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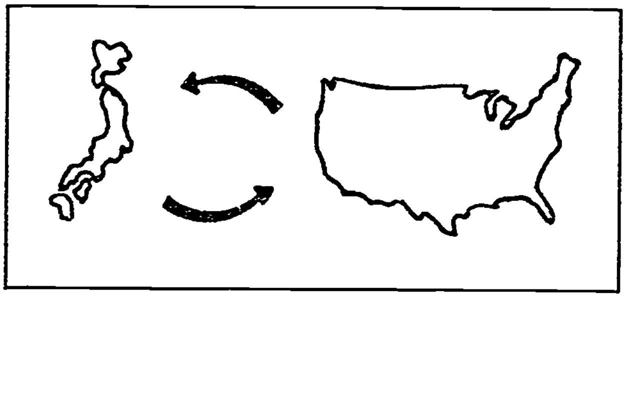
ABSTRACT

A delegation of 22 mathematics educators sponsored by the Illinois Council of Teachers of Mathematics visited Japan September 23-October 11, 1988. The group included teachers from elementary through university levels. The group visited 16 schools, 2 teacher training colleges, a scientific university, and 2 juku schools in the four major population centers of Tsukuba, Tokyo, Nagoya, and Osaka. Visits also included bookstores, factories, and cultural sites. School visits included briefing, classroom visits, and discussions with teachers. Similarities and differences between Japanese and American societies and education are identified. The Japanese mathematics curriculum content for grades 1 through 12 is described. Other topics included are Japanese educational change, uses of technology, and implications for mathematics teaching in the United States. Four tables and 22 references are included. (DC)

ICTM

Illinois Council of Teachers of Mathematics Serving Teachers of Mathematics and Computer Science

Mathematics Teaching in Japanese Elementary and Secondary Schools — A Report of the ICTM Japan Mathematics Delegation (1988)



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MATHEMATICS TEACHING IN JAPANESE ELEMENTARY AND SECONDARY SCHOOLS -

A REPORT OF THE ICTM JAPAN MATHEMATICS DELEGATION (1988)

Report Prepared

By

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ICTM Japan Mathematics Delegation at Osaka International Airport

Back Row: Gary Phillips, Sam Ritchie, Steve Victora, Tim Halloran, Greg Gruenkemeyer, Randy Pippen
Second Row: Neal Foland, Dick Hedke, Karen Strabel, Jane Schleeter, Sandy Dawson, Fran Thoele, Diane Moore
First Row (Standing): Jerry Becker, Ruth Reynolds, Donna DeMuth, Kenji Inouye, Pat Nichols
Front Row (Seated): Julie Weithorn, Cheryl Martin, Mark Wall, Sue Eddins

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A Motivating Spirit

"Gambare!"
"Persevere!"
"Endure!"
"Fight on!"
"Don't give up!"
"Do your best!"

These exclamations are symbolic of the motivating spirit of the Japanese people, a spirit which grows from homelife, continues throughout all of schooling, and extends into the adult, working lives of the Japanese. Gambare is synonymous with being Japanese and it means "to persist in any business or enterprise undertaken, in spite of counter influences, opposition, etc.; to pursue steadily any design or course once begun, to be steadfast in purpose with determination."[1]

Some General Observations and Impressions

- Japan's population is 122,000,000 - about half that of the U.S. Its land area is about that of the state of Montana.

  Japan's land mass is 80 percent mountainous, so that nearly all the population lives on 20 percent of the land area.

  Analogy: Think of the entire U.S. population residing in the state of Illinois!

- Japan is nearly the world's most densely populated country. The "living space" of the Japanese is very different (smaller) from the U.S.

- There is a pervasive uniformity or homogeneity in Japanese life - race, language, diet, customs, and education.

- All jobs are considered important in Japan.

- Japan has an extremely non-mobile population. The Japanese generally live in the same area for life - maybe even the same neighborhood - and work for the same company.

- Loyalty to the group is a dominant characteristic of the Japanese. It transcends all layers of society.[2] It is the stuff of "being Japanese."

- Great respect is accorded to the elderly.

- Japanese life expectancy is about 80 years for women and 74 years for men.[3]

- Seniority and loyalty are the underpinnings of advancement in a career.

- There is very little crime directed towards people in Japan. One is safe anywhere in Japan.

- There is little outward display of affection in everyday life, a kind of "formal feeling" in behavior.

- Women play a major role in the home and in the education of children.

- The Buddhist and Shinto religions are predominant in Japan (only 1.5 million Christians).

- The ethics of Confucius is a fundamental part of Japanese life.

- Japan and the U.S. account for 50 percent of the free world's GNP. Aside from Canada, Japan ranks first in volume of trade with the U.S. The two economies are now fundamentally interrelated.[4]

- Japan has a parliamentary form of government: Legislative (House of Representatives and House of Councillors), Executive (Prime Minister and Cabinet), and Judicial. There are forty-seven prefectures (states) with elected officials, but with no independent authority and finance as in the U.S.[5]

- There are 9 years of compulsory schooling: grades 1-6 (elementary) and 7-9 (junior high). Nearly 93 percent of students enter and complete senior high (grades 10-12). Higher education: 4 years university, 2 years junior college, 5 years technology college. There are no public kindergartens, but 44 percent of children aged 3-5 attend private kindergartens.[6] About 32 percent of high school students are admitted to university study.

- The central authority in education is the Ministry of Education, Science, and Culture (Monbusho). Each of the 47 prefectures has a 5-person board of education and each of more than 3,000 municipalities has a 5-person local board of education.

- Each school establishes its own curriculum based on Monbusho's National Course of Study (NCS).

- We saw no Japanese flag displayed in classrooms.

- Nearly all school textbooks are commercially published, but only after evaluation to adhere to Monbusho's NCS.

- All school-level teachers are certified by prefectural boards of education and are educated in universities or junior colleges.

- Mathematics, along with the National Language, holds a central place in the school curriculum. Mathematics has a long and important tradition in Japan.

- Parents, teachers, and administrators have high expectations of students in schools.

- Teachers frequently stay with the same class for two years in elementary schools.

- Students are taught how to behave in moral education classes, beginning in kindergarten and grade 1.

- Students attend school from April 1 to March 31 the following year, in three terms: April-July, September-December, January-March. There are at least 35 weeks of school, 6 days per week (half days on Saturday), in all schools.

- By the time of graduation, Japanese school students have attended school at least the equivalent of one of our school years longer than our students.[7]

- All students take mathematics the first 10 years and most take 1-2 years more, even if enrolled in a vocational senior high school.

- Japanese teachers spend more time on verbal explanations of the material than may be typical in American classrooms. They rely less on lengthy seat work assignments and worksheets with many problems of the same type.

- Students stand and bow to teachers at the beginning and end of each class. Teachers are accorded great respect.

- Class sizes range from 40 students upwards.

- In contrast to harsh sounding bells to begin and end classes in our schools, a pleasant gong is broadcast over the public address system in Japanese schools. The gongs have a visibly calming effect on students.

- The teaching of geometry and algebra begins in elementary school and nearly all of our algebra and geometry is taught by the end of grade 9.

- Examinations, of which mathematics is a major part, are required in grade 9 for entrance to senior high school and in grade 12 for entrance to college/university study.

- Homework is required which involves a small number of problems, almost all of which are more complicated than our own, and is checked regularly.

- Students begin keeping a notebook in grade 1 for each course.

- There is no wide-spread use of calculators in Japanese schools. Computers are more common, but much less than we expected.

- Mental arithmetic is an integral part of the elementary mathematics curriculum.

- The Japanese abacus (Soroban) is still taught, beginning in grade 3, for addition and subtraction. The Soroban is commonly studied by students outside the regular public schools and is still commonly used in business enterprises.

Background

The Japan visit was sponsored by the Illinois Council of Teachers of Mathematics (ICTM). The delegation was formed following an invitation from the Japan Society For Mathematics Education (JSME) which is Japan's national organization for mathematics teaching at all school levels as well as teacher education. It serves as the prime agency for coordination and stimulation of efforts to improve mathematics education. Our international travel arrangements were handled by B & A Travel in Carbondale, IL and our local travel by the Japan Travel Bureau in Tokyo. Several key Japanese mathematics educators assisted in all professional aspects of our visit.

The delegation included school teachers from the elementary and secondary levels, community college teachers, university professors, an Educational Service Center person, and a fluent Japanese speaker (Mr. Kenji Inouye, who did an admirable job of translation). Members represented many and varied aspects of mathematics teaching. The delegation met for an orientation at a hotel near O'Hare Airport before departure. Professor Hiroki Kato of Northwestern University provided information to delegates on Japanese culture and education in a superb manner. This was followed by a discussion of literature provided to delegates in the months before departure. Two books - The Japanese School by Benjamin Duke and Japanese Education Today by Robert Leestma and others - were provided to delegates before the trip. Both are strongly recommended to the reader. Other authoritative reference materials were presented to the delegation by Japanese counterparts. All these materials, together with information from classroom visits and discussions, serve as the basis for this report.

The group visited 16 schools, 2 teacher training colleges, 1 scientific university, and 2 juku schools in four major population centers (Tsukuba, Tokyo, Nagoya, and Osaka) and their environs. It also visited the National Institute For Educational Research (NIER) and bookstores, with side visits to the famous Fujitsu computer factory, a Toyota automobile assembly plant, and various cultural sites in Kyoto and Nara.

A typical school visit began with a short briefing by an official (usually the Principal or Head Teacher), then visits to classrooms where we observed entire lessons of 45-50 minutes duration. We were free to move about the room while students worked. Though we moved about a great deal, students seemed undisturbed by it all. After class we met for discussion with the teacher(s) whose class(es) we observed. These were open discussions and we could ask any questions we chose. In cases where the class did not proceed smoothly and we inquired about it, teachers were not defensive but frankly commented that they could handle the lesson better next time. These were informal discussions, accompanied by tea and snacks, with communication in English or through translation.

The delegation's main priority was visits to classrooms and discussions with teachers. There was a healthy exchange of ideas and teaching practices as well as exchanges of teaching materials donated by American publishers and manufacturers. The Japanese, in turn, provided similar materials to the delegation. Two magnificent receptions were given for the delegation by the JSME and the Study Group in Mathematics Education in Osaka Kyoiku University - our hosts were classroom teachers and numerous outstanding Japanese scholars and leaders in mathematics education.

The delegation had informal discussions of what it observed and learned during the September 23-October 11, 1988 visit. There followed a "summing up" session in Osaka and a meeting several weeks later in Urbana, IL in which the visit was discussed and this report prepared. Since Japan's achievements in mathematics education are at the center of interest in the U.S., the delegation wants to share its findings with mathematics teachers both in Illinois and throughout the country.

Rhythm of Life

As we travelled in Japan, we found ourselves immersed in the movement of people, especially in the cities where such a large part of the Japanese reside. We travelled by foot, car, bus, train (and bullet trains) and subways.

People and Movement People are everywhere in the cities. It is quite an experience to travel by train and subway, especially during the rush hour, and is hard to imagine, without experiencing it, how so many thousands of people move so efficiently on the trains, subways, and buses. Highly disciplined, the Japanese queue up for transportation and move and walk as though they "have a mission." There is relatively little pushing and shoving except as necessary to, in an orderly manner, cram twice as many people into a train car as would realistically fit! The subway and trains operate on time - remarkable! Riding by car or bus, at times though, is not recommended due to the enormous congestion of vehicles on the streets. For this reason we frequently moved by subway and train led by a trusty guide - teacher, professor, or JTB representative - and, at times, on our own.

There are well-dressed, slim people everywhere with men in suits and ties and women in smart dresses and heels - adults going to or from work or shopping as well as children (even very young). Elderly people are accorded special consideration on the streets as well as on mass transportation.

Personal Space The Japanese sense of "personal space" is very different from our own in many respects. Many vehicles, side streets, rural fields, apartments, shops, and hotel rooms are commonly smaller than our own. Corporate or public space is the responsibility of all and it is common to see children and adults cleaning windows, bathrooms, trimming and weeding gardens, and sweeping and scrubbing hallways, sidewalks, or entrances to buildings. There is a seeming orderliness and cleanliness everywhere. Viewing graffiti was rare throughout our travel. Public parks and gardens are beautiful, meticulously maintained and one sees "balance in imbalance" in the garden layouts and architecture. They are enjoyed by large numbers of people. Garbage and vandalism do not seem to be a problem.

No matter where we travelled, among large numbers of people or few, we had an overriding sense of security. We felt completely safe on any form of transportation at any time - day or night. It is common to see young people and even small children (first and second graders) riding the subway by themselves, even at night. It is common, for example, for students to go long distances to and from school by themselves by foot or public transport.

One also perceives a sense of independence and personal responsibility in children. For example, it is more common to see parent and child walking together than children being carried. Children commonly carry their own belongings in a back pack. Children riding bicycles to school are seen wearing "hard hats" in case of accident. Student crosswalks are carefully monitored by adults.

This sense of personal responsibility extends beyond children and school, for the country is seemingly filled with "workaholics." We saw office lights and workers into the night. Men frequently work until evening (e.g., 8 p.m.) and then dine out or have a drink with fellow workers, not returning home until later (e.g., 10 p.m.). It is not considered proper for a worker to go home while there is work to be finished, and no manager leaves before the employees. Great pride is taken to contribute towards the success of a business or company - personal commitment to the welfare of one's group is paramount!

Since Japan was opened to the West in the 19th century and, especially following WWII, the Japanese see themselves as a people on a mission to "catch up" to the countries of the West. It is the duty of citizens, whether worker, housewife, or student, to contribute towards achieving this goal. Despite recent stunning successes in business and education, the Japanese still see themselves as working towards that goal.

Due to the close proximity in which people live, they do not appear so "spontaneous" or to do things "on impulse." We saw little public display of affection (PDA) - some hand-holding, but virtually nothing more. There is little sexual interaction between boys and girls and we saw no "flirting," in sharp contrast to the typical PDA in the halls of American schools.

People were incredibly kind to us. Store clerks often left their stations to guide us to what we were looking for. People helped us on subways and, if they did not understand English, they quickly sought someone who could. In one instance, a man closed his palm reading stand to show us to a restaurant; in another, a man left his store to provide a car ride to a destination!

Food and Restaurants There is an immense array of restaurants in the cities, mostly Japanese, of course, but also "foreign food" restaurants. And they do a spirited business. Contrary to common belief, it is possible to get a reasonably priced meal, but only outside the hotels. Our group enjoyed scrumptious meals. We experimented with dishes of fish (raw and cooked), meat, noodles, vegetables, rice, fruits, breads, and a host of other really fine foods and desserts. One commonly selects the dish(es) from a permanent glass-enclosed display outside the restaurant before entering.

Open-air markets are common for food shoppers. But one of the most exciting places to be is in a "supermarket" on the basement floor of department stores. There are enormous displays of fresh, packaged, and canned foods of all kinds - meats, fish, vegetables, fruits, seaweed, pastries, desserts and sweets, liquors and beverages, teas, coffees,...the list goes on and on. It is exciting to see throngs of people shopping here. Free samples are available in numerous areas of such supermarkets and one can snack on regular as well as exotic foods such as pickled vegetables, vegetable chips, tiny fish and many unidentified items. An adventuresome palate helps a great deal.

Themes in Japanese Society and Education

As interesting as Japan is to the visitor, it is not easy to get an understanding of its complex of heritage, culture, social system, and education, all of which are intertwined. Nevertheless, there are some rather prominent "themes" which emerged from our study of Japan.

There is a strong home support for education. This begins in pre-school and continues at every level during schooling with mothers showing steady and strong support for children in studying and learning. One of the main consequences is that Japan has a highly literate, numerate, and well-educated citizenry. High expectations by teachers and administrators, combined with that of parents, is another contributing factor. There is an extensive system of profit-making schools outside the regular public schools, called juku ("joo-koo" or "jook"), which complements and extends learning opportunities for students. By design, students not only learn a great deal of curricular content in school, but very early learn a great deal about attitudes, skills, caring, and knowledge prerequisite to functioning as a good citizen in society as well as a productive member of the work force.

There is a clear uniformity in Japanese society - a common national language, culture, race, and educational system. While individuality is emphasized in some ways, there are nevertheless clear roles or distinctions among its people based on age, social background, gender, and education. Behavior is more formal throughout society, in contrast to the U.S., and the Japanese possess, to a high degree, common values imparted through moral education courses in school and family learning. A good education carries with it likelihood of economic security. Success in school implies a successful (happy) adult life.

Working cooperatively with others as opposed to emphasis on individual aspirations is something easily seen in schools. The objective is to seek harmony in striving for common goals. This characteristic is prominent in elementary schools where children, in a systematic fashion, become members of a group - the classroom of children (kumi) and even subgroups within the class (hans). Children learn to be responsible to their han, kumi, school and, later, business or corporation where they work. In a highly disciplined manner, children learn to practice loyalty and a smooth cohesion in group activities. In virtually all activities, Japanese children can be overtly seen to minimize competition in favor of working towards group goals and conforming behavior. Students learn that such behavior is correct and moral.

