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ABSTRACT

A large proportion of college students majoring in science are unable to translate even simple sentences into algebraic equations. Given the following sentence, "There are six times as many students (S) as professors (P) at this university," 37% of 150 freshmen engineering students in a study conducted in 1981 by Clement, Lockhead, and Monk wrote the following equation: 6S=P, referred to as the reversal error. It is plausible to suggest that in order to overcome the reversal error students need to operate in a hypothetico-deductive manner. The objective of this study was to investigate the relation between student ability to translate sentences into equations, equations into sentences, and student performance in the following variables: formal operational reasoning; proportional reasoning; and achievement in an introductory freshman level chemistry I course. Selected results show that: (1) as the student ability to translate sentences into equations and equations into sentences increases, their mean scores in chemistry I, formal operational and proportional reasoning increase; and (2) ability to translate an equation into a sentence does correlate with student scores in chemistry I while the reverse translation does not. This study reports support for the hypothesis that students who lack formal operational reasoning (hypothetico-deductive reasoning) may experience more problems in the translation of algebraic equations. (MVL)

TRANSLATION OF ALGEBRAIC EQUATIONS AND ITS RELATION TO FORMAL OPERATIONAL REASONING

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INTRODUCTION

The ability to translate a problem usually expressed in words into a algebraic equation and retranslate an equation back into words, is an important part of science and mathematics courses at the secondary and college level. Clement, Lochhead and Monk (1981) have shown the inability of a large proportion of science major college students to translate even simple sentences into algebraic equations. The authors along with other problems used the Students and Professors Problem (presented below) in a study based upon 150 freshmen engineering students and found that 37% missed the problem.

Students and Professors Problem:

Write an equation using the variables S and P to represent the following statement: "There are six times as many students as professors at this university". Use S for the number of students and P for the number of professors.

Roughly half of the students in the study were given the hint: "Be careful. Some students put a number in the wrong place in the equation". It was found that the group given the hint, scored 3% higher. Clement, Lochhead and Monk (1981) further reported that two-thirds of the errors took the form of a reversal of variables such as $6S = P$, instead of the correct $S = 6P$. These results led the authors to observe that in secondary school mathematics courses students are rarely asked to "construct" a formula or to interpret one in a significant way, and to conclude that student difficulty in translation problems was deep seated and not due to carelessness. Rosnick and Clement (1980) further point out that in the Students and Professors Problem, subjects confuse the use of letter as a variable with the use of letter as a label or unit. After making the reversal error 6S = P the subjects read the equation as "six students for every professor", i.e., identified the letter S as a label standing for "students" rather than making the correct interpretation that S means the "number of students". Finally, the authors conclude that the reversal error in translating algebraic equations is a resilient one and not easily taught away.

Clement (1982) has conducted protocol analysis in order to determine the cognitive events underlying the different approaches utilized by the students to solve the Students and Professors Problem, and other similar problems. The following three approaches were identified from the protocols: a) Word order matching approach: It involves the direct mapping of the words of English into the symbols of algebra, which leads to a reversed solution. For example: There are 6 times as many students as professors

$$6 \cdot S = P$$

b) Static comparison approach: Students using this approach do comprehend that there are more students than professors. The protocols show that some of the students even wrote the correct answer, considered it, and finally switched to the reversed equation, i.e., 6S = P. Clement, Lochhead and Monk (1981) consider this approach as an important first step in the translation process (as it goes beyond the word order matching approach) which eventually leads to the use of a semantic approach dependent on the meaning of the problem. c) Operative approach: This requires the comprehension of the static comparison approach, together with the ability to invent an operation, such as, increasing the number of professors six times. Thus S = 6P describes an equivalence relation that would occur if one were to perform a particular hypothetical operation, i.e., making the group of professors six times larger than it really is.

The reversal error appears to reveal a stable and persistent problem concerning the meaning of variables and equations. Clement, Lochhead, and Soloway (1979) have shown that translation reversal difficulties appear consistently even in different symbol systems, i.e., in translation from pictures to equations, data tables to equations and equations to sentences.

Clement (1982) further suggests that most of the reversal errors were due to a difficulty in translating words into algebraic equations, rather than a difficulty with simple algebraic manipulation skills (for example, solve for X; when 5X = 50) or with simple ratio reasoning of the type: Jones uses 3 gallons of gas for driving 60 miles. How many gallons would he use for driving 90 miles?

Wollman (1983) has studied the sources of error in translation from sentence to equation and found that the principle sources for the reversal error were: haste, failure to think of checking the equation, failure to base the equation on the meaning of the sentence, and use of non-algebraic symbols. Wollman (1983) suggests the following monitoring processes that could help improve student performance in translation problems: 1) Always check any answer; 2) Check the equation; 3) Compare the equation to the sentence; 4) Find the greater quantity in the equation and the greater quantity in the sentence and compare the two quantities; 5) Find numbers that satisfy the sentence and substitute them into the equation.

