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ABSTRACT

Computer technology may be a powerful support for teaching through guided inquiry, but this process still depends on teachers who often find it difficult to carry it out in classrooms. This article examines the efforts of a group of secondary school geometry teachers to shift their instruction toward guided inquiry with the use of a computer software program called the "Geometric Supposers." The study focuses on the evolution of the teachers' concerns, and the curricular and pedagogical dilemmas they faced during their work with this innovative approach. The paper analyzes several themes in these teachers' experiences which are likely to reappear whenever teachers try to shift from the predominant recitation mode of "teaching as telling" to the widely recommended, but difficult, process of joining students in a process of constructing and critiquing. The five teachers who participated in this study were from three different schools. (YP)

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FROM RECITATION TO CONSTRUCTION:

TEACHERS CHANGE WITH NEW TECHNOLOGIES

Technical Report

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Educational Technology Center
Harvard Graduate School of Education
337 Gutman Library Appian Way Cambridge MA 02138
(617) 495-9373

FROM RECITATION TO CONSTRUCTION: TEACHERS CHANGE WITH NEW TECHNOLOGIES

Martha Stone Wiske

Richard Houde

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TABLE OF CONTENTS

INTRODUCTION 1
- Goals of the Study 1
- Understanding the Metacurriculum 2
- Teaching the Metacurriculum 3
- Methodology 4

EVOLUTION OF TEACHERS' CONCERNS 5
- Personal 5
- Managerial 6
  - Designing problem sets and exercise 6
  - Managing teacher/student interactions 7
  - Lesson design 8
  - Course architecture 9
- Consequences 10
  - Time 10
  - Assessment 11
  - Authority 12
- Refocusing 13

DILEMMAS OF GUIDED INQUIRY IN THE CLASSROOM 14
- Curriculum Dilemmas 14
- Pedagogical Dilemmas 15
- Resolutions 16

CONCLUSIONS 17

TABLE 1 20

REFERENCES 21

INTRODUCTION

Mathematics education has been widely criticized for concentrating too much on rote memorization of facts and algorithms, failing to teach students how to pose and solve problems (NCTM, 1988; NRC, 1985; NSBC, 1983). Recently computer technology has been widely hailed as a valuable tool to help shift mathematics education away from recitation of ready-made knowledge toward active inquiry and construction of knowledge. Equipped with appropriate software, computers can enable users to gather, manipulate, and represent mathematical data in ways that are impractical or impossible with traditional technologies like paper, pencil, and chalkboards. In this way, computers can help students and teachers gain access to a broader range of mathematical forms and ideas. Computers can also become learning stations where students work independently or in small groups, enabling teachers to circulate among the stations helping to guide students' inquiry (U.S. Congress, 1988; Hawkins & Sheingold, 1986). Thus computer technology can support a more constructive, student-centered approach than the traditional teacher-centered recitation format.

While this may be a tantalizing vision, making it a classroom reality is not easy. Purchasing hardware and software does not automatically change classroom practice and curriculum. Close observation of efforts to integrate technology-enhanced guided inquiry into classrooms reveals that this process may entail profound shifts in educational goals, practice, curriculum, and classroom roles and structures (Amaral, 1983; Martin, 1987). We must understand the nature and difficulty of making these shifts if we are to provide the resources needed to incorporate new technologies into improved mathematics instruction.

This article examines the efforts of a group of secondary school geometry teachers to shift their instruction toward guided inquiry with the use of a computer software program called the *Geometric Supposers*. The study focuses on the evolution of the teachers' concerns, especially the curricular and pedagogical dilemmas they faced during their second year of working with this innovative approach. Finally, the paper analyzes several themes in these teachers' experiences which are likely to reappear whenever teachers try to shift from the predominant recitation mode of "teaching as telling" to the widely recommended, but difficult, process of joining students in a process of constructing and critiquing

Goals of the Study

The *Geometric Supposers* (Schwartz, et al., 1985-1987) were designed to help teachers and students construct knowledge of geometry inductively, developing conjectures and testing them empirically. Through a fairly simple menu, the software allows the user to construct a geometric figure (circle, quadrilateral, or triangle), draw additional elements (e.g. tangent, angle bisector, median), measure entities (e.g. angles, line segments, areas, and perimeters), and compute relationships among quantities. Having made a construction, the user can then repeat it on another figure of the same type, either specified by the user or randomly generated by the program. The "repeat" feature of the menu enables the user to see whether visual or numerical patterns hold across cases and facilitates the testing of conjectures about geometric relationships. The *Geometric Supposers*' developers hoped the software would enable teachers and students to build their knowledge of geometry through empirical investigations, recognizing the limits of particular cases as evidence for general truths, and appreciating the need for formal proofs to establish theorems. In short, they hoped this innovation, meaning both the technology and ways of using it, would facilitate an instructional approach that integrated inductive and deductive reasoning.

This approach to geometry approximates the way geometers work, indeed the way mathematics is made, but it differs radically from the way geometry and other mathematics are usually taught in schools. Most textbooks present geometry as a tight deductive system of theorems built elegantly from an initial set of definitions and postulates. Teachers of secondary school geometry normally organize their course around their textbook. They present topics in the same order as they appear in the text, often believing (Lampert, 1988a) that this sequence both reflects the logical structure of the subject matter and conveys this structure to students in the most coherent way. Teachers commonly present material in class and assign related readings and problems from the textbook as homework. Tests require students to recall definitions, and theorems, and to use these to produce formal proofs. The formal two-column proof which many teachers regard as the backbone of their geometry course is, for many students, a fragile construct of partially memorized rules whose relationship to empirical evidence is poorly understood at best. Like most of school mathematics, geometry is usually taught as a body of knowledge to be rehearsed, memorized, and parroted (Sirotnik, 1983). As Judah Schwartz, one author of the *Geometric Supposers*, is fond of saying, "If English were taught this way students would be required to memorize passages from Shakespeare, Donne, Emerson, and Hemingway, but never asked to write a word."

