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ABSTRACT

This paper describes a framework for making sense of the relationship between emotions, cognition, mathematical activity, and mathematical beliefs. Subjects were participants in a special mathematics anxiety program. Beliefs are classified as instrumental and relational approaches to a situation. Emotion is also distinguished as a two-faceted construct which includes the physiological arousal (emotion as state) and the cognitive construction (emotion as acts). Perceptions of participants are described in detail to illustrate the nature of shifts in beliefs about mathematics and to elaborate on a framework for interpreting the relationship between mathematical beliefs and emotional acts. Some cases of individuals who made no change and became more anxious as the program progressed are discussed. Anxiety is not an inherent response to mathematics, but is based on an individual's beliefs about mathematical activity. (YP)

A CONSTRUCTIVIST PERSPECTIVE ON THE RELATIONSHIP BETWEEN MATHEMATICAL BELIEFS AND EMOTIONAL ACTS

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A Constructivist Perspective on the Relationship Between Mathematical Beliefs and Emotional Acts

Math is a dark alley that one walks down or passes by. Surprises jump out at you, and convince you that you didn't really know what you thought you did after all. Numbers and more numbers that won't always jump through a hoop, no matter what you try.

Someone once said, "Hey, math is fun! I can get these figures to do whatever I say!"

Well, math seems to measure things and help people build things. To me, math is terror! I want to know why these things make those things possible.

I guess overall, math is addition, subtraction, division, multiplication and formulas.

Annette

Introduction

When developing an explanation for an observed phenomenon it is the discrepant event, the one that does not follow anticipated patterns, that gives occasion to pause and investigate further. It is precisely this type of discrepancy that we have observed as we analyzed results from a program designed to help individuals (such as Annette) who experience strong negative emotional responses to mathematics to deal with the "terror" of mathematical activity. Most of the participants in our program made major positive changes in their emotional responses to mathematics and their success in meeting their goals in mathematical situations. A few, however, did not. These "discrepancies" forced us to examine the program and develop a framework for making sense of the relationships among emotion, cognition, mathematical activity and mathematical beliefs. This analysis helped us interpret both the changes we observed and why we did not observe positive changes in other participants. In developing the framework, it was necessary to make explicit and to refine our view of the nature of emotion, to examine the influence of the social contexts of mathematical activity, and to examine how norms of socially acceptable mathematical behavior are established. This paper will describe this framework.

Context of the study

The Success With Math/Overcoming Math Anxiety program was an eight week noncredit program offered through Continuing Education at a large research university. The program, offered for eight semesters, was designed to help participants overcome the inhibiting emotional responses to mathematical situations often labeled "mathematics anxiety."

The bulk of the data for this study was collected during four semesters from 1983-1988. Participants in the program ranged from high school juniors to retirees. For the most part these participants had identified themselves as "mathematics anxious" and voluntarily participated in the program because they wanted to overcome their negative emotional responses. Program participation required no specific background in mathematics. The range of participants' last formal mathematics preparation was wide, often spanning a spectrum from elementary school mathematics to college calculus. There were approximately equal numbers of male and female participants each semester.

The program, based on a constructivist view of learning (Confrey, 1987; Von Glasersfeld, 1984), was designed to encourage participants to become aware of their beliefs about mathematics and their emotional responses, and to promote reflection on these facets of their mathematical world view. We wanted participants to make themselves aware of their contexts of mathematical activity. Thus, we encouraged reflection not only on their mathematical activity but on emotional reactions as well. We also attempted to provide a setting that would give participants opportunities for restructuring their beliefs and opportunities that would allow them to experiment with and concretize new potentially more powerful conceptions of mathematics and emotional responses to mathematical activity.

For this examination and reflection to occur, it was essential to establish a supportive, non-threatening atmosphere for the program sessions. We believed, and participants occasionally commented, that it would be impossible to establish this atmosphere in the presence of taping equipment or of researchers who were not an active part of the program. Thus, data from the sessions consists of our field notes, copies of participants' writing, problem solving work, autobiographies and responses to often open-ended questionnaires or scales. We also interviewed many participants at the beginning and end of the program. These interviews were audiotaped.

