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ABSTRACT

Reading to learn mathematics forges a new synthesis of the traditional basics of reading and mathematics which aims at fostering critical thinking and may provide an instructional context within which students and teachers can work out meaningful conceptions of mathematics. Benefits of this synthesis of reading and mathematics include: (1) contributing to better learning and understanding of mathematical content; (2) developing new learning strategies useful in new learning situations; and (3) developing a deeper understanding of mathematics as a discipline. Reading to learn mathematics may be able to play a role in bringing about much needed reform of the mathematics curriculum and to a reconceptualization of the role of the traditional "basics" in educating students as critical thinkers. A review of the research on the reading process shows how the concept of reading as a transaction contributes to the attainment of these goals for mathematics instruction. (Sixty-eight references are attached.) (RS)

"Reading to Learn Mathematics": A New Synthesis of the Traditional Basics

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Introduction

What is more basic to schools than reading and mathematics? And yet, the traditional definitions of these "basics" have been assailed as inadequate when measured against the goal of educating students as critical thinkers (Kuhn, 1986; NAEP, 1985; Carnegie, 1986; Holmes, 1986). Schooled to "solve" ready-made, well-defined problems, students are unprepared to deal with the novel, ill-structured situations that characterize thinking in context. Educators have begun to argue that the traditional sense of basic skills as algorithms for acquiring stable bodies of facts has created rote thinkers. If, instead, students are to become critical thinkers, then a new view of the "basics" must be forged.

This paper attempts to do just that by exploring the possibility of a new synthesis of two traditional basics--reading and mathematics--aimed at fostering critical thinking. Our perspective on critical thinking is grounded in the work of John Dewey who characterized critical thinking as a process of inquiry motivated by doubt and ambiguity. As such, critical thinking is a generative activity involving both the creation and evaluation of knowledge. This view of critical thinking challenges the goals of most current mathematics and reading curricula, since schools have traditionally interpreted both disciplines as specialized forms of information processing.

Recently, however, there have been movements within mathematics education calling for a reconception of the goals of the current mathematics curriculum in ways which are more in line with a critical thinking perspective. Parallel developments have occurred in the area of reading, where the most recent theoretical models have challenged the view that reading comprehension is essentially the transfer of information encoded in a text. The new integration of reading and mathematics we present takes advantage of these developments. In brief, we suggest that reading can play a unique role in mathematics instruction as a mode of learning which promotes the kind of reflective, skeptical thought characteristic of critical thinking.

Building on relevant research in critical thinking, mathematics education, and reading comprehension, we first of all identify educational goals and instructional approaches for mathematics and reading which support critical thinking. A framework for using "reading to learn mathematics" is then developed through a synthesis of these interpretations of mathematics and reading instruction, and an illustrative example of "reading to learn mathematics" presented so as to flesh out the framework introduced. We conclude with a discussion of the educational significance of this synthesis.

Conceptual Framework

In this section we present the foundations for the new synthesis of reading and mathematics instruction outlined in the introduction. We define critical thinking and then explain how the goals of the mathematics curriculum must be reconceptualized if schools are to produce students who think critically in the context of mathematics. Finally, research on the reading process is reviewed to show how the concept of reading as a transaction contributes to the attainment of these new goals for mathematics instruction.

Critical Thinking Defined

Critical thinking has become a buzzword in educational circles. Everyone agrees there should be more of it in schools but there is less agreement on just what it means to think critically. One commonly held definition is based on the work of Ennis (1962) who argued that critical thinking is a matter of correctly assessing the truth value of statements. Drawing heavily on the rules of informal logic, Ennis identified a list of 12 "aspects" of critical thinking which he claimed could serve as ways to "avoid pitfalls" in assessing statements. Critical thinking thus came to be thought of as a set content- and context-free procedures which could be used to evaluate the form of an argument, a notion that promotes the teaching of isolated skills in the name of critical thinking (e.g., Beyer).

Critics claim that this characterization of critical thinking as a particular set of "skills" fails to adequately capture the actual practice of critical thinking in context. Evaluating statements and arguments is not just a matter of form but involves thinking <u>about</u> something <u>in</u> a particular context. Hence, there can be no set of critical thinking skills that can guarantee "right reasoning" independent of the particular domain of knowledge and circumstances of use (McPeck, 1981). Moreover, the most important (and possibly the most difficult to teach) aspects of critical thinking are the initial decision to engage in critical thinking (Sternberg and Martin, 1988) and the subsequent framing of the problem.

In contrast to a "basic skills" approach, current scholarship defines critical thinking as "informed skepticism" (Cornbleth, 1985) or "reflective skepticism" (McPeck, 1981) and requires knowledge of the relevant domain. Scholars have drawn on the work of John Dewey to work out a perspective on critical thinking that captures its complex and fluid nature. According to

Dewey (1933), critical thinking is reflective thought involving both the correction (in the sense of looking back at the bases of an hypothesis) and the generation (in the sense of looking forward to potential consequences) of knowledge. Ambiguity plays a central role in his description of critical thinking since reflective thought involves: "(1) a state of doubt, hesitation, perplexity, mental difficulty, in which thinking originates, and (2) an act of searching, hunting, inquiring, to find material that will resolve the doubt, settle and dispose of the perplexity (p. 12)." Critical thinkers raise questions about what is taken for granted and consider new possibilities in light of their knowledge of the particular knowledge domain and the context of situation. In short, these definitions suggest that critical thinking is more an attitude of inquiry than a collection of skills (Siegel and Carey, in press).

