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ABSTRACT

The Van Hiele theory asserts that there exist five hierarchical levels of geometric thinking that a successful learner passes through. The purpose of the study described in this paper was to investigate the geometric understanding and misconceptions in students in the fourth through eighth grades who have been identified as gifted. The students were selected based on intelligence quotient or standardized test scores, teacher recommendations, and other instruments chosen by school districts. Group 1 received 20 hours of instruction in geometry using LOGO. Group 2 attended a 1-week summer camp having 7 hours of geometry instruction and activities. The change in the Van Hiele level between pretest and posttest was significant for both groups. Analysis of the protocols from interviews indicates three categories of reasoning, the relationship between reasoning and knowledge of definitions, and reasoning involving parts of figures. Several implications for mathematics instruction are discussed. Seven references are listed. (YP)

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GEOMETRIC UNDERSTANDING AND MISCONCEPTIONS AMONG GIFTED FOURTH-EIGHTH GRADERS

A paper presented to the Annual Meeting of the American Educational Research Association

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Dutch educators P. M. van Hiele and Dina van Hiele-Geldof proposed a linearly-ordered model of geometric understanding. The van Hiele theory asserts that there exist five hierarchical levels of geometric thinking that a successful learner passes through: Basic Level - visualization, Level 1 - analysis, Level 2 - abstraction, Level 3 - deduction, and Level 4 - rigor. According to the van Hieles' model, the learner cannot achieve one level without passing through the previous levels. Progress from one level to the next is more dependent on educational experiences than on age or maturation, and certain types of experiences can facilitate (or impede) progress within a level and to a higher level (Fuys, 1984).

Previous research tends to support the hierarchical nature of the van Hiele levels within several populations. Joanne Mayberry (1981) found sufficient evidence among 19 undergraduate preservice elementary teachers to support this aspect of the theory but she rejected the hypothesis that an individual demonstrated the same level of thinking in all areas of geometry included in the school program. The van Hiele levels of her subjects were quite low: they did not recognize squares as rectangles and did not perceive relationships between classes of figures.

In examining high school sophomores, Usiskin (1982) found that over 80% of these students can be assigned a van Hiele level by means of a paper-and-pencil test, but students may be in transition between levels and therefore difficult to classify. Burger and Shaughnessy (1986) found mainly level 0 thinking for subjects in grades K-8. They described the levels as dynamic rather than static and more continuous than discrete. Fuys, Geddes and Tischler (1985) utilized instructional modules in geometry with sixteen sixth graders and sixteen ninth graders. They found entry levels of 0 and 1, but several students, especially those deemed above average in mathematics ability prior to instruction, exhibited level 2 behavior by the completion of the six hours of clinical interviews and instruction. They also reported several misconceptions or errors found among these sixth and ninth graders. Among the examples cited were thinking "sides" refers only to vertical segments, using the phrase "straight lines" when referring to parallel lines, and thinking that a parallelogram has to have oblique angles (Fuys, Geddes and Tischler, 1985, p. 199). Hershkowitz (1987) found several geometric misconceptions displayed by students in grades 5 through 8. Examples include misidentification of right triangles, isosceles triangles, quadrilaterals and altitudes in various types of triangles.

Research indicates that gifted, average, and retarded children all follow the same pattern of progression through the Piagetian stages (Roeper, 1978; Weisz & Zigler, 1979; Carter & Ormrod, 1982). Gifted students showed superiority on Piagetian tasks over students of normal intelligence at every age level tested. Piaget proposed that the transition to the formal operational stage occurs at ages 11 to 12. Carter and Ormrod (1982) found that the majority of subjects of average intelligence were still transitional to formal operations even as late as age 15. They also found that the gifted subjects entered formal operations successfully by 12-13 years of age (p. 114). Does the gifted students' ability to operate abstractly earlier than other students affect the linearly ordered development hypothesized by the van Hieles?

