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ABSTRACT

Students make errors in mathematics for a variety of reasons. It is the job of the educator to identify the source of these errors and to assist students in correcting them. The purpose of this monograph is to help identify, analyze, and evaluate students' computational proficiency. This monograph is divided into four sections: "Whole Numbers"; "Fractions"; "Decimals"; and "Percents." Each section is arranged according to specific computational skills. Each computational skill is examined via the following components: (1) skill (giving the specific computational skill to be examined; (2) example (offering a sample problem that could lead to typical student errors); (3) entry conditions (listing the mathematical prerequisites for mastery of the skill being examined); (4) correct solution (showing correct answer to the given sample); (5) errors, identification and analysis (highlighting typical student errors and analyzing their possible source); and (6) practice (providing problems similar in scope to the example). (YP)

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Eve Lewis, Senior Marketing and Acquisitions Manager
David Otte, Editor

FOCUS ON COMPUTATIONAL ERRORS for Teachers of Basic Skills Mathematics Programs

by

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TABLE OF CONTENTS

INTRODUCTION

Students make errors in mathematics for a variety of reasons. Students who are deficient in basic mathematical skills will be plagued by this disability at home, at school, in the workplace, and at the marketplace. It is the job of the educator to identify the source of these errors and to assist students in correcting them. The purpose of this monograph is to help you to identify, analyze, and evaluate your students' computational proficiency.

This monograph is divided into four sections:

1. Whole Numbers
2. Fractions
3. Decimals
4. Percents

Each section is arranged according to specific computational skills. Each computational skill is examined via the following components:

1. **Skill**—gives the specific computational skill to be examined.
2. **Example**—offers a sample problem that could lead to typical student errors.
3. **Entry Conditions**—lists the mathematical prerequisites for mastery of the skill being examined.
4. **Correct Solution**—shows correct answer to the given sample.
5. **Errors: Identification and Analysis**—highlights typical student errors and analyzes their possible source.
6. **Practice**—provides problems similar in scope to the example.

This monograph, when used in conjunction with a comprehensive, motivating basic mathematics curriculum, will help your students improve their skills and appreciate the usefulness of these skills.

SECTION 1 WHOLE NUMBERS

Skill 1.1 Addition of Whole Numbers

Example

$$\begin{array}{r} 39 \\ +\ 47 \\ \hline \end{array}$$

Entry Conditions Single-digit addition facts, place value.

Correct Solution

$$\begin{array}{r} {}^{1}39 \\ +\ 47 \\ \hline 86 \end{array}$$

Errors: Identification and Analysis

A.

$$\begin{array}{r} 39 \\ +\ 47 \\ \hline 716 \end{array}$$

By not regrouping 16 ones into 1 ten and 6 ones, the student appears to lack an understanding of place value. Reteach the concept of place value and then use vertically lined paper or graph paper for addition problems. This organizational technique helps the students to align place values.

B.

$$\begin{array}{r} {}^{6}39 \\ +\ 47 \\ \hline 131 \end{array}$$

This error results from either carelessness or a lack of understanding of place value. The student has reversed the tens' and the ones' places in the regrouping (carrying) process. The remedies mentioned in A would also apply here.

Practice

(1)	(2)	(3)	(4)	(5)
$\begin{array}{r} 67 \\ +\ 38 \\ \hline \end{array}$	$\begin{array}{r} 84 \\ +\ 29 \\ \hline \end{array}$	$\begin{array}{r} 95 \\ +\ 56 \\ \hline \end{array}$	$\begin{array}{r} 36 \\ +\ 87 \\ \hline \end{array}$	$\begin{array}{r} 21 \\ +\ 49 \\ \hline \end{array}$
(105)	**(113)**	**(151)**	**(123)**	**(70)**

Skill 1.2 Subtraction of Whole Numbers with Regrouping

Example

$$\begin{array}{r} 87 \\ -\ 28 \\ \hline \end{array}$$

Entry Conditions Single-digit subtraction facts, place value, regrouping (borrowing).

Correct Solution

$$\begin{array}{r} \overset{7}{\cancel{8}}{}^{1}7 \\ -\ 28 \\ \hline 59 \end{array}$$

Errors: Identification and Analysis

A.

$$\begin{array}{r} 87 \\ -28 \\ \hline 61 \end{array}$$

The student instinctively subtracts the smaller digit from the larger digit regardless of their positions in the subtraction problem. Encourage the student to estimate the difference before subtracting. An error of this type also indicates a lack of understanding of place value and regrouping.

B.

$$\begin{array}{r} 8\,{}^{1}7 \\ -28 \\ \hline 69 \end{array}$$

The student recognizes a need for regrouping but incorrectly completes the process. The error made here usually results in an answer that is ten greater than it should be. Give the student practice in regrouping and estimating differences. For example, 80 + 7 = 70 + 17. By not decreasing the tens' place in the regrouping process, the student has incorrectly implied that 87 = 80 + 17.

Practice

(1) $\begin{array}{r} 43 \\ -37 \\ \hline \end{array}$	(2) $\begin{array}{r} 52 \\ -29 \\ \hline \end{array}$	(3) $\begin{array}{r} 74 \\ -58 \\ \hline \end{array}$	(4) $\begin{array}{r} 92 \\ -27 \\ \hline \end{array}$	(5) $\begin{array}{r} 65 \\ -59 \\ \hline \end{array}$
(6)	**(23)**	**(16)**	**(65)**	**(6)**

Skill 1.3 Subtraction of Whole Numbers Involving Zero

Example

$$\begin{array}{r} 130 \\ -\ \ 18 \\ \hline \end{array}$$

Entry Conditions Single-digit subtraction facts, place value, regrouping.

