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ABSTRACT

This publication is a compilation of abstracts and critical comments for 12 published investigations in mathematics education. Information for each study includes: purpose; rationale; research design and procedures; findings; and interpretations. For each study also, abstractor's comments provide a brief critique. The 12 studies are: (1) "Mathematical Problem-Solving Performance of Eighth-Grade Programmers and Nonprogrammers"; (2) "Three Decade Comparison of Elementary Teachers' Mathematics Courses and Understandings"; (3) "Students' Miscomprehension of Relational Statements in Arithmetic Word Problems"; (4) "Sex Differences in Learning Mathematics: Longitudinal Study with Item and Error Analysis"; (5) "Effects of CAI with Fixed and Adaptive Feedback on Children's Mathematics Anxiety and Achievement"; (6) "Structuring and Adjusting Content for Students: A Study of Live and Simulated Tutoring of Addition"; (7) "Usefulness of Tables for Solving Word Problems"; (8) "Early Developments in Children's Use of Counting to Solve Quantitative Problems"; (9) "Mathematics Classrooms in Japan, Taiwan, and the United States"; (10) "Calculators and Instruction in Problem Solving in Grade 7"; (11) "Integers as Transformations"; and (12) "Learning Mathematics from Examples and by Doing". Also provided are lists of mathematics education research studies reported in journals as indexed by "Current Index to Journals in Education" and "Resources in Education" for July through September of 1987. (RT)

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INVESTIGATIONS IN MATHEMATICS EDUCATION

Winter 1988

Blume, G. W. and Schoen, H. L. MATHEMATICAL PROBLEM-SOLVING PERFORMANCE OF EIGHTH-GRADE PROGRAMMERS AND NONPROGRAMMERS. Journal for Research in Mathematics Education 19: 142-156; March 1988.

Abstract and comments prepared for I.M.E. by MARY KIM PRICHARD, University of North Carolina at Charlotte.

1. Purpose

The purpose of this study was to investigate the problem-solving processes used by eighth-grade students with and without computer-programming experience. The researchers hypothesized that the students with programming experience would (a) use more systematic approaches solving problems, (b) use more planning processes, (c) make more use of variables and equations, and (d) find and correct errors more effectively. They also hypothesized that students who previously had shown higher achievement in problem solving would exhibit greater effects from the programming experience.

2. Rationale

Research indicates that processes and activities used while designing and refining a computer program are similar to those used during a problem-solving episode. This assumption provides a framework for the researchers to study the effects of computer-programming experience on problem-solving performance. Those processes (mentioned above) that are conspicuous in the process of developing a computer program are hypothesized to transfer to a nonprogramming mathematical problem-solving experience.

3. Research Design and Procedures

The researchers invited eighth-grade students in three fall semester and three spring semester BASIC programming classes to participate in the study. The students were grouped into four levels based on their performance on the Iowa Test of Basic Skills

Problem-Solving Test. The students were randomly chosen from the four levels so that from each level the same number of students participated from the fall semester classes as from the spring semester clases. A total of 54 students was selected.

Three instruments were used to collect data on students' problem-solving performance: (a) five interview problems, (b) a multiple-choice word problem test, and (c) a logic test (10 multiple-choice questions and eight open-ended items).

During the last week of the fall semester, the students in the fall semester programming classes were tested. The students who were taking the programming classes in the spring were tested during the first week of the semester before instruction in programming began.

The students' problem-solving processes and outcomes exhibited during the interviews were coded. The primary solution method each student used to solve each problem was also identified. The classroom teacher administered two paper-and-pencil tests to each class as a group. (The same teacher taught each class.)

Seventeen of the most frequently occurring processes and outcomes were dependent variables in a MANOVA test. Univariate ANOVAs were also run on each of these 17 variables. Two analysis of variance tests were used with the scores from the paper-and-pencil tests. The test scores were the dependent variables and the Group (Programming or Nonprogramming) and Level (one through four) were the independent variables.

4. Findings

The main methods used by the students to solve the interview problems, as well as the effectiveness of their methods, differed by problem. Across all the problems, the students with programming experience (Group P) used systematic trial more often than the students with no programming experience (Group NP). Among those students wh used equations, Group P did so more effectively than did Group NP.

Along with the main methods, the planning processes, looking-back processes, and solution outcomes were analyzed for the interview problems. No statistical differences were found between the groups in their use of planning processes and in their solution outcomes. The univariate tests showed significant differences at the .05 level between the groups for use of systematic trial, checking, and correcting errors. Group P used these processes more often than Group NP across the levels and problems. One Group x Level interaction for frequency of impasses was significant at the .05 level. The groups did not differ in frequency of impasses at Levels 1 and 2; at Level 3, the Group P frequency was greater than that of Group NP; and at Level 4, the Group NP frequency was greater than that of Group P.

5. Interpretations

The results from this study support the hypothesis that the systematic approaches and the frequent checking used for computer programmers may transfer to the activity of solving non-routine mathematical problems. However, the hypothesis that the use of planning processes might also transfer was not supported. This might be explained by the inability of eighth-grade students to plan well for computer programming and problem solving.

There were several processes used more often by Group P than Group NP; however, Group P did not produce more correct answers. According to the authors, this might be explained by the nature of certain processes, such as systematic trial, which introduces more steps where computational errors may occur.

Although not all of the researchers' hypotheses were supported, "there seems to be sufficient evidence to warrant continued investigation of the connection between computer-programming experience and mathematical problem solving" (p. 155).

Abstractor's Comments

This is a very interesting study that I found both timely and significant. I will address three points specifically: (a) the relevance of the work with respect to previous research on programming and problem solving, (b) the procedure and statistical methods used, and (c) directions for further research.

Previous Research

Blume and Schoen in just a few pages have provided an excellent bibliography for research in this area. They cite reviews of research on the effects of computer-programming experience, as well as research studies that deal with BASIC programming and Logo programming. Their brief but concise literature review provides a framework and rationale that demonstrates that this study addresses questions that are relevant to our concerns in mathematics education.

Procedure and Statistical Methods

The statistical methods used by the researchers to analyze the data were very appropriate. I applaud their ability to organize the interview data from 54 students, each of whom solved five problems, into a manageable, readable report. The combined use of qualitative and quantitative methods is very effective. The descriptions of student methods are interesting, and the tables provided are very helpful.

Further Research

I agree with the authors that this study provides evidence that further research is needed in the relationship between computer programming and problem solving. I want to comment on two areas discussed by the authors.

The researchers' findings supported their hypothesis that programmers use more "systematic approaches" than nonprogrammers. During the coding and analyzing process, they accepted trial and error processes as evidence of systematic approaches in the solution stage. What is it in the programming experience that "transferred" to the problem-solving experience? Was it the systematic nature of coding the steps of an algorithm? Or was it the trial-and-error approach many students take when debugging a program? Run the program, fix it; run it, fix it again; and continue this process until the desired outcome or an impasse is reached.

Questions are also raised about the planning processes used by programmers and nonprogrammers. The authors make the statement: "Perhaps eighth-grade students, even bright ones, are not yet able to plan properly for either computer programming or problem solving" (p. 154). What does "plan properly" mean? I have observed eighth-grade students successfully write programs and solve problems. Some type of planning process was used. Is it possible that the nature of the planning processes are different enough for the two activities that transfer does not take place? Is there evidence of transfer with older students?

This well-planned and thoughtfully carried-out study leads to many questions and avenues for further study.

Ginther, John L.; Pigge, Fred L.; and Gibney, Thomas C. THREE DECADE COMPARISON OF ELEMENTARY TEACHERS' MATHEMATICS COURSES AND UNDERSTANDINGS. School Science and Mathematics 87: 587-597; November 1987.

Abstract and comments prepared for I.M.E. by OTTO BASSLER, Vanderbilt University.

1. Purpose

The purpose of this study is to assess the mathematical understandings of elementary teachers who had varying numbers of high school and college mathematics courses. Also, the achievement of teachers in this 1983-85 study were compared with teachers from two previous studies, in 1967-69 and 1975-77.

2. Rationale

The authors argue that due to increasing public demand for quality education, it should follow that elementary teachers today need to be better prepared and score higher on a test of mathematical understandings than teachers of previous decades. The results of previous studies, one from the 1960s and one from the 1970s were:

a. more years of high school mathematics implied greater understandings
b. more courses of college mathematics implied greater understandings
c. a greater proportion of the 1975-77 group completed four or more years of high school mathematics than in the 1967-69 group
d. a greater proportion of the 1975-77 group completed three or more college mathematics courses than in the 1967-69 group
e. the 1975-77 group scored higher on the mathematics understandings test than did the 1967-69 group.

Trends from these previous studies should continue if teachers today are better prepared than those of previous decades.

3. Research Design and Procedure

Data were collected from 755 elementary teachers who were either undergraduate or graduate elementary education majors at Bowling Green State University, The University of Toledo, or Eastern Michigan University. Approximately 20% of the sample were inservice teachers and 80% were preservice teachers.

The same test as used in the 1967-69 and 1975-77 studies was used in the 1983-85 study. This test measured understandings on "basic principles of sets, numeration systems, fundamental operations on the whole numbers, number theory, fractional numbers, structural properties of the whole number system, and geometry." Reliability, assessed using the Kuder-Richardson Formula 20, was .91 and each item of the test was judged by the authors to have content validity prior to the 1967-69 study.

4. Findings

The results from all three studies were reported. Within each study there were significant mean differences in the mathematics understandings of elementary teachers who had none, one year, two years, three years, and four or more years of high school mathematics. Almost all post hoc pairwise comparisons were significant and these indicated a linear improvement in understandings as the number of years of high school mathematics increased. A similar finding was reported for the number of college courses; that is, students with more college mathematics courses did progressively better on the test of mathematical understandings.

When comparing results from the 1967-69, 1975-77, and 1983-85 studies for number of courses in high school, it was found that at all levels except "no mathematics courses" the 1975-77 group outperformed the 1967-69 and 1983-85 groups, and there were no significant differences between the 1967-69 and 1983-85 groups. At the college level there were no differences between the three groups for no courses, the 1975-77 group scored higher than the 1967-69 group and for two or more than three courses, and the 1967-69 and 1975-77 group scores were significantly higher than the 1983-85 group for more than one co rse.

