DOCUMENT RESUME

ED 312 139 SE 050 909

TITLE	Problem Solving Strategies. Inservice Presentation. Bulletin 1788.
INSTITUTION	Louisiana State Dept. of Education, Baton Rouge. Bureau of Elementary Education.
PUB DATE	Jul 86
NOTE	126p.; For other presentations in this series, see SE 050 896, SE 050 908.
PUB TYPE	Guides - Classroom Use - Guides (For Teachers) (052)
EDRS PRICE	MF01/PC06 Plus Postage.
DESCRIPTORS	Elementary Education; Elementary School Teachers; *Inservice Teacher Education; Instructional Materials; Mathematical Applications; *Mathematics Materials; *Mathematics Skills; Mathematics Teachers; *Problem Solving; Teacher Improvement; Teacher Workshops

ABSTRACT

This document contains an inservice presentation for teaching common problem solving strategies in mathematics word problems. Sections included are: (1) "Introduction"; (2) "Common Word Problem Strategies"; (3) "Break into Parts"; (4) "Guess and Check/Test"; (5) "Working Backwards"; (6) "Look For a Pattern"; (7) "Make or Construct a Table, Chart, or List"; (8) "Logical Thinking"; (9) "Make a Graph"; (10) "Act It Out"; (11) "Draw a Picture"; (12) "Solve a Similar/Simpler Problem"; (13) "Write an Equation"; and (14) "Suggestions for Teaching Problem Solving." Transparencies, handouts, and a list of five references are appended. (YP)

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PROBLEM SOLVING STRATEGIES

INSERVICE PRESENTATION

DEVELOPED BY THE

LOUISIANA STATE DEPARTMENT OF EDUCATION

BUREAU OF ELEMENTARY EDUCATION

BULLETIN 1788

July, 1986

This public document was published at a total cost of $652.45; 300 copies of this public document were published in this first printing at a cost of $652.45. The total cost of all printings of this document, including reprints, is $652.45. This document was published by the Louisiana Department of Education, Bureau of Elementary Education, Post Office Box 94064, Baton Rouge, Louisiana 70804, to provide leadership for continuous development, coordination, and improvement of education in the State of Louisiana under authority of R.S. 17:21. This material was printed in accordance with the standards for printing by state agencies established pursuant to R.S. 43:31.

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PROBLEM-SOLVING STRATEGIES

Introduction

(Transparency #1): Show transparency and state goal of workshop.

Goal: Teachers will learn common problem-solving strategies.

A great deal of the emphasis of our mathematics curriculum has traditionally been placed on developing arithmetic skills. The back-to-the-basics movement has certainly supported that emphasis. Students are required to memorize their basic facts and to do drills on pages of arithmetic problems, often in situations where they're encouraged to write answers as fast as possible. After a unit of addition, there is a section of addition word problems. After subtraction, subtraction word problems are provided. It's all very neat and seems logical. The sad fact is that the approach is basically backwards.

We teach the abstract process of computation first and then hope that children will learn to use these processes to solve problems. That's the backwards part. The emphasis on problems must come first; it's the starting place for developing arithmetic understanding and for establishing the need for computation. The child needs to see that developing computational skills serves a purpose--that computation skills are tools for solving problems. Mathematics needs to be taught so that it makes sense to the child.

Many children think that addition, subtraction, multiplication, or division is taking numbers and doing something to them to get an answer. They think this way because the operations are introduced as recipes for working with figures to produce answers.

The effective teaching of mathematics should teach children "how to think." To accomplish this, teachers should practice certain methods of teaching the strategie^ for problem solving.

A problem is a proposed question for which the answer is not known immediately. For example 8 + 7 is a problem to most children in kindergarten, but should not be a problem for a child in the third grade.

Children should be exposed to problems when they are very young, to problems that require thinking beyond a rote response.

The world children will live in as adults will require that they acquire the skills to solve problems and to ask good questions.

To be effective in helping children sharpen their skills of solving problems, teachers must change their teaching methods. The teachers can no longer be the fountain of knowledge at the front of the room who can illustrate all the steps of a solution to an exercise with numbers, like compound multiplication.

Instead, teachers should present relevant problems and guide children toward solutions rather than telling them answers.

Teachers should give children time to solve problems. This is called "wait time", the period in which the teacher does not talk but allows children the opportunity to think or talk to each other.

Teachers also need to learn to ask good questions. These are questions that help children go forward toward solutions, but don't tell the children how to solve the problems.

In order to help children find methods which can be used to solve problems, we will now look at 12 problem-solving strategies. Though this list is not comprehensive, it is more than enough for a beginning.

(Show transparency #2 and briefly review the 12 strategies.)

Say: These strategies serve as a basis for problem solving in school and in life. We are going to discuss and experience six of these strategies. Research has shown that when students were taught these strategies, their problem-solving abilities greatly improved and their scores were three times higher.

Strategies are not specific to particular problems or to particular areas of the mathematics curriculum, but can be applied alone or in combination with other strategies to solve a wide variety of problems. Gaining familiarity with several strategies, seeing them modeled, and then trying to apply them can provide students with useful tools for tackling problems.

Common Word Problems Strategies

Some strategies are used at all grade levels. These common strategies require the least effort on the part of children, but they are the strategies used most often in solving word problems. Solving word problems in textbooks is only a small part of learning how to be a good problem solver.

The four strategies most often used in solving word problems are as follows: (SHOW TRANSPARENCY #3.)

1. Write or ask a question.

2. What is missing?

3. What is the operation?

4. Write a problem.

WRITE OR ASK A QUESTION

The question is very important in solving word problems. To emphasize the importance of the question, children should practice writing or asking questions.

ACTIVITY I

Primary

(Show transparency #4.)

Say: Given: 17 boys and 14 girls

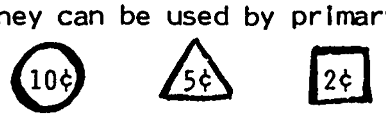
Ask/write a question. (Wait and elicit responses)

Ex: How many boys and girls in all?

How many more boys than girls?

Facts using money can be used by primary children to ask questions.

Given:



Use the facts about the cost of the shapes to ask some questions. (Wait and elicit responses.)

How much more does a circle cost than a triangle?

What is the cost of both a circle and a square?

(Show transparency.)

UPPER GRADES

Given: A movie costs $4 for adults and $2 for children under 16.

Ask/write a question. (Wait and elicit responses.)

Ex: How much do tickets cost for two adults and one child?

How much more does an adult ticket cost than a childs ticket?

Facts can also be shown in an advertising format.

Given:

Bananas	Grapefruit	Potatoes
3 lbs. for 85¢	3 for $1.00 40¢ each	5 lbs. for $1.50 or 35¢ a lb.

Use the information in the ads to ask or write questions. (Wait and elicit responses.)

Ex: How much is saved if you buy one 5 lb. bag of potatoes instead of five 1 lb. bags?

How much do two grapefruit cost?

How much do six pounds of bananas cost?

ACTIVITY II

(Show transparency #5 "What's the Question." Allow time for participants to complete and discuss.)

The second strategy commonly used to solve word problems is to find:

WHAT IS MISSING?

Sometimes problems do not have enough information to be solved. Children need to tell which fact is missing.

ACTIVITY III

Given: Bob weighs 62 kilograms. How much heavier is Bob than Mike?

Missing Fact: Mike's weight (allow time for participants to respond)

Given: Joe made $52 painting fences. How much did he earn for each hour he worked?

Missing Fact: The number of hours Joe worked (allow time)

Given: The girl's club made $340 selling bags of popcorn. How much did each bag of popcorn cost?

Missing Fact: The number of bags of popcorn sold (allow time)

ACTIVITY IV

(Show Transparency #6 "What's Missing:" Allow time for participants to complete and discuss.)

The third strategy is to determine:

WHAT IS THE OPERATION?

After reading a problem, children need to decide whether to add, subtract, multiply, or divide. There are four general rules that can help students make a decision.

(Show transparency #7.)

A. When you combine two or more things, you add.

B. When you find the difference between numbers or amounts or which is larger or taller, you subtract.

C. When you repeat one number several times, you multiply.

D. When you separate something into groups of the same amount, you divide.

To practice picking out the operation, often the best activity is to circle the correct symbol needed to solve the problem. It is not necessary to do the computation.

ACTIVITY V

(Show Transparency #8 and discuss one at a time.)

Given: Mike is 62 inches tall, and Sam is 58 inches tall. How much taller is Mike than Sam?

+ − x ÷

Given: A camping trip for five boys cost $70.45. How much did each boy pay?

+ − x ÷

Given: Sue baked 142 cookies. Mary baked 86 cookies. Ann baked 108 cookies. How many cookies did they bake in all?

+ − x ÷

Given: There are eight cans in a carton. How many cans are there in nine cartons?

+ − x ÷

WRITE A PROBLEM

Children need the experience of writing problems, to think of a question and the facts to make up the problem. Teachers can give an answer and then have the children write the problem. Sometimes, children may be amazed at how many different problems can have the same answer.

ACTIVITY VI

Primary

(Show transparency #9.) Ask that participants think of a question and the facts to make a problem. Allow time and discuss orally.

(In the following activity, give the answer and allow time for participants to write their problems. Note how different problems can have the same answer.)

ACTIVITY VII

Given: The answer is $7.50.

Ex: Problem: I have $10. I buy a book for $2.50. How much change do I receive?

Given: The answer is 6 robots.

Ex: Problem: Ruth has 24 robots. Tom has 18 robots. How many more robots does Ruth have than Tom?

Given: The answer is 300 miles.

Ex: Problem: The Smith family traveled 85 miles on Monday, 150 miles on Tuesday, and 65 miles on Wednesday. How many miles did the family travel in the three days?

ACTIVITY VIII

(Show transparencies #10 and #11 "Make up the Problem." Allow time to complete and discuss.)

After children have written problems, they should solve problems others have written.

The teacher needs to be a good model of a problem solver. Teachers will find that once children have success in solving one problem, they are willing to risk tackling other problems.

The arrangement of the class can be different for problem-solving sessions. It's easier for children to exchange ideas while sitting in groups than in rows. The role of the child changes in problem-solving situations. They are not spectators but participants. Each child should be encouraged to contribute ideas and to accept the ideas of others as having value. It is important for children to know that much can be learned from what appear to be wrong answers.

BREAK INTO PARTS

The next strategy that we will discuss in this session is Break into Parts.

According to research, in problem solving children have much more trouble with two-step and multi-step problems than with one-step problems. In a multi-step problem, the solver is required to obtair some preliminary information from the problem itself before proceeding on to the final answer.

(Show transparency #12.)

There are five cages in the science lab, and each cage has three white mice. If one mouse eats 15 grams of food each week, how many grams of food do the mice eat each week?

The question to be answered is "How many grams of food do the mice eat each week?" Since the amount eaten by each mouse is given, the preliminary information needed is "how many mice there are in the lab." This is the "Hidden Question" or "Hidden Step."

Multi-step problems present greater difficulty to students because they require this "in-between" step, and because most of their problem-solving experiences have been with one-step problems.

Let's find some hidden questions.

(Transparency #13)

"Mike bought a hamburger for $1.79 and a soft drink for 79¢, tax included. How much change did he receive from a $5 bill?" What is the hidden question? (What is the cost of the hamburger and cola? $2.58) What is the final answer? ($2.42)

Let's try another.

"Linda bought four record albums at $6.49 each. She paid $1.82 in sales tax. How much did she spend altogether?" What is the hidden question? How much did the four records cost before the tax was added? ($25.96) What is the final answer? ($27.78)

Again, let's try another.

A Technical Lab has an inventory of 30 cases of calculators packed six to a case and nine cases of adding machines packed two to a case. If a case of calculators costs $81.00 and a case of adding machines costs $128.98, what is the value of Technical Lab's inventory?

What are some hidden questions?

What is the cost of the calculators? ($2,430.00)

The adding machines ($1,160.82)

What is the total of both? ($3,590.82)

It there any information given that is unnecessary? (the number packed in each case).

Let's try this one.

The current balance in Allyson Button's savings account is $2,375.10. If the bank adds 0.5% interest to her account every month, and she makes no deposits or withdrawals, to the nearest cent what will be the balance in her savings account at the end of four months? (answer $2,422.62)

What are some hidden questions?

What is the interest each month? ($11.88)

Let's try a problem now which contains more than one hidden question. In your group, list those questions. Do not be concerned with the final answer.

"The length of a rectangle is 12 inches and its perimeter is 34 inches. What is the area of the rectangle?" (1. What is the sum of the known sides? 2. What is the difference between that sum and 34? 3. What is the length of one of the unknown sides?)

(Allow time for groups to work. Then have groups share. Questions may vary somewhat from those I have listed.)

ACTIVITY

(Show transparency #14 "Hidden Questions." Allow time to complete and then review.)

Summarizing

Many times the problem is not difficult, it is just complex. In teaching this technique, first show the pupils how to use this approach with a number of examples. Then have them practice the technique with a similar problem.

(Ask for questions or comments.)

GUESS AND CHECK/TEST

Probably the most commonly used strategy in our everyday lives is guess and test or check. This strategy involves guessing at an answer and testing or checking to see if the guess works. Guess and test is a reliable method in most areas of math and science. Hypotheses (the guess) are made and then verified by testing them (the test) in order to draw a conclusion. But this strategy is contrary to traditional teaching, in which guessing is not encouraged. Have you ever heard or made the statement, "Do you really know, or are you just guessing?" In order to discourage blind or wild guessing, children need to learn how to make an educated guess, test the guess, and if the guess is not correct, use that information to make another guess. Children need to learn that finding a solution may take several guesses, but that it's important to continue until a solution is found.

A prerequisite to making an educated guess is estimation. Children should be given many opportunities to estimate and then to check for accuracy. Have students estimate a measurement of any kind, then check to see what the correct answer is. How many inches tall is that person? How many meters is it between those two telephone poles? How many pounds does that book weigh? How many more pounds do you think the dictionary weighs? By providing opportunities for children to guess and check, you are providing a frame of reference from which they can make educated guesses.

How good are you at guessing? (Pass out Guess Test and follow instructions on the test. After about five minutes say: Now discuss you answer with your small group giving your reasons for your answers and see if your answers change after the small group discussion. As a large group, quickly discuss the answers and their reasonableness. Ask: "How important are numerous experiences to making an educated guess?")

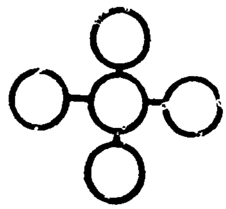
Now let us look at various activities which will provide children opportunities to learn to guess and check. (Choose only a few of the activities which best meet the needs of the group with which you are working. The entire group of activities can be presented as a handout at the end of the inservice session.)

ACTIVITY I

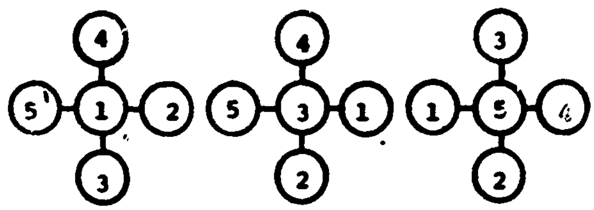
Primary

Number Puzzles (Transparency #15 and #16)

Use the numbers 1, 2, 3. 4, and 5 to make to sum of eight in both directions on the following puzzle.



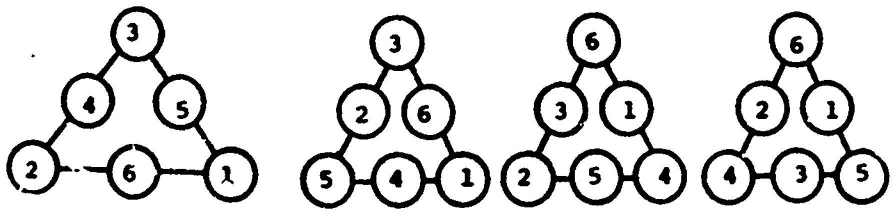
This activity can also be done using discs with the number 1, 2, 3, 4, and 5 written on them. The child can move the discs around until the sum of 8 is in both directions. Have the child make a record of his results. After he finds the sum of 8, have him find the sums of 9 and 10.



Sum 8 Sum 9 Sum 10

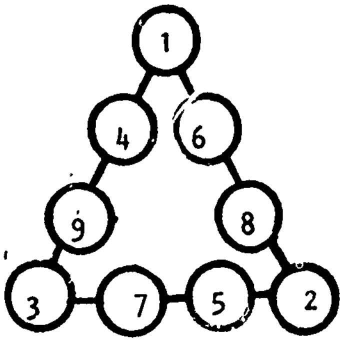
Number Puzzles-Upper Grades

Use the numbers 1, 2, 3, 4, 5, and 6 to make the sum of 9 in all three directions. After he finds the sum of 9, have him find the sums of 10. 11, and 12.

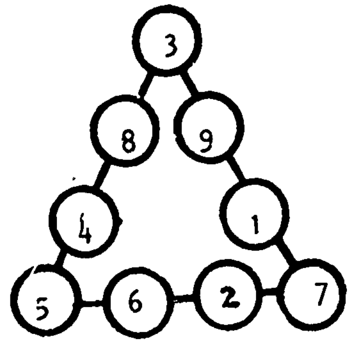


Sum 9 Sum 10 Sum 11 Sum 12

Place the digits from 1-9 in the triangle so that each side has a sum of 17, 20.



Sum 17



Sum 20

ACTIVITY II

Sum of Consecutive Numbers (Transparency #17)

The sum of three consecutive numbers is 45.
What are the numbers?

1st Guess: _____ + _____ + _____ Test: _____

2nd Guess: _____ + _____ + _____ Test: _____

3rd Guess: _____ + _____ + _____ Test: _____
(14 + 15 + 16 = 45)

The example above shows that using the first and second guesses is helpful in finding the solution by the third try. It is possible that a child will need only two guesses or more than three will be needed. The point to stress is that guessing is one way to find a solution and that one needs to learn to use the guess to help solve problems.

Given: The area of a rectangle is 36 sq. in. The perimeter is 40 inches. What is the length and width of the rectangle?

1st Guess: Length _____ Width _____ Test: _____

2nd Guess: Length _____ Width _____ Test: _____

3rd Guess: Length _____ Width _____ Test: _____
(2 x 18)

ACTIVITY III

Computation Puzzle

Find as many ways to make this true using the numbers 1-9.

```
   OOO
 + OOO
 -----
   OOO
```

The discs from the number puzzle activity could also be used for this activity. Have the child record his results. (There are 320 possible solutions. Digits in sum will equal 18.)

ACTIVITY IV

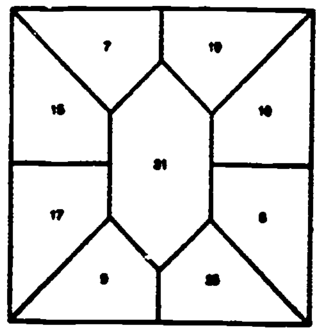
Coin Problem

There are exactly 10 coins with a total value of $1. If three of the coins are quarters, what are the remaining coins? (2 dimes, 5 pennies)

ACTIVITY V

Dartboard Math (Transparency #18)

Mary hit the dartboard below with four darts. Each dart hit a different number. Her total score was 60. Find the different ways she could have scored 60.



