DOCUMENT RESUME

ED 309 988 SE 050 830

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TITLE	A Constructivist Program for College Remedial Mathematics at the University of Massachusetts, Amherst.
PUB DATE	Aug 89
NOTE	14p.
PUB TYPE	Viewpoints (120) -- Reports - General (140)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	*Cognitive Development; *College Mathematics; Course Descriptions; Educational Strategies; Higher Education; *Instructional Design; Mathematics Education; *Mathematics Instruction; *Metacognition; Remedial Instruction; *Remedial Mathematics; Remedial Programs
IDENTIFIERS	*Constructivism

ABSTRACT

The recent increase in enrollments in college remedial mathematics courses makes clear the need for the development of pedagogy and curriculum which is tailored to this relatively new class of service courses. Besides the need for courses which teach basic skills in arithmentic, general mathematics, and elementary and intermediate algebra, a growing concern for better thinking skills among college students has added an important new dimension to the teaching of remedial mathematics. The Basic Math Program of the University of Massachusetts, Amherst, has instituted two new courses for empowering math-weak students to think critically and quantitively. Two fundamental perspectives guide the design of the program: (1) the constructivist epistemology of Piaget advocates the active role of the learner who must invent concepts rather than passively receive "knowledge transmissions" from authorities; and (2) recent studies on the role of metacognition demonstrate the importance of reflecting on one's thought processes to facilitate learning and generally to increase self-awareness and self-confidence. Students spend the majority of class-time solving problems in pairs and in small groups. Teachers interact with their students more as coaches and clinical interviewers than as lecturers. Diversity, debate, controversy, and consensus replace authority in the quest for understanding in mathematics. (Author)

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ED309988

A CONSTRUCTIVIST PROGRAM FOR COLLEGE REMEDIAL MATHEMATICS AT THE UNIVERSITY OF MASSACHUSETTS, AMHERST

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August, 1989

The recent increase in enrollments in college remedial mathematics courses makes clear the need for the development of pedagogy and curriculum which is tailored to this relatively new class of service courses. Besides the need for courses which teach basic skills in arithmetic, general mathematics, and elementary and intermediate algebra, a growing concern for better thinking skills among college students has added an important new dimension to the teaching of remedial mathematics. The Basic Math Program of the University of Massachusetts, Amherst, has instituted two new courses for empowering math-weak students to think critically and quantitively. Two fundamental perspectives guide the design of the program: 1) the constructivist espistemology of Piaget advocates the active role of the learner who must invent concepts rather than passively receive "knowledge transmissions" from authorities; 2) recent studies on the role of metacognition demonstrate the importance of reflecting on one's thought processes to facilitate learning and generally to increase self-awareness and self-confidence. Students spend the majority of class-time solving problems in pairs and in small groups. Teachers interact with their students more as coaches and clinical interviewers than as lecturers. Diversity, debate, controversy, and consensus replace authority in the quest for understanding in mathematics.

Submitted to the Practitioner's Hall of Fame Summer Institute, Palm Springs, CA.

SE 050 830

Introduction

Mathematics education, especially remedial mathematics education, will be influenced by future educators who respond to the following two observations: 1) the dramatic rise in enrollment in remedial arithmetic, general mathematics and algebra indicates the need for training in basic skills in content areas which are considered prerequisite not only for college but for the vocations also; 2) even students who have taken as many as three years of college preparatory mathematics do not demonstrate an understanding of the material sufficient to use it. From 1960 to 1980, college enrollment at universities and four-year colleges in remedial arithmetic, general mathematics and algebra increased by 165% (Usiskin, 1985). Two-year colleges report that 42% of mathematics enrollments in 1980-81 were in remedial courses. Dealing with remediation was described as the biggest problem facing two-year college mathematics faculty (Fey, Albers, and Fleming, 1981). As colleges continue to insist that minimum competency in mathematics be demonstrated for graduation, the enrollment in remedial mathematics courses can be expected to remain high.

Perhaps more dramatic than the rise in enrollment is the poor performance of students who have already had several years of college preparatory mathematics. Results from the First Mathematics Assessment of the National Assessment of Educational Progress [NAEP] (Carpenter, et al. 1978) indicate that just 50% of 17-year-olds and 60% of adults were able to correctly answer word problems with decimals and percents, perhaps the most frequently applied topic in mathematics today. Similar error rates were identified in the interpretation of algebraic equations. Moreover, the lack of understanding of elementary algebra concepts is not confined to high school and college remedial mathematics courses. Recent studies show that about seventy percent of college freshmen in calculus were unable to translate a simple linear relationship into an equation (Clement, 1982). Comparable error rates were reported for high school and college faculty when asked to translate a linear equation into a sentence (Lochhead, 1980). Apparently conceptual understanding is not assured through academic success.

