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AUTHOR Connell, Michael L.
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ABSTRACT

The mathematical experiences of elementary students often focus on memorizing facts and rules as opposed to making sense of the subject and developing problem solving skills. Students spend large amounts of time processing, memorizing and sorting collections of data which are tasks well performed by computer technology. To correct this situation, this paper describes an instructional model for problem solving. The learner proceeds through four problem types (manipulations, sketches, mental pictures, and abstractions) using the phases memory/recall, instructor-posed problem, and self-posed problem for each problem type. A sample application is given showing computer-assisted instruction. Included are 42 references. (DC)

Microcomputer referents in elementary mathematics: A sample approach.

Michael L. Connell
University of Illinois at Urbana-Champaign

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The view is becoming widely accepted that consequential knowledge does not include rote memorization. As Sternberg (1984a, 1984b) puts it, consequential knowledge involves deciding what information is important to learn and then incorporating that information into the individual's already existing knowledge base. Yet, in looking at the mathematical experiences of elementary aged students Davis (1974), Erlwanger (1973), and more recently Peck, Jencks, and Connell (in press) have found that instructional focus has typically been placed upon memorizing facts and rules as opposed to making sense of the subject. Rather than developing skills requiring the use of information in a meaningful way, students in elementary mathematics spend large amounts of time processing, memorizing, and sorting collections of data - the very tasks performed so well by computer technology . Quite often the problem solving experiences which are offered consist of problem specific heuristics with little instruction in generating generalizable strategies for manipulating information in goal-oriented situations. Students taught in this fashion come to view mathematics as a system of rules to be memorized and retrieved (Erlwanger, 1973; Schoenfeld, 1983; Garofalo & Lester, 1985). In their minds successful mathematical thinking in the classroom becomes either rote recitation of tables or case specific utilization of rules, memorized facts, or miscellaneous data. Lacking the ability to make decisions based on their own judgment, verifying the correctness of an answer or a process is left to a source outside of themselves.

Peck, Jencks and Connell (1985) suggest that a primary cause for these difficulties in elementary mathematics lies in application of a rote memorization teaching methodology wherein students are routinely required to memorize and practice facts and procedures isolated from each other and from any real world referent. In addition to the fragmentation effects this has upon the curriculum, such an approach effectively sidesteps development of a real world referent base. Keil (1984) points out that humans are capable of engaging in complex chains of problem solving when the problem solving is embedded within, and done in reference to, a specific knowledge structure or referent base. However, when conceptual referents are not present for the mathematical symbols being manipulated, Schoenfeld (1983, 1985) and others suggest that people construct undesirable models concerning knowledge and their role in acquiring it. These models subsequently block the development of unifying structures for the information they possess and contribute to the problem of inert knowledge as described by Whitehead (1929). For example, problems are viewed as always having unique and specific answers which are wholly determined not by the logic of the problem but by the answer book, a neighbor, or the teacher. As a result, problems in mathematics are approached from unproductive viewpoints, with greater emphasis placed upon recalling memorized rules than in analyzing the situation to be evaluated.

What is needed is a change of emphasis in mathematics education. Taking a clue from Keil (1984), structuring knowledge with respect to real world referents should play a substantially more important role than rote mastery of arbitrary rules governing algorithms and procedures. When students possess such a reality base they are able to recombine features into new, successful relations in the course of problem solving. Without such a mapping from the abstract symbols to the real world, it is difficult to apply even elementary mathematical metacognitive techniques (Campione, Brown, and Connell, in press). Lacking concrete referents, students are unable to identify when the problem situation causes misapplication of their developed rules.

In spite of the mechanical and computational focus of traditional mathematics curricula, there emerge groups of children who seem to naturally organize their thinking in ways that are conducive to problem solving. Kachuk (April, 1987), Connell (1988a), Connell and Harnisch (in press), and others report that the thought processes and structuring strategies these students utilize are markedly different from other students in the same classroom settings, even though their peers may be considered equally capable in other respects. Students who are good problem solvers possess many linkages relating the subject matter to elements of their real world experiences (Peck and Jencks, 1979; Connell and Harnisch, in press). These linkages provide a referent base, allowing them to assume ownership over their work and to readily address questions such as "How can you tell?" or "What would happen if ...?" in regard to their final answer, the process by which the answer was obtained, or the underlying premises upon which the choice of procedures were based.

