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ABSTRACT

Secondary school geometry students have been found to have misconceptions concerning parallel lines. Correctional teaching programs did not seem effective in changing these misconceptions. This research describes an attempt to use teaching strategies which take into account current learning theory and which encourages students to be actively involved in constructing a new understanding of parallelism using their own misconceptions as a starting point. Materials developed by the investigators were used by 2 teachers and 12 students. Discussion of the investigation includes teacher and student perceptions of the program and learning outcomes. (DC)

STUDENTS' AND TEACHERS' PERCEPTIONS OF THE COGNITIVE AND AFFECTIVE OUTCOMES OF SOME LESSONS IN GEOMETRY.

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INTRODUCTION

In secondary school geometry an understanding of parallel lines is important in terms of defining shapes and proving theorems. Teachers classify quadrilaterals either as parallelograms or not and they derive the properties of parallelograms from those of parallel lines. Teachers also demonstrate the angle relationships created when a transversal intersects parallel lines and they may use a parallel construction to prove that the angle sum of a triangle is 180^{o}.

Our initial research, referred to as Phase One in this paper, (Mansfield and Happs, 1987) documented a number of misconceptions about parallel lines that students bring to lessons, the more obvious ones being that parallel lines may be curved and that parallel segments must be aligned. Furthermore, it was found that some students believed that the presence or absence of arrows or dots on the ends of lines affects whether or not those lines are parallel. Such misconceptions have been identified at the grade 6, 8 and 10 levels and amongst final year college students in the U.S.A. and in Western Australia.

Conventional teaching programs do not seem to be effective in changing these misconceptions to the commonly-accepted mathematical view and this research describes an attempt to call upon teaching strategies which take into account current learning theory and which encourage students to be actively involved in constructing a new understanding of parallism using their own misconceptions as a starting point for conceptual change.

CONSTRUCTIVISM AND CONCEPTUAL CHANGE

The monitoring of students' and teachers' perceptions of the teaching program in general was undertaken to determine whether or not a constructivist approach to learning and teaching about parallelism was viable and acceptable by those engaged in the exercise at the classroom level.

A constructivist approach to learning, unlike the developmental approach (Shayer and Adey, 1981) and the behaviourist approach (White, 1973) adopts the view that the learner's existing ideas (which often incorporate misconceptions) are important as learners respond to incoming information from teacher and/or text. Thus, according to Osborne and Wittrock (1985) constructivism assumes that:

(1) All knowledge is constructed by the individual as he or she interacts with the environment and tries to make sense of it; and

(2) All knowledge is acquired not by the internalisation of some outside given meaning but by construction from within, of appropriate representation and interpretation.

(Osborne and Wittrock, 1985, p61)

Constructivism then acknowledges the importance of the learner's conceptual schemes and the application of these in attempts to understand new situations (Driver, 1982).

Much of what happens in mathematics classrooms suggests that students are treated as retainers of information, frequently being called upon to rote learn and recall rather than demonstrate understanding. The research reported here has been driven by constructivist theory and the view of learning as relating what the student experiences to his/her prevailing ideas. We have endeavoured to monitor the ways in which new information about parallel lines interacts with individuals' existing misconceptions and how that new information might be incorporated into (or rejected by) existing knowledge frameworks.

Often, if students have views that differ from the common mathematical view, they will reject the mathematical view and their own views will persist, at least for use in out-of-school contexts. However, if students are confronted with their own views and those of others, their own views are more readily modified and, if still held some months after initial teaching, then the modified views are likely to persist.

The teaching materials and strategies employed within this investigation were developed from a cognitive perspective on learning, refuting that students can be provided with new information such that this will simply be added on to the learner's existing store of knowledge.

THE INVESTIGATION

The first phase of our investigation involved the identification, documentation and comparison of misconceptions about parallel lines held by students in the U.S.A. and Western Australia.

The second phase of our teaching experiment involved the design of strategies that focussed on group work, the recognition and resolution of conflict, and the articulation by students of their own views about parallel lines. This approach has been used successfully in previous research (Hewson, 1982; Happs ,1983) and we found that our strategies could bring about conceptual change that lasted six months beyond the time of initial teaching. In this phase the two investigators taught and monitored the trial lessons.

In the third phase we further developed our teaching materials on the topic parallel lines to incorporate the strategies we found to be effective and these were used by two grade 8 teachers who were not involved in the development of those materials. The purpose of this phase being to see whether our materials might be adapted and used by teachers without any special training, to seek student and teacher perceptions of learning outcomes at this stage and to see whether the lessons brought about any long-term conceptual change (Mansfield and Happs, 1988).

