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AUTHOR	Wynn, Karen
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ABSTRACT

Three studies examined the abstractness of children's mental representation of counting, and their understanding of the cardinality principle, namely, that the last number word used in a count tells how many items there are. In the first experiment, 24 toddlers of 2-3 years counted objects, actions, and sounds. Findings revealed that children counted objects best; most were also able to count actions and sounds. This finding suggests that at a very young age, children begin to develop an abstract mental representation of the counting routine. When asked, "How many?" after counting, only older children gave the last number word used in the count a majority of the time. This suggests that children younger than 3.5 years do not understand the cardinality principle. In the second experiment, children were asked to give a puppet 1 through 6 items from a pile. Older children succeeded at this task. They spontaneously counted how many items they gave, thus showing a clear understanding of the cardinality principle. Younger children succeeded only at giving 1, and sometimes 2, items, and never spontaneously counted. In experiment 3, 18 toddlers were asked several times for 1 through 6 items. Results indicate that children learn the meanings of smaller number words before larger ones within their counting range up to number 3 or 4. (RH)

Children's Understanding of Counting

Karen Wynn

Massachusetts Institute of Technology

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This material is based on work supported under a National Science Foundation Graduate Fellowship. All correspondence should be sent to the author at The Department of Brain and Cognitive Sciences, E10-105, Massachusetts Institute of Technology, Cambridge, MA, 02139 (E-mail address: *wynn@psyche.mit.edu*).

Running Head: CHILDREN'S UNDERSTANDING OF COUNTING

Abstract

This study examines the abstractness of children's mental representation of counting, and their understanding that the last number word used in a count tells how many items there are (the cardinality principle). In the first experiment, 24 2- and 3-year-olds counted objects, actions, and sounds. Children counted objects best, but most were also able to count actions and sounds. This shows some ability to generalize counting to novel situations, suggesting that at a very young age, children begin to develop an abstract, generalizable mental representation of the counting routine. When asked "how many" following counting, only older children (mean age 3:6) gave the last number word used in the count a majority of the time, suggesting that before this age children do not understand the cardinality principle. In the second experiment, the same children were asked to give a puppet 1, 2, 3, 5, and 6 items from a pile. The older children succeeded at this task for all the numerosities, and spontaneously counted how many items they gave, showing a clear understanding of the cardinality principle. The younger children succeeded only at giving 1 and sometimes 2 items, and never spontaneously counted; this suggests that they were simply recognizing the correct number when asked for 1 or 2 items, not using the cardinality principle. In the third experiment, 18 2- and 3-year-olds were asked several times for 1, 2, 3, 5, and 6 items, to determine the largest numerosity at which each child could succeed. Results indicate that children learn the meanings of smaller number words before larger ones within their counting range, up to the number *three* or *four*. They then appear to make a general induction that *every* number word within their counting range refers to a specific numerosity. This induction occurs in conjunction with children's acquisition of the cardinality principle at about 3-and-a-half-years of age.

Children's Understanding of Counting

Because counting is one of the very earliest number-related activities, it may shed light on the young child's concept of number. Much research currently revolves around whether domain-specific knowledge of number underlies children's counting, or whether it is through general cognitive capacities that children learn to count, and thus come to understand number. The two main theories regarding this question are outlined below.

'Principles-Before' Theory

There has been growing interest over the past 15 years in a domain-specific theory of the basis of counting, developed by Gelman and her colleagues (e.g. Gelman & Gallistel, 1978, Gelman & Greeno, 1987; Gelman & Meck, 1983; Gelman, Meck & Merkin, 1986; Greeno, Riley & Gelman, 1984). The claim (outlined in detail in Gelman & Gallistel, 1978) is that there are innate, number-specific principles that underlie children's ability to count. The following "How-to-count" principles define the counting procedure: The *One-to-one correspondence* principle states that there must be a one-to-one correspondence between things to be counted and number words (or, more generally, between things to be counted and members of the set of symbols used for counting); the *Stable-Order* principle states that the set of symbols used to count with (e.g., number words) must have a fixed order in which they are consistently used; and the *Cardinality* principle stipulates that the last number word used in a particular count represents the numerosity, or cardinality, of the collection of items counted.[1] These principles exist before children have any experience with counting, and do two main things for children. First, they define the equivalence class of correct counting behavior, allowing children to recognize and group together all instances of counting as a specific kind of activity. Second, they serve as guidelines for counting, so that children can initiate, monitor, and correct their own counting.

'Principles-After' Theory

Briars and Siegler (1984), Fuson (1988), and Fuson and Hall (1983), among others, are advocates of the following account of children's counting: Young children first learn counting as a routine activity. Counting is modeled for them by parents, teachers, etc., and they begin to imitate it. At this point it is an activity without meaning, much like a game of patty-cake. Children do not distinguish between different components of the counting routine; all components are equally essential. They must learn different routines for different counting contexts; counting objects arranged in a circle, for example, entails a different procedure than counting objects in a line. Once they have learned many such routines, children eventually generalize over all these routines, abstracting out what all have in common -- namely, the counting principles. Only after this has happened do children have principled knowledge.

On this view, how do children come to understand that counting is related to numerosity? Children cannot learn *why* we count (to determine numerosity) just by learning *how* we count. *Subitization*, the ability to recognize some numerosities without having to count them, may be at the root of this accomplishment. Adults and children can subitize small numerosities, up to four or five for adults and two or three for 3-, 4-, and 5-year-olds (Chi & Klahr, 1975; Schaeffer, Eggleston & Scott, 1974; Silverman & Rose, 1980). It has been suggested (Klahr & Wallace, 1973, 1976) that by applying the initially meaningless counting routine to sets of items within the subitizing range, children come to associate words (e.g. "one", "two", "three") with those numerosities they can recognize (e.g. one, two, three), and thus come to relate the counting activity with the concept of numerosity.

At issue in the above debate is the very young child's mental representation of the counting routine. If it is represented in terms of abstract components, such as the counting principles, as the Principles-Before theory states, then it can potentially be generalized to a wide variety of dissimilar tasks. If, however, it is represented as a specific procedure, in terms of a series of precise, concrete steps as the Principles-After theory states, then it will not be widely generalizable because different tasks require different procedures. Examining the extent to which children can generalize their counting to new contexts can therefore shed light on the form of representation of the counting routine.

It has been shown that children as young as 3-and-a-half can catch and correct genuine errors that a puppet makes in counting, in which the puppet violates one-to-one correspondence, fails to use the standard stably-ordered count list, or gives a number other than the last tag used when asked, after a count, how many objects there are. More interestingly, they can distinguish these genuine errors from unusual but correct counts by the puppet which, while differing significantly from a 'normal' counting routine, remain true to the counting principles (e.g., starting a count in the middle of an array of objects, proceeding to the end of the array, then going back and counting the remaining objects; or counting every alternate object, then turning around and counting back the other way, getting the ones missed) (Gelman & Meck, 1983; Gelman et al., 1986). Thus children are applying knowledge of counting to these apparently novel counting situations. However, these results have not been consistently obtained (Briars & Siegler, 1984). Furthermore, although the "correct but unusual" ways to count were chosen to be novel to the children, it is quite possible that children may have seen things counted out of order at one time or another. In fact, the counting contexts in these experiments were quite similar to those children encounter when being shown how to count; they all consisted of counting objects arranged linearly, of roughly the same size and proximal distance, with merely the order of counting altered. Finally, the mean age of the younger children in these experiments was over 3-and-a-half. Some children start learning to count as early as their second birthday. It is therefore possible that these experiments reflect knowledge *learned* about counting rather than knowledge *underlying* the learning of counting.

As a more stringent test of children's counting abilities under novel circumstances, consider all the types of entities other than objects that are countable: sounds, actions, abstract entities such as thoughts or mental representations, properties of objects, etc. There have been only a few studies of children's counting of entities other than objects. To determine how much of the list of number words their subjects could recite, Schaeffer et al. (1974) asked children to count to taps of a drum, saying a number word in time with each tap, for up to 10 taps. If the child increased the pace of saying the numbers, the experimenter increased the pace of drum tapping. The mean number of taps that their younger two groups of children (mean ages 3:5 and 3:8) counted to was about 6.5. In comparison, these children counted sets of 1 to 4 objects correctly about 74% of the time, and sets of 5 to 7 objects correctly 43% of the time. They thus appear to count drum taps and objects about equally well, which suggests that they have an abstract, generalizable representation of counting. However, it is not clear what children considered the task to be -- were they actually counting the drum taps, or were they simply reciting a list in time to the beat of the drum?

In another set of experiments, children's counting of parts and properties of objects was examined (Shipley & Shepperson, 1988). When some of the objects in the array of, e.g., cars, to be counted were cut in half, children under about 5 years of age, when asked to count the cars, tended to count each individual item rather than each complete car. When children (mean ages 3:1 and 3:8) were asked to count attached parts of objects, such as the total number of ears on several toy bears, they frequently either counted the number of ears separately for each bear ("one, two, one, two, ..."), or counted the number of bears rather than ears. When children (mean age 3:11) were asked to count properties of objects (the number of different colors, sizes, or kinds of objects), they instead predominantly counted the individual objects, though performance improved with tutoring. Children thus showed a strong bias to count physically separate entities and/or discrete whole objects rather than parts or properties of objects. This could be because children's counting routine is usually applied to separate discrete objects, and children may represent this as an integral component to the counting routine and are therefore unable to generalize their counting to entities other than separate objects. Alternatively, it could be that young children have a general bias to interact with discrete physical objects that is not limited to nor derived from counting. Further experiments (Shipley & Shepperson, 1988) support the hypothesis that children have a general discrete object bias, so children's poor counting in these experiments is not necessarily due to lack of an abstract, generalizable representation of the counting routine.

