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AUTHOR	Bisanz, Jeffrey
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ABSTRACT

The cutoff method was used on longitudinal data in more than one content domain in a study attempting to determine whether the effects of schooling are general or limited. Conservation of number, an informally acquired skill, and mental addition, a formally acquired skill, were evaluated among older kindergarten children, younger 1st-grade children, and older 1st-grade children. Older kindergarten children and younger 1st-grade children were about the same age; younger and older 1st-grade children had the same amount of schooling. The conservation data indicated that improvements in performance were due primarily to age rather than schooling, and improvements in the accuracy of mental addition were due primarily to schooling rather than age. This pattern of results has three implications: (1) young and old children are equally ready to benefit from the kinds of counting and arithmetic operations that are emphasized in school; (2) schooling seems to have relatively specific effects on performance; and (3) the cutoff method can be quite useful for discovering which kinds of skills are acquired as a function of schooling or age. and for addressing questions about children's readiness for acquiring certain kinds of knowledge. It is concluded that as results accumulate, priority should be given to the construction of a coherent picture of the effects of early schooling on cognitive and academic development. (RH)

Development of Arithmetic Computation and Number Conservation Skills

Jeffrey Bisanz

University of Alberta

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Development of Arithmetic Computation
and Number Conservation Skills

The "cutoff" method developed by Morrison and illustrated in the previous two papers is useful for evaluating the effects of age and schooling on cognitive development and achievement. When we refer to effects of "age", we really mean effects associated with age; we do not mean to imply that age per se causes anything. The cutoff method allows us to separate effects associated with age from the effects associated with schooling if we accept the assumption that children with birthdates one month after the cutoff date for entry into school do not differ significantly in cognitive development from children with birthdates one month before the cutoff date. This assumption seems reasonable, and it allows us to investigate several important questions about cognitive development and schooling.

One such question is whether the effects of schooling are general or whether they are limited to specific skills or content domains. The issue of general versus domain-specific learning is central in developmental and instructional research. By using the cutoff method and examining longitudinal data in more than one content domain, as we are doing in this paper session, we can begin to address this issue.

In the research I have conducted in collaboration with Fred Morrison, Lisa Smith, and Maria Dunn, we have used the cutoff method to investigate the effects of early schooling on the development of elementary mathematical skills. Some of these skills are acquired informally, whereas others are taught directly or practiced under supervision. We might expect that formally taught skills will develop primarily as a function of schooling, or that schooling and age might interact in a way that reflects a lack of "readiness" in younger children. Expectations about the effects of age and schooling on informally acquired skills are less obvious. These skills might be acquired as a function of age independently of schooling. Alternatively, formal instruction in school might generalize in such a way that makes acquisition of these skills possible. These possibilities were assessed in our study.

Two types of early quantitative skills were examined in "old" kindergarten children, "young" Grade 1 children, and "old" Grade 1 children. (I apologize if "old kindergarten and Grade 1 children" sounds like an especially ridiculous oxymoron; the label is convenient, if somewhat difficult to get used to.) First, conservation of number was selected as an example of a elementary skill that is acquired informally. Children show markedly different solution strategies, including counting-based solutions, logical and nonnumerical solutions, and solutions based on irrelevant aspects of the task. Moreover, the reasoning that is appropriate for conservation problems typically is not taught directly in school and develops over an extended time period.

Second, mental addition was selected as an example of an elementary skill that is taught formally. Children often show considerable improvement in the speed and accuracy of performance in the early school years, as well as changes in the procedures used. Moreover, mental addition is a skill that often is taught directly or practiced under close supervision.

To briefly review the method we used, we tested 17 to 19 children in each of three groups, as indicated in Figure 1. The old Kindergarten children and the young Grade 1 children were approximately the same age, and the young Grade 1 children and the old Grade 1 children had the same amount of schooling. The children were tested once in the Fall and once in the Spring. In each case, 25 addition problems were presented in which the addend and augend varied from 1 to 5. Children were asked to solve each problem and to describe how they got the answer. Near the middle of this set, two conservation-of-number problems were administered, and another two were presented at the end of the set. I shall begin by describing the conservation task and results.

