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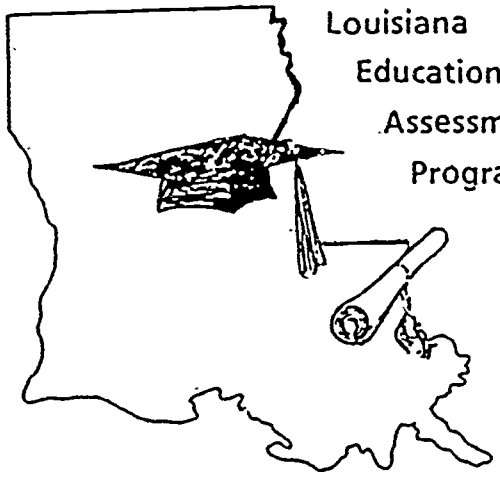
ABSTRACT

The Louisiana Educational Assessment Program (LEAP) Grade 11 Test is designed to measure proficiency in four subject areas including English, mathematics, social studies, and science. This guide for mathematics is intended to provide a description of the way in which specific skill areas are assessed on the LEAP test and instructional considerations in promoting proficiency of these target skill areas. Skill areas included are: (1) fractions and operations; (2) decimal numbers and operations; (3) percent, ratio, and proportion; (4) measurement; (5) geometry; (6) graphs, probability, and statistics; (7) pre-algebra; and (8) algebra. Specific skills to be assessed are described at the beginning of each section. In each skill area, sample items, descriptions of test questions, descriptions of answer choices, and instructional analyses are provided. (YP)



LEAP

INSTRUCTIONAL STRATEGIES GUIDE

Grade 11
MATHEMATICS

Louisiana Department of Education

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Introduction

Recent efforts by the Louisiana Department of Education (LDE) to upgrade the state's competency-based educational plan include the establishment of a criterion-referenced high school exit level testing program. The Louisiana Educational Assessment Program (LEAP) Grade 11 Test is designed to measure proficiency in four subject areas: English/language arts, mathematics, social studies, and science.

In accordance with R.S. 17:24.4 (Act 146 of the 1986 Regular Session of the Louisiana Legislature), the Grade 11 criterion-referenced test items will be piloted on an approximate 5 percent sample of public school eleventh-grade students during the weeks of April 11-15 and April 18-22, 1988. The legislation requires statewide implementation of the test in 1988-89.

State Board of Elementary and Secondary Education (SBESE) policy, Standard 2.099.00, Bulletin 741, Louisiana Handbook for School Administrators, states that the test will be used as a graduation requirement. The 1990-91 school year was established by the SBESE as the effective date to require satisfactory performance on the test in order to receive a high school diploma. Students who fail to pass the Grade 11 test will be offered retake opportunities. In addition, R.S. 17:24.4 states that those students who fail to meet required proficiency levels on the state-administered criterion-referenced tests of the LEAP shall receive remedial education programs that comply with regulations adopted by SBESE.

Developmental activities for the LEAP Grade 11 Test have been substantial. Specific target skills and skill areas were selected by subject area advisory committees after a thorough review of appropriate textbooks and LDE curriculum standards. These skills represent the most salient, testable skills emphasized in the curriculum.

Test item specifications, test blueprints, and test items were developed by IOX Assessment Associates of Culver City, California, under the direction of both the LDE and subject area advisory committees composed of local education agency curriculum specialists.

The mathematics section of the LEAP examination will consist of approximately 45 multiple-choice items.* The items were developed under the readability restriction that no words used exceed an eleventh-grade vocabulary level. The LEAP Grade 11

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*The exact number of mathematics items to appear on the LEAP Grade 11 Test has yet to be determined.

Test addresses eight mathematics skill areas. These skill areas and the approximate percent of items in the LEAP mathematics item pool represented by each skill area are listed below:*

Fractions and Operations	12%
Decimal Numbers and Operations	12%
Percent, Ratio, and Proportion	13%
Measurement	12%
Geometry	12%
Graphs, Probability, and Statistics	12%
Pre-Algebra	11%
Algebra	16%

The specific content eligible for testing under these skill areas was adapted from the following curriculum guides:

K-8 Mathematics Curriculum Guide, Bulletin 1609
Introduction to Algebra Curriculum Guide, Bulletin 1802
Algebra I Curriculum Guide, Bulletin 1580

Because of the high stakes associated with the LEAP Grade 11 Test, the LDE is making available to Louisiana educators instructional strategies guides that focus on the examination. These guides are intended to provide: (1) a clear description of the way in which specific skill areas are assessed on the LEAP test and (2) instructional considerations that might be used by Louisiana educators in promoting proficiency of these target skill areas.

Components of the Instructional Strategies Guide

The instructional strategies guide for Mathematics is organized into separate sections describing each of the eight skill areas. At the beginning of each section is a description of the specific skills to be assessed. This description is followed by the specific components which are briefly described below.

Sample item. An illustrative item (or items) is presented that is representative of the LEAP test items that will be used to assess students' proficiency in a given skill or skill area.

Description of test questions. The essential elements in the questions used in test items, such as format and content, are listed.

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*The percentages refer to the item pool as it existed prior to field-testing.

Description of answer choices. Basic characteristics of an item's answer choices are presented. In addition to noting the general nature of the correct answer choice, descriptive information is provided for the categories of incorrect answer choices.

Sample item answer choice descriptions. The correct and incorrect answer choices for the sample item(s) are identified. The particular incorrect-answer category represented by each incorrect answer choice is specified.

Instructional analysis. Instructional considerations relevant to preparing students for the various skill areas tested on the LEAP examination are described. These instructional analyses are neither comprehensive nor prescriptive. Rather, they are intended to provide suggestions to Louisiana educators for instructional strategies that might be used to further student competence in the LEAP target skill areas.

Use of the Instructional Strategies Guide

This guide has been developed by experienced educators and revised by Louisiana teachers and curriculum specialists. Its purpose is to provide Louisiana educators with information regarding assessment strategies used on the LEAP Grade 11 Test as well as suggested instructional approaches to enhancing student proficiency in the skill areas covered on the examination. Use of this guide should assist Louisiana educators in providing students with effective, on-target instruction in these critical skill areas.

The materials in this guide will be useful in a variety of settings, including regular classes and LEAP-focused review/ remedial classes. The guide is organized in a manner that permits flexible use. Therefore, its sections dealing with individual skill areas are self-contained to facilitate use by educators who choose to focus on a specific skill.

SKILL AREA: FRACTIONS AND OPERATIONS
(Domain IV, Standards CD)

The skill area testing fractions and operations includes two standards:

Computes with Fractions: Uses operations with whole numbers, fractions, and/or mixed numbers with like or unlike denominators, expressing results in simplest terms (with or without regrouping).

Solves Word Problems Involving Fractions: Solves appropriate word problems involving fractions, using the problem-solving process and strategies.

SAMPLE ITEMS:

Computes with Fractions

$3\frac{1}{2} \times 2\frac{2}{3}$

A. $6\frac{1}{6}$

B. 9

C. $9\frac{1}{3}$

D. $6\frac{1}{3}$

Solves Word Problems Involving Fractions

Kelly had 8 yards of material. She used $3\frac{1}{8}$ yards of material to make a costume for the school play. How much unused material did she have after making the costume?

A. $4\frac{3}{4}$ yards

B. $4\frac{7}{8}$ yards

C. $3\frac{7}{8}$ yards

D. $5\frac{3}{4}$ yards

DESCRIPTION OF TEST QUESTIONS:

1. The student will be presented with numeric or word problems involving operations with fractions.

2. Numbers will include whole numbers, mixed numbers, and fractions. All values will be positive. Whole numbers and whole number portions of mixed numbers will be no larger than two digits. Fractions will be presented in lowest terms. Denominators will be limited to the following: 2, 3, 4, 5, 8, 10, and 16.

2. Addition problems will require the student to add a maximum of three numbers. All other operations will involve a maximum of two numbers.

3. For test items that require solving problems in a numeric format:

   a. Addition and subtraction problems will be presented in a horizontal or a vertical format.

   b. Multiplication and division problems will be presented in a horizontal format.

4. For test items that require solving problems in a word problem format:

   a. No more than two operations will be required to find the correct solution.

   b. Irrelevant numeric information will not be included.

DESCRIPTION OF ANSWER CHOICES:

1. The correct answer choice will be the fraction, whole number, or mixed number that represents the correct solution to the problem presented.

2. Fractions will be presented in simplest terms.

3. Answer choices for word problems will be labeled in appropriate units.

4. An incorrect answer choice will be a number that is obtained as a result of one or more of the following types of errors:

a. computation error: an error in the addition, subtraction, multiplication, or division of two numbers;

b. common denominator error: an error in selecting the lowest common denominator or in renaming fractions in terms of the lowest common denominator;

c. incorrect simplification: incorrectly simplifying a fraction;

d. missing operation: not operating on the data or omitting one of the required operations; or

e. incorrect operation: performing an operation other than the required one.

SAMPLE ITEM ANSWER CHOICE DESCRIPTIONS:

Computes with Fractions

A. incorrect operation

B. computation error

C. correct

D. incorrect simplification

Solves Problems Involving Fractions

A. incorrect simplification

B. correct

C. missing operation

D. computation error

INSTRUCTIONAL ANALYSIS:

Computing with Fractions

Proficiency in computing with fractions involves the mastery of two important subskills: (1) simplifying a fraction, and (2) finding the least common denominator. You may wish to review these two subskills with students before focusing on the steps involved in addition, subtraction, multiplication, and division of fractions.

Simplifying a Fraction

For LEAP items involving fractions, answer choices will be presented in simplest terms. A fraction that is reduced to simplest terms has a numerator and a denominator that are not divisible by the same natural number (except 1). For example, $\frac{4}{24}$ is not expressed in simplest terms; both parts of the fraction can be divided by 2 or 4. To simplify a fraction, find the largest number that divides evenly into both the numerator and the denominator. Then divide both parts of the fraction by that number.

$$\frac{4}{24} \div \frac{4}{4} = \frac{1}{6}$$

To make sure that this "reduced" form of the fraction is in simplest terms, check that the new numerator and denominator are not evenly divisible by any number except 1.

Point out to students that dividing any number by 1 does not change the number and that dividing both parts of the fraction by the same number is the same as dividing the whole fraction by 1. This is why a fraction does not change in value when it is reduced to simplest terms.

Determining the Least Common Denominator

In order for fractions to be added or subtracted, they must have the same denominator. Thus, before fractions with different denominators can be added or subtracted, they must be renamed in terms of the same denominator, or common denominator. The most efficient way to rename fractions is in terms of the least common denominator (LCD).

The LCD is the smallest number that is a multiple of each of the denominators of the fractions to be renamed. The LCD can be found by using prime factors. The steps used to determine the LCD are illustrated below for the fractions $\frac{3}{4}$ and $\frac{1}{6}$.

(1) Rewrite each denominator as the product of its prime factors.

(2) List each prime factor that appears in any of the denominators along with the greatest number of times that the given factor appears in any one denominator.

2: two times
3: one time

(3) The LCD is the product of these prime factors and the greatest number of times each factor appears in any one denominator.

$$LCD = 2 \times 2 \times 3 = 12$$

Now the original fractions can be renamed with respect to the LCD. Since 12 ÷ 4 = 3 and 12 ÷ 6 = 2,

$$\frac{3}{4} \times \frac{3}{3} = \frac{9}{12} \qquad \frac{1}{6} \times \frac{2}{2} = \frac{2}{12}$$

After the original fractions have been converted to equivalent fractions with the same denominator, the fractions can be added or subtracted.

Listed below are the steps necessary to solve each type (addition, subtraction, multiplication, division) of fraction computation problem. You may want to review these steps with students as you work through several sample problems. Because addition and subtraction of fractions involve very similar steps, they are presented together.

