DOCUMENT RESUME

ED 302 893		EA 020 548

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TITLE	Policy Implications of the Three R's of Mathematics Education: Revolution, Reform, and Research.
PUB DATE	Apr 88
NOTE	36p.; Paper presented at the Annual Meeting of the American Educational Research Association (New Orleans, LA, April 5-9, 1988).
PUB TYPE	Speeches/Conference Papers (150) -- Information Analyses (070)
EDRS PRICE	MF01/PC02 Plus Postage.
DESCRIPTORS	*Educational Change; Educational Policy; Educational Quality; Educational Researchers; *Educational Trends; Elementary Secondary Education; *Mathematics Education; *Mathematics Instruction; *Mathematics Teachers

ABSTRACT

This paper examines the recent growth and changes in the discipline of mathematics, as well as current trends in the research on the teaching and learning of mathematics. The focus of this paper is revolution, reform, and research and their effects on school mathematics. The first section deals with revolution with regard to the teaching of mathematics and the pressure for reform as a result of inadequate schooling in this area. The second section summarizes the typical reactions of mathematics educators, educational researchers, and educational policymakers to the calls for reform. The third section discusses the importance of reliable knowledge and the information and insights that can be derived from research. A summary of key concepts is presented as well as recommendations. Appended are 56 references. (SI)

POLICY IMPLICATIONS OF THE THREE R's OF MATHEMATICS EDUCATION: REVOLUTION, REFORM, AND RESEARCH

by

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Paper prepared for the annual meeting of the American Educational Research Association, New Orleans, Louisiana, April 1988.

Revised, May 1988

INTRODUCTION

This paper's primary audience is comprised of "educational policy makers," those men and women at the federal, state, and local levels who establish the rules and regulations, allocate the resources, and make many of the decisions that influence the way the American educational system operates. In addition, this audience includes the advisees and lobbyists, who provide information to the decision makers. My aim is to convince these individuals to consult and listen to the advice of the mathematical sciences education community as they make decisions about the teaching and learning of mathematics in our schools. In constructing my argument, I have examined the recent growth and changes in the discipline of mathematics, as well as current trends in the research on the teaching and learning of mathematics. Thus, a second audience for this paper is the mathematicians, mathematics educators, and mathematics teachers--the entire mathematical sciences education community --who have been examining the developments within mathematics. Finally, this paper addresses those educational researchers (other than those in mathematics education) who use mathematics in their studies, including psychologists, cognitive scientists, and sociologists.

It is assumed here that the members of each of the three groups--policy makers, mathematics educators, and educational researchers--are interested in and concerned about what happens in schools. While each group is influenced by the current social and economic revolution, however, each has responded to the pressures for reform in school mathematics from a unique perspective. As a result, despite their common good intentions, each of the groups has, on occasion, been ignorant, critical, and even intolerant of others' perspectives. My argument is based on a consideration of three R's--Revolution, Reform, and Research--<u>and</u> their effects on school mathematics. My hope is that the three groups will learn to cooperate with one another so that their shared goal of real reform can be accomplished.

REVOLUTION AND THE PRESSURE FOR REFORM

Pressure for school reform is a product of the social and economic revolution that is today wreaking havoc in Western society. Upheavals in industry and the economy have spurred public awareness that we are moving into a new industrial age, variously called "the information age" (Bell, 1973; Naisbitt, 1982; Toffler, 1985), "the post-industrial age" (Bell, 1973), or "the super-industrial age" (Toffler, 1985). Integration of the telephone, television, and computer now permits instant transfer of information among people anywhere in the world. This, in combination with the geometric growth of knowledge, particularly in the mathematical sciences, is at the heart of this revolution whose impact promises to be as dramatic as the shift that transformed an agrarian society of the 1800s into an industrial society of the 20th Century. Economically, the consequences of this revolution are only now being realized as market rivalries shift from within-country competition to between-country competition. American auto manufacturing is a case in point: Today, individual manufacturers compete not only with their American counterparts, but with manufacturers from a variety of countries (Japan, Germany, Korea, Yugoslavia, etc.) in a world market. Jennings (1987) has recently argued this economic competitiveness represents "the Sputnik of the 80s" in educational reform.

In terms of schools, one must first understand that the salient feature of this revolution is that information is the new capital and the new raw material. Like urbanization, which is defined as a population shift that brought more than 50% of Americans to live in urban areas, identification of a predominantly information-based economy usually is linked to the point in time at which more than 50% of the population was earning its living through the sensible linking and exchange of information. The validity of this statistical definition is open to question; the concept is not (Naisbitt, 1982).

The works of several authors (Naisbitt, 1982; Shane & Tabler, 1981; Toffler, 1985;

Yevennes, 1985; Provenzo, 1986) suggest the attributes of the new age. Naisbitt's (1982) key points characterize the shift to an information society:

1. It is an economic reality, not merely an intellectual abstraction.

2. The pace of change will be accelerated by continued innovation in communications and computer technology.

3. New technologies, which will be applied first to traditional industrial tasks will soon generate new processes and products.

4. Basic communication skills will become more important than ever before, necessitating a literacy-intensive society. Information has value only if it can be controlled and organized for a purpose. To tap the power of computers, it is obligatory first to communicate efficiently and effectively, to be both literate and numerate. In addition, in an environment of accelerating change, the approach of training for a lifetime occupation must be replaced by learning power, which also depends on the abilities to understand and to communicate.