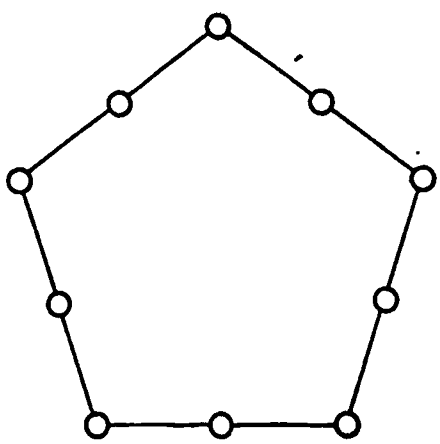
(This problem has more than one solution. However, the solution must contain four odd numbers in order to have an even sum. Two possible solutions are:
19 + 17 + 15 + 9 = 60   31 + 15 + 9 + 5 = 60)

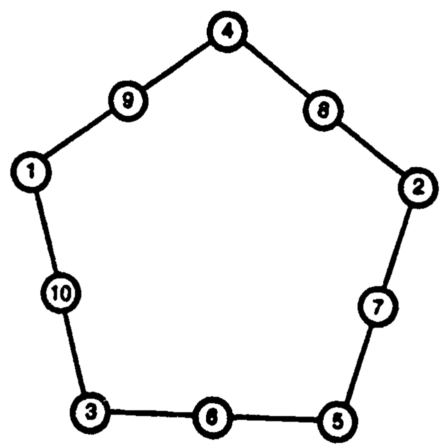
ACTIVITY VI

Pentagon Math

Fill in the empty circles on each side of the pentagon with the numbers 1-10, so that the sum of the numbers on each side is 14.

Here is one solution:





ACTIVITY VII

Guess the Number (Transparency #19)

If 17 is subtracted from four times the number, the result is 35. What is the number? (13)

ACTIVITY VIII

Problem: (Transparency #19)

A certain bag of marbles can be broken up into 2, 3, 4, 5, or 6 separate but equal piles with one marble left over each time. It can also be separated into seven piles evenly with none left. How many marbles are in the bag?

Solution:

We know the answer must be a multiple of 7. If divided into two equal parts with one left over, we know the number cannot be even. If divided by five with one left over, we know the number must end in either a one or a six. Since six would produce an even number, we know the number must end in one. Find a multiple of seven ending in one that is divisible by 3, 4, 5, and 6 with one left over each time. The number is 301.

Problem:

In an effort to motivate his daughter to do her mathematics homework quickly and accurately, a father offered to pay her 8¢ for each correct problem, but to "fine" her 5¢ for each incorrect one. After doing 26 problems, they found that neither owed the other any money. How many problems did the daughter solve correctly?

Solution:

One set of numbers the students might guess could be 5 correct (+ 40¢) and 8 incorrect (- 40¢). This does yield a solution where no one owes any money. However, it only has a total of 13 problems. Doubling this to 10 correct (+ 80¢) and 16 incorrect (- 80¢) give the solution to the problem.

Problem:

Leroy mailed some letters and postcards which cost him a total of $3.85 in postage. If each letter costs 20¢ and each postcard costs 13¢, how many of each did he mail?

Solution:

This problem can be simplified by using refined guessing. Note that $3.85 implies that the number of postcards mailed must end in a "5." Thus the only possible answers for the number of postcards Leroy mailed would be 5, 15, or 25. Testing these guesses yields solutions for both 5 and 25 postcards.

As you can see, there is real value in teaching students how to make an educated guess and then check. They must understand it's all right not to get the correct answer on the first guess, but it's important to teach how to use that information to help them get closer to the correct answer.

ACTIVITY IX

Build a Sum

Choose a sum. Using all the digits 0-9, build the sum.

(Transparency #19)

144	0	8451	135
	1		249
	23		8067
	95		8451
	6		
	7		
	8		
	4		
	144		

ACTIVITY X

A. There was a group of children and their dogs playing in the park. Twelve heads and 38 legs could be counted. How many dogs and children were there? (7 dogs, 5 children) (Transparency #20)

B. A farmer has hens and rabbits. These animals have 50 heads and 140 feet. How many hens and how many rabbits has the farmer? (20 rabbits, 30 hens)

ACTIVITY XI

Alphanumerics

Each letter represents a specific digit from 0 to 9. There are many solutions to these particular problems. (One solution is provided for each.)

(Transparency #20)

Add	One	621	Subtract	Five	7862
	Two	836		- Four	-7410
	+ Five	9071		One	452
	Eight	10,528			

Adam	8384
and	803
Eve	626
on	50
a	8
Raft	9871

WORKING BACKWARDS

Another strategy we often use when the outcome of a situation is known but the initial conditions are unknown is working backwards. When working backwards, we reverse the original operations. That is, subtraction will replace addition, and division will replace multiplication. Let's look at several activities that demonstrate this strategy.

(Below are a number of activities which can best be solved by working backwards. Choose several of the activities to teach the strategy to the group. Do not use all of the activities with a group.)

ACTIVITY I (Transparency #21)

Multiply a given number by 4; add 3; the answer is 31. What is the number?

Explanation: To find the number, each operation is reversed, starting with the last operation, and moving up to the first operation. Instead of adding 3, subtract 3. Instead of multiplying by 4, divide by 4.
(31 - 3 = 28 ÷ 4 = 7)

Add a given number to 5; multiply by 2; subtract 1; the answer is 21. What is the given number? (21 + 1 = 22 ÷ 2 = 11 - 5 = 6)

Subtract 4 from a given number; multiply by 3; and 5; the answer is 29. What is the given number? (29 - 5 = 24 ÷ 3 = 8 + 4 = 12)

ACTIVITY II

If children know how to find an average by adding the numbers and then dividing by the number of numbers, they can do the next examples by working backward.

Examples: (Transparency)

Problem:

Bill made an average score of 83 on four spelling tests. His papers show the results of three tests were 72, 85, and 85. He wants to know the score of the fourth test. Find the score.

Solution:

To find the average score of 83, some number was divided by 4. The reverse of division is multiplication. 83 x 4 = 332. The four test scores add up to 332. The papers he has add up to 242. Then, 332 - 242 = 90. The score of the fourth test is 90.

Problem:

The three hottest cities of the USA averages a temperature of 107 degrees. Phoenix was 115 degrees. Palm Springs was 105 degrees. The third city was Santa Fe, but the paper lost the temperature. What was the temperature in Santa Fe?

Solution:

The solution is similar to the problem above. Multiply by 3, then add and subtract to find the solution.

Problem:

The local library fine schedule for overdue books is as follows: 10¢ per day for each of the first three days, 7¢ per day thereafter. Sally paid a fine of $1.00. How many days was her book overdue?

Solution:

We know the final outcome was a $1 fine. Sally paid 30¢ for the first three days. This leaves 70¢ for the remainder of the fine. At 7¢ a day this is 10 days. Her book was 13 days overdue. To attack this problem directly, the order of operations would be the multiply 3 x 10¢ and add that to the product of 10 x 7¢. However, in the working backwards mode, we multiply 3 x 10¢, subtract, and then divide.

Problem: (Transparency)

Jimmy was trying a number trick on Sandy. He told her to pick a number, add 5 to it, multiply the sum by 3, then subtract 10 and double the result. Sandy's final answer was 28. What number did she start with?

Solution:

We begin with Sandy's answer, 28. Since she doubled the result divide to get 14. She subtracted 10, so we add 10 (to get 24). Divide by 3 (8), subtract 5. Her starting number was 3.

Problem:

After receiving her weekly take-home pay, Marcy paid her roommate the $8 she owed for her share of the telephone bill. She then spent one-half of what was left on clothes, and then spent one-half of what was left on a concert ticket. She bought six stamps for 20¢ each, and has $12.10 left. What is her weekly take-home pay?

Solution:

Begin with the $12.10 that Marcy had left. Add the $1.20 she spent on stamps ($13.30). Since she spent half on a concert ticket, multiply by two ($26.60). She spent half on clothes; multiply by two ($53.20). She paid $8 for the telephone bill, so we add $8. Her take-home pay is $61.20.

Problem:

Alice, Beth, and Carol decide to play a game of cards. They agree on the following procedure: when a player loses a game, she will double the amount of money that each of the other players already has. First Alice loses a hand, and doubles the amount of money that Beth and Carol each have. Then Beth loses a hand and doubles the amount of money that Alice and Carol each have. Then Carol loses a hand and doubles the amount of money that Alice and Beth each have. They then decide to quit and find that each of them has exactly $8. How much did each of them start with?

Solution:

We represent the action with a list, while working backwards:

Alice	Beth	Carol	
$ 8	$ 8	$ 8	
4	4	16	
2	14	8	
13	7	4	(The amounts each started with)

Problem:

Sally and the Peanuts (Transparency #22)

Activity: On the way home from school, Sally McCrackin likes to eat peanuts. One day, just as she was reaching into her sack, a hideous, laughing creature jumped into her path, identified itself as a pigs eye, and grabbed her sack. It stole half of her peanuts plus two more. A bit shaken, Sally continued home. Before she had a chance to eat even one peanut, another horrid creature jumped into her path and also stole half of her peanuts plus two more. Upset, she continued on. (What else could she do?) But before she had a chance to eat even one peanut, another of these tricksters jumped out and did the very same thing--took half of her peanuts plus two more.

Now there were only two peanuts left in Sally's sack. She was so despairing, she sat down and began to sob. The three little pig eyes reappeared, feeling some sense of remorse, and told her they would return all her peanuts to her is she told them how many she had when she started. How many peanuts had been in Sally's sack? (Suppose Sally had been left with 3 peanuts? Or there had been four nasty pig eyes? Can you find a way to predict how many peanuts Sally had in her sack to start with regardless of how many she was left with, or how many pig eyes stole peanuts from her?)

Solution:

Begin with the two peanuts Sally had left. Double and multiply by two (8). Add two and multiply by 2 (20). Add two and multiply by 2 again. The number of peanuts originally in the bag was 44.

The more practice one has with this strategy, as with any of the strategies, problem solving whether routine or nonroutine becomes much easier. It is important that we as teachers provide numerous experiences and opportunities for students to practice each of the problem-solving strategies so that they become second nature when a problem is presented.

LOOK FOR A PATTERN

Patterns and relationships are very important to children understanding mathematics. Children should be encouraged to think, seek, and discover patterns for themselves.

*<u>Primary Grades</u>--(For upper grades skip to Lorton's quote.)
For younger children, recognizing and using patterns are basic skills for them to develop in their math learning. There are ways to give children experience with patterns at their level. There are ways to provide these experiences in a variety of ways--visually, verbally, and physically.

Look at the dots on the transparency. One simple way to begin work on patterns is on a series of dots.

<u>Dot Patterns</u> (Transparency #23)

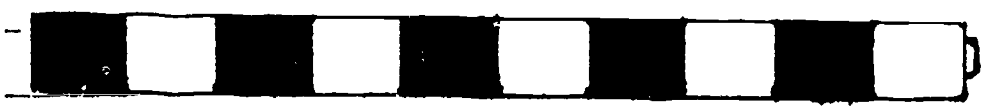
bump, line, bump, line, ...
tunnel, road, tunnel, road, ...
bridge, river, bridge, river, bridge, ...

Encourage the children to verbalize the pattern in many different ways.

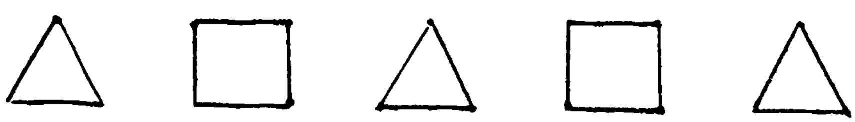
Teachers also use rhythmic clapping.

clap, clap, stamp, clap, clap, stamp, ...
or
snap, snap, clap, clap, snap, snap, clap, ...

Patterns can also be interpreted with materials such as Unifix cubes or pattern blocks. (Transparency #24)

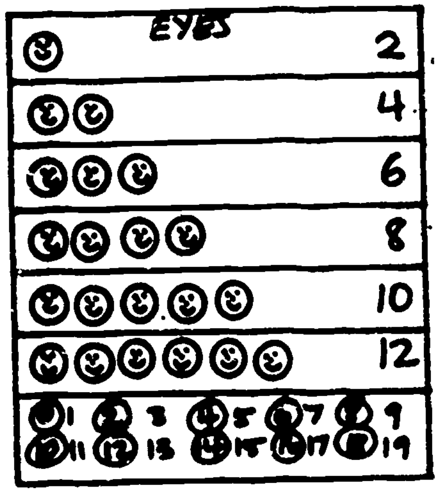


red, yellow, red, yellow, red, yellow, ...



triangle, square, triangle, square, triangle, square, ...

It is important to provide children opportunities to experiment using real experiences. For example: The Eyes Pattern. Have one child stand up. Ask the class how many eyes one child has. Have another child come up also. Count how many eyes two have. Continue. Record as shown below. (Use transparency #25.) Children can make pattern records like this on individual recording sheets.



Patterns are plentiful. Mary Baratta Lorton offers an explanation of the importance of the study of patterns.

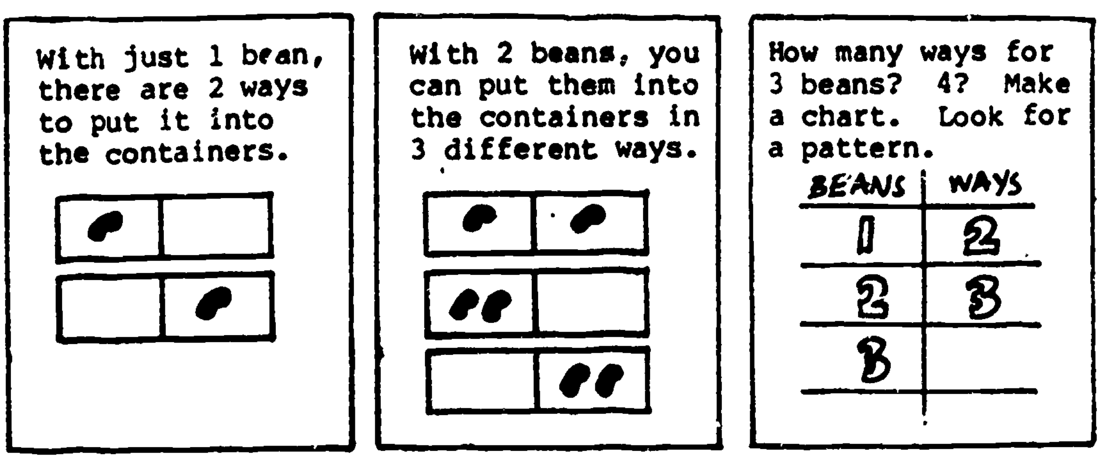
> "Looking for patterns trains the mind to search out and discover the similarities that bind seemingly unrelated information together in a whole ... A child who expects things to make sense looks for the sense in things and from this sense develops understanding. A child who does not see patterns often does not expect things to make sense and sees all events as discrete, separate, and unrelated."

ACTIVITY I

Primary

Beans and Ways (Transparency #26)

Suppose you had 25 beans (or pennies or other counters) and 2 containers (small cups or squares drawn on paper). How many ways could you put the 25 be

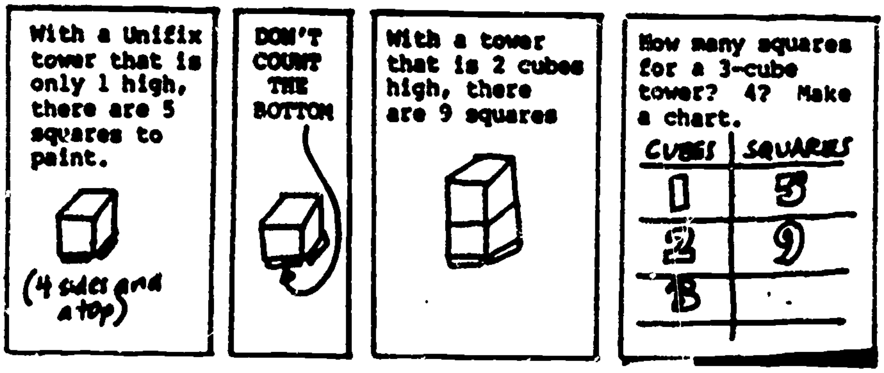


ACTIVITY II

Primary

Unifix Towers (Transparency #27)

Suppose you build a Unifix tower that was 99 cubes high. And suppose you had to paint each square on the tower. How many squares would you have to paint?



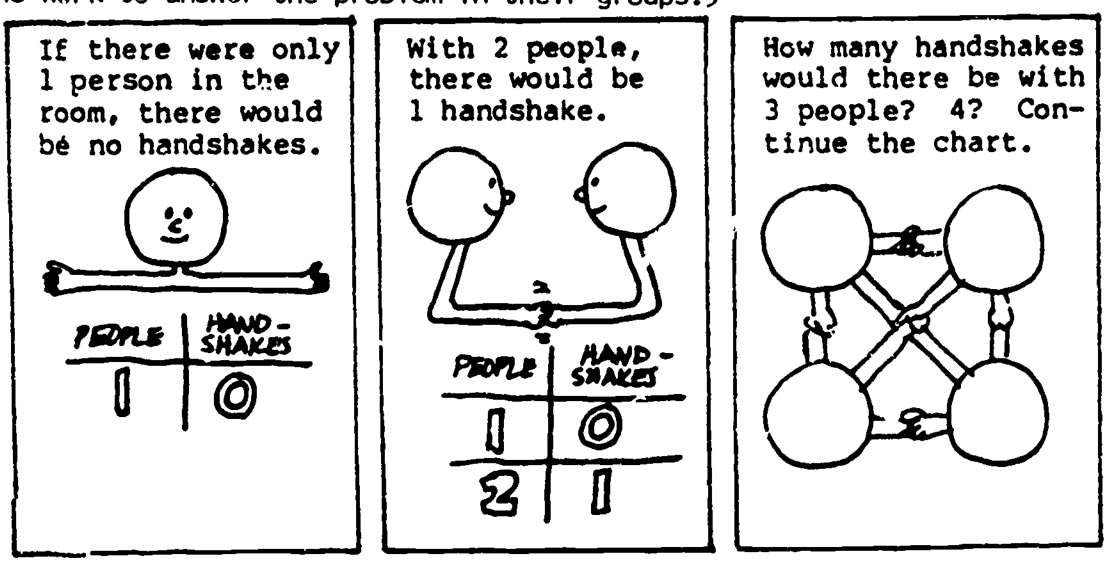
ACTIVITY III

Primary

The Handshake Problem (Transparency #28)

Suppose everyone in this room were to shake hands with every other person in the room. How many handshakes would that be?

(Have students come up and act this out.. After you've completed the chart for five people, have them describe patterns they see. Have them complete the work to answer the problem in their groups.)



UPPER GRADES

ACTIVITY I--Unifix Cubes (Transparency)

ACTIVITY II--Handshake Problem (Transparency)

ACTIVITY III--Number Patterns (Transparency)

Write the following patterns on the transparency. Allow the group time to determine the pattern. Ask the group to tell you what rule should be followed to continue the pattern.

Examples (Transparency #29)

Find the Rule. Complete the series.

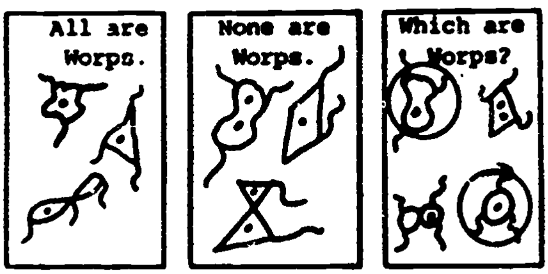
Given: 3, 6, 12, 24, 48, 96
Rule: Multiply by 2.

