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ABSTRACT

This document contains a workshop presentation for elementary mathematics teachers. The purpose of the workshop is to demonstrate methods for teaching computational concepts beginning with concrete materials and moving to expressing these concepts abstractly with numerical symbols. Section headings are: (1) "Introduction"; (2) "Sorting and Classifying"; (3) "Beginning Number Concepts"; (4) "Place Value" (providing activities about how the base ten number system works); (5) "Addition"; (6) "Subtraction"; (7) "Multiplication"; and (8) "Division." For each concept the format for teaching is provided. The appendices contain transparencies and five references. (YP)

TEACHING COMPUTATION THROUGH
CONCEPT DEVELOPMENT

INSERVICE PRESENTATION

DEVELOPED BY THE
LOUISIANA STATE DEPARTMENT OF EDUCATION
BUREAU OF ELEMENTARY EDUCATION

BULLETIN 1789

July, 1986

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"TEACHING COMPUTATION THROUGH CONCEPT DEVELOPMENT"

INTRODUCTION

Traditionally, if students are asked, "Do you like math?" the majority response will be negative. When asked "Why not?" they will usually give reasons like, "It's too hard; I don't understand it; I'm not good in math"; or "It's boring." These are the attitudes we are seeking to change in order to improve students' computational and problem-solving abilities.

In many cases the textbook not only determines the math curriculum but also the methodology. Charles Silberman says that educators have the idea that they have a "moral contract" to teach the text and lose sight of their real responsibility to teach students.

Our traditional methods of teaching math have been great motivators. They have motivated students to take as few math courses as possible. They have emphasized mastery of small isolated skills. They have neither allowed nor encouraged students to discover and become independent, confident problem solvers.

It is not to say that textbooks and traditional methods of instruction are all bad and should be thrown out. They certainly have their place in the curriculum. They just don't provide the starting point for discovering mathematical relationships. They only provide the means for expressing those relationships symbolically.

Based on National Assessment Results, students are learning much of their arithmetic by rote, without the needed understanding to apply these concepts to solve problems. Teaching rules by rote without developing understanding results in students who rely on recipes rather than thinking through the process. They are then unable to make generalizations and apply the skills in problem-solving situations.

Concepts have the best chance of being understood when developed through experiences with tangible materials. Manipulatives allow students to become active learners. They are motivational as well as producers of involvement, understanding, and achievement. Concrete experiences allow the student to move toward a level of understanding at which generalizations can be discovered. Then, at that point, it is appropriate to attach the abstract numerical symbolization.

The purpose of this workshop is to demonstrate methods for teaching computational concepts beginning with concrete materials and moving to expressing these concepts abstractly with numerical symbols. The format for teaching each of the concepts will follow a process which was developed by the late Mary Baretta-Lorton and is known as Concept/Connecting/Symbolic.

Traditionally, we begin math instruction with the symbolic as in a textbook; but we will begin by developing an understanding of the concept by using concrete materials to solve problems. At this level, you will be introduced to a concept with a variety of experiences using a variety of materials. In this first stage, we will use no numerical symbolization. Symbols often get in the way of discovery and developing understanding.

When students become comfortable and confident with the materials and concept, the concrete experience will be connected to its abstract, numerical symbolization. At the connecting level, the teacher only models the connection; this forms a link between the learner's experience and the written abstraction. The students continue to use manipulatives to express relationships and solve problems while the teacher records what they are doing with symbols. This connection doesn't automatically happen; it has to be taught.

The final level, which is traditionally where math instruction begins, is the symbolic. At this level, the student has developed an understanding of the concept and is ready to do his own recording of symbols that represent the concept. When math instruction begins at this level, the student is just being taught "to do the page" instead of developing an understanding of the concept.

The majority of our time, as it should be in the classroom, will be spent at the Concept level. Rushing through this first step usually results in the need to backtrack. It has been recommended that 60% of time be spent at the concept development level, 30% at the connecting level, and only 10% at the symbolic level.

As you participate in this workshop, allow yourself to assume the learner's role. You will be interacting with peers and verbalizing your activities. This will enable you to internalize and communicate the concept as it is "seen." This will aid in clarifying your thinking and help you discover other ways of seeing and doing.

We can change attitudes toward math, greatly improve math scores, and help students become independent problem solvers when we are able to change our approach and methods of instruction. It is our responsibility to try to bring about the kind of change that will ultimately make a difference.

Remember, one of our goals as educators is to teach students how to think - to stretch their minds.

SORTING AND CLASSIFYING

In this session we are going to work on some ideas that can be used in the classroom for teaching sorting and classifying.

Place a junk bag on each table--buttons, screws, keys, etc.)

First, I would like for you to work in groups of four or five.

Next, please empty your junk bags. I would like for you to sort your junk into two different ways. (Ex. round, square, etc. Give a few minutes.)

(Ask for volunteers or ask someone from each group to share. Discuss.)

Now, I would like for you to sort your objects again in a different way. (Give a few minutes. Discuss.)

This time we are going to sort our objects in a different way. When you finish, write your answer on this sheet of paper and turn it over so that no one can see your answer.

Next, I would like for you to visit each others tables and try to guess how each group sorted their objects. When you think you know the answer, check by looking at what was written on the answer sheet.

Try to come up with a good way to sort your junk in order to out-smart the other groups.

(Go around to each table and discuss sorting. Discuss with the group the language development that will evolve by using this activity.)

This activity brings about a lot of language development that is relative to math and reading. What are some of the words we keep hearing in our discussion of this activity?

(Let the participants respond.)
Examples: large, small
whole, half
big, little
long, short

I need everyone to come to the middle of the room. We are going to sort and classify ourselves into groups. We will take turns sending each group out of the room. Upon their return, they will have to guess how we have sorted and classified ourselves.

Examples: open toed shoes/closed toed shoes
shirts in/shirts out
earrings/no earrings
ties/no ties
buttons/no buttons

(Give out blocks--different sizes and colors.)

We will use blocks for our next activity. In your bags you have four different shapes. (circles, squares, diamonds, triangles)

I want you to sort your blocks into two groups.

Examples: straight sides/not straight sides
points/no points
circles/not circles

(Discuss how there are lots of different ways to describe the same group. Examples: points/no points would be the same as circles/not circles which would be the same as straight/not straight sides. You want the students to understand that there is more than one way to describe something or more than one way to do something.)

(Another activity would be to ask a student to pick up the large, green block and show it to their classmates. This lets you know that the students understand the meaning of terms such as large and green. Actually ask the participants to do this activity.)

Another idea is for the teacher to pick up a block and ask the students to describe how it looks. Example: small, blue, square

I would like for each person to pick up one block. (Let the participants take turns describing their blocks.)
Examples: large, yellow block
big, green block
small, red circle
big, red square

Again, this is a good language development activity for reading as well as mathematics.

To extend this activity and for further language development, you may ask a student to describe a block and then have other students describe the same block.

Now we will work on what we term the "Train Activity." I would like for you to use your blocks to make a train. Listen to your directions.

All of the blocks must touch, including the beginning block and the ending block. The touching blocks can only have one difference. For example: You could put a large, green block and a large, yellow block together because there is only one difference. (color) Another example: You can't put a small, yellow block with a large, red block because that would be two differences. (color, size)

(Give them time to make a train. Ask the participants to visit each other's tables. Look at the trains and discuss.)

The activities that we have discussed up to this point have been to give you ideas concerning how sorting and classifying can be taught to your students. Now we are going to move on to the Beginning Number Concepts.

BEGINNING NUMBER CONCEPTS

Students need a lot of concrete experiences before they move to abstract math. In this session the concept/connecting/symbolic approach will be used for developing number concepts. Let's review these three levels of mathematics instruction.

(1) At the Concept Level, students experience concepts through the use of concrete materials. No symbolic representation of the concept is introduced. At this level, it is important that students interact with a concept in a variety of ways, using a variety of materials.

(2) At the Connecting Level, the concept, as concretely experienced by the student, is connected to the mathematical symbolization that represents the concept. The teacher does the connecting, and the student must already have a firm grasp of the concept.

(3) At the Symbolic Level, the students themselves write the mathematical symbols to represent the concepts they have learned. They may still use concrete materials; they will drop them when they are ready. Remember, the students should be the ones to push the curriculum, not the other way around. Also, at this level the symbols are not the vehicle for teaching the concept. They are used for recording concepts that have already been learned.

OVERVIEW

This session focuses on development of beginning number concepts utilizing the Concept/Connecting/Symbolic approach. Included are concepts of numbers from 0-9. The goals for teachers are both to learn how to assess children's number understandings and how to teach number concepts in a way that is understandable to students. The approaches presented in this session are based on the work of Mary Baratta-Lorton in Mathematics Their Way and Workjobs II. These books offer fuller development of the concepts presented in this session.

Time	Activities	Materials
	Number Stations	beans spray-painted on 1 side, jewels, flat toothpicks, junk, Unifix cubes, pattern blocks, 6"x9" black and white newsprint and construction paper, paper pattern block shapes, 12"x18" newsprint, glue, small cups, crayons, stapler
	Assessing Number Concepts	counters
	Additional Activities	6"x9" construction paper for working space papers, beans, cubes, margarine tubs, numeral cards,

To do before the session:

Collect materials.
Set out materials for number stations.
Make working space papers from 6" x 9" construction paper by drawing 10 dots in 2 rows of 5 each on one side.
Make numeral cards

Initial Discussion

In this session, we will explore ways to help students develop the beginning concepts of number. These activities surround students with number concepts using a variety of concrete materials. The goal is for students to learn to deal flexibly with numbers and to discover and develop understanding of the mathematical relationships between numbers. The Concept/Connecting/Symbolic approach will be used in all the activities.

There are three segments to this session. First is the presentation of the Number Station approach for exploring number. During this time, your responsibility is to get involved in depth with each material you explore. The more you explore the materials as learners, the more you will see the potential in the experience for students. Following this, you will learn specific ways to assess children's number understanding. Finally, a collection of additional activities will be presented. The goal is for you to feel the limitless possibilities for presenting number concepts to students in ways that respond to how they learn.

Six different stations have been set up, each with a different material for you to explore. You are to explore the number 5 at each station. You can choose where you want to work, and move from station to station as you wish. Try to make 8 to 10 designs at each station. (Model briefly at each station.)

Also, try to think about what the student would be intuiting, and what is the potential for mathematical discovery in the experience. We will discuss the experience afterwards as it relates to children's development of number concepts. (Have the teachers explore for about 15 minutes.)

By working with these materials in this way, students have many experiences with the concepts of counting, invariance, and the combinations of the number. The reason for having them use a variety of materials is so that the student does not associate number with any one material, but with the underlying concepts in all materials.

While the students are working, your job is to circulate and ask questions to encourage language development.

(Sketch a toothpick design on the chalkboard or use some design on a table if all can see it.)

Here are examples of the kinds of questions it is important to ask:

What does this design remind you of?
I see 2 and 3. Who thinks they know what I'm looking at?

How would you describe this design with numbers? What is another way? Another way?

Concept Level

You'll now make records of the designs at the number stations. (Explain briefly how to do this at each station.) Again, work where you like. Symbols have not been used so far; this is still exploration at the Concept Level. You may not have time to get to each station, but we will discuss what to do with each afterwards. (Allow them about 8 to 10 minutes for this. Then collect examples of recording for pattern blocks, toothpicks, and junk. You may want to bring some of your own to have available for this.)

Connecting Level

It is the teacher's job to connect the children's experiences to the mathematical symbols. This is the Connecting Level.

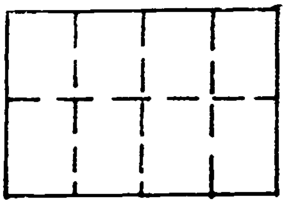
(Spread examples of recordings from the pattern blocks, toothpicks, and junk on the floor. Have the group gather around. Ask questions. Lead them from word descriptions to number combinations. You record their responses for designs on 6" x 9" newsprint, placing your recording on top of the design described. When all have been described, gather them up and staple them into a book. Read the book together.)

Symbolic Level

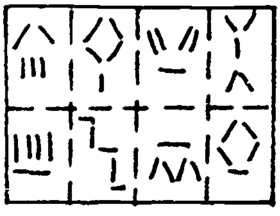
At the Symbolic Level, the students once again rebuild at the number stations, but also record using the correct mathematical symbols.

Using the recordings from the Unifix cube, sprayed beans, and jewels stations, students can cut their recordings into separate pieces, record the number sentence, and then staple them into tiny books. (Model making one for the teachers.)

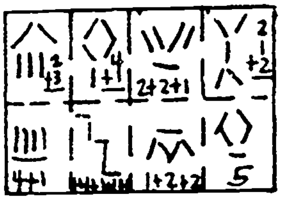
Fold a piece of 12" x 18" newsprint into eights.



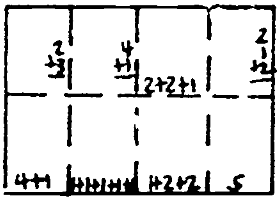
Build a design in each area using one material.



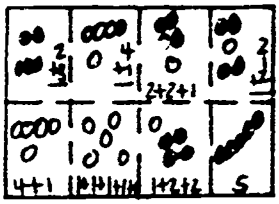
Record a number combination for each design.



Clear your paper.



Rebuild each number combination using a different material. Then clear your paper and build again, or exchange and build on someone else's paper.



Review

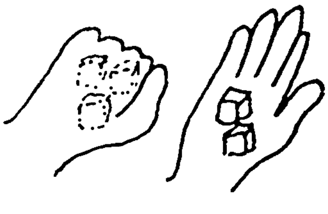
What is the value of number stations for students? They develop a flexible view of number, familiarity with number combinations, experience with the invariance of number, counting reinforcement.

What are the differences in the Concept, Connecting, and Symbolic Levels?

The experiences that students receive through involvement in the number stations goes beyond merely counting to 5 or making pretty designs. The number stations also present problem solving situations. For example:
How many different designs can you make with 5 toothpicks? How many different combinations of 5 can you make with the jewels? Does any particular combination of sprayed beans come up more often than others? Why? In dealing with number concepts, you also are developing ideas in the other strands as well.

Assessment

BEGINNING NUMBER CONCEPTS (0 - 9)

Teacher	Child	Interpretation
"Count 5 blocks into my hand." 	Child is able to do so.	Appropriate level. Shows ability to count with 1-to-1 correspondence.
	Child is unable to do so.	Inappropriate level. Try it with 3 blocks. If this is not possible, do not go on.
"How many blocks do I have in my hand?"	Child says "5" without counting.	Appropriate level. Child conserves for 5. Ask next question.
	Child must recount.	Inappropriate level. Child doesn't conserve. Try with 3 blocks.
Hide some blocks in one hand and show the others. Ask: "How many am I hiding?" 	Child answers instantly, correctly, and confidently.	Appropriate level. Check several other combinations, and then assess for 6. As long as the level is appropriate, continue assessing up to 10.
	Child guesses wrong, or cannot guess, or does not know instantly with confidence.	Try several other combinations to make sure level is inappropriate. If so, try again with 3 blocks.

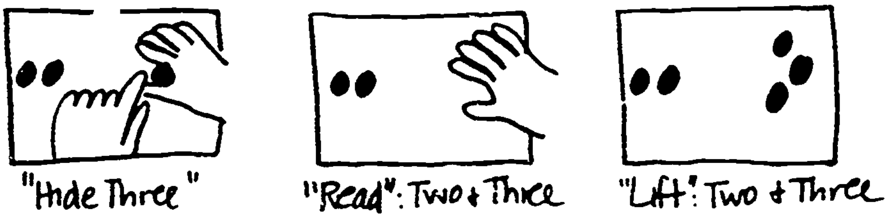
(This is adapted from Mathematics Their Way by Mary Baratta-Lorton, p. 187, © 1976, Addison-Wesley Publishing Company.)

Additional Activities

These activities are additional ways to help students develop number concepts. They are suitable to do with small groups of 6 to 8 students who have been assessed at the same level. They will all be presented first at the Concept Level, with no numerical symbols used. The Connecting and Symbolic Levels will be presented afterwards.

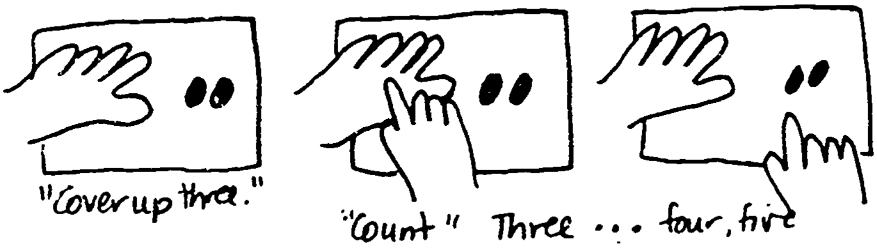
Activity I The Cave

Students put blocks on the blank side of their working space papers in a straight line. They make a cave with their right hands, and slide the objects with their other hands. They read the combinations with the cave in place, and then they lift and read the combination again.