Understanding the Metacurriculum

An overarching goal of this study was to clarify what is entailed in taking advantage of this new technology to incorporate inductive reasoning into a high school geometry course. This was seen as a step toward helping students understand what mathematics is and how it is made. We call this instructional agenda and approach a metacurriculum because it aims to teach students *about* the usual curriculum--about how geometry knowledge is constructed and how they can participate in that process. During the course of this study, participants became clearer about what this metacurriculum includes. The clarification was largely implicit, however, and subject to variation among individual participants. At the risk of suggesting more clarity and consensus about the metacurriculum than actually existed, we describe it here to orient the reader.

The metacurriculum for this innovation included several elements. Students were to understand how the processes of inductive and deductive reasoning weave together in making geometry. So, for instance, teachers wanted students to learn to gather data, to figure out productive ways of recording and displaying data to facilitate analysis, to notice patterns and form conjectures, to verify conjectures with empirical data, to evaluate apparent counterexamples and to appreciate their disconfirmatory power, to view diagrams as exemplars representing a set of cases, and to recognize the limits of particular cases in establishing general truths--these are some among many examples. To list these aspects of mathematical reasoning may suggest erroneously that the process boils down to a fixed sequence of discrete strategems. In fact the process of logical thinking requires an interweaving of examining, conjecturing, verifying, and communicating combined in an indeterminate sequence. Effective inquiry entails an artful combination of reasoning skills guided by an acquired taste for elegant, powerful ideas.

A second element of the metacurriculum is the recognition that mathematics is the product of human intellect, developed through an arduous but potentially exhilarating process of expending mental energy. Students often believe that mathematics is a c llection of right answers waiting to be found by very smart people (Schoenfeld, 1985). This view fails to acknowledge the tentative, evolving nature of the field and undermines a third element of the metacurriculum. This is the realization that students themselves can make and critique mathematical knowledge. They and their teachers can take the risk of investigating new ideas to figure out things they do not know, relying on their own judgment to decide whether an idea is important or persuasive. Helping students develop the confidence, judgment, and skill to participate in arguments about mathematical ideas is an important goal of the metacurriculum.

The participan ; in this project had not themselves been explicitly taught this metacurriculum, nor had they previously articulated their own version of it. The process of defining the metacurriculum was one of gradually making implicit understanding explicit, while struggling not to squeeze the life out of a process by pinning it down with false precision. Undoubtedly one source of difficulty in teaching this metacurriculum stemmed from its implicit, evolving, variously interpreted nature.

Teaching the Metacurriculum

The *Geometric Supposers* can be used in a variety of ways to support teaching geometry. For example, a teacher might use the *Supposers* with a large monitor or projection device to illustrate geometric ideas during a classroom presentation. Alternatively, students might be encouraged to use the software outside of classtime as an aid in completing homework assignments. In the project discussed here, teachers were encouraged to hold a portion of their geometry classes in a computer laboratory where students worked, either singly or in pairs, on exercises designed to be explored with the *Geometric Supposers*. Subsequent class sessions were devoted to discussion of the data, conjectures and proofs students developed from their lab exercises. These *Supposer*-related inquiries in the laboratory and post-lab discussions were interspersed with more traditional, text-based presentations by teachers.

Designing and conducting *Supposer*-based lessons and integrating them into the structure of the traditional deductive high school geometry course was the challenge teachers faced in this project. The goal of the research was to understand how teachers managed this challenge, tracing the evolution of their concerns and focusing especially on the curricular and pedagogical dilemmas they encountered as they attempted to teach this metacurriculum.

Methodology

The five teachers who participated in this project taught in three different high schools in Massachusetts[1]. They volunteered to participate in a study conducted by researchers at the Educational Technology Center (ETC) to learn about the implementation of technology-enhanced guided inquiry in the classroom. In the summer of 1986, the teachers attended several meetings where they were introduced to the *Geometric Supposers* software, were given sample problems that they might use with their students, and given opportunities to work through some of these problems. These meetings were led by Richard Houde and Daniel Chazan. Houde, a high school mathematics teachers, had worked with the *Geometric Supposers* developers to refine the software, create appropriate problems, and design methods of teaching geometry inductively with this technology. He had taught high school geometry courses with the *Supposers* for several years, and continued to do so while working part-time on this research project. Chazan, a former mathematics teacher, had previously taught with the *Supposers* and had served as an advisor and researcher in a study with other teachers learning to teach with the *Supposer*. After their initial introduction, the five teachers were given computers, software, and problem sets to take home during the summer. During the 1986-87 academic year, Houde served as an advisor to the teachers, observing them regularly in their classrooms and meeting with them to plan lessons, prepare materials, and discuss their questions about classroom practice and management. The teachers also met regularly as a group with Houde and Chazan to share problems, ideas, materials, and encouragement for their complicated innovative effort. Their first year experiences are reported elsewhere (Lampert, 1988a; Wiske et al., in press).

At the end of the first year in Spring, 1987, we decided to follow these teachers as they continued to work with this approach during a second year. Recognizing that the first year of teaching with a complex new technology may be largely taken up with logistical challenges, we anticipated that second year teachers would be able to focus more on the curricular and instructional aspects of this innovation. Throughout the 1987-88 academic year Houde continued to observe and consult with teachers at their schools, serving both as advisor and researcher. As an advisor he helped teachers plan lessons and design and prepare materials. He also observed their classes--both computer laboratory sessions and subsequent discussion sessions--and consulted with teachers about teaching strategies. As a researcher, he prepared detailed notes after each meeting, recording particular events from his classroom observations and summarizing the concerns raised by teachers. Houde and Wiske, a researcher interested in the process of teacher and educational change, met regularly to review Houde's observations, clarifying themes and relating them to other research on teacher development. As the year progressed, Houde's notes focused on the way the innovation challenged teachers' accustomed curriculum and practice and how teachers dealt with these challenges at many levels of lesson design. Both authors met several times throughout the year with the teachers as a group, to discuss teachers' concerns and approaches and to share our emerging analyses with them.

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1 The schools were: a small rural high school, a large comprehensive high school in a middle class suburb, and an alternative school within a large, comprehensive urban high school.

