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**ABSTRACT**

This document contains two inservice presentations. Each presentation has a full script with transparencies, handouts, graphs, and a list of materials needed for the presentation. The first presentation describes eight ideas common to both mathematics and reading namely, that: (1) a comprehensive instruction program is needed; (2) educators teach students, not mathematics or reading; (3) mathematics/reading should be made relevant to the learners; (4) mathematics/reading is a thinking process; (5) mathematics/reading is a language communication process; (6) the learning of mathematics/reading proceeds from the concrete to the abstract; (7) mathematics/reading is seeing relationships, and (8) involvement in the learning process is critical to success in mathematics/reading. The second presentation discusses eight recommendations by the National Council of Teachers of Mathematics. Six references are listed. (YP)

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BUILDING A MIND SET FOR MATHEMATICS  
 AND  
 CURRENT TRENDS AND ISSUES IN MATHEMATICS

INSERVICE PRESENTATIONS

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 LOUISIANA STATE DEPARTMENT OF EDUCATION  
 BUREAU OF ELEMENTARY EDUCATION

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**PART I**

**BUILDING A MIND SET FOR MATHEMATICS**

## BUILDING A MIND SET FOR MATHEMATICS

### Anticipatory Set

(As participants arrive, have them complete three graphs:

- (1) Relating teaching experience and attitude toward mathematics
- (2) How long a line will the average pencil draw?
- (3) The 3 Sacks Problem)

(Ask:) Is graphing a reading skill or a mathematics skill? (Guide participants to see that it is a skill common to both mathematics and reading.)

(Say:) Graphing is one area that mathematics and reading have in common. In this workshop, we will look at eight major concepts or ideas common to both mathematics and reading. By relating mathematics to reading, we hope to capitalize on a principle of learning--building from the known to the unknown.

### Objectives (On transparency #1 and #2)

(Say:) Our objectives are. . . (What) The participant will be aware of eight concepts or ideas common to both mathematics and reading.

- (1) A comprehensive instructional program is needed in mathematics and reading, and the same process may be used to achieve either.
- (2) We teach students, not mathematics or reading.
- (3) Mathematics/reading should be made relevant to the learners.
- (4) Mathematics/reading is a thinking process.
- (5) Mathematics/reading is a language communication process.
- (6) The learning of mathematics/reading proceeds from the concrete to the abstract.
- (7) Mathematics/reading is seeing relationships.
- (8) Involvement in the learning process is critical to success in mathematics/reading.

(Transparency #3) The participant will have background information for developing a statement of beliefs about mathematics regarding the following:

- (1) What is mathematics?
- (2) Why should we teach mathematics?
- (3) What mathematical abilities do students need to be productive citizens?
- (4) How can we provide students with these abilities?
- (5) Should mathematics be enjoyable? If so, what is the joy in mathematics?
- (6) How can we promote positive attitudes of teacher/students toward mathematics?

(Transparency #4) (Why) The participant will feel more comfortable in working to upgrade mathematics. (Elaborate as appropriate to the group.)

## Input, Modeling, Checking for Understanding

(Transparencies will be used to introduce the eight unifying concepts or ideas. After a brief discussion of each topic, participants will engage in an activity or activities illustrative of the topic and relevant to both mathematics and reading.)

Since this is part of the Mathematics Improvement Effort, the first thing we would like to do today is to brainstorm in small groups what you think needs to be done to improve mathematics in Louisiana.

(Wait. Have groups report list, naming only those things not previously given.)

You have just participated in a mini-version of the process of program improvement that has proven successful in upgrading reading. It is a process of collaborative planning and problem solving and is based on the belief that the answers to education's problems lie within you, the educators.

As we have discussed before, the process of program improvement is an ongoing process which is constantly open to new evidence.

This is the first in a series of three workshops which we hope will help you examine and formulate your views on the teaching of mathematics.

As you listen to the eight commonalities which have been identified for reading and mathematics, see if they support or alter your beliefs about mathematics improvement. (Transparency #5)

1. A comprehensive instructional program is needed in mathematics and reading, and the same process may be used to achieve either.

The first step in the instructional improvement process is to assess needs--where you are in relation to the ideal. In mathematics as in reading there is no one blueprint for a comprehensive mathematics program. The important thing is that you define your program so that all people are working toward common goals, and students are engaged in a program characterized by consistency, coordination, and continuity from grade to grade and school to school. Direction for the development of a comprehensive program is provided by the development of a statement of beliefs regarding the content area being addressed.

(Transparency #6)

2. We teach students, not mathematics or reading.
  - a. Students are students, and learning is learning whether it be the learning of mathematics or the learning of reading.

The concepts we'll be discussing today apply to elementary and secondary students, including regular and special education students. These concepts have been reviewed by experts in reading, mathematics, elementary, and secondary education, as well as special education.



- b. Individual differences exist and must be accommodated.  
Individual differences in learning rate, for example, have implications for grouping, scheduling, and recordkeeping. Differences in learning styles necessitate a variety of teaching methods, e.g., the use of visuals, manipulatives, etc.
- c. Likewise, principles of learning and effective teaching behaviors apply to all content areas.  
Research on the effective school and the effective teacher have identified a set of generic teaching behaviors that cut across all curricular areas. These include teaching behaviors such as those identified by Madeline Hunter. (Distribute a summary of these behaviors.)

The effective teacher, be it math or reading, teaches to an objective that is appropriate for the learners. In order to do this the teacher must know the student--whether he or she has the requisite knowledge or skills critical to achieving the objective as well as personal needs that might facilitate or impede learning. During instruction the focus must be on the student. The effective teacher monitors student performance to see if he/she is learning and makes appropriate adjustments. The teacher's mind set is: "You didn't understand? Let's try it again another way, a better way." Always the teacher is guided by the principles of learning and consciously uses those factors to enhance students' motivation, rate and degree of learning, retention, and transfer. Take a few moments to skim over the principles of learning. (Wait.) These factors have implications for a comprehensive math program and should be considered when assessing the effectiveness of your program.

For example, in regard to motivation--

- . Are students highly motivated to learn mathematics?
- . Is math a pleasant experience?
- . Are students placed where they can be successful yet challenged?
- . Are teaching methods used which stimulate the student's curiosity and interest?
- . Does the student get specific and rapid feedback?

A relevant follow-up activity might be to go back to your school(s) and see if these things are happening. If not, brainstorm ways to build these into the math program.

These factors might also be used to assess lessons or activities. When you arrived today, you completed three graphs. Assess the potential effectiveness of this activity in terms of the principles of learning. (Solicit responses from the group phrasing the principles of learning as questions. If not volunteered, point out the following) Graphs are a vehicle for actively involving a group in learning. They are also a way to present statistics in a concise, visual way that makes it possible to see relationships in the information more easily. Incorporating graphing into the math program is a way to bring a use of numbers into the classroom as they would be used in the real world.

(Add:) Graphs can also be a very useful tool for getting to know students' interests and needs. (Refer to the graph on their attitude toward mathematics, asking the following questions:)

- (1) What can you say about the math attitudes in this group?
- (2) How many people who have taught grades K-2 are in this group?
- (3) What fraction of this group has taught grades K-2?
- (4) Is there some relationship between the grade taught and attitude toward math?

(Have the group brainstorm other questions that are appropriate for graphing that might help us get to know our students better.)

- (1) Where were you born (state)?
- (2) Position of birth in your family.
- (3) Time you went to bed last night.
- (4) Hours you usually sleep each night.
- (5) Did you eat breakfast today?
- (6) Favorite drink.
- (7) Favorite school subject.
- (8) How many TVs in your house.
- (9) How many hours you watched TV yesterday.
- (10) How many people live in your home.
- (11) What do you call your mother?
- (12) What do you call your father?
- (13) Favorite hamburger place.
- (14) Do you have a pet?
- (15) What kind of house do you live in? (Brick, etc.)
- (16) Who decides when you get your hair cut?
- (17) How many books do you read each month?
- (18) How do you get to school each morning?
- (19) Least favorite school subject.

(Transparency #7)

3. Mathematics/reading should be made relevant to the learners.

a. Individuals learn when they have a "compelling why" or need to know. Reading and mathematics are skills or tools which cut across all content areas and are needed to function effectively in everyday living as well as on the job. Therefore, it is important that we help students relate and apply what's learned to the real world and that learning grow out of real life situations. Why do students need to know mathematics?

b. To illustrate the everyday uses of mathematics, please...

- (1) List all the situations for which you've used mathematics during the past month outside of job responsibilities. (Allow time.)
- (2) Beside each one, list the method usually used to do each task: calculator, paper and pencil, figure mentally.
- (3) Determine which method is used most/least often. (Allow time.)
- (4) Then review the list to see whether each task requires estimation or accuracy and the percentage of each.

(Wait and then point out the following:) Using a calculator and doing arithmetic mentally are the most frequently reported methods for doing arithmetic. There is a 50-50 split, usually, between the need for accuracy and estimation. Paper and pencil is not the usual choice, yet more than 75% of math time is spent on paper and pencil drill, with students practicing arithmetic skills in isolation from problem solving situations. In real life, problems do not present themselves on worksheets, ready for people to compute answers.

Deciding what to do is the important first step before computation. "Facts" must be mastered and practice is important, but it is important that computation be taught in the context of problem solving so it can assume its proper role as a tool to use in solving problems.

- c. The number of students who are attending colleges and universities has been increasing for several years. We must relate to students the importance of becoming good problem solvers to keep the doors to further educational opportunities open and making employment opportunities more predictable.
- d. On the job, the ability to solve problems take on increased significance. Ask any employer the one thing he values most in a potential employee, and he will usually say, "I want someone who can reason and solve problems." The more technologically advanced the field, the more critical this ability becomes.

Computation is to mathematics as decoding is to reading.

Just as comprehension is the goal of reading, the ability to solve problems is the goal of mathematics. This is best achieved by teaching mathematics in the context of relevant problem solving situations which grow out of real life.

(To illustrate, formulate word problems based on the graphs.)

#### Pencil Graph

- (1) What is the most common opinion?
- (2) What is the range of opinions?
- (3) How many more think \_\_\_ than \_\_\_ ?
- (4) What fraction of the group think \_\_\_ ?
- (5) Can you tell who in the group has what opinion?
- (6) Does this graph give you information about what most people think? If so, what can you say about the opinions of this group?
- (7) How are these questions which are based on the graph relevant to you as a learner? (Your opinion is valued, pencils are used daily, reveals the wealth of opportunities in our everyday world for explaining mathematics.)

(Show Transparency #8)

4. Mathematics/reading is a thinking process.

This fact has numerous implications for instruction:

- a. First, the emphasis of instruction should be on thinking rather than on the answer. Getting the right answer is not synonymous with understanding.

In Durkin's classic study on reading comprehension, she found that teachers and texts were testing comprehension rather than teaching comprehension. A related finding in mathematics based on National Assessment Results is that students are learning much of their arithmetic by rote, without the needed understanding to apply these concepts to solve problems. Teaching rules by rote without developing understanding results in students who rely on recipes rather than students who think through the process.

In both reading and math the focus must be on understanding. In reading, understanding cannot be evaluated on the students' ability to answer a question, but on their ability to reconstruct the thought processes used to arrive at the answer so that they can apply those processes in comprehending other written materials. Likewise, children's understanding of mathematics cannot be evaluated merely from their ability to apply procedures to numbers and arrive at correct answers. Understanding is evaluated by their ability to analyze real-world situations and recognize what process is called for.

(Transparency #9) Teachers focus on the process by:

- (1) modeling and labeling the process for students, which simply stated means.. "Tell them what you're gonna tell them, show them while you tell them, then tell them what you told them."
  - (2) encouraging students to explain "how" they arrived at the answer or solved the problem and allowing students to listen, question, and learn from each other in small groups.
  - (3) focusing on the process used as well as the answer when grading papers, and by
  - (4) analyzing error patterns as clues to students' reasoning.
- b. This leads us to a second implication for instruction: Answers are not random. By analyzing wrong answers in mathematics and reading, we gain valuable clues into students' reasoning. As a matter of fact, it was an analysis of mistakes made in paragraph reading that led Thorndike to conclude back in 1917 that reading was reasoning. He stated that "understanding a paragraph is like solving a problem in mathematics. It consists of selecting the right elements of the situation and putting them together in the right relations, and also with the right amount of weight or influence or force for each." Thorndike found some common error patterns.

Analysis of responses on mathematics tests, likewise, reveal some common error patterns. (Pass out the handout on common errors and have participants describe the incorrect thinking exhibited. After discussion, continue by stating...) (See Common Errors Insert.) (Transparency #10)

The errors of many students follow a pattern because what they have actually learned is incorrect - is based on faulty or incompletely learned mathematical concepts. This incorrect procedure, however, has some logic for the student, even though the logic is incorrect. When performing isolated arithmetical calculation, there is no way for students to check their logic against the real world.

The elementary math program should help students learn to organize their ideas and understand what they learn. Much of informal logical thinking is a matter of common sense, but unfortunately, Voltaire said, "Common sense is not so common." Regular practice can help students strengthen their thinking abilities. In the words of Robert Barratta Lorton, "Students' ability to think logically increases in direct proportion to practice. Ongoing experiences in logical thinking provide students with skills that are useful in formulating solutions to problems they confront in any area of mathematics."

It is important to engage students in thinking where the emphasis is on the process of thinking rather than on the answer.

In such a setting, wrong answers are viewed as opportunities for learning. Stress, which often accompanies emphasis on the right answer is reduced, and thinking is facilitated. Abstract thinking cannot occur under stress.

- c. A third thing to remember in dealing with reading and mathematics as a thinking process is that telling answers or insistence on one way of solving problems stops thinking.

In mental processes as in life, there is more than one way to get the desired result (ex. ironing a shirt).

Mentally double 38. In a moment, I'll ask you to describe the method you used to get your answer. I'm not as concerned with the actual answer as with what went on inside your head.

Ex. 1.	$\begin{array}{r} 38 \\ +38 \\ \hline 76 \end{array}$	2.	$\begin{array}{r} 38 \\ \times 2 \\ \hline 76 \end{array}$	3.	$\begin{array}{r} 40 \\ +40 \\ \hline 80 \\ -4 \\ \hline 76 \end{array}$	4.	$\begin{array}{r} 30 \quad 8 \\ +30 \quad +8 \\ \hline 60 \quad 16 \\ +16 \\ \hline 76 \end{array}$	5.	$\begin{array}{r} 35 \\ +35 \\ \hline 70 \\ +6 \\ \hline 76 \end{array}$
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It is usual for a variety of methods to be reported when a group of adults do this mental computation. However, in school, calculations are taught with an emphasis on one particular process, not on a sensible approach to arriving at an answer. The implication to the student is that there is one right way to do arithmetic.

Recent research on learning styles, particularly that on left brain, right brain, emphasizes the importance of giving students optional ways of processing information.