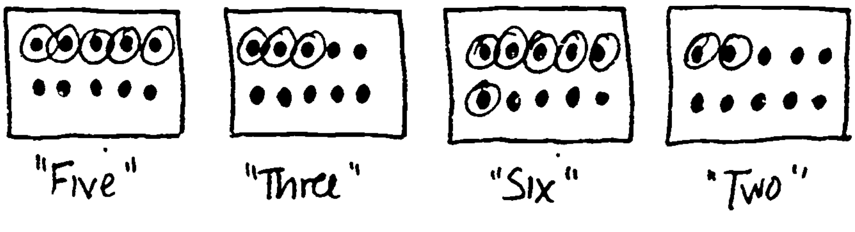
Activity II Cover Up

Students put objects on a line on the blank side of their working space papers. You tell how many objects you want them to cover up with their left hand. Then they count, starting with the number they covered up.



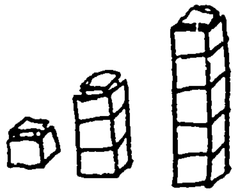
Activity III Grow and Shrink

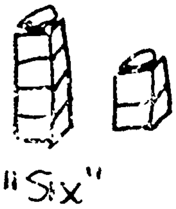
You say numbers. Students make sure that many objects are on their working space papers. They use the side with the dots, and place the objects from left to right.



Activity IV The Tower Game

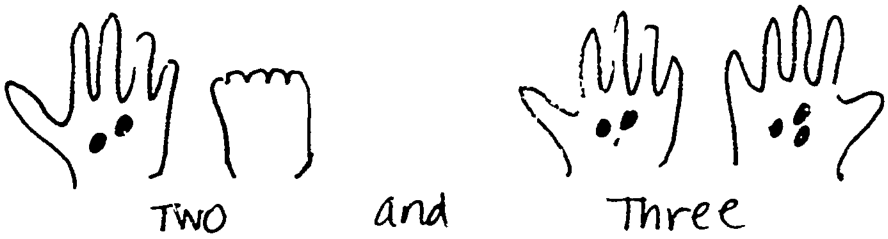
Have the students each build 5 towers using Unifix cubes. Each tower should be of all one color and 1, 2, 3, 4, and 5 long. You call out numbers, and the students show it using the towers needed.





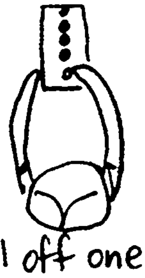
Activity V The Hand Game

The students all take the same number of objects in their hands. They shake their hands so some objects are in one and some in the other. They open one hand at a time and read. The word "and" is the signal to open the second hand.

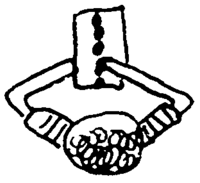


Activity VI The Wall Game

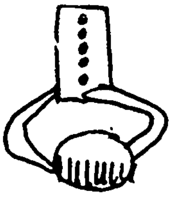
Students line up their objects. They wall off the number you say, using their hand for the wall. Then they say the combination.



"Wall off one."



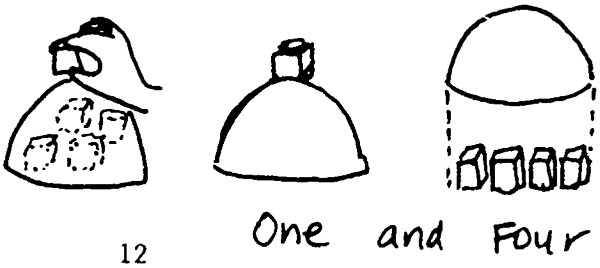
"Wall off two."



"Wall off zero."

Activity VII The Bowl Game

Students each have a bowl and the same number of blocks. They put the blocks under the overturned bowl, and then take some out and put them on top. They say the combination of blocks on top and underneath. The word "and" is the signal to lift the bowl.



Summary

Not only are the ideas presented in this session designed to give you practical suggestions to take back to your classroom, they aim to open up new ways for you to look at your children's learning. In working with these ideas, the search is always to find ways to teach students their way, learning to observe them to get the information you need to know when and how to move along. The students should be the impetus for moving along in the curriculum; the curriculum should not be pushing the students.

PLACE VALUE

Understanding place value is the key to understanding the structure of our number system. Using manipulatives, offer students models to help them see and understand the patterns in the structure of numbers.

Our number system relies on grouping objects into 10's and using those groupings to represent larger numbers of things. Though the number 10 is key to the grouping in our number system, the concepts of grouping and counting groups are basic to understanding place value. That is, the important concepts are making groups and counting groups instead of counting individual objects. These concepts, in turn, need to be connected to the appropriate symbolization.

When learning a new concept, most students need a concrete model to help plant the concept in their minds. When students manipulate concrete materials, the structure becomes part of the learners. Rote learning, however, is not so lasting because there is so little foundation behind the learning.

To provide concrete experiences, teachers need to know the materials to use and the appropriate activities for the learners to perform.

Learning can be hindered when the teacher provides the wrong materials or presents inappropriate activities.

Before base 10 is established, kindergarteners and first graders may form many kinds of bases or sets. For example, given 11 objects, the teacher may ask how many sets of 3 and how many leftovers. The students make 3 sets of 3, with 2 leftovers. This should be repeated with many groups of objects, making sets of 2, 3, 4, and 5. Some should have no leftovers. All examples should be recorded.

The purpose is for students to see that objects can be grouped in many different ways.

Bundling is a good way of developing the concept of Base Ten with young students.

First, students must learn the symbols for the numbers 0-9. Then they learn that when another object is joined to 9 objects, they form a bundle of 10.

By bundling sticks or straws, students are involved in making sets of 10. Any time the student needs to check, the rubber bands may be removed to show 10 ones in each set of 10. Students now begin to count. As they are counting, they need to see that sets of 10 are counted the same way they count ones, i.e., "1 ten, 2 tens, 3 tens." Now students need to learn the words; ten, twenty, thirty, etc.

After they are comfortable with counting tens, they should begin to count tens and ones.

After they are comfortable counting tens and ones, they should learn the words; eleven, twelve, etc.

The same pattern should be used for teaching students to count with hundreds.

Doing this, students develop a pattern that is necessary for understanding later work with large numbers. Students begin to see that each time there is ten of a set, a new unit is formed. This is the pattern.

Let me share a true situation with you. (Read the following.)

Alice Lang was excited about the new school year. After seven years as a kindergarten and first-grade teacher, she'd wanted a new challenge, and this year she would teach fifth graders. With the grade's math instruction grouped by ability into three classes, Alice had asked for the slower group because she had a talent for working patiently with youngsters and making tough subjects understandable.

Three weeks into classes, she reached the review in the math textbook on multi-digit addition, subtraction, and multiplication. The blank faces of her students told her that few of them had any idea of place value. A problem like 46 + 55 + 32 = ___ put them in left field. They were confused when a number had to be carried over from one column to the next.

Alice scouted around the classroom for materials. In the back cupboard were the expected stacks of blank paper and several boxes of mimeograph masters--but no counting sticks, base ten blocks, not even a abacus to show youngsters how to work with units of one, ten, and 100.

Undaunted, Alice decided to improvise. She asked each student to bring in a box of paper clips while she bought some sandwich bags from the supermarket. Each student interlocked the paper clips in chains of 10 and counted off 10 of the chains to put into a sandwich bag representing 100.

When finished, every student had a sandwich bag of 100 paper clips, some loose chains of 10, and leftover single clips to stand for ones. For four days, Alice demonstrated the place value of whole numbers by having the students count out the clips as they did addition problems. They finally caught on.

As she was about to move on to regrouping with two and three-digit problems, the math supervisor dropped by. As Alice recounted the place-value incident, the supervisor gave her a long, hard look before speaking. Then she began, "The Iowa Test of Basic Skills comes in just three or four weeks. Forget place value and concentrate on decimals and long division. And it might be a good idea also to run by fractions, even if time is short. At least the students will have been exposed to them."

The key word in the supervisor's vocabulary was "exposure," and the overriding concern was to prepare students for next year's mathematics, and Alice sadly concluded. It didn't seem to matter if the students had not yet learned their earlier math. "Forget second-grade math," said the supervisor firmly, "and teach fifth-grade."

Alice was disheartened. Not one to make waves with administration, she acquiesced to the supervisor's advice--kidding herself that place value lessons might be reinforced when it came time to do computation with decimals. The supervisor needn't know. In the end, though, she just followed orders.

Many teachers feel uncomfortable with the mathematical approaches or strategies suggested in the teacher's guide. They sometimes rationalize that these approaches are far too advanced for their classes or that it just "isn't their style." Often these strategies involve teaching the "why" behind the procedure. Alice Lang's review of place value with paper clips illustrated such a strategy.

Because many teachers have learned only how to do computation and not the mathematics behind the procedure, they avoid teaching the supportive reasoning to their students. These teachers instead opt to present mathematics as a series of rules and procedures to be tediously followed without explanation.

The activity that we are now going to do will show you a sequence for introducing the concept of place value to students.

Place Value - Model I

Materials: counters, cups, Unifix cubes, place value boards

Say: The goal for place value instruction is to give students a concrete understanding of how our base ten number system works. In order to achieve this goal, students need to explore grouping in bases other than ten and examine the patterns that result. Once they understand the patterns with smaller groupings, they can understand the same patterns in base ten.

I'll model how to introduce this to students while you participate with the materials.

We are going to play a special counting game. In this game we can't say "four." We have to make up a new word that means four and that doesn't mean anything else. Who can give us a word. (Have them pick one-"glub.") Here's how we count: one, two, three, "glub." Let's count this table and make some "glubs." (Do this with several things in the room.) Let's now do this with your beans. This time, every time you make a "glub," put it in the cup. (Do a few together.)

On your place value board, the shaded side is for glubs, and the other side, on the right, is for loose objects. Each time I say "plus one," put a bean on the right side of your board.

Let's try it: "Plus one." Now there are zero glubs and one on your board. Once more: "Plus one." How many now? (zero glubs and two) "Plus one." How many? (zero glubs and three) "Plus one." Now you have enough for a glub. Put the beans in a cup and move it to the shaded side of your board. What do you have now? (1 glub and zero)

(Continue in this way up to 3 glubs and 3. Then start saying "minus one." Continue up and down.)

When doing this with students, play the same game over until they are comfortable and can anticipate what comes next. Then the students need to go through the same experience with other groupings, choosing other nonsense words for different numbers. Alternate using counters and cups, and using Unifix that get snapped together.

At the Connecting Level

At the Connecting Level, you help the students see the number patterns that evolve from counting and grouping. Play the counting games as you have, but this time record each time they report what is on their boards. Let's try it with Unifix cubes, using a nonsense word for the number five. What nonsense word should we use? (Use their word; this is written using "zurkle.") How much is on your board now? (Zero "zurkles" and zero. You record.) Plus one. How many now? (Zero "zurkle" and one. Record. Continue to four "zurkles" and four. Loop the patterns on each side of your strip.)

(On the overhead make a recording that looks like this:

0	0
0	1
0	2
0	3
0	4
1	0
1	1
1	2
1	3

With the students, on succeeding days, you would repeat this activity using other groupings and changing the material. Also do it for minus one, starting with four "zurkles" and four on the board, or three "glubs" and three.

Also, at the Connecting Level, repeat the same procedure, but record on a grid instead of in a column. Use a 4 x 4 grid for groupings by 4 a 5 x 5 grid for groupings by 5, and so on. A chart for "glubs" would look like this:

(Show transparency.)

00	01	02	03
10	11	12	13
20	21	22	23
30	31	32	33

At the Symbolic Level

Once the students understand, they can do their own recordings on strips of adding machine tape or on grids. Students should do the "Plus 1" and "Minus 1" versions with "fours," "fives," and "sixes." They can then extend their explorations to "Plus 2," "Plus 3," and so on, as well as exploring groupings of 7, 8, and 9.

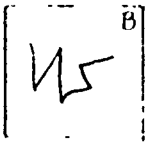
After sufficient experience with other groupings, grouping by tens becomes the focus. At this time, move quickly through the teacher-directed lessons as you did with glubs and zurkles. The students then explore the patterns in base ten by adding and subtracting by ones and recording the patterns. Then explore other patterns by adding twos, threes, fives, and so on.

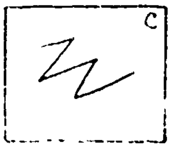
To reinforce and provide drill and practice with place value, let's do an activity called:

Measuring Lines

Materials: Task cards and Unifix cubes

Guess how long it would take to fit along the lines. Then place the cubes on the lines and group them by tens to count. Record on a worksheet.





(On an overhead, explain the worksheet.)

Guess	# of cubes
A. ____	____
B. ____	____
C. ____	____

Place Value - Model II

Materials: Base 10 blocks, FLU boards, dice

In this activity we will use base 10 blocks to introduce the concept of place value.

The first step in instruction is for students to have the opportunity to explore the materials. If possible, they should be available for students to handle and build with before starting any formal instruction. When showing students the materials, identify the names. The smallest is called a unit. Then there is a long. Ask them to predict how many units you'd have to stick together to make a long; then have them check their predictions. Show the flat and have them figure out how many longs it takes to make a flat and how many units, also.

There is a rationale for using the names unit, long, and flat at this time instead of one, ten, and hundred. The idea is to keep the emphasis on the concrete materials. If you say ten, you're likely to trigger the symbolic image of 10, the 1 and the 0. The word long, however, describes the material without the symbolic interference.

At the Concept Level

The FLU boards are place value boards that serve as organizers for the materials. Students need to know where the units, longs, and flats are to be put on the board. This is obvious to them once they're told.

Three games that help students get needed experience with grouping and exchanging are introduced at the Concept Level. They can be played with partners or in a small group. Each player needs a FLU board, access to the materials; each group needs a pair of dice.

Game #1: Race for a Flat

Each player in turn rolls the dice. The sum tells how many units they get from the central supply. They take that many units, put them on their board in the correct place, and do any exchanging they can. Then they pass the dice to the next player. The first to get a flat wins. (Stop and give them time to try this game.)

Game #2: Clear the Board

All start with 1 flat, 1 long, and 1 unit on their board. The idea is to be the first to clear the board. Again, take turns rolling the dice. The sum tells them how many units you can remove from your board. You need to clear your board exactly. If there are 7 units left and that's all, you can't win by rolling a sum larger than a 7. You must roll 7 exactly, or less. Players can elect to roll just one die if they wish. (Give them time to try this.)

Game #3: What's in the Bag?

This works best for two players. One hides some of the blocks in a bag and the other guesses what is hidden. The player who hides the blocks gives some clue; the other guesses by placing blocks on the FLU board. Here are some samples: (Make sacks for each of these.)

I have made all possible exchanges. My blocks are worth 31. (3 longs and 1 unit)

I have made all possible exchanges. I have 4 blocks, including 3 different sizes. They are worth less than 113. (1 flat, 1 long, 2 units)

I have not made all possible exchanges. I have 14 blocks. They are worth 23. (1 long and 13 units)

If students write their clues, you'll have a class set of task cards to use.

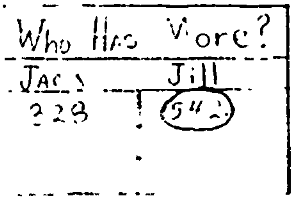
At the Connecting Level

When the students are involved with the games, no symbolism is used. The focus is entirely on exchanging with the materials. Introduce the symbolism in the context of additional activities that still involve using the materials.

The following two activities are designed to give students additional experience with place value while linking their activities to the appropriate symbolism.

Who Has More?

Two students work together. One rolls one die three times. The first tells how many flats to get; the second tells longs; the third tells units. When all have their blocks on their FLU board, they each record their number on a joint recording sheet. Then they circle which one is more.

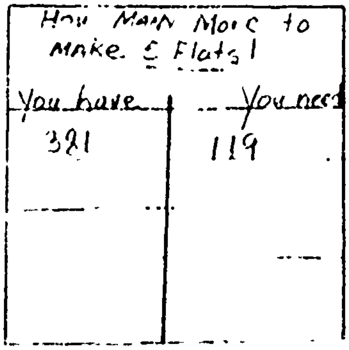


Who Has More?

Jack	Jill
328	542

How Many More?