If critical thinking is a generative activity in which knowledge is created and refined through a reflective process motivated by doubt and ambiguity, then schools can no longer teach mathematics as the "discipline of certainty." The goal of fostering critical thinking in schools can only be achieved if the goals of the mathematics curriculum are reconceived.

A Reconceived View of the Goals for School Mathematics

The call for reconceptualizing the goals of the mathematics curriculum has come from researchers and practitioners alike (Carnegie, 1986; NCTM, 1980; Ralston, 1987). Though the debate is still raging on the issue of "how much" factual knowledge and computational skills should be required in the K-12 mathematics curriculum of the future, mathematics educators agree that there is a need to complement this component with new goals for learning and understanding mathematics. We will focus here on the following issues: (1) the concern with equipping students with general strategies which will allow them to successfully approach novel tasks in the future; (2) the need for helping students develop conceptions of mathematics which are more realistic and conducive to the learning of the discipline; (3) the need to deal effectively with affective as well as cognitive issues which can affect students' learning of mathematics.

The first goal has been widely recognized (see, for example, NCTM [1980]), and has already motivated a shift toward greater emphasis on the process doing mathematics rather than on the product of mathematical activity or factual learning. The numerous research studies and curricular projects geared to making problem solving the focus of school mathematics have helped move mathematics education in this direction, making teachers aware of the importance of setting new goals for school mathematics such as the development of problem solving heuristics (Polya, 1957), and metacognitive abilities (Schoenfeld, 1985a; Silver, 1985). The more recent concern for creating "critical thinkers" has added new goals to this list, such as developing skills for "framing" or "redefining" a given problem as well as the generation of new questions and avenues for inquiry (Brown, 1984, 1986; Brown and Walter, 1983).

Since Perry's study on the development of students' conceptions of knowledge (Perry, 1971), several studies (Borasi, 1986c; Brown, 1982; Buerk, 1981; Cooney, 1985; Copes, 1974; Meyerson, 1977; Oaks, 1987; Schoenfeld, 1985a, 1985b) have shown that students (and mathematics teachers as well!) often hold a view of school mathematics which is at odds with the one held by mathematicians, and which may have a negative effect on students' attitudes towards the discipline and their approach to its learning. These studies suggest that most students perceive mathematics as a dualistic, rigid discipline, where results are always univocally determined and there is no space for personal judgment, values and taste. As a result, they interpret their role as mathematics students as essentially acquiring (i.e., memorizing) facts and algorithms that can be immediately applied to the solution of problems; few students expect mathematics to be a meaning making process, and even fewer see it as a creative undertaking. We are thus in need of strategies which will help students understand mathematics as a more humanistic discipline; students are usually unaware of the struggles and difficulties which have marked the creation of mathematical knowledge, as reflected, for example, in Kline's account of the history of mathematics (Kline, 1980); the uncertainty and limitations still existing in some areas of mathematics, as in the case of the notions of infinity (Borasi, 1984, 1985) and provability itself (Hofstadter, 1979); and the role played by anomalies and errors in mathematical growth (Borasi, 1986a, 1987; Kuhn, 1970; Lakatos, 1976), all of which point to the need to teach mathematics as critical thinking.

Studies on "math anxiety", particularly in the case of women, (Buerk, 1982; Leder, 1982; Resek and Rupley, 1980; Tobias, 1978) and on self-systems (McLeod and Adams, 1980; Oaks, 1987) have also pointed out the important role feelings and attitudes play in successful learning experiences in mathematics. A recognition of the emotional difficulties met by many students and explicit efforts to make students realize that mathematics is indeed something they can be a part of and that mathematicians are not a special breed may be essential for success in the discipline.

The new goals for school mathematics identified above will require the development of new teaching strategies which engage students in experiences calling for critical thinking as defined earlier. The recent shift in theories from reading as information processing to reading as the active construction of meaning provides a basis for developing instructional strategies that emphasize reflective inquiry in an atmosphere of meaning-making and risk-taking.

A Reconceived View of Reading

During the last 15 years the field of reading education has experienced an explosion of knowledge about the reading process which has led to a proliferation of theoretical models of reading. The most recent model, from which our synthesis is drawn, looks at reading as a "transaction" in which new meanings arise from the negotiation of reader, text, and context. To better understand the radical character of this model, and its consequences for implementing the goals of mathematics education and critical thinking discussed above, it may help first of all to contrast it to its predecessors — the information transfer and interactive models. These models have dominated not only reading instruction but most of the current attempts to integrate reading and mathematics.

Both information transfer and interactive models draw on the computer as a metaphor; thinking is processing information and so reading becomes a matter of transforming visual information into meaning. Information transfer models of reading adhere closely to this metaphor, portraying reading as a linear process involving the mechanical connection of visual input with a lexicon of letter and word codes (e.g. Gough, 1972; LaBerge and Samuels, 1976). These models view the reader as passive and comprehension as something that happens automatically once word meanings have been established; from this perspective, reading is simply a matter of transferring information from the text to the reader (cf. Harste, 1985).