- d. A fourth implication is that activities should be provided which encourage logical thinking. The "Three Sacks Problem" is illustrative of this. (Use graph and discuss)

1. Most common opinion
2. How would you convince someone else of your point of view?

(Ask different people to share the thought process they went through to decide which bag they would reach into.) Say: Even though not all students in a class will be able to reason this through, it's beneficial for all to hear the discussion. The emphasis here should be on the process--looking at how students think, not on whether the student "gets the right answer." The value of having the actual sacks is that the verification for the thinking is in the material, not from the teacher or an answer book.

(Show Transparency #11)

5. Mathematics/reading is a language/communication process.

- a. Both mathematics and reading have written symbols which allow man to communicate over time and space. Math symbols have an advantage over English symbols in that they are universal, but the same principle of learning applies to both:
- b. In mathematics, as in reading, concept development and oral language must precede dealing with written symbols. The text is not the proper starting place for learning. The text focuses on the symbolic which is much too abstract. As in the language experience approach to reading, concepts must first be developed through concrete experiences. At the concept level, there is doing and talking. The teacher's role is to make the connection between the concept and the symbol. As in reading, meaning is within the student, not in the text. It is suggested that teachers spend 60% of the time on conceptual development and oral language, 30% of the time connecting concept and symbol, and 10% of the time dealing with symbols. Drill and practice do not build concepts.

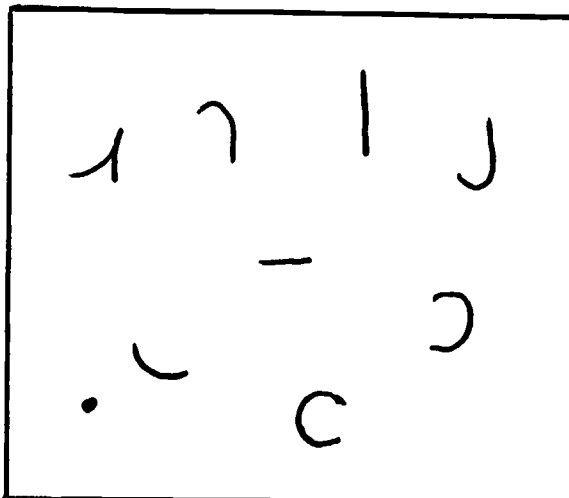
To give us added insight into the importance of connecting concepts with symbols in mathematics, we are going to "learn" a new set of symbols for mathematics.

(Distribute excerpts from the Math Teakst Buk)

For this experience, we'll use the alphabet for counting instead of 1, 2, 3. The alphabetical sequence is one that is familiar to you just as the number sequence is familiar to most students when they enter school. This sequence is useful to have you experience a child's process in learning concepts above the rote auditory sequence. Try to approach this in the same manner as a learner who does not already possess number concepts as you do.

If you count "a, b, c," how many fingers am I holding up? Hold up "e" fingers. Hold up "h" fingers. (Wait.) Some people need to count from "a" up to "h" to find out how many "h" are. It is important to remember that even though a child learns the counting sequence, this does not necessarily mean they know what the numbers mean.

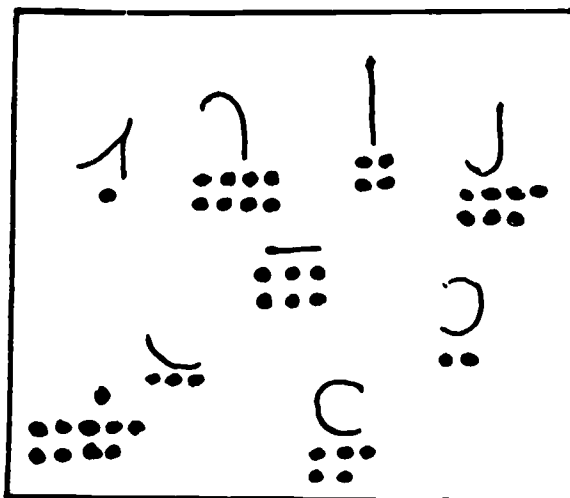
Count backward from "h" to "a." Do this now, in your head. (Ask someone to describe how they did it.) Young children do not have the luxury of having learned the written symbols. Learning to count backward is a different skill from learning to count forward. The sequence "c, b, a," feels comfortable for most. If you were asked to learn to count backward from "h" to "a," a reasonable way is to add one additional letter to the sequence you already know--"d, c, b, a" and extend the sequence in that way.



(Ask group to identify the symbols as you point to them.)

What are you learning? What are you feeling? There is nothing in the symbols themselves that have meaning--that convey "c-ness" or "e-ness." Assigning meaning to symbols is arbitrary. What I'm doing here by asking you to recall the numerals is testing, not teaching. Here is how I can shift to teaching, offering you assistance to learning.

(Add dots to the symbols. Then ask them to identify the symbols.)



(Give the symbol for 0. )

(Hand out the worksheet.)

Written with the hand you usually do not write with, write your name at the top of the page. Writing with your nondominant hand helps you to experience what children feel when their motor coordination isn't well developed.

(Have someone read the directions out loud and do the first problem together. Ask the group to complete the page. Tell them they may talk with each other.)

(Ask for reactions to the experience.)

What insights into children's learning did you have as you worked the page?

How did the number line help?

Notice you could do the exercises merely by noting the position of the numbers without paying attention to their values. No number understanding is developed in this way.

How did you feel about talking with or helping your neighbors? (Summarize responses with the following.) Children's symbolic work with numbers needs to follow concrete experiences that develop concepts. These concepts then must be connected to the symbolism before the students are expected to work independently with the symbols.

- c. Because mathematics is a language/communication process, talk solidifies thinking, and writing brings precision to the thinking process. Verbalization allows one to internalize and communicate what is "seen." Interaction with peers clarifies thinking and helps one discover other ways of seeing and doing. To illustrate the value of verbalization and interaction with others, we are going to work in groups of four to complete an activity--the Consecutive Sums Activity. As you complete the activity, analyze your thinking and the benefits of interaction.

Consecutive Sums (Cooperative Learning, reinforcement for addition, search for patterns)

For this activity, you are to use consecutive numbers only--numbers that go in order such as 1, 2, 3, 4. Are 11, 12, 13 consecutive? (Yes.) Are 16, 17, 19 consecutive? (No.)

Tell me a way to write 9 as the sum of consecutive numbers.

$$9 = 4 + 5 \quad \text{and} \quad 9 = 2 + 3 + 4 \quad (\text{Record.})$$

Your group task is to find all the ways to write the numbers from 1 through 25 as the sum of consecutive numbers. Some can be done only one way; some can be done more than one way; some are impossible. Look for patterns as you explore. Decide how to divide up the work and prepare one group record of results. On that record, write summary statements to describe what patterns you find. (Wait.)

What patterns have you been able to discern from your work? (Let groups share.)

Which numbers were written as sums in two ways? Three ways? Is there a pattern?

Describe the "Impossible numbers."



Examine the prime numbers and describe what you notice.

During this kind of problem-solving, the focus is not on mastery of skills but on students using their skills in problem-solving situations. Having students work in groups gives them the chance to learn from one another and to benefit from each other's thinking. Writing the summary statements provides structure and reinforcement for the thinking process.

- d. A final observation concerning mathematics as a language/communication process is that in some ways, learning the symbols of mathematics is like learning a foreign language. Students sometimes need guidance in translating from English into mathematics. To illustrate, can you translate the following English sentence into a mathematical equation? Example: Mary is four years older than John. (Wait for response:  $MA = JA + 4$ .) Algebraic equations, like English sentences, must have a subject and a verb. According to Marilyn Burns, the true test of math understanding is the ability to translate from real-life situations to math symbols and from math symbols to real life situations. The following activities allow for this translation to happen: If  $C$  stands for the cost of one record, how would you represent the cost of 9 records? ( $9C$ )

If you are going to cut a piece of lumber 12 feet long into two unequal sections, how could you represent the lengths of the two pieces? ( $x$ ,  $12 - x$ )

If  $x$  represents the cost of one stamp, what does the expression  $\$1.00 - 4x$  represent? (The change you would get for  $\$1.00$  after purchasing 4 stamps.)

The field of mathematics grew out of the need of man to solve problems. Mathematical symbols can be manipulated to show real-life situations without having situations present. Ex: 300 people and 400 people can be joined together as a group of 700 people. It is much easier to add  $300 + 400$  than to gather the actual people to show the resulting group.)

(Show Transparency #12)

6. The learning of mathematics/reading proceeds from the concrete to the abstract.

As previously discussed, textbooks focus on mathematical ideas symbolically. Often, this abstract representation of the concepts is the beginning and main focus of the mathematics instruction students receive. However, students need to start with concrete experiences, that, once they are learned, are connected to the symbolism. We often assume that the symbols on textbook pages have meaning embedded in them that is obvious to students. It is possible for students to learn to do textbook pages--learn the mechanics--without understanding the concepts underlying what they are doing. Ex: Teakst Buk experience. Learning skills in this way is not an effective strategy for applying those skills in problem-solving situations.

The arithmetical operations must be taught so that students see the logic in the operations while learning to get the correct answers. They should be taught in the context of word problems so that students see the logic in the operations while learning to get the correct answers. They should be taught in the context of word problems so that word problems are linked with the process and do not appear as something that comes afterward.

Marilyn Burns says that the traditional method of teaching mathematics--progressing from computation to word problems--is a pedagogical version of putting the cart before the horse. This might imply the need to reorder textbook lessons. Word problems are usually found at the end of the lesson. In a problem-solving approach, the word problems which represent real-life situations should become the starting point.

To illustrate, we shall do a regrouping activity.

For this activity, it would be necessary that students have had ongoing experiences with word problems. This means they are comfortable with hearing stories and writing equations about them. It would also be critical that students have had the prerequisite place value experiences necessary for the concept of regrouping. (Distribute place value boards, beans, and cups.)

### Concept Level

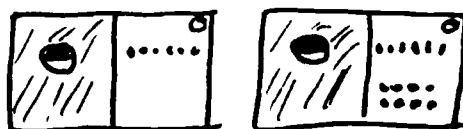
We're going to use the place value boards, beans, and cups to solve some problems. Here's a problem: (Be sure the problem is relevant to the group.)

A meeting was planned for (day-date) at (time). At 7:30, 16 people arrived at the meeting place.

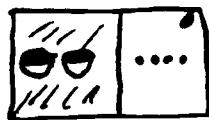
By 8:00, 8 more people had arrived at the meeting place. How many people had arrived for the meeting by 8:00?

What is the equation for the problem? (Record.)

Remember that at 7:30, 16 people arrived. Show 16 on your place value boards. By 8:00, 8 more people had arrived. Put 8 more on your boards, but put them below the others so they are not mixed.



Now you need to find out how many people had arrived by 8:00. Push the materials together. Do you need to exchange? (Regroup, rename, etc.) If so, do it. Now tell what you have on your board. you have 2 tens and 4 ones, and that is 24. (Record.) The answer is in the materials, not in symbols.



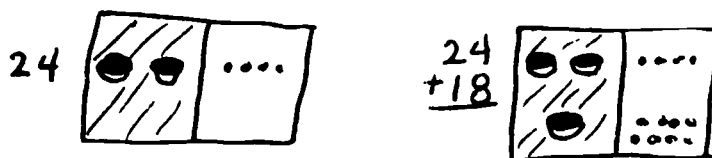
### Connecting Level

This time I will model how to record the symbolism for the regrouping when we solve a problem.

Mrs. Hebert, a TA, works at different schools. At one school she works with 24 teachers. At another school she works with 18 teachers. How many teachers does she work with at these schools?

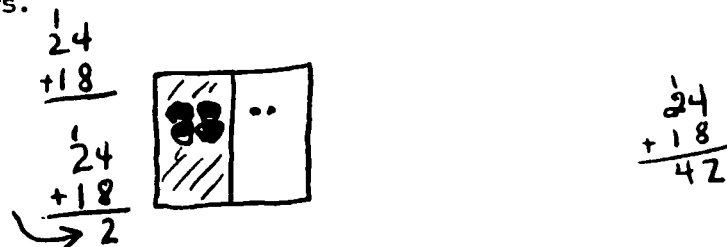
What's the equation for this story? (Record.) We'll do this with your materials, but this time I'm going to show you a way to keep track of your exchanges. Even though you can solve this easily, work along with me and see if you understand what I'm doing.

Using your cups and beans, put 24 on your board to show the teachers at one school, and then 18 underneath to show the teachers at the other school. Notice that I consistently link the materials back to the problem.



Combine the materials. (Wait.)

Did you exchange ones to make another ten? (Yes.) Look at how I record that on the problem. I write the extra ten above the other tens. How many ones do you have? I'll write that here. How many tens are not in your board? (4) I'll write that here. So altogether, Mrs. Hebert works with 42 teachers.



### Symbolic Level

At the symbolic level, the students deal with the concepts by doing their own recording of the symbols. They may still use concrete materials and will drop them when they are ready.

(Show Transparency #13)

### 7. Mathematics/reading is seeing relationships.

- a. Plaget tells us that from the time the child is born, he attempts to find order, structure, and relationship in the world around him. Learning is facilitated when we capitalize on this natural tendency of students. Einstein reminds us that all things are relative.

Relationships inherent in reading make it possible for the reader to make predictions. Bill Martin, Roach Van Allen and others have used the patterns in language to help children predict and eventually connect sounds and symbols. (Read the example on transparency #14 as well as a brief excerpt from Bill Martin's book, Monday, I Like Monday.) This Book, in addition to demonstrating patterns in reading, demonstrates patterns in mathematics.

- b. All of mathematics makes sense because of relationships or patterns. The ability to recognize patterns is the key to mathematical thinking. Searching for patterns is a way of thinking that is essential for making generalizations, seeing relationships, and understanding the logic and order of mathematics. Patterns make it possible to predict what is supposed to happen, rather than seeing the teacher's answer book as the only source for verification of thinking.

(Quote from Mary Barratta-Lorton) "Looking for patterns trains the mind to search out and discover the similarities that blend seemingly unrelated information together in a whole. A child who expects things to "make sense" looks for the sense in things and from this sense develops understanding. A child who does not see patterns often does not expect things to make sense and sees all events as discrete, separate, and unrelated."

We must help students see relationships and use them in making reasonable predictions. Students who do not see relationships fail to notice when they have ridiculous answers. We must encourage students to make sense out of things.

While we can create learning situations that will help students see relationships, each student must come to see the relationship internally.

- c. Activities such as the following help students learn by doing.  
Beans and Ways.

Here is the problem: You are to find how many ways 25 beans can be put into two containers or piles? This may seem overwhelming when given in this way. However, one problem solving strategy is to work a simpler or similar problem. Let's begin with a simpler problem.

With just one bean, there are two ways to put into the piles, either in the left one or the right one. (Record.) See how many different ways there are to put two beans into the piles.

(Discuss and record.)

Continue this pattern to find how many ways you could arrange 25 beans into two piles. Make sure everyone in your group agrees. (Wait.)