Model this with a small group. Once they get the idea, they can work together on a worksheet. Here's a sample: Put out 3, 8, and 1 on your board. That's 3 flats, 8 longs, and 1 unit. 381. Record on the board: 381. Ask: What additional blocks would you need so you'd have exactly 5 flats on your board after you've made all exchanges possible. Notice the variation in the language. Interchanging the various forms helps to keep students flexible.



How Many More to Make 5 Flats!

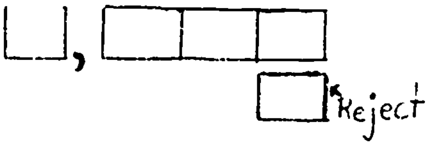
You have	You need
381	119

At the Symbolic Level

At this level, students deal with the concepts symbolically, without the use of materials. Again, games are useful ways to keep students motivated while providing the needed reinforcement. We'll try two games suitable for this purpose.

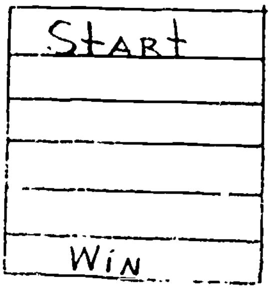
Game #1: The Largest Wins

For two players, each draws a gameboard as shown. They need a 0-9 spinner. (A die can also be used.) They take turns spinning the spinner. For each number that comes up, they each write it in one of the blanks on their gameboard. Once a number is written, it cannot be changed. They continue until they've filled in all blanks. Whoever has the larger number wins, providing he/she can read it. (Stop and play this with the class.)



Game #2: Roll to Win

For two players, each draws a gameboard as shown. They also need three dice or three 0-9 spinners. For a first turn, each player rolls or spins all three and uses any two of the numbers to make a two-digit number that he/she writes in the START box. Then on each subsequent turn, players roll or spin all three and use any two numbers to make a two-digit number that is greater than the previous one on their list. If they cannot make a number that is greater, or choose not to, they skip that turn. The first to fill in all the boxes wins.



Assessing Place Value

There are several ways to assess your students' understanding of place value. (Show transparency.)

1. Give a student 24 or so counters and ask him or her to put them into groups of 10. When they've done this, ask three questions:

    1. How many groups do you have?
    2. How many extras?
    3. How many counters are there altogether without counting them up one by one?

If the student cannot tell without counting, that indicates he or she is not making use of the grouping by 10's and needs more concept work.

2. Check to see that the student can count by 10's up to 100. Up to 150.

3. Give the student three numerals on small cards. Ask him or her to make the largest number possible from arranging these three numerals. Ask for the smallest.

4. Dictate several numbers. The ones that will pick up lack of understanding are those with zeros in them: 107,2003, 4020, and so on.

5. Show the student various three- and four-digit numbers and ask him or her to read them.

Even for older students, it's important that understanding of place value be checked. Confusion in this area can cause confusion in dealing with operations on numbers. The base 10 materials are used extensively to introduce all the operations with whole numbers as well as for decimals. Even if your students have the concepts, they will need to become familiar with the materials in order to use them effectively. The activities suggested are useful for that, regardless of their level of understanding. Students with well-developed concepts may need less work with the activities, but they should not be entirely skipped if the concepts are new to them.

ADDITION

This workshop session will be devoted to the mathematical operation of addition. Before getting into the addition process, students need to understand exactly what addition means. Addition is simply the joining, combining, or uniting of sets. Students need a lot of concrete experiences to help them understand the operation while learning how to get correct answers.

Two approaches for teaching addition will be demonstrated in this session. The first approach is from Marilyn Burns and the second approach is from Lola May. In order to model these approaches, the presentation will be made as if it were being done with students. You will play the role of the students as learners.

Materials: Place value boards, counters and cups, Unifix cubes, ganizer, and numeral cards.

The Concept Level (Marilyn Burns method)

We're going to use the place value boards and materials to solve some problems.

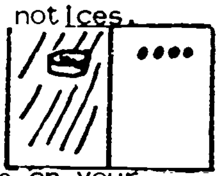
Problem:

A field trip was planned for the class. All the students were given field trip notices. On Tuesday, 14 students returned them. On Wednesday, 9 more students returned their slips. How many field trip notices were returned on these two days?

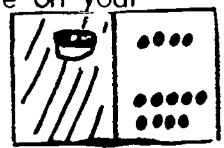
What is the equation for this problem? (Ask for a response. Record the problem on the board, then continue.)

$$\begin{array}{r}14\\+9\\\hline\end{array}$$

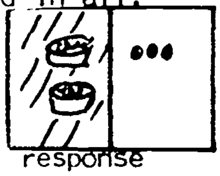
Remember that on Tuesday, 14 students returned their field trip notices. Show 14 on your place value boards.



On Wednesday, 9 more students returned their slips. Put out 9 more on your boards, but put them below the others so they don't get mixed up.



Now you need to find out how many field trip notices were returned in all. Push the materials together. How much do you have?



(Point the following out to the teachers.) You may get an initial response of 1 ten and 13 ones. If so, record that and ask the students if they have 113 on their boards. Tell them to exchange to make all the tens possible when finding out how many you have altogether when adding. Then you have 2 tens and 3 ones, and that is 23. Record that as well.

$$\cancel{\begin{array}{r}14\\+9\\\hline 113\end{array}} \qquad \begin{array}{r}14\\+9\\\hline 23\end{array}$$

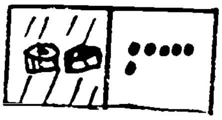
(Do another problem with the teacher.)

Students were selling raffle tickets to raise money. Gary sold 26 raffle tickets the first week. The next week, he sold 17 more. How many did Gary sell altogether?

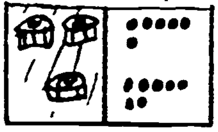
What is the equation for this problem? (Write it on the board.)

$$\begin{array}{r}26\\+17\\\hline\end{array}$$

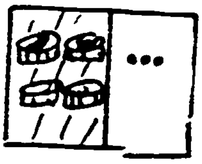
Let's solve it with our materials. Gary sold 26 raffle tickets the first week. Put 26 on your boards.



The next week, he sold 17 more. Put 17 more on your board so they don't get mixed up with the others.



What do you need to do with the materials to find out how many tickets Gary sold altogether? (Take responses: push them together or combine them or some such.) Yes, combine the materials. Do you have enough 1's to exchange for another 10? Yes. Do the exchange and see what you get. (4 tens and 3 ones) Yes, 26 plus 17 is 43. (Record the result.)



In the class, you would continue doing problems like this until all the students can follow along. Vary problems that require exchanging with those that do not. Notice that there is no attention to how to record the exchange. That is introduced at the Connecting Level. At the Concept Level, the focus is entirely on the process of the regrouping concretely, not on the symbolism that represents it.

$$\begin{array}{r}26\\+17\\\hline 43\end{array}$$

Reinforcement at this level is best done with students working in pairs to monitor each other. They take turns, one using the materials and the other writing the result. Provide this practice both with word problems and without. When giving students isolated examples to work without story contexts, have them each choose one of the examples and write a story that describes it. This way, you consistently link the process to real situations.

The Connecting Level

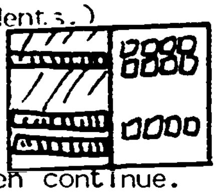
This time, I'll model how to record the symbolism for the regrouping when we solve a problem.

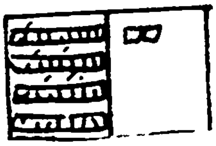
Problem:

Carrie cleaned out her toy drawer and found a sack of marbles. She dumped out the marbles and counted the solid colored ones. She found she had 18. Then she counted the others and counted 24 more. How many marbles were there in Carrie s sack?

What is the equation for this story? (Record it on the board and continue.) We'll do this with your materials, but this time I'm going to show you a way to keep track of your exchanges on your problem. Even though you can solve this easily, work along with me and see if you understand what I'm doing.

Put out the 18 to show the solid colored marbles, and then 24 underneath to show the others. (Notice how I consistently link the materials back to the problem. This reinforcement is invaluable for the students.)





Combine the materials. Wait for them to do this, then continue. Did you exchange ones to make another 10? Yes. Look at how I record that on the problem. I write the extra 10 above the other 10's. 1

How many ones do you have? (2) I'll write that here. 18

How many 10's altogether on your boards? (4) I'll write that here. + 24

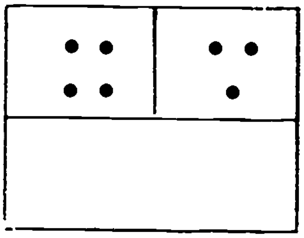
So altogether, Carrie had 42 marbles 42

Continue doing this until the students are comfortable and confident. On subsequent days, have the students write as you do so they can practice recording and check it against yours. They can practice in pairs as they did at the Concept Level.

The Symbolic Level

Continue, but have the students do the recording first for problems, and then you do it on the board so they can check. Eventually, they should be able to write independently without having you for a constant check. Again, have students work in pairs on problems so they can get support and monitor each other.

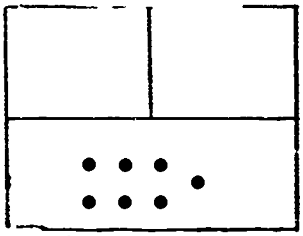
Allow the students to use the materials until they are comfortable enough to drop them on their own. You can try, from time to time, to do problems with individual or small groups of students without the materials, having them try to visualize what they would be doing on their place value boards. You'll be able to gauge from their responses if this is the appropriate time to do that. Don't rush it if it isn't easily received. Remember, let the students push the curriculum, not the other way around.



(Lola May approach to teaching addition)

Ganizer

It is helpful to use a ganizer, a frame divided into three parts. The two small spaces represent parts. The larger area represents the whole.



Each student should have a ganizer and chips or counters.

The teacher should tell a story and do the problem on the chalkboard or overhead projector as the students work with their ganizers and chips.

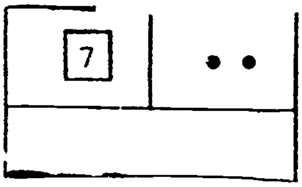
Teacher: "The balloonman has 4 red balloons." (Each student puts 4 chips in one small space, or part.)

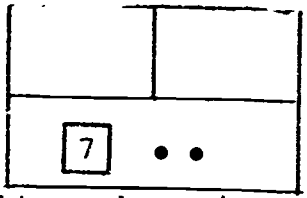
Teacher: "The balloonman has 3 blue balloons." (Each student puts 3 chips on the other small space. How many altogether?)

(Children move the chips down to the whole section and count.)

The teacher should repeat this activity many times, using as many as nine objects for the sums. During the activity the teacher needs to have the students use the ganizer to help them understand, for example, that 3 + 4 is the same as 4 + 3, that 0 + 7 is the 7, and that 7 + 0 is 7.

The next step with the frame is to have the students use both numeral cards and objects for the parts.





Teacher: "The balloonman has 7 green balloons." (Children place the 7-card.) "He has 2 red balloons." (Children place two chips.) "How many balloons does he have? Students count, beginning with 7; 7, 8, 9.

Using both numeral cards and objects allows the students to count for only one of the parts. The students start with 7, and count 8, 9, for the two objects.

After many activities using both numeral cards and objects, the teacher needs to ask:

Teacher: "If you were to solve 3 + 6 = ?, which numeral card would you use, the 6 or the 3?"

The students should know that it is easier to use the 6 card and 3 objects, rather than the 3 card and 6 objects.

As students practice the facts, they can use the ganizer frame for a model of addition.

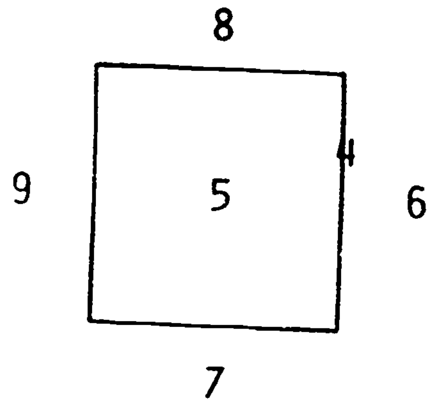
Another model for addition can be presented using Unifix cubes.

Unifix Cubes

To solve 5 + 3 = ?, the students take 5 cubes and 3 cubes, then join them to form a train of 8 cubes.

For the remainder of the session, we will participate in some motivational addition activities that provide students with mathematical practice.

The first activity is called Numbers with a Square. This activity was created by Dr. Lola May. (Show Transparency 1.)



This is an addition activity. First, we want to identify the position of each number shown on the diagram.

What number is above the square? (8) Below? (7) In? (5) On? (4)
To the right? (6) To the left? (9)

Once the students are acquainted with the position of the numbers, they are ready to begin the activity.

You must listen very carefully to the directions, then add, and then respond with the sum total.

Example: What is the sum of in plus on?

The participants should respond 9.

Example: Right, left, top?

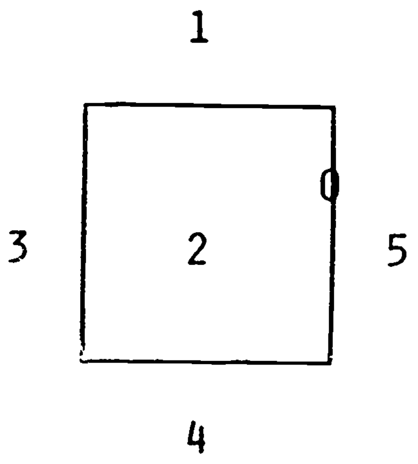
The participants should respond 23.

This activity has two objectives. One is to give students practice in following directions and the second is to give students practice in addition.

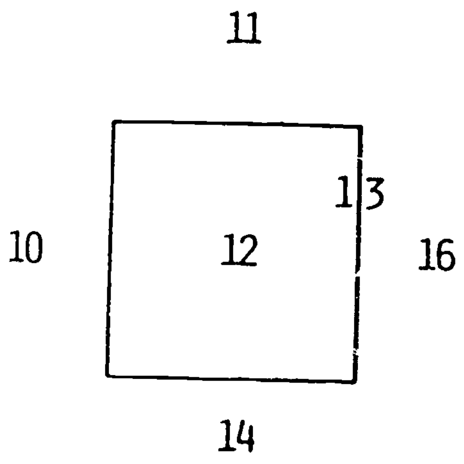
The activity can be made more difficult by changing the directions. For example: The teacher can ask, "Find the directions for the sum 20." Students must look at the numbers and decide which numbers will make 20. Some possible solutions are:

1. in, in, in, in
2. on, on, on, on, on
3. over, under, in
4. left, in, right

To give variety, the teacher can change the numbers around, in, and on the square to give practice with other combinations. For primary grades, the numbers can be smaller. For example: (Show Transparency 2.)



For the upper grades, the numbers can have two digits. For example:



Word Value Problems

There are many activities for providing students with addition practice. This section of the workshop deals with activities for finding word values. (Show Transparency 3.)

1	2	3	4	5
A	B	C	D	E
F	G	H	I	J
K	L	M	N	O
P	Q	R	S	T
U	V	W	X	Y-Z

For example, this chart may be used for students in the lower primary grades. Place the alphabet letters in columns and limit the number value of the columns to a maximum of five. Next, ask the students to find the value of words such as cat, dog, moon, etc.

When students become bored with this activity, increase the level of difficulty by asking the students to find words that are worth exactly 25 points.

The objective for this activity is to give students experience in writing numbers in columns and addition practice.

(Show Transparency 4.)

This activity provides an even greater level of difficulty because of the higher number values for the letters of the alphabet. A number is assigned to each letter of the alphabet.

Ask the participants the find the "cost" of their first name. Extend the activity by asking the students to find the cost of their middle name, last name, and entire name.

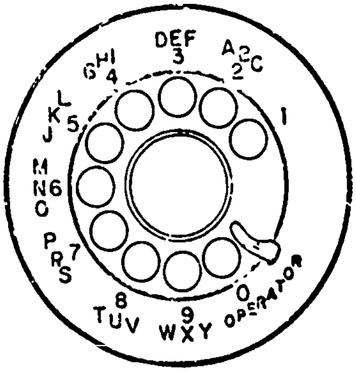
Some additional ideas that can be used with this format are:

1. Find words that have a value between 40 and 60.
2. Find the greatest score for a word that has only two vowels.
3. Find the five-letter word that has the greatest value.
4. Find the six-letter word that has the least value.

(Have the participants work in groups to find solutions to these four problems. Ask the groups to share their results.)

Some other ideas for work value activities can be found in the mathematics magazine "Arithmetic Teacher."

(Show Transparency 5.)



A telephone dial can be a very useful and motivational idea for providing students with addition practice. Again, students can be asked to find word values by finding the alphabet letter and corresponding numbers. Also, students may be asked to find words that are equal to 10, 20, or 28 points. This gives more variety and a higher level of difficulty.