EVOLUTION OF TEACHERS' CONCERNS

The widely-cited Stages of Concern model (Hall & Loucks, 1978; Hall & Hord, 1987) describes a sequence of considerations expressed by teachers as they deal with innovations. (See Table 1--Stages of Concern). According to this model, teachers initially gather general information about the characteristics of an innovation and consider how it will address their personal concerns, then focus on the task of managing the new approach in their own settings, and finally progress to assessing its impacts and considering refinements to enhance its effects. Like Hall and Hord, we found that teachers evinced several kinds of concerns at once rather than proceeding in a lock-step fashion through this progression. Nevertheless, this model offers an overall framework that effectively categorizes the kinds of concerns faced by teachers in this study and points out the sequence in which particular issues tended to preoccupy them. Examining the nature and progression of these concerns clarifies the kinds of resources and assistance teachers need as they incorporate this kind of innovation into their practice.

Personal

Teachers in this project reported they were drawn to the *Geometric Supposer* innovation for one or more of several reasons: they were eager to explore the potential of new technologies for mathematics education, they wanted to enliven geometry instruction with more active inquiry, they wanted to engage students who learned mathematics better through manipulation of objects and visual data than through formal, symbolic representations. These overlapping considerations focused on technology, pedagogy, and learning mathematics, respectively.

The focus of a teacher's initial attraction seemed to shape the teacher's on-going orientation to the project. For example, teachers initially drawn by the computer *per se*, tended to remain more focused on the technology than on the changes its use implied for teaching and curriculum. Those drawn by the prospect of giving students more opportunity to discover and create knowledge for themselves were attracted to the computer laboratory, but slow to recognize the value of this technology for teacher-led demonstrations and lectures. The teachers most interested in helping students see the visual side of mathematics tended to concentrate on extending this aspect of the innovation. Despite these differences in teachers' particular interests and styles, they did seem to move through similar stages of concern.

As Hall and Loucks predict, during the first year of work with the *Supposer* teachers worried initially about how the innovation would affect them personally. Teachers wondered whether they would have the time, interest, and inclination to deal with the technology, the instructional approach, and the demands of researchers. They questioned whether their participation in the project would help or hinder their work at school. While these kinds of concerns are common and deserve attention, in this study we focused on the concerns about management and consequences that preoccupied teachers after they began to incorporate the innovation into their practice.

Managerial

In Hall and Hord's (1987) model , managerial concerns encompass more than just classroom management or logistical issues. They recognize this stage as including concerns about the best use of materials, time, information, and other resources to carry out the innovation in one's own setting. We viewed these concerns even more broadly. Like Lampert (1985), we saw teachers balancing multiple, often conflicting agendas as they made myriad decisions about designing and conducting their lessons. In managing to teach, they faced a set of considerations at various levels of lesson architecture such as, designing exercises and problems sets for students, managing interactions with students during a lesson, planning a sequence of lessons to address a particular topic, and mapping the overall structure of the course syllabus. At every level teachers confronted questions about how to integrate the new technology and inquiry approach into their accustomed curriculum and practice.

Particular examples illustrate the nature of teachers' concerns and the complexity of dealing with them. Descriptions of the advisor's interventions reveal the kinds of assistance that teachers may need as they work an innovation like this into their practice.

Designing problem sets and exercises

The selection and presentation of problems is a crucial aspect of the guided inquiry approach to teaching. Teachers in this project were offered many sets of problems designed to be investigated with the *Geometric Supposers*. The problems instructed students to make a geometric construction (e.g. a triangle with medians drawn to each side) and to investigate the geometric relationships among elements of the construction. In selecting, modifying, and presenting problems to their students teachers had to take a range of considerations into account. Problems must a) address appropriate subject matter about geometry, b) provide enough guidance about how to investigate the problem without directing the students so specifically that they were not required to make any intellectual leaps, c) be arranged on the page so that students could record diagrams or other data, and d) be worded clearly and accurately so that students were not confused. One typographical error, such as labeling an angle ACB rather than ABC, could create major confusion. [See Yerushalmy, Chazen, & Gordon (1988) for a more detailed discussion designing and posing good problems.]

In designing and presenting a problem to the class the teacher makes multiple judgment calls based on educational goals, preferred teaching style, and student needs. For example, one teacher assigned a problem to be investigated in the computer lab that required students to reflect points over the sides of a triangle, then gather data and make conjectures about a range of relationships among elements of the resulting figure. She wanted students to then prove one or more of their conjectures as a homework assignment. The problems that arose illustrate the interplay of judgments. First, some students did not understand the term "reflect". Second, some students did not know how to record their data in a systematic way that would facilitate their making conjectures. When the advisor recommended that the teacher suggest a format for collecting the data, the teacher responded that she thought the students would learn more by discovering the need for a system and inventing one on their own. Clearly, the decision required a judgment about the appropriate amount of structure to provide, given multiple agendas and perhaps alternative pedagogical approaches. Finally, several students were not familiar enough with the geometry raised by the problem to figure out which conjectures they might be able to prove. Addressing multiple goals, in ways both consistent with one's pedagogical preferences and tailored to students' level of knowledge and skill is a tricky business.

Managing teacher/student interactions

When teachers lecture to their class they confront relatively few decisions about managing interactions with students. In shifting to a guided inquiry approach they invite students to generate and share ideas which greatly increases the number of occasions when a teacher must decide how to respond. On such occasions, teachers must make choices shaped by a multiple agendas. The teacher's goals might include: a) teaching certain subject matter about geometry, b) developing students' abilities to conduct and critique inductive reasoning, c) fostering students' confidence and skill as members of a productive intellectual community. For example, the teacher might want students to learn that the sum of a triangle's interior angles equals 180 degrees, that verifying a conjecture with several cases is not the same thing as proving the conjecture, and that one's idea need not be correct to be worth expressing. Another goal might focus on helping students develop and express their own ideas, particularly if the teacher believed that students learn best by making sense of their own observations and that in order to do this in school they must overcome a heavily reinforced belief that their role is simply to memorize the methods teachers give them.