Adding/Subtracting Fractions and Mixed Numbers

1. Check the denominators. If they are the same, proceed to step 4.
2. If not, find the LCD.
3. Rename the fractions with respect to their LCD.*
4. Perform the addition/subtraction, keeping the denominator the same.
5. Simplify the fraction, if possible.

Example 1: $2\frac{5}{6} - \frac{1}{6}$

Step 1: denominators are the same

*If a subtraction problem involves a whole number and either a fraction or a mixed number, the whole number must be renamed to a mixed number having the same denominator as the fraction or mixed number in the problem.

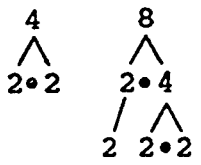
Step 4:
$$\begin{array}{r} 2\frac{5}{6} \\ -\ \frac{1}{6} \\ \hline 2\frac{4}{6} \end{array}$$

Step 5: $\frac{4}{6} = \frac{2}{3}$; $2\frac{4}{6} = 2\frac{2}{3}$

Example 2: $1\frac{3}{4} + \frac{1}{8}$

Step 1: denominators are different

Step 2: Since 2 is the only factor of 4 and 8, and the greatest number of times 2 appears as a factor in any denominator is 3,



the LCD is 8.

Step 3: $\frac{3}{4} = \frac{6}{8}$

Step 4:
$$\begin{array}{r} 1\frac{6}{8} \\ +\ \frac{1}{8} \\ \hline 1\frac{7}{8} \end{array}$$

Step 5: $1\frac{7}{8}$ cannot be simplified.

Many students find adding and subtracting more difficult than multiplying fractions. Watch for common student errors in renaming fractions, such as failing to determine the LCD correctly.

Multiplying Fractions and Mixed Numbers

1. Convert mixed numbers to improper fractions.
2. Multiply the numerators together and the denominators together. (Stress to students that denominators need not be the same.)
3. Simplify, if possible.

Example: $1\frac{1}{7} \times \frac{7}{9}$

Step 1: $1\frac{1}{7} \times \frac{8}{7}$

Step 2: $\frac{8}{7} \times \frac{7}{9} = \frac{8 \times 7}{7 \times 9}$

Step 3: $\frac{8 \times (7)}{9 \times (7)} = \frac{8}{9}$

Be sure to emphasize to students that multiplying fractions involves multiplying both the numerators and the denominators. This is in contrast to adding or subtracting fractions, where denominators are not added or subtracted.

Dividing Fractions and Mixed Numbers

1. Convert mixed numbers to improper fractions.
2. Replace the divisor by its reciprocal and replace the division sign with a multiplication sign.
3. Multiply the numerators together and the denominators together.
4. Simplify, if possible.

Example: $2\frac{2}{5} \div 1\frac{1}{10}$

Step 1: $2\frac{2}{5} = \frac{12}{5}$; $1\frac{1}{10} = \frac{11}{10}$

Step 2: The reciprocal of $\frac{11}{10}$ is $\frac{10}{11}$

Step 3: $\frac{12}{5} \times \frac{10}{11} = \frac{12 \times 10}{11 \times 5}$

Step 4: $\frac{12 \times 10}{11 \times 5} = \frac{12 \times 2 \times (5)}{11 \times (5)} = \frac{24}{11} = 2\frac{2}{11}$

Thus, dividing fractions is identical to multiplying fractions, with the exception that the divisor is replaced by its reciprocal and the division sign is changed to a multiplication sign. Explain to students that the divisor is replaced by its reciprocal because dividing by a number is equivalent to multiplying by its reciprocal.

Solving Word Problems

As with word problems in general, the key to success with word problems involving fractions is knowing which operations are required and in what order. You may wish to review the following steps with your students:

1. Read the problem carefully. Determine what question is to be answered.

2. Decide which operation(s) is needed. If more than one operation is needed, decide the order in which they must be performed.

3. Perform the computations.

4. Reread the problem and decide if the answer is reasonable.

The sample problem can be used to illustrate the above steps.

1. The problem asks how much material will be left after making a costume.

2. Because the problem asks how much will remain, subtraction is required.

3. $$\begin{array}{rl} 8 = & 7\frac{8}{8} \\ -\ 3\frac{1}{8} = & -\ 3\frac{1}{8} \\ \hline & 4\frac{7}{8} \end{array}$$

4. Using estimation, we see that $8 - 3 = 5$. Thus, our answer seems reasonable.

SKILL AREA: DECIMAL NUMBERS AND OPERATIONS
(Domain V, Standards BCD)

The skill area testing decimals and operations includes three standards:

Expresses Decimal Equivalencies: Changes an appropriate common fraction to an equivalent terminating decimal or vice versa, or identifies decimal-number and mixed-number equivalents.

Computes with Decimals: Uses operations to solve problems involving whole numbers and decimal numbers with no more than three places to the right of the decimal point.

Solves Problems Involving Decimal Numbers: Solves appropriate word problems involving decimal numbers, using problem-solving processes and strategies.

SAMPLE ITEMS:

Expresses Decimal Equivalencies

Change $1\frac{3}{8}$ to a decimal number.

A. 1.3

B. 1.25

C. 1.375

D. 1.5

Computes with Decimals

83.765 + 14.29 =

A. 97.55

B. 98.055

C. 85.194

D. 98.955

Solves Problems Involving Decimal Numbers

Jenny must buy 16 cases of juice for the school picnic. Each case costs $5.59. Assuming no sales tax, how much will Jenny spend on the juice?

A. $91.44

B. $39.13

C. $89.44

D. $21.59

DESCRIPTION OF TEST QUESTIONS:

1. The student will be presented with a numeric or word problem that requires one of the following:

   a. changing a decimal number to a fraction or mixed number;

   b. changing a fraction or mixed number to a decimal number;

   c. computing with decimal numbers; or

   d. solving problems involving rounding of decimal numbers.

2. Decimal numbers will be terminating. They will contain no more than three digits to the right of the decimal point and no more than two digits to the left of the decimal point. Decimal numbers to be changed to equivalent common fractions will be limited to those that are equivalent to meaningful fractions such as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{8}$, and $\frac{1}{10}$.

3. Fractions will represent terminating decimal numbers. They will be presented in simplest terms. Eligible denominators will be limited to 2, 4, 5, 8, and 10. The whole number portion of a mixed number will be no larger than two digits.

4. Decimal computation problems will be presented in horizontal or vertical format for addition, subtraction, or multiplication and in horizontal format only for division. Addition problems will require the student to add a maximum of three numbers. All other operations will involve a maximum of two numbers.

5. For items involving word problems:

   a. Some test items will direct students to round the answer to the nearest tenth or hundredth place.

   b. No more than two operations will be required.

   c. Irrelevant numeric information will not be included.

DESCRIPTION OF ANSWER CHOICES:

1. The correct answer choice will be the fraction, mixed number, or decimal number that correctly solves the problem. For test items that require the student to round an answer, the correct answer choice will be the correct number rounded to the specified place value. The number 5 will always be rounded up.

2. Answer choices specifying amounts of money will be aligned by place value. Other answer choices will not be aligned by place value.

3. Decimal numbers will contain no more than three digits to the right of the decimal point.

4. Decimal numbers will have varying numbers of digits to the right of the decimal point, except in word problems involving money.

5. Fractions will be presented in lowest terms.

6. Answer choices for word problems will be labeled in appropriate units.

7. An incorrect answer choice will be a number that is obtained as a result of one or more of the following types of errors:

   a. computation error: an error in the addition, subtraction, multiplication, or division of two numbers;

   b. incorrect simplification: incorrectly simplifying a fraction;

   c. misplaced decimal point: placing the decimal point in an incorrect position on either a correct solution or an incorrect solution;

   d. misaligned digit: (1) adding or subtracting a digit from one place to or from a digit in an adjacent place or (2) misaligning partial products;

e. reciprocal error: a number that is the reciprocal of the correct answer;

f. incorrect conversion: incorrectly converting a fraction to a decimal number or a decimal number to a fraction;

g. missing operation: not operating on the data or omitting one of the required operations;

h. incorrect operation: performing an operation other than the required one;

i. incorrect rounding: rounding incorrectly to the specified place value; or

j. incorrect place value: rounding correctly or incorrectly to a place value other than the one specified.

SAMPLE ITEM ANSWER CHOICE DESCRIPTIONS:

Express Decimal Equivalencies

A. computation error

B. incorrect conversion

C. correct

D. incorrect conversion

Computes with Decimals

A. computation error

B. correct

C. misaligned digit

D. computation error

Solves Problems Involving Decimal Numbers

A. computation error

B. misaligned digit

C. correct

D. incorrect operation

INSTRUCTIONAL ANALYSIS:

Converting Fractions to Decimal Numbers

This operation involves dividing the denominator of the fraction into the numerator:

$$\frac{3}{4} = 3 \div 4 = 4\overline{)3.00} \quad .75$$

$$\begin{array}{r} 28 \\ \hline 20 \\ 20 \\ \hline 0 \end{array}$$

Emphasize to students the need to add zeros to the dividend and to continue dividing to the desired place value or until the quotient terminates evenly. (Note: All conversion problems on the LEAP test will terminate by the thousandths place.) You may also want to review division with decimals and the fundamentals of place value.

Converting Decimal Numbers to Fractions

To convert decimals into fractions, students need to possess a solid understanding of place value. The conversion process can be summarized as follows:

1. Read the decimal number. Note the place value of the last digit in the decimal.

2. Drop the decimal and place the number over the above-noted place value (for example, place the number over a 10 if the last digit is in the tenths place).

3. Simplify, if possible.

Example: .35 = _____

Step 1: .35 (This number is read 35 hundredths because 5 is in the hundredths place.)

Step 2: $\frac{35}{100}$ (The decimal is dropped and 35 is placed over 100.)

Step 3: $\frac{35}{100} = \frac{5 \times 7}{2 \times 2 \times 5 \times 5} = \frac{7}{2 \times 2 \times 5} = \frac{7}{20}$

Computing with Decimal Numbers

A whole number can be written as a decimal number with zeros to the right of the decimal point. Therefore, performing operations on decimal numbers is very similar to performing operations on whole numbers. The steps required to solve problems involving addition, subtraction, multiplication, and division with decimal numbers follows. You may wish to review these steps with students as you present sample problems.

Adding and Subtracting with Decimals:

1. Write the problem in vertical format, aligning decimal points.

2. To prevent alignment errors, add 0's so that all numbers in a problem have the same number of digits to the right of the decimal point.

3. Perform the computation. Align the decimal point in the answer with the decimal points in the problem.

Example A: 34.2 + 7.65

Step 1:

$$\begin{array}{r} 34.2 \\ +\ 7.65 \end{array}$$

Steps 2 and 3:

$$\begin{array}{r} 34.20 \\ +\ 7.65 \\ \hline 41.85 \end{array}$$

Example B: 36.8 - 2.91

Step 1:

$$\begin{array}{r} 36.8 \\ -\ 2.91 \end{array}$$

Steps 2 and 3:

$$\begin{array}{r} 36.80 \\ -\ 2.91 \\ \hline 33.89 \end{array}$$

Multiplying with Decimals:

1. Write the problem in vertical format, aligning digits from the right side.

2. Perform the multiplication as with whole numbers.

3. Place the decimal point ir the answer so that the number of places to the right of the decimal is equal to the sum of the number of places to the right of the decimal points in the factors.

Example: 4.71 x .6

Step 1:
$$\begin{array}{r} 4.71 \\ \times\ .6 \end{array}$$

Step 2:
$$\begin{array}{rl} 4.71 & \text{(2 places to right of decimal)} \\ \times\ .6 & \text{(1 place to right of decimal)} \\ 2826 & \end{array}$$

Step 3: 2.826 There are a total of 2 + 1, or 3, places to the right of the decimal in the factors.