Interactive models of reading (Adams and Collins, 1979; Rumelhart, 1977) were developed in order to address the inadequacies of information transfer models (cf. Samuels and Kamil, 1984). These models also draw on the computer metaphor but give readers an active role and allow for interactions among the knowledge that serves as input (e.g. graphic, syntactic, semantic, and world knowledge); researchers hypothesize that interactions are possible because knowledge is represented in the form of schemata, structures which represent the relations implied by a concept. When readers approach a text they use relevant schemata to make inferences; comprehension is said to occur when connections between new information (presented in the text) and the reader's knowledge base are made (Anderson and Pearson, 1984). Research conducted from this perspective has explored the relationship between text factors and reader factors: researchers have examined the effect of text structure on readers' comprehension (Meyer and Rice, 1984), the effect of background knowledge on comprehension (Anderson and Pearson, 1984), and the processes (e.g. inferencing, summarizing, allocation of attention) readers use to comprehend (see Crismore [1985] for a review of this research). Implicit in interactive reading models, however, is the belief that the reader essentially reconstructs the author's message; the reader is active and strategic but only in the service of recovering the meaning represented by the text. These models regard reading as a purely cognitive process, that is, the interactive models ignore the context of the reading event and thus omit social and affective dimensions from consideration. While these models have far more explanatory power than information transfer models, they still fail to capture the full complexity and generativity of the act of reading.

Transactional models (Carey and Harste, 1985; Eco, 1979; Rosenblatt, 1978) portray reading comprehension as a learning process inasmuch as the reader is said to transform the text in the act of reading. The text is seen as a springboard for generating meanings rather than a template against which a reader's understanding is measured. The reader brings not only background knowledge to the text but also a socio-cultural orientation, a personal history of reading, and beliefs and feelings, all of which are mediated by the context of the reading event. Because transactional models are models of reading in context, the process is described as fluid: comprehension changes when the reader or the context changes, that is, comprehension varies across readers and contexts; the same reader will generate different meanings for a text in different contexts and at different points in time. Researchers working within the framework of a transactional model of reading are less likely to study the relationship between separate variables (e.g., text features and reader characteristics), focusing, instead, on studying the "whole cloth" of reading in home and school contexts (Bloom and Green, 1984). These studies typically make use of naturalistic research designs, yielding detailed portraits of the ways readers make meanings (see, for example, Rowe, 1986; Siegel, 1984; Tefft Cousin, 1988).

A transactional model of reading not only affects the way reading research is approached, but the way the reading curriculum is conceptualized and implemented in classrooms. In contrast to information transfer and interactive reading models, which emphasize direct instruction in text factors (e.g. vocabulary, identifying text structures) and reader factors (e.g. making inferences, using background knowledge, generating main ideas), reading educators working from a transactional perspective have developed instructional strategies that call for the invention and transformation of meaning. Strategies calling for transmediation (i.e. the transformation of a written text into other media such as art [Siegel, 1984] or drama [Grumet, 1985]) and the creation of analogies (Hayes and Tierney, 1982), for example, support comprehension by engaging readers in the sort of reflective, skeptical thought associated with critical thinking. The transactional view of reading as a process of making meaning also supports the learning of specific content and disciplines. Reading educators have therefore developed curricula which uses reading activities (along with other thinking and learning processes such as writing, discussions, and so on) as modes of learning which foster critical thinking.

A New Synthesis:

"Reading to Learn Mathematics" for Critical Thinking

Prologue

Notice how the shifts in the ways researchers have conceptualized reading comprehension as a transaction, and the new concern for developing problem generation strategies and a humanistic conception of mathematics are essentially shifts toward critical thinking. As a prologue to the new synthesis which follows, let us highlight, here, a few points that may further clarify some important parallels between a transactional view of reading and the reconceived goals set for mathematics instruction in the 1990's:

- ambiguity is always present to some extent in the material students encounter, and acts as a positive force in that it creates doubts which motivate the student to marshal his or her resources and search for understanding;
- the personal background and knowledge of the individual students as well as the context in which the reading or mathematics instruction occurs are an integral part of the learning process in both fields;
- both reading comprehension and understanding mathematics require the student to create his or her own personal meaning of the material presented;
- creating personal meaning for a written text or a mathematical concept requires some active and generative effort on the part of the student, which transforms the original material.
- the representation and transformation of a message into different media play a key role in both mathematics (cf. Janvier, 1987) and reading

(Siegel, 1984).

These parallels suggest that research on reading as a transaction could help mathematics educators reconceive their approach to mathematics instruction. At the same time, the introduction of transactional reading experiences could be a valuable addition to current instructional practices in mathematics classes since reading is a mode of learning and so far school mathematics has taken little advantage (if any!) of written texts to foster learning.

"Reading to Learn Matheamtics" for Critical Thinking

"Reading to learn mathematics" introduces reading experiences which encourage students to be active and generative learners so as to develop aspects of learning and understanding that are crucial but often neglected in the traditional mathematics curriculum--such as the nature of mathematics, the process of doing mathematics, affective as well as cognitive aspects of the learning of mathematics, and the contextualized nature of mathematical work. The introduction of "reading to learn mathematics" in classrooms requires the integration of the following three components:

(1) <u>A rich variety of mathematical texts</u>. To help students gain a better understanding of mathematics, we need reading materials which address not only technical content, but also issues in the history and philosophy of mathematics, applications of mathematics to real life, accounts of the strategies used to frame and solve specific problems, and biographies and anecdotes which can provide insight into the more affective and humane aspects of mathematical discoveries. The concern for clarity and organization which characterizes textbooks and technical essays, for example, leads

their authors to present only the end product of mathematical activity, ignoring the process that brought it to that point as well as the possibility of alternative solutions. A range of mathematical texts would portray mathematical thinking in a variety of textual formats. At the same time, a transactional perspective on reading is appropriate for more commonly used texts. These would include regular textbooks as well as sheets of exercises and examples which may not contain a single word. Computer programs and the printout of their output could also be thought of as texts in need of interpretation.