"To have a spirit of independence
To have a strong will
To have an active and creative mind
To have good judgment
To be open-hearted and sincere
To be cooperative and responsible
To be considerate about the feelings of others"

- English translation of a poster in a junior high school

As indicated in the first lines of this report, students learn that success in learning is due more to diligent, hard work than to innate ability. Gambare is the Japanese word for this attitude and behavior. A certain optimism pervades parental and teacher thinking for students to learn in this context. What could be more important in a person's life than to confront a difficult or challenging situation, and then overcome it through gambare? Such an attitude is believed to contribute towards building strong, moral character. But this is not possible unless a student has a desire to approach learning like this. Thus, it is a main responsibility of parents, teachers, and the school to provide an environment in which students are encouraged to persevere or "do your best" at all times. But individual students also provide encouragement to their peers, thus we see the importance of a strong group identity.

The Japanese are able to inculcate these attitudes in children because their society as a whole developed and maintains a consensus that education organized with these goals in mind is important. Various writers refer to this consensus as an invisible basis for Japan's productive educational system. Would our own educational system be more productive if the entire country developed a consensus regarding school objectives, and then pursued it with great vigor?

Perhaps we can summarize this section by noting that the two overriding values are egalitarianism and uniformity. It is not difficult to see that the educational system is designed to and functions in trying to achieve these goals.

Structure of Education

Schooling is structured similarly to the U.S., with elementary (grades 1-6), junior high (7-9), and senior high (10-12). Grades 1-9 are free and compulsory for all students and 99.9 percent attend. Though neither compulsory nor free, about 94 percent of students go on to full-time senior high school - academic (about 70 percent) or vocational (about 30 percent) - and about 2 percent continue on a part-time basis. Japan is a world leader in school retention. Pre-elementary (kindergarten, ages 3-5, or day care, ages 1-5) is also neither compulsory nor free, but about 45 percent of children attend. Approximately 33 percent of senior high graduates enter junior college or university study from which most graduate. Another 25 percent of high school graduates enter various vocational programs of study. There are also technical colleges that span senior high and two-year junior college. In addition, there are schools for handicapped students of various kinds.

During compulsory education (grades 1-9), students study the same Monbusho NCS mathematics content, taught at about the same rate and at about the same time. Compulsory education is strictly egalitarian. There is very little distinction among students on the basis of ability. Thus, there are no tracking, electives, or remedial opportunities for students. Promotion is automatic. Schools have very similar facilities - throughout the country - and in the schools we visited, the facilities were excellent. There are both public and private schools for compulsory education but the public are predominant. Private schools become more numerous at the senior high and, especially, at the college/university levels. There are two types of public schools: national, funded by the national government and local, funded by either the prefectural, municipal or national governments.

Elementary and secondary schools are in session 240 days per year, which includes half-days on Saturday. Monbusho requires at least 210 days of instruction but Saturday half days reduces this to a full-time equivalent of at least 195 full days. Local boards may, and often do, require more. Thus, it is common for school administrators to provide 30 days for sports and cultural festivals (which were underway in full force when we were in Japan and which involve families of students, teachers, and administrators - exciting!), short and long field trips (some classes go on trips to other parts of the country, or even abroad - it is interesting to see these groups in a highly disciplined fashion waiting to board trains in stations), graduation ceremonies, and other activities.

Japanese students have a shorter summer vacation than ours, go to school one-half day more each week, and also attend evening or extra-curricular schools (to be addressed later). Overall, students come to regard schooling as a full time activity. Evidence of this is easily seen by being in their schools, visiting bookstores, and travelling throughout the country. Japanese television has a significant amount of educational programming, some of which is used throughout the country in elementary schools. Our observations confirm that Japan's society is a learning society.

Elementary and Secondary Schools

Schools and Classrooms Japanese schools and classrooms are lively and give the appearance of places of serious learning and physical fitness. One may see flowers and plants on approach to a school and in schools. There may be a statue reflecting learning and thinking in a courtyard. Students and adults (including visitors) remove shoes and get into slippers on entrance to school buildings which are often immaculately clean and orderly. Unlike our schools, lockers are uncommon and hallways are devoid of students while classes are in session. There are also no vending machines for candy, soda, and the like in school buildings, but there is often a small "store" where students can purchase school supplies.

Students can be seen erasing blackboards, cleaning erasers, sweeping floors, washing windows, and taking out garbage at designated times of the day (usually a cleaning schedule is mounted on classroom bulletin boards). Most schools have a swimming pool and both indoor and outdoor athletic facilities. Physical fitness ("a healthy mind in a healthy body") is emphasized throughout schooling.

Classrooms are equipped with a large blackboard at the front and an overhead projector and screen and a small desk (podium) for the teacher. Some blackboards have a curtain that can be drawn to preclude chalk dust blowing into the air. Erasers are cleaned by students using a little vacuum machine at the front of each room. Extensive use is made of colored chalk by teachers

> "The way students behave today is a result of how they were raised yesterday; the way they behave tomorrow will be a reflection of the attitudes they have today."
>
> - A quote heard in a discussion with mathematics teachers

and their boardwork is excellent, using our letters and Hindu-Arabic numerals in addition to Japanese script and Chinese characters. There are bulletin and poster boards that usually provide information to students or examples of student work, but few colorful displays or posters. Rooms are well-lighted by a bank of windows (often open) and artificial light and some rooms are heated by a stove.

Students usually sit in a boy-girl configuration in desks with benches (sometimes rows of single desks). Forty or more students in a class is common. Little blackboards (e.g., 2' x 2' or 2' x 3') are used by teachers (and students) which can be hung from the top of the big blackboard. These frequently have problems or solutions written on them for all to see. Teachers also commonly use different sized posterboard paper held to the blackboard by magnetic buttons. They also have small-sized pieces of transparencies on which students write solutions for projection accompanied by student or teacher explanation.

Students show respect for the teacher - they quickly quiet down when the teacher signals attention and, of course, there is the ubiquitous rising and bowing before and after a class lesson. Students are prepared for class with notebooks and a little box which contains pencils, sharpener, pen, eraser, and straightedge. They are well-mannered, sit erect and display politeness in behavior.[8] But Japanese students are kids, and kids will be kids. There is laughter, playing, and "horsing around" - but not during class and students "tone down" this behavior when a teacher is around. Our group rarely observed a student sitting in a slouched position, with feet on a chair or desk, leaving class for any reason, falling asleep, or showing disrespectful or disruptive behavior - either in the classrooms we visited [illegible] other classrooms seen by delegates as they parted from the group and "[illegible]d" the school during a visit.

<u>Typical Student School Day</u> The day begins at about 8:30 a.m. with a class meeting led by a student. There follow two classes and then a 20-25 minute break. Then two more classes before a lunch break and recess at about 12:30 p.m. By about 1:40 p.m. students begin a period of cleaning the hallways and classrooms. Then there follow two more classes and a ten-minute class meeting. Students then pack up (nothing is left in school) and make their way to school club activities, extra help from teachers, or home. Saturdays are different, with three class periods ending about noon (four for senior high whose schedule is slightly longer).

A lunch program is common. Students serve lunch to classmates (responsibility for this passes from students to students) and the teacher remains for lunch with the class. As in the U.S., parents pay for the required lunch program.

Professional Life of Teachers Teachers arrive in their fairly large teacher rooms in which they have their own desk, supplies, etc. by about 8 a.m. There may follow a teacher meeting of 10-15 minutes before classes begin. Generally, teachers move between classrooms, not the students.

In the elementary schools, a Principal (sometimes a university professor) and Head Teacher administer the school (nearly all are men and highly experienced in both teaching and administration (e.g., 55 years of age)). Head Teachers, in practice, handle the school on a daily basis though nearly all also teach regularly. A teacher is in charge of each class and responsible for all subjects. Teaching duties take up about 22 hours per week, but teachers also have considerable responsibility outside daily teaching. About 60 percent are women.

Elementary teachers do not teach at the same grade level each year. This allows them to stay with the same group of students for more than one year. In the process, they also get a good understanding of the entire elementary curriculum and schooling process. Only experienced, excellent teachers handle the early grade levels in order to better ensure emphasis on careful development of attitudes and learning habits of students. Teachers at each grade level meet each week to plan and schedule activities for their students. This work is usually led by a grade-level Head Teacher and these groups commonly develop a newsletter for parents which includes curricular content to be covered, schedules of activities, and encouragement for parents to support and help their children.

Fairly large teachers' rooms have a desk for each teacher, supplies, blackboard, and a variety of equi. .ent for teaching.

It is the teacher's responsibility to not only teach content, but to also impart a feeling that all students are capable of learning the content, that hard work and diligence lead to learning, and that good behavior, attention to detail, and classroom routine are part of learning. Teachers are required to plan instruction according to the textbook and teachers' manual which closely adhere to Monbusho's NCS. Table 1 shows that emphasis is placed on mathematics and students learn that it also plays an important role in entrance to junior and senior high school. Though tests are commonly given by teachers, there is no "pressure" from the well-known entrance examinations ("exam hell") which come years later.

Excited Elementary Students Greeting
the Delegation Before a Class Visit

The organization of elementary classes in Japan is quite different from the U.S. Teachers partition the class (called a kumi) into small groups (of 4-6 students, called a han) with mixed ability for purposes of cooperative learning, performing chores, and discipline. Students, in pairs, also assume the position of classroom monitors on a rotating basis. The monitors assist the teacher in various tasks. Together, the han and monitors provide opportunities for students to grow as leaders and learn the value of cooperation for the good of the whole.[9] This organization of the classroom and its ramifications are detailed in the excellent book by Benjamin Duke, the essence of which is to inculcate loyalty to the group, free the teacher from a lot of management, and to locate discipline in the han of which each student is a part.[10] For example, teachers rarely publicly reprimand a student; rather, the teacher may comment that a particular han "is not prepared" - thus, the student's han-mates then exert pressure to bring the inattentive or misbehaving student to proper behavior.

The junior high schools are organized similarly to the elementary schools, but there are some differences. Only about one-third of teachers are women. Teacher instructional time is about the same - 22 hours per week, but in some cases only about 16 hours per week. Each class remains together all day and, in addition to a teacher, has an advisor called a tannin. A tannin acts as counsellor (academic and social), handles behavior problems, and meets with the class only during daily and weekly "homeroom" periods.

Junior high teachers frequently approach instruction from a "lecture" point of view closely following the NCS. If the elementary instruction is student-centered, then the junior high is more teacher-centered. At this level, students wear uniforms and good manners and appearance are emphasized. Further, the same basic organization of the kumi continues but the han no longer exist. Mathematics along with the National Language is emphasized and English is required as can be seen in Table 2.

There is increased emphasis on academics in the junior high. Instruction is departmentalized and the majority of teachers teach only one subject, but sometimes two. Literature we read before going to Japan represented teaching as placing a premium on mastery of factual material, drill and memorization, and one right answer. Yet, in the schools we visited, we found numerous very different class lessons with emphasis placed on exploration, conjecturing, pattern-finding, and discussion of different approaches to solutions of problems. Perhaps this is an indication of changes occurring in Japanese mathematics teaching, due to recommendations of professors and professional organizations. But, in general, the movement through the NCS is intense and has been characterized as "roughly equivalent to the fast track in a good suburban school system in the U.S."[11]

In grade 9 students take an entrance examination for the senior high school they wish to attend. Mathematics is a crucial part of this examination. Table 3 shows the mathematics problems on one such entrance examination. The five problems, each with 2-3 parts and which are to be solved in fifty minutes, give the reader an idea of the difficulty level of such entrance examination problems.

Organization of the senior high schools is very similar to the junior high. Approximately 83 percent of all teachers are men, with women teaching mostly in the areas of home economics and (girls) physical education. Like our own schools, there are department heads, counsellors and personnel who look after matters of student discipline. Sometimes these are teachers with lighter teaching loads. The teachers' rooms, characteristic of Japanese schools, are situated close to those of counsellors and grade level Head Teachers. Students in each grade are assigned to different classes each year and teachers "move up" with students to graduation. For mathematics teachers, there are seminars with teachers from other schools, and teachers thereby become acquainted with many other mathematics teachers. Many teachers have homeroom assignments, in addition to teaching, and thereby play a role in advisement for university study and handling students' personal and behavior problems.

The pace of instruction in mathematics increases at this level since the all-important university entrance examinations occur in grade 12. In private universities, attended by the majority of students in higher education, the examinations are prepared by individual universities. For public universities, they are prepared by a committee representing all public universities. Teachers cover a large amount of mathematics and straightforward lecturing is the mode - clearly teacher-centered. It is not different from our own university mathematics courses in the view of members of our delegation. Student questions are rare and teachers exhibit an exceptional command of their subject matter and frequently show exceedingly interesting in's and out's of the concepts being taught. If academics become more important in junior high, it is even more the case in the senior high.

Continuing education is characteristic of all Japanese education in general, and mathematics teaching in particular. All beginning teachers are required to attend at least 20 days of inservice education their first year. Much of the inservice work is not only initiated by mathematics teachers, but is also carried out by them, sometimes in cooperation with professional organizations such as the JSME and professors in colleges and universities. Much inservice work occurs in the teachers' schools led by experienced teachers on leave of absence from their own positions. We saw demonstration lessons by several such teachers which were excellent. As an aside, we viewed classroom lessons of teachers with a variety of experiences: student teachers, novice teachers, experienced teachers, and master or demonstration teachers. Some schools have the distinction of providing demonstration lessons two times a year to teachers who come from all parts of Japan. It is also common for teachers in a school or municipality to teach classes in front of their colleagues and master teachers, with reactions, suggestions, and discussions following. This strikes us as a very admirable characteristic of professionalism among Japanese mathematics teachers.

Each prefecture has an inservice teacher education center. These centers provide an array of inservice programs, including some for mathematics, staffed by master teachers. Teachers commonly spend 1-5 days a year at these centers for purposes of updating subject matter knowledge, teaching methods, and uses of technology in teaching. We were told that inservice programs are generally judged by teachers to be very useful.

Mathematics teaching at all school levels in Japan carries with it important rewards, both in terms of social status and economic security. Teachers tend to be held in high esteem and this is especially true of mathematics teachers. Salaries are high enough so as to provide strong incentive for many young people to seek teaching jobs (there are five times more applicants than positions available). Even beginning teachers earn higher salaries than, say, a white collar worker in a business or corporation with a similar educational background; in fact, it is higher than for a beginning engineer with a B.S. degree! At mid-career, the differences are minimal, but beyond age 50 teachers again begin to earn more than other professional white collar workers. With the accumulation of years of service and fringe benefits, there are very strong incentives to enter and remain in education.

There are salary schedules for four categories of teachers: kindergartens, elementary and junior high schools; senior high schools; technical colleges; and junior colleges and universities. Educational attainment and years of service are basic factors. But the salary of a mathematics teacher also depends on many other factors (allowances) such as bonuses (all teachers), family allowance (teachers with dependents), COLA (teachers living where living costs are high), commutation (teachers who commute to their school), housing (teachers who pay high rent or own their own house), day-and-night duty (teachers who have these responsibilities), teachers in isolated areas allowance, multi-grade classes allowance (teachers in small schools), vocational education allowance, teachers of part-time and correspondence school allowance, and other special allowances.[12] There are also good medical and survivor insurance benefits, paid partly by the teacher, a lump sum retirement payment (more than two years' salary at age 60), and an annual pension (at age 60).[13] While it is not easy to make comparative purchasing power statements for American and Japanese teachers, in 1983-84 "The average teacher in Japan could buy a significantly larger share of his country's goods and services than could the average teacher in the U.S."[14]

The Mathematics Curriculum

We visited one kindergarten in Japan. It was connected to a university (there are 48 of these in Japan) and enrolled about 170 children of ages 3, 4, and 5. The main philosophy of the kindergarten is to provide an environment in which children learn to be strong, cheerful, play and work hard, and get along with others. There is only a little group instruction and the main focus is on individual children as they engage in many different activities of play. In particular, there is no instruction in arithmetic (or writing). Children arrive between 8:30-9:00 a.m. and leave about 11:30. Older children on some days remain until 2:00 p.m. There is substantial amount of free play in more contexts using more materials than we could possibly list or describe here. Children choose the activities they want and then there is an adult to watch, help, and guide. We saw many aspects of the Montessori approach in children's play, with some aspects of mathematics (e.g., geometric shapes, different sizes of objects).

The formal mathematics curriculum begins in grade 1 and students have textbooks. Japanese mathematics textbooks are consumable paperbacks which measure 21 cm. x 15 cm. and are much thinner than our own. Students in compulsory education receive the books free, paid for by Monbusho, even in private schools. Senior high students must buy them in bookshops. Students in part-time or correspondence courses also receive them free.