Dividing with Decimal Numbers:

1. Write the problem using this format: $x\overline{)xxx}$.
2. Make the divisor a whole number by moving the decimal point as many places to the right as necessary.
3. Move the decimal point in the dividend the same number of places to the right of the decimal point as was moved in the divisor. Add zeros as needed to retain place value.
4. Perform the division as with whole numbers.
5. Place the decimal point in the quotient directly above the decimal point in the dividend.

Example: 82.18 - 1.4

Step 1: $1.4\overline{)82.18}$

Steps 2 and 3: $14.\overline{)821.8}$

Steps 4 and 5:
$$\begin{array}{r} 58.7 \\ 14.\overline{)821.8} \\ \underline{70} \\ 121 \\ \underline{112} \\ 98 \\ \underline{98} \\ 0 \end{array}$$

Solving Word Problems Involving Decimals

Aside from computation, there are no skills uniquely associated with word problems involving decimal numbers as opposed to whole numbers. The important steps that can be used

to solve word problems are described below. The sample problem is used to demonstrate each step. You may want to review this problem with your students.

1. Read the problem carefully. Determine what question is to be answered.

   - The problem asks for the total amount of money that Jenny will spend on juice.

2. Determine the necessary operation(s). If two operations are needed, decide the order in which they must be carried out.

   - Multiplication is indicated because the problem asks for a total cost based on a multiple number of items at the same cost.

3. Perform the computations.

$$\begin{array}{r} \$\ 5.59 \\ \times\ 16 \\ \hline \$\ 89.44 \end{array}$$

4. Reread the problem, and decide if the answer is reasonable.

   Using estimation: $6.00 x 10 = $60.00;
   $6.00 x 6 = $36.00;

   $\triangle$ $60.00 + $36.00 = $96.00

The real answer should be somewhat smaller than this, because the actual factor is $5.59, not $6.00. The answer, $89.44, is slightly smaller than $96.00 and is, therefore, reasonable.

SKILL AREA: PERCENT, RATIO, AND PROPORTION
(Domain VI, Standards ABCDE)

The skill area testing percent, ratio, and proportion includes five standards:

Demonstrates a Working Knowledge of Percent: Converts (1) a decimal to a percent, (2) a percent to a decimal (3) a percent to a fraction, (4) a common fraction to a percent, or (5) an improper fraction to a percent.

Calculates Rate, Base, and Percentage: Finds a percent of a given number; determines what percent one number is of another; or finds a number when a percent of it is known.

Understands Ratio and Proportion: Solves for the missing term of a proportion.

Understands Consumer Terms Involving Percents: Computes price and sales tax, discount, rate of discount, and simple interest.

Solves Problems Involving Percent, Ratio, and Proportion: Solves appropriate word problems involving percent, ratio, or proportion, using problem-solving processes and strategies.

SAMPLE ITEMS:

Demonstrates a Working Knowledge of Percents

Change 0.7 to a percent.

A. .07%

B. 0.7%

C. 7%

D. 70%

Calculates Rate, Base, and Percentage

What is 90% of 30?

A. 30

B. 27

C. 3

D. 28

Understands Ratio and Proportion

Solve the proportion $\frac{4}{9} = \frac{x}{36}$

A. 16

B. 4

C. 9

D. 81

Understands Consumer Terms Involving Percents

Wilma borrows $600.00 from a bank at [illegible] interest. If she pays the money back in one year, [illegible] interest will she have to pay?

A. $6.00

B. $36.00

C. $30.00

D. $100.00

Solves Problems Involving Percent, Ratio, and Proportion

Of 75 students in Maplewood School, 27 take music lessons. What percent of the students take music?

A. 36%

B. 27%

C. 48%

D. 40%

DESCRIPTION OF TEST QUESTIONS:

1. For items involving conversions among decimal numbers, fractions, mixed numbers and percents:

   a. The student will be required to change (1) a decimal number, fraction, or mixed number to a percent or (2) a percent to a decimal number, fraction, or mixed number.

   b. Fractions will be presented in simplest terms and will have denominators no greater than 100.

   c. Percents will range from 1% to 500%.* Percents may or may not contain decimal points. For items that require changing a percent to a fraction, eligible percents are limited to 25%, 50%, 33.3%, and 75%.

   d. Decimal numbers will contain no more than three digits to the right of the decimal point and no more than two digits to the left of the decimal point.

2. For items that require determining the percentage, rate, or base of a specified number:

   a. The question will be one of the following:

      (1) percentage: What is [given %] of [given number]?

      (2) rate: [Given number] is what percent of [given number]?

      (3) base: [Given number] is [given %] of what number?

*Percentages will exceed 100% for conversion problems only.

b. Whole numbers, decimal numbers, and percentages will be no greater than 100.

3. For items that require solving a proportion:

   a. The question will direct the student to complete the proportion.

   b. The proportion will be in the following form: $\frac{3}{4} = \frac{x}{16}$

   c. Any term of the proportion may be missing.

   d. The proportion will contain no values greater than 100.

4. For items that require solving word problems concerning percent, ratio, or proportion:

   a. The question will be a word problem that requires one of the following:

      (1) conversion of percents, decimals, fractions, or mixed numbers;

      (2) computation of base, rate, or percentage; or

      (3) completion of a proportion.

   b. Numbers will be limited to those described as eligible for corresponding numeric problems, with the exception of numbers specifying amounts of money. These will have no more than three digits to the left of the decimal point.

   c. Items may refer to consumer terms (for example: simple interest, price and sale price, tax, discount, rate of discount, straight salary, and commission).

   d. No more than two operations will be required.

   e. Irrelevant numeric information will not be included.

DESCRIPTION OF ANSWER CHOICES:

1. The correct answer choice will be the exact solution to the problem presented.

2. The answer choices will be whole numbers, percents, decimal numbers, fractions, or mixed numbers as appropriate to the question.

3. Answer choices specifying amounts of money will be aligned by place value. Other answer choices will not be aligned by place value.

4. Decimal numbers will have no more than three digits to the right of the decimal point.

5. Fractions will be presented in simplest terms.

6. An incorrect answer choice will be a number that is obtained as a result of one or more of the following types of errors:

   a. computation error: an error in the addition, subtraction, multiplication, or division of two numbers;

   b. misplaced decimal point: placing the decimal point in an incorrect position in either a correct solution or an incorrect solution;

   c. reciprocal error: a number that is the reciprocal of the correct answer;

   d. incorrect conversion: incorrectly converting a decimal number to a percent, a percent to a decimal number, a percent to a fraction, or a fraction to an equivalent fraction;

   e. common denominator error: an error in selecting the lowest common denominator or in renaming a fraction in terms of its lowest common denominator;

   f. incorrect simplification: incorrectly simplifying a fraction;

   g. missing operation: not operating on the data or omitting one of the required operations; or

   h. incorrect operation: performing an operation other than the required one.

SAMPLE ITEM ANSWER CHOICE DESCRIPTIONS:

Demonstrates a Working Knowledge of Percents

A. misplaced decimal point

B. missing operation

C. misplaced decimal point

D. correct

Calculates Base, Rate, and Percentage

A. missing operation

B. correct

C. incorrect operation

D. computation error

Understands Ratio and Proportion

A. correct

B. missing operation

C. missing operation

D. incorrect conversion

Understands Consumer Terms Involving Percents

A. missing operation

B. correct

C. computation error

D. incorrect operation

Solves Problems Involving Percent, Ratio, and Proportion

A. correct

B. missing operation

C. incorrect operation

D. computation error

INSTRUCTIONAL ANALYSIS:

Conversions Among Decimal Numbers, Fractions, Mixed Numbers, and Percents

Each type of conversion eligible for testing on the LEAP examination requires a different set of steps. These steps are presented below. In presenting the various types of conversion processes, provide practice in determining which particular conversion process would be required by a given problem.

Converting percents to decimals:

1. Drop the percent sign.
2. Move the decimal point two places to the left (in other words, divide by 100).

EXAMPLE: 70% = .70

Converting decimals to percents:

1. Move the decimal point two places to the right (in other words, multiply by 100).
2. Add a percent sign.

EXAMPLE: .70 = 70%

Converting a percent to a fraction in simplest terms:

1. Drop the percent sign.
2. Place the number in a fraction with a denominator of 100.
3. Reduce to simplest terms.

EXAMPLE: $70\% = \frac{70}{100} = \frac{7}{10}$

Converting common fractions to percents:

1. Divide the numerator by the denominator.
2. Move the decimal point two places to the right (in other words, multiply by 100).
3. Add a percent sign.

EXAMPLE: $\frac{7}{10} = 7 \div 10 = .70 = 70\%$

Converting improper fractions or mixed numbers to percents:

1. If a mixed number, convert to an improper fraction.

2. Divide the numerator by the denominator.

3. Move the decimal point two places to the right (in other words, multiply by 100).

4. Add a percent sign.

EXAMPLE: $1\frac{7}{10} = \frac{17}{10} = 1.70 = 170\%$

Calculating Rate, Base, and Percentage

The calculations of rate, base, and percentage involve some of the conversion processes described above. Emphasize to students that they need to read a given problem carefully to determine whether it is asking for calculation of rate, base, or percentage. Described below are the steps required in calculating each. Have students pay close attention to correct placement of the decimal point, particularly when working with percents greater that 100.

Find a percent of a given number (percentage):

EXAMPLE: What is 90% of 30?

Review what happens when a decimal less than 1 is multiplied by a number.

1. Change the percent to a decimal number (drop the percent sign and move the decimal point two places to the left, or divide by 100).

   90% = .90

2. Multiply the decimal number by the given number.

$$\begin{array}{r} 30 \\ \times\ .90 \\ \hline 27.00 \end{array}$$

Encourage students to check that the answer is reasonable. Given that 27 is close to 30, and 90% is close to 100%, the answer seems reasonable.

Find what percent one number is of another (rate):

EXAMPLE: 20 is what percent of 80?

1. Divide the number that represents a percent of the other number by that other number. (Divide the part by the whole.)

   20 80 = .25

2. Change the quotient to a percent.

   .25 = 25%

Find a number when a percent of it is known (base):

EXAMPLE: 30 is 40% of what number?

1. Change the percent to a decimal.

   40% = .40

2. Divide the given number by the percent

   30 .40 = 75

Solving for the Missing Term of a Proportion

Solving for the missing term of a proportion requires an understanding of how equalities work. Therefore, you may wish to review the properties of equalities before examining with students the steps necessary to solving these types of items, as shown below:

EXAMPLE: Solve the proportion $\frac{4}{9} = \frac{x}{36}$

1. Find the cross products of the two ratios.

   $4(36) = 9x$

   $144 = 9x$

2. Solve the equation by dividing each side by 9.

   $\frac{144}{9} = \frac{9x}{}$

   $16 = x$

Encourage students to check their answer by substituting it into the problem:

$$\frac{4}{9} = \frac{x}{36}$$

$$\frac{4}{9} = \frac{16}{36}$$

$$4(36) = 9(16)$$
$$144 = 144$$

Solving Word Problems Involving Percent, Ratio, and Proportion

Word problems in this skill area may involve any of the percent, ratio, or proportion computations described earlier in this section. Therefore, you may wish to review each of these processes in the context of word problems. You may also wish to focus instruction on computations related to personal finances. Some items in this skill area will involve personal aspects of finance such as banking (interest), taxes (sales and income), and purchasing (rate of discount).

Regardless of the types of computations required, the steps involved in solving word problems are essentially the same, as shown below. The problem-solving sample item is used to illustrate these steps.

1. Read the problem carefully. Determine what question is to be answered.

   - The sample problem asks how much interest will accrue over a year.

2. Determine the necessary operation(s). If more than one operation is needed, decide the order in which they must be performed.

   - The two operations required are (1) changing a percent to a decimal number and (2) multiplying.

3. Perform the operations.

$$6\% = .06; \quad \begin{array}{r} 600.00 \\ \times \quad .06 \\ \hline 36.0000 \end{array} = \$36.00$$

4. Reread the problem, and decide if the answer is reasonable.

   - Estimating that 10% of \$600.00 is \$60.00, it seems reasonable that 6% of \$600 could be \$36.00.