It would be interesting to see whether 2- and 3-year-old children can generalize their counting to novel situations, by comparing children's counting of objects (a familiar counting context) with their counting of actions and sounds (novel counting contexts). The counting of, e.g., rings of a bell is different from counting objects arranged in a row in several important respects. Objects have material existence and can be seen, touched, and pointed to, while sounds are not visually but aurally accessible. Objects have continuous existence and are separated from each other in space, while sounds have a momentary existence and are separated from each other in time; one can therefore choose the order in which to count objects in a way that one cannot for sounds, which are already temporally ordered. Perhaps most importantly, one has perceptual access to the entire set of objects to be counted simultaneously, while one has perceptual access to only one element at a time of the set of sounds to be counted, and cannot anticipate the final element in the set. Thus the procedural requirements for counting sounds (and actions) are very different from those for counting objects. If children can count sounds, then, they must be representing the counting routine at a relatively abstract level.

However, even if it turns out that young children do have an abstract representation of the counting routine that honors principles of counting, they may not have a set of *number-specific* counting principles. Children may be capable of representing some abstract components of the counting routine, such as one-to-one correspondence, without having any knowledge of number. Children as young as 2 years old honor one-to-one correspondence in many situations -- they can point to each picture in a page exactly once when asked to (Potter & Levy, 1968), can give one cookie to each person in the room, can name each person in a photograph exactly once while pointing, e.g., "Mummy, Daddy, me!", can put one sock on each foot or one spoon in each dish, can learn turn-taking games, etc. In some sense it appears that children *know* that they must point to each picture on the page exactly once; that they must name everybody, and that once is enough; and that everyone must get exactly one cookie. As these kinds of tasks vary so widely and occur so frequently in children's activities, it is very likely that children have a general cognitive ability to recognize and represent a one-to-one correspondence between two sets of entities. Similarly, children exhibit sensitivity to a stable ordering of entities every time they recite the alphabet, learn a stable ordering of actions such as, say, a game of patty-cake, learn the list of the days of the week or the months of the year, or learn a nursery rhyme. Children are capable of easily and quickly recognizing and representing stable orderings of words or events at a very young age. Thus, studying whether young children represent the one-to-one correspondence and stable-order principles in counting will distinguish between the Principles-Before theory and the Principles-After theory only if it turns out that children do *not* represent them; if children do represent these principles as part of counting, it could be either because they have unlearned knowledge of counting, or because of their ability to recognize and represent these principles as components of many activities.

The cardinality principle, however, is qualitatively different from the other two How-to-count principles. It is relevant only to counting, not to other activities; this is true by definition of the cardinality principle. An understanding of the cardinality principle depends on an understanding of the significance of the counting activity, that counting determines *numerosity*. Thus the ultimate test of the two theories must concentrate on whether very young children do in fact understand cardinality. It is entirely possible that young children mentally represent one-to-one correspondence as part of the count:ng routine, and know the correct order of the counting word list, but do not at first connect this routine with any concept of numerosity. If children understand the cardinality principle, however, they must be granted number-specific knowledge of counting.

It has been considered evidence of possession of the cardinality principle (Gelman & Gallistel, 1978) if children (a) emphasize the last tag used in a count, (b) repeat the last tag used in a count, (c) state the correct numerosity of a set without counting after that set has been counted earlier, or (d) respond with the correct number word without counting when asked how many items there are. It has been found that most children as young as 2-and-a-half display one or more of these behaviors at least sometimes when counting sets of 2 or 3 items (Gelman & Gallistel, 1978). However, repeating or emphasizing the last tag, or stating the correct numerosity without counting after the set has been counted previously, could result simply from imitating adult counters, who tend to emphasize, repeat, and otherwise direct attention to the last tag when instructing children in counting. Emphasis of the last tag could also simply be signalling the end of the routine. Just giving the correct number word when asked how many items there are could indicate that a child associates a particular number word with the perception of that numerosity (i.e., that a child has subitized the set without counting), which does not require an understanding of the cardinality principle (e.g., Fuson & Hall, 1983). Also, children often produce wrong number words when asked how many items there are. Given that the sets were of only 2 or 3 items, and that children are more familiar with number words that refer to small numbers and are therefore more likely to produce them, if children were responding with random number words one would expect that occasionally they would respond with the right one. Thus none of these behaviors is a clear indication of possession of the cardinality principle.

Some studies, however, do offer more conclusive evidence of children's possession of the cardinality principle. In one experiment, children were shown cards with different numbers of items on them and asked how many things there were on each card. They used words for larger numbers to describe larger numerosities, even if the number of items on a card was too large (up to 19 items) or the exposure time of the card too short (as little as 1 second) for the child to accurately determine the numerosity (Gelman & Tucker, 1975). These children thus apparently understood that a number word's position in the number word list is related to the number of items that word refers to, which suggests an understanding of the cardinality principle. It has also been shown that children can correct a puppet who gives a response other than the last number tag used in a count when asked, after the count, how many items there are (Gelman & Meck, 1983). In another experiment, a puppet counted a set of items twice and was asked "how many" after each count. Both times, the puppet gave the last tag used for that count, but in the second count it had surreptitiously made an error, so that the last tag differed in the two counts. Most of the children said the puppet's second response was wrong, even though they had not caught the puppet's surreptitious error. When asked to justify their judgement, children often expressed the belief that the answer should still be the same as the puppet gave the first time (Gelman et al., 1986). However, the mean age of the youngest children in each of these three experiments was over 3-and-a-half, so all that can safely be concluded from these studies is that by age 3-and-a-half, children understand the cardinality principle.

As evidence for the claim that children do *not* start out with an understanding of the cardinality principle, children were asked to put 1 to 7 candies in a cup from a large pile of candies, and to tap a drum 1 to 7 times (Schaeffer et al., 1974). It was found that the youngest two age groups (mean ages 3:5 and 3:8) gave the correct number of candies only about 45% of the time, and the correct number of drum taps only about 25% of the time. Children were especially poor on the larger numbers, and did not in general count aloud while responding on any of the trials. This suggests that they were using subitization to obtain the correct number when giving the correct amount rather than using the cardinality principle. However, it is not clear exactly what children's strategies were -- they may have been estimating the number of candies they took or drum taps they made, and just not getting the exact number. Also, these same groups of children in another task *on average* only recited the number word list to "five" or "six". Thus many of the children were sometimes being asked for more candies or taps than they could reliably count to, so one would expect them to do poorly on these numbers. These results therefore do not clearly indicate whether or not children possess the cardinality principle.

It has been found that many children will recount a set of objects when asked "how many" following a count, rather than repeating the last tag used in the count, thus apparently not applying the cardinality principle (Fuson & Mierkiewicz, 1980). However, children may simply view the question as a request to count, whether or not they understand the cardinality principle. After all, one way of asking a child to count a set of things in the first place is to ask how many there are. To test this hypothesis, Schaeffer et al. (1974) asked children "how many" after covering up the set of objects the child had just counted, thus preventing the child from recounting the set. They found that most 3-year-olds did not respond with the last tag used in the count, suggesting that they lacked the cardinality principle. However, asking a "how many" question just after a child has counted a set may be pragmatically strange; after all, the child has *already* indicated how many there are, by counting. Children may take the question as an indication that their first result was wrong, and change their answer. Furthermore, hiding the objects and then asking "how many" may seem strange to the child; if the adult wants to know how many there are, why cover them up?

Experiment 1 studies the generalizability of children's counting routine by comparing children's counting of objects, actions, and sounds. Experiments 1, 2, and 3 study children's understanding of the cardinality principle. In Experiment 1, children's responses to "How Many?" questions following the counting of objects, actions, and sounds are examined. In Experiment 2, the same children are tested for the cardinality principle in a different way, to rule out alternative explanations for children's performance in Experiment 1. Experiment 3 examines how children learn the meanings of the number words, and the relationship between their understanding of number words and their understanding of the cardinality principle, using a variation of the task given in Experiment 2.

Experiment 1: The "Novel Entities" Study

In this experiment, children's performance in the counting of objects (a familiar counting context) is compared to their performance in the counting of actions and sounds (novel counting contexts). Having children count objects, actions and sounds also allows an interesting test of their understanding of cardinality; children cannot construe a "how many" question about a set of sounds or actions just counted as a request to recount, because the set is no longer available. To reduce the pragmatic effect of asking "how many" right after the child has just counted, children were introduced to a puppet who "had forgotten how to count", and were encouraged to help show the puppet how. The assumption is that the question is more natural when the child is in the role of teacher to an ignorant puppet.

Method

Subjects

Subjects were 8 2-and-a-half-year-olds (mean 2:7; range 2:4 - 2:8), 8 3-year-olds (mean 3:0; range 2:10 - 3:2), and 8 3-and-a-half-year-olds (mean 3:5; range 3:4 - 3:7), labelled Age I, Age II, and Age III, respectively. There were roughly equal numbers of girls and boys in each group. They were from day-care centers in the greater Boston area with a predominantly middle-class population.

Procedure

There were four conditions; Object, Cave, Jump, and Sound (described below), each with four trials, one each of set sizes 2, 3, 5, and 6. The set sizes 2 and 3 were chosen to be within young children's subitizing range, those of 5 and 6 to be outside. The two smaller set size trials were always given first, in counterbalanced order, followed by the trials with set sizes 5 and 6, again counterbalanced. The order of conditions was counterbalanced between children within each age group. Roughly two-thirds of the trials in each condition were randomly designated as "How-many" trials, in which children were asked "how many" after counting. (It was felt that to designate all the trials as such might make the children worry too much about having to justify their responses.)