The complexities of balancing multiple instructional goals were particularly apparent when the teachers led discussions of the conjectures students had made regarding a problem they had investigated with the *Geometric Supposers* in the computer lab. As the advisor watched teachers lead discussions, he often found himself thinking, "What a missed opportunity!" Sometimes this happened when a student offered a conjecture that the advisor recognized as related to the teacher's content agenda, but the teacher responded, "We won't be getting to that until next month." To see the connection, the advisor had in mind a broad mental map of the content domain indicating multiple paths among topics, whereas the teacher seemed to map the subject as a fixed linear sequence of topics. Sometimes the advisor perceived a missed opportunity when a student's comment related to his meta-agenda (e.g., teaching students how to reason inductively), but the teacher attended only to the geometry content of the remark. In these cases, it was often difficult to tell whether the teacher simply did not understand or care about the advisor's metacurriculum, was too preoccupied with the demands of a new approach to notice the opportunity, or consciously chose to attend to a different agenda in fashioning a response.

Clearly, leading such a discussion is a very complicated affair. It can easily lose coherence unless a teacher is capable of keeping track of multiple agendas at once. The advisor developed a strategy to help simplify the complexity which he called "the three-board technique". He used one board to write down students conjectures as they called them out. By writing them in the students' words he credited their ideas and spared himself the cognitive demand of translating them into more standard vocabulary. When no new conjectures were forthcoming he shifted to the second board where he rewrote the students' conjectures, grouping them into categories and translating them into language that led toward the points he wanted to make. During this process he explained how the conjectures related to each other, modeled the process of analyzing and evaluating conjectures, and wondered aloud about whether any of the conjectures could be proved. The third board he used to work out proofs for selected conjectures. He found this technique useful in balancing attention to both content and process, and synthesizing students' ideas with his own agenda.[2]

Lesson Design

This innovation challenged teachers on several fronts at once, necessitating a redesign of lesson architecture at several levels. The innovation involved integrating a new technology into the teaching and learning process, shifting pedagogical approach from primarily teacher-centered lecture mode to a more student-centered and problem-focused inquiry mode, and weaving a new inductive reasoning curriculum into the standard deductive curriculum. In attempting to accommodate all these changes, teachers found themselves rethinking the structure of their lessons, the sequence of lessons within a particular topic, and the overall syllabus of their courses.

In designing lessons, teachers first needed to consider overall teaching format. A topic might be taught in the regular classroom through teacher-led lecture, demonstration, or discussion. It might be taught in a computer laboratory where the teacher must decide when to address the whole class versus circulating among stations helping to guide students' inquiry as they worked independently in pairs or by themselves. Or a topic might be dealt with in a homework assignment, that either introduced a topic to be examined later during class or that reviewed material previously discussed in class.

The *Geometric Supposers* technology was amenable to a variety of lesson formats. Used with a large monitor or display screen, the *Supposer* could serve as a sort of dynamic chalkboard, augmenting a teacher-led presentation or discussion with the whole class. Several teachers, who had used the *Supposer* only in the laboratory where students worked with the software directly, found using the *Supposer* in front of the class a bit awkward at first. They had to become fluent with the new technology before its advantages outweighed those of the accustomed technology of chalkboard, compass, and straightedge.

Computer laboratory sessions offered the advantage of engaging students in active inquiry, but they required both teachers and students to learn new skills and practice new roles. Teachers found that while students learned to use the software fairly quickly, they took longer to learn how to read and interpret a problem set, to collaborate on making constructions, to make observations, to gather visual and numerical data, to record their findings, and to form conjectures. Teachers gradually learned how to introduce a problem, walking a line between explaining so much that the challenge of inquiry was eliminated versus leaving students too confused to work productively on their own. While the laboratory-based lesson format offered many advantages, it might also be a very time-consuming way to cover a topic. Teachers had to learn to be selective about the lessons they

2 See Lampert (1988b) for a more extensive discussion of the factors that constrain teachers' efforts to connect students' ideas with their own agendas and of some strategies they employ to accomplish this difficult feat.

chose to teach through computer-based inquiry in order not to slight important topics in their syllabus.

Selecting homework assignments and weaving them into the course was also problematic. Sometimes teachers assigned homework that either introduced or followed up exercises students investigated in the computer laboratory. Designing homework to follow lab sessions was tricky if students did not all proceed with lab work at the same pace. Collecting, grading, and incorporating these assignments into class discussions was a challenge. Faced with these challenges in integrating home and lab work, teachers frequently took the more familiar path of assigning textbook problems as homework. These problems were often structured and worded in a way that made them difficult to integrate with the inductive approach to the lab material, however. Integrating homework and classwork is always challenging, but particularly so when students set their own pace in class and when connections must be forged between computer-based lessons in class and text-based homework assignments.

Course architecture

This discussion of lesson design issues reveals another set of dilemmas at the level of course architecture. The inductive approach to geometry with the *Supposers* sometimes prompted teachers to rearrange the sequence in which they presented certain topics and even called into question their accustomed sequence of units within the course. Two examples help illustrate thi point.

1. The *Supposers'* measure menus allow students easily to measure the sizes of angles, lengths of segments, and areas of shapes. Most textbook-based geometry syllabi, however, do not examine the topic "area" until the fourth or fifth month of the school year. Teachers in this project faced a dilemma: teach a unit on area at the very beginning of the school year and thus open rich opportunities for students to investigate area relationships or tell students to ignore the *Supposers'* area options until the textbook chapter on area had been formally studied. Most teachers opted for the former choice and modified their curricula accordingly.

2. The geometry textbooks used by teachers in this project presented chapters on quadrilaterals followed by chapters on similiarity. After teachers had students investigate quadrilateral problems with the *Supposers* during the first year of the project, however, teachers discovered that the richness of their quadrilateral units would be significantly enhanced if students had previously learned concepts related to similarity. Thus, during the second year of the project, two teachers revised their previous year's curriculum sequences and taught units on similarity prior to teaching units on quadrilaterals.