It probably wasn't necessary for you to try all the different number of beans all the way to 25. Once you saw the pattern, it served to give you the result you needed.

You've recorded your pattern in a table. It's also possible to write it as a formula. If B stands for beans and W stands for ways, the relationship can be written as  $B + 1 = W$ . It is critical for students to see that abstract notation came from ideas that can be represented concretely.

This activity is a good introduction to the study of probability and to higher levels of patterning.

(Show Transparency #15)

8. Involvement in the learning process is critical to success in mathematics/reading.

- a. Learning is not a spectator sport. Teaching by telling does not work. You cannot tell a student to understand because learning is an internal process that happens in individual ways and on individual time schedules. In order for learning to occur, students must get involved. They need to grapple with problems, search for strategies and solutions on their own, make their own mistakes, and to learn how to evaluate their own results. To do this we must convince students that ... (Read the poem.)

The Road to Wisdom

The road to wisdom is simple and easy to express:

To err

and err

and err again,

But less

and less

and less.

Piet Hein

Though the teacher needs to be very much present, the primary focus needs to be on students' own thinking processes. One way to organize the classroom to establish an environment that is safe and supportive for problem solving is to have students work in small groups. A small group structure has the potential to maximize the active participation of each student and reduce individual's isolation. When organized into small groups, more students have the opportunity to offer their ideas for reaction and receive immediate feedback. This provides a setting that values social interaction, a needed element of students' learning.

A summary by Johnson and Johnson of 122 studies between 1924 and 1984 on cooperative learning reveals the following benefits: (Transparency #16)

- (1) Cooperative learning experiences tend to promote higher achievement than do competitive and individualistic learning experiences. These results hold for all age levels, for all subject areas, and are critical for tasks involving concept attainment, verbal problem solving, categorization, spatial problem solving, retention and memory, motor performance, and guessing-judging-predicting.
- (2) Cooperative learning promotes the use of higher reasoning strategies and greater critical thinking competencies.
- (3) Cooperative learning experiences promote more positive attitudes toward both the subject area and the instructional experience, as well as more continuing motivation to learn more about the subject area being studied.
- (4) Cooperativeness is positively related to a number of indices of psychological health, namely: emotional maturity, well-adjusted social relations, strong personal identity, and basic trust in and optimism about people.
- (5) Cooperative learning experiences promote considerably more liking among students. This is true regardless of differences in ability level, sex, handicapping conditions, ethnic membership, social class differences, or task orientation. Students who collaborate on their studies develop commitment toward and caring for one another regardless of their initial impressions of and attitudes toward one another.
- (6) Cooperative learning experiences result in stronger beliefs that one is personally liked, supported, and accepted by other students, and that other students care about how much one learns, and that other students want to help one learn.
- (7) Cooperative learning experiences promote higher levels of self-esteem.
- (8) Cooperative learning experiences not only affect relationships with other students, they also affect relationships with adults in the school. Students participating in cooperative learning experiences like the teacher better and perceive the teacher as being more supportive and accepting academically and personally.

When in this workshop have you been involved in cooperative learning? (Wait for audience response.) In what other ways have you been actively involved? (Wait for audience response and list as they are given: marking the graph, thinking about responses to the graph, answering questions, brainstorming questions on graphing, brainstorming uses of mathematics, analysis of student error patterns, mental arithmetic, sharing of logical thinking process, use of "Math Teakst Buk," consecutive sums activity, Beans and Ways, use of concrete manipulatives.)

### Guided Practice

Through your involvement in these activities we hope you have "learned by doing."

We have discussed and "experienced" eight concepts that reading and mathematics have in common. Professors of mathematics have reviewed them and state that these concepts are the heart of an effective mathematics program at any level, from kindergarten through graduate school.

- (1) A comprehensive instructional program is needed in mathematics and reading and the same process may be used to achieve either.
- (2) We teach students, not mathematics or reading.
- (3) Mathematics/reading should be made relevant to the learners.
- (4) Mathematics/reading is a thinking process.
- (5) Mathematics/reading is a language communication process.
- (6) The learning of mathematics/reading proceeds from the concrete to the abstract.
- (7) Mathematics/reading is seeing relationships.
- (8) Involvement in the learning process is critical to success in mathematics/reading.

Now, we'd like for you to use these eight concepts to answer the six questions posed at the beginning of this session.

- (1) What is mathematics?  
(The art of solving problems. Arithmetic computation - a branch of mathematics.)
- (2) Why should we teach mathematics?  
(To make productive citizens who can cope with everyday life situations and function effectively on the job.)
- (3) What mathematical abilities do students need to be productive citizens?  
(Ability to think, process, and solve problems.)
- (4) How can we provide students with these abilities?  
(By incorporating the eight concepts)
- (5) Should mathematics be enjoyable? If so, what is the joy in mathematics?  
(Joy is discovery, finding order, and structure, involvement, enlightenment.)
- (6) How can we promote positive attitudes of teachers/students toward mathematics?  
(By applying these eight concepts, particularly the principles of motivation.)

These questions are critical in developing a statement of beliefs. (Divide into six groups, distribute the handout including the eight concepts and the six questions, and have each group write a response to one assigned question. Allow 10 to 20 minutes for group work. A group may choose to work on more than one question. Have each group share its response.)

### Independent Practice

As a follow-up to this session, meet with your Task Force or Mathematics Committee to review the training and formulate the system's statement of beliefs regarding these six key questions. We also request that you use the principles of learning to assess their intergration into the math program. (Distribute assessment instrument.)

In closing, we would like to leave you with these thoughts of Dr. Lola May. "After all, who is a pupil? A child of God, not a tool of the state. Who is a teacher? A guide, not a guard. Who is a principal? A master of teaching, not of teachers. What is learning? A journey, not a destination. What is discovery? Questioning the answers, not answering the questions. What is the process? Discovering ideas, not covering content. What is the goal? Open minds, not closed issues. What is the test? Being and becoming, not remembering and reviewing. What is the school? Whatever we choose to make it."



SUMMARY OUTLINE

## BUILDING A MIND SET FOR MATHEMATICS

- I. A comprehensive mathematics/reading program
  - A. Developing a statement of beliefs defining the ideal program
  - B. Assessing needs in light of the ideal program
  - C. Establishing goals for achieving the ideal program
- II. We teach students, not mathematics or reading.
  - A. Students are students and learning is learning.
  - B. Individual differences exist and must be accommodated.
    1. Implications for grouping, scheduling, recordkeeping
    2. Differences in learning style
  - C. Principles of learning and effective teaching behaviors apply to all content areas.
    1. Teaching to an objective
    2. Teaching at the correct level of difficulty
    3. Monitoring and adjusting
    4. Facilitating use of principles of learning
      - a. Motivation
      - b. Rate and degree of learning
      - c. Retention
      - d. Transfer
- III. Mathematics/reading should be made relevant.
  - A. Related to student experiences
  - B. Related to everyday life
  - C. Related to further educational opportunities
  - D. Related to job success
- IV. Mathematics/reading is a thinking process.
  - A. Emphasis on thinking rather than "the answer"
    1. Focus on understanding
    2. Focus on process
      - a. Modeling and labeling
      - b. Explaining by students
      - c. Considering process and answer in grade assignment
      - d. Analyzing error patterns
  - B. Answers are not random.
    1. Errors follow a pattern.
    2. Logical thinking practice strengthens thinking abilities.

- C. Telling answers/insistence on one way of solving problems stops thinking.
  - D. Providing logical thinking activities
- V. Mathematics/reading is a language/communication process.
- A. Has written symbols
  - B. Concept development and oral language precede symbols.
  - C. Talk solidifies thinking.
  - D. Writing brings precision to the thinking process.
  - E. Math symbols are universal.
    - 1. Translate math to English
    - 2. Algebraic equations have subject and verb.
- VI. The learning of mathematics/reading proceeds from the concrete to the abstract.
- A. Concept level--concrete experiences
  - B. Connecting level--teacher responsibility
  - C. Symbolic level--abstract
- VII. Mathematics/reading is seeing relationships.
- A. Search for order, structure and relationships.
  - B. Patterns are key to mathematical thinking.
  - C. Internalization of concept is with student.
- VIII. Involvement in the learning process is critical to success in mathematics/reading.
- A. Learning is an internal process.
  - B. Learning is individualistic.
  - C. Learner must be involved
    - 1. Grapple with problems
    - 2. Search for strategies, patterns, and solutions
    - 3. Evaluate own results

- D. Cooperative learning promotes student learning.
1. Higher achievement
  2. Greater reasoning and critical thinking competencies
  3. More positive attitudes toward the subject area and instruction
  4. Psychological health
  5. Commitment and caring for fellow students
  6. Peer support and acceptance
  7. Higher self-esteem
  8. Improved relationships with school personnel

TRANSPARENCIES  
AND  
HANDOUTS

# Objectives

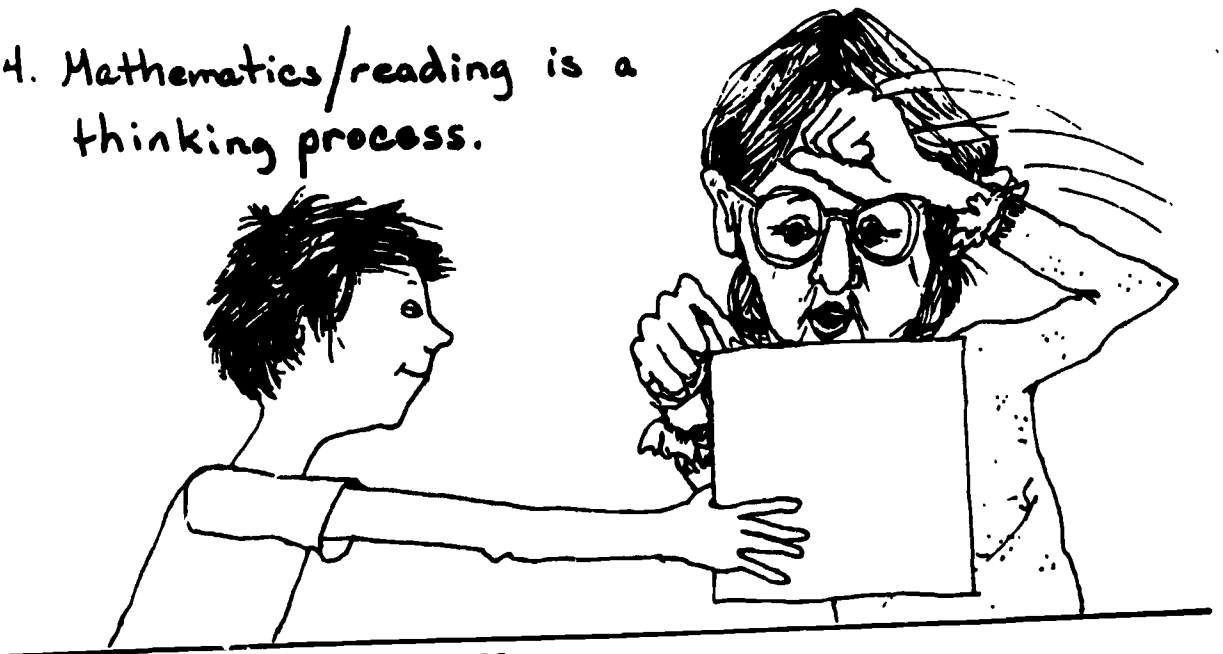
The participant will be aware of eight concepts or ideas common to both mathematics and reading.

1. A comprehensive instructional program is needed in mathematics and reading, and the same process may be used to achieve either.

2. We teach students, not mathematics or reading.

3. Mathematics/reading should be made relevant to the learners.

4. Mathematics/reading is a thinking process.





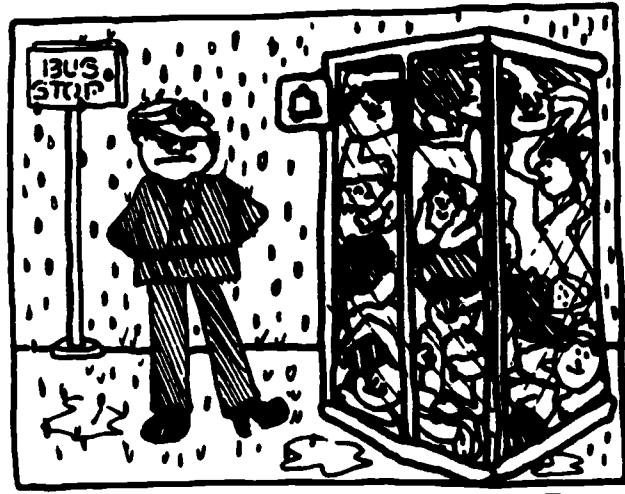
5. Mathematics/reading is a language/communication process.
6. The learning of mathematics/reading proceeds from the concrete to the abstract.
7. Mathematics/reading is seeing relationships.
8. Involvement in the learning process is critical in mathematics/reading.

The participant will have background information for developing a statement of beliefs about mathematics.



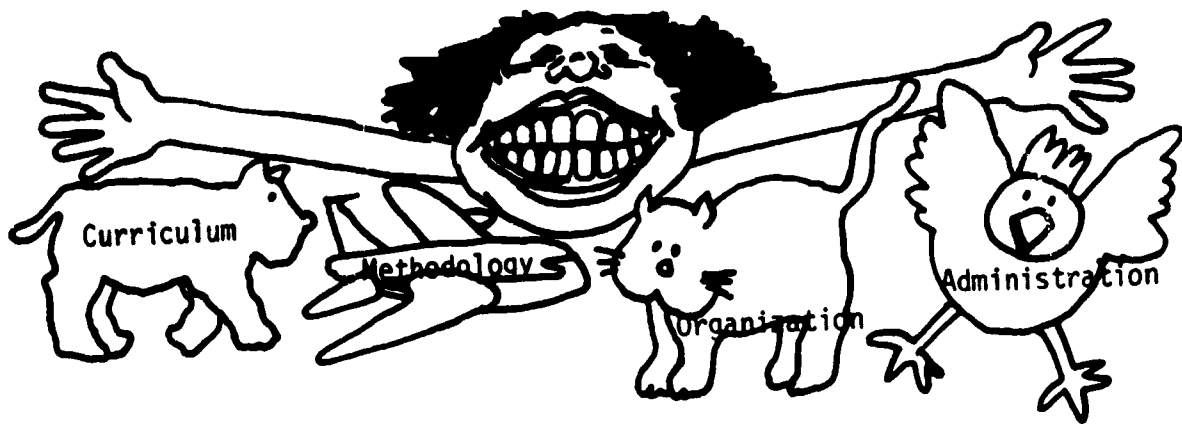
1. What is mathematics?
2. Why should we teach mathematics?
3. What mathematical abilities do students need to be productive citizens?
4. How can we provide students with these abilities?
5. Should mathematics be enjoyable? If so, what is the joy in mathematics?
6. How can we promote positive attitudes of teachers/students toward mathematics?