Correct Solution

$$\begin{array}{r} \scriptstyle 2\ \ 1\ \\ 1\cancel{3}0 \\ -\ \ 18 \\ \hline 112 \end{array}$$

Errors: Identification and Analysis

$$\begin{array}{r} 130 \\ -\ \ 18 \\ \hline 128 \end{array}$$

This common error occurs when there is a zero in the ones' place of the subtrahend. To the student, it is easier to subtract zero from a number than it is to subtract a number from zero. Have the students estimate the difference and review their knowledge of place value and regrouping.

Practice

(1)	(2)	(3)	(4)	(5)
$\begin{array}{r} 340 \\ -\ \ 88 \\ \hline \end{array}$	$\begin{array}{r} 470 \\ -\ 127 \\ \hline \end{array}$	$\begin{array}{r} 100 \\ -\ \ 67 \\ \hline \end{array}$	$\begin{array}{r} 780 \\ -\ 543 \\ \hline \end{array}$	$\begin{array}{r} 20 \\ -\ \ 8 \\ \hline \end{array}$
(252)	**(343)**	**(33)**	**(237)**	**(12)**

Skill 1.4 Multiplication of Whole Numbers Using Partial Products

Example

$$\begin{array}{r} 38 \\ \times\ 54 \\ \hline \end{array}$$

Entry Conditions Basic multiplication and addition facts, place value.

Correct Solution

$$\begin{array}{r} {}^{4\,3}38 \\ \times\ 54 \\ \hline 152 \\ 190 \\ \hline 2052 \end{array}$$

Errors: Identification and Analysis

A.

$$\begin{array}{r} 38 \\ \times\ 54 \\ \hline 152 \\ 190 \\ \hline 342 \end{array}$$

The student mistakenly believes that the product can be determined by multiplying 38 by 4 and 38 by 5 and then adding those two products together. Here the student does not recognize that multiplying by the digit 5 is in fact a multiplication by 50. The resulting partial product must end in a zero since the product of any number and a multiple of 10 always has a zero in the ones' place. The student may believe that the zero is merely a placeholder and not recognize its significance in the multiplication process.

B.

$$
\begin{array}{r}
38 \\
\times\ 54 \\ \hline
1232 \\
1540 \\ \hline
16632
\end{array}
$$

By not regrouping (carrying), the student appears to lack an understanding of place value. Reteach the concept of place value and then use vertically lined paper or graph paper for multiplication problems. This organizational technique helps the students to align place values.

C.

$$
\begin{array}{r}
{}^{2}38 \\
\times\ 54 \\ \hline
143 \\
154 \\ \hline
1683
\end{array}
$$

This error results from either carelessness or a lack of understanding of place value. The student has reversed the tens' and the ones' places in the regrouping (carrying) process. The remedies mentioned in **B** would also apply here.

Practice

(1)	(2)	(3)	(4)	(5)
$\begin{array}{r} 67 \\ \times\ 81 \\ \hline \end{array}$	$\begin{array}{r} 26 \\ \times\ 13 \\ \hline \end{array}$	$\begin{array}{r} 79 \\ \times\ 26 \\ \hline \end{array}$	$\begin{array}{r} 23 \\ \times\ 89 \\ \hline \end{array}$	$\begin{array}{r} 156 \\ \times\ 45 \\ \hline \end{array}$
(5427)	**(338)**	**(2054)**	**(2047)**	**(7020)**

Skill 1.5 Division of Whole Numbers

Example

$$7\overline{)7147}$$

Entry Conditions Basic division, multiplication, and subtraction facts; knowledge of place value.

Correct Solution

```
   1021
7)7147
  7↓↓ |
  014 |
   14 ↓
    07
     7
     0
```

OR

```
   1021
7)7147
```

Errors: Identification and Analysis

```
   121
7)7147
  7
  014
   14
    07
     7
     0
```

The student shows an inability to estimate that the quotient should be close to 1,000. Offer the student a simulated distribution of 7,147 dollars among 7 people. After the 7 one-thousand-dollar bills have been distributed (each person getting 1), the student will see that the single one-hundred-dollar bill is useless and cannot be distributed until it is changed or regrouped into 10 ten-dollar bills. No one received a hundred-dollar bill, and this is represented by a zero in the hundreds' place of the quotient. When students have a realistic referent they are less likely to make this common division error. Also point out to the students that there should be a number in the quotient directly above each number of the dividend into which they are dividing.

Practice

(1) 3035 – 5 = **(607)**

(2) 5663 – 7 = **(809)**

(3) 1204 – 4 = **(301)**

(4) 840 8 = **(105)**

(5) 8136 – 9 = **(904)**

Skill 1.6 Division of Whole Numbers by Two-Digit Divisors

Example

$$16\overline{)1360}$$

Entry Conditions Basic division, multiplication, and subtraction facts; knowledge of place value.

