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AUTHOR	Merrill, William L.
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ABSTRACT

This unit is designed to acquaint middle school students with Pascal's Triangle. The unit is intended to be completed in small groups with a minimum of teacher direction. Students complete the activities by using manipulatives, calculators and computers and then report the results to the teacher and the class. The activities include: (1) patterns and puzzles with colored rods; (2) "random" walks with a street map; (3) coin toss investigation; (4) explorations with subsets; (5) explanation of Pascal's Triangle; (6) probability with dice; and (8) extensions on some of the activities. Teacher's notes, with answers, and a reference list accompany the unit. (PK)

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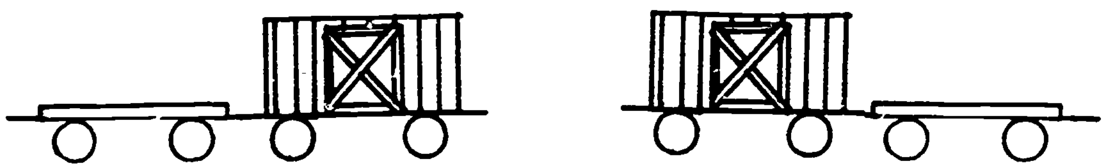
d	
w	y
r	c
g	g

Are there more that can be made? Let's think about a real train for a few minutes. Does a flat car followed by a box car look the same as a box car followed by a flat car?



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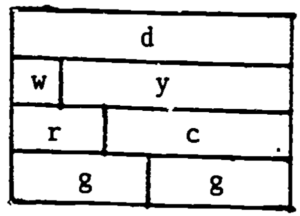
d	
w	y
r	c
g	g
c	r
y	w

Why wasn't the light green-light green train reversed? Because no matter how you arrange two light green rods, they look the same. Just like in a real train, two box cars look the same no matter how they are placed on the track.

Now you can try the activity. On Activity Sheet 1 there is a table which you will fill out using the colored rods to make trains of



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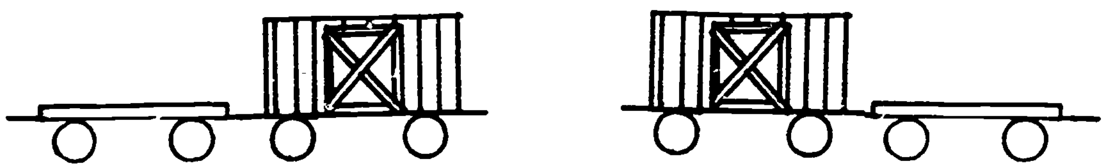

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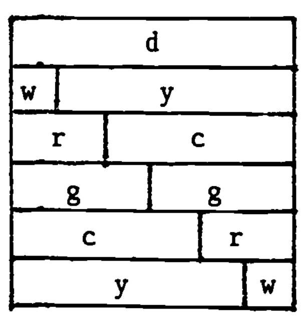

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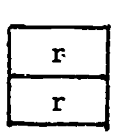
Now you can try the activity. On Activity Sheet 1 there is a table which you will fill out using the colored rods to make trains of different lengths and different numbers of cars. Down the side of the table you should see the colors of the rods. These represent the length of the trains to be made. Across the top of the table you should see the numbers 1 through 10 and "Total." These represent the number of cars you should put in each train.

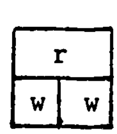
To understand what is wanted on this table, we can fill in the first few lines together. For the box where the white line and the 1 column intersect you should ask the question: "How many different one car trains can we make which will be the same length as a white rod?" Try it with your rods. You should see that there is only one, one car train which is the same length as a white rod. That is the white rod itself.

For the box where the white line and the 2 column intersect you should ask the question: "How many different two car trains can we make which will be the same length as a white rod?" Is this possible? What can you say about all of the rest of the boxes in the white row? Don't forget the total of each row.

We can now move on to the red row. See if you can ask the questions that go with each of the boxes on this row and make the trains which go with them.