Although not tested for significance, it was noted that numbers of students with three or more years of high school mathematics increased from 55 percent in 1967-69 to 60 percent in 1975-77 to 66 percent in 1983-85. Similar results were found for students taking three or more college courses where the data were 13 percent in the 1967-69 study, 42 percent in the 1975-77 study and 58 percent in the 1983-85 study.

5. Interpretations

The results clearly show that as the number of years of high school mathematics and the number of courses in college mathematics increases, the mathematical understandings of elementary teachers increase.

The findings also show that the mathematical understandings of elementary teachers increased from 1967-69 to 1975-77 and then decreased from 1975-77 to 1983-85. This decrease cannot be attributed to students taking less years of mathematics at the high school level or at the college level since a greater percent of students enrolled in more courses in each succeeding decade.

Abstractor's Comments

Studies that investigate problems in mathematics education over time are important and provide valuable information about trends and progress or lack thereof. In this case the results indicated an increase in the number of mathematics courses in each succeeding decade but a decrease of mathematics understandings for elementary teachers. It is disturbing that understandings decreased and this certainly warrants further study. The authors recognize this and pose several interesting and important questions which relate to applicability of the content, the capabilities of elementary teachers, and the regional nature of the sample.

Some additional limitations of the study deal with the test. It is possible that the items were valid in the 1967-69 study but are not valid for the 1983-85 study. One example might be number bases, which are not currently emphasized to the same degree that they were in the 1960s.

Lewis, Anne Bovenmyer and Mayer, Richard E. STUDENTS' MISCOMPREHENSION OF RELATIONAL STATEMENTS IN ARITHMETIC WORD PROBLEMS. Journal of Educational Psychology 79: 363-371; December 1987.

Abstract and comments prepared for I.M.E. by FRANCES R. CURCIO, Queens College of the City University of New York, Flushing.

1. Purpose

Two experiments were designed to test a consistency hypothesis-- "comprehension errors will be more likely to occur when the structure of the presented information does not correspond to the problem solver's preferred format" (pp. 363-364).

The purpose of "Experiment 1 was to identify factors contributing to students' miscomprehension of relational statements in word problems" (p. 365).

The purpose of Experiment 2 was "to determine students' preferences for unmarked versus marked relational terms" (p. 368).

2. Rationale

Two studies, one conducted by Huttenlocher and Strauss (1968) which focused on young children's interpretation of relational statements, and the other conducted by Clark (1969) which examined the principle of lexical marking, were used as a framework for this study. Based upon the former study, an example of a relational statement which "define(s) one variable in terms of another" (p. 364) is "'Gas at Chevron is 5 cents more per gallon than gas at ARCO'" (p. 363).

Based upon the principle of lexical marking, such "positive" terms as "more than," "taller than," and "higher than" are unmarked and are believed to be stored in memory in a less complex form than terms which are their marked opposites -- "less than," "smaller than," and "shorter than."

Other research studies which examined problem comprehension and compare problems were cited, also. Problem comprehension "includes (a) translation of each sentence of the problem into an internal representation and integration of the information to form a coherent structure" (p. 363). Compare problems are "problems concerning a static numerical relation between two variables" (p. 363). The two types of compare problems examined in this study are (1) consistent language problems in which "the unknown variable ... is the subject of the second sentence, and the relational term in the second sentence (e.g., more than) is consistent with the necesary arithmetic operation (e.g., addition)" and (2) inconsistent language problems in which "the unknown variable is the object of the second sentence, and the relational term (e.g., more than) conflicts with the necessary operation (e.g., subtraction)" (p. 363).

The results of the studies cited indicate "that children make more errors on inconsistent language problems than on consistent language problems" (p. 363), and "word problems that contain relational statements are more difficult for children to solve than word problems that do not contain relational statements" (p. 364). Some studies indicate that adults have similar difficulties.

3. Research Design and Procedures

Experiment 1. The subjects for this study were "96 college students, ranging in age from 18 to 21, who were recruited from the psychology subject pool" at a university (p. 365). The subjects, who were tested in groups ranging from 2 to 20, were given an instruction sheet and test book which contained 24 arithmetic word problems, one on each page. Of the 24 problems, 16 were filler problems and 8 were target problems.

> Each target problem involved two steps: a compare problem as the first step and a direct variation problem as the second step. The compare portions of the eight target problems corresponded, respectively, to eight problem

types: consistent addition (e.g., requires adding two numbers and the problem says more), inconsistent addition (e.g., requires adding two numbers and the problem says less), consistent subtraction (e.g., requires subtracting the second number from the first and the problem says less), inconsistent subtraction (e.g., requires subtracting the second number from the first (and) the problem says more), consistent multiplication (e.g., requires multiplication of two numbers and the problem says n times as many), inconsistent multiplication (e.g., requires multiplication of two numbers and the problem says 1/n as many), consistent division (e.g., requires division of two numbers and the problem says 1/n as many), inconsistent division (e.g., requires division of two numbers and the problem says n times as many). (pp. 365-366)

Thirty-two different test booklets resulted from using Latin squares to generate eight different orderings of these problem types with respect to four cover stories (i.e., problem topics).

The subjects solved the 24 problems within the allotted time of 35 minutes. The written solutions for the target problems were either correct ("if the proper numerical answer was given") or incorrect (categorized as reversal errors, arithmetic errors, or goal monitoring errors) (p. 366). An error was identified as a reversal error if the inverse operation was used incorrectly (e.g., addition for subtraction). Computational errors were identified as arithmetic errors. If the second step of the problem was omitted, it was identified as a goal monitoring error.

Experiment 2. The 26 college students "recruited from the same psychology subject pool as in Experiment 1" (p. 368), were randomly assigned to one of two booklet treatment groups (each containing 13 of the subjects). The two booklet treatments differed in the value of the quantity presented in the second statement of each problem situation in relation to the value of the quantity in the first statement (i.e., in one case the quantity in the second sentence was larger than that of the first sentence; in the other case, vice versa).

The test booklets contained 8 sheets of paper, each of which contained two sentences that assigned a value to a variable. "These sentence sets corresponded to...the compare step, in each of the eight cover stories for the target problems in Experiment 1" (p. 368).

The subjects, tested in groups ranging from 5 to 8, were instructed to express in writing the numerical relationship between the quantities presented in the eight different problems. All subjects completed the problems within 8 minutes.

"The data were the eight sentences produced by each of the 26 subjects. Each sentence was scored as belonging to one of the following categories: (a) unmarked addition/subtraction..(b) marked addition/subtraction..(c) unmarked multiplication/division..(d) marked multiplication/division..and (e) miscellaneous" (p. 369).

4. <u>Findings</u>

<u>Experiment 1</u>. To analyze error frequencies as a means to identify factors contributing to miscomprehension, a one-way ANOVA was employed. The two major results were:

(1) "Subjects are more likely to miscomprehend a problem and therefore commit a reversal error when the problem is presented in an inconsistent language form" (p. 367).

(2) "The tendency to miscomprehend an inconsistent language problem is even greater when the necessary operation is addition or multiplication rather than subtraction or division. This interaction may be due to the presence of marked terms in inconsistent language addition (e.g., <u>less than</u>) and multiplication (e.g., <u>1/n as many</u>" (p. 367).

Experiment 2. Since there was no significant difference between the results of the two booklet treatment groups, the proportion of subjects who produced each type of relational statement for each set was collapsed over booklet treatments. The "subjects had a strong preference for unmarked addition/subtraction terms such as more, older, or taller rather than for marked addition/subtraction terms such as less, younger, or shorter..there was a strong tendency to prefer unmarked terms for multiplication/division, such as n times as many or unmarked terms for addition/subtraction, such as more rather than marked terms" (p. 369).

Based upon the results of z tests, it was noted that "the proportion of subjects generating unmarked terms was significantly greater than the proportion of subjects generating marked terms" (p. 369).

5. Interpretations

"The results of Experiment 1 highlight the important role of students' comprehension schemata and processes for arithmetic word problem solving" (p. 367). The authors provide "a model of schemata and translation procedures for representing compare problems" (p. 368).

The results support the findings of Huttenlocher and Strauss (1968) in that "the probability of a reversal error increases when the subject of the second sentence is the same as the subject of the first sentence" (p. 368). Supporting the work of Clark (1969), these results were found to be more prominent "when the relational term is linguistically marked" (p. 368).

The results suggest that the comprehension phase of problem solving is more difficult than the solution phase. Rather than focusing on the computational aspects of a problem, the level of problem difficulty might be reflected more accurately in terms of comprehension and representation procedures needed for solving the problem. Instruction should focus on problem representation, with an emphasis on the representation of relational statements (p. 370).

Abstractor's Comments

This research study is a contribution to the problem-solving literature. The authors have analyzed students' answers to typical, routine problem-solving tasks in relation to linguistic components. In particular, the language instructors use and the way instructors use language to communicate in mathematics is a current issue of concern (Commission on Standards of the NCTM, 1987). Cross-disciplinary studies (e.g., linguistics and mathematics education) provide a different perspective when examining certain constructs. More work in this area is needed.

For instructional purposes if might be advantageous to present problems to students using their "preferred" format (i.e., unmarked and consistent language), but this alone will not facilitate transfer to problems presented in "unpreferred" formats (i.e., marked and inconsistent language). It seems more practical to guide children in analyzing problems based upon not only the literal aspects of the problem, focusing on the topic and the explicit information given, but also analyzing the underlying mathematical relationships implicit in the problem situation.

Although not an explicit intent of this study, the results support instruction that does not emphasize the "clue" or "key" word approach to problem solving. Often, with all good intentions, elementary school children are told to look for clue words as a means to determine which arithmetic operation to employ. Children become so concerned about finding the "clue" word to determine the operation that they do not "read" the problem to focus on the situation which defines the relationship between (or among) the quantities presented.

This "clue" word approach is contrary to the basic idea behind problem solving. Consequently, the "clue" word approach leads to erroneous solutions (Krulik and Rudnick, 1986).