Making decisions at this level of course architecture was yet another aspect of the management concerns generated by this innovation. Given the range and depth of these concerns, it is little wonder that teachers stuggled with them throughout the period of this study.

Consequences

As teachers faced and coped with personal and management concerns, they gradually began to worry more about the consequences of this innovative approach. Their concerns settled on several aspects of their multi-faceted responsibilities: students' learning of the traditional course content as measured by standard achievement tests, teacher's coverage of the traditional course materials as laid out in the textbook and perhaps in required curriculum guides, and students' learning of the new "metacurriculum", i.e. learning how to reason inductively and understanding and participate in the process of making mathematics.

These concerns imbued the dilemmas described earlier with particular urgency. Choosing how to respond to a student remark, selecting problems, and designing lesson architecture all involved tradeoffs between sometimes conflicting priorities. Three themes recurred in teachers' discussions of the consequences of this innovation: time, assessment, and authority.

Time

Time is the engine that runs life in secondary schools. The day is segmented into periods signaled over the loudspeaker system. Both teachers and students carry little schedules marked off in a grid that determines their activities during each period, five days a week. Except in the rare cases when teachers can schedule a double period, they cannot continue an activity past the end of the period without tangling all the class members' subsequent engagements. Such a rigid schedule is much better suited to a teacher-controlled lesson than to a student-centered, inquiry-oriented one. Once students become engrossed in working on a problem the teacher loses control over the focus and pace of their thinking. A student may have just caught the scent of a powerful proof when the teacher gives the five minute warning before the bell rings. Teachers in this project often remarked that standard periods were too short for computer lab sessions. Students needed time to take their seats and arrange their materials, work through the exercise enough to understand the shape of it and embark on several lines of inquiry, study their findings, and decide which conjecture to pursue. The inflexible schedule laid a troublesome crosshatch over the unpredictable trains of thought in multiple centers of inquiry in the class.

The larger blocks of time into which the school year is divided also conflicted with the rhythm of the inquiry-oriented approach used in this project. Teachers were accustomed to devoting a certain number of weeks to a pre-determined set of topics: for example, a week for parallel lines, followed by three weeks on congruence, then three weeks for quadrilaterals. Especially in one school involved in this study, where a year of geometry was intertwined with a year of algebra in a two-year course sequence, the schedule of topics was elaborately set. Teaching with the *Supposers* made teachers want to reduce or expand the amount of time they devoted to some topics and reconsider the sequence in which they could be taught most effectively. But these deliberations must be made within the framework of the pre-determined school year taking account of the impact of scheduled events like vacations, field trips, and exams.

Finally, teachers remarked on the difficulty of finding time anywhere in their week, let alone during the school day, to prepare for this innovation. Teaching this way required time for a whole host of activities. They must invent or modify exercise sheets and then try out problems themselves as a way of proof-reading the sheets and anticipating students' reactions. They needed time to design the lesson architecture described earlier, orchestrating an effective combination of lesson formats. They needed time to think, for example, after leading a discussion, about the points students had raised and decide how they might weave them into a later class or pursue them if they came up another time. They needed time (among other things discussed below) to grade student papers that included widely varying drawings and arguments rather than standard short answers.

Assessment

Assessment issues were another recurring theme in teachers' discussion of the consequences of this innovation. Teachers wondered what their students were learning, how to make this determination, and how to conduct assessments in a way that reinforced their purposes in the course. With respect to computer-based assignments, teachers needed to articulate criteria for assessing student work. As the teachers themselves became clearer about the skills and knowledge they wanted students to master, they became better able to design tests that helped reveal and support such mastery.

During the year discussed here teachers used three types of lab tests reflecting progressively more advanced stages of development that they wanted students to achieve with the *Supposers*. During stage one, which usually occurred during the first two months of the year, teachers administered tests that required students only to state conjectures and exhibit data to support them. Teachers were primarily concerned that students learn how to use the *Supposers* to gather empirical evidence and placed little emphasis on asking students to write logical arguments to support their conjectures. As the year progressed and students learned more about the meaning of proof, teachers then had the option to require students to defend their conjectures with logical arguments or formal proofs (stage two). Some teachers insisted that students report all their data, state their conjectures, and write just one or two proofs while other teachers simply asked students to state conjectures. Teachers were continually forced during this stage to decide how much emphasis to place on proof in computer lab-based testing situations. During the third and final stage, teachers de-emphasized or omitted the need for reporting empirical data and asked students to concentrate on using the *Supposers* to verify their ideas and develop logical arguments or proofs to defend them. No two teachers ever administered exactly the same test, but they all led their students through these stages.

Teachers also needed to develop strategies for grading *Supposer*-based assignments. They found themselves buried in papers that required much more thought and commentary than they had time to provide. With a mounting backlog they sometimes gave up trying to discuss lab assignments with the class or even skipped reading some papers. The advisor recommended that they grade only portions of each paper, devoting thoughtful attention to selected problems rather than superficial treatment at best to the entire assignment.

Weighing student mastery of the new curriculum in relation to mastery of the old curriculum constituted another dilemma. During the first year, teachers in one school discovered that students figured out they could make good enough grades on their computer lab assignments to earn a passing grade for the course even if they failed the examinations. The teachers found this unacceptable, especially because the examinations more nearly mirrored the material included on standardized achievment tests that other mathematics teachers would expect students to have learned in a geometry course. In the second year they redesigned their grading system to better reflect the balance of their course goals.

The lack of appropriate assessment instruments and strategies both reflected and hampered the teachers' ability to articulate and legitimize the new curriculum. The advisor offered to help teachers design *Supposer*-based tests. He recognized that such tests helped to define and legitimate the *Supposer*-based curriculum: they helped teachers specify the aims of this curriculum and demonstrate their commitment to it, they helped students chart their own progress, and they enabled teachers to monitor and demonstrate what students had learned.

Authority

Finally, teachers' deliberations about the consequences of this innovation revealed that it had occasioned a shift in authority in the geometry classroom. The basis of authority broadened from the textbook and the teacher's interpretation of the standard curriculum to encompass the judgment of an intellectual community made up of the teacher and students. Like paradigm shifts in the history of ideas, this change was gradual, multi-faceted, and marked by fits and starts and reversions to the old ways.