SKILL AREA: MEASUREMENT
(Domain VII, Standards DEFG)

The skill area testing measurement includes four standards:

Understands and Uses Linear Measures: Adds or subtracts measurements of length, using either the customary system (inches, feet, yards) or the metric (SI) system (centimeters, meters).

Understands and Uses Measures of Weight (Mass): Adds or subtracts measurements of weight, using either the customary system (ounces, pounds) or the metric (SI) system (grams, kilograms).

Understands and Uses Measures of Capacity: Adds or subtracts measurements of capacity, using either the customary system (pints, quarts, gallons) or the metric (SI) system (milliliters, liters).

Solves Problems Involving Measurement: Solves problems involving measurement, using problem-solving processes and strategies.

SAMPLE ITEMS:

Understands and Uses Linear Measures

6 feet 7 inches
- 2 feet 11 inches

A. 3 feet 6 inches

B. 4 feet 8 inches

C. 3 feet 8 inches

D. 4 feet 4 inches

Understands and Uses Measures of Weight (Mass)

8 kilograms 450 grams
+ 9 kilograms 680 grams

A. 17 kilograms 40 grams

B. 17 kilograms 30 grams

C. 18 kilograms 140 grams

D. 18 kilograms 130 grams

Understands and Uses Measures of Capacity

28 liters 60 milliliters
+ 43 liters 50 milliliters

A. 71 liters 110 milliliters

B. 72 liters 10 milliliters

C. 71 liters 120 milliliters

D. 71 liters 10 milliliters

Solves Problems Involving Measurement

Ellen is mailing a book to her friend in a mailing box that weighs 9 ounces. When the book is in the box, the entire package weighs 3 pounds, 1 ounce. How much does the book weigh?

A. 2 pounds 8 ounces

B. 2 pounds 4 ounces

C. 2 pounds 7 ounces

D. 3 pounds 10 ounces

DESCRIPTION OF TEST QUESTIONS:

1. The student will be presented with a numeric or word problem that requires either addition or subtraction of measurements. Each measurement will consist of no more than two units. For example, "2 feet 6 inches" is an eligible measurement, whereas "3 yards 2 feet 6 inches" is not.

2. Only whole numbers will be used. Numbers in customary measurement problems will be no larger than two digits per unit measure; numbers in metric measurement problems will be no larger than three digits per unit measure. Numbers will be labeled in appropriate units.

3. Addition operations will involve a maximum of three measurements. All other operations will involve a maximum of two measurements.

4. Some problems will require a conversion between units within one system of measurement. A conversion table will not be presented.

5. A test question will not include both metric and customary units and will not require conversions between the two measurement systems.

6. Eligible units of measure will be:

METRIC	CUSTOMARY
Length	
millimeter	inch
centimeter	foot
meter	yard
kilometer	
Weight/Mass	
milligram	ounce
gram	pound
kilogram	
Capacity	
milliliter	pint
liter	quart
	gallon

7. Numeric problems will be presented in vertical format.

8. For items involving word problems:

   a. No more than two operations will be required.

   b. Irrelevant numeric information will not be included.

DESCRIPTION OF ANSWER CHOICES:

1. The correct answer choice will be the measurement that represents the accurate sum or difference for the given problem.

2. Each answer choice will be a measurement labeled in the appropriate unit(s). Units in the answer choices may differ from those in the test question.

3. An incorrect answer choice will be a measurement that is obtained as a result of one or more of the following types of errors:

   a. computation error: an error in the addition or subtraction of two numbers;

   b. conversion error: using an incorrect conversion constant or regrouping between units incorrectly; or

   c. incorrect operation: performing an operation other than a required one.

SAMPLE ITEM ANSWER CHOICE DESCRIPTIONS:

Understands and Uses Linear Measures

A. conversion error

B. conversion error

C. correct

D. computation error

Understands and Uses Measures of Weight (Mass)

A. computation error

B. conversion error

C. computation error

D. correct

Understands and Uses Capacity

A. correct

B. conversion error

C. computation error

D. conversion error

Solves Problems Involving Measurement

A. correct

B. conversion error

C. computation error

D. incorrect operation

INSTRUCTIONAL ANALYSIS:

A LEAP measurement item will require basic addition or subtraction. In some cases, conversion between units within the same measurement system will be required; however, students will not be asked to perform conversions between customary and metric measurement systems. Because conversion tables will not be provided during the test, students will need to memorize eligible (within-system) conversions.

For measurement items involving addition, students will need to recognize when simplification is required. Consider the sample item for Understands and Uses Measures of Weight. Explain to students that kilograms should be added separately from the grams. Upon obtaining the sum 17 kilograms, 1130 grams, students should recognize that 1000 grams equals 1 kilogram. Therefore, the correctly simplified answer is 18 kilograms, 130 grams, as shown below.

```
  8 kilograms  450 grams
+ 9 kilograms  680 grams
 17 kilograms 1130 grams = 18 kilograms 130 grams
```

With respect to subtraction, present students with problems that require conversion, such as the sample item for Understands and Uses Linear Measures. To solve this problem, first change 6 feet to 5 feet, 12 inches. Then add 7 inches to 5 feet, 12 inches to obtain 5 feet, 19 inches. Now the subtraction can be performed, as follows:

$$\begin{array}{r} 6 \text{ feet } 7 \text{ inches} \\ -\ 2 \text{ feet } 11 \text{ inches} \end{array} = \begin{array}{r} 5 \text{ feet } 19 \text{ inches} \\ -\ 2 \text{ feet } 11 \text{ inches} \\ \hline 3 \text{ feet } 8 \text{ inches} \end{array}$$

Word problems require the same conversion skills as the numeric measurement problems, but they also require the use of problem-solving strategies. You may wish to review the steps below with your students; the steps are described in relation to the sample word problem.

1. Read the problem carefully. Determine what question is to be answered.

   - The question asks for the weight of the book.

2. Determine the necessary operation(s). If more than one operation is needed, decide the order in which they must be performed.

   - To isolate the weight of the book from the weight of the entire package, subtraction is the appropriate operation.

3. Perform the computations.

   - $$\begin{array}{r} 3 \text{ pounds } 1 \text{ ounce} \\ -\quad 9 \text{ ounces} \end{array} = \begin{array}{r} 2 \text{ pounds } 17 \text{ ounces} \\ -\quad 9 \text{ ounces} \\ \hline 2 \text{ pounds } 8 \text{ ounces} \end{array}$$

4. Reread the problem and decide if the answer is reasonable.

   - Given that the box and book together weight 3 pounds, 1 ounce, the obtained answer, 2 pounds 8 ounces for the weight of the book alone seems reasonable.

SKILL AREA: GEOMETRY
(Domain XIII, Standards CDE)

The skill area testing geometry includes three standards:

Determines Perimeter and Area of Plane Figures: Computes (1) the perimeter of any polygon; (2) the area of squares, rectangles, triangles, and circles; (3) the circumference of circles; and (4) the length of a side of a right triangle, using the Pythagorean rule.

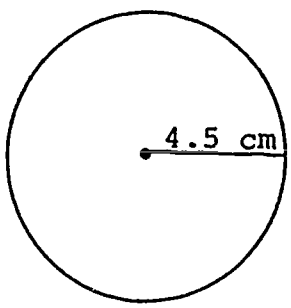
Recognizes and Measures Spatial Figures: Computes the volume of cubes and rectangular solids.

Solves Problems Involving Geometry: Solves word problems involving plane and spatial figures, using the problem-solving process and strategies.

SAMPLE ITEMS:

Determines Perimeter and Area of Plane Figures

Find the area of the circle below:



A. 4.5π cm^2

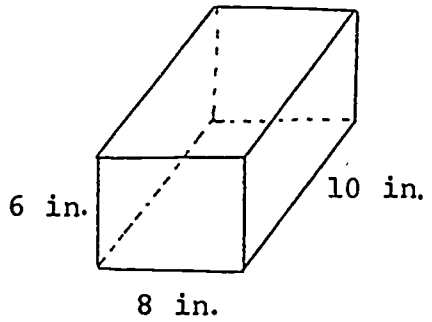
B. 9.0π cm^2

C. 20.05π cm^2

D. 20.25π cm^2

Recognizes and Measures Spatial Figures

What is the volume of this figure?



A. 24 $in.^3$

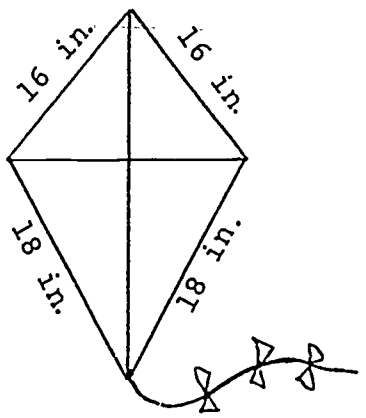
B. 80 $in.^3$

C. 480 $in.^3$

D. 560 $in.^3$

Solves Problems Involving Geometry

Sandra bought a new kite. She wanted to determine the perimeter of the new kite in order to compare it to her old kite. What is the perimeter of Sandra's new kite as pictured below?



A. 34 in.

B. 68 in.

C. 76 in.

D. 288 in.

DESCRIPTION OF TEST QUESTIONS:

1. The student will be presented with a geometric figure showing labeled measurements for given dimensions and/or a word problem that requires one of the following:

   a. calculation of the perimeter, area, circumference, or volume of a specified figure; or

   b. application of the Pythagorean rule.

2. Numbers will be whole numbers or decimal numbers with no more than two digits to the left of the decimal point and no more than one digit to the right of the decimal point. Numbers will be labeled in appropriate units. All measurement units may be abbreviated (that is, in., ft., yd., mm, cm, m), with the exception of liter.

3. The test question may require the use of a formula to calculate one of the following:

PROPERTY	GEOMETRIC FIGURE
perimeter	any polygon
area	square rectangle triangle circle
circumference	circle
volume	cube rectangular solid

4. The correct formula for each item will not be provided in the test item. Instead, a list of eligible formulas will be provided only for the following properties or dimensions: area of a rectangle, triangle, or circle; circumference of a circle; volume of a rectangular solid; and the Pythagorean rule.

5. A word problem will require no operation other than those required in the application of one eligible geometric formula.

6. Irrelevant numeric information will not be included.

DESCRIPTION OF ANSWER CHOICES:

1. The correct answer choice will be the accurate measure of the geometric property in question.

2. Each answer choice will be labeled in appropriate units.

3. The answer choices for a test item involving the area or circumference of a circle will be stated in terms of π (for example, 4.5π cm^2).

4. An incorrect answer choice will be a number that is obtained as a result of one or more of the following types of errors:

   a. computation error: an error in the addition, subtraction, multiplication, or division of two numbers;

   b. misplaced decimal point: placing the decimal point in an incorrect position on either a correct solution or an incorrect solution;

   c. missing operation: not operating on the data or omitting one of the required operations;

   d. incorrect operation: performing an operation other than a required one; or

   e. incorrect formula: using an incorrect formula to calculate a dimension or property of the figure.

SAMPLE ITEM ANSWER CHOICE DESCRIPTIONS:

Determines Perimeter and Area of Plane Figures

A. missing operation

B. incorrect formula

C. computation error

D. correct

Recognizes and Measures Spatial Figures

A. incorrect formula

B. missing operation

C. correct

D. computation error

Solves Problems Involving Geometry

A. missing operation

B. correct

C. computation error

D. incorrect formula

INSTRUCTIONAL ANALYSIS:

For this skill area, students must be able to select the appropriate geometric formula and solve for the properties (perimeter, area, circumference, or volume) of common geometric figures. A list of eligible geometric formulas (excluding the formulas for perimeter of a polygon and area of a square) will be provided on the LEAP test. Although memorization of formulas is not required (except for perimeter of a polygon and area of a square), students will need to be familiar with all the eligible formulas and know when and how to use them.