(2) Transactional reading strategies. To fully exploit the educational potential of these texts to foster critical thinking, reading strategies which foreground reading as a mode of learning should be employed. Research in reading has made us aware that it is not only WHAT students read, but also HOW they read that can make a difference in their learning. Students can use transactional reading strategies to learn from any kind of mathematical texts. These strategies engage readers in critical thinking in the sense that textual meanings are constructed through reflective thought motivated by ambiguity.

Several examples of transactional strategies may clarify their role in the learning process. A "say something" strategy (Harste, Pierce, and Cairney, 1985) encourages students to pair up with a peer and read the text together, stopping at various points to make predictions, formulate and discuss questions they have, and explore possible consequences of the matherial being read. This strategy supports critical thinking in several ways. First, it encourages students to value and take ownership of their reading/learning experiences by allowing them to select the points in the text that merit further study and discussion. Second, the "say something" strategy demonstrates to students that social interaction between peers can support their attempts to work out a meaning for the text. Finally, this experience promotes reflective skepticism and an attitude of inquiry. In talking their way through the text, students may define some textual anomalies worth exploring. These anomalies may then set the stage for the construction and discovery of new understandings about the material read.

A "cloning the author" strategy (Harste, Pierce, and Cairney, 1985) may also be used to support critical thinking and learning. In this strategy, readers are invited to read the text with a set of blank 3x5 cards in hand. When they come to an important concept or something they don't understand, they are to record it on a card. Later, students sort through their cards and identify one as the central theme of the piece; all other cards are laid out so as to represent the relationships among ideas, as constructed by the reader. This experience helps the reader (or readers, the strategy can easily be adapted for group work) relfect on what they've read, taking advantage of anomalies they've identified in the course of reading. Further, this strategy, like other transactional strategies, encourages the reader to transform the text into his or her own. This helps keep the reading process form becoming a routinized activity in which the sole concern is recovering the author's meaning.

(3) <u>A curriculuar framework</u>. If we do not want reading experiences to become isolated events perceived as curricular "frills," they must be embedded in an overall curricular framework that creates a context for critical thinking, one which values risk-taking over right answers, process as well as product, and problem formation as well as problem solving. The authoring curriculum (Harste, Woodward, and Burke, 1984; Rowe and Harste,

1986) is a good example of this kind of curriculum and serves as the basis for our conceptualization of "reading to learn mathematics." This curriculum is based on the idea that meaningful learning occurs when students use different communication systems (e.g., language, art, mathematics, drama, etc.) to generate meanings about a topic in a purposeful context. These different ways of knowing help learners take new perspectives which, in turn, help them form as well as refine problems. Unlike most curricular frameworks, in which <u>teachers</u> define the problems and students solve them (Short, 1985), the authoring curriculum assumes that students must experience ownership of this process. Without this experience, students may learn how to process information and "fill in the blanks" without thinking critically. Ambiguity and doubt are therefore hallmarks of reading strategies used in the authoring curriculum.

At the core of this curriculum is a set of beliefs about how to create a context for "authoring." One of the most important is that learning is social and every person in the classroom is a teacher as well as a learner. Another is that meaning-making must be experienced, demonstrated, and valued in classrooms. This means that in addition to planning specific activities, teachers must act in ways that consistently demonstrate the value they place on meaning-making and critical thinking.

The educational potential of this new synthesis can best be understood by looking at an example of "reading to learn mathematics." In the section that follows, we present a set of reading experiences on the topic of "factorization" so as to illustrate the proposed synthesis.

An Illustration: Factoring Trinomials

To illustrate the scope and potential of "reading to learn mathematics," we have chosen what many perceive as a "typical" mathematical topic--factoring trinomials. Factoring trinomials, is an important topic in the current algebra curriculum and presents a stumbling block for many students. It may be useful, therefore, to see how reading could be incorporated into a "typical" mathematics topic and how this integration could contribute to a reconceptualization of the goals of the mathematics curriculum. What we hope to illustrate is the radical shift in the definition of "the basics" from information processing to critical thinking.

Performing factoring of trinomials with success and reasonable speed is, in fact, a major goal of the current curriculum--as shown by the emphasis on these elements in most exams and textbooks. The fact that we already possess technology that allows computers, and even some calculators, to perform symbolic manipulations, including factoring, with speed and accuracy, may suggest a need to rethink the goals of teaching this topic. Rather than striving to make students skillful performers of factoring, mathematics teachers may want to give priority to goals such as developing (a) a conceptual understanding of the process itself, and (b) the ability to decide when it is appropriate and worthwhile to factor, and what to do with the result of factoring. In addition, we may want to exploit the potential of "factoring trinomials" to lead students to discover some of the more humanistic aspects of mathematics and to challenge some unrealistic expectations students have with respect to mathematics.