Insert Figure 1 about here

Conservation

Four conservation problems were administered to each child. An example is provided in Figure 1. In this case, five objects were presented in each of two rows, as shown at Time 1, and the child was asked whether each row had the same, or whether one had more than the other. Then the interviewer transformed one of the two rows so that it became longer, as shown at Time 2. Again the question about the relative quantity of the two rows was asked. Set size and transformation were combined orthogonally so that half the problems had 5 objects and half had 9, and half involved spreading or extending one row and half involved scrunching or condensing one row. As it turns out, these manipulations had no effect on any measure of performance. Children's responses were videotaped so that we could analyze children's judgments of conservation and the justifications they used.

Insert Figure 2 about here

Several different criteria can be used for judging whether children show conservation. The first and most lenient criterion is whether children answered "same" to the question about relative quantity <u>after</u> the spreading or scrunching transformation. Children who were not influenced by this irrelevant transformation responded "same", just as they did to the first question, and so they were judged to have shown conservation of number. If a child responded "different" to the second question, he or she failed to conserve number. The implication is that the child's judgments of number were influenced by a nonquantitative transformation. Such children often responded on the basis of the width or density of the row, aspects that are irrelevant to number. Note that with this criterion, we did not care how children justified their judgments; we considered only the judgment itself.

The results obtained with this criterion are illustrated in Figure 3. Time of test is on the abscissa, and proportion of "same" responses, independent of the justification, is on the ordinate. O1 refers to old Grade 1 children, Y1 refers to young Grade 1 children, and OK refers to old Kindergarten children.

Insert Figure 3 about here

Two aspects of these data are of special interest. First, old Grade 1 children performed better than the other two groups, and these latter two did not differ at all. This pattern is what would be expected if age, rather than schooling, were the critical factor: The old Grade 1 children were about a year older than children in the other two groups, who were about the same age as each other.

Second, the improvement associated with time of test was quite striking. At first glance, this result might imply that schooling is indeed an important influence on performance, but we must remember that children, like the rest of us, grow older between Fall and Spring. The fact that age is the more important factor is evident in Figure 4, in which the same six data points from Figure 3 are plotted as a function of median age. The open symbols are Fall data points, the dark symbols are Spring data points, and the shapes correspond to the three groups. These data correspond with age very nicely; in fact, in a linear regression age accounts for 94% of the variance. Adding time of test to the regression equation accounts for an additional and nonsignificant 2% of the variance, and adding grade accounts for an additional and nonsignificant 3% of the variance. Thus performance on number conservation improves linearly with age; schooling is relatively unimportant.

Insert Figure 4 about here

The conclusion is much the same when we use a more stringent criterion for assessing conservation of number. With the lenient criterion described above, it is possible for a child to respond "same" without really understanding the logical basis for conservation of number. (See Figure 2.) For example, a child might not be sure whether the number of objects in the two rows at Time 2 are the same after a transformation. To make a judgment, the child might <u>count</u> the two rows and respond accordingly. If the child counts correctly, he or she would respond "same" and, by the lenient criterion, this judgment would be classified as a conservation response. According to Piaget and others, however, if a child really understands conservation of number, then counting or any other computational procedure is completely unnecessary; a conserver is able to infer logically that the number has not changed because he or she knows that the transformation cannot affect number. When these children justify their judgments, they do not refer to counting or appearances; instead, they explain their answers in terms of compensation (e.g., that one row looks longer but the objects are just spaced out more) or inversion (e.g., that the two rows were the same before, and so that must be the same now). According to Piaget, these two types of justification reflect reversible thinking, which is the hallmark of concrete operational thought.

To determine whether using this more stringent criterion of conservation would influence our conclusions, we examined "same" responses that were accompanied by reversibility justifications. The results are presented in Figure 5.

Insert Figure 5 about here

The pattern of results using the more stringent criterion is very similar to the results with the more lenient criterion. Note that the old Grade 1 children perform better than the other two groups, which are indistinguishable from each other. Again, age seems to be the critical factor, and again this point is most evident when performance is plotted by age, as in Figure 6. In this case, age accounts for 97% of the variance. Adding grade to the equation accounts for an additional and nonsignificant 1% of the variance.

Insert Figure 6 about here

The conclusion, then, is that acquisition of number conservation is associated with age and is largely independent of schooling.

Mental Addition

I now discuss the mental addition data, but only briefly. One attraction of conducting this research is to examine changes in the addition procedures used by young children over the course of a year of school, but developing a coding scheme to identify children's procedures reliably has turned out to be more difficult than we anticipated. At this point I am only willing to discuss children's accuracy, but in the near future we will be addressing more interesting questions about the solution procedures children select.