Given: 6, 10, 7, 11, 8, 12, 9
Rule: Add 4, subtract 3.

Given: 2, 2, 4, 6, 10, 16, 26, 42
Rule: Add two consecutive numbers.

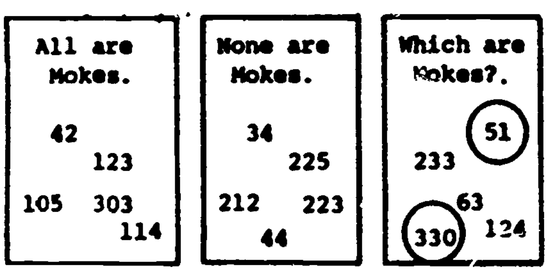
ACTIVITY IV (Transparency #30)

A. Find the common property. Circle the answers.



Worps have 1 dot and 3 tails. The children need to find the common properties. (Transparency #31)

B.



Mokes are numbers whose digits add to 6.

ACTIVITY V

Hidden Rules (Transparency #32)

23	38	71	18
437	583	910	381

Look at all four cards. The number in the middle is formed from the two-digit number in the corner. The same rules are applied to each of the four cards. Children need to analyze the data. Then, they test the rules on the cards below by writing the three-digit number on each one.

16	40	63	27

The rule is: First add 20 to the two-digit number. Write the result on the card. This number is the first two digits of the three-digit number. Then, add the two digits. This number is the third digit of the three-digit number. If the sum of the two digits is more than 9, then record only the number in the one's place. For example, look at the second card at the top of the page.

38 + 20 = 58. 5 + 8 = 1<u>3</u>. The three-digit number is 583.

To sum up

There are several things we need to remember about patterns.

(Transparency #33)

1. Patterns are the basis of how our number system is structured.

2. Children think of math as a set of rules and steps to follow, to which getting the right answer is the goal. Children do not try to look for the underlying order and logic of mathematics.

3. If children do not learn to use patterns as a basic approach to understanding, learning is much more difficult than it should be.

4. Patterns make it possible to predict what is supposed to happen in math, rather than seeing the teacher's answer book as the only source for verification of thinking.

MAKE OR CONSTRUCT A TABLE, CHART, OR LIST

Putting information in a table, chart, or list is an excellent way to organize data. Once information is organized in this way, solutions to problems can be more easily found.

Now we are going to do several problems that may be easier to solve by making or constructing a table, chart, or list.

ACTIVITY I

Primary (Transparency #34)

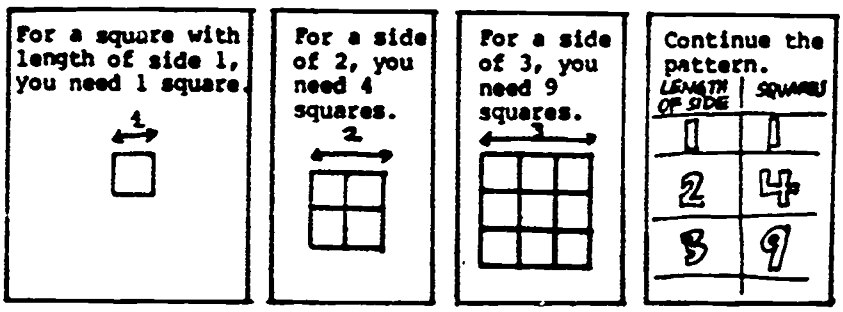
Given: You have dimes, nickels, and pennies. How many ways can you make 10¢ using dimes, nickels, and pennies?

10¢	5¢	1¢
1	0	0
0	2	0
0	1	5
0	0	10

ACTIVITY II

Squares from Squares (Transparency #35)

If you build bigger and bigger squares from small squares, how many squares will you need to build one that measures 12 on a side?



ACTIVITY III (Transparency #35)

Review with the group the Handshake Problem, Unifix Towers, and Beans and Ways. Discuss with the group how arranging information into a chart made the solution to the problem easier to find.

ACTIVITY I

Upper Grades (Transparency #36)

Given: You have quarters, dimes, nickels, and pennies. How many ways can you make 25¢?

25¢	10¢	5¢	1¢
1	0	0	0
0	2	1	0
0	2	0	5
⋮	⋮	⋮	⋮
0	0	0	25

ACTIVITY II (Transparency #37)

Given: There are the same number of bicycles and wagons. There are 30 wheels altogether. How many of each are there?

Bicycles	Wagons	Total Wheels
1	1	6
3	3	18
4	4	24
5	5	30

By entering the information in a table, the solution of 5 bicycles and 5 wagons is easily found.

ACTIVITY III

The manager of a racquetball club would like to know the average amount paid per week by each player using Court 1. From previous reports, she noticed that Mary and John each play 8.5 hours a week; their daughter, Cammy, plays 9 hours; Brad plays 7 hours; his son, Ken, plays 5 hours; Brad's daughter, Darsi, plays 12 hours; Yolanda plays 9.5 hours; and her son, Vernon, plays 11 hours. If each of the adults pays $4 an hour for court time and the children pay half of the adult fee, what is the average amount paid per week be each player?

Solution:

One way to work with the information in this problem is to organize it in a table. Copy and complete the table. (Transparency #38)

	Hours played each week	Court fees
Mary	8.5	$34
John	8.5	$34
Cammy	9	$18
Brad	7	$28
Ken	5	$10
Darsi	12	$48
Yolanda	?	?
Vernon	?	?
	TOTAL	?

1. To find the average amount paid per week by each player, add the weekly court fees and ___ the total by the number of players?

2. The average amount paid per week by each player is ___?

LOGICAL THINKING (REASONING)

The process of reasoning is basic to all mathematics. Mathematics is first and foremost a way of thinking, rather than a body of facts. It may be said that children will be better logical thinkers if they practice logical reasoning. Getting children to stop and think, to consider the consequences of their actions, to plan ahead, and to consider multiple alternatives is another important area of problem solving.

At all grade levels, children benefit from experiences that help them gain clarity, precision, and conciseness in their thought processes. The beginning approach to logical thinking needs to be informal. There should be many opportunities for children to explore and manipulate concrete objects, to identify likenesses and differences, to classify and categorize, and to state generalizations. Oral experiences using terms that are integral to logical thinking need to be provided. These terms include <u>all</u>, <u>some</u>, <u>none</u>, <u>if</u>, <u>then</u>, etc. In this way, children can gain familiarity with this language of logic in non-mathematical contexts.

Activity I

Pretest

Now let's see how logical you are. (Pass out the pre-test "^re You Logical?" Allow time to complete and review by giving correct answ .s.)

(Answers)

1. Figure 5, 10 points

2. Figure 2, 10 points

3. Figure 4, 10 points

4. 12, 20 points

5. 8 o'clock plus 5 hours is 1 o'clock, 20 points

6. your right elbow, 10 points

7. nine more balls, 20 points

Your Score

100-85 Excellent

80-60 Average

50-20 Weak

10-0 Very Poor

Activity II (Transparency #39)

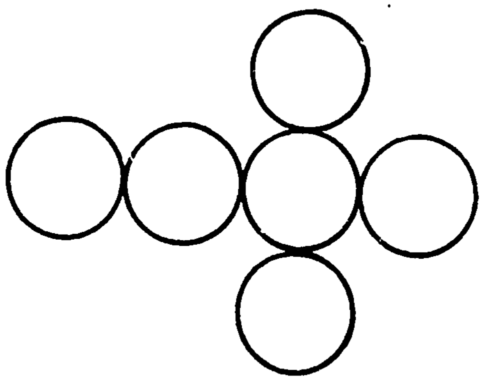
This next activity will provide experience in identifying likenesses and differences and categorizing objects by their characteristic features. In groups of 4, complete the activity entitled "All, Some, None." (Allow time. Discuss and ask for volunteer statements.)

Activity III

In order to provide an experience using concrete objects, let's work the following activity entitled "Rows of Money."

ROWS OF MONEY

Six quarters form two rows, one row with three quarters, one row with four quarters. Move one coin to make two rows each having four quarters.



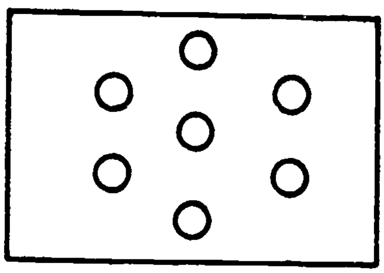
(Allow time and ask for a volunteer to explain the solution.)

Activity IV

Now let's look at another problem requiring logical reasoning.

PEN THE PENNIES!

With three straight lines separate all the pennies. (Transparency #40)



(Allow time to experiment and discuss.)

Activity V

This activity is a word game used to provide experience using logical thinking to solve a problem.

GUESS MY WORD

One player thinks of a three-letter word for the other to guess. Whenever the person guesses, the player who thought of the word tells whether the guess comes "before" or "after" the word in the alphabet.

(Play a sample game with the class first, and then allow partners to play the game several times.)

Activity VI

Now let's take a look at a word problem that involves logical reasoning.

(Show transparency #41, "Logical Breakfast," allow time to work through the problem, and then discuss.)

LOGICAL BREAKFAST

Steven, Doreen, and Jay each ate something different for breakfast. One had granola, one had scrambled eggs and toast, one had a banana split. (The last one was because the parents were away on vacation and there was a very lenient babysitter.)

Jay did not have scrambled eggs and toast or a banana split.
Doreen did not have scrambled eggs and toast.

Whose parents were on vacation? (Doreen's)

Activity VII

Let's try one that requires more logical reasoning.

(Show transparency #42, "Has Teaching Come to This," allow time for problem-solving, then discuss.)

HAS TEACHING COME TO THIS?

Ruth, Maria, Katy, and Jane are all teachers having trouble making ends meet. They all hold part-time jobs after school. One is a carpenter's apprentice, one is a cook, one is a delivery person for a pizzeria, and one teaches judo. Ruth has nothing to do with food. Maria and the cook are sisters. Katy and Maria are allergic to wood working. Jane, the delivery person, and the judo teacher are strangers. Match up each woman with her work.

Activity VIII

Now that we have experienced several logic activities, let's see if our logic skills have improved.

(Pass handout entitled "You're Logical?" Allow time, then review answers.)

MAKE A GRAPH

(As participants enter room, ask them to mark the graphs posted.)

Needed: Newsprint
Markers
Tape

Graphing is a way to present data in a clear, concise, and visual way that makes it possible to see relationships in the data more easily. In order to learn to interpret graphs and use them as a problem-solving strategy, children benefit by first making their own. Making graphs requires collecting and then sorting and classifying data. Experiences can be provided at all grade levels.

For younger children, graphing experiences best begin concretely. The possibilities for things to graph are best taken from the interests of the children and experiences that occur in the classroom. After children become more familiar with graphs, they should be able to tell you information without being prompted with questions.

There are three levels of graphs, from the concrete to the abstract. They are Real Graphs, Picture Graphs, and Symbolic Graphs.

Now let's talk about each of these. Real Graphs use actual objects to compare, and they build on children's understanding of more and less. Starting with real graphs is essential for young children and is useful as well to help older children see that abstract information can be related to the real world. The simplest of real graphs sorts objects into two groups. (Example: shoes with laces and shoes without laces.)

Picture Graphs use pictures of models to stand for real things. Children can draw pictures to represent the actual object or cut pictures from magazines. (Show transparency #43.)

Symbolic Graphs are most abstract because they use symbols to represent real things. Graphs where you make a tally mark or color in a square are examples of symbolic graphs. (Show transparency #44.)

There are also three categories of graphs: graphs that ask for opinions, graphs that require factual reporting of information, and graphs that require some processing before recording the information needed.

Now let's look at the graphs we have marked. (Discuss each. Ask the type and category of each and seek questions that could be asked about the graphs.)

Divide in groups of four and ask each group to construct a graph using newsprint and markers. The graphs should be designed to find out information about the people in this workshop.

Examples are:

1. How old are you?

2. What is your favorite color?

3. What make of car do you drive?

4. Is you hair straight, wavy, or curly?

5. What is your favorite morning beverage?

As each group completes its graph, post the graphs so that they may be marked by all participants. Discuss each graph and ask for questions that can be formulated from each graph.

ACT IT OUT

(Transparency)

Some problems lend themselves to experimentation or "acting out." Students can sometimes solve a problem best by "doing it." Acting out problems will force the student to understand the nature of the problem. In most cases if a child can act out a problem, we can feel sure the child understands the problem. For example, if a student is asked to determine how many times an unsharpened pencil can be placed end-to-end on the longest side of a rectangular table, the answer can be found by actually lining an unsharpened pencil across the table.

Another example would be to determine how many times you can pat your foot in 30 seconds. Again, this can be acted out to find the answer.

Word problems or story problems often lend themselves to the problem-solving strategy of "acting out." Many word problems are provided in the mathematics textbooks. However, children should be given lots of experiences for acting out stories before working with textbook word problems. At first, emphasis should be placed on making sense of the situation and then applying the appropriate arithmetic. Mary Baratta-Lorton has published in her book Mathematics Their Way some very good activities for young children. A number of these activities can be acted out.

Let's do an activity called "The Octopus Story."

(Put Pepperidge Farm goldfish crackers in a cup for each participant.)

Put the cup of goldfish crackers in front of you.

THE OCTOPUS STORY

This is a story about an octopus. He was a big octopus and he liked to eat little fish. In this story you are going to pretend that you are the octopus.

This is the ocean. (The teacher holds out her/his hand.)

Who is the octopus?
Let me see your ocean.

One beautiful, sunny day the octopus was swimming in the deep, blue ocean. Suddenly he saw three fish swim by. Show me three fish. He thought, "I'm hungry; I believe I'll eat two fish for my morning snack." So he did. Show me what happened. How many fish are left in your ocean. (1)

A little later five more fish swam by. How many fish in your ocean now? Mr. Octopus thought, "It's getting close to lunch time. I will eat four fish for lunch." So he did. How many fish do you have left? (2)

Some mathematics experts suggest that teachers present fantasy scripts for children to act out. Each time the teacher makes up a problem; language should be varied so that the same words do not become cues that the children look for. For example, altogether, sum of, total, in all. Also, the problems should require a combination of several operations.

For example: (Transparency #45)

_____, _____, _____, and _____ are candles on a birthday cake.

_____ blew three of them out. How many are now burning?

_____, _____, _____, and _____ are birds flying in the sky.

_____ and _____ are more birds flying in the sky. How many birds are there in all?

Upper Grades

By the time students are in the upper elementary grades, they've had a variety of experiences with word problems and with isolated arithmetic practice. Research indicates that their computational abilities surpass their ability to apply these skills to word problems. It's important that we start with what children "do" know and build from there.

The following exercise is helpful for experiencing the kind of difficulty children encounter when faced with word problems.

Handout (Transparency #46)

DEALING IN HORSES

A man bought a horse for \$50 and sold it for \$60. He then bought the horse back for \$70 and sold it again for \$80. What do you think was the financial outcome of these transactions?

__ Lost \$20 __ Earned \$10

__ Lost \$10 __ Earned \$20

__ Came out even __ Earned \$30

__ Other (describe) ____________________

Explain your reasoning: ____

____

Discuss this problem with your neighbors. Check the correct answer. Explain why you chose that answer.

(Let several people share why they selected a certain answer.)

(Put the participants in groups according to the answer they chose. For example, everyone who chose the answer earned $10 should be in a group, everyone who answered lost $10 should be in a group, etc.)

(Ask for volunteers to help act out the problem.)

(You will need three volunteers. One to be the owner, one to be the buyer, and one to be the horse. You will also need play money--$20 bills/ $10 bills.)

Some other quick ideas that you may be interested in using are as follows: (Handout)

1. How long does it take for a book to fall from the desk to the floor?

2. How many pennies laid next to each other are needed to measure exactly one foot?

3. How many checkers will fill a one pound coffee can?

4. How many seats are in the auditorium of your school?

5. Are there more boys or more girls in your class?

6. How many people in your class were born in April?

7. Which has more pages, your dictionary, or your local telephone directory?

8. How many scoops of ice cream can you get from one-half gallon of vanilla ice cream?

9. Which rock group is the favorite in your class?

10. How long does it take to read a page of print?

11. Can you run a 100-yard dash faster than a car can drive one-half mile at 55 miles an hour?

12. Place 20 pennies on the table in a row. Replace every fourth coin with a nickel. Now replace every third coin with a dime. Now replace every sixth coin with a quarter. What is the value of the 20 coins now on the table?

13. Place 20 pennies on the table in a row with heads up. Now "flip" every coin to show tails. Now "flip" every other coin beginning with the second penny (2,4,6,...). Now flip every third coin, beginning with the third penny (3,6,9,...). Next flip every fourth penny, beginning with the fourth coin (4,8,12,..). Continue this procedure for 20 trials. Which pennies now have the heads up?

I think we can sum up by saying that the problem-solving strategy "acting out" provides students with another way to find solutions to problems as well as the opportunity to develop a deeper understanding of the mathematics process.

USING OBJECTS

Using objects is another problem-solving strategy. One way teachers give students experience using this strategy is through telling stories and having the children act out by using cubes or other objects to represent cars, cookies, children, etc. For example, in the goldfish problem we participated in earlier we used the little goldfish as objects for helping us to solve problems.

Let's try another short story activity that can be used with younger children.

(Have the participants use the unifix cubes on their tables to act this problem out.)

Problem

We will be using the unifix cubes to do the next activity. There were four horses in the pasture. Show me four horses by using the cubes. Two more horses wandered into the pasture. How many horses were there altogether?

Now let's move on to some other activities that give children experience with using objects to solve problems. The first activity is called "Spill the Beans."

SPILL THE BEANS

You need six beans each painted on one side. If you spill them and record how many painted sides come up each time, do you think you'll get one result more often than any other? If so, which result will it be? Then try it. Spill the beans at least 25 times and record. Try the experiment with other numbers of beans as well.

(Hand out Recording Sheet.)

Now we are going to do a couple of activities using dice.

WILL THEY EVER BE THE SAME?

If you roll just one die, it's equally likely for any of the six numbers to come up. If you roll a die many times, will all of the numbers actually come up the same number of times?

To go further:

1. Each child rolls one die 10 times recording what comes up each time. Then each child finds the average of the 10 throws. Post and discuss.

Ask children to roll the die five or 10 times and set up a class chart for them to record on. Or do it once a day as a whole class activity, with one child rolling and another recording. Keep this going for a series of days and see what happens over time. Discuss: Will all of the columns ever be the same? What do you think will happen tomorrow? Why do you think that number is ahead?

(Transparency #47)

HOW MANY ROLLS TO GET A 1?

Roll one die until a one comes up. Record how many rolls it took. Do this five times. Put your five results on the class chart.

Before getting started, here are some ideas for discussion questions: Is it possible to get a 1 on your first roll? Do you think it's possible to roll it 100 times and still not get a 1? Is that likely? What do you feel is likely? What do you think the results will look like?

A group chart can be made like this. Children can enter their results with tallies. Discuss the results when they've all entered their data.

1	11	21
2	12	22
3	13	23
4	14	24
5	15	25
6	16	26
7	17	27
8	18	28
9	19	29
10	20	30 or more

Suppose the question was: How many rolls to get a 6? Would the results be similar or different? How could you find out?

