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ABSTRACT

From 1984 through 1988, the authors worked with teachers using an inquiry approach to teach high school geometry courses with the aid of the "Geometric Supposers." Problems are a critical component of the approach because they focus attention and energy and guide students in the application, integration, and extension of knowledge. Inquiry problems differ from traditional, single-answer textbook problems in that they must leave room for student initiative and creativity. The observations presented in this paper about the delicate balance between specifying too much instruction and too little, which is part of creating and posing inquiry problems, are based on careful examination of students' inquiry problem papers. The paper also discusses speculations on whether these observations suggest general lessons for those seeking practical and successful strategies to introduce student inquiry into classrooms, with the hope of stimulating interest in and discussion of such strategies. Appendices include solutions to 12 problems and examples of students' answers. (Author/YP)

TR88-21

POSING PROBLEMS: ONE ASPECT OF BRINGING INQUIRY INTO CLASSROOMS

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Posing Problems:
One Aspect of Bringing Inquiry into Classrooms

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ABSTRACT

From 1984 through 1988, the authors worked with teachers using an inquiry approach to teach high school geometry courses with the aid of the GEOMETRIC SUPPOSERS. Problems are a critical component of the approach, as they are of any instructional process, because they focus attention and energy and guide students in the application, integration, and extension of knowledge. Inquiry problems differ from traditional, single-answer textbook problems in that they must leave room for student initiative and creativity. The observations presented in this paper about the delicate balance between specifying too much instruction and too little, which is part of creating and posing inquiry problems, are based on careful examination of students' inquiry problem papers. The paper closes with speculations on whether these observations suggest general lessons for those seeking practical and successful strategies to introduce student inquiry into classrooms, with the hope of stimulating interest in and discussion of such strategies.

INTRODUCTION

Our image of how students should learn mathematics is bound up with the word "inquiry," a word with a long history in educational contexts and with many connotations. In using "inquiry," we want to bring to the reader's mind the process of learning employed by creative people at the forefront of their fields — people interested in a particular area and continuously motivated to learn more about it, who set themselves problems, design methods to explore them, and then try to create solutions. (This characterization is based on Schwab's (1962) description of "pure inquiry.")

More specifically, inquiry teaching in mathematics might mean that students should learn mathematics by choosing a topic, posing problems, creating approaches to the problems, and recreating historical discoveries. We believe that approach is neither realistic nor practical; students are not expert mathematicians. In our image of inquiry teaching, teachers organize inquiry experiences for their students by posing inquiry problems to explore. A central activity of this approach is inquiry by individual students or in small groups. Typically in our work, student inquiry occurred in a computer lab, with other sessions in a regular classroom. Tools such as the GEOMETRIC SUPPOSERS[1] act as intellectual "amplifiers," and inquiry facilitators helping students explore in the manner of experts.

As the designers of the GEOMETRIC SUPPOSERS note, inquiry teaching is uncommon in high school mathematics classrooms:

> There is something odd about the way we teach mathematics in our schools. We teach it as if we expect that our students will never have occasion to make new mathematics. We do not teach language that way. If we did, students would never be required to write an original piece of prose or poetry. We would simply require them to recognize and appreciate the great pieces of language of the past, the literary equivalents of the Pythagorean Theorem and the Law of Cosines[2].

This sort of inquiry teaching is at odds with common school practice, for several reasons. First, an inquiry approach is potentially replete with doubt, confusion, dead ends, frustrations, and wild goose chases. Common conceptions of teaching suggest that teachers should not willingly lead students into such difficult terrain, but, rather, eradicate or smooth over confusion. Second, if teachers have indeed led their students into difficult terrain, they should figure out how to help students resolve confusions. Experts faced with confusion rethink and recategorize, they stand back and reexamine. In contrast, teachers are

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[1] The GEOMETRIC SUPPOSERS developed by Judah L. Schwartz, Michal Yerushalmy, and Education Development Center are published by Sunburst Communications, Inc. This paper describes the SUPPOSERS only as inquiry tools, but they can be used also as demonstration tools or for verification activities.

[2] Schwartz and Yerushalmy, 1987, p. 293.

taught to reduce confusion by atomizing material into smaller chunks, for example, the way factoring quadratics or long division are taught. Third, current evaluation procedures do not test students on inquiry tasks; they do not measure inquiry skills such as testing conjectures, finding counterexamples, or posing new problems. Fourth, inquiry is time-consuming, and time is precious in schools where teachers must cope with a host of conflicting demands on their time and energy.

We encountered these conflicts between an inquiry approach and current school practice, between our image of how we would like students to learn mathematics and the constraints of school settings, in implementing our approach to teaching geometry. This paper presents observations about strategies for designing inquiry teaching materials, indicating which helped to preserve an "inquiry" spirit and to defuse conflicts between traditional school practice and our approach and which did not, in the hope that such strategies will be helpful to others planning to implement inquiry approaches in schools.[3]

This paper is primarily intended for researchers interested in inquiry learning and in the use of computers to promote inquiry and for mathematics educators interested in implementing inquiry approaches in schools (see footnote 3). Although not explicitly about the evolution of an innovation during implementation or about collaboration between teachers and researchers, it can be read as the result of such a collaborative evolution.

INQUIRY PROBLEMS IN GENERAL

Our first major compromise with the "pure inquiry" model was to include as a central tenet of our approach that teachers, as students' guides, take responsibility for providing questions or problems for students to explore.[4] Thus, "problem" assumes a different meaning from that commonly used in mathematics classrooms. Inquiry problems are not tidy textbook problems easily solved and with only one answer; they are "real" problems — though not necessarily "real world" problems — resembling those an expert would explore. They are related to the teacher's agenda, they are open-ended, can be approached in many ways, and have many solutions; in short, they are worth exploring. Students and teachers must understand the differences between inquiry problems and textbook problems and the differences in appropriate student performance in solving each kind of problem.

Creating or finding problems worth exploring is not easy. Once one exists, good ways to pose or communicate it in an open-ended manner must be created. The challenge for the problem poser is to communicate the problem in a way that provides sufficient guidance so the *Task* of the problem is clear, while at the same time not providing so much that all inquiry converges to a single point. Communication of inquiry problems becomes a locus of tension between open-ended inquiry and structured school work. The challenge is

[3] Footnotes throughout provide specific information for those particularly interested in posing inquiry problems for use in high school Euclidean geometry courses using the GEOMETRIC SUPPOSERS.

[4] We have observed schools, not those in this study, where teachers rely on students to pose problems before going into the lab. This approach places a heavy burden on teachers and students and may not be realistic for wide application.

compounded because each person (teacher or student) views the balance between too much and too little instruction differently.

In this paper we examine materials used to communicate to students both the particular problem used as a basis for inquiry and our expectations for appropriate inquiry. We chose to focus on strategies for designing materials because comparing and contrasting such written materials is relatively easy, while acknowledging that many other less tangible ways exist to communicate expectations to students. (Perhaps the most important and effective way to communicate these new expectations is to model the process of working on inquiry problems, a complex endeavor which is hard to evaluate. This paper reports part of a larger ongoing project to understand how to integrate inquiry into the teaching of high school geometry. Future research will focus on modeling inquiry skills.)

INQUIRY PROBLEMS IN GEOMETRY: AN ANALYTIC FRAMEWORK

From 1984 through 1988, we worked closely with high school students and teachers in the Boston area in 23 classes in Euclidean geometry taught using the GEOMETRIC SUPPOSERS in an inquiry approach. We observed classrooms and met with teachers monthly. Many conversations with teachers centered on actual inquiry problems. Reactions to the problems varied from teacher to teacher and from class to class: different kinds of students and different kinds of teachers needed different problems. We encouraged the teachers to modify the problems to fit their perception of their classes' needs.

From the experience of creating and posing inquiry problems, we learned to preserve the spirit of inquiry in activities practical for classrooms and created an analytic framework of what we call "considerations" for designing inquiry problems in geometry. The pedagogical strategies chosen to address these considerations dictate the degree of the spirit of "inquiry" a problem retains. The reader looking at the list of six considerations should imagine a teacher creating and posing an inquiry problem for a geometry class. Initially, the teacher chooses the geometric content of the problem, and, **before drafting a written statement of the problem,** might consider:

- *Kind of problem*
- *Size or scope of the problem*
- *Students' ability or background*

Then, **while drafting a written statement of the problem,** the teacher might consider how to word:

- *A statement of the goal of the problem*
- *A description of any constructions in the problem*
- *Process instructions*

Below, we offer observations based on empirical evidence about the success or failure of pedagogical strategies used by teachers in three geometry classes during 1985-86[5] (Yersuhalmy et al., 1987) for addressing these considerations. The evidence was collected by examining students' work, observing classrooms once every two weeks throughout the year, and meeting with teachers once every three weeks. (For a detailed description of our methods of data collection, see Yerushalmy et al., 1987.) For problems to be considered successful they had to meet all of the following criteria:

- Preserve the spirit of "inquiry"
- Be enjoyed by students
- Be the catalyst for significant student work by almost all students
- Be considered successful by teachers in reaching their goals

The evidence suggests that success or failure of the inquiry problems was determined by the strategies teachers used to address the six considerations listed above and that these strategies determined both the clarity of the problem and the extent of student inquiry.