Unfortunately, this ability to attack and solve problems often appears to have developed independently of school experiences. Evidence suggests that many educational experiences in traditional settings contribute to the formation of barriers which inhibit further conceptual growth. As Spiro, Vispoel, Schmitz, Samarapungavan, and Boerger (in press) discuss, instructional problems develop when students routinely memorize facts without opportunity to relate these facts to the real world and its intricacies. As discussed earlier, this situation is particularly problematic in elementary mathematics. Klahr (1984) notes that children often display the ability to produce and manipulate symbols well before they can demonstrate understanding concerning what the symbols represent. Furthermore, students routinely make verbal assertions which make perfect syntactic sense, yet lack a semantic referent. Cobb (1987) describes this clearly in his analysis of a group of second graders' understandings of 10. When the child says that the **4** in **48** means four tens, the child is often merely demonstrating verbal fluency based upon the right and left positional labels. The child often does not recognize that the 4 represents 40 objects. To the child, such symbols bear no relationship to recognizable facets of the world; hence, the child fails to perceive the underlying ideas and concepts. Nonetheless, a demonstrated ability in the production and manipulation of symbols, however arbitrary or nonsensical these symbols may appear to the student, may lead to instruction proceeding before the student has grasped the concepts that the symbols represent. Given these perceptions and strategies, it is little surprise that students do not come to value problem solving skills and do not learn them well. They lack the conceptual building blocks with which to link their memorized data into meaningful structures.

In order to correct this situation a substantially different curriculum base, presentation schemata, system of psychological rewards, and setting of instruction should be provided. A first step in facilitating this goal is placing children in situations emphasizing problem solving skills, requiring them to develop and apply their own logic structures. A legitimate problem in this setting would involve working on concepts that are within reach given currently possessed knowledge structures. The problem should be new, but within the conceptual grasp and existing analytic powers of the students. Problem solving strategies involving posing questions, analyzing situations, translating results, illustrating results, drawing diagrams, and using trial and error should be developed and utilized to develop a referential base for later application. These problem solving activities should take place at a concrete level, with problems designed to achieve curriculum goals using elements familiar to the child. As Bruner (1978) points out, we must try to make students at least as self conscious about their strategies of thought as they are about their efforts to commit things to memory. In addition to this goal, however, the problem situations must contain the opportunity for children to assume ownership over issues of correctness of result and process.

This paper uses an instructional model proposed by Connell (1986), as adapted from Robert Wirtz's (1979) model of mathematical problem solving.

Instructional Model

	Memory/Recall	Instructor-Posed Problem	Self-Posed Problem
Manipulatives			
Sketches			
Mental Pictures			
Abstractions			

As the above figure shows, a learner would proceed from initial use of manipulatives through abstraction via four transitional problem types. For the purposes of discussion we shall refer to these problem types as:

1) *Manipulatives*

2) *Sketches*

3) *Mental Pictures*

4) *Abstraction*

An example of physical manipulatives in this model might be a pile of pebbles used to illustrate elementary addition. A sketch would then be drawn recording the actual pile of pebbles. A mental picture would consist of an internal representation of the external sketch. Abstraction would occur when addition is no longer described in terms of countable piles of pebbles, but in terms of pure number.

1) Manipulatives. The power of a physical manipulative lies in the structures which can be built upon it, the linkages it enables in the mind of the student, and its power in explaining concepts. The merit of a manipulative is that it can be used to simplify information, generate new propositions, and increase the manipulability of a body of knowledge.

In thinking of manipulatives it is important to remember that all problems have their origins in the real world about us. The symbolism adopted derives as a result of formal attempts to solve those problems. Although there is certainly a single correct answer for the majority of problems, Hogben (1983) points out that much of what we accept as the correct method for solving a specific problem has resulted from accidents of notation which have little to do with the underlying logic or mathematics of the situation. By focusing upon the logic that lies beneath the rules, however, we can expand the role of conscious control significantly.