You should have found the following:



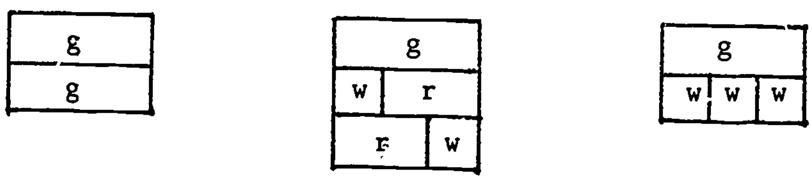


And your chart should look like this for the first two rows:

	1	2	3	4	5	6	7	8	9	10	Total
w	1	0	0	0	0	0	0	0	0	0	1
r	1	1	0	0	0	0	0	0	0	0	2

The light green row will be the last one we do together. Once again, see if you can ask the right questions and make the trains which go with each box in the row. Don't forget that the trains need to be different.

You should have found the following:



And your chart should look like this for the first three rows:

	1	2	3	4	5	6	7	8	9	10	Total
w	1	0	0	0	0	0	0	0	0	0	1
r	1	1	0	0	0	0	0	0	0	0	2
g	1	2	1	0	0	0	0	0	0	0	4

Now try to complete the rest of the rows. It is important that you use the rods since it is clear now that 1 is not the answer which goes in all of the boxes.

Here are a few questions which you should consider as you complete the chart:

1) Are there rows or diagonals that you can predict without making the trains?

2) Is it possible to predict the total of each row?

Number of Cars in the Train

Length of Train

	1	2	3	4	5	6	7	8	9	10	Total
w											
r											
g											
c											
y											
k											
n											
u											
o											

Activity Sheet 1

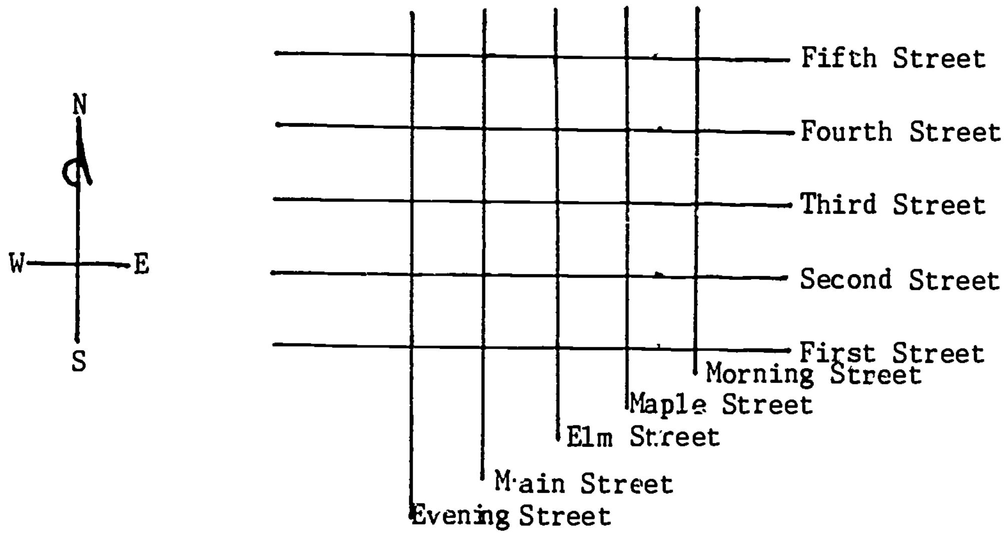
3) Can you tell which boxes will contain 0 trains?

4) What other patterns can you find in this table?