Some clarification is needed in the description of the method for the two experiments. In Experiment 1, it is not clear why so many filler problems were required. Two-thirds of the 24 problems were filler problems. In Experiment 2, it is not clear whether any of the 26 students who "were recruited from the same psychology subject pool" (p. 368) participated in both experiments. Any experience from Experiment 1 could have influenced the results in Experiment 2.

Although the results suggest that preference for unmarked terms in routine word problems facilitates comprehension, some of the reading literature suggests that expectation may also influence comprehension (Royer and Cunningham, 1978; Stein, 1978). Further study relating expectations as well as preferences to problem comprehension should be considered.

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Abstract and comments prepared for I.M.E. by IAN D. BEATTIE, University of British Columbia, Vancouver.

1. Purpose

The stated purposes of this study were: (1) to compare children's performance in mathematics in third grade with their performance in sixth grade, (2) to identify and compare the strengths and weaknesses of boys and girls at each of these grades, and (3) to identify and compare the errors made by boys and girls.

2. Rationale

Studies to date have not addressed the issues of whether there are distinguishable and constant differences in mathematics performance which can be attributed to sex, or whether these differences, if any, change as children mature. Answers to these questions require a longitudinal approach. Two premises underlying this study are: (1) group comparisons should be based on comparison of individual items since comparison on the basis of total test scores can mask important differences and (2) analysis of incorrect responses can provide important information about how groups differ.

3. Research Design and Procedures

The subjects in the study were the same group of children enrolled in third grade in California in 1980 and in sixth grade in 1983. The tests were developed and the data gathered by the California Assessment Program of the California Department of Education. The third-grade test comprised 360 items over 30 separate test forms, while the sixth-grade test comprised 480 items over 40 separate test forms. Items in both tests fell into six categories: computation problems, counting problems, visual problems, geometry and measurement problems, traditional word problems, and non-traditional word problems. Computation and counting items accounted for 56 percent and 40 percent of the total items at third and sixth grade respectively. It appears, though it is not explicitly stated, that all items were multiple choice. Approximately 300,000 children responded to the tests at each grade level. Numbers of boys and girls were similar.

Data obtained were used by the authors to:

1. analyze total scores (percent correct) in each of the above categories by grade and sex.

2. compare the difficulty level of each category by grade and sex.

3. compare performance by sex and grade on individual items using two-tests for two individual proportions with x = .05.

4. compare responses by grade and sex on matched pairs of items (one item tests acquisition of a skill, the other application of the same skill; e.g. an addition computation and an addition word problem), where the set of response alternatives is the same, using regression analysis.

5. analyze the 50 most difficult items for each sex.

6. analyze distribution of incorrect answers over distractors by sex and grade level.

7. identify error types at each grade level (19 at third grade and 15 at sixth grade) and compare performance on error types by grade and sex.

Except as indicated above, the specific statistical test used was not identified.

4. Findings

The analyses above produced the following findings:

1. Boys and girls were very similar in their abilities to solve all categories of problems. Differences were significant at third grade only in computation and non-traditional items, and at sixth grade only in computation. These differences favored girls.

2. There were three distinct difficulty levels for both boys and girls at third-grade level, with visual items being the easiest and word problems the most difficult for both boys and girls. For sixth-grade there were only two levels of difficulty for boys and four levels for girls. For boys, all categories were of similar difficulty, with the exception of word problems, the most difficult. For girls, computations were the easiest and word problems the most difficult.

3. Of the items on the third-grade test, 56 percent showed significant differences by sex. Girls performed better on 41 percent of the items while boys performed better on 1[illegible] percent of the items. Girls demonstrated better performance on every category of item except word problems. Of the items on the sixth-grade test, 57 percent of the items showed significant differences by sex. Boys performed better than girls on 27 percent of the items and girls performed better on 30 percent.

4. At both grades both sexes were significantly better at solving acquisition items than at solving the related application items. Boys and girls did not differ significantly from each other on either type of item. At both grades, for both sexes, there was a significant linear relationship between application of a skill and the correct acquisition of it.

5. A large majority of the items that caused difficulty for one sex caused difficulty for the other. At third-grade 7 items were more difficult for one sex than the other and 10 items at sixth grade were more difficult for one sex. Eleven of the 16 most difficult items for boys but not for girls were computation items. The items more difficult for girls than boys were fairly evenly spread over all categories, with the highest number in a category (word problems) being 4.

6. At third grade 181 items (50 percent) of the items had significant sex differences for errors, but 80 of the items had non-significant differences in percentage correct for boys and girls. At sixth grade 191 items (42 percent) had significant sex differences for errors, and 71 of them had non-significant differences in percentage correct.

7. With regard to categories of error, there were significant differences by sex in two categories. Boys were more likely to use erroneous rules, and girls were more likely to make association errors. With regard to types of error, at third grade 9 of the 19 error types showed significant sex differences, while 7 of 15 error types at sixth grade showed significant differences. Of 13 error types common to both grades, 9 produced the same result at both grade levels.

5. Interpretations

Based on analysis of correct performance, the authors concluded that girls in the third grade performed better than boys in all areas except word problems. There was a shift in grade six that tended to equalize boys' and girls' performance, with girls losing ground in areas of strength and falling seriously behind in word problems and geometry/measurement problems. Over both grades girls were more successful in performing computations and boys more successful in solving word problems. Girls were more likely to solve acquisition items and less likely to solve matching application problems, leading the authors to conclude that application involves important factors other than computational skills. For both boys and girls, the authors concluded that students know how to use many arithmetic skills, but do not know when to apply them.

Based on analysis of errors, the following conclusions were drawn. Boys and girls made different types of errors on a large number of items. Both girls and boys tended to make the same type of error in sixth grade that they made in third grade, with girls more likely to make association errors and boys were likely to use erroneous rules. These erroneous rules errors were systematic applications of incorrect algorithms.

The authors put forward a number of possible explanations for the results observed. There may be developmental differences. In third grade girls may be more likely than boys to have the ability to automatize procedures, to sit still, and to perform tasks requiring small muscle coordination. It may be that the skills required for success in third grade are no longer needed in sixth grade. There may be differneces in the way knowledge is stored and retrived, with girls seeking to construct automatic rules for solving word problems in the same way as for computations. Girls may develop a network of conceptual understanding not related to the automatic rules of computation. The errors that boys make are more apparent to the teacher, and hence boys are more likely to received remedial, differential instruction. The superiority of boys in problem solving may be due to the fact that, provided the correct procedure is selected, faulty computation may still enable them to select the correct answer from the options provided. Lastly, the authors note that patterns of error develop early and persist.

Abstractor's Comments

This is an interesting and provocative study. The item and error analyses provide an opportunity to gather additional information from an existing data pool and the conclusions drawn and methods used will undoubtedly encourage further research in this area. The methodology is sound, the discussion section is excellent and the organization of the report, given the complexity of the analysis, is also very good. However, there are a number of areas in which greater clarity or additional information is needed, as described below.

Minor details: (1) It is not at all clear how many students participated in the study. The researchers mention "the same group of students," yet the number at each grade level differs. The inference one draws is that only data from students who were tested in both third and sixth grade were used in the study, but this does not appear to have been the case. (2) The title and the body of the report discuss differences in learning mathematics, but the study is (as the authors' abstract acknowledges) concerned with differences in performance. (3) The study was not a longitudinal study over three years; it examined performance after an interval of three years. (4) The use of the word "problem" in conjunction with computation and counting is inappropriate. (5) It is not clear until the latter part of the report that all items were multiple choice.

Analysis: The statistical tests used in the study were not in most instances identified, and the levels of significance varied greatly (.01, .05, .10, .05 < p < .10) with no rationale being offered. Considering the number of responses to each item (over 10,000 at third grade and 6,000 at sixth grade) the number of errors analysed is incredibly small (387 at third grade and 293 at sixth grade). When doing analyses of computational errors of individual classes, my experience has been that most of the errors are made by a relatively small number of individuals and one must wonder whether this would hold for this study. An indication of the number of students would be useful in the present study.

Conclusions: The authors present fairly strong conclusions regarding the sex differences in performance observed. With regard to overall performance, the actual differences is small and, given the size of the sample and the level of significance chosen, statistical differences are relatively easy to obtain. Given the small number of errors analyzed, care must be taken in making generalizations. Statistical significance must be put in the context of educational significance. No discussion is offered as to why, with the overwhelming differences in favor of girls on item analysis, the overall percentage differences is small. Could it be that there are huge differences in favor of boys on the items where boys did signficantly better or that there were marginal differences in favor of boys on many of the nonsignificant items?

Several of the conclusions are worthy of further investigations. The explanation that boys may be better in problem solving because the type of errors they tend to make make is more likely for them to choose the correct answer in multiple choice items may well be true, but it depends upon the distractors chosen. The conclusion that students know how to use skills, but not when to apply them, may also be true, but is not justified based on the data presented. Again, the hypothesis that the errors made by boys are more likely to be noticed by teachers and hence that boys receive more differential instruction may also be true, but again has no basis in the data. Still, they raise interesting questions which are worthy of further investigation.

Mevarech, Zemira R. and Ben-Artzi, Sigal. EFFECTS OF CAI WITH FIXED AND ADAPTIVE FEEDBACK ON CHILDREN'S MATHEMATICS ANXIETY AND ACHIEVEMENT. Journal of Experimental Education 56: 42-46; Fall 1987.

Abstract and comments prepared for I.M.E. by JAMES H. VANCE, University of Victoria.

1. Purpose

The purpose of the study was to investigate the effects of computer-assisted instruction with fixed and adaptive feedback on students' mathematics anxiety and achievement.

2. Rationale

There has been little research on the effects of computer-assisted instruction (CAI) for drill and practice in mathematics on children's mathematics anxiety. Since a high success rate tends to reduce anxiety, one should expect a lower level of mathematics anxiety in a CAI setting which matches the difficulty level of the problems to the students' ability than in a non-CAI classroom. The quantity of feedback received from significant adults is also related to mathematics anxiety. High-anxious learners are more sensitive to evaluative feedback than are low-anxious learners. It was therefore hypothesized that positive feedback would enhance the effects of the high success rate, particularly in the case of high-anxious learners. Previous research indicated that corrective feedback adjusted to the levels of errors the student made resulted in greater learning than fixed feedback. This study was designed to examine the differential effects of CAI with fixed and adaptive feedback on anxiety and achievement of high- and low-anxious students.