In a recent study Lawson (1986) has explored the relationship between student misconceptions (Driver, 1981) and reasoning patterns, and suggests that for students to overcome their misconceptions, they must be able to logically "see" how the evidence supports the scientific conceptions and contradicts the naive misconception. Logically "seeing " this requires the use of formal operational reasoning, which is necessary to evaluate alternative scientific conceptions in a hypothetico-deductive manner. Thus it is not surprising that the operative approach (Clement, 1982) mentioned above, necessary to solve the Students and Professors Problem correctly, requires the students to operate in a hypothetico-deductive manner, i.e., performing a hypothetical operation which makes the group of professors six times larger than it really is (S = 6P).

The objective of the present study is to investigate the relation between student ability to translate sentences (Spanish) into equations, equations into sentences (Spanish), and student performance in the following variables: a) Formal operational reasoning; b) Proportional reasoning; and c) Introductory freshmen level chemistry course.

METHOD

The subjects (Ss) were 54 freshmen science major students (Mean age = 18.1 years; SD = 1.1) at the Universidad de Oriente, Venezuela, who had registered for Chemistry I. All Ss were pretested at the start of the semester, using the followong tests:

a) Lawson (1978) Classroom Test of Formal Reasoning (a slightly modified version). The test includes 15 items requiring the Ss to isolate and control variables, and use proportional, probabilistic, combinatorial and conservation reasoning. A split-half reliability coefficient of 0.75 was obtained for the present sample.

b) Proportional reasoning. The four items of proportional reasoning in the Lawson Classroom Test of Formal Reasoning were used to compute student scores in proportional reasoning. Two of the items involve pouring of water from a wider to a narrow cylinder and the other two are based on equilibrium in the balance beam.

c) Test A. Using symbols write an equation which represents, each of the following sentences:

Item A1: There are six times as many students as professors at this university.

Item A2: In a classroom there are five times as many boys as girls.

Item A3: A country sells four times as much wheat as corn.

Item A4: A store sells six times as many shirts as pants.

Item A5: Density is defined as mass per unit volume and could be expressed in units such as: grams and litres.

Item A6: Molarity is defined as the number of moles of solute dissolved in one litre of solution.

d) Test B

Item B1: In an experiment it was found that: X = 0.2 Y

(where X = volume of gold; Y = volume of aluminium)

It could be concluded that:

a) The volume of gold is greater than that of aluminium.

b) The volume of aluminium is greater than that of gold.

c) The volume of aluminium is equal to that of gold.

d) None of the previous.

Item B2: Given the following equation: Y = 4X

(where X = Number of ties sold by a store; and

Y = Number of socks sold by the same store)

It could be concluded that:

a) The store sells more ties than socks.

b) The store sells more socks than ties.

c) The store sells the same number of socks as ties.

d) None of the previous.

Items B3, B4 and B5: A super market sells four times as much milk as juice. Please answer the following questions:

Item B3: If the super market sold 120 litres of milk, then how many litres of juice did it sell?

Item B4: If the super market sold 50 litres of juice, then how many litres of milk did it sell?

Item B5: According to the sentence, does the store sell more milk or more juice.

The students were asked to justify their answers in all the tests. All items were scored 0, 1. The Lawson test, Proportional reasoning test, Test A, and Test B, had a maximum score of 15, 4, 6 and 5 respectively. Test B was administered separately, two weeks after the Test A. One important difference from Clement, Lochhead and Monk (1981) and Wollman (1983) studies should be noted in the manner of formulating the questions. The first group of authors used the following hint for roughly half of the students: "Be careful. Some students put a number in the wrong place in the equation". On the other hand, Wollman (1983) used the following item format: "Item: A store sells six times as many volleyballs as footballs. Instructions: Let V stand for the number of volleyballs and F stand for the number of footballs. The equation should relate F, the number of footballs, to V, the number of volleyballs. The total number of footballs and volleyballs has nothing to do with the question". Items A1, A2, A3, and A4 were adapted from Clement (1982) and Wollman (1983). A split-half reliability coefficient of 0.64 was obtained for the six items of Test A. Item B1 was adapted from Niaz (1985, 1988). Items B2, B3, B4, and B5 were adapted from Wollman (1983). A split-half reliability coefficient of 0.58 was obtained for the five items of Test B. Student performance in Chemistry I was evaluated during the semester by administering three class tests (multiple-choice and computational items), with a maximum score of 30 points, based on the following topics: Fundamental Laws, Stoichiometry, Solutions, and Gases. Evaluation in Chemistry I formed part of the grades obtained by the students.

