac{1}{3}$

c. $y = \pm\sqrt{3}$

d. $y = \pm 1$

e. Solution to this system has no real coordinates

27. A thief slipped into an apple orchard and stole 72 apples. On the way out the first guard caught him and took half of the apples and 2 more besides; the second guard stopped him and took half of the remaining apples and 3 more besides; the third and last guard stopped him and took half of the remaining apples and 5 more bsides. How many apples did the thief end up with?

a. $\frac{1}{36}$ of the initial number

b. $\frac{1}{12}$ of the initial number

c. no apples

d. 10 apples

e. 10 less than $\frac{1}{8}$ of the original number

28. Evaluate $\log_2 \sqrt{2}$.

a. 0 b. -1 c. 2 d. $-\frac{1}{2}$ e. $\frac{1}{2}$

29. Simplify $\sqrt[x]{x^{x^2-x}}$ where x is a whole number, $x > 0$.

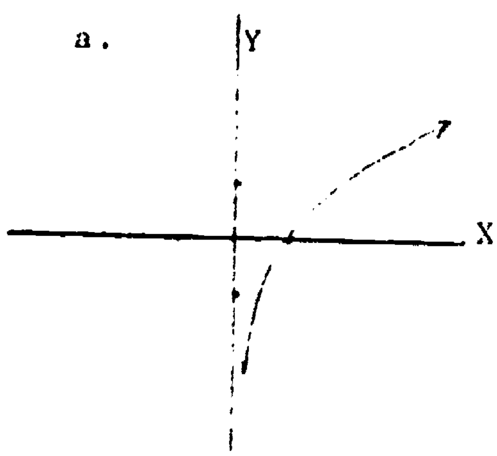
a. $x^2 - x$ b. $x - 1$ c. $x^{x^2} - x$ d. x^{x-1} e. $\frac{1}{x}$

30. Solve for x: $\ln(x + 2) - 4 = \ln(x - 5)$

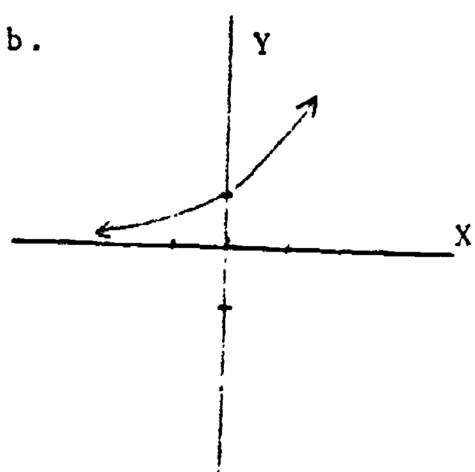
a. $x = \frac{5e^4 + 2}{e^4 - 1}$ b. $x = \frac{3}{2}$ c. $x = \frac{3}{e^4 - 1}$ d. $x = 0$ e. no solution exists

31. Which of the following represents the graph of $y = \left(\frac{1}{3}\right)^x$?

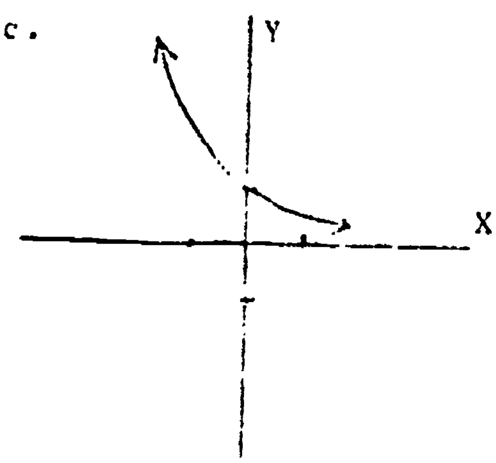
a.



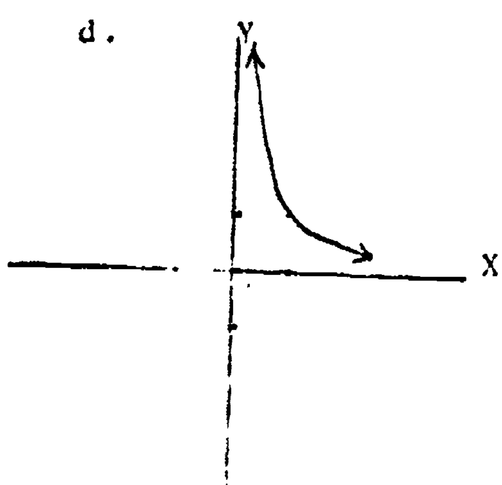
b.



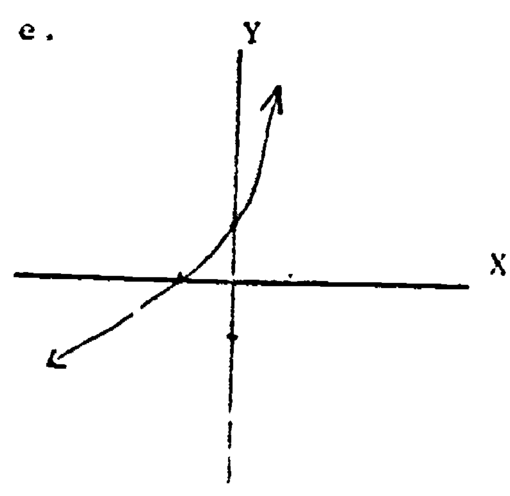
c.



d.



e.



32. The value of $16^{\log_{64}8}$ is:

a. 8 b. 2 c. $\frac{1}{256}$ d. ± 8 e. 4

33. Find value of x in the solution of the system:

$$\begin{cases} 2x - 4y + 2z = 1 \\ 3x + y + 3z = 5 \\ x - y - 2z = -8 \end{cases}$$

a. $-\frac{3}{2}$ b. 3 c. $\frac{1}{2}$ d. $\frac{3}{2}$ e. -3

34. Which of the following equations has a graph which is perpendicular to the graph of $3x - 4y = 5$?

a. $y = \frac{3}{4}x - 2$ b. $3y - 4x = 0$ c. $y = \frac{4}{3}x + 1$

d. $4x + 3y = 3$ e. $6x - 4y = 5$

35. If z varies jointly with x and the square of y, what happens to z if both x and y are doubled?

a. z is doubled b. z is multipled by 8

c. z is divided by 2 d. z is multipled by 6

e. z is multipled by 4

36. Which of the following is true about the solutions of

$$\begin{cases} x = y - 2 \\ x = \dfrac{6}{y + 3} \end{cases}$$

a. (2,4) is a solution and (-5,-3) is not a solution
b. (-6,-4) is the only solution
c. (-4,-1) and (3,1) are both solutions
d. (1,3) and (-6,-4) are both solutions
e. none of the above are true

37. Simplify $\sqrt[3]{16} + \sqrt[3]{250} - \sqrt[3]{81}$

a. $4\sqrt[3]{3}$ b. $3\sqrt[3]{3} + 7\sqrt[3]{2}$

c. $7\sqrt[3]{2} - 3\sqrt[3]{3}$ d. $-5 + 5\sqrt{10}$

e. $4\sqrt[3]{2}$

38. Find the slope of the line with equation $10y - 3x = 17$.

a. $\frac{10}{3}$ b. $\frac{3}{10}$ c. $-\frac{3}{10}$ d. $-\frac{10}{3}$ e. $\frac{17}{10}$

39. Evaluate $-\frac{1}{4}x^3y^{\frac{3}{2}}z^{\frac{1}{3}}$ for $x = -2$, $y = 16$, $z = 27$

a. -1296 b. 384 c. $24\sqrt[3]{12}$ d. -512 e. none of these

40. Which of the following is equivalent to $\dfrac{a^4 - b^4}{a^4 + 2a^2b^2 + b^4}$?

a. $\dfrac{a - b}{a + b}$ b. $\dfrac{-1}{2a^4b^4}$

c. -1 d. $\dfrac{(a - b)^2}{(a + b)^2}$

e. $\dfrac{(a - b)(a + b)}{a^2 + b^2}$

1988

STATE HIGH SCHOOL MATHEMATICS CONTEST

SPONSORED BY THE INDIANA COUNCIL OF TEACHERS OF MATHEMATICS

GEOMETRY

THIS TEST WAS PREPARED BY THE FACULTY OF
UNIVERSITY OF SOUTHERN INDIANA.