The basis for authority shifted in several ways. One of them was that teachers took on more authority for exercising their own judgment about what and how geometry should be taught, rivaling the textbook and curriculum guide as shapers of these decisions. Teachers wavered as they took on this role, but the progression was clear in several cases.

For example, during the first year of the project one teacher taught a unit on concurrency that closely followed the dictates of his textbook. He taught students how to copy segments and angles and construct angle bisectors, altitudes, and perpendicular bisectors of line segments with compass and straightedge. He presented lessons and assigned homework exercises that followed the text's recommended guidelines. Although he had students use the *Supposer* to do a lab assignment, it did not significantly change his manner of teaching the unit. Midway through the second year of the project, this teacher told the advisor he was unhappy with his previous approach and intended to teach concurrency using the *Supposer* as the primary technology and his text only for selected exercises. His new unit included sets of problems that students investigated in the computer lab, followed by classroom discussions of these exercises. Homework exercises emerged from lab work and class discussions. The teacher's agenda and students' findings shaped the unit while the textbook served merely as a reference.

Another teacher radically changed her methods for teaching elementary geometry concepts. She abandoned her former approach of assigning pages in the textbook which instructed students in basic definitions and substituted a series of computer-based problems which helped students discover similar concepts. She led discussions about students' lab results to summarize the important understandings she wanted students to remember and used the textbook problems to reinforce lab work.

A second kind of shift occurred as students exercised more authority as makers and critiquers of knowledge rather than passive receivers and reciters of knowledge made by others. In this process they were supported by the *Supposers* as an aid in providing empirical evidence. With the *Supposers* they could check to see whether their conjectures held true for multiple cases, investigating the limits of generality of a pattern or relationship. This tool helped to put empirical verification within reach and, for some students, seemed to help them recognize the limits of empirical evidence. As a member of the class accumulated confirmatory examples in arguing for a conjecture, students might challenge, "Prove it!". The ready access to empirical evidence seemed to help students gain confidence in their conjectures, and, in some cases, apparently helped students develop a more sophisticated intolerance for the particular case as a basis for proof.

A third type of shift in authority stemmed from the structure of the *Supposers*. Its menu and facilities seemed to place the software in a position to rival the textbook as a curriculum authority. For example, the *Supposers* Circles disc menu includes the word "inversion", a fascinating topic rarely taught in traditional high school geometry courses. The *Supposers* make this topic readily accessible. One of the teachers experimented with this option, following the *Supposers'* directions while inverting a series of points, but was unable to discover the meaning of the term. The advisor shared some knowledge of the topic and responded to the teacher's request for suggestions about an exercise to culminate her unit on circles. He suggested that she might assign some *Supposer*-based problems involving inversion. The teacher became so interested in the topic that with the advisor she designed a set of lab problems on inversion for her students to investigate.

Refocusing

Hall and Hord (1987) discern that once teachers have figured out how to manage an innovative approach and have thought through its consequences for teaching and learning, their concerns often turn toward rethinking the innovation itself. In this process they may seek out opportunities to collaborate with colleagues in revising or extending the innovation. While teachers dealt with this range of concerns, they did not address them in a lock-step linear sequence.

At the end of the first year of the project, several of the participating teachers decided to introduce the *Supposer* and inquiry-oriented geometry to their colleagues. In three schools they proceeded to use their own positions and their system's idiosyncratic norms and structures to devise ways of encouraging the spread of this innovation beyond their own classrooms. Their efforts are described in Shepard & Wiske (in press). During this process, the teachers opted to balance dependence on their own wisdom and resources with support and guidance both from ETC researchers and from their fellow teachers on the project. In many respects, their way of dealing with the innovation parallelled the guided inquiry process they carried out with their students. The teachers appeared to learn best when guidance from the research and development team was combined with opportunities for them to invent and extend ideas on their own and to examine their experiences with fellow learners confronting similar challenges.

As teachers supported the spread of the innovation within their own schools, they also reconsidered how to modify and extend the innovation. Some of the adjustments resulted from wrestling with concerns about management and consequences, reconsidering goals, and finding ways to retune problems, strategies, and techniques to address these goals more successfully. Other adjustments grew out of the experience of introducing the innovation to other teachers, and recognizing the range of interpretations that might be made to accommodate various student needs, teaching styles, and course syllabi. Debates continued throughout the year among the teachers and ETC staff about, for example, appropriate ways to modify *Supposers*-based and inquiry-oriented lessons to suit students of varying academic achievement levels. Teachers also recognized that their colleagues varied in their pedagogical preferences and instructional goals. Whereas one teacher was deeply committed to letting students discover ideas on their own, another instinctively directed students so that they would learn the "textbook" definitions. As the veteran teachers became advisors to their colleagues, they experienced the dilemmas their own advisor had faced in balancing his advice with respect for his fellow professionals' goals, preferences, and expertise.

DILEMMAS OF GUIDED INQUIRY IN THE CLASSROOM

As discussed earlier in the paper, the instructional goals of this innovation included teaching students how to reason inductively and helping them understand and participate in the process of making mathematics. The instructional approach included guidance from teachers as students investigated problems, worked to make sense of their findings, and formulated arguments and proofs regarding their results. The examples provided in the preceding section illustrate the multiple challenges teachers face in teaching this curriculum through a guided inquiry approach.

This type of innovation requires most teachers to make a fundamental paradigm shift. Most teachers operate within a recitation paradigm, assuming that knowledge is specifiable, that the teacher's role is to transmit information, and that the students' role is to absorb, remember, and repeat what they are taught. Teaching for understanding through guided inquiry reflects a constructivist paradigm, assuming that knowledge is personal and problematic, that the teacher's role is to help students build on their own ideas to construct understanding, and that the students' role is to participate actively in making knowledge not just to memorize what they are told. In shifting from one paradigm toward the other teachers do not suddenly and totally transform their knowledge, behaviors, and beliefs. They confront myriad decisions and dilemmas, choosing between alternative approaches, shifting the balance between priorities.