The accuracy data are presented in Figure 7. In contrast to the conservation data, accuracy on addition problems appears to be almost entirely a function of schooling: Grade 1 children, old and young, do considerably better than kindergarten children. Thus the performance of young Grade 1 children does not appear to have been hampered significantly as a result of entering school at an early age.

Insert Figure 7 about here

Note that all three groups show about the same rate of change during the course of the school year. We must be cautious about interpreting the apparent similarity of slopes here, however, because differences in accuracy rates are not necessarily equivalent at all points of the scale. For example, the 14% increase shown by old kindergarten children involves mastery of problems that are, in general, easier than those mastered by the old Grade 1 children who showed a nearly equivalent increase of 16%. So in one sense it would be inappropriate to conclude that the two age groups made equivalent progress during the year. Moreover, the performance of old Grade 1 children in the Spring is nearly perfect, so that any potential for greater improvement could not be detected.

Conclusions

To summarize, the conservation data show that improvements in performance are due primarily to age rather than to schooling, whereas improvements in the accuracy of mental addition are due primarily to schooling rather than to age. This pattern of results has three implications.

First, it appears that young and old children were equally "ready" to benefit from the kinds of counting and arithmetic operations that are emphasized in school, as indicated by the data on accuracy of addition. Whether this same conclusion holds for the actual procedures children use remains to be seen. It is possible, for example, that the three groups differed in how advanced their procedures were in solving these problems. We should have answers to this question soon.

Second, schooling seems to have relatively specific effects on performance, as indicated by the fact that conservation skills improved independently of schooling. This result contrasts markedly with Smith's data, which showed clear effects of schooling and little or no effect of age. Our finding with conservation is somewhat surprising because many researchers believe that counting and arithmetic operations provide a basis for understanding the conservation of number. It appears, then, that instruction does not generalize effectively to a related but different domain of skill.

Third, the cutoff method clearly can be quite useful for discovering which kinds of skills are acquired as a function of schooling or age, as well as for addressing questions about children's readiness for acquiring certain kinds of knowledge. As more results accumulate, the important task will be to construct a coherent picture of the effects of early schooling on cognitive and academic development.