5. Concurrent with the move from an industrial society to a society based on information is awareness of the change from a national economy to a global economy. This change is accompanied by the perception that the United States and other advanced societies of the West are losing their industrial supremacy. Mass production is more cheaply accomplished in the less developed parts of the world. Toffler (1985) envisioned the change as a series of waves, in much the same framework as Frederick Jackson Turner characterized the westward movement of the frontier in North America. Thus, just as industrial society replaced agrarian society and then began to push out, so the new post-industrial age will replace industrial society in the West and gradually expand.

Unfortunately, schools and the mathematics taught in their classrooms are products of the past relics of the Industrial Age. This paper proposes that <u>schools and the mathematics taught in them need to be changed</u>. In fact, radical changes in mathematics, as well as teaching approaches and organization, are necessary if students are to become productive citizens in the Information Age. All students must have an opportunity to learn the practical mathematical concepts and skills needed for everyday life, for intelligent citizenship, for vocations, and for human culture in an age centered on information rather than industrialization.

Why Current Schooling is Inadequate

In the industrial society of the late 19th century, it may have been frugal to educate the populace in accordance with the structure of the economy. An elite, well-educated group established policy, directed the government, managed industry, and advanced the scientific and technological base; the remainder of the population provided the physical labor for production and services, and was educated only to the level required for reliable performance. As a result, a dual school system of "high literacy" and "low literacy" schools evolved (Resnick & Resnick, 1977). When the U.S. was formed, its village schools reflected the notions of literate citizenship appropriate to the new nation. The educational system that evolved during the 19th century focused largely on elementary schooling, producing the sharp distinction between elementary and secondary education that persists today. Most children attended school for up to eight years, but few went on to high school. The "low literacy" curriculum at the elementary school focused on the basic skills of reading, writing, and computation; in fact, the foundation of this mathematics curriculum was "shopkeeper" arithmetic.

The political conditions under which mass education developed encouraged the routinization of basic skills and standardized teaching. Standardization was a means of insuring that a minimal curriculum standard would be met, that teachers would be hired on the basis of professional competency rather than political or familial affiliation, and that those responsible for the expenditure of public funds could exercise orderly control over the process of education. This notion of standardization also jibed well with prevailing Industrial

Age theories about the efficiency and effectiveness of routinization (Bobbitt, 1924; Charters, 1924; Rice, 1913).

At the other extreme, the academic high school maintained a tradition of scholarly, cultural, and scientific high literacy. Students were prepared for academe and policy making. This is not to suggest that all were educated to the level of advanced academic study; it did mean that all were rigorously educated, to whatever level elected, in such a way as to prepare for that option.

This dual educational system may have served the Industrial Age of the 19th and early 20th centuries reasonably well. Unfortunately, many of the traditions embedded in this dual system have been resistant to change and persist today, including age-graded classrooms, differential schools, primary-grade tracking, licensure of general teachers, competence at paper-and-pencil arithmetic, general mathematics as the terminal course for non-college intending students, and pre-calculus mathematics for college intending students.

The most serious failing of this dual educational system is that many students are denied the opportunity to study any mathematics except arithmetic. Today, such basic mathematics training is not enough. In particular, many groups frequently denied equal participation in our society--including women, blacks, Hispanics, and those of low socioeconomic background--must be encouraged to study more mathematics. Because our society will need the full participation of all of its citizens if it is to enter a post-industrial era, educational innovations must create the conditions by which such traditionally excluded groups can be included. Absent such concerted effort, the role of mathematics as a filter that excludes groups from prestigious positions in our society will heighten as the importance of mathematical literacy increases.

Bleak national performance data are yet another indication of the inadequacy of current educational practice. Results from the National Assessment of Educational Progress (NAEP) in mathematics (Carpenter et al., 1987), for example, show that while most students are reasonably proficient in computational skills, the majority do not understand many basic concepts and are unable to apply the skills they have learned in even simple problem-solving situations. Add to this the fact that our students do not fare well when compared with students in other industrialized nations, particularly those in the Orient (McNight et al., 1987). We expect less of our students, they spend less time studying mathematics, and fewer are enrolled in advanced mathematics than are students in other countries.

In addition, the "shopping mall" high school (Powell, Farrar, & Cohen, 1985), which is intended to provide all students an opportunity for education until they are 17 or 18 years of age, has been criticized as having a curriculum that has been homogenized, diluted, and diffused to the point that the courses no longer have a central purpose (National Commission on Excellence in Education, 1983). The "shopping mall" high school is inadequate to the needs of the Information Age: while society is becoming increasingly heterogeneous (Steen, 1986), such schools do not provide most students with adequate preparation for productive citizenship.

Finally, in too many schools, routine instruction and scheduling demands, as well as teachers' general working conditions, make it difficult to provide students with diverse mathematical experiences. In most schools, mathematics class lasts about 43 minutes, about 20 minutes of which is devoted to written work. A single text is used in whole-class instruction. The text is followed fairly closely, but students are likely to read, at most, one or two of the five pages of textual materials other than problems For students, the text is primarily a source of problem lists (Conference Board of Mathematical Sciences, 1975, p. 77). The daily sequence of activities has been described as follows:

> First, answers were given for the previous day's assignment. The more difficult problems were worked at the chalkboard. A brief explanation, sometimes none at all, was given of the new material, and problems were assigned for the next

> day. The remainder of the class was devoted to students working independently on the homework while the teacher moved about the room answering questions. The most noticeable thing about math classes was the repetition of this routine (Welch, 1978, p. 6).