The remainder of this paper outlines observations about strategies for designing inquiry materials that are clear and also leave room for student inquiry. Since these observations concern strategies used to address the six considerations, the considerations are used to organize the discussion.

CLASSROOM OBSERVATIONS

A. BEFORE DRAFTING A WRITTEN STATEMENT OF THE PROBLEM

Kind of problem

Our first two observations relate the success of a problem to two aspects of its nature:

Whether it is:

- A construction problem (see Problem 1)
- A conjecture problem (see Problem 2)

Its instructional role:

- To help students discover theorems
- To familiarize students with relationships in a construction
- For students to apply concepts already learned

The first observation concerns the relative success of construction and conjecture problems at different points in the year:

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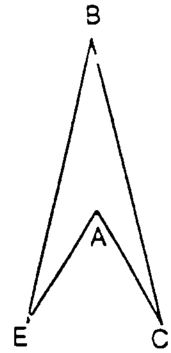
[5] Although, as explained in RECOMMENDATIONS AND CONCLUSIONS, our observations presented here stem from that year's work, they have been confirmed by our ensuing experience.

Problem 1

Task: To develop a procedure for reproducing this figure.



Procedure:

- Make a drawing similar to this figure.
- Collect data.
- Describe below the procedures for reproducing this figure.
- State your conjectures.
-

Drawings & Data

Conjectures

Procedure for reproducing figure:

___

___

___

For quadrilateral ACBE above, under what conditions will ∠CAE be a 90° angle?

___

___

___

From Geometry Problems and Projects: Triangles (1987), M. Yerushalmy, R. Houde, and the Center for Learning Technology, Education Development Center, Inc.. Publisher: Sunburst Communications, Inc., Pleasantville, NY.

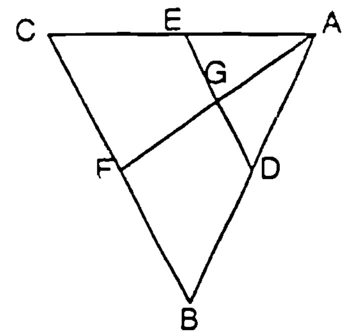
Problem 2

One Midsegment

Task: To explore figures formed by drawing one midsegment in a triangle.

Procedure:

- Construct any $\triangle ABC$.
- Draw midsegment $\overline{DE}$ connecting $\overline{AB}$ and $\overline{AC}$.
- Label the midpoint of $\overline{BC}$ with point F.
- Draw $\overline{AF}$ and label the intersection of $\overline{DE}$ and $\overline{AF}$ with point G.
- Measure the elements of the figure.
- Record your data.
- State your conjectures.



Drawings & Data

Conjectures

From Geometry Problems and Projects: Triangles (1987), M. Yerushalmy, R. Houde, and the Center for Learning Technology, Education Development Center, Inc.. Publisher: Sunburst Communications, Inc., Pleasantville, NY.

Observation: Mechanisms for checking results (to know when they were correct) were an important element in the structure of successful inquiry problems.

Construction problems were especially successful early in the year, for several reasons. First, the GEOMETRIC SUPPOSERS make trial and error strategies easy to carry out. Students were motivated to try a variety of different constructions. They had a tool to use to test their strategies. Second, in most construction problems there is a level of solution that does not require generalization. Most students interpreted the construction problems as "Construct a figure that looks like this (or that has these properties)...."[6] In such problems, the characteristics of the solution are known from the start; students are given criteria to judge their solutions and know when they have a correct answer. They can check their methods of construction by making measurements on the resulting diagram to determine if their method works. The problem does not dictate a particular construction method. If the construction is not too elementary and is adequately described, then the underlying *task* will not be vague yet will allow room for open-ended inquiry.

In contrast to a problem whose solution does not require generalization, a problem that does require a general answer does not provide all the characteristics of its solutions. Students can be sure of their answers only when they have a proof for their conjecture. Before students learned how to devise proofs, they found problems that required general solutions and conjecturing more difficult than those that allowed for specific solutions.

The success of problems also depended on their pedagogical role, in that:

Observation: The pedagogical roles of problems had implications for the amount of structure required.

Teachers assigned problems at least three different roles (discovering theorems, familiarizing students with relationships in a construction, applying concepts already learned).

In some cases, when teachers wanted students to discover the theorems, postulates, and definitions of geometry, they gave inquiry problems before the concepts were introduced. The teachers had specific agendas — they wanted certain conjectures to appear — and these problems typically relied on charts to organize students' data collection and focus their attention. See **Problem 3**, a discovery problem.

A second pedagogical role for problems was to familiarize students with a set of relationships in a particular construction before the teacher taught a theorem based on that construction. When problems were used in this way, the teachers were less concerned about the production of the particular theorem; in some cases they preferred that, rather than discover the theorem, students should understand and become familiar with the

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[6] In another paper (Yerushalmy and Chazan, in preparation), we examine how these students viewed diagrams. At the beginning of the year, they treated diagrams as specific instances, not as models of a class of figures; later, they treated them as general models and were able to argue that specific characteristics of a given diagram were not representative of the whole class.

Problem 3

T 2
Angle Measurements

Task: To explore the relationship among the interior angles in different types of triangles.

Procedure:
- Construct a triangle.
- Measure each angle.
- Draw the triangles and record the angle measurements on the chart below.
- Repeat this procedure on five other triangles.
- On the following page, state conjectures about your findings.

Triangle Drawings	∠ABC	∠BCA	∠CAB
1.			
2.			
3.			
4.			
5.			
6.			

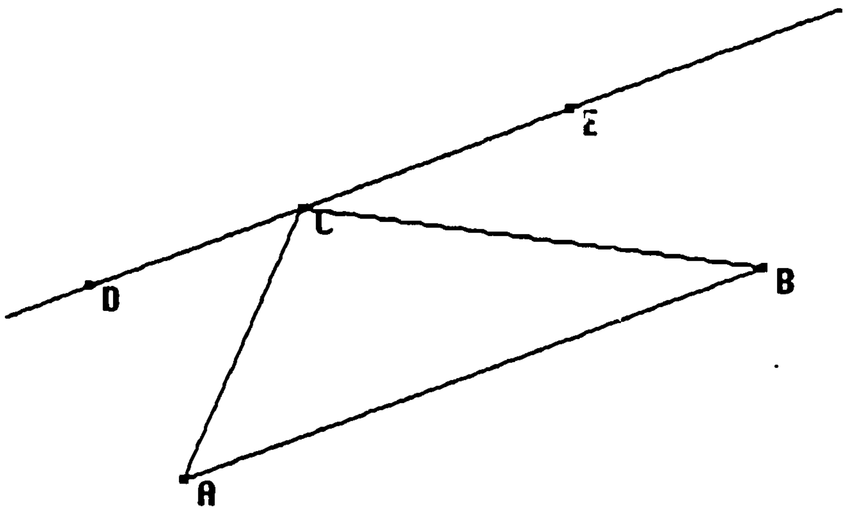
From Geometry Problems and Projects: Triangles (1987), M. Yerushalmy, R. Houde, and the Center for Learning Technology, Education Development Center, Inc.. Publisher: Sunburst Communications, Inc., Pleasantville, NY.

relationships in the construction. The theorem to be taught was extracted from their understanding of those relationships. For example, when teaching about the sum of the interior angles in a triangle, instead of using **Problem 3**, a discovery problem, a teacher might ask students to explore the construction in **Problem 4**:

Problem 4

Task: To explore angle relationships in the following construction

PROCEDURE: Draw any triangle ABC. Through C, draw a segment parallel to AB. Record a diagram and measurements. Repeat this process on other triangles. State your conjectures below.