2) Sketches. The sketches follow the form of the original manipulatives as closely as possible. Ideally, the mapping from manipulative to sketch, sketch to mental picture, and finally mental picture to abstraction should be as smooth as possible. If we select an appropriate manipulative, the subsequent sketch draws much of its descriptive power from it. One way of insuring that mapping from manipulative to sketch will occur naturally is to tie the presence of a recording scheme reflecting the real world nature of the manipulative, in sketch form, at the earliest levels of manipulative problem solving (Peck, 1979).

3) Mental Pictures. In developing a mental picture the student must internalize the informational structure encoded in the sketch. At this time there are many conflicting theories concerning the mechanisms behind the creation and utilization of mental imagery as reflected in the work of Cooper & Shephard (1984), Sawyer (1964), Jencks & Peck (1972) Tweney (1987) and others. They agree, however, that whatever is going on in the brain when we have an image produces a representation possessing useful functional properties in structuring and organizing information. In a quote attributed to Albert

Einstein it is said that he arrived at the theory of relativity by "visualizing.. effects, consequences, and possibilities" through "more or less clear images which can be 'voluntarily' reproduced and combined." (Cooper & Shephard, 1984) In applying this model one should exercise care, lest familiarity with a sketch be confused with possession of the underlying mental representation. A sketch is based upon one instantiation of a specific problem type; a mental representation corresponds to a more generalized and broadly applicable knowledge structure.

4) Abstraction. The final step lies in the ongoing mental structuring of experience into a more abstract and formal setting. At this point the student has completed the sequence of internalizing the real world problem into justifiable processes by which it may be solved. This setting can then be used in future problems and as a stepping stone towards independent investigations. If we are successful in following the steps outlined in this model the student will possess not just a single answer schema, but an entire structural linkage to be utilized by the student under varied circumstances. The student will have developed conceptual building blocks which may be used in later, more complex, endeavors in problem solving.

In an effort to apply this research base to actual classroom teaching experience has suggested that the four problem types should each be presented in three phases (Connell 1981, 1984, 1988)

1) *Memory/Recall*

2) *Instructor Posed Problem*

3) *Self Posed Problem.*

1) Memory/Recall This phase consists of committing to memory the symbolism of the referent and assorted terms with which it may be labeled. In a very real sense we are attempting to provide the basis for a common 'language' to be used by both students and teachers in talking about problem situations. In terminology, terms are kept to a minimum with essential terms often given in the natural language of the child. For elementary mathematics, it is of prime importance that the language be clearly presented, defined, and mutually understood (Davis, 1974, 1979; Cobb, 1988). It is equally important that the child is comfortable with the symbolism being suggested (Confrey, 1988).

Once the teacher is sure that the basic terminology and symbolism is clear to the students the second phase is entered.

2) Instructor Posed Problems As may have been surmised from the information presented thus far, there is much important work which must be done prior to actual presentation of a problem in the classroom. In most traditional models, however, instruction begins at this point. Failure to set the stage at this point is a primary cause of many of the blockages reported in the introduction. Provided the students have been properly prepared, the instructor should to pose problems which relate to the referent provided and lead to internalization of the concepts presented in the problem situation.

Problems at this level have the added virtue of being soluble by the student's usage of the referent itself. In these cases, the referent becomes the gauge of correctness of the child's work. It is true that the teacher must still correct the student, but only in a manner that enables the learner to assume ownership of correctness. This ownership is assumed by the student's reliance upon and use of the knowledge structures created through referent use.

3) Self-Posed Problems It is unfortunate that in many classrooms the instruction cycle is complete with the teacher's presentation of sample problem types. If we are to be successful in teaching problem solving, we must allow the students to pose problems. Bruner puts this very well when he states

> A body of knowledge, enshrined in a university faculty and embodied in a series of authoritative volumes, is the result of much intellectual activity. To instruct someone in these disciplines is not a matter of getting him to commit results to mind. Rather, it is to teach him to participate in the process that makes possible the establishment of knowledge
> (Bruner, 1978. p. 72)

In this model of instruction we allow students the opportunity to use the developing referent to pose and investigate problems of their own. It is extremely important that this be allowed, as it is at this point the children develop the essential linkages which later serve to tie their data into useful problem solving structures (Case, Kurland, Daneman & Goldberg, 1982). It is during these independent investigations that we can best promote the development of self accounting. This self accounting then enables the student to progress beyond adaptive behavior to the conscious application of logic and reasoning. Furthermore, it is in independent investigation that the child begins to develop a sense of ownership over their problem solving strategies (Jencks & Peck, 1973; Peck & Jencks, 1981). This ownership establishes self-rewarding sequences and becomes an incentive towards further learning. When a student is capable of posing and solving problems this becomes a reinforcement for further problem solving attempts in the future.