These observations suggest that a mathematics teacher's work is largely managerial or procedural in that the "job is to assign lessons to their class of students, start and stop lessons according to some schedule, explain the rules and procedures of each lesson, judge the actions of students during the lesson, and maintain order and control throughout" (Romberg, 1985, p. 5). In such situations, the teaching of mathematics involves too little care or reflection. Too many teachers feel obligated to cover the book. Too few teachers recognize that student mastery of mathematical methods and their application to problem solving is the primary goal of instruction. It has long been the case that most mathematics teachers at the elementary school level have had inadequate mathematical training. Today, however, growing numbers of teachers at the secondary school level also are under-prepared. To meet current shortages, many teachers now are being licensed with minimal preparation --a problematic situation that can only worsen over the next decade if current trends in teacher education continue. Furthermore, teachers tend to be isolated in their own classrooms, with little opportunity to share information with other staff members and little access to new knowledge (Tye & Tye, 1984).

The most important feature of schools is that schooling is a collective experience: For the student, being in school means being in a crowd; for the teacher, being in school means being responsible for a group of students. Thus, the issue of how a small number of adults can organize and manage a large number of students is the central institutional problem of schools. Furthermore, despite social resources adequate to the task, the education and training of all students has not been a top priority in the nation's school systems; instead, schools seem designed to relieve the home of its school-age children for several hours each day and to train the children to keep quiet. Timid supervisors, bigoted administrators, and ignorant school boards often inhibit real teaching, and commercially debauched popular culture makes learning disesteemed. The academic curriculum has been mangled by the demands of both reactionaries and liberals. Attention to individual students is out of the question, and all the students--the bright, the average, and the dull--are systematically retarded one way or another, while the teacher's hands are tied.

In summary, the Holmes Group (1986) described this view of instruction as "passing on' a substantive body of knowledge. . . . 'planning, presenting, and keeping order.' . . . The teachers' responsibility basically ends when they have told students what they must remember to know and do" (pp. 27-28). This succinct characterization of the role and work of teachers stemmed from an objective view of truth; an iconic, formalistic view of knowledge; the notion of the learner as a product; a stimulus-response view of learning; and the need to efficiently prepare the majority of students to fit smoothly into a mass-production economy.

Each of these assumptions has changed. Truth is now regarded as a social construction; the prevailing view of knowledge is constructivist rather than formal; the learner is viewed as an active participant rather than a product; psychology has progressed beyond behaviorism to cognitive science and models of how information is processed and knowledge is constructed; social progress depends upon a citizenry that can continue to learn and adapt to rapidly changing circumstances in order to produce new knowledge. In sum, values have changed, and notions of effective instruction also must follow suit. Such a practical and ideological transformation is critical in the face of the personal, national, and global problems that are an inevitable by-product of the social and economic revolution currently underway, a revolution whose success and survival depend upon an adult population able to learn and adapt as a result of their confidence in their own personally created and firmly founded understandings.

REFORM

Next, let me summarize briefly the typical reactions of the three groups--mathematics educators, educational researchers, and educational policy makers--to the calls for reform. Admittedly, these descriptions are not specific enough to reflect the variations in response that actually are occurring. However, what emerges is the clear difference in perspective in the groups' reactions to the revolution and process of reform.

Reactions by Mathematics Educators

As trained mathematicians, members of the mathematical sciences education community have reacted to the revolution and its concurrent calls for reform by considering the impact they have had on the discipline of mathematics and the uses to which mathematics has been put. They reject the notion that mathematics is a static collection of an overly fragmented set of concepts and skills, and characterize it instead as a dynamic, growing, and changing discipline. In fact, with the possible exception of the impact of word processing on writing, no other discipline has been as profoundly effected by the computer and the calculator. Lynn Steen (1988), currently Chairman of the Conference Board of Mathematical Sciences, argues that

> The rapid growth of computing and applications have helped cross-fertilize the mathematical sciences, yielding an unprecedented abundance new methods, theories, and models.... No longer just the study of number and space, mathematical science has become the science of patterns, with theory built on relations among patterns and applications derived from the fit between pattern and observation. (p. 611)

Given this explosion of knowledge, it is essential that school mathematics focus explicitly on the fundamental knowledge needed for contemporary mathematics. For school programs, the content must be selected carefully and must emphasize fundamental knowledge needed for contemporary mathematics. To illustrate the changes occurring in mathematics and their potential impact on school programs, let us look at changes in technology and applications.

Technology. Most current school mathematics programs fail to reflect the impact of the technological revolution affecting our society. The availability of low-cost calculators, computers, and related new technology have already dramatically changed the nature of business, industry, government, sciences, and social sciences. Unfortunately, most students are not educated to participate in this new society. Despite the advancements that have brought untold computational and graphical power to our fingertips, in-school hours are spent drilling on computational procedures in arithmetic, algebra, statistics, and even calculus --despite the fact that any step-by-step procedure involving the manipulation of mathematical symbols according to a fixed set of rules can be accomplished by a calculator or computer. Some procedures are simple enough that they are best done mentally or by hand; others are more complex, or take time to work manually and should be done by machine, as they are in the world of work.

Applications. Complementing the influence of technology on mathematics is the fact that the use and application of mathematics have expanded dramatically. Quantitative and logical techniques have permeated almost all intellectual disciplines and change has been particularly great in the social and life sciences. The computer's ability to process large sets of information has made quantification possible in such areas as business, economics, linguistics, biology, medicine, and sociology. Furthermore, the fundamental mathematical ideas pertinent to such research are not necessarily those studied in the traditional algebra-geometry-precalculus-calculus sequence.

In summary, changes in technology, a broadening of the areas in which mathematics is applied, and growth in mathematics itself have transformed the problems important to mathematics and the methods mathematicians use to investigate problems. These changes must be echoed by changes in the school curriculum so that students have an opportunity to learn the skills and knowledge that are likely to be fundamentally important in their lives

Mathematical Literacy. Upon reflection, mathematics educators have reacted to the pressures for reform by proposing a response labeled "mathematical literacy," which challenges the traditions of current mathematics instruction. This response is reflected in such documents as the NCTM Curriculum and Evaluation Standards (1987) and the AAAS project 2061 report, What Science is Worth Knowing (1987).