Once students recognize the angle relationships present in this construction, even if none of them conjectures that the sum of the interior angles of a triangle is 180° or can prove this conjecture, then the teacher can use the relationships they do find to prove it.[7]

The third pedagogical use of problems was to help students apply the concepts they had already learned. In this situation, since they were not asked to discover basic concepts, their individual conjectures were less important than in discovery problems, and

[7] The construction in Problem 4 was created from the construction in Problem 3 by adding the auxiliary line necessary for the proof of the sum of the interior angles theorem. Note that Problem 4 differs from Problem 3 in several ways. The construction in Problem 4 is more complicated, allowing for more conjectures, and that the presence of the auxiliary line makes the proof more accessible to students, providing greater flexibility for their exploration.

teachers were less concerned with every possible conjecture. Following a discussion of similarity, for example, **Problem 5** can serve as a concept application problem.

Using these three pedagogical roles of problems, teachers' levels of need for particular student results dictated different levels of teacher guidance and directions. For problems intended to fulfil the discovery role the instructions were directive and leading, to enable students to discover the results most important to the teacher. In problems intended to fulfil the roles of familiarizing students with relationships in a construction and applying concepts the instructions were less directive and leading, and, not surprizingly, these types of problems were more successful in preserving the "inquiry" spirit. The different roles for problems were more or less successful according to our criteria (see p. 4). For example, much to their teachers' consternation, students had trouble both in finding patterns in their data in **Problem 3** and in developing conjectures for problems like **Problem 6**, where the *Task* is to chart exterior angles. Students may find it difficult to discover the one relationship central to the curriculum which the teacher is seeking. They may discover and be distracted by other relationships less central to the course and of less interest to the teacher. The process instructions may also contribute to the relative ineffectiveness of such problems, an idea examined below in the discussion of *Process instructions* (see p. 14).

Size or scope of the problem

Theoretically, the size o an inquiry problem cannot be gauged apart from the learner, because it depends on the learner's effort and creativity in exploring the problem. This assertion can be illustrated by examination of a problem that one student selected as the most trivial and uninteresting of all: "Investigate the sum of the interior angles of a triangle." Yet even this problem can be the starting point for a long and interesting exploration, if students do not equate it with the written statement just given. A creative inquirer might explore polygons with more than three sides and find a pattern relating the number of sides and the sum of the measures of the interior angles; might look at the sum of two angles in any triangle and explore how the sum varies by the type of triangle; might examine exterior angles; or might even generalize to three dimensional objects. These explorations of the initial problem were created using Brown and Walter's (1983) "What if not" strategy.

We did not, however, observe this kind of expansion of problems by students:

> **Observation:** Students explored problems as they were written. They did no more than the instructions suggested. Therefore, we were able to define the size of problems by examining the instructions on the page.

In the classroom, the size of an inquiry problem is a function of the expectations of members of the class. The students did not go beyond the instructions given in the problem, equating it with the statement on the materials and were unwilling to expand or change a problem without specific instructions to do so. For example, in **Problem 7**, below, students did not generalize the number of triangular sections; that is, they did not attempt to divide the triangle into 2, 3, 4, 5... n equal triangular areas.

Problem 5

Reflecting a Point to Create Triangles - Part A

Task: To explore the figure formed by reflecting the intersection point of the altitudes in each side of a triangle and connecting the three image points.

Procedure:

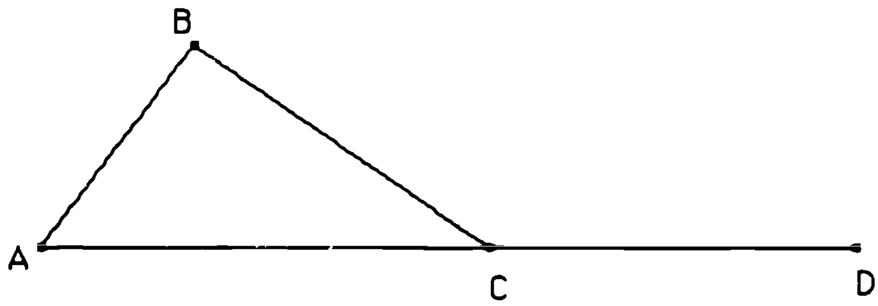
- Construct an acute ΔABC.
- Draw the three altitudes.
- Label G as their point of intersection.
- Reflect point G in each of the three sides of ΔABC producing points H, I, J.
- Draw ΔDEF and ΔHIJ.
- State your conjectures about the relationships among the points, elements, and triangles.
- Repeat the procedure for other types of triangles.

Drawings & Data

Conjectures

From Geometry Problems and Projects: Triangles (1987), M. Yerushalmy, R. Houde, and the Center for Learning Technology, Education Development Center, Inc.. Publisher: Sunburst Communications, Inc., Pleasantville, NY.

Problem 6



<BCD is called an exterior angle for ΔABC. Conjecture how the measure of exterior <BCD is related to the measures of the interior angles of the triangle.

Drawings of ΔABC	m<BCD	m<BAC	m<ACB	m<CBA
Acute ΔABC				
Obtuse ΔABC				
Right ΔABC				
Isosceles ΔABC				
Others				

Conjectures

Problem 7

Try to split a triangle into triangular sections that have the same area. First, try to get four sections with equal area. When you have a solution, make sure it works in all kinds of triangles. Record a drawing of your solution and explain what kind of lines you added to the triangle to get the drawing.

The observation noted above is clearly disappointing, although not surprising (for a similar finding in a different setting, see Jensen, 1986). In our view, an important part of exploring a problem is changing the formulation, asking the question, "What if not?" (Brown and Walter, 1983) about some aspect of it. No matter how a problem is posed, this question can open up new fields of inquiry, yet in many schools, such changes are often considered inappropriate, by both teachers and students[8].

For the most part, teachers felt that the SUPPOSER problems were too large and vague. They rewrote them and relied on process instructions (for a definition, see *Process instructions*, p. 14) to clarify problems and break them into manageable parts. As a side effect, parts of problems were jettisoned and problems became smaller (Lampert, 1988; Yerushalmy et al., 1987). For example, **Problem 8**, below, which teachers revised and simplified into **Problem 7**, is in our view larger because it explicitly asks students to generalize their findings to any number of triangular sections of equal area.

Problem 8

One segment can cut or divide any triangle into triangular sections. For example, an angle bisector divides any triangle into two sections, and three medians divide any triangle into six sections. Give the type(s) and number of line segments that divide any triangle into 2, 3, 4, 5,... sections all having the same area.

An additional observation related to the size and scope of problems is the following:

Observation: Students did not like working on one problem for an extended amount of time. Though, in general, problems with few solutions were less successful than those with a large number of avenues of exploration and possible solutions, which were the most successful.

[8] Even though this finding is discouraging, there are ways, besides building wider exploration into the statement of the problems, to make generalization part of students' repertoire for solving problems. One way to do so is to model such behavior in the classroom and make clear that changing and manipulating the elements of a problem are part of the task of doing a problem. In teaching with the GEOMETRIC SUPPOSERS the teachers had already taken on a large and difficult task, and for that reason we did not spend a lot of time on modeling the students' behavior; other issues had greater priority.

This observation and the previous one both support and contradict the teachers' intuitive decision to make problems smaller. If *size* is viewed as purely the amount of time and work students need to complete a problem, then the larger problems were unsuccessful After three or four consecutive days' work, students tired of working in the computer lab[9]. When they were interviewed after a year of using the SUPPOSERS, some, though not all, said they preferred to learn in ways that required less work. Such students preferred a traditional classroom, where the teacher does a few examples of one kind of problem in class and the homework consists of a set of similar problems (Yerushalmy et al., 1987)[10].

If *size* is instead defined by the number of avenues of exploration or solutions available, then larger problems were more successful than smaller ones. Since one of our criteria for the success of an inquiry problem is that it have many solutions, this reasoning may seem tautological. Yet, problems with many solutions were successful also according to our other criteria — that is, significant work from most students, student enjoyment, and teacher satisfaction[11]. For example, one reason both teachers and students enjoyed construction problems was the range of different solutions. (Students also enjoyed conjecture problems with different solutions). At the same time, small problems created early in the year to lead students to basic concepts, which had one preferred solution and suggested the measurements students should make, were not successful (for examples, see Problem 3 or 6). As noted, in the discussion of Problems 3 and 4, adding an auxiliary line can produce a problem that generates a larger number of conjectures and brings the proof of the desired theorem into students' range. We used this strategy more generally to change unsuccessful small problems into larger problems with a large number of possible conjectures with easier proofs.