3. Research Design and Procedure

The subjects were 245 sixth-grade students in three Israeli schools. One school did not have a CAI curriculum and the 71 students served as the control group. The CAI with fixed feedback school (n = 82) used a successfully validated program call TOAM. It covered number concepts, the four basic operations with whole numbers, fractions, decimals, equations, measurement, negative numbers, and word problems. The system continually adjusted the level of the practice problems to the students' ability level. For correct responses on the first, second, and third tries, the messages "very good", "good", and "correct" were given. The message for incorrect responses was "You made a mistake. Please try again." After three failed attempts, the correct answer was given and the student could continue. An 80 percent mastery criterion for movement to the next level was adopted. The numbers of correct first-try responses and incorrect responses were provided at the end of each session. The CAI with adaptive feedback school (n = 92) used the same CAI curriculum except that the feedback was adjusted to the type of problems solved. Problems found to raise anxiety (word problems and fractions) gave the messages "Superb job, David!", "Very fine work!", and "You got it!" for correct answers on the first, second, and third tries. Messages for the other problems were the same as in the fixed feedback treatment. Messages for incorrect responses were "Think again" or "Try again." Summary reports gave only the number of problems answered correctly on each attempt and omitted the number of incorrect answers.

The three schools were comparable in terms of size, sex, SES, GPA, curriculum, and mathematics instructional time. The CAI groups had three periods a week of traditional instruction and two 20-minute computer drill-and-practice sessions. The control school had four periods of traditional instruction each week.

The Mathematics Anxiety Scale used in the study consisted of 37 items selected from three previously developed instruments for assessing mathematics anxiety and some additional items on attitudes relating to learning mathematics with a computer. Students responded on a four-point Likert scale to the 37 statements, first in the fall before implementation of the treatments and again in the spring after almost a year of CAI treatments. The items were subjected to factor analysis using a varimax rotation. Six factors accounting for 58 percent of the variance were identified: Worries about Learning Mathematics, Fear of Failure in Math Class, Physiological Arousal in Math Situations, Perception of Difficulties in Solving Math Problems, Attitudes Toward Learning Mathematics with Computers, and Math Test Anxiety. Cronbach alpha reliability coefficient of the instrument was .91.

The Arithmetic Achievement Test was constructed by the Israeli Ministry of Education. It consisted of 25 items covering the behavioral objectives of the sixth-grade mathematics curriculum and was administered to all students at the end of the school year. The KR-21 reliability coefficient was .76.

The six anxiety factors were analyzed by a two-way (three treatments and two initial anxiety levels) multivariate analysis of covariance, followed by separate analysis of covariance for each factor. High and low anxiety levels were determined by pre-treatment scale scores (above and below the median). A two-way univariate analysis of covariance was used on the achievement test scores. Students' GPA in mathematics was used as a covariate. The Scheffe procedure was used to compare treatment means.

4. Findings

The multivariate analysis of the anxiety data indicated that the interaction was not significant, but significant ($p < .001$) main effects were found for both treatment and anxiety level. The treatment effect resulted mainly from the univariate effects of two anxiety factors: Worries About Learning Mathematics ($p < .05$) and

Attitudes Toward Learning Mathematics with Computers ($p < .03$). Scheffe analyses revealed significant differences between the CAI groups and the control groups (favoring CAI), but no differences between the two CAI groups.

For the achievement data, the interaction and the treatment main effect were not significant while the initial anxiety effect was "marginally significant" ($p < .09$).

5. Interpretations

CAI as compared to traditional instruction produced lower scores on two of six aspects of mathematical anxiety, but did not improve achievement.

Within CAI settings the provisions of adaptive or fixed feedback made little difference in students' anxiety and achievement in mathematics.

Low- and high-anxious learners benefited equally from the two CAI treatments and traditional instruction.

Abstractor's Comments

The report of this interesting year-long study on the effects of CAI is very well written and contains needed information regarding the method, data analysis, and conclusions. It should be noted at the outset that only the drill-and-practice aspect of using computers in learning mathematics was investigated. The particular CAI program used presents a problem and allows the student three chances to give the correct answer before moving to another problem. Furthermore, there is no direct indication in the description provided of the classroom instruction (teacher presentation of new rules, student practice, test) of the emphasis on teaching for understanding, the use of concrete models, or the development of thinking skills and problem-solving strategies. These are important considerations in looking for ways of improving achievement and reducing anxiety in mathematics.

The investigators were surprised that the CAI treatments did not produce higher mathematical achievement than the traditional instruction and suggested two factors that may have contributed to this finding. First, the teachers and students may have had difficulty adapting to the new technology. Second, the test used assessed a limited number of skills compared to the CAI curriculum; the test focused on fractions and decimals while many students spent a fair amount of time practicing problems with whole numbers because of their initial level of achievement. A further possible explanation is that, within the context of this study, the traditional treatment really is at least as effective in helping students learn sixth-grade content. To start with, the students in this group had four periods of instruction with a teacher each week as compared to three for the CAI students. Therefore, teachers of traditional classes had 33 percent more time to provide meaningful instruction and diagnostic and remedial teaching. They could insure that students were practicing problems they had recently been taught how to do and give immediate help to students experiencing difficulty. The CAI program was designed to have students practice problems they already knew how to do, but it did not provide any instruction or feedback other than indicating a right or wrong answer. During the two CAI sessions each week, students were likely often practicing problems not directly related to the content being taught in class at that time. Had the study been designed so that the treatment groups did each day's practice work on a computer while the control group did paper-and-pencil worksheets on the same skills, the value of using CAI for drill and practice could have been assessed more fairly.

The CAI treatments did produce lower scores on two of the six math anxiety factors. The results for Attitudes Toward Learning Mathematics with Computers makes sense, since the control group did not use CAI. The authors attributed the lower scores on Worries About Learning Mathematics to the high success rate insured by CAI. The authors attributed the lower scores on Worries About Learning Mathematics to the high success rate insured by CAI. They suggested that the insignificant treatment effect on factors relating to problem solving and test anxiety may have occurred because of the amount of drill and practice and testing in all treatments. It would be of interest to know how the pre and post anxiety scores compared for the whole group and each treatment. Did they remain the same, increase, or decrease?

One of the main purposes of the study was to determine possible differential effects of fixed and adaptive feedback in CAI. The authors stated, "The insignificant differences between the two CAI treatments were not expected" (p. 45). I think it would have been more amazing if differences had been found. Simply altering slightly the feedback expressions for correct and incorrect answers on two of the (ten or eleven?) strands should not by itself be expected to produce greater achievement or less anxiety.

Another variable in the study was the initial level of anxiety. As would be expected, high-anxious learners (above the median on the pre-treatment scores) showed higher levels of mathematics anxiety than low-anxious learners on the posttest. This was true for each of the six factors for each of the three treatments. To better detect any possible treatment-by-anxiety interaction effect, groups of students representing the two extremes on the instrument (upper and lower quartiles) should have been compared. On the achievement test the interaction was not significant, but there was a marginally signficant anxiety main effect. Examination of the data reveals that within both CAI treatments the low-anxious students performed better than the high-anxious students, while the high-anxious students scored slightly higher under traditional instruction. Any findings of this analysis would be difficult to interpret, since it is generally accepted that there is an optimal level of anxiety associated with good test performance. In order to investigate the relationships among treatments, anxiety level, and achievement, the scores of students at the two extremes and in the middle of the range of initial anxiety scores should be compared (rather than those above and below the median). Furthermore, even if it was determined that high-anxious learners benefited from stronger and more positive reinforcement statements, would it follow that the same program would not be equally beneficial for less anxious students?

Putnam, Ralph T. STRUCTURING AND ADJUSTING CONTENT FOR STUDENTS: A STUDY OF LIVE AND SIMULATED TUTORING OF ADDITION. American Educational Research Journal 24: 13-48; Spring 1987.

Abstract and comments prepared for I.M.E. by SANDRA PRYOR CLARKSON, Hunter College of CUNY.

1. Purpose

Teachers are often urged to use a diagnostic approach to provide information to allow them to individualize a student's learning. But how does the teacher gather such information and how does this information influence instruction? This study was originally designed to determine how experienced teachers gather and use information about individual students during tutoring to individualize their instruction. However, it soon became evident that the selected teachers did not have diagnosis as a clear priority. The study then shifted "from describing specific diagnostic strategies to examing how the teachers structured and sequenced subject-matter content for their students" (p. 13). This report was based on the author's doctoral dissertation.

2. Rationale

In working with individual students, the teacher has an idea of the student and his needs, her own set of goals, subgoals, and activities for lessons, and her own knowledge of the subject matter. The information gathered from the student during a session might influence the teacher to move faster or slower through the material, to review something just covered or to move ahead in the material, or to change the sequence of topics in the tutoring lesson. In a model such as this, the teacher uses cues about an individual's knowledge to drive the lesson. This model, however, did not describe the instructional moves made by the teachers participating in this study. Rather than having the lesson driven by student knowledge and student needs, it was motivated primarily by "the teacher's knowledge of goals for teaching addition skills." The teacher appeared to have a "curriculum script" that varied little from student to student.

3. Research Design and Procedure

Six female public school teachers were chosen for the study--two first-grade and four second-grade teachers, each with more than 10 years of teaching experience. Teachers were selected with varied backgrounds and approaches to teaching; the teachers were paid for their participation. Each teacher tutored a student they had taught in two 20-minute tutoring sessions. They also tutored in six computer simulations. The teachers were given written directions for the tutoring task and a live briefing session lasting about 30 minutes. The teachers were "to teach the student to add two numbers, each having up to three digits, with renaming (carrying)" (p. 18). The teachers were told to bring any written or manipulative materials to the session that they thought appropriate. They were to adjust the instruction to an appropriate level of difficulty for the student. Teachers were videotaped during the sessions and interviewed after viewing their own videotapes. During the simulated tutoring sessions, teachers were to "teach" the computer students the addition algorithm by "testing" with problems generated by the teacher herself and by selecting from a list of instructional procedures. The computer student "learns" from the instruction and indicates this by performing correct additions on the teacher-generated problems. The errors made by computer students and the procedures that "work" to correct these errors were based on research.

