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ABSTRACT

Because little is presently known about changes in children's knowledge of the logical principles of arithmetic and, more specifically, about how children's developing knowledge is reflected in the use of solution procedures, two types of three-term arithmetic problems were presented for solution to 6-, 7-, 9-, 11-, and 20-year-olds. Problems were to be solved mentally. One type could be solved only by calculating sums and differences. The other type could also be solved without computation by using a procedural shortcut based on the logical principle of inversion, i.e., a + b - b must be equal to a. Analyses of latencies and verbal reports revealed that: (1) some children as young as 6 years of age used inversion-based shortcuts spontaneously; (2) there were marked individual differences among elementary school children in the use of inversion-based shortcuts; (3) the use of procedural shortcuts based on knowledge of logical principles increased markedly between 9 and 20 years but changed little from 6 to 9 years; and (4) some 6-year-olds used a functional shortcut, the negation procedure, that may be a precursor to inversion-based procedures. It is concluded that the results indicate that early improvements in arithmetic may reflect changes in computational skill rather than in knowledge and use of the logical principles that characterize the domain of arithmetic. (Author/RH)

Developmental Changes in the Use f Logical Principles in Mental Arithmetic

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Abstract

Knowledge of a domain often includes an understanding of the logical or semantic principles that define the structure of the domain. Little is known about changes in children's knowledge of the logical principles of arithmetic and, more specifically, about how this developing knowledge is reflected in the use of solution procedures. Two types of three-term arithmetic problems were presented to adults and to 6-, 7-, 9-, and 11-year-olds. One type could be solved only by calculating sums and differences. The other type could be solved without computation by using a procedural shortcut based on the logical principle of inversion (i.e., $a + b - b$ must be equal to a).

Analyses of latencies and verbal reports revealed that (a) some children as young as six years of age used inversion-based shortcuts spontaneously, (b) individual differences were substantial, such that some individuals at all ages used procedural shortcuts and some did not, (c) the use of procedural shortcuts based on knowledge of logical principles increased markedly between 9 and 20 years but changed little from 6 to 9 years, and (d) some 6-year-olds used a functional shortcut that may represent an early form of logical understanding. The results indicate that early improvements in arithmetic may reflect changes in computational skill rather than in knowledge and use of the logical principles that characterize the domain of arithmetic.

Introduction and Method

Understanding the logical principles that define the structure of a domain is an important aspect of cognitive development. In arithmetic, changes in skill or knowledge have been described primarily in terms of (a) the use of more effective procedures and (b) increasing ability to retrieve arithmetic facts (e.g., 2 + 3 = ?) quickly and correctly. Less well understood is the development of knowledge about the logical principles that define the structure of arithmetic. We investigated changes in the use of solution procedures that reflect knowledge of the logical principle known as *inversion*.

To demonstrate, we suggest that you solve the problems below as quickly as you can without paper and pencil. Try to remember *how* you solved each problem.

$$18 + 57 - 32 = ?$$
$$29 + 26 - 26 = ?$$

One way to solve all three problems is to successively add and subtract numbers, a procedure that is consistent with the method taught in school. For example, on the first problem you probably added 18 and 57 and then subtracted 32.

In some cases, however, you are able to use your knowledge of the logical structure of arithmetic to enable a more efficient solution. For example, the second problem above can be solved without adding or subtracting at all by noticing that the problem is subject to the logical rule of inversion, namely, that adding and subtracting the same number results in no change to the original number. Successive addition and subtraction also would have worked, but it would have been much slower and probably less accurate.

Two types of three-term arithmetic problems were presented to 7-, 9-, 11-, and 20-year-olds. Standard problems could only be solved by successive addition and subtraction. Inversion problems could be solved either by successive addition and subtraction or by an inversion-based shortcut. Some examples are provided below.

Inversion	Standard
4 + 5 - 5 = ?	4 + 5 - 7 = ?
2 + 9 - 9 = ?	2 + 9 - 7 = ?

Students stated the answer to each problem aloud and solution latencies were recorded. After these problems, students were asked to solve extra examples of each type of problem and to describe how each was solved.

In mental arithmetic, solutions tend to be longer in problems with larger numbers. This "problem-size effect" should be evident on Standard problems because they can be solved only by computing sums and differences. That is, latencies should be an ascending function of problem size. Students who do not use an inversion-based shortcut should show a similar problem-size effect on Inversion problems. In contrast, students who use an inversion-based shortcut on Inversion problems should show no such effect because they are not adding and subtracting.

A group of 6-year-olds were tested with similar but simpler problems. After finishing these problems, these children were asked to talk aloud as they solved a few extra problems.