DIRECTIONS FOR TEST:

DO NOT open this booklet until you are told to do so.

This is a test of your competence in high school geometry. For each of the 44 problems there are listed 5 possible answers. You are to work each problem and determine which is the correct answer, and indicate your choice by filling in the circle in the correct place on the separate answer sheet provided. A sample follows:

1. If $x + 30^0 = 90^0$, then x equals

   A. 120^0

   B. 3^0

   C. -60^0

   D. 60^0

   E. none of these

	A	B	C	D	E
1.	①	②	③	●	⑤

The correct answer for the sample is "60^0", which is answer D; therefore, you should answer this question by filling in the circle D as indicated above.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any question. If you are unable to work any particular problem, it is to your advantage to guess at the answer rather than leave it blank. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet and begin work. When you have finished one page, go on to the next page. The working time for the entire test is 90 minutes.

DIRECTIONS FOR ANSWER SHEET:

Fill in your name in the blanks provided. Above your name write the name of your school and the city where it is located, including zip code.

Along the side of your name indicate your sex and grade by filling in the circle provided. A sample follows:

SAMPLE: Mary A. Brown, who goes to Western High School in Muncie and is in the tenth grade, would write across the top and fill in along the side.

WESTERN HIGH SCHOOL - MUNCIE, INDIANA 47306

NAME (Last, First, M.I.)														
B	R	O	W	N		M	A	R	Y		A			

SEX
Ⓜ
●

GRADE
⑨
●
⑪
⑫

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.

1. How many triangles are there in the figure below?

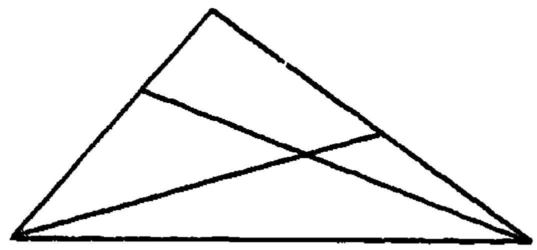
   a. 9

   b. 4

   c. 6

   d. 7

   e. 8



2. Which statement is the inverse of " If I dropped the book, then it did not fall."?

   a. If the book did not fall, then I dropped it.

   b. If I did not drop the book, then it did not fall.

   c. If the book did not fall, then I did not drop it.

   d. If I did not drop the book, then it did fall.

   e. If I dropped the book, then it did fall.

3. In the figure, $\overline{AE} \parallel \overline{BD}$, AB = 9, BC = 3, BD = 4.

   What is the measure of $\overline{AE}$?

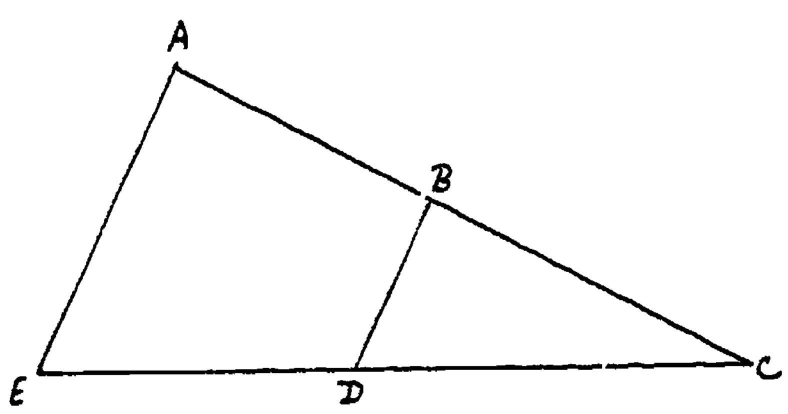
   a. $8\frac{1}{4}$

   b. $9\frac{3}{4}$

   c. 16

   d. 12

   e. 8



4. Given: $\overline{DA} \perp \overline{AB}$, $\overline{AC} \parallel \overline{DE}$, and $\overline{AC} \cong \overline{AB}$