Correct Solution

$$\begin{array}{r} 85 \\ 16\overline{)1360} \\ \underline{128} \\ 80 \\ \underline{80} \\ 0 \end{array}$$

Errors: Identification and Analysis

$$\begin{array}{r} 715 \\ 16\overline{)1360} \\ \underline{112} \\ 24 \\ \underline{16} \\ 80 \end{array}$$

An arithmetical error is compounded by the student's failure to estimate that the quotient should be somewhat less than 100. After subtracting 7 groups of 16 from 136, 24 is still left. The student should then realize that 16 will go into 136 a total of 8 times instead of 7.

Practice

(1) 2596 – 11 = **(236)**

(2) 14448 – 24 = **(602)**

(3) 5598 – 18 = **(311)**

(4) 8890 – 35 = **(254)**

(5) 16983 – 17 = **(999)**

SECTION 2 FRACTIONS

Skill 2.1 Simplifying Fractions

Example

Simplify (in lowest terms) $\frac{24}{36}$

Entry Conditions Basic division and multiplication facts, factors and factorization, prime numbers, greatest common factor, relatively prime numbers.

Correct Solution

$$\frac{\cancel{24}^{\,2}}{\cancel{36}_{\,3}} = \frac{2}{3}$$

Errors: Identification and Analysis

$$\frac{\cancel{24}^{\,4}}{\cancel{36}_{\,6}} = \frac{4}{6}$$

The student recognizes that 24 and 36 share the common factor of 6, and correctly simplifies the fraction by removing the common factor of 6 from each. But the student has not rewritten this fraction in its simplest form. Students frequently stop here because they do not understand the concept of "relatively prime" and how it relates to simplifying a fraction to lowest terms. A fraction is simplified when the numerator and the denominator are said to be "relatively prime"; that is, they share only the common factor of 1. In this case, the problem is not in its simplest form because the numbers 4 and 6 still share a common factor of 2. Encourage students to write out a factor tree for both numbers so that they are able to see the factors that are held in common.

Practice

Rewrite each as an equivalent fraction in lowest form:

(1) $\frac{18}{36} =$ $(\frac{1}{2})$

(2) $\frac{20}{25} =$ $(\frac{4}{5})$

(3) $\frac{35}{49} =$ $(\frac{5}{7})$

(4) $\frac{14}{21} =$ $(\frac{2}{3})$

(5) $\frac{19}{38} =$ $(\frac{1}{2})$

Skill 2.2 Multiplication of Fractions by a Whole Number

Example

$$\frac{2}{3} \times 4 =$$

Entry Conditions Basic multiplication facts, factoring, finding common factors, finding the greatest common factor.

Correct Solution

$$\frac{2}{3} \times \frac{4}{1} = \frac{8}{3} = 2\frac{2}{3}$$

Errors: Identification and Analysis

$$\frac{2}{3} \times 4 = \frac{8}{12}$$

The student has multiplied both the numerator and the denominator by 4. In effect, $\frac{2}{3}$ has been multiplied by $\frac{4}{4}$ or 1. The student's result is a fraction that is equivalent to $\frac{2}{3}$. Encourage the student to estimate a solution. (Four groups of $\frac{2}{3}$ is more than 4 groups of $\frac{1}{2}$ and less than 4 groups of 1; therefore, the correct answer should be somewhere between 2 and 4.) Whenever such a problem is given, the student should immediately rewrite the whole number as a fractional equivalent. Any fractional equivalent will do ($4 = \frac{8}{2}, \frac{12}{3}$, etc.), but the simplest equivalent is $\frac{4}{1}$.

Practice

(1) $\frac{5}{10} \times 7 =$ $(3\frac{1}{2})$

(2) $\frac{2}{5} \times 8 =$ $(3\frac{1}{5})$

(3) $\frac{3}{18} \times 6 =$ (1)

(4) $\frac{7}{11} \times 11 =$ (7)

(5) $\frac{13}{24} \times 8 =$ $(4\frac{1}{3})$

Skill 2.3 Multiplication of Fractions Using Cancellation

Example

$$\frac{3}{5} \times \frac{6}{18} =$$

Entry Conditions Basic multiplication facts, factoring, finding common factors, finding the greatest common factor.

Correct Solution

$$\frac{\cancel{3}^{1}}{5} \times \frac{\cancel{6}^{1}}{\cancel{18}_{\cancel{3}\,1}} = \frac{1}{5}$$

Errors: Identification and Analysis

$$\frac{3}{5} \times \frac{6}{18} = \frac{\cancel{3}^{1}}{5} \times \frac{\cancel{6}^{2}}{18} = \frac{\cancel{3}^{1}}{5} \times \frac{\cancel{6}^{\cancel{2}^{1}}}{\cancel{18}_{9}} = \frac{1}{45}$$

The student has incorrectly simplified the fractions. Students should first be given a variety of problems to show them that

$\frac{3}{5} \times \frac{6}{18}$ is the same as $\frac{3}{18} \times \frac{6}{5}$

Common factors can be divided out along the diagonal as if those two numbers were numerator and denominator of a single fraction. Therefore, students should realize that they can remove any common factors as long as one of the factors is in the numerator and the other is in the denominator. The most common error occurs when students remove common factors from pairs of numerators and/or pairs of denominators. Show students that by so doing, they have created a completely different pair of fractions.