The last problem we will do today for demonstration of how objects can be used to help solve problems is called "The Stacks Problem." For doing this problem you will need 15 Unifix cubes.

(Transparency #48)

THE STACKS PROBLEM

Show the ways that 15 Unifix cubes can be put into four stacks so that each stack has a different number of cubes, and there is at least one cube in each stack. Suppose you tried this, putting the 15 cubes into three stacks instead. Would there be more ways or fewer ways and how many?

DRAW A PICTURE

Another strategy students can use to help solve problems is to draw a picture. Everyday many of you draw pictures to help you solve problems that you encounter. For example, if a teacher asked you for some suggestions as to how she could arrange her classroom to include a chair for each child, three learning centers, a work table, and a teacher's desk, the first thing you would do is have the teacher describe the size of her room. Next, you would probably sketch out several pictures to illustrate possible arrangements.

Many mathematics problems can be solved by drawing pictures. Let's look at a few problems.

Problem

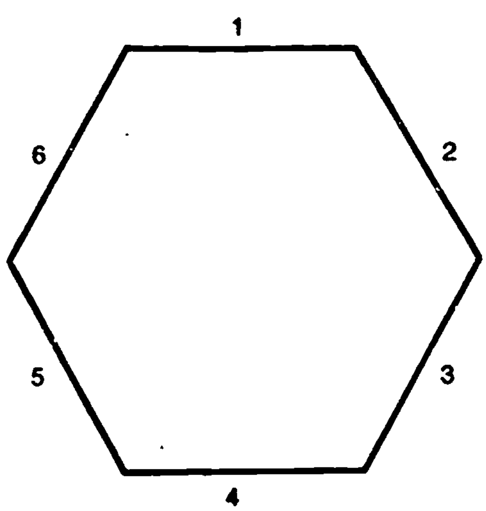
Jim is in line at the bridge waiting to pay his toll. He counts four cars in front of him and six cars behind him. How many cars are there in line at the bridge?

Work with your neighbors to solve this problem.

(Let several people share how they solved the problem.)

Problem (Transparency #49)

The six students in Mr. Smith's biology class were arranged numerically around a hexagonal table. What number student was opposite number 4?



Work with your neighbor to find the solution to this problem.

(Ask for volunteers to share how they solved the problem.)

Problem (Transparency #50)

A log is cut into four pieces in nine seconds. At the same rate, how long would it take to cut the log into five pieces?

Notice the key phrase "at the same rate."

(Let the participants work in groups to solve the problems.)

In many cases, students react to this problem as an exercise in proportion:

$$\frac{4}{9} = \frac{5}{x}$$

If you drew a diagram, you quickly saw that the solution process should focus on the number of cuts needed to get the required number of pieces, rather than on the actual number of pieces.

Once the picture has been interpreted, students should realize that they do not need proportions. They see that 3 cuts produced 4 pieces, and thus they need 4 cuts to produce 5 pieces. Since 3 cuts were made in 9 seconds, each cut required 3 seconds. Therefore the necessary 4 cuts will require 12 seconds.

(Let volunteers share their answers.)

Problem (Transparency #51)

Suppose you put 10 dots on a circle. If you drew lines connecting every dot to every other dot, how many lines would you draw?

Again, ask the participants to share how they arrived at the solution to the problem.

SOLVE A SIMILAR/SIMPLER PROBLEM

Realistic problems often contain very large numbers. These tend to obscure the procedures and processes needed for solving the problem. The problem can be simplified or reduced but remains mathematically unaltered if smaller numbers are substituted or number of cases reduced. The solution of a simpler problem will aid the student in seeing patterns and provide insight toward the solution of the more complex original problem.

Here are some problems in which reduction and/or simplification should be used:

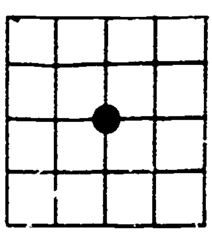
I. Problem

How many squares are there on a standard 8 x 8 checkerboard?

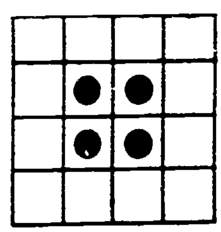
(Discussion)

The students' first reation to this problem is to say "64 squares." To prepare your students for this kind of problem solving, have them fold a square piece of paper in half vertically and horizontally twice without unfolding. While the paper is still folded, ask them what they "see" as the unfolded figures. Have them look for 1-by-1, 2-by-2, 3-by-3, and 4-by-4 squares in their mental pictures. To be sure all are actively involved, have them estimate the total number of all such squares and write down their guesses on the still-folded square piece of paper. Once the paper is unfolded, every student has a physical model to help in the counting and visualizing process. To aid in systematically counting and tabulating all the squares, locate the centers of the squares of various sizes as in the diagrams below. This geometric approach should clue the student into the role squares will play in the final answer for the 8-by-8 chessboard.

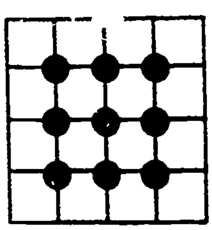
(Transparency #52)



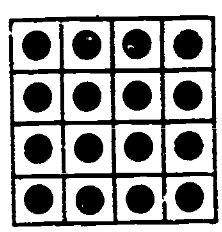
One 4-by-4 square



Four 3-by-3 squares



Nine 2-by-2 squares



Sixteen 1-by-1 squares

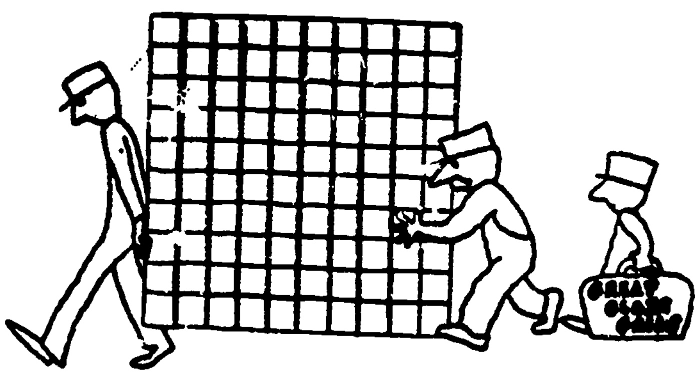
It may also be helpful to record the number of squares by making a table as below.

Repeat the problem and allow students to solve the larger problem based on this experience with a simplified problem.

Number of Squares

Size board	1x1	2x2	3x3	4x4	5x5	6x6	7x7	8x8	Total
1 x 1	1	___	___	___	___	___	___	___	1
2 x 2	4	1	___	___	___	___	___	___	5
3 x 3	9	4	1	___	___	___	___	___	14
4 x 4	16	9	4	1	___	___	___	___	30
5 x 5									
6 x 6									
7 x 7									
8 x 8									

A follow up or extended activity would be to see how many squares there are on a 10-by-10 grid.

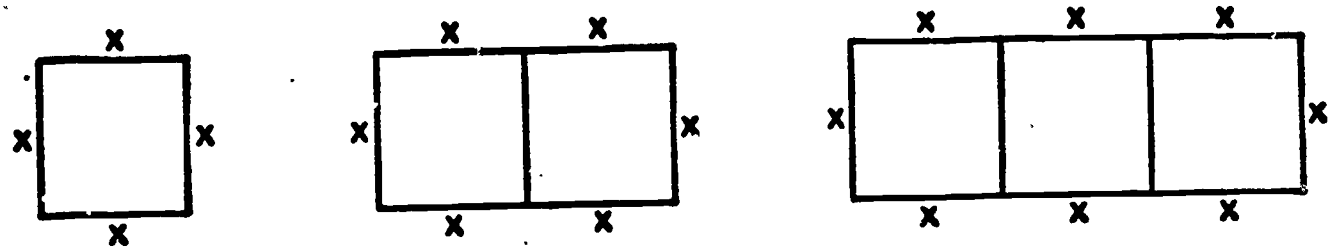


II. Problem (Transparency #53)

Twelve couples have been invited to a party. The couples will be seated at a series of small square card tables, placed end to end so as to form one large long table. How many of these small tables are needed to seat all 24 people?

(Discussion)

Students should draw a picture of what the situation looks like for 1 table, for 2 tables, for 3 tables, etc. then record the data on a chart.



Number of tables	1	2	3	4	.		n
Number of guests	4	6	8	10	.	.	24

Suppose we limit the number of guests to a smaller number, say 16. Let's see if our pattern holds true. Extend the chart.

Number of tables	1	2	3	4	5	6	7
Number of guests	4	6	8	10	12	14	16

Since the pattern seems to be holding true for 16 guests, we can continue to add 1 table for every 2 additional guests until we reach our required number of 24 guests. We thus add 4 additional tables for the additional guests (16 + 8 = 24). It will take 11 tables to accommodate 24 guests.

III. Problem (Transparency #54)

A Polaris Commuter has seats for 108 passengers. On a flight to Memphis, there was 1 empty seat for every 2 passengers actually on board. How many passengers were on the flight?

(Discussion)

A simpler problem would be one that considered a plane with seats for 12 passengers.

E X X
E X X
E X X
E X X

Empty	People	Total
1	2	3
2	4	6
3	6	9
4	8	12

The pattern revealed by the table leads to $x + 2x = 108$.

$3x = 108$ $x=36$ $36 + \boxed{72} = 108$

43

IV. Problem (Transparency #55)

The new high school has just been completed. There are 1,000 lockers in the school and they have been numbered from 1 to 1,000. During recess, the students decide to try an experiment.

When recess is over, each student will walk into school, one at a time. The first student will open all of the locker doors. The second student will close all of the locker doors with even numbers. The third student will change all the locker doors with numbers that are multiples of three. (Change means closing lockers that are open and opening lockers that are closed.) The fourth student will change the position of all locker doors numbered with multiples of four; the fifth student will change the position of the lockers that are multiples of five, and so on. After 1000 students have entered the school, which locker doors will be open?

(Discussion)

It seems rather futile to attempt this experiment with 1000 lockers, so let's reduce the number to 20 lockers and 20 students and try to find a pattern.

Locker #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Student 1	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O
2		C	O	C	O	C	O	C	O	C	O	C	O	C	O	C	O	C	O	C
3			C	C	O	O	O	C	C	C	O	O	O	C	C	C	O	O	O	C
4				O	O	O	O	O	C	C	O	C	O	C	C	O	O	O	O	O
5					C	O	O	O	C	O	O	C	O	C	O	O	O	O	O	C
6						C	O	O	C	O	O	O	O	C	O	O	O	C	O	C
7							C	O	C	O	O	O	O	O	O	O	O	C	O	C
8								C	C	O	O	O	O	O	O	C	O	C	O	C
9									O	O	O	O	O	O	O	C	O	O	O	C
10										C	O	O	O	O	O	C	O	O	O	O
11											C	O	O	O	O	C	O	O	O	O
12												C	O	O	O	C	O	O	O	O
13													C	O	O	C	O	O	O	O
14														C	O	C	O	O	O	O
15															C	C	O		O	O
16																O	O	O		O
17																	C	O	O	O
18																		C	O	O
19																			C	O
20																				C

In our smaller illustration, the lockers with numbers 1,4,9, and 16 remain open, while all others are closed. Thus we conclude that these lockers with numbers that are perfect squares will remain open when the process has been completed by all 1000 students. Notice that a locker change corresponds to a divisor of the locker number. An odd number of "changes" is required to leave a locker open. Which kinds of numbers have an odd number of divisors. Only the perfect squares!

V. Problem (Transparency)

Of 200 coins, 199 are the same weight and one is lighter than the others. Given a balance with two pans for comparing weights, what is the least number of weighings needed to determine which coin is light?

(Discussion)

To simplify the problem, reduce the number of coins to seven; six are the same weight and one is lighter than the others. Given the same balance, explain how the light coin may be identified in no more than two weighings.

Weigh six of the coins--3 and 3. If they balance, the lighter coin is the one of the seven not weighed. Otherwise, weigh two of the three coins from the lighter side of the balance. Once again, if it balances, the lighter coin was the one of the three not weighed. If it doesn't balance, the lighter coin can easily be identified.

Students need to see this strategy modeled regularly so that it will become a meaningful and workable strategy.

WRITE AN EQUATION

Write an equation is a strategy often used in mathematics to solve problems. Early experiences with this strategy should be those which will guide students to translate from English to mathematical terms. An example of this would be to express the following statement as a mathematical equation: Mary is four years older than John (M = J + 4). The true test of math understanding is the ability to translate from real-life situations to math symbols and from math symbols to real-life situations. The emphasis should continue to be placed on the importance of understanding in working problems and not merely on getting right answers.

Many of the problems we have worked in previous strategies can also be solved by writing an equation. Once the understanding is developed through the use of another strategy, the student can be shown how an equation can also be used to solve the problem.

We will now work several problems in which equations help to clarify the problem.

I. Problem (Transparency #56)

A certain number added to itself 3 times is the same as 2 times 9.

$$\quad + \quad + \quad = 2 \times 9$$

Three students the same age added their ages and found the sum to be 2 more than the age of the principal who was 34. How old was each student?

$$\quad + \quad + \quad = 34 + 2 \quad \text{or} \quad 3 \times \quad = 34 + 2$$

Find at least one rectangle where the number of square inches of area is the same as the number of inches in the perimeter.

$$L \times W = 2 \times (L + W)$$

II. Problem (Transparency #57)

Sara averages 12 kilometers an hour riding her bike to the state park against the wind. She averaged 18 kilometers an hour riding home with the wind at her back. If it took her 4 hours to return, how long did it take for the total trip?

(Discussion)

Rather than "let x equal what we're trying to find" and write and solve the equation (a somewhat mechanical approach that isn't as useful for this problem), encourage students to use a process like the following:

- Study the problem and make a sketch (if useful).

12 km/h

18 km/h (4 hours)

- List the quantities involved in the problem (known and unknown):

  rate (against wind)
  rate (with wind)
  time (with wind)
  time (against wind)
  distance (home to park)
  distance (park to home)

- Replace the words with equal expressions containing known data. Use variables for data that are not known.

$12 \times t = 18 \times 4$ (distance = rate x time; let t = time it takes to ride to the park).

- Solve the equation and check: $12t = 72$, so $t = 6$. It took 6 hours for Sara to ride to the park. The total time for her trip is $4 + 6$, or 10 hours.

Notice how important it is to check the solution to the equation in terms of the original problem. Without this check, many students would give 6 as the answer.

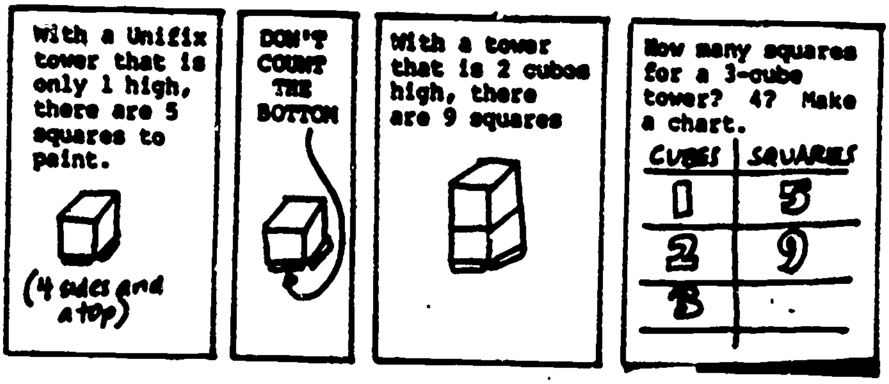
Special emphasis should be placed on looking for quantities that are equal in a problem. To develop this skill, give students a problem that they are not to solve but only to study and state which quantities are equal.

Experiences using a process such as the one described above can be of great value to students later when they use the "write an equation" strategy to solve more difficult problems. As they learn to write an equation, praise them when they really think about the conditions of the problem.

III. Problem (Transparency #58)

Suppose you build a Unifix tower that is 99 cubes high. And suppose you had to paint each square on the tower. How many squares would you have to paint?

(Discussion)



In another strategy we solved a part of this problem by working with objects and looking for a pattern. In order to extend the problem to 99 cubes, see if you can find an equation that will express the problem in mathematical terms and allow you to solve the problem without building a 99 cube model. (Allow "think" time.)

Let "n" represent the number of cubes. Each cube has four sides, so n x 4 would represent the number of side squares to be painted. But each time a cube is added there will also be one top square to paint; therefore, the equation (n x 4) + 1 will help us solve the larger problem.

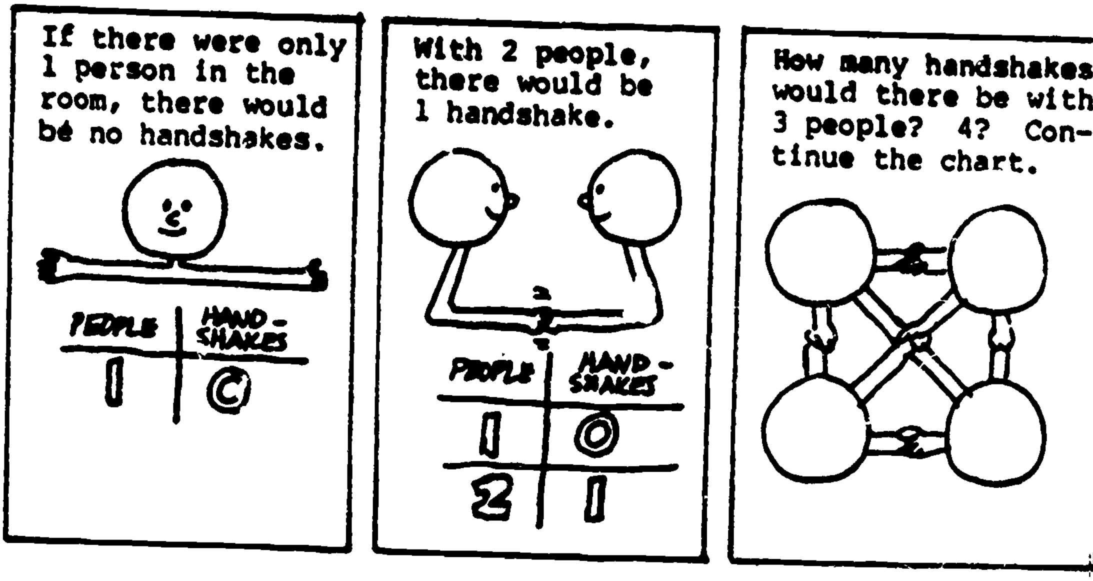
$(n \times 4) + 1$
$(99 \times 4) + 1 = 397$ squares

IV. Problem (Transparency #59)

Suppose everyone in this room were to shake hands with every other person in the room. How many handshakes would that be?

(Discussion)

In the pattern strategy, you solved this problem by acting it out and looking for a pattern.



Without extending the chart, can you find an equation that will help you solve the problem? (Allow "think" time.)

Let "n" represent the number of people in the room. Multiply the number of people in the room times the number of handshakes each person will make--(n - 1) because the person shaking hands will not shake his own hand. Then divide by two because one handshake represents two people.

$$\frac{n\ (n - 1)}{2}$$

SUGGESTIONS FOR TEACHING PROBLEM SOLVING

1. Provide a wholesome emotional climate for problem solving.

2. Teach various problem-solving strategies.

3. Emphasize the method of solution rather than the solution.

4. Encourage experimentation, trial and error, estimation, intuition, guessing and hunches to suggest a method of solution.

