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AUTHOR	Konold, Clifford
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ABSTRACT

Results of the most recent administration of the National Assessment of Educational Progress (NAEP) suggest that the majority of secondary students believe in the independence of random events. In the study reported here, a high percentage of high school and college students answered similar problems correctly. However, about half of the students who appeared to be reasoning normatively on a question concerning the most likely outcome of five flips of a fair coin gave an answer on a follow-up question that was logically inconsistent. It is hypothesized that these students are reasoning according to an "outcome approach" to probability in which they believe they are being asked to predict what will happen. This finding has implications for both test development and curriculum design. (Author)

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AN OUTBREAK OF BELIEF IN INDEPENDENCE?

Clifford Konold

Scientific Reasoning Research Institute
University of Massachusetts
Amherst, MA 01003

Results of the most resent administration of NAEP suggest that the majority of secondary students believe in the independence of random events. In the study reported here, a high percentage of high-school and college students answered similar problems correctly. However, about half of the students who appeared to be reasoning normatively on a question concerning the most likely outcome of five flips of a fair coin gave an answer on a follow-up question that was logically inconsistent. It is hypothesized that these students are reasoning according to an "outcome approach" to probability in which they believe they are being asked to predict what <u>will</u> happen. This finding has implications for both test development and curriculum design.

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AN OUTBREAK OF BELIEF IN INDEPENDENCE?

The belief that successive outcomes of a random process are not independent (the so called "gambler's fallacy") is supposedly one of the most common misconceptions about probability. An example of this misconception is the belief that a long run of heads in coin flipping increases the likelihood that the next trial will produce a tails.

One of the possible explanations for the gambler's fallacy is that people reason about such situations according to a "representativeness heuristic" (Kahneman & Tversky, 1972). According to this heuristic, the likelihood that a given sample comes from a particular population is judged on the basis of the degree of similarity between salient features of the sample and the corresponding features of the parent population. After a run of four Hs, and given a choice between the two possible samples HHHHH and HHHHT, the latter is judged as the more likely via the representativeness heuristic since it is closer to the ideal population distribution of 50% Hs.

Use of the representativeness heuristic is often elicited by asking people to choose among possible sequences the most likely to occur. In the case of five flips of a fair coin, all possible ordered sequences are in fact equally likely, the probability of each being $.5^5$. Given a choice among several options, people reasoning according to the representativeness heuristic will chose THHTH as being more likely than THTTT or HTHTH. Kahneman and Tversky (1972) argue that this choice is consistent with the representativeness heuristic in that it reflects both the fact that heads and tails are equally likely and the belief that random series should be "mixed up."

The belief that non-normative expectations such as the gambler's fallacy are widely held has inspired the development of probability and statistics instruction to counter such beliefs. Curriculum designed by Shaughnessy (1977) and Beyth-Maron and Dekel (1983) include units intended to confront and correct judgments based on informal judgment heuristics. However, results on problems involving probability on the most resent administration of the National Assessment of Educational Progress (NAEP) suggest that the majority of secondary students in the United Stated believe in the independence of random events. Asked for the most likely outcome of a fair coin given four successive trials on which the coin landed with tails up, 47% of the 7th graders and 56% of the 11th graders selected the correct alternative. The percentage of responses that were incorrect but consistent with the representativeness heuristic was 38% for the 7th graders and 33% for the 11th graders (Brown, Carpenter, Kouba, Lindquist, Silver, & Swafford, 1988). Given that probability is infrequently taught at the secondary level, these data suggest that a concept of independence is more prevalent than non-normative reasoning even prior to formal probability instruction.

In the study reported here, high-school and college students performed even better on similar problems. However, given their inconsistent responses to a follow-up question, it appears that nearly half of the students who answered the problem correctly were reasoning according to a non-normative construct of probability, the "outcome approach" (Konold, 1989; in press).