Let's take a few minutes to work these two activities. (Hand-out)

Once the participants have completed the activities, discuss the possible solutions.

Money Problems

Another way to give students practice in addition is to give them problems involving money.

(Show Transparency 6.)

Look at this activity. The circle is worth 10¢, the triangle is worth 5¢, and the square is worth 2¢.

I would like for you to draw a picture that is worth 30¢, use only circles. Next, draw a picture worth 25¢ using triangles and circles. Last, draw a picture worth 35¢ using circles, triangles and squares.

(Discuss the answers with the participants.)

(Show Transparency 7.)

This is a more difficult problem that requires students to add. Let's do the first problem together. Can you name six coins that are worth 50¢? (4 dimes, 2 nickels)

(Divide the participants into groups. Write the problems on cards. Give each group two problems to solve.

Let each group share the solutions they found.)

Punch-out arithmetic is introduced as an addition activity. (Show Transparency 8.)

128	64	32	16	8	4	2	1	**Given Number**
								7
								50
								100
								110

In Punch-out Arithmetic, each student punches out the numbers on the chart that add to make a given sum, using the fewest number of punches. Each number may be used only once to show the given number.

For example, when the teacher asks a student to find the sum 7 using the numbers on the chart, the student places an X in the columns for 4, 2, and 1.

$7 = 4 + 2 + 1$

This answer meets the requirements: The student has used the fewest number of punches and has used each number only once.

The teacher should stress that the fewest number of punches must be used and that each number may be used only once, or a student might answer by using the number 1 seven times.

To make 50, start with 32, then 16, and finally, 2.

For variety, students, rather than the teacher, can give the numbers to be found on the chart. This way students discover that not just certain numbers can be made, but all numbers through 255 can be made.

To vary the activity, the teacher can change the numbers at the top. The rules must be adjusted as well.

For example, multiples of 3 may be used, with the numbers 1, 3, 9, 27, 81, and 243 placed at the top of the chart. With this variation, the same number may be used twice. One rule remains the same: Use the fewest number of punches.

Now, to make the number 7, a student places two X's in the three-box and one X in the one-box.

Another variation of the activity uses the numbers 1, 4, 16, 64, 256, and 1,024 at the top of the chart. This time, any number can be used as many as three times.

Trouble Spots in Addition

By: Lola May

1. Addition

The teacher needs to look at each student's results on a problem at the highest level of the algorithm. If the problem is done correctly, the teacher can assume that the student has mastered addition.

Consider this problem:

(Show Transparency 9.)

$$\begin{array}{r} 6\,4,\,8\,9\,7 \\ 9,\,5\,3\,6 \\ +\qquad 6\,7\,8 \\ \hline \end{array} \; 13$$

Usually, a student's first step in solving the problem will be to add 7 and 6. To do so, the student needs to know that the sum of 7 and 6 is 13.

Thus, the teacher is aware that, after students understand the meaning of addition, they must master the addition facts.

Each student needs to have an automatic response to every addition fact. If some of the facts are not under control and the student is still counting, this trouble spot must be eliminated.

A student in the sixth grade who still needs to count to obtain some of the facts is operating with a great handicap.

The addition facts must be memorized.

A fact is memorized by writing the whole fact, 7 + 6 = 13, and by saying the whole fact, "Seven plus six equals thirteen."

One hundred facts cannot be mastered at one time. Rather, the teacher should have the students start with two or three facts. After the students have learned them, they can go on to a few more.

It should be noted that there are 36 facts that have either 0 or 1 as one part, or addend. Very few students have trouble with these.

There are 14 facts that have 2 as one addend. These are also easy for students to master.

Eight more facts are doubles. Doubles seen easy for students, probably because only one number is involved.

Now the teacher needs to find out which of the remaining facts are causing difficulty for each student. Then the student must learn these facts.

Diagnosing the specific facts to be mastered and having the students memorize these facts will help the students avoid trouble in the future.

The next step in the given problem is to add 13 + 8. The <u>13</u> is not seen.

$$\begin{array}{rl} 64,897 & \searrow \\ 9,536 & \nearrow\ 13 \\ +\quad 678 & \\ \hline \end{array}$$

The learner is adding an unseen number to a seen number.

This is a trouble spot for many students. The only help the teacher can give is to offer the students many opportunities to practice mental math.

<u>Mental Math Practice</u>

Teacher: "Start with <u>2</u>, add <u>4</u>, add <u>10</u>, subtract <u>3</u>. Where are you?"

Students can respond by holding up number cards so that the teacher can see the response of each student.

Each student is given a set of <u>10</u> cards with the numbers <u>0</u> through <u>9</u> printed on them. Now, each student can use one or two hands to show one- or two-digit answers in response to the teacher's questions.

When students "tell" their answers by holding up the number cards, all the students have time to respond to the problem.

Mental math should be part of every math program. It should be practiced for at least five minutes each session, and at least twice a week.

In the example given, 13 + 8 calls for adding a two-digit number to a one-digit number without counting.

$$\begin{array}{r} 37 \\ +\ 8 \\ \hline \end{array} \qquad \begin{array}{r} 57 \\ +\ 8 \\ \hline \end{array} \qquad \begin{array}{r} 77 \\ +\ 8 \\ \hline \end{array} \qquad \begin{array}{r} 27 \\ +\ 8 \\ \hline \end{array}$$

Each of the examples above uses the basic fact, 7 + 8. In each example, the answer in the ones place is <u>5</u>. In each of the examples, the answer in the tens place place in one more than the digit in the problem.

Practice needs to be provided in which students are trained to look for the basic facts and then give the sum without counting.

This is a special step and can be a trouble spot for those who have not learned to give an automatic response.

$$\begin{array}{r} 2 \\ 64,897 \\ 9,536 \\ +\quad 678 \\ \hline 1 \end{array}$$

When the sum of a column of numbers is greater than nine, a number is carried to the next column.

This carrying process is not called renaming, regrouping, or trading. The trouble spot in carrying is that the students often carry the wrong digit.

Primary teachers need to use concrete materials and to involve the students in trading.

For example, in the problem given, after adding 7 + 6 + 8, sum 21, 20 ones are traded for 2 tens. Students now have 2 tens and 1 one. The 2 tens are placed in the column with the other tens.

Often, older students who have difficulty with mathematics have forgotten these trading experiences.

$$\begin{array}{r} 5 \\ 48 \\ +27 \\ \hline 1 \end{array} \qquad\qquad \begin{array}{r|l} \textcircled{1}\leftarrow & \\ 4 & 8 \\ +2 & 7 \\ \hline 7 & \textcircled{1}5 \end{array}$$

Common Error Remedial Work

In this example, the student may add 8 + 7 and get the sum 15, then make the mistake of carrying the 5, not the 10. This is common error.

If the teacher gives exercises in which the sum is written first, and the 1 is carried to the tens, the student learns to carry the correct number.

In adding the remaining columns in the problem, all the steps are repeated.

The need is to master each step. Then, several numbers of any size can be added correctly.

In the middle grades, teachers can use a short diagnostic test. The results can give the teacher an idea where students need help.

(Show Transparency 10.)

ADDITION DIAGNOSTIC TEST

1. $\begin{array}{r} 45 \\ +34 \\ \hline \end{array}$

2. $\begin{array}{r} 65 \\ +32 \\ \hline \end{array}$

3. $\begin{array}{r} 6 \\ 7 \\ +8 \\ \hline \end{array}$

4. $\begin{array}{r} 48 \\ +\ 9 \\ \hline \end{array}$

5. $\begin{array}{r} 37 \\ +46 \\ \hline \end{array}$

6. $\begin{array}{r} 438 \\ +357 \\ \hline \end{array}$

7. $\begin{array}{r} 297 \\ +455 \\ \hline \end{array}$

8. $\begin{array}{r} 609 \\ +297 \\ \hline \end{array}$

9. $\begin{array}{r} 4,896 \\ +3,548 \\ \hline \end{array}$

10. $\begin{array}{r} 26,159 \\ 9,476 \\ +\ \ 857 \\ \hline \end{array}$

11. Write the numbers in a column and add.

$8,176 + 39 + 946 =$

The class should have no more than 10 minutes to complete the test.

The teacher needs to look at the results of this test carefully. Much can be learned by noting <u>how</u> errors were made.

Problems	Diagnosis
1. 45 2. 65 +34 +32	Facts with sums less than 10.
3. 6 7 +8	Column addition of one-digit numbers.
4. 48 +9	Bridging, adding a two-digit and a one-digit number.
5. 37 +46	Carrying once.
6. 438 +357	Carrying once, with three-digit numbers.
7. 297 +455	Carrying twice.
8. 609 +297	Carrying twice, with zero as one of the digits.
9. 4,896 +3,548	Carrying three times.
10. 26,159 9,476 + 857	Ragged addition, not all numbers have the same number of digits, carry 4 times.
11. 8,176 + 39 + 946 = ?	Writing numbers in a column, 1's under 1's, 10's under 10's, and so on.

If there are errors on the test, the teacher can check further and provide remedial work with the focus on each student's deficiencies.

This short test also can help with grouping the students for instruction.

SUBTRACTION

The meaning of subtraction is whole, part, find the missing part. Subtraction is the inverse operation of addition and is more difficult to learn.

The subtraction facts need to be under control. Subtraction of larger numbers requires mastery of the facts. Students who need to count to get the answer to a basic fact are not ready to subtract large numbers.

Knowing the basic facts is a prerequisite of subtraction of larger numbers.

Let's look at a problem:

(Show transparency.)

$$\begin{array}{r} 4,025 \\ -1,987 \\ \hline \end{array}$$

To begin this problem, the first step is to look at the digits in the ones place. Students look and then decide if the numbers can be subtracted.

Teacher: "In the set of whole numbers, can you subtract 7 from 5?

The answer is, "No."

This is a trouble spot. Students need to practice looking at digits before they start to subtract.

To eliminate this trouble spot, exercises like this one should be practiced.

(Show transparency.)

Write N for no, if you cannot subtract.

Write Y for yes, if you can subtract.

$$\begin{array}{rrrrrrr} 0 & 8 & 2 & 7 & 9 & 3 & 2 \\ -7 & -3 & -9 & -2 & -4 & -8 & -6 \\ \hline \text{N} & \text{Y} & \text{N} & \text{Y} & \text{Y} & \text{N} & \text{N} \end{array}$$

Students should be given many opportunities to practice looking at whole numbers and deciding whether or not they can subtract.

The next step is to look at two-digit numbers. If the digits in the ones place cannot be subtracted, write N. If the digits in the ones place can be subtracted, do the whole problem.

(Show transparency.)

$$\begin{array}{rrrr} 80 & 79 & 73 & 86 \\ -27 & -23 & -49 & -32 \\ \hline \text{N} & 56 & \text{N} & 54 \end{array}$$

Remember, any subtraction problem solved without borrowing is a special case. Most problems require some borrowing.

When the numbers in the ones place cannot be subtracted, students need to borrow.

The terms renaming, regrouping, and trading are sometimes used in place of borrowing.

At the <u>Concept Level</u>

As with addition, you should start subtraction with a word problem. It is important that problems with and without regrouping be presented together so students learn to sort out when it's necessary to exchange.

Let's start with a word problem.

A school assembly was planned for all the 342 students to attend. On the day of the assembly, 25 students were absent. How many attended the assembly?

$$\begin{array}{r} 342 \\ \underline{-25} \end{array}$$

Do you need to add or subtract to solve this problem? (Subtract.) What is the equation? (Record it on the board.) We'll use our materials to solve this. Follow along with me. Put out 342 on your boards to show all the students in the school.

How many students were absent on the day of the assembly? (25) Start with the units. How many <u>units</u> does that mean you have to subtract? (5) What's the problem? (There aren't enough.) Then you'll have to exchange one of your <u>longs</u> for 10 units. Do that and stop.

How many <u>units</u> do you have now? (12) Do you have enough to subtract 5? (Yes.) Subtract them.

Look back at the equation. We subtracted 5, but the problem says to subtract 25. How much more do you need to remove? (20 or 2 longs) Can you do that? (Yes.) Then remove <u>2 longs</u>. What do you have left on your boards? (3 flats, 1 long, and 7 units) How many students attended the assembly? (317)

Notice that at this stage, you record the result. You do not symbolically represent the exchanges. You're presenting the process conceptually, not symbolically.

Do not move to the next level (Connecting Level) until students can function with ease, confidence, and accuracy.

At the Connecting Level

At this level, you connect their concrete method to the standard algorithm. You do the recording while the students work with the materials. We'll try the procedure now.

We're going to do a subtraction problem. Even though you can do this easily, I want you to do it step-by-step with me so I can show you how to record what you do as you do it. Every time you either subtract some blocks or make an exchange, you'll have to stop, and I'll show you how I record what you've done. The problem we'll try is:

$$\begin{array}{r} 312 \\ \underline{-145} \end{array}$$

Put out 312 on your boards.

Start with the units. How many do you need to subtract? (5) What's the problem? (There aren't enough.) OK, you'll have to exchange. Exchange and STOP.

Now, I'll record what you did and what the materials show now.

Do you still have two units? (No, there are 12.) Record.

Now you have enough to subtract. Subtract the five units and STOP. I'll record what you have left. (7)

Now we'll go to the longs and repeat the procedure. Remember, you STOP for me to record at two different times -- whenever you EXCHANGE and whenever you SUBTRACT.

Continue. Here's how the problem will look when you're finished:

$$\begin{array}{r} {}^{10} \\ \cancel{0}12 \\ \cancel{3}\cancel{1}2 \\ \underline{-145} \\ 167 \end{array}$$

At the Symbolic Level

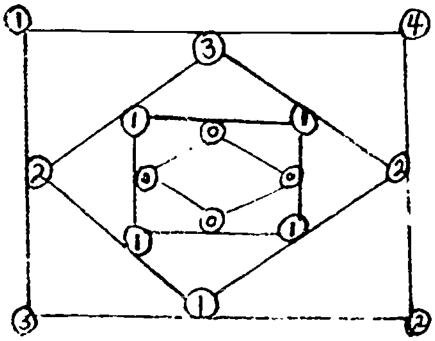
At the symbolic level, students do problems without the materials. When helping them to make this transfer, encourage them to visualize the materials and talk about the numbers in terms of units, longs, and flats. They may still need to go back and check their abstract work with the materials. Allow them to do this for verification as they need to. There is no rushing this process.

Now that we have experienced the three levels of instruction -- Concept, Connecting, and Symbolic -- let's look at some activities that provide drill and practice to reinforce subtraction skills.

Diffy

This is a subtraction game for two players. One player is the square and the other is the diamond.

1. The square starts by filling in all four circles with numbers. (This is on the outer square.)
2. The diamond inside and its circles get drawn, and the diamond player writes the difference between each of the square numbers in the circles.
3. The square player goes again, drawing a square and circles and figuring the differences.
4. A player loses when all four circles come up with zeros.



Get to Zero

This is a subtraction game for 2 or 3 players. They need 3 dice. Each player needs a sheet of paper with all the players' names and 999 under each, as shown.

1. Players take turns rolling the 3 dice. They arrange the 3 numbers that come up in any order (235,352,532, etc.) and subtract that number from 999. The others subtract also as a check.
2. The players continue subtracting, trying to get to zero.
3. First to zero wins, but they must get to zero exactly.
4. At any time, a player may choose to roll 1 or 2, instead of 3 dice. If he/she can't subtract because the number left is too small, the next player plays.

Shelby	Sandy	Helen
999	999	999

The Peculiar Number--6174

Follow this subtraction procedure:

1. Pick 4 different numbers from 0-9.
2. Arrange them to make the largest number possible.
3. Now arrange them to make the smallest number possible.
4. Subtract the small number from the large one.
5. Now, take the answer and arrange those numbers to make the largest number possible.
6. And the smallest.
7. Subtract.
8. Keep doing this until you get to 6174.

2137

7321
-1237
6084

8640
- 468
8172

8721
-1275
7443

7443
-3447
3996

9963
-3699
6264

6642
-2466
4,176

7641
-1467
6174!

For different combinations of four numbers, try to find a way to predict how many times you'll have to rearrange and subtract before you get the peculiar number.

Double and Subtract

This is a game for two players. They start with 1000.

1. The first player subtracts any number he chooses.
2. The second player may subtract up to twice what the first player subtracted.
3. The first player then can subtract up to the double of what the second player subtracted on her turn.
4. Whoever gets to zero wins.

Example:

1000
−200 Player A
800
−400 Player B
400
−400 Player A
0

What's the Difference?

This is a subtraction card game for two players. Remove the face cards from a deck. Aces are worth 1. Each student needs an equal pile of counters.