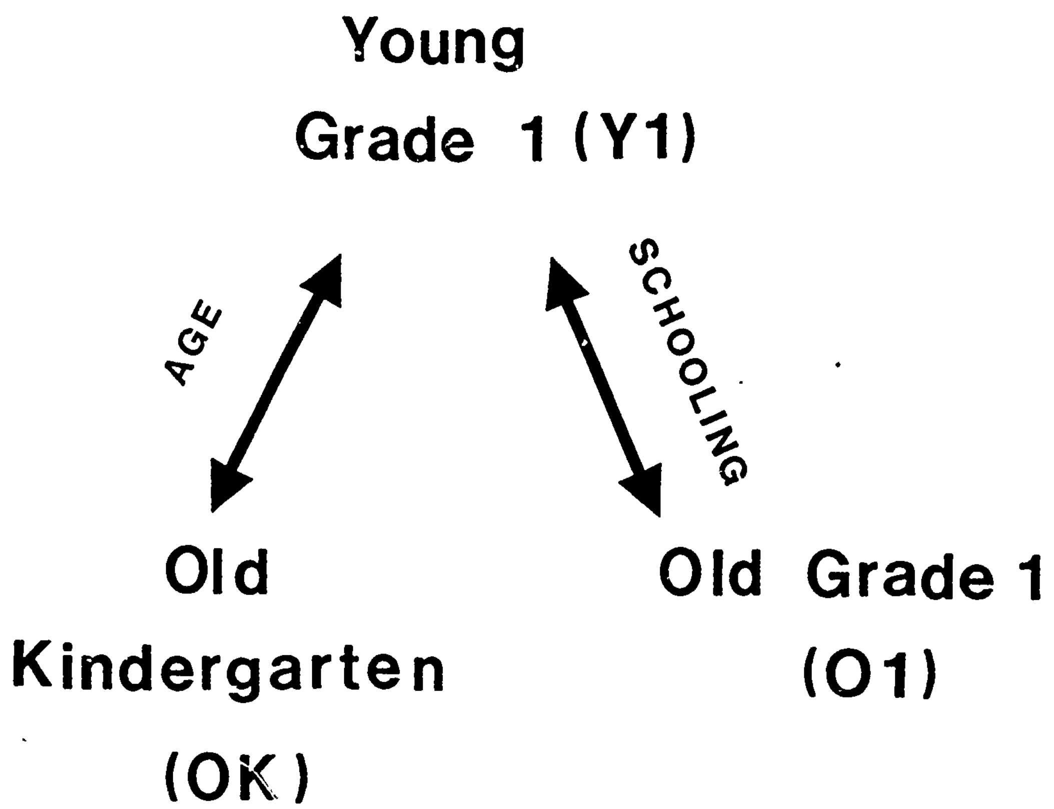

Figure 1

Figure 1

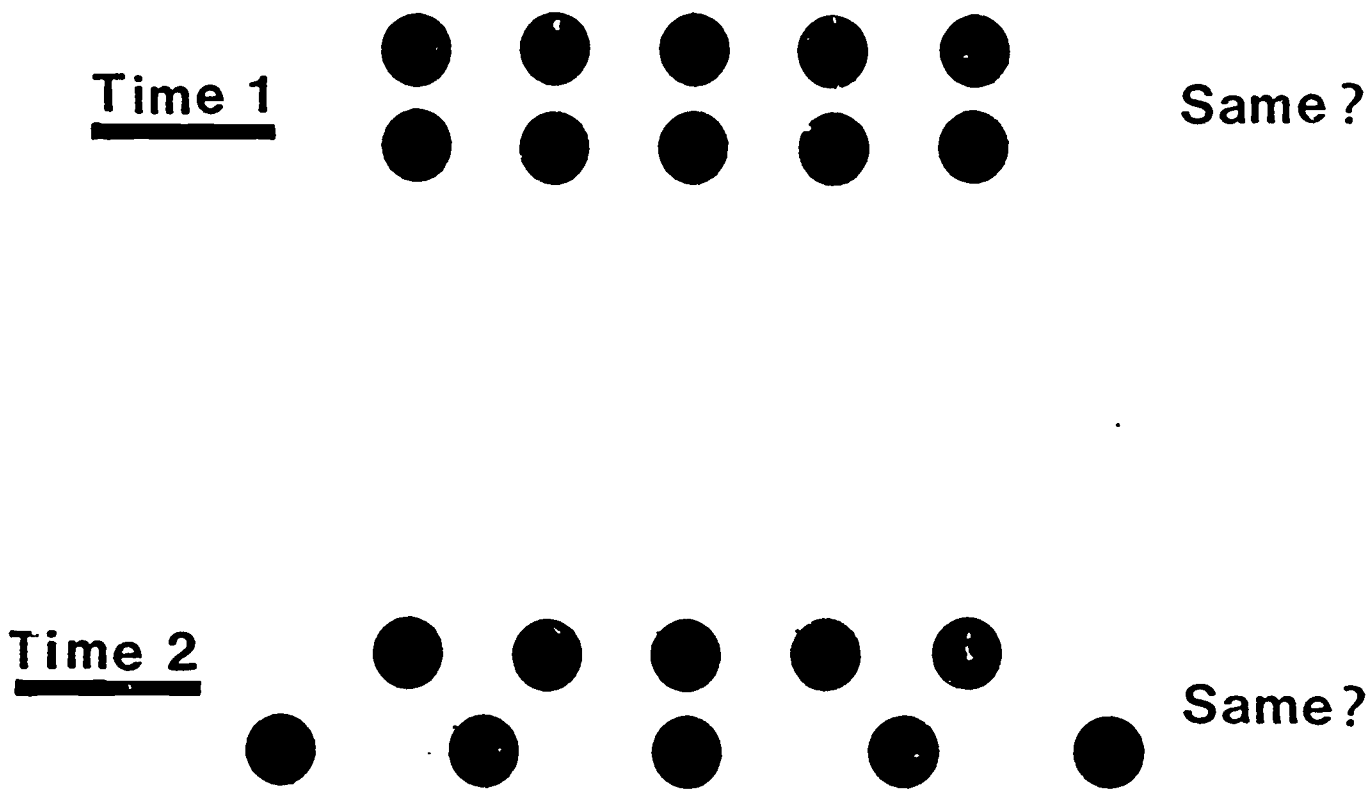

Figure 2

Mean Proportion of Same Judgments
with Any Justification