4. Findings

Analysis of written protocols of the live tutoring sessions were used to "focus" the inquiry. The three foci were "(a) how the teachers responded to student errors and deficient responses, (b) how teachers elicited student responses that provided student performance cues and structured the content of the tutoring sessions, and (c) the teacher's overall goals for the tutoring sessions." Hypotheses generated based on the live tutoring were confirmed and supported by data from the computer tutoring sessions.

Response to errors: Teachers rarely tried to discover the cause of the student's error prior to correction of it. In only 8 of 107 (7%) errors did a teacher attempt to determine the cause of an error. Following 62/107 errors (58%) the major goal of the teacher seemed to be getting the answer to that particular problem correct--nothing more. In 41 percent of the errors (44/107), the teacher appeared to have an additional strategy of correcting the student's knowledge of addition, not simply correcting the problem itself. Once the particular error was corrected, with satisfactory student performance, the teacher moved on to another problem. Student errors were not used as information for the teacher in planning instruction. Teachers were often not able to identify or remember the error that the student was making.

Eliciting student responses: Teachers typically asked students to work addition problems or asked questions about addition itself. Teachers appeared, for the most part, to be moving through sequences of problems that remained somewhat constant from student to student even though the actual problems themselves might vary. This led to the researcher's idea of a "curriculum script" (p. 35) followed by the teachers. In addition, each teacher seemed to have preference for certain question "types" while working with students.

Teachers' overall goals: The teachers' overall goals were centered, for the most part, on teaching the student to carry out the addition algorithm, with or without deep understanding. Teachers varied on their emphasis on anything other than the mechanical aspects. Diagnosis did appear to be one of their goals.

5. Interpretations

Because of observations during the live and simulated tutoring, the author determined that the teachers were guided in their instruction not so much by diagnosis of student errors as by a "curriculum script" that set out goals and actions designed to be followed, with only minor modifications, in a specific order. After going through such a script, the student was expected to perform the addition algorithm without errors.

Abstractor's Comments

It appears that teachers either will not or cannot assimilate information on the errors a student makes and use that information to structure individualized remedial instruction for that particular student. Possibly, the directions given to the teachers did not suggest strongly enough that they attempt to do this. They may have assumed that the major task was to teach the student to get the right answer, not merely determine where the student was and help him progress in his understanding and competence. In remedial instruction, that is sometimes as far as we get--progress but not mastery in a topic. Since the amount of tutoring time available was minimal, teachers may have felt that teaching the entire sequence of appropriate moves was the most efficient means of instruction. That could have led to their choosing a "script" over a true "diagnosis." In addition, choosing problems that can quickly and unambiguously sort out student errors is not a trivial matter. Teachers may have felt much more comfortable using problems that were pre-sequenced. It would be interesting to know where they obtained such sequences.

The study was interesting, meaningful, and the article was well written, with a number of excellent diagrams and tables.

Reed, Stephen K., and Ettinger, Michael. USEFULNESS OF TABLES FOR SOLVING WORD PROBLEMS. Cognition and Instruction 4(1): 43-59; 1987.

Abstract and comments prepared for I.M.E. by LARRY SOWDER, San Diego State University.

1. Purpose

The purpose of the two studies reported here was to sharpen our understanding of the role of a table in the solution of conventional algebra story problems. The studies were part of an ongoing sequence of studies dealing with algebra story problems.

2. Rationale

Earlier work had shown that students can make little use of a detailed solution of an algebra story problem in solving even quite similar problems. The use of tables could lead to better performance by providing a form for the important representational step between problem presentation and equation composition. The second study attempted to improve transfer by giving explicit instructions to try to use earlier, solved problems.

3. Research Design and Procedures

Experiment 1 involved two sequences of this form: the presentation of a solved algebra story problem, folowed by two parts of similar test problems, another solved problem of the same genre but in a different context (as a measure of transfer), and then a pair of test problems similar to this last one. Table-based solutions of the similar problems required more than just a mimicking of the table entries in the detailed solutions; for example, a time of t+3 rather than t might have been needed. The 35 control group Ss received only the detailed solutions to the sample problems; the 35 experimental group Ss were given these solutions, were also to fill in a given table for the first problem in each pair, and then had available a completed table for the second problem in the pair. Subjects were students in a university college algebra course.

Experiment 2, also carried out with college algebra students (29 experimental and 24 control), followed the same general pattern as the first experiment, but included instructions for both groups "to try to remember" the cited, most pertinent problems in the earlier work.

4. Findings

The results in both experiments indicated that having the students fill in a table had only "limited value" in their construction of correct equations for the problems, whereas giving the students tables already filled in did given an equation-writing performance superior to that of the control group. In Experiment 1 the data did not support the superiority of the tables group on the transfer problems, but in Experiment 2, there was "mixed support" for the superiority of the tables group. The experimental students table entries erred often by inappropriate imitation of entries in tne sample solutions.

5. Interpretations

"...asking students to fill in a table had little impact on their ability to construct equations," whereas giving students completed tables did give results superior to those of the control group. (The authors' ongoing work centers on getting students to fill in tables correctly.) Telling students to use an earlier solution seemed to help their performance in a transfer situation, but "...encouraging students to use analogous solutions may be a necessary, but not a sufficient condition, for obtaining transfer."

Abstractor's Comments

It is pleasing to see researchers carry out a focused sequence of studies, each flowing from its predecessors. Progress can be painfully slow, however, as the current two illustrate. And, even though we no doubt shall continue to expect students to solve the conventional algebra story problems, it is less clear how the old-fashioned make-a-table-then-imitate algorithm, no matter how polished, fits into the broader view of a problem-solving curriculum as envisioned today.

A few remarks seem in order. First, the studies here suggest that writing an equation, given a model solution and a table, may not be a difficulty for college algebra students. It would have been interesting if the analyses reported whether the students in the study did indeed write equations that would have been correct if their table entries had been. Second, the brief "instruction" contained in the example solutions might have encouraged a somewhat rote imitation of the sample solution; would longer instruction highlighting, rather than just stating, general principles like "total number of grams = sum of numbers of grams in parts" be more productive? Finally, the diverse backgrounds of the students in the usual college algebra class in a university makes one wish that some sort of pre-treatment measure had been used.

Sophian, Catherine. EARLY DEVELOPMENTS IN CHILDREN'S USE OF COUNTING TO SOLVE QUANTITATIVE PROBLEMS. Cognition and Instruction 4: 61-90; (2) 1987.

Abstract and comments prepared for I.M.E. by TERRY GOODMAN, Central Missouri State University, Warrensburg.

1. Purpose

The purpose of the study was to evaluate preschool children's use of counting to solve different kinds of quantitative problems. Three uses of counting were considered: (a) counting to quantify a single set, (b) counting to compare two sets, and (c) counting to generate a set of a specified numeriosity.

2. Rationale

Learning to count can be viewed as a major step in early mathematical development. Previous research indicates that children use counting in learning basic addition and subtraction facts as well as in solving word problems. Knowing when to count is viewed as being as important as learning how to count. It was pointed out that the research evidence with respect to knowing when to count is rather limited.

The approach used in this study was to characterize developments in children's knowledge about when to count by examining the range of problems to which children of different ages apply their counting skills. Counting to compare two sets is more complex than counting a single set, while counting to generate a set of a specified size may be an intermediate step.

3. Research Design and Procedures

Two experiments were conducted, with the first focusing on the use of counting to compare the quantities of two sets and the second examining children's use of counting to generate a set with a given numerosity. In the first experiment, thirty-six 3 and 3 1/2-year olds responded orally to two comparison tasks. The paired-sets task did not require counting but served as a control. For the separate-sets task, counting was a potentially useful strategy.

The children were tested individually in a single session lasting about 30 minutes. Sets of small toys were used to present four different situations to each child; paired equal sets, paired unequal sets, separate equal sets, and separate unequal sets. In each setting, children were asked to decide whether there were enough objects in the first set for every object in the second set to get one. Each child responded to eight experimental tasks. At the end of the study, each child took a counting test to determine the largest set, between 2 and 9, the child could count accurately.

For experiment 2, fifty-five 3 1/2, 4 and 4 1/2-year-olds were tested. In the first part of this experiment, children's use of counting to compare two sets was reassessed using a task in which the child was asked to construct a second set equivalent in numerosity to a given set rather than to compare two existing sets. In the second part of the experiment, children were asked to quantify existing sets and to generate a set of a specified quantity.

As in experiment 1 children were tested individually in a single session lasting about 30 minutes. Each child received six experimental tasks. Each task was preceeded by two warm-up problems designed to familiarize the child with the task. Problems were varied with respect to the sizes of the sets involved.

4. Findings

Experiment 1

The children did not often count during the paired-sets problems where the two sets could be compared directly. On the separate-sets problems, the number of children who counted increased with the age of the child. Children were also more likely to count when dealing with smaller sets. Finally, the children tended to count only one of the two sets to be compared.

Using analysis of variance it was found that both age groups performed above chance in the paired-sets condition, but the younger children did not perform above chance with large sets in the separate-sets condition. Children also performed better when working with equal sets than with unequal sets.

The counting test revealed significant improvement in counting skills with age. There was a rather large number (17) of the children who could count at least four items on the counting test and yet never counted on the separate-sets tasks.

Experiment 2

Children's performance on the two make-equal tasks was consistent with the performance of the children in Experiment 1. Again, children rarely counted when they could compare the two sets directly. When the sets could not be compared directly, the 4-year-olds counted more often than the 3 1/2-year-olds. When counting was used, generally only one of the two sets was counted. Overall, performance increased with age; it was better when the sets could be compared directly and when the sets were smaller.

On the other four tasks in Experiment 2, the older children again counted more often than the younger children. Two forms of counting were observed; generative and static counting. Generative counting involves counting objects as they are placed in a set. In static counting a number of objects is put into a set and then counted. Generative counting was the more common method used by the children in this study.

Possible sequences in children's use of counting were examined by considering individual children's patterns of performance. There was an ordinal relationship between the quantify-set and count-up tasks and the other three tasks.