These new goals call for new instructional experiences as well. In most schools today, the procedure for factoring trinomials (i.e., looking at the integer divisors of the constant term, etc.) is introduced by the teacher and followed by practice in which students apply this procedure to a variety of exercieses. The traditional approach, therefore, assumes that the most efficient and effective way to teach students how to factor trinomials is a clear explanation by the teacher followed by student practice. If, instead, critical thinking were the goal of instruction, teachers might plan experiences that engaged students in the discovery of an original solution for factoring tasks, before any "official" procedure had been presented. The apparent "clarity" of a ready-made solution would be abandoned in favor of complexity of problem-finding on the part of students.

In other words, the students themselves, rather than the teacher, would engage in the generation and definition of specific problems to be solved with the ultimate goal of finding a general procedure for factoring trinomials. Thus, students might be asked to produce their own examples of trinomials to be factored, on the assumption that the judicious choice of such examples plays a key role in creating a general procedure. One might also expect the students to evaluate the appropriateness of applying the "factoring" procedure discovered for specific trinomials, depending on the context in which one is operating. It seems important, in fact, to realize that while it is certainly worth trying to factor numerator and denominator in the expression $\frac{x^2-3x+2}{x^2+3x+2}$ in the hope of simplifying it, it may be a waste of time to factor the trinomial in y=x-3x+5 if all we want to know is whether the parabola opens up or down, or if we need to take the derivative of the expression.

With these premises in mind, we need, first of all, to envisage the teaching of "factoring trinomials" as part of an overall reconceptualization of the algebra curriculum, which emphasizes the meaning and rationale for algebraic expressions and their manipulations. "Reading to learn mathematics" activities could be used throughout this curriculum to help achieve these goals. The following examples illustrate this point:

1. Essays discussing applications of algebra in various domains, and/or reporting on the historical development of algebra could be read using transactional strategies similar to those described earlier. For example, the following excerpt from an essay by Neal Koblitz can provide a starting point for sensitizing students to the fact that equations can be used inappropriately to represent relationships.

> ONE NIGHT SEVERAL YEARS AGO WHILE WATCHING TV, I WAS SURPRISED TO SEE A MATHEMATICAL EQUATION MAKE AN APPEARANCE ON THE ``TONIGHT SHOW." PAUL ERLICH, AUTHOR OF THE POPULATION BOMB, WAS ARGUING THAT THE SOLUTION [TO ENVIRONMENTAL PROBLEMS], AS ALWAYS, WAS IN POPULATION CONTROL. HE TOOK A PIECE OF POSTERBOARD AND WROTE IN LARGE LETTERS FOR THE TV AUDIENCE:
>
> $D=N \times I$.
>
> "IN THIS EQUATION," HE EXPLAINED, "D STANDS FOR DAMAGE TO THE ENVIRONMENT, N STANDS FOR THE NUMBER OF PEOPLE, AND I STANDS FOR THE IMPACT OF EACH PERSON ON THE ENVIRONMENT. THIS EQUATION SHOWS THAT THE MORE PEOPLE, THE MORE POLLUTION. WE CANNOT CONTROL POLLUTION WITHOUT CONTROLLING THE NUMBER OF PEOPLE.
>
> WHO CAN ARGUE WITH AN EQUATION? AN EQUATION IS ALWAYS EXACT, INDISPUTABLE. CHALLENGING SOMEONE WHO CAN SUPPORT HIS CLAIMS WITH AN EQUATION IS AS POINTLESS AS ARGUING WITH YOUR HIGH SCHOOL MATH

TEACHER. HOW MANY OF JOHNNY CARSON'S VIEWERS HAD THE SOPHISTICATION NECESSARY TO QUESTION ERLICH'S EQUATION? IS ERLICH SAYING THAT THE "I" FOR THE PRESIDENT OF HOOKER CHEMICALS (OF LOVE CANAL NOTORIETY) IS THE SAME AS THE "I" FOR YOU AND ME? PREPOSTEROUS, ISN'T IT? BUT WHAT IF THE VIEWER IS TOO INTIMIDATED BY A MATHEMATICAL EQUATION TO APPLY SOME COMMON SENSE? ERLICH KNEW HOW TO USE HIS TIME ON THE SHOW WELL.

From: Koblitz, N. (1981). Mathematics as Propaganda. In L. Steen (Ed.), Mathematics tomorrow (pp. 111-120). New York: Springer-Verlag.

2. Excerpts from classical mathematical texts written before the introduction of algebraic symbolism could be read asking students to transform the mathematical content into their own words, using current algebraic symbolism as much as possible. Such an activity could powerfully convince students of the advantages of using symbols, and make them realize the invaluable role that algebraic symbolism played in the growth and dissemination of mathematical knowledge.

3. Computer programs written to perform algebraic algorithms could be read generatively. Students could be asked to describe the procedure used in their own words and to explain and comment on its application to appropriate examples, which the students themselves have generated. The same approach could be applied to the description of algorithms reported in the textbook.

4. Students may be asked to create word-problems and stories illustrating the application of a learned concept and/or procedure. These stories could

be exchanged among the students, and read with the task of suggesting clarifications, improvements, and further elaborations on the problem or story to the authors.