The mathematics education community has appealed to teachers, state boards of education, local administrators, test writers and text book publishers to make problem solving and conceptual understanding of mathematics foundational to the mathematics curriculum. The two foremost recommendations from the National Council of Teachers of Mathematics (1980) were:

> 1. That problem solving be the focus of school mathematics in the 1980's.
>
> 2. That basic skills in mathematics be defined to encompass more than computational facility.

Project EQuality of the College Board (1983) echoes this appeal by declaring that mathematics understanding is prerequisite to competency in all areas of college education. In addition to the ability to use calculators and computers, the report makes clear the need for conceptual understanding and the ability to demonstrate that understanding in problem solving. Another widely cited government report, "A Nation At Risk" (National Commission on Excellence in Education, 1983), states that the teaching of mathematics in high school should equip students with an understanding of basic mathematics concepts which will enable them to solve problems in college or in the workplace. Clearly the stated emphasis in mathematics education has shifted from rote computational facility and the manipulation of algebraic symbols toward an understanding of the concepts of basic mathematics which can be demonstrated through problem solving.

Educators and education researchers are beginning to acknowledge that algebraic symbol manipulation, the area most emphasized in high school and college algebra curriculum, is quite useless if students are unable to express quantitative ideas precisely.

Critical Thinking and Remedial Mathematics

Remedial mathematics has traditionally served to "remediate" student mathematics deficiencies by drilling students in arithmetic computation and algebra symbol manipulation in order to prepare them for their next mathematics course. Such courses are attempts to "fill the gaps" or help students "brush up" by presenting one arithmetic or algebra technique after another in quick succession. Textbooks present topics with only the most terse description or "proof", followed by example problem solutions which students subsequently practice in "exercises". Word problems are generally included though not emphasized, and they are not used to teach concepts but to illustrate how certain techniques may be applied. If the concepts have not been understood, the difficulties students experience in solving word problems can be insuperable. It is not surprising that word problems are one of the least appreciated features of mathematics courses for students (Lester & Garofalo, 1982).

Remedial mathematics courses can do much more than just fill gaps in content. Critical thinking skills can be developed within the context of a mathematics curriculum designed to teach the concepts of arithmetic and introductory algebra. Word problem solving can serve as a vehicle for helping students develop conceptual understanding rather than serve as a post hoc test of their understanding. For this shift to occur the pedagogy of proof, example, drill and test needs to be subordinated to a pedagogy based on the discovery of concepts by students who need those concepts to solve problems. The fundamental difference in the two approaches lies in their respective epistemologies. The traditional approach views ideas as the currency of instruction; teachers teach ideas by presenting them clearly, while students demonstrate their learning by performing rote computations and manipulations. Contrary to the view of knowledge as "ideas proved true", the constructivist approach considers ideas to be idiosyncratic mental constructions; teachers cannot teach ideas, but they can ask questions so that their students can construct ideas for themselves. The highest value of this "constructivist" approach to education is the intellectual autonomy of the student.

The goal of constructivist remedial mathematics is to empower the student to think mathematically and critically.

The content of remedial mathematics courses frequently includes topics in analytic geometry, algebra and trigonometry. The program at the University of Massachusetts is not quite as extensive. The Basic Arithmetic course instructs topics in whole number operations, fractions, decimals, percents, exponents, ratio and proportion, simple linear equations, and very basic geometrical notions about length, area, and volume. The second semester course, Basic Algebra, stresses the use of algebra as a tool for interpreting and representing quantitative relationships, particularly linear and quadratic relationships. The intent of the curriculum is not only to prepare students for subsequent courses in calculus and statistics, but mainly to prepare students to reason quantitatively and to apply mathematics to a variety of problems which they may encounter in their daily experiences.

Method of Instruction

The method of instruction incorporates two key notions: constructivism; the idea that students must construct knowledge for themselves, and metacognition; the supposition that the vehicle for the construction of knowledge is self-reflection, or in Piagetian terms, reflective abstraction.

The problems which students solve to develop conceptual understanding of mathematical content are not in themselves sufficient for learning. The various relationships between concepts and ideas which comprise the conceptual web are best discerned and integrated within a social context (Von Glasersfeld, 1988). Though knowledge is constructed individually, it is corroborated largely through consensus, and consensus-achieving is a social activity.

The implication that education should be conducted in a socially interactive environment has been supported by research. Cooperative learning has been effective in aiding student understanding of texts (McDonald, et. al., 1985; Spurlin, et. al., 1984; Ross and DiVesta, 1976), and also for concept learning and problem solving in mathematics and science (Dees, 1985; Sharan and Hertz-Lazarowitz, 1980; Webb, 1978).