The purpose of this study was to investigate the geometric understanding and misconceptions in students in the fourth through eighth grades who have been identified as gifted. In particular, how well the cognitive structures and developmental levels of these gifted students conformed to what was predicted by the van Hiele theory of geometric understanding was examined.

METHOD

Subjects.

The subjects were students in the fourth through eighth grades who have been identified as gifted based on IQ or standardized test scores, teacher recommendations, and other instruments chosen by their individual districts. The population included two distinct groups of subjects: the first group included 11 fourth graders, 12 fifth graders, and 11 sixth graders who were participants in a science and math pull-out program for the gifted in a small rural district in Illinois.

Procedure

The van Hiele level of all subjects was determined both prior to and at the conclusion of approximately 20 hours of instruction in geometry via Logo using a 15 item subset of the paper and pencil test developed by the Cognitive Development and Achievement in Secondary School Geometry Project (CDASSGP) (Usiskin, 1982). In addition, they participated in a 30-45 minute interview based on Mayberry's questions. The second population consisted of 1 fifth grader, 15 sixth graders, 10 seventh graders, and 5 eighth graders attending a one week summer camp for the Academically Talented. These subjects completed the full 25 item test developed by the CDASSGP prior to and following the camp which contained approximately seven hours of geometry instruction and activities. Additionally, selected questions of particular interest from the interview were administered in written form to these students. The paper and pencil and interview questions focused on the concepts of square, isosceles triangle, right triangle, circle, parallel lines, similarity, and congruence.

Results

CDASSGP Test

The distribution of written tests scores for the first group (combined fourth, fifth and sixth grades) is may be seen in Table 1. This distribution of the pretest levels is consistent with the findings of previous researchers. Disregarding the subjects who were unclassifiable on at least one test, the van Hiele level is significantly different between the pretest and the posttest ($t = 1.90$, $df = 24$, $p < .05$). The posttest levels tend to support Dina van Hiele's statement that 20 hours of structured intervention may facilitate movement from one level to the next.

Table 1
% of Subjects in Group 1 at Each van Hiele Level as Determined by the CDASSGP Test

	van Hiele Level				
Test	0	1	2	above 2	?
Pretest	34%	56%	3%	0%	6%
Posttest	27%	27%	21%	12%	12%

The distribution of written test scores for the second group (combined fifth, sixth, seventh and eighth grade campers) is displayed in Table 2. The change in van Hiele level is significant for this group as well ($t = 2.11$, $df = 30$, $p < .025$). A number of the unclassifiable scores were related to students showing mastery of Level 4 type problems when they had not mastered Level 3. The scores on both the tests and interviews also seemed to be influenced in part by the gifted students' experience with and talent for analyzing questions and taking tests.

Table 2
<u>% of Subjects in Group 2 at Each van Hiele Level as Determined by the CDASSGP Test</u>

	van Hiele Level					
Test	0	1	2	3	4	?
Pretest	13%	42%	13%	7%	0%	26%
Posttest	3%	36%	19%	10%	3%	29%

<u>Interviews</u>

Analysis of the protocols from the interviews indicates several patterns not apparent in the written tests.

<u>Types of Reasoning Observed.</u> Three categories of reasoning can be detected in the subjects' identification of geometric figures.

1. Appearance. The student is operating at Van Hiele Level 0. For example, "It's a triangle because it looks like a shark's fin on the top." or "These are all triangles because they have the shape of a triangle." or "Line segment DE is three centimeters long because it looks like about half of the line segment labeled eight centimeters." This type of reasoning sometimes led to correct answers.

2. Reasons based on noncritical attributes, usually those of the prototype. Figures such as squares, rectangles, right triangles, isosceles triangles, and circles are defined in terms of critical attributes and examples. Certain of these examples may be considered prototypes such as squares and rectangles with sides parallel to the bottom of the page or blackboard. A prototype right triangle has its base parallel to the page bottom and the other side vertical and looks like it's pointing toward the right (as contrasted to a left triangle). A prototype isosceles triangle also has its base parallel to the page bottom. It points upward with its base shorter than the two congruent sides.