Students' ability or background

There are at least three interrelated aspects to the consideration of students' ability: their general mathematical ability and achievement; their knowledge of geometry; and their inquiry skills (cf. Kruteskii, 1969a, b). Although, the effect of students' general mathematical ability could not be carefully examined, our experience suggests that the amount of structure and direction students need from written materials varies according to their general mathematical ability and according to the school's expectations for self-directed activity. Concerning the other two aspects of ability, we observed the following:

[9] Splitting one large construction problem about reflection into a series of connected subproblems (all construction problems) proved a successful strategy. The students' work on the subproblems was similar, in a positive way, to their work on the "smaller" construction problems.

[10] This attitude coincides with Schoenfeld's (1988) observations about students' five-minute theory of problems. The students he observed gave up on any problem that they could not solve in five minutes. They seemed to think a problem, by definition, should be solved quickly.

[11] DiSessa's (1985) claims about small problems in physics complement this indication of the success of problems with many possible solutions and avenues for exploration. He argues that small problems "can hardly establish the context for inventing a technique for solving a class of problems" (p. 113). In mathematics, he points out, small problems can never motivate students to invent definitions in the way a mathematician does.

Observation: As students' knowledge of geometry and their repertoire of inquiry skills grew, inquiry problems became more successful.

For example, students' ability to produce conjectures improved with increasing geometric knowledge. They attack a problem, we believe, by developing an initial conjecture and, then, refining that conjecture in light of further experience. In the beginning of the year, when students had little knowledge of geometry, their conjectures were a hit-or-miss business. As they learned more geometry, they derived their first conjectures from deductive geometric knowledge and used the SUPPOSERS to elaborate and verify them. Thus, as students gained geometric knowledge, they made more conjectures. Further, as their inquiry skills developed and they learned, for example, to appreciate generalization and to identify fruitful situations for exploration, they used numerical manipulations and found interesting conjectures even in "dull" problems (for a detailed description of this development with a different sample of students, see Yerushalmy, 1986).

B. WHEN DRAFTING A WRITTEN STATEMENT OF THE PROBLEM

The preceding discussion of three considerations a teacher might take into account before drafting a written statement of an inquiry problem indicates the bind in which we found ourselves. While students responded poorly to small-size problems created to lead them to basic concepts, which had a single preferred answer and suggested the measurements students should make, their teachers were uncomfortable with large-size, open-ended problems, which they felt were too vague for the students. The need to find how to pose large but specific problems led us to develop another set of considerations when drafting a written statement of an inquiry problem: a statement of the goal, a description of the construction, and process instructions. These considerations are individually indicated in Problem 9.

A statement of the goal of the problem

Greeno suggests that "When a problem has an indefinite goal, the problem solver cannot know what the solution state will be like until it is achieved" (1976, p. 480). As the construction problems described above illustrate, however, it is possible to pose a problem with a definite goal (e.g., "Draw a rectangle") that provides students with information about the solution without specifying the exact solution.

Conjecture [12] : A statement of the goal of an inquiry problem will help students work productively.

Originally, when we wrote inquiry problems, we did not include a statement of the goal of the problem for students (see, as examples, Problems 7 and 8). It seems, however, that we can write such statements which can help students. Because we found this practice helpful, it has become our common practice. See, for example, the format used in the three

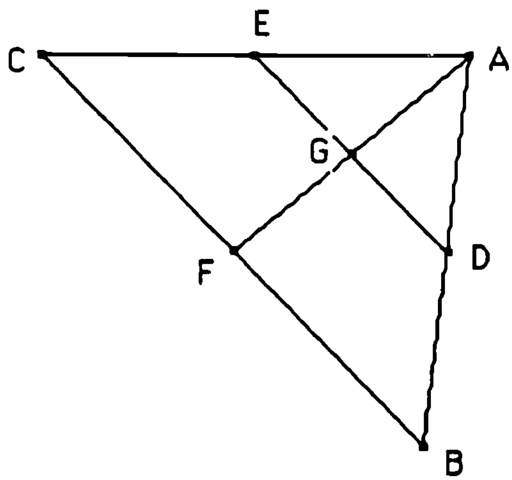
[12] We use the term conjecture, as opposed to observation, because we have no direct data to support this statement.

Problem 9

STATEMENT OF THE GOAL

Task: To explore figures formed by drawing one midsegment in a triangle.

Procedure:



CONSTRUCTION INSTRUCTIONS

-Construct any ΔABC.
-Draw midsegment DE connecting AB and AC.
-Label the midpoint of BC with point F.
-Draw AF and label the intersection of DE and AF with point G.

PROCESS INSTRUCTIONS

-Measure the elements of the figure.
-Record your data.
-State your conjectures.

Adapted from Geometry Problems and Projects: Triangles (1987), M. Yerushalmy, R. Houde, and the Center for Learning Technology, Education Development Cente~, Inc.. Publisher: Sunburst Communications, Inc., Pleasantville, NY.

GEOMETRIC SUPPOSERS Problems and Projects books published by Sunburst Communications, Inc. **Problem 10** shows how the addition of a statement of goal to Problem 7 might help students make a more thorough investigation of the problem by providing some indication of the desired direction of inquiry. The task is specified and the phrase "different numbers of" has been added to provide further direction.

Problem 10

(Another version of Problem 7)

> *Task*: Split a triangle into different numbers of triangular sections of equal area.
>
> PROCEDURE: First try to get four sections of equal area. When you have a solution, make sure it works in all kinds of triangles. Record a drawing of your solution and explain what kind of lines you added to the triangle to get the drawing.

As this problem illustrates, a sentence that directs students to the goal makes the problem less vague by answering the question, "What are we trying to do?" (Although such a statement seems valuable, sometimes it may be difficult to write without, on one hand, specifying more of the solution than actually desired or, on the other, being too vague; see Problem 4 for a vague task statement.) In Problem 10, the question of whether triangles of equal area must be congruent provides a connection to the curriculum; the statement of the goal includes a mathematical property or relationship that links the lab problem to class lessons. The statement of the goal of a problem may therefore not only explain to the students what they should be doing, but by linking lab to class may also clarify why a particular lab problem was assigned.

A description of the construction in the problem

In all the SUPPOSER problems we used, a construction was described. In construction problems, the construction is itself the goal; in conjecture problems, the construction was what students explored. This section evaluates methods of describing constructions, specifically the effectiveness of diagrams as vehicles for stating the specifications of a desired construction.

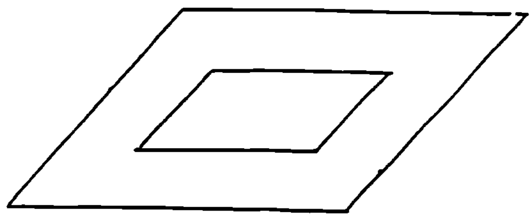
> **Observation:** An unaccompanied diagram was not a sufficient description of the construction to be made. A diagram accompanied by a written description proved a more successful method of describing the construction to be made.

Problem 11 asks students to describe methods for drawing a construction and provides a diagram as an example of the construction desired. (The students had learned about similarity and properties of quadrilaterals and triangles before this problem was assigned.) In this case, the use of only a diagram to specify the desired construction was unsuccessful:

students did not produce solutions to this problem and, according to teachers' reports, had difficulty understanding the assignment.

Problem 11

Describe methods for drawing the following shape



In contrast, Problem 12, which was assigned somewhat earlier in the year and includes written specifications as well as a diagram, was successful.[13] The problem asks students to construct a triangle within a triangle and defines the relationship between the two triangles (similarity). Next, the problem is broken down into parts. Students are asked to construct particular types of similar triangles to be drawn, and diagrams serve as models for the written descriptions.

Students produced many solutions to Problem 12, and they described their solutions in clear geometric terms, not in step-by-step lists of the keys to press in order to make the construction. The contrast between their difficulty with Problem 11, which presented only a diagram to specify the construction desired, and their productive response to Problem 12 suggests that they needed written descriptions to help them identify and isolate key attributes of a diagram. This interpretation makes sense, because a diagram is a specific member of a class and has a large number of attributes. It is hard to know simply by examining a diagram which of its attributes must be reproduced.[14] For example, in Problem 11, the diagram does not specify clearly enough which configurations will be considered correct solutions. The diagram appears to be a parallelogram, but the student is not told whether the desired construction also must be a parallelogram. In order to evaluate thoroughly the use of diagrams as specifications of a construction, one needs to understand how students view diagrams, a large issue we explore elsewhere (Yerushalmy and Chazan,

[13] In some problems, particularly conjecture problems, diagrams can be dispensed with altogether and the description of the construction presented as procedure, or list of steps, for the student to carry out. We do not suggest that all descriptions of a construction must include diagrams. Indeed, because initially students treat diagrams as specific instances, not as models, avoiding diagrams early in the year may be preferable.