Practice

(1) $\frac{4}{12} \times \frac{7}{28} =$ $(\frac{1}{12})$

(2) $\frac{3}{22} \times \frac{11}{30} =$ $(\frac{1}{20})$

(3) $\frac{16}{48} \times \frac{22}{44} =$ $(\frac{1}{6})$

(4) $\frac{6}{18} \times \frac{5}{30} =$ $(\frac{1}{18})$

(5) $\frac{2}{3} \times \frac{15}{25} =$ $(\frac{2}{5})$

Skill 2.4 Multiplication Involving Mixed Numbers

Example

$3\frac{1}{2} \times 4\frac{2}{3} =$

Entry Conditions Basic multiplication facts, multiplication of fractions, simplifying fractions.

Correct Solution

$$3\tfrac{1}{2} \times 4\tfrac{2}{3} = \frac{7}{\cancel{2}_{1}} \times \frac{\cancel{14}^{7}}{3} = \frac{49}{3} = 16\tfrac{1}{3}$$

Errors: Identification and Analysis

$$3\tfrac{1}{2} \times 4\tfrac{2}{3} = 12\tfrac{1}{3}$$

The student has multiplied the whole number parts and the fractional parts separately. Show the students the following:

$$3\tfrac{1}{2} \times 4\tfrac{2}{3} = (3 + \tfrac{1}{2}) \times (4 + \tfrac{2}{3})$$

likewise, $14 \times 12 = (10 + 4) \times (10 + 2)$

If you follow the student's line of reasoning with the second problem, the product of 14 x 12 will be 100 + 8, or 108. But it isn't! Encourage students to rewrite all mixed numbers as improper fractions and then follow the rules for multiplication of fractions.

Practice

(1) $5\tfrac{1}{4} \times 2\tfrac{1}{7} =$ **($11\tfrac{1}{4}$)**

(2) $3\tfrac{2}{7} \times 5\tfrac{3}{5} =$ **($18\tfrac{2}{5}$)**

(3) $6\tfrac{1}{4} \times 8\tfrac{4}{5} =$ **(55)**

(4) $9\tfrac{4}{5} \times 3\tfrac{1}{7} =$ **($30\tfrac{4}{5}$)**

(5) $4\tfrac{6}{7} \times 5\tfrac{1}{6} =$ **($25\tfrac{2}{21}$)**

Skill 2.5 Division of Proper Fractions

Example

$$\frac{3}{8} - \frac{5}{16} =$$

Entry Conditions Basic multiplication and division facts, multiplication of fractions, simplifying fractions, finding a reciprocal.

Correct Solution

$$\frac{3}{8} \div \frac{5}{16} = \frac{3}{\cancel{8}_1} \times \frac{\cancel{16}^2}{5} = \frac{6}{5} = 1\frac{1}{5}$$

Errors: Identification and Analysis

A.

$$\frac{3}{8} \times \frac{5}{16} = \frac{15}{128}$$

Here, the student remembers to change the division sign to a multiplication sign, but forgets to take the reciprocal of the divisor. As a result, the student has answered a totally different problem. This error is frequently made when the divisor is a whole number. In these instances, encourage students to immediately rewrite the whole number as an equivalent fraction with a denominator of 1 and then follow the rules for division of proper fractions.

B.

$$\frac{3}{8} \div \frac{5}{16} = \frac{\cancel{8}^1}{3} \times \frac{5}{\cancel{16}_2} = \frac{5}{6}$$

The student remembers the "invert and multiply" rule, but inverts the dividend rather than the divisor.

Practice

(1) $\frac{5}{8} - \frac{6}{8} =$ $(\frac{5}{6})$

(2) $\frac{2}{3} - 8 =$ $(\frac{1}{12})$

(3) $\frac{7}{10} - \frac{4}{5} =$ $(\frac{7}{8})$

(4) $\frac{2}{7} - 5 =$ $(\frac{2}{35})$

(5) $\frac{1}{2} - \frac{7}{8} =$ $(\frac{4}{7})$

Skiil 2.6 Addition of Fractions with Like Denominators

Example

$$\frac{5}{11} + \frac{2}{11} =$$

Entry Conditions Basic addition facts.

Correct Solution

$$\frac{5}{11} + \frac{2}{11} = \frac{7}{11}$$

Errors: Identification and Analysis

$$\frac{5}{11} + \frac{2}{11} = \frac{7}{22}$$

The student has added both numerators and denominators, yielding an answer that has a denominator of 22. Use manipulatives that show parts of a whole or color in sections of pies to picture the addition to help the student to under-
d this error.

Practice

(1) $\frac{5}{8} + \frac{3}{8} =$ (1)

(2) $\frac{6}{10} + \frac{2}{10} =$ ($\frac{4}{5}$)

(3) $\frac{7}{14} + \frac{11}{14} =$ ($1\frac{2}{7}$)

(4) $\frac{8}{9} + \frac{3}{9} =$ ($1\frac{2}{9}$)

(5) $\frac{9}{20} + \frac{4}{20} =$ ($\frac{13}{20}$)

Skill 2.7 Addition of Fractions with Unlike Denominators

Example

$$\frac{1}{6} + \frac{3}{4} =$$

Entry Conditions Basic addition facts, fractional equivalences, addition of fractions with like denominators.

Correct Solution

$$\begin{array}{rcl} & \frac{1}{6} = & \frac{2}{12} \\ + & \frac{3}{4} = & \frac{9}{12} \\ \hline & & \frac{11}{12} \end{array}$$

Errors: Identification and Analysis

A.

$$
\begin{array}{r}
\frac{1}{6} \\
+\ \frac{3}{4} \\
\hline
\frac{4}{10}
\end{array}
$$

This error is the same as the one in Skill 2.6 (page 17). The student does not understand the basic concepts involved in addition of fractions and should be given practice with visual aids.