Activity 2

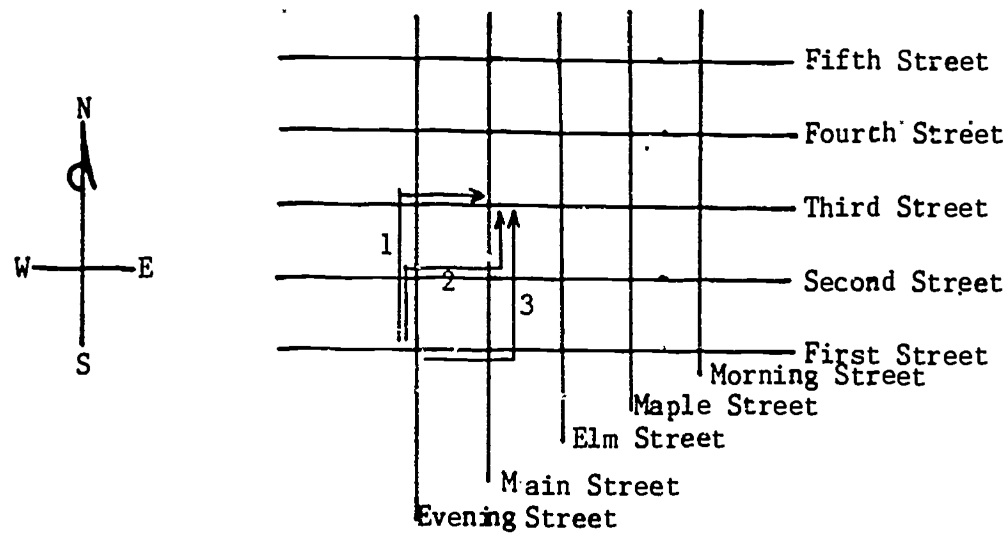
Random Walks

I live in a town that has 10 streets. Five of them run north/south and five run east/west. A map of my town is shown below:



I live in a house located at the intersection of First Street and Evening Street. I also have friends living at each of the intersections in the town. I want to know how many different ways I can walk TO each of my friend's house by walking the shortest distance possible. That is, I can walk only north or east, I can never walk south or west. I am also a good neighbor so I never cut through a neighbor's back yard. I walk only on the sidewalks.

Below are all the shortest ways that I can walk to the corner of Third Street and Main Street. Notice I only walk north and east, never south and west, and I stay on the sidewalks:



Now try to find out how many ways to walk to each of my friends' houses. Try to find patterns which will help you find all of the routes.

Activity 3

Coin Tosses

If you have one coin and you toss it in the air, what could happen? Think hard. There are four possible outcomes (not just two). I'll also warn you that we consider two of these outcomes to be impossible on the Earth.

If we only count the possible outcomes, the coin could land either heads up or tails up. And further more, there is only one way it can land with heads up and one way it can land tails up.

One Coin H - 1
T - 1

Now let's look at all the possible things that can happen if we flip two coins at the same time. The coins can land: 1) both being heads; 2) with the first being heads and the second being tails; 3) with the first being tails and the second being heads; or 4) both being tails.

Two Coins

HH - 1

HT - 1 ⟩ 2

TH - 1

TT - 1

Therefore, there is one way they can both be heads, one way they can both be tails and two ways they can land as one head and one tail.

Now try to find all of the ways that 3 coins can land. It might help if you lay coins down to show all the possibilities and remember, just as in the Train Game, order is important.

Once you have finished with three coins, try 4, 5, 6, 7, and 8 coins. Do each separately. Once again, these questions might help you:

1) What patterns can you see to help you figure out all the possibilities?
2) Can you tell how many ways there are to get 9 heads when flipping 9 coins? (without actually working out the problem)
3) Can you tell how many ways there are to get 7 heads and 2 tails when flipping 9 coins? (without actually working out the problem)
4) What is special about the totals of all of the possible ways for each number of coins? (The total for 1 coin is 2, the total of 2 coins is 4, the total of 3 coins is ?)

Activity 4

Subset Mania

In your mathematics class you have studied sets and subsets. If we start with a set which has one member in it, how many subsets can we make which contain no members? With one member? Remember, every set has the empty set and itself as subsets. Try to figure out the number of the subsets for different sets shown below:

	Subsets with 0 members	Subsets with 1 member	Subsets with 2 members	Subsets with 3 members	Subsets with 4 members	Subsets with 5 members
1-member set	1	1				
2-member set						
3-member set						
4-member set						
5-member set						

You might need to get out a number of different items from which to make sets and subsets. Try to look for patterns which will help you find all of the different subsets for each of these sets. Can you extend this table to include 6-, 7-, and 8-member sets without actually working out the different number of subsets?

Activity 5

What Have We Been Doing?

So far we have done four activities, Train Game, Random Walks, Coin Tosses, and Subset Mania. There is one important pattern that you could have found in all four of the activities. Did you see it? It should have helped you hypothesis what was coming next in each problem.

Did you see a triangle of numbers like the one below?

1 1

1 2 1

1 3 3 1

1 4 6 3 1

This is only the first four lines of the triangle. Can you complete the next four lines? Can you find this triangle in all the four problems? What are the totals of each of the rows in the triangle? This triangle is called Pascal's Triangle and was developed by a French philosopher, mathematician, and scientist, Blaise Pascal. Pascal spoke of the triangle in a paper written in 1654 entitled "The Arithmetic Triangle."