- *Object* condition: Children were asked to count objects linearly arranged. Toy dinosaurs were used, about 4 cm long, glued to a board with about 3 cm space between each one. A different kind of dinosaur was used for each trial; within each trial, only one kind of dinosaur was used.

- *Cave* condition: Children were asked to count toy dinosaurs as they were moved from one cardboard box into another box which they could not see inside of, a "dinosaur cave" with a small hole in the lid. For each trial, the experimenter moved one dinosaur at a time from the first box into the cave, at a rate of about 2 seconds per dinosaur. This condition is procedurally similar to the counting of sounds. It thus controls for the possibility that children might perform poorly on the Sound condition not because they haven't the correct procedures at their disposal, but because they do not consider sounds as countable entities. Here, children are still counting objects which they can see and which have a permanent existence.

- *Jump* condition: Children were asked to count the jumps of a puppet (Big Bird from the children's TV show "Sesame Street"). The experimenter made the puppet jump so that each jump took one-half to 1 second, with roughly an additional second between jumps. This condition is more novel than the Cave condition; although children can see the jumps, each jump exists only for an instant in time.

- *Sound* condition: Children were asked to count sounds played on a tape recorder. Four sounds, one per trial, were used: an "elephant" roaring (actually a person's voice); a single-chime doorbell ringing; a splash in a bathtub; and the beep of a computer. Sounds occurred about 2 seconds apart. This condition is the most novel, as children cannot even see the items they are counting.

At the beginning of the experiment, the child was introduced to Big Bird and told: "Big Bird has a problem. He's forgotten how to count, and he needs someone to show him how. Would you like to help Big Bird, and show him how to count? Can you help him count his toys?" Then the child was presented with the first trial, e.g.: "Look what Big Bird has, some dinosaurs! Can you show Big Bird how to count how many there are?" Children were frequently reminded that Big Bird did not know how to count and needed help. A trial was started over again if a child was obviously distracted in the middle of a trial, i.e., if the child started telling a story, ran off, or otherwise interrupted the trial. On the "how many" trials, children were asked after counting, "So how many dinosaurs are there/how many times did it ring?" etc. Sometimes the puppet asked the question; sometimes the experimenter did, often adding "can you tell Big Bird how many?". This was to make it seem that the object of the question was to help inform the ignorant puppet. The experiment was usually conducted over two sessions for each child (sometimes one session); sessions were typically between 1 and 3 days apart. Big Bird's problem was explained at the beginning of each session.

Results and Discussion

Counting of novel entities

In the early stages of counting, many children consistently use an idiosyncratic list of the counting words, e.g., "One, six, seven, eleven" (e.g., Fuson & Mierkiewicz, 1980; Gelman & Gallistel, 1978). The fact that children have such lists is consistent with the claim that children understand the stable-order principle. Even if children start out with the counting principles, they must still *learn* the number word sequence. For this reason, children who used a different list than the standard number word list were not automatically considered to be counting incorrectly. A set was considered correctly counted if:

- the child started the count with the first number word in his or her own stably-ordered list (for almost all children, the first number word was "one");
- for sets of 3, 5, and 6 items, the trial contained no more than one of the following mistakes: double-counting or skipping an item (one-to-one correspondence mistakes), or skipping or repeating a tag in the child's own stably-ordered list (stable-order mistakes);
- for sets of 2 items, the trial contained *no* one-to-one correspondence mistakes, and no more than one stable-order mistake.

A 3(Age group) by 4(Condition) by 2(Set Size -- Large or Small) ANOVA was performed. There was a significant Age effect, $\underline{F}(2, 21) = 11.017$, $\underline{p} < .001$. A contrast analysis on the three age groups showed a strong linear trend for older age groups to count more trials correctly (mean number of correct trials out of two was 0.67 for Age I, 1.22 for Age II, and 1.64 for Age III), $\underline{t}(21) = 4.681$, $\underline{p} < .0001$, one-tailed. Set Size was also significant (smaller set sizes were easier to count -- the mean for small Set Size was 1.33 and for large Set Size was 1.02), $\underline{F}(1, 21) = 13.636$, $\underline{p} < .001$. There was a Condition effect, $\underline{F}(3, 63) = 4.684$, $\underline{p} < .005$. It might be asked whether there was a trend for children to perform more poorly the more novel the condition (from Objects to the Cave condition to Jumps to Sounds). This was so; the mean number of successful trials (out of 2) for each condition (over all children and both set sizes) was 1.42, 1.29, 1.10, and 0.90 respectively. A Newman-Keuls post-hoc means comparison showed that performance in the Object condition and performance in the Cave condition were both significantly better than performance in the Sound condition, $\underline{p} < .05$. There was a marginally significant Age X Size interaction: $\underline{F}(2, 21) = 3.045$, $\underline{p} = .069$; for older children there was less difference in performance on large versus small set sizes. Figure 1 shows the percentage of trials which were correctly counted in each condition for each age group.

INSERT FIGURE 1 ABOUT HERE

The Condition effect indicates that children are worse at counting in quite novel circumstances. However, even if children do have an abstract counting routine, one would not expect them to perform *equally* in both familiar and novel contexts. Practice in applying the counting routine in different contexts will improve performance. For instance, children who have counted objects but not sounds will have had practice coordinating their words with their pointing, but will not have had practice coordinating their words with entities that occur one after another in time. Thus, if children have an abstract, generalizable counting routine, they ought to apply it in novel contexts, but not necessarily with equal facility.

Almost all of the younger children could count the non-Objects to some extent. Only the 3 youngest children (mean age 2:5) failed to count at all in any of the non-Object trials. They seemed not to understand what was being asked of them, appearing bewildered, even though they counted and appeared comfortable during the Object condition. All of the other children counted for the non-Object trials, and all except one child counted at least 25% of the non-Object trials correctly. The percentage of correctly counted non-Object trials increases steadily with age (the 5 Age I children who counted on the non-Object trials had a mean of 38% correct non-Object trials per child; Age II children's mean was 60% per child; Age III children's mean was 79% per child). This suggests that very early on, children begin to develop an abstract, generalizable representation of the counting routine that they can apply to new counting situations. However, the fact that the 3 youngest children did not count at all for the non-Object trials suggests that children may not start out with a generalizable counting routine. But what about those children who counted at least some of the novel entities correctly, thus showing that they have developed at least the beginnings of an abstract mental representation of counting? This representation may or may not encompass an understanding of the significance of counting -- that counting determines the numerosity of a set. The "How-many" test for the cardinality principle addresses this question.

"How-many" cardinality task

If children understand the cardinality principle, they should repeat the last number word used in a count when asked, after counting, how many items there are. However, they may be less likely to give the last tag used if they have little confidence that they obtained the correct value when counting, because in an incorrect count the last number word does *not* indicate the cardinality of the set. Therefore, except where stated otherwise, only "how many" responses following *correct* counts (as determined by the criteria above) are included in the analyses.

Children in the younger two age groups, when asked "how many" after counting a set of physical objects, preferred recounting the set to saying the last tag used in the count. However, there was no consistent trend to increase the likelihood of cardinality responses in the non-Object conditions where recounting was not possible. Table 1 shows the proportions of "how many" responses where children repeated the last tag used in counting, and the proportion of responses in the Object condition where children recounted (indicated in square brackets).

INSERT TABLE 1 ABOUT HERE

The percentage of responses to "how many" questions which were cardinality responses was determined for each child (one of the Age I children was not asked "how many" after any correct counts because almost all of her counts were incorrect, so is not included in this analysis). The mean of children's percentages of cardinality responses was 26% for Age I, 25% for Age II, and 53% for Age III. Children in Age III did not all perform equally well; 4 gave cardinality responses more than 50% of the time (mean 78%) while the other 4 did not (mean 28%). There was a significant difference between the Age I and Age II children's mean percentages and the Age III children's ($\underline{t}(21) = 2.194$, $\underline{p} < .05$, one-tailed). These results suggest that children learn the cardinality principle at about 3-and-a-half years of age, and that the younger children's ability to apply their counting routine to the non-Object conditions is not due to unlearned knowledge of counting.

It could be objected that perhaps children learn by observing adults that the correct response to a "how many" question is just to repeat the last tag used, without understanding that this tag refers to the numerosity of the set; older children may have just learned this better than younger children. This account would explain the results without having to credit any of the children with an understanding of cardinality. However, there is a wealth of independent evidence that 3-and-a-half-year-olds do understand the cardinality principle (e.g., Gelman & Meck, 1983; Gelman et al, 1986; Gelman & Tucker, 1975).

Alternatively, it could be that even the younger children understand that counting is about determining the cardinality of a set, but simply do not understand that the question "how many" is asking about cardinality. There is a large body of research indicating that young children do not understand until quite late terms such as "less", "more", "the same", and other quantifiers (e.g., Carey, 1982; Clark & Clark, 1977; Donaldson & Balfour, 1968; Palermo, 1973), and it is plausible that they also have trouble with "many" and/or "how many". Finally, although the experiment was designed to reduce the possible pragmatic effects of asking how many items there were immediately after the children have just counted them, this might not have been completely successful. Experiment 2 was designed to address all of these concerns.

Experiment 2: The "Choose" Study

Method

Subjects

Subjects were the same 24 children who participated in Experiment 1. They were given the Choose task at the end of their final session of Experiment 1. Two children, one in Age I and one in Age II, did not want to finish this task and were dropped from Experiment 2, leaving 22 subjects.