The constructivist classroom is conducted almost entirely within the context of group problem solving, with approximately thirty students working together in dyads and in larger groups. Lecture is kept to a minimum. The teacher serves as a coach, moving from one student group to another, listening to their discussions, asking selected group members to summarize the group's solution path, and probing student solutions and conceptions with questions rather than answering questions.

A student's answer is not acknowledged as either right or wrong. Instead, the instructor listens to the reasons for the answer and either agrees that the reasons make sense, asks for more elaboration, or asks more questions to help the student to think about the problem in a different way. Often instructors ask other students in the group to explain another student's solution and to comment on whether or not they agree with it and for what reasons.

The method of instruction in the Constructivist classroom is derived from a combination of managerial techniques and clinical interview methodology. Ten graduate student TAs, from various academic departments, are trained in a two-week intensive training course each summer. They conduct and analyze clinical interviews of students solving mathematics word problems to help them to better understand student thought processes. The interviews also serve to train instructors in the questioning and listening skills which are fundamental to their teaching in the prog. am. Although a prepared text is required for the course (Narode, et. al., 1985), the instructors are free to select the problems which they feel will be most effective to instruct a particular concept or heuristic. Most important, they train their students to communicate their ideas, thus helping them to develop conceptual understanding through reflection during problem solving.

The observed allocation of class time is divided among four main activities, administrative (9%), lecture/discussion (15%), problem solving (56%), and quiz taking (18%) (Konold, 1986). Ideally, instructors and experts feel that students should spend more time problem solving than they in fact do. By finding methods to streamline administrative and quiz taking activities, more time can be permitted for problem solving.

Promoting Metacognition

The model for cooperative problem solving in the Basic Math classroom is the pair problem solving method of Whimbey and Lochead (1986) which is itself modelled after the clinical interviews used by Piaget. The approach requires that one student solve a problem by reading it aloud to the other student (the listener) and verbalizing all thoughts on the problem as they occur. The problem solver does all the writing and all of the talking about the problem. Meanwhile, the listener must suspend solving the problem him/herself so that complete concentration and attention is devoted towards understanding the problem solver's solution. The problem solver is responsible for articulating all ideas as they occur, whereas the listener has a somewhat more difficult task. The following instructions to the listener were developed for students in the course: Listen carefully, ask the speaker to repeat statements if needed, or to slow down. Encourage vocalization, ask, "What are you thinking?" and "Can you explain what you are writing?" Ask for clarification, for example, "What do you mean", and "Can you say more about that?" Check for accuracy by asking, "Are you sure about that? Several warnings are offered in the form of "do nots", do not give hints, do not solve the problem yourself, do not tell the solver how to correct an error.

By encouraging students to verbalize their thoughts, they are forced to examine their ideas as they communicate. They must evaluate those ideas in the light of another person's interpretation of what they are saying. Requests for clarification and repetition often help students to catch and correct their errors as well as helping to reinforce ideas that they may have held only tentatively. By exchanging roles of problem solver and listener, students have the opportunity to learn the related skills of problem solving aloud and listening for meaning.

Instructors also use the clinical interview method in their teaching. By assuming the role of the listener, the instructor promotes metacognitive activity through the use of questioning strategies which require students to reflect on their thought processes. Four such strategies are (Confrey, 1985): 1) ask students to discuss their interpretation of the problem, 2) ask the students to describe precisely their methods of solution, 3) ask students to defend their answer and their solution, 4) ask students to retrace the steps in their solution so as to review the process they engaged in to solve the problem. The student/teacher interaction is characterized by the focus on language used by the student and by the teacher's acceptance of the student's vision of a solution path.

In addition to listening to students, teachers should model expert problem solving for their students. Students rarely see adults solve problems, much less hear them solve problems aloud. By describing one's thought processes aloud, instructors demonstrate both the process of thinking aloud and their thought processes with all of the dead ends, mistakes, and corrections which characterize real problem solving.

Conclusion

The students in college remedial mathematics courses are at risk of never learning the basic rudiments of arithmetic and algebra to satisfactorily apply them in their daily lives. However, all of these students in fact possess knowledge and understanding of mathematics by virtue of having lived in a society where notions about quantity abound. The constructivist mathematics program begins with the students' prior knowledge and uses it to challenge them to solve applied problems whose solutions demonstrate powerful mathematical concepts. Students create their own meaning when they work collaboratively in pairs and in groups. In the process, they learn to reflect on their thought processes so that they may judge for themselves whether their solutions do or do not make sense. They learn to think critically as well as quantitatively. Their anxieties about learning mathematics yield to a new confidence.

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