B.

$$
\begin{array}{rcl}
\frac{1}{6} & = & \frac{1}{12} \\
+\ \frac{3}{4} & = & \frac{3}{12} \\
\hline
 & & \frac{4}{12}
\end{array}
$$

Here, the student has recognized the need for common denominators in addition of fractions, but simply retains the old numerator in each of the new fractions. The student needs to practice building equivalent fractions by multiplying both numerator and denominator by the same number.

Practice

(1) $\frac{4}{5} + \frac{7}{10} =$ **($1\frac{1}{2}$)**

(2) $\frac{3}{8} + \frac{8}{12} =$ **($1\frac{1}{24}$)**

(3) $\frac{2}{3} + \frac{5}{6} =$ **($1\frac{1}{2}$)**

(4) $\frac{7}{9} + \frac{2}{15} =$ **($\frac{41}{45}$)**

(5) $\frac{10}{25} + \frac{9}{15} =$ **(1)**

Skill 2.8 Addition of Mixed Numbers

Example

$$2\frac{3}{4} + 3\frac{2}{3} =$$

Entry Conditions Basic addition and multiplication facts, writing mixed numbers as equivalent improper fractions, addition of proper fractions, finding a common denominator.

Correct Solution

$$\begin{array}{rcl} 2\frac{3}{4} & = & 2\frac{9}{12} \\ +\ 3\frac{2}{3} & = & 3\frac{8}{12} \\ \hline & & 5\frac{17}{12} = 6\frac{5}{12} \end{array}$$

Errors: Identification and Analysis

$$
\begin{array}{rcl}
2\frac{3}{4} & = & 2\frac{9}{12} \\
+\,3\frac{2}{3} & = & \underline{3\frac{8}{12}} \\
 & & 6\frac{7}{12}
\end{array}
$$

This is probably one of the most common errors that students make when adding mixed numbers. Students treat the addition as if it were an addition of whole numbers by regrouping the tens' place rather than writing the improper fraction as a mixed number and combining. Practice with manipulatives and other visual aids helps the students to understand the reasoning behind the correct answer and why the solution achieved through "carrying" is incorrect.

Practice

(1) $5\frac{4}{5} + 6\frac{3}{4} =$ **($12\frac{11}{20}$)**

(2) $2\frac{9}{10} + 3\frac{7}{8} =$ **($6\frac{31}{40}$)**

(3) $6\frac{5}{7} + 8\frac{6}{11} =$ **($15\frac{20}{77}$)**

(4) $1\frac{1}{2} + 10\frac{9}{10} =$ **($12\frac{2}{5}$)**

(5) $3\frac{8}{9} + 4\frac{9}{10} =$ **($8\frac{71}{90}$)**

Skill 2.9 Subtraction of Fractions

The errors made here are similar to those made in addition of fractions. Manipulatives and visual aids—ones that help students see the subtraction—assist in the remediation process. The most common error of this skill type occurs when the problem involves the subtraction of mixed numbers.

Example

$$5\frac{1}{3} - 2\frac{3}{4} =$$

y Conditions Basic subtraction and multiplication facts, subtraction of ›r fractions, finding a common denominator.

Correct Solution

$$\begin{array}{rcrcr} 5\frac{1}{3} & = & 5\frac{4}{12} & = & 4\frac{16}{12} \\ -\underline{2\frac{3}{4}} & = & \underline{2\frac{9}{12}} & = & \underline{2\frac{9}{12}} \\ & & & & 2\frac{7}{12} \end{array}$$

Errors: Identification and Analysis

$$\begin{array}{rcr} 5\frac{1}{3} & = & 5\frac{4}{12} \\ -\underline{2\frac{3}{4}} & = & \underline{2\frac{9}{12}} \\ & & 3\frac{5}{12} \end{array}$$

The student has reversed the subtraction sequence in order to get a positive answer. This has changed the entire problem. Lead students through the solution in the following way:

Step 1	Step 2	Step 3	Step 4

$$\begin{array}{rcrcrcr} 5\frac{1}{3} & = & 5\frac{4}{12} & = & {}^{4}\cancel{5}\,1\frac{4}{12} & = & 4\frac{16}{12} \\ -\underline{2\frac{3}{4}} & = & \underline{2\frac{9}{12}} & = & \underline{2\frac{9}{12}} & = & \underline{2\frac{9}{12}} \\ & & & & & & 2\frac{7}{12} \end{array}$$

Practice

(1) $4\frac{1}{3} - 3\frac{9}{15} =$ $(\frac{11}{15})$

(2) $5\frac{1}{2} - 3\frac{7}{8} =$ $(1\frac{5}{8})$

(3) $6\frac{3}{5} - 5\frac{2}{3} =$ $(\frac{14}{15})$

(4) $16\frac{3}{7} - 10\frac{9}{10} =$ $(5\frac{37}{70})$

(5) $12\frac{4}{5} - 3\frac{6}{7} =$ $(8\frac{33}{35})$

SECTION 3 DECIMALS

Skill 3.1 Comparing Decimals

Example Which is the largest?

(a) .3 (b) .29994 (c) 1 (d) .0796

Entry Conditions Place value.