The participant will feel more comfortable in working to upgrade mathematics.



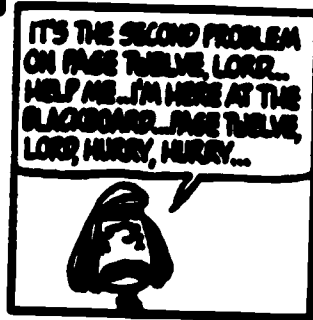
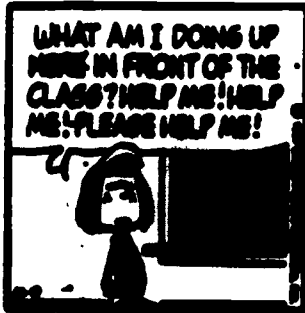


A comprehensive instructional program is needed in mathematics.

- \* Develop a statement of beliefs defining the ideal program.
- \* Assess needs in light of the ideal program.
- \* Establish goals for achieving the ideal program.

# We teach children - not mathematics

## PEANUTS



By CHARLES SCHULZ

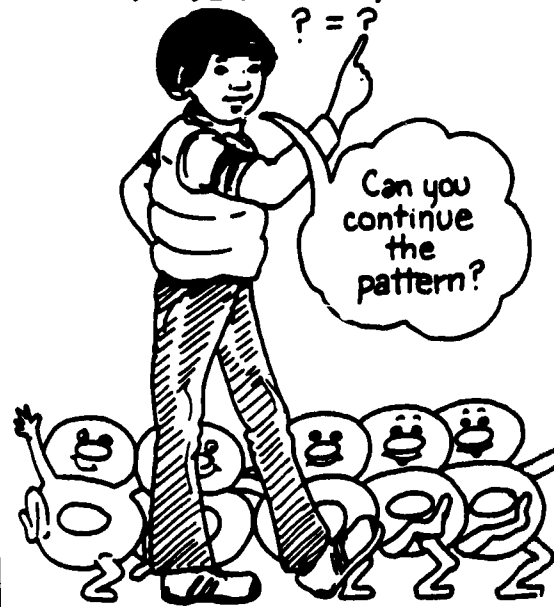


Mathematics should be made relevant to the learner.



Everyday Uses

$$9 \times 9 + 7 = 88$$
$$9 \times 98 + 6 = 888$$
$$9 \times 987 + 5 = 8888$$
$$? = ?$$



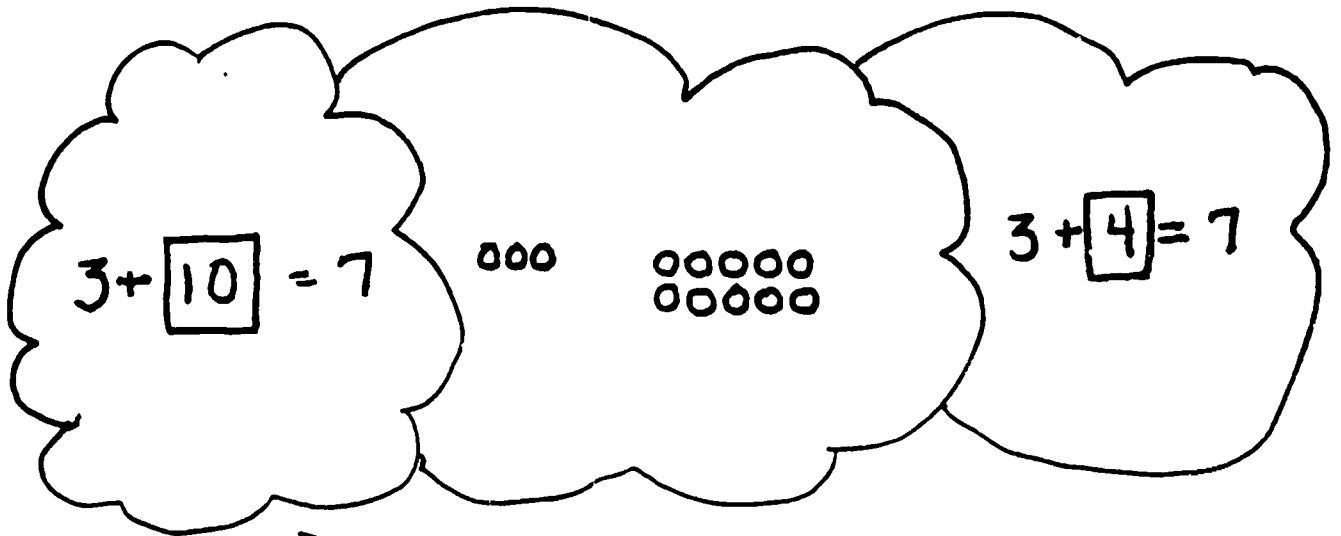
Future educational opportunities

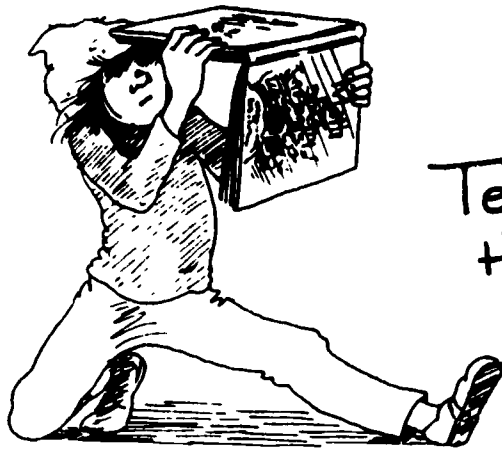


On the job

# Mathematics

is a thinking process.





Teachers focus on the process by:

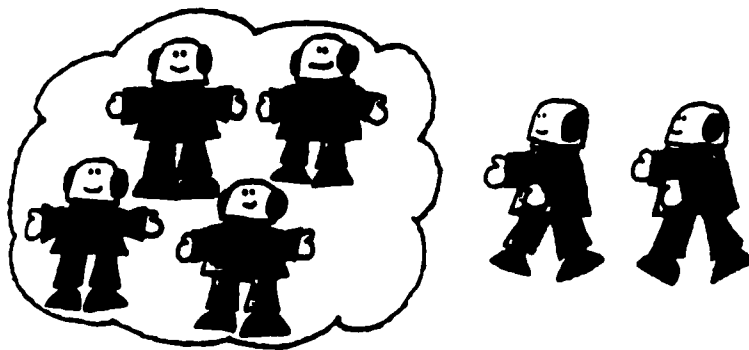
1. modeling and labeling the process for students;
2. encouraging students to explain "how" they arrived at the answer or solved the problem and allowing students to listen, question and learn from each other in small groups;
3. focusing on the process used as well as the answer when grading papers;
4. analyzing error patterns as clues to students' reasoning.

## Common Arithmetic Errors

<p>1.</p> $3 + \boxed{10} = 7$	<p>2.</p> $\begin{array}{r} 67 \\ + 18 \\ \hline 715 \end{array}$
<p>3.</p> $\begin{array}{r} 54 \\ - 19 \\ \hline 45 \end{array}$	<p>4.</p> $\begin{array}{r} 399 \\ 400 \\ - 238 \\ \hline 161 \end{array}$
<p>5.</p> $15 \overline{)3450} \begin{array}{l} 23 \\ \hline \end{array}$	<p>6.</p> $\frac{4}{5} + \frac{2}{3} = \boxed{\frac{6}{8}}$
<p>7.</p> $1.07 + 2.5 + .45 = \boxed{1.77}$	<p>8.</p> $\begin{array}{r} \$3.50 \\ \times .12 \\ \hline \$42.00 \end{array}$

# Mathematics

is a language/communication  
process.



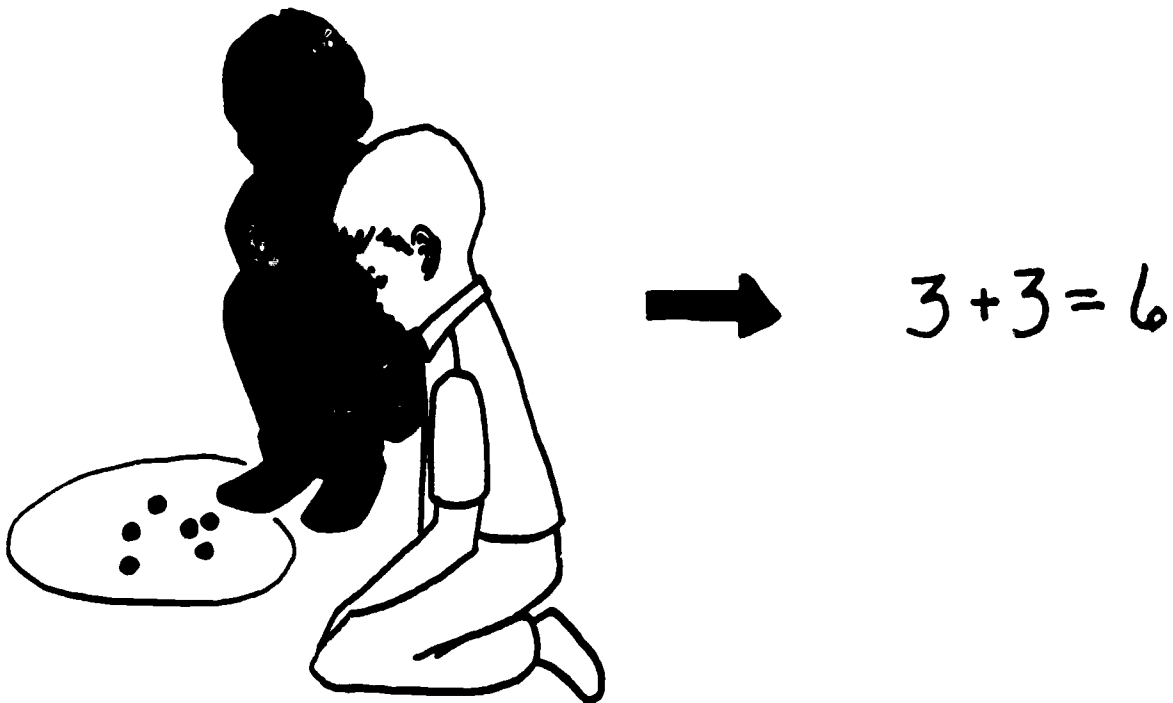
Four children are playing in  
the snow. Two more children  
join them. How many children  
are now playing in the snow?

Four plus two is 6.

$$4 + 2 = 6$$













The learning of mathematics  
proceeds from the concrete  
to the abstract.



# Mathematics

is seeing relationships.

Eyes

					2
					4
					6
					—

**Monday,  
Monday,  
I Like Monday**  
*by Bill Martin, Jr.*

Sunday, Sunday,  
I like Sunday,  
Sunday, the first day  
of the week.

Monday, Monday,  
I like Monday,  
Monday, the second day  
of the week.

Tuesday, Tuesday,  
I like Tuesday,  
Tuesday, the third day  
of the week.

Wednesday, Wednesday,  
I like Wednesday,  
Wednesday, the fourth day  
of the week.

Thursday, Thursday,  
I like Thursday,  
Thursday, the fifth day  
of the week.

Friday, Friday,  
I like Friday,  
Friday, the sixth day  
of the week.

Saturday,  
Saturday,  
I like Saturday,  
Saturday,  
the seventh day  
of the week.

Sunday, Monday,  
Tuesday, Wednesday,  
Thursday, Friday,  
Saturday, a week.

Saturday, Friday,  
Thursday, Wednesday,  
Tuesday, Monday,  
Sunday, a week.

First day, second day,  
third day, fourth day,  
fifth day, sixth day,  
seven days a week.

1 day, 2 days,  
3 days, 4 days,  
5 days, 6 days,  
7 days a week.

Some days – school days,  
Some days – play days,  
Some days – holy days,  
Seven days a week.

Some days – sunny days,  
Some days – rainy days,  
Some days – snowy days,  
Seven days a week.

Some days – work days,  
Some days – play days,  
Some days – holidays,  
Seven days a week.

Involvement in the learning process is critical to success in mathematics.



# Benefits of Cooperative Learning



1. Higher student achievement
2. Higher reasoning and greater critical thinking competencies
3. More positive attitudes
4. Psychological health
5. Commitment and caring for fellow students
6. Peer support and acceptance
7. Higher self-esteem
8. Improved relationships with school personnel.

# Common Arithmetic Errors

1. $3 + \boxed{10} = 7$	2. $\begin{array}{r} 67 \\ + 18 \\ \hline 715 \end{array}$
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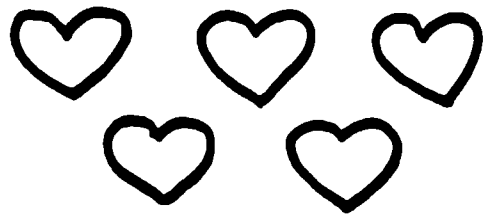
←————→  
 2 1 3 2 1 2 - 3 2 1 2

Whow mened

1



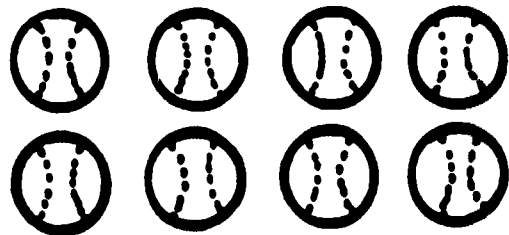
3 2 1 2



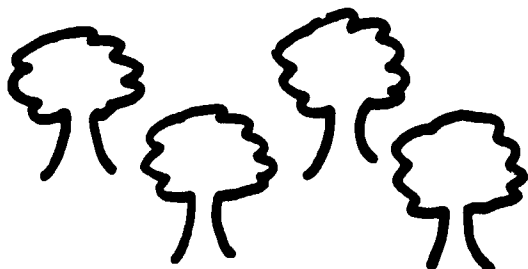
2 1 2 -



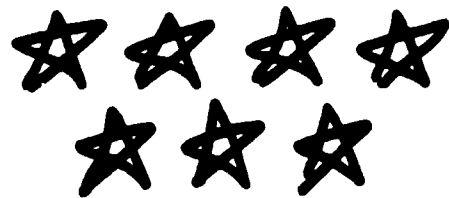
1 3 2 2



1 2 - 3 2



3 2 1 2



2 - 3 2

GRAPHS



# Math Attitudes

Make a tally in the appropriate box.

Grade Level	I love math and am good at it.	I'm average in math and don't mind it.	I'm not very good in math and am fearful of it.	I hate math and am terrible at it.
K-2				
3-5				
6+ up				
Resource				
Aide				
Other				

The three sacks have Red and Blue cubes in them. One has just Red and one has just Blue; the other contains a mixture. Each of the sacks has been deliberately mislabeled. Your job is to figure out which sack is really which. You get one clue.