Changes in beliefs and emotional response

As instructors we considered the Success With Math/Overcoming Mathematics Anxiety program amazingly successful. Whether gauged by our observations of participants while solving mathematical problems, the interviews, participants' self-evaluation, changes in career or coursework plans, or responses to scales such as the Math Anxiety Rating Scale (Suinn, 1972) or the Mathematical Beliefs Scale (Yackel, 1984), most participants made major changes in their emotional responses to mathematics and in their beliefs about mathematics. We will discuss these often quite intense shifts in belief and in the accompanying emotional reaction as analogous to Kuhn's (1970) discussions of scientists' changes in world view. One participant will be described in detail to illustrate the nature of these shifts and elaborate a framework for interpreting the relationship of mathematical beliefs to emotional acts. We will also discuss the small number of individuals who seemed to make no lasting changes in their relationship to mathematics -- and, the individual who became increasing math-anxious as the program progressed.

BELIEFS

Beliefs constitute knowledge about the world for the individual. This knowledge includes perspectives on the nature of mathematics; the origins of mathematical knowledge; how one learns mathematics; what it means to do mathematics; one's ability to do mathematics; why and how mathematics should be taught; how one goes about solving mathematical problems; and what constitutes an acceptable mathematics problem or solution strategy. These beliefs serve to establish the "psychological context" (Schoenfeld, 1985) for learning and problem solving. As Cobb (1986) notes:

> [Beliefs] are a crucial part of the assimilatory structures used to create meaning and establish overall goals that limit general contexts. The act of formulating a goal immediately delimits possible actions; the goal, as an expression of beliefs, embodies implicit anticipations and expectations about how a situation will unfold. For example, students who have constructed instrumental beliefs about mathematics (Skemp, 1976) anticipate that future mathematics experiences will "fit" these beliefs. They intend to rely on authority as a source of knowledge, they expect to solve tasks by employing procedures that have been explicitly taught, they expect to identify superficial cues when they read problem statements, and so forth."

Emotional acts can be viewed as expressions of beliefs and, from the observer's perspective, are valuable sources of insight into the possible nature of those beliefs (Cobb, 1986; Smith, 1978). Since beliefs also influence mathematical actions, beliefs provide a link between the worlds of emotional acts and mathematical actions. Skemp's (1976) terms, instrumental and relational understanding, provide a useful, albeit simplified language for discussing a continuum of mathematical beliefs. Instrumental approaches to mathematics focus on "rules without reasons." Relational learners try to understand both the hows and the whys of mathematics.

Individuals with strongly instrumental beliefs can be thought of as rule and procedure-driven. Because mathematics is a series of rules, to learn mathematics means one must learn the rules, usually through memorization. One must also learn the situations to which the rules apply. This perspective implies that there is an authority, such as the teacher or textbook author, who knows the rules and transfers these rules to the learner. Learning is successfully receiving knowledge, which is then reproduced without processing. When the student is confused or unsure of how to proceed, the only alternative is to consult an authority because students cannot generate or derive mathematical knowledge on their own. An answer is correct when it agrees with the authority's answer. If the learner does not know what to do in a situation, either the learner's memory has failed or the teacher has failed because the teacher has not covered the situation or the appropriate rule. The goal of an instrumental learner is to produce "the right answer," not to develop a greater understanding of mathematics. Since mathematics is rules and algorithms, instrumental learners may view mathematics as many unrelated topics and as a rigid, uncreative discipline.

Relational learners view learning in terms of understanding. Knowledge is constructed in integrated conceptual networks. New concepts are supposed to make sense in light of previous knowledge. Knowledge construction is the responsibility of the student, not a body of procedures transferred from some external authority. Relational learners are free to explore problems, not merely reproduce algorithms. The relational learner would not necessarily consider an unfamiliar problem unfair, or a situation that will automatically lead to failure. Relational learners tend to view mathematics as a group of related concepts and ideas. They can use their creativity in solving problems and generating mathematical knowledge. Producing "right answers" is not as important as constructing new mathematical relationships, or understanding.

Belief structures are much more complex than this discussion of an instrumental/relational dichotomy indicates. Work in political science (Wilker & Milbraith, 1972), chemistry (Carter, 1987), teacher thinking (Brickhouse, 1988) and mathematics (Frank, 1985) describe beliefs as intricately related systems which are not simply and cleanly categorized. Skemp's discussion is quite useful, however, because of ties between beliefs about mathematics and work in adult epistemological development such as that by Perry (1970), Buerk (1981) and Belenky, Clinchy, Goldberger & Tarule (1986). These studies report that mathematical (and scientific) knowledge is often perceived as much more dualistic, instrumental, authoritarian, and rigid than knowledge in other domains. Certainly this was the perspective of many of the participants in the Overcoming Mathematics Anxiety program.