"Reading to learn mathematics" activities specific to factoring trinomials could also be devised. The following are some examples:

1. Students could be asked to skim the textbook, searching for every instance of the instruction to "simplify an algebraic equation." Working in pairs or small groups, students could then categorize each instance of this instruction and construct a definition for this phrase. It is likely that students will discover that the meaning of "simplifying an algebraic equation" is ambiguous and needs to be clarified by taking the context of use into consideration. In particular, students may be asked to focus on the use of factoring, pointing out circumstances where "simplifying" calls for factoring and others where "simplifying" instead calls for multiplying. Hence, they will once again see the importance of personal judgment and context in mathematical thinking (two crucial elements of our interpretation of critical thinking).

2. To assist the students in achieving a better understanding of the procedure of factoring, a preliminary exercise could involve reading a page showing the multiplication of monomials, such as:

$(x-1)(x-2) =$	$(x+1)(x+2) =$
$x^2 - 2x - x + 2 =$	$x^2 + 2x + x + 2 =$
$x^2 - 3x + 2 =$	$x^2 + 3x + 2 =$

etc.

Though consisting of mathematical symbols rather than English words, this page still represents a written text to which students can apply transactional reading strategies, resulting in a generative approach to mathematics as well as reading. A valuable transformation of this text could consist of using writing to describe the relationships illustrated by specific mathematical examples; that is, recognizing and putting into words patterns such as the relationship between the coefficients of the trinomial and its roots.

3. Given the tentative nature of the procedure for factoring trinomials, it is very difficult to express it clearly in words. Students could be asked to describe in their own words the steps involved. In small groups, students could then compare their descriptions with their classmates' as well as with the description presented in the textbook; the goal would be to arrive at a description of factoring trinomials upon which all members could agree.

4. Factoring trinomials requires the use of an "educated" trial-and-error procedure since there is no algorithm available. The procedure usually taught to factor quadratic trinomials before the quadratic formula is introduced (i.e., searching among the integer divisors of the constant term for a pair whose sum is the opposite of the linear coefficient), guarantees success in factoring only trinomials with integer roots. Discussing the limitations and advantages of this procedure may be very valuable for students since it may help them appreciate the role played in mathematics by an "educated" trial-and-error approach and by heuristic rather than algorithmic procedures.

To help students think about the use of "educated guesses" in mathematics, a Sherlock Holmes story by Sir Arthur Conan Doyle could be read, paying close attention to the way Holmes reasons his way through the case. Though Holmes claims to be the master of deductive logic, a number of philosophers have argued that Holmes is using a form of logic that calls for the generation of hypotheses that might explain a surprising result—a somewhat risky strategy not unlike making an educated guess. Students could then be asked to write an autobiographical essay describing their own use of educated guesses in mathematics; they would also be asked to examine their expectations regarding factoring in particular and mathematics more generally and think about whether making educated guesses fits their expectations for doing mathematics.

Obviously, we are not suggesting that these reading experiences should constitute all that students would do on factoring in an algebra course. Rather, the experiences described above should be integrated with other types of mathematical activities which may not require the support of written texts yet still embody the spirit of generativity and "reflective skepticism" characteristic of our approach to critical thinking.

Within the constraints created by the present curriculum, standardized exams, as well as parental and administrative expectations, one might reasonably argue that most mathematics teachers would not have the time for all the activities sketched above, nor would they have the power to radically reconceive the goals of the algebra curriculum and consequently redesign their teaching of it. We suggest, however, that a selection of the above activities could still be employed and contribute (though clearly to a lesser extent) to the students' learning of factoring.

For example, the activity of reading a page of examples, recognizing patterns and redescribing it in terms of general rules, and reading generatively the procedure for factoring as it is described in the textbook and by classmates, could certainly contribute to a better understanding of the procedure--and also, one would hope, to students' successful performance.

A teacher may also consider the value of devoting some time to the reading of an essay describing instances in mathematics where algorithms are not available. Though a discussion of these issues is not strictly a part of the current curriculum, we would like to argue that it would not be a digression or "icing on the cake." Some research (see, for example, Oaks [1987]) suggests that students' lack of success in factoring may, in fact, stem from their inability to accept a procedure that is not algorithmic. They do not trust the "trial-and-error" procedure taught, feel insecure about their performance, and often nurture the belief that they have not really understood the procedure and/or that the teacher is keeping back the "real" method. Time devoted to dispelling these dysfunctional conceptions may eliminate a stumbling block which impedes some students' progress in mathematics, and thus may turn out to be time well spent even in the context of the current instructional goals.

Conclusion

In conclusion, we believe "reading to learn mathematics" forges a new synthesis of the traditional "basics" that may provide the following benefits:

- "Reading to learn mathematics" experiences built around some basic concepts may contribute to a better learning and understanding of that mathematical content. These activities may help students see different aspects and facets of this content (see the process at work, become aware of historical development, see how these concepts can be applied in new theoretical and everyday contexts). These reading activities have the potential to help students arrive at a personal understanding of the concept by taking an active role and transforming the presented concepts into concepts meaningful to them.

- The value of these reading experiences for learning mathematics may go beyond the specific concepts and topics, and help students develop valuable learning strategies that can be used in new learning situations. We hope to encourage an approach to reading any math text, including textbooks, in a generative way. Since "reading to learn mathematics" activities emphasize learning mathematics as an active and constructive process, we hope that being an active reader will encourage students to be active mathematicians as well.