1. Place the deck face down.
2. Each student draws a card. They show one another.
3. Whoever has the larger gets the difference between his number in counters from the opponent.
4. Play continues until the deck is used up. The player with the most counters is the winner.

Students need to be taught the processes of addition and subtraction in ways in which the processes make sense. Using concrete materials gives them a way to link the processes to the physical world. Incorporating word problem instruction at the same time links the processes to situations that call for their application. Though it is important that students learn to perform the operations abstractly, it is equally important that they learn to do this with firm conceptual understanding. Learning recipes or rules for manipulating numbers has little to do with understanding the processes of these two operations.

Drill and practice will reinforce skills the students already have. It does not teach the concept. Be sure that you do not expect more from the drill than it can provide. At all times, keep in mind that the purpose of learning the algorithms for addition and subtraction is to be able to use those skills to solve problems.

MULTIPLICATION

Students should be introduced to multiplication through a variety of materials and approaches. Before opening a textbook, students should have a well-developed concept of multiplication. For older students who are still having problems with multiplication, it is important to go back to the concept development stage. Although this appears to be a waste of valuable time when you have so many other skills to cover, the student will continue to be frustrated and grow to hate math simply because the understanding of the concept was never sufficiently developed. Once the understanding of a concept is in place, building on that concept progresses much faster. Multiplication, like any other operation, should be presented in real life problem situations to give relevance, a reason, for learning to multiply.

There are many ways to introduce the concept of multiplication without attaching it to any symbolization. First, the student must learn the process in order to think through a situation and understand what is actually happening before trying to memorize facts.

Concept:

First, we present the concept using concrete objects which allows students to discover the relationship of numbers. Then we present the same types of activities and connect them to their appropriate symbolism. This should all be done before turning to the abstract or total symbolism found in a textbook. For the sake of time, I will be presenting activities at the concept level and will show you how to make the connection to the symbolic at the same time.

The most concrete way of demonstrating multiplication is through acting out situations. (Have participants dramatize problems.)

(Show transparency.)

> ____ has four students in her first reading group. She gave each student two books. How many books did she pass out?
>
> ____, ____, and ____ were playing cards. ____ dealt four cards to each student. How many cards were dealt?

When connecting this to symbols, it should be shown as both repeated addition and then as multiplication. (__+__+__+__=__;__x__=__)

Another way of introducing the multiplication concept in a concrete method is by using materials to represent problem situations. (beans, cubes, bottle caps, etc. Present problems similar to the following and have participants use the materials to represent the problems.)

> A tricycle has 3 wheels. The Smiths have 2 tricycles. How many tricycle wheels do they have?
>
> ____ had a birthday party and invited 5 friends. Each friend was given 3 goldfish to take home as a party favor. How many goldfish were given as favors?

When connecting this to symbols, the terminology of factors and products should be developed.

Robert Wirtz developed intersecting lines as a method of exploring multiplication relationships.

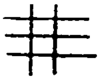
Draw 2 parallel lines on the board. Ask the students to put down 2 sticks going in the same direction.

Draw 1 line through the 2 parallel lines on the board. Ask the students to put 1 stick across the 2 sticks.

"How many intersections do you have? What is 1 times 2?"

Draw another line through the 2 parallel lines on the board. Ask the students to put another stick across the 2 sticks. "How many intersections? What is 2 times 2?"

Draw another line through the 2 parallel lines on the board. Ask the students to put another stick across the 2 sticks. "How many intersections? What is 3 times 2?"



Continue this procedure through 9 x 2, and then, using 3 sticks across, do the multiples of 3.

As an extension, with the sticks set up, ask the students to suppose another stick has been laid down. "How many intersections?"

Example:

"Suppose we put a sixth stick across the 3 sticks, how many intersections would you have? What is 6 x 3?"

Another explanation could be to describe the lines as roads that cross each other. Suppose there was a traffic light at each intersection. How many lights would there be?

Students can practice drawing the lines and marking the "traffic lights" with dots. An extension of this activity is to draw lines and cover over the dots. Allow the students to guess the number of dots.

The multiplication concept can also be modeled by using materials to build groups. This is a good activity for letting students discover the communitative property.

Instruct participants in the following way:

a. Using the materials in front of you, make 4 rows with 3 in each row. (Wait for response.) Now make 3 rows with 4 in each row.

b. Make 5 groups with 2 in each group. (Wait.) Now make 2 groups with 5 in each group.

Exercises like these allow students to discover the communitative property of multiplication. Understanding the communitative property will reduce the number of multiplication facts a student must memorize.

Building rectangular arrays is another way of modeling the multiplication concept. Have the students take 12 cubes, blocks, or tiles and arrange them in a solid rectangle. Look around the room at the various results. Point out that one has 3 on one side and 4 on the other. Another has 2 on one side and 6 on the other, or one may have 1 on one side and 12 on the other. Let them record how many different possibilities there are by drawing the arrays on graph paper.

Connecting:

After fully developing the multiplication concept, it is time to connect it to the symbolization. The recording should first be modeled by the teacher as it relates to a specific activity of concept development.

Model recording the symbolization, using the same kinds of activities as were used for developing the concept. Then pass out strips of paper with multiplication facts on them. Have the students match the written fact with problems from each of the activities. For example, given a written fact, have the student draw lines and crossing points that express the fact; have the student make up and record a story problem with the written fact; build an array, etc.

Symbolic:

In the next step, the student records the symbolization. Unfortunately, this is the step we usually begin with because of our textbooks. As you can see by developing the concept concretely you are developing understanding and not just teaching students to do something magical and mysterious to numbers.

I have demonstrated at the connecting level that multiplication is a short method of adding when the addends, or parts, are all the same in number.

It is important for students to see that if the numbers to be added are the same, then the operation of multiplication can be used. If the addends are different, only addition can be used.

An exercise which will stress this idea and help students distinguish between problems which should either be added or multiplied is as follows:

Write Y for yes if it can be multiplied or N for no if it can't be multiplied. If it can be multiplied rewrite the addition problem as a multiplication problem. (Show transparency.)

7	6	5	3	6
7	6	4	3	8
7	6	9	+3	+6
+7	+6	+3		

After the multiplication concept has been developed, then time can be spent on memorizing facts. As students memorize the facts, it is important that they repeat the fact and not just say the answer. The mastery of facts must precede the introduction of the multiplication algorithm or compound multiplication.

Drill can take place without paper and pencil. Dr. Lola May recommends each student having a set of 10 cards, each card with a number 0 through 9 printed on it. The teacher will then call out a problem. To keep some students from shouting out their answers before the others have time to respond, the teacher can have the students "tell" their answers by holding up cards with the correct response.

0	1	2	3	4
5	6	7	8	9

Another type of activity involves dividing the class into two or three teams. Each team earns points as its members find solutions to the problems.

Chanting and singing enable some students to memorize multiplication facts more easily.

Students are often overwhelmed and frustrated by the number of facts to memorize. This frustration can be eased by demonstrating the following activity using a multiplication chart. We will color code and analyze the facts to be memorized.

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

(Pass out multiplication charts.)

1. The 0, 1, 2, 5, and 10 tables are the easiest, so we'll shade those products red.

2. Since the communitative property tells us many facts we already know, shade one of every two matching facts. For example, since 3 x 6 = 6 x 3, shade the 6 x 3 box blue.

3. Since the doubles are easy, shade the ones that haven't been shaded with yellow.

Now the students can see that the actual number of facts to be memorized has been greatly reduced. The difficult facts for students are usually: 6x7, 4x8, 6x8, 3x7, 7x8, 4x7.

These six facts could easily be drilled orally throughout the school day as a sponge activity.

The nine facts are easy to learn because of several patterns or tricks. One pattern is that the digits in the products always add to 9. (Example: 6 x 9 = 54--5 + 4 = 9) (Transparency)

Another nines pattern is demonstrated as directed below:

Write 0-9 in a column going down, and then repeat, going up.

0	9
1	8
2	7
3	6
4	5
5	4
6	3
7	2
8	1
9	0

A third "trick" to aid in memorizing the nines facts is demonstrated by holding up the fingers on each hand in front of you. If the fact is 4 x 9, lower the fourth finger on your left hand. There are three fingers to the left of the lowered finger and six fingers to the right. This represents the product of 36 (4x9=36).

At the back of this packet are activities that may be used as drill and practice to aid students in memorizing the multiplication facts.

After the facts have been mastered, introduce missing factors. The language of factors and product should have been developed throughout the initial concept development stage. This skill is a prerequisite to division and also an essential prealgebra skill.

To introduce missing factors, write a multiplication fact on the board such as 3 x 9 = 27. Ask the following questions:

What is the product of 3 x 9?
How many 3's make 27?
How many 9's make 27?

Then write the example as 3 x __ = 27. Ask, "What is the missing number?" Explain that this is called a missing factor. Repeat this activity with other multiplication facts.

It is important to develop language in mathematics. Ask about missing factors in different ways such as the following:

Eight times what number is 48?
What times 3 is 24?
Six times what number is 36?
What times 5 is 45?

A multiplication chart may also be used to identify missing factors. Ask questions such as, "What number times 3 equals 12?"

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

A problem solving activity which students enjoy requires a set of nine cards or papers with the numbers 1 through 9 written on them. Pin a card on the backs of two students. Have the class multiply the factors and say the product. Have the two students look at each other's back and try to find their own factor. (Act out)

Multiplication Algorithm:

After the concept of multiplication has been developed and the facts mastered, the student is ready for the multiplication algorithm. An algorithm is a procedure for solving a mathematical problem.

The first step in teaching the multiplication algorithm is to relate it to concrete experiences and link to word problem situations before worrying about paper and pencil. In teaching the multiplication algorithm, we will follow the same Concept/Connecting/Symbolic process.

I'll begin by showing you how to solve a problem using multiplication with graph paper and Base 10 materials. (Transparency)

> Everyone in the eighth grade received 3 free circus tickets. There were 24 students in Mr. Miller's class. How many ticket were issued in the class?

What is the equation for the problem? (24x3=__? record on overhead r board.) On the grid, I will draw a rectangle 24 by 3. I will now fill in the rectangle with FLU materials, using the largest pieces I can first. Next, I will count up how much I've used to fill up the rectangle. That's how many tickets the class received altogether. The answer to the multiplication problem is called the product. The product of this problem is 72.

A concrete experience can also be used to develop understanding of a two digit multiplier.

> Mr. Smith decided to replace the floor in his family room. He bought one-foot-square tiles. His family room measured 24 feet long by 13 feet wide. How many tiles did he need to buy?

What is the equation for the problem? (Record on overhead 24 x 13 = ___) On your grid, draw in the rectangle. Fill in the rectangle with FLU materials, beginning with the largest possible. Then count up how much you've used to fill up the rectangle. (Allow students time to count and solve the problem.)

Now you are ready to connect this concrete experience to the algorithm. We will also show where the partial products come from in the work you've been doing. At the connecting level the teacher records the partial product as well as the answer.

> The center section of the school auditorium has 14 seats. There are 22 rows. How many seats are in the center section?

Draw a rectangle that is 22 by 14 on grid paper. Fill in with FLU materials as before. Split the rectangle (10 x 22; 4 x 22 as shown on transparency) and figure each side separately. Record how much is in the right-hand side first, and then how much is in the left-hand side. Add these two together for the final answer. It is easier to keep track of the problem this way rather than counting up all the blocks in the rectangle at once. This method of teaching the multiplication algorithm enables you to link it to a concrete understanding so the student can visualize the logic behind the method.

When students transfer to the symbolic level, errors still occur. One error trouble spot occurs when the student has to multiply and then add.

This error can be eliminated if students practice multiplying, then adding before they start to do compound multiplication at the symbolic level.

(3 x 7) + 5 = ____ (6 x 8) + 9 = ____ (7 x 6) + 3 = ____

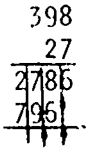
Another trouble spot occurs when recording the second partial product. (Demonstrate on overhead

$$\begin{array}{r} 398 \\ \underline{\times 27} \end{array})$$

To avoid this error, present problems in which a space is shaded under the ones column as a reminder that there are no ones in the second partial product.

$$\begin{array}{r} 398 \\ \underline{\times 27} \\ 2786 \\ \underline{796\blacksquare} \\ 10{,}746 \end{array}$$

In the initial symbolic stage, vertical lines to help record partial products are often helpful. Often errors aren't conceptual, they're just careless.



It may help to visualize partial products in yet another way.

$$\begin{array}{rl} 46 & \\ \underline{\times\ 32} & \\ & 2 \times 46 \\ \underline{\qquad} & 30 \times 46 \end{array}$$

It is always wise for students to check their work to know immediately if their answers are correct. Students do not like to remultiply, and, if they do remultiply, they could repeat their original error. To help students over this trouble spot, teachers can teach them to cast out nines to check multiplication.

Casting Out Nines

First: Practice adding digits until a one-digit number occurs.
4783 ----- 22 ---- 4
3652 ----- 16 ---- 7

It is important that the students feel comfortable with adding digits before they begin to check problems by casting out 9's.

Next: Practice casting out any digits that add to 9.

~~8~~ ~~1~~ 3 4 --- 7

The 8 and 1 add to 9. Cast them out.

Add 3 and 4. 7 is left.

~~5~~ ~~4~~ ~~3~~ ~~6~~ --- 0

The 5 and 4 add to 9. Cast them out

The 3 and 6 add to 9. Cast them out

0 is left.

~~2~~ 3 ~~7~~ 4 ~~9~~ --- 7

Cast out the 2 and 7.
Cast out the 9.

Add 3 and 4. 7 is left.

When the skills of adding digits and casting out nines is learned, students can check multiplication problems by casting out nines in the two factors and in the product.

Example:

~~4~~7~~5~~ = 7
32 = 5
15,200 = 8 35 = 8

The factor, 475, casts out to 7.

The factor, 32, casts out to 5.

Their product, 7 x 5, is 35, which casts out to 8.

The product of the problem, 15,200, casts out to 8.

After students become skilled at casting out nines, they can do it quickly and will not mind checking their multiplication problems to see if their computation is correct.

The following activities may be used to make drill and practice of multiplication facts fun and enjoyable.

Multiplication Activities:

1. <u>Buzz</u>

Choose a number from 1 to 9, say 4. They are to go around the circle counting from 1. When a number comes up that is a multiple of 4, the student whose turn it is must say "buzz" instead of the number--for example, "1, 2, 3, buzz, 5, 6, 7, buzz," and so on. If a student names the number instead of saying "buzz," he/she is eliminated from the game. Repeat the game with multiples of 2, 3, 4, 5, 6, 7, 8, and 9.

2. <u>Concentration</u> <u>Multiplication: Products to 81</u>

Players: 2

Materials: Cards in two colors, 8 of each color

Write these facts on 8 cards of the same color.		Write these products on 8 cards of the other color.	
9 x 7	8 x 7	63	56
7 x 6	8 x 6	48	42
9 x 6	9 x 8	54	72
7 x 4	8 x 3	28	24

How to Play:

One player shuffles all the cards of one color and places them, face down, in 4 rows with 2 cards in each row. Then he shuffles the other cards and places them, face down; 2 cards at the end of each row already down to form a 4 by 4 square.

The other player goes first by picking up a card of each color. If the fact and the product he picks up match he keeps the two cards and takes another turn.

If the cards do not match he places them back where they came from, and it is the other player's turn.

Play continues in this manner until all the cards are picked up.

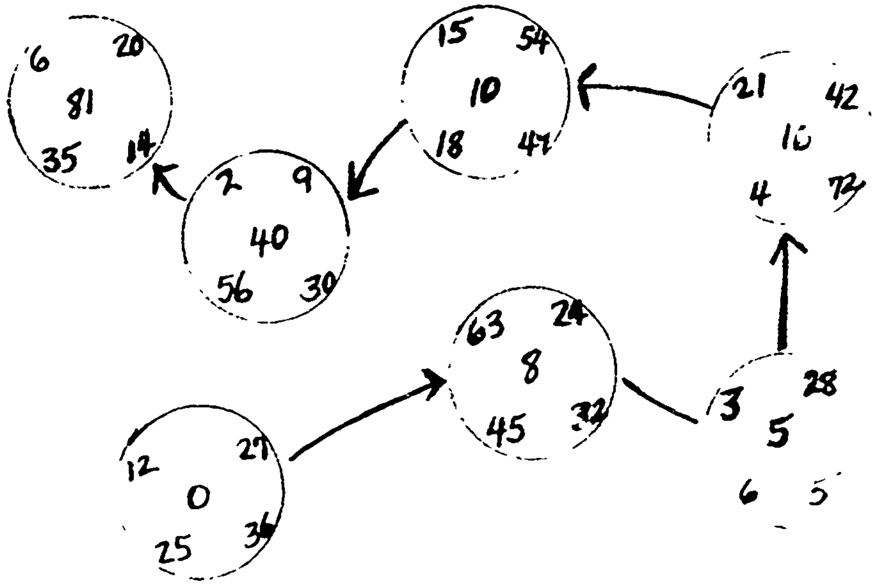
The player with the most cards wins.