<u>Anxiety as an "appropriate" response to mathematical activity.</u>

An appropriate emotion is "one which is warranted because it is generated by true beliefs and/or accurate evaluations concerning the evoking object'." (Armon-Jones, 1986, p. 67). For an individual who learns mathematics instrumentally, the emotion act commonly termed anxiety can be thought of as a very appropriate response to a problematic mathematical situation. If the way one typically solves a mathematics problem is by quickly responding with a previously internalized solution path, when such a path or algorithm is missing, an appropriate interpretation of the emotion aroused by that gap is anxiety, for the solver is stuck and without recourse. For an individual who views mathematics as relational, however, anxiety is not an appropriate response. If exploration and bringing to bear one's conceptual knowledge in new ways is part of an individual's expectations when solving problems, the emotional responses often related to creativity and challenge become appropriate. Thus, the solver's beliefs about the nature of mathematics are intimately related to his or her emotional response to a mathematical situation.

CHANGES IN BELIEFS

As part of the program we used the process of examining often widespread mathematical beliefs to encourage individuals to reflect on their own beliefs. For example, participants were asked to respond briefly to the question "What is mathematics?".

The quote at the beginning of this paper is one individual's written response to the question. Annette's perceptions are, to some degree, typical of those of participants in the program. The following examples illustrate how an individual's responses to the question "What is mathematics?" are indicators of their often tacit beliefs about mathematical activity. Our interpretation of these sample responses is necessarily influenced and corroborated by an ensuing group discussion of responses, by our observations of the participants while they were involved in problem solving, and written records such as diaries and self-reports of their interactions with mathematics.

These examples, taken from a student's response to the question on the first and last meetings of the program illustrate quite contrasting beliefs about the nature of mathematics. At the first class session Jill wrote:

> Math seems to be a subject that I have had difficulty with most of my life. It's hard for me to understand how people can take variables (a, b, c) and make them solve a problem. Sometimes I feel my mind cannot comprehend such abstract thought. Even easy (?) math confuses me at times. I really don't know what it is!

For Jill mathematics is a hard, calculating, incomprehensible subject. Doing mathematics involves arcane manipulations with abstract, meaningless symbols. Mathematics is a barrier to understanding, one she doubts her ability to overcome.

Contrast Jill's perspective with her response to the same question eight weeks later. This time, instead of labeling her paper "What is mathematics?" she wrote "What mathematics means to me"

> Mathematics is a subject that helps society change and grow. It helps us understand certain concepts and to measure things. I used to think it was a real stress issue because of my unbelief in my ability to do it. Now I can feel for math in a positive way and try to understand it's a good, useful, necessary subject for the betterment of life.

Now, mathematics is a human endeavor. Moreover, instead of providing a barrier, it is useful in assisting social change and growth and for improving the human condition. This belief is already reflected in the revised title of her paper. Mathematics has changed from a mysterious personal barrier to a societal asset and to a way of knowing or of understanding the world. Moreover, she now believes she can use mathematics, not only be confused by it.

Commenting on her last statement later during the session in which she wrote it, Jill expressed disbelief that she could have produced such a description. She stated that she had to read her definition more than once before she realized that she actually could have written about mathematics in this way. She now planned to enroll in a mathematics class. The mathematical world she now lived in was not the same world she inhabited previously. Her old responses to the world did not fit her new reality.

Lynette, an artist, also applied the depersonalized label "subject" to mathematics.

> Math is a subject I have always had to hurry to keep up with. I believe that a good understanding of mathematics

> will help you to be able to deal with business in this world. . . . I don't want to view math as just a subject that I'm OK in anymore. I want to use it as another important tool in my life.

For Lynette, mathematics was school-related. She knew that other people (those who are able to understand mathematics) were able to see mathematics as useful tool, however. Eight weeks later she wrote:

> Math is a way of thinking. It deals with logic, organization, orderliness. I find when it's time to relate to a math problem that I have to really switch gears in my way of thinking and that can be hard for me. Maybe because I'm an artist and think abstractly most of the time. but then, math can be abstract as well.

The stark contrast in the statements "Math is a subject . . . " and "Math is a way of thinking . . ." reflect strikingly differing underlying beliefs. "Math is a way of thinking" reflects a view of mathematics as an activity, a way of making sense of things. She went on to ask "Should I be trying to work out a math problem with the logical (left-brain) or the abstract (right-brain)?". This new question indicates that Lynette is now struggling with her view of the role of creativity in mathematics. Is math creative, like art? Or is it logical? Earlier math merely "was". Instead of having to deal with mathematics in rigid, prescribed ways she could address which was best <u>for her</u> to solve mathematics problems. The very questions she could ask about mathematics had changed.