Find: the measure of $\angle C$

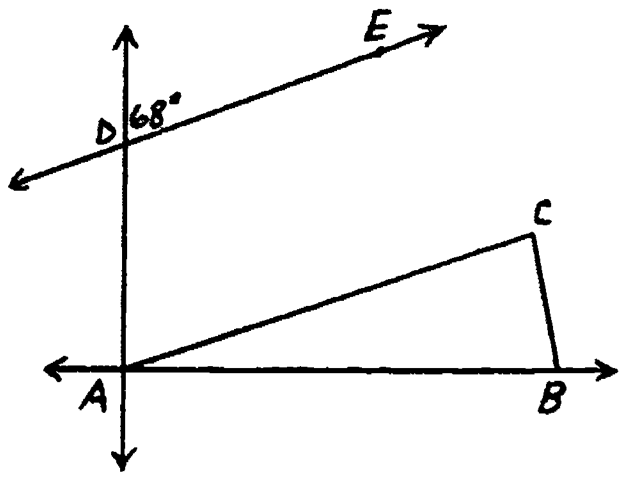
a. 79°

b. 74°

c. 68°

d. 84°

e. 64°



5. For any circle, π is exactly equal to:

a. $\frac{22}{7}$

b. 3.14

c. $\frac{\text{circumference}}{\text{diameter}}$

d. $\frac{22}{7}$ and 3.14

e. $\frac{22}{7}$, 3.14 and $\frac{\text{circumference}}{\text{diameter}}$

6. A line with slope $\frac{5}{2}$ contains point (3,1) and (7, _?_)

a. $3\sqrt{3} + 1$

b. 3

c. 11

d. 12

e. -9

7. If the measure of an angle is 30° less than four times that of its supplement, then its measure is:

a. 66°

b. 150°

c. 78°

d. 75°

e. 138°

8. In the figure $\overline{AC} \parallel \overline{BD}$.
Find the measure of ∡ ACB.

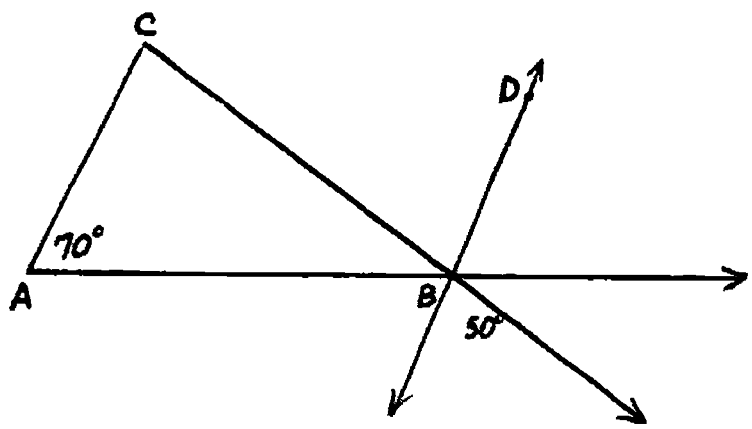
a. 60°

b. 50°

c. 70°

d. 40°

e. 30°



9. If the measure of one angle of an isosceles triangle is 64°, which of these is possible?

I. Another angle has a measure of 64°
II. Another angle has a measure of 52°
III. Another angle has a measure of 58°

a. I only

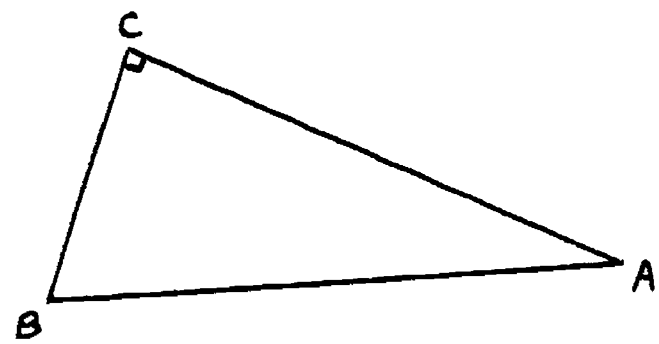
b. II only

c. I and III only

d. I and II only

e. I, II, or III

10. Given $\triangle$ ABC is a right triangle, the measure of $\angle B = 60°$ and $AC = 4$
Find: $\overline{AB}$



a. $4\sqrt{3}$

b. $\frac{4\sqrt{3}}{3}$

c. $4\sqrt{2}$

d. 8

e. $\frac{8\sqrt{3}}{3}$

11. The radii of two circles are in the ratio 2 to 3. If the area of the larger circle is 54π, find the area of the smaller circle.

a. 6π

b. 24π

c. 36π

d. 81π

e. 18π

12. Which of the following sets of numbers may not represent the lengths of the three sides of a triangle?

a. 8, 12, 13

b. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{3}{4}$

c. 2.4, 4.1, 3.1

d. 3, 4, 7

e. 6, 8, 10

13. In the plane of two parallel lines, the locus of points equidistant from the two lines is:

a. a point

b. a line

c. a pair of lines

d. three lines

e. the empty set

14. In the figure $\overrightarrow{BD}$ bisects ∠ ABE, $\overrightarrow{BE}$ bisects ∠ ABG, the measure of ∠ EBF = 34° and the measure of ∠ CBG = 48°, find the measure of ∠ DBF.

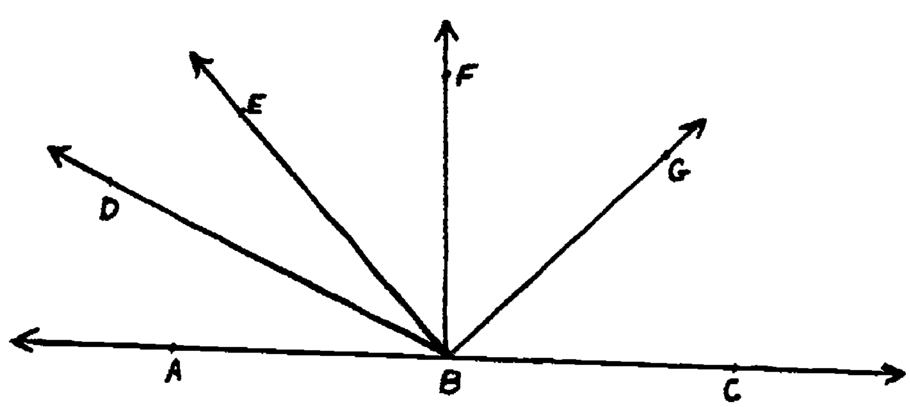
a. 67°

b. 33°

c. 32°

d. 66°

e. 72°



15. If $\triangle$ ABC $\sim$ $\triangle$ JOT,

a. $\frac{AC}{JT} = \frac{AB}{OT}$

b. $\frac{AB}{JT} = \frac{AC}{JO}$

c. $\frac{AB}{OT} = \frac{BC}{OJ}$

d. $\frac{AC}{JT} = \frac{BC}{OT}$

e. None of the above

16. Consider the cube at right with sides of length one. The length of $\overline{AG}$ is:

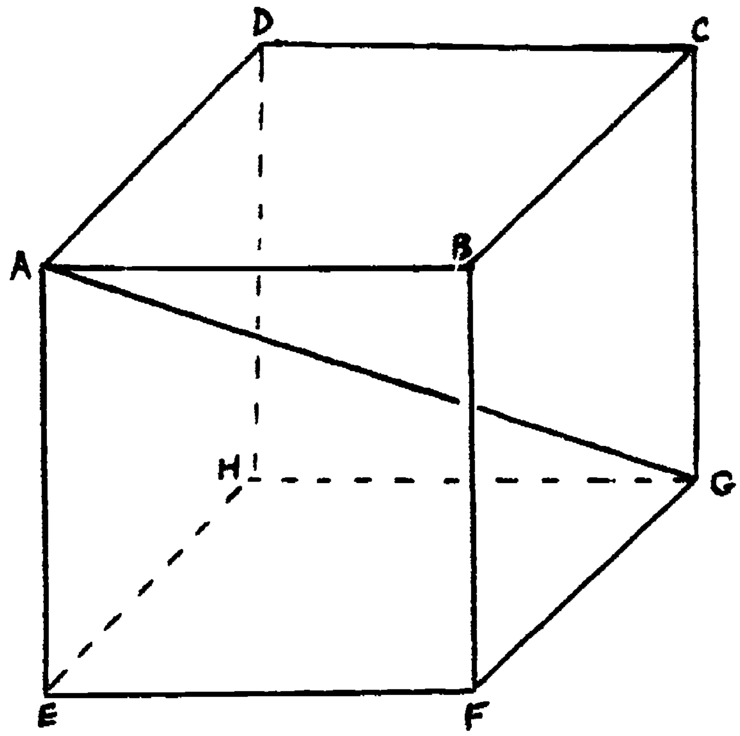
a. 1

b. $\sqrt{2}$

c. $\sqrt{3}$

d. 3

e. $\sqrt{5}$



17. Name all possible intersections of two different planes.

a. a line and a point

b. a line

c. a line and a plane

d. a line and the empty set

e. none of these

18. Which of the following is true?

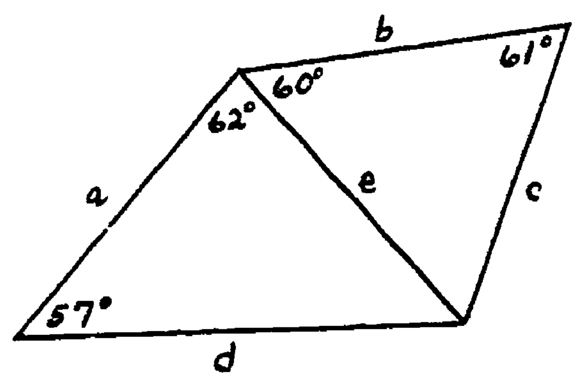
a. d > a > e > c > b

b. d > e > c > b > a

c. e > a > b > c > d

d. d > a > e > b > c

e. e > a > c > b > d



19. Which of the following is <u>not</u> a property of all rhombi?

a. The diagonals are congruent.

b. The diagonals bisect each other.

c. Each diagonal bisects a pair of opposite angles.

d. The diagonals are perpendicular.

e. The diagonals form four congruent triangles.

20. Given $\overline{AD}$ bisects $\overline{BE}$, what must be shown to prove that the triangles are congruent by the AAS method.