Since teachers and textbooks tend to use them more frequently, these prototypes are usually the first learned. They appear to persist with the students failing to differentiate between critical and non-critical attributes in a figure. For example, orientation of the shape was a factor in grades 4-6 with 1/7 of the subjects failing to identify a square rotated 45° from parallel to the sides of the page (◇) as a square. As one fourth grader said "I think it might be a square if you tilted it." The reference point does not seem to be the sides of the paper but their own position. For example, another fourth grader said that the sides of a square "have to be straight up and straight across."

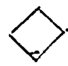
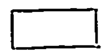
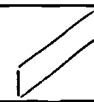
Several of the fourth graders could identify a square printed on a file card with its sides parallel to the sides of the card as a square when it was presented to them so that the sides appeared "straight ahead and straight across" but when the card was then rotated 45° in their view, they said the figure as no longer a square. These results are summarized in Tables 3 and 4.

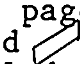
During the instructional phase of the project, the subjects wrote LOGO procedures for drawing squares when required that they incorporate four 90° angles and four sides of equal lengths to create their squares. They used their procedures to draw squares with different orientations both on the computer monitor and on paper on the floor with a computer-controlled robot turtle. They sorted and classified cut-out shapes, some of which were squares and in the process viewed the shapes from different orientations. However, four subjects still failed to identify the "rotated square" as a square at the end of the study.

Table 3
<u>% of Students Identifying Various Shapes as Squares</u>

	Shape					
Grade	□	◇	◊	▭	▯	▱
8	100.0	100.0	20.0	0.0	0.0	0.0
7	100.0	100.0	11.1	0.0	0.0	0.0
6	96.3	92.6	7.4	7.4	7.4	0.0
5	100.0	87.5	25.0	37.5	37.5	0.0
4	100.0	100.0	20.0	0.0	0.0	0.0

Table 4
<u>% of Students Identifying Various Shapes as Rectangles</u>

	Shape					
Grade						
8	60.0	60.0	0.0	100.0	100.0	20.0
7	11.1	11.1	0.0	100.0	100.0	11.1
6	70.4	59.3	7.4	96.3	96.3	25.9
5	37.5	37.5	12.5	100.0	100.0	75.0
4	0.0	0.0	0.0	100.0	100.0	80.0

This orientation factor did not seem to affect the recognition of non-square rectangles when the rectangles did not appear with their sides parallel to the page edges. However, over 30% of the students identified  as a rectangle. Many of these same students included "has four right angles" or "has four 90° angles" as part of their definition of a rectangle. However, the four right angles did not appear to be the attribute they were focusing on. Rather these subjects were focusing on the non-critical attribute for a rectangle of have two long sides and two short sides which was not always mentioned in their definitions.

This type of reasoning based on noncritical attributes, usually those of the prototype, will usually result in incorrect responses.

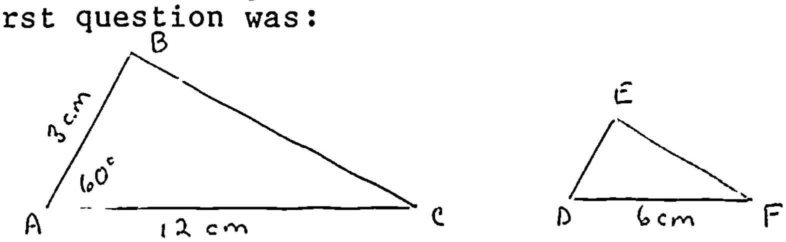
3. Reasons based on critical attributes. This reasoning type will usually led to correct responses, given that the definitions employed are correct. For example, "I guess that a right triangle could be isosceles because you could have the right angle at the top and the two sides of it, meeting in the middle, be equal."