- These "reading to learn mathematics" activities may also help students develop a deeper understanding of mathematics as a discipline and, in doing so, help them redefine school mathematics. For instance, it's important for students to see that mathematics is a product of human activity mediated by personal values and judgments since their conceptions and attitudes on the nature of mathematics have a considerable impact on how students learn and use mathematics. Developing a deeper understanding of mathematics as a discipline, however, cannot be accomplished through a single, separate unit on the topic; it requires ongoing discussion and demonstrations of these concepts through daily learning experiences. Teachers can't just tell students what mathematics is all about; students must experience it. "Reading to learn mathematics" experiences may provide an instructional context within which students and teachers can work out meaningful conceptions of mathematics.

Beyond the development of specific mathematical knowledge, strategies, and concepts, however, "reading to learn mathematics" proposes that "the basics" can no longer be thought of as the 3 R's, no matter how sophisticated the description may be. Instead, educators will need to define "the basics" in a way that avoids the artifical clarity so characteristic of thinking in schools today. What is basic to schools in the future may well be the unity of knowledge and attitude of inquiry central to the new synthesis of reading and mathematics presented in this paper. "Reading to learn matheamtics" may therefore be able to play a role in bringing about a much needed reform of the mathematics curriculum and to a reconceptualization of the role of these traditional basics in educating students as critical thinkers.

References

Adams, M., & Collins, A. (1979). A schema-theoretic view of reading (Tech. Rep. No. 32). Urbana, IL: University of Illinois, Center for the Study of Reading.

Anderson, R.C., & Peterson, P.D. (1984). A schema-theoretic view of basic process in reading. In P.D. Pearson (Ed.), Handbook of reading research (pp. 255-291). New York: Longman.

Beyer, B. (1984, March). Improving thinking skills-defining the problem. Phi Delta Kappan, 486-490.

Bloom, D., & Green, J. (1984). Directions in the sociolinguistic study of reading. In P.D. Pearson (Ed.), Handbook of reading research (pp. 395-421). New York: Longman.

Borasi, R. (1984). Reflections and criticisms on the principle of learning concepts by abstraction. For the Learning of Mathematics, 4 (3).

Borasi, R. (1985). Errors in the enume[illegible]n of infinite sets. FOCUS: On Learning Problems in Mathematics, 7 (3-4), 77-90.

Borasi, R. (1986a). On the educational uses of of mathematical errors: Beyond diagnosis and remediation. Unpublished doctoral dissertation, SUNY at Buffalo, NY.

Borasi, R. (1986b). On the nature of problems. Educational Studies in Mathematics, 17 (2), 125-141.

Borasi, R. (1986c). Behind the scenes. Mathematics Teaching, 117, 38-39.

Borasi, R. (1987). Exploring mathematics through the analysis of errors. For the Learning of Mathematics, (forthcoming).

Brown, S.I. (1982). On humanistic alternatives in the practice of teacher education. Journal for Research and Development in Education, 15 (4), 1-14.

Brown, S.I., & Walter, M.I. (1983). The art of problem posing. Franklin Institute Press.

Brown, S.I. (1984). The logic of problem generation: From morality and solving to deposing and a rebellion. For the Learning of Mathematics, 4 (1), 9-20.

Brown, S.I. (1986). Liberal education and problem solving: Some curriculum fallacies. In D. Nyberg (Ed.), Proceedings of the Philosophy of Education Society (pp. 299-312). Normal, IL.

Buerk, D. (1981). Changing the conception of mathematical knowledge in intellectually able, math avoidant women. Unpublished doctoral dissertation, SUNY at Buffalo, NY.

Buerk, D. (1982). An experience with some able women who avoid mathematics. For the Learning of Mathematics, 3 (2), 19-24.

Carey, R., & Harste, J. (1985). Comprehension as context: Toward reconsideration of a transactional theory of reading. In R. Tierney (Ed.) Understanding readers' understanding. Hillsdale, NJ: Lawrence Erlbaum Associates.

Carnegie Forum on Education and the Economy. (1986). A nation prepared: Teachers for the 21st century. New York: Carnegie Corporation.

Cooney, T. (1985). A beginning teacher's view of problem solving. Journal for Research in Mathematics Education, 16 (5), 324-326.

Copes, L. (1974). Teaching models for college mathematics. (Doctoral dissertation, Syracuse University, 1974), Dissertation Abstracts International, 1976, 36 (10), 6501A.

Cornbleth, C. (1985). Critical thinking and cognitive processes. In W. Stanley (Ed.), Review of research in social studies education (1976-1983 (pp. 12-64). Washington D.C.: National Council for the Social Studies.

Cousin, P. Tefft. (1988). The social construction of learning problems: Language use in a special education resource room. Unpublished doctoral dissertation, Indiana University, Bloomington, IN.

Crismore, A. (Ed.) (1985). Landscapes: A state-of-the-art assessment of reading comprehension research (1974-1984) (USDE-C-300-83-0130). Washington, D.C.: Department of Education.

Dewey, J. (1933). How we think. Boston: D.C. Heath.

Eco, U. (1979). The role of the reader. Bloomington, IN: Indiana University Press.

Ennis, R. (1962). "A concept of critical thinking," Harvard Educational Review, 32 (1), 81-111.

Gough, P. (1972). One second of reading. In J.F. Kavanagh & I.G. Mattingly (Eds.), Language by ear and by eye (pp. 331-358). Cambridge, MA: MIT Press.

Grumet, M. (1985). Bodyreading. Teachers College Record, 87(2), 175-193.