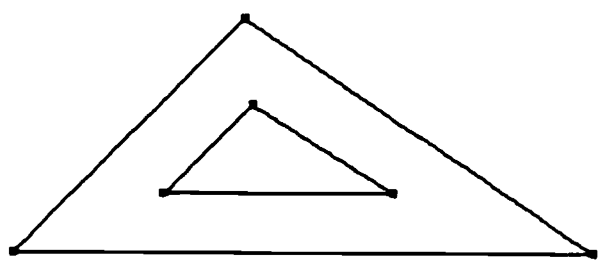
[14] A sequence of diagrams can be used to aid students in determining which attributes should be reproduced, but the diagrams must be carefully chosen; students may find common features the problem poser did not intend.

Problem 12

A) This problem asks you to describe different methods for drawing a triangle similar to but inside ΔABC such that the two triangles share no points in common. Provide data to verify that your methods work.

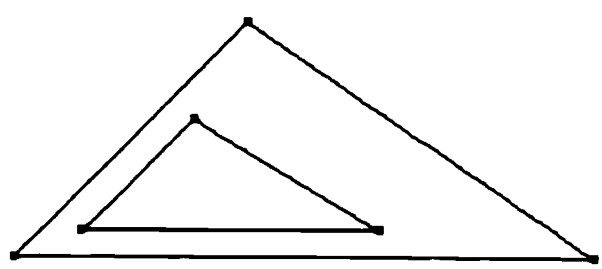
B) Draw the sides of the new triangle inside ΔABC such that they are equidistant from the corresponding sides of ΔABC.

Example:



C) The new triangle is located anywhere inside ΔABC.

Example:



in preparation). Early in the school year, students in this study made construction tasks into "Draw this specific figure...." They treated diagrams as specific instances, not general models. Had Problem 11 been presented to them early in the year, they might have assumed that the final product must include two parallelograms, maybe even parallelograms with the same rotational orientation as those in the diagram. As the year went on, they learned that the diagrams accompanying their instructions were models for a class of figures. Thus, while diagrams can be helpful models for specific instances of a construction, they are inappropriate for communicating the characteristics of a desired construction especially for beginning students.

In many problems where the construction is specified by a written description (with or without an accompanying diagram), the steps of the construction are described by labels which refer to the points in the resulting diagram. For example, in the diagram in Figure 1, A, B, C, and D are the labels[15].

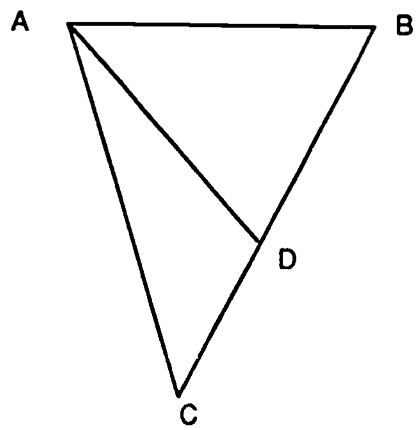
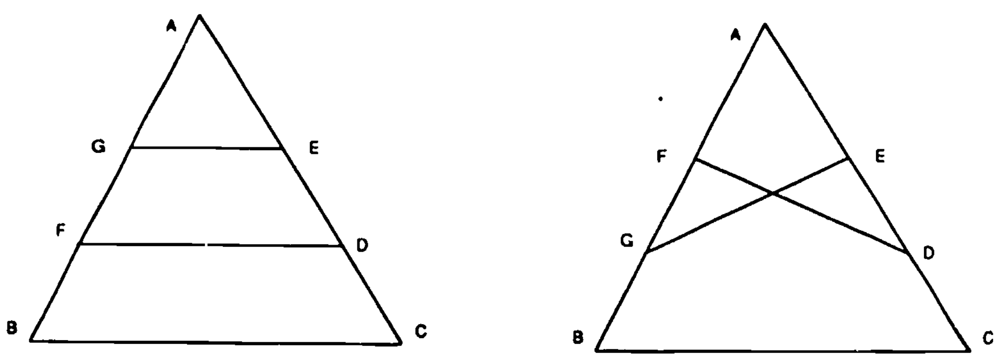


Figure 1

[15] It might seem that "talking in labels" would become easier with the SUPPOSERS, for the following reasons: every new triangle is labelled ABC; right angles in right triangles and obtuse angles in obtuse triangles are always at vertex A; and, when the construction is repeated, the labels remain the same — only the picture changes. Even with the SUPPOSERS, however, labels do not always produce an unambiguous description of a construction; e.g., two different kinds of diagrams can result from the following instructions: Draw triangle ABC. Subdivide seegment AC into three segments. Subdivide segment AB into three segments. Draw segments EG and DF. See figure below.



Observation: When labels were included in the problems, students wrote their conjectures using labels, not geometric language.

When a diagram is presented accompanied by written instructions which use labels, the labels may distract students from the geometric relationships in the construction. For example, for a problem about exterior angles where the columns of data were identified by labels (see Problem 6), students conjectured that the number in one column added to the number in the next column was always the number in the third column. They did not mention that the relationship they described indicates that the sum of two remote interior angles in a triangle is equal to the exterior angle at the third vertex.

Process instructions

Process instructions tell students what they need to do beyond the construction that must be made. They may suggest the measurements to make or the types of figures to explore, or they may remind students to test conjectures on other triangles and to write their conjectures. More general instructions or metacognitive hints can also be considered as process instructions. For example, one teacher wrote the instructions for a large assignment shown in Figure 2. The instructions are sufficiently general to be used with many problems, and they help students step through a model of the kind of inquiry desired by the teacher.

Observation: Explicit written process instructions helped our students.

Geometric Supposer Project

Directions:

1. You will select one of the problems to work on.
2. You may work with a partner or alone. If you choose a partner indicat his/her name under your name. Each must turn in his/her own work.
3. You will make a brief restatement of what the problem asks for.
4. You will make an outline of the steps you think necessary to explore and solve your problem.
5. You will collect, examine and study the data you think will help you to make conjectures about the relationships in the problem. This data collection is for you. (Use log sheets for this purpose.) Ahthough you may enclose these sheets, I will not grade them. You will have 4 days in the lab to collect data. If you need more time you will have to go to the math lab on your own to complete your collection of data.
6. A conjecture sheet will be due ____________. This conjecture sheet must include a diagram and a list of your conjectures, clearly numbere This summary sheet will be graded and returned in order that you will be able to complete the last and most important part of the project.
7. The last segment of your project is to prove "formally" as many of your conjectures as possible, but no fewer than 3. More credit will be given for more proofs.

Figure 2

In a problem posed earlier in two different ways — **Problems** 7 and 8 — the key difference between the two ways of writing the problem lies in the process instructions. Regardless of which way the problem is written, students do not need to analyze a diagram to discover the construction to be completed. The process instructions in Problem 7 state the general goal and then detail the steps to be taken in exploring it, which helps students to organize their inductive work. The process instructions in Problem 8 provide examples and ask for a generalization focused on the number of triangular section, without describing the work to be done or separating different cases.

Even though students using **Problem 7** did not generalize the number of triangular sections (p. 7), clear and detailed process instructions did not prevent them from making disparate generalizations of different kinds other than the number of sections, e.g., that the vertex of origin can be changed or that the medians can be used from different vertices in the subtriangles; see Figure 3 (Generalizations of different kinds for Problem 7). **Problem 8**, which was focused on generalization, kept students from carrying out a systematic analysis of the cases. Those who restricted their investigation to one type of triangle (e.g., isosceles) were more successful in making generalizations. The explicit process instructions given in problem 7 were, therefore, helpful to students doing this problem.

Our next observation concerns the nature of process instructions:

> **Observation:** When students were given charts that dictated which data should be collected, the problems were not successful. When students were asked to record data in charts they created and labelled, charts proved a useful way to help them inquire systematically.

Using charts and tables that dictate which measurements should be made proved an unproductive strategy for giving process instructions. Despite its attractiveness as a technique for organizing data, overemphasis of this type of instructions may paralyze students' ability to direct their own inquiry. For example, in a class where tables and charts were used frequently, students ignored written instructions and turned directly to them, limiting their inquiry to the headings specified in the charts. When interviewed about their work with the SUPPOSERS, they reacted negatively to this method of giving process instructions. As one said:

> On the worksheets the problem is all mapped out for you. The problem that they just gave us, you have to find the solution and find the work. They give you step by step, number one, number two, number three" (Yerushalmy et al., 1987).