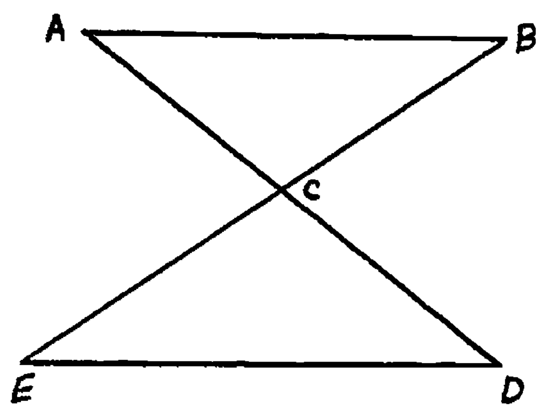
a. $\angle B \cong \angle D$

b. $\angle A \cong \angle E$

c. $\angle A \cong \angle D$

d. $\angle B \cong \angle E$

e. $\overline{BE}$ bisects $\overline{AD}$



21. $\overline{AB}$ is a diameter of a circle. Find the center and radius of the circle.

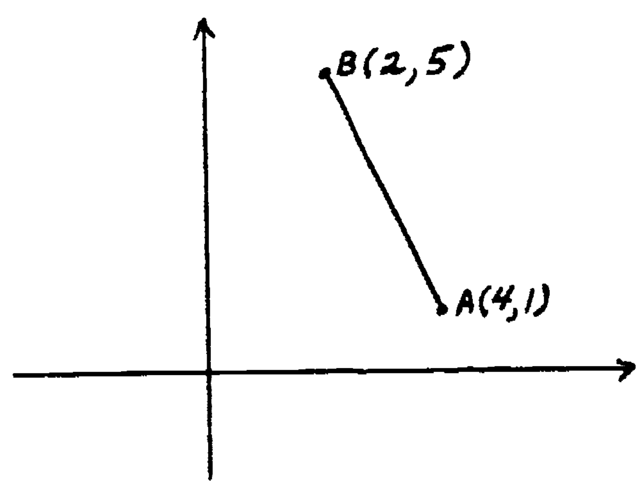
a. Center (2,4); radius $\sqrt{5}$

b. Center (3,3); radius $\sqrt{5}$

c. Center (2,4); radius $2\sqrt{5}$

d. Center (3,3); radius $2\sqrt{5}$

e. Center (3,3); radius $\frac{5}{2}$



22. Which of the following is <u>not</u> sufficient to conclude $\triangle ABC \sim \triangle DEF$

a. $\measuredangle A \cong \measuredangle D$, $\measuredangle C \cong \measuredangle F$

b. $\frac{AB}{DE} \cong \frac{BC}{EF}$, $\measuredangle B \cong \measuredangle E$

c. $\frac{BC}{EF} \cong \frac{BA}{ED} \cong \frac{CA}{FD}$

d. $\frac{AB}{DE} \cong \frac{BC}{EF}$, $\measuredangle A \cong \measuredangle D$

e. All of the above are sufficient.

23. Quadrilateral ABCD $\sim$ quadrilateral HGFE

Find the perimeter of ABCD.

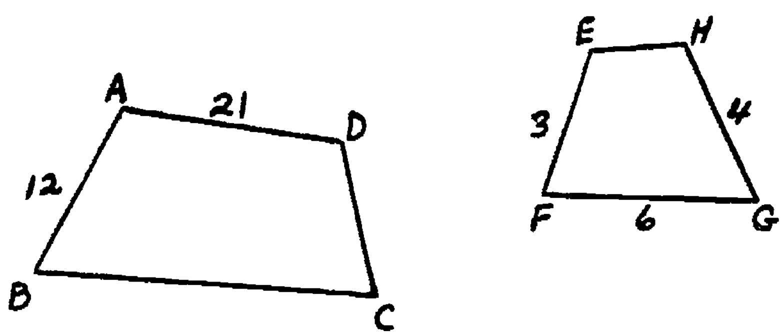
a. 70

b. 60

c. 66

d. 73

e. 80



24. The measure of the angle at each vertex of a regular pentagon is:

a. 140°

b. 108°

c. 120°

d. 72°

e. $128\frac{4}{7}^\circ$

25. Which of the following is false:

a. Some quadrilaterals are rectangles.

b. All trapezoids are quadrilaterals.

c. Some rectangles are parallelograms.

d. Some parallelograms are squares.

e. All rectangles are squares.

26. Given: $\sin 30° = 0.5$, $\sin 56° = 0.8$, and line L is perpendicular to plane M. Find x.

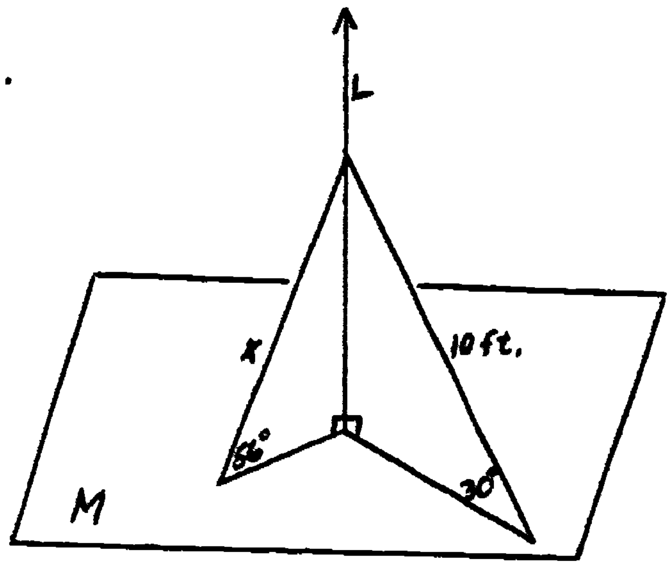
a. 16 ft.

b. $4\sqrt{5}$ ft.

c. 0.625 ft.

d. 6.25 ft.

e. none of these



27. Which of the following sets of conditions is **not sufficient** to conclude that $\triangle PQR \cong \triangle MNO$

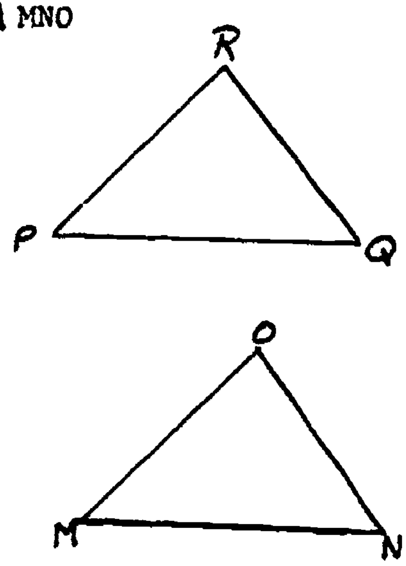
a. $\overline{PQ} \cong \overline{MN}$, $\overline{QR} \cong \overline{NO}$, $\overline{RP} \cong \overline{OM}$

b. $\overline{PR} \cong \overline{MO}$, $\angle P \cong \angle M$, $\overline{RQ} \cong \overline{ON}$

c. $\overline{PQ} \cong \overline{MN}$, $\overline{QR} \cong \overline{NO}$, $\angle Q \cong \angle N$

d. $\overline{PR} \cong \overline{MO}$, $\angle R \cong \angle O$, $\angle P \cong \angle M$

e. $\overline{RQ} \cong \overline{ON}$, $\angle R \cong \angle O$, $\angle P \cong \angle M$



28. Which of the following is not a property of all parallelograms.

a. Opposite sides are congruent.

b. Opposite angles are congruent.

c. The diagonals bisect each other.

d. The diagonals are congruent.

e. A diagonal divides the figure into two congruent triangles.

29. Given triangle ABC is equiangular, $\angle DAC \cong \angle DCA$ and $CD = \sqrt{3}$

Find the length of $\overline{CE}$.