<u>Reasoning and Definitions.</u> The reasoning ability of the students was far beyond what may have been anticipated, given their lack of knowledge of basic definitions and concepts. Some students had incorrect definitions of various terms. For example, an isosceles triangle has exactly two congruent sides and a rectangle has two long sides and two short sides. During the initial interviews, many students were unsure of the definitions of the mathematical terms such as isosceles, congruent and hypotenuse. For terms such as similar, they attempted to use the English language definition of the word to apply to

the mathematical problem. For example, if they had not been specifically taught the mathematical meaning of the term "similar" students used definitions such as "It's like the same, but there might be a very slight little difference.", "Two figures look a little, sorta like each other.", or "Congruent means exactly. Similar, you know, means maybe like this much off and stuff." or "They're a lot alike."

Most of these gifted students attempted to deduce the definitions of terms they were unsure of from the context of the question. They would then base their answers upon their conjectured meanings. For example, on the initial interview, one sixth grader, when asked if two squares with a 10 cm side are always, sometimes or never congruent, replied "That depends on what congruent means. If it means the same, then those will always be congruent. And if it means different, they never will be." In many cases the students would build valid logic structures based upon their conjectured definitions. This type of thinking is indicative of Level 2, but has been accomplished without knowledge of specific definitions or geometric content.

Reasoning Involving Parts of Figures. One difficulty which many students exhibited became apparent in their answers to two questions requiring the subjects to focus on an angle which was part of another figure.
The first question was:



Triangle ABC is similar to triangle DEF.
How long is $\overline{ED}$? How do you know?
What is the size of <EDF? How do you know?

Two categories of reasoning can be discerned in the answers to "How long is $\overline{ED}$?": answers involving a comparison between the two figures, triangle ABC and triangle DEF (characteristic of Level 2 thinking) and those depending on comparisons within triangle DEF only (characteristic of Level 1 thinking). (See Figure 1.) Approximately two-thirds of the students gave 4 centimeters as the answer supported by reasons such as "because the base of triangle ABC is 12 and the base of DEF is 6. And the side AB is 8 centimeters and everything else is halves, so DE has to be half of that" or "The ratio in ABC is 2 to 3 so if this is 6 certimeters then the side must be 4." or "The proportion of the lengths of the first triangle to the second is 2 to 1." As can be seen in Table 3, the older the subject, the more likely s/he was to answer in this manner. An additional 4% also compared the two figures, but gave 8 centimeters as their answer using reasoning such as "Because it says that ABC is similar to DEF and the measurements are on ABC and they have to be the same measurements." or " Because they said that "ABC is similar to DEF and if it was similar, they'd be about the same length." These subjects apparently ignored the 6 cm label on $\overline{DF}$ and treated "similar" as "congruent". However, later in the interview, they defined similar as "pretty close to the same", saying that they are the same shape, could be the same size, but could be different sizes. This definition is not what they based their previous answer of 8 centimeters on.

None of the seventh and eighth graders depended on comparisons only within triangle DEF for their answers, but approximately 30% of the fourth - sixth graders did, with the younger subjects doing so more frequently. (See Table 5.) In all cases, the subjects were apparently estimating the length of $\overline{DE}$. The most frequent answer given was 3 centimeters with reasons such as "because it looks like it is half of the 6 centimeters." or "Because if the bottom is 6 centimeters, this looks like it's half of that one."