RESULTS AND DISCUSSION

Table 1 shows that the difficulty level of items A1, A2, A3, and A4 (translation of a sentence into a equation) and item B1 (translation of a equation into a sentence) is about the same. It can be further observed that in items A1, A2, A3, and A4, almost three-fourths of the errors took the form of a reversal of variables. Item A5 deals with the concept of density, fairly common for science major students. It was found that 21 (38.9%) students wrote the following equation: $d = m \cdot V$ (d=density, m=mass, V=volume) and 5 (9.3%) students wrote the following equation: $d = m \cdot V$ = grams/litres. It is interesting to note that there is a tendency to translate the sentence "literally" into an equation.

In Item B1, 22 (40.7%) students responded: a) The volume of gold is greater than that of aluminium. Some of the students reasoned in the following manner: $X = 1$; $Y = 0.2$; $\therefore\ X > Y$. It is important to note that the contextual cue (aluminium is lighter than gold, and thus would occupy more volume) did not help the students.

It is interesting to note that in Item B2, 24 (44.4%) students responded: a) The store sells more ties than socks, and justified the answer by arguing that the store sells 4 ties for every pair of socks. Wollman (1983) uses the same item in his Study 2, with the difference that he asked three questions of the type: 1) If $X = 28$, what is Y? 2) If $Y = 36$, what is X? 3) Which must be greater, X or Y? The format used by Wollman led to an almost 100% performance by the students, whereas, the multiple-choice reasoning format used in this study led to a considerably low performance of 25.9%. The format used in this study seems to be preferable as it avoids plugging of values in the equation, which the student acquires as a manipulative skill in secondary school mathematics courses.

Out of the 47 students who responded correctly in Item B3, 14 responded by explicitly formulating the following reasoning strategy:

4 litres of milk ------> 1 litre of juice

120 litres of milk ------> $X = 120/4 = 30$ litres of juice.

The same 14 students responded correctly in Item B4, by explicit use of the following reasoning strategy:

1 litre of juice ------> 4 litres of milk

50 litres of juice ------> $X = 4.50 = 200$ litres of milk.

It is interesting to note that all these 14 students responded to Items A1, A2, A3, and A4 by committing the reversal error. Apparently these students do understand the problem to a certain extent, and can reason adequately, but still fail to write an algebraic equation, which could represent the sentence. Eight students responded correctly in Item B3 by formulating an algebraic equation of the form: Y = 4X; and 6 of these 8 students formulated the same algebraic equation to solve correctly Item B4. Out of the 8 students who responded correctly by formulating an algebraic equation in Item B3, 7 committed the reversal error while solving Items A1, A2, A3 and A4. It is possible that Item B2, in which the students are presented an equation, could have helped the students to later formulate an equation of the form Y = 4X, in Item B3. The results, however, are not conclusive on this point, in view of the fact that out of the 8 students who formulated the equation correctly in Item B3, only 4 had previously responded Item B2 correctly.

Table 2 shows the relation between the ability to translate algebraic equations and student performance in Chemistry I, Formal and Proportional reasoning. It can be concluded that as the total score in Items (A and B) decreases, the possibility of committing the reversal error is greater. The results obtained provide support for the hypothesis that students who lack formal operational reasoning skills (hypothetico-deductive reasoning) may have more difficulty in translating algebraic equations.

It is important to note that although Clement (1982) found that simple ratio reasoning was not important for translation of sentences into equations, this study shows that those students who lack proportional reasoning, obtain a lower score in Items A and B. Table 3 shows a Pearson correlation coefficient, $r = 0.57$ ($p < 0.05$) between the total scores in Items (A and B) and Items of proportional reasoning.

Similarly it can be observed from Table 3 that:

a) Correlation coefficient between total (A and B) and Formal operational reasoning, $r = 0.51$ ($p < 0.05$).

b) Out of the 4 items requiring translation of a sentence into an equation three (Items A1, A2 and A3) have a significant correlation with Formal and Proportional reasoning.

c) Items B1 and B2 requiring the translation of an equation into a sentence have a significant correlation with scores in Chemistry I, Formal and Proportional reasoning.

d) It is interesting to note that the ability to translate sentences into equations (Items A1, A2, A3 and A4) does not correlate with student scores in Chemistry I. On the other hand the ability to translate an equation into a sentence (Items B1 and B2) does correlate significantly with student scores in Chemistry I.

e) Correlation coefficient ($r = 0.29$; $p < 0.05$) between Item A6, which indicates the ability to memorize a definition and student scores in Chemistry I is significant.

CONCLUSIONS

This study has found support for the hypothesis that students who lack proportional and formal operational reasoning (hypothetico-deductive reasoning) may experience greater difficulty in the translation of algebraic equations. Apparently in order to overcome the reversal error students require the generation of alternative hypotheses and their test through experimentation, data collection, and hypothetico-deductive reasoning. In view of the fact that formal operational reasoning enables the students to evaluate alternative hypotheses in a logical manner, concrete operational students are expected to have more problems in the translation of algebraic equations.