The mathematics textbooks are "tightly organized" and provide little or no repetition or review, unlike our own which contain so much repetition.[15,16] Teachers present concepts and development of particular skills just once and cover the entire book according to Monbusho's guidelines. In contrast, our books may contain several chapters which may be skipped or never taught by any teachers. The teacher, of course, plays a key role in introducing and elaborating material, and teachers throughout the country proceed through the texts at about the same pace. Students have a major responsibility in learning: taking notes, working on assignments, and "doing their best" to master understanding and skills outside of class. Some textbook publishers sell study guides with extra worksheets and practice tests, and these are used by students at home and frequently under parental supervision. With this system, students are believed to be equally ready for work the following year.

Elementary students spend a little more time in mathematics than their American counterparts. But the major difference is in the amount of time and effort put into study outside class and in the more efficient use of class time. During grade one, four forty-five minute periods per week are used for arithmetic instruction. The number of periods increases to five per week for the students in grades two through six. Accuracy in computation is emphasized, and there appears to be no great emphasis on estimation skills. Many concepts are introduced at an earlier grade level in Japan than in the United States. An outline of the mathematics curriculum is as follows:

> In grade 1 children learn the concept of number and to compare numbers (correspondence), count and express the number and order of objects; know the size and order of numbers, sequence them, and express them on a number line; know place value of two-digit numbers; add and subtract (inverse operation) one- and two-digit numbers and know when to do each; count objects by grouping, and dividing them into equal parts and classifying them; understand length, area, and volume and their measurement by comparing their sizes; simple clock reading; recognize shapes of objects and their features, construct various shapes and divide them into more basic shapes; express the position of an object such as before and behind, right and left, or top and bottom. Symbols for the one's place, ten's place, addition, subtraction, and equals are introduced. Children are guided to think developmentally by thinking from different viewpoints, based on operational activities, and after getting a familiarity with figures, to abstract concepts from concrete objects.
>
> In grade 2, children learn to rearrange objects into groups of the same size and classify them; to know Hindu-Arabic numeration and represent numbers up to four digits and know and order numbers through activities; add and subtract two- and three-digit numbers using the basic facts; know the properties of addition and subtraction and their use in developing algorithms or checking answers; to know the meaning of multiplication and when it may be applied, its properties (e.g., when multiplier increases by one, commutativity, etc.) and to multiply one-digit numbers; to use algebraic expressions such as equality and inequality and read them; to extend understanding of length, area, and volume and to make measurements - know units of measurement (e.g., mm., cm., m., dl, and l.); to know day, hour, minute and their relationships; to know geometrical figures (faces, sides, vertices), compose them and make objects the shape of a box; to know squares, rectangles, right triangles, and quadrilaterals, and to represent positions of objects in a plane; learn the symbols unit, straight line, right angle, x, >, <, ( ), and represent objects in tables or graphs and

read them; tessellating the plane using congruent squares or rectangles.

In grade 3, children learn place value to ten thousands, sizes of 10 times, 100 times, 1/10 of a whole number, and how to indicate them; further work with addition and subtraction and use of their properties; multiplication of whole numbers and properties (when multiplier increases by one, commutativity, etc.) and their use in multiplication; multiplication of two- and three-digit numbers by one- and two-digit numbers; to use the column form of multiplication; to know the meaning of division and when it can be applied, the relationship between multiplication and division, or division and subtraction, and to use these relationships in calculation, deriving algebraic expressions, and checking results; to know the meaning of a remainder; to use the column form of division, with a one-digit divisor; decimal fractions and the notation for decimals and fractions; addition and subtraction of decimals and fractions; to know the Soroban (abacus) - setting numbers and addition and subtraction; to know the units and measure in weight (gm. and kg.); to estimate length and know km.; reading time; to understand isosceles and equilateral triangles, angles of geometrical figures; center, radius and diameter of a circle and diameter of a sphere in relation to a circle; to represent mathematical relations using algebraic expressions and to read the expressions - formula and representing a variable quantity as " $\square$ " and to consider values placed in it; to represent data in simple graphs and read them; to classify and arrange data by time, day, place, etc. and put in a table; to read and make bar graphs. Terms and symbols introduced are whole number, number line, decimal point, place of 1/10 (tenth), numerator, denominator, second, signs of equality and inequality, and $\div$. Emphasis is placed on mental computation of addition of two-digit numbers and subtraction as the inverse operation, multiplication of two- and one-digit numbers, and division as the inverse operation of multiplication. Ruler and compasses are used in making figures and checking them. Students are expected to read graphs by using scales of 2, 5 or 20, 50, etc.

In grade 4, children learn numbers such as hundred, million, billion, and trillion and the characteristics of the base ten system; rounding numbers and purpose (e.g., 0.5 rounds to 1); continue multiplication of whole numbers and introduce the division algorithm by a two-digit number and (dividend) = (divisor) x (quotient) + (remainder); to represent decimal fractions and adding and subtracting them; multiplication and division, including when the quotient is a decimal; to represent fractions and their meaning and equivalent fractions; addition and subtraction of fractions with the same denominator; summary of meanings and properties of the four fundamental rules of arithmetic and use them in application and checking concrete situations; to know the relations between the four fundamental operations and commutativity, associativity, and distributivity; extend the meaning of unit and measure of area ($cm.^2$, $m.^2$, and $km.^2$) and to find areas of squares and rectangles; to learn degree, for measuring angles, and half rotation, full rotation, etc.; to get a better understanding of parallelograms, trapezoids, rhombuses, etc., and to learn parallelism and perpendicularity of lines; to learn about solid figures and space: cube, rectangular parallelopiped, and position of an object in space; to represent and study the relation between two varying quantities (ordered pairs in a table), broken-line graph and to read its features of variation; to know the meaning of expressions with the four operations and parentheses, and to compute; using formulas, and representing variables using $\square$ and $\triangle$ and placing values in them; gathering, classifying, and arranging data - checking for omissions and duplication, representing in bar-graphs or broken-line graphs and interpreting graphs. Terms and symbols introduced are sum, difference, product, quotient, mixed fraction, proper fraction, improper fraction, parallel, perpendicular, diagonal line, and plane. Where multiplication involving three-digit numbers is involved, children are guided to themselves devise the algorithm, extrapolating from the case of two-digit multiplication. Emphasis is placed on use of the Soroban and students are encouraged to make sketches when working in three dimensions.

In grade 5, students learn subsets of the whole numbers (evens, odds), divisors and multiples, and that division can always be represented by a single number using fractions; to know that whole numbers and decimal fractions are decimal numbers; multiples of a number by 10, 100, 1/10, or 1/100 by moving the decimal point; multiplication and division of decimal numbers and properties of whole numbers extended to decimal numbers; transform whole numbers and decimals to fractions and vice-versa; comparing sizes of fractions and equivalence when top and bottom are multiplied by the same number; addition and subtraction with different denominators and multiplication and division of fractions when multipliers and divisors are whole numbers; rounding numbers and its use in estimating products and quotients; finding areas of triangles, parallelograms, trapezoids, etc., and area of polygons by partitioning into triangles; area of circle; the concept of volume, units, and the measurement of one: $cm.^3$ and $m.^3$, finding volume of a cube and rectangular parallelopiped, and knowing the meaning of capacity; estimating length, area, and volume and sizes of quantities by rough measurement; to know the meaning of mean (average) and its use; expressing the ratio of two quantities, "per unit," and knowing the meaning of speed, its representation, and calculation; to know congruence of geometrical figures, correspondence of vertices, sides, and angles; to know what elements determine the shape and size of geometrical figures; investigating and constructing figures by finding the simple properties of fundamental figures; meaning of ratio of circumference of a circle to its diameter (use 3.14 as the constant); draw polygons and investigate properties by using circles; to know the meaning of percentage and use it; investigating mathematical relations represented by simple expressions by paying attention to correspondence between two quantities and variation; to know that the relationship in a formula holds true whether the domain is the whole numbers or decimal fractions; to know that "x" can be used for $\square$, $\triangle$, or words standing for quantities; classifying data according to an objective and representing data in circle- or tape-graphs.' Terms and symbols: reduction, reduction to a common denominator, common divisor, common multiple, congruence, sector, central angle, percent. The idea of ratio is used in percentage and, though algebraic expressions are emphasized, it is done somewhat informally. Graph paper, divided into 10 or 100 equal parts, is used in making circle graphs.

In grade 6, students learn the meaning of and operations of multiplication and division of fractions, including the divisor as a fraction; to know division as multiplication by the reciprocal, reforming multiplication and division of decimal numbers as calculations of fractions and to express all calculations comprising multiplication and division as one fraction; to know the mutual relations among whole numbers, decimals, and fractions; to know that equivalence and difference relations in size hold in all numbers; to more thoroughly understand the number line; to know that numbers are used to represent the measure of probability of occurrence of indefinite events; to measure efficiently using proportional relationships, to understand the metric system and relations among its units and to use them, and to know area (a), hectare (ha.), ml., kl., mg., and ton (t); to know the meaning of line and point symmetry and use the concepts in considering figures; understanding shape and size of a figure; read and draw simple scale drawings and extended figures; to know the relationships between fundamental figures; to know fundamental prisms, circular cylinders, pyramids, and circular cones - and represent and construct them; to better understand direct and inverse proportion, and to know their features by using algebraic expressions and graphs; to know that the proportional relation has important applications; investigate the distribution of data and use frequency tables and graphs; to know that the tendency of the population is conjectured through the ratio gained from statistical data; to select tables and graphs according to purpose and to devise useful ones, and to arrange, in order, the possible cases concerning simple affairs; terms and symbols: reciprocal, axis of symmetry, center of symmetry, value of a ratio, more than or equal to, less than. Complex operations using mixed numbers are avoided. Only right prisms, circular cylinders, circular cones, and pyramids are considered and further, only those cases are treated where the student can draw the figures. Students are gradually drawn to consider continuous quantities and the domain relating to graphs of direct and inverse proportion.

Regarding teaching of the content above, teachers are reminded both of the importance of mastery of understanding and intentional and purposeful skills development. Further, the content of grades three to six are to be deliberately used in the teaching of other domains as the opportunity arises. Terms and symbols are used in direct relation to the content of each grade. Teachers have some latitude from fifth grade onwards to use the Soroban or calculators in calculation, but not in a way which disturbs the development of abilities such as rough estimation.[17]

Since the elementary school is at the core of children's development in thinking, attitudes and habits, we want to briefly describe some of the other curricular areas. Only the Japanese language receives more emphasis than mathematics (see Table 1). In grades one through four, eight forty-five minute periods per week are devoted to the Japanese language. In grades five and six, the time is reduced to six periods per week. The study of Japanese includes reading, writing, and speaking the language. We were told that, at the first grade level, students learn about 76 Chinese characters, at the second grade another 145, and at the third grade about 195 more. Each subsequent year more than 200 characters are learned until students have mastered about 2,000 characters by the end of ninth grade.

Like American schools, social studies is included in the Japanese elementary curriculum. The emphasis is placed on the interdependence of society and the responsibility of the individual for the collective welfare of society. At the first grade level, topics include the child's school and family. In the second grade the focus is broadened to include the community. During the following grades the emphasis is on the city, the prefecture, the nation, and then foreign countries. In the sixth grade the topics are Japanese history and the Japanese political system. Geography, understanding maps, graphs, and tables are stressed throughout the elementary grades.

In science, students are taught to observe, conduct experiments, and to appreciate nature. Scientific areas are restudied at each grade level with increasing detail. The core topics are biology, matter, energy, and the earth and universe. The first two grades have two periods of science per week. Three periods per week of science instruction are given in grades three through six.

Singing, instrumental performance, and appreciation of Western and Japanese music are the core of the music curriculum. Keyboard, wind, string, and percussion instruments are used in the elementary music classroom. The students are also exposed to Bach, Handel, Beethoven, and Schubert. Two forty-five minute periods per week are given to the music curriculum in each of the six grades.

The arts and handicrafts curriculum includes drawing, painting and sculpture. First graders are introduced to printmaking. The teaching of perspective, depth, dimension, shadow, light, and composition begin in the third grade. Art is taught two periods per week in all six grades.

Instruction in many physical education activities is included during three forty-five minute periods per week. The goals are to learn to enjoy physical exercise, develop strength and perseverance, and knowledge of good health. Activities include track and field, marching, soccer, basketball, gymnastics, dance, swimming, ice skating, and skiing. The health curriculum emphasizes nutrition, traffic safety, and a healthy lifestyle. Fifth graders also study physical growth and the changes of puberty.

A major difference between the American and Japanese elementary curricula is the inclusion of a course in moral education, which meets one period per week at each grade level. This curriculum has twenty-eight themes that are taught and reinforced during daily activities. Some of the topics are the importance of order, routine, cooperation, and thoughtfulness; endurance, hard work, and high aspirations; freedom, justice, fairness, rights, duties, trust, and conviction; the individual's place in groups; harmony with nature; and the need for rational and scientific attitudes toward human life. Since there is no prescribed textbook, there is considerable variation of the teaching techniques used.

Special activities is another difference between Japanese and American elementary curricula. The first, second, and third grades have one hour per week of special activities, and the fourth, fifth, and sixth grades have two hours per week. These activities include sports events, cultural festivals, excursions, and ceremonies, as well as club or class meetings.

Another curricular difference occurs in the fifth and sixth grades - the inclusion of homemaking. Both girls and boys receive two hours per week in basic homemaking skills. This includes meal planning and cooking, taking care of clothing, and simple sewing and embroidery.

The junior high mathematics curriculum in the U.S. is different from Japan. Our curriculum is generally characterized as a review of arithmetic with a great deal of repetition of elementary school arithmetic. In fact, it is the end of mathematics study for many students, including some who may eventually end up entering elementary school teaching! But the Japanese curriculum has more new mathematical content which centers around algebra and geometry.

> In <u>grade 7</u>, students learn integers, their properties, an integer as a product of primes, and the properties of divisor and multiples; positive and negative numbers are introduced along with the four operations on them; multiplication and division of algebraic expressions are taught along with addition and subtraction; solutions of linear equations in one variable and approximate value are also taught. Terms and symbols: natural number, factor, greatest common measure, least common multiple,

sign, absolute value, term, coefficient, similar terms, <, and >. Functions are introduced - change and correspondence and variable and domain; meaning of coordinates and functional relations in a table, graph, and formula; graphs of direct and inverse proportions, lines and planes in space, making solid figures by movement of plane figures, and cutting, projection, and expansion of solid figures; bisector of an angle, perpendicular bisector of a segment; constructing a figure as a set of points satisfying certain conditions; length of an arc and area of a sector; surface area; volume of a cylinder, conical solid and a sphere. Terms and symbols: body of revolution, arc, chord, π, $\parallel$, $\perp$, $\angle$. Care is taken so that the teacher does not delve deeply into the topics of section and projection.

In <u>grade 8</u>, students learn computation using formulas and the four operations; addition and subtraction of simple monomials, multiplication and division of monomials, multiplication of a monomial and polynomial, and division of a polynomial by a monomial; inequalities and solutions of inequalities using their properties; linear equations with two variables and their solutions; solutions of simple simultaneous linear equations and linear inequalities; describing phenomena using linear functions, the linear equation in two variables expressing functional relations between two variables; graphs and linear functions; the ratio of changes in the values of corresponding variables of a linear function is constant; displacements by parallelism, symmetry, and rotation; properties of parallels; conditions of congruence for triangles; meaning of similarity and conditions of similarity for triangles; properties of the ratio of segments of parallel lines; properties of triangles and parallelograms. Terms and symbols: opposite angle, interior angle, exterior angle, center of gravity, $\angle R$, $\triangle$, $\equiv$, $\backsim$. Students learn the meaning of frequency distribution and interpreting a histogram; meaning of relative or cumulative frequency; meaning of mean value and range. Terms and symbols: frequency, class.

In <u>grade 9</u>, students learn the meaning of square root of a number and computation in formulas involving square roots of numbers; multiplication of simple linear expressions, expansion and factorization of an expression by use of $(a+b)^2 = a^2 + 2ab + b^2$, $(a-b)^2 = a^2 - 2ab - b^2$, $(a+b)(a-b) = a^2 - b^2$, $(x+a)(x+b) = x^2 + (a+b)x + ab$; quadratic equations and their solutions (real numbers), using factorization to solve quadratics and the formulas for solution; Terms and symbols: radical sign, rational number, irrational number, and $\sqrt{\ }$. Various phenomena and their functions; a function which is in proportion to a square; a function which is in inverse proportion to a square; the ratio of change in the value of a function; sets and function, domain and range; properties of a circle and a straight line and the properties of two circles; relationship of the angle of circumference to the central angle; Pythagorean theorem and applications; height and distance as an application of similarity,

similarity of simple solid figures, and the relationships between the ratios of length, area, and volume in similar figures; Terms and symbols: tangential line, point of tangency. Students learn the meaning of probability and computing probability in simple cases, population and sample, mean value and ratio in a sample; events that can be classified with the aid of tree diagrams and dealing with experiments and observations.

Teachers are given the latitude of using the Soroban, slide rule, or calculators as the situation warrants.[18]

There are two parts of the senior high school mathematics program: Part I - Mathematics Program For General Students, and Part II - Mathematics Program in the Science-Mathematics Course of the Upper Secondary School.