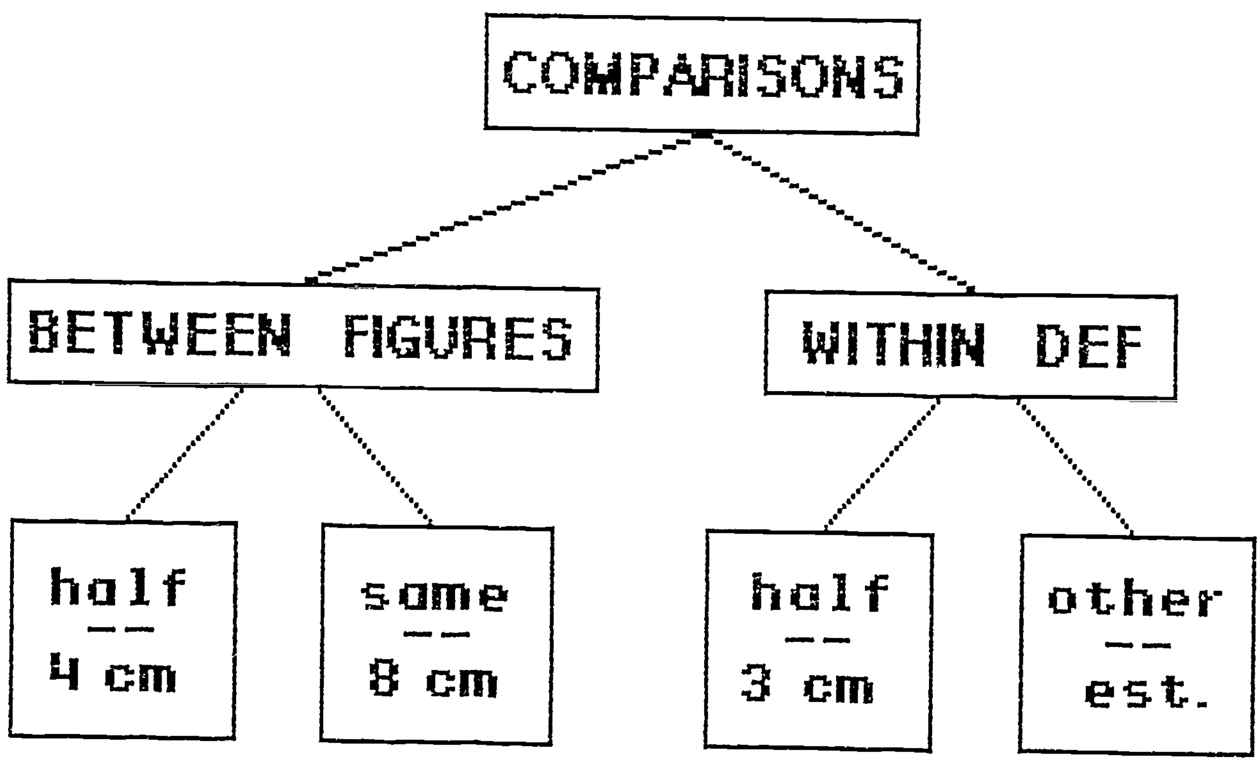


Figure 1. Categories of Reasoning Discerned in Answers to "How long is ED?"

Table 5
Answers Given for Length of Line Segment DE in Per Cents

Grade	Answer Given: 4 cm	8 cm	3 cm	other	blank
8	100.0	0.0	0.0	0.0	0.0
7	88.9	0.0	0.0	0.0	11.1
6	66.7	7.4	11.1	7.4	7.4
5	50.0	0.0	37.5	0.0	12.5
4	20.0	0.0	40.0	40.0	0.0
Total	66.7	3.7	14.8	7.4	7.4

Reasoning displayed in answering the question "What is the size of <EDF?" can also be categorized in the same manner: answers involving a comparison between the two figures, triangle ABC and triangle DEF (characteristic of Level 2 thinking) and those depending on comparisons within triangle DEF only (characteristic of Level 1 thinking). (See Figure 2.) Less than 40% of the subjects employed comparisons between the two figures when considering the size of <EDF. 70% of the subjects did so when determining the size of ED. (See Table 6.)