Hearing another's metaphor for mathematics -- one that was new for her -- led one participant, Bonnie, to enroll in the program. During the first session she wrote about her reaction to a mathematics teacher's answer to the question "What is calculus?"

> His Answer: "A high-powered tool for solving problems".
>
> That was five years ago and I often remember this experience and must say (even if borrowed from someone else) that mathematics is a tool for solving problems.
>
> Before that question/answer time, I believed it to be an important skill, but needed in advanced form only by those who directly worked in specific vocations.

While an authority (her son's math teacher) had told Bonnie that mathematics was a tool, and she, at one level, believed this authority, she had not had experiences that would be useful to her in concretizing the concept of mathematics as tool. The utility of mathematics in daily life was still an abstraction. By the end of the program, she saw the utility of mathematics as one of the critical features of the discipline from her own perspective, not just on the word of an authority. Mathematics became an extremely empowering tool. She wrote:

> Mathematics is first computation and finding answers to computation problems in order to do one's work. Each person should (needs) to do a certain amount of computation to be in charge of ones personal life.
>
> Mathematics is also a tool, powerful at that, to enable one to solve problems of a more advanced state than daily computation. If learned in an open, creative exploring environment, mathematics can empower an individual to develop his/her potential to the fullest. It is on the same level as history, art, philosophy and science. Very exciting!

Mathematics is now a way of solving problems, not a problem in itself. It is exciting -- a source of strength, a way of evoking one's potential. In the second instance her use of the metaphor for mathematical activity reflected a more powerful, exciting way of thinking. Although Bonnie used the same metaphor in both instances, at the beginning of the program "mathematics as a tool" was a "borrowed" metaphor taken on the word of an authority. Nevertheless her use of it indicates that is may have served to provide an orientation for her mathematical activity and she later was able to take this metaphor for her own and elaborate upon it.

These descriptions of mathematics as creative, a way of knowing as a useful social tool would not be so surprising if we, as instructors, had explicitly addressed these issues or had defined mathematics in these terms. We did not, however, and were somewhat astonished upon our first comparisons of the responses. Upon reflection, this new language seemed a natural outgrowth of the experiences the participants had during the problem solving portions of the program.

These quite typical new descriptors such as creativity, utility, and empowerment reflect changes in underlying belief structures, changes that were concretized by the problem-solving activities. Mathematics changed and the vocabulary and metaphors used in describing mathematics had to change as well. The old language no longer fit. We will utilize a discussion of one participant, Gene, to analyze such changes in belief in greater detail.

BELIEFS AND PARADIGM SHIFTS

One of the most striking facets of the program and one of the most difficult to convey, was the emotional intensity that accompanied these changes in belief. These sudden changes can best be described as conversion experiences. The program participants were often making major changes in world view. Some of these changes happened early in the program, some did not occur until much later. A few participants did not make changes at all. However, when the changes occurred they were fairly sudden switches. Kuhn's (1970) discussion of scientists' changes in world view (or paradigm) provides an analogy of the changes that occurred in program participants.

Kuhn describes scientific revolutions as "noncumulative developmental episodes in which an older paradigm is replaced in whole or in part by an incompatible new one." One world view is replaced by another which provides an entirely new way of seeing. As Kuhn notes "During revolutions scientists see new and different things when looking with familiar instruments in places they have looked before." The mathematics that course participants such as Jill, Lynette and Bonnie were seeing was no longer the same mathematics as they had seen before. They were"living in a different world". The language Kuhn uses to describe the impact of atomic theory on chemistry seems analogous to the revolution in mathematical world view experienced by many of the participants in the program.

> What chemists took from Dalton was not new experimental laws but a new way of practicing chemistry. . . . As a result, chemists came to live in a world where reactions behaved quite differently from the way they had before."

What program participants who made such shifts took from the program was not a set of new strategies for problem solving or new knowledge about mathematical concepts, but a new way of thinking about mathematics itself.

Gene was one participant who made such a shift. A product of the New York State Regents system, Gene had experienced some difficulty with high school mathematics and in response to the pressure and rigidity he perceived as part of that system had developed a strong negative emotional response to mathematics. He described it as "a psychological barrier holding me back from learning math properly." When he enrolled in the program as a university freshman, he was failing his mathematics course. At that time mathematics was "strict formulas and rules". Mathematics was not creative, it was a rigid, logical, rule-oriented discipline. During the third week of the program Gene experienced a major change in his beliefs. He suddenly came to regard mathematics as a creative enterprise, one where he could draw on his intuition. His life and his experiences with mathematics suddenly changed. His grades improved dramatically. More than this, however, his views of mathematics and himself as a doer of mathematics changed. When asked to describe his mathematical ability at the end of the program, he stated:

> I have to say average, presently, because my grades reflect that. However, I feel I've developed an above-average sense of intuitions, and it goes beyond the math book. . . . More importantly, it's how it applies outside of class. I think I can probably do better than average. I feel I'm above average in that sense. In being intuitive or something.