Student success in an introductory freshman level Chemistry I course depends more on the translation of a sentence like the one in Item A6, which requires the memorization of a definition, than on the translation of sentences (A1, A2, A3, and A4) requiring the generation of a hypothetical operation. This finding leads one to the observation made by Rosnick and Clement (1980) that large number of students may be slipping through their education with good grades and little learning. That so many science major students are confused at the interface between algebraic symbols and their meaning, is an indicator of an educational system that focusses primarily on manipulative skills. It is a cause for concern that most of our students can manipulate equations, find derivatives, and pass exams, but yet fail to comprehend verbal descriptions of real world every day problems.

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TABLE 1

STUDENT RESPONSES AND REVERSAL ERRORS IN DIFFERENT ITEMS OF TEST A AND TEST B (N=54)

Item	No. of Correct Responses	(%)	No. of Reversal Errors	(%)
A1	3	(5.6)	42	(77.8)
A2	3	(5.6)	40	(74.1)
A3	3	(5.6)	41	(75.9)
A4	2	(3.7)	38	(70.4)
A5	25	(46.3)	-	-
A6	48	(88.9)	-	-
B1	4	(7.4)	-	-
B2	14	(25.9)	-	-
B3	47	(87.0)	-	-
B4	49	(90.7)	-	-
B5	51	(94.4)	-	-

TABLE 2

RELATION BETWEEN TOTAL SCORE IN ITEMS OF TESTS (A & B) AND STUDENT PERFORMANCE IN CHEMISTRY I, FORMAL AND PROPORTIONAL REASONING (N=54)

Total Score		Chemistry I		Formal Reasoning		Proportional Reasoning	
Items (A & B)	N	Mean	SD	Mean	SD	Mean	SD
1-2	4	1.25	0.89	1.15	0.75	0.00	-
3-4	17	6.47	4.79	3.37	2.59	0.62	0.83
5-6	29	11.15	7.85	4.27	3.26	1.10	1.43
7-8	2	7.88	1.38	3.45	1.85	1.75	0.25
9-10	2	11.50	0.50	7.55	0.75	3.25	0.25

TABLE 3

PEARSON CORRELATION COEFFICIENTS FOR ITEMS OF TESTS (A & B) AND CHEMISTRY I (Chem), FORMAL (FR) AND PROPORTIONAL REASONING (PR)

	A1	A2	A3	A4	A5	A6	Total A	B1	B2	B3	B4	B5	Total B	Total (A&B)	FR	PR	Chem
A1	1																
A2	1.00*	1															
A3	1.00*	1.00*	1														
A4	0.81*	0.81*	0.81*	1													
A5	0.09**	0.09**	0.09**	0.01**	1												
A6	-0.17	-0.17	-0.17	-0.24	0.21**	1											
Total A	0.79*	0.79*	0.79*	0.64*	0.61*	0.24**	1										
B1	0.24**	0.24**	0.24**	-0.05	-0.12	0.10**	0.12**	1									
B2	0.41**	0.41**	0.41**	0.33**	-0.04	-0.06	0.29**	0.48**	1								
B3	0.09**	0.09**	0.09**	0.08**	0.25**	0.21**	0.26**	0.11**	-0.02	1							
B4	0.08**	0.08**	0.08**	0.06**	0.17**	0.29**	0.23**	0.09**	0.19**	0.64**	1						
B5	0.05**	0.05**	0.05**	0.04**	-0.01	0.24**	0.10**	0.06**	0.12**	0.22**	0.61*	1					
Total B	0.32**	0.32**	0.32**	0.19**	0.08**	0.21**	0.35**	0.57**	0.66**	0.60**	0.76*	0.53*	1				
Total (A&B)	0.69**	0.69**	0.69**	0.52**	0.43**	0.28**	0.84*	0.41**	0.57**	0.52**	0.59*	0.37**	0.80**	1			
FR	0.28**	0.28**	0.28**	0.20**	0.07**	0.26**	0.33**	0.33**	0.41**	0.30**	0.32**	0.22**	0.52**	0.51**	1		
PR	0.39**	0.39**	0.39**	0.24**	0.22**	0.14**	0.44**	0.29**	0.48**	0.26**	0.25**	0.15**	0.49**	0.57**	0.85**	1	
Chem	0.05**	0.05**	0.05**	0.02**	0.17**	0.29**	0.21**	0.39**	0.36**	0.40**	0.34**	0.21**	0.56**	0.46**	0.67**	0.67**	1

*$p \geq 0.1$ **$p \leq 0.05$