Table 6
Answers Given for Size of Angle EDF in Per Cents

Grade	Answers Given: 60^o	30^o	other <	12-18 cm	10 cm	other cm	blank
8	60.0	0.0	0.0	0.0	20.0	0.0	20.0
7	22.2	22.2	0.0	22.2	11.1	0.0	22.2
6	29.6	7.4	22.2	14.8	7.4	3.7	14.8
5	0.0	25.0	0.0	37.5	0.0	12.5	25.0
4	0.0	20.0	0.0	40.0	0.0	40.0	0.0
Total	24.1	13.0	11.1	20.4	7.4	7.4	16.7

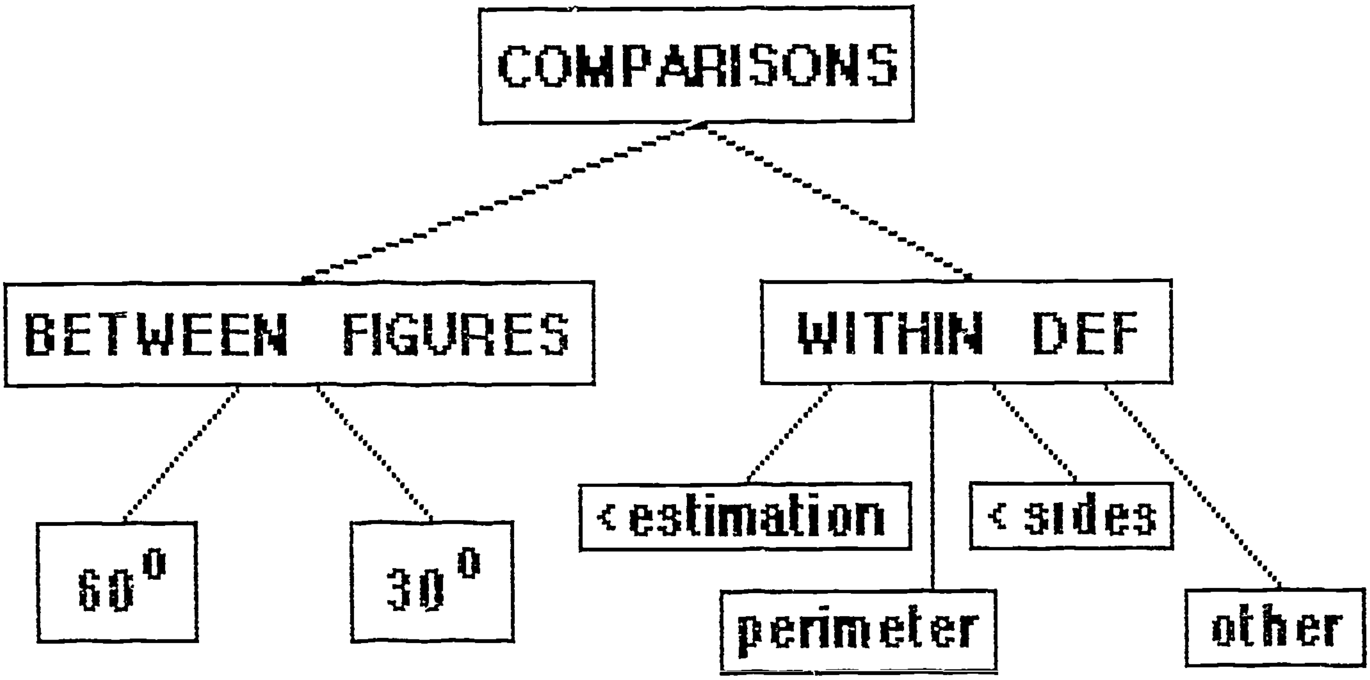


Figure 2. <u>Categories of Reasoning Discerned in Answers to "What is the size of <EDF?"</u>

Subjects employing comparisons between the two triangles gave one of two answers. Approximately a quarter of the subjects correctly answered 60° as the size of <EDF, giving such reasons as "They're similar triangles. Even though the sides are longer, it's still the exact same angle as the first figure." No fourth or fifth graders answered correctly during the initial interviews. As might be expected, a number of the students answered 30°, citing reasons such as "I think it's 30° because <BAC's 60 and I think EDF is half scale, which would be 30."