Pascal is best known for his triangle which is used mostly for the study of probability and chance. Probability is the study of measuring the likelihood that something will happen. Even though Pascal is known for his triangle, he accomplished other important things. Investigate at least one of the following topics concerning Pascal:

1) Find out more about Pascal. Write a report or create a bulletin board to show what you learned.
2) Pascal invented a type of computer called a "Calculating Machine." Report on this invention and make a model of the machine.
3) Report on Pascal's Law and its practical applications today.
4) Explore how Pascal is remembered today. What books have been written about him? Is he on any stamps issued in the United States or in other countries? What else can you find which commemorates this gifted man?
5) Find out when Pascal's birthday is and throw him a birthday party. It might not be possible to have a birthday party during the school year on his real birthday. If you can't, choose a day and party!

Activity 6

Probability With Dice

We have already explored one application using probability and Pascal's Triangle, namely Coin Tosses. You should be able to reproduce Pascal's Triangle and tell what the probability of results is when any number of coins are tossed.

We can also tell what the probability of throwing different numbers on a pair of dice. If there are 11 different numbers which can be formed by throwing two dice (2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12) and you can read the probability from a line in Pascal's Triangle, then which line should you use?

When we read probability, we say how many chances there are of something happening out of the total number of opportunities. In other words, the probability of flipping a head when one coin is flipped is 1 out of 2, since a head can only come up one way and there are 2 different things that can happen (a head and a tail). What is the total of the row you are using to determine the probability of rolling two dice? Answer the following questions using this information:

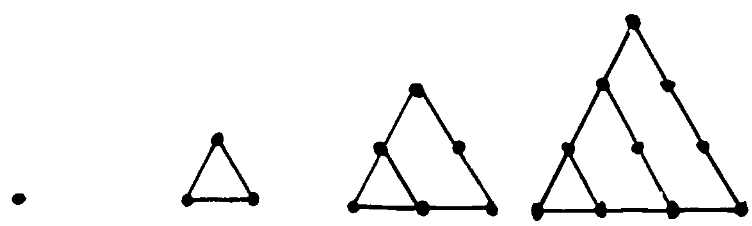
1) What is the probability of rolling a 4? a 2? a 10?
2) What number should come up the most when you roll two dice? Get two dice and roll them 10 times keeping track of the sum of the two dice. What happened? Keep rolling the dice another 50 times and see how the results change. What would happen if you rolled the dice 1024 times? Do you think that your results would be close to the probability predicted by Pascal's Triangle? Try it on the computer using this program. (Apple version)

```
10 HOME
20 INPUT "HOW MANY TIMES DO YOU WANT TO ROLL THE DICE";X
30 LET A = INT (6 * RND (1)) + 1
40 LET B = INT (6 * RND (1)) + 1
50 IF A + B = 2 THEN LET C = C + 1
60 IF A + B = 3 THEN LET D = D + 1
70 IF A + B = 4 THEN LET E = E + 1
80 IF A + B = 5 THEN LET F = F + 1
90 IF A + B = 6 THEN LET G = G + 1
100 IF A + B = 7 THEN LET H = H + 1
110 IF A + B = 8 THEN LET I = I + 1
```

Activity 7

Triangular Numbers

There are numbers which mathematicians call "Triangular Numbers." They are found by making triangles with dots as shown below:



1   3   6   10

Therefore, the numbers 1, 3, 6, 10,... are called triangular numbers. Draw pictures and find the next three triangular numbers. What are they?

1) Where are the triangular numbers in Pascal's Triangle?
2) What are the first 10 triangular numbers?
3) What is the pattern used to find the triangular numbers?
4) What is the 15th triangular number? the 25th?
5) How many dots are on the side of the 15th triangular number? on the 25th triangular number?

Activity 8

Additional Investigations

There are additional projects you can do using Pascal's Triangle.

Choose at least one of the following to investigate:

1) How could the program in Activity 6 be changed in order to roll more than two dice? Try it and then find the line in Pascal's Triangle which goes with the new number of dice. List the probability for each number occurring and then test it using the program.

2) Pascal's Triangle is used to find the coefficients of expanded binomials in algebra. I know what is written here makes little sense, but find an algebra book and see if you can figure out how the triangle is used.