The last phase of our investigation made available our teaching strategies and materials to teachers and students in ten diverse secondary schools so that a wider and more detailed assessment of the teaching program might be obtained

In the case of the third and last phases within our teaching experiment, the teaching materials and plans for four lessons were provided for the teachers. The materials included the relevant overhead transparencies and worksheets, as well as resources such as shapes for tessellations. A brief written rationale for the study was provided as well as a brief written rationale for each lesson. The lesson plans outlined the procedures to be followed but were not scripted, thus allowing the teachers to use their own preferred explanations, styles of teaching and classroom management.

No instruction in the use of the teaching package was provided and it was stressed that we were interested in evaluating the effectiveness of our materials and not the teachers' performance. We wanted to know whether the teachers found our materials intelligible and if they were comfortable with the teaching strategies. The fact that we had already taught with the materials in our initial investigation appeared to lend us greater credibility and all teachers stated that they were happy to participate. The teaching program in each of the two classes in the third phase lasted for four seventy-five minute lessons whilst teachers from the ten schools involved in the final phase of the investigation tailored their teaching according to the time available to them.

The lessons included the following steps.

1. Students sorted ten items into groups that they considered showed parallel lines, lines that were not parallel and items of which they were unsure. The students' responses and reasons for those responses were recorded and the teacher introduced the mathematical view of each item. Students compared their own view with those of mathematicians. A class definition of "parallel" was constructed.

2. Students were introduced to coplanarity of parallel lines through a consideration of parallel lines and skew lines drawn as edges on a box.

3. Students were introduced to the use of file-cards to measure the distances between lines. In our observations, during initial interviews, we noted that some students had difficulty using rulers to measure distances.

4. Students identified sets of parallel lines in tessellation patterns that they constructed.

5. Students identified "ladder" and "zigzag" patterns in pictures of real-life examples and in sets of parallel lines and discovered through comparison that corresponding angles and alternate angles are equal. The "ladder" and "zigzag" approach in this lesson was drawn from the Van Hiele research undertaken by Fuys, Geddes and Tischler (1984).

6. Students used corresponding and alternate angles to develop an argument that the angle sum of a triangle is 180^{o}.

7. Students used their knowledge of angle properties to solve problems involving parallel lines.

8. Students in groups summarised properties of parallel lines and of angle relationships found in sets of parallel lines.

Prior to teaching the program in the third phase of the investigation twelve students (six from each class) were identified as being of "average ability" and were randomly selected by their teacher. These students were interviewed individually, being asked to give a definition of parallel lines and to cite any real-life examples of parallel lines that they could think of.

They were then shown the ten items used in step 1 of the teaching program (see Appendix A) and asked to say whether the items showed parallel lines and to give reasons for their responses. Following the teaching program, during which each investigator sat in with and monitored the six selected students in each class, students were again asked for a definition of parallel lines and were shown the same ten items and an additional thirteen items which also assessed their ability to identify parallel lines. Students were also asked to identify corresponding and alternate angles within test items and were also asked to determine the third angle of a triangle given the other two. Similarly structured interviews were conducted with those twelve students following a wait-period of six months after the teaching program was completed.

The third phase of the study provided the investigators with an opportunity to observe the implementation of the teaching program by two classroom teachers without any intervention from the investigators. It also allowed specific groups of students to be observed in order to see how they interacted with the materials and how their understanding of aspects of geometry changed over time. Outcomes are reported later in this paper.

In phase four the teaching program and materials, with some further minor modifications, were distributed to mathematics teachers from ten diverse secondary schools. These teachers had been made aware of our investigation through a local mathematics association meeting and were willing participants in the larger-scale trialling process. No preliminary training was given and the teachers used the program at times convenient to themselves.

Student performance was evaluated essentially by use of concept mapping exercises before and after the teaching program (Novak and Gowin,1986) and results from these are currently being assessed.

The ten teachers involved in the last phase were each asked to complete a questionnaire designed to evaluate the program. Statements and results from this questionnaire are discussed later in the paper.

TEACHERS' PERCEPTIONS OF THE PROGRAM : PHASE THREE

Both of the teachers involved in this third phase of the study were fairly recent graduates. Tim was in his second year of teaching and Alan was in his fifth.

Both teachers were very surprised that their students displayed so many misconceptions about parallel lines at the start of the program.