Among the subjects considering only triangle DEF to determine the size of <EDF, those subjects who gave an answer in degrees appeared to do so by estimating the size of <EDF, usually estimating 45° or 90°.

Most of the subjects could trace the angle with their finger and had at least a basic understanding of what an angle was. All had previously used protractors to measure angles. Surprisingly, over 35% of all subjects gave the size of <EDF in centimeters. Three categories of answers were discernible. About 60% of these subjects estimated the perimeter of triangle DEF, about 20% added together the lengths of the two sides of angle EDF, and about 20% gave

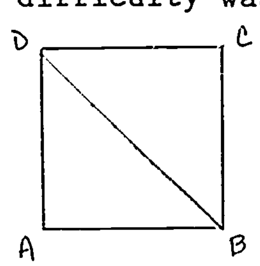
another answer in centimeters.

Various types of explanations were given for giving the perimeter: "I think the size of angle EDF means like what's the area and stuff like that. Like how much is there in the whole thing. ... This ($\overline{EF}$) would be about... let's say 4 centimeters to 6 ($\overline{DF}$ is labeled 6 cm.). That would be about 10, then 13 (She had previous said $\overline{DE}$ was 3 centimeters). It looks about 13 centimeters. 6 plus 4 plus 3. 13 centimeters." or "If you had asked me what the size of angle D is, I would say 60^{o}. But you didn't. You asked what the size of angle EDF is. That means I have to add up the three sides that enclose angle D."

Two types of reasoning appeared to occur among those students who added together the length of the two sides of angle EDF (6 cm + 4 cm) to obtain the size of angle EDF (10 cm). One group of subjects believed that you can measure an angle by measuring the lengths of its sides. The other group seemed to be following the old rule of when you have two numbers and don't know what to do, add them. ($\overline{DF}$ was labeled 6 cm and they had just determined that $\overline{DE}$ was 4 cm.)

Among those subjects who gave other answers, some just gave 6 cm because it was the only label on the figure. Others tried to estimate the amount of opening of the angle. For example, "You measure an angle by how much it's open. So angle EDF is 5 centimeters. Like when you look at it, it's kind of... Well, in the third grade, my teacher told me a little bit about measuring. She said that your fingernail, you thumb nail is about the same as a centimeter and when you picture the size of you thumb nail or a centimeter that you have seen in books. Like from there (E) to there (F), that's about 5 centimeters.

The second question involving parts of figures which caused difficulty was:



ABCD is a square.
$\overline{BD}$ is a diagonal.
Name an angle congruent to <ABD.
How do you know?

Many subjects gave expected answers arrived at through valid reasoning such as "The square is cut in half so <DBC would be congruent because the corners make a 90 degree angle so if they're cut in half it would be 45 degrees and they would be congruent." or "<CDB. Square ABCD was divided into two congruent triangles placed next to each other with one reversed, so opposite corners were equal."

Some subjects were having difficulty with the notation or were unable to focus on the angle as part of the whole figure, interpreting <ABD as triangle ABD. In these cases,

their answers were reasonable since they identified the other triangle in the figure and could justify their answer by such reasons as "The diagonal cuts the square into two equal pieces." In each case, it was pointed out to the students that the question read <u>angle</u> ABD, not triangle ABD. In most cases, they could even properly trace the angle with their finger, but still seemed to be interpreting the question as referring to the shape rather than the angle. Over 30% of the students answered <BCD in this manner, and another 15% answered <CDB or <CBD correctly, but justified their choices in such a way that it was obvious that they were considering congruent shapes rather than angles. This inability to distinguish part of a figure from the rest of the figure may also have caused many of the errors in the previous problem where subjects estimated the perimeter of the triangle when asked to give the angle measure.