It is helpful to bear in mind that each of these three phases can occur in a single instructional period. Robert Wirtz (1979) has reported that: "At a single setting children can move from one cognitive level to another -- from remembering experiences, to solving problems, to making independent investigations."

In using these problem types and sequencing suggestions it is helpful to observe two trends which occur as children gather experience in problem solving. First, they become more nearly exhaustive in their processing of information presented in the problem, and consider all or almost all of the information presented (Sternberg 1984a, 1984b). Secondly, they spend relatively more time in planning how to go about solving a problem, and less time in actually solving it (Chi & Glaser, 1985).

A sample application.

The power and potential of the microcomputer can play a much greater role than is currently utilized in helping students make the leap from the sketch to the abstraction. The tremendous flexibility of the microcomputer makes it possible to create learning environments and micro-worlds utilizing the very presentation schemata, system of rewards, and instructional settings hinted at earlier (Bransford, Hasselbring, Littlefeild and Goin, in press). In an effort to address this need, a microcomputer based interactive icon processor was developed for use in helping students construct a referent base for solving systems of simultaneous linear equations (Connell and Ravlin, 1988). This icon processor makes use of a well developed and flexible mental representation in the form of user-controllable graphic objects (icons) which are in turn based upon plausible physical manipulatives. The current program is written in the IBM Handy authoring language and is implemented on a 640K IBM PC AT with an EGA graphics card. Initial efforts concentrated upon creating a flagpole world within which problems involving two equations and two unknowns may be addressed and solved. Future work will be done in expanding this system to handle larger systems of equations using the HyperCard on the Apple MacIntosh series computer.

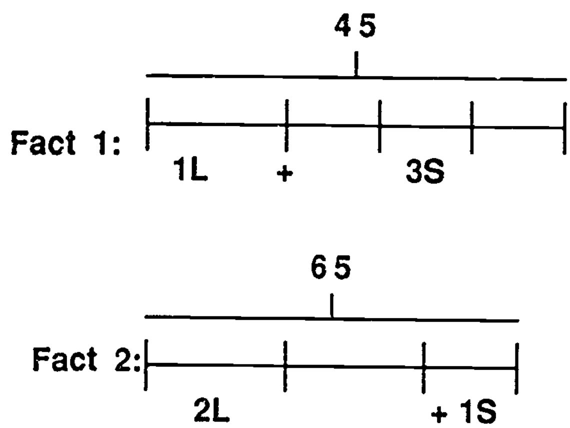
In the flagpole world as currently constructed, flagpoles are constructed graphically on the computer screen using various numbers of labeled long and short flagpole sections, corresponding to variables in formally presented algebraic equations. The student is initially presented with a problem and a graphic representation showing the lengths of two distinct flagpoles formed from integer combinations of long and short sections. These initial flagpoles correspond to a consistent system of two equations, each of which has two unknowns. The student's goal is to use the icon processor to derive the lengths of the long and short sections. Various operations are available to the student for manipulating the flagpoles in working towards a solution. For instance, flagpoles may be made longer by integral coefficients, corresponding to the elementary row operation of multiplying a single equation by a constant; flagpoles can be compared and the difference computed, corresponding to the elementary row operation of subtracting one equation from another; and so on. At each stage of this process the newly created flagpoles are displayed graphically together with their associated values, if known, and this information is available for further use by the student. Comparing strategically constructed flagpoles leads to derivation of the lengths of the component sections, equivalent to solving the system for each unknown.