Guiding student inquiry by providing a chart that dictates the measurements to be made reduces student inquiry to an unthreatening rote collection of data. It is easy to specify clearly in the problem exactly what needs to be done to create a successful solution — but, as the student's comment above shows, it is uninteresting. However, we are *not* arguing that charts are never appropriate tools in the inquiry process. They were useful after students had determined which measurements and drawings were to be collected and had organized them in their own charts (see Appendix I). In a critical sense, the act of organizing a table or chart is an important part of the inquiry process itself.

Try to split a triangle into triangular sections that have the same area. First try to get four sections with equal area.

When you have a solution, make sure that it works in all kinds of triangles.

Record a drawing of your solution and explain what kind of lines you added to the triangle to get the drawing.

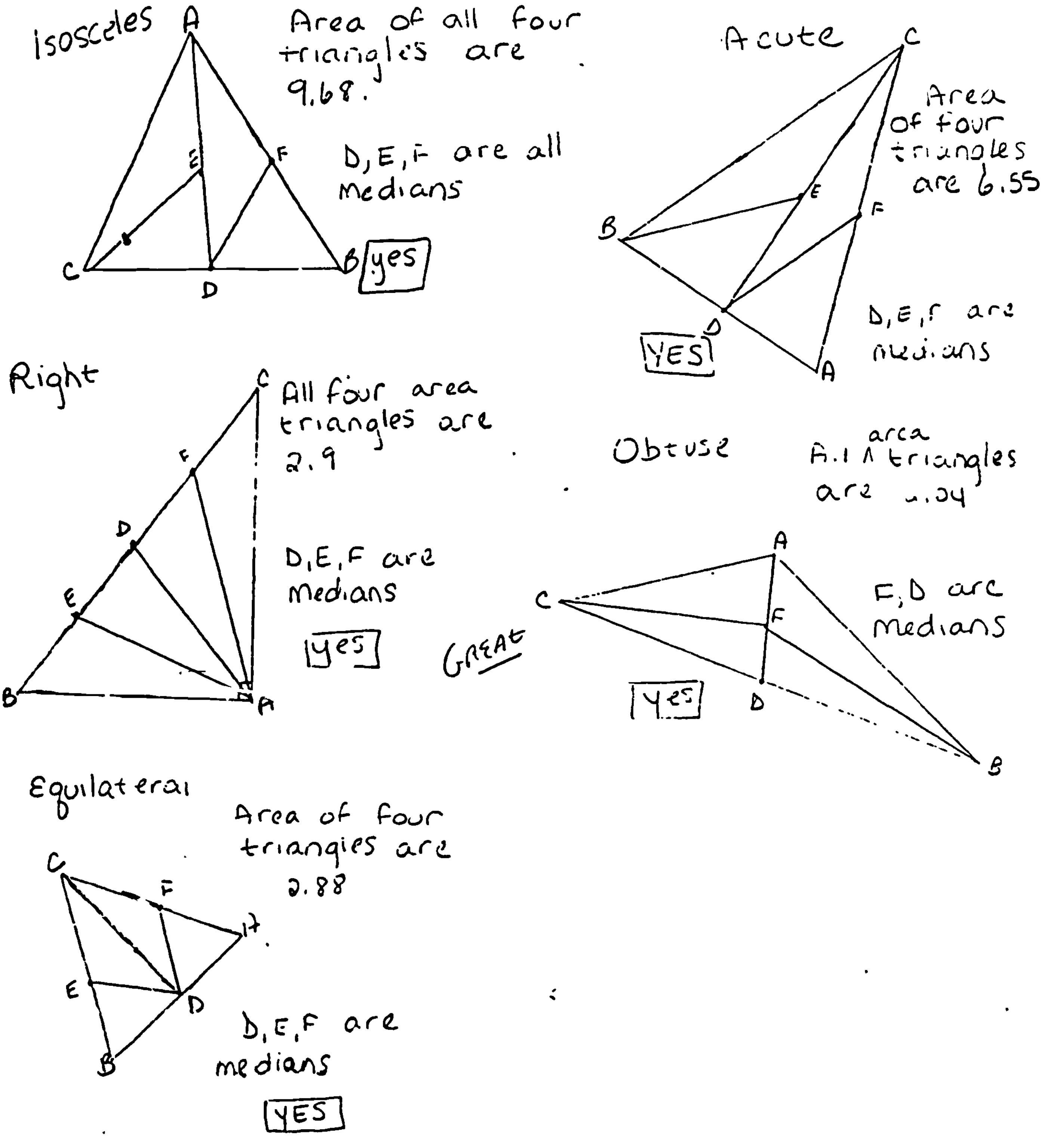


Figure 3

Our third observation concerns changes during the schoolyear:

> **Observation:** As the year progressed, students were able to work on their own with less direction (that is, less explicit process instructions). This phenomenon was observed both with work on the SUPPOSERS and other classroom activities.

For example, at the end of the year, an observer visited a class working with one of a type of geometry construction devices known as *Miras*, a clear plastic drawing tool for investigating, or looking at, reflections (thus, "mira"), as well as compass and straightedge. That students could do this work with little direction from the teacher indicates how comfortable they were with construction tools and their willingness, absent at the beginning of the year, to try their own constructions. The following excerpt is taken from the notes of the classroom observer:

> I was impressed by the students' ability to make constructions and follow directions without asking millions of question .... Students were recording the constructions neatly on their papers.... There was alot of commotion in the room. Students were talking most of the time, although a high percentage of the talk was on geometry.... The kids seemed comfortable working with tools, even one (*Mira*) that they had never worked with before.

RECOMMENDATIONS AND CONCLUSIONS

Our goal was to create materials that describe problems clearly and unambiguously so students know what needs to be done, while still leaving room for student inquiry. The observations presented here are based on our experience. To summarize, we present the following recommendations for posing inquiry problems with the SUPPOSERS.

First, we recommend three strategies for writing clear materials for the SUPPOSERS: state the goal of the problem at the top of the page; provide explicit process instructions to remind students of what they should do as inquirers; and, once students understand that diagrams can be models, use diagrams to exemplify written construction instructions.

Second, small-scale problems that use charts to tell students which measurements to make in order to discover a single conjecture desired by the teacher should be avoided. Students told which data to collect have difficulty seeing numerical patterns in the data and do not necessarily discover the particular relationship that interests the teacher. They also find this process of collecting data and doing the problem uninteresting, because it precludes individual creativity. We recommend enlarging such problems and using them in a different manner. For example, one might ask students to add the auxiliary lines necessary to write a proof, which allows a more complex initial construction to be created. If the students cannot find the particular conjecture desired, they can at least find conjectures the teacher can use to derive the one desired. At the same time, each student will be able to explore more freely and, theoretically, at least, will have an opportunity to develop a proof of the desired conjecture. Keep in mind however that conditions for effective inquiry go well beyond problems and how they are written.

The level and evolution of students' knowledge and abilities are critical factors. When students do not have the necessary background in geometry to make deductive arguments, they do not have mechanisms for checking general answers. With such students, it is best to start with construction problems and other problems with specific solutions. Since early in the course students may not be able to distinguish between diagrams used as specific instances and those used as models, avoiding diagrams as models early in the year may be helpful.

The written word is not sufficient for transmitting expectations. Modeling inquiry strategies and parts of the inquiry process is important. For example, the "what if not" strategy should be modeled explicitly to help students become adept at changing aspects of the written statement of the problem.

LOOKING AHEAD: INQUIRY, PROBLEMS, SOFTWARE, AND DEVELOPERS

Beyond the SUPPOSER geometry contexts, our work suggests more general recommendations for introducing inquiry teaching into schools. At the opening of this paper, we suggested that the utility and power of a software tool environment, i.e., a program designed around a set of capabilities (rather than an explicit curricular content or instructional framework), become apparent only in the context of a problem. The observations included here indicate that well-crafted problems can clarify the instructional approach, define the relationship between the use of the software and the curricular content, and provide students with direction. Furthermore, our observations make clear that there was a relationship between the formulation of the materials and the success of the problems.

We believe that the formulation of inquiry problems will be important to the successful development of guided inquiry approaches using other tool-based software environments in other domains. Our scheme of considerations is an initial framework for such formulation. With the exception of the description of the construction, the considerations seem general enough to apply to other environments; clearly, the development of other software like the GEOMETRIC SUPPOSERS and further research on the use of this sort of tool in the curriculum are essential for their evaluation and refinement. Already we have some indication that our concern about the formulation of problems is warranted and fruitful. In working with a piece of software of a similar, "toolish" nature in algebra, we have found that teachers' foremost concern and difficulty is the creation of problems which exploit the power of the program. Our experience analyzing the formulation of inquiry problems for the SUPPOSERS has helped us in this endeavor.