5. Expose students to many problems and to varied problems so that they develop flexibility in problem-solving behavior.

6. Provide sufficient time for discussion, practice, and reflection on problems and problem-solving strategies.

7. Have students construct their own problems.

8. Attempt to find the source of the students' difficulty and use various instructional techniques to remove these difficulties.

9. Insist on persistent effort and on concentrated and sustained attention.

10. Provide very frequent short sets of problems on which the students experience absolute success.

11. Promote problem solving through the use of mathematical games and other activities.

12. Have students work together in small groups.

13. Attempt to establish and maintain students' motivation.

14. Show the learner how to ask himself questions.

15. Give conscious attention to reading skills.

16. Use problem situations to discover new mathematical concepts, principles, or relationships.

17. Use problem situations as a basis for practice and as a substitute for isolated drill exercises.

18. Model good problem-solving behavior.

TRANSPARENCIES

#1

GOAL: TEACHERS WILL LEARN
COMMON PROBLEM-SOLVING
STRATEGIES.

#2

PROBLEM-SOLVING STRATEGIES

I. APPLYING WORD PROBLEM STRATEGIES

II. BREAK INTO PARTS

III. LOOK FOR A PATTERN

IV. MAKE OR CONSTRUCT A LIST/TABLE/CHART

V. GUESS AND CHECK

VI. WORK BACKWARDS

VII. ACT IT OUT/USING OBJECTS

VIII. MAKE A GRAPH

IX. DRAW A PICTURE

X. LOGICAL THINKING (REASONING)

XI. SOLVE A SIMILAR OR SIMPLER PROBLEM

XII. WRITE AN EQUATION

#3

1. WRITE OR ASK A QUESTION.

2. WHAT IS MISSING?

3. WHAT IS THE OPERATION?

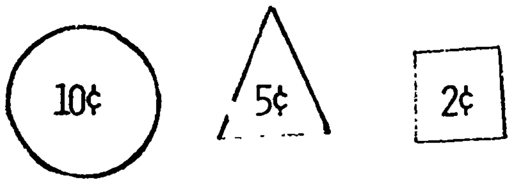
4. WRITE A PROBLEM.

#4

PRIMARY

GIVEN: 17 BOYS AND 14 GIRLS

GIVEN:



UPPER GRADES

GIVEN: A MOVIE COSTS $4.00 FOR ADULTS AND $2.00 FOR CHILDREN UNDER 16.

GIVEN:

BANANAS	GRAPEFRUIT	POTATOES
3 LBS. FOR 85¢	3 FOR $1.00 OR 40¢ EACH	5 LBS. FOR $1.50 OR 35¢ A LB.

WHAT'S THE QUESTION

Directions:

In each of the following, you are presented with a problem situation. Supply the question that makes each situation into a problem.

1. Marie and her father went to the baseball game last Sunday. They each ate two hot dogs and two soft drinks. The hot dogs cost 95¢ each, while the soft drinks cost 45¢ each. The admission to the ball park was $4.40 for adults and $2.20 for children. There were 45,000 fans at the game.

What's the Question? ____

2. Keri and Kim spent all day Saturday shopping in the local shopping center. Keri bought a pair of blue jeans for $22.60 and a shirt for $11.95. Kim had $23.50 to spend, but he only bought a shirt for $14.95.

What's the Question? ____

3. Sam and Dave drove to Georgia from their home, a distance of 832 miles. They bought 18 gallons of gasoline on the first day. They drove 388 miles the second day. They spent $22 for food on each day.

What's the Question? ____

4. George Johnson can type a paper at the rate of 45 words per minute, while a word-processor can produce 250 words per minute. An assignment was made in class to write a 5,000 word term paper.

What's the Question? ____

5. Lucille is planning to sell her collection of baseball cards which contains 1,250 cards. She has cards representing all the teams and all positions. She has 278 cards of pitchers, of which 113 are left handed. She has 31 catchers and 531 infielders.

What's the Question? ____

6. The WHAT was giving a live performance at the fairgrounds. The concert began at 1:00 p.m. and lasted until 4:45 p.m. The group took two twenty-minute breaks. The concert consisted of 31 different selections.

What's the Question? ____

#6

WHAT'S MISSING?

Directions:

Sometimes a problem does not give you enough information. You must be able to tell what fact is missing. Read each of the following problems carefully. From the three statements given, select the one that is necessary to complete the solution.

1. Pete's Pet Palace is having a 25% sale on tropical fish. Janet buys guppies for her fishtank. How many can she buy for $2.85?

What's Missing?

_______ The number of guppies on sale
_______ The amount of money Janet has to spend
_______ The price of a guppy •

2. Manuel bought dog food for his pet boxer, Bud. The dried dog food costs $8.95 for a twenty-five pound bag, while the moist dog food comes in 16-ounce cans. Manuel bought one twenty-five-pound bag and seven cans of the moist dog food. How much did he spend altogether?

What's Missing?

_______ The number of cans Manuel bought
_______ The price he paid for the dried dog food
_______ The price he paid for the cans of moist dog food •

3. The most expensive coin ever sold at auction was sold for $272,000. The commission paid on the sale was $42,000. How much profit did the owner of the coin make on the sale?

What's Missing?

_______ The price the coin was sold for at the auction
_______ The price the owner paid for the coin when he originally bought it •
_______ The amount of the commission

4. The Rams and the Cougars had a post-season tournament. The Rams established a new record by scoring 112 points. By how many points did they win?

What's Missing?

_______ How many points the Cougars scored •
_______ How many points the Rams scored
_______ Where the game was played

5. The local donut shop is filling an order for the junior class prom, which will be held on Saturday evening. The donuts are packed 12 to a box. How many boxes are needed?

What's Missing?

_______ The price of each donut
_______ The weight of each donut
_______ The number of donuts ordered •

WHAT IS THE OPERATION?

A. WHEN YOU COMBINE TWO OR MORE THINGS, YOU ADD.

B. WHEN YOU FIND THE DIFFERENCE BETWEEN NUMBERS OR AMOUNTS, OR WHICH IS LARGER OR TALLER, YOU SUBTRACT.

C. WHEN YOU REPEAT ONE NUMBER SEVERAL TIMES, YOU MULTIPLY.

D. WHEN YOU SEPARATE SOMETHING INTO GROUPS OF THE SAME AMOUNT, YOU DIVIDE.

#8

GIVEN: MIKE IS 62 INCHES TALL, AND SAM IS 58 INCHES TALL. HOW MUCH TALLER IS MIKE THAN SAM?

\+ - X ÷

GIVEN: A CAMPING TRIP FOR FIVE BOYS COST $70.45. HOW MUCH DID EACH BOY PAY?

\+ - X ÷

GIVEN: SUE BAKED 142 COOKIES. MARY BAKED 86 COOKIES. ANN BAKED 108 COOKIES. HOW MANY COOKIES DID THEY BAKE IN ALL?

\+ - X ÷

GIVEN: THERE ARE EIGHT CANS IN A CARTON. HOW MANY CANS ARE THERE IN NINE CARTONS?

\+ - X ÷

#9

Problem: ______________________________

____

MAKE UP THE PROBLEM

Directions:

Most of the time in school, the teacher or the textbook gives you the problem, and you have to supply the answer. Let's change things around. Here are the answers. For each of these answers, you make up a problem.

1. The answer is 103. *Problem:* ______________________

2. The answer is 7 carrots. *Problem:* ______________________

3. The answer is 45 miles per hour. *Problem:* ______________________

4. The answer is the Los Angeles Dodgers. *Problem:* ______________________

5. The answer is David. *Problem:* ______________________

MAKE UP THE PROBLEM

Jeff got all his word problems correct. His work is shown below. Can you think of word problems that match his work? Write your problem in the space provided next to each solution.

JEFF'S WORK	YOUR WORD PROBLEM
1. 258 − 194 = 64. My answer is 64 miles.	
2. 25 × 12: 50, 250, 300. My answer is 300 eggs.	
3. 81 + 93 + 78 = 252; 252 ÷ 3 = 84. My average is 84	
4. 288 ÷ 36 = 8; 288, 0. My answer is 8 buses.	
5. $1.16 + .83 + 1.79 + .72 + .18 = $4.68; $5.00 − 4.68 = .32. She received 32¢ change.	

#12

THERE ARE FIVE CAGES IN THE SCIENCE LAB, AND EACH CAGE HAS THREE WHITE MICE. IF ONE MOUSE EATS 15 GRAMS OF FOOD EACH WEEK, HOW MANY GRAMS OF FOOD DO THE MICE EAT EACH WEEK?

"MIKE BOUGHT A HAMBURGER FOR $1.79 AND A SOFT DRINK FOR 79¢, TAX INCLUDED. HOW MUCH CHANGE DID HE RECEIVE FROM A $5 BILL?"

"LINDA BOUGHT FOUR RECORD ALBUMS AT $6.49 EACH. SHE PAID $1.82 IN SALES TAX. HOW MUCH DID SHE SPEND ALTOGETHER?"

TECHNICAL LAB HAS AN INVENTORY OF 30 CASES OF CALCULATORS PACKED 6 TO A CASE AND 9 CASES OF ADDING MACHINES PACKED 2 TO A CASE. IF A CASE OF CALCULATORS COSTS $81.00 AND A CASE OF ADDING MACHINES COSTS $128.98, WHAT IS THE VALUE OF TECHNICAL LAB'S INVENTORY?

THE CURRENT BALANCE IN ALLYSON BUTTON'S SAVINGS ACCOUNT IS $2,375.10. IF THE BANK ADDS 0.5% INTEREST TO HER ACCOUNT EVERY MONTH, AND SHE MAKES NO DEPOSITS OR WITHDRAWALS, TO THE NEAREST CENT WHAT WILL BE THE BALANCE IN HER SAVINGS ACCOUNT [illegible] THE END OF 4 MONTHS?

"THE LENGTH OF A RECTANGLE IS 12 INCHES, AND ITS PERIMETER IS 34 INCHES. WHAT IS THE AREA OF THE RECTANGLE?"

HIDDEN QUESTIONS

Directions:

Sometimes there is a problem within a problem. That is, the problem contains a Hidden Question which must be answered in order to complete the solution. The problems below each contain a Hidden Question. State the Hidden Question, find its answer, and then complete the solution of the problem.

1. Luisa bought a hamburger for $1.65 and a soft drink for 69¢. How much change did she receive from a $5 bill?

   Hidden Question: ______________________________

   Final Answer: ______________________________

2. Jan bought five record albums at $6.95 each. She paid $1.15 in sales tax. How much did she spend altogether?

   Hidden Question: ______________________________

   Final Answer: ______________________________

3. Which is the better buy, a five-pound bag of potatoes for $1.59, or a 20-pound bag of potatoes for $6.00?

   Hidden Question: ______________________________

   Final Answer: ______________________________

4. A 12-ounce can of orange juice concentrate is used together with three cans of water to make orange juice. Is a 42-ounce container large enough to hold the mixture?

   Hidden Question: ______________________________

   Final Answer: ______________________________

5. The Noise rock group recorded 15 songs last year. Of these, 20% became million sellers. How many records that they recorded last year did not become million sellers?

   Hidden Question: ______________________________

   Final Answer: ______________________________

6. A leaky faucet drips one drop of water every second. If it takes ten drops of water to make a milliliter, how much water is lost in a 24-hour period of time?

   Hidden Question: ______________________________

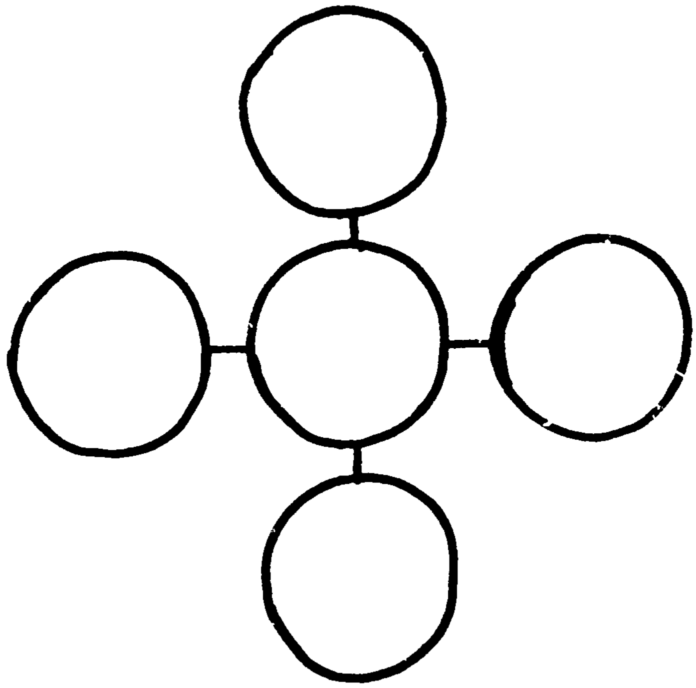
   Final Answer: ______________________________

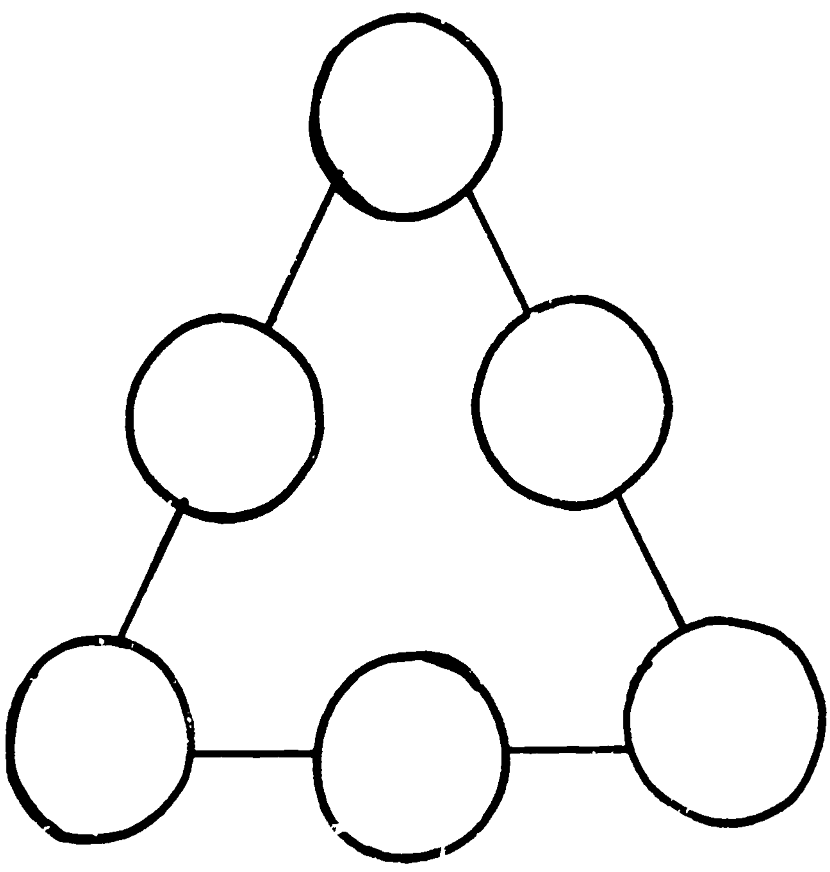
7. An airplane takes off from Philadelphia Airport for Los Angeles at 9:00 a.m. and travels at a ground speed of 550 miles per hour. At the same time, another plane takes off from New York and flies to Las Vegas, at 525 miles per hour. The distance from Philadelphia to Los Angeles is 2560 miles, while the distance from New York to Las Vegas is 2150 miles. Which plane lands first?

   Hidden Question: ______________________________

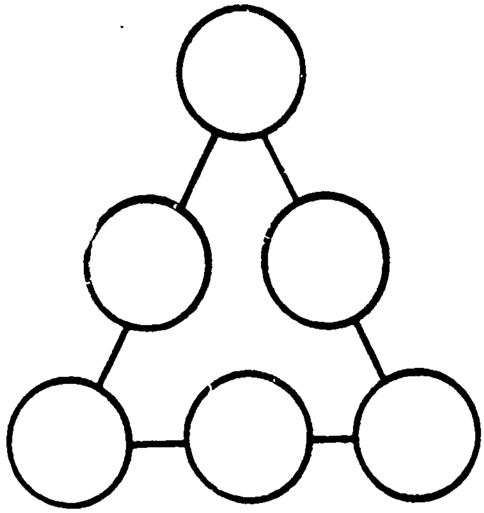
   Final Answer: ______________________________

#15 Number Puzzles:

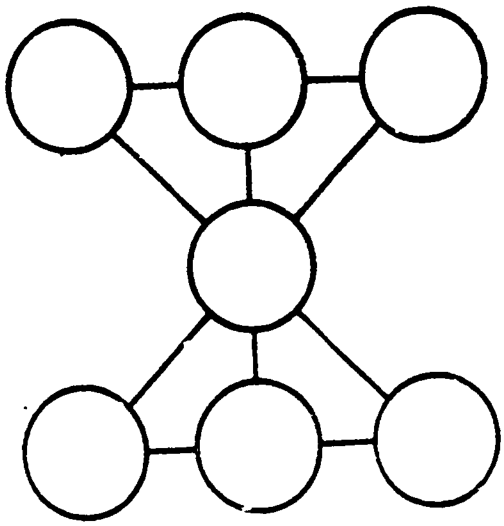




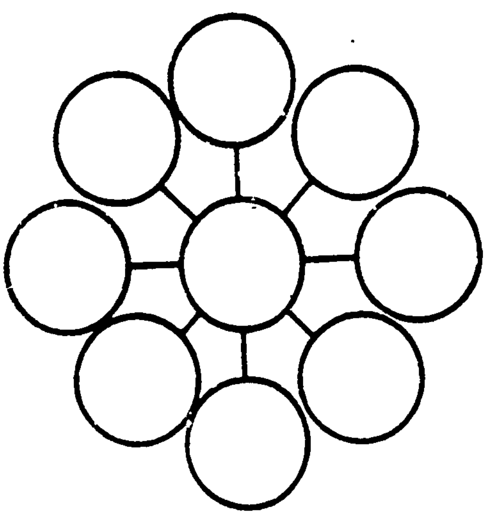
#16



1. Use the numbers 3,4,5,6,7, and 8. Make the sum 15.
2. Use the numbers 3,4,5,6,7, and 8. Make the sum 16.
3. Use the numbers 3,4,5,6,7, and 8. Make the sum 17.



1. Use the numbers 1,2,3,4,5,6, and 7. Make the sum 12.
2. Use the numbers 2,3,4,5,6,7, and 8. Make the sum 15.
3. Use the numbers 3,4,5,6,7,8, and 9. Make the sum 18.



Use the numbers 1,2,3,4,5,6,7,8, and 9. Make the sum 15.

Sum of Consecutive Numbers

The sum of three consecutive numbers if 45.
What are the numbers?

1st Guess: ________ + ________ + ________ Test: ________

2nd Guess: ________ + ________ + ________ Test: ________

3rd Guess: ________ + ________ + ________ Test: ________

Given: The area of a rectangle ' 36 sq. in. The perimeter is 40 inches.
What is the length and width of the rectangle?