Correct Solution Find the correct answer by aligning the decimals and adding zeros:

```
 .3              .30000
 .29994          .29994
1.              1.00000 ←largest
 .0796           .07960
```

Students can read the numbers with the decimal point disregarded if they can't see which is largest (or smallest). Students should also be able to use this technique to put the decimals in order.

Errors: Identification and Analysis Students often think the number with the most digits is the largest. Some students think any number with "lots of 9s" must be large. Still others pick the answers haphazardly. The key in this example is the placement of the decimal point to the right of any whole number.

Practice Put these sets of decimals in descending order·

(1)	.245	.24	.204	2
	(2, .245, .24, .204)			
(2)	.3	.03	.303	2.1
	(2.1, .303, .3, .03)			
(3)	.997	1	.9204	.7
	(1, .997, .9204, .7)			
(4)	6.63	.603	.063	6.3
	(6.63, 6.3, .603, .063)			
(5)	3.4	3.49	3.094	3.494
	(3.494, 3.49, 3.4, 3.094)			

Skill 3.2 Addition of Decimals

Example

$$6.51 + 19.6 =$$

Entry Conditions Place value, addition facts, estimation.

Correct Solution The key is to line up the decimal points. Zeros can be used as placeholders.

$$\begin{array}{r} {}^{1}6.51 \\ +\,{}^{1}19.60 \\ \hline 26.11 \end{array}$$

Have students estimate the answer in advance. They should expect an answer somewhere near 25, since 19 + 6 = 25.

Errors: Identification and Analysis Students may forget to line up the decimal points:

$$\begin{array}{r} 6.51 \\ +\,19.6 \\ \hline .847 \end{array}$$

They may use the rule for multiplication of decimals when placing the decimal point in the answer. An advance estimation helps students realize that certain answers are illogical and they should search for a mistake. Students must remember to always place the decimal point to the right of any whole number.

Practice

(1) 6.33 + 89 = **(95.33)**

(2) .75 + 6.3 = **(7.05)**

(3) 9.421 + 86 + 3.2 = **(98.621)**

(4) 400 + .8 + .16 = **(400.96)**

(5) 84 + 8.4 + .084 + .84 = **(93.324)**

Skill 3.3 Subtraction of Decimals

Example

$$256 - 4.2 =$$

Entry Conditions Place value, subtraction facts, addition of decimals, estimation.

Correct Solution The key is to place the decimal in the whole number correctly and then line up the decimal points:

$$\begin{array}{r} 25\overset{5}{\cancel{6}}.{}^{1}0 \\ -\quad 4.2 \\ \hline 251.8 \end{array}$$

The correct answer, 251.8, seems logical to a student who estimated the answer 252 (from 256 - 4) in advance.

Errors: Identification and Analysis

A. The most common error involves not placing a decimal point to the right of 256:

$$\begin{array}{r} 256 \\ -\ 4.2 \\ \hline 21.4 \end{array}$$

The answer of 21.4 is not reasonable when compared to a correct estimate.

B. Another common error occurs when the student forgets to regroup:

$$\begin{array}{r} 256.0 \\ -\quad 4.2 \\ \hline 252.2 \end{array}$$

This answer *is* reasonable when compared to the estimate, so the student committing this error may not notice any discrepancy.

Practice

(1) 375.1 - 16.3 = **(358.8)**

(2) 39.85 - 5.2 = **(34.65)**

(3) 605 - 98.12 = **(506.88)**

(4) 89.312 - 6.4 = **(82.912)**

(5) 924.1 - 30 = **(894.1)**

Skill 3.4 Multiplication of Decimals

Example

$$6.4 \times 3.9 =$$

Entry Conditions Place value, addition and multiplication facts, estimation, rounding.

Correct Solution

$$
\begin{array}{r}
{}^{1}\cancel{{}^{2}}6.4 \\
\times\ 3.9 \\
\hline
576 \\
{}^{1}192 \\
\hline
24.96
\end{array}
$$

Students may use a "0" or an "x" to hold the ones' place while finding the second partial product. Some students just leave a blank space. Count the number of places to the right of the decimal point in both the multiplier and the multiplicand to find the number of decimal places in the product. An estimation in advance (in this case, 6 × 4 = 24) is advisable.

Errors: Identification and Analysis

$$
\begin{array}{r}
6.4 \\
\times\ 3.9 \\
\hline
576 \\
192 \\
\hline
249.6
\end{array}
$$

The students may use the "bring down the decimal point" rule that is used in addition and subtraction. Advance estimates will help students find errors; in this example, 249.6 is nowhere near the advance estimate of 24.

Practice

(1) 7.2 × 9.6 = **(69.12)**

(2) 50.3 × 8.7 = **(437.61)**

(3) 85 × 3.6 = **(306)**

(4) 1.23 × .2 = **(5.116)**

(5) .12 × .6 = **(.072)**

Skill 3.5 Division of Decimals with Whole-Number Divisors

Example

$$6\overline{)721.86}$$

Entry Conditions Place value, subtraction, multiplication and division facts.

Correct Solution

Long Division:

$$\begin{array}{r} 120.31 \\ 6\overline{)721.86} \\ \underline{6} \\ 12 \\ \underline{12} \\ 018 \\ \underline{18} \\ 06 \\ \underline{6} \\ 0 \end{array}$$

Short Division:

$$\begin{array}{r} 120.31 \\ 6\overline{)72^{}1.^{1}86} \end{array}$$

Once the student has lined up the decimal point in the quotient with the decimal point in the dividend, the problems in division of decimals are often the same as the problems in division of whole numbers.