3) Below is a BASIC program to keep track of a single coin flip.

```
 10 HOME
 20 INPUT "HOW MANY TIMES DO YOU WANT TO FLIP THE COIN";F
 30 FOR X = 1 TO F
 40 C = INT (RND (1) * 2) + 1
 50 IF C = 1 THEN H = H + 1
 60 IF C = 2 THEN T = T + 1
 70 NEXT X
 80 PRINT
 90 PRINT "HEADS "H
100 PRINT
110 PRINT "TAILS "T
120 END
```

Test this program out using just one coin. Does it come close to matching the Pascal's Triangle's prediction of 1 head and 1 tail for every 2 flips of the coin? Try changing the program so it will flip more than one coin at the same time. Try to see if the predicted

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Test this program out using just one coin. Does it come close to matching the Pascal's Triangle's prediction of 1 head and 1 tail for every 2 flips of the coin? Try changing the program so it will flip more than one coin at the same time. Try to see if the predicted probability from Pascal's Triangle matches what you find when you run the program.

4) Most scientific calculators have a key which will give you the number of combinations in a particular set. The key looks like this: [nCr] . When you use this key as follows: [6] [nCr] [4] , you are asking the calculator, how many subsets containing 4 members can be formed from a set containing 6 members? Can you construct any row of Pascal's Triangle using this key? Try the 25th row. the 30th row.

5) Using a checkerboard, choose two squares along opposite edges of the board. The two squares must be the same color. Now, moving only one checker on the board from the first square to the second on the opposite side, how many different paths could you use? Do the edges of the board present any particular problems when you are solving this problem? Does it matter which two squares are chosen? If so, why? and if not, why not? What happens if you choose squares in the middle of the board?

Reference List

Buckeye, D.A. & Ginther, J.L. (1971). Creative mathematics. San Francisco: Canfield Press.

Denholm, R.A. (1970). Mathematics: Man's key to progress. Chicago: Franklin.

Haskell, R. (1982). Apple basic. Englewood Cliffs, NJ: Prentice-Hall.

National Council of Teachers of Mathematics (1987). Teaching with... student math notes. Reston, VA: NCTM.

Schultz, J.E. (1982). Mathematics for elementary school teachers. Columbus, OH: Merrill.

Sobel, M.A. & Maletsky, E.M. (1988). Teaching mathematics: A sourcebook of aids, activities, and strategies. Englewood Cliffs, NJ: Prentice-Hall.

TEACHER NOTES

The ability to see patterns and problem solve are important skills for the future study of mathematics. This unit on applications of Pascal's Triangle encourages the seeking of patterns and general problem solving skills. The historic 1 study of mathematics is also evident in this unit. The children should not be told that this is a study of Pascal's Triangle at the beginning of the unit since the initial emphasis of the unit is to encourage pattern seeking and problem solving skills. Some of the activities listed in the unit could also take more than one day to complete.

Activity 1

Train Game

Answer to Activity Sheet 1

	1	2	3	4	5	6	7	8	9	10	Total
w	1										1
r	1	1									2
g	1	2	1								4
c	1	3	3	1							8
y	1	4	6	4	1						16
d	1	5	10	10	5	1					32
k	1	6	15	20	15	6	1				64
n	1	7	21	35	35	21	7	1			128
u	1	8	28	56	70	56	28	8	1		256
o	1	9	36	84	126	126	84	36	9	1	512

Answers to questions:

1) Students should be able to predict the 1, 2, and possibly the 3 columns (and the corresponding diagonals).

2) Students should see, after doing several rows, that the totals are doubling each time.