Part I: Mathematics Program For General Students

The following courses are taught in the senior high school:

Mathematics I	4 credits
Mathematics II	3
Algebra/Geometry	3
Basic Analysis	3
Differential and Integral Calculus	3
Probability/Statistics	3

Note: 1 credit is 35 periods of 50 minutes each throughout the year.

Mathematics I: Numbers and algebraic expressions: numbers and sets, integers, rational numbers, real numbers, polynomials, rational expressions, including zero and negative integers; equations and inequalities: quadratic equations, simple equations of higher degree (factor theorem for third and fourth degree equations), simultaneous equations (two variables of first and second degree); quadratic inequalities; algebraic expressions and proof (including necessary and sufficient conditions and contraposition); functions: quadratic and simple rational and irrational functions including functions such as $y = ax+b$; $y = \frac{ax+b}{cx+d}$ and geometric figures; trigonometric ratios - sine, cosine, tangent, sine theorem, cosine theorem; plane figures and equations: points and coordinates, equations of straight lines, equations of circles, regions represented by inequalities.

Mathematics II: Probability and statistics: permutations and combinations, probability, statistics; vectors: vectors and their operations, applications of vectors; differentiation and integration: meaning of differential coefficient, derivatives and applications, meaning of integration and indefinite and definite integrals; sequences: arithmetic and geometric sequences; various functions: exponential, logarithmic and trigonometric functions; computer and flow charts: functions of computers, flow charts, and algorithms, including experience in writing programs, running them and analyzing the results.

Algebra/Geometry: Conics: parabola, ellipse and hyperbola (normal form, rough sketches, and foci included); vectors in a plane: vectors and their operations, inner products of vectors, applications; matrices: matrices and their operations (2x2), inverse matrix, linear transformations and mappings; space figures: points, straight lines and planes in space (built around the relations between each position about parallelism and perpendicularity and the theorem of three perpendiculars), coordinates in space, vectors in space.

Basic Analysis: sequences: simple sequences - arithmetic, geometric, etc. (limited to sums of each and $\{n^2\}$), mathematical induction; functions: exponential, logarithmic (excluding computation with logarithms), trigonometric functions (generalized angle and circular measure method), trigonometric functions and their periodicity, addition theorems for trigonometric functions; variation of values of a function: meaning of differential coefficient, derivatives and their applications (derivatives of sums, differences and products multiplied by real numbers or functions, tangent, increase and decrease of values of function, velocity, etc.), integration and applications (indefinite integral, definite integral, area, etc.).

Differential and Integral Calculus: limit: limit of a sequence (confined to infinite geometric series), limit of a value of a function; derivatives: differential calculus of product and quotient of functions, composite and inverse functions (simple functions such as $y = x^k$ (k rational), $y = \sqrt{ax+b}$, and $y = \sqrt{ax^2+b}$,) and derivatives of trigonometric, exponential and logarithmic functions (the mean value theorem approached intuitively and used to make clear the relation between changes in the value of a function and its derivative); applications of derivatives: tangent, increase and decrease of values of a function, velocity, acceleration, etc.; integral calculus: meaning of integration, integration by substitution and by parts in simple cases (include functions such as $ax+b = t$ or $x = a \sin\theta$; integration by parts (limited to those cases where only a single application is required concerning simple functions), integration of various functions, applications of integrations such as area, volume, length of path, etc., and meaning of differential equations, including solving such equations as $\frac{dy}{dx} = ky$.

Probability/Statistics: arrangement of data: distribution of the variate, representative values and measures of dispersion; number of cases of possibilities; permutations, combinations, binomial theorem; probability: probability and its basic laws, independent trial and its probability, conditional probability; probability distribution: random variable and its probability distribution, binomial distribution, normal distribution; statistical inference: population and sample, idea of statistical inference through concrete examples.

Mathematics I is required of all students in tenth grade. The other courses are optional and generally follow Mathematics I. Differential and Integral Calculus is usually studied after Basic Analysis. The use of computers is encouraged, as well as other mechanical aids such as the calculator.

Part II: Mathematics Program in the Science-Mathematics Course in the Senior High

The Science-Mathematics Course is specialized and includes two subjects: Science Mathematics and Integrated Mathematics. Credits awarded are decided by administrators.

Science Mathematics Numbers and algebraic expressions, equations and inequalities, functions, geometric figures, common logarithms (including extension of index), electronic computer (writing programs, running them, and analyzing the results).

Integrated Mathematics Algebra/geometry: conics, vectors in a plane, matrices, space figures; analysis: functions, sequences, differential calculus and its applications, integral calculus and its applications; probability and statistics: arrangement of data, number of cases of possibilities, probability, probability distribution, population and sample; project study.

The content for Integrated Mathematics comes from Algebra/Geometry, Basic Analysis, Differential and Integral Calculus, and Probability/Statistics described earlier, but the content is further developed. The teaching is fast-paced and involves student reading and exercises.[19]

Mathematics Lessons

Our delegation observed from one to four mathematics lessons in each of the sixteen schools we visited, at times dividing ourselves into smaller groups and observing different classrooms. We viewed many excellent lessons. Afterwards, as mentioned earlier, we met the teachers for a discussion of their lessons.

A typical mathematics lesson was organized as follows:

*Students rise and bow

*Review previous day's problems or introduce problem solving topic . . . . 5 minutes

*Understanding the problem . . . . 5 minutes

*Problem solving by students, working in pairs or small groups (cooperative learning). . . . . 20-25 minutes

*Comparing and discussing (students put proposed solutions on small or large (front) blackboards . . . . . .10 minutes

*Summing up by teacher . . . . 5 minutes

*Exercises (only 2-4 problems)

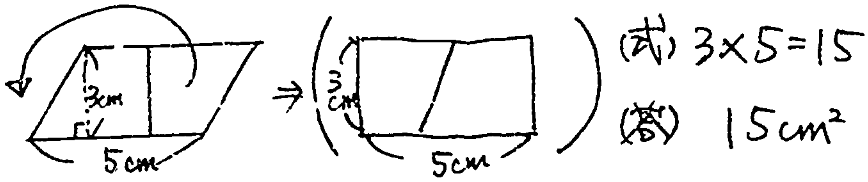
*Soft gong sounds, indicating end of class and students rise and bow.

The same basic organization of instruction was evident in most classes we visited. It is important to note that, when the gong sounds, the teacher may elect to use more time (of the 10 minutes before the next class) to summarize and finish the lesson. We thought this a very commendable characteristic of Japanese schools which shows the importance placed on instruction and the authority the teacher has to use more time if necessary. Further, the public address systems are not used during class, in contrast to this common practice in our schools.

Below we give some examples of lesson plans and descriptions of lessons we observed.

I. Fifth grade lesson on "Determining the Area of a Figure by Transforming It"

Process	Students' Activities
Understanding the problem	(1) Pose a problem and discuss as below



Process	Students' Activities
Problem solving by students	(2) Students work on 10 problems given on a worksheet [Note: only 6 are shown here]

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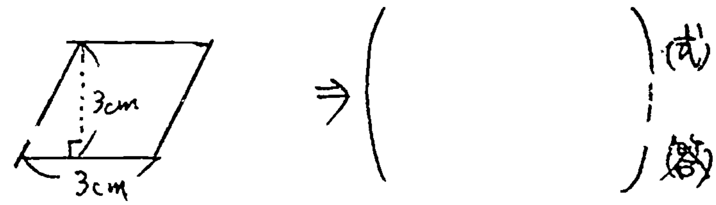

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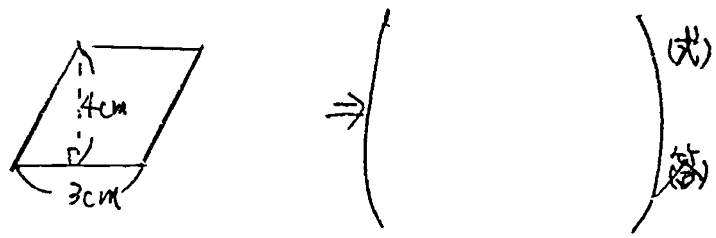

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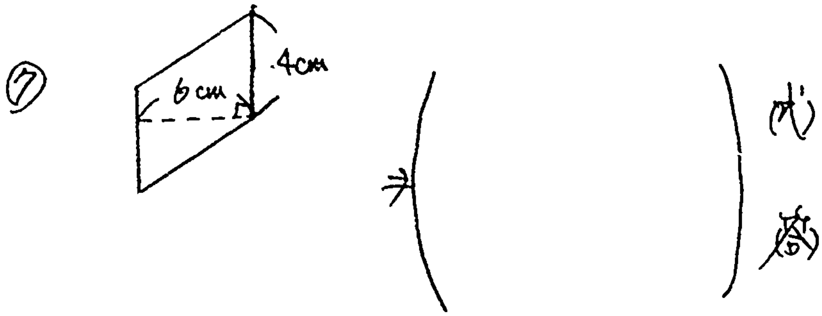
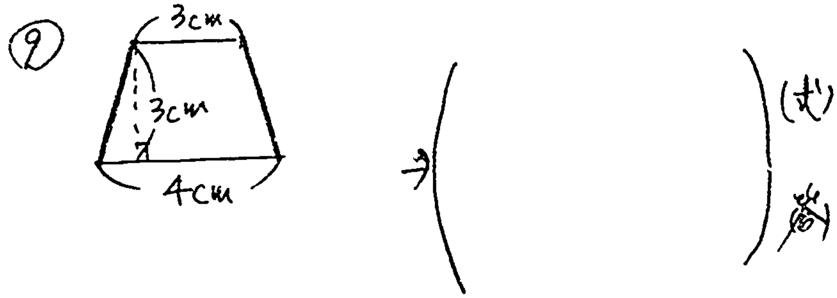
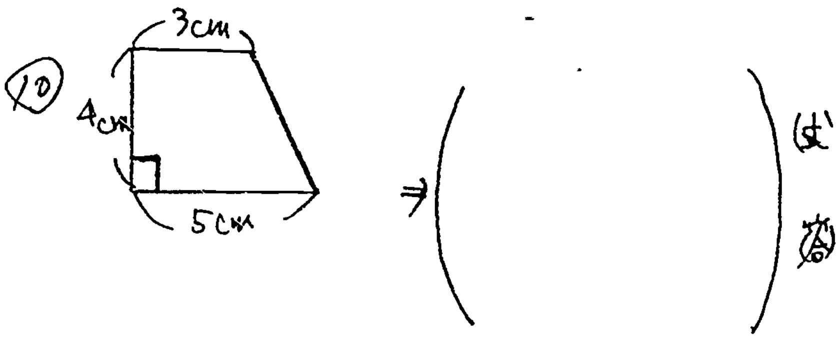
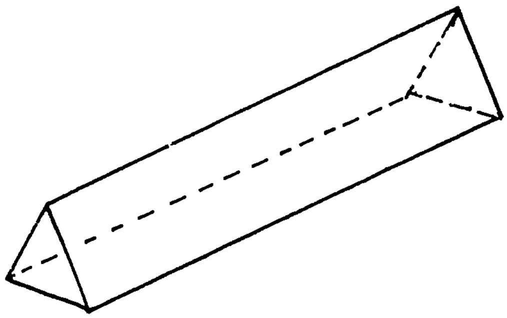






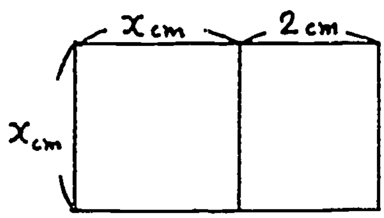
Comparing and discussing	(3)	Students explain their solutions on the blackboard
Summing up	(4)	Grasping the idea that the figures can be transformed to a rectangle and the area then determined.

II. Sixth grade lesson on "Making a Three-Dimensional Model by Folding"

Process	Students' Activities
Understanding the problem	(1) Here is a 3-dimensional model (large, bright red model held up by teacher for students to see):  <u>Objective</u>: Have students sketch on paper how the model can be made by folding.
Problem solving by students	(2) Students work in small groups, sketching the "unfolded" model on paper.
Comparing and discussing	(3) Students go to blackboard and sketch their "solutions." [Note: We counted 11 different ways given by students.] For each solution, the teacher opens the 3-dimensional model, which magnetically sticks to the blackboard, to show each student "solution" and asks students to indicate which edges "match up" to make the solid figure.
Summing up	(4) Help students grasp the idea that the process can be carried out in many ways, emphasizing "visualization" in three dimensions.

III. Ninth grade lesson on "How to Solve Quadratic Equations"

Process		Students' Activities
Review	(1)	How to solve $ax^2+c=0$, $(x+d)^2=k$
Understanding the problem	(2)	Problem: There is a square whose side is x cm long. If we make a rectangle stretching the width by 2 cm, the area of the rectangle is 8 cm^2. How long is the side of the original square?

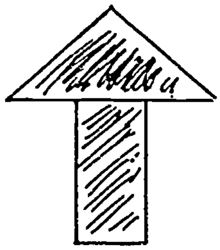


Problem solving by students	(3)	Solve the problem: Students try to solve the problem by manipulating pieces of square and rectangular pieces of paper (made in class the day before). Algebraic expression: $x(x+2)=8$ or $x^2+2x-8=0$ Way to solve: 1) substitute suitable numbers 2) transform to $(x+1)^2=9$ 3) factorize to $(x+4)(x-2)=0$
Comparing and discussing	(4)	Grasp the idea to solve the problem
Problem solving by students	(5)	Solve quadratic equations by transformation to $(x+d)^2=k$. 1) $x^2+4x-12=0$ 2) $x^2+6x-5=0$
Summing up	(6)	Summarize how to solve quadratic equations $x^2+bx+c=0$
Exercises	(7)	1) $x^2+2x-4=0$ 2) $x^2-4x-5=0$

IV. Ninth grade lesson in probability on "Using the RND-function, Instead of a Die on the Microcomputer (using BASIC)"

Process		Students' Activities
Review	(1)	A die can be used to determine the "count" of 1 ... 6 in many trials.
Understanding the problem	(2)	We want to ascertain the irregularity and uniformity of the RND-function and, later, write a program using it.
Solving by students	(3)	Make a frequency table for the integer n using RND, in this case the integer 10. [Note: Using the microcomputer, students fill in the table provided on a worksheet, with students divided into six groups.]
Comparing and discussing	(4)	The results of all six groups are summarized in a table on the blackboard.
Summing up	(5)	Results of students work are discussed with respect to the results of using the RND-function.
Exercise	(6)	Students write a program using the RND-function.

Not all class lessons are organized as above, as would be expected. In a first-grade lesson, the teacher slowly uncovered the figure below, from the top downwards. The teacher asked students to indicate what they thought it looked like in "real life" as she uncovered it. Afterwards, students used tangrams at their desks to construct the figure, making their construction the same shape and size.



In a third-grade lesson, the teacher asked students to write down a number on their worksheet. Then he asked them to write the number formed with the digits reversed and add the two. He had them continue this process until each had a palindromic number. The number of steps was used to classify the original numbers. Students then created their own palindromic numbers with a designated number of digits. There was great interest in the lesson and later, during discussion, the teacher told us that he had earlier surveyed the students and found that sixteen of forty-three students disliked arithmetic at the end of second grade. The lesson was an example of activities he organized in an attempt to make arithmetic more interesting to students. He planned to use magic squares in subsequent lessons.

An eleventh-year differential calculus class began with some students immediately going to the board and solving the following problems given by the teacher:

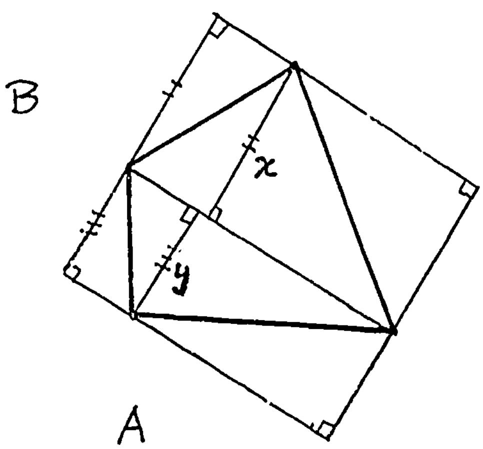
(1) $\lim\limits_{x\to 2} \dfrac{x^3-10x+2}{x^2-4}$ (2) $\lim\limits_{x\to -3} \dfrac{x^3+5x^2+7x+3}{x^2+2x-3}$

(3) $\lim\limits_{h\to 0} \dfrac{(x+h)^2-(x-h)^2}{h}$ (4) $\lim\limits_{h\to \infty} \dfrac{(x+2h)^3-x^3}{h}$

A discussion of each followed, including the use of synthetic division, and, later, derivatives and the solution of a system of equations in solving a problem.

In a tenth-grade class we saw the teacher review the graphs for $y=x^2$ and $y=6/x$ and then, on worksheets, had students, for example, find the value of "a" when $y=x^2-4x+a$ intersects the x-axis and to determine the value of "a" for which $x^2-2ax+2a^2+2a-3<0$. The results of the lesson were summarized by the teacher at the end.

