DOCUMENT RESUME

ED 300 263 SE 050 108

AUTHOR	Balfanz, Robert
TITLE	Elementary School Quality: The Mathematics Curriculum and the Role of Local Knowledge.
PUB DATE	Mar 88
NOTE	25p.; Paper presented at the Annual Meeting of the Comparative and International Education Societies (Atlanta, GA, March 19, 1988). Partial funding for this paper was received from the University of Chicago Mathematics Project-Elementary Component.
PUB TYPE	Reports - Research/Technical (143) -- Speeches/Conference Papers (150)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	*Cognitive Development; Curriculum Development; Educational Research; Elementary Education; *Elementary School Mathematics; Learning; Learning Activities; *Mathematics Curriculum; *Mathematics Instruction; *Number Concepts; Primary Education; Research Utilization; *Social Influences
IDENTIFIERS	Mathematics Education Research

ABSTRACT

This report considers how the mathematical knowledge children develop on their own outside of formal school instruction can be used to increase the distribution and level of mathematical knowledge attained by students in grades K-3. Included are preliminary results of an investigation of the counting and calculating abilities brought to instruction by students in grades K-3 in Chicago (Illinois) and similar research from a number of developing countries. This demonstrates the dangers both of ignoring the knowledge children develop outside of formal school instruction and of becoming too dependent upon it. It is argued that a key means by which elementary school quality can be improved is to begin with the knowledge students develop on their own and transform it though pedagogic and curricular intervention into a set of portable intellectual skills. The point is also made that children around the world bring more knowledge to school with them today than students 50 years ago did, yet curricular materials fail to account for this. This knowledge depends not so much on where they live, what their sex is, or who their parents are, but on the level and complexity of mathematical experience their out-of-school activities provide. (MNS)

Elementary School Quality: The Mathematics Curriculum and the Role of Local Knowledge

Paper presented as the Comparative and International Education Societies Annual Meeting, Atlanta, Ga., March 19, 1988

Robert Balfanz
Comparative Education Center
University of Chicago

Partial funding for this paper was recieved from the University of Chicago Mathematics Project-Elementary Component.

In recent years there has been an upsurge of interest in the quality of elementary schooling. In the United States this interest has been fueled by concerns over declining scholastic performance and continued educational inequities. In the developing world it has been spurred on by the realization that the quantitative expansion of schooling alone will not achieve national educational goals and a desire to further indigenize national schooling practices. In both cases policy makers have been the most active reformers. In the United States, state legislators have mandated increased testing, longer periods of instruction and generally supported a back to the basics educational program. In the developing world , international agencies, notably the World Bank, and national governments have taken an active role in promoting and supporting the spread of textbooks and improved teacher training. Against these trends have stood a number of academics and teachers who believe that the answer to improved elementary school quality does not lie so much in tests, texts or teachers but in a greater understanding of the knowledge which children bring to school, a greater respect for cultural variation, and a greater interest in the relevance of the knowledge which children learn (D'Ambrosio 1985).

This paper will address this debate by examining how the mathematical knowledge children develop on their own outside of formal school instruction can be used to increase the distribution and level of mathematical knowledge obtained by students in grades K-3. It will use preliminary results of an investigation I am participating in, which is examining the counting and calculating abilities brought to instruction by K-3 students in Chicago, and similar research from a number of developing countries to demonstrate the dangers of both ignoring the knowledge children develop outside of formal school instruction and becoming too dependent upon it. In the process it will hope to take a small step in reconciling those who believe improved school quality lies in improving the technology of teaching and those who believe it lies in a deeper understanding of the mental life of children. It will argue that a key means by which elementary school quality can be improved, if quality is mea ured by the number of elementary graduates who have acquired a solid foundation of knowledge they can use creatively, confidently and productively, is to begin with the knowledge students develop on their own and transform it through pedagogic and curricular intervention into a set of portable intellectual skills.

Recent research indicates children begin acquiring mathematical knowledge by the time they are a year old, if not from the moment they are born (Strauss, Curtis 1984). By four or five most children seem to have acquired both the conceptual foundation needed for the acquisition of complex mathematical knowledge and a fair degree of mathematical prowess. They can count, calculate and measure. This appears to be true in both high and low SES environments in the United States (Ginsburg, Russell 1981, Fuson, Richards, Briars 1982, Fuson, Hall 1983) and with the exception of some remote hunting and gathering tribes, most environments in the world (Posner 1^32, Lancy 1983, Yoshida, Kuriyama 1986 , Watson 1987).

