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Hispanic and Anglo Students' Misconceptions in Mathematics. ERIC Digest.

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Math teachers and researchers are beginning to agree on the importance of a series of new findings. According to new research, many students have misconceptions about mathematics--sometimes called "naive theories"--that can turn them into clumsy learners. This digest describes misconceptions in math--what causes them and why they interfere with learning. Next it considers common mathematical misconceptions among Anglos and Hispanics. It concludes with a discussion of techniques to help students overcome their misconceptions in math.

THE NATURE OF MISCONCEPTIONS

Students do not come to the classroom as "blank slates" (Resnick, 1983). Instead, they come with theories constructed from their everyday experiences. They have actively constructed these theories, an activity crucial to all successful learning. Some of the theories that students use to make sense of the world are, however, incomplete half-truths (Mestre, 1987). They are "misconceptions."

Misconceptions are a problem for two reasons. First, they interfere with learning when students use them to interpret new experiences. Second, students are emotionally and intellectually attached to their misconceptions, because they have actively constructed them. Hence, students give up their misconceptions, which can have such a harmful effect on learning, only with great reluctance.

What do these findings mean? They show teachers that their students almost always come to class with complex ideas about the subject at hand. Further, they suggest that repeating a lesson or making it clearer will not help students who base their reasoning on strongly held misconceptions (Champagne, Klopfer & Gunstone, 1982; McDermott, 1984; Resnick, 1983). In fact, students who overcome a misconception after ordinary instruction often return to it only a short time later.

SOME COMMON MATHEMATICAL MISCONCEPTIONS AMONG ANGLO STUDENTS

There are many misconceptions in elementary mathematics. In fact, there are so many that researchers have developed a catalog of them (Benander & Clement, 1985). Here, the discussion considers some common misconceptions that are hard to change.

A very prevalent misconception surfaces in the "students and professors" problem:

Write an equation using the variables S and P to represent the following statement: "There are six times as many students as professors at a certain university." Use "S" for the number of students and "P" for the number of professors.

The most common error in this problem (committed by about 35% of college engineering majors!) is to write "6S = P." (The correct equation, of course, is "S = 6P.")

This misconception grows from the interplay of two factors. First, students thoughtlessly translate the words of the problem from left to right. Second, as they give in to that temptation, they confuse the idea of variables and labels. Using the left-to-right strategy, students interpret the "S" and the "P" in the equation as labels (verbal shorthand) for the terms "students" and "professors." They fail to apply the idea that variables stand for numerical expressions.

A more complex problem also illustrates this type of difficulty:

I went to the store and bought the same number of books as records. Books cost two dollars each and records cost six dollars each. I spent $40 altogether. Assuming that the equation 2B + 6R = 40 is correct, what is wrong, if anything, with the following reasoning?

2B + 6R = 40. Since B = R, we can write,

2B + 6B = 40, and therefore,

8B = 40

This last equation says 8 books cost $40, so one book costs $5.

In this problem students may interpret the letter "B" to mean "books," "the number of books," "the price of books," "the number of books times the price," and other even more imaginative combinations.

"B" is indeed the number of books in the initial equation, but it is also the number of records in the substitution "B = R." Solving for "B" in the final equation yields the price of the average item, not the price per book. "B" is a variable--it stands for an unknown number--not for the word "books."

Teachers will recognize evidence of other misconceptions, as well:

Students often mistake the way in which an original price and a sale price reflect one another. They often incorrectly calculate the original price from a sale price by applying the discount to the known sale price, instead of to an unknown original price.

Students may misconceive the independent nature of chance events. For example, after getting heads on four consecutive tosses of a coin, they may claim that tails are more likely than heads on later tosses.

Most college students in remedial math courses want to subtract in the following problem: "Margaret had 2/3 of a gallon of ice cream. She ate 1/4 of it. How much ice cream did she eat?" Their misconception shows that they want to save multiplication for computing increases.

MATHEMATICAL MISCONCEPTIONS AMONG HISPANIC STUDENTS

The few studies that have investigated mathematical misconceptions among Hispanic students show that their error patterns are nearly always the result of differences in language or culture. The discussion that follows presents some of the unique difficulties experienced by Hispanics, as described in two recent studies (Mestre, 1982; 1986):
In the "students and professors" problem, Hispanics sometimes write the answers, "6S = 6P" and "6S + P = T." In the former case, they reason that the phrase "as many students as professors" implies there is an equal number of each. In the latter case, students claim that their equation (in which T = total number of students and professors) combines everyone in the correct proportions. In both cases, students' misconceptions come from language differences.

Some Hispanics wrote the answer "9 x 28 - 7" to the following problem: "In an engineering conference, 9 meeting rooms each had 28 participants, and there were 7 participants standing in the halls drinking coffee. How many participants were at the conference?" The students assumed "participant" referred only to people in the meeting rooms and that the coffee drinkers came out of the rooms. This misconception, too, has its roots in language differences.

Some Hispanic students did not grasp this problem: "A carpenter bought an equal number of nails and screws for $5.70. If each nail costs $.02 and each screw costs $.03, how many nails and how many screws did he buy?" They believed it meant the carpenter spent an equal amount of money on nails and screws. Again, the influence of language is clear.

The number of unique errors among Hispanics resulting from linguistic difficulties is, however, small. In general, they cause Hispanics to commit the same types of errors as Anglos, but with a higher frequency. Cocking and Mestre (1988), in an edited book, deal with this topic among various ethnic and cultural groups. For example, some chapters discuss the unique mathematical difficulties experienced by Native Americans and by Oksapmin aborigines of Papua, New Guinea.

IDENTIFYING AND HELPING STUDENTS OVERCOME MISCONCEPTIONS

Simply lecturing to students on a particular topic will not help most students give up their misconceptions. Since students actively construct knowledge, teachers must actively

help them dismantle their misconceptions. Teachers must also help students reconstruct conceptions capable of guiding their learning in the future.
Lochead & Mestre (1988) describe an effective inductive technique for these purposes. The technique induces conflict by drawing out the contradictions in students' misconceptions. In the process of resolving the conflict--a process that takes time--students reconstruct the concept.

The following discussion illustrates the three steps of this technique with the "students and professors" example:

1. PROBE FOR QUALITATIVE UNDERSTANDING. Keep on the look-out for misconceptions. A simple, well placed question can show if a student's difficulty comes from linguistic confusion, naive misconceptions, or both. In the "students and professors" problem, a good question to ask is "Are there more students or professors in the university?"

2. PROBE FOR QUANTITATIVE UNDERSTANDING. If students understand that there are more students than professors, the next step is to ask a question (for example, "Suppose there are 100 professors at the university. How many students would there be?") In most cases, students will give the answer, "600 students."

3. PROBE FOR CONCEPTUAL UNDERSTANDING. Next, ask students to write an equation, and look for common error patterns. Now is the time to induce conflict. For example, with the reversal error, "$6S = P$," the teacher might ask, "What would happen if you substituted $S=600$ in your equation? Would you get $P=100$, as before?"

With this inductive approach, the classroom can serve as a forum for some heated discussions among students who will disagree on an answer. Note that the teacher does not tell students "the right answer." Instead, the teacher guides them toward constructing it. In this way, students' most important and most effective learning has to do with concepts, not just correct numbers. An active classroom discussion, with the teacher serving as guide, helps students air their misconceptions and, together, truly overcome them.

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