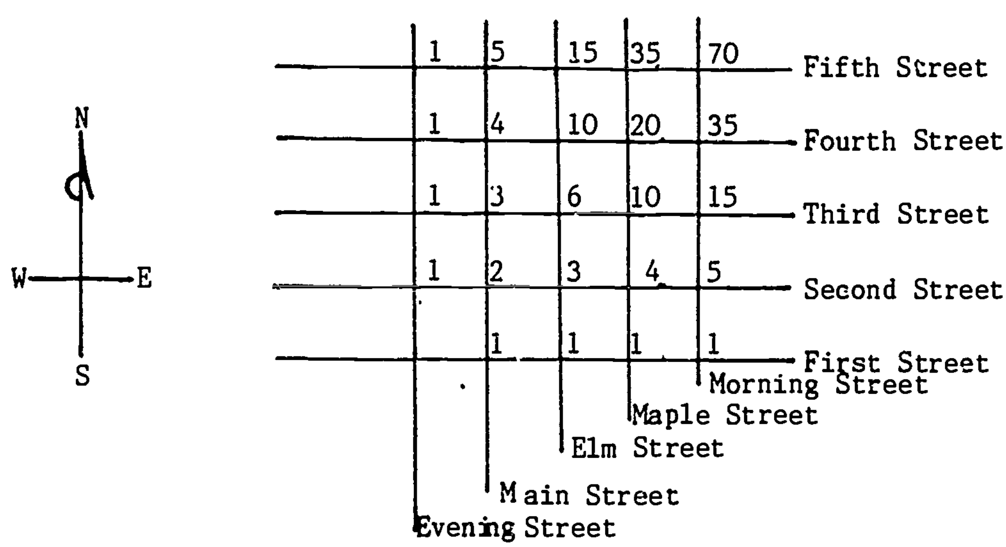
3) Rods which are shorter than the number of cars in the train will show a 0 in that box.

4) Among others, the students should see the pattern of Pascal's Triangle; the number above and to the left add to produce the number below.

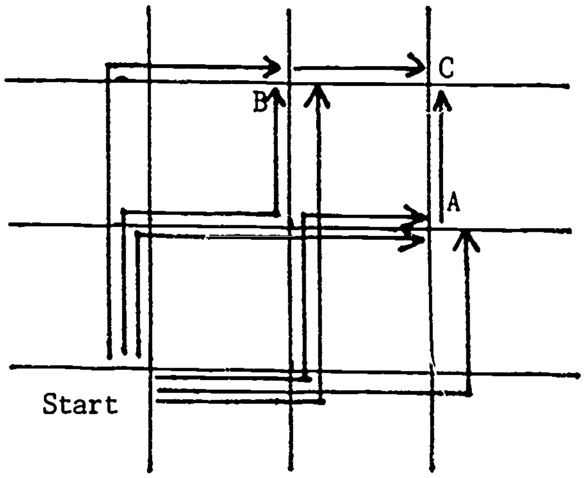
Activity 2

Random Walks

Answer to the random walk problem:



Students should also look for the following pattern:



If there are three ways to walk to intersection A and three ways to walk to intersection B, then there must be 6 ways to walk to intersection C. This is because there is one way to get from A to C so you can attach that one way to each of the ways to get to C. Likewise, there is one way to get from B to C so you can attach that one way to each of these ways to get to C. Therefore, you can add the two intersections together to find how to get to the next intersection. This is the pattern of Pascal's Triangle.

Activity 3

Coin Tosses

Flipping three coins: 1 way to get 3 Heads; 3 ways to get 2 Heads and 1 Tail; 3 ways to get 1 Head and 2 Tails; and 1 way to get 3 Heads.

Flipping four coins: 1 way to get 4 Heads; 4 ways to get 3 Heads and 1 Tail; 6 ways to get 2 Heads and 2 Tails; 4 ways to get 1 Head and 3 Tails; and 1 way to get 4 Tails.