Our experience also highlights the challenges and difficulties of bringing inquiry approaches into classrooms. The students and the teachers we worked with had some difficulties with both the new expectations and the roles they had to adopt in an inquiry approach, difficulties that we believe are not unique to our approach and will be found with most inquiry approaches (cf. Kaput, 1986). Hardware, software, and sources of inquiry problems all are not yet sufficiently available. In addition, students need modeling and support from teachers to take on new expectations and to leave behind the security of learning without the exercise of inquiry, and teachers need modeling and support as they explore innovations that require new teaching skills, especially the skill of saying enough without saying too much.

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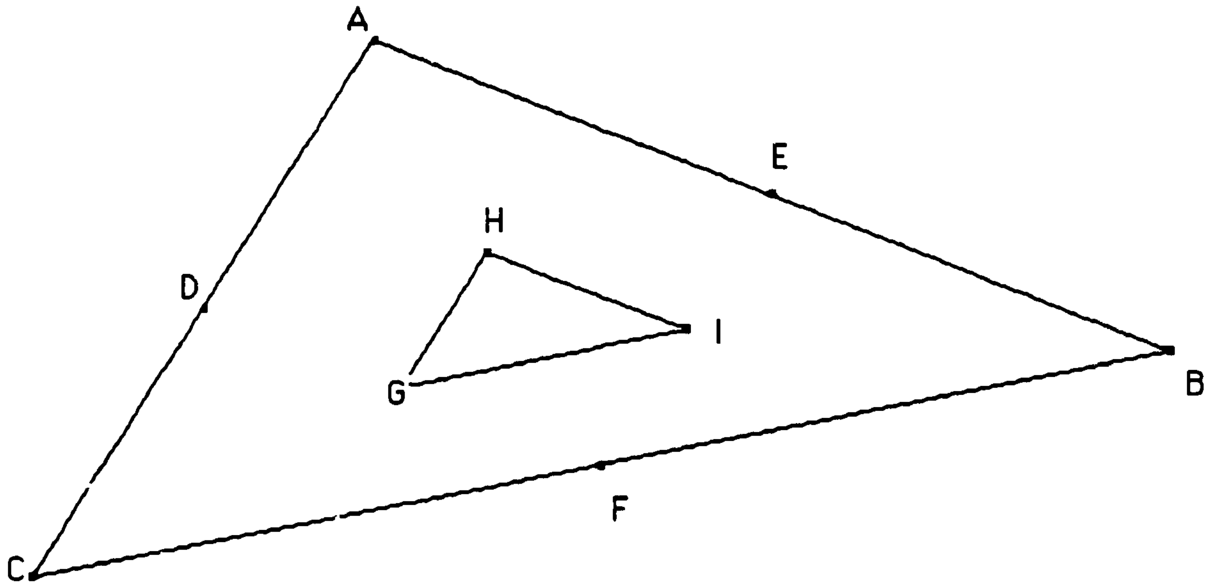
Appendix 1

The following solutions to Problem 12 were recorded during one session of a tenth grade geometry class in an urban high school.

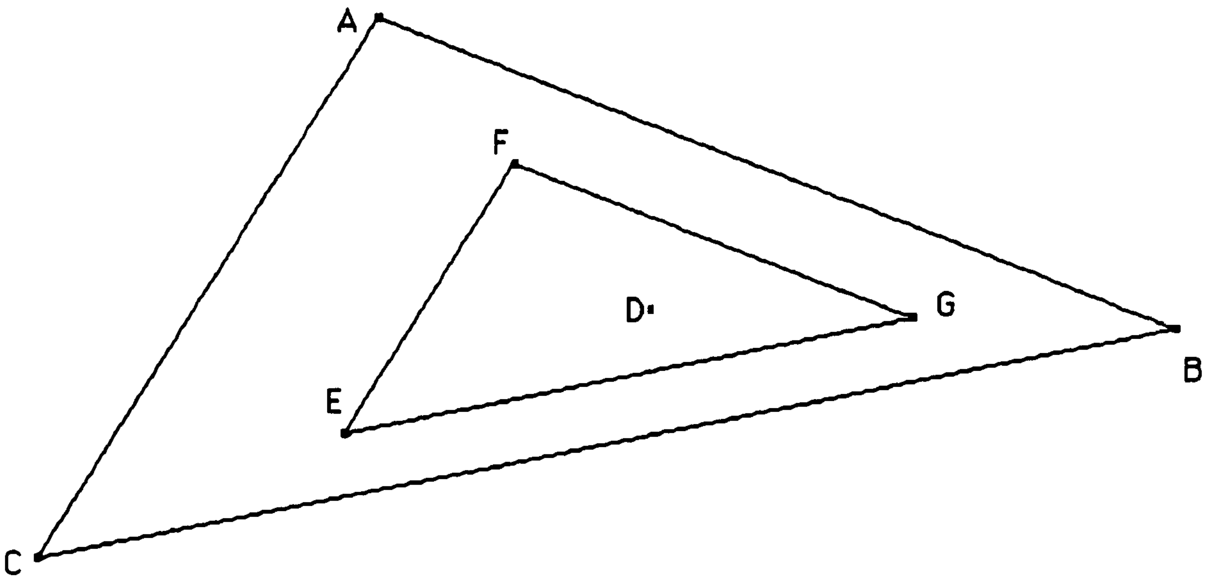
Solution 1:

The students labelled the midpoints of triangle ABC to get points D, E, and F. They then labelled the midpoints of segments DF, DE, and EF (not shown in the picture below) to get point G, H and I. Triangle HIG is similar to triangle ABC.



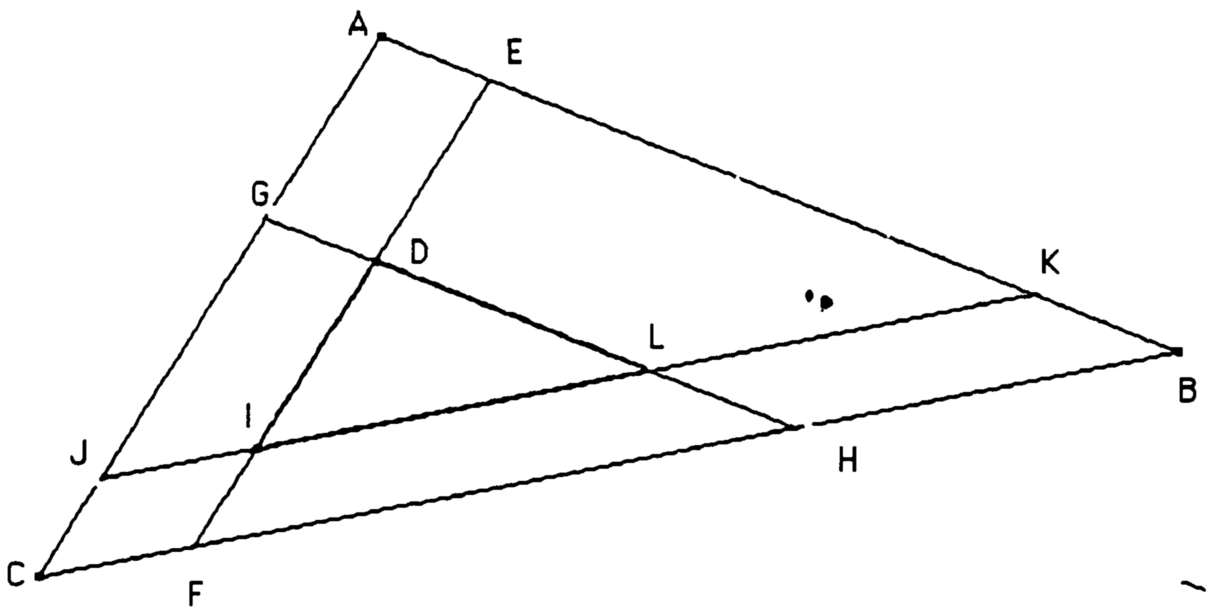
Solution 2:

The second solution starts from a point inside triangle ABC. Different pairs of students got this initial point (D) in different ways. Some students labelled the intersections of the three medians of triangle ABC, the three altitudes, or the three angle bisectors. Others used the center of the circle inscribed in triangle ABC or the center of the circle circumscribed about triangle ABC. Once point D was identified, they bisected segments CD, AD, and BD (not drawn in the picture below) to get points E,F and G. Triangle FGE is similar to triangle ABC.