Results

Analyses of latencies and verbal reports yielded clear evidence for developmental changes and individual differences in the use of solution procedures. Verbal reports were used to distinguish subjects who used inversion-based shortcuts ("Users") from those who simply computed sums and differences ("Nonusers"). The latency data confirmed this distinction. Consider, for example, the latency data for 7-year-olds in Figure 1. Both Users and Nonusers showed a problem-size effect on Standard problems, as would be expected. On Inversion problems, however, Users showed no such effect, indicating that they used an efficient shortcut. Nonusers showed a significant problem-size effect on Inversion problems, indicating either that they used successive addition and subtraction or that they used a shortcut inefficiently or inconsistently. Similar differences were found among 9- and 11-year-olds.

The more detailed probes used with 6-year-olds allowed us to identify the use of a non-inversion procedure that we call Negation. Given 4 + 3 - 3, for example, Negation users initially counted 4 + 3 fingers. When they encountered -3, instead of removing 3 fingers sequentially from the total, they simultaneously collapsed the 3 fingers they had just counted. That is, these children simply negated the last operation, rather than subtracting in the normal manner by removing one finger at a time. A Negation procedure should result in an intermediate problem-size effect on Inversion problems because it is more efficient than successive adding and subtracting but less efficient than an inversion-based procedure. As indicated in the center panel of Figure 2, children who used Negation showed a clear problem-size effect for Standard problems but less of an effect for Inversion problems. Nonusers, who added and subtracted successively, showed nearly identical problem-size effects for both types of problem right panel, and children who used an inversion-based procedure showed the problem-size effect only on Standard problems (left panel).

The percentage of students who used an inversion-based shortcut was relatively constant from 6 to 9 years of age and increased thereafter, as indicated in Table 1. This finding is somewhat surprising because 9-year-olds have had much more practice than 6- and 7-year-olds with arithmetic, and presumably 9-year-olds also would have more knowledge about arithmetic. One possible explanation is that, as a result of massive amounts of practice with simple arithmetic problems, 9-year-olds automatically invoke a familiar computational procedure even when a shortcut would be appropriate and more efficient. That is, 9-year-olds may be capable of using an inversion-based procedure spontaneously, but they tend to use a familiar procedure rather than a more appropriate shortcut. Alternatively, 9-year-olds simply may not understand inversion well enough to influence performance on arithmetic tasks. Perhaps the type of arithmetic encountered by children in early grades is not conducive for inferring logical principles of arithmetic, such as inversion. That is, even though children can execute computational procedures and retrieve facts competently, they may understand little about the logical structure of arithmetic.

Summary

Four conclusions emerge from this study on the use of computational shortcuts in arithmetic.

1. Some children as young as six years of age spontaneously use problem-solving shortcuts that are based on the logical principle of inversion.

2. During the elementary school years, there are marked individual differences among children within age groups in the use of inversion-based shortcuts.

3. The proportion of children who use inversion-based procedures is stable from 6 to 9 years but increases thereafter. This finding may reflect a lack of understanding of inversion or a tendency to use familiar algorithms even when more efficient procedures could be used.

4. Some children use a functional shortcut, the Negation procedure, that is not clearly based on inversion but that may be a precursor to inversion-based procedures.

Thus knowledge of logical principles in the domain of arithmetic varies both between and within age groups. Although 9-year-olds are generally more competent than 6-year-olds in arithmetic, this improvement during the early school years may well reflect changes in computational skill rather than in knowledge of the logical principles that underlie the domain of arithmetic.