If we adopt Steen's (1988) position that mathematics is the "science of patterns," then mathematical literacy involves learning to communicate in accordance with the terms, signs, symbols and rules for use of a language. All students must learn to read, write, speak, and listen to messages in that language in order to communicate with others about its patterns Furthermore, they must understand that the origins of the signs, symbols, and rules, as well as their development, are situation dependent. To illustrate, we must first realize that mathematical knowledge arose from rudimentary ideas acquired through perception of situations in the complex physical world. Many millennia ago, our ancestors planted the seed of mathematics by observing several quantitative and spatial regularities. From these humble beginnings, mathematics has flowered into the impressive body of knowledge we have been fortunate enough to inherit. Thus, from its origins, mathematics was an empirical science. Its fundamental terms, signs, symbols, and rules are merely abstractions and inventions created to represent properties observed in the environment. Thus, numbers were created to represent the numerosity of sets of familiar objects, signs such as "+" were invented to represent the quantity found by the joining of sets, and terms such as parallel and perpendicular were introduced to name spatial properties. The purpose of such a language is communication with others; the terms of the language are useful only when their meanings are shared. Thus, mathematics is a language created by man and a set of rules for the use of that language. Its origins are to be found in the regularities of the world in which we live. Furthermore, like any language, mathematics grows and changes as a result of empirical investigation. To be literate, all students must confront a rich array of problem situations from which the empirical language of mathematics, its notation, and its rules can gradually be built, and they must come to understand a shared meaning for those terms.

In addition, a good deal of mathematical knowledge has been created through the investigation of this empirically based language and set of rules. By observing the properties of numbers, operations, and spatial figures, for example, humans have applied abstraction and invention to create another set of terms, signs, symbols, and rules. Some are generalizations of empirical procedures; the creation of computational algorithms for empirical processes, for example, has made mathematics applicable to many seemingly unrelated problem situations. In addition, no longer bound by perceptual reality, man has extended mathematics by asking "What if...?" questions. For example, while the creation of an equation for the shortest distance between two points on a plane surface has empirical origins, the generalization of this formula to two points in n-dimensions does not. And while multiplication of whole numbers has its roots in the grouping of objects empirically, more abstract multiplication algorithms do not; instead, they grew out of observations about properties of exponents such as $a^x \cdot a^y = a^{x+y}$ and the fact that any decimal number can be expressed as an exponent through use of powers of 10. Thus, mathematics involves the study of abstract systems that grow as a result of investigating other problem situations. Hence, to be mathematically literate, all students must have the opportunity to explore the properties of the empirically derived mathematics and to understand the relationships, rules of transformations, extensions, and structures derived from these investigations.

Another critical aspect of mathematics is the set of mathematical methods or thinking skills mathematicians use when developing conjectures, reasoning about phenomena, building abstractions, validating assertions, and solving problems. For example, no proposition is considered a mathematical product until it ha: been validated. Justifications may initially be built upon empirical evidence since they are based only on our perceptions, but, proof of an assertion by rigorous, logical argument has become the hallmark of abstract mathematics. For example, no geometer who had measured the base angles of an isosceles triangle would conclude that they were congruent based solely on a demonstration no matter how accurate the measurements, although such measurements may have formed the basis of a conjecture about their congruence. Mathematicians demand that this result be deduced from the fundamental concepts of geometry. Furthermore, the discipline grows by applying these mathematical methods to a wide variety of problem situations. To be mathematically literate, the students need to make conjectures, abstract properties from problem situations, explain their reasoning, follow arguments, validate assertions, and communicate results in a meaningful form.

Finally, the power of mathematical knowledge is that it is useful in many problem situations quite removed from those to which it was originally applied. For example, while paper-and-pencil computational algorithms for such tasks as addition and subtraction are no longer the central focus of applied mathematics, the decision sequence involved in algorithms is an important conceptual tool that helps address structural properties of operations, which has led to the study of operator algebras. In turn, the study of such operations has made it possible to simulate computaticnally a vast array of complex problems, such as flow of blood through an artificial heart valve, the trajectory of a hurricane as it approaches a coastline, and tomographic images of the mantle of the earth. In fact, the building of mathematical models and the computational simulation of complex situations are now commonplace. As the sciences move increasingly toward computational methods, so too must the mathematics curriculum.

This picture of mathematical knowledge, related to what it means to be mathematically literate, is quite different from the experiences of students in typical classrooms, where they have most often worked on sets of mathematical exercises to get correct answers. Mathematics is a foundation discipline for other disciplines and grows as a result of its utility. To be mathematically literate, students must be confronted with a variety of problems from other disciplines and have an opportunity to build mathematical models, structures, and simulations.

In summary, the mathematical sciences community has responded to the pressures for change by reconsidering the content of school mathematics. Current proposals suggest that all students need to study more and somewhat different mathematics, and to encounter and work on problems in considerably different ways.

Reactions by Educational Researchers

Like mathematicians who reacted to the revolution in terms of its effects on mathematics, researchers trained in other disciplines have reacted in terms of its impact on their theories, and methods of investigation. For example, since the turn of the century, educational psychology in the U.S. has been dominated by a behavioral, "black box" view of how humans process information. Instructional tasks (stimuli) and observable behaviors (responses) have been emphasized, with little attention to the ways in which information presented in the tasks is actually processed. Common metaphors included "the mind as a muscle needing exercise," "learning as absorption," "shaping behavior by reinforcement," and "fixed mental abilities." The picture of current mathematics instruction in schools reflects this behavioral orientation.