Errors: Identification and Analysis Students often forget to enter the decimal in the quotient.

$$\begin{array}{r} 12031 \\ 6\overline{)721.86} \end{array}$$

Encourage students to use multiplication to check their answers. Be on the alert for common errors in dividing whole numbers (see Skill 1.5, p. 8).

Practice Have the students carefully examine the answers to these practice problems; they emphasize decimal placement in the quotient.

(1) 46714 - 4 = **(11678.5)**

(2) 4 6714 - 4 = **(1.16785)**

(3) 46.714 - 4 = **(11.6785)**

(4) 467.14 - 4 = **(116.785)**

(5) 4671.4 - 4 = **(1167.85)**

Skill 3.6 Division of Decimals with Decimal Divisors

Example

$$.4\overline{)68.136}$$

Entry Conditions Place value, subtraction, multiplication and division facts, division of decimals with whole-number divisors.

Correct Solution Although the advent of calculators has reduced the necessity of tackling this problem by hand, there is a correct procedure. It requires moving the decimal point an equal number of places to the right in the dividend and in the divisor.

$$.4\overline{)68.136}$$

This example is now converted to a problem of division by a whole-number divisor, which the student should know before trying division with decimal divisors. The above problem becomes:

$$
\begin{array}{r@{}l}
 & 170.34 \\
4 & \overline{)681.36} \\
 & \underline{4} \\
 & 28 \\
 & \underline{28} \\
 & 013 \\
 & \underline{12} \\
 & 16 \\
 & \underline{16} \\
 & 0
\end{array}
$$

The decimal point is placed in the quotient *after* the decimal points in the divisor and dividend have been moved. The quotient 170.34 is equivalent to the quotient found when 68.136 is divided by .4, which was the original problem.

Errors: Identification and Analysis

A. Some students ignore the decimal in the divisor and treat the problem as if it had a whole number divisor:

$$\begin{array}{r} 17.034 \\ .4\overline{)68.136} \end{array}$$

The decimal point has been incorrectly placed in the quotient. Have students check their answers by multiplying the quotient by the divisor.

B. Other students move the decimal points and create whole numbers in *both* the dividend and divisor:

$$\begin{array}{r} 17034 \\ .4\overline{)68.136} \end{array}$$

These students do not understand the rationale for the correct procedure. They view it as arbitrary, and thus may remember only certain aspects of the correct procedure.

C. Look for mistakes rooted in division of whole numbers that are not based on decimal-point placement.

Practice This practice sequence highlights the placement of the decimal point, without getting bogged down in repeated whole-number-division examples. Obviously, students should enter this topic with a command of whole-number division.

(1) 68.136 − 4 = **(17.034)**

(2) 68.136 − .4 = **(170.34)**

(3) 68.136 − .04 = **(1703.4)**

(4) 681.36 − 4 = **(1703.4)**

(5) 6813.6 − .4 = **(17034)**

SECTION 4 PERCENTS

Skill 4.1 Changing Percents to Equivalent Decimals

Example Change each of the following percents to an equivalent decimal:

(a) 3% (b) 30% (c) 300%

Entry Conditions Place value, percents.

Correct Solution Place the decimal point in the percent expressions and then move it two places to the left:

$$3\% = 3.\% = .03$$

$$30\% = 30.\% = .30$$

$$300\% = 300.\% = 3.00$$

This sequence and similar sequences can be used to highlight the subtle but significant differences in these problems.

Errors: Identification and Analysis Students must realize that percent means "per hundred" so they can relate percents to hundredths. This can be developed through a series of developmental lessons when the topic is first undertaken. This knowledge base will help the students avoid these common errors:

A. Decimal point is simply placed "in front of the number":

$$3\% = .3$$

$$30\% = .30$$

$$300\% = .300$$

Students often make this error because it works for two-digit percents, which are the most common.

B. Decimal point is moved two places to the right of the number instead of to the left:

$$3\% = 3.\% = 300$$

$$30\% = 30.\% = 3000$$

$$300\% = 300.\% = 30000$$

This student is confusing the rule for changing decimals to percents with the rule for changing percents to decimals.

Practice Change the following percents to equivalent decimals:

(1) 53% **(.53)**

(2) 5.3% **(.053)**

(3) .53% **(.0053)**

(4) 530% **(5.3)**

(5) 126% **(1.26)**

Skill 4.2 Changing Fractions to Equivalent Percents

Example Express $\frac{3}{4}$ as an equivalent percent.

Entry Conditions Division of decimals, place value, changing decimals to percents.

Correct Solution Students must divide and then move the decimal point:

$$\begin{array}{r} .75 \\ 4\overline{)3.00} \\ \underline{2\,8}\downarrow \\ 20 \end{array}$$

$$\frac{3}{4} = .75 = 75\%$$

Errors: Identification and Analysis The problems in this two-step procedure are usually rooted in either an inability to change a decimal to a percent or an inability to change a fraction to a decimal.