> "I thought that it was very interesting that they had so many misconceptions. I wouldn't have thought that would have been the case."
> (Tim, Year 8 teacher)

In the first step of the program, where students anticipated the views about parallel lines, resolved their conflict and reached a consensus about each item, both teachers considered this step to be particularly successful in changing student ideas:

> ". . . others in the group would say "No, they can't be parallel because of this and this or whatever." And there'd be a discussion and they'd say "Oh yeah." and modify their views accordingly . . . . they were discussing it . . . putting their views into words, which I think is always a good idea."
>
> (Alan, Year 8 teacher)

Clearly, for these two teachers, the classroom organisation that we requested meant a substantial change in teaching style since students worked in groups to facilitate their discussion. Neither teacher had used a group organisation previously in teaching and this was not always a comfortable situation for them.

Both teachers would have preferred more numerical examples to be used in the program. In step 5 corresponding and alternate angles were introduced through an examination of real-life examples and through "ladder" and "zigzag" patterns in tessellations. In steps 6 and 7, in which students developed an argument that the angle sum of a triangle is 180° and solved problems involving parallel lines, the exercises used did not involve numerical questions. Our approach was to use examples familiar to students (a prominent Perth building and the Australian Bicentennial Logo) to help them to visualise relationships between sets of angles rather than to present more traditional calculation exercises. The comment was made:

> "Because the groundwork's been laid so well the kids that have got any idea at all should have no trouble crossing over from parallel lines to numerical examples where they can work out angle sizes."
>
> (Alan, Year 8 teacher)

Both teachers felt that our program demanded more time than they would normally spend in this area :

> "There's no doubt in my mind about the superior method that you've developed there. It's just the time in the actual classroom whether it's feasible or not."
>
> (Alan, Year 8 teacher)

Both teachers did concede that when the topic had been taught previously to Year 8 students, and the same students were taught again at the Year 9 level,they appeared to know very little about parallel lines. Furthermore, both teachers considered that our teaching program on parallel lines was likely to lead to long-term retention and thus alleviate the need to reteach the topic in subsequent years.

Both teachers claimed that the rationales were useful and easy to comprehend and that the strategies were also intelligible and easy to implement :

"I thought it was very straightforward.
I read it through once and I had the basic
idea of what you were trying to get at."
(Tim, Year 8 teacher)

Given that we provided these two teachers with no instruction in the use of the program we were encouraged by their positive responses as to the usefulness of the program.

STUDENTS' PERCEPTIONS OF THE PROGRAM : PHASE THREE

All students, in both groups, indicated that they had enjoyed the lessons and audible groans ensued when they were told that lesson four was the last one in the program. The lessons in both classes were characterised by animated discussion amongst students and a general willingness to articulate their own ideas and sometimes modify them when challenged. The students appeared to enjoy the lessons and indicated that they had learned a lot from them. Post-program interview responses reflected this enjoyment and Amanda's comments were typical of those from the other eleven students. She stated that she had enjoyed all of the lessons and particularly the discussions. She was clearly surprised that other people had different ideas about parallel lines.

Interviews and assessment items allowed the investigators to determine if the learning outcomes were as favourable as student and teacher comments suggested and the results of a pre-test, immediate post-test and a delayed (six months) post-test are shown in Table 1.

LEARNING OUTCOMES : PHASE THREE

The individual interviews were structured such that students were initially asked to provide their definition of "parallel" prior to attempting to identify items 1-10 as being parallel or not parallel. Students were also asked to give reasons for their responses. Follow-up interviews and tests involved the same items 1-10 and a new set of items 1-13.

Table 1 shows the item scores for the six monitored students in each of the two classes following their pre-test, post-test and delayed (six months) post-test:

CLASS 1	pre-test (1-10)	post-test (1-10)	6 monthly post-test (1-10)	post-test (1-13)	6 monthly post-test (1-13)
BRETT	3	10	10	13	12
AARON	5	10	10	12	9
TANIA	0	10	10	13	13
AMANDA	* 7/8	10	10	13	13
CHARMAYNE	6	8	8	8	11
JASON	6	10	10	13	13
CLASS 2					
BILLY	4	9	9	13	12
RICHARD	5	10	10	12	10
JASON	6	1 0	9	13	13
MONIQUE	7	10	9	12	11
TOTAM	5	10	10	13	12
MIRANDA	3	9	10	13	13

Table 1 : Number of correct responses of students to items in pre-test, post-test and delayed post-test.

* Amanda thought that the straight line in item 10 was parallel whilst the curved line was not parallel.

It seems evident from the pre-test scores that only Amanda (class 1) appeared able to correctly classify the items yet, when asked to give reasons for her responses she suggested that parallel lines were any <u>single</u> lines that were straight. Consequently she said that all items showing curved lines were not parallel

Prior to the teaching program, the students displayed a wide variety of ideas in their responses to the ten items used in the pre-test. Tania (class 1) had never heard of the word "parallel" before and was unable to respond to any of the items.