Sources of the mathematical knowledge which children bring to school vary considerably. They range from participation in trade and herding to the playing of electronic games and the use of microwave ovens (Balfanz 1988). Consequently this knowledge is not universally the same. Different environments and cultures promote different conceptual understandings of quantification (Lancy 1983, Watson 1987), different number systems and different means of calculation (Zaslavsky 1970) . In Papua New Guinea, for example,the counting systems of 225 languages, 30% of the nations total, were investigated. 12% used a upper body-part counting system based on 24 to 29 numbers. 15% used a base 2-5 tally system. 40% used a counting system with bases at 5 and 20 and the remaining 33% were divided among people who approximated a base ten system and those who used an unique counting systems of their own (Lancy 1983). In a similar vein, surveys of calculation practices in Africa (Zaslavsky 1970, Ginsburg, Posner,Russell 1981, Brenner 1985), South America (Vasco, 1986. Carraher, Carraher, Schliemann 1987), and my own work in Chicago have turned up many unique and ingenious procedures. The Yoruba of Nigeria for instance, might represent 315 as (200 x 2) - (20 x 4) - 5, and 525 as (200 x 3) - (20 x 4) + 5 and solve 315 plus 525 as (200 x 5) - (20 x 8) = 840 (Saxe, Posner 1983) , while a second grader I interviewed in Chicago, solved 44 plus 37 by saying (45 + 35) = 80, 80 -1=79, and 79 + 2 = 81.

In some form, however, mathematical knowledge appears to be possessed by most children before they enter school. Furthermore, it is known that the knowledge children acquire outside of formal school instruction can exert a positive influence on mathematical performance throughout a child's elementary schooling and into adulthood (Brownell 1941, Scribner 1986, Balfanz 1988).

Despite this, children's environmentally acquired mathematical knowledge is recognized by few curriculums in the world. In practice, elementary mathematics is taught as a global language which children are ignorant of until the day they enter school (Vasco 1986). This notion is wrong on both accounts. The question of ignorance has been addressed and mathematics is no more a global language today than French was in the nineteenth century or English is today. The evidence is compelling that mathematical procedures can be carried out in a variety of tongues. For proof one need look no further than the number of counting systems and calculation procedures previously mentioned.

The danger of recent educational reforms which focus on the technology and organization of teaching (texts and tests) rather than on the mental life of children is that they may exacerbate the problems and missed opportunities which result from ignoring the mathematical knowledge which children bring to school. We are aware of the difficulties which are caused when a child's home language is different from the language of school instrcution. What has not yet been recognized is that similar problems of manufactured inequality, poor academic performance and loss of confidence can occur when the mathematical procedures used at school are different from the mathematical procedures used outside of sc'ool by a child (D'Ambrosio 1985). Although it has not been empirically documented, it is logical and consistent with what is known about learning that children who environmentally acquire and practice counting systems and calculation procedures which are the same as the procedures used in school, will do better in school than those children who do not. This will be especially true when instruction is limited or poorly delivered.

In the United States, reform efforts which call for increased testing and a back to the basics approach by necessity and tradition promote the use of written algorithms. Most children's environmentally acquired knowledge, however, is obtained orally and developed mentally, without the use of writing. Children learn to add and subtract, multiply and divide outside of school by manipulating, trading, and sharing objects. First they physically count them one by one, then they may represent them on their fingers or by other means and later they carry out complex calculations in their head (Gelman,Galliste 1978, Desforges, Desforges 1980, Fuson, Hall 1983, Carraher, Carraher, Schliemann 1987).

When children are taught that written algorithms are the only appropriate means by which to calculate, they appear unable to apply much of what they know. They also lose touch with their common sense. As a result, mathematics often ceases to be a thinking process and becomes a mechanical one. In the course of my research in Chicago a sample of 40 second graders attending a private school were asked to mentally solve the problem 40 take away 18. They had been taught how to solve carry addition problems using column arithmetic but had not yet recieved instruction in column subtraction. Their most common answer was 38. They arrived at this answer by solving the problem in column form from left to right. Four take away one is three and since they believe 0 take way 8 can not be done, they misapplied the notion of communitivity and took 0 away from 8 for an answer of 38. Fewer but still a significant number of children used the same thinking when presented with the problem 300 take away 1 or 300 cents take away 1 cent and answered 301 or 200.