Curriculum Dilemmas

Berlak and Berlak (1981) describe these dilemmas of teaching in ways that illuminate the choices faced by the geometry teachers in this study. In rethinking educational goals, teachers must decide what is worthwhile to teach. Among the curriculum dilemmas they face are the following:

- knowledge as content vs. knowledge as process

Should teachers concentrate on teaching facts, concepts, and theories or should they help students experience and understand the process of thinking, reasoning, and critiquing ideas? Should geometry teachers focus on Euclidian geometry theorems and their use in solving real world problems? Or should they emphasize that geometry is the study of ideas whose relationships are determined by a rigorous, axiomatic, deductive system leading to formal proofs?

Many school systems in effect favor teaching the content of Euclidian geometry by endorsing either locally-developed or nationally-standardized tests that focus almost entirely on this material. Meanwhile, many geometry teachers believe that the power of a geometry course lies in its ability to hone students' logical thinking skills. This project promoted an approach that placed more emphasis on learning to reason, but the process encompassed inductive as well as the deductive reasoning that geometry teachers usually emphasize. Teachers in the project tended to vary in their judgments about the proper balance to strike in teaching geometry content, deductive reasoning, and inductive reasoning. Their judgments seemed to depend on their own teaching preferences, their assessment of their students' ability, and the priorities of their school system.

- personal knowledge vs. public knowledge

Should teachers concentrate on transmitting to students the information in their textbooks and the wisdom the teacher poses? Or should they focus more on helping students make sense of their own ideas and build their own personal understanding? This dilemma arises from questions about the basis for intellectual authority in the classroom. How much ought teachers to rely on the textbook or curriculum guide or other codification of "the geometry to be taught" rather than looking to their own judgments about what to teach or building on students' ideas? It lies at the heart of teachers' decisions about balancing their guidance with opportunities for students to pursue their own inquiries.

Pedagogical Dilemmas

Along with curriculum dilemmas, teachers faced poignant pedagogical dilemmas. Of those mentioned by Berlak and Berlak (1981) the following appeared particularly salient for teachers in this project:

- learning is social vs learning is individual

Is learning a private matter between child and text or child and teacher or is it a more social affair whereby understanding is developed through argument, collaboration, and corroboration with other people?

Teachers in this project wondered whether students learned well and efficiently when they helped each other analyze exercises. They were also concerned about assessing students' individual learning and felt obligated to structure at least some assignments as individual rather than collaborative projects.

- teacher control vs. student control of time, operations, and standards

Should teachers direct the classroom agenda and schedule or allow students' to exercise control over the focus and pace of their learning? Should teachers prescribe how and in what order students learn their lessons or should students be encouraged to invent and discover their own approaches? Should teachers determine the standards for intellectual and social performance or should students participate in deciding what, for instance, is a persuasive argument, which of several conjectures is worth trying to prove, whether a remark is worth making to the whole class?

If teaching a course is conceived as building a community of scholars who share responsibility for teaching and learning, then students must be helped and allowed to take more control in the classroom. Teachers in this project found that deciding when to relinquish control was difficult. They also puzzled over ways to help students' develop the judgment, self-discipline, and confidence to exert such control responsibly.

Resolutions

Teachers' efforts to manage these dilemmas appeared to reflect their preferred teaching styles, their assessments of their students' level of ability and learning needs, and the norms and values of their schools systems.[3] In addition, it appears that teachers' views and choices change as they became more familiar with the innovation and more adept at weaving guided inquiry into their accustomed practice. The advisor often noticed opportunities as he watched the teachers' classes when they could have managed to resolve some of these dilemmas.

For example, he noticed opportunities when teachers could develop students' inductive reasoning skills by modeling, labeling, and reinforcing these skills during class. Using terms like "conjecture", "verify", and "prove" consistently and carefully established a form of discourse in the classroom which emphasized and legitimized a set of activities. In this way teachers were able to attend to their inductive reasoning agenda without always having to take time in class for explicit instruction on this topic.

Other opportunities arose when students made conjectures or asked questions related to the teachers' agenda. Recognizing and seizing these opportunities depended on the teacher's having certain knowledge, skills, and beliefs. As Lampert (1988b) has noted,

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[3] See Shepard & Wiske (in press) for a fuller discussion of the ways school structures and values shape teachers' ways of dealing with this kind of innovation. They suggest that in schools where students are expected to do what the teacher tells them and teachers are expected to do what the administration decides that both teachers and students are disinclined to exercise the kinds of intellectual authority that a guided inquiry approach calls for. By the same token, this approach seems to be encouraged by schools systems which expect teachers be active learners and to participate in decisions about curriculum, assessment, and school administration.

connecting students' ideas with the teacher's agenda is a complicated endeavor. The teacher must see the connection between the student's ideas and his or her own lesson plan. This may require teachers to have in mind a map of the subject matter domain in which topics are connected by multiple links rather than laid out in the single linear sequence that textbook chapters and curriculum guides suggest. To build on students' ideas might also require the teacher to revise his or her lesson plan on the spot, taking up an idea or an activity that was not scheduled. Achieving this flexible spontaneity without sacrificing coherence is particularly difficult when a teacher is first using a new technology or approach. As the innovation becomes more familiar teachers may free up the mental energy to hear students' ideas more clearly and to discern connections to their own agenda. Making such connections also requires teachers to respect and trust students' ideas as legitimate grist for the intellectual mill of the classroom. These beliefs are easier to espouse as both students and teachers become more accustomed to being active participants in constructing knowledge (Lampert, 1988a). Such connections reduce or resolve the problem of choosing between poles of several of the dilemmas described earlier.

Our findings suggest that as teachers became more knowledgeable about the goals of the curriculum associated with this innovation, and developed more materials and strategies for teaching this curriculum, they were more able to resolve or at least manage some of the curricular and pedagogical dilemmas it posed. This is not to say, however, that with sufficient experience the dilemmas disappear entirely. While teachers might become better able to recognize and use opportunities to integrate the old curriculum with the new, at times they still had to make difficult choices.