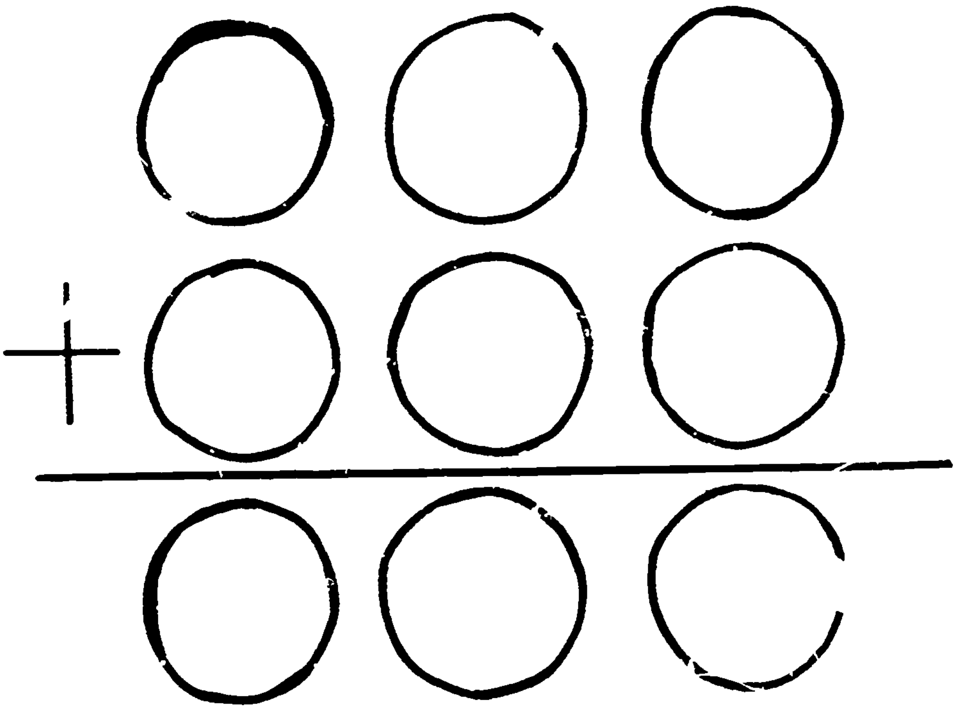
1st Guess: Length _____ Width _____ Test: _____

2nd Guess: Length _____ Width _____ Test: _____

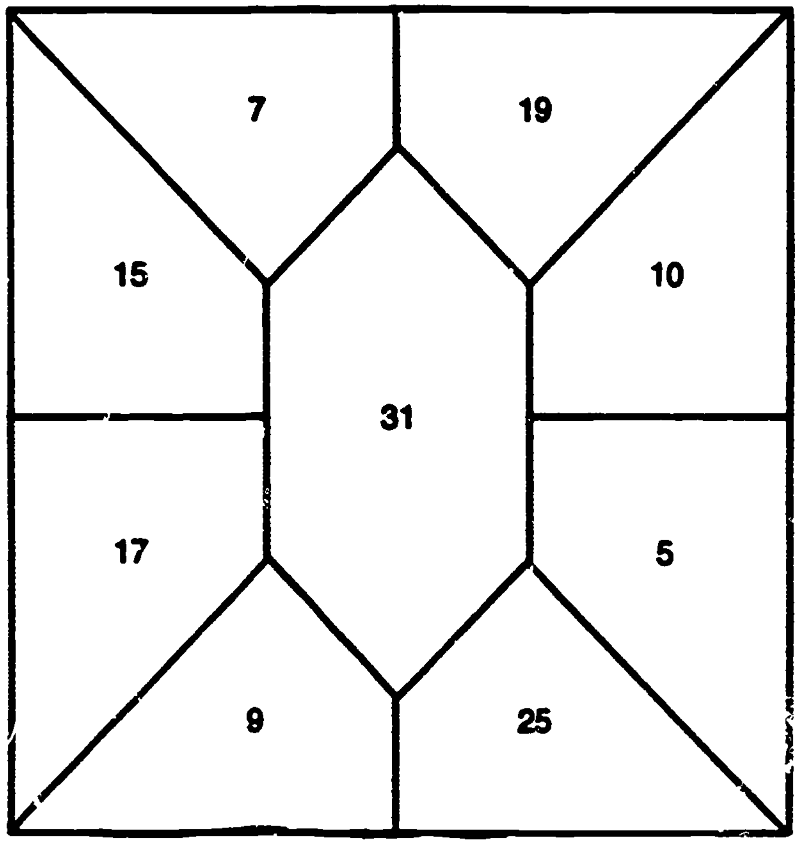
3rd Guess: Length _____ Width _____ Test: _____

Computation Puzzle

Find as many ways to make this true using the numbers 1-9.

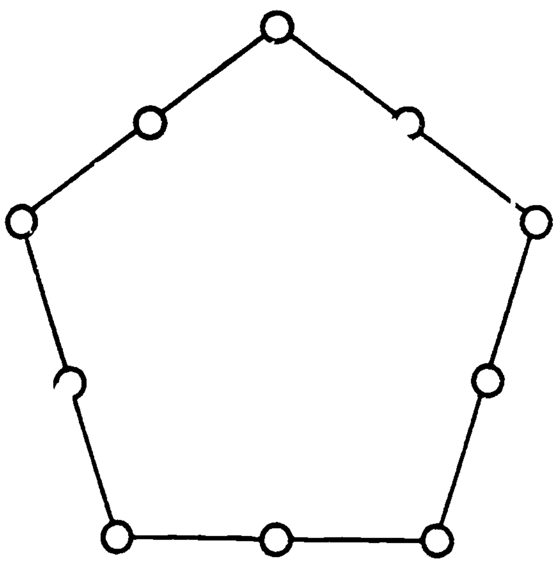


#18



Dartboard Math

Mary Hit the dartboard below with four darts. Each dart a different number. Her total score was 60. Find the different ways she could have scored 60.



Pentagon Math

Fill in the empty circles on each side of the pentagon with the numbers 1-10, so that the sum of the numbers on each side is 14.

Bu.ld a Sum

Choose a sum. Using all digits 0-9, build the sum.

144

Guess the Number

If 17 is subtracted from four times the number, the result is 35. What is the number?

Problem:

A certain bag of marbles can be broken up into 2, 3, 4, 5, or 6 separate but equal piles with one marble left over each time. It can also be separated into 7 piles evenly with none left. How many marbles are in the bag?

#20

Coin Problem

There are exactly 10 coins with a total value of $1. If three of the coins are quarters, what are the remaining coins?

There were a group of children and their dogs playing in the park. Twelve heads and 38 legs could be counted. How many dogs and children were there?

Alphanumerics

Add One
Two
Five
Eight

#21

Working Backwards

Examples:

Number Selected:	?	Solve: Number	___
Multiply by 4	↓	Divide by 4 ↑	___
Add 3	↓	Subtract 3 ↑	___
Answer is 31	↓	Answer ↑	___
What is the number?		Start	

Original Number ↓	↑ Original No. = ___
Add 5 ↓	________ 5 = ___
Multiply by 2 ↓	________ by 2 = ___
Subtract 1 ↓	________ 1 = ___
Answer is 21 ↓	Answer = 21

Original Number ↓	↑ Original No. = ___
Subtract 4 ↓	________ 4 = ___
Multiply by 3 ↓	________ by 3 = ___
Add 5 ↓	________ 5 = ___
Answer is 29 ↓	Answer = 29

Problem: Jimmy was trying a number trick on Sandy. He told her to pick a number, add 5 to it, multiply the sum by 3, then subtract 10 and double the result. Sandy's final answer was 28. What number did she start with?

Problem: Bill made an average score of 83 on four spelling tests. His papers show the results of three tests were 72, 85, and 85. He wants to know the score of the fourth test. Find the score.

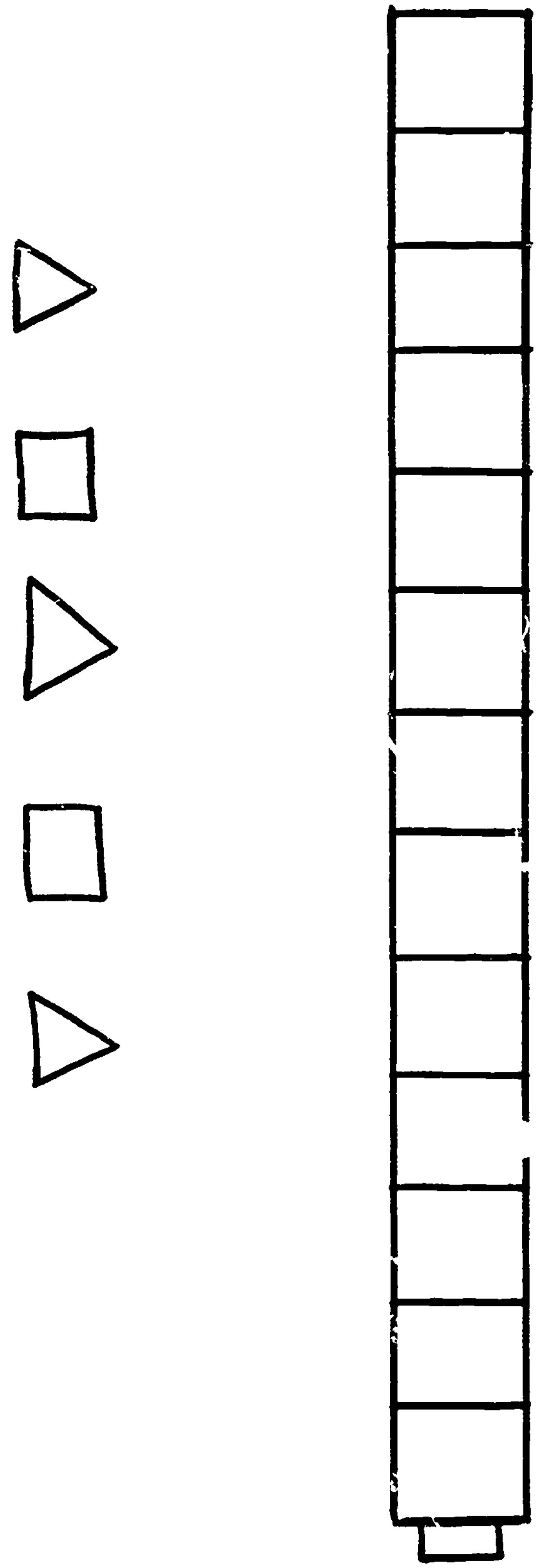
Sally and the Peanuts

Activity: On the way home from school, Sally McCrackin likes to eat peanuts. One day, just as she was reaching into her sack, a hideous, laughing creature jumped into her path, identified itself as a pigs eye, and grabbed her sack. It stole half of her peanuts plus two more. A bit shaken, Sally continued home. Before she had a chance to eat even one peanut, another horrid creature jumped into her path and also stole half of her peanuts plus two more. Upset, she continued on. (What else could she do?) But before she had a chance to eat even one peanut, another of these tricksters jumped out and did the very same thing--took half of her peanuts plus two more.

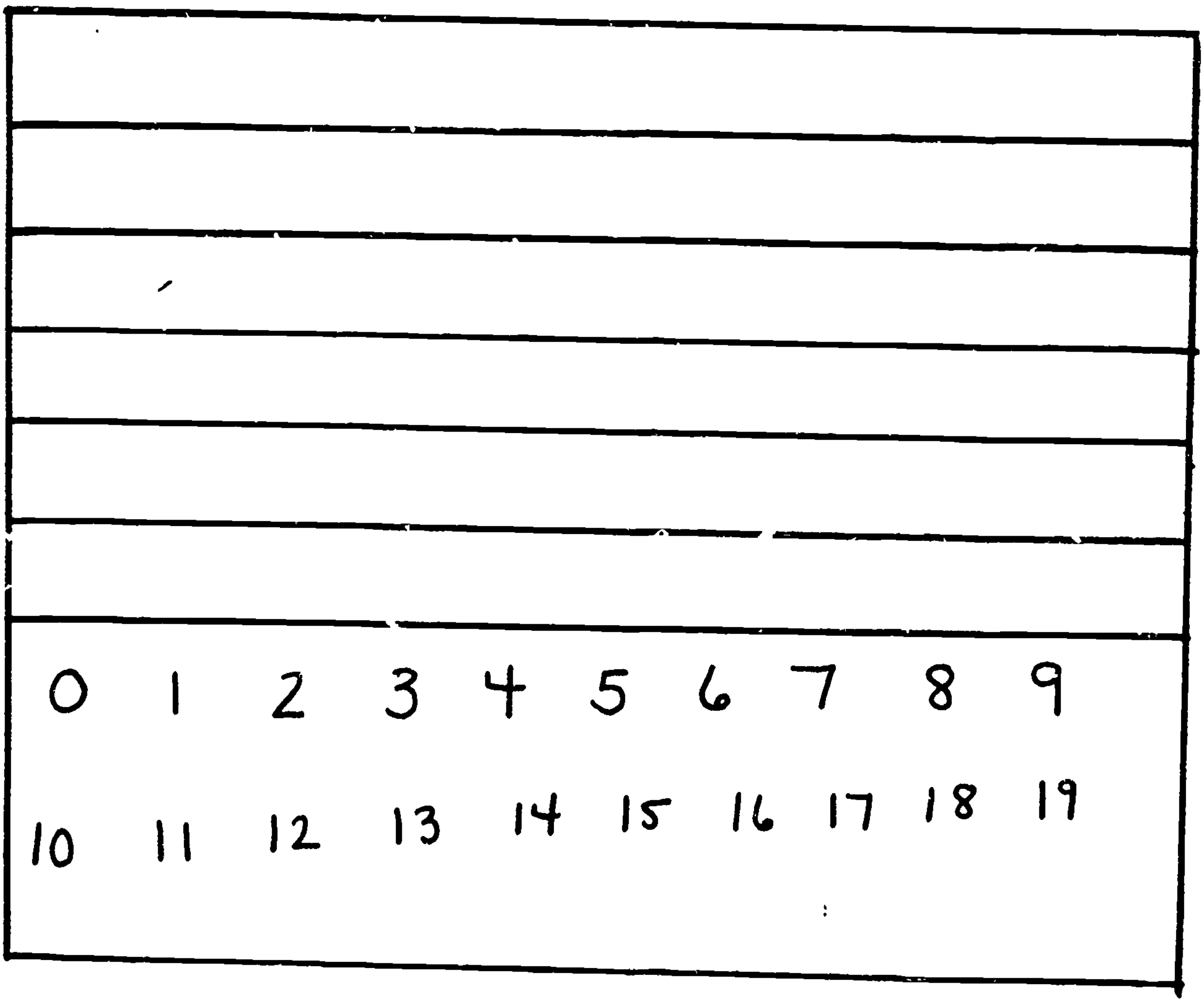
Now there were only two peanuts left in Sally's sack. She was so despairing, she sat down and began to sob. The three little pig eyes reappeared, feeling some sense of remorse, and told her they would return all her peanuts to her is she told them how many she had when she started. How many peanuts had been in Sally's sack?

#24

2



Eyes



#26

BEANS AND WAYS

BEANS	WAYS

#27

UNIFIX TOWERS

CUBES	SQUARES

#28

THE HANDSHAKE PROBLEM

PEOPLE	HANDSHAKES

FIND THE RULE. COMPLETE THE SERIES.

3, 6, 12, 24, 48, 96

RULE:

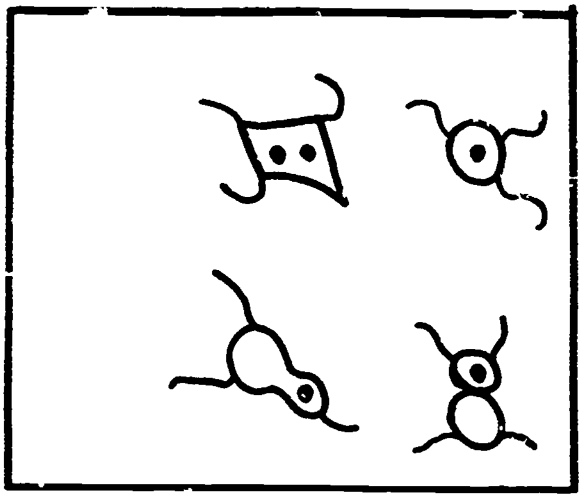
6, 10, 7, 11, 3, 12, 9

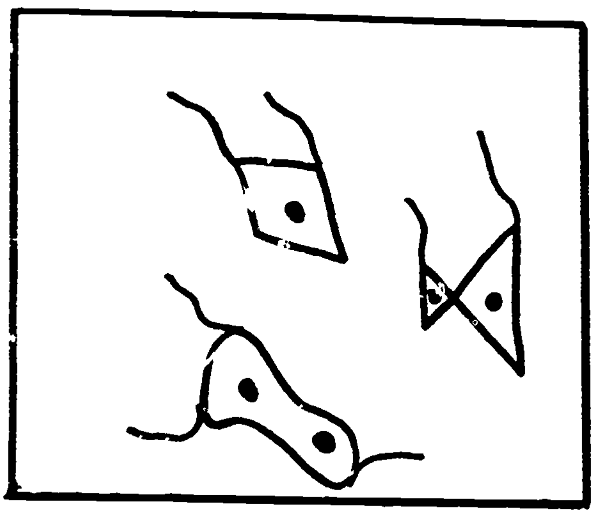
RULE:

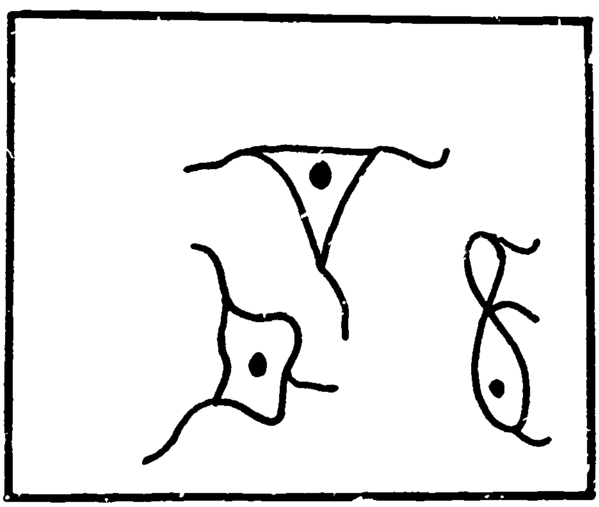
2, 2, 4, 6, 10, 16, 26, 42

RULE:

#30







All are Mokes.

42

123

105

303

114

None are Mokes.

34

225

212

223

44

Which are mokes?

51

233

63

330

124

82

23 437	38 583	71 910	18 381
16	40	63	27

#33

SUMMARY

1. BASIS OF NUMBER SYSTEM.
2. LOOK FOR ORDER OR LOGIC.
3. USE PATTERNS TO HELP UNDERSTANDING.
4. USE PATTERNS TO PREDICT.

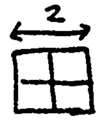
#34

GIVEN: YOU HAVE DIMES, NICKELS, AND PENNIES. HOW MANY WAYS CAN YOU MAKE 10¢ USING DIMES, NICKELS, AND PENNIES?

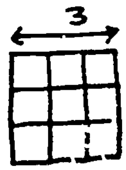
10¢	5¢	1¢
1	0	0
0	2	0
0	1	5
0	0	10

For a square with length of side 1, you need 1 square.

For a side of 2, you need 4 squares.



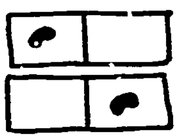
For a side of 3, you need 9.



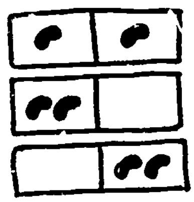
Continue the pattern.