Similar results have been obtained with other samples of students in the U.S. and in Brazil (Carraher et al. 1987, Siegler 1986). Moreover, it has been shown that while students believe this incorrect method of calculation will lead them to the right answer on a written problem at school, and in my experience defend it with great zeal, they are quite aware that if they had forty objects and gave away 18 they would not be left with just two less than they started with (Carraher et. al. 1987). In fact, if you were to suggest this to them, they would likely tell you that you were crazy.

Prior to instruction in written algorithms, these problems would largely be solved mentally through the use of counting, (the children would count up from the smaller number to the larger number or vice versa, first by ones and then by larger units) or by some means of distributive subtraction based on a set of known number facts (40 take away 10 is 30, 30 = 20 + 10, 10 - 8 = 2, 20 + 2 = 22). My research in Chicago, as well as research in Brazil, indicates that these methods lead to higher accuracy rates and errors which are much closer to the correct answer than the use of school-taught written algorithms. In Chicago, 24 first and 23 second graders attending a private school performed approximately 20% better when oral subtraction problems were presented in a form which encouraged the use of counting instead of column arithmetic. Moreover, when counting was encouraged, the errors made were much more likely to be within 1 or 2 of the correct answer than when column arithmetic was used. In Brazil, 16 third graders ranging in age from 8 to 13 and randomly selected from two public schools also performed approximately 20% better on a set of addition, subtraction, multiplication and division problems when they were encouraged to use local counting and regrouping strategies, than when they were not. Even more dramatically, the students performed 35% better on subtraction and 46% better on division problems when they solved the problems orally and mentally using local means of calculation than when they solved similar written problems using school-taught algorithms.

One solution to this problem is to drill children in the correct written algorithm until the overwhelming majority of them get it right. This, however, is time consuming and research indicates that even with substantial drilling a significant number of children will still apply faulty written algorithms (Brown Burton 1978).Moreover, where teaching and drilling are insufficient, children will be left using incorrect procedures. The alternative solution is to demonstrate to the children how they can apply and expand the knowledge they already know, be it finger counting, counting up from the small number to the larger number or distributive subtraction, and afford them the opportunity to use this knowledge along with standard arithmetic procedures. My preliminary research in Chicago indicates that the classes which add and subtract the best are the classes in which a variety of solution strategies are allowed, developed and encouraged. Similar results have been obtained in a study conducted in Liberia (Brenner 1985). This direction, however, is not being encouraged by many elementary school reform movements in the U.S.

In the developing world the spread of textbooks with their codified means of solving mathematical problems may have the same effect as overreliance on written algorithms in the United States. If textbooks are used to establish correct procedures rather than just as an instructional tool, they may deprive students of their local knowledge. This may have even increased significance because of the older age and probable greater environmental experience with mathematics of elementary students in some developing countries (Ginsburg, Posner, Russell 1981, Vasco 1986, Balfanz 1988). Secondly, and perhaps even more damaging, it has been argued by D'Ambrosio (1985) among others that poorly learned school procedures only serve to weaken students' confidence in their local ways and offer nothing better in return. Thus it is possible to foresee a situation in which the blind spread and use of textbooks could actually lower the quality of elementary education for some students.

On this point a comparison of practices and performance in Liberia and Brazil is instructive. In the Liberian schools examined by Brenner (1985), teachers promoted the use of indigenous methods in concert with school-taught procedures. The students who performed best in the class were the students who used the greatest variety of procedures. In the Brazilian schools examined by Carraher et al (1986, 1987), children were expected to use only school-taught algorithms. The result was that children from the urban slums consistently got problems wrong which they solved every day in a different form on the street. One reason why the Liberian teachers may have allowed and encouraged indigenous methods is that without a sufficient supply of textbooks they may not have felt bound by its teachings.