He no longer saw mathematics as rule-driven, and described this rule driven view as a constraint faced by many of his classmates. He commented that mathematics teachers should not stress strict rules because:

> I think there's many different ideas. There's a lot of basic ideas to follow, in a problem or mathematics. I think when they say strict rule, I think it makes people limit themselves. They think "Oh I've gotta follow this strict rule." Then they don't really understand the concepts of it. So, when you think according to the strict rule, you fail to understand the main idea of what's going on. You just apply the formula they give you, or the rule, and you fail to understand what it really means.

When he moved away from "mathematics as rules", his perspective of what it means to do mathematics and of himself as a doer of mathematics also changed. Gene remarked:

> I think that some people, even I, had the notion that mathematics is never gonna come to me. Cause its something I have, a mental block inside my head and I'm not gonna be able to work around it. . . . I think if you work hard at it and you think "Why am I doing this? What are you gonna accomplish out of it?" you can learn mathematics.

Kuhn argues that a new paradigm is most often successful if "the new paradigm can solve problems that led the old one to crisis". Gene's crisis was his negative emotional response to mathematics and his failure to be successful in his mathematics course because of this response. Viewing mathematics as a creative domain allowed him to resolve these problems.

> I think math is a lot, involves a lot of creative thinking. When I started thinking of math in the creative sense, I started doing better. When I was thinking about it as strict formulas and rules and stuff I really had problems understanding it. But once you start thinking about it in creative ways it benefitted me more.

Creativity was the central theme of his new view of mathematics -- a view incompatible with his former perceptions.

> I think that looking at problems differently means you understand the problem really well. I think being original with the problem . . . I kinda really have been trying to work that way through things. I'm not trying to go against what the teacher says, but I really try to look at the problem and try and understand the way the teacher found the answer. Then I look at it my own way, approach it from my angle, and find it a lot easier.

Gene moved from the view that there was one way or one best way to solve mathematics problems to a view that there were many equally good ways to work problems. He was now free to use his quite powerful creativity and problem solving skills in his mathematics courses, something he had not been able to do before. The psychological context of mathematics no longer excluded these types of reasoning. Instead conceptual relations and understanding, not just memorization and algorithms, were mathematics. Whereas earlier he had to rely upon the teacher as the authority who showed one how to do mathematics, he was now his own source of authority.

It was only after he made this transformation in his relationship to mathomatical knowledge that Gene could begin to develop a conceptual knowledge of mathematics. Paralleling Kuhn's view of scientists:

> **Just because it is a transition between incommensurables, the transition between competing paradigms cannot be made a step at a time, forced by logic and neutral experience. Like the gestalt switch, it must occur all at once (though not necessarily in an instant) or not at all.**

Gene could not live in both worlds at once. Mathematics was rules or it was creative, the two were incompatible to him -- and so were the learning and problem-solving strategies connected with the two modes of learning.

ADVANTAGES OF RETAINING INSTRUMENTAL BELIEFS

As Kuhn has noted, "there are losses as well as gains in scientific revolution". Similarly, a shift from instrumental to relational perspectives on mathematics can create problems which did not exist under the old set of beliefs. In the case of Percy, a program participant, viewing mathematics as relational generated negative emotional reactions to mathematics where none had existed before. In the language we used at the time, Percy became "math anxious" during the course of the program. As with many program participants, the context of the program led Percy to reflect upon his mathematical beliefs and to make some changes in them. However, instead of finding these new views empowering, to Percy they were quite threatening.

Percy initially had quite positive emotional responses to mathematical activity. He enrolled in the program because, at age 65, he was retiring and was planning to return to college. He thought that the program would provide a non-threatening introduction to the university system. Percy had always believed he had a fairly good understanding of mathematics because he had always been successful when doing the mathematics necessary for his job or for daily life. His mathematics was similar to Gene's initial view of mathematics, an algorithmic and rule-oriented discipline. Percy believed that mathematics was handed down or transmitted from authorities to others, but he had been successful in using this transmitted knowledge so far in life. To a certain extent he identified with these authorities because of his success and his beliefs about the difficulty of mathematics.