**CLUE** Later you can reach into any one sack and draw one cube at random.

Mark an X to show which sack you would reach into.

Red	
Blue	
Red and Blue	

How long a line do you think an average pencil will draw before it is used up?  
(You can sharpen it as you go.)

Make a tally to show your opinion.

0 - 5 miles	
6 - 10 miles	
11 - 15 miles	
16 - 20 miles	
21 - 25 miles	
26 - 30 miles	
31 - 35 miles	
36 - 40 miles	
41 - 45 miles	
46 - 50 miles	
Over 50 miles	

## MATERIALS NEEDED

1. Graphs
  - a. Attitude
  - b. Pencil
  - c. 3 sacks
2. OH Pens
3. Tape
4. Blank Transparencies
5. Large Paper
6. Cups
7. Beans
8. Place value boards
9. Bill Martin book

## HANDOUT SHEETS

1. Error Patterns
2. Teakst Buk page

PART II.

CURRENT TRENDS AND ISSUES IN MATHEMATICS

CURRENT TRENDS AND ISSUES  
IN MATHEMATICS

(As participants arrive, have them complete the three graphs.)

1. Value of Your Name
2. Mirror Symmetry
3. String Measurement

Say: In this workshop, we will take a look at some current trends and issues in mathematics. We know that change is part of life. An institution that does not meet new environmental challenges will eventually disappear, and this need to change is never-ending.

(SHOW TRANSPARENCY #1.)

Today our objectives are to acquaint you with the Trends and Issues in Mathematics for the 1980's by:

1. Presenting the "Agenda for Action."
2. Defining the Ten Basic Skill Areas, as cited by the National Council of the Supervisors of Mathematics.

The National Council of Teachers of Mathematics

Presents

An Agenda for Action

The National Council of Teachers of Mathematics has made eight critical recommendations for school mathematics for the 1980's.

Let's take 2 or 3 minutes to read these recommendations; then we will discuss each in a little more detail.

(SHOW TRANSPARENCY #2 )

An Agenda for Action

Recommendations for

School Mathematics of the 1980's

The National Council of Teachers of Mathematics recommends that....

1. problem solving be the focus of school mathematics in the 1980's;
2. basic skills in mathematics be defined to encompass more than computational facility;
3. mathematics programs take full advantage of the power of calculators and computers at all grade levels;
4. stringent standards of both effectiveness and efficiency be applied to the teaching of mathematics;
5. the success of mathematics programs and student learning be evaluated by a wider range of measures than conventional testing;

6. more mathematics study be required for all students and a flexible curriculum with a greater range of options be designed to accommodate the diverse needs of the student population;
7. mathematics teachers demand of themselves and their colleagues a high level of professionalism;
8. public support for mathematics instruction be raised to a level commensurate with the importance of mathematical understanding to individuals and society.

#### RECOMMENDATION I

(SHOW TRANSPARENCY - REFER TO #1 ON TRANSPARENCY 2.)

#### PROBLEM SOLVING BE THE FOCUS OF SCHOOL MATHEMATICS IN THE 1980'S

First, we should define problem solving. Problem solving is a process, not a step-by-step procedure or an answer to be found; it is a journey, not a destination.

The principal reason for studying mathematics is to learn how to solve problems. A major goal for the mathematics program is to help students acquire the knowledge they need to solve problems not only at school but also in the outside world. Teaching math just for the sake of students making good grades or successfully working math assignments in school is just not enough. It will take more than students doing well on skills they have studied in the classroom to succeed in the complex situations of life.

Students are missing the connection between their learning in school and learning for real life. Problems in real life just do not tend to appear in an orderly paper-and-pencil format as they do in school. Therefore, we must provide problem-solving situations within the school experience. Students must be given time to approach problem solving-situations.

At this time, it seems that students are learning much of their arithmetic at a rote level without the understanding they need for applying those concepts to solve problems.

We need to take an in-depth look at the mathematics curriculum. At this point the curriculum emphasizes computational skills apart from their application. Mastery of computational skills is a very important concern. However, it is also important for children to be able to call upon their skills

in dealing with problems faced in their personal, professional, and daily experiences. If children cannot apply what they have learned, they will not be able to function successfully when it comes to solving problems in their day-to-day living. NCTM is emphasizing problem solving, but, not at the expense of computation. The curriculum should maintain a balance between application of math and the fundamental concepts.

What should problem solving include?

(SHOW TRANSPARENCY #3.)

It has been recommended that these be the major components of problem solving.

- A. Routine and nonroutine word problems (Example: Routine-typical textbook problems--Nonroutine-Consecutive Sums problem)



B. Process problems

Examples: logic as in The 3-Sacks Problem  
graphs as in the Mirror Symmetry graph  
deductive thinking (thinking that goes from general  
to specific)

C. Real-life problem-solving activities (as in balancing a checkbook)

One serious mistake that we make is limiting problem-solving activities to strictly "word problems." Again, problems in real life are rarely presented in the structured way of story problems. The emphasis in using word problems is usually to find one right answer. Problems in life are rarely presented this way and may have several solutions.

There are many ways teachers can help their students become good problem solvers.

First, a classroom environment must be created that allows for problem solving. Problem solving is a creative activity. Allow the children time to question, experiment, estimate, explore, and suggest explanations. Second, the teacher needs to help pupils develop the processes or attitudes of mind needed for becoming good problem solvers. Third, the teacher must be a good model. She must help the students to understand that a good problem solver must take chances, be determined and persevere as well as develop the necessary skills and knowledge needed for solving problems.

To solve problems students must be able to:

(SHOW TRANSPARENCY #4.)

(Read.)

Formulate key questions;

Analyze and conceptualize problems;

Define the problem and the goal;

Discover patterns and similarities;

Seek out appropriate data;

Experiment;

Transfer skills and strategies to new  
situations;

Draw on background knowledge to apply  
mathematics.

Say: Now, you have been patiently listening all morning. I would like for us to stop here and put ourselves into a problem-solving situation.

Get yourself a partner. (Wait.)

Here is the problem. Has Teaching Come to This? (Give each person a copy of the problem.) I want you to work this problem with your partner. Next, I want you to think about how many of these things listed on the transparency you had to do before you could solve the problem and make some notes for discussion.

(SHOW TRANSPARENCY #5 and discuss. Ask for volunteers to explain what they did to solve the problem.)

To summarize, it is evident that both lay and professional people consider problem solving a priority for the 1980's. As educators, we need to give immediate attention to how we can help children become good problem solvers. We may need to do many things in order to accomplish this very important task.

(SHOW TRANSPARENCY #6.)

- (1) Reorganize the mathematics curriculum,
- (2) Create classroom environments conducive to problem solving,
- (3) Develop problem solving materials, and
- (4) Make sure children are given the opportunity to be actively involved in problem-solving situations.

After all, our major goal is to make sure that students are prepared to meet life's challenging situations.

#### RECOMMENDATION 2

(SHOW TRANSPARENCY--REFER TO #2 ON TRANSPARENCY 2.)

#### THE CONCEPT OF BASIC SKILLS IN MATHEMATICS MUST ENCOMPASS MORE THAN COMPUTATIONAL FACILITY

Everyone these days is talking about the "basics." However, parents, educators, and mathematicians have not come to a common understanding and acceptance of exactly what these basic skills should comprise.

Some groups have limited the "basics" to routine computation at the expense of understanding, application, and problem solving. This leaves little hope of developing the functionally competent student that is desired. Rather than

fostering a return to some acceptable common threshold of performance, the back-to-basics movement tends to place a low ceiling on math competence. This is a problem since we have moved into a new era that requires more diverse uses of mathematics than ever before.

Also, there is much pressure on teachers today to devote a lot of time and energy to minimal skills. Teachers are afraid to deviate from the minimal skills because this is where they are judged. There seems to be a lot of overdoing on the minimal target areas even though it yields little productive achievement. Even if improvement in rote computation takes place, a student who cannot analyze real-life situations to the point of knowing what computations must be made to solve real-life problems will not be able to function as successfully as needed for productive citizenship.

There must be an acceptance of the full spectrum of basic skills and recognition that there is a wide variety of such skills beyond mere computation. The full scope of what is basic must include those things that are essential to meaningful and productive citizenship. A student's future is of the utmost importance. We must keep in mind that we want students to leave after 12 or 13 years of schooling with as much knowledge and as many positive experiences as possible.

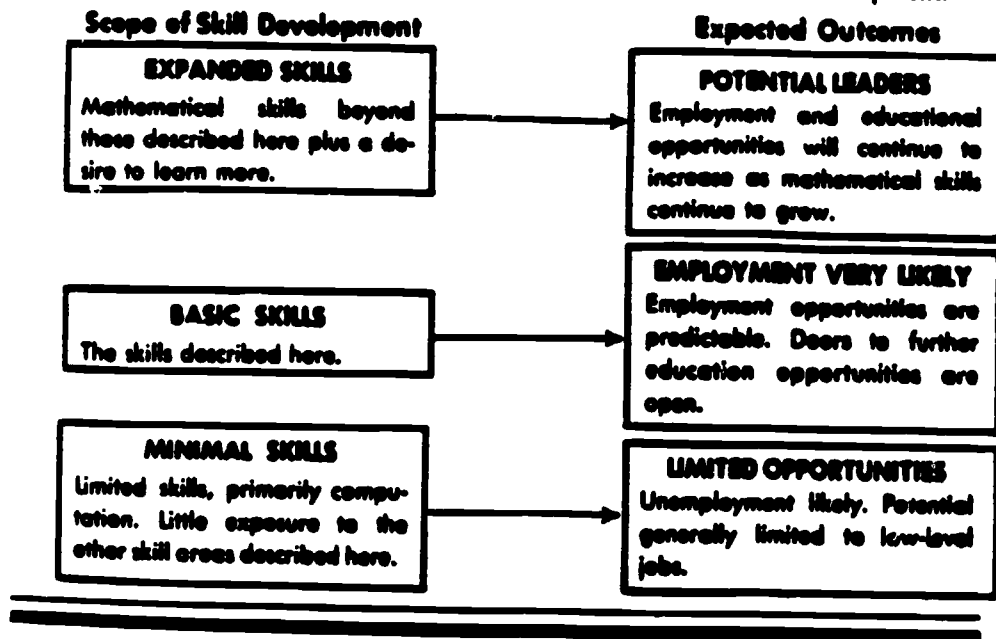
When considering basic skills, we need to consider the "door-opening/door-closing" implications of the list you see on this transparency.

(SHOW TRANSPARENCY #7 AND DISCUSS.) (SEE NEXT PAGE.)

## Basic Skills and the Student's Future

Anyone adopting a definition of basic skills should consider the "door-opening/door-closing" implications of the

list. The following diagram illustrates expected outcomes associated with various amounts of skill development.



National Council of Supervisors of Mathematics. "National Council of Supervisors of Mathematics Position Paper on Basic Skills." Arithmetic Teacher, 25, (October, 1977): 19-22.

The National Council of Supervisors of Mathematics has identified in its position paper on basic skills 10 basic skill areas. These areas will now be outlined for you.

#### TEN BASIC SKILL AREAS

The popular slogan "Back to the Basics" has become a rallying cry of many who see a need for certain changes in education. The major concern, however, is the danger that this movement might eliminate teaching for mathematical understanding. It will do no good for citizens to have the ability to compute if they do not know what computations to perform when they meet a problem. The use of the hand calculator emphasizes this need for understanding: one must know "when" to push "what" button. Computational skills in isolation are not enough; we must address skills, but we must address them within a total mathematics program.

In a total mathematics program, students need more than arithmetical skill and understanding. They need to develop geometric intuition as an aid to problem solving, and they must interpret data. Without these and many other mathematical understandings, citizens are not mathematically functional.

Leaders in mathematics education have expressed a need for clarifying the basic skills needed by students who hope to participate successfully in adult society.

In response to the need for basic skills, the National Council of Supervisors of Mathematics has defined Ten Basic Skill Areas.

The first area is:

Problem Solving (SHOW #1 ON TRANSPARENCY #8.)

We know that learning to solve problems is the principal reason for studying mathematics.

Problem solving is the process of applying previously acquired knowledge to new and unfamiliar situations.

Solving word problems in texts is one form of problem solving, but students should also be faced with solving non-textbook problems.

Problem solving involves using various strategies. Some of these strategies include:

(SHOW TRANSPARENCY #9.)

- 1) posing questions
- 2) analyzing situations
- 3) translating results
- 4) illustrating results
- 5) drawing diagrams
- 6) using trial and error

In solving problems, students need to be able to:

- 1) apply logic to arrive at valid conclusions
- 2) determine facts that are relevant
- 3) be unfearful of arriving at tentative conclusions

When solving word problems in textbooks, a student's task is to decide what arithmetical operations fit a particular situation, to set up the arithmetic sentence, and to solve it. Word problems focus on the meaning of the arithmetical operations in a variety of situations.

However, students also need to solve problems for which choosing an operation is not the main issue, but instead the emphasis is on using one or more operations to explore a situation. In problems of this kind there may be more than one suitable way to solve the problem; there may be more than one solution; there may be many preliminary solutions before arriving at the final answer. The Consecutive Sums problem that you did in the last session is an example of this type.

Say: Today we will experience a problem that has many different possible answers.

Using the alphabet system presented on the graph, you can figure out the value of any word. (Refer to graph - Value of Your Name.)

Find the value of the word INFLATION. (\$1) (Wait.) Inflation is worth exactly \$1.00.

Now working in groups of four find as many words as you can that are worth exactly \$1. To help you get started, here are some hints to help you find \$1. words: a day of the week, a Thanksgiving word, a Halloween word, a United States coin, an astrological sign.

(SHOW TRANSPARENCY #10.) (Answers: Wednesday, turkey, pumpkin, quarter, Taurus) After you find the dollar words given on the transparency, please continue finding others until time is called. In your groups, discuss the methods you used for finding \$1. words and record these strategies. (Wait.)

(Have groups share their answers and any other \$1. words that were found. Discuss any strategies that were found.) More than 500 \$1. words have been found by students across the United States. Some teachers have organized class or school \$1. word contests.



In classrooms students often learn that it's the quick right answer that is valued. They also learn to depend on the faster thinkers to provide answers and on teachers for finding out if they are right or wrong. Children need problem-solving experiences in which they are expected to decide on strategies to use, apply them, and evaluate their results, while using the skills they've been learning.

Students should be able to solve word problems. However, word problems do not embody the total approach to problem solving. As in the \$1. word problem you just had, the focus is not on mastery of skills, but on students using their skills in problem-solving experiences.

The second area is:

Applying mathematics to everyday situations (SHOW #II ON TRANSPARENCY #8.)

The use of mathematics is interrelated with all computational activities. Students should be encouraged to take everyday situations, translate them into mathematical expressions, solve the mathematics, and interpret the results. In our first session you were given oral problems that you translated into mathematical expressions. Remember: Mary is 4 years older than John ( $M=J+4$ ) and if  $C$  stands for the cost of one record, how would you represent the cost of 9 records? ( $9c$ )

The third area is:

Alertness to the reasonableness of results (SHOW #III ON TRANSPARENCY #8.)