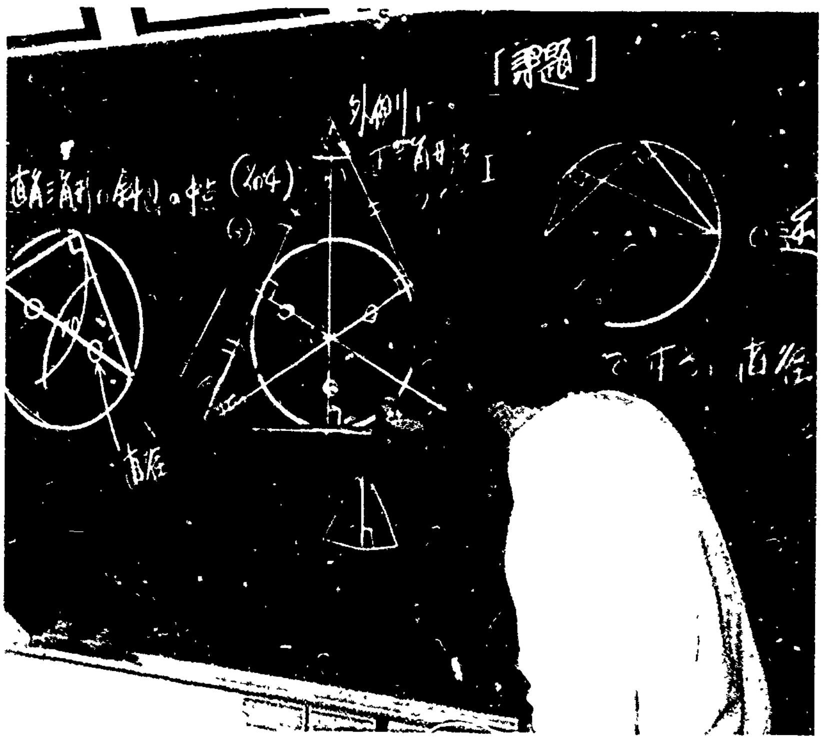
In a sixth-grade lesson that we thought was remarkable for its elegant mathematics, thinking, excited student involvement and drama, we saw the teacher show a quadrilateral and then lead students to see a method for determining its area, as below:



$x + y = B$

$B \times A \div 2 = \text{Area}$

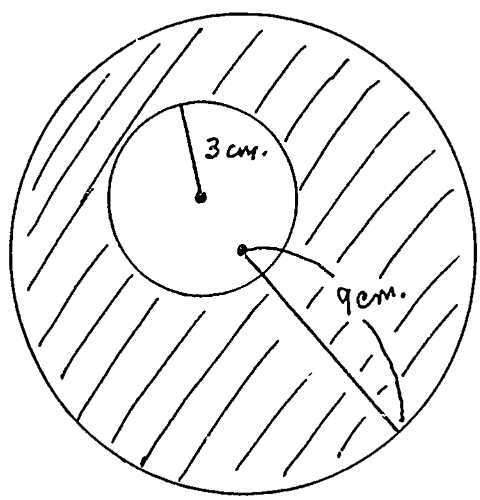
We also saw a remarkable lesson in "open-ended" problem solving. Here the teacher provided students with a worksheet on the top of which was a circle (no center indicated) and the question "How can we determine the center of the circle?" Students were asked to indicate all methods they could think of on their worksheets, working in small groups. Numerous methods provided by



Given a circle with no center indicated, how can we determine the center of a circle?

students were later discussed by the teacher which involved folding the paper to get intersecting diameters (4 students); drawing a chord, making perpendiculars at the endpoints and then connecting opposite vertices (2 students); using a right angle square to make inscribed right angles (8 students); circumscribing an equilateral triangle about the circle, and then connecting tangent points to opposite vertices (5 students); making two chords and their perpendicular bisectors (14 students), and a couple others. Each method was discussed for its advantages and disadvantages (one student also suggested placing a "stable" circle on her fingertip to see where it would balance). This lesson, we were to learn in later discussion, laid the basis for the next textbook chapter dealing with circles: perpendicular bisectors of chords pass through the center, tangents and points of tangency, inscribing circles in triangles, circumscribing triangles about a circle, and so forth. The preliminary "open-ended" work used four class periods.

In another sixth grade lesson, the teacher put the following figure on the blackboard.



Students were asked to find the area of the shaded portion in as many different ways as they could. Students worked in small groups and were permitted to use calculators. Approximately twenty minutes later, students put their solutions on the blackboard, including the following:

1. $9 \times 9 \times 3.14 - 3 \times 3 \times 3.14$

$= (9 \times 9 - 3 \times 3) \times 3.14$

$= (81 - 9) \times 3.14$

$= 226.08$ $\underline{226.08 \text{ cm.}^2}$

2. $9 \times 9 = 81$

$3 \times 3 = 9$

$81 \div 9 = 9$

$3 \times 3 \times 3.14 - 28.26$

$28.26 \times 9 = 254.34$

$254.34 - 28.26 = 226.08$ $\underline{226.08 \text{ cm.}^2}$

3. $9 \times 9 \times 3.14 : 3 \times 3 \times 3.14$

$9 \times 9 : 3 \times 3 = 81 : 9$

$2 : 3 = (2 \times 3) : (3 \times 3)$

$81 - 9 = 72$

$81 : 72 = 9 : 8$

$9 \times 9 \times 3.14 \times 8/9 = 9 \times 3.14 \times 8$

$= 72 \times 3.14$

$= 226.08$ $\underline{226.08 \text{ cm.}^2}$

4. $9 \times 9 : 3 \times 3 = 81 : 9,\ 81 \div 9 = 9$

$3 \times 3 \times 3.14 \times 9 = 254.34$

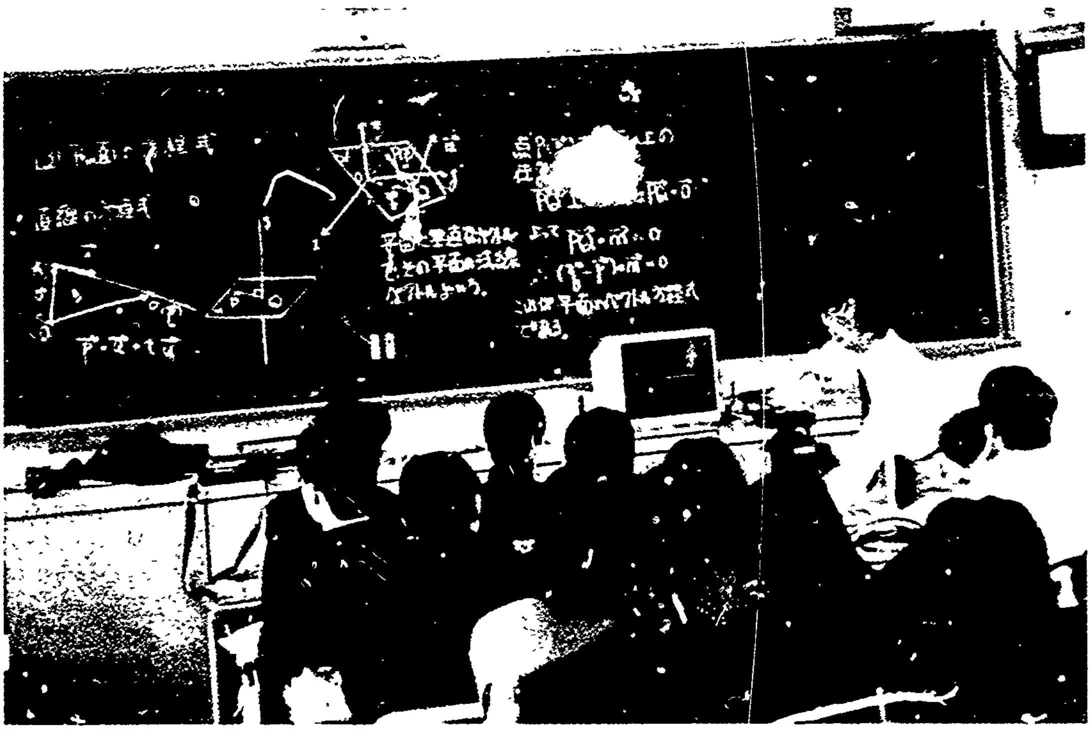
$254.34 - 3 \times 3 \times 3.14 = 226.08$ $\underline{226.08 \text{ cm.}^2}$

5. $3 \times 3 \times 3.14 \times (9-1) = 226.08$ $\underline{226.08 \text{ cm.}^2}$

The relationships among the student solutions were discussed with a summary by the teacher.

In a lesson similarly organized in another school, a ninth-grade class was given two fixed points A, B on a worksheet and the "variable" angle APB of 60°. The task was to make a circle which was the locus of the variable point P. Then a theorem followed which was proved: ABN is a straight line. If C and P are points on the same side of AB and if $\angle ACB = \angle APB$, then C, P, A, B are points on a circle, center O. A problem which was an application of the theorem was then given.

Though the use of microcomputers in mathematics classes we observed was not common, we viewed an excellent use of the computer in an eleventh-grade class. The objective of the lesson was to deduce the equation of the plane



Using computer graphics to illustrate an idea
in eleventh grade math class

from a normal vector and to help students visualize that the linear equation represents the plane, using computer graphics. Here the teacher began (lecture approach) by reviewing the equation of a straight line $\vec{p} = \vec{a} + t\vec{u}$ and having the students realize that a straight line consists of a set of points. Then new ma' erial was presented: the normal line in a plane is orthogonal to all the lines in a plane. Therefore, to have students see that given a normal vectc $\vec{n}$ and a point Q ($\vec{q}$), the point P ($\vec{p}$) in the plane satisfied $\vec{n}\cdot(\vec{p}-\vec{q})=0$. By computer graphics, the teacher had students watch that the set $\{\vec{p} \mid \vec{n}\cdot(\vec{p}-\vec{q})=0\}$ is the plane. Then the equation of plane $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$ was deduced from the component representation in vectors where $\vec{n} = (a,b,c)$, $\vec{p} = (x,y,z)$ and $\vec{q} = (x_1,y_1,z_1)$. By computer graphics, the teacher had students watch that the set $\{(x,y,z) : a(x-x_1)+b(y-y_1)+c(z-z_1)=0\}$ is the plane. After summarizing, the teacher provided a worksheet (in English) with seven problems and asked students, in each case, to draw the plane which has a normal vector $\vec{n}$ through a point A and give the equation (e.g., A(2,0,0) and $\vec{n} = (1,0,0)$).

Preservice Teacher Training

Teacher training in Japan changed drastically following World War II. Prefectural normal schools (for elementary teachers) and higher normal schools (serving larger areas) were reorganized into four-year teacher's colleges and teacher training was also incorporated into university programs. The system of teacher training today has two main characteristics: (1) it is based on a certification system, which involves a prefectural examination; and (2) it is carried out primarily in colleges or universities, not necessarily colleges of education (called the "open system"). Three new universities have recently been established with the sole function of teacher education: Hyogo (1978), Joetsu (1978), and Naruto (1981). General Education, Teacher Education, and Professional Education were the main ingredients in the programs established after the war.

Teacher licenses are given by prefectural boards of education. A regular teacher certificate is valid in all prefectures and for life. There are both first class and second class certificates but, since the vast majority of teachers hold a first class one, we will not dwell on the latter.

For kindergarten and compulsory school certificates (grades 1-9), the basic requirement is a Bachelor's Degree. For senior high school teachers, more advanced work is required - a Master's Degree or completion of a non-degree course for graduates. By law a minimum number of credits must be taken in university study, in addition to other course requirements, in the Major Subject Area (MSA) and Education-Related (ER) subjects - kindergarten: 8 in MSA, 18 in ER; elementary: 16 in MSA, 32 in ER; junior high: 32 in MSA, 14 in ER; senior high (Master's prerequisite or one-year non-degree prerequisite): 16 in MSA, 14 in ER.[20] [Note: one credit equals a class of 15 hours, with 30 hours of student preparation; up to one half of the total minimum number of credits for Education-Related subjects may be substituted for credits in the major subject (e.g., mathematics).] Education Related subjects include Principles of Education, Educational Psychology, Studies of Moral Education, and others.

The areas of the teacher training curriculum include cultural, social, and natural science (36); foreign language (8); health and physical education (4); special subjects for teacher training (76) (Major Subject Area and

Education-Related subjects). A total of at least 124 cred'ts must be earned, requiring at least four years of study. Within this general framework, institutions may organize their own program and, of course, students develop their programs of study in accordance with their goals for certification.[21] Student teaching is required of all those who seek certification, rangin from about 5-6 weeks duration for kindergarten and elementary to about 3-5 weeks for junior and senior high teachers; however, individual colleges/universities may require somewhat less or more. In general, however, preservice education involves only about half as much time in student teaching as in the U.S. There is no certification for teachers at the college or university levels.

It appears that most teachers in Japan, especially the elementary and junior high school teachers, complete more course work in mathematics than teachers in our country. In particular they have studied more mathematics before entering college. One reason for this is due to the large amount of mathematics required for the entrance exams for post-secondary study.

The preservice training of teachers in Japan is carried out in some 800 post-secondary institutions. This includes some 460 universities of which only 65 are Colleges of Education. New graduates from the colleges of education are for the most part employed at the elementary level (for example, approximately two-thirds of the new elementary teachers, one-third of the new lower secondary teachers, and one-tenth of the new upper secondary teachers are graduates of colleges of education).

Many preservice teachers do their student teaching in schools attached to universities or colleges of education. These schools also provide opportunities for research in teaching practices and curriculum development.

There are some 260 schools (of all levels K-12 and handicapped) attached to national universities in Japan.

Before a person can begin teaching, a teaching license (certification) must be secured. The first step is to complete the prescribed course of study which is usually done by completion of a Bachelor's or Master's degree. Thereafter, the prospective teacher must make a satisfactory score on a prefectural examination of which mathematics is an important part. The first part of this is written and covers the special teaching area, general education, and tests of skill in areas such as physical education, music and art. Junior high aspirants must also pass a physical fitness test. Interviews then follow. Successful aspirants are very often under age 30, and we were told that adults older than 30 are not admitted into teacher training programs. With far more applicants than positions, prefectural boards of education can be quite selective. After six successful months in a teaching position, tenure is earned and, from then onwards, teaching is a lifetime career with seniority as the sole determiner of promotion - merit pay is nonexistent.[22] We were surprised to learn that, within prefectural public schools, it is common practice to rotate teachers among schools.

Though we did not get complete information regarding the mathematical training of teachers, we feel it is substantial. Junior high teachers, for example, study algebra, geometry, analytic geometry, analysis, applied mathematics, survey, topology, and additional topics from several other areas of mathematics. Senior high teachers study calculus, linear algebra, abstract algebra, probability, statistics, differential equations, differential geometry, and real and complex analysis. Combined with the high school backgrounds in mathematics, secondary teachers appear to have a solid background in mathematics with which to tackle their teaching responsibilities. Though we do not have solid information, we have the impression that elementary teachers receive a good grounding in courses of mathematics pedagogy, perhaps more so than their American counterparts. This is an area about which we would like to learn more.

Juku

Outside the regular schools there is another very large group of schools called juku ("joo-koo" or "jook") by the Japanese. Any consideration of Japanese mathematics teaching must take juku into account. Though the juku are private, profit-making schools run independently of the regular schools, they still play an important role in developing scholarship in students, and they depend in various ways on publicly oriented schools.[23]

Juku provide opportunities for tutorial, remedial, and enrichment as well as preparation for entrance examinations. Students attend juku outside regular school hours (i.e., afternoon, evening, or weekends). Juku exist throughout the country with heavier concentration in large urban areas. Insight into the relationship between juku and regular schools is provided by the Japanese scholar Kazuyuki Kitamura:

> The dominant values of the Japanese public primary school are egalitarianism and uniformity: Pupils are not classified according to their academic ability because all pupils are supposed to keep up with the progress of the class. There they are taught by means of a nationally controlled, uniform curriculum. Despite its principles of egalitarianism and uniformity, however, the school inevitably must produce high achievers and low achievers. The school and its teachers are unable to counter these disparities because they are bound by the two mandatory principles. So...high achievers who are dissatisfied with the progress of the school class...attend a...school...where they can take more advanced classes, while...[students with learning problems can attend]

> another type of...school offering remedial classes. Then, thanks to the existence of these...supporting institutions, the formal school can continue to function according to the principles of egalitarianism and uniformity.[24]

This characterization may be considered an overstatement, but nevertheless, it gives a good perspective on this very important trait of Japanese education.