To successfully solve problems with geometric formulas, students must know the basic geometric shapes and their properties. Students should also know related terminology such as length, width, base, radius, and hypotenuse. Knowledge of these terms is essential for appropriate use of the geometric formulas. Students should be given sufficient practice solving problems using the various geometric formulas. You may find that application of the Pythagorean rule warrants particular emphasis. Test items involving the Pythagorean rule may require students to solve for the length of any side of a right triangle.

Some items on the test will display a geometric figure with appropriately labeled dimensions and direct the student to determine the property in question. For such items, the student simply needs to select the appropriate formula and perform the computations. (Students don't need to multiply by a value for π; answers will be presented in terms of π, such as 10.5π.)

Other items will be word problems that require that students find the area, perimeter, circumference, or volume of a figure. A word problem may or may not display a figure for use in solving the problem. Students may have to determine what geometry property is being asked for by the context of the problem; for example, a problem asking how much a container will hold requires solving for the volume of that container. Thus, in some word problems, the geometric property in question must be identified before the appropriate formula can be selected. To help students clarify what is being asked for in word problems you may want to encourage students to incorporate the information presented in the problem into drawings that will allow visualization of the geometric figure described.

You may wish to review with students the steps necessary to solve geometry word problems, as presented below. The sample word problem is used to demonstrate each step.

1. Read the problem carefully. Determine the geometric property that needs to be computed. Sketch the described figure and dimensions, if necessary.

   - The problem asks for the perimeter of Sandra's kite. (There is no need to sketch the kite and dimensions because they are displayed in the problem.)

2. Decide which formula to use.

   - $P = S_1 + S_2 + S_3 + S_4$ (Note: The formula for perimeter will <u>not</u> be provided on the LEAP test.)

3. Perform the computations.

   - $P = 16 + 16 + 18 + 18 = 68$

4. Reread the problem and decide if the answer is reasonable.

   - Using estimation:

     $P = 20 + 20 + 20 + 20 = 80$

     Because each side is actually smaller than 20 inches, the answer seems reasonable.

SKILL AREA: GRAPHS, PROBABILITY, AND STATISTICS (Domain IX, Standards ABC)

The skill area testing graphs, probability, and statistics includes three standards:

Constructs and Interprets Graphs: Interprets and uses information presented in maps, charts, and tables.

Uses Probability and Statistics: Computes the average of not more than five numbers in the context of a word problem.

Solves Problems Involving Graphs, Probability, and Statistics: Solves word problems involving graphs, using problem-solving processes and strategies.

SAMPLE ITEMS:

Constructs and Interprets Graphs

Use the chart to answer the question that follows.

Consumption of Major Food Items in the U.S.
(in pounds per person)

Food Item	1975	1980	1985
Beef	84.0	76.5	77.2
Eggs	39.1	34.6	33.6
Chicken	40.4	50.1	51.7
Cheese	11.5	17.6	18.0
Fresh fruit	78.9	85.7	87.3

For which food item did per-person consumption drop from 1980 to 1985?

A. eggs

B. chicken

C. cheese

D. fresh fruit

Uses Probability and Statistics

In Marsha's math class there is a test given every week. Each test has the same number of possible points. Marsha's scores on the first 5 tests are shown below.

Test:	1	2	3	4	5
Score:	39	31	45	26	34

What is Marsha's average score for these 5 tests?

A. 44

B. 35

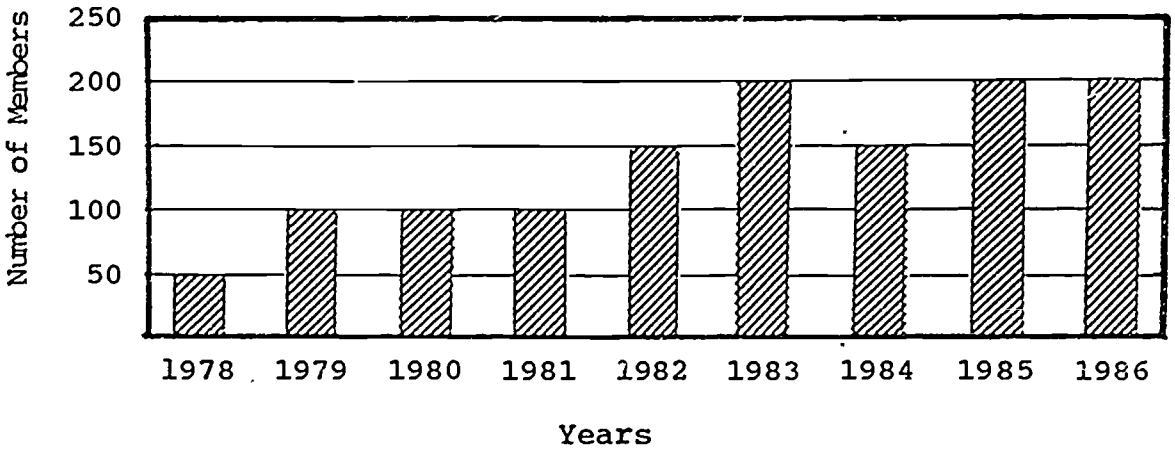
C. 45

D. 31

Solves Problems Involving Graphs, Probability, and Statistics

Use the graph to answer the question that follows.

Membership in Chatsworth High School Vocal Club for 1978-1986



How many more members did the vocal club have in 1984 than in 1981?

A. 0 members

B. 50 members

C. 100 members

D. 150 members

DESCRIPTION OF TEST QUESTIONS:

1. For test items involving graphs, charts, or tables:

   a. The student will be presented with a pictograph, bar graph, circle graph, line graph, chart, or table. One to three questions will appear below each display. Each question will require the student to either interpret or operate (perform addition, subtraction, multiplication, or division) on the data in the display.

   b. A display will have the following characteristics:

      (1) Each display will have a title describing the information presented. Both axes of bar and line graphs will be labeled, as will be the columns and rows of tables and charts, and the horizontal axes of pictographs.

      (2) Numbers represented on displays will be whole numbers or decimal numbers consisting of no more than 6 digits. Decimal numbers will have no more than 2 digits to the right of the decimal point.

      (3) A pictograph will indicate the value of a given element on the graph (for example, each 💻 represents 100 computers). There will be no more than 10 elements represented. One half of an element may be shown.

      (4) In a bar graph, bars may be represented vertically or horizontally. A bar graph will present one or two relationships and will have no more than 12 bars. Students may be required to estimate a graphed value.

(5) A circle graph will have no more than eight segments. Each segment will be labeled with a title and a percentage. Percentages will be limited to whole numbers.

(6) A chart or table will have between 5 and 20 numeric entries.

(7) A line graph may show one or two straight or dotted lines, permitting comparisons of data points or trends. Students may be required to estimate a graphed value.

c. Irrelevant numeric information may be included.

d. For test items involving interpretation of information in a display, one of the following will be required:

(1) an accurate reading of a single data point or element represented in the display;

(2) an accurate interpretation of the relationship between points on the display; or

(3) an accurate count of the number of points below or above a given value on a graph.

e. For items involving operating on information in a display, up to two operations will be required.

2. For test items involving computation of a mean:

a. The student will be presented with a word problem that presents up to five numeric values of a variable followed by a question or statement that specifies that the average be calculated for that set of values.

b. Values in the problem will be natural numbers, each with a maximum of three digits.

c. Irrelevant numeric information will not be included.

DESCRIPTION OF ANSWER CHOICES:

1. For test items involving graphs, charts, or tables:

a. The correct answer choice will be the exact solution to the problem, labeled in the appropriate unit.

b. An incorrect answer choice will be a number that is obtained as the result of one or more of the following types of errors:

(1) computation error: an error in the addition, subtraction, multiplication, or division of two numbers;

(2) missing operation: not operating on the data or omitting one of the required operations;

(3) incorrect operation: performing an operation other than the required one;

(4) incorrect order: performing the required operations in the wrong order; or

(5) inaccurate information: selecting incorrect information from a table, chart, or graph.

2. For test items involving computation of a mean:

a. The correct answer choice will be the average (mean) of a set of values labeled in appropriate units.

b. An incorrect answer choice will be a number that is obtained as the result of one or more of the following types of errors:

(1) computation error: an error in the addition, multiplication, or division of two numbers;

(2) incorrect divisor: using an incorrect divisor due to incorrect counting;

(3) missing operation: not operating on the data or omitting one of the required operations; or

(4) incorrect operation: performing an operation other than the required one.

SAMPLE ITEM ANSWER CHOICE DESCRIPTIONS:

Constructs and Interprets Graphs

A. correct

B. inaccurate information

C. inaccurate information

D. inaccurate information

Uses Probability and Statistics

A. incorrect divisor

B. correct

C. missing operation

D. computation error

Solves Problems Involving Graphs, Probability, and Statistics

A. inaccurate information

B. correct

C. inaccurate information

D. missing operation

INSTRUCTIONAL ANALYSIS:

Graphs, Charts, and Tables

To help students successfully interpret and operate on data from graphs, charts, and tables, you may wish to focus instruction on how to read information presented in various types of displays. Present examples of graphs and tables from magazines and newspapers and have students construct their own displays based on information of their choice. Be sure that students are familiar with the format and purpose of the different kinds of displays, which are briefly described below.

Bar Graph: A bar graph typically compares data; for example, the number of books read by different people or the heights of various buildings. The bars, displayed vertically or horizontally, can be compared according to their relative lengths or heights.

Pictograph: In a pictograph, which is also used to compare sets of data, a symbol is used to represent a specified number of items. For example, the symbol ♀ could be used to represent 100 members in a pictograph comparing the respective memberships of various organizations.

Line Graph: A line graph typically shows changes in data; for example, the amount of sales a salesperson makes over a period of several months or a child's height over years.

Table: A table (or chart) is used to organize and present, in rows and columns, large amounts of numeric information.

Provide students with sufficient practice solving problems with graphs, charts, and tables. Begin with simple questions that require, for example, the reading of a specific data point. Then introduce questions that require interpretation or deductive reasoning, such as in the sample item for constructing and interpreting graphs. Finally, present questions that require operating on displayed data, such as the sample-item question for problem-solving involving graphs. Point out that these displays contain essential and nonessential information with respect to any potential test question. Therefore, it is important to first isolate what the question is asking, and then examine the display for the relevant information.

In presenting practice items to students, you may also wish to review a sequence of steps that can be used to solve word problems involving graphs. These steps are described below with respect to the problem-solving sample item.

1. Read the problem carefully. Determine what question is to be answered.

   - The question asks for the difference in vocal club membership in 1984 and 1981.

2. Determine the necessary operation(s). If more than one operation is needed, decide the order in which they must be performed.

   - Subtraction is indicated.

3. Locate the displayed information that is needed to answer the question.

   - 1984: 150 members
     1981: 100 members

4. Perform the computation.

   - 150 - 100 = 50 members

5. Reread the problem and decide if the answer is reasonable.

   - The answer appears reasonable given the question and the heights of the data bars displayed on the graph.

Computing an Average (Mean)

Point out to students that the concept of "average" is used frequently in everyday life. Common types of averages include grade-point average, batting average, average life expectancy, and average earnings. Explain that the term "mean" is used synonymously with average. The average (or mean) is the sum of all relevant values (for example, course grades, batting scores, individual lifespans, or specific amounts earned) divided by the number of values. The following steps, illustrated with the sample item presenting Martha's test grades, are used in calculating an average:

1. Sum the values to be averaged.

   - 39 + 31 + 45 + 26 + 34 = 175

2. Count the number of values that were summed in Step 1, including zeros.

   - There are five test scores to average.

3. Divide the sum obtained in Step 1 by the total number of values obtained in Step 2.

   - 175 ÷ 5 = 35

You may want to reinforce the concept of average by having students compute averages based on their own experiences; for example, the average of their last five test scores in a particular class or the average amount of time they spend per night watching television.