3. Move Around

Multiplication: Products to 81

Players: 2 or more

Materials: Move Around board (Shown below.)
Chips- a different color for each player.
Multiplication Flash Cards



How to Play:

Cards are shuffled and placed face down in a pile. Players in turn pick a card. If the product for their facts is in the first circle they move to first circle. They move from circle to circle in this manner, only moving ahead if the product for the fact drawn is in the circle they are going to. If not, they lose a turn. The first player to get to the last circle wins.

4. Greatest Product

Find the greatest product.

Materials: digit cards 0-9.

Have the students make this pattern:

☐ ☐ ☐
× ☐ ☐

Have a student draw a digit card and decide where he/she wants the digit in the pattern. Then the second card is drawn and so on until five cards are drawn. Students then multiply. The student having the greatest product wins.

5. Two-operation Drill

SUM	12		15	17	24
PART	7	8			
PART		6			
PRODUCT			56	72	140

The objective of this activity is to practice two operations, addition and multiplication.

Students are given two numbers, and they give two numbers back. For example, students are given the sum 12 and one part, 7. To give back two numbers, they must determine the other part of the sum, or 5, and the product, 35. 7 x 5 = 35.

Students are give 8 and 6 and asked to find the sum, 14 and the product, 48.

The teacher indicates that the sum of two numbers is 15 and that the product of the same two numbers is 56. Students must find the numbers 7 and 8.

Students are given the sum 17 and the product 72 and are asked to find the two parts, 9 and 8.

Here is a more difficult challenge for students:

The sum is 24 and the product is 140. What are the two numbers?

When students have difficulty with this problem, the teacher should tell the, "The hint is in the zero. The hint is in the zero in the number 140."

This problem is for students in fourth grade and higher who have been taught that any number, multiplied by 10, has a product that ends in zero. When students see that a product ends with zero, they should know that one of the factors is 10.

A good example of a braintwister is to give the students the sum 20 and the product 36. The students have to think about the factors of 36 to get one large number and one small number.

What two numbers add to 20 and have a product of 36?

The answer is: 18 and 2

When students get this answer, they usually consider it quite a victory because the problem is a difficult one. They have met the challenge. They also have met the objectives of the activity. They have practiced addition and multiplication.

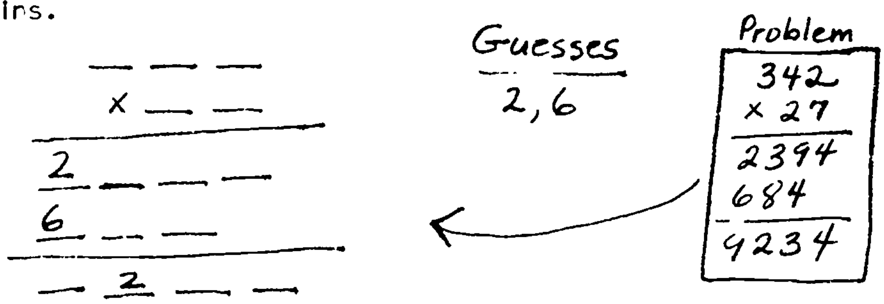
6. The Greatest Wins

This game is for two or more players. They need four dice. The object of the game is to get the largest product possible. Players take turns rolling the four dice. They each use the four numbers they individually rolled to make a multiplication problem. Their problem can either be a 2-digit times a 2-digit, or a 3-digit times a 1-digit. Winner of that round is the person who gets the largest product from his dice.

Extension: Given any four numbers, devise a strategy that will consistently give you the largest product.

7. Multiplication Hangman

This is a two-person game. One player makes up a multiplication problem and draws it in blank form for the opponent. That person guesses numbers. When a correct number is guessed, the player who made up the problem writes it wherever it goes in the problem. The opponent may add other numbers he likes without asking. The person with the problem does not tell him if his extra numbers are correct or not, but just responds to guesses. Score how many guesses he had to make before he completed the entire problem correctly. Low score wins.



8. The Game of 110

This is a card game similar to blackjack, but involving multiplication as well. The idea of the game is to get as close as possible to 110 without going over it. It appeared in the March, 1981, issue of The Mathematics Teacher. It can be played by a group of four using an ordinary deck of playing cards. All cards are kept face up during play so all check one anothers' scores.

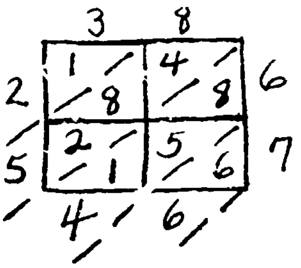
1. Two cards are dealt faceup to each player.
2. Players multiply the values of their cards. (Picture cards are worth 10.)
3. If a student elects another card (a hit), the value of this card is added to the score.
4. If a student elects another hit, this value becomes a multiplier for the last card selected.
5. If a student elects another hit, this value is added to the previous total. Steps 4 and 5 are repeated for further hits.

Example:

1. A student is dealt a 7 and a K.
2. Her hand is worth 70. (7x10)
3. She elects a hit and is dealt another 7. Her hand is now worth 77.
4. She elects another hit and is dealt a 4. She recomputes her score: 7x10 stays the same for the first two cards and 7x4 is her score for the next two. Her total is 70 + 28 or 98.
5. She gets another hit, this time a 9. Her score is now 98 + 9, or 107. She stops there because the next card will have to be multiplied by the 9 and then added to the 98, and will go over 110.