Discussion

Several patterns of thought and misconceptions in geometry were identified in this study. The influence of prototype figures in the identification of geometric figures is strong. Textbook publishers need to illustrate figures so that their sides are not always parallel to the sides of the page. Teachers should be careful not to use prototype figures exclusively when making drawings on the blackboard or overhead projector and in other work. They should vary the size and orientation of their figures. Experiences with cutouts of shapes and LOGO may also be beneficial in addressing this problem of orientation. Games in which students sort shapes according to attributes and classify figures may be useful. Creating a "family tree" of shapes will also focus attention on critical attributes.

Subjects also exhibited difficulty in focusing on an angle embedded in a triangle or square. Additional work is needed on the identification and labelling of angles and triangles.

Analysis of the clinical interviews confirmed Mayberry's rejection of the hypothesis that an individual demonstrates the same level of thinking in all areas of geometry included in the school program.

The cognitive structures and developmental levels of these gifted students seem different from what would be predicted by the van Hiele theory of geometric understanding. Their reasoning is typical of at least Level 2 and, in some cases, Level 3 and Level 4 thinking, but many of them have not mastered Level 0 or Level 1. Deduction is meaningful to most of them. In fact, many of them can manipulate symbols without referents according to the laws of formal logic. However, they have not been exposed to the "rules of the game" and so do not know how to construct an acceptable proof. In addition, they do not know the role of axioms and definitions and the meaning of necessary and sufficient conditions.

Criteria which distinguish a figure are determined artificially by general agreement on the required defining attributes possessed by the figure rather than on something innate or inherent in the figure. That is, definitions are based on invented criteria rather than innate attributes. For example, it is as reasonable to define isosceles triangles as triangles having <u>exactly</u> two congruent as it is to define them as triangles having <u>at least</u> two congruent sides. Many of these gifted subjects had not been exposed to or did not remember what the critical defining attributes of various figures were. However, they tended to look for similarities and differences in figures (a characteristic of subjects who have attained at least Piaget's Stage 2) and deduce what the defining attributes might be. In other words, not knowing what the previously established criteria were, they tried to discover innate attributes which could provide them with a working definition. The students would then base their reasoning on these conjectured meanings, no matter how conceptually inadequate they might be. Generally, the subjects were consistent, given the definition they were basing their thinking on, and often quite sophisticated in their reasoning. When they were faced with a contradiction or inconsistency, they would generally fault their definition. Some students would simply give up at this point, while others would attempt to change or refine their definition. They enhanced their definitions as they detected additional information in the context of subsequent questions as well.

A similar process occurred with terms the students were not familiar with terms such as congruent. Most subjects who were unfamiliar with a term attemptted to construct a meaning out of contextual clues. Once they encountered the symbol for congruence ($\cong$) most students assumed congruent meant equal. Many appeared to not even notice the tilde. Students also had a tendency to skip over symbols like $\triangle$ (as in $\triangle$ ABC), < (as in <ABC), and - (as in $\overline{EF}$) when reading them aloud as if the symbols weren't even there.

In dealing with terms they had an existing schema for, such as the English language definition of "similar", students had a tendency to persist in their definitions. Apparent inconsistencies were often ignored.

Generally, these students were capable of handling inclusion relationships if they had suitable definitions of the elements involved, a characteristic of Piaget's Stage 3 as well as van Hiele's Level 2. But an equilateral triangle can not be an isosceles triangle if you think that an isosceles triangle has <u>exactly</u> two congruent sides. A square isn't a rectangle if a rectangle has two long sides and two short sides.

While the cognitive structures of these gifted subjects do not seem to be described well by the van Hiele theory of geometric understanding, they do need Level 1 and Level 2 experiences in order to provide a foundation for their reasoning, so that they do not have to deduce the meaning of the terms they encounter and the relationships. Provided with this additional background, gifted seventh and eighth graders should be capable of a proof oriented geometry course.

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