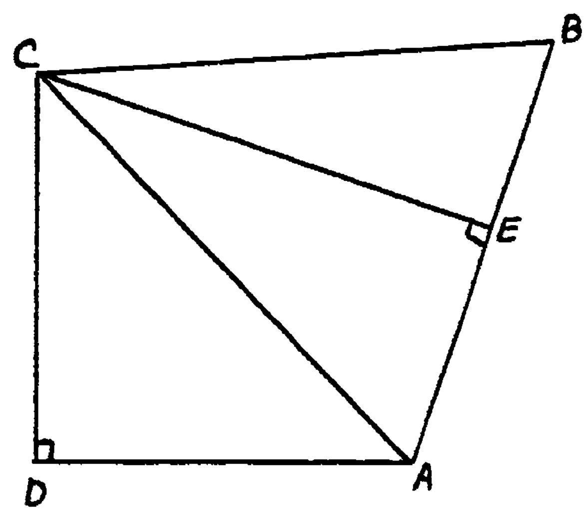
a. $\sqrt{6}$

b. $\frac{\sqrt{6}}{2}$

c. $\frac{3\sqrt{2}}{2}$

d. $3\sqrt{2}$

e. $2\sqrt{6}$



30. Which of the following relations does not have the transitive property?

a. "Is perpendicular to" for lines

b. "Is congruent to" for angles

c. "Is similar to" for triangles

d. "Is equal to" for measure of angles

e. "Is parallel to" for lines

31. Quadrilateral ABCD is inscribed in a circle.

the measure of ∡ A = 100°. Find the measure of ∡ B

a. 80°

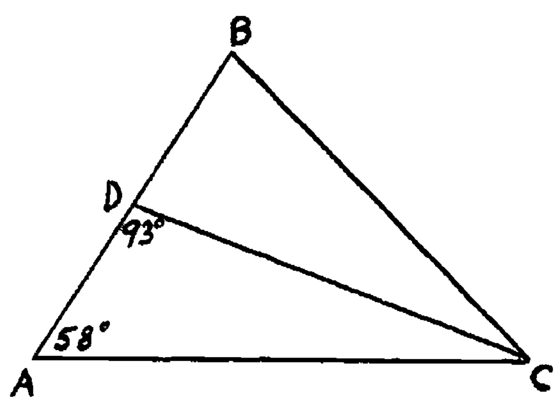
b. 90°

c. 100°

d. equal to the measure of ∡ D

e. 180° - measure of ∡ D

32. Given triangle ABC as shown with ∡ ACD ≅ ∡ BCD. List the lengths of sides $\overline{AB}$, $\overline{AC}$, and $\overline{CD}$ in order starting with the smallest.



a. CD < AB < AC

b. AB < CD < AC

c. AC < AB < CD

d. AB < AC < CD

e. CD < AC < AB

33. Given ∡ A ≅ ∡ E, ∡ B ≅ ∡ D, $\overline{BC}$ ≅ $\overline{DF}$, What conclusion can be made?

a. Δ ABC ≅ Δ EDF by AAS

b. Δ ABC ≅ Δ DEF by ASA

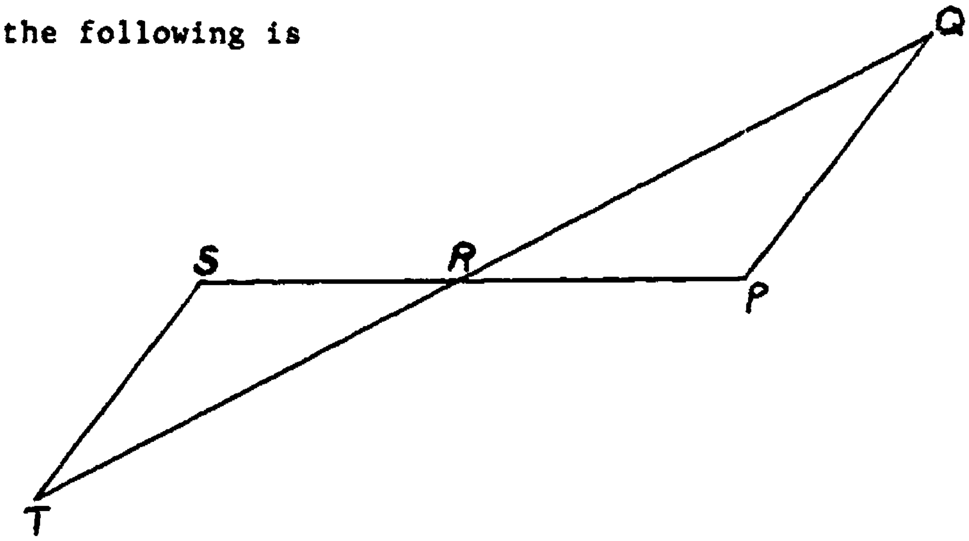
c. Δ ABC ≅ Δ DEF by SAS

d. Δ ABC ≅ Δ DEF by AAS

e. Δ ABC ≅ Δ EFD by ASA

34. Referring to figure at right,

$\overline{ST} \parallel \overline{PQ}$, which of the following is **not** true?



a. $\frac{RT}{RQ} = \frac{ST}{PQ}$

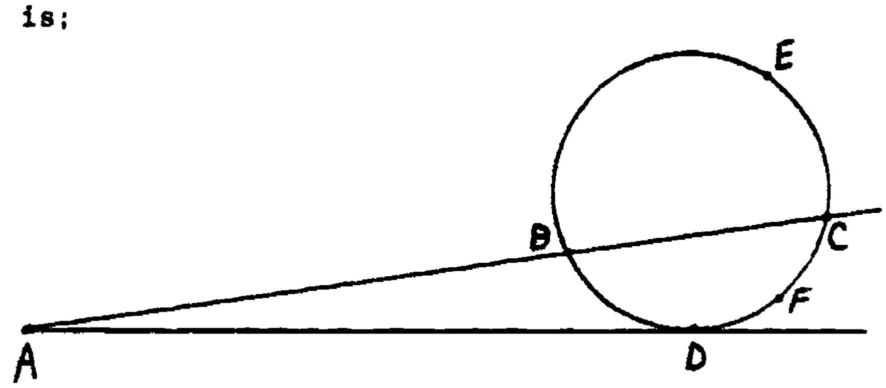
b. $\frac{SR}{RT} = \frac{RQ}{PR}$

c. $\frac{SR}{PR} = \frac{TR}{QR}$

d. $\frac{TR}{ST} = \frac{QR}{QP}$

e. $\frac{TR}{QR} = \frac{RS}{RP}$

35. In figure at right, the measure of $\widehat{BEC}$ = 200° and the measure of $\widehat{DFC}$ = 10C´ and $\overline{AD}$ is tangent to the circle. The measure of ∡ BAD is:



a. 60°

b. 40°

c. 30°

d. 20°

e. 50°

36. In space, the geometric figure(s) formed by all the points one foot from a given line is:

a. a pair of planes

b. a circle

c. a cylindrical surface

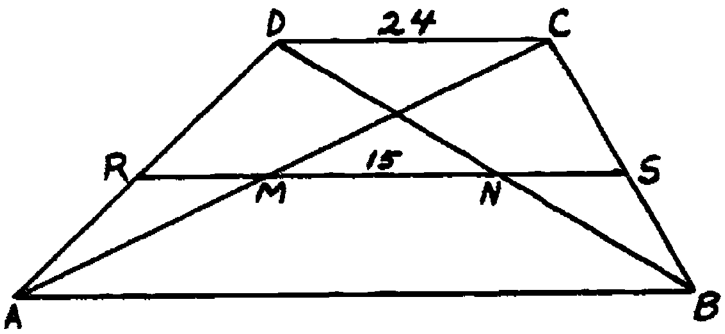
d. a pair of lines

e. a rectangular prism without ends

37. ABCD is a trapezoid with bases $\overline{AB}$ and $\overline{CD}$;

R and S are the midpoints of $\overline{AD}$ and $\overline{BC}$ respectively.