Jason (class 2) thought that parallel lines must join and so item 9 (intersecting curves) and item 6 (concentric circles where the circles were seen as lines "joining themselves") were the only items to be classified as parallel.

Most students thought that curved lines can be parallel and "railway lines" was a typical example of parallel lines brought to mind by most students.

Other ideas were that parallel segments must be aligned, that parallel segments must have equal length and that parallel lines must have a specific direction such as travelling horizontally or diagonally.

Students were much more successful in their responses to the same ten items in the post-test (see Table 1) than they had been in the pre-test. With the exception of Charmayne (class 1), errors made by the students were due largely to errors in their measuring technique or a total failure to measure the distance between lines at various points. In the post-test, Charmayne stated that parallel lines must be straight but later said :

> "They can be wiggly as long as the distance between them at the ends are the same."

Charmayne subsequently measured between the end points of the curved lines and judged them on the basis of her measurements, ignoring other factors such as their intersection or the distances between other pairs of points. She was the only student at the end of the teaching program who still considered that curved lines might be parallel.

Clearly the teaching program was not successful in changing Charmayne's views about parallel lines which seemed to be well entrenched by the time she had completed the delayed post-test.

Results from the delayed (6 months) post-test were encouraging in that 7 out of the 12 students monitored were able to correctly identify the parallel and non-parallel items on the 1-10 test. Only Charmayne had not shifted her understanding of parallel towards the mathematician's perspective and, although only 2 out of the 12 students were able to respond correctly to all items on the 1-13 test, four students (Brett, Billy, Jason and Monique) made only one error on this test.

Brett provided a sound definition of "parallel" during the delayed post-test interview, indicating that parallel lines are:

> "Two straight lines which are equal in distance apart and on the same plane."

Later in the interview Brett considered that lines not opposite each other aren't parallel.

Billy also provided a good definition of "parallel", mentioning that they must be straight, equal distance apart and on the same plane, yet he failed to check item 2 carefully with his ruler.

Jason (class 2) erred because he felt no necessity to check the distance between the lines in any of the items and Monique was also reluctant to measure the distance between lines even though pencil, ruler and file cards were made available. These students felt that they intuitively 'knew" whether or not lines were parallel.

Charmayne re-iterated her view that parallel lines "don't have to be straight if they are the same distance apart" yet her score improved on the delayed post-test for items 1-13.

Aaron appeared to regress from scoring 12 out of 13 on the 1-13 item post-test to a score of 9 out of 13 on the delayed test although this apparent decline was not due to his lack of understanding of parallel lines, which he defined as :

> "The same distance from one line to another all the way through and they have to be straight."

Aaron failed to use the file cards provided and went on to incorrectly use a ruler to measure the distance between lines in items 4,6,7 and 13.

It appeared from the results of the delayed post-tests that the majority of the 12 monitored students had retained the mathematician's view of parallel lines whilst the technique of using a file card to check the distance between straight lines was often overlooked or ignored.

TEACHERS' PERCEPTIONS OF THE PROGRAM : PHASE FOUR

Following the implementation of the teaching program at ten diverse secondary schools (9 in the Perth Metropolitan Area and 1 rural school), teachers were asked to respond to a questionnaire designed to assess their perception of the program. Nine completed questionnaires were returned and the Likert Scale responses were quantified (1 for strongly disagree to 5 for strongly agree) so that they might be more easily understood and compared. Additional comments were provided by teachers and these have been coded alphabetically as shown below each quotation.

Statement 1 : I found the lesson plans easy to use over the 4 sessions.

A mean score of 3.5 was returned and this mid-range score suggested that not all teachers found the lesson plans easy to use. Some comments were most positive:

> "Well presented, step by step lesson guide was excellent."
> (teacher A)

and

> "Extremely pleased - could not fault them."
> (teacher C)

It was apparent that teachers in some other schools were not so convinced:

> "I found lesson 3 disjointed and hard to teach. There seemed too many handouts."
> (teacher G)

and

> "Too complex and generally found the lessons contained too much material to get through.

Suggest that the handouts be stapled and given out in the first lesson."

(teacher H)

Statement 2 : I found the rationale explained and introduced the lessons well.

A mean score of 4.2 was returned for this statement, suggesting that the nine teachers readily understood and followed the lesson rationales provided. No negative comments were received about the rationale whilst numerous comments about the value of the rationale were offered:

"Lesson 1 rationale was very useful (for sorting out teachers' misconceptions as well.)

(teacher H)

Statement 3 : The students generally found the materials easy to use.

This item returned a high mean score of 4.2 and all comments were very favourable:

"All students participated."

(teacher B)

and

"The materials themselves were very good, suitable and easy to use."