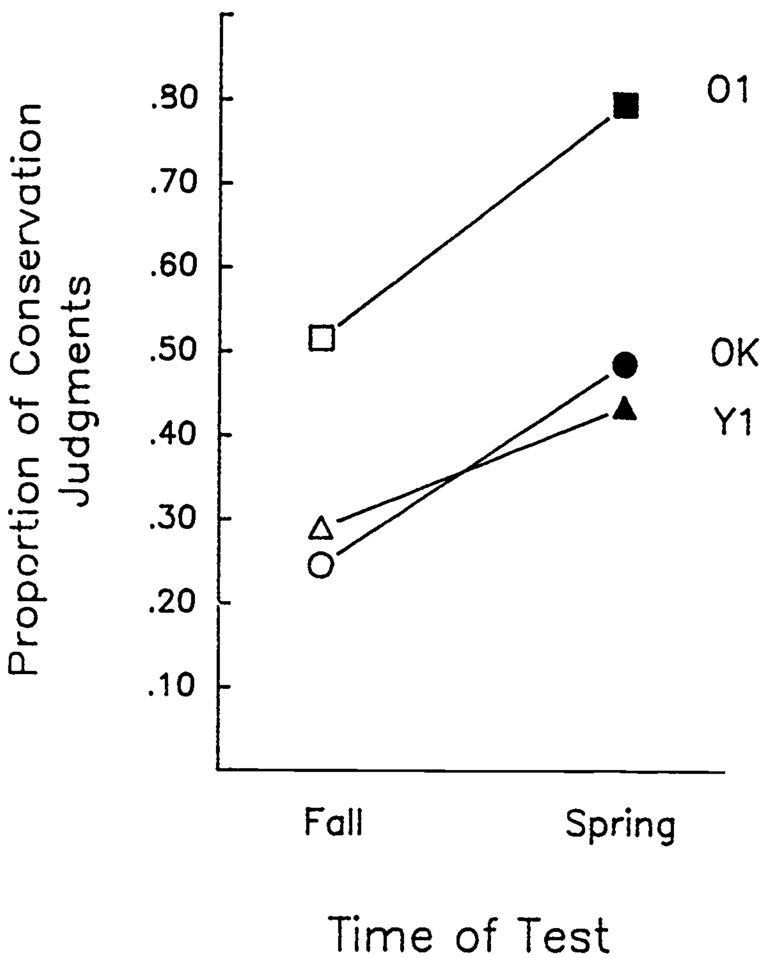

Figure 3

Mean Proportion of Same Judgments with Any Justification as a Function of Age

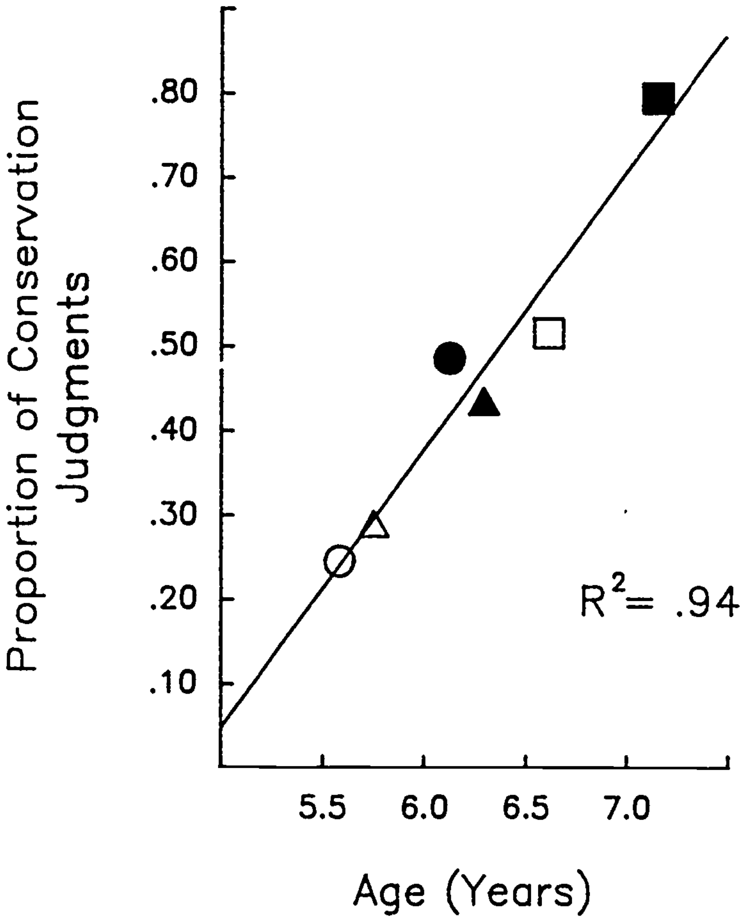

Figure 4

Mean Proportion of Same Judgments