LENGTH OF SIDE	SQUARES
1	1
2	4
3	9
4	

With just 1 bean, there are 2 ways to put it into the containers.



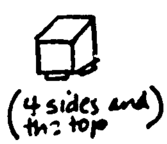
With 2 beans, you can put them into the containers 3 different ways.



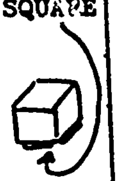
How many ways for 3 beans? 4? Make a chart. Look for a pattern.

BEANS	WAYS
1	2
2	3
3	

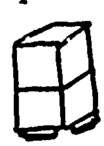
With a Unifix tower that's only 1 high, there are 5 squares to paint.



DON'T COUNT THE BOTTOM SQUARE



With a tower that's 2 cubes high, there are 9 squares to paint.



How many squares for a 3-cube tower? 4? Make a chart.

CUBES	SQUARES
1	5
2	9
3	

#36

YOU HAVE QUARTERS, DIMES, NICKELS, AND PENNIES.
HOW MANY WAYS CAN YOU MAKE 25¢?

25¢	10¢	5¢	1¢
1	0	0	0
0	2	1	0
0	2	0	5
⋮	⋮	⋮	⋮
0	0	0	25

#37

THERE ARE THE SAME NUMBER OF BICYCLES AND WAGONS. THERE ARE 30 WHEELS ALTOGETHER. HOW MANY OF EACH ARE THERE?

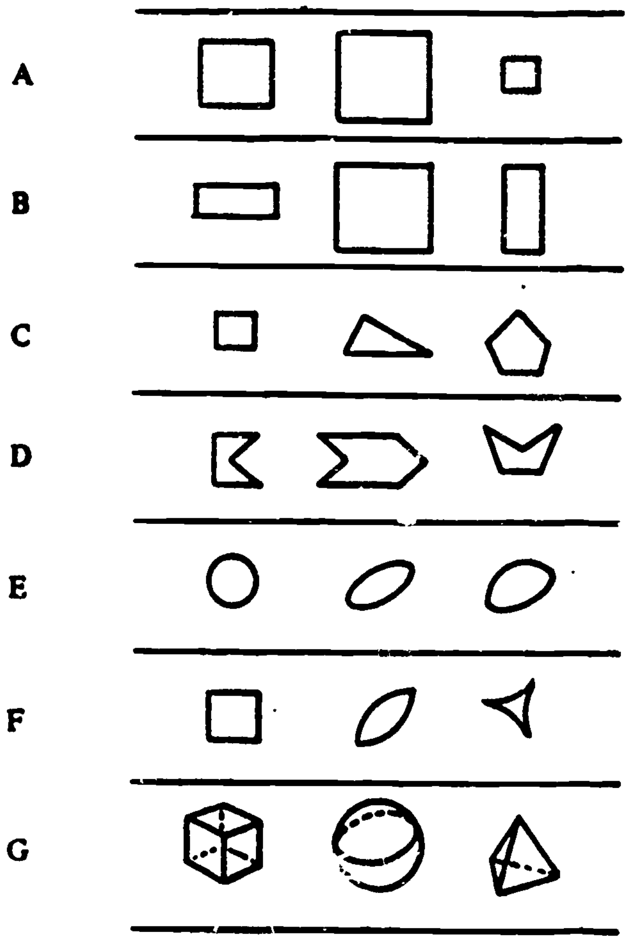
Bicycles	Wagons	Total Wheels
1	1	6
3	3	18
4	4	24
5	5	30

#38

	Hours played each week	Court fees
Mary	8.5	$34
John	8.5	$34.
Cammy	9	$18
Ara	7	$28
Ken	5	$10
Darsi	12	$24
Yolanda	?	?
Vernon	?	?
	TOTAL	?

ALL, SOME, NONE

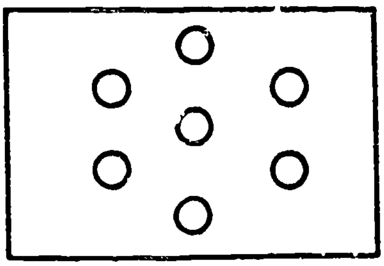
Write all, some, and none statements about the geometric figures in each of the following sets.



#40

PEN THE PENNIES!

With three straight lines separate all the pennies.



LOGICAL BREAKFAST

STEVEN, DOREEN, AND JAY EACH ATE SOMETHING DIFFERENT FOR BREAKFAST. ONE HAD GRANOLA, ONE HAD SCRAMBLED EGGS AND TOAST, ONE HAD A BANANA SPLIT. (THE LAST ONE WAS BECAUSE THE PARENTS WERE AWAY ON VACATION AND THERE WAS A VERY LENIENT BABYSITTER.)

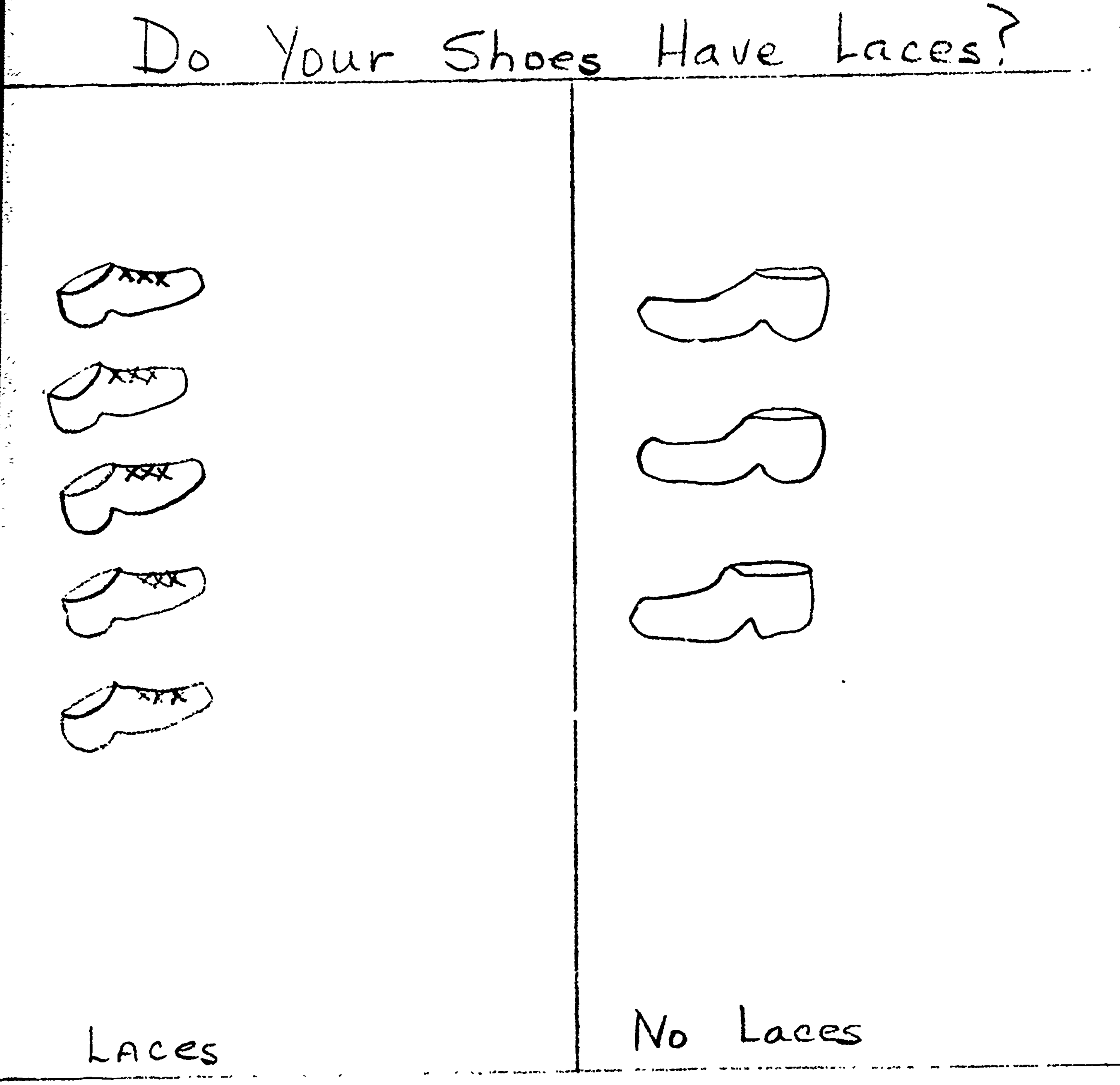
JAY DID NOT HAVE SCRAMBLED EGGS AND TOAST OR A BANANA SPLIT. DOREEN DID NOT HAVE SCRAMBLED EGGS AND TOAST.

WHOSE PARENTS WERE ON VACATION?

HAS TEACHING COME TO THIS?

RUTH, MARIA, KATY, AND JANE ARE ALL TEACHERS HAVING TROUBLE MAKING ENDS MEET. THEY ALL HOLD PART-TIME JOBS AFTER SCHOOL. ONE IS A CARPENTER'S APPRENTICE, ONE IS A COOK, ONE IS A DELIVERY PERSON FOR A PIZZERIA, AND ONE TEACHES JUDO. RUTH HAS NOTHING TO DO WITH FOOD. MARIA AND THE COOK ARE SISTERS. KATY AND MARIA ARE ALLERGIC TO WOOD WORKING. JANE, THE DELIVERY PERSON, AND THE JUDO TEACHER ARE STRANGERS. MATCH UP EACH WOMAN WITH HER WORK.

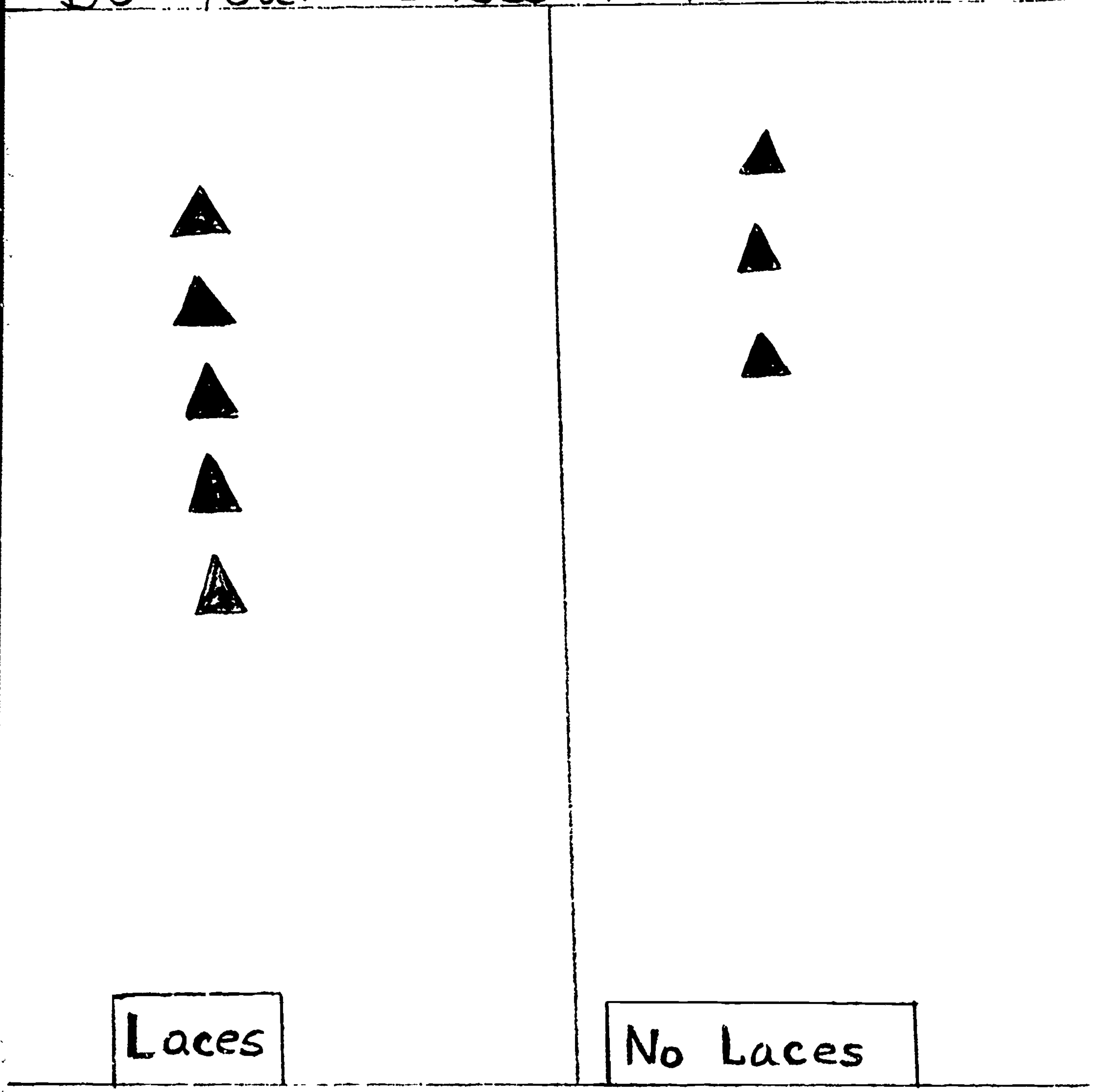
#43



"Picture"

#44

Do Your Shoes Have Laces?



"Symbolic"

_______, _______, _______, and _______ are candles on a birthday cake. _______ blew three of them out. How many are now burning.

_______, _______, _______, and _______ are birds flying in the sky. _______ and _______ are more birds flying in the sky. How many birds are there in all?

DEALING IN HORSES

A man bought a horse for $50 and sold it for $60. He then bought the horse back for $70 and sold it again for $80. What do you think was the financial outcome of these transactions?

__ Lost $20

__ Lost $10

__ Came out even

__ Earned $10

__ Earned $20

__ Earned $30

__ Other (describe) ______________________

Explain your reasoning: ____

____

#47

HOW MANY ROLLS TO GET A 1?

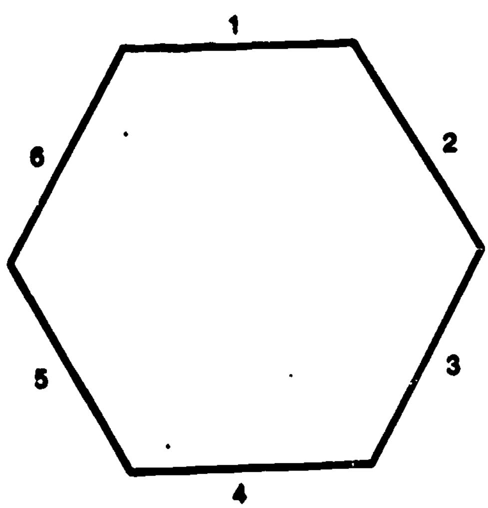
1	11	21
2	12	22
3	13	23
4	14	24
5	15	25
6	16	26
7	17	27
8	18	28
9	19	29
10	20	30 OR MORE

THE STACKS PROBLEM

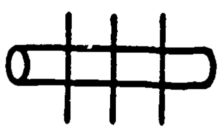
SHOW THE WAYS THAT 15 UNIFIX CUBES CAN BE PUT INTO 4 STACKS SO THAT EACH STACK HAS A DIFFERENT NUMBER OF CUBES, AND THERE IS AT LEAST ONE CUBE IN EACH STACK.

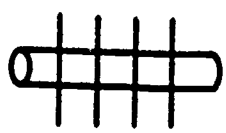
#49

THE SIX STUDENTS IN MR. SMITH'S BIOLOGY CLASS WERE ARRANGED NUMERICALLY AROUND A HEXAGONAL TABLE. WHAT NUMBER STUDENT WAS OPPOSITE NUMBER 4?



A LOG IS CUT INTO 4 PIECES IN 9 SECONDS. AT THE SAME RATE, HOW LONG WOULD IT TAKE TO CUT THE LOG INTO 5 PIECES?

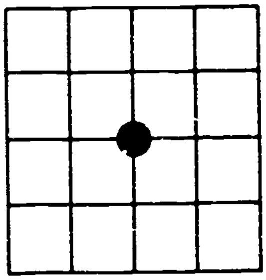




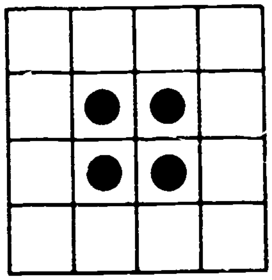
SUPPOSE YOU PUT 10 DOTS ON A CIRCLE. IF YOU DREW LINES CONNECTING EVERY DOT TO EVERY OTHER DOT, HOW MANY LINES WOULD YOU DRAW?

#52

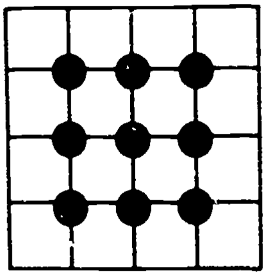
SOLVE A SIMILAR/SIMPLER PROBLEM



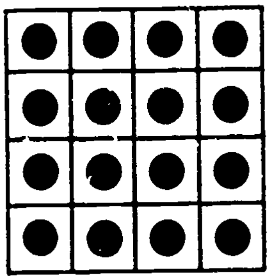
One 4-by-4 square



Four 3-by-3 squares



Nine 2-by-2- squares

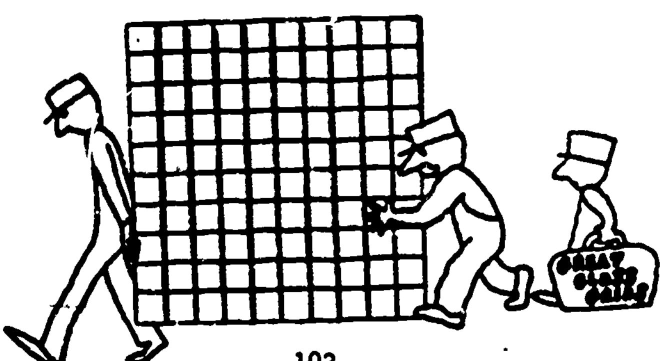


Sixteen 1-by-1 squares

Number of Squares

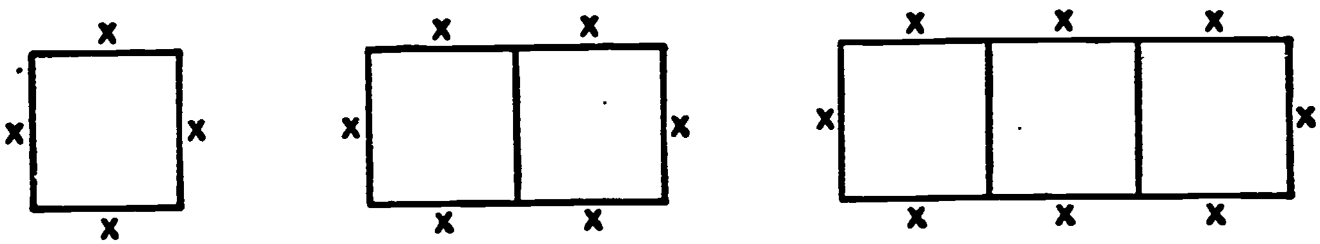
Size board	1x1	2x2	3x3	4x4	5x5	6x6	7x7	8x8	Total
1 x 1	1	___	___	___	___	___	___	___	1
2 x 2	4	1	___	___	___	___	___	___	5
3 x 3	9	4	1	___	___	___	___	___	14
4 x 4	16	9	4	1	___	___	___	___	30
5 x 5									
6 x 6									
7 x 7									
8 x 8									

A follow up or extended activity would be to see how many squares there are on a 10-by-10 grid.