The information revolution has irrevocably altered this viewpoint. Current models of information processing and learning, which have been labeled "cognitive science," are based on the metaphor that the mind is similar to a computer in that information is received, stored, and processed by humans in ways analogous to computer functions.[1] This metaphor

[1]This is not the place for a review of those models. Howard Gardner's book, The Mind's New Science (1985), is a thorough discussion of the history of this development and Richard Anderson's treatise, The Architecture of Cognition (1983), is an excellent example

of the mind as a computer has proven to have great heuristic power for psychological research. Those that are of particular importance to the learning of mathematics include: memory storage, cognitive modeling, domain specific knowledge, and instructional objectives.

Memory. The storage of information in memory is important in this context because current research suggests that learning occurs when information entering the senses is actively processed and related to previously learned information stored in a permanent semantic and factual knowledge base. New information is fitted or assimilated into existing cognitive structures in such a way as to provide a meaning, an explanation, an order, or a logic for the experiences being witnessed or reflected upon by the learner. A consequence of this assimilation process is that each individual's knowledge is uniquely personal. Individuals process and link new information in unique ways and, hence, develop cognitive structures that reflect a variety of perspectives of the same reality. Hewson and Posner (1984) hypothesized three conditions necessary for the assimilation of new information: First, the learner must understand the new information; second, the new information must be reconcilable with existing conceptions; and third, the resulting accommodated structure must be useful. The potential for learning exists only when these conditions are satisfied. In summary, these views about how information is stored, retrieved, and modified are not reflected in mathematics classrooms.

Cognitive modeling. By reflecting on the capabilities of human problem solvers, educational researchers have developed descriptive models of how information is processed when solving complex problems, as well as computer simulations of that processing. These models have been developed by reflecting on the capabilities of human problem solvers. Although current mathematics instruction does not emphasize problem solving, there is consensus that it should (NCTM, 1980, 1987). By examining think-aloud problem-solving protocols of subjects solving simple kinematics problems, for example, it is possible to

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of current theorizing in the field.

design a production system reflecting their problem-solving behavior. Both strategy and sequencing considerations can be built into the system. Production systems can be tested to substantiate the degree to which the system reflects performance, by comparing the protocols of the subjects with those of the production system on a wide variety of problems within the capabilities of the system. Researchers have obtained remarkable similarities between the protocols of individuals and their corresponding production systems (Simon & Simon, 1978; Anderson, Greeno, Kline, & Neves, 1981); these matches suggest the production system's ability to model at least some cognitive behaviors. Furthermore, by making slight modifications or additions to the production system of a novice, it is often possible to model the problem-solving behavior of experts. Differences in the cognitive structures of experts and novices can then be studied by examining the modifications made in the production system.

Domain specific knowledge. There is increasing consensus that improvement of the capability to learn is inseparable from the specific domain of application. Again, rather than the fragmentation of mathematical knowledge reflected in current texts and tests, the interrelationship of ideas in specific domains is critical. Understanding in a complex domain requires a great familiarity with its connections (Rissland, 1985); "good thinking almost always involves articulation between knowledge and strategies" (Pressley, 1986, p. 144). The purpose for which knowledge was created and the process by which it was acquired are as essential as the formal structure of the ideas (di Sessa, 1979) because the mathematical meaning of the parts of a situation is often derived entirely from the situation of which they are a part (Lesh, 1985). More specifically, learning to solve problems embedded in a situation is important because "effective thinking is the result of conditionalized knowledge --knowledge that becomes associated with the conditions and constraints of its use" (Glaser, 1984, p. 99). The claims are that, if the situations are familiar, conceptions are created from objects, events, and relationships in which operations and strategies are well understood.

From this understanding, students construct a framework of support that can be drawn on in the future, when rules may well have been forgotten but the structure of the situation remains embedded in memory, as a foundation for reconstruction (Brainin, 1985).

Instructional Objectives. Greeno (1987) has argued that formulating objectives of instruction based on cognitive models that simulate performance in school tasks should now be possible. Hayes (1976) has articulated this notion:

> Cognitive objectives in education [are] intended to replace the more traditional behavioral objectives. To specify a behavior objective for instruction, we state a particular set of behaviors we want the students to be able to perform after instruction, e.g., to solve a specified class of arithmetic problems or to answer questions about a chapter in a history text. To specify a cognitive objective, we state a set of changes we want the instruction to bring about in the students' cognitive processes, e.g., acquisition of a particular algorithm for division or the assimilation of a body of historical fact to information already in long-term memory. (pp. 235-236)

Greeno (1987) also maintains that recent research on instructional objectives has offered new insights into meaningful learning and the ways in which conceptual systems change; this research emphasizes a more active role played by learners, and holds that learning involves construction of knowledge, rather than its passive acquisition. Environments that encourage the construction of knowledge include (a) collaborative settings in which teachers and students work together to construct meanings and ideas; (b) settings in which teachers or tutors function as coaches and models of the activities the students are learning to engage in; and (c) settings in which students engage in exploration of ideas and environments.

Today, most educational psychologists view information processing as an essential aspect of human behavior, and learning as an interactive process involving the assimilation of new information with what is already known. This "cognitive science" approach has provided powerful insights into cognitive processing, and memory capacity and storage. And while it is premature to argue that findings from this work provide a definitive basis for redesigning mathematics instruction, the results suggest new directions for a reform program.

Other educational researchers (e.g., sociologists, anthropologists, historians) have reacted similarly to the revolution. Schools and schooling practices have increasingly become the targets of many of these scholars; such topics as the "hidden" curriculum, differential distribution of knowledge, managerial constraints to teaching, teacher deskilling, and "street" math are under investigation. Furthermore, the revolution has prompted scholars to pose new questions based on new models, and to investigate new ideas using innovative methods. In summary, educational researchers from many disciplines are reacting to the information revolution and its impact on their disciplines and their methods of inquiry--and many are now studying schooling practices.