DC = 24 and MN = 15. Find AB.



a. 39

b. $19\frac{1}{2}$

c. 54

d. 48

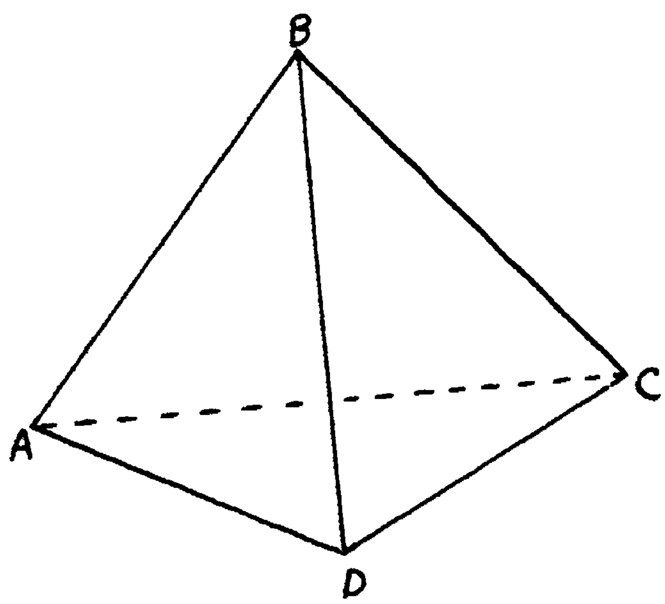
e. None of the above

38. Given the following pyramid with

AB = DB = CB = 5 and

AD = DC = CA = 6.

Find the total area.



a. 26

b. 36

c. 63

d. $45\sqrt{3}$

e. $36 + 9\sqrt{3}$

39. Assume that the total area of a rectangular prism is 40 square inches and its volume is 16 cubic inches. Suppose that all dimensions of this prism are doubled. Find the new total area and the new volume.

a. 80 sq. in.; 64 cu. in.

b. 80 sq. in.; 96 cu. in.

c. 160 sq. in.; 64 cu. in.

d. 160 sq. in.; 96 cu. in.

e. 160 sq. in.; 128 cu. in.

40. Which of the following conditions guarantee similar figures?

I. Any two squares
II. Any two rhombi with one pair of congruent angles
III. Any two parallelograms with pairs of adjacent sides proportional

a. I only

b. II only

c. III only

d. I and II only

e. I, II and III

41. A regular square pyramid has a 6-inch base edge and a 5-inch altitude. The volume of the pyramid is

a. 45 cu. in.

b. 75 cu. in.

c. 90 cu. in.

d. 60 cu. in.

e. 180 cu. in.

42. A regular pentagon has a perimeter of 50 cm and an area of 100 cm. What is the length of the apothem?

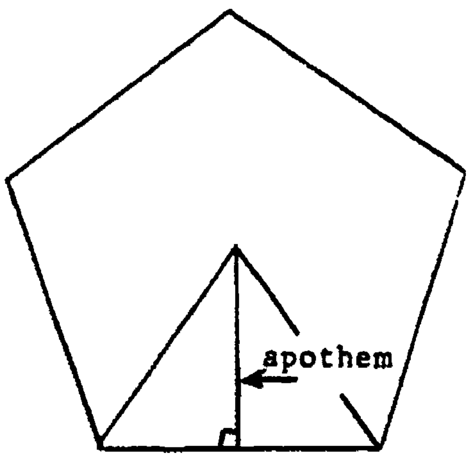
a. $5\sqrt{3}$ cm

b. 2 cm

c. 4 cm

d. 5 cm

e. 10 cm



43. A cube has symmetry with respect to how many planes?

a. More than 7

b. 7

c. 5

d. 3

e. 2

44. A solid metal cylinder with a 3-cm radius and a 10-cm altitude is melted and recast into solid right circular cones, each with a 1-cm radius and a 2-cm altitude. The number of cones formed is

a. 30

b. 45π

c. 90

d. 45

e. 135

1 9 8 8

STATE HIGH SCHOOL MATHEMATICS CONTEST

SPONSORED BY THE INDIANA COUNCIL OF TEACHERS OF MATHEMATICS

C O M P R E H E N S I V E

THIS TEST WAS PREPARED BY THE MATHEMATICS AND MATHEMATICS EDUCATION FACULTY OF INDIANA UNIVERSITY AT KOKOMO.

DIRECTIONS FOR TEST:

DO NOT open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each of the 28 problems there are listed 5 possible answers. You are to work each problem and determine which is the correct answer, and indicate your choice by filling in the circle in the correct place on the separate answer sheet provided. A sample follows:

1. If $x + 2 = 6$, then x equals

   A. 8

   B. 3

   C. $\frac{1}{3}$

   D. 4

   E. none of these

	A	B	C	D	E
1.	①	②	③	●	⑤

The correct answer for the sample is "4", which is answer D; therefore, you should answer this question by filling in the circle D as indicated above.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any question. If you are unable to work any particular problem, it is to your advantage to guess at the answer rather than leave it blank. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet and begin work. When you have finished one page, go on to the next page. The working time for the entire test is 90 minutes.

DIRECTIONS FOR ANSWER SHEET:

Fill in your name in the blanks provided. Above your name write the name of your school and the city where it is located, including zip code.

Along the side of your name indicate your sex and grade by filling in the circle provided. A sample follows:

SAMPLE: Mary A. Brown, who goes to Western High School in Muncie and is in the twelfth grade. would write across the top and fill in along the side.

WESTERN HIGH SCHOOL - MUNCIE, INDIANA 47306

NAME (Last, First, M.I.)															
B	R	O	W	N		M	A	R	Y		A				

SEX: Ⓜ ●

GRADE: ⑨ ⑩ ⑪ ●

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.

1. The graphs of the equations $X + 3Y = 3$, $7X + Y = 41$, and $3X - Y = 9$ intersect to form a right triangle. What is the slope of the hypotenuse?

A. -1/3
B. 7
C. 3
D. -3
E. -7

2. $\frac{\cos 2x}{2 \cos x} + \frac{\sin x}{\sin 2x} = ?$

A. $\sec x - 1$

B. $\sec x$

C. $3 \sec x$

D. $\sin^2 x \sec x$

E. $\cos x$

3. If $x^4 + y^4 = z^4$, then

A. $1 + \log_x y = \log_x z$

B. $\log_x (x+y) = \log_x z$

C. $\log_x y = \log_x (z-x)$

D. $\log_x (z-y) + \log_x (z+y) + \log_x (z^2+y^2) = 4$

E. $2 \log_x (z-y) - 2 \log_x y + \log_x (z^2+y^2) = 4$

4. How many real number solutions are there for the equation

$\cos x + 2 = \frac{1}{x}$?