In the program we spent much time on non-routine problem-solving in small group settings. The problems we used did not lend themselves to ready applications of formulas or algorithms. We also stressed multiple methods of solving problems and encouraged individuals to reflect back on previously completed mathematical activity. For most of the participants in the program this was an entirely new approach to mathematics -- and a new perspective that could only lead to improvement. Percy saw the situation somewhat differently. Suddenly the mathematics he had been successful with was no longer "real mathematics" and did not necessarily lead to ready solutions to the mathematical tasks at hand. He had to confront the beliefs, knowledge and strategies that had worked for him until now -- in an atmosphere where others were finding these "new" beliefs about mathematics a powerful improvement over their former perspectives. He was threatened by this problematic situation, and began experiencing negative emotional responses that had previously not been present.

Research in student misconceptions and conceptual change in science has noted that changes such as those Percy experienced are threatening on several levels. These changes can challenge an individual's self-perception or self image and their knowledge structure, which can lead to a questioning of all beliefs or may put the individual into conflict with others who may not share these beliefs (Osborne & Wittrock, 1985). Percy initially perceived himself as someone who could do mathematics, at least up through algebra. This perception was important because of the benefits that "it (mathematics) can do for the mind." His perception of his mental ability was related to his ability to do the mathematics he needed to do. Now his belief in mathematics, and thus of his abilities, was being shaken. He was no longer sure what he knew. As a result, he developed negative emotional responses where they did not exist before.

EMOTION AS STATE AND EMOTION AS ACT

While we have described changes in mathematical belief, we have not yet discussed a rational for understanding why beliefs should relate to emotional responses. To enter into this discussion, it is productive to think of what is typically termed "emotion" as a two-faceted construct. These two aspects may be described as a physiological reaction and a cognitive appraisal of a situation (Harré, 1986; Mandler, 1986). As cross-cultural studies have observed (Harré, 1986) the cognitive appraisal of the physiological reaction occurs in relation to a social and cultural context. These two facets, the physiological arousal and the cognitive construction correspond to the distinction between emotion as state and emotion as acts (Cobb, Yackel, & Wood, in press). By emotional acts we mean that there is an appraisal of the physiological reaction relating to some internal standard or value. Emotion words (anger, frustration, joy, embarrassment, pride, anxiety, etc.) do not refer to distinct physiological states, since the same physiological agitation can be part of many different emotions (Bedford, 1986). The emotion act is the interpretation of this emotion state. For example, one of the authors enjoys doing mechanical problem-solving. When her new car broke down, she had a physiological reaction. She could have interpreted the reaction as excitement because there was a new problem to solve. However, frustration was the emotional act that occurred because the breakdown was a violation of her expectations of the reliability of a new car. Similarly, when confronted with a mathematical problem solving situation, an individual may experience a physiological reaction. In interpreting these reactions, the individual makes a cognitive appraisal of the emotional response deemed by that individual to be socially appropriate to the situation. This 'socially acceptable' interpretation may be anxiety or frustration if the individual's expectation is that mathematical problems are quickly and easily solved by application of an algorithm. This same reaction might be interpreted as excitement because there is a new challenge to be met if the individual's expectation is that mathematical problem solving requires exploration and generation of new mathematical relationships. It is the accompanying expectation that evokes the interpretation of the physiological response. Thus, emotion acts are influenced by the expectations, which in turn reflect the individual's beliefs. Beliefs, however cannot be considered without attention to the operative social norms, as will be discussed later.

The distinction between emotion as act and emotion as state is consistent with traditional anxiety literature (Gifford & Marston, 1966) which indicates that performance is often enhanced by small to moderate amounts of the physiological state termed "anxiety". According to this literature, as the anxiety level increases, performance decreases, however. While traditional anxiety literature does not make this distinction, we believe that perhaps the emotion act termed low to moderate "anxiety" may better be interpreted as excitement or anticipation because of a challenge. The physiological arousal that is termed "high anxiety" may be interpreted as a result of the expectation that the task may not be completed. The emotion state is the physiological state. The emotional act differs in these two situations because it is an interpretation based on expectations. Traditional methods for measuring the degree of physiological arousal may not make this distinction, because the physiological response is the same in both instances. The interpreta[illegible] of the response differs, however.

Expectations and "appropriate" apprai[illegible]s

An individual's expectations are derived from experiences in social interactions and as such are part of the culture within which the individual lives and carries out activities such as mathematics. Just as all learning takes place in and is influenced by the social context of that learning, the construction of emotions occurs and is influenced by that social context. As Cobb, et al. (in press) note:

> A student's realization that he or she has failed to fulfill an obligation can itself give rise to a problematic situation for the student. To the extent that the student wants to fulfill the obligation, situations of this sort can precipitate strong emotions.