with Reversibility Justifications

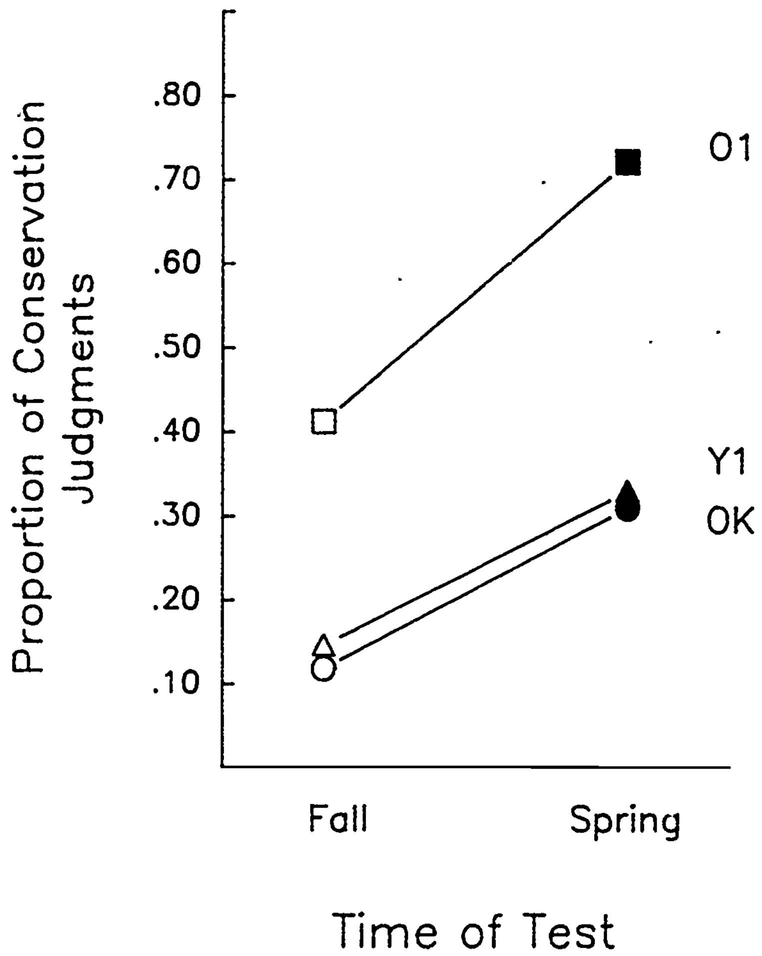

Figure 5

Mean Proportion of Same Judgments with Reversibility Justifications as a Function of Age