5. Interpretations

It was proposed that these results support the conclusion that young children have only a limited understanding of the applications of counting to different kinds of quantitative problems. Young children will readily use counting to quantify a set of objects but they are much less likely to use counting to generate a set of a specified number or to compare two sets of objects.

Three hypotheses were proposed and discussed relative to the effect that task factors might have with respect to children's use of counting. These three hypotheses focused on the complexities of coordinating two sets, taking objects from a larger set, and counting while keeping in mind a large number. It was concluded that the data do not support any simple processing explanation for counting failures.

Further discussion focused on what the children's conception of counting might be, how children develop from a prequantitative to a quantitative understanding of counting, and implications for classroom instruction.

Abstractor's Comments

This study focuses on an interesting and important aspect of young children's mathematical understanding, use of rational counting. It seems clear that an understanding of counting as well as its use as a computational tool play major roles in arithmetic learning. The emphasis of this study on children's understanding of when to count and their ability to use counting in a variety of settings is quite relevant. There is evidence that suggests that for many young children counting involves memorization of number names in order with little understanding of the underlying concepts. Consequently, many children are able to use counting in a rather limited way. This study succeeds in providing further information about young children's understanding of counting.

A very positive aspect of this study was the use of two experiments with the second being a revision of the first. This provides a more detailed study and serves as a good example of how an experiment can be built from a preceding experiment. More studies of this type are needed.

There are several suggestions that can be made relative to this study. First. since both male and female children were used in the study it might be useful to investigate whether there are any sex differences in the children's understanding and use of counting.

Second, since preschool children were used in the study, it might be important to know something about their prior experience with counting. Most of this experience is likely to be rather informal. Had those children that did exhibit a clearer understanding of counting had any formal or informal instruction? What experiences can/should preschool children have in order to enhance their understanding of counting?

Finally, while this study did speak briefly to implications for instruction there are other questions that could be considered here. What do kindergarten/first grade teachers need to know about the children's understanding of counting prior to entering kindergarten/first grade? What counting experiences should children have in these early grades? How can teachers in these early grades build upon children's preschool understanding and background?

Stigler, James W.; Lee, Shin-Ying; and Stevenson, Harold W. MATHEMATICS CLASSROOMS IN JAPAN, TAIWAN, AND THE UNITED STATES. Child Development 58: 1272-1285; October 1987.

Abstract and comments prepared for I.M.E. by RANDALL SOUVINEY, University of California at San Diego.

1. Purpose and Rationale

In light of persistent reports of poor mathematics performance by United States school children relative to their peers in other countries (McKnight et al., 1985), the research reported in this article is particularly informative and timely. The authors have previously argued that deficits in mathematics performance of American's secondary students reported in other studies can be observed at the elementary level as well. The research reported in this article attempts to account for student performance differentials by observing classroom practices.

2. Research Design and Procedures

Recognizing the difficulties inherent in cross-national comparisons, the authors attempted to limit variability by matching the research sites according to several criteria. Schools were selected from similar-sized, ethnically-homogeneous cities in each of the three countries. The three cities selected (Minneapolis, U.S.A.; Sendai, Japan; and Taipei, Taiwan) are each relatively homogeneous in ethnic representation and provide a range of socioeconomic neighborhoods. Based on recommendations of local school authorities, 40 representative classrooms were selected from approximatley 10 schools in each city. Using a randomized time-sampled observation schedule, trained observers coded student and teacher behavior over a period of two to four weeks in each classroom. Student behaviors coded included: the subject matter studied; individual, small group, or whole class organization; who was leading the activity; on-task behaviors; and off-task behaviors. Teacher behaviors coded included: instructional group size; type of teacher talk; type of academic feedback; and type of feedback relative to student behavior. Two groups of six students were observed on alternate days in each classroom. Although 1600 hours of observation were planned for each city (yielding total observations of 33 minutes per student and 120 minutes per teacher), due to scheduling difficulties and illness the actual observation time was 1600 hours in Taipei, 1200 hours in Sendai, and 1353 hours in Minneapolis. Coded data were summarized as percents of total observations by category, and submitted to analysis of variance with the classroom as the unit of analysis. Correlations between mathematics achievement and student in-class behavior were also computed.

3. Findings and Interpretations

The results showed striking differences between the amount of on-task time Chinese and Japanese children spent engaged in academic activity pooled across all subject areas compared to that of their American peers. The first-grade differential of 10-15 percent increased to 20-25 percent at the fifth-grade level. As the authors pointed out, these results take on greater importance when considering that fifth-grade children attend school an additional seven hours per week in Japan and fourteen hours per week in Taiwan, and school is in session in these two countries more than ten weeks longer than in the United States. These results would project that on a yearly basis, Minneapolis fifth-grade children were engaged in academic activities less than one-half the number of hours accomplished by their Asian counterparts. Smaller, though equally dramatic differences, are reported for first-grade children.

The number of hours committed to language arts instruction in the United States was more than double that for mathematics instruction. The total number of hours of language arts per week was found to be roughly equal across the three countries. Since about 60 percent of the total instruction time at all sites was devoted to mathematics and language arts, the authors concluded that the Minneapolis schools stressed language arts activities to the detriment of mathematics instruction.

Additional classroom observation results indicated that American students were five times as likely to be out of their seats and engaged in irrelevant activities (5 percent versus less than 1 percent for Asian children). Asian students spent about twice as much time learning as a whole class than the American children. American children spent more than half their class time doing independent seat work as compared to about 20 percent for the Asian students. American children spent less than half their time in teacher-lead activities in contrast to 90 percent in Taiwan and 74 percent in Japan. American teachers were more than twice as likely to address questions to individuals than were Asian teachers.

Excessive transition time, teacher attention to individual students, a student or no one in charge of the class, non-academic activities, negative feedback by the teacher, and asking non-academic questions produced significant negative correlations with mathematics achievement for American fifth-grade students. Except for a significant negative correlation between achievement and total inappropriate activities, correlations for Taiwan classrooms were not significant. The positive correlations between mathematics achievement of the Japanese children and the amount of time spent attending and volunteering in class were significant. The authors were unable to justify the significant positive correlations between achievement and the frequency of punishment and incidence of non-academic questions for Japanese classrooms.

Abstractor's Comments

The research methodology of the reported study limits the generalizability of the results. Although reasonable care was taken to insure consistency across sites, the coding techniques employed offer only gross measures of behavior. For example, the reader is unable to determine the quality of a whole class, small group, or individualized lesson. One can imagine a range of lesson excellence displayed in each of the three instructional groupings. Ethnographic case studies of selected classrooms are needed to express, more fully, the richness of teacher/student interactions (Erlwanger, 1973; Mehan, 1979). It would be particularly interesting, for example, to interview the three teachers in Minneapolis who were never observed teaching mathematics. Although it is possible that this finding was the result of sampling error, such flexibility in curriculum scheduling on the part of teachers, if true, warrants further investigation. Likewise, several teachers in all three countries taught mathematics about 40 percent of the school day. It would be informative to contrast the quality of instruction of these teachers with those who commit, say, less than 15 percent of class time to mathematics.

Such concerns not withstanding, there is much reported in this article that demands consideration. The vast differences in mathematics instructional time between the American and Asian schools are daunting. Part of the difference can be attributed to a longer school week and school year in Japan and Taiwan. The remainder, it seems, is the result of relatively little emphasis being placed on mathematics instruction in the American classrooms and the qualitative difference in the attention paid to academic learning by these students. While it has been argued that quality small group and individualized instruction can lead to significant achievement gains (Slavin, 1983, 1987; Kagan, 1986), poorly structured group activity and uninspired seat work are unlikely to prove superior to a well-planned and executed whole-class lesson. The current study provides a basis for further ethnographic, cross-national investigation of mathematics teaching practices.

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Szetela, Walter and Super, Doug. CALCULATORS AND INSTRUCTION IN PROBLEM SOLVING IN GRADE 7. Journal for Research in Mathematics Education 18: 215-229; May 1987.

Abstract and comments prepared for I.M.E. by J. PHILIP SMITH, Southern Connecticut State University.

1. Purpose

The major purpose of the study was to examine the effectiveness of a year-long grade 7 curriculum which emphasized selected Polya-type problem-solving strategies. Secondary purposes were to measure the effect of sex differences and supplementary calculator instruction on problem-solving performance.

2. Rationale

The study was developed in light of other research, some supporting and some raising doubts, about the efficacy of teaching general problem-solving strategies. Although broad support for the teaching or problem solving exists, serious disagreements surface concerning whether or not and in what ways various problem-solving skills can be taught.

Although the preponderance of research evidence now favors, at least weakly, the use of calculators in mathematics learning and problem-solving instruction, some studies have raised doubts about the value of calculators in higher-level conceptual learning. Similarly, the matter of sex-related differences in mathematics achievement, particularly with regard to higher-level skills, is still a research issue.

3. Research Design and Procedures

The sample consisted of 42 grade 7 classes, widely scattered throughout an "urban-rural" school district of about 200,000 population. Of the 42 teacher-volunteered clases, 24 comprised two problem-solving groups and 18 formed a control group. Fourteen teachers of the 24 problem-solving classes volunteered to use calculators in their classes; thus, the experimental groups consisted of 14 classes (290 students) in a problem-solving-strategies-supplemented-by-calculators group and 10 classes (195 students) in a problem-solving-strategies-without-calculators group. (These groups are known, respectively, as the CP group and the P group.) The 18 control group classes (338 students) used no calculators and received no special problem-solving instruction. A pretest indicated no significant differences among the three groups in knowledge of whole number operations.

Tests with reasonable reliability coefficients were selected or constructed to examine differences among the treatment groups on the following criterion variables: (1) solving "translation" problems (i.e., typical textbook word problems, solvable by a routine application of a single arithmetical operation); (2) solving "process" problems (i.e., problems "requiring strategies more general than simply choosing correct arithmetical operations"); (3) solving more complex process problems (i.e., "problems requiring more extensive analysis, development of strategies, and persistence" than those in category 1 or 2); (3) attitude toward problem solving; and (4) computational skills.