Expectations and obligations, and what conditions constitute a failure to meet these expectations and obligations, constitute the operant social norms. These norms, in turn, are negotiated in conjunction with the individual's beliefs about the nature of the activity. These beliefs, part of the culture of mathematical activity, both influence and are influenced by "typical" school mathematics. The social system of the classroom, interacting with the wider set of societal beliefs about mathematics serve to create these expectations.

As depicted in surveys, written responses, interviews and classroom discussions, the expectations of the participants in the program at the outset are derived almost entirely from a view of mathematics as received knowledge and mathematical problem solving as rapid, correct reproduction of algorithms. These expectations included getting right answers, knowing what to do the first time and not making mistakes. Implicit in these descriptions was the message that "if you are a 'good' student, i.e., have paid attention, followed along, and listened to the teacher, then you will be able to do the assigned tasks and not make mistakes, get them right and do it without stumbling." These individual expectations are not the only operant influences, however. Participants commonly described situations in which they felt obligated to meet standards set by some authority such as a teacher as well. Timed tests, competitive math games, and solving problems in public view (such as in front of one's boss) are examples of situations where individuals felt required to meet external standards of performance. Failure to meet one's obligations leads to an interpretation of the situation as one warranting negative emotion, perhaps guilt, embarrassment or anxiety. In this sense an individual's emotional response is strongly influenced by the obligations and expectations of society and so is tied to the local social order.

EMOTION ACTS AND SOCIAL NORMS

Not only are social norms, with their concomitant expectations and obligations central to emotion acts, but occurrence of emotion acts serves to reinforce and sustain the social norms. Norms are not static entities; they are continually subject to renegotiation and are kept alive in response to reinforcements such as negative emotional acts. The process acts analogously to a feedback loop in an operational amplifier with each outcome feeding into the system again. For example, suppose an individual believes that mathematics problems should be solved quickly, and in a logical, linear fashion. If the individual is not able to solve a problem quickly, the emotional act accompanying problem solving may be anxiety for that individual is not meeting his/her expectations. Further, if in the local social setting, individuals are rewarded for solving problems quickly and logically, quick, logical solutions become part of the expectations and obligations of solving mathematics problems and the interpretation that leads to the negative emotion act is reinforced as appropriate in this setting. The anxiety response itself perpetuates the expectation that gave rise to the response. The next time the individual approaches a problem if he/she cannot do it quickly easily, he/she expects to experience anxiety. The individual has constructed an emotion act for interpreting the emotion state. If, however, there had been an intervention that indicated that anxiety was not an appropriate response, that intervention would challenge the expectations (and obligations) and provide an opportunity for an alternative interpretation and consequently for an alternative emotional act. For an alternative interpretation to be made it is important for it to be compatible with the local social order. In this sense, structuring a social setting for mathematical activity in which alternative expectations and obligations are operative is one way in which to make such an intervention and so break the cycle.

In the program we, as instructors, initiated and guided the negotiation of social norms which included:

- persisting on a challenging problem
- explaining how one interpreted and attempted to complete the activities
- attempting to understand explanations given by others
- working with group partners to reach a consensus when completing activities
- respecting others' problem solving efforts.

The negotiation of these norms took place over the course of the eight week program as we interacted with the participants and as they interacted with each other. As participants interpreted their mathematical activity in light of the operant expectations and obligations they had an opportunity to reorganize their beliefs about mathematics and mathematical activity. This was reflected in a decrease in the frequency of negative emotional acts. For example, situations such as not knowing how to begin when first reading a problem, completing only one problem when others completed several, or developing an idiosyncratic solution procedure did not evoke negative emotional acts as the program progressed.