Flipping five coins: 1 way to get 5 Heads; 5 ways to get 4 Heads and 1

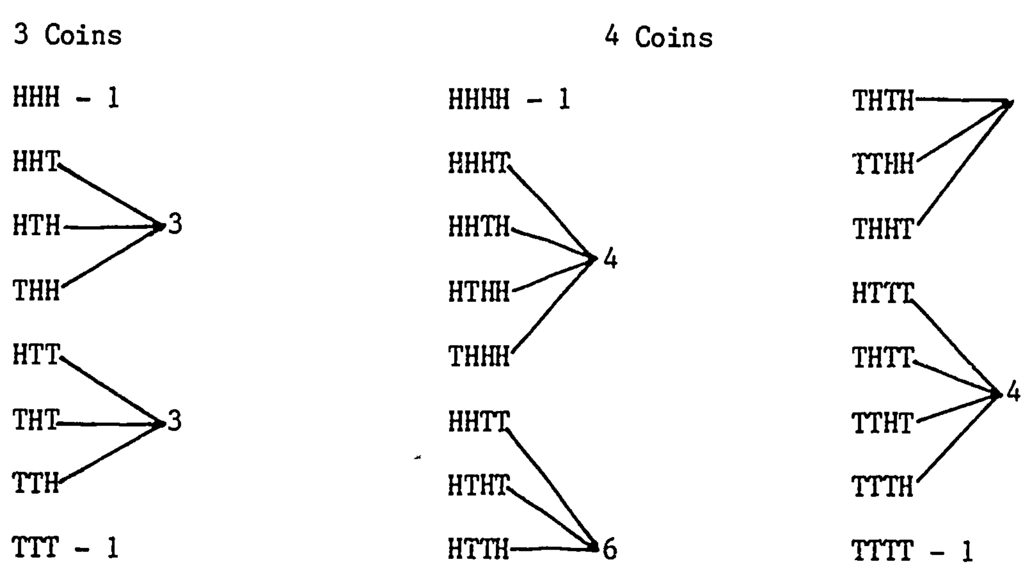
Tail; 10 ways to get 3 Heads and 2 Tails; 10 ways to get 2 Heads and 3 Tails; 5 ways to get 1 Head and 4 Tails; and 1 way to get 5 Tails.

Flipping six coins: 1 way to get 6 Heads; 6 ways to get 5 Heads and 1 Tail; 15 ways to get 4 Heads and 2 Tails; 20 ways to get 3 Heads and 3 Tails; 15 ways to get 2 Heads and 4 Tails; 6 ways to get 1 Head and 5 Tails; and 1 way to get 6 Tails.

Flipping seven coins: 1 way to get 7 Heads; 7 ways to get 6 Heads and 1 Tail; 21 ways to get 5 Heads and 2 Tails; 35 ways to get 4 Heads and 3 Tails; 35 ways to get 3 Heads and 4 Tails; 21 ways to get 2 Heads and 5 Tails; 7 ways to get 1 Head and 6 Tails; and 1 way to get 7 Tails.

Flipping eight coins: 1 way to get 8 Heads; 8 ways to get 7 Heads and 1 tail; 28 ways to get 6 Heads and 2 Tails; 56 ways to get 5 Heads and 3 Tails; 70 ways to get 4 Heads and 4 Tails; 56 ways to get 3 Heads and 5 Tails; 28 ways to get 2 Heads and 6 Tails; 8 ways to get 1 Head and 7 Tails; and 1 way to get 8 Tails.

Two examples would be:



Answers to questions:

1) Students should, by now, be looking for the addition of numbers in the coins before.
2) There is only one way.
3) There are 36 ways to get 7 Heads and 2 Tails
4) Each time a coin is added the total number of possibilities doubles.

Activity 4

Subset Mania

The subsets of different number sets forms Pascal's Triangle.

Activity 5

What Have We Been Doing?

To find the numbers in Pascal's Triangle, add the two numbers above to get the number below.

Encourage students to explore the history of mathematics through the study of Pascal. Creativity should be encouraged

Activity 6

Probability With Dice

Students should use the tenth line for two dice since this is the one which has eleven members.

1 10 45 120 210 252 210 120 45 10 1

Answers to questions:

1) The probability of rolling a 4 is 45 out of 1024

rolling a 2 is 10 out of 1024

rolling a 10 is 45 out of 1024

2) The number 7 should come up the most when rolling 2 dice.

Activity 7

Triangular Numbers

The first 7 triangular numbers are 1, 3, 6, 10, 15, 21, and 28.

Answers to questions:

```
        1   1
      1   2   1
    1   3   3   1
  1   4   6   4   1
 1   5  10  10   5   1
1  6  15  20  15  6  1
```

1) The triangular numbers are shown above.

2) 1, 3, 6, 10, 15, 21, 28, 36, 45, and 55

3) start at 0 and add the next successive whole number.

4) 120, 325

5) 15, 25

Activity 8

Additional Investigations

Five additional investigations are provide to encourage students to explore the applications of Pascal's Triangle in more depth, search for patterns, and solve problems in a creative and meaningful manner. Activity Number 2 is beyond the understanding of all but a small handful of sixth grade students and therefore, should not be used with all students. A BASIC programming guide is needed in order to do Numbers 1 and 3. (There are many good guides including the one in the reference list.) Students who have programmed before will be the most successful with this activity, although others can do equally well. To complete Number 4 the students will need access to a scientific calculator with a combination key.