A. 0
B. 1
C. 2
D. 3
E. more than 3

5. Given that x is in radians, which of the following are solutions, correct to 4 significant figures, for the equation

$$\cos^4 x - 4\cos^2 x = -2\ ?$$

A. 0.6992, 2.442
B. 40.06, 139.9
C. 69.93, 110.1
D. 1.221, 1.921
E. none of the above

6. Solve for Z:

$$(1/2)(X^2+Y^2)^{-1/2}(2X+2YZ) - 6X^2Y^3 - 9X^2Y^2Z + 4YZ - Z = 8$$

A. $$\frac{-8 + X(X^2+Y^2)^{-1/2} + 6X^2Y^3}{Y(X^2+Y^2)^{-1/2} - 9X^2Y^2 + 4Y + 1}$$

B. $$\frac{8 + X(X^2+Y^2)^{-1/2} - 6X^2Y^3}{Y(X^2+Y^2)^{-1/2} - 9X^2Y^2 + 4Y - 1}$$

C. $$\frac{8 + X(X^2+Y^2)^{-1/2} - 6X^2Y^3}{Y(X^2+Y^2)^{-1/2} - 9X^2Y^2 + 4Y}$$

D. $$\frac{8 - X(X^2+Y^2)^{-1/2} + 6X^2Y^3}{Y(X^2+Y^2)^{-1/2} - 9X^2Y^2 + 4Y}$$

E. $$\frac{8 - X(X^2+Y^2)^{-1/2} + 6X^2Y^3}{Y(X^2+Y^2)^{-1/2} - 9X^2Y^2 + 4Y - 1}$$

7. The polynomial $Y^3 + Y^2 + mY - 7$ has the same remainder when it is divided by either Y-1 or Y+1. The remainder must be:

A. m
B. 6m
C. -m
D. 7m
E. none of the above

8. If X and Y are two real numbers satisfying the equations $2^X = 8^{Y+1}$ and $9^Y = 3^{X-9}$, then the SUM of X and Y is:

A. 16
B. 18
C. 22
D. 27
E. 126

9. The expression $\frac{(3)(X^2-1)^{1/3} - (1/3)(3X-6)(2X)(X^2-1)^{-2/3}}{(X^2-1)^{2/3}}$ can be simplified to:

A. $\frac{X^2 + 4X - 3}{(X^2-1)^{4/3}}$

B. $X^2 + 4X - 3$

C. $\frac{X^2 + 4X + 3}{(X^2-1)^{4/3}}$

D. $\frac{X^2 - 4X - 3}{(X^2-1)^{4/3}}$

E. none of the above

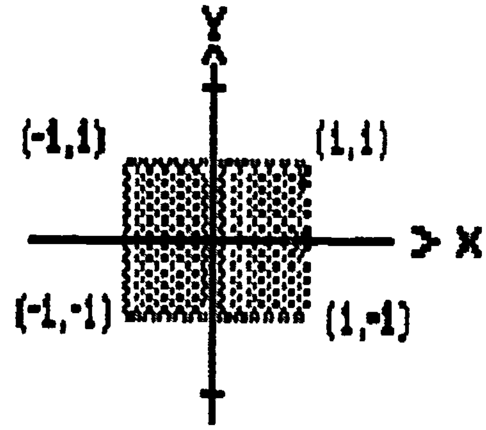
10. If $(1 - 1/a)^6 = A + B/a + C/a^2 + D/a^3 + E/a^4 + F/a^5 + G/a^6$, then the value of E + F + G is:

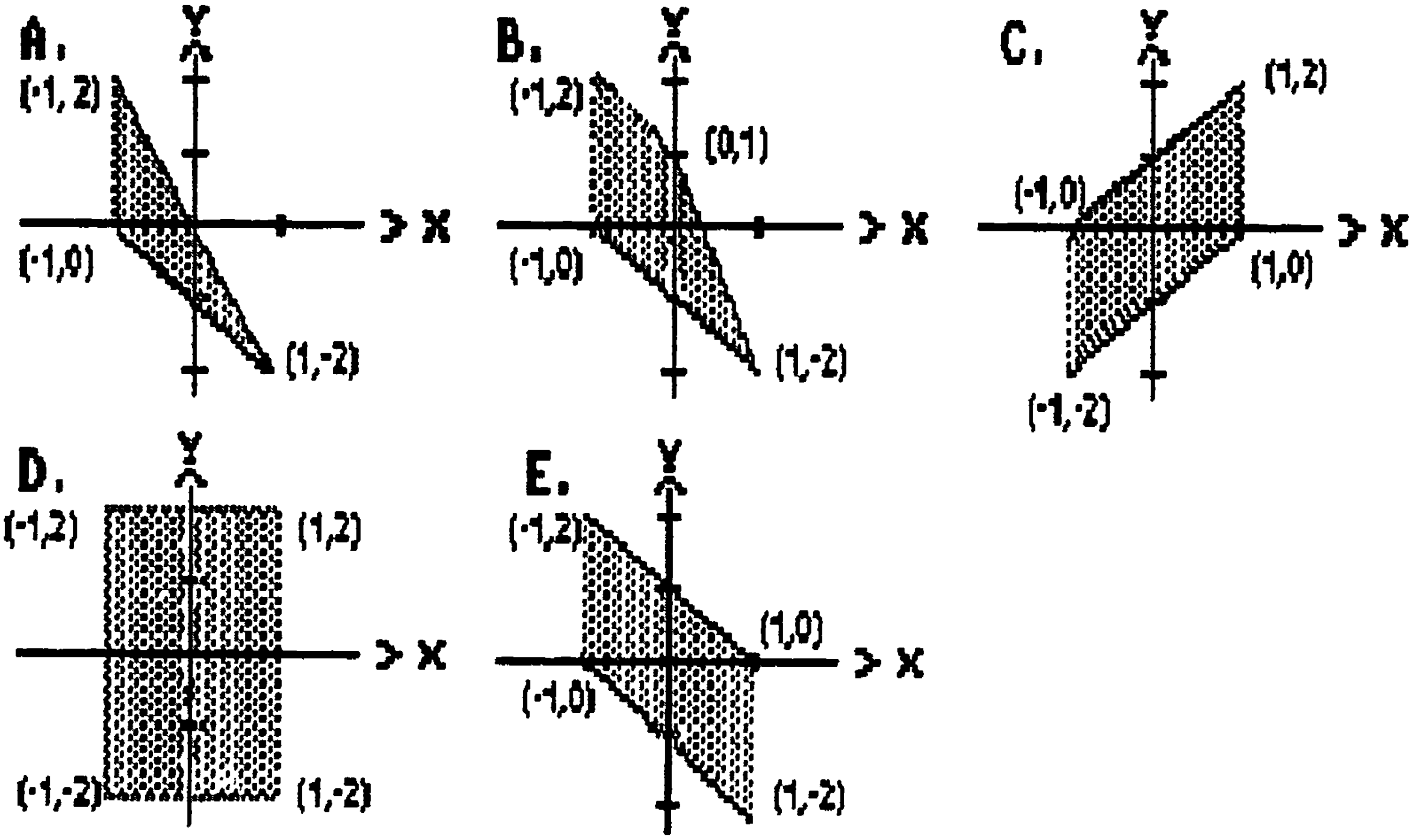
A. -15
B. 15
C. 10
D. -10
E. 5

11. $\frac{100!}{99!} - 99 = ?$

A. 0
B. 1
C. 99
D. 100
E. 99!

12. If S is the set of all points in the shaded square, which of the following shows the set consisting of all points (X,Y-X) where (X,Y) is a point in S?





13. If $f(n) = (-1)^n n^2$ where n is an integer, then $f(2n) + f(2n+1) = ?$

A. $8n^2 + 1$

B. $4n + 1$

C. $-4n - 1$

D. $8n^2 + 4n + 1$

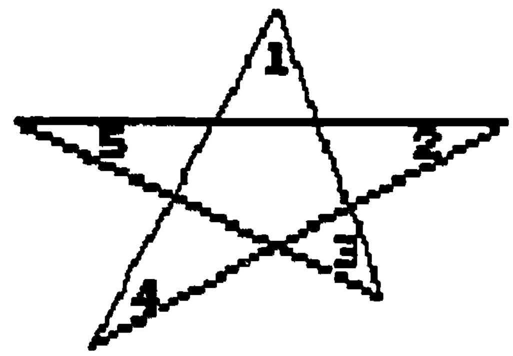
E. $-8n^2 - 4n - 1$

14. What is the value of $[\log_{10}(5 \log_{10}100)]^2$?

A. 3.40
B. 25
C. 10
D. 2
E. 1

15. The sum of the measures of angles 1,2,3,4, and 5 of the star figure is:

A. 270°
B. 180°
C. 360°
D. 150°
E. 225°



16. If $2X - 3Y - Z = 0$ and $X + 3Y - 14Z = 0$, $Z \neq 0$, the numerical value of $\frac{X^2 + 3XY}{Y^2 + Z^2}$ is:

A. 7
B. 2
C. 0
D. -20/17
E. -2

17. The isosceles trapezoid ABCD is inscribed in a semicircle with radius 6 and center at the origin. Find the area of ABCD as a function of p, the x-coordinate of the point C.