A second problem which results from policy makers' lack of interest in the environmentally acquired knowledge which children bring to school is that they are likely promoting curriculums which may significantly underestimate the abilities of entering students. In work done by Bell and Bell(see Fuson et al, 1982, Balfanz 1988) on the counting abilities of entering kindergarteners and first graders in Chicago, some differences were found in the performance of low and high SES students. But these differences pale in comparison to the differences I found when I compared Bell and Bell's results to comparable surveys which have been taken from 1930 onwards(Balfanz 1988). The comparison indicates that there has been at least a 20 to 30 percent increase in the number of entering first graders who can count through 20 in the past fifty years and that today it is close to a universal ability. It is also appears that there has been even greater growth in children's ability to count through 100.

Despite this fact, however, many first grade arithmetic texts spend the bulk of their time teaching students about the first twenty numbers. These books are based on conservative interpretations of research conducted in the 1940's and 1950's. They also serve as the basis for many of the texts used in the developing world (Vasco 1986,Altbach 1987). Research conducted in West Africa and Papua New Guinea, also indicates that entering students in those areas may have a greater than acknowledged counting ability (Posner,Baroody 1979 , Lancy, 1983). Furthermore, my preliminary findings on the mental calculation abilities of first and second graders in Chicago indicates a unrecognized and comparable improvement in these skills over the last fifty years. Thus it is quite likely that the level of mathematical knowledge many early elementary students have or are the slightest amount of instruction away from acquiring is not reflected in current texts. To promote these texts then, may be to promote ignorance.

These concerns should not be interpreted as an attack on the spread of textbooks or testing. Both are necessary to increase the quality of elementary schooling . Rather they are aired to point out that the textbooks and tests must be used wisely as learning tools and not allowed to become the sole arbitrators of correct mathematical procedures. The most correct procedure is always the one which allows the student to obtain the right answer. As we have seen, this will not always be the procedure promoted in the text or proscribed by the test.

Secondly, they are aired to broach the subject of the - quantity trade-off in the provision of texts and increased class time. It may well be that it is more learning effective to provide fewer but freshly written texts to a class which reflect students' current level of knowledge and promote the use of indigenous means along with standard mathematical procedures than to provide the whole class with a second-hand text which underestimates and constricts their knowledge. The same may be true for increases in instructional time. Some of the increased class-time might be better spent training teachers how to allow and promote the use of multiple calculation strategies, as opposed to using all the time to drill students in procedures which do not take account of the knowledge they already have.

Having outlined some of the dangers which arise from ignoring the mathematical knowledge which children bring to school, it is time to say a few words about the dangers of depending on it too heavily. In the United States, overdependence can be seen in the movement by some academics and teachers to base calculation instruction on student developed algorithms. Spurred on by over a decade of research which has highlighted students' ability to invent successful means of calculation (Siegler 1986), this movement concludes that, left to their own devices, students will be their own best teachers. While student developed algorithms can play a substantial role in the acquisition of calculation knowledge, the results of my Chicago research suggest that total reliance on student self-development can be counter-productive.

In the course of interviewing or surveying 40 first grade and 65 second graders from an upper-middle class private and middle class public school on how they solved a range of addition and subtraction problems, it became clear that there was a basic difference between the knowledge they had developed outside of school and the knowledge they extrapolated from what they had learned in school. The first was usually correct, the second was often wrong. As I have already noted, many of the children discovered outside of formal instruction that subtraction could be simplified by counting up from the smaller number to the larger number and performed this operation with considerable skill on problems beyond grade level. An equal number of children, however, believed they had found a better way. They developed a range of column subtraction procedures based upon what they had learned about column arithmetic at school. These procedures were more often than not erroneous.

The reason for this is still unclear but a good possibility is that knowledge developed outside of school is empirically tested, while knowledge developed in school often is not. Outside of school, children either operate with the aid of physical objects which can be manipulated to test their calculations , use their knowledge in an exchange situation in which the exchangee expresses an opinion as to its correctness, or present it to a more knowledgeable friend, sibling or adult who acts as a censor against erroneous information and a conveyor of correct procedures.