People who reason according to the outcome approach do not see their goal in uncertainty as specifying probabilities that reflect the distribution of occurrences in a sample, but as predicting results of a single trial in a yes/no fashion. Given the probability of some event, such as "70% chance of rain tomorrow," outcome-oriented individuals adjust the probability value to one of three decision points: 100%, which means "yes," 0%, which means "no," and 50%, which means "I don't know." Thus, the number in the forecast "70% chance of rain" is adjusted up to 100%, after which the forecast is interpreted as "It will rain tomorrow." If it fails to rain, the forecast was "wrong." Given this orientation, a forecast of 50% chance of rain suggests total ignorance on the part of the forecaster about the outcome.

On problems involving coin flipping, the outcome-oriented individual infers from the known probability of "50/50" that there is "no way to know" the outcome of a trial, or series of trials. Although this conclusion appears correct, for the outcome-oriented individual in will be shown to involve a contradiction. When those who reason according to the outcome approach are eliminated from the pool of correct responders in this study, there no longer appears to be an outbreak of belief in independence.

METHOD

Problems and Procedure

This study includes student performance on the following two items:

Four-heads problem. A fair coin is flipped 4 times, each time landing with heads up. What is the most likely outcome if the coin is flipped a fifth time?

a. Another heads is more likely than a tails.
b. A tails is more likely than another heads.
c. The outcomes (heads and tails) are equally likely.

H/T Sequence problem. 1) Which of the following is the most likely result of 5 flips of a fair coin?

a. H H H T T
b. T H H T H
c. T H T T T
d. H T H T H
e. All four sequences are equally likely.

2) Which of the above sequences would be least likely to occur?

These two items were included on questionnaires along with other items on probability and statistics. Each item appeared on a separate page, and students were instructed not to return to a page once it had been turned.

Students

Summermath. Both items were administered as part of a nine-item pretest to 16 high-school girls on the first day of a workshop on probability. This workshop was offered in 1987 as part of "Summermath," a six-week residential program sponsored by Mount Holyoke College. Summermath recruits nationwide, and participants represent a range of mathematical ability.

Remedial math. Twenty-five undergraduate students enrolled in the Spring 1987 semester of a remedial-level mathematics course at the University of Massachusetts, Amherst, volunteered to participate in a study on probabilistic reasoning. Probability was not a topic covered in this course. The Four-heads and H/T Sequence problems were among 14 items they completed.

Graduate statistics. Both items were administered as part of a pre-course survey for a graduate-level statistical methods course in the College of Education, University of Minnesota, in the Fall of 1987. This course is the first of a three-semester methods sequence required of all advanced-degree candidates in psychology and education. Dr. Joan Garfield was the instructor, and administered the survey.

RESULTS AND DISCUSSION

Overall, 86% of the students answered the Four-heads problem correctly. Not surprisingly, the performance of the Remedial students was the poorest (70% correct) and the Graduate students the best (96% correct). The most popular alternative answer was the one consistent with the gambler's fallacy, that a tails is more likely after a run of heads. This option was selected by 22% of the Remedial students, 19% of the Summermath students, and 4% of the Graduate students. These results parallel the NAEP results cited earlier and suggest that even without instruction, the majority of students do not commit the gambler's fallacy on this particular problem.

Performance on the H/T Sequence problem is summarized in Table 1. The percentage of students who chose each option as the most likely is listed under the heading "Most." The majority of students (72% overall) correctly chose option e. The alternative f in the table was written in as the correct option by 1 Remedial and 3 Graduate students. They indicated that options a, b, and d were equally likely and that option c was least likely to occur.

Table 1. Performance on H/T Sequence Problem

	Group					
	Remedial		Summermath		Stat Methods	
N=	23	23	16	15	47	41
Sequence	Most	Least	Most	Least	Most	Least
a. HHHTT	17%	9%	0	7%	0	7%
b. THHTH	13%	4%	25%	0	2%	2%
c. THTTT	4%	9%	0	33%	2%	27%
d. HTHTH	0	43%	6%	40%	11%	17%
e. Equal	61%	35%	69%	20%	79%	46%
f. a,b,d	4%				6%	

The most interesting result is the percentage of correct responses to the question of which sequence is least likely. These percentages are listed in Table 1 under the heading "Least." Overall, only 38% of the students responded that all four sequences were equally unlikely. Thus, roughly half of the students who selected the correct option e (all equally likely) for the question regarding the most likely sequence went on to select one of the sequences as least likely rather than respond in a consistent manner that all four sequences were also equally unlikely. This contradiction suggests that even though students may respond correctly to the question of which sequence is most likely, their answers may not be based on normative reasoning.