Procedure

After the final condition of Experiment 1, 15 toy dinosaurs were placed in front of the child in a pile, and the puppet and child began to play with them. The child was then asked to give the puppet a certain number of dinosaurs. Children were first asked to give 1, then 2 and 3 in counterbalanced order, and then 5 and 6, also counterbalanced. The request was of the form: "Could you give Big Bird *two* dinosaurs to play with, just give him *two* and put them here (experimenter pats a place in front of Big Bird, within easy reach of the child), can you get *two* dinosaurs for Big Bird?" The request was repeated until the child responded (usually children responded right away).

After responding, children were given "follow-up" questions. They were asked to "check and make sure" that they'd given the correct number, and were reminded how many had been asked for. Any child who did not spontaneously count the objects was prompted to count them, e.g. "Can you count and make sure there are two?" Children who counted and obtained a different number than what they had been asked for were then prompted, e.g. "But Big Bird wanted two. How can we make it so there's two? How can we fix it so that Big Bird has two?" Children's further responses were followed up until the child seemed to be getting bored or uncomfortable.

Results and Discussion

Children were divided into two groups on the basis of the initial strategies they used. "Counters" were those who, on at least four of the five trials, responded in one of the following three ways:

- They counted the items as they gave them, using their own stably-ordered list, and stopped at the number word asked for (they were allowed to make a single one-to-one correspondence mistake when counting);
- They silently gave the correct number of items *one by one*. What was considered the "correct" number of items was determined by the child's own stably-ordered list. For example, if a child regularly counted "one, three, four, five, six", then, when asked for "three dinosaurs", a correct response would be to give 2 one at a time; when asked for "six dinosaurs", a correct response would be to give 5 one at a time, as the word "six" is the fifth word in the child's list;
- They spontaneously counted the items they had grabbed and, if necessary, adjusted them (within plus or minus one) to the number asked for, according to their own stably-ordered list.

All other children were "Grabbers", and their strategy was generally to grab and give a handful all at once, or, occasionally, to give some other number of items silently one at a time.

The results are divided into three sections. First, children's initial performance on the Choose task is analyzed. Second, the Grabbers' performance in Experiment 1 is compared with that of the Counters. Finally, children's responses to the follow-up questions are examined.

Initial performance on Choose task

There were 4 Counters, all in Age III. All used the standard count word list. Every Counter gave a correct response on all five trials. In three cases Counters, when checking how many items they had given, counted a number different than that asked for; in all of these cases they either added or took away items as appropriate, and then counted again, repeating if necessary until they counted the number asked for. Thus the Counters showed a clear understanding of the cardinality principle.

All of the children in the younger two age groups, and half those in the oldest age group, were Grabbers, and tended either to simply grab a handful of items and give them all, or to give an apparently random number of items one at a time. However, it is possible that despite their poor strategy, Grabbers could be grabbing the number asked for. Table 2 shows the number of Grabbers who gave each number when asked for 1, 2, 3, 5, and 6 items. There were only 3 Grabbers whose stably-ordered lists differed from the standard list. Two of them alternated between the correct word list, and that list with one word omitted ("two" was omitted half the time in one case, "five" in the other). On no trial did either child give a number of items that would be correct according to their list with the omission -- their responses were either the correct number according to the standard list, or incorrect by both lists. The third child counted correctly up to five, but had no consistent list beyond that. He gave 5 items when asked for 6 (and 8 items when asked for 5). Thus, these children's data do not distort the results in Table 2.

INSERT TABLE 2 ABOUT HERE

All Grabbers gave the correct number when asked for 1 item, while tending not to give 1 when asked for more than 1 item. Thus *one* seems to be a number readily identified by children as young as 2:4. Grabbers also gave 2 more often when asked for 2 than when asked for 3, 5, or 6. However, this was not significant, by a Wilcoxon Signed Ranks test comparing the percentage of time individual Grabbers gave 2 items when asked for 2, with the percentage of time they gave 2 when asked for 3, 5, or 6 items.

Grabbers did not give 3, 5, or 6 items more often when asked for one of those numbers than when asked for other numbers. Furthermore, Grabbers do not even appear to be approximating the number of items they grab, when asked for 3, 5, or 6 items. The correlation between the number asked for (3, 5, or 6) and the number given is $r = .12$ (NS) for Age I, $r = .03$ (NS) for Age II, and $r = .42$ (NS) for Age III Grabbers. Table 3 shows the mean number of items initially given by Grabbers and Counters of each age group.

INSERT TABLE 3 ABOUT HERE

The problem is not that the children could not count this high. For each Grabber, the largest correctly counted set of objects in the Object condition was determined. The mean largest correct trial for Age I was 4.5 objects, for Age II was 5.0 objects, and for Age III was 5.8 objects. Thus even the youngest group of children can count to four or five accurately, yet they fail even at giving 3 items. Thus, except for 1 and possibly 2 items, Grabbers do not give the number asked for, nor do they appear even to be approximating the number asked for. Even in those cases where Grabbers did give the correct number of items when asked for 2 or 3, they never counted the items aloud as they gave them. This suggests that they were not applying the cardinality principle to get the right number, but were either subitizing the correct number, or getting the right number by chance. This indicates that, before around 3-and-a-half years of age, children do not understand the cardinality principle.

Grabbers' vs Counters' performance in Experiment 1

If it is true that the Counters possess the cardinality principle while Grabbers do not, this should be reflected in their performance on the How-many task in Experiment 1 -- Counters should give more cardinality responses than Grabbers. This is in fact the case. There were 17 Grabbers who had "how many" responses following correct counts (the 18th was not asked "how many" following a correct count because almost all of her counts were incorrect). The mean percentage of individual Grabbers' cardinality responses was only 25%, while the mean percentage of Counters' cardinality responses was 78% ($t(19) = 4.147$, $p < .0005$, one-tailed). Figure 2 shows the mean of individuals' percentages of cardinality responses given by Counters versus Grabbers on the How-many task in Experiment 1. There is no increase with age in the proportion of cardinality responses to "how many" questions. Rather, there is a very sudden shift from a cardinality response rate of one-quarter of the time to a cardinality response rate of over three-quarters of the time, that coincides with the shift from Grabber to Counter.

INSERT FIGURE 2 ABOUT HERE

Yet another prediction can be made about Grabbers' versus Counters' performance on the How-many task in Experiment 1. If it is indeed the case that Counters understand the cardinality principle while Grabbers do not, one would predict that Counters would be less likely to give the last tag used in the count in cases where they were uncertain of the accuracy of their counting. Thus they should give cardinality responses less often after incorrect counts than after correct counts. Grabbers, on the other hand, should not understand the relationship between accuracy of counting, and likelihood that the last tag in the count represents the cardinality of the set, so there should be no difference in their rate of cardinality responses following correct versus incorrect counts.

This prediction too is borne out by the data. Fourteen of the Grabbers and 3 of the 4 Counters had "how many" responses following both correct and incorrect counts. Figure 3 shows the means of these individuals' percentages of cardinality responses following correct versus incorrect counts, for Counters and Grabbers.

INSERT FIGURE 3 ABOUT HERE

Grabbers were actually slightly *less* likely to give cardinality responses after correct counts than after incorrect counts, though this difference is not significant. In comparison, Counters were *three times* as likely to give cardinality responses following correct counts (84% of the time) as they were following incorrect counts (28% of the time), $t(4) = 3.503$, $p < .05$, one-tailed. This indicates that Counters understand the relationship between accuracy of counting, and likelihood that the last tag in the count indicates the cardinality of the set. Grabbers appear not to appreciate this relationship, further supporting the conclusion that Grabbers do not understand the cardinality principle while Counters do. These findings provide a firm basis for the claim that children do not understand the cardinality principle before about 3-and-a-half years of age.

Age III Grabbers and Counters were equally successful at counting the novel entities in

Experiment 1. Counters gave correct counts on 83% of the non-Object trials, while Age III Grabbers gave correct counts on 75% of the non-Object trials; this difference does not approach significance. Thus, children appear to acquire considerable skill at counting before understanding that counting determines the numerosity of a set.

Responses to follow-up questions

As previously stated, Counters, when prompted to check how many items they had given, always counted them and, if necessary, added or took away items as appropriate. Grabbers, on the other hand, revealed several strategies that reinforce the conclusion that they do not understand the cardinality principle. In most cases, the experimenter succeeded in getting Grabbers to count what they'd given. There were several ways they attempted to reconcile the discrepancy between what they had been asked for, and what they counted:

1) Naming or Tagging the Last Item With the Number Asked For: Seventeen responses by 9 Grabbers (4 Age I children, 3 Age II children, 2 Age III children) were to count so that a number word said (usually the last one) was the number asked for. For example, a boy who had given 2 items when asked for 6 counted them "one, six!". A girl who gave 2 items when asked for 5 counted them, "five, five!". Three responses by 2 children clearly appeared to be *naming* particular items with the number tags, e.g. the following interaction between a child and the experimenter after the child was asked to give 5 objects, and had grabbed and given 3:

Experimenter: So how many are there?

Adam: (Counting the 3 objects which are in a triangular arrangement) One, two, five!

E: So there's five here? (pointing towards the 3 items)

A: No, *that's* five (pointing to the item he'd tagged "5"). One, two, five (counting them in the original order).

E: So there are five altogether?

A: No, one, two, five (counting them again in the original order).

E: So does Big Bird have five?

A: Yeah, this is five (pointing to the item always tagged "5" in his counting). One, two, five (counting again, in the same order as before).