9. Lattice Multiplication

```
  38
x 67
2546
```



Add on the diagonal

```
   607
  x 49
29,743
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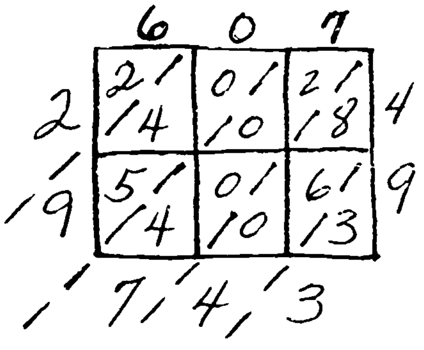


10. Race to 23,000

Do ten problems using the digits 0-9. See how close you can come to 23,000.

DIVISION

Before teaching the division algorithm, the concept of division must be introduced and internalized by the student. We enable this to happen by presenting real-life problem situations, acting them out and/or using concrete objects. Word problems that involve division can be introduced even in the primary grades as long as the concept is introduced through concrete materials with no numerical symbolization.

After presenting ways to introduce the concept at both primary and intermediate grades, I will show you a method for teaching the division algorithm, using the Concept/Connecting/Symbolic approach.

Division can be separated into two types of situations. The first situation occurs when you divide objects into several groups to find out how many are in each group. (Example: If you have a dozen eggs and six people eating breakfast, how many eggs can each person have?)

The other division situation occurs when you make certain sized groups of objects and find out how many groups result. (Example: If each taxicab can seat five people and there are 20 people waiting for a cab, how many taxicabs will be needed to accommodate all the people?)

Concept/Connecting/Symbolic:

Introducing division through word problems in situations that use both the students and props found in the classroom is a good starting point. The following are examples of this at both the primary and intermediate levels and using both types of division. (Transparency)

1(P). ______ has 8 books. She puts them into 4 piles of the same size. How many books are there in each pile? (An example of how many in each group)

2(P). The teacher gives 12 crayons to _____. He is told to give 3 crayons to each student. How many students receive crayons? (An example of how many groups.)

1(I). There are 12 pencils on the teacher's desk to be given to honor roll students. If there are 4 students on the honor roll, how many pencils will each student receive? (How many in each group?)

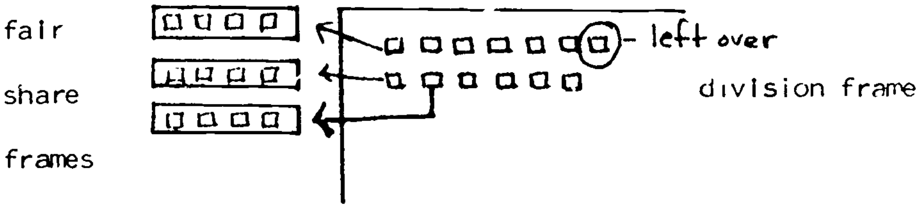
2(I). The Mother's Club contributed a box of 48 cookies to the class. If each student received 2 cookies, how many students received cookies? (How many groups?)

Students need to understand that most of the time the answer to a division problem does not come out even. When it does not come out even, there is a whole number and a part left over which is called the remainder. It's important to develop the vocabulary as you develop the concept.

The students should be exposed to many problem solving situations which may be acted out. An alternate method is the use of concrete objects to show the problems. Sample problems are as follows: (Transparency)

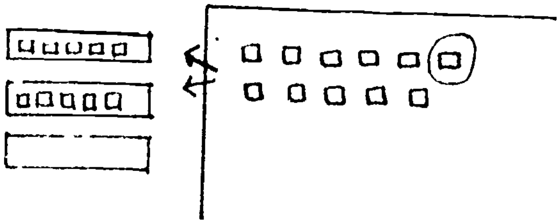
> ____, ____, ____, ____, ____, and ____ are frogs in a pond. There are 3 lily pads in the pond. The same number of frogs climb on each pad. How many frogs are on each pad?
>
> Twelve students are lined up to go on the ferris wheel. Three can fit in each car. How many cars will they fill up?
>
> I have 11 pieces of candy to share with my best friend. If we each get the same number of pieces of candy, how many will we each get?
>
> The secretary gave you 15 pencils and asked you to put them in groups of 4. How many groups will you have?
>
> Our class is planning a field trip. Parents will provide transportation in cars. Each car can carry four students. How many cars will be needed to carry 28 students?
>
> The library was giving away 40 old books that were no longer needed. A student could choose 3 books. If each student chose 3 books, how many students received free books?

Division frames are another way of developing the division concept. (Show transparency. Have participants draw a division frame and fair share frames. Use beans, cubes, or bottle tops for shares.)

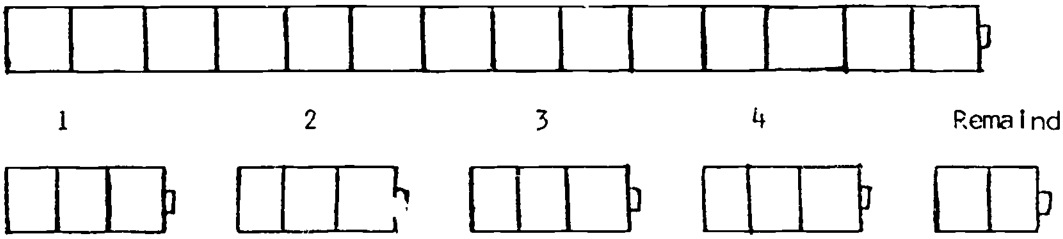


Beginning with 13 objects inside the frame, take the objects and make 3 fair shares. After the student has moved 12 objects, he will have 3 fair shares with 4 in each share and 1 left over which is called the remainder, only a part of a fair share. (An example of how many in each group)

To demonstrate the other type of division situation with division frames, tell the students to put 11 objects in their division frame. Now tell them to place 5 objects in each set. How many fair shares do they have? How many are left over?



Division can also be shown as repeated subtraction. For example, 14 divided by 3. Students can use unifix cubes to show a rod of 14. They can take off cubes in sets of 3. Then they find they have 4 sets of 3 with 2 left over or a remainder of 2. This problem demonstrates the division situation of how many groups there are.



After students have developed an understanding of the division concept, the teacher should begin making the connection to numerical symbolization using the same kinds of problem-solving situations we have used at the conceptual level.

The teacher should model both ways to record the problem mathematically.

$$14 \div 3 \qquad 3\overline{)14}$$

When students are ready to do their own recording, give them a division equation and allow them to write their own problem situations. (Write an equation on the overhead $5\overline{)24}$ $24 \div 5$ and allow participants to write a problem to go with it.) Then have them choose their own equations and write problem situations for each other to solve. These can then be maintained as a set of division task cards.

Students should continue to be allowed to solve problems using concrete or semi-concrete objects even when they're functioning at the symbolic level.

After moving through the connecting and symbolic stages using the division frames, students should be able to find a missing number in division by re-creating the action they have performed on the division frame.

Examples:

$$\begin{array}{r} 5\ r\ 2 \\ \underline{\textcircled{3}}\overline{)17} \end{array} \qquad \begin{array}{r} \textcircled{6}\ r\ 3 \\ 4\overline{)27} \end{array} \qquad \begin{array}{r} 3\ r\ 2 \\ 6\overline{)\underline{\textcircled{20}}} \end{array}$$

Find the missing numbers by using division frames.

When students find the missing quotient, divisor, or dividend, they show they really understand the process of division.

Once the division concept is fully developed but before the algorithm is introduced, students should understand the relationship between multiplication and division. It is important that students see that they can use what they know about multiplication to help solve division problems.

Ask students to write a multiplication equation to solve the following problem. (Transparency)

> There are 8 students in the group. Each student needs 4 sheets of paper. How many sheets of paper does the group need?

Now use the same situation to pose a different but related problem:

> Suppose I had 32 sheets of paper and wanted each student to have 4 sheets. How many students do I have enough paper for?

A division equation fits this situation: $32 \div 4 = \underline{\ ?\ }$ Ask: How can I use what I know about multiplication to help me solve this division equation?

Students need many of these word problem experiences for them to see the relationship between the two operations.

Students need to feel comfortable with the symbolic expression of the relationship between multiplication and division. For every division or multiplication equation, there are three other related equations. (Show transparency.)

$21 \div 3 = 7$
$21 \div 7 = 3$
$7 \times 3 = 21$
$3 \times 7 = 21$

Demonstrate using their knowledge about relationships to solve the following problem:

> If I have 21 cookies and would like to give each person 3 cookies, how many people will get cookies?

$3\overline{)21}$ or $21 \div 3 = \underline{\qquad}$

If I didn't know the answer and wanted to solve it without the use of materials, I could use what I know about multiplication. What is the related multiplication equation? $3 \times \underline{\quad} = 21$

Sometimes writing out a multiplication table will help.

3 x 1 person = 3 cookies
3 x 2 persons = 6 cookies
3 x 3 persons = 9 cookies
3 x 4 persons = 12 cookies
3 x 5 persons = 15 cookies
3 x 6 persons = 18 cookies
3 x 7 persons = 21 cookies

These are all examples of various kinds of activities to which a student should be exposed before you teach the standard division algorithm.

Remember, there are various rates of learning and styles of learning. By presenting concepts in several ways, you're attempting to accommodate these.

Once the student has developed an understanding of the concept, it is time to introduce the algorithm using manipulatives. As in multiplication, we will teach the algorithm using the Concept/Connecting/Symbolic process.

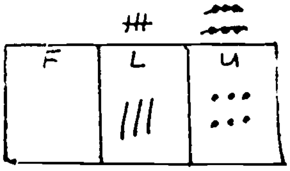
Division Algorithm:

Concept:

Once again, I will use a problem-solving situation using Base 10 materials. Initially, at the conceptual level, we will only <u>record the results</u> and <u>not the process</u>.

> The class was going on a hike. There were 36 students in the class. The teacher asked them to get into groups of 3 so she could keep track of them more easily. How many groups were there?

This problem can be written mathematically as $3\overline{)36}$. I'll show you how to use the FLU materials to find the answer. Put 36 on your board.



The problem is to put 36 into groups of 3. Start with the longs. Make a group of 3 longs, and put the group up above the board. How many groups did you put up there? (1) Notice where I write that.

$$\begin{array}{r} 1 \\ 3\overline{)36} \end{array}$$

Now move to the units. Make a group of 3 and move it above the board. You also have enough to make another group of 3; move that above the board also. How many groups do you have? (2)

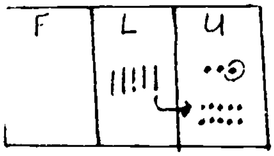
$$\begin{array}{r} 12 \\ 3\overline{)36} \end{array}$$

The solution is 12. That means there is 1 group of longs and 2 groups of units in 36 when we group by 3s.

Sometimes exchanging is necessary to solve a problem:

Let's try another problem: $2\overline{)53}$ Put 53 on your board.

Group the longs into groups of 2. How many groups? (2) Notice where I write that.



There is a long left over. It's not enough to make another group, but you can exchange it for units and then group them. How many units can you exchange a long for? (10) Do that and stop.

$$\begin{array}{r} 2 \\ 2\overline{)53} \end{array}$$

How many units do you have altogether? (13) Group them into 2s. How many groups? (6) I'll record that in the problem.

$$\begin{array}{r} 26 \\ 2\overline{)53} \end{array}$$

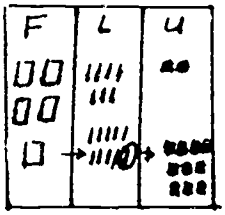
You still have a unit left on your board. That isn't enough to make another group. It's called the remainder. Notice how I record that: 1/2. This shows that there was 1 left, and we were grouping by 2s.

$$\begin{array}{r} 26\ r\,\frac{1}{2} \\ 2\overline{)53} \end{array}$$

Problems with a three-digit dividend are done with the same procedure:

> At a banquet, guests were seated 4 to a table. There were 572 guests in all. How many tables were needed?

This problem can be written as $4\overline{)572}$. Put 572 on your boards.



Start with the flats. Make a group of 4 and move them above your board. How many groups? (1)

Notice you still have a flat left on your board. That isn't enough to make another group of 4, so you'll have to exchange that for longs. Do that.

$$\begin{array}{r} 1 \\ 4\overline{)572} \end{array}$$

Now on to the longs. How many groups? (4) I'll record that.

$$\begin{array}{r} 14 \\ 4\overline{)572} \end{array}$$

Notice there's a long left over. You can't just leave it there, and there aren't enough longs to make another group. Exchange it for units. How many units can you exchange the long for? (10) Do that; exchange and stor

$$\begin{array}{r} 143 \\ 4\overline{)572} \end{array}$$

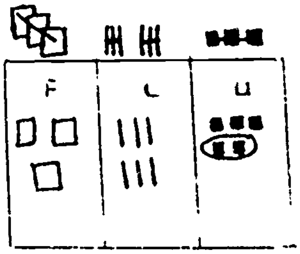
Now group the units into 4's. How many groups? (3) Any remainder? (No) So 572 divided by 4 gives a result of 143.

Once you feel that the students understand this, have them work in pairs for reinforcement at the Concept Level. It's sufficient for each of the pairs to do a total of six problems a day for practice, alternating who does it with the materials, and who records the answer. Be careful with the problems you choose. If you limit them so there is only 1 block to exchange, you're not as likely to run into a materials problem.

Connecting:

At this stage, you begin to connect what the students are doing concretely with the standard algorithm. Begin this stage only when the students are truly confident, even bored, with the work at the Concept Level. Don't push it; you'll only have to reteach later. Remember, the student should set the pace of the curriculum, not the other way around.

I'm going to give you a problem that is easy for you to do now. But I want you to do it step-by-step with me because I want to add something new--a way to show more about what you're doing with the blocks in the written problem.



There are 365 days in a calendar year. If the year was divided into 3 equal parts, how many days would be in each third?

Start by putting 365 on your board. Here's the new twist. Whenever you put some blocks into groups and move them above your board, you have to STOP so I can record what you did. Start with the flats. Group them into 3s and STOP.

Now I'm going to record what you did by asking you three questions. You answer each of the questions and watch how I record each answer.

① ←Question #1
3|365
300 ←Question #2
65 ← Question #3

Question #1: How many groups? (1)

Question #2: How much did you put up? That means how much you just moved up above your board. Flats first. How many flats did you move up? (3) Longs? (0) Units? (0)

Question #3: How much is left on your board? Flats? (0) Longs? (6) Units? (5)

OK, you've finished with the flats and you're ready to go on. Do you need to exchange? (No) Then group the longs into 3s and STOP so I can record the answers to the 3 questions.

1② ←Question #1
3|365
300
65
60 ← Question #2
5 ← Question #3

Question #1: How many groups? (2)

Question #2: How much did you put up? Longs? (6) Units (0) Since we were done with the flats, I didn't have to check on them again.

Question #3: How much is left on your board? Longs? (0) Units? (5)

Any leftovers to exchange? (No) Then on to the units. Group and STOP.

Question #1: How many groups? (1)

Question #2: How much did you put up? (3 units)

12① ← Question #1
3|365
300
65
60
5
3 ← Question #2
2 ← Question #3

Question #3: <u>How much is left on your board?</u>
(2 units)

Record your remainder. That's it.

At this point, several examples will be necessary. Do as many as you think are needed for the students to be able to do it in pairs, independently of you. It helps to write out these three questions so they can refer to them during this reinforcement time.

<u>Symbolic</u>

After the students can confidently, easily, and accurately do the recording with their partners, it's time to move on to the Symbolic Level. At this level, the students go through the same procedure and language, but try doing it without the actual manipulatives. Do a problem with them writing on a chalkboard. Encourage them to visualize the blocks and boards as you're writing and talking.

Before students finish their work with one-digit divisors, they need to lea short division.

Short division requires mental work. Students do multiplication and subtaction without writing down the numbers.

$$\begin{array}{r} 9\;6\;3 \text{ r } \frac{5}{6} \\ 6\overline{)5\;7^{3}8^{2}3} \end{array}$$

In this example, <u>54</u>, 6 x 9, is subtracted from <u>57</u>. The <u>3</u> is written in front of the <u>8</u>.

Next, 6 x 6 = 36. The <u>36</u> is subtracted from <u>38</u>, and the <u>2</u> is written in front of the <u>3</u>.

Then, 3 x 6 = 18. The <u>18</u> is subtracted from <u>23</u>. The remainder is <u>5</u>.

The next step is teaching how to divide with a two-digit divisor. Students need to master division by multiples of ten before they work with other two-digit divisors.

The reason for working with multiples of ten is that it is almost as easy to multiply by <u>40</u> as it is to multiply by <u>4</u>.

Dividing by other two-digit numbers should not be taught until students have grasped the principles of dividing by multiples of ten. If this step is mastered, it should help the student avoid trouble with two-digit divisors. Many of these types of problems can be used throughout the day as sponge activities. Mental multiplication requires practice.

The hidden prerequisite in division with two-digit divisors is mental multiplication.

Students need to learn to multiply a two-digit number by a one-digit number mentally before they start division with two-digit divisors. Mentally multiply 23 x 7.

How did you multiply 23 x 7? (Allow time for brief discussion.)

One might think of 23 as 20 and 3, then multiply 3 x 7. Hold that product, 21, mentally. Then, multiply 20 x 7, add that product, 140, to the first product, 21, for a total of 161.

$$\begin{array}{rcrcr} 23 & \quad & 20 & + & 3 \\ \underline{x7} & & \underline{x7} & & \underline{x7} \end{array}$$

To save students from the struggle and frustration of working with larger numbers in division, the process can be broken down into steps.

Exercises like these will help.

Students are asked to write the first digit in the quotient. Then they write the number of digits in the quotient. It helps to have the starting number.

$$\begin{array}{r} 3xx \\ 27\overline{)9346} \end{array}$$

Find the beginning digit. (3) Now find how many digits will be in the quotient. (3)

$$\begin{array}{r} 7xxx \\ 18\overline{)137,864} \end{array}$$

The first digit is 7.
There are 4 digits in all.

Find the beginning digit and the number of digits in the quotient.

The procedure for teaching the algorithm of problems with two-place divisors is exactly the same as with one place divisors. If you have a sufficient amount of materials, you can do any problem. However, the difficulty for students when the numbers are larger often is their lack of familiarity with the numbers, not the process. What is important is that they have a firm conceptual base with simpler problems and are introduced to longer ones at the symbolic level when their number facts and understandings can handle it. This requires your judgment.

One recommendation for two-digit divisors is to have them work out the "facts" for that number first. An example is as follows: (Show transparency.)

$13\overline{)1987}$

13	13	13	13	13	13	13	13	13
x1	x2	x3	x4	x5	x6	x7	x8	x9
13	26	39	52	65	78	91	104	117

Checking Division:

There are two ways for students to check to see if division problems are correct. Casting out nines is one method of checking. Also, division can be checked the long way by multiplying the divisor times the quotient, then adding the remainder.

Examples of these two methods follow:

To check the problem by casting out nines, first cast out nines in each factor, multiply the results, and add the remainder. Then cast out the nines in that sum. Now, cast out the nines in the dividend. The two numbers should be the same.

$$\begin{array}{r} 152 \text{ r } 11 \\ 13\overline{)1987} \end{array}$$

Factors

$$\begin{array}{rr} 152 \rightarrow & 8 \\ 13 \rightarrow & \times 4 \\ \hline & 32 \\ & +11 \\ \hline & 43 \rightarrow ⑦ \end{array}$$

Dividend

$1\cancel{9}\cancel{8}7 \rightarrow ⑦$

In the following example, the problem is checked the long way. The students multiply 13 times 152, then add the remainder of 11.

$$\begin{array}{r} 152 \\ \times 13 \\ \hline 456 \\ 152 \\ \hline 1976 \\ +11 \\ \hline 1987 \end{array}$$

When developing any concept, it's important that students experience the concept in a variety of ways. Even though they have reached the symbolic level, it helps to return to some concrete experiences from time to time so they can reexperience that in light of their new understandings. Our overall goal is for students to become confident calculators as well as understanding the use of the operations in a variety of situations.

Division Activities

Game #1 Concentration--Make a Ten Division: Facts to Products of 81

Players: 2

Materials: 16 division cards as shown below.

27 ÷ 3	8 ÷ 8	24 ÷ 6	12 ÷ 3
42 ÷ 6	21 ÷ 7	10 ÷ 5	56 ÷ 7
63 ÷ 9	18 ÷ 6	54 ÷ 9	28 ÷ 7
48 ÷ 8	36 ÷ 9	72 ÷ 9	14 ÷ 7

How to Play:

One player shuffles the cards and places them face down if 4 rows with 4 cards in each row. The other player picks up 2 cards. If the sum of the answers for the 2 cards is 10, the player keeps the cards and takes another turn. If the answers do not add up to 10, the cards are placed face down, in the same places they came from. The next player takes 2 cards and plays in the same manner. Play continues until all the cards are used. The person with the most cards at the end is the winner.

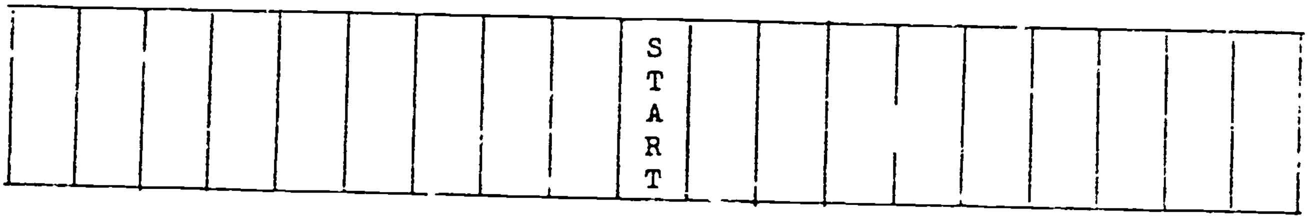
Variation:

Make up a set of 16 cards whose answers will pair off to make a sum of 12.

Game #2 Tug of War Division: Facts to Products of 81

Players: 2

Materials: Tug of War Board



Game #3 Bump It

Players: 2

Materials: Chips--two different colors, one color for each player.
Game Board (Shown below)
A set of 7 cards for each of the numbers 4, 5, 6, 7 and 8.

42 - 7	24 + 3	36 - 9	30 + 5	30 - 6
20 + 4	54 - 9	56 + 8	12 - 3	42 - 6
[illegible]	35 [illegible]	48 - 8	[illegible]5 - 7	49 - 7
32 - 8	72 + 9	15 - 3	36 + 6	63 - 9
64 + 8	28 + 7	48 - 6	45 - 9	56 7

How to Play:

One player shuffles the number cards and places them face down in a pile. Each player, in turn, picks a card and places a chip on a fact that has that answer. If a player draws an answer that has a fact covered by the other player, he can bump that chip off the board and put his own chip on instead. Keep playing until the board is covered with chips. The player with the most chips on the board wins.

Game #4 Krypto

Chosen Numbers	Target Number
3 7 11 14 15	32

The game Krypto was invented in California. It provides an opportunity for students working in teams to use the mathematical operations they have been taught to meet the challenge of problem solving.

In Krypto, students in the class call out five numbers from 1 through 20. In the example above, the chosen numbers are 3, 7, 11, 14, and 15.

The teacher puts these numbers in a frame on the chalkboard.

After the five numbers are chosen, a sixth student calls out any number from 21 through 50. This is the target number. In the example above, the target number is 32.

The teacher writes the target number on the chalkboard. The class is then divided into teams.

Students may use any operation--addition, subtraction, multiplication, and division. They may use square root or factorial if they know how. They can use any operation they have been taught.

The goal is to get the target number, using three or more of the chosen numbers.

The teams earn points as follows:

Three chosen numbers--100 points

Four chosen numbers--500 points

Five chosen numbers--1000 points

Each chosen number can be used only once. students cannot add 3 + 3 to get 6, because only one 3 can be used.

Let's consider the chosen numbers 3, 7, 11, 14, and 15 and the target number 32.

Three chosen numbers may be used to earn 100 points: 11 + 7 + 14 = 32. Many students will get this answer, worth 100 points for their team.

Some students will work hard to use more than three chosen numbers. Some may come up with 3 x 7 + 11 x (15-14).

Let's take a look at these choices:

First, 3 x 7 = 21
Now, 21 + 11 = 32

Next, the numbers in parentheses: 15- 14 = 1. The students will use this 1 to multiply by 32.

All five numbers have been used.

This activity can continue for some time, and many solutions can be found. Much variety is provided because of the children's selection of numbers.

Before students start this activity, the teacher should tell the class:

> "When numbers are selected at random, as you students will do when you choose five numbers and the target number, computers have shown that in 86 percent of the cases, using only the operations of addition, subtraction, multiplication, and division, all five numbers can be used to get the target number."

It is important that the students understand this so that, after a minute or two, if the target number is not found using all five numbers, they won't give up too easily.

After all, a chance to earn 1000 points is worth working for.

Sometimes the chosen numbers are difficult to work with, and it takes a long time to find many solutions.

When this activity is presented five or ten minutes before the end of a period, students can keep trying to find solutions for homework. They can bring in their solutions the next day, and the scores can be added to the team score.

Another way to present the activity is to limit the time given, taking only the solutions found in that time for the teams' scores.

Game #5 Baby Krypto

Another version of this activity is Baby Krypto, an excellent activity for students in the second, third, and fourth grades.

Chosen Numbers	Target Number
2 3 4 7 9	14

The rules are the same: The students call out five numbers, then a target number, which the teacher writes on the chalkboard.

In Baby Krypto, the chosen numbers are from 1 through 10, and the target number is from 11 through 20.

Again, the class is divided into teams. The scoring is the same, 100 points for three chosen numbers, 500 points for four chosen numbers, and 1000 for all five chosen numbers.

Students can use addition, subtraction, multiplication and division. If the students do not know how to multiply and divide, they use addition and subtraction only.

Three chosen numbers: 3 + 4 + 7 = 14

This solution is worth 100 points.

Four chosen numbers: 9 - 4 + 7 + 2 =14

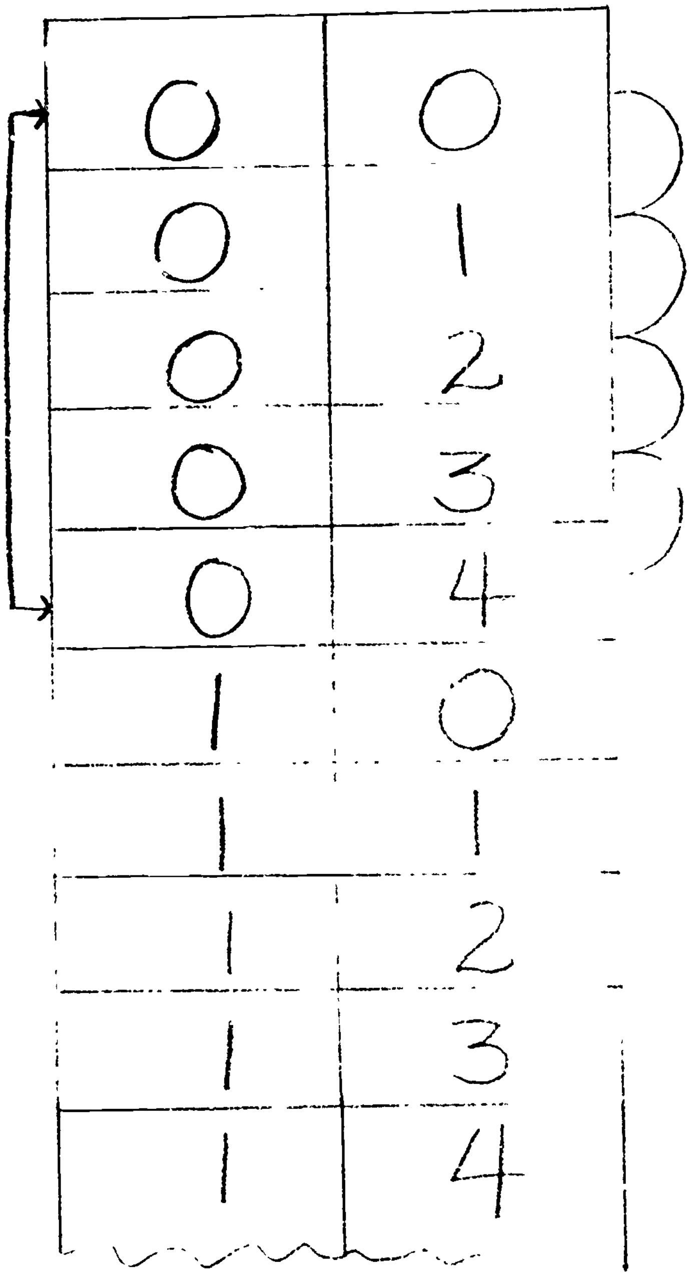
This is a 500 point solution.

Perhaps someone will think of a solution using five chosen numbers: 4 7 + 9 - (2 x 3) = 14.

This five-chosen-number solution is worth 1000 points.

With Baby Krypto, computers show that a five-number solution can be found in more than 80 percent of the cases. The teacher should explain this to the students to encourage them to keep trying for a five-chosen-number solution.

TRANSPARENCIES



0	0
0	1
0	2
0	3
0	4
1	0
1	1
1	2
1	3
1	4

00	01	02	03
10	11	12	13
20	21	22	23
30	31	32	33

PV-T7

Guess	# of cubes
A.	
B.	
C.	
D.	

1. GIVE A STUDENT 24 OR SO COUNTERS AND ASK HIM OR HER TO PUT THEM INTO GROUPS OF 10. WHEN THEY'VE DONE THIS, ASK THREE QUESTIONS:

   1. HOW MANY GROUPS DO YOU HAVE?
   2. HOW MANY EXTRAS?
   3. HOW MANY COUNTERS ARE THERE ALTOGETHER WITHOUT COUNTING THEM UP ONE BY ONE?

IF THE STUDENT CANNOT TELL WITHOUT COUNTING, THAT INDICATES HE OR SHE IS NOT MAKING USE OF THE GROUPING BY 10'S AND NEEDS MORE CONCEPT WORK.

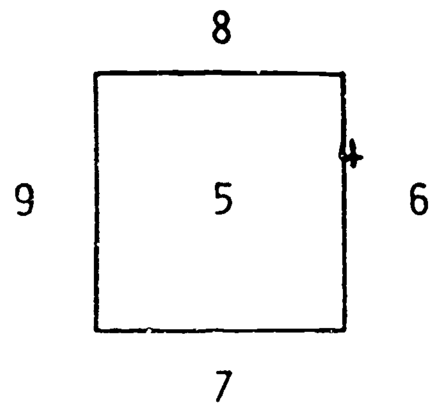
2. CHECK TO SEE THAT THE STUDENT CAN COUNT BY 10'S UP TO 100. UP TO 150.

3. GIVE THE STUDENT THREE NUMERALS ON SMALL CARDS. ASK HIM OR HER TO MAKE THE LARGEST NUMBER POSSIBLE FROM ARRANGING THESE THREE NUMERALS. ASK FOR THE SMALLEST.

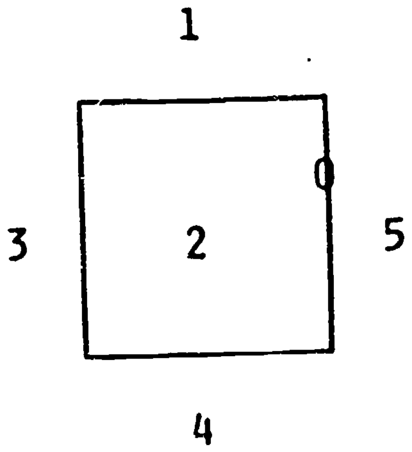
4. DICTATE SEVERAL NUMBERS. THE ONES THAT WILL PICK UP LACK OF UNDERSTANDING ARE THOSE WITH ZEROS IN THEM: 107, 2003, 4020, AND SO ON.

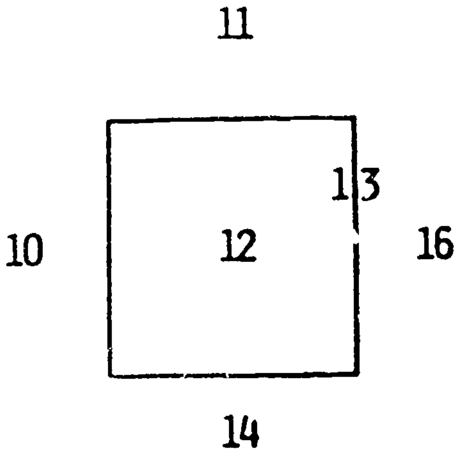
5. SHOW THE STUDENT VARIOUS THREE- AND FOUR-DIGIT NUMBERS AND ASK HIM OR HER TO READ THEM.

PV-TA


8
9
5
4
6
7

PRIMARY





A - T2

PRIMARY

1	2	3	4	5
A	B	C	D	E
F	G	H	I	J
K	L	M	N	O
P	Q	R	S	T
U	V	W	X	Y-Z

BALLOON

BOAT

DOG

COLOR

A-T3

UPPER

Value of Words

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13

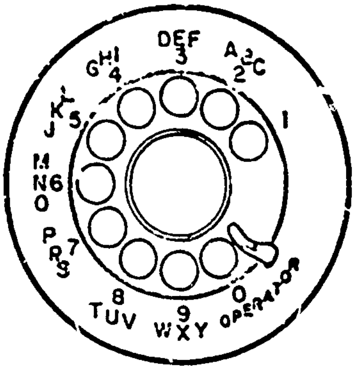
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
14	15	16	17	18	19	20	21	22	23	24	25	26

1. Find words that have a value between 40 and 60 points.

2. Find the greatest score for a word that has only two vowels.

3. Find the five-letter word that has the greatest value.

4. Find the six-letter word that has the least value.



If you add the value of each letter (M= 6, A=2, and so on), then MATHEMATICS is worth 52.

Find these:

APPLE =____

NUMBER =____

ADDITION =____

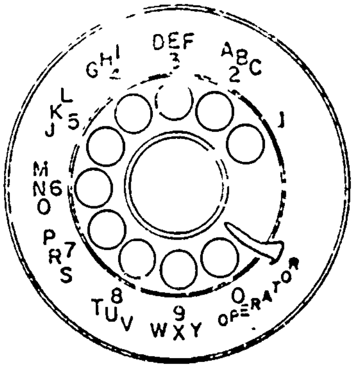
SCHOOL =____

RECESS =____

SUNSHINE =____

Write your first name below and find its value.

10 20 28



A-T5

Money

Circles, 10¢

Triangles, 5¢

Squares, 2¢

1. Draw a picture worth 30¢.
   Use only circles.

2. Draw a picture worth 25¢.
   Use triangles and circles.

3. Draw a picture worth 35¢.
   Use circles, triangles, and squares.

6 coins worth 50¢	21 coins worth 25¢
8 coins worth 60¢	16 coins worth 25¢
4 coins worth 30¢	16 coins worth 50¢
10 coins worth $1.00	26 coins worth 60¢

128	64	32	16	8	4	2	1	Given Number
								7
								50
								100
								110

$$\begin{array}{r} \left.\begin{array}{r} 64,897 \\ 9,536 \end{array}\right\rangle \textcircled{13} \\ + \quad 678 \quad\quad \\ \hline \end{array}$$

ADDITION DIAGNOSTIC TEST

1. 45
+34

2. 65
+32

3. 6
7
+8

4. 48
+ 9

5. 37
+46

6. 438
+357

7. 297
+455

8. 609
+297

9. 4,896
+3,548

10. 26,159
9,476
+ 857

11. Write the numbers in a column and add.
8,176 + 39 + 946 = ?

A - T10

$$\begin{array}{r} 4{,}025 \\ -\,1{,}987 \\ \hline \end{array}$$

0	8	2	7	9	3	2
-7	-3	-9	-2	-4	-8	-6
N	Y	N	Y	Y	N	N

S-T2

$$\begin{array}{r} 80 \\ -27 \\ \hline N \end{array} \qquad \begin{array}{r} 79 \\ -23 \\ \hline 56 \end{array} \qquad \begin{array}{r} 73 \\ -49 \\ \hline N \end{array} \qquad \begin{array}{r} 86 \\ -32 \\ \hline 54 \end{array}$$

____ HAS FOUR STUDENTS IN HER FIRST READING GROUP. SHE GAVE EACH STUDENT TWO BOOKS. HOW MANY BOOKS DID SHE PASS OUT?

____, ____, AND ____ WERE PLAYING CARDS. ____ DEALT FOUR CARDS TO EACH STUDENT. HOW MANY CARDS WERE DEALT?

A TRICYCLE HAS 3 WHEELS. THE SMITHS HAVE 2 TRICYCLES. HOW MANY TRICYCLE WHEELS DO THEY HAVE?

____ [illegible] BIRTHDAY PARTY AND INVITED 5 FRIENDS. EACH FRIEND WAS GIVEN 3 GOLDFISH TO TAKE HOME AS A PARTY FAVOR. HOW MANY GOLDFISH WERE GIVEN AS FAVORS?

WRITE Y FOR YES IF IT CAN BE MULTIPLIED OR N FOR NO IF IT CAN'T BE MULTIPLIED. IF IT CAN BE MULTIPLIED, REWRITE THE ADDITION PROBLEM AS A MULTIPLICATION PROBLEM.

7	6	5	3	6
7	6	4	3	8
7	6	9	+3	+6
+7	+6	+3		

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

ONE DIGIT MULTIPLIER -

EVERYONE IN THE EIGHTH GRADE RECEIVED 3 FREE CIRCUS TICKETS. THERE ARE 24 STUDENTS IN MR. MILLER'S CLASS. HOW MANY TICKETS WERE ISSUED IN THE CLASS?

TWO DIGIT MULTIPLIER · CONCEPT

MR. SMITH DECIDED TO REPLACE THE FLOOR IN HIS FAMILY ROOM. HE BOUGHT ONE-FOOT-SQUARE TILES. HIS FAMILY ROOM MEASURED 24 FEET LONG BY 13 FEET WIDE. HOW MANY TILES DID HE NEED TO BUY?

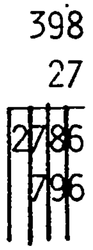
TWO DIGIT MULTIPLIER - CONNECTING

THE CENTER SECTION OF THE SCHOOL AUDITORIUM HAS 14 SEATS. THERE ARE 22 ROWS. HOW MANY SEATS ARE IN THE CENTER SECTION?

M-T5

AVOIDING TROUBLE SPOTS