II. Problem

Twelve couples have been invited to a party. The couples will be seated at a series of small square card tables, placed end to end so as to form one large long table. How many of these small tables are needed to seat all 24 people?



Number of tables	1	2	3	4	.	.	.	n
Number or guests	4	6	8	10	.	.	.	24

Number of tables	1	2	3	4	5	6	7
Number of guests	4	6	8	10	12	14	16

#54

Problem

A Polaris Commuter has seats for 108 passengers. On a flight to Memphis, there was 1 empty seat for every 2 passengers actually on board. How many passengers were on the flight?

A simpler problem would be one that considered a plane with seats for 12 passengers.

E	X	X
E	X	X
E	X	X
E	X	X

Empty	People	Total
1	2	3
2	4	6
3	6	9
4	8	12

The pattern revealed by the table leads to $x + 2x = 108$.

$3x=108$ $x=36$ $36+\square=108$

#55

The new high school has just been completed. There are 1,000 lockers in the school and they have been numbered from 1 to 1,000. During recess, the students decide to try an experiment.

When recess is over, each student will walk into school, one at a time. The first student will open all of the locker doors. The second student will close all of the locker doors with even numbers. The third student will change all the locker doors with numbers that are multiples of three. (Change means closing lockers that are open and opening lockers that are closed.) The fourth student will change the position of all locker doors numbered with multiples of four; the fifth student will change the position of the lockers that are multiples of five, and so on. After 1000 students have entered the school, which locker doors will be open?

Locker #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Student 1	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O
2		C	O	C	O	C	O	C	O	C	O	C	O	C	O	C	O	C	O	C
3			C	C	O	O	O	C	C	C	O	O	O	C	C	C	O	O	O	C
4				O	O	O	O	O	C	C	O	C	O	C	C	O	O	O	O	O
5					C	O	O	O	C	O	O	C	O	C	O	O	O	O	O	C
6						C	O	O	C	O	O	O	O	C	O	O	O	C	O	C
7							C	O	C	O	O	O	O	O	O		O	C	O	C
8								C	C	O	O	O	O	O	O	C	O	C	O	C
9									O	O	O	O	O	O	O	C	O	O	O	C
10										C	O	O	O	O	O	C	O	O	O	O
11											C	O	O	O	O	C	O	O	O	O
12												C	O	O	O	C	O	O	O	O
13													C	O	O	C	O	O	O	O
14														C	O	C	O	O	O	O
15															C	C	O	O	O	O
16																O	O	O	O	O
17																	C	O	O	O
18																		C	O	O
19																			C	O
20																				C

Write an Equation

I. Problem

A certain number added to itself 3 times is the same as 2 times 9.

$\square + \square + \square = 2 \times 9$

Three students the same age added their ages and found the sum to be 2 more than the age of the principal who was 34. How old was each student?

$\square + \square + \square = 34 + 2$ or $3 \times \square = 34 + 2$

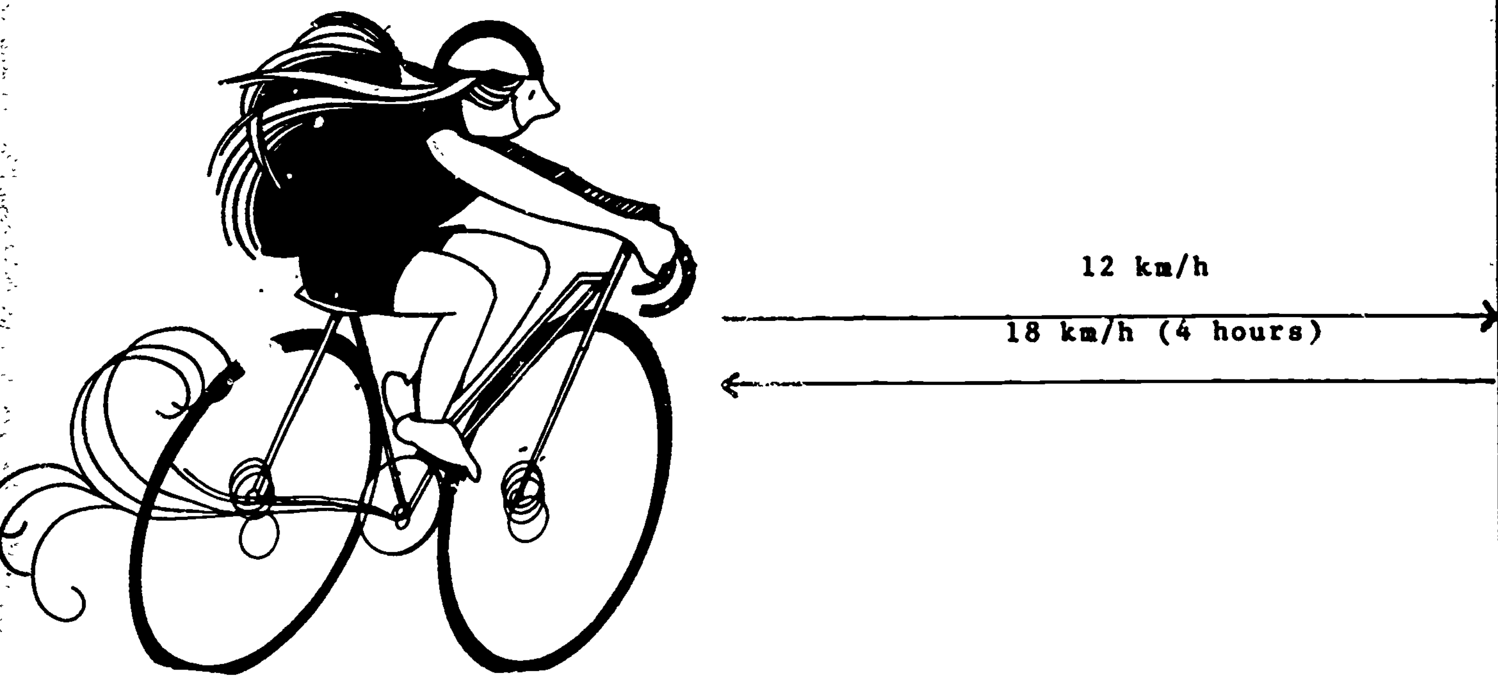
Find at least one rectangle where the number of square inches of area is the same as the number of inches in the perimeter.

$L \times W = 2 \times (L + W)$

#57

II. Problem

Sara averages 12 kilometers an hour riding her bike to the state park against the wind. She averaged 18 kilometers an hour riding home with the wind at her back. If it took her 4 hours to return, how long did it take for the <u>total</u> trip?



* List the quantities involved in the problem (known and unknown):

rate (against wind) ___
rate (with wind) ___
time (with wind) ___
time (against wind) ___
distance (home to park) ___
distance (park to home) ___

□ × □ = □ × □

□ = □

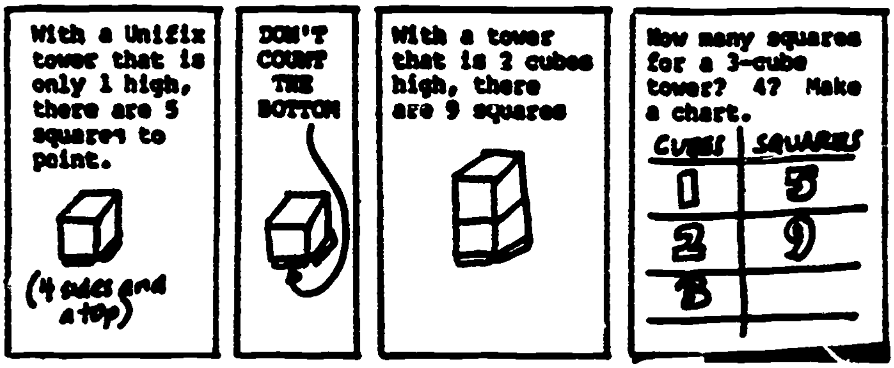
□ = □

□ + □ = □

#58

III. Problem

Suppose you build a Unifix tower that is 99 cubes high. And suppose you had to paint each square on the tower. How many squares would you have to paint?

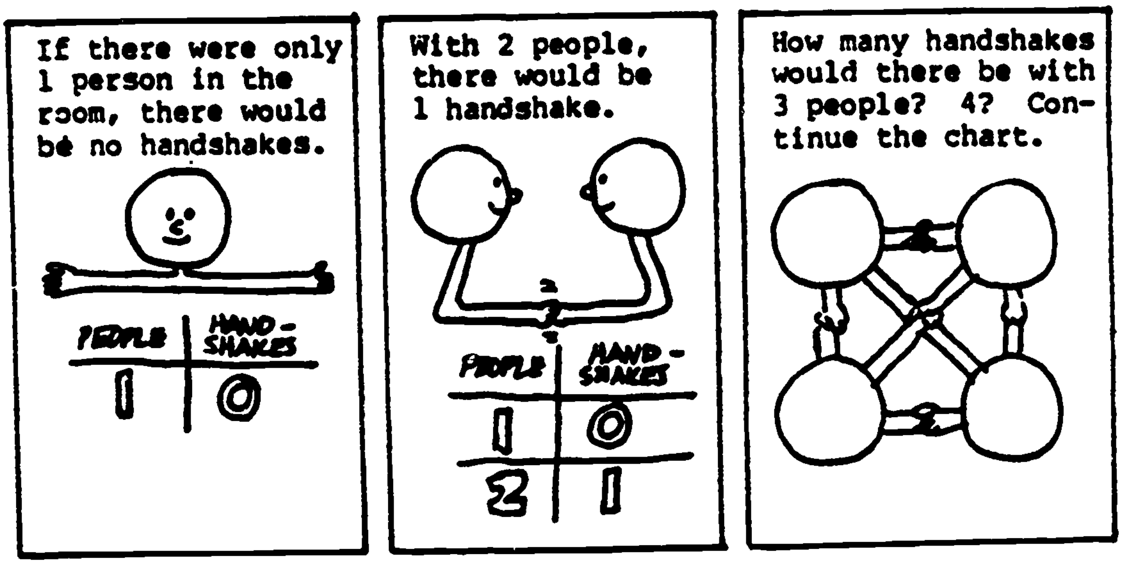


(__ x __) + __

(__ x __) + __ = ____

IV. Problem

Suppose everyone in this room were to shake hands with every other person in the room. How many handshakes would that be?



$$\frac{\_\_\,(\_\_ - \_\_)}{\_\_}$$

HANDOUTS

GUESS TEST

Directions:

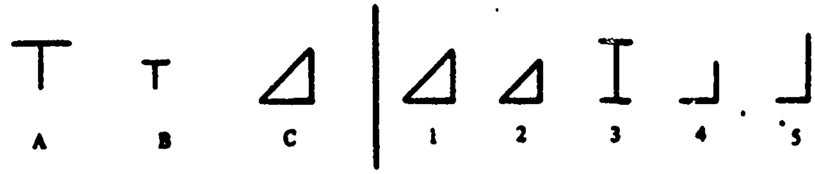
How good are you at guessing? Record your guesses for each of the following items.

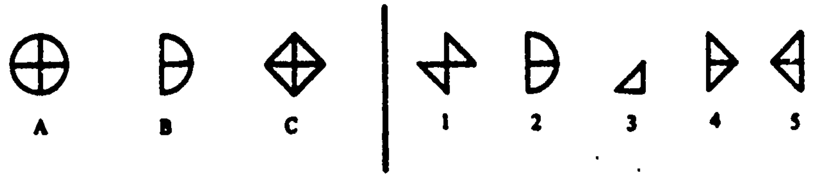
1. How many times does your heart beat in one day?
2. What was the orbiting speed of the space shuttle Columbia?
3. How long does a light bulb burn?
4. How long is a new pencil?
5. What is the weight of a Datsun 210?
6. How many seats are there in the school auditorium?
7. What is the length of the longest beard ever grown by a man?
8. How many words are there on a page in your dictionary?
9. What part of a typical 60 minute television show is devoted to commercials?
10. How many times does a wheel on a car revolve as it travels for one mile?
11. How far does the needle on a record player travel when a 12 inch LP record plays?

Pre-test **ARE YOU LOGICAL?**

1 - 3 Below are three panels of figures. Study figures A and B in each line. Whatever is done to figure A in order to change it to figure B must be done to figure C. What is the number of the correct figure in each line (1 through 5)?

1) 

2) 

3) 

4) 3 is to 9 as 4 is to ________?

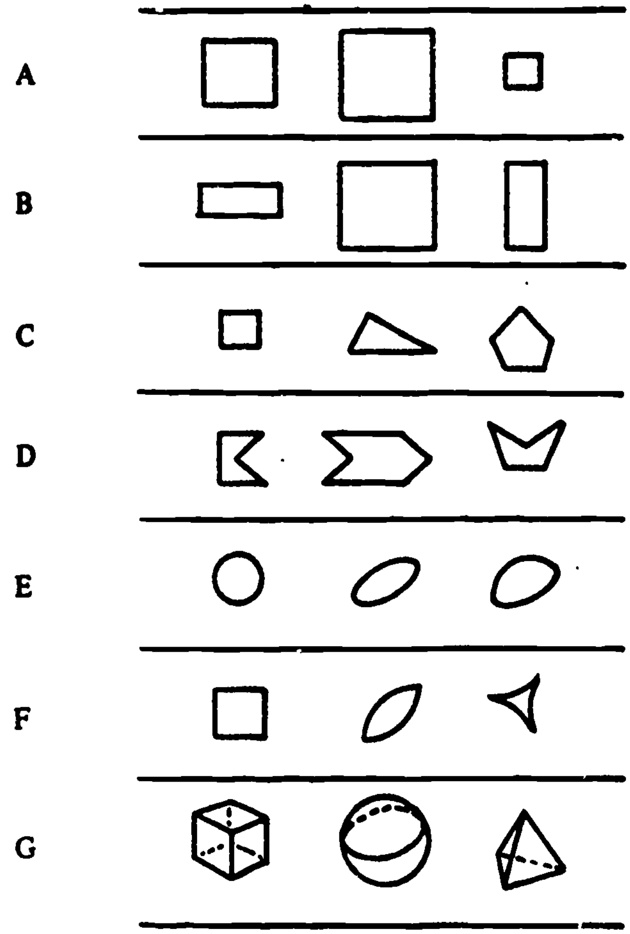
5) When does 8 plus 5 equal 1?

6) What can you put in your left hand that you cannot put in your right hand?

7) How many additional ping pong balls do you need so that all the balls touch a center ball? (Do not count the center ball.)

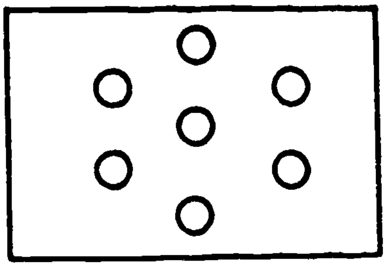
ALL, SOME, NONE

Write all, some, and none statements about the geometric figures in each of the following sets.



PEN THE PENNIES!

With three straight lines separate all the pennies.



THE MANAGER OF A RACQUETBALL CLUB WOULD LIKE TO KNOW THE AVERAGE AMOUNT PAID PER WEEK BY EACH PLAYER USING COURT I. FROM PREVIOUS WEEKLY REPORTS, SHE NOTICED THAT MARY AND JOHN EACH PLAY 8.5 HOURS A WEEK; THEIR DAUGHTER, CAMMY, PLAYS 9 HOURS; BRAD PLAYS 7 HOURS; HIS SON, KEN, PLAYS 5 HOURS; BRAD'S DAUGHTER, DARSI, PLAYS 12 HOURS; YOLANDA PLAYS 9.5 HOURS; AND HER SON, VERNON, PLAYS 11 HOURS. IF EACH OF THE ADULTS PAYS $4 AN HOUR FOR COURT TIME AND THE CHILDREN PAY HALF OF THE ADULT FEE, WHAT IS THE AVERAGE AMOUNT PAID PER WEEK BY EACH PLAYER?

Pro-test

YOU'RE LOGICAL?

1. If you went to bed at 8 o'clock at night and set the alarm to get up at 9 o'clock in the morning, how many hours sleep would this permit you to have?

2. Do they have a 4th of July in England?

3. How many birthdays does the average man have?

4. Why can't a man living in Winston-Salem, N.C. be buried west of the Mississippi River?

5. If you had only one match and entered a room in which there was a kerosene lamp, an oil heater, and a wood-burning stove, which would you light first?

6. Some months have 30 days, some have 31. How many months have 28 days?

7. If a doctor gave you 3 pills and told you to take 1 every half hour, how long would they last you?

8. A man builds a house with 4 sides to it, and it is rectangular in shape. Each side has a southern exposure. A big bear comes wandering by. What color is the bear?

9. How far can a dog run into the woods?

10. What four words appear on every denomination of U.S. coins?

11. What is the minimum number of active baseball players on the field during any part of an inning? How many outs in each inning?

12. I have in my hand two U.S. coins which total 55 cents in value. One is not a nickel. (Please keep this in mind.)

13. A farmer had 17 sheep. All but 9 died. How many did he have left?

14. Divide 30 by ½ and add 10. What is the answer?

15. Two men played checkers. They play five games and each man wins the same number of games. How can you figure this out?

16. Take two apples from three apples and what do you have?

17. An archeologist claimed he found some gold coins dated 46 B.C. Do you think he did?

18. A woman gives a begger 50 cents. The woman is the beggar's sister, but the beggar is not the woman's brother. How come?

19. How many animals of each species did Moses take aboard the Ark with him?

20. It is legal in North Carolina for a man to marry his widow's sister?

21. What word is misspelled in the test?

16 correct- Genius

10 correct- Normal

8 correct- Subnormal

5 correct- Idiot

SPILL THE BEANS

1	6	11	16	21
2	7	12	17	22
3	8	13	18	23
4	9	14	19	24
5	10	15	20	25

WILL THEY EVER BE THE SAME?

1	6
2	7
3	8
4	9
5	10

ACT IT OUT

IDEAS

1. How long does it take for a book to fall from the desk to the floor?

2. How many pennies laid next to each other are needed to measure exactly one foot?

3. How many checkers will fill a one pound coffee can?

4. How many seats are in the auditorium of your school?

5. Are there more boys or more girls in your class?

6. How many people in your class were born in April?

7. Which has more pages, your dictionary, or your local telephone directory?

8. How many scoops of ice cream can you get from one-half gallon of vanilla ice cream?

9. Which rock group is the favorite in your class?

10. How long does it take to read a page of print?

11. Can you run a 100-yard dash faster than a car can drive one-half mile at 55 miles an hour?

12. Place 20 pennies on the table in a row. Replace every fourth coin with a nickel. Now replace every third coin with a dime. Now replace every sixth coin with a quarter. What is the value of the 20 coins now on the table?

13. Place 20 pennies on the table in a row with heads up. Now "flip" every coin to show tails. Now "flip" every other coin beginning with the second penny (2,4,6,...). Now flip every third coin, beginning with the third penny (3,6,9,...). Nest flip every fourth penny, beginning with the fourth coin (4,8,12,..). Continue this procedure for 20 trials. Which pennies now have the heads up?

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