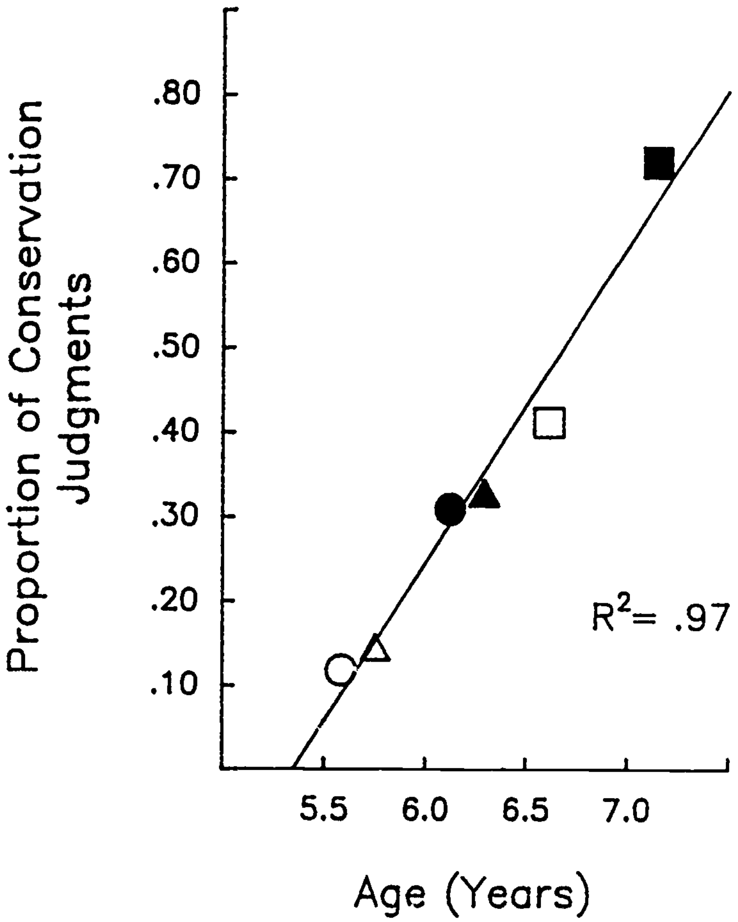

Figure 6

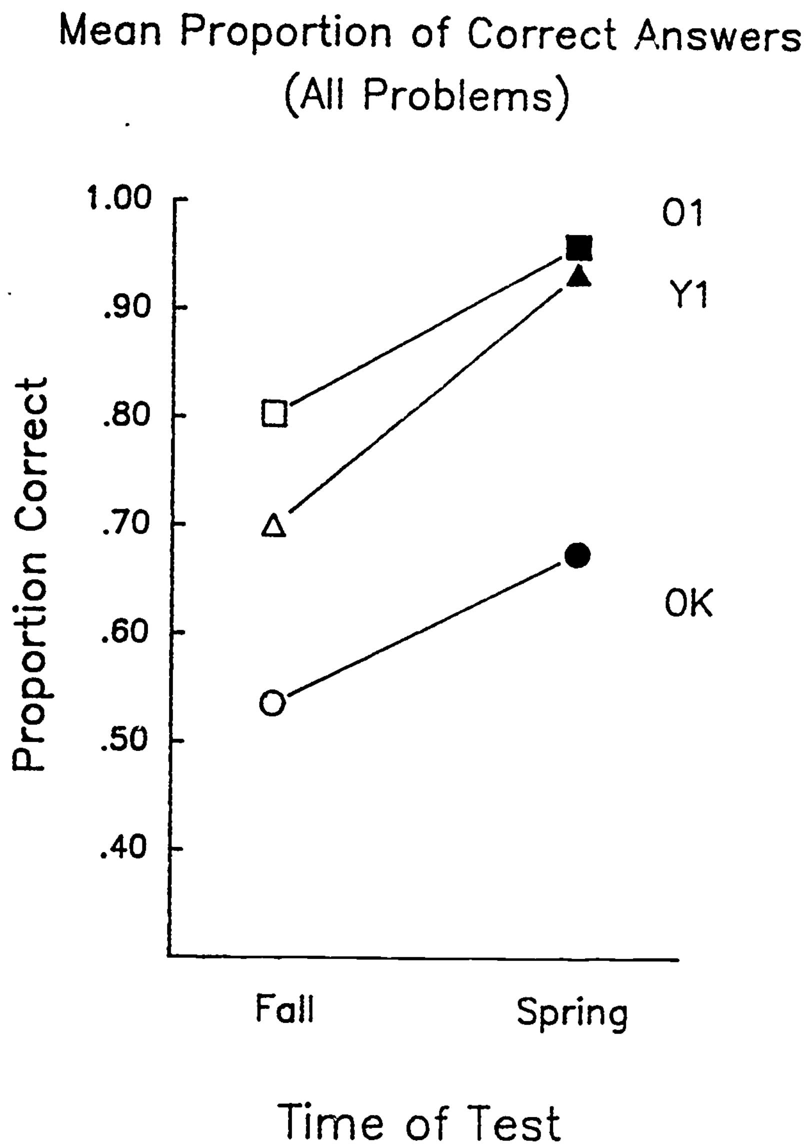
Mean Proportion of Correct Answers
(All Problems)
1.00
.90
.80
.70
.60
.50
.40
Proportion Correct
O1
Y1
OK
Fall
Spring
Time of Test

Figure 7