(teacher H)

Statement 4 : The 4 lessons integrated well into my intended teaching scheme.

A mean score of 3.2 suggested some variation in response to this statement and teachers' comments were mixed:

"Yes, but I would have probably spent less time on activities and more time on consolidation and practice."

(teacher H)

One comment reflected the feelings that the teachers involved in Phase 2 had expressed about this kind of approach:

> "Not a method I would readily choose."
> (teacher G)

Statement 5 : I felt that the time spent on the topic was appropriate.

This statement returned a mean score of 3.0 which, although in itself suggested some ambivalence by teachers towards the statement, was accompanied by mainly positive comments:

> "Yes and the class was heterogenous, i.e. not streamed."
> (teacher B)

Two negative comments were received:

> "Too long on some of the introductory concepts."
> (teacher E)

and

> "Too long spent on the ideas covered in lessons 2,3 and 4."
> (teacher A)

Statement 6 : I generally enjoyed teaching the series of lessons.

A mean score of 3.9 suggested that most teachers generally enjoyed teaching the lessons and this was borne out by the comments provided, all of which were positive:

> "This was made so easy by your preparation that it was very enjoyable."
> (teacher C)

"Enjoyment" didn't necessarily mean "easy" for all teachers:

> "Yes, although they were a little more demanding than most."
>
> (teacher B)

<u>Statement 7 : The students generally appeared to enjoy the lessons.</u>

A mean score of 3.8 for this statement also suggested that positive comments might be forthcoming and this was the case:

> "Students were interested to know when the next lesson was."
>
> (teacher B)

One negative comment was offered concerning the pace of the lessons:

> "Better ability students found it too slow in parts - too much distribution of materials - stop/start."
>
> (teacher E)

<u>Statement 8 : I feel that other mathematics teachers could easily use the materials.</u>

A mean score of 3.0 suggested some difference of opinion concerning this statement and conflicting responses emerged:

> "Anybody could teach this the way you prepared it."
>
> (teacher C)

and

> "Inexperienced teachers could experience difficulties."
>
> (teacher F)

Statement 9 : The materials are better suited to high-ability students.

This statement resulted in the lowest returned mean score (2.0) and associated comments suggested that the program was well-suited to less able students:

> "Materials were needed fc: less able students to grasp concepts. Many bright students already had mastery of the issues."
>
> (teacher B)

Statement 10 : The materials are better suited to middle-ability students.

A mean score of 3.2 was accompanied by only one (favourable) response:

> 'Certainly the materials are suited to the middle and low-ability students."
>
> (teacher H)

Statement 11 : The materials are better suited to low-ability students.

A mean score of 2.9 indicated a slight tendency towards disagreement with this statement although most of the subsequent comments were favourable:

> "My class was middle to low-abilities. They seemed to find the work presented at the correct levels.
>
> (teacher F)

and

> "Any and all could benefit from this. I don't think abilty level would matter."
>
> (teache, C)

One teacher was adamant that some aspects of the program would prove demanding for low-ability students:

> "Some parts would be too difficult,

e.g. workshop for lesson 4."
(teacher A)

Statement 12 : I would readily use the teaching materials again.

A mean score of 3.2 and the many varied associated comments suggested that some teachers found the materials more useful than others:

"Certainly"
(teacher C)

and

"Clearly some of them."
(teacher E)

compared with:

"I would modify them slightly
to suit our own teaching styles.
The lessons gave us some very
good ideas."
teacher H)

and

"Unfortunately, time would need to
be spent in front of a photocopier
to reproduce materials."
(teacher B)

Statement 13 : The teaching materials were useful for students in my class.

A mean score of 3.8 pointed to a positive response to this statement. Only one (negative) comment was received:

"Too many materials on the whole." (teacher F)

Statement 14 : I was surprised at the extent of revealed student misconceptions.

A mean score of 3.2 was accompanied by comments which suggested that some teachers were very surprised with the revealed misconceptions whilst others responded as if they were expecting these:

> "Most surprised since initially I was doubting a couple of them myself. I had never thought of some of the examples with respect to parallel lines."
> (teacher C)

compared with

> "No because I expected them."
> (teacher F)

Statement 15 : I frequently use group work/discussions in my mathematics class.

A mean score of 2.9 suggested once again how different teachers use different organisational strategies and teaching styles within the mathematics classroom:

> "Probably more on a whole class basis rather than in small groups."
> (teacher H)

compared with:

> "Occasionally"
> (teacher E)

and

> "most of the time. The first lesson on parallel lines was one of the best I have seen."
> (teacher C)

Statement 16 : I felt comfortable using group work in these 4 lessons.