One hypothesis about why some students respond in contradictory fashion is that these students are reasoning according to the outcome approach. As mentioned in the introduction, outcome-oriented individuals, when asked the probability of some event, interpret the request as one to specify what will happen. In the case of the Four-heads and H/T Sequence problems, they think they are being asked what will happen on the fifth trial, or which sequence will occur, respectively. The 50% probability associated with coin flipping, however, suggests to them that no prediction can be made. Thus they choose the answer "equally likely," and by this they mean they have no basis for making a prediction of what will happen. However, the question in the H/T Sequence problem about which sequence is least likely cannot sensibly be interpreted as, "Which sequence will not occur?" (since none of them may occur). In this case, the outcome oriented individual may switch from a yes/no- to a more probabilistic interpretation of the question and choose the option that they think is least likely.

In addition to choosing an option, Remedial and Summermath students were asked for each problem to "give a brief justification" for their answer. These justifications provide further evidence that a few of the students were reasoning as described above. The responses of four students whose answers to the H/T Sequence problem were inconsistent are given below. Each excerpt is preceded by a code that specifies whether the student was from the Summermath (S) or Remedial (R) group. The answers the students gave for the most likely and the least likely sequences are given in parentheses.

> S15: (e,a). [For e] Anything can happen with probability. The chances of some of the examples are least likely to occur (a,c), but it can happen. [For a] This chance is least likely to occur because they happen the same side in a row.

> S16: (e,c). [For e] They all could occur. [For c] Because it is least likely to occur when you have almost a perfect score.

> R2: (e,a). It's a chance game. Receiving 3 heads in a row seems unlikely, but could very well occur. No skill is involved, therefore all could likely occur by chance.

> R14: (e,d). One never knows which way the coin will drop.

CONCLUSION

These results have some fairly direct implications for curriculum development and testing in probability. The belief that the majority of novices faced with these type of problems will commit the gambler's fallacy has helped to shift the focus in probability instruction away from computational skills towards conceptual development (cf. Garfield & Ahlgren, 1988). This shift has been accompanied by curriculum aimed at the development of concepts such as independence and randomness and the design of items to test for conceptual understanding. Given this focus, problems like those used in this study are likely to become standard fare on course and national exams of mathematical achievement. The results of this study suggest that a sizeable percentage of correct responses to such problems are spurious and reflect an approach to uncertainty that is perhaps more pernicious than the gambler's fallacy. Problems need to be developed that can discriminate individuals who reason according to what has been described here as the outcome approach from those with a normative concept of independence.

The development of probability and statistics curricula for the secondary and even elementary levels has become part of the agenda of current efforts to reform mathematics education in the United States. As mentioned above, one of the directions of new curricula being developed is to help students overcome various of the well-documented misconceptions about probability and statistics. Having students analyze data from actual trials or computer simulations holds some promise of helping them overcome misconceptions based on judgment heuristics -- data that contradict their expectations can lead them to question their beliefs. The outcome orientation, however, may not as easily be challenged through simulations or experiments with objects composed of equally-likely outcomes, because virtually every result would appear to support the expectation that "anything can happen." In fact, the variation in results of replications might serve to strengthen rather than undermine the outcome approach. What may prove effective in the case of the outcome approach is the simulation of phenomena composed of non-equally likely alternatives, especially where one of those alternatives is associated with a very high or low probability. Outcome-oriented individuals predict that events with low probabilities will not occur, and to discover that they do occur, and with about the same relative frequency as their probability, may lead them to alter their belief. As one of the Summermath students observed with some surprise after conducting such simulations, "Even if there is a 1% chance, it could happen!"

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