There are non-academic and academic juku. The former include instruction in such areas as music (e.g., vocal, instrumental), Soroban (abacus), calligraphy, the arts, and physical education. Elementary school children commonly attend these juku. The academic juku are more numerous and the number of children attending increases by grade level: 40% for grade 5, 47% for grade 6, 49% for grade 7, 55% for grade 8, 67% for grade 9 and, for the entirety of Japan, for example, seventh-grade students attend juku 2.1 hours per week.[25] There are also prep schools for students who do not pass the university entrance examinations the first time. The reasons for the steadily increasing numbers of students in juku from upper elementary into junior high school are that (1) juku provide help to students who fall behind in regular schools; (2) high parental expectations are somewhat relieved by juku; (3) preparation for the high school entrance examinations in grade 9.[26]

The academic juku generally provide supplementary schooling in mathematics, science, English, and the national language. A special juku provides preparation for entrance tests to senior high school and the universities. Juku may exist in a one-room physical layout or in a chain of such one-room facilities. There are major corporate chains enrolling as many as one million students. There has been major growth in the size of juku in the last 10-15 years, and we were told there are now more than 35,000 academic juku in Japan. Juku teachers are mainly part-time and very few are regular school teachers - many are university students or certified teachers not working in regular schools.

We visited two juku. The first, called the Kumon Institute of Education, has about 900,000 students studying mathematics, about 400,000 studying Japanese, and about 150,000 studying English. About 5 percent are pre-school students, about 80 percent elementary, and about 15 percent junior-senior high.

We were given sample mathematics materials to examine. The Kumon approach is a self-study one involving individual, self-paced learning, with emphasis on speed and accuracy in computation and algebraic manipulation. The content ranges from arithmetic concepts through high school topics in mathematics. Children come to their one-room Kumon juku, usually located near their home, check in, are given materials (worksheets) to work on, have their work checked, and return home. There is no formal instruction. Children may stay only a few minutes or longer, depending on their level and mastery. The cost is about ¥5000 per month (about $40) for elementary children and ¥6000 per month (about $50) for secondary students. The Kumon juku is now international and maintains some activities in the United States.

Aside from the individual, self-study approach, which we thought a positive characteristic, we found the materials to be aimed primarily at lower level thinking skills, which is not contrary to Kumon's stated objectives. We should comment that the Kumon staff received our delegation in a very cordial and professional manner. In addition to providing a briefing and answering our questions, we were also provided transportation to a small, one-room juku where we viewed children diligently working on their tasks. In a short period of time, we saw several children come and go.

Our second visit was to a Soroban (Japanese abacus) Juku, called the League For Soroban Education of Japan. Here we observed what can only be described as remarkable computation on the part of students, ranging in age from third to tenth grade. While students "warmed up" with a few exercises, members of our delegation were given calculators for use in checking the results of computation students were to carry out. Several of our delegation volunteered to go to the front blackboard and, using magnetic tabs with digits on them, place complicated problems on the board - the students could not see the problems. On signal, the students turned and began their computation. Below are a few examples of groups of computational problems the students carried out and which we timed ourselves.

<u>4th graders</u>: (53 seconds to complete)

621 x 958 = 594,918
968 x 459 = 444,312
791 x 443 = 350,413
905 x 916 = 829,800 (wrong, but quickly corrected!)

<u>5th graders</u>: (60 seconds to complete)

6121484 ÷ 9537 = 641 R = 8267
3989494 ÷ 7970 = 500 R = 4494
1099558 ÷ 8656 = 127 R = 246

<u>6th graders</u>: (1 minutes 17 seconds to complete)

6121 x 4984 = 30507064
6668 x 6590 = 43942120
7593 x 5437 = 41283141
9091 x 9879 = 89809989
7467 x 8756 = 65381052

<u>7th graders</u>: (21 seconds to complete)

6191484
-3982594
+1099548
-9971071
+7597683
+3659037
4594087

Note: Each number was very quickly spoken out loud by Mr. Kouzi Suzuki, our host, and students listened and computed - the pace was so fast we could not write down the numbers as they were spoken.

There followed other exercises such as 458,565,072 ÷ 4816 (10 seconds), 30,392,060 ÷ 157 (10 seconds) and 80,939,273 ÷ 905,361 (9 seconds). Still other demonstrations were made, this time by Mr. Suzuki, for determining the square roots of numbers (e.g., $\sqrt{1849} = 43$).

As we watched students doing the computations, we noticed some were seemingly writing with their right index finger on the palm of their left hand. During the discussion, we learned that these students were, in fact, "manipulating" the beads of the "mental" Soroban (i.e., they did not have to use a real Soroban, everything is done mentally - both computation and recording of results).

Like other juku, students come to small neighborhood rooms like the one we were in, which are located throughout Japan (and in the U.S. too), two-three times per week for, say, one hour. Here they learn and drill on various mathematical processes. We learned that there are ten classes of Soroban proficiency - one of our Japanese professor hosts had earned a "second class" Soroban rating (nearly the best). Actually, there are also two or three special classes of proficiency above first class.

We had learned earlier that the teaching of the Soroban in public schools would be increased in the near future. Beginning in grade three, students will study addition and subtraction two periods per week. But since the Soroban is considered such an important part of Japanese culture, many parents want their children to be able to use the Soroban with good proficiency. The juku, therefore, is the place for thousands of students to learn this and parents are willing to pay for this out-of-school instruction.

As with the Kumon visit, we were received in an extremely friendly and professional manner at the Soroban Juku. After the demonstrations, we had our questions answered and then were treated to a magnificent dinner at a very nice local restaurant. The Soroban League also provided transportation to and from this juku school, consistent with the very nice reception we received throughout our Japan visit.

Due to the rigorous schedule we adhered to during our Japan visit and the demands of other social and diplomatic functions, we had no time to visit any other juku. For sure such visits to academic and other juku would have been interesting and worthwhile but, as several delegates commented, we'll just have to return to Japan again.

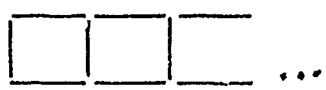
The realities of juku and their relative merits and demerits are topics of discussion and present research in Japan. In general though, teachers in regular schools do not approve of juku and there are, at present, very few conclusive research results available. But the research is continuing to assess how the regular schools and juku interface. This research is taking place in a context in which the whole of Japanese education is being reassessed as well as the organization of Japanese society.

Open-ended Problem Solving

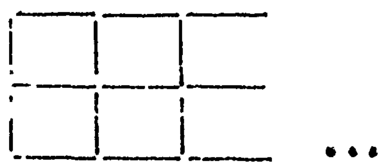
As indicated earlier, our delegation observed several problem solving lessons of an "open-ended," nonroutine nature. Using this approach, the teacher poses a problem for which several or many approaches to a solution exist. After posing the problem, students w[illegible] to solve the problem individually or in small groups. Later the teacher provides time to examine and discuss the various approaches used by students, including the advantages and disadvantages of each approach or their "mathematical quality." Such problems provide opportunities for problem extensions and generalizations. Implicit in this approach is mathematical inquiry and the potential for student insight into the depth and complexity of the mathematics involved.

One example of an "open-ended" problem is the following:[27]

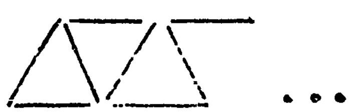
Squares are made by using matches as in the figure below. When the number of squares is 5, how many matches are used?



After examining different approaches students use to find the solution, the problem can be extended to say 10 squares, 35 squares, etc. A converse problem can also be posed, namely, given, say, 70 matches, how many squares can be made? Suppose the number of matches is 72? What about the following situation - how many matches are used in forming 10 squares?



What if we consider triangles as in

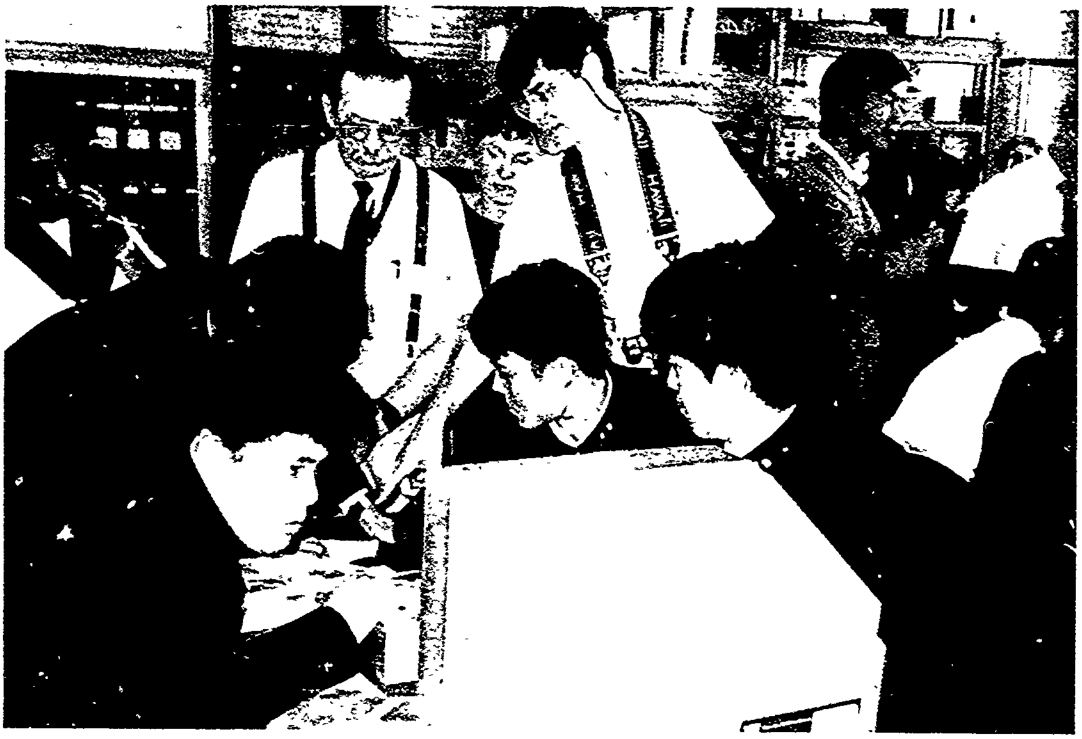


or parallelograms, pentagons, etc.?

This "open-ended" approach to problem solving is clearly related to some of the priorities given in NCTM's new Curriculum and Evaluation Standards for School Mathematics (1989). There is experimentation with this approach underway in Japan now which we think will be of interest to American teachers. Though we have teachers who emphasize conjecturing, problem posing, and extensions in their teaching (i.e., Polya's approach), we feel that this approach may have potential for more widespread use in our own teaching. The approach is in sharp contrast to the "closed-ended" problems that seem so characteristic of our present textbooks. Student experience with such problem solving may help to alleviate the belief so many of our students have that mathematics is a closed system, static, and a set of rules to be memorized. "Open-ended" problems would appear to place students at the heart of the problem solving process and has them doing mathematics.

Technology (Calculators and Microcomputers)

In such a highly developed technological society as Japan's, we expected to see calculators and computers used more commonly in mathematics teaching. Even though Monbusho's NCS provides opportunity for teachers to use technology, we saw only minimal use of calculators in classes but learned, through discussions, that some teachers use them and that studies are underway to assess the merits and demerits of their use in mathematics teaching. We were briefed on this during our visit at the National Institute For Educational Research (NIER) and were given some very useful reading material. Similarly, we saw no wide scale use of computers; but where we saw it, they were used well. Both excellent hardware and software exist, but we were told that more and better software is needed. We were also told that microcomputers are used, at this time, more for demonstrations as opposed to students working on them or students using them for exploring ideas.



Though the use of microcomputers was not common in the classes we observed, when they were used, they were used well.

It is common in Japanese education, where strongly differing viewpoints are held by teachers, to research the potential of newly proposed innovations before implementation. In the absence of consensus no changes are made. Data are collected through tryouts and differing viewpoints are heard and discussed in meetings throughout the country. Once the various views are mediated and if consensus emerges, the Japanese move very quickly to implement proposed changes. This process is now underway regarding technology. Thus, we strongly expect changes to occur in the use of technology in the near future.

Changes in Japanese Education

Though Japanese education has been on the "leading edge" in mathematical achievement productivity for several years and in spite of its stunning recent successes in international comparisons, educational reform is now a source of concern and debate. But if significant educational reform evolves in Japan in the near future, it is something not new to Japanese society. There was a major reorganization of Japanese education based on Western systems in the latter part of the 1800s (called the Meiji Reform) and, following this, rapid movement towards industrialization. Japan's educational system seems to have produced excellent results in providing people to meet the country's needs in manpower during those years.

As is well known, there was another major reorganization of education following World War II in which the new system was modeled after our own educational system. Fitting our system into the Japanese educational, cultural, social, and industrial context was not an easy matter. But the Japanese succeeded in this enterprise and education seemed, once again, to produce a skilled and educated workforce to feed a rather large economic and industrial growth in the era of the 1950s and 60s. These accomplishments in aligning educational expansion and economic growth are rather significant, given that education is among any country's most conservative institutions, and certainly Japan's.

Though education is a very stable institution in Japan, especially at the compulsory schooling level, there is increasing dissent among students and virtually all parts of society are now engaging in a reexamination of the educational system. Evidence of this stems from student unrest during the 1970s in universities which has already spread down into the senior high schools. There are aspects of this dissent with which mathematics educators will have to deal. For example, during our first meeting, at Tsukuba University, two researchers showed us some interesting data. In the mid-70s, "liking" mathematics, in relation to other school subjects, was at about the middle in grade 1, dropped to last in grade 2, and moved steadily up to mid-level again by grade 6. In the 80s "disliking" mathematics, in relation to other school subjects, steadily moves up from mid-level in grade 1 to the top spot (most disliked!) in grade 6. Elementary students like mathematics less now than ten years ago. These results are based on data from the elementary level. Perhaps mathematics is "liked" even less as students near late junior high and late senior high school where the formidable university entrance examinations await them. But it is interesting to note that while Japanese students seem to dislike mathematics, they still do very well at it (does gambare account for the high achievement?).

Among the sour s of conflict are the entrance examinations for secondary schools and universities and certification examinations for various professions. This process of "credentialism" seems to have served Japan well for a long time, but is now questioned on the grounds that it impedes young people's inspiration and drive towards education and seeking a successful career and happy adult life. Tied to the entrance "examination hell" is the rapid growth of the juku (also called "cramschools") as a large scale economic enterprise which is now flourishing.

Developments have evolv to the point at which the major objective in senior high education is to pass the entrance examinations for university study. Though school , has been and still is oriented towards upward mobility, the race to get into the best schools and universities has become very harsh - perhaps too harsh - and students are rebelling. They now seem to be joined in this dissent by a growing number of parents. It now appears that "Ironically, the more emphasis the teachers and the schools place on examination-centered curriculums and the harder they push children to study to beat the competition for entrance to better schools, the lower children's aspirations are becoming."[28]

There are many aspects of the educational reform movement in Japan. All are being addressed by the National Council on Educational Reform which was established in 1984 by an initiative of Prime Minister Nakasone. Three reports have now been issued by the Council, one in 1985, another in 1986, and a final report in August 1987.[29] The report deals with transition to a lifelong learning system; reform of elementary, secondary and higher education; coping with internationalization; coping with the information age; changing the school year; and educational administration and finance.[30]

It seems clear that there is a changed set of values and attitudes among Japanese youth. At the same time, the established social and educational order of Japan's "older generation" has changed very little. Thus, the issues have been raised, studied and are now being debated. The process that is underway is typical of the Japanese: no change until there is consensus about what to do; but once consensus is achieved, then change may evolve fairly rapidly.

There are a couple of very interesting observations to be made, regarding American and Japanese views on our present respective educational systems. The first is that when compared to our own social and educational problems, those of the Japanese may be considered far less severe; nevertheless, they are now being addressed as serious problems by the Japanese in a very serious manner. The second is that Japan has long been on a charted course to "catch up" with the West and has used Western models in change; however, the Japanese have now caught up and, perhaps, surpassed the West. Thus, there is now no model for them to adopt or follow. Perhaps it will be the case that present Japanese reform efforts will provide a model from which other countries will learn. Finally, we observe that in the U.S. we have local and state control of school education and a very open and diverse system of higher education in contrast to Japan's. The Japanese consider our educational system learner-oriented as opposed to teacher-oriented and to provide considerable freedom in choosing curricula. It appears that the Japanese are moving in the direction of ours, while we are seriously considering moving in the direction of theirs with recent recommendations coming from many diverse groups for more uniformity at all levels in teaching, curricula, and achievement standards. It is ironic, therefore, that the impetus for U.S. change comes, at least in part, from the stunning achievement results of Japanese education on various international comparison studies, which are due to a system that has led to serious problems.

Some Implications For U.S. Mathematics Teachers

Our delegation visited schools and numerous classrooms in Japan for nearly three weeks. We observed class lessons in mathematics at many different grade levels. Though the schools we visited were among the better schools, even in the words of our Japanese hosts, still we feel many of our observations might very well be characteristic of mathematics classrooms in general. That this is so can be seen by considering the widely acknowledged uniformity in Japanese education and society, the national course of study, uniformity in teacher training, the common practice of moving teachers from school-to-school in prefectures, the entrance examination system, and the generally highly productive educational system. Whatever the case though, it is clear to us that Japanese students are learning a great deal of mathematics and, while doing so, are also enjoying school and life a great deal. There is no doubt in our minds that, at any school grade level, the students we observed appear well ahead of our own both in terms of mathematical topics covered and their mastery of them.