```
   398
    27
  2786
  796■
10,746
```



```
46
x32
      2 x 46
     30 x 46
```

M-T6

1. (P) _____ HAS 8 BOOKS. SHE PUTS THEM INTO 4 PILES OF THE SAME SIZE. HOW MANY BOOKS ARE THERE IN EACH PILE? (AN EXAMPLE OF HOW MANY IN EACH GROUP.)

2. (P) THE TEACHER GIVES 12 CRAYONS TO _____. HE IS TOLD TO GIVE 3 CRAYONS TO EACH STUDENT. HOW MANY STUDENTS RECEIVE CRAYONS? (AN EXAMPLE OF HOW MANY GROUPS.)

1. (I) THERE ARE 12 PENCILS ON THE TEACHER'S DESK TO BE GIVEN TO HONOR ROLL STUDENTS. IF THERE ARE 4 STUDENTS ON THE HONOR ROLL, HOW MAY PENCILS WILL EACH STUDENT RECEIVE? (HOW MANY IN EACH GROUP?)

2. (I) THE MOTHER'S CLUB CONTRIBUTED A BOX OF 48 COOKIES TO THE CLASS. IF EACH STUDENT RECEIVED 2 COOKIES, HOW MANY STUDENTS RECEIVED COOKIES? (HOW MANY GROUPS?)

D-TI

____, ____, ____, ____, ____, AND ____ ARE FROGS IN A POND. THERE ARE 3 LILY PADS IN THE POND. THE SAME NUMBER OF FROGS CLIMB ON EACH PAD. HOW MANY FROGS ARE ON EACH PAD?

TWELVE STUDENTS ARE LINED UP TO GO ON THE FERRIS WHEEL. THREE CAN FIT IN EACH CAR. HOW MANY CARS WILL THEY FILL UP?

I HAVE 11 PIECES OF CANDY TO SHARE WITH MY BEST FRIEND. IF WE EACH GET THE SAME NUMBER OF PIECES OF CANDY, HOW MANY WILL WE EACH GET?

THE SECRETARY GAVE YOU 15 PENCILS AND ASKED YOU TO PUT THEM IN GROUPS OF 4. HOW MANY GROUPS WILL YOU HAVE?

OUR CLASS IS PLANNING A FIELD TRIP. PARENTS WILL PROVIDE TRANSPORTATION IN CARS. EACH CAR CAN CARRY FOUR STUDENTS. HOW MANY CARS WILL BE NEEDED TO CARRY 28 STUDENTS?

THE LIBRARY WAS GIVING AWAY 40 OLD BOOKS THAT WERE NO LONGER NEEDED. A STUDENT COULD CHOOSE 3 BOOKS. IF EACH STUDENT CHOSE 3 BOOKS, HOW MANY STUDENTS RECEIVED FREE BOOKS?

D-TZ

Fair Shares

$$\begin{array}{r} 5\ r\ 2 \\ \_\_\_\ \overline{)\ 1\ 7} \end{array}$$

$$\begin{array}{r} \_\_\_\ r\ 3 \\ 4\ \overline{)\ 2\ 7} \end{array}$$

$$\begin{array}{r} 3\ r\ 2 \\ 6\ \overline{)\ \_\_\_\_} \end{array}$$

THERE ARE 8 STUDENTS IN THE GROUP. EACH STUDENT NEEDS 4 SHEETS OF PAPER. HOW MANY SHEETS OF PAPER DOES THE GROUP NEED?

SUPPOSE I HAD 32 SHEETS OF PAPER AND WANTED EACH STUDENT TO HAVE 4 SHEETS. HOW MANY STUDENTS DO I HAVE ENOUGH PAPER FOR?

IF I HAVE 21 COOKIES AND WOULD LIKE TO GIVE EACH PERSON 3 COOKIES, HOW MANY PEOPLE WILL GET COOKIES?

$3\overline{)21}$ OR $21 \div 3 =$ ___

$21 \div 3 = 7$
$21 \div 7 = 3$
$7 \times 3 = 21$
$3 \times 7 = 21$

SOMETIMES WRITING OUT A MULTIPLICATION TABLE WILL HELP.

3 X 1 PERSON = 3 COOKIES
3 X 2 PERSONS = 6 COOKIES
3 X 3 PERSONS = 9 COOKIES
3 X 4 PERSONS = 12 COOKIES
3 X 5 PERSONS = 15 COOKIES
3 X 6 PERSONS = 18 COOKIES
3 X 7 PERSONS = 21 COOKIES

THE CLASS WAS GOING ON A HIKE. THERE WERE 36 STUDENTS IN THE CLASS. THE TEACHER ASKED THEM TO GET INTO GROUPS OF 3 SO SHE COULD KEEP TRACK OF THEM MORE EASILY. HOW MANY GROUPS WERE THERE?

AT A BANQUET, GUESTS WERE SEATED 4 TO A TABLE. THERE WERE 572 GUESTS IN ALL. HOW MANY TABLES WERE NEEDED?

THERE ARE 365 DAYS IN A CALENDAR YEAR. IF THE YEAR WAS DIVIDED INTO 3 EQUAL PARTS, HOW MANY DAYS WOULD BE IN EACH THIRD?

QUESTION 1 - HOW MANY GROUPS?

QUESTION 2 - HOW MUCH DID YOU PUT UP?

QUESTION 3 - HOW MUCH IS LEFT OF YOUR BOARD?

$13\overline{)\,1987}$

13	13	13	13	13	13	13	13	13
x1	x2	x3	x4	x5	x6	x7	x8	x9
13	26	39	52	65	78	91	104	117

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