The study ran for a full September-to-June school year. The CP and P teachers received 2 1/2 days of problem-solving instruction. One such day occurred in the May of the preceding school year and the other day and a half took place during the first month of the study. The instruction familiarized teachers with Polya's four-state problem-solving model and provided practice with widely available grade 7 problem-solving materials. The heuristics "understanding the problem" and "looking back" were particularly emphasized, as were matters such as motivation, classroom atmosphere, and the importance of solving problems by more than one method and discussing solution strategies.

Five of Polya's strategies were emphasized during the year-long problem-solving instruction. "Guess and test" and "make a systematic list" were introduced during the first two months, followed in subsequent months by the strategies "look for a pattern," "make a simpler problem," and "draw a diagram."

The CP and P group teachers were provided with suggested topics and problems for the first two months. Some of the participating teachers, together with the investigators, developed topic sequences and problems for the rest of the year. Because two grade 7 textbooks were in use among the classes of the study, topical sequences and problems were not identical for all groups. Topical content and the number of problems covered were the same for both groups, however.

Teachers of the control group classes received no special instruction and no problems for classroom use beyond any that might be contained in the regular textbooks.

Attempts were made to monitor classroom activity in the CP and the P groups. Graduate assistants visited all teachers during the fall and the investigators visited each teacher in the late fall and winter. Some teachers requested and received demonstration lessons.

One version of the Translation Problems Test was administered to all 42 classes in February, and a version of the Process Problems Test was given in March. At the end of the school year in June, alternate versions of these tests were administered, along with the Complex Problems Test. In April, all students took the Whitaker Attitude Test, an instrument designed to measure attitude toward problem solving. In May, all students took the Rational Numbers Test, selected primarily to determine if students in the calculator group would show relatively weaker performance on questions involving rational number computations and concepts.

All teachers involved in the study completed a Teacher Questionnaire toward the end of the school year in March. The questionnaire was designed to assess perceptions of problem solving itself and (for the CP and the P group teachers) of the experimental program in particular.

4. Findings

Test differences were analyzed by means of partially nested analyses of covariance with treatment by sex nested within class. The covariate was the pretest on whole-number operations. The unit of analysis by this method is the class, and when significant F ratios were obtained, the authors used the Tukey test to determine how the groups differed.

Scores on the two versions of the Translation Problems Test, the two versions of the Process Problems Test, the Operations with Rational Numbers Test and the Complex Problems Test showed, in general, small but sometimes significant differences, with the control group scoring below the two experimental groups in all cases but one. The exception occurred on the Complex Problems Test, where the differences were not significant but the control group mean fell between the CP group low and the P group high.

Significant ($p < .05$) results are as follows. The P group scored higher than the control group on both midyear tests on translation problems and process problems. The CP group scored higher than the control group on both the midyear and the year-end versions of the Translation Problems Test. The CP group scored higher than both other groups on the problem-solving attitude test.

Sex differences within treatment groups were not significant at the .05 level. When treatment groups were combined, small but significant differences favoring boys appeared on the 'dyear test for translation problems and the year-end test on process problems.

The Teacher Questionnaire revealed extremely positive reactions on the part of the CP and the P teachers about participating in the program. Teachers in the CP group perceived calculators as helpful in learning. In response to the question, "How much involvement (is there) in problem solving activities in your class?" the CP teachers had an average response rating of 3.92 on a scale from 1 (low) to 5 (high). The P teachers mean response was 3.56 and that of the control teachers was 2.89. When asked to estimate the percent of time spent on problem solving, the CP, P, and control teachers responded, respectively, 38, 34, and 32. When asked, "How much attention was given to looking back?" mean responses for the CP and the P groups were, respectivel , 2.50 and 3.22 on the 1-to-5 scale.

5. Interpretations

The authors concluded that instruction in problem-solving strategies accompanied by practice in solving numerous problems appropriate for those strategies was slightly more successful than "traditional" instruction in problem solving. Because of attitude scores favoring the CP group with no apparent loss of pencil-and-paper skills, the authors suggested "that resistance to the use of calculators in schools may be delaying opportunities to learn mathematics." Sex differences are viewed as "very few or nonexistent."

Why were there not greater gains in test scores favoring the problem-solving groups? The investigators suggested several reasons: perhaps the low attention paid to the looking-back strategy was a crucial error; perhaps the control group teachers themselves spent more time than the investigators anticipated on problem-solving instruction; perhaps domain-specific knowledge rather than general problem-solving strategies is the chief determinant of success in the problem-solving arena.

Abstractor's Comments

Given the complex nature of problem solving and problem-solving instruction, the authors of the present study may well have been attempting the impossible. Nonetheless, problem solving is one of those phenomena that can profitably be attached on more than one level, and it is nice to see someone with enough courage to attempt a large classroom study.

In the present case, although the evidence appears slightly to favor problem-solving instruction, we really need to know more about what actually happened in those classrooms if we are to make anything much of the results. To visit teachers twice during the school year is simply not sufficient to enable us to say with much confidence just what the treatment really was. Some evidence suggests that the teachers did not focus on problem solving in ways that the investigators encouraged. Could this account for the small differences obtained? Further, in the present case, we are informed that the school district participating in the study was emphasizing problem-solving instruction in its schools. Apparently no one visited the control group teachers. What was the nature of their problem-solving instruction? Perhaps the apparent small differences would have been larger had the control group not itself been concerned with problem solving. After all, Polya's model and the problem sources used in the study were readily available to any of the control group teachers who read professional publications.

(We note in passing the extreme difficulty of conveying to teachers in a short period of time just what one means by advice such as "look back." In fact, "look back" is not a piece of advice, but rather an umbrella term under whose shadow many different strategies lie. Different teachers can take "looking back" to mean different things unless great care and much time is devoted to a discussion of the term.)

The report speaks of the control group's "traditional instruction in problem solving." What is such "traditional instruction"? My impression is that such instruction is not so traditional and that any such "traditions" are probably changing fairly rapidly thanks to textbook attempts at incorporating problem solving and to the publicizing efforts of groups such as NCTM.

As the investigators note, the effect of all the experimental group teachers being volunteers is highly likely to affect results, certainly on the attitude items in the questionnaire and perhaps elsewhere as well. Nonetheless, it is nice to see the experimenters working with teachers in devising the year's problem-solving curriculum, and one might reasonably argue that if we wish a practical, realistic study then we should deal with volunteers. That's often the approach taken in trying new methods.

In summary, although this study has many flaws as a tightly designed, controlled piece of research, it does raise some interesting questions and suggest a few answers. It is also important to read if you are working on a district level and seeking to implement problem-solving instruction.

Thompson, Patrick W. and Dreyfus, Tommy. INTEGERS AS TRANSFORMATIONS. Journal for Research in Mathematics Education 19: 115-133; March 1988.

Abstract and comments prepared for I.M.E. by CHARLEEN M. DERIDDER, Knox County Schools, Tennessee.

1. Purpose

The investigation in this study was an attempt to determine if elementary school students could construct operations of thought for (1) integers as transformations, (2) addition of integers as the composition of transformations, and (3) negation as a unary operation upon integers and integer expressions. The study was meant to answer the question of whether or not it is possible to organize instruction to facilitate students' development of mental operations that directly parallel components of algebraic thinking traditionally accepted as important.

2. Rationale

This investigation emphasized the conception of integers as transformations based on a theory proposed by Vergnaud (1982) which features a system of mental operations for analyzing quantitative relationships. Vergnaud contended that children needed to perceive the addition of integers as the addition of transformations of quantities instead of cardinal quantities. Several studies of algebraic concepts were cited concerning research on concepts of expressions.

3. Research Design and Procedures

The method of investigation was based on the constructivist teaching experiment in which selected concepts are introduced to the student for the first time, followed by an analysis of student understanding which, in turn, serves as a basis for planning for the next instructional session. The microworld, INTEGERS, used in this study presents integers as unary translation operators acting on positions on a number line. Two middle-ability sixth-grade female students (Kim and Lucy) were the subjects, receiving 40-minute sessions twice weekly during a five-week period and once during the first of the six weeks. A research assistant conducted these sessions, each of which was audiotaped.

4. Findings

With respect to (1) integers as transformations, Kim early distinguished between ideas of position and change of position, and began to consolidate an integer as a direction-displacement pair during the third session. Lucy confused the ideas of position and change of position and, although some notion of distinction occurred after a few sessions, such clarity was not consistent or reliable. Both quickly understood the process for negating numbers using the software. Concerning (2) addition of integers as a composition of transformations, both had difficulty determining the result if both a positive and negative addend were used. Success with this concept seemed to occur in the eighth session. With (3) negation of expressions, both initially experienced difficulty, but understood the semantics of a negated expression by the ninth session. Their ability to negate expressions was not high, but they did seem able to reason in terms of equivalencies of expression and composition (net effect). Kim was able to do this with more complex problems, but Lucy was not. The investigators observed that the subjects were considerably confused during the eleventh session with thought operations involving equating an expression and its net effect, and even more so when the expression was negated.

5. Interpretations

Although the investigators felt they had shown that sixth graders "can conceive of integers in nontrivial ways" (p. 130), they felt further research is needed to determine if such conceptual development makes a difference in the learning of algebra. They concluded that the difficulty Kim and Lucy had in constructing the concepts of transformation, composition and expression negation "in the semantically rich environment of INTEGERS and in the course of 11 highly individualized lessons--calls into question the depth of understanding we may expect from students studying integers under more traditional instruction" (p. 130). They felt that the instability of the subjects' conceptualizations was remarkable. The conjecture is offered that a comparable study over a longer duration would result in more stable and reliable knowledge. It is also mentioned that models other than INTEGERS, although they are less versatile, might be used in further research. They recommend that further research within traditional instruction should focus on operations of thought that generalize to elementary algebra and skill with addition and subtraction algorithms.