To see how this might be done, consider the following example from Connell and Ravlin (1988):

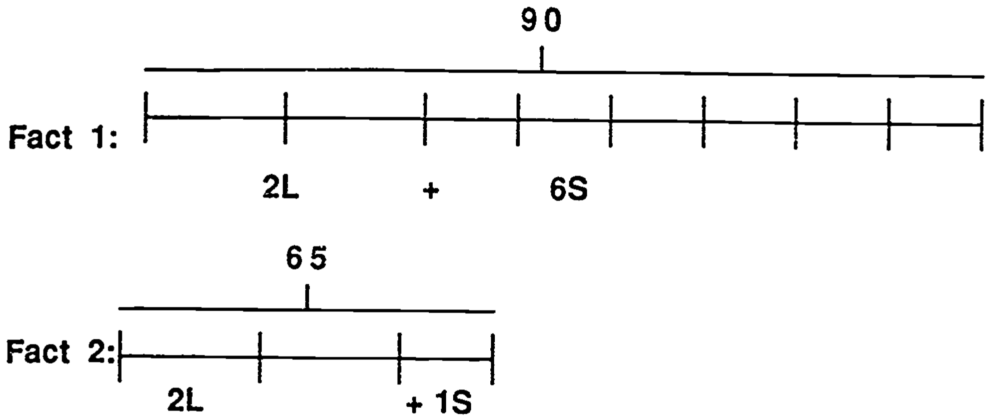
> A flagpole factory has two different machines used to manufacture flagpole sections. These machines can be set to make any length of section, but once set they cannot be changed until the next dat. On Monday there were two different lengths of flagpole sections, one longer than the other. The length of three of the shorter sections and one of the longer is forty-five feet. The length of two of the longer sections and one of the shorter is sixty-five feet. What are the lengths of each flagpole section?

Solution Steps:

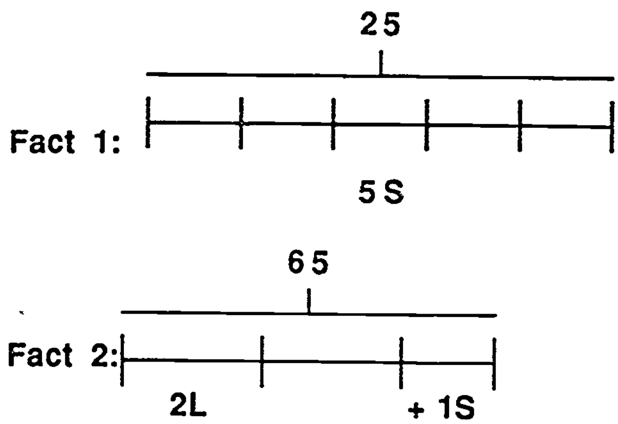
1) Draw the flagpoles to represent the problem facts:



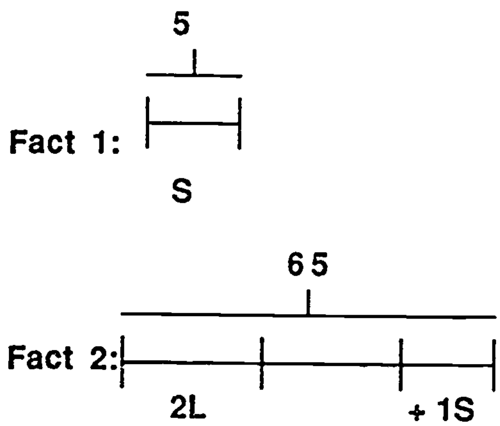
2) Make **Fact 1** twice as long to get a common number of long sections (this corresponds to the elementary row operation of multiplying an equation by a non-zero constant).



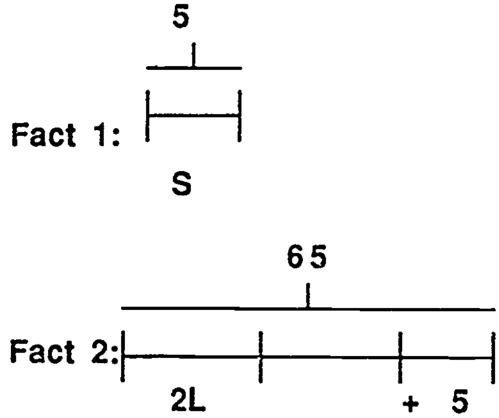
3) Remove **Fact 2** from **Fact 1** (this corresponds to subtracting one row in the matrix from another).