The mean score of 3.8 was perhaps higher than we expected since statement 15 pointed to only <u>some</u> teachers using group work. Comments associated with statement 16 suggested that many of the teachers could adapt to group work when it was deemed appropriate:

"I found group work worked well."
(teacher H)

and

"Not a problem."
(teacher C)

One teacher felt that group work generally did not prove advantageous:

"I felt some (students) were easily sidetracked."
(teacher F)

DISCUSSION

The four phases of action research described in this paper were planned responses from our awareness that conventional teaching strategies, which tend to emphasise expository methods, might not be successful in bringing about long-term understanding of mathematical concepts for the majority of students. The investigation involved the development of geometry teaching materials by the two investigators and trialling with twelve secondary school mathematics teachers and approximately four hundred Year 8 students.
Feedback, in terms of cognitive and affective learning outcomes was sought from these teachers and students.

It is important in this kind of curriculum development that any teaching materials and strategies devised for classroom use should be intelligible, useful and enjoyable to both teacher and learner. The broad-based survey employed in phase 4 of the investigation showed that teachers generally considered that they and their students found the materials easy to use (statements 2 and 3), useful (statement 13) and "enjoyable" (statements 6 and 7).

Some doubts were expressed about the amount of time allocated to teaching aspects of parallel lines, corresponding and alternate angles (statement 5) and associated comments brought into focus the competition between an expanding mathematics syllabus and the need to teach less content in mathematics with a greater depth of understanding.

The use of group work/discussion played an important part within the overall teaching program and although teachers generally stated that they felt comfortable with this approach (statement 16) it was evident that some teachers had rarely been involved with group work (statement 15) whilst those who had readily acknowledged that other teachers might experience some difficulties with group work (statement 8).

The notion of regression in learning is not new (Strauss, 1982; Happs, 1985) and the development of specific teaching materials and strategies described in this investigation was influenced by the need to address this problem. Tim and Alan, the teachers involved in phase 3 of the project felt that students they had taught previously by "conventional" means appeared to have remembered little of their geometry . Both teachers intuitively felt that this constructivist approach would be more likely to result in long-term retention and our delayed (6 months) post-testing appeared to substantiate this. Students involved in phase 3 considered that they had "learned a lot" from their experience and cognitive outcomes from the ten classes involved in phase 4 of the investigation are currently being examined . Preliminary analysis appears encouraging.

Current learning theory strongly suggests that both the learner's prior knowledge (in the form of conceptual frameworks) and the complex information-processing abilities which they are likely to draw upon when attempting to understand new information means that learners will strive to generate links between new information and their existing knowledge and understanding. Unfortunately those links and resultant learning outcomes are not always the ones teachers aim for.

Despite the many positive perceptions documented from teachers and students throughout this investigation it is evident that learning outcomes, following the deployment of any teaching strategies, will be diverse and not always favourable. In phase 3 of our investigation Charmayne hardly changed her understanding at all whilst Brett reverted back to thinking that parallel lines have to be directly opposite one another.

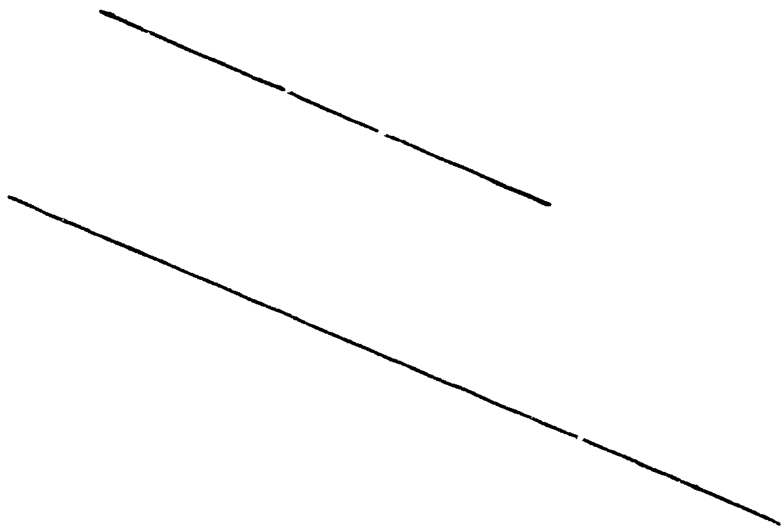
Billy and Jason continued to "identify" parallel lines via intuition about distances between lines rather than accurate measurement whilst Aaron was reluctant to adopt the newly introduced file-card technique for measurement. Consequently, Aaron continued to incorrectly use his ruler to measure the distances between straight lines.