Table 1
Percentage of Students Who Did or Did Not Use Inversion-Based Shortcuts

Age	User	Nonuser
6	25	75[a]
7	29	71
9	29	71
11	58	42
20	94	6

[a] Including 43% who used Negation

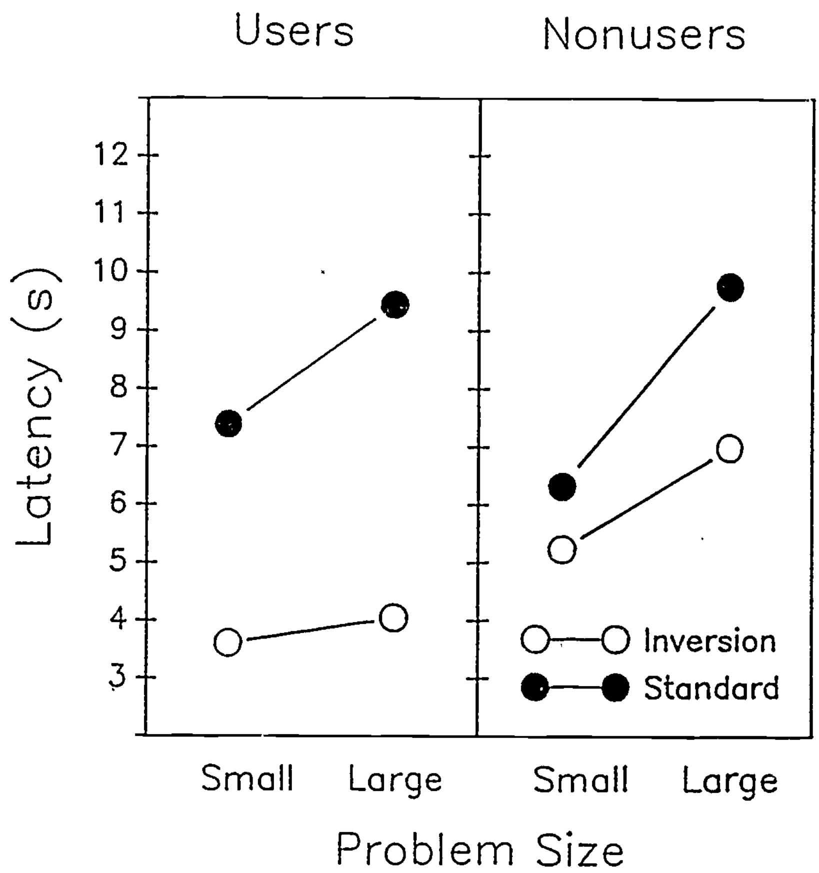

Figure 1

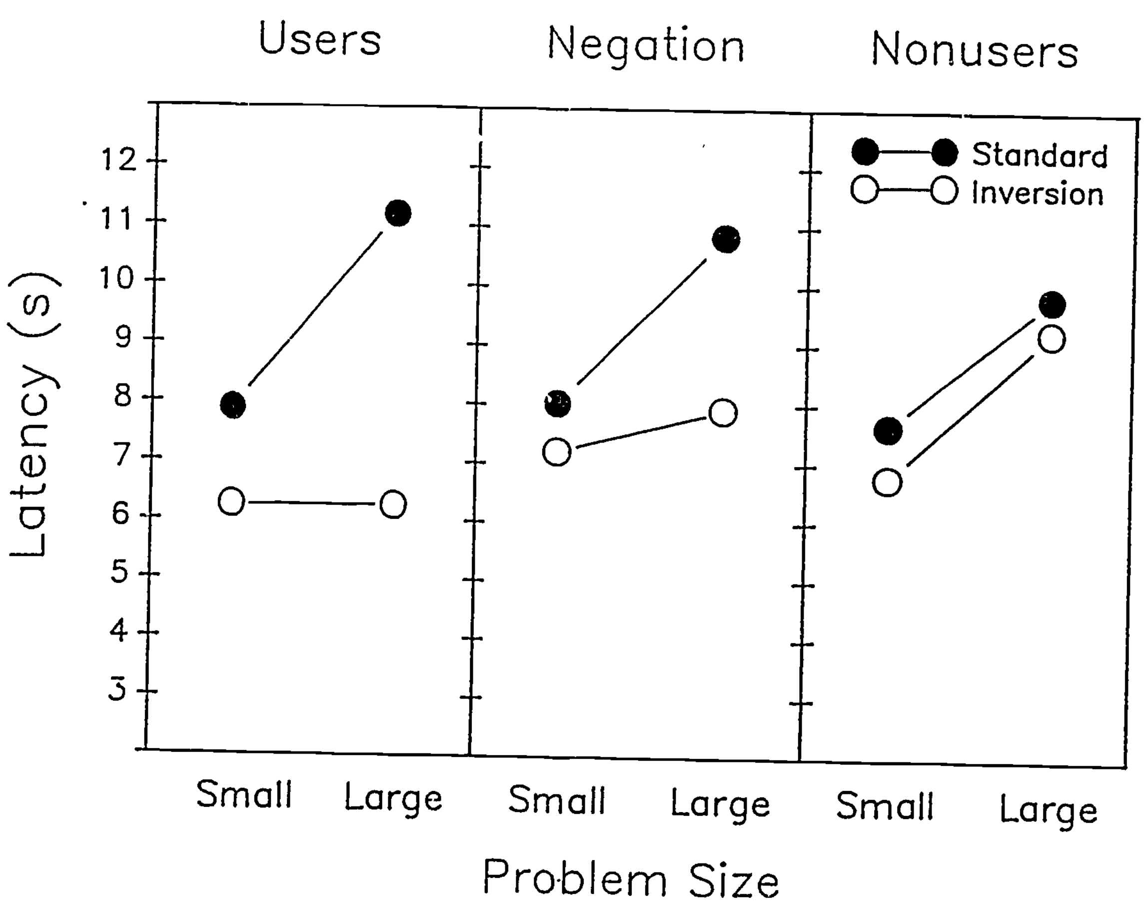

Figure 2