CONCLUSIONS

Computer technology may well be a powerful support for teaching through guided inquiry, but this process still depends on teachers who often find it extremely difficult to carry out in classrooms. The recitation paradigm is reinforced by curriculum guides, textbooks, and tests that direct teachers to "cover" a prescribed sequence of many topics and that assess how well students can recover this material on demand. The recitation paradigm leads to a didactic instructional approach that is much better suited to teaching whole classes, many periods a day, on tightly structured schedules. As a rarely examined, only partially articulated, pervasive educational philosophy, the recitation paradigm shapes both teachers' and students' assumptions about what they should do and expect of each other in the classroom. Even more broadly this paradigm is reflected in school policies and procedures that favor discipline and central authority over individual initiative and invention. The entrenched recitation paradigm reinforces deeply rooted beliefs and patterns of behavior in schools. The constuction paradigm and the process of guided inquiry pose major intellectual, emotional, and moral challenges as well as technological and practical ones for teachers in classrooms.

This paper has attempted to map the dimensions of these challenges in order to clarify the kinds of support that teachers need as they face them. Our study demonstrates that incorporating technology-enhanced guided inquiry into their practice was a protracted process for teachers. The process raised a predictable sequence of concerns for teachers which called for varied and evolving forms of implementation assistance. Initial concerns focused on personal issues such as learning to use the technology and finding the time and other

other resources in one's life to tackle the innovation. Once embarked on the new approach teachers faced a range of concerns about managing the incorporation of the innovation into accustomed curriculum and practice. These concerns arose at many levels of lesson architecture, from the design of exercises and teaching materials, to the management of class discussions, to the structure of curriculum units and course syllabi. As teachers found ways of incorporating the new instructional approach into their repertoire they faced concerns about impact and tradeoffs. For teachers in this project these concerns focused on questions about time, about effective means of assessing the impact of the innovation, and about a new basis for intellectual authority in the classroom encompassing the teacher, the students, and the software. Eventually, teachers turned to reassessing the innovation itself, rethinking how they wished to use it, and wanting to help spread it to their colleagues. In the course of supporting colleagues to extend the innovation, teachers readdressed issues of management and consequences in light of their deepening understanding of the innovation.

As teachers dealt with these evolving concerns, they needed a range of types of implementation assistance. Initial training in the use of the software and exposure to sample problems had to be supplemented with ongoing consultation. As they worked through a range of managerial concerns teachers needed assistance both from an experienced advisor and from each other. The advisor and their fellow innovators helped them think through the innovation at all levels of lesson architecture, and to develop exercises, lessons, curriculum units, and tests. The advisor also observed the way they put their plans into effect and offered feedback and suggestions.

The support of colleagues engaged in the same innovative effort helped teachers identify and deal with the logistical, curricular, and pedagogical dilemmas they faced and the emotional strains these created. They confronted choices about the balance of attention to teaching geometry content versus thinking skills, to "covering the standard curriculum" versus developing students' abilities to reason inductively. They also faced pedagogical dilemmas regarding the appropriate balance of teacher control versus student control over the focus, pace, and assessment of instruction.

Teachers' responses to these choices appeared to reflect their preferred teaching styles, their assessments of students' needs, and the prevailing priorities and expectations of their school systems regarding curriculum and teacher/student roles. It also appeared that teachers' reaction to these apparent choices changed as they become more experienced in weaving a guided inquiry approach into their standard courses. As teachers become more clear about the goals of the innovation and more adept at integrating the innovative methods into their practice, they found more ways to integrate the old and new approaches rather than having to choose between them.

If our conjecture is correct that incorporating guided inquiry into standard secondary school courses requires a fundamental shift in educational paradigm, we should expect that most teachers will need the same sort of extended multi-faceted assistance that teachers in this study received. If the innovation entails not merely learning how to use a new technology to do the same wo' . in the same ways as usual, but a change in educational goals and in deeply-rooted behaviors and beliefs, then making the change will require considerable time, thought, courage, and practice. Our study suggests that beyond investing these resources teachers also need help to plan, prepare, and think through the management, consequences, and extensions of such innovations. This help appears to be most useful if it combines assistance from experienced advisors with opportunities for collegial exchange among fellow innovators. Such a combination may be thought of as the equivalent of guided inquiry in implementation assistance. From this perspective the form of the implementation assistance serves to model and engage teachers in a process like the one they are trying to create with their students. If teachers are expected to share authority with their students in constructing understanding, it makes sense for the process of supporting this shift to include a similar sharing between leaders and learners.

TABLE I

Stages of Concern about the Innovation

6 REFOCUSING: The focus is on exploration of more universal benefits from the innovation, including the possibility of major changes or replacement with a more powerful alternative. Individual has definite ideas about alternatives to the proposed or existing form of the innovation.

5 COLLABORATION: The focus is on ccordination and cooperation with others regarding use of the innovation.

4 CONSEQUENCE: Attention focuses on impact of the innovation on student in his/her immediate sphere of inflence. The focus is on relevance of the innovation for students, evaluation of student outcomes, including performance and competencies, and changes needed to increase student outcomes.

3 MANAGEMENT: Attention is focused on the processes and tasks of using the innovation and the best use of information and resources. Issues related to efficiency, organizing, managing, scheduling, and time demands are utmost.

2 PERSONAL: Individual is uncertain about the demands of the innovation, his/her inadequacy to meet those demands, and his/her role with the innovation. This includes analysis of his/her role in relation to the reward structure of the organization, decision making, and consideration of potential conflicts with existing structures or personal commitment. Financial or status implications of the program for self and colleagues may also be reflected.

1 INFORMATIONAL: A general awareness of the innovation and interest in learning more detail about it is indicated. The person seems to be unworried about himself/herself in relation to the innovation. She/he is interested in substantive aspects of the innovation in a selfless manner such as general characteristics, effects, and requirements for use.

0 AWARENESS: Little concern about or involvement with the innovation is indicated.

[see Hall and Hord, 1987, p. 60.]

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