A. Sometimes the divisor and the dividend are reversed:

$$\begin{array}{r} 1.3\bar{3} \\ 3\overline{)4.00} \end{array}$$

$$\frac{3}{4} \approx 1.33 = 133\%$$

B. Sometimes the division is done correctly, but the decimal is converted into a percent incorrec'ly:

$$4\overline{)3.00}\quad .75$$

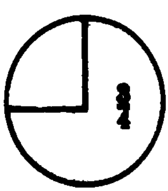
$$\frac{3}{4} = .75\%$$

C. Errors A and B may be combined with mistakes in basic operations to produce other incorrect solutions. Examine the students' scrapwork and not just their answers.

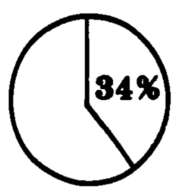
D. Some students forget the rules altogether and use an arbitrary rule:

$$\frac{3}{4} = 34\%$$

This practice of using the numerator and denominator to convert a fraction to a percent is common. Pie charts help students see that this can't be correct:



$\frac{3}{4} \neq 34\%$



$\frac{3}{4}$ is more than half

34% is less than half

Practice Change the following fractions into equivalent percents:

(1) $\frac{2}{5}$ **(40%)**

(2) $\frac{1}{4}$ **(25%)**

(3) $\frac{7}{8}$ **(87.5%)**

(4) $\frac{1}{2}$ **(50%)**

(5) $\frac{5}{8}$ **(62.5%)**

Skill 4.3 Changing Mixed-Number Percents to Decimals

Example Express $8\frac{1}{4}\%$ as an equivalent decimal without a percent sign.

Entry Conditions Place value, changing percents into decimals.

Correct Solution The fraction is converted to a decimal, the decimal point is moved two places to the left, and the percent sign is removed.

$$4\overline{)1.00}\ \ .25$$

$$8\tfrac{1}{4}\% = 8.25\% = .0825$$

Errors: Identification and Analysis The errors commonly made are errors in converting either fractions to decimals or percents to decimals. This is a two-step problem, but students often stop after one step.

A. Student forgets to move the decimal point:

$$4\overline{)1.00}\ \ .25$$

$$8\tfrac{1}{4}\% = 8.25$$

B. Student doesn't convert the percent to an equivalent decimal:

$$8\tfrac{1}{4}\% = 8.25\%$$

C. The variety of errors that can occur in division of decimals and percent-fraction-decimal conversions may combine in these examples, so a careful look at the student's scrap paper is recommended.

Practice Express the following percents as equivalent decimals without percent signs:

(1) $7\frac{1}{2}\%$ (.075)

(2) $16\frac{1}{4}\%$ (.1625)

(3) $90\frac{3}{4}\%$ (.9075)

(4) $17\frac{1}{8}\%$ (.17125)

(5) $6\frac{1}{5}\%$ (.062)

Skill 4.4 Finding a Percent of a Number

Example Find 16% of 500.

Entry Conditions Place value, multiplication of decimals, changing percents to equivalent decimals.

Correct Solution Students must convert 16% to a decimal and multiply:

$$\begin{array}{r} 500 \\ \times\ .16 \\ \hline 3000 \\ 500 \\ \hline 80.00 \end{array}$$

Errors: Identification and Analysis This procedure is common in problems involving money. Students need to know *when* to perform this function by their interpretation of such concepts as sales tax, discounts, and interest. Mistakes in computation are different from mistakes in interpretation.

A. Because students view this problem as "taking a part of sometning," they are often inclined to divide:

$$.16\overline{)500}$$

B. Errors in multiplication and addition can plague a student who starts out with the correct procedure.

C. Decimal placement can become a problem, especially if the multiplier and the multiplicand have digits to the right of the decimal point.

D. Incorrect answers can compound themselves in many consumer math applications that require this answer to be used in subsequent steps. Inspect how a student carries through an initial error. Only then can the specific diagnosis be made.

Practice

(1) Find 62% of 380. **(235.6)**

(2) Find 8% of 65. **(5.2)**

(3) Find 14% of 400. **(56)**

(4) Find 6% of 65.50. **(3.93)**

(5) Find 8% of 150.75. **(12.06)**

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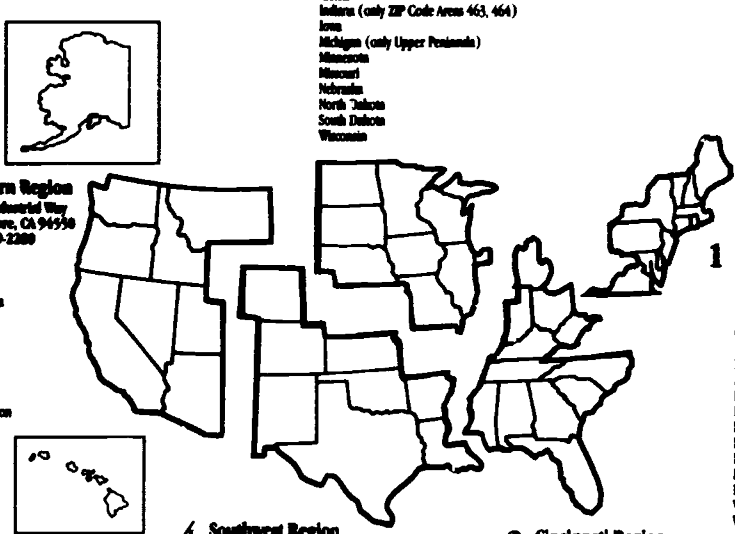

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