BELIEFS AND PARADIGM SHIFTS

Changes in beliefs about mathematics and mathematical activity, however, could be specific to the setting provided by the program. We had no control over the operant social norms outside the program setting. As program instructors, our role was limited to fostering a context which promoted changing beliefs and helping to provide strategies for functioning in a world where traditional expectations of mathematical activity exist. Program participants operated in two different mathematical worlds. To the extent that they viewed these as two completely different contexts, the program was ineffective in impacting on negative emotional acts related to their traditional mathematical settings, such as the mathematics classroom. To the extent that they attempted to relate the two worlds, it was possible for them to try to interpret traditional mathematical experience from a new perspective. Nevertheless such attempts could be problematic, especially for students concurrently enrolled in mathematics courses. For example, we focused on problem-solving and de-emphasized reliance on algorithms. Evaluation in college math courses often utilizes tests and quizzes with time limits which do not permit exploration of a problem. The more traditional expectations of mathematics courses often are based on an implicit view of learning as transmission of algorithms from teacher to student and success is evaluated by the degree to which students return these algorithms to the teacher. These traditional experiences and expectations are powerfully pervasive. While we could encourage individuals to reinterpret the context of their classroom mathematics experiences, each individual had to make that decision based on their beliefs, goals and motivations. As Cobb et al. (in press) note, "in the last analysis however, it is the students who have to make construals that constitute a cognitive basis for delight." While there have always been students with relational beliefs about mathematics in the most instrumental of mathematics classrooms, adopting a non-traditional perspective of mathematical activity requires a strong commitment to one's view of the nature of learning in mathematics and of the nature of mathematical activity. Merely providing a safe atmosphere to do mathematics in the context of the program is insufficient. Participants must make the "gestalt switch" Kuhn describes before lasting changes in their mathematical beliefs may take place.

One program participant illustrated the situation that can arise when these two worlds are taken as distinct contexts.

Dennis, a high school senior, is the son of a prominent physicist. He had a strong desire to please his parents by success in mathematics and science courses. Throughout high school he had been placed in accelerated or honors courses where he was always one of the poorer students in the group. Dennis had never been allowed to excel in mainstream courses. Instead, he had always been at the "bottom of the top". During program sessions Dennis engaged in meaningful mathematical activity and experienced little if any negative emotional reaction. In the program setting he interpreted his activity in terms of the local social order. His activity in the specific local context of the program differed sharply from that in his mathematics classroom, however. He did not reinterpret his classroom experiences, he continued to follow his typical patterns of instrumental learning and lack of success. His behavior outside the specific local context of the program indicates a different set of mathematical beliefs. His beliefs about mathematics were highly context-dependent.

Research on "everyday cognition" (Rogoff & Lave, 1984) or on student beliefs and/or conceptions in science (ex. Carter, 1987; Driver & Oldham, 1986; Driver & Erickson, 1983; Pines & West, 1986; McCloskey & Rohl, 1983; Hewson, 1986) describes the often quite separate domains of formal and informal problem-solving. For example, students may be quite skilled in doing classroom classical mechanics problems, but do not apply those skills when solving problems that do not evoke those formalized skills.

Dennis was quite successful in the context of the program sessions, but perceived his day to day interactions with mathematics quite differently. As noted earlier, breaking down these contextual barriers can be quite difficult.

TAKE HOME LESSON

While most of the participants in our program made highly positive changes, the participants who did not make such changes provided a greater source of illumination into the relationships among beliefs, norms and emotion acts than those who did. By considering these individuals in detail we conceptualized a framework that accounts both for the changes we observed, and those we expected, but did not observe. Critical considerations include beliefs social norms -- and expectations and obligations. In the Overcoming Mathematics Anxiety program "cognitive dissonance" or the need to explore a problem was a part of what was expected as mathematical activity -- an expectation that is not part of typical classroom mathematics. Establishing these new expectations and obligations facilitates paradigm shifts because they provide a way of concretizing new beliefs about the nature of mathematics. These new beliefs in turn served to reinforce the expectations of mathematical activity. Merely utilizing "nice" teachers, or talking about beliefs lacks the aspect of concretization to mathematical situations necessary to reorganize beliefs. Beliefs are not changed by dialogue or direct instruction, but by solving a problem that calls existing beliefs into question. This concretization, a reflection of the constructivist emphasis of the program, seems essential to promote conceptual change. The program proved successful in so many instances not because the specific mathematical activities were different, but because interpretations of mathematics changed. Anxiety, appropriate if mathematics is instrumental, became inappropriate to relational views of mathematics. Beliefs determine emotional acts and occurrence of these acts gives opportunity for a renegotiation of beliefs.

Does "mathematics anxiety" make sense?

Is mathematics anxiety a meaningful term? While we believed so at one time, we now question our use of the language. Certainly, anxiety is not an inherent response to mathematics, but is based on an individual's beliefs about mathematical activity. The physiological response is definitely present and may be quite strong, but the construction of "anxiety" arises through an interpretation of this physiological response because emotions are constructed in relation to the social order. The physiological response occurs when the individual realizes that there is a gap, or when personal or societal expectations are violated. The emotion act of anxiety is an individual construction developed through social interaction. Usage of the term "mathematics anxiety" can obscure the cognitive basis of these emotional acts.

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