E: What if you counted this way, one, two, five? (experimenter counts the objects in a different order than Adam has been doing)

A: No, *this* is five (pointing to the one he has consistently tagged "5").

E: That one's five? (pointing to the one Adam called "five").

A: Yes.

E: Why is this one five?

A: Because, one, two, five (counting them once again, in the same order).

2) Denial: Eight responses by 8 Grabbers (2 Age I children, 4 Age II children, 2 Age III children) were to count what they'd given, and deny that there was any discrepancy. For example, one girl gave 3 when asked for 6. When asked to "count and make sure" there were 6, she counted them correctly, saying, "one, two, three. That's six!" This is a direct violation of the cardinality principle.

3) Changing the Number of Items: Eighteen responses by 10 Grabbers (2 Age I children, 7 Age II children, 1 Age III child) were to change the number of the items, in all but one case by adding more to what they'd given. In only 10 of these cases had the children first counted the number of items they'd given or indicated that they believed they'd given an incorrect number (e.g. by saying "that's not five!"). In six of these cases the children changed the number in the right direction; in the other four, the children changed the number in the wrong direction (e.g. giving and counting 4 items after being asked for 3, and then adding another item).

4) Silence/No Justification: Twenty-nine responses by 14 Grabbers (5 Age I children, 6 Age II children, 3 Age III children) were to remain silent, or say it was the number asked for without justification, when prompted to check what they'd given.

The first strategy is particularly interesting. It strongly suggests that some children have a sort of "cardinality *rule*". They understand that the last number word is the answer to "how many" there are, but do not understand "how many" in the same way we do. They do not yet have the cardinality *principle*, i.e., they do not yet understand that the last tag indicates the numerosity of a set, or even that the last tag refers to some property of the entire set as a whole rather than to a particular member of the set. They simply have the heuristic that "the last number word in a count is 'how many' there are". Thus, if counting items the usual way does not end with the correct number word, they 'count' the items in a way that *does* end with the correct number word.

One would expect that if these Grabbers did have a "cardinality rule" which states that "the last number word is how many items there are", they should apply it in the How-many task in Experiment 1. In particular, they should respond, more often than the other Grabbers, with the last number word in the count when asked "how many". This appears to be the case. Eight of the 9 Grabbers who employed Strategy 1 had "how many" responses following correct counts (the 9th was not asked "how many" following any correct counts, because almost all of her counts were incorrect). They gave cardinality responses on the How-many task on average 34% of the time, while the other 9 Grabbers gave cardinality responses on average only 18% of the time (this is a marginally significant difference: $t(15) = 1.414$, $p = .089$, one-tailed). Thus, about half the Grabbers do appear to have such a "cardinality rule".

However, they did not give cardinality responses to the extent that the 4 Counters did. The difference between their mean of 34% and the Counters' mean of 78% is significant, $t(10) = 2.995$, $p < .01$, one-tailed. This is consistent with the interpretation that these Grabbers are giving cardinality responses for a different reason than are the Counters, who appear to possess, rather than a meaningless "rule", a *principled* reason for stating the last tag when asked "how many". Fuson (1988) has also proposed that some children have such a "cardinality rule", on the basis of several empirical results. For example, when asked to count very large sets, some children gave a very small number as a response to a "how many" question, if that number word was the last said in the count; e.g., a child counted a set of 26 items, "one, two, three, six, seven, eight, nine, one, two." When asked "how many", she responded, "two". In another study, after counting *n* soldiers, 2- and 3-year-olds were asked to choose between the last soldier and all the soldiers in response to either the question "Is this the soldier where you said *n*?" or the question "Are these the *n* soldiers?". Children tended to choose the last soldier more often than all the soldiers in answer to both questions, often saying things like, e.g., "This one's the five soldiers", while pointing to the last one.

These results indicate that before about 3-and-a-half years of age, children do not understand the cardinality principle. Grabbers' responses to the Choose task show that they have no understanding that the last number tag refers to the *numerosity* of the set, or even to some property of the set as a whole.

The fact that all the children gave 1 item when asked for 1, and most gave 2 when asked for 2, suggests that children learn the meaning of the word "one" very early, followed by the word "two". It may be that in general children learn the meanings of smaller number words before larger ones, even for number words well within their counting range. At some point, children must realize that every counting word refers to a distinct numerosity -- this is the adult competence. But children may learn this for particular number words before making a general induction. Experiment 3 tests this hypothesis, and begins to explore when children do make the induction that every counting word refers to a specific numerosity.

Experiment 3: "Choose" Follow-up

In this study, children are asked several times for each of a number of animals (1, 2, 3, 5, and 6), so the consistency of individual children's responses for different numerosities is obtained. If children do learn the meanings of smaller number words before those of larger number words within their counting range, then two things should occur. First, individual children should succeed when asked for a number of items up to a certain numerosity, and then fail for all higher numerosities (e.g., there should be no children who consistently succeed at 1 and 3 but not at 2). Second, different children should have different numerosities at which they start to fail. There should be some children who succeed consistently only when asked for 1, and others who succeed consistently when asked for 1 and 2. Whether there are children who succeed when asked for 1, 2, and 3 items but not more, or even 1, 2, 3, and 5 items but not 6, depends at what point children make the general induction that *all* the counting words (within their counting range) refer to particular numerosities. Once they make this generalization, children should succeed at all the numerosities within their counting range.

This study also examines the relationship between children's understanding of the cardinality principle and their understanding of the meanings of number words, and tests children's ability to perform some of the number-irrelevant task demands of the Choose task.

Method

Subjects

Subjects were 18 2- and 3-year-olds (mean age 3;3; range 2;4 - 4;0) from schools and day care centers in the Greater Boston area. About half were girls. Five more children were tested but not used as subjects; one had a hearing impairment and had trouble understanding the experimenter, three stopped before the end of the experiment, and the data from one child were lost due to equipment failure.

Procedure

There were three tasks given to the subjects:

- *"Sticker game"*: This was a variation of the Choose task. Children were asked to give 1, 2, 3, 5, and 6 animals to the puppet, and were given a sticker to put on a piece of paper after each trial to keep them motivated (each sticker was different). The numbers 1, 2, and 3 were chosen because they are within children's subitizing range; 5 and 6, because they are outside the subitizing range. The number 4 was not asked for because to do so would have risked boring children before obtaining a sufficient number of trials of each numerosity. The goal was to determine the maximum numerosity each child could succeed at. Thus the exact procedure differed for different children. All children were first asked for 1 item and then for 2 items. Depending on their success, they were then asked for 3 items, or asked again for 1 or 2 items. What children were asked for on a trial depended partially on their success in the previous trial. Children who failed on a trial were then asked for a numerosity at which they had previously succeeded. This served two purposes; to determine the consistency of a child's performance on a particular numerosity, and to avoid discouraging children. All children were asked at least twice for 2 and 3 items. The experimenter concentrated on the highest number a child succeeded at reliably and on the next highest number, so children got more trials for these than for other numbers. At some point in the task, however, all children were asked for the larger numerosities at least once (usually twice). The exact number of trials depended largely on a child's willingness to keep playing. The experimenter followed up children's responses by asking questions such as, "Is that three?", "Can you count and make sure?" etc. The number of follow-up questions for a particular trial depended on the child's willingness to continue that trial.
- *"Give-some-pigs" task*: Children were asked to give the puppet Big Bird some pigs (or other kind of animal) from a pile containing four kinds of animals, from 4 to 10

of each kind. The kinds of animals were easily recognized and named even by 2-year-olds (pigs, dogs, dinosaurs, horses). Many of the same task demands are present in the Choose task: children must (a) construct a subset of the entire pile of animals, and (b) give that subset to the experimenter. No particular animal need be included in the subset for either task; for example, any three animals are okay for the Choose task, and any of the pigs are okay for the Give-some-pigs task. Children who succeed at the Give-some-pigs task but fail at the Choose task cannot be failing from an inability to construct a subset and give it to someone. They must be failing due to an inability to *construct a set of a certain numerosity.*

- *"Count/How-many" task*: Children were asked to count 3, 2, 5, and 6 linearly arranged items (the same stimuli used in the Object condition in Experiment 1), in that order. After counting each set, children were asked how many items there were.

All children played the Sticker game last. About half the children received the Give-some-pigs task first, half the Count/How-many task first. When given the Count/How-many task, children were told that Big Bird had "forgotten how to count" and were asked to help him count his toys. At the beginning of the Sticker game, children were first given a sticker to put on a piece of paper, and then told that: "The way this game goes is that Big Bird is going to ask you for a certain number of animals, and when you give him the right number, you get another sticker. So it will go like this: Big Bird is going to say, 'Can you give me *one* animal?'" This was the child's first trial. The experimenter repeated the question in different ways until the child gave one or more animals. Though told they would be given a sticker after giving "the right number", children were actually given a sticker after every trial.

Results and Discussion

Only 1 child (age 2:4) failed at the Give-some-pigs task. He also failed to count correctly on any of the four Count/How-many trials, and failed on all numerosities in the Sticker game. This child is not included in the following analyses. All of the other children carefully picked out some or all members of the kind of animal asked for in the Give-some-pigs task; they never just grabbed a handful of animals.