SKILL AREA: PRE-ALGEBRA
(Domain X, Standards BCD)

The skill area testing pre-algebra includes three standards:

Uses Exponents: Writes the exponential form of a power of ten given in standard form or writes the standard form of a number given in exponential form.

Identifies and Uses Integers and Rational Numbers: Adds, subtracts, multiplies, and divides integers.

Operates on Real Numbers: Performs multiple operations on real numbers, with and without grouping symbols.

Uses Exponents

SAMPLE ITEM:

Simplify: 6^3

A. 216

B. 36

C. 18

D. 186

DESCRIPTION OF TEST QUESTIONS:

1. The student will be presented with a test item that requires one of the following:

   a. selecting the equivalent exponential form of a number that is presented in standard form; or

   b. selecting the equivalent standard form of a number presented in exponential form.

2. For test items that present a number in standard form, the given number will be a power of 10 and will not exceed 1000.

3. For test items that present a number in exponential form, the base will be a whole number from 0 to 9 and the exponent will be a whole number from 0 to 3.

DESCRIPTION OF ANSWER CHOICES:

1. For test items that present numbers in standard form:

   a. The correct answer choice will be the number in exponential form that is equivalent to the number given in the test question.

   b. An incorrect answer choice will be a number in exponential form that is a power of ten not equal to the number presented in the test question.

2. For test items that present numbers in exponential form:

   a. The correct answer choice will be the whole number that is equivalent to the exponential number given in the test question.

   b. An incorrect answer choice will be a number that is obtained as a result of one or more of the following types of errors:

      (1) computation error: an error in the multiplication of two numbers;

      (2) incorrect operation: performing an operation other than the required one, specifically, adding rather than multiplying the base;

      (3) incorrect power: multiplying the base fewer or more times than indicated by the exponent; or

      (4) reversal: reversing the base with the exponent and multiplying accordingly.

SAMPLE ITEM ANSWER CHOICE DESCRIPTIONS:

A. correct

B. incorrect power

C. incorrect operation

D. computation error

INSTRUCTIONAL ANALYSIS:

This standard requires that students be familiar with exponential notation. In describing exponential notation to students, distinguish between the base and the exponent. For

example, in the exponential number 5^4, 5 is the base and 4 is the exponent. The exponent indicates the number of times to use the base as a factor. Thus, $5^4 = 5 \times 5 \times 5 \times 5 = 625$.

Present some general rules of exponents for students to learn. For example, for any number x:

$x^0 = 1$ (except when x is 0);
$x^1 = x$; and
$1^x = 1$.

Thus, $5^0 = 1$, $5^1 = 5$, and $1^5 = 1$.

You may wish to explain reading conventions for exponential numbers to students. For example, the expression 6^4 is usually read "six to the fourth power." When the exponent is 2, as in the expression 8^2, the number is usually read "eight squared." Likewise, when the exponent is 3, as in the expression 7^3, the number is usually read "seven cubed."

Proficiency with exponential notation simply requires practice. As students work with exponential notation, steer them away from common errors such as mixing up the base and the exponent ($2^3 \neq 9$) or multiplying the base by the exponent ($2^3 \neq 6$).

Identifies and Uses Integers and Rational Numbers

SAMPLE ITEM:

Simplify: 36 - (-8)

A. 28

B. 46

C. 44

D. 34

DESCRIPTION OF TEST QUESTIONS:

1. The student will be presented with a mathematical expression that requires a single operation (that is, addition, subtraction, multiplication, or division) between integers. Integers will be no larger than two digits. At least one integer will be negative.

2. Addition problems will be limited to two addends. Multiplication problems will be limited to two factors. For division problems, the dividend and divisor will be chosen to provide a quotient with no remainder.

DESCRIPTION OF ANSWER CHOICES:

1. The correct answer choice will be an integer that represents the accurate simplification of the given mathematical expression.

2. An incorrect answer choice will be an integer that is obtained by one or more of the following types of errors:

   a. computation error: an error in the addition, subtraction, multiplication, or division of two numbers; or

   b. incorrect sign: using the wrong sign in either the calculations or the response, that is, using a positive sign when a negative sign is indicated or vice versa.

SAMPLE ITEM ANSWER CHOICE DESCRIPTIONS:

A. incorrect sign

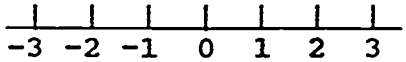
B. computation error

C. correct

D. computation error

INSTRUCTIONAL ANALYSIS:

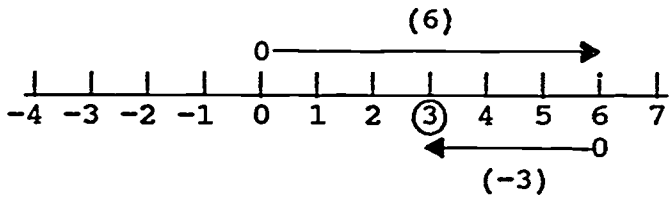
Test items for this standard involve basic operations with integers. Students should be aware that integers consist of whole numbers and their opposites (..., -3, -2, -1, 0, 1, 2, 3, ...). Use a number line to demonstrate the relative location of positive and negative integers.



Negative numbers start to the left of 0 with -1, -2, -3, and so on. Positive numbers start to the right of 0 with 1, 2, 3, and so on. A minus sign may be used to denote a negative number, whereas a plus sign may or may not be used to denote a positive number.

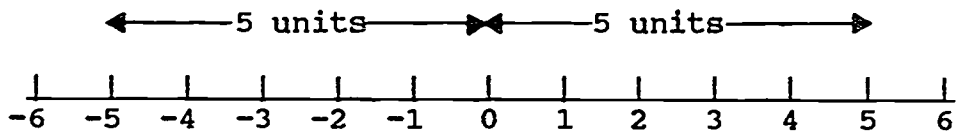
To add integers using the number line, begin at 0, the origin of the number line. Then move to the right for a positive number or left for a negative number. For example, given the expression "6 + (-3)," start at 0 and move 6 places to the right. Then, move 3 places to the left. As shown below, the correct answer is 3.

Step 1



Step 2

Introduce the concept of absolute value to students. Explain that absolute value is the number of units that a number is from 0 on the number line. Thus, as illustrated below, the absolute value of both -5 and 5 is 5.



Students can use basic rules involving absolute value to compute successfully with integers. Review these rules with students, using examples as follows:

Adding Integers

1. To add integers with the same sign, add their absolute values. The sum has the same sign as the addends.

   examples: 9 + 3 = 12; -9 + (-3) = -12

2. To add integers with different signs, find the difference of their absolute values. The sum has the same sign as the addend with the greater absolute value.

   examples: 9 + (-3) = 6; -9 + 3 = -6

Subtracting Integers

1. To subtract an integer, add its opposite.

   examples: $9 - 3 = 9 + (-3) = 6$;
   $9 - (-3) = 9 + 3 = 12$

Multiplying Integers

1. The product of two integers with the same sign is positive.

   examples: $9 \times 3 = 27$; $-9 \times (-3) = 27$

2. The product of two integers with different signs is negative.

   examples: $9 \times (-3) = -27$; $-9 \times 3 = -27$

Dividing Integers

1. The quotient of two integers with the same sign is positive.

   examples: $9 \div 3 = 3$; $-9 \div (-3) = 3$

2. The quotient of two integers with different signs is negative.

   examples: $-9 \div 3 = -3$; $9 \div (-3) = -3$

Present various examples demonstrating these basic rules of operating on integers. Students will need to memorize these rules in order to compute with integers.

Operates on Real Numbers

SAMPLE ITEM:

Simplify: $5(2-6)$

A. 20

B. -15

C. 4

D. -20

DESCRIPTION OF TEST QUESTION:

1. The student will be presented with a mathematical expression containing integers that must be simplified by use of multiple operations. The expressions may or may not contain grouping symbols ( ).

2. Each expression will contain at least two <u>different</u> operations, but no more than four operations.

3. Integers will be no larger than two digits.

DESCRIPTION OF ANSWER CHOICES:

1. The correct answer choice will be the integer that represents the accurate simplification of the given mathematical expression.

2. An incorrect answer choice will be an integer that is obtained as a result of one or more of the following types of errors:

   a. <u>computation error</u>: an error in the addition, subtraction, multiplication, or division of two numbers;

   b. <u>incorrect sign</u>: using the wrong sign in either the calculations or the response, that is, using a positive sign when a negative sign is indicated or vice versa;

   c. <u>missing operation</u>: not operating on the data or omitting one of the required operations;

   d. <u>incorrect operation</u>: performing an operation other than the required one; or

   e. <u>incorrect order</u>: performing the required operations in the wrong order.

SAMPLE ITEM ANSWER CHOICE DESCRIPTIONS:

A. incorrect sign

B. computation error

C. incorrect order

D. correct

INSTRUCTIONAL ANALYSIS:

Students are expected to know how to perform multiple operations on integers, with and/or without grouping symbols. Explain to students that when more than one operation is specified in a mathematical expression, grouping symbols are often used to clarify the order in which the operations are to be carried out. The most frequently used grouping symbols are parentheses ( ), brackets [ ], and braces { }. The operation(s) within the grouping symbols should be performed first. For example:

$$
\begin{aligned}
& (6 \cdot 4) + 7 \\
= \quad & 24 + 7 \\
= \quad & 31
\end{aligned}
$$

whereas

$$
\begin{aligned}
& 6 \cdot (4 + 7) \\
= \quad & 6 \cdot 11 \\
= \quad & 66
\end{aligned}
$$

[Note: This expression is commonly written as $6(4 + 7)$.]

Thus, different values are obtained depending on where the grouping symbols are placed in a mathematical expression.

Explain to students that in the absence of grouping symbols, expressions should be simplified according to the following order of operations:

1. Simplify products and quotients in order from left to right.

2. Simplify sums and differences in order from left to right.

Demonstrate this order of operations using various examples such as the following:

$$
\begin{aligned}
& 12 - 5 \cdot 4 + 3 \\
= \quad & 12 - 20 + 3 \\
= \quad & -8 + 3 \\
= \quad & -5
\end{aligned}
$$

$$
\begin{aligned}
& 14 + 16 \div 8 - 9 \\
= \quad & 14 + 2 - 9 \\
= \quad & 16 - 9 \\
= \quad & 7
\end{aligned}
$$

Knowing how to group operations in evaluating expressions with integers will help prepare students for evaluating expressions with variables, as is required in algebra.

SKILL AREA: ALGEBRA
(Domain XI, Standards BCDE)

The skill area testing algebra includes four standards:

Understands and Uses the Language of Algebra: (1) Evaluates algebraic expressions given a replacement set for the variable(s) and (2) uses properties of equality to solve linear equations in one variable.

Solves and Uses First Degree Equations and Inequalities: (1) Solves linear equations that contain variables in both members of the equation, (2) Solves inequalities that contain variables in one member of the inequality, and (3) Uses linear equations to solve word problems involving number relations.

Understands and Operates on Polynomials: Adds, subtracts, multiplies, and divides monomials and polynomials.

Factors Polynomials: Factors special types of polynomials, that is, a polynomial that is factorable into a monomial and a polynomial, a difference of two squares, a trinomial square, and a quadratic trinomial.

Understands and Uses the Language of Algebra

SAMPLE ITEM:

If $a = 3$ and $b = 5$, then $ab^2 - 3b = ?$

A. 10

B. 35

C. 60

D. 90

DESCRIPTION OF TEST QUESTIONS:

1. The student will be presented with a problem that requires either:

   a. evaluating algebraic expressions given a replacement set for the variable(s); or

b. using properties of equality to solve linear equations in one variable.

2. For test items that require the evaluation of algebraic expressions given a replacement set for the variable(s):

   a. An open-ended question will be presented in the following form:

      If a = (number) [and b = (number)], then (algebraic expression) = ?