Because of arithmetical errors and other mistakes, results of mathematical work are sometimes wrong. Students should learn to inspect all results and to check for reasonableness in terms of the original problem. With the increased use of calculating devices, this skill is essential.

The next area is:

Estimation and approximation (SHOW #IV TRANSPARENCY #8.)

Students should be able to carry out rapid approximate calculations by first rounding off numbers. They should acquire some simple techniques for estimating quantity, length, distance, weight, etc. It is also necessary to decide when a particular result is precise enough for the purpose at hand.

The fifth basic skill is:

Appropriate computational skills (SHOW #V ON TRANSPARENCY #8.)

Students should be proficient in addition, subtraction, multiplication, and division with whole numbers and decimals. Today we must recognize that long, complicated computations will usually be done with a calculator. However, knowledge of single-digit number facts is essential and mental arithmetic is a valuable skill.

Say: Let's do a classroom activity that helps reinforce mental arithmetic.

(Pass out cards.) Now I will begin this activity by reading my card, and the person who has the correct answer should read his card. We will continue until everyone has had a turn. (Play the game.)

Let's not forget that there are everyday situations which demand recognition of and use of simple computation with common fractions and percentages. The ability to recognize and use these skills should be developed and maintained.

Area six is:

Geometry (SHOW #VI ON TRANSPARENCY #8.)

Students should learn the geometric functions they will need to function effectively in the three-dimensional world. They should have knowledge of concepts such as point, line, plane, parallel, and perpendicular. They should know basic properties of simple geometric figures, especially those properties which relate to measurement and problem-solving skills. They must also be able to recognize similarities and differences among objects.

Say: Now let's look at our graph on Mirror Symmetry. What kind of graph is this? (fact, processing, or opinion) What are some questions that can be answered from the graph? (Wait and seek possible questions.)

1. Which category has the most different letters?
2. How many more letters have both symmetry than have just vertical?
3. Which letters of the alphabet are not on the graph?
4. Where would they fit?

In the elementary grades, students' involvement with geometry should be informal, as presented in this graph. Children should be encouraged to observe natural and man-made objects in the environment and have the opportunity to use concrete materials from which they can begin to develop geometric concepts. Studying geometry offers opportunities for investigating patterns, seeing applications in the real world, and for creative and logical thinking.

The seventh area is

Measurement (SHOW #VII ON TRANSPARENCY #8.)

A minimal skill, students should be able to measure distance, weight, time, capacity, and temperature. Measurement of angles and calculations of simple areas and volumes are also essential. Students should be able to perform measurement in both metric and customary systems using the appropriate tools.

Say: Now let's look at the graph about how many times you think a string equal to your height will wrap around your head. What type of graph is this (fact, opinion, processing).

What are some questions we can ask from the graph? (Wait.)

1. What is the most common opinion?
2. How many more people thought \_\_\_\_ than \_\_\_\_?
3. What percentage of the class thinks their height is more than four times the distance around their head?)

The easiest way to find out is to measure. But first discuss your opinions at your table and see if you can come up with a group consensus and tell why you think that. (Wait.)

(Ask for group responses. To show the answer measure around your head with string the same length as your height.) (Answer: about three times)

Measurement tools and skills have a variety of uses in everyday life. Being able to measure connects mathematics to the environment. The ability to use measuring tools--rulers, thermometers, scales, etc--is a necessary skill for children to develop.

Basic skill number eight is:

Reading, interpreting, and constructing tables, charts, and graphs (SHOW #VIII ON TRANSPARENCY #8.)

Students should know how to read and draw conclusions from simple tables, charts, maps, and graphs. They should be able to condense numerical information into more manageable or meaningful terms by setting up simple tables, charts, and graphs.

Graphing is a way to present statistics in a concise, visual manner that makes it possible to see relationships in the information more easily.

The ninth basic skill area is:

Using mathematics to predict (SHOW #IX ON TRANSPARENCY #8.)

Students should learn how elementary notions of probability are used to determine the likelihood of future events. They should learn to identify situations where immediate past experience does not affect the likelihood of future events. Also they should become familiar with how mathematics is used to help make predictions such as election forecasts.

Say: We will now experience a probability experiment in which you make a prediction and then test it with real data.

I will pass out a slip of paper, quickly circle one of the numbers on it and turn your paper face down. Do not discuss this in your group. (Pass out slips and wait.)

Individually (to yourself) answer these two questions:

1. Do you think any one number will be circled lots more than any other in the class and, if so, which one?
2. Why do you think that?

Take a few minutes. Write down your responses. This way you will be using the math lesson to provide reinforcement in writing. (Wait.)

Now in your group discuss your individual predictions and come up with a group answer to both of these questions. Also, choose a group spokesman to report your group's prediction. (Wait.) (Ask for each group's prediction.)

(Explain.) Individually, you had a very small sample of information on which to base your prediction. It's important to make decisions on large enough samples. You may have found that your individual prediction changed when additional data were added in your group.

A column in a Sunday paper reported that when given a slip of paper as you were, four out of five people circled the 3.

- Ask:
1. Do our results match what the newspaper reported?
  2. Does it match your predictions?
  3. How many of you circled 3?

It's good for children to engage in activities in which there isn't a set right answer that is predetermined. The teaching of probability should stem from real problems. Children's intuition needs to be challenged first. Once they sense what "should happen" in a situation, they need to carry out an experiment to test their predictions.

The final area is:

Computer literacy (SHOW #X ON TRANSPARENCY #8.)

It is important for all citizens to understand what computers can and cannot do. Students should be made aware of the many uses of computers in society. The increasing use of computers by government, industry, and business demands an awareness of computer uses and limitations.

This will be elaborated on in Recommendation 3.

The 10 basic skill areas are now on the overhead. The order of their listing should not be interpreted as indicating either a priority of importance or a sequence for teaching and learning. As society changes our ideas about which skills are basic will also change. For example, today our students should learn to measure in both metric and customary systems, but in the future the significance of the customary system will be mostly historic. There will also be increasing emphasis on when and how to use hand-held calculators and other electronic devices.

One individual difference among students is style or way of learning. In offering opportunities to learn the basic skills, options must be provided to meet these varying learning styles. The present "back-to-basics" movement may lead to an overemphasis on drill and practice. But let us remember it is only one of many possible ways to reinforce learning and to provide motivation and create interest. To help students fully understand basic mathematical concepts, teachers should use the full range of activities and materials available.

The learning of basic mathematical skills is a continuing process which extends through all of the years a student is in school. A tendency to emphasize computation while neglecting the other nine areas must be avoided.

Now that you are aware of the 10 basic skill areas, one other recommendation that NCTM made concerning basic skills is that there be decreased emphasis on some activities.

The activities are as follows:

(SHOW TRANSPARENCY #11.)

- . Isolated drill with numbers apart from problem context
- . Performing paper-and-pencil calculations with numbers more than two digits
- . Mastering highly specialized vocabulary--this includes vocabulary that is not useful later either in mathematics or in daily living (Example: commutative property, distributive property, multiplicand)
- . Converting measures given in one system to corresponding measures in another system



- . Working with tables whose usefulness has been supplanted by calculators and other technological aids.

RECOMMENDATION 3

(REFER TO #3 ON TRANSPARENCY.) Briefly review Recommendations 1 and 2. Say:  
Now we will look at recommendation 3.

MATHEMATICS PROGRAMS MUST TAKE FULL ADVANTAGE OF THE POWER OF CALCULATORS AND COMPUTERS AT ALL GRADE LEVELS.

The Prism study conducted in mathematics has revealed some interesting results on the use of calculators and computers in the classroom.

(SHOW TRANSPARENCY #12.)

Strong support was given for the use of calculators and computers in the classroom in the following ways:

Elementary level--strong support for:

Calculators

1. checking answers
2. solving word problems
3. homework
4. doing a chain of calculations

Computers

1. availability of computers
2. some interaction with computers
3. integration of computer literacy topics in the math curriculum

High School level--strong support for:

1. computing area
  2. trigonometry
  3. solving equations using geometric formulas
1. one required course for college-bound students
  2. teaching about the role of computers in society
  3. integration of computer literacy topics in the math curriculum

Little support was given for using calculators:

1. to take tests
2. to learn basic facts
3. to replace paper-and-pencil algorithms

Students just do not seem to be getting the "understanding" they need. Students must understand algorithms in order to generalize and apply them to new situations.

(SHOW TRANSPARENCY #13.)

For example, look at the problem on the transparency.

(Read--A man has 1,310 baseballs to pack in boxes which hold 24 baseballs each. How many baseballs will be left over after the man has filled as many boxes as he can?)

Students were allowed to use a calculator to help them solve this problem if they so desired. The students had more trouble solving the problem with a

calculator than without. Why? Because the students do not seem to demonstrate that deeper understanding of numbers and operations that is needed in order to make alternative interpretations. This understanding must take place before students are allowed to use calculators as an aid in working math problems. It is unthinkable in today's world to operate a store without a cash register and an office without a typewriter. It is fast becoming unthinkable to operate a business without a computer. We have moved beyond a mere acquaintance with the role of computers and calculators in society. It is necessary for students to obtain a working knowledge of how to use them.

Of utmost importance is the fact that computers and calculators are here to stay and students must be knowledgeable of them. Therefore, we need to think about proper training of teachers in computer literacy, in selecting appropriate hardware/software, and being knowledgeable about the various kinds of calculators.

As professional educators we need to start at the beginning stages of computer literacy and assess where we are in our knowledge about computers/calculators and what our needs are in this area. We also need to put some effort into becoming knowledgeable for selecting appropriate computer hardware/software.

(Give handout.) Microcomputer Vocabulary Test

Let's take a look at this handout. Take a few minutes to answer these multiple choice questions.

Once all students have completed this handout, ask them how they think they fared.

(Discussion)

One important thing we need to keep in mind is the elimination of misuses associated with the use of calculators and computers in the classroom. Calculators are not to be used before students have learned their basic facts and can successfully perform paper-and-pencil algorithms.

Do not use the computer to do tasks that can be done just as well in other ways. Example: Multiple choice questions, games without educational merit, drill/practice without immediate feedback, and watching the computer without enough interaction.

The questions concerning computer involvement in education are not going away. We must take time to consider how we think computers could be purchased and used for educational purposes as well as how training can be managed for teachers and students.

Also, consideration must be given to where you think calculators fit into the math program.

#### RECOMMENDATION 4

(REFER TO #4 ON TRANSPARENCY.)

STRINGENT STANDARDS OF BOTH EFFECTIVENESS AND EFFICIENCY MUST BE APPLIED TO THE TEACHING OF MATHEMATICS.

The teacher is the key to education. The teacher is charged with the awesome task of motivating and maintaining students' interest in mathematics. As the times change, the teaching profession must change as well. (Read quote.)

(SHOW TRANSPARENCY #14.) (READ QUOTE.)

Education which is not modern  
shares the fate of all organic  
things which are kept too long.

Alfred North Whitehead

There are many ways that teachers can become more effective and efficient in the teaching of mathematics.

Time is a major factor to consider in teacher effectiveness and efficiency. Many feel that too much time is being wasted on students working one drill sheet after another when in actuality the students have already mastered some of the skills.

It seems that most teachers are having a hard time providing enough time for mathematics because of the very tight and busy schedules they must meet each day. NCTM feels that time spent on the drill/practice worksheets could be lessened in order to devote more time to involving students in problem-solving activities. Explanation, practice, and directive teaching are important but should not eliminate the time necessary for children to work on problem solving.

(SHOW TRANSPARENCY #15.) (SEE NEXT PAGE.)

Activity	Percent Responding			
	Age	Often	Sometimes	Never
<b>Student-centered</b>				
Mathematics tests	9	44	46	9
	13	61	37	1
	17	63	33	3
Mathematics homework	9	43	45	12
	13	67	29	3
	17	57	36	8
Worked mathematics problems alone	9	71	22	7
	13	81	17	9
	17	80	19	1
Worked mathematics problems on the board	9	39	54	7
	13	33	58	9
	17	27	60	13
Used a mathematics textbook	9	75	18	6
	13	81	14	5
	17	87	11	3
Worksheets	9	71	27	2
<b>Teacher-centered</b>				
Listened to the teacher explain a mathematics lesson	9	85	11	3
	13	81	18	2
	17	78	19	2
Watched the teacher work mathematics problems on the board	9	78	19	4
	13	76	21	2
	17	79	18	3
Received individual help from the teacher on mathematics	9	21	67	11
	13	17	71	10
	17	18	70	11

Carpenter, Thomas P. et al. "II. National Assessment." In Mathematics Education Research: Implications for the 80's, edited by Elizabeth Fennema. Alexandria, Va.: ASCD, 1981.

It may be of interest to look at the chart you see on the transparency. This chart was developed from the information collected from the National Assessment. It centers around what 9, 13, and 17-year-olds consider their involvement in mathematics to be. As you see, the chart is divided into student-centered activities and teacher-centered activities. It shows that students spent a lot of time listening and watching the teacher work and explain math problems. Also, they spent a lot of time either working on mathematics problems from the textbook or worksheets. Students perceive their role in the mathematics classroom as passive.

The students also indicated that they had little opportunity to interact with classmates.

These findings are in direct conflict with NCTM recommendations. More active involvement on the part of the student is needed. NCTM recommends that approaches being used currently be changed to encourage more student involvement in learning.

(SHOW TRANSPARENCY #16.)

NCTM recommends that the following action steps be taken concerning teacher effectiveness and efficiency in the teaching of mathematics.

1. Involving students in meaningful activities
2. More effective use of mathematics time
3. School administrators and parents support the teacher's efforts to engage students in more effective learning tasks
4. Teachers use of diverse instructional strategies, materials, and resources

RECOMMENDATION 5 (REFER TO #5 ON TRANSPARENCY.)

THE SUCCESS OF MATHEMATICS PROGRAMS AND STUDENT LEARNING MUST BE EVALUATED BY A WIDER RANGE OF MEASURES THAN CONVENTIONAL TESTING.

For years we as teachers have held to the tradition of paper-and-pencil evaluation of mathematics. Maybe this is true because of our own school experience, emphasis on standardized testing, mechanical grading techniques, and public stress on computation and accountability.

What is the purpose for testing?

1. To judge the effectiveness of instruction
2. To adjust curriculum
3. To adjust a student's instruction

Too many times one test score is just accepted. This is placing too much confidence in the results of one particular test. These tests may not generate the information needed for determining how much a student knows.

Alternatives need to be considered.

Say: Let's brainstorm for a moment. What are some ways you think would be appropriate for evaluating student learning?

(Write participants ideas on clean transparency.)

Examples: criterion referenced test

competency test

observation

open-ended assessments

interviews

manipulative tasks

problem-solving task



It is important to remember that the goals of the math program must dictate the nature of the evaluation. Too often the reverse is true--tests dictate the program.