For each of the 17 remaining children, the numerosities at which they succeeded consistently in the Sticker game were determined. The criterion for "consistent success" on a certain numerosity was as follows:

- On at least two-thirds of a child's trials for that numerosity, the child's final response was either the correct number according to his or her own stably-ordered count list, or the correct number plus or minus one if the child had counted aloud from the pile to the number word asked for, but had erred in the counting by either double-counting or skipping one item. Two-thirds was chosen as the criterion for success because many children were given three trials of a particular numerosity, so it is a natural cut-off point. Children's final responses were used rather than their initial responses because children occasionally corrected a wrong response, but rarely changed a response that was initially correct.
- The child responded with that number when asked for other numerosities no more than half as often, percentage-wise, as he or she did when asked for that number itself. For example, a child who gave 2 items 80% of the time when asked for 2, was scored consistently correct on 2 only if he or she gave 2 items no more than 40% of the time when asked for 1, 3, 5, and 6 items. This was to prevent children who had a preference for giving, e.g., 2 items no matter what they were asked for, from being considered to know the meaning of the word "two".

Children fell into five groups according to the numerosities at which they succeeded. Table 4 shows the number and ages of children in each group, and how high the children in each group could count (determined by averaging children's highest correct counts in the Count/How-many task). The criterion for a correct count in the Count/How-many task was the same as that used in Experiment 1: Children had to start the count with the first element in their own stably-ordered list, and were allowed a single one-to-one correspondence or stable-order mistake on sets of 3, 5, and 6 items, and a single stable-order mistake on sets of 2 items.

INSERT TABLE 4 ABOUT HERE

It can be seen that each child succeeded up to a certain numerosity, and then failed for all higher ones.[2] Children's failures are not a result of not being able to count that high. All of the 10 children that failed at some numerosities could correctly count set sizes larger than they could correctly give when asked. (Only one child had a count word list differing from the standard count list; she omitted the word "four" from her list. She did not succeed for any of the numerosities, and when asked for "five" and "six" items, did not give 4 and 5 respectively, which would be correct by her list, but gave 1 item.) These results support the hypothesis that children learn the meanings of smaller number words before those of larger ones, even when they use those larger words capably in counting.

The pattern of children's ages also supports this hypothesis. Children who succeeded at larger numerosities are in general older than those who succeeded only at smaller numerosities. The correlation between children's ages in months and the highest numerosity they succeeded at is $\underline{r} = .64$ ($\underline{t}(15) = 3.218$, $\underline{p} < .005$, one-tailed). Thus children appear to learn the meanings of the number words one at a time, for progressively larger numbers. However, this pattern of learning does not continue indefinitely. All 7 children who succeeded at giving 5 items also succeeded at giving 6 items, while 4 children succeeded at giving 3 items but not more. This suggests that by the time children learn the meaning of the word "five", but after they have learned the meaning of "three", they make the general induction that *all* the number words within their counting range refer to specific numerosities.

In order to further examine children's understanding of the cardinality principle, and its relationship to their understanding of the meanings of number words, children were divided into Counters and Grabbers on the basis of their strategies in the Sticker game when asked for 3 or more items. (For 2 items, many children just gave 1 in each hand, and it was impossible to determine whether they had counted them silently or not; for 1 item, almost all children just gave 1.) "Counting strategies" were to count the items while giving them and stop at the number word asked for, to silently give the correct number of items (determined by children's own stably-ordered lists) one by one, or to spontaneously count what was given and correct it if necessary to within plus or minus one of the correct number.

There was a bimodal distribution in children according to how often they performed Counting strategies. Ten of the children (mean age 3:1; range 2:7 - 3:8) applied a Counting strategy on 0% to 38% of their trials (the mean was 13%). These children were classified as Grabbers. The other 7 children (mean age 3:7; range 2:11 - 4:0) applied Counting strategies on 86% to 100% of their trials (the mean was 96%). These children were the Counters. A t-test on Grabbers' versus Counters' individual percentages of Counting strategies was significant, $t(15) = 13.019$, $p < .0001$, two-tailed. The difference in the ages (in months) of children in the two groups was also significant ($t(15) = 2.366$, $p < .05$, one-tailed).

This sharp division of children according to their strategies, with a dramatic increase in the use of Counting strategies, indicates a sudden shift, occurring at about 3-and-a-half years of age, in children's approach to the task. The 2 Grabbers who consistently succeeded at giving 2 items did not tend to count out items aloud from the pile when giving 2, and the 4 Grabbers who consistently succeeded at giving 3 items did not tend to count out items aloud from the pile when giving 2 or 3. These children did so, on average, only 6% of the time. This suggests that their strategy was to subitize in order to give the right number. In contrast, the Counters counted items aloud from the pile when asked for 2 or 3 items, on average, 45% of the time ($t(11) = 2.666$, $p < .05$, one-tailed). Thus, children appear to be abandoning one successful strategy for giving 2 or 3 items in favor of another. This suggests that a major conceptual change occurs in children's understanding of counting at this age.[3]

There was again a relationship between being a Counter, and giving cardinality responses when asked "how many" following counting. Counters gave cardinality responses an average of 61% of the time following correct counts on the 4 Count/How-many trials, while Grabbers gave cardinality responses an average of 22% of the time ($t(15) = 2.345$, $p < .05$, one-tailed). Since there were so few incorrect counts, a comparison of Grabbers' and Counters' responses following correct versus incorrect counts could not be made.

The 7 children who were consistently correct on all the numerosities in the Sticker game, and thus appear to have made a general induction that all the counting words within their counting range refer to specific numerosities, are also all Counters. They are thus the only children who were clearly applying the cardinality principle to obtain the number of items asked for. This suggests that children's acquisition of the cardinality principle coincides with their making the general induction that all the counting words refer to particular numerosities.

It would not have to be this way. Children could learn the cardinality principle first for those numerosities whose number words they know the meanings of, before making a general induction. If that were so, it would be expected that some of the children who only succeeded on numerosities of 3 or less would have counted items out aloud from the pile to obtain the correct number, when asked for 2 or 3 items. The fact that this did not occur suggests that children's understanding of the cardinality principle, even for numbers within the subitizing range, is intimately connected with their understanding that *all* the counting words refer to numerosities.

General Discussion

In this section I will consider three main issues. First, I discuss children's understanding that counting establishes numerosity of a set. Second, I examine the possibility that children have a concept of number independent of the counting routine and the number words, and relate it to the results of this paper. This is followed by a discussion of the abstractness of children's mental representation of counting.

Results of the How-many and Choose tasks and the Sticker game strongly suggest that children do not understand the cardinality principle, or the relationship between counting, cardinality, and numerosity, until about 3-and-a-half years of age. Taken together, these results indicate that the Principles-Before theory is incorrect -- children do not start out with a set of principles which guide their counting behavior and constitute an understanding of the significance of counting.

The conclusion that children do not understand cardinality until about 3-and-a-half may at first appear to conflict with conclusions from other studies (e.g., Gelman & Gallistel, 1978; Gelman & Meck, 1983; Gelman et al., 1986; Gelman & Tucker, 1975). However, the mean age of the youngest children tested in these experiments was over 3-and-a-half, so their results are consistent with the claim that children learn the cardinality principle at around 3-and-a-half. Results from the Schaeffer et al. (1974) cardinality study, in which children were asked to put 1 to 7 candies in a cup and to tap a drum 1 to 7 times, support the conclusion that children learn cardinality some time after their third birthday. While the first two groups of children (mean ages 3:5 and 3:8) succeeded on the candy placement about 45% of the time and on the drum tapping about 25% of the time, these numbers for the third group of children (mean age

4:2) are 87% and 75% respectively, a dramatic improvement.

It could be argued that the poor performance of the younger children on the Choose task and the Sticker game reflects performance demands, rather than competence. There are several ways children can fail at a task. They can fail due to lack of conceptual understanding (in this case, lack of knowledge of the cardinality principle). They can fail even if they have conceptual competence, by not having learned appropriate procedures that instantiate their conceptual competence in a particular context. Different counting situations require different counting procedures in order to honor the counting principles. The counting principles are not themselves procedures; children must learn appropriate procedures for different situations. Finally, even if children have at their disposal procedures which are appropriate to a particular counting context, children might not know which of their procedures to utilize, and fail at the task (see Greeno et al., 1984, for a detailed discussion of what they term conceptual, procedural, and utilizational competences; see also Gelman & Greeno, 1987). However, there is strong support for the claim that children's failure in the Choose task/Sticker game is due to lack of knowledge of the cardinality principle. This comes from the finding that success or failure in this task is a good predictor of several things: (a) whether a child will respond a majority of the time with the last number word used in a count when asked "how many" following counting; (b) whether a child will give the last number word more often after correct than incorrect counts when asked "how many"; and (c) whether a child will tend to count out items aloud from a pile, when asked for a number that she or he is generally successful at giving. The How-many task is procedurally very different from the Choose task/Sticker game, so it is unlikely that there is a procedural requirement common to both tasks with which the Grabbers were having difficulty. It is at the conceptual level that the two tasks are similar, and therefore children's failure is almost certainly due to lack of conceptual competence.

The general conclusion that young children do not have unlearned knowledge of counting at their disposal also appears to conflict with studies suggesting that children do represent one-to-one correspondence as a component of the counting routine, and are sensitive to the stable ordering of the counting words. Children will say that a puppet has counted wrong and will often correct the puppet, when it violates one-to-one correspondence in some way (Gelman & Meck, 1983). Again, however, the mean age of the youngest children for which this has been shown was over 3-and-a-half, so these results could reflect knowledge learned about counting rather than knowledge underlying counting. Furthermore, as noted above, given that young children appear to represent one-to-one correspondence as a component of so many of their daily tasks, it is plausible that they have a general ability to quickly recognize when it is part of an activity. Showing that very young children represent one-to-one correspondence as a part of the counting routine, then, does not by itself show that they did not *learn* this knowledge. It has also been found (Fuson & Mierkiewicz, 1980; Gelman & Gallistel, 1978) that children as young as 2-and-a-half use consistently ordered lists of number words when counting, even though these lists may not follow the standard order of the counting words (e.g.; a child may consistently count, "one, two, six, eight, eleventeen."). This indicates that children are sensitive to the fact that counting uses a stably ordered list of words. However, as argued above, children of this age are sensitive to many other stable orderings as well, such as the alphabet. There is no evidence that children represent the (necessary) stable-ordering of the counting words any differently than the (arbitrary and nonessential) ordering of the letters of the alphabet.