      The value for all variables (a or b) will be given.

   b. The algebraic expression will contain no more than two variables and no more than two terms. The expression may require adding, subtracting, multiplying (including squaring), or dividing the variables and/or numerals within a term. However, operations between terms in the expression will be limited to addition and subtraction.

   c. Variables will be integers. Variable coefficients and constants will be integers, fractions, or exponents (squares only). Integers, the bases of exponents, and the numerators and denominators of fractions will not exceed two digits.

3. For test items that require using properties of equality to solve linear equations in one variable:

   a. The student will be directed to solve for a variable in a linear equation that contains only one variable. The variable will appear only once in the equation.

   b. The equation may require multiple steps to solve.

   c. Variable coefficients and constants will be positive or negative fractions in lowest terms or integers. Integers as well as numerators and denominators of fractions will be no larger than two digits.

DESCRIPTION OF ANSWER CHOICES:

1. For test items that require the evaluation of algebraic expressions given a replacement set of variables:

   a. The correct answer choice will be the integer that accurately solves the algebraic expression.

b. An incorrect answer choice will be a number that is obtained as a result of one or more of the following types of errors:

(1) computation error: an error in the addition, subtraction, multiplication, or division of two numbers;

(2) incorrect sign: using the wrong sign in either the calculations or the response, that is, using a positive sign when a negative sign is indicated or vice versa;

(3) wrong value: using the wrong value for a given variable;

(4) missing operation: not operating on the data or omitting one of the required operations;

(5) incorrect operation: performing an operation other than the required one; or

(6) incorrect order: performing the required operations in the wrong order.

SAMPLE ITEM ANSWER CHOICE DESCRIPTIONS:

A. missing operation

B. computation error

C. correct

D. incorrect operation

INSTRUCTIONAL ANALYSIS:

In order for students to learn how to evaluate algebraic expressions and solve simple linear equations, they must be familiar with basic algebraic vocabulary and concepts. Explain to students that in algebra we use symbols to represent numbers. Such symbols, usually shown as letters, are called variables. A mathematical phrase consisting of variables, numerals, and operation signs is called an algebraic expression. Variables in an algebraic expression can be replaced with numbers. These numbers represent the replacement set for a given variable. Numbers in the replacement set of a variable are known as the values of the variable. For example:

Suppose the replacement set for variable x consists of the value 5. Substituting for the variable in the algebraic expression 10 + x, we get 10 + 5 = 15.

Repeat this process with students, using various replacement sets and simple algebraic expressions. As students grasp the concept, introduce more complex algebraic expressions with more than one variable or more than one number in the replacement set. Explain that this process of (1) replacing each variable in an expression with the numeral for a given value of the variable and (2) simplifying the result is known as evaluating the algebraic expression for the given values.

Once students have achieved proficiency in evaluation of algebraic expressions, they will be ready to manipulate an algebraic equation to solve for a variable. Explain that an algebraic equation is a mathematical statement with an equality symbol between numerical and/or variable expressions. For example:

$x + 5 = 12$ $\quad 9 - x = 2$ $\quad 2x - 3 = 11$

To solve an equation means to find a value for the variable that will make the equation a true statement. This is achieved by replacing the equation by equivalent equations and/or simplifying the equation so the variable is alone on one side of the equation and its value can be determined easily. Demonstrate how the following properties of equality can be used to set the variable on one side of the equation in order to solve the equation.

Addition Property of Equality

Adding the same number to each side of an equation forms an equivalent equation. For example:

$$x - 5 = 11$$

and

$$x - 5 + 5 = 11 + 5$$

thus

$$x = 16$$

Subtraction Property of Equality

Subtracting the same number from each side of an equation forms an equivalent equation. For example:

$$x + 8 = 20$$

and

$$x + 8 - 8 = 20 - 8$$

thus

$$x = 12$$

Multiplication Property of Equality

Multiplying every term on each side of an equation by the same number forms an equivalent equation. For example:

$$\frac{1}{4}x = 10$$

and

$$4(\frac{1}{4}x) = 10(4)$$

thus

$$x = 40$$

Division Property of Equality

Dividing every term on each side of an equation by the same number forms an equivalent equation. For example:

$$3x = 30$$

and

$$\frac{3x}{3} = \frac{30}{3}$$

thus

$$x = 10$$

Students will be required to use one or two of these properties to solve each problem. Begin by giving students practice with applying one of these properties, as in the examples provided above. Then extend this instruction to the use of two properties, such as in the solving of the following equation:

$$\begin{aligned} 2x + 5 &= 13 \\ 2x + 5 - 5 &= 13 - 5 \\ 2x &= 8 \\ \frac{2x}{2} &= \frac{8}{2} \\ x &= 4 \end{aligned}$$

Be sure that students "check" their answer by substituting it into the equation.

Example:

$$\begin{aligned} 3x + 4 &= 22 \\ 3x &= 18 \\ x &= 6 \end{aligned}$$

Check by substituting 6 for x.

$$\begin{aligned} 3(6) + 4 &= 22 \\ 18 + 4 &= 22 \\ 22 &= 22 \end{aligned}$$

To review, students should:

1. Use the equality properties (one at a time) to set the variable alone on one side of the equation; and

2. Do the necessary computations to solve for the variable.

Solves and Uses First Degree Equations and Inequalities (One Variable)

SAMPLE ITEM:

Solve for x:

$x + 4 = 3x - 8$

A. $x = 3$

B. $x = 2$

C. $x = 6$

D. $x = 7$

DESCRIPTION OF TEST QUESTIONS:

1. The student will be presented with one of the following:
   a. a linear equation;
   b. a linear inequality; or
   c. a word problem that requires solving a linear equation.
2. For test items that present a linear equation or inequality:
   a. The student will be directed to solve for the variable.
   b. The direction will be followed by a linear equation or inequality. The variable (x) will appear on both sides of an equation, but only on one side of an inequality.
   c. Variable coefficients or constants will be positive or negative fractions in simplest terms or integers. Integers as well as numerators and denominators of fractions will not be larger than two digits.
   d. Multiple steps may be required to solve the problem.
3. For test items consisting of word problems:
   a. The student will be presented with a word problem that involves number relations (e.g., number, consecutive integer, geometry, distance, coin, and age problems).

b. Numbers will be limited to positive and negative fractions or integers. Integers as well as numerators and denominators of fractions will be no larger than two digits.

c. Multiple steps may be required in order to solve the problem.

d. Irrelevant numeric information will not be included.

DESCRIPTION OF ANSWER CHOICES:

1. For test items that require solving linear equations or inequalities:

   a. The correct answer choice will be a number that accurately solves the linear equation or inequality.

   b. The answer choices will be of statements in the form "x = (number)." Numbers may be integers, or positive or negative fractions in simplest terms.

2. For test items that require the use of linear equations to solve word problems:

   a. The correct answer choice will be the number that is the accurate solution to the word problem.

   b. The answer choices will be integers, or positive or negative fractions in lowest terms labeled in appropriate units.

3. An incorrect answer choice will be a number that is obtained as the result of one or more of the following types of errors:

   a. computation error: an error in the addition, subtraction, multiplication, or division of two numbers;

   b. incorrect sign: using the wrong sign in either the calculations or the response, that is, using a positive sign when a negative sign is indicated or vice versa;

   c. reciprocal error: using a number that is a reciprocal of the correct number;

   d. missing operation: not operating on the data, omitting one of the required operations, or failing to perform the same operation on both sides of the equation; or

e. incorrect operation: performing an operation other than the required one on one or both sides of the equation.

SAMPLE ITEM ANSWER CHOICE DESCRIPTIONS:

A. incorrect operation

B. missing operation

C. correct

D. computation error

INSTRUCTIONAL ANALYSIS:

Linear Equations

Unlike the linear equation items described in the previous standard, linear equation items in this standard will have the variable appear on both sides of the equation and, therefore, may require more steps to solve. Because the process involved in solving either type of equation is essentially the same, however, you may wish to review the equality properties described in the previous "Instructional Analysis" section when presenting students with these slightly more advanced linear equations.

Linear Inequalities

In teaching students to solve linear equalities, point out that the process is very similar to that for solving linear equations. A primary difference is that in solving linear inequalities, the direction of the inequality must be taken into consideration. Specifically, students need to know the following principles that are used in solving linear inequalities:

1. Adding or subtracting the same positive or negative number to or from both sides (members) of an inequality results in an equivalent inequality.

2. Multiplying or dividing each side of an inequality by the same positive number results in an equivalent inequality.

3. Multiplying or dividing each side of an inequality by the same negative number and reversing the direction of the inequality results in an equivalent inequality.

To reinforce students' understanding of the concept of inequality, you may want to have them graph their solution sets for practice items.

Word Problems that Require Solving Linear Equations

In teaching students how to solve word problems that require the use of linear equations, emphasize the translation from words to algebraic equation. Tell students that they must first determine how the numbers relate to each other in order to write an appropriate equation that, when simplified, solves the problem. You may want to review a step-by-step procedure such as the one outlined below.

1. Read the problem carefully and determine the unknown number(s).

2. Choose a variable to represent the unknown number(s).

3. Write an equation based on the facts given in the problem.

4. Solve the equation.

5. Check the answer by substituting the answer for the variable in the equation and by examining the answer in light of the facts given in the problem.

Illustrative items such as the one below should be used to demonstrate each of the five steps:

> Jeremy has 70 stamps in his collection. He calculated that he has 6 less than 4 times the number of stamps that are in Ben's collection. How many stamps does Ben have in his collection?

Step 1. The problem asks for the number of stamps in Ben's collection.

Step 2. Let x = the number of stamps in Ben's collection.

Step 3. $70 = 4x - 6$

Step 4.
$$\begin{aligned} 70 &= 4x - 6 \\ 70 + 6 &= 4x \\ 76 &= 4x \\ 19 &= x \end{aligned}$$

Step 5. $70 = 4x - 6$
$70 = 4(19) - 6$
$70 = 76 - 6$
$70 = 70$

The answer, 19 stamps in Ben's collection, appears reasonable because the problem states that Jeremy, who has 70 stamps, has almost four times as many stamps as Ben ($4 \times 19 = 76$).

Understands and Operates on Polynomials

SAMPLE ITEM:

Simplify:

$(8x^5 + 3x^2 + 11) + (6x^5 - 7x^2 + 9x)$

A. $14x^{10} - 4x^4 + 9x + 11$

B. $14x^5 - 10x^2 + 9x + 11$

C. $14x^5 - 4x^2 + 20x$

D. $14x^5 - 4x^2 + 9x + 11$

DESCRIPTION OF TEST QUESTIONS:

1. The student will be presented with a polynomial expression that requires simplification by addition, subtraction, multiplication, or division.

2. Exponents will be integers from 0 to 9.

3. Coefficients will be integers no larger than two digits.

4. For problems that involve only addition or subtraction of polynomials:

   a. Polynomials will be limited to no more than 6 terms.

   b. Only one variable will be used in each item, although the variable may appear more than once and may have different exponents.

   c. Terms will be arranged in random order within the polynomial. The exception will be terms inside parentheses, which will be arranged with the exponents of the terms appearing in descending order.

5. Problems that involve multiplication of polynomials will require the multiplication of two or more monomials, a polynomial by a monomial, or a polynomial by a polynomial.

   a. For test items that require multiplying two or more monomials:

      (1) No more than one monomial may have an exponent.

      (2) There will be no more than three monomial factors in any item.

   b. For test items that require multiplying a polynomial by a monomial:

      (1) The polynomial will contain no more than three terms.

      (2) Monomials may have exponents.

   c. For test items that require multiplying two polynomials:

      (1) Each polynomial will contain only two terms.

      (2) An item may require squaring a two-term polynomial.

6. Test items that require division will be limited to the division of two monomials.

DESCRIPTION OF ANSWER CHOICES:

1. The correct answer choice will be the monomial or polynomial obtained by accurately operating on the terms in the test item.