On the other hand, several studies have shown that knowledge developed in school may only be used in school (Greenfield and Lave 1982, Carraher et. al. 1987) where students are often deprived of the information they need to understand why their invented methods are wrong. In the course of schooling, a student may be told that his approach led to an incorrect answer, but he is seldom told why this is the case. On an individual basis, this would be difficult for a teacher of twenty and close to impossible for a teacher of 60. Thus, the student is left not knowing what he did wrong and may continue to operate on false information (Brown, Burton 1978). This is why allowing students to develop their own calculation knowledge in the absence of instruction will not lead to general improvement across a class. When this teaching strategy is adopted, only those students who are able to get sufficient empirical feedback on their inventions will be successful. The constant one-on-one or small group interaction children engage in outside of the classroom is difficult, if not impossible, to reproduce within it.

In parts of the developing world, another form of overdependence can be seen in the tendency of some academics and policy makers to promote local mathematical knowledge as a replacement for school based mathematical knowledge(D'Ambrosio 1985). Established local methods are viewed as equal to or better than the abstract mathematics of the standard school curriculum, which is attacked as either lacking in relevance or as a chauvinistic brand of mathematics. There is some truth to both charges but the answer to the problem does not lie in entirely basing the early elementary curriculum on local mathematical methods. The main reason for this is that work by Greenfield, Lave, (1982) and Petitto (1982) among others indicates that local mathematical methods can lack portability. They are often grounded in the tasks they were designed to facilitate. Lave, for instance, has shown that a group of tailors she interviewed did not transfer the mathematical skills they developed for tailoring to non-tailoring tasks.

Greenfield and Lave in turn have shown that this might be related to the manner in which the mathematical knowledge was learned. When children learn a mathematical skill in the context of some larger operation, they have a ready means by which to test the accuracy of their knowledge. At the same time, however, they may not see that the knowledge which they develop can be extended beyond the task for which it was acquired. Thus, to teach only local methods invites disappiontment if one of the goals of schooling remains the development of portable intellectual skills.

A final problem inherent in any attempt to use the mathematical knowledge which children bring to school is identifying it. It does not appear possible to establish this knowledge a priori or through limited samples. The mathematical knowledge children bring to school does not depend so much on where they live, what their sex is or who their parents are but the level and complexity of mathematical experience their activities provide. This experience in turn can not be simply discerned. There appears to be no consistent difference across SES, sex, age, or urban/rural living environments. Nor does it seem possible to establish society-wide experiences. Three short examples should suffice. In 1968, an all India survey of mathematical achievement found that, in most districts, boys at the elementary level performed at a higher level than girls(Kulkarni et. al. 1970). In four districts, however, the opposite was true. In-depth analysis of the girls' superior performance in one district revealed that many of them came from a matriarchal society in which women controlled family finances, and basic mathematical knowledge was passed from mother to daughter (Kulkarni, Naidu, Arya 1969). The larger Indian study also found cases where, counter to expectations, rural and over-aged students in a grade outperformed urban and normal-aged students. It also found no general effect of SES or father's occupation, which stands in contrast to studies in both the U.S.(Yando et al. 1979) and Taiwan (Chalip,Stigler1986) which have found small but significant effects .

Second, a study conducted by David Lancy and others in Papua New Guinea in the late 1970's examined the impact of different environmental and economic areas on children's mathematical knowledge. It found that no clear pattern could be developed. The authors concluded that "both cities and villages offered a panorama of opportunities to children". Farming villages which on some measures were more societally complex offered fewer learning opportunities to children who spent the majority of their working hours hoeing, than some simpler fishing villages in which children participated in a range of economic activities. Children from the urban slums, however, demonstrated the same relatively low level of knowledge as children from remote villages, while children from some villages showed a high level of knowledge equal to that of expatriate's children who lived in the capital.

Finally, a failed hypothesis from my own work in Chicago is revealing. Initially it was surmised that one factor behind the growth of children's counting abilities over the past 50 years was increased experience with money of larger amounts. Whereas 50 years ago, elementary aged children may have lived in a nickel or quarter economy, it now seemed reasonable that they might live in a dollar or five dollar economy. It was further surmised that this environmental experience with money of larger denominations would give the children increased experience with addition and subtraction.