It is also important that school systems keep parents informed about the kinds of tests being used to evaluate their children. If parents know from the beginning how testing will be done and why; this may eliminate problems down the road.

#### RECOMMENDATION 6

(REFER TO TRANSPARENCY.)

MORE MATHEMATICS STUDY MUST BE REQUIRED FOR ALL STUDENTS AND A FLEXIBLE CURRICULUM WITH A GREATER RANGE OF OPTIONS SHOULD BE DESIGNED TO ACCOMMODATE THE DIVERSE NEEDS OF THE STUDENT POPULATION.

NCTM is strongly urging that more time and study be required for mathematics. Competence in mathematics is necessary to achieve a productive life. NCTM would like to see at least three years of required mathematics courses in grades 9-12. Also, it is recommended that more time be devoted to math at the elementary school level. They suggest around 5 hours per week for the primary grades and up to 7 hours per week for the upper grades. Some of this time may be gained through application of math skills in other subjects such as science and social studies.

Louisiana now requires in the elementary grades a minimum of 50 minutes a day in mathematics and a minimum of 3 units of mathematics in high school, specifically:

- 1) Algebra I

- 2) Geometry and Algebra II, or
- 3) Algebra II, or Geometry and one of the following: Advanced Mathematics, Calculus, Consumer Mathematics, or Business Mathematics.

Besides additional study and time, a more flexible range of options is needed to meet the interests, abilities, and goals of students. In the past math programs have been set up with the high school student taking one required course. The only way for a student to receive more mathematical knowledge was to take an elective course in math.

At the college level the trend is to broaden mathematical study to "the mathematical sciences." This would include not only math but also the use of mathematical ideas and tools to solve real-life problems. Students need more knowledge of mathematics not only if they plan to attend college, but also if they plan to enroll in technical and vocational training because of the more diverse mathematical backgrounds that are needed in these areas.

The actions steps recommended by NCTM are as follows:

(SHOW TRANSPARENCY #17.)

1. A more flexible curriculum that permits a greater number of options is needed at the secondary level.
2. Teachers, school officials, counselors, and parents should encourage a positive attitude toward mathematics and its value to the individual learner.
3. Special programs stressing problem-solving skills should be devised for special categories of students--that is, the gifted, handicapped, and others who have difficulty understanding algorithms.

4. School systems should increase the amount of time students spend in the study of mathematics.
5. The curriculum that stresses problem solving must pay special heed to the developmental sequence best suited to achieving process goals, not just content goals.

#### RECOMMENDATION 7

(REFER TO #7.)

#### MATHEMATICS TEACHERS MUST DEMAND OF THEMSELVES AND THEIR COLLEAGUES A HIGH LEVEL OF PROFESSIONALISM.

Teaching is a profession that is constantly changing. As our society changes, education has to adapt to meet new challenges. Many times educators need additional training in order to be properly prepared to meet the needs of our changing world.

The teacher is the key to education--the major factor in motivating student's interest in the areas of mathematics, science, and technology. Teachers have a direct influence in the course of students' lives now and for the future. Justice Oliver Wendell Holmes, Jr., in explaining his ability to give such outstanding public service said, "through our great good fortune in our youth, our hearts were touched with fire." Because today the general public views mathematics as a cold subject, it is difficult to light such a fire of interest.

Clearly, there is an indication that teachers must develop and maintain teaching competence. This mandate for both preservice and inservice help is one that should aid in implementing the seventh recommendation.

RECOMMENDATION 8 (REFER TO #8.)

PUBLIC SUPPORT FOR MATHEMATICS INSTRUCTION MUST BE RAISED TO A LEVEL COMMENSURATE WITH THE IMPORTANCE OF MATHEMATICAL UNDERSTANDING TO INDIVIDUALS AND SOCIETY.

A teacher's job is very complex these days, much more so than 10 years ago. Students are undisciplined, parents have negative attitudes, governmental agencies are not always supportive, government regulations are sometimes ambiguous and vacillating, and school officials are kept so busy with the day-to-day operations that little time is left for working to improve what is happening in the classrooms. Besides these problems we are also faced with not being able to find qualified math teachers and the decreased number of students going into the field of education.

Even though all these problems are tough we cannot use them for an excuse for inaction. In the long run students will be the losers if the math program is not improved. If students leave school without the education they need the ultimate loser will be society.

(SHOW TRANSPARENCY #18.)

NCTM recommends several action steps concerning public support.

1. Society must provide incentives that will attract and retain competent, fully prepared, qualified mathematics teachers.
2. Parents, teachers, and school administrators must establish new and higher standards of cooperation and teamwork.
3. Government at all levels should operate to facilitate, not dictate, the attainment of goals agreed on cooperatively by the public's representatives and the professionals.

For solutions to be reached in such a complex task, commitment and cooperation among all segments of society are required. One cannot pull this off alone. There must be involvement of all.

In closing, let us remember the words of William Ralph Inge:

"There are two kinds of fools: those who say 'this is new and therefore better' and others who say 'this is old and therefore good'."

TRANSPARENCIES  
AND  
HANDOUTS

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# FOCUS IN

## OBJECTIVES

1. PRESENT THE "AGENDA FOR ACTION"
2. DEFINE THE TEN BASIC SKILL AREAS (NATIONAL COUNCIL OF THE SUPERVISORS OF MATHEMATICS)

# **An Agenda for Action**

## **Recommendations for School Mathematics of the 1980s**

The National Council of Teachers of Mathematics recommends that—

- 1. problem solving be the focus of school mathematics in the 1980s;**
- 2. basic skills in mathematics be defined to encompass more than computational facility;**
- 3. mathematics programs take full advantage of the power of calculators and computers at all grade levels;**
- 4. stringent standards of both effectiveness and efficiency be applied to the teaching of mathematics;**
- 5. the success of mathematics programs and student learning be evaluated by a wider range of measures than conventional testing;**
- 6. more mathematics study be required for all students and a flexible curriculum with a greater range of options be designed to accommodate the diverse needs of the student population;**
- 7. mathematics teachers demand of themselves and their colleagues a high level of professionalism;**
- 8. public support for mathematics instruction be raised to a level commensurate with the importance of mathematical understanding to individuals and society.**



- A. ROUTINE AND NONROUTINE WORD PROBLEMS
- B. PROCESS PROBLEMS
- C. REAL-LIFE PROBLEM SOLVING ACTIVITIES





**FORMULATE KEY QUESTIONS**

**ANALYZE AND CONCEPTUALIZE PROBLEMS**

**DEFINE PROBLEM**

**DISCOVER PATTERNS AND SIMILARITIES**

**SEEK OUT APPROPRIATE DATA**

**EXPERIMENT**

**TRANSFER SKILLS AND STRATEGIES TO NEW SITUATIONS**

**DRAW ON BACKGROUND KNOWLEDGE TO APPLY MATHEMATICS**

### HAS TEACHING COME TO THIS?

RUTH, MARIA, KATY, AND JANE ARE ALL TEACHERS HAVING TROUBLE MAKING ENDS MEET. THEY ALL HOLD PART-TIME JOBS AFTER SCHOOL. ONE IS A CARPENTER'S APPRENTICE, ONE IS A COOK, ONE IS A DELIVERY PERSON FOR A PIZZERIA, AND ONE TEACHES JUDO. RUTH HAS NOTHING TO DO WITH FOOD. MARIA AND THE COOK ARE SISTERS. KATY AND MARIA ARE ALLERGIC TO WOOD WORKING. JANE, THE DELIVERY PERSON, AND THE JUDO TEACHER ARE STRANGERS. MATCH UP EACH WOMAN WITH HER WORK.



1. REORGANIZE MATHEMATICS CURRICULUM
2. CREATE CLASSROOM ENVIRONMENTS CONDUCIVE TO PROBLEM SOLVING
3. DEVELOP PROBLEM SOLVING MATERIALS
4. STUDENT INVOLVEMENT IN PROBLEM SOLVING SITUATIONS



### Basic Skills and the Student's Future

Anyone adopting a definition of basic skills should consider the "door-opening/door-closing" implications of the

list. The following diagram illustrates expected outcomes associated with various amounts of skill development.

#### Scope of Skill Development

**EXPANDED SKILLS**  
Mathematical skills beyond those described here plus a desire to learn more.

**BASIC SKILLS**  
The skills described here.

**MINIMAL SKILLS**  
Limited skills, primarily computation. Little exposure to the other skill areas described here.

#### Expected Outcomes

**POTENTIAL LEADERS**  
Employment and educational opportunities will continue to increase as mathematical skills continue to grow.

**EMPLOYMENT VERY LIKELY**  
Employment opportunities are predictable. Doors to further education opportunities are open.

**LIMITED OPPORTUNITIES**  
Unemployment likely. Potential generally limited to low-level jobs.

## TEN BASIC SKILL AREAS\*

- I. PROBLEM SOLVING
- II. APPLYING MATHEMATICS TO EVERYDAY SITUATIONS
- III. ALERTNESS TO THE REASONABLENESS OF RESULTS
- IV. ESTIMATION AND APPROXIMATION
- V. APPROPRIATE COMPUTATIONAL SKILLS
- VI. GEOMETRY
- VII. MEASUREMENT
- VIII. READING, INTERPRETING, AND CONSTRUCTING TABLES,  
CHARTS AND GRAPHS
- IX. USING MATHEMATICS TO PREDICT
- X. COMPUTER LITERACY

\*AS DEFINED BY THE NATIONAL COUNCIL OF THE SUPERVISORS  
OF MATHEMATICS

## STRATEGIES USED TO SOLVE PROBLEMS

1. POSING QUESTIONS
2. ANALYZING SITUATIONS
3. TRANSLATING RESULTS
4. ILLUSTRATING RESULTS
5. DRAWING DIAGRAMS
6. USING TRIAL AND ERROR

## STUDENTS NEED TO:

1. APPLY LOGIC
2. DETERMINE FACTS
3. ARRIVE AT TENTATIVE CONCLUSIONS



\$1.00 WORD HINTS

1. DAY OF THE WEEK
2. A THANKSGIVING WORD
3. A HALLOWEEN WORD
4. A UNITED STATES COIN
5. AN ASTROLOGICAL SIGN





DECREASED EMPHASIS ON:

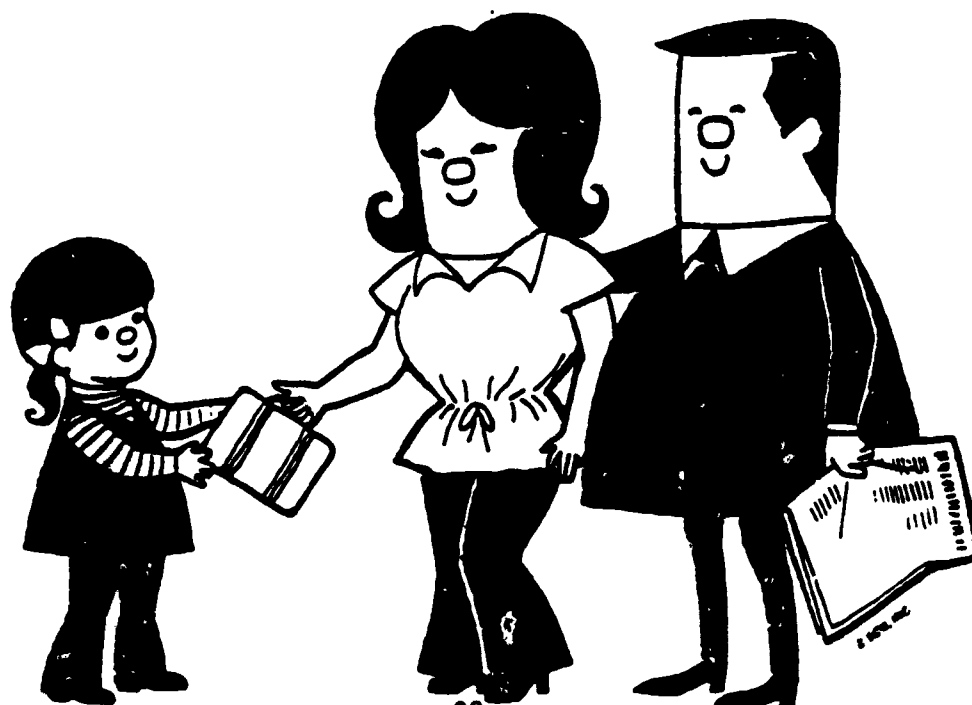
ISOLATED DRILL

PAPER-AND-PENCIL CALCULATIONS

VOCABULARY

CONVERTING MEASURES

OUTDATED TABLES



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92

98

ELEMENTARY--STRONG SUPPORT FOR:

CALCULATORS

1. CHECKING ANSWERS
2. SOLVING WORD PROBLEMS
3. HOMEWORK
4. DOING A CHAIN OF CALCULATIONS

COMPUTERS

1. AVAILABILITY
2. INTERACTION
3. CHANGE MATH CURRICULUM

HIGH SCHOOL--STRONG SUPPORT FOR:

1. COMPUTE AREA
2. TRIGONOMETRY
3. SOLVE EQUATIONS USING GEOMETRIC FIGURES

1. 1 REQUIRED COURSE FOR COLLEGE-BOUND
2. ROLE OF COMPUTERS
3. COMPUTER LITERACY



A MAN HAS 1,310 BASEBALLS TO PACK IN BOXES WHICH HOLD 24 BASEBALLS EACH. HOW MANY BASEBALLS WILL BE LEFT OVER AFTER THE MAN HAS FILLED AS MANY BOXES AS HE CAN?



EDUCATION WHICH IS NOT MODERN SHARES THE FATE OF ALL  
ORGANIC THINGS WHICH ARE KEPT TOO LONG.