Abstractor's Comments

This study deals with a significant problem and the narrative indicates careful documentation of the treatment sessions as well as objective assessment of the data. The study is limited in that it deals with only two subjects. Also, according to the conclusion, there is a tacit assumption made that the use of the microworld INTEGERS in eleven sessions with a graduate assistant is superior to what a student might experience in traditional instruction with a qualified teacher. With respect to the software, the fact that the turtle on the screen always turns around to face the right on the screen after a negative move, but remains facing to the right after a positive move, is most apt to cause a conceptual imbalance concerning, if not intrude a value judgment upon, positive and negative integers. In examining the thought processes of these sixth graders, all forms of stimuli need to be considered as to their potential impact on the subject's thinking. It is not my conviction that the use of this software, as described in the study, provides a "semantically rich environment." Because of the logistics of the procedures, it, in fact, could prove to be an obstruction to clarity in the development of a concept. In session nine, the interviewer asks, "Now, what is the net effect of negative fifty thirty?" (p. 128) and proceeds to type in -50 30 without pressing the return. Both students respond with "negative twenty" which is acknowledged as correct. Then in session ten, the interviewer, following previous work on -z as being the undoing of net effect z, says, "Try negative thirty sixty" (p. 129) and types -30 60 without pressing the return. One subject responds with "thirty" and the other with "ninety." When the return is pressed, the correct answer is shown to be negative thirty. Apparently the first reverted back to the previous work on composition, while the other was attempting the undoing of the net effect using only the first integer. But please note that the interviewer's words have an identical format in two conceptually different situations. The subjects' confusion can be understood.

It would be truly enlightening to compare the results of this study with one conducted with the same number of students and sessions by an experienced teacher who utilized the strategies of drawing on the subjects' current understandings and existing mathematical frames of reference, instead of INTEGERS, to develop the same new concepts.

Zhu, Xinming and Simon, Herbert A. LEARNING MATHEMATICS FROM EXAMPLES AND BY DOING. Cognition and Instruction 4: 137-166; (3) 1987.

Abstract and comments prepared for I.M.E. by JOHN R. KOLB, North Carolina State University.

1. Purpose

Experiments were conducted to determine the feasibility and efficiency of children learning school subjects from worked-out examples or from learning by doing, in contrast to conventional means of direct instruction or lectures. This report analyzes the depth of understanding and the nature of what is learned in the thinking-aloud protocols of twenty students learning factorization of quadratic trinomials by the two methods in one experiment.

2. Rationale

The authors assert:

> Our interest in the prospects of learning from examples was sparked by incidents such as: A student who was late for class missed the teacher's lecture, but at the end of the class, he looked at the problems worked by another student. When the tardy student was tested, we were surprised to see that he worked the test problems correctly. Apparently he had learned by studying the worked-out examples. How generalizable is the result? How efficient is the process?

Worked-out examples have long been an essential element in textbook presentations and teacher demonstrations of new materials. Diligent students have long known that studying worked-out examples often provides enough information to learn the procedure without additional instruction. Students appear to be able to identify salient characteristics and infer generalizations from examples, thereby internalizing them as rules and applying them to new situations.

Learning by doing is closely related to learning from examples. Through feedback and persistence, the student finds a path to a solution of an exercise that serves as a worked-out example. The internalization of the process of finding a solution path may produce an algorithm that can be applied to new exercises.

If a student can learn school subjects with efficiency and understanding with these methods, then they may be adapted to design tutoring systems and instructional displays in computer assisted instruction.

3. Research Design and Procedures

The authors refer to several experiments that used tasks involving simplifying fractions, manipulating terms with exponents, and solving several geometry problems as well as a two-year study involving the standard curriculum in the Chinese middle schools. However, only cursory data are given for these studies and the focus of this abstract will center on the one experiment with factoring quadratic trinomials that is more fully reported in this article.

Instructional materials. The task to be learned was factoring quadratic expressions of the form $x^2 + ax + b = (x+c)(x+d)$ in which a, b, are integers. They constructed a global unsequenced task analysis for this skill that they call a "production system". Their production system consists of seven principles (rules) which are assumed to approximate the skills that a student must acquire to factor the given expressions. One of the rules asserts that the numbers c, d that are sought must be a factorization of b. A second rule states that when the correct values for c and d are found, then the result is written as $x^2 + ax + b = (x + c)(x + d)$. Four of the rules relate the signs of c and d to the combination of signs on a and b.

Two versions of the learning materials were prepared; one version for the learning from examples group and a second version for the learning by doing treatment. In both conditions, the materials are arranged in three parts: Part I involves factoring where a and b are positive, Part II pairs instances that are identical except that in the first a and b are positive and in the second a is negative and b is positive, and in Part III instances are paired that are the same except a is positive and b is negative in the first pair but both are negative in the second pair.

The two treatments are compared in the Figure 1 below which is taken from Part II of the materials. Notice that in both versions of Part II as in Parts I and III there is a section denoted as examples followed by a section called exercises. The materials used in the two treatment groups differ only in the section labeled examples. In the learning by doing group, the students had to fill in the correct values for c and d in Part I and had to fill in the signs before c and d in Part II and Part III.

LEARNING FROM EXAMPLES	LEARNING BY DOING
Examples	Examples
(1)$x^2 + 5x \div 6 = (x + 2)\ (x + 3)$	(1)$x^2 + 5x + 5 = (x\underline{\ \ }2)\ (x\underline{\ \ }3)$
(2)$x^2 - 5x + 6 = (x - 2)\ (x - 3)$	(2)$x^2 - 5x + 6 = (x\underline{\ \ }2)\ (x\underline{\ \ }3)$
(3)$x^2 + 7x + 6 = (x + 1)\ (x + 6)$	(3)$x^2 + 7x + 6 = (x\underline{\ \ }1)\ (x\underline{\ \ }6)$
(4)$x^2 - 7x + 6 = (x - 1)\ (x - 6)$	(4)$x^2 - 7x + 6 = (x\underline{\ \ }1)\ (x\underline{\ \ }6)$
Exercises	Exercises
(1)$x^2 + 9x + 18 = (\quad)\ (\quad)$	(1)$x^2 + 9x + 18 = (\quad)\ (\quad)$
(2)$x^2 - 9x + 18 = (\quad)\ (\quad)$	(1)$x^2 - 9x + 18 = (\quad)\ (\quad)$
(3)$x^2 - 11x + 18 = (\quad)\ (\quad)$	(3)$x^2 - 11x + 18 = (\quad)\ (\quad)$
(4)$x^2 + 11x + 15 = (\quad)\ (\quad)$	(4)$x^2 + 11x + 15 = (\quad)\ (\quad)$

FIGURE 1. Comparison of Part II of learning task for each treatment group.

Tests. Two tests were prepared. The first pretest consists of six statements that paraphrase the rules for factoring stated in the production system. The student fills in blanks by selecting the correct word from a list of alternatives associated with each blank. The second test served as a second pretest and also as the posttest. It consists of five exercises to be factored.

Procedure. The participants were 118 middle school students from an average school in Beijing, China enrolled in the first-year algebra course and the material studied was a part of the standard curriculum. Ninety-eight students, in two clases, were taught in a whole group setting and all were in the learning from examples condition. No comparison group using conventional methods or learning by doing was formed. The remaining 20 students studied the learning material individually and verbal protocols were recorded. Half studied the learning from examples material and the rest used the learning by doing material.

In all cases the procedure was: 1) take the first pretest; 2) study some review material on a) finding factors of a number, b) identifying constant, linear, and squared term in a trinomial, c) multiplying two binomials, and d) factoring as the inverse of multiplying; 3) take the second pretest; 4) learn from examples or by doing; and 5) take the posttest.

4. Findings

Classroom group. Of the five problems on the posttest, 83% of the students solved all five, 8.5% solved three, and 8.5% solved two or fewer after a single class session spent on the review and the learning from examples material. The results from the pretest were used to exclude four students from the results given above since they could solve the problems before instruction.

Protocol group. None of the students could solve any of the pretest problems or recognize the rules for factoring. After spending 25 minutes working in one or the other conditions, all 20 students could solve the posttest problems.' Fifteen of the 20 could verbalize correct procedures in response to the interviewers questions.

In both conditions, students progressed in a similar way. Most students focused first upon finding numbers that add to the coefficient of the linear term. Also, in the beginning, students did not tend to check whether their numbers satisfied both conditions; that is, satisfied the linear term and constant term. These errors were rare in the latter stages. Students noticed their errors when they began to check their work by multiplying or in the case of the worked examples group when they compared their work to similar worked examples. A misconception that prevailed to the end was that the order of the factors make a difference (e. g., (x-3) (x+5) versus (x +5) (x-3).

There was little difference in the performance of the two groups. Any differences were most evident at the outset. While the learning from examples group can study the worked examples and extract a procedure to apply to the unworked examples, the learning by doing student often struggled at the beginning to produce a correct example. All the student has available is the principle that factoring is the reverse of multiplying. Invoking this principle is essential to get started but once it is successfully used in one or two examples the student then proceeds successfully. While learning by example students rely heavily on the examples to provide a model they do try to generalize from the examples before going on to complete exercises. As the students progressed to Part III, they showed less evidence of how they searched for number pairs or determined the correct signs. They seemed to work correctly and automatically and only verbalized the final answer. Recognizing nx1 as a factorization of n took longer than other combinations.

5. Interpretations

While the two groups differed in the way in which the learning process began, both conditions result in most students mastering the skill of factoring quadratics within a short time after given knowledge that factoring is the inverse of multiplication.

For a few students, understanding was limited largely to the ability to factor, but from the protocols it is evident that most have the ability to formulate the rules, not from rote memory, but in their own words as a product of generalization of factoring by showing its relationship to multiplication when they check their results.

Based on this experiment and the brief mention of several other experiments, one of which involves teaching two years of algebra and one year of geometry, the authors concluded that their work provides substantial support for the success of learning from examples in comparison to learning from more conventional methods.

Abstractor's Comments

The obvious outcome of this study is that we once again see the power of examples in learning mathematics. Students do learn from examples ... students who succeed in mathematics often achieve because they learn to identify and model the strategy of a worked example to the current exercise they have before them.

In this study, we have not just worked examples but a carefully constructed sequence of examples that is intended to maximize a learner's chances of attending to the salient features in the factoring pattern. The mastery level achieved in the short time was impressive. The brief results given from other experiments that replicate these findings are promising. However, until these are reported in sufficient detail to be scrutinized, we must delay full acceptance of them.

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