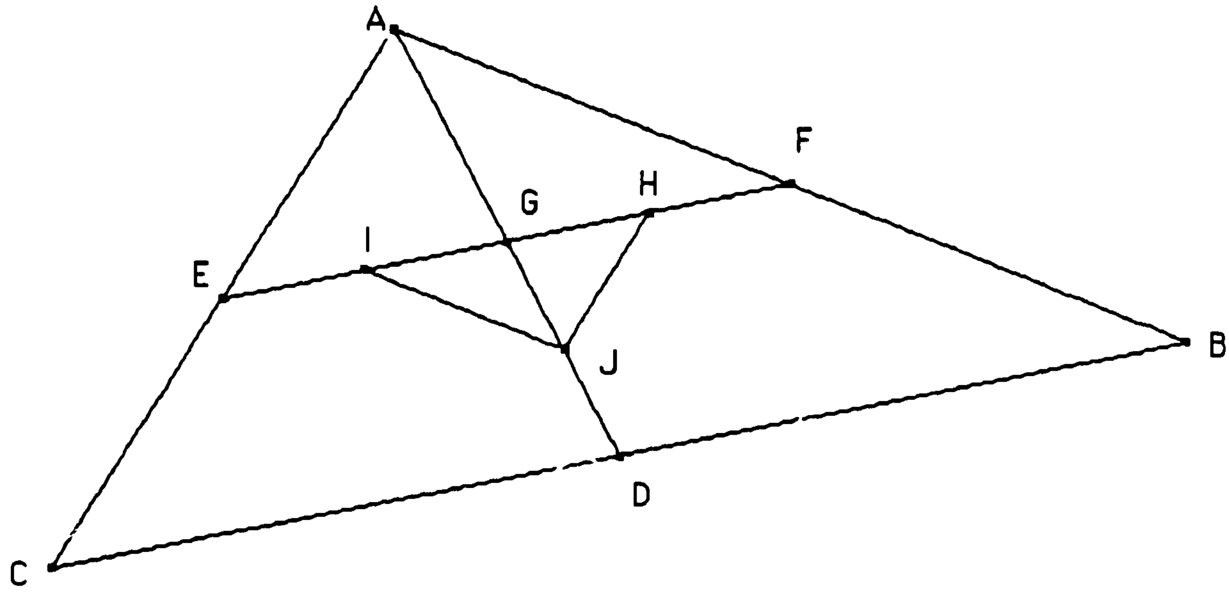
Solution 3:

Students put a random point D inside triangle ABC. They drew a parallel to AC through D to intersect sides AB and BC. They then drew a parallel to AB through D to intersect AC and BC. They then put a random point, I, on segment DF and drew a parallel to BC through I They then labelled the unlabelled intersection of GH and JK with the point L. Triangle DLI is similar to triangle ABC.



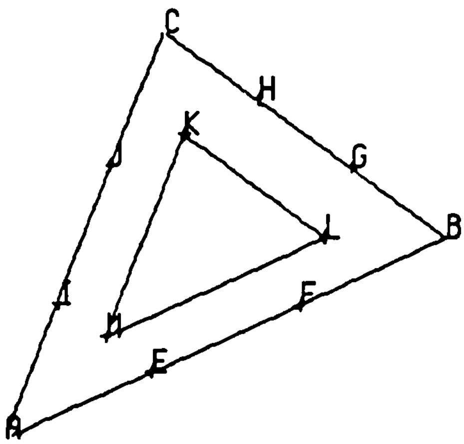
Solution 4:

Students drew median AD in triangle ABC from vertex A and the midsegment EF connecting sides AC and AB. They labelled the intersection of AD and EF with the letter G. They then found the midpoints of GD, GE, and GF. The points J, I, and H make a triangle which is similar to triangle ABC.



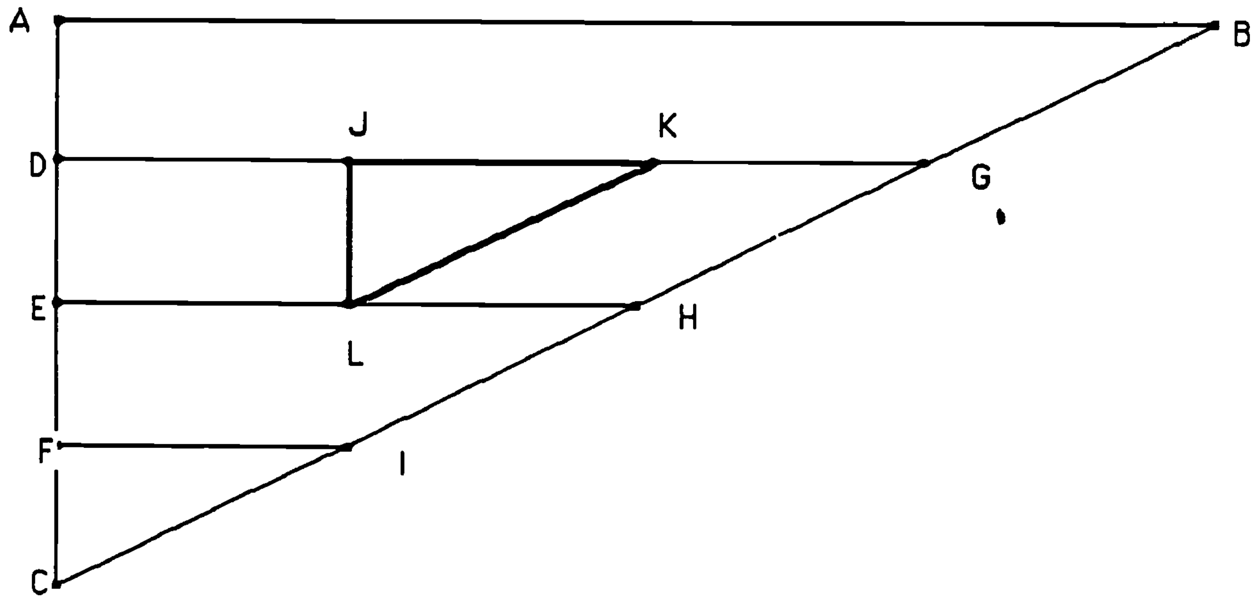
Solution 5:

Students subdivided AB, AC and BC into three equal parts. They connected points J and H, F and G, and E and I (These segments are not pictured below). They found the midpoints of each of the new segments and connected these points to make triangle KLM which is similar to triangle CBA.



Solution 6:

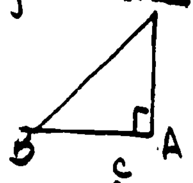
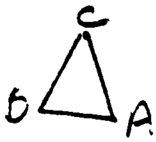
This solution was devised by a group of students who claimed that it only works in right triangles. They subdivided sides AC and BC into four equal parts. They then connected the subdivisions as pictured below. They then divided segment DG into three equal parts and segment EH into two equal parts. Triangle JKL is similar to triangle ABC.



Appendix 2

IV. Use the SUPPOSER to find under what conditions three numbers represent the lengths of the sides of a triangle.

a) Report data in a tabular form.

	Diagram of Δ	AB	AC	BC
①		3	4	5
②		9	3	5
③	no triangle	4	6	1
④		1	1	1
⑤		6	2	6
⑥	no triangle	5	7	2

b) State a conjecture.

when high numbers and low numbers are put into one triangle, the triangle cannot be formed.

IV. Use the SUPPOSER to find under what conditions three numbers represent the lengths of the sides of a triangle.

a) Report data in a tabular form.

Δ 1, 2, 3	No	6, 5, 10	Yes
Δ 2, 3, 4	Yes	1, 3, 5	No
Δ 3, 4, 5	Yes	3, [illegible], 7	[illegible]
Δ 4, 5, 6	Yes	5, 6, 9	Yes
Δ 5, 6, 7	Yes	7, 4, 9	Yes
Δ 6, 7, 8	Yes	2, 5, 8	No
Δ 7, 8, 9	Yes	1, 5, 9	No
Δ 8, 9, 10	Yes	2, 6, 10	No
Δ 2, 4, 6	No	1, 4, 7	No
Δ 4, 6, 8	Yes	2, 7, 12	No

b) State a conjecture.

THE only TRIangles THAT don't exist are THE ones THAT or when THEY are put TOGETHER THEY draw a line or THEY don't [illegible] like 1, 3, 5

Note: The Triangles THAT Have [illegible] number in common and THEY start with 1 or 2 THEY don't exist like (1, 2, 3) (2, 4, 6) (2, 5, [illegible]) (1, 3, 5, (1, 5 [illegible]