However, these conclusions ought not be taken as evidence that children have no understanding of number at all. There is some evidence that children as young as 2-and-a-half do recognize at least small numerosities and can perform inductions on them. In one experiment, children of this age were shown two plates, one with three toy mice on it identified as the "winner" plate, and one with two mice identified as the "loser" (Gelman, 1977). The plates were covered and shuffled, and children had to identify which was the winner plate. Children could do this task. Then, surreptitiously, either the number of mice on the winner plate was reduced to two mice, or a number-irrelevant transformation was made with the winner plate such as replacing one of the mice with a soldier, or changing the spatial arrangement of the three mice. When the covers were removed after shuffling, in the number-irrelevant transformations children still chose the three-item plate as the winner, while in the number-relevant transformations, children often declared that there was no winner, and in some cases even fixed one or both plates to be winner plates by adding an extra mouse. They had thus evidently represented the number of the array. There is also evidence that even infants have some very basic knowledge of the numerosities *two* and *three*. When habituated to many different pictures of two arbitrary objects, 3-month-olds will dishabituate when shown a picture of three objects, and vice-versa (Starkey & Cooper, 1980). More impressively, when shown two pictures simultaneously, one of two objects and one of three objects, and played a sound recording of either two knocks or three knocks, 7-month-old infants show preferential looking at the picture with the same number of objects as the number of knocks heard (Starkey, Spelke, & Gelman, 1983).

These studies suggest that young children do have some basic concept of number, or at least of smaller numbers. This in turn suggests that the child's task may be one of mapping already existing concepts of *oneness*, *twoness*, and *threeness* with the number words "one", "two", and "three", and with the counting activity. Results from the Choose task and Sticker game indicate that children map smaller numbers onto their number words before achieving such a mapping for larger numerosities. This is plausible. The word "one" occurs much more frequently than other number words, and in many special contexts (e.g. "I want another one", "give your brother one of those", "get me the little one", "which one do you want", etc.). It may even be that children first learn the word "one", not as a number word but rather as a pronoun that picks out a single individual, similar to "he", "she", or "it".[4] Children then map the word "two" onto its corresponding numerosity, followed by the word "three". Further evidence that children map number words onto their corresponding numerosities in order of increasing numerosity comes from the subitization literature; when shown small numerosities and asked to tell "how many" there are, children's rate of correct response decreases as the number increases (e.g., Gelman & Tucker, 1975; Silverman & Rose, 1975). It appears that, after acquiring the meanings of the words "one", "two", and "three", children perform a general induction over these instances, at about 3-and-a-half years of age, that *all* the words in the counting list (at least within their counting range) refer to distinct numerosities. The child's acquisition of the cardinality principle appears to occur in conjunction with this general induction.

It should be stressed that in the "Novel Entities" experiment, most of even the youngest children were able to generalize their counting routine to sounds and actions, showing that children have the ability to develop very quickly an abstract and sophisticated mental representation of the counting routine. This is suggestive of strong powers of abstraction in young children, and may point to unlearned abilities more general than knowledge of counting. It has been proposed (Shipley & Shepperson, 1988) that certain general cognitive abilities, or *'operating principles'*, may underlie the development of the one-to-one correspondence principle, such as: a bias to operate on discrete whole objects rather than on parts or properties of objects; a tendency to exhaust a set of things being operated on (e.g. to throw *all* the toys out of the playpen); and an ability to match elements of one set to elements in another. Further analyses of this sort could lead to an explanation of how children are able to so quickly develop an abstract representation of counting. That many of the older children performed at or near ceiling in counting both objects and other entities, while failing the cardinality tasks, suggests that there may be several levels of abstraction and representation of the counting routine.

It appears that the development of children's understanding of counting is complex and piecemeal. Infants may have some concept of *one*, *two*, and *three*, which they must map onto the correct number words. Counting is at first a meaningless activity, something like a game of patty-cake, from which children abstract out certain properties earlier, others later. One property that some children learn is that the last number word used in a count is the answer to "how many" items there are, even though they do not at first understand that "how many" refers to *numerosity*. Children first map the word "one" onto its numerosity, achieving this next for the word "two" and then the word "three". At around 3-and-a-half years of age, most children induce that there is such a mapping for every word in the counting list. At the same time they also come to understand that the last number word used in a count represents the numerosity of the set, and thus learn the significance of the counting activity.

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Footnotes

[1]Gelman & Gallistel (1978) posit two further principles which specify a *lack* of constraints on the application of the counting procedure. *Order-Irrelevance principle:* The same result will obtain regardless of the order in which a set of entities is counted. *Abstraction principle:* Any entities can be grouped together for a count. For example, we can count the number of eyes, sneezes, and ideas in this room between 1:00 and 2:00 pm as a single count.

[2]Three children, after giving 1 item when asked for 1, were asked to "count to make sure". Because it is an odd request to make for 1 item. children's responses to this question were not included in the analysis. If responses to this question are included. then 1 child who was successful at 2 was not consistently correct when asked for 1: for two trials she gave 1, and upon being asked to count it she added some more items and counted them all. On the other three trials in which she was asked for 1, her response was to give 1. If this child is considered as going against the hypothesis, then a total of 16 out of 17 children performed in the expected manner -- still a highly significant result.

[3]All the strategies employed by Grabbers in the Choose task in Experiment 2 were also used by Grabbers in the Sticker game. Many children counted so that the last number word said was the number asked for, and some children named an item the number asked for. For example, one boy gave 2 when asked for 6. When asked to count them, he pointed to each of them while saying, "One, this is six". The experimenter then asked him "How many are there altogether?", to which he replied, pointing, "This is six, and this is six. They're *both* sixes!" Some children tried to "fix it" in new ways. One girl gave 2 when asked for 6. She then counted them "six, six". When asked to count them "the normal way", she counted them correctly. "one, two". She was then asked, "How can we fix it so there's six?" She picked them up, turning them around and switching their places, and put them back down saying, "Maybe this way". Some children, when asked to "fix it" to the correct number, used what could be called "magic" strategies. A boy who had given 4 when asked for 6, picked up a toy dog and touched each of the 4 animals with it, saying, "There! The dog fixed it!". Another child "tickled" each of 3 animals to "make it 5". Thus, Grabbers again show that they do not understand. that counting determines the numerosity of a set. The children in these examples also did not

appear to understand that they had been asked for a certain *number* of items; hence, the number-irrelevant operations they performed in efforts to fix what they'd given to what had been asked for.

[4] I am grateful to Paul Bloom for suggesting this possibility to me.

TABLE 1

Proportion of cardinality and recount responses [recount responses indicated by square brackets].

Age Group	Object	Cave	Jump	Sound	Mean
Age I (2:7)	.10 [.20]	.44	.57	.00	.31
Age II (3:0)	.30 [.50]	.24	.18	.18	.22
Age III (3:5)	.42 [.33]	.69	.53	.50	.56

TABLE 2

No. Grabbers Giving 1, 2, 3, 5, and 6 Items,

by No. Asked For

	No. Asked For				
No. Given	1	2	3	5	6
1	18	1	1	0	0
2	0	13	8	7	8
3	0	1	5	5	3
5	0	0	3	0	2
6	0	1	0	1	0

TABLE 3

Mean No. of objects first given by Grabbers and Counters.

	Number asked for				
Age Group	1	2	3	5	6
Age I Grabbers	1.0	2.6	2.9	3.4	3.3
Age II Grabbers	1.0	3.6	2.9	3.3	2.9
Age III Grabbers	1.0	1.7	2.7	4.7	4.5
Age III Counters	1.0	2.0	3.0	5.2	6.0

TABLE 4

Patterns of Success in Sticker Game

Success Pattern 1	2	3	5	6	No. of Children	Mean Age	Counting Ability Mean	Range
-	-	-	-	-	1	2:8	3.00	(3 - 3)
+	-	-	-	-	3	3:0	4.67	(3 - 6)
+	+	-	-	-	2	2:11	4.50	(3 - 6)
+	+	+	-	-	4	3:5	5.75	(5 - 6)
+	+	+	+	+	7	3:7	6.00	(6 - 6)

(Note: "+" indicates success on a numerosity; "-" indicates failure.)