There are a number of observations we want to make which we feel will be of interest to American mathematics teachers or which have implications for their classroom teaching. Before doing so, we want to comment that it is our purpose to neither criticize nor praise Japanese mathematics education. Rather, we went to Japan with the view of learning what we could through reading, observations, and discussions, and then sharing it on return. This report is an attempt to do this and we do it consistent with the following outlook:

> There is little to be gained, and much to be lost, by comparing national systems in an "our-system-can-beat-your-system" frame of mind. The important gains are to be had only if each nation tries to learn from the other...One road toward (that) improvement may be the careful study of what other nations do, in the hope of learning from them.[31]

Here are some observations stemming from our Japanese visit:

1. Mathematics holds a central place in the curriculum and students study it virtually every year. We feel that we might well elevate the importance of mathematical studies in our schools to a much higher status.

2. Parents, teachers, administrators, and society, in general, have high expectations of students in schooling. If we could combine this characteristic with the notion that all students are capable of learning mathematics, our schools could produce far more mathematically literate

students which, in turn, would lead to a more highly literate and numerate society.

3. Students at the primary level are treated with special care in terms of nurturing their development in mathematics. For example, only the better, more experienced teachers handle students in the early grades. Perhaps this is a policy we might consider in the U.S. for it is at the early grade levels that creativity and thinking abilities both exist in children and can be nurtured in a constructive manner.

4. All teachers, including especially those of mathematics, are accorded great respect. They are also paid well and meaningful inservice, renewal education is characteristic of the profession.

5. There is more emphasis on topics from geometry, algebra, and statistics in the elementary schools. Perhaps this is an orientation we should pursue more vigorously, as well as placing more emphasis on development of mathematical thinking.

6. The mathematics curriculum treats decimal fractions before common fractions, unlike our curricular approach. Perhaps we should consider this approach more seriously, perhaps even more so now that a wide use of calculators in the schools is "just around the corner."

7. Japanese teachers give less homework, but the homework problems are more complicated and mathematically challenging than our own. We feel this is an aspect of teaching which needs to be addressed by our teachers.

8. Students have a larger responsibility in the learning process than ours, and they try harder with seemingly fewer social diversions. We need to work towards encouraging our students to accept a larger responsibility in understanding the subject matter. Further, this kind of diligence can pay large dividends in achievement.

9. Mental arithmetic is more strongly emphasized than in the U.S. There is no question in our minds that our students can be similarly challenged and educated to rely less on paper and pencil computation, and more on mental computation.

10. "A healthy mind in a healthy body" is something which is emphasized throughout schooling. Similar to the 1987 ICTM China Delegation's comments, in contrast to Japanese students, ours seemingly carry the burdens of modern society with them into the classroom. We refer to drugs, alcohol, and the likes of overindulgence in sugar and fast foods, not to mention the breakdown of the traditional family, single-parenting, latch-key children and the anguish of dealing with too many options...as well as societal emphasis on style and appearance, popularity, money, and romance.[32] We in the U.S. need to develop a national consensus to restore the development of sound character, values, and proper behavior in school students.

11. Japanese teachers see their jobs and role in society as important and this, in turn, may play a role in student learning. The teaching of mathematics in the U.S. should be improved and elevated in the eyes of the general public, and teachers should be given a significant role to play in improving the profession.

12. The Japanese maintain a national consensus regarding mathematics education: the mathematics curriculum is firmly set, teachers teach it, and the standards are high. We feel we need a national consensus on which to base a revitalization of mathematics in all its aspects: content, curricula, its teaching, teacher preparation, and evaluation of learning.

13. In virtually all the mathematics classes we visited, we saw a clear, single objective implicit in the instruction. Further, the lesson plans given to us clearly reflected this sort of purposeful classroom learning environment. We feel this aspect of Japanese teaching is one our teachers should emphasize more.

14. In many classrooms, especially at the compulsory school level, we observed students working together in small groups on learning tasks nicely organized by the teacher. Later, the teacher "pulls together" for the entire class the results of group learning. We viewed numerous instances of how this instructional approach can be organized in a manner that works, and we feel our own teachers should more widely adopt this approach to teaching and learning.

15. We viewed classroom lessons in which the teacher, after group learning activities, purposefully provided opportunities for different students or groups to share their many different approaches to thinking about a problem or solving it. We think this to be an excellent manner in which to foster development of this aspect of mathematical thinking. But it was clear to us, that classroom management must carefully provide for this objective in a lesson.

16. Implicit in virtually all lessons we saw was a certain intensiveness in the mathematics curriculum. To a large extent, topics are covered just once, with the expectation that students will spend considerable effort in follow-up work on their own, including out-of-class help from teachers, parents, or tutors. This is a curriculum characteristic which, we feel, demands more attention in the U.S.

As has been noted in many sources, the Japanese show very good achievement on international comparison measures. But, their accomplishments need to be viewed in terms of their culture, which is very different from our own. Thus, the Japanese way is not necessarily the American way to reasonably move towards improving mathematics education. At the same time, however, we feel there are many aspects of Japanese mathematics education worthy of our study, research, and possible implementation. This is the spirit in which we have prepared our report, and it is the spirit in which it should be viewed.

Final Comments

As mentioned earlier, Japan was a participant in both the First and Second International Mathematics Studies (FIMS and SIMS) and its achievement results were very good. Patricia Horvath has written that the FIMS and SIMS studies tend to confirm to her what she had earlier observed (emphasis added) in Japanese classrooms and concluded from interviews with students, teachers, and counsellors; further, like Horvath, we feel "that more than minor adjustments are needed if the level of American junior and senior high school mathematics achievement is to improve."[33] The same holds true for elementary school mathematics.

As seen earlier in this report, the Japanese have a common nucleus of mathematical content for all students in compulsory education. Is it not possible for the U.S. to similarly develop a core, and then vigorously pursue its teaching? Even at the senior high school level we should be able to do the same, not only for the more highly motivated or talented students, but for a much larger percent of the student population including minority students. Anyone who doubts this should get familiar with the work of Uri Treisman, Bill Johntz, John Fallon, Jaime Escalante (i.e., "Stand and Deliver"/Garfield High School in Los Angeles), Angelo Dillavicencio, and others who have demonstrated excellent learning results with minority students, even in advanced topics in mathematics.[34] They downplay offering "general," "consumer" or remedial mathematics courses taught by less senior teachers having minimal expectations of students. Instead, they recommend just the opposite - more solid mathematics courses taught by excellent teachers who have high expectations of their students. Less emphasis on lower level skills (i.e., routine computation) and more emphasis on higher level skills (i.e., mathematical thinking) are clear implications of their work.

In the end, no matter what the content of school mathematics courses, it is the teachers on whose shoulders rests the responsibility for upgrading mathematics teaching. We saw excellent teaching in Japanese schools and we know there are also many excellent teachers in U.S. classrooms. What such successful teachers have in common is a good command of the mathematics they teach, a love of students and teaching, years of experience, and solid support of their schools' communities. Somehow it should be possible for these teachers to set the standards and help show the way for a multitude of other teachers to change their school environments, emphasize the important role of mathematics in our society, and, in general, show leadership in the overall improvement of mathematics teaching throughout the country.

We have two final comments. We learned in Japan of the enormous role played by entrance examinations as determiners of mathematics teaching, particularly at the senior high school level where the tests drive the curriculum. In the U.S., we know that standardized tests influence the mathematics curriculum and its teaching. Mathematics teachers in both countries have the same reservations, if not outright opposition, to such "measures of evaluation." The visit to Japan helped to magnify our concern about this aspect of our own educational system. It also raised our consciousness of the need to change it.

Finally, some of the most interesting classes we visited were concerned with problem solving in which teachers actively engaged their students in mathematical thinking. The "open-ended" approach to teaching that we observed, especially at the intermediate grade levels, represented prime examples of developing mathematical thinking in students. It was interesting to see this and to realize, at the same time, that this is an area of teaching practice and research of common interest to mathematics teachers in the two countries. Perhaps this is an area in which useful cross-cultural interaction and research can be carried out. Already there has been an exchange of videotapes of classroom lessons using this approach, as well as the beginning of organized cross-cultural research on students' problem solving behaviors.[35]

Acknowledgements

Our trip to Japan was a great professional learning experience made possible only by the efforts of many people. We wish to thank Professor Hiroki Kato of Northwestern University for his excellent talk and discussion with delegates at the Orientation before departure. We also want to thank Professors Yoshitomo Matsuo, President of the Japan Society of Mathematics Education (JSME), and Professor Tatsuro Miwa, JSME's International Mathematics Education Representative, for their invitation to ICTM to form a delegation to visit Japan. We also thank B and A Travel (Mr. Dave Coracy/Carbondale, IL) and the Japan Travel Bureau (Ms. Chie Sato/Tokyo) for handling all our travel arrangements in an expert and professional manner.

We also thank the ICTM, who through three Presidents (Cathy Cook, Sandy Spalt, Marvin Doubet) and Boards of Directors, sanctioned and encouraged our Japan visit, though no ICTM funds were used. We also thank the following

publishers and manufacturers who not only generously donated a substantial number of educational materials for the delegation to take to Japan, but donated exactly what we requested:

Texas Instruments [Hilde Uribe] (generous donation of calculators used in American classrooms and copies of the Math...you can count on it videotape)

Hewlett-Packard (USA) [Kent Henscheid and Clain Anderson] and Yokokawa Hewlett-Packard (Japan) [Nobuhi o Shigiya] (generous donations of scientific calculators and instructional books)

Addison-Wesley Publishers [Bob Brede] (elementary and secondary textbooks and supplementary materials)

D. C. Heath [Mary Jacobsen] (elementary and junior high school textbooks)

Charles E. Merrill Publishing Company [Renee Keown] (elementary and secondary textbooks)

Harcourt-Brace-Jovanovich Publishers [Dick Wallace] (elementary and secondary textbooks)

Houghton Mifflin Company [Jim Vandiver] (elementary and secondary textbooks)

W. H. Freeman (Computer Science Press) [Rod Gavin] (numerous computer-related textbooks)

Holt, Rinehart, and Winston Publishers [Paul Peterson] (elementary and secondary textbooks)

McDougal-Littell and Company [Allen Hull] (geometry textbooks and supplementary materials)

Janson Publications [Barbara Janson] (numerous booklets dealing with discrete mathematics)

National Council of Teachers of Mathematics [James Gates] (numerous brochures, pamphlets, monographs, books, and yearbooks)

Mathematical Association of America [Al Wilcox and Walter Mientka] (books and mathematics contest materials)

National Research Council [Marcia Sward] (copies of Improving Indicators of the Quality of Science and Mathematics Education (K-12) and MSEB information)

Creative Publications [Marne Coggan] (numerous teaching aids and books)

Dale Seymour Publications [Dale Seymour] (numerous teaching aids and books)

Cuisenaire Company of America [Jeff Sellon] (numerous teaching aids and books)

Illinois Council of Teachers of Mathematics (brochures and mathematics contest materials)

Lawrence Erlbaum Publishers [Joe Petrowski] (authoritative research books on problem solving, thinking skills, and cognitive science)

The delegates, themselves, who individually and with their schools donated various materials: student yearbooks, mascot insignias, "pen pal" letters from students, brochures, school folders, school pennants, school belt buckles, school pins, coffee mugs and a host of other things. They also brought books about the life and culture of Illinois (Land Between the Rivers) and the U.S. (A Day in the Life of America).

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We also wish to thank Mr. Toshiyuki Asami for arranging our visit to Toyota City and the Toyota Corolla and Tercel car assembly plant where we viewed the entire car assembly process, including watching robots weld car body parts and get an explanation of the "KANBAN" "just in time" delivery (of parts) and production system. Our visit was more than a "tourist" activity for we learned about the social-educational background which makes the "KANBAN" system not only possible, but also a highly organized and efficient one.

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arranged and coordinated, with his colleagues and graduate students, numerous and excellent school visits which were an important part of the Japan visit. He also organized an excellent one-day mathematics education conference and a magnificent farewell party for our delegation at which Professor Kiyoshi Yokochi spoke and greeted delegates. Mrs. Okamori kindly presented each delegate with a gift which will always remind us of this memorable visit.

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A P P E N D I X

Notes

1 Webster's New Universal Unabridged Dictionary 1979, p. 1327.

2 Duke, B. (1986) The Japanese School - lessons for industrial America. New York: Praeger Publishers, p. 25.

3 Leestma, R., et al. (1987) Japanese education today. Washington, DC: Superintendent of Documents, U.S. Government Printing Office, p. 91.

4 Ibid., p. 91.

5 Ibid., p. 91.

6 National Institute For Educational Research (NIER), (March, 1988) Basic Facts and figures about the educational system in Japan, Tokyo: p. 2.

7 Leestma, op. cit., p. 10.

8 Becker, J.P., et al. (1988) Mathematics Teaching in the Japanese Elementary and Junior High Schools. Mimeograph. 24 p.

9 Leestma, op. cit., p. 27.

10 Duke, B. (1986) The Japanese School - lessons for industrial America. New York: Praeger Publishers, pp. 25-51.

11 Leestma, op. cit., p. 35.

12 NIER, (March, 1988) Basic facts and figures about the educational system in Japan, Tokyo: National Institute for Educational Research.

13 Leestma, op. cit., p. 20.

14 Ibid., p. 20.

15 Flanders, J. (1987) How much of the content in mathematics textbooks is new? Arithmetic Teacher, 35 (1), 18-23.

16 Fuson, K., et al. (1988) Grade placement of addition and subtraction topics in Japan, Mainland China, the Soviet Union, Taiwan, and the U.S., Journal for Research in Mathematics Education, 19 (5), 449-456.

17 Hashimoto, Y. and Sawada, T. (1979) Mathematics Program in Japan, Tokyo: National Institute for Educational Research (Note: the program was implemented in 1980.)

18 Ibid., pp. 25-36.

19 Hashimoto, op. cit., pp. 37-51.

20 Committee for Facilitating Research (1986) Teacher Training in Japan, Japan Association of Universities of Education. Tokyo: Dai-ichi Hoki Publishing Co., Ltd. pp. 17-18.

21 Ibid., p. 20.

22 Leestma, op. cit., p. 17.

23 Sawada, T., and Kobayashi, S. (translated by Patricia Horvath) (1986) An analysis of the effect of arithmetic and mathematics education at juku. Tokyo: National Institute for Educational Research (Compendium #12/March).

24 Leestma, op. cit., p. 11.

25 Sawada, op. cit., p. 2.

26 Sawada, op. cit., p. 2.

27 Becker, J. P. and Miwa, T., (1987) Proceedings of the U.S.-Japan Seminar on Mathematical Problem Solving, Carbondale, IL: Southern Illinois University, 481 p.

28 Amano Ikuo (1986) The dilemma of Japanese education today. The Japan Foundation Newsletter XIII (5) p. 8 (Tokyo).

29 National Institute for Educational Research (NIER) (1988) The Recommendations of the National Council on Educational Reform - A Summary, Tokyo: NIER (32 p.).

30 Ibid., p. i.

31 Davis, R. B., Romberg, T. A., Rachlin, S. and Kantowski, M. G. (1979) An analysis of mathematics education in the USSR. Eric Clearinghouse for Science, Mathematics, and Environmental Education: The Ohio State University, Columbus, OH, p. 29.

32 Becker, J. P., et al. (1987) The Great China Adventure (1987) - A Report of the ICTM China Mathematics Delegation. Illinois Mathematics Teacher, 38 (3) and 38 (4) [pp. 4-14 and 3-27] p. 32.

33 Horvath, P. J. (1987) A look at the Second International Mathematics Study Results in the U.S.A. and Japan. The Mathematics Teacher, 80 (5), p. 367.

34 Treisman, V., Hart, P., Dillavicencio, A. Talks given at the Success for Everyone II Conference, University of Illinois at Chicago, January 28, 1989.

35 Becker, J. B., and Miwa, T., op. cit., 1987.

Table 1
Required Number of Class Periods Per Week and Year in Each Subject at Each Elementary Grade Level*

Subject	Grade					
	1	2	3	4	5	6
Japanese	8 (272)	8 (280)	8 (280)	8 (280)	6 (210)	6 (210)
Social Studies	2 (68)	2 (70)	3 (105)	3 (105)	3 (105)	3 (105)
Arithmetic	4 (136)	5 (175)	5 (175)	5 (175)	5 (175)	5 (175)
Science	2 (68)	2 (70)	3 (105)	3 (105)	3 (105)	3 (105)
Music	2 (68)	2 (70)	2 (70)	2 (70)	2 (70)	2 (70)
Art & Handicraft	2 (68)	2 (70)	2 (70)	2 (70)	2 (70)	2 (70)
Homemaking	-	-	-	-	2 (70)	2 (70)
Physical Education	3 (102)	3 (105)	3 (105)	3 (105)	3 (105)	3 (105)
Moral Education	1 (34)	1 (35)	1 (35)	1 (35)	1 (35)	1 (35)
Special Activities	1 (34)	1 (35)	1 (35)	2 (70)	2 (70)	2 (70)
Total Number of required class periods	25 (850)	26 (910)	28 (980)	29 (1015)	29 (1015)	29 (1015)

*Each class period is 45 minutes long. Numbers in parentheses are number of class periods required per school year.