Although the preliminary results from this investigation might be regarded with some optimism in terms of learning outcomes from the strategies employed, it is readily acknowledged by the investigators that these results have not been directly compared with learning outcomes via other teaching strategies. Such comparisons might be inappropriate since curriculum developers and teachers hope that learning outcomes from innovative teaching strategies will prove beneficial to the majority of students yet, time and again, it is found that such outcomes are rarely homogeneous or predictable.

Clearly, different students are likely to respond in various (and often unanticipated) ways to different teaching strategies because of the complexity of the teaching/learning situation and the many factors involved in the individual's construction of meaning. We need to better understand these factors and take them into account when planning teaching strategies.

APPENDIX A

item 1



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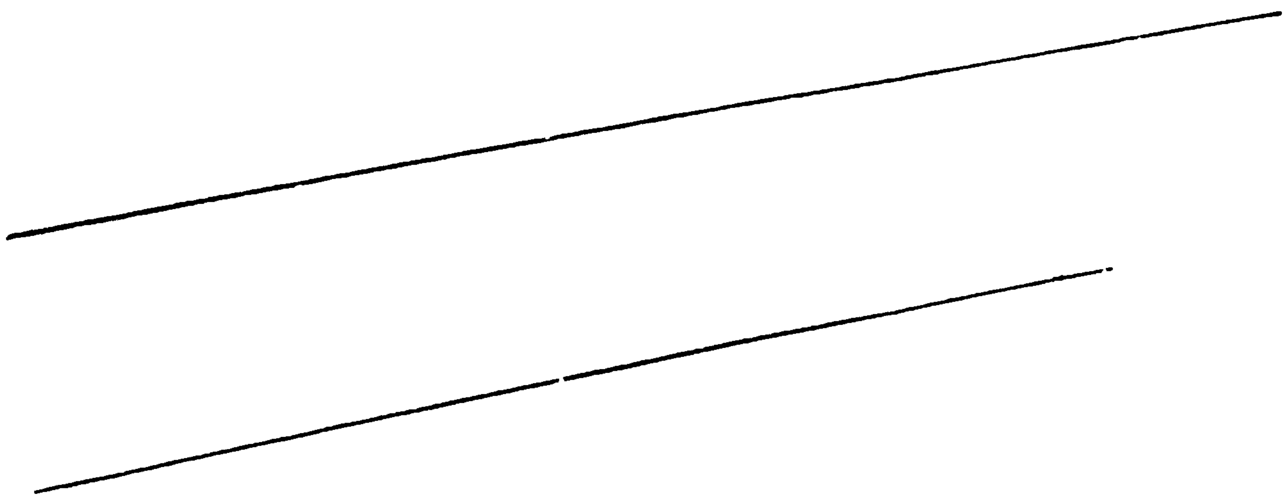
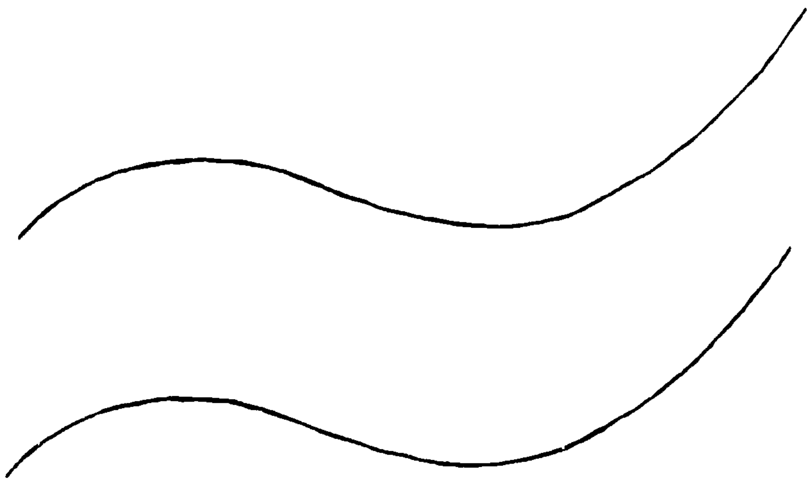