ALFRED NORTH WHITEHEAD

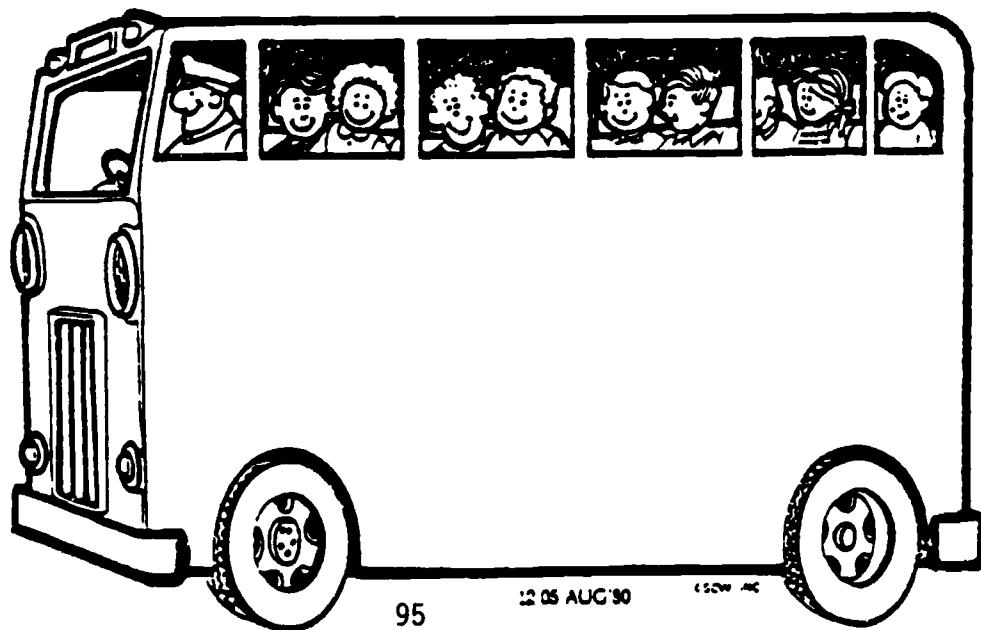
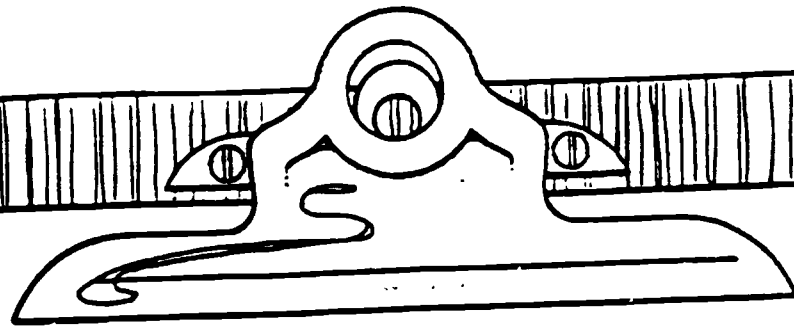


Figure B-8. Ratings of Frequency of Selected Classroom Activities

Activity	Percent Responding			
	Age	Once	Sometimes	Never
<b>Student-centered</b>				
Mathematics tests	9 13 17	44 61 69	46 37 38	9 1 3
Mathematics homework	9 13 17	49 67 67	46 29 38	12 3 6
Worked mathematics problems alone	9 13 17	71 61 66	28 17 19	7 9 1
Worked mathematics problems on the board	9 13 17	39 33 27	54 58 69	7 9 13
Used a mathematics textbook	9 13 17	75 61 67	16 14 11	6 8 3
Worksheets	9	71	27	2
<b>Teacher-centered</b>				
Listened to the teacher explain a mathematics lesson	9 13 17	66 61 78	11 16 19	3 2 2
Watched the teacher work mathematics problems on the board	9 13 17	78 76 79	19 21 16	4 2 3
Received individual help from the teacher on mathematics	9 13 17	21 17 16	67 71 70	11 10 11

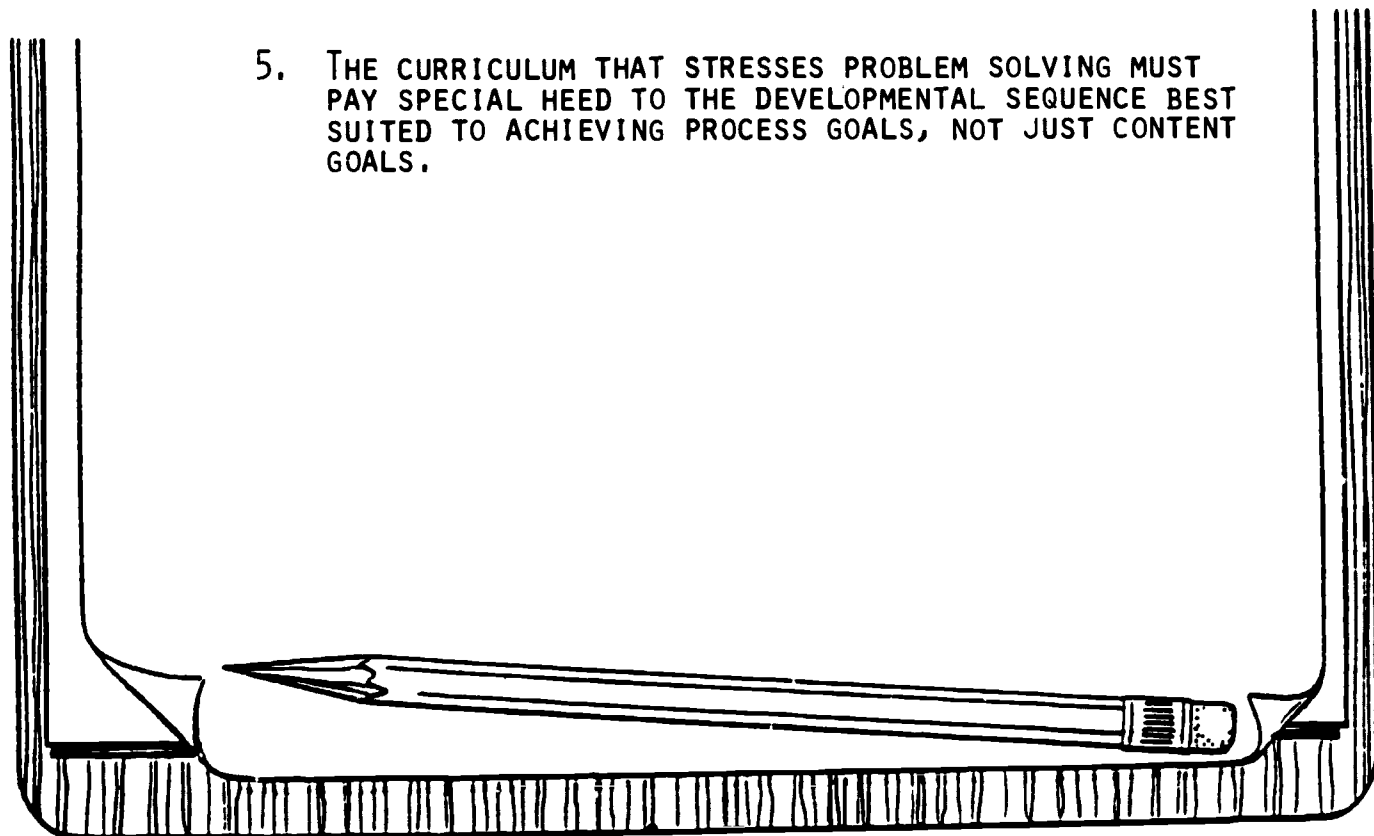


**NCTM RECOMMENDS THAT THE FOLLOWING ACTION STEPS BE TAKEN CONCERNING TEACHER EFFECTIVENESS AND EFFICIENCY IN THE TEACHING OF MATHEMATICS.**

- 1. INVOLVING STUDENTS IN MEANINGFUL ACTIVITIES**
- 2. MORE EFFECTIVE USE OF MATHEMATICS TIME**
- 3. SCHOOL ADMINISTRATORS AND PARENTS SUPPORT THE TEACHER'S EFFORTS TO ENGAGE STUDENTS IN MORE EFFECTIVE LEARNING TASKS**
- 4. TEACHERS USE DIVERSE INSTRUCTIONAL STRATEGIES, MATERIALS, AND RESOURCES**

THE ACTION STEPS RECOMMENDED BY NCTM ARE AS FOLLOWS:

1. A MORE FLEXIBLE CURRICULUM THAT PERMITS A GREATER NUMBER OF OPTIONS IS NEEDED AT THE SECONDARY LEVEL.
2. TEACHERS, SCHOOL OFFICIALS, COUNSELORS, AND PARENTS WOULD ENCOURAGE A POSITIVE ATTITUDE TOWARD MATHEMATICS AND ITS VALUE TO THE INDIVIDUAL LEARNER.
3. SPECIAL PROGRAMS STRESSING PROBLEM-SOLVING SKILLS SHOULD BE DEvised FOR SPECIAL CATEGORIES OF STUDENTS.
4. SCHOOL SYSTEMS SHOULD INCREASE THE AMOUNT OF TIME STUDENTS SPEND IN THE STUDY OF MATHEMATICS.
5. THE CURRICULUM THAT STRESSES PROBLEM SOLVING MUST PAY SPECIAL HEED TO THE DEVELOPMENTAL SEQUENCE BEST SUITED TO ACHIEVING PROCESS GOALS, NOT JUST CONTENT GOALS.



ACTION STEPS

1. SOCIETY MUST PROVIDE INCENTIVES
2. ESTABLISH NEW AND HIGHER STANDARDS OF COOPERATION AND TEAMWORK
3. SHOULD OPERATE TO FACILITATE, NOT DICTATE





1 2 3 4

1 2 3 4

1 2 3 4

1 2 3 4

1 2 3 4

1 2 3 4

1 2 3 4

1 2 3 4

MENTAL ARITHMETIC CARDS FOR A GROUP OF 20

<p>I have 1. Who has this plus 5 plus 6?</p>	<p>I have 2. Who has this plus 4 plus 3?</p>	<p>I have 3. Who has this plus 6 plus 4?</p>
<p>I have 4. Who has this plus itself?</p>	<p>I have 5. Who has this times 4?</p>	<p>I have 6. Who has half of this?</p>
<p>I have 7. Who has this plus 5 plus 4?</p>	<p>I have 8. Who has this plus 9?</p>	<p>I have 9. Who has this plus itself?</p>
<p>I have 10. Who has this plus 9?</p>	<p>I have 11. Who has this minus 9?</p>	<p>I have 12. Who has this minus 7?</p>
<p>I have 13. Who has this minus 9?</p>	<p>I have 14. Who has half of this?</p>	<p>I have 15. Who has this less 14?</p>
<p>I have 16. Who has this minus 10?</p>	<p>I have 17. Who has this less 2?</p>	<p>I have 18. Who has this minus 4?</p>
<p>I have 19. Who has this minus 8?</p>	<p>I have 20. Who has this minus 10?</p>	<p>101</p>

### Microcomputer Vocabulary Test

Draw a circle around the letter of your answer like this: (d)

1. Which one of the following is the most powerful?
  - a) computer
  - b) minicomputer
  - c) microcomputer
  - d) calculator
  - e) don't know
2. The Apple is a
  - a) computer
  - b) minicomputer
  - c) microcomputer
  - d) calculator
  - e) don't know
3. The monitor is what you
  - a) press
  - b) listen to
  - c) switch on
  - d) look at
  - e) don't know
4. What you type on the keyboard of the Apple is called the
  - a) input
  - b) output
  - c) cursor
  - d) monitor
  - e) don't know
5. A floppy disk looks like a
  - a) soft cricket ball
  - b) small phonograph record
  - c) milk bottle top
  - d) Frisbee
  - e) don't know
6. Programs can be stored on
  - a) floppy disks
  - b) cassettes
  - c) both floppy disks and cassettes
  - d) monitors
  - e) don't know
7. A disk drive is used to
  - a) switch the Apple on
  - b) make the Apple work
  - c) show a picture on the Apple's screen
  - d) feed a program into the Apple
  - e) don't know
8. The cursor is
  - a) a flashing signal
  - b) the return key on the Apple
  - c) the printed instruction sheet
  - d) the TV screen on the Apple
  - e) don't know
9. The graphics mode shows
  - a) lines of writing
  - b) pictures
  - c) a flashing signal
  - d) words in color
  - e) don't know
10. A copy of what is showing on the TV screen can be obtained by using the
  - a) disk drive
  - b) monitor
  - c) cursor
  - d) printer
  - e) don't know

Collis, Betty and Geoffrey Mason. "A Coordinate Graphing Microcomputer Unit for Elementary Grades." In An Agenda In Action, edited by Gwen Shufelt. Reston, Va.: NCTM, 1983.

## HAS TEACHING COME TO THIS?

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GRAPHS

104

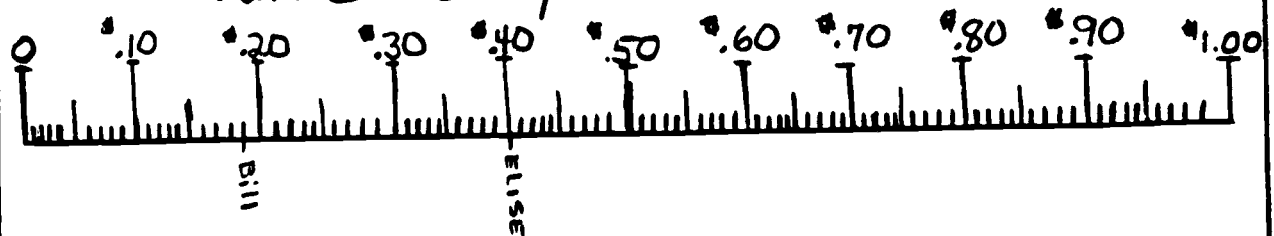
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# GRAPHS

## I VALUE of YOUR NAME

WHAT IS THE VALUE OF YOUR NAME?

If  $a = \$.01$ ,  $b = \$.02$ ,  $c = \$.03$ , etc., what is the value of your first name worth?



## II MIRROR Symmetry

Some letters (like A) have a vertical axis of symmetry; some have horizontal axis (like K); some have both (X); some have neither (F)

Write the first letter of your first name where it fits.

Vertical	Horizontal	Both	Neither

# GRAPHS

## III. STRING Measurement

How many times do you think a string equal to your height would wrap around your head? Mark an X.

2																				
3																				
4																				
5																				
6																				
7																				
8																				
9																				
10																				
11																				
12																				

NEEDED FOR WORKSHOP

GRAPHS

- I. VALUE OF YOUR NAME
- II. MIRROR SYMMETRY
- III. STRING MEASUREMENT

HANDOUTS

1-2-3-4 SLIPS

MENTAL ARITHMETIC CARDS

MICROCOMPUTER VOCABULARY TEST

HAS TEACHING COME TO THIS?

MATERIALS

STRING

SCISSORS

OH PENS

BLANK TRANSPARENCIES

TAPE



## BIBLIOGRAPHY

An Agenda for Action, Recommendations for School Mathematics of the 1980's. Reston, Va.: The National Council of Teachers of Mathematics, 1980.

Burns, Marilyn. Meeting the Math Challenge. Palo Alto, California: Pitman Learning, Inc., 1981.

Carpenter, Thomas P. et al. "III. National Assessment." In Mathematics Education Research: Implications for the 80's, edited by Elizabeth Fennema. Alexandria, Va.: ASCD, 1981.

Collis, Betty and Geoffrey Mason. "A Coordinate Graphing Microcomputer Unit for Elementary Grades." In An Agenda In Action, edited by Gwen Shufelt. Reston, Va.: NCTM, 1983.

National Council of Supervisors of Mathematics. "National Council of Supervisors of Mathematics Position Paper on Basic Skills." Arithmetic Teacher, 25, (October, 1977): 19-22.

Priorities In School Mathematics. Reston, Va.: The National Council of Teachers of Mathematics, 1981.