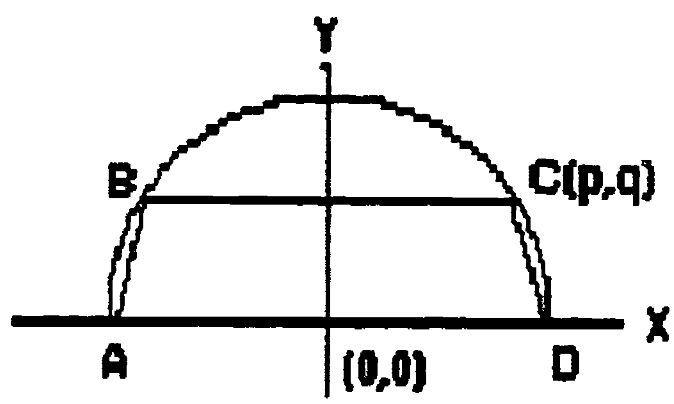
A. $(6+p)\sqrt{36-p^2}$

B. $6\sqrt{36-p^2}$

C. $(1/2)\pi p^2 -$ ([illegible])$\sqrt{36-p^2}$

D. $(1/2)(6+p)\sqrt{36-p^2}$

E. $12(36-p^2)$



18. How many real number solutions are there fo: ..e equation

$(x^{\log x})(5x^3) = 1$, where the base for log x is 10?

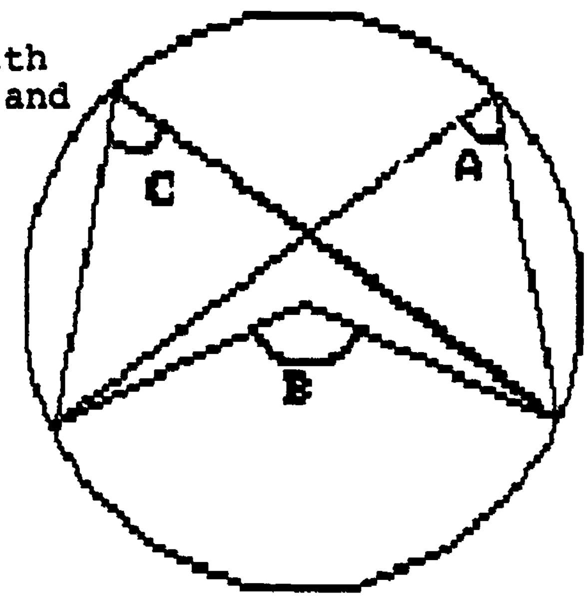
A. 0
B. 1
C. 2
D. a finite number greater than 2
E. infinitely many

19. The fourth power of $\sqrt{1 + \sqrt{1 + \sqrt{1}}}$ is:

A. $1 + 2\sqrt{2}$
B. 3
C. 5.81
D. $3 + 2\sqrt{2}$
E. $17 + 12\sqrt{2}$

20. The figure is a circle with inscribed angles A and C and central angle B as drawn. If C = 29.5°, find A+B.

A. 29.5°
B. 59°
C. 88.5°
D. 118°
E. 147.5°



21. A water tank with its base on the ground has the shape of a right circular cylinder surmounted by a hemispherical dome. If the diameter of the hemisphere is 20 meters, the height of the tank from the base to the top of the hemispherical dome is 40 meters, and paint costs 90 cents per square meter, what is the cost of the paint for the tank?

A. $12,252.21
B. $ 5,654.87
C. $ 2,827.43
D. $ 2,261.95
E. $ 1,696.46

22. If $f(x) = 10^x$ where x is real and if the inverse function of f is denoted by f^{-1}, then what is $\frac{f^{-1}(a)}{f^{-1}(b)}$, where a > 1 and b > 1 ?

A. $\log_{10}a - \log_{10}b$

B. $\log_{10}(a-b)$

C. $\frac{10^b}{10^a}$

D. $\log_b a$

E. none of the above

23. The graph of the equation $3X^2 - 4Y^2 + 12X + 8Y + 8 = 0$ is:

A. an ellipse
B. a circle
C. a hyperbola
D. two lines
E. a point

24. Let f(x) satisfy $[f(x)]^2 \leq |x-3|$ for all real numbers. What is true about f(3)?

A. $f(3) > 0$
B. $f(3) < 0$
C. $f(3) = 0$
D. $f(3) > 3$
E. none of the above

25. A rectangular piece of paper 6 inches wide is folded as in the diagram so that corner B touches the opposite side at B'. The length in inches of the crease L in terms of angle A is:

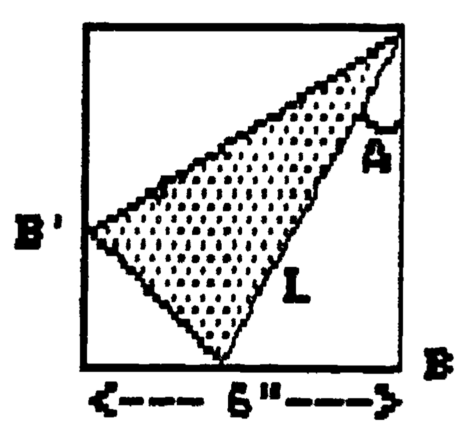
A. $3 \sec^2 A \csc A$

B. $3 \csc^2 A \sec A$

C. $3 \cos^2 A \sin A$

D. $6 \cos^2 A \sin A$

E. $6 \sin^2 A \cos A$



26. Which could be an equation for the graph?

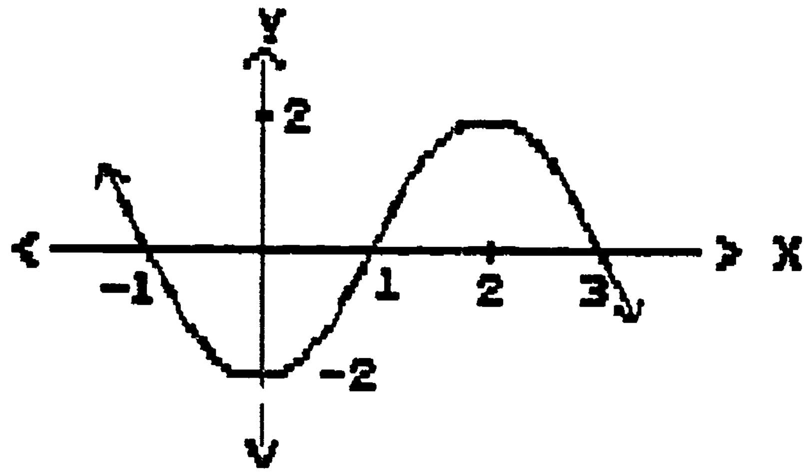
A. $Y = -2\cos[(\pi/2)(X-2)]$

B. $Y = 2\cos[(\pi/2)(X-2)]$

C. $Y = 2\sin[(\pi/2)(X+2)]$

D. $Y = 2\cos[\pi(X+1)]$

E. $Y = -2\sin[\pi(X+1/2)]$



27. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, then $f\left(\frac{2x}{1+x^2}\right) = ?$

A. $[f(x)]^2$
B. $2\,f(x)$
C. $\log(4x)$
D. $\log 4$
E. none of the above

28. Which of the following best represents the graph of $|3X - 5| < Y$?