2. For test items that require addition or subtraction of polynomials:

   a. The answer choices will have from one to six terms.

   b. Polynomials will be arranged with the exponents of the terms appearing in descending order.

3. For test items that require multiplication or division of monomials or polynomials:

   a. The answer choices will each have from one to four terms.

b. Polynomials will be arranged with the exponents of the terms appearing in descending order.

4. An incorrect answer choice will be a monomial or polynomial that is obtained as a result of one or more of the following types of errors:

   a. polynomial computation error: incorrectly adding, subtracting, multiplying, or dividing polynomials or terms in a polynomial. Common errors of this type include incorrectly combining similar or dissimilar terms in adding polynomials, failing to multiply each coefficient by -1 in subtracting a polynomial, and operating on variables separately from constants when multiplying or dividing monomials and/or polynomials;

   b. incorrect sign: using the wrong sign in either the calculations or the response, that is, using a positive sign when a negative sign is indicated or vice versa; or

   c. missing operation: omitting one of the required operations, terms, or coefficients.

SAMPLE ITEM ANSWER CHOICE DESCRIPTIONS:

A. polynomial computation error

B. incorrect sign

C. polynomial computation error

D. correct

INSTRUCTIONAL ANALYSIS:

For this standard, students will need to know the principles that apply to adding, subtracting, multiplying, and dividing polynomials. Before explaining these principles, be sure that your students are familiar with the terminology associated with operating on polynomials.

monomial: an algebraic expression containing one term

Examples: 3; $2x^4$; $-5ab$

binomial: an algebraic expression containing two terms

Examples: $3x + 7$; $-8x^2 - x$

trinomial: an algebraic expression containing three terms

Examples: $4x^7 + 6x^2 + 3x$; $16x^3 - 4x + 2$

polynomial: an algebraic expression containing one term or the sum of two or more terms

Examples: $3x^8 + 12x^2 + 11$;
$-3x^4 + (-7x^3) + 16$

term: either (1) a monomial or (2) one of the numerals and/or variables separated by plus or minus signs in a polynomial

Example: In the polynomial $4x^3 + 12x^2 - 4x + 6$, the terms are $4x^3$, $12x^2$, $-4x$ and 6 all are terms.

coefficient: in a term, the numeral by which the variable is multiplied

Example: In the term 10x, 10 is the coefficient.

degree: The degree of a term with one variable is its exponent; the degree of a polynomial is its greatest exponent.

Example: The term $5x^4$ has a degree of 4; the polynomial $2x^6 + 4x^4 + 5x^3$ has a degree of 6.

leading term: in a polynomial, the term with the highest degree

Example: In the polynomial $8x^6 - 3x^4 + 2$, the leading term is $8x^6$.

leading coefficient: the coefficient of the leading term

Example: In the polynomial $4x^8 - 16x^3 + 7$, the leading coefficient is 4.

Addition of Polynomials

Explain to students that addition of polynomials consists of combining like terms. Stress that exponents are neither added nor subtracted. For example:

$$(3x^8 + 4x^3 - 2x^2 - 12) + (5x^8 + 2x^3 + 8)$$
$$= (3x^8 + 5x^8) + (4x^3 + 2x^3) + (-2x^2) + (-12 + 8)$$
$$= 8x^8 + 6x^3 - 2x^2 - 4$$

Subtraction of Polynomials

Tell students that subtraction of polynomials also involves collecting similar terms and can be done in horizontal or vertical format. However, to subtract one polynomial from another, you must remember to add the opposite (additive inverse) of each term you are subtracting. For example:

$$(3x^8 + 4x^3 - 2x^2 - 12) - (5x^8 + 2x^3 + 8)$$
$$= (3x^8 + 4x^3 - 2x^2 - 12) + [-(5x^8 + 2x^3 + 8)]$$
$$= (3x^8 + 4x^3 - 2x^2 - 12) + (-5x^8 - 2x^3 - 8)$$
$$= (3x^8 - 5x^8) + (4x^3 - 2x^3) + (-2x^2) + (-12 - 8)$$
$$= -2x^8 + 2x^3 - 2x^2 - 20$$

Multiplication of Monomials/Polynomials

For the test, students will be required to perform three types of multiplication problems: (1) multiplying two monomials; (2) multiplying a polynomial by a monomial; and (3) multiplying two binomials. Demonstrate each type of multiplication problem to students with examples such as the following:

Multiplying two monomials

To multiply two monomials, multiply the coefficients and the exponents separately. Exponents are multiplied by adding their powers.

For example: $(6x^2)(5x^4) = (6 \cdot 5)(x^2 \cdot x^4)$
$$= 30x^{2+4}$$
$$= 30x^6$$

Multiplying a polynomial by a monomial

To multiply a polynomial by a monomial, multiply the monomial by each term of the polynomial and add the resulting products.

For example: $5x^4(2x^3 + 3x^2) = 5x^4(2x^3) + 5x^4(3x^2)$

$= 10x^{4+3} + 15x^{4+2}$

$= 10x^7 + 15x^6$

Multiplying two binomials

To express the product $(w + x)(y + z)$ as a polynomial, first distribute $(y + z)$ over w and x as follows:

$w(y + z) + x(y + z)$

Then apply the distributive law to each product:

$wy + wz + xy + xz$

The resulting polynomial can sometimes be simplified, as in the following example:

$(2x + 3)(4x + 7) = 2x(4x + 7) + 3(4x + 7)$

$= (2x \cdot 4x) + (2x \cdot 7) + (3 \cdot 4x) + (3 \cdot 7)$

$= 8x^2 + 14x + 12x + 21$

$= 8x^2 + 26x + 21$

Division of Monomials

Division problems on the LEAP test will be limited to division of two monomials. Show students that to divide two monomials, simply divide the coefficients and exponents separately. Exponents are divided by subtracting their powers. For example:

$30x^8 \div 6x^2 = (30 \div 6)(x^8 \div x^2)$

$= 5x^{8-2}$

$= 5x^6$

Students will need ample practice with problems that require operating on polynomials before they proceed to the next polynomial skill, factoring polynomials.

Factors Polynomials

SAMPLE ITEM:

Factor completely:

$(x^2 - 36)$

A. $(x + 6)(x - 6)$

B. $(x - 4)(x + 9)$

C. $(x - 6)(x - 6)$

D. $(x - 12)(x - 3)$

DESCRIPTION OF TEST QUESTIONS:

1. The student will be presented with a polynomial preceded by the direction to "factor completely."

2. The test item will consist of one of the following special types of polynomials:

   a. a polynomial that is factorable into a monomial and polynomial;

   b. a difference of two squares;

   c. a trinomial square; or

   d. a quadratic trinomial.

3. Integers will not be larger than two digits.

4. Quadratic trinomials will have leading coefficients of 1.

DESCRIPTION OF ANSWER CHOICES:

1. The correct answer will represent the appropriate factors of the given polynomial. It will be either (1) a pair of polynomials or (2) a pair comprised of a monomial and a polynomial.

2. An incorrect answer choice will be factors that are obtained by one or more of the following types of errors:

   a. incomplete factoring error: incompletely factoring a polynomial;

b. common monomial error: removing the greatest common monomial factor from one term of a monomial and polynomial pair, but not from the other; or

c. incorrect factor: identifying factors in the given polynomial that are not factors of the complete given polynomial; or

d. incorrect sign: using the wrong sign in either the calculations or the response, that is, using a positive sign when a negative sign is indicated or vice-versa.

SAMPLE ITEM ANSWER CHOICE DESCRIPTIONS:

A. correct

B. incorrect factor

C. incorrect sign

D. incorrect factor

INSTRUCTIONAL ANALYSIS:

Explain to students that factoring polynomials is the reverse of multiplying polynomials. To factor a polynomial is to express it as a product of polynomials. In the LEAP test, four special types of polynomials may be presented for factoring: (1) a polynomial that is factorable into a monomial and a polynomial, (2) a difference of two squares, (3) a trinomial square and (4) a quadratic trinomial. Before presenting these special factoring problems to students, you may wish to review factoring of integers and monomials.

A Polynomial That is Factorable into a Monomial and a Polynomial

To factor this type of polynomial find the greatest common factor (GCF) from the terms of the polynomial and then apply the distributive axiom. For example, to factor $72x^2 - 45x$:

1. Find the GCF of the numerical coefficients of $72x^2$ and $45x$.

$$72 = 9 \cdot 8 = 3^2 \cdot 2^3$$

$$45 = 9 \cdot 5 = 3^2 \cdot 5$$

The GCF of the coefficients is the greatest factor which $3^2 \cdot 2^3$ and $3^2 \cdot 5$ have in common. The number is 3^2 or 9.

2. Compare the powers of each variable that is a factor of both terms, and select the power in which the exponent is least.

   Compare x^2 and x; choose x.

3. The GCF is the product of the constant identified in Step 1 and the power(s) of the variable(s) selected in Step 2.

   The GCF is $9x$.

4. Use the distributive axiom to complete the factoring.

$$72x^2 - 45x = 9x(8x) - 9x(5)$$
$$= 9x(8x - 5)$$

The factorization is complete because the polynomial is expressed as the product of a monomial and a polynomial that cannot be reduced, that is, 1 is the greatest common monomial factor.

A Difference of Two Squares

A binomial that is a difference of two squares consists of two terms that are both squares and are separated by a minus sign. The following rule can be used to factor a polynomial consisting of the difference of two squares:

$$a^2 - b^2 = (a + b)(a - b)$$

Example: Factor $x^2 - 9 = x^2 - 3^2 = (x + 3)(x - 3)$

A Trinomial Square

A trinomial square is a polynomial with three terms that can be factored into the square of a binomial. A trinomial square commonly takes either of the following patterns:

$$a^2 + 2ab + b^2 = (a + b)^2$$
$$a^2 - 2ab + b^2 = (a - b)^2$$

Examples: $x^2 + 4x + 4 = (x + 2)^2$

$x^2 - 18x + 81 = (x - 9)^2$

Note that when the sign of the middle term in the given polynomial is positive (+), the terms in the factors of the binomial are added. If the sign of the middle term in the given

polynomial is negative (-), the terms in the factors of the binomial are subtracted.

A Quadratic Trinomial

A quadratic trinomial has degree two and contains a single variable. If a quadratic trinomial can be factored, the factorization pattern is as follows:

$$x^2 + (y + z)x + yz = (x + y)(x + z)$$

For example, $x^2 + 2x - 8$ is a quadratic trinomial. The term of degree two, x^2, is known as the quadratic term. The term of first degree, $2x$, is called the linear term, and the numeric term, -8, is the constant term.

The factors of a quadratic trinomial are two binomials. There are three criteria that help in identifying the correct binomial factors of a quadratic trinomial (with leading coefficient 1). These three criteria can be demonstrated using the previous example of a quadratic equation: $x^2 + 2x - 8$

(1) The product of the linear terms in the factors must be x^2. Therefore, we start with: (x) (x)

(2) The product of the constant terms in the factors must be the numeric term of the trinomial (-8). The only factors of (-8) are:

(-4, +2)
(+4, -2)
(+8, -1)
(-8, +1)

(3) The coefficient of the linear term in the trinomial (+2) is the sum of the two constant terms in the factor. The only pair that satisfies this criterion is +4, -2. Thus the correct factors are $(x + 4)(x - 2)$.

General Guidelines for Factoring Polynomials

You may wish to present this list of guidelines to students to help them in factoring polynomials.

1. Always look for a common factor.

2. Consider the number of terms.

   If there are two terms, determine whether this is a difference of squares.

   If there are three terms, determine whether this is a trinomial square.

   If the polynomial represents neither a difference of squares or a trinomial square, factor by trial and error.

3. Always factor completely. If a factor with more than one term can be factored even further, be sure to do so.

4. Check your answer by multiplying to see if you get the original polynomial.