Reactions by Policy Makers

Of the three target audiences for this paper, I am least familiar with the group composed of policy makers. Until I chaired the conference, "School Mathematics: Options for the 1990s" for the Department of Education and the National Council of Teachers of Mathematics in 1984 (Romberg, 1984), my contacts with policy makers were casual. Since then, I have been thrust by NCTM into the policy arena to serve on several occasions as a spokesperson for school mathematics. Whether they be elected officials, administrators, bureaucrats, or union officials at the federal, state, or local level, policy makers have the legal and fiscal responsibility of making decisions about how schools operate.

At least six aspects of their task merit consideration. First, their work centers on decision making: Budgets must be determined, school rules and regulations must be established, credentials and requirements for administrators and teachers must be set, and so forth. Often these decisions are made in the face of conflicting values, demands, and advice. Second, educational policy in America is diversified, because, by its very nature, the educational system is local, rather than national. Since the Constitution of the United States omitted reference to education, thereby leaving decisions about schooling to the states, control of schools has, for the most part, been assigned to local communities with locally elected school boards. These boards hire administrators and teachers and approve programs. Today, 15,248 school districts operate in the United States; shared state and local control and shared state and local taxes that support schools have created vast differences in the quality of programs, facilities, staff and teachers, both across and within states. There is no national curriculum, no national set of standards for the licensing or retention of teachers, no common policies for student assessment of progress or admission to higher education. Decisions are made by a variety of persons at several levels. One consequence of such diversity is that change is difficult. Third, policy decisions often are made quickly to alleviate a perceived problem. The political context in which policy makers operate require them to foster a public image of being in charge and on top of problems in response to citizens' demands. Unfortunately, initial solutions often are no more than "band-aids" that fail to address the underlying causes of the problem; with time, however, carefully considered programs often follow such spontaenous responses. Fourth, information--whether drawn from personal experience, judgments of colleagues or confidants, staff reviews, or opinion polls (i.e., how will constituents react to alternatives)--is essential as each decision is made. Even so, decisions will be made, based on whatever information is available even if the quality of the information is suspect. Finally, all information is judged in view of the political environment in which policy makers operate. Legislators filter input in terms of their party's position and the concerns of voters, administrators listen to legislative concerns, bureaucrats attend to what administrators say, and so on. Enmeshed in the system are the administrative staffs, foundations, interest groups, and lobbyists who attempt to influence decisions by providing information about both the decision to be made and the concerns of their constituents. A working knowledge of these key characteristics of policy makers' activities and decision-making processes is critical if mathematics educators and educational researchers are to exert influence on policy decisions.

The responses of this community to the revolution can be analyzed as a series of waves, each unique in terms of immediacy of response and the information used to justify those responses. First wave responses during the past five years involved reacting quickly to the intense political pressure created by such documents as A Nation at Risk (National Commission on Excellence in Education, 1983). Legislators, chief state school officers, and school boards called for increased requirements for graduation, more time spent on mathematics and science instruction, more rigorous testing of students, testing teacher competence, and so forth. The justifications for these actions were based largely on the belief that the educational system needed more "effective and efficient" management. As McNeil (1987) put it, this first wave was "the twenties revisited." For example,

> At the district level, there was a real concern of unevenness, of equity, and that algebra at this high school isn't really the same as algebra over here. So let's have a standard district curriculum... Make all teacher lessons plans, confirm to a system, and at the end of the semester, we at the district level will send out the proficiency exam to see if teachers have covered their proficiencies that were numbered and organized and rationalized in their curriculum. (pp. 11-12)

She goes on to argue that many states and districts

> are legislating the teacher test, the student test, the curriculum based on the student test--not done by curriculum people, but done by research and evaluation officers who have some tests they can pull out of their folders. And for the first time, even more than in the original social efficiency days, the first wave reforms are locking in a structure that is depersonalizing teaching. (pp. 12-13).

Apple (1987) arguess that this first wave of response has emphasized educational accountability rather than reform. While some of the recommendations for change have merit, the trend has been toward increasing external control of schools.

In fact, these initial responses are likely to perpetuate an outdated mathematics curriculum in the manner described in the classic satire, The Saber-Tooth Curriculum (Peddiwell, 1939). Without change, we will continue to train shopkeepers who can perform some procedural skills that are now better accomplished by calculators and computers everywhere in society except in its schools. The first wave, even if based on a narrow conception of education, brought attention to the fact that the educational system is in trouble. To make it a viable system for the future, major investments of time, money, and effort will be needed.

The first wave was followed by a series of waves and counterwaves distinguished not by chronology, but by such external considerations as funding source, or political agenda. Given the diversity of the educational system, this is not surprising. At the federal level, the second reaction was to commission a number of groups to study various aspects of the educational system and to make recommendations for action. For example, the National Science Foundation and the Department of Education have funded a variety of conferences and commissions since 1983, related to school mathematics instruction. They include:

1. Two conferences on the mathematical sciences community's general response to the calls for reform. These conferences produced complimentary reports, New Goals for Mathematical Sciences Education (CBMS, 1984), and School Mathematics: Options for the 1990s (Romberg, 1984).

2. A survey of mathematics and science teaching (Weiss, 1987).

3. Three projects that studied the ways in which a better monitoring system could be developed. Reports of these groups are Improving Indicators of the Quality of Science and Mathematics Education in Grades K-12 (Murnane & Raizen, 1988); Indicator Systems

for Monetary Mathematics and Science Education (Shavelson, McDonnell, Oakes, & Carry, 1987), and The Monitoring of School Mathematics: Background Papers (Romberg & Stewart, Eds., 1987).