Figure Captions

Figure 1: *Percent of Successful Counts, by Age*

Figure 2: *Grabbers' vs Counters' Mean % Cardinality Responses*

Figure 3: *Mean % Cardinality Responses Following Correct vs Incorrect Counts*

Figure 1

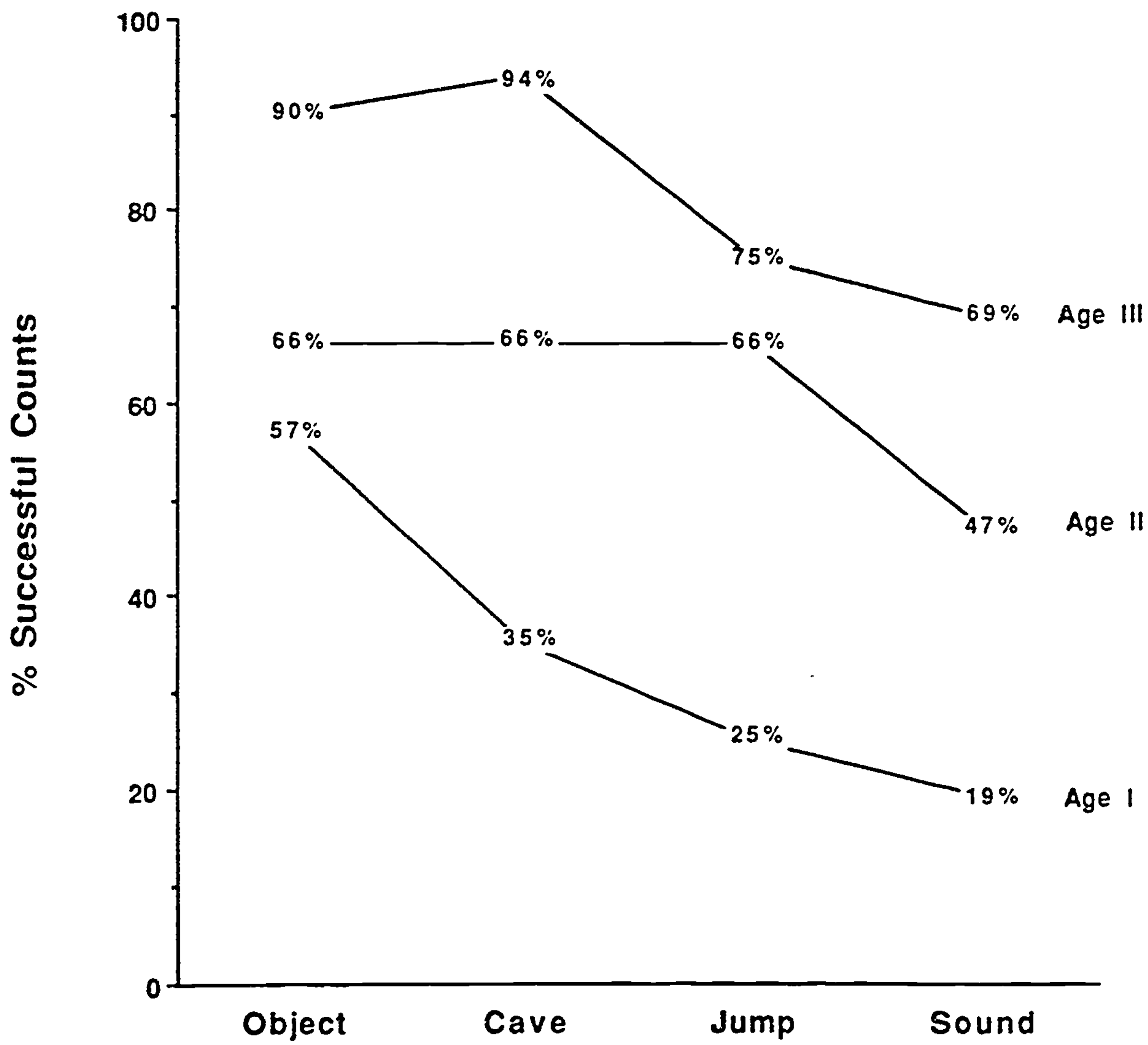
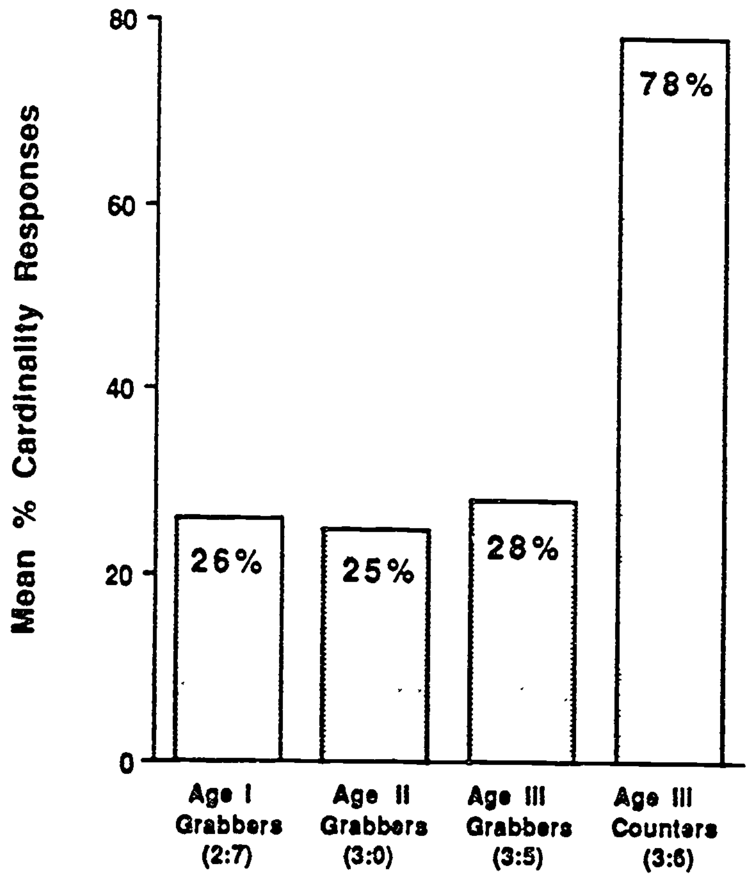
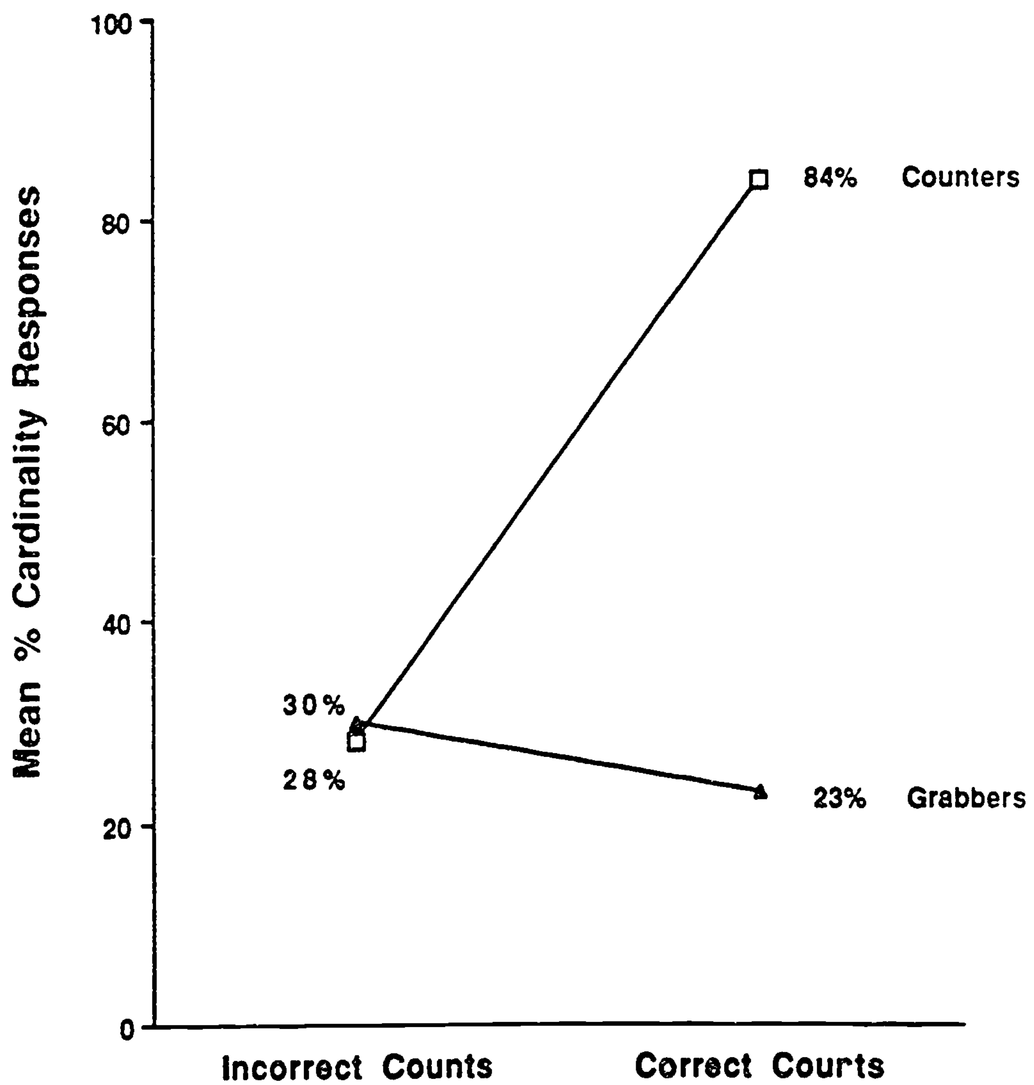

100
80
60
40
20
0
% Successful Counts
90%
94%
75%
69%
Age III
66%
66%
66%
47%
Age II
57%
35%
25%
19%
Age I
Object
Cave
Jump
Sound

Figure 2



Mean % Cardinality Responses
80
60
40
20
0
78%
26%
25%
28%
Age I Grabbers (2:7)
Age II Grabbers (3:0)
Age III Grabbers (3:5)
Age III Counters (3:6)

Figure 3



Mean % Cardinality Responses
100
80
60
40
20
0
84% Counters
30%
28%
23% Grabbers
Incorrect Counts
Correct Courts