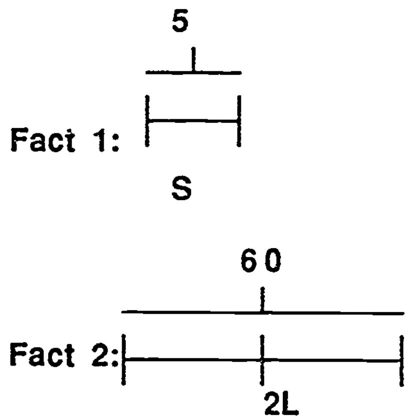
4) Divide **Fact 1** by **5**.



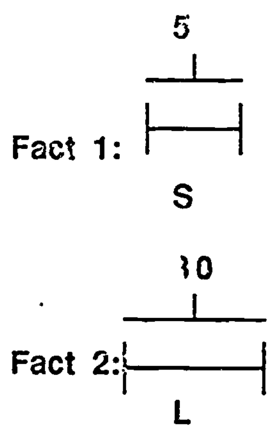
5) Enter the length found for the short section into the other flagpole (i.e., backsubstitute for the value of the known variable).



6) Remove the length found for the known section from **Fact 2** (i.e., eliminate the effect of the known variable).



7) Divide **Fact 2** by 2.



8) The length of each type of flagpole section (i.e., the value of each variable is now determined. Long sections (L) are 30 feet long; short sections (S) are 5 feet long.

Such a micro-world consisting of dynamically changing configurations of graphics objects, each with an associated set of properties, is ideally implemented on a microcomputer. The icon processor developed thus far is capable of manipulating flagpoles according to the user's directions, although more is planned for it in future versions. The program has the potential to lead the learner to understand concepts underlying linear algebraic algorithms. For example, when the student uses a multiple of one flagpole and subtracts this from another flagpole to remove all sections of a certain type the student has, in a formal sense, accomplished the elimination of one of the variables. Once the length of the remaining section type is determined, this value can then be used to determine the length of the other section. As all work is done from within a graphical representation, this is done quite intuitively by using the program to compare developed icons and to perform interesting manipulations of these quantitative objects. When view this sequence of operations is viewed from a formal perspective, however, the student is building a flexible referent for later formal concepts such as Gaussian elimination and back-substitution.

For a computer-assisted instructional program to be useful, intelligible feedback must be provided by the program to the student. Furthermore, for instructional purposes, we must provide more than a black-box which always gives the right answer; the box itself must be transparent and its methods analogous to those ultimately desired of the student. The developed expert capable uses thought processes similar to those found in a skilled user (Peck and Jencks, in press) and is available to act as an advisor when help or feedback is needed. As currently implemented the expert is responsive to individual differences in problem solving strategy. Even in simple two by two systems of equations, multiple solution paths are possible. Either variable may be solved for first, although given the configuration of the problem it may be more economical in terms of the number of operations to be performed, to choose one variable over the other. Although there may be a unique optimal solution, what is more important to reinforce is the general strategy by which any system may be solved. The expert does not force a single solution path upon the student. If the student is determined to solve for the long section first and asks the expert for help the next step in that solution path will be provided, even if it might be easier in the particular system to solve for the short section first. If the student has no idea as to how to proceed, the expert will present the easiest solution path (i.e., the one requiring the fewest steps). Furthermore, if the student has developed from the original problem facts any flagpole which might lead him or her closer to the solution, the expert will use that flagpole in suggesting how to proceed, rather than constructing a new flagpole which might lie on a different solution path and possibly confuse the student.

In addition to providing expert feedback, the program can play other important roles in the educational process. The computer keeps a chronological record of the operations a student , as well as the number, content, and sequencing of hints given. This trace can illustrate differences in the approach utilized during successful and unsuccessful attempts at problem solving. Such information can be invaluable in determining the domain knowledge, heuristics, and the control strategies utilized by the students which, as argued by Collins, Brown and Newman (in press), is critical in the design of effective learning environments.

Acknowledgments

(1) Handy is an experimental language for writing interactive educational software currently under development at the IBM T. J. Watson Research Center, Yorktown Heights, NY. We wish to acknowledge the cooperation Don Nix and Brad McCormick, Handy's designer and implementer, respectively, in providing us with a test version of the language to use in our research.

(2) Original inspiration for the flagpole world came from an adaptation done by Donald M. Peck and Stanley M. Jencks of W. Warwick Sawyer's (1964) Man and Sons problem. Peck and Jencks (in press) provide excellent examples of the use of this micro-world in a non-computer setting.

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