In the course of interviewing 50 second graders(1 upper-middle class private school class, 1 middle-class public school class) , however, two things became clear. First, it appears that the most affluent among them had the most limited or constricted experience with money. Secondly, it was found that three levels of environmental experience with money could be established. At the bottom were a significant number of children who reported that they never bought anything by themselves, and had trouble distinguishing between a quarter and a nickel. The bulk of the children were in the middle. They bought things on their own, but usually this amounted to only one or two items which were constantly purchased. For these items, most of the children had learned their cost and the various coin and paper combinations which could be used for their purchase, but this was the extent of their contact with money . At the upper end of the spectrum were those children who bought a variety of goods and had the level and complexity of environmental experience with money to develop calculation knowledge through its use. They, however, were in the minority. My sample was small and limited to middle and upper middle class city children. A larger and more diverse sample may have shown a different pattern, but what it does point out is that logical deduction is an insufficient means of identifying the mathematical knowledge which children bring to school.

How then can the mathematical knowledge which children bring to school be identified and used to improve the quality of elementary mathematical instruction? Based on what is known, no explicit answers can yet be given but some guidelines can be suggested. WIth regards to identification, in lieu of national samples, it seems wise to concentrate on the knowledge which has the largest implications and most versatile uses. In K-3 mathematics this will likely be the children's counting knowledge. If a child can count, he or she can calculate and measure. The more sophisticated a child's counting ability is, the more complex calculations and measurements he or she will be able to perform. Thus it is important for teachers to know the level and type of counting knowledge which children bring to school.

One means of identifying this knowledge is to create a classroom environment in which students feel free to demonstrate and use what they know. Variations in number systems should be allowed, as should deviations from the number system used in the text, as long as they produce accurate results. It is also important for teachers to allow methods of calculation which are based on simple and complex counting. Finally, it should be recognized that the use of local knowledge and local methods within the classroom may be particularly helpful to those students who do not demonstrate as strong an ability with school based methods. The fact that some students may be able to learn school based methods easily should not be allowed to give local methods a secondary status. Rather it should be stressed that all methods which are productive are equally valid.

To actively use the knowledge which children bring to school to improve the quality of mathematics education , teachers ,textbooks and tests should, in part, take on the role of translators. They specific mathematical knowledge children bring to school needs to be developed into a more general tool. Schooling should neither divorce students from the knowledge they already know nor propagate narrow skills. In terms of calculation, this can be accomplished by stressing the development of multiple solution strategies. Research has indicated that local mathematical knowledge is adaptive (Saxe 1982) and my own work indicates that children have a sense of efficiency when they are selecting solution strategies. Thus, by encouraging and helping students to find more than one way to solve calculation problems, it should be possible to use, generalize, develop and expand upon their local knowledge and in the process increase the learning effectiveness of schooling.

An additional benefit of this approach is that it makes elementary mathematics a thinking process. The development, expansion and selection of calculation strategies, in a small way, teaches invention and evaluation. If combined with the development of measurement strategies, this approach could be used to teach the basics of scientific thought i.e. moving from the known to the unknown through hypothesis, epxerimentation and analysis, solving a hard problem by looking for ways to simplify it and thinking by analogy. In this way, using and expanding upon the local knowledge which children bring to school may not only lead to a greater distribution and higher level of mathematical ability but in places where resources are limited, serve as the foundation of an elementary science curriculum.

Overall, skilled management of the knowledge which children bring to school has a considerable potential to improve the quality of elementary schooling. Yet for this to occur the limitations of this knowledge must be recognized. Children are neither ignorant of the mathematical world nor always correct in their independent mathematical thinking. Texts and tests will have their greatest impact when they reflect students' current level of environmentally acquired knowledge and aid teachers and students in using this know-how to create portable intellectual skills. For this to occur that fact that children live in dynamic worlds which provide them with an abundance of mathematical knowledge prior to, and outside of, formal schooling needs to be recognized and research aimed at discovering this knowledge needs to be supported.

On the other hand, advances in elementary school quality can not be achieved by solely focusing on the mental life of children, their culture and local environmental experience. Formal elementary schooling is a mass institution which is charged with producing basic literacy and numeracy in all its students. For fiscal and organizational reasons it is not possible to fully localize curriculums or pedagogic approaches. Nor would this be desireable. A central responsibilty of elementary schooling is to take children from different backgrounds and develop within them a common langauge which gives them access to the full range of opportunties within a society. Thus, research which focuses on the mental life of children and their environmental experience must look for knowledge which can be distributed via the mass printing of texts and centralized teacher training. In the pusuit of school quality one hand has to help the other.

End Notes

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