4. A study group which examined national assessment practices and produced the report The Nation's Report Card (Alexander & James, 1986).

The intent of these study groups has been to provide educational policy makers with information. While it is premature to judge the impact of these reports, three related problems have emerged at the federal level. First, while more reasoned than the first wave responses, these reports make recommendations focused on segments of the educational system rather than on the system as a whole. Second, part of this fragmentation reflects the fact that these reports lack a coherent vision about schooling. In fact, the National Science Foundation was criticized in a recent review of its educational policies for its failure to synthesize and articulate its educational goals (Knapp, Stearns, St. John, & Zucker, 1987). Third, most of the reports have failed to take into account the impact of the information revolution. In a recent review of the "indicators" reports, Anne Zarinnia (1987) found that none adequately reflected the social and economic implications of the Information Age. At the same time, Secretary of Education, William J. Bennett's vision of ideal schooling (as described in James Madison High School, 1987) reflects a value system rooted in the practices of the schools of the present, thereby failing to realize that those institutions also are in need of change.

Another wave reflects the perspective of reform of the foundations, labor unions, and professional organizations involved in education. While these groups also have commissioned studies of the nation's educational system that have included recommendations for policy action, they have not been constrained by the same political considerations as those that influence studies commissioned by the federal government. Important reports related to mathematics instruction include:

1. Tomorrow's Teachers (1986) prepared by the Holmes Group, and

2. A Nation Prepared: Teachers for the 21st Century (1986) prepared by the Carnegie Forum on Education and the Economy, both of which examine the problems of teacher preparation and licensure.

3. What Science is Most Worth Knowing (American Association for the Advancement of Science, 1987) written by several groups of scientists. This document outlines the basic knowledge the scientific community believes will be important in the next century.

4. Curriculum and Evaluation Standards for School Mathematics (NCTM, 1987) prepared by the National Council of Teachers of Mathematics. This working draft is a position statement that reflects the kind of changes the mathematics education community thinks are possible in the near future.

These reports present positions that are more academic, less constrained by practical realities, and more radical in their recommendations than those articulated in the federal reports. In addition, often they have tried to portray a more coherent vision about the future. As a result, some of the recommendations conflict with those found in the federally sponsored documents. Here again, it is premature to judge the impact of these national reports.

Quite different waves of reform have occurred at the state and local levels. For example, Michaels (1988) argues that the second wave of reform at the local level involves:

> the individual school as the unit of decision making; development of a collegial, participatory environment among both students and staff; flexible use of time; increased personalization of the school environment with a concurrent atmosphere of trust, high expectations, and sense of fairness; a curriculum that focuses on students' understanding what they learn--knowing "why" as well as "how"; and an emphasis on higher-order thinking skills for all students. (p. 3)

Note that this is a markedly different agenda from that of the first wave. It focuses on the school staff and the creation of a more personalized instructional environment, not on management or accountability.

At least one counterwave--the "neo-Ludd ' reaction to technology--should be noted. The workers who destroyed machines during the early phases of the Industrial Revolution in England were known as Luddites, apparently because a series of letters threatening English factory owners were circulated in 1811 under the name Ned Ludd (Thomis, 1970). Provenzo (1986) used the term to describe the opposition that has developed among many people to the proliferation and increasingly widespread use of calculators and microcomputers in our culture. However, this negative response is more than just a superficial reaction to change: It embodies a reaction against the implicit cultural imperialism of the Information Age. Bell (1979) has argued that "just as capital and labor have been the central variables of industrial society, so information and knowledge are crucial variables of post-industrial society" (p. 168). If this is so, the control of knowledge will be the key to political and social power in the next century. The development of computer systems implies the development of data banks and systems for the organization of knowledge as well as hardware. While this may, at first, seem a trivial issue, in fact it represents what may be a significant form of cultural domination and imperialism. In a society in which computer literacy will be an essential prerequisite to successful social functioning access to instruction and to the use of computers will be critical. Microcomputers are expensive and relatively complex tools; they are more readily available to the rich than to the poor. In fact, it is distinctly possible that computer availability will perpetuate and enlarge existing patterns of racial, sexual, and social inequity.

In summary, policy makers at all levels of control within the diverse American educational system have reacted in various ways to the information revolution and the calls for reform in the system as a whole, and particularly in school mathematics.

RESEARCH

This document is replete with references to research. In fact, its content thus far is founded upon a large body of research that has documented current deficiencies in the schooling system, described barriers to reform, and suggested alternatives to current practices related to the teaching and learning of mathematics. At this point, however, I want to emphasize the development of new research agendas that are "facilitators of the reform movement;" they are a necessary component of the education community's response to the revolution. In so doing, two aspects of research merit mention--the importance of reliable knowledge and the information and insights that can be derived from research--if research agendas are really to serve this role in the reform movement.

Reliable Knowledge

Given the broad consensus that changes are needed, a proliferation of claims and counterclaims about the most appropriate actions, programs, and policies to be followed seems inevitable. Larrabee (1945) argued that anyone who has surveyed the long history of man's claims about knowing is "struck by the discrepancy between the pretentiousness of most knowledge-claims and the small amount of evidence actually available with which to back them up" (p. 82). Researchers make every effort to circumvent this stereotype by admitting their ignorance, expending considerable effort gathering evidence so that whatever information is acquired is reliable, and marshalling the evidence into well-argued briefs to justify their assertions.