Sources: Okuda, Shinjo. "The Curriculum and its Contents in Secondary Education." Paper presented at International Seminar on Educational Reform, National Institute of Multimedia Education, Kyoto, Chiba, October 14-17, 1985, and Ministry of Education, Science, and Culture. Course of Study for Elementary Schools in Japan. Tokyo: Monbusho, 1983, p. 122. (From Japanese Education Today, January, 1987, p. 28)

Table 2
Required Weekly Class Periods per Subject, by Grade Level.*

Subject	First Year (Grade 7)	Second Year (Grade 8)	Third Year (Grade 9)
Japanese language	5	4	4
Social studies	4	4	3
Mathematics	3	4	4
Science	3	3	4
Music	2	2	1
Fine arts	2	2	1
Health and physical education	3	3	3
Industrial arts or Homemaking	2	2	3
Moral education	1	1	1
Special activities	2	2	2
Elective (usually English)**	3	3	3
Additional elective hour***	0	0	1
Total	30	30	30

*Each class period is 50 minutes long

**Electives are assigned at the principal's discretion. The entire grade level is required to take the same elective. English is the almost universal elective.

***The additional elective hour in the 3rd year is typically assigned by the principal to one of the more difficult subjects such as mathematics or English.

Source: Ministry of Education, Science, and Culture, Japan. Education in Japan: A Graphic Presentation. Tokyo: The Ministry, 1982, p. 59. (From Japanese Education Today, January, 1987, p. 34)

Table 3
Mathematics Problems on a Senior High School Entrance Examination
(50 minutes)

NOTE: Put all answers on separate answer sheet.

Calculations may be done anywhere on test booklet.

Note: Leave answers in radical form.

1. Solve problems [1]-[4] below.

[1] $\frac{12}{\sqrt{3}} - \sqrt{(-2)^2} + (-2 + \sqrt{3})^2$

[2] Solve the compound inequality $\frac{3x + 1}{2} < \frac{4x + 1}{3} \leq \frac{7x + 5}{4}$

[3] In the quadratic equation in x: $x^2 - (a+1)^2x + (2a^2+2a-3)=0$, a is a constant and one of the roots of the equation is 2. Find the value of a and the equation's other root.

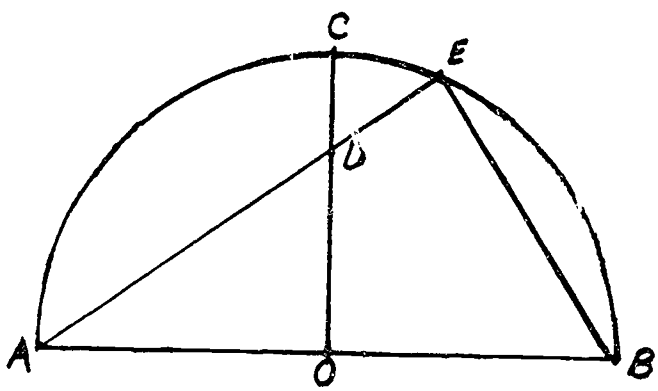
[4] a, b, and c are integers such that a is greater than 1 and less than 9 and b and c are each greater than 0 and less than 9. Find all triples (a, b, c) which satisfy the equality $100a + 10b + c = 5(12a + 5b + c)$.

2. There is a semicircle with point O as center, line segment AB as diameter, and a radius of 3 cm. Take radius OC perpendicular to AB and consider a point D on OC such that OD:DC = 2:1. The line segment AD extended intersects the semicircle at E. Solve the problems.

[1] Find AE:EB.

[2] Find the length of BE.

[3] Find the area of $\triangle$CDE.



3. In isosceles triangle ABC, AB = AC = 1 and $\angle CAB = 36^\circ$. From a point D on side BC a perpendicular DE to side AC was drawn. When a perpendicular EF was drawn from E to side AB, $\angle FDB$ turned out to be a right angle.

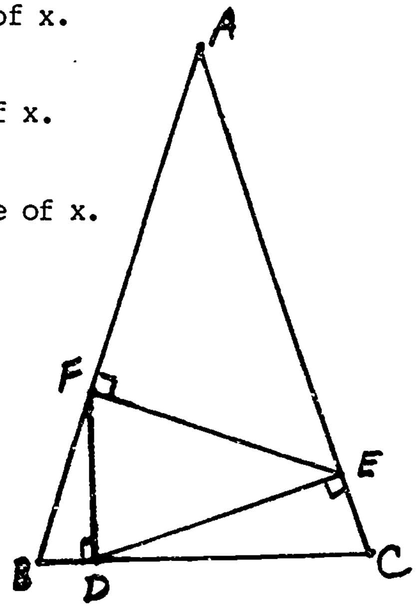
Let BC = 2x and solve the following problems.

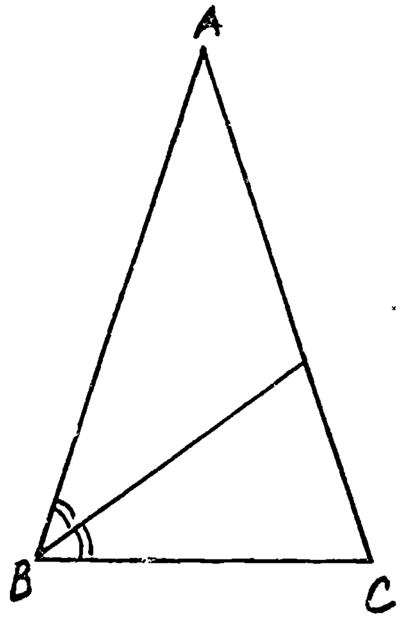
[1] (1) write an expression for $\frac{BD}{DF}$ in terms of x.

(2) Write an expression of $\frac{BD}{DC}$ in terms of x.

[2] Using the bisector of $\angle ABC$, find the value of x.

[3] Find the value of $\frac{DB}{DC}$



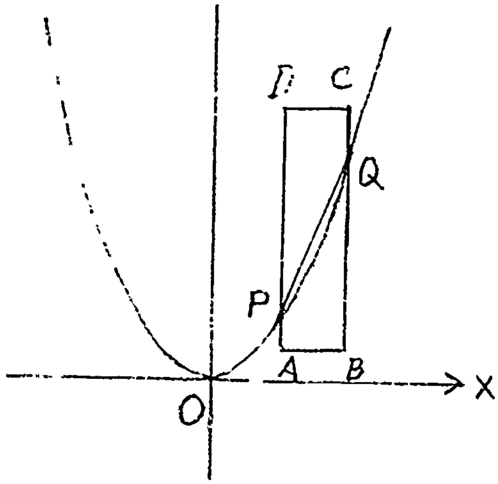


4. Rectangle ABCD has vertices A(2,1), B(4,1), C(4,8) and D(2,8). The parabola $y = ax^2$ (a>0) intersects the rectangle at two distinct points P and Q. Solve the following problems.

[1] Find the range of values a can take under these conditions.

[2] Find the value of a for which the line segment PQ bisects the area of rectangle ABCD.

[3] If P lies on AD and Q on BC and line $\overline{PQ}$ passes through (0,-3), find quadrangle ABQP:quadrangle CDPQ



5. Since for four consecutive integers n-1, n, n+1, and n+2 it is true that (n-1) + (n+2) - n - (n+1) = 0, it is possible, for example, to divide the consecutive integers 1, 2, 3, and 4 into two groups whose sums are equal, using each integer once and only once. Expressing this division as an equality, we can write 1 + 4 = 2 + 3.

For squares of consecutive integers, do problems [1] and [2].

[1] For eight consecutive integers n-3, n-2, n-1, n, n+1, n+2, n+3, and n+4, compute:

$$A = (n-3)^2 + n^2 - (n-2)^2 - (n-1)^2$$

$$B = (n+1)^2 + (n+4)^2 - (n+2)^2 - (n+3)^2$$

Using these results we can divide the consecutive integers 1, 2, 3, 4, 5, 6, 7, and 8 into two groups such that, using each integer once and only once, the sum of the squares of the integers in one group is equal to the sum of the squares of the integers in the second group. Express this division as an equality.

[2] For 12 consecutive integers n-5, n-4, n-3, n-2, n-1, n, n+1, n+2, n+3, n+4, n+5 and n+6, compute:

$$C = (n-4)^2 + (n-2)^2 - (n-5)^2 - (n-3)^2$$

$$D = (n+1)^2 + (n+2)^2 - (n-1)^2 - n^2$$

$$E = (n+4)^2 + (n+6)^2 - (n+3)^2 - (n+5)^2$$

Using these results we can divide the consecutive integers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12 into two groups such that, using each integer once and only once, the sum of the squares of the integers in one group is equal to the sum of the squares of the integers in the second group. Express this division as an equality.

Translated by K. Inouye

Table 4
Schools/Universities Visited, Principal Person at Each Institution, and Contact Person
PROFESSIONAL VISIT OF ICTM DELEGATION TO JAPAN
(September 23 - October 10, 1988)

School & others	Address	Principal Person (Contact Person)
University of Tsukuba, Institute of Education	Tennodai 1-1-1, Tsukuba-shi Ibaraki 305 JAPAN	(Tatsuro Miwa) (Nobuhiko Nohda)
Tsukuba Onan Elementary School	Sasagi 875, Tsukuba-shi Ibaraki 305 JAPAN	Shogoro Okubo (Miwa, Nohda)
Meikei Gakuen Lower & Upper Secondary School	Akatsuka 737, Tsukuba-shi Ibaraki 305 JAPAN	Minoru Okamoto (Miwa, Nohda)
Tsuchiura Daiichi Upper Secondary School	Manabe 4-4, Tsuchiura-shi Ibaraki 300 JAPAN	Masao Mori (Miwa, Nohda)
Tokyo Gakugei University Upper Secondary School	Shimouma 4-1-5, Setagaya-ku Tokyo JAPAN	Takehisa Aida (Yoshishige Sugiyama) (Toshiakira Fujii)
National Institute for Educational Research	Shimomeguro 6-5-22, Meguro-ku Tokyo JAPAN	Isao Suzuki (Toshio Sawada)
Kumon-Juku	Kumon Kyoiku Kaikan 6-6, Nishinakajima 5-chome Yodogawa-ku Osaka 532 JAPAN	Masazumi Hayakawa (Toshio Sawada)
University of Tsukuba Elementary School	Otsuka 3-29-1, Bunkyo-ku-shi Tokyo 112 JAPAN	Tsuneo Takano (Tomio Taki) (Miwa, Nohda)
University of Tsukuba Lower Secondary School	Otsuka 1-9-1, Bunkyo-ku Tokyo 112 JAPAN	Takayoshi Kaneko (Mikio Masuda) (Miwa, Nohda)
Japan Society of Mathematical Education	Koishikawa PO Box 18 Bunkyo-ku, Tokyo 112 JAPAN	Yoshitomo Matsuo (Toshio Sawada)
Fujitsu Co. Numazu Factory	140 Miyamoto, Numazu-shi Shizuoka, 410-03 JAPAN	Kunihiko Yoshii Toru Tanaka (Mr. Haruo Kakuta)

School & others	Address	Principal Person (Contact Person)
The Japan Federation of Soroban Education League	Shitaya 2-chome 17-4, Taito-ku Tokyo 110 JAPAN	Hisao Takashima (Kouzi Suzuki) (Nohda)
Nagoya Takaharidai Lower Secondary School	3-801 Segobo, Meito-ku, Nagoya, 465 JAPAN	Akira Kanou (Rokuji Shibata)
Toyota Automobile Co.	1, Toyota-cho, Toyota Aichi, 471 JAPAN	Toshiyuki Asami (Rokuji Shibata)
Aichi University of Education Department of Mathematics	Hirosawa 1, Igaya-machi, Kariya-shi, Aichi JAPAN	Rokuji Shibata
Osaka Tennoji Lower Secondary School	6-20 Kitakawahori-cho Tennoji-ku, Osaka 543 JAPAN	Takeshi Sekai (H. Okamori)
Osaka Kyoiku University Hirano Lower & Upper Secondary School	Nagare-cho 2-1-24 Hirano-ku, Osaka JAPAN	Minoru Hanatoku (H. Okamori)
Osaka Kyoiku University Hirano Kindergarten	79-1-2 Nagaremachi Hirano-ku, 547 Osaka JAPAN	Kazuhiro Nakanishi Ms. Akiro Ueda (H. Okamori)
Osaka Otemon Gakuen Elementary School	Kyobashimaeno-cho 2-chome Higashi-ku, Osaka JAPAN	Seichi Sugishita (H. Okamori)
Seifu Upper Secondary School	Ezashi-cho 6-chome, Tennoji-ku, Osaka JAPAN	Hidenobu Hiraoka (H. Okamori)
Osaka Kyoiku University Tennoji Lower & Upper Secondary School	Minamikawahori-cho 4-88, Tennoji-ku, Osaka 543 JAPAN	Okura Matsuda (Junior High) (H. Okamori) Noboru Shimomura (Senior High) (H. Okamori)
Osaka Kyoiku University	Minamikawahori-cho 4-88, Tennoji-ku, 543 Osaka JAPAN	H. Okamori

References

Amano Ikuo (1986) The dilemma of Japanese education today. The Japan Foundation Newsletter XIII (5), Tokyo.

Azuma, H., Hakuta, K., and Stevenson, H. (1986) Child Development and Education in Japan. New York: Freeman Publishers.

Becker, J. P. et al. (1987) The Great China Adventure (1987) - A Report of the ICTM China Mathematics Delegation. Illinois Mathematics Teacher, 38 (3 and 38 (4) [pp. 4-14 and 3-27]

Becker, J.P. et al. (1989) Mathematics Teaching in the Japanese Elementary and Junior High Schools. Mimeograph, 25 p.

Becker, J. P. and Miwa, T., (1987) Proceedings of the U.S.-Japan Seminar on Mathematical Problem Solving, Carbondale, IL: Southern Illinois University, 481 p.

Committee for Facilitating Research (1986) Teacher Training in Japan, Japan Association of Universities of Education. Tokyo: Dai-ichi Hoki Publishing Co., Ltd.

Davis, R. B., Romberg, T. A., Rachlin, S. and Kantowski, M. G. (1979) An analysis of mathematics education in the USSR. Eric Clearinghouse for Science, Mathematics, and Environmental Education: The Ohio State University, Columbus, OH.

Duke, B. (1986) The Japanese School - lessons for industrial America. New York: Praeger Publishers.

Flanders, J. (1987) How much of the content in mathematics textbooks is new? Arithmetic Teacher, 35 (1), 18-23.

Fuson, K., et al. (1988) Grade placement of addition and subtraction topics in Japan, Mainland China, the Soviet Union, Taiwan, and the U.S., Journal for Research in Mathematics Education, 19 (5), 449-456.

Hashimoto, Y. and Sawada, T. (1979) Mathematics Program in Japan, Tokyo: National Institute for Educational Research (Note: the program was implemented in 1980.)

Horvath, P. J. (1987) A look at the Second International Mathematics Study Results in the U.S.A. and Japan. The Mathematics Teacher, 80 (5), p. 367.

Leestma, R., et al. (1987) Japanese education today. Washington, DC: Superintendent of Documents, U.S. Government Printing Office.

McKnight, C. C., et al. (1987) The underachieving curriculum - assessing U.S. school mathematics from an international perspective. Champaign, IL: Stipes Publishing Co. (127 p.)

National Institute For Educational Research (NIER) (March, 1988) Basic Facts and figures about the educational system in Japan, Tokyo: NIER.

National Institute for Educational Research (NIER) (1988) The Recommendations of the National Council on Educational Reform - A Summary, Tokyo: NIER.

Sawada, T., and Kobayashi, S. (translated by Patricia Horvath) (1986) An analysis of the effect of arithmetic and mathematics education at juku. Tokyo: National Institute for Educational Research (Compendium #12/March)

Stevenson, H. W., Lee, S. Y., and Stigler, J. W. (1986) Mathematics achievement of Chinese, Japanese, and American children, Science, 231, 693-699.

Stigler, J. W. (1988) The use of verbal explanation in Japanese and American classrooms. The Arithmetic Teacher, (October), 27-29.

Stigler, J. W., Lee, S. Y., Lucker, G. W., and Stevenson, H. W. (1982) Curriculum and achievement in mathematics: a study of elementary school children in Japan, Taiwan, and the United States. Journal of Educational Psychology, 74, 315-322.

Treisman, U., Hart, P., Dillavicencio, A. Talks given at the Success for Everyone II Conference, University of Illinois at Chicago, January 28, 1989.

Webster's New Universal Unabridged Dictionary (second edition) (1983) Cleveland: Dorset and Baber.