Harste, J. (1985). Portrait of a new paradigm: Reading comprehension research. In A. Crismore (Ed.), Landscapes: A state-of-the-art assessment of reading comprehension research 1974-1984. Washington D.C.: Department of Education.

Harste, J., Pierce, K., & Cairney, T. (Eds.) (1985). The authoring cycle: A viewing guide. Portsmouth, NH: Heinemann.

Harste, J., Woodward, V., & Burke, C. (1984). Language stories and literacy lessons. Portsmouth, NH: Heinemann.

Hayes, D., & Tierney, R. (1982). Developing readers' knowledge through analogy. Reading Research Quarterly, 17, 256-280.

Hofstader, D.R. (1979). Godel, Escher, Bach: An eternal golden braid. New York: Vintage Books.

Holmes Group. (1986). Tomorrow's teachers. East Lansing, MI: The Holmes Group, Inc.

Janvier, C. (Ed.). (1987). Problems of representation in the teaching and learning of mathematics. Hillsdale, NJ: Lawrences Erlbaum Associates.

Kuhn, D. (1986). Education for thinking. Teachers College Record, 81 (4), 495-512.

Kuhn, T. (1970). The structure of scientific revolutions. The University of Chicago Press.

LaBerge, D., & Samuels, S.J. (1976). Towards a theory of automatic information processing in reading. In H. Singer & R. Ruddell (Eds.), Theoretical models and process in reading (2nd ed.) (pp. 548-579). Newark, DE: International Reading Association.

Lakatos, I. (1976). Proofs and refutations. Cambridge: Cambridge University Press.

Leder, G. (1982). Mathematics achievement and fear of success. Journal for Research in Mathematics Education, 13, 124-135.

McLeod, D., & Adams, V. (1980). Locus of control and mathematics instruction: Three exploratory studies. Journal of Experimental Education, 94-99.

McPeck, J. (1981). Critical thinking and education. New York: St. Martin's Press.

Meyer, B.J.F., & Rice, G.E. (1984). The structure of text. In P.D. Pearson (Ed.), Handbook of reading research (pp. 319-351). New York: Longman.

Meyerson, L. (1977). Conception of knowledge in mathematics: Interaction with and applications to a teaching methods course. (Doctoral dissertation, SUNY at Buffalo, 1977), Dissertations Abstracts International, 1978, 39 (2), 733A.

NAEP. (1985). The reading report card. Princeton, NJ: National Assessment of Educational Progress and Educational Testing Service.

National Council of Teachers of Mathematics. (1980). An agenda for action.

Reston, VA: National Council of Teachers of Mathematics.

Oaks, A. (1987). The effect of the interaction of mathematics and affective constructs on college students in remedial mathematics. Unpublished doctoral dissertation, University of Rochester, Rochester, NY.

Perry, W. (1971). Intellectual and ethical development in the college years: A scheme. New York: Holt, Rinehart, and Winston.

Polya, G. (1957). How to solve it. Princeton, NJ: Princeton University Press.

Ralston, A. (1987, October). A framework for the future of the K-12 mathematics curriculum in the US. Paper presented at the meeting of the Association of Mathematics Teachers of New York State, Rochester, NY.

Resek, D. & Rupley, W. (1980). Combatting 'mathophobia' with a conceptual approach toward mathematics. Educational Studies in Mathematics, 11.

Rosenblatt, L. (1978). The reader, the text, the poem. Carbondale, IL: Southern Illinois University Press.

Rowe, D. (1986). Literacy in the child's world: Young children's explorations of alternative communication systems. Unpublished doctoral dissertation, Indiana University, Bloomington, IN.

Rowe, D., & Harste, J. (1986). Reading and writing in a system of knowing. In M. Sampson (Ed.), The pursuit of literacy (pp. 126-144). Dubuque, IA: Kendall Hunt.

Rumelhart, D. (1977). Toward an interactive model of reading. In S. Dornic (Ed.) Attention and performance VI. Hillsdale, NJ: Lawrence Erlbaum Associates.

Samuels, S.J., & Kamil, M. (1984). Models of the reading process. In P.D. Pearson (Ed.), Handbook of reading research (pp. 185-224). New York: Longman.

Schoenfeld, A. (1985a) Mathematical problem solving. Orlando, FL. Academic Press.

Schoenfeld, A. (1985b). Students' beliefs about mathematics and their effects on mathematical performance: A questionnaire analysis. Paper presented at the annual meeting of the American Educational Research Association, Chicago, IL.

Short, K. (1985). A new lens for reading comprehension: Comprehension processes as critical thinking. In A. Crismore (Ed.), Landscapes: A state-of-the-art assessment of reading comprehension research 1974-1984. (USDE-C-300-83-0130). Washington, D.C.: Department of Education.

Siegel, M. (1984). Reading as signification. Unpublished doctoral dissertation, Indiana University, Bloomington, IN.

Siegel, M., & Carey, R. (in press). Critical thinking: A semiotic perspective. Bloomington, IN: ERIC/RCS; Urbana, IL: National Council of Teachers of English.

Silver, E.A. (1985). Teaching and learning mathematical problem solving: Multiple research perspectives. Hillsdale, NJ: Lawrence Erlbaum Associates.

Sternberg, R., & Martin, M. (1988). "When teaching thinking does not work, what goes wrong?" Teachers College Record, 89 (4), 555-578.

Tobias, S. (1978). Overcoming math anxiety. W. W. Norton & Co.