By reliable knowledge, I mean any claim to know that is substantiated as trustworthy for some given purpose. The gathering of evidence and the construction of an argument are the means by which researchers substantiate conjecture. This is an arduous and endless task that requires a substantial amount of training and effort; in the more complex cases, it taxes the patience and ingenuity of the most gifted thinkers. Nor does it, once achieved, stay finished and complete; it must be continually re-achieved, since both what constitutes a reasonable argument and its given purposes change continually.

What can be learned from research

A report of research or a set of studies provides its readers with three fundamental types of information: (a) the researcher's view about a phenomenon, (b) the way evidence has been collected and organized about conjectures, and (c) the work's findings or conclusions. The primary purpose of any research program is to make sense of a complex phenomenon. The first step in such a program is to develop some model (framework, metaphor, etc.) designed to capture what are important features of the phenomenon; all such models are of necessity incomplete. Nevertheless, they are fundamental to the investigations that follow, for it is from the model that conjectures are derived. Second, a research program is established to systematically gather and report evidence to substantiate or refute those conjectures. In this sense, every research result is descriptive in that its findings are about its model. Finally, it is hoped that such research findings provide us with some understanding of the phenomenon.

It must be noted that most past and contemporary research addresses a traditional vision of schooling and literacy. For example, much research on effective teaching has been based on predicting residualized-mean-gain scores on standardized tests. Such research does not support the pursuit of reliable knowledge about the new vision of mathematical literacy. New questions are now being posed with expectations of outcomes different from those assessed on such tests. Research agendas for the future must address questions and embody methods of gathering information pertinent to the reform.

In summary, research can provide reliable knowledge about important aspects of school reform; the results, stated as principles, should provide an information base for the reform movement. Current school mathematics operates within a coherent system; reform will happen only if an equally coherent system replaces it. Information to be gathered via research must be related to the new conceptions of how mathematics is learned and taught, as well as what it means to know mathematics and how mathematical knowledge can be assessed.

SUMMARY OF IMPLICATIONS

The views articulated in this paper inhere a number of important implications for its three audiences. Five key concepts should be reiterated:

1. The information revolution is not a myth, but a reality. As the world changes, so must our schools and the mathematics that is taught in them.

2. Because of the revolution, the fundamental mathematical concepts and procedures that students should learn also have changed.

3. The three audiences (as well as with other groups) need first to listen to one another, and then to cooperate to bring about the needed reform in school mathematics. In particular, the three groups--mathematics educators, educational researchers, and policy makers--can no longer operate in ignorance of the other groups' ideas and concerns. Mathematics educators need to listen to researchers and policy makers. For example, the most current mathematics curriculum development projects have paid scant attention to the psychological literature; similarly, several recent projects directed by educational psychologists have failed to consider any changes in mathematics. They involve good ideas from the psychological revolution but have approached mathematics from a perspective garnered from current textbooks and classroom practice. In the same vein, policy makers should not base their recommendations for change on traditional notions of either mathematics, psychology, or sociology of schools. Without efforts to adjust our perceptions to jibe with the ongoing information revolution, the economic and social decline of this country is likely to become a fact.

4. The mathematical sciences education community must build coalitions at the national and state levels to make apparent to all the needed changes in school mathematics.

At the national level, the Mathematical Sciences Education Board fulfills this role, and all mathematics educators should support and help it to develop a coherent argument for change; it is left to educators to establish similar boards at the regional or state level. Only through the efforts of such groups will the public be alerted to the need for change and learn to turn to these coalitions for information and advice. These key concepts are synthesized in the recommendations that follow:

Recommendation 1. A reasoned vision of the direction the country needs to take in school mathematics needs to be developed. Mathematics professionals must cooperate to develop a cohesive, reasoned vision of the reform in school mathematics necessary to prepare students to be productive citizens in the 21st century.

Currently, for example, the vision set forth in James Madison High School (Bennett, 1987) is incompatible with that described in the NCTM Curriculum and Evaluation Standards (1987). In fact, it is apparent that policy makers, mathematics educators, and educational researchers hold different visions about the goals for school mathematics. Reform is possible only if there is consensus on goals among all concerned parties.

Recommendation 2. A systemic framework to accommodate and organize reform efforts toward this common vision must be developed.

Change that focuses on single elements (e.g., texts, tests, or teachers)--regardless of its potential or soundness--is bound to fail. Policy makers also must recognize that systemic effort will require them to address the related political infrastructure of schooling. It will be particularly important to identify and remove barriers to reform; already it is clear that reform is impossible without improvements in the status of teachers and the working conditions in schools.

Recommendation 3. A change plan based on the vision and systemic framework must be developed.

The need for fundamentally different school mathematics is expressed in the vision and systemic framework. A change strategy involves coordination of efforts toward achieving the vision. The precise rise, tread, and shape of the steps to be taken will vary in multiple ways, depending on fiscal and human resources and inclinations. A vision strategy, a systemic framework, and identifiable steps toward the vision's attainment are critical. From this perspective, school reform is a matter of timely, manageable, effective, and well-coordinated steps toward an agreed upon vision, rather than a simplistic choice between polarized options.

In conclusion, the emphasis of school mathematics must shift from drill in paper-and-pencil computations to experience in examining patterns and learning to communicate about them if our students are to be prepared to live in the next century. This shift will require a fundamental restructuring of the educational environment from the current "transmission of knowledge" model into one based on "stimulation of learning" The transition will involve fundamental changes in content, modes of instruction, teacher education, and methods of assessing student progress. The reform programs should make it possible for all students to learn more and somewhat different mathematics. This can be accomplished only if the groups identified here are able to work collaboratively on the important issues identified in an agreed-upon long-range plan. The need for reform is understood; our challenge is to make it happen.

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