item 3

item 4



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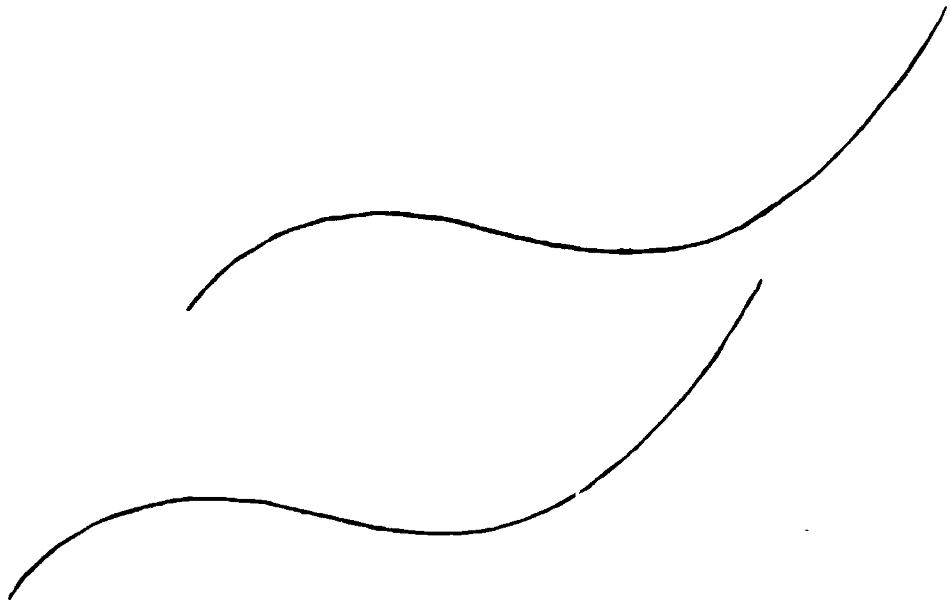

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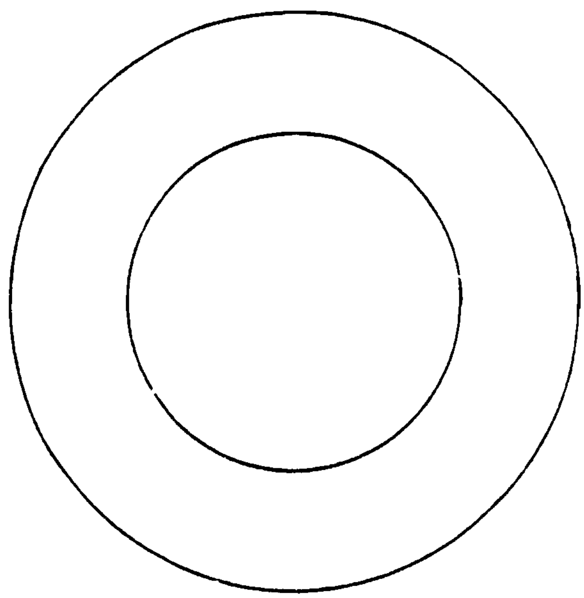
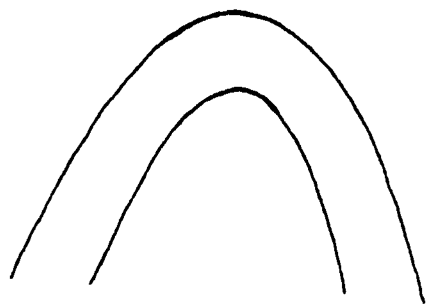


item 7



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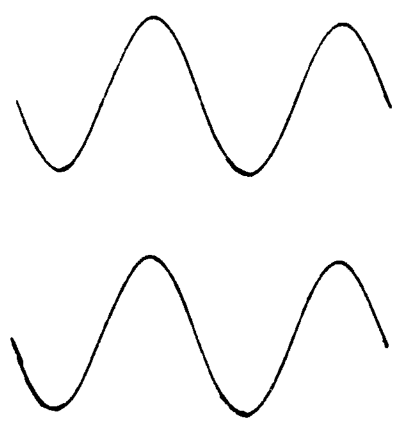

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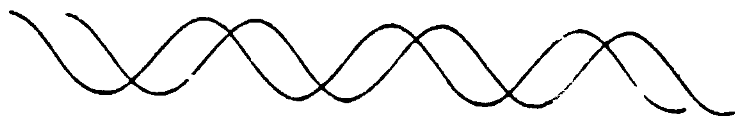

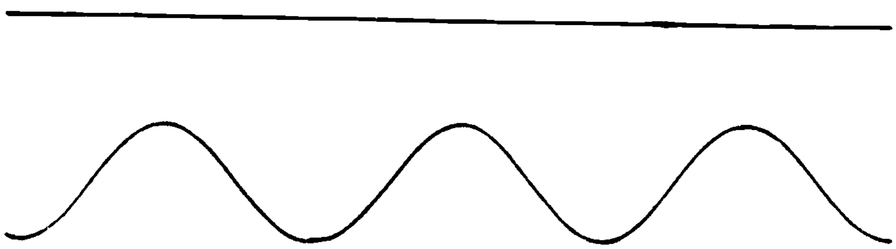

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