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ABSTRACT

The idea that cognitive science can provide useful guidance to the teaching of physics has been met by some with skepticism. One argument is that the current understanding of cognition is too crude to be helpful; another, that any scientific approach to education stifles the art of teaching. Some feel that art and science need not be incompatible. This paper offers several illustrations of ways in which cognitive science can be used to refine the art of physics teaching. For a theory to be pedagogically useful it should help teachers see things they might not otherwise have noticed, provide an overall organizing scheme for instruction and suggest specific activities that can improve instruction. This paper focuses on the physics problem. Examples have been drawn from introductory level college physics classes. (CW)

CAN COGNITIVE THEORY HELP US TEACH PHYSICS?

by

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The idea that cognitive science can provide useful guidance to the teaching of physics has been met with skepticism. One argument is that the current understanding of cognition is too crude to be helpful; another, that any scientific approach to education stifles the art of teaching. Several months ago an article in this journal by Davis Hestenes addressed these arguments and suggested that art and science need not be incompatible.[1] In that same spirit we offer several illustrations of the ways in which cognitive science can be used to refine the art of physics teaching.

For a theory to be pedagogically useful it should: (a) help teachers to see things they might not otherwise have noticed: (b) provide an overall organizing scheme (philosophy) for instruction; and (c) suggest specific activities that can improve instruction. In this paper we focus on one aspect of physics teaching: the physics problem. Most of our examples will come from a two-semester introductory calculus-based course for engineering students which R.L.G. has taught for the past five years (current enrollment, 500 students). We will also use examples from a small, 20-35 student, pre-physics course which has been taught by J.L.

Cognitive Development and the Direction of Learning

The essential insight of modern cognitive science is that human intelligence is not a fixed entity but that it develops gradually from birth through a series of increasingly powerful levels of function.

The developmental sequence has been described differently by Piaget[2], Bruner[3], Perry[4], Vygotsky[5], and others, but there is general agreement on certain aspects. The direction of growth is from the perceptual to the conceptual, from the reflexive to the reflective. The mental activity of a newborn infant is dominated by direct sensory perceptions and reflex actions. Only later in life does the child acquire the ability to organize its perceptions via conceptual structures and to modify reflexive actions by reflection.

The direction is evident not only through the life span but also during the acquisition of new concepts. This natural flow of learning, from the concrete to the abstract, is reflected in the Piagetian based high school curricula which Lawson describes in his contribution to this issue. If, for example, one wants to teach students about momentum, one usually starts with perceptual examples of objects possessing momentum. This is the function of the demonstration lecture and the laboratory. However, as the subject matter becomes more abstract it becomes increasingly difficult to maintain this approach. The study of advanced topics can only begin at the conceptual level. Hamiltonians just are not perceivable objects, and as we shall show at the end of this paper, neither are accelerations. For this reason one major function of the college level calculus-based physics course should be to help students modify their approach to learning so that it can begin with abstract concepts.[6] The fact that this involves undoing a twenty-year-old, well established pattern is precisely why physics is so difficult to teach.

What is the Instructional Purpose of a Question or Problem?

If one role of physics instruction should be to teach students conceptual thought, how well do current physics problems measure up to that objective? Arons sees a major shortcoming:[7]

> "One of the weakest links in our chain of instruction consists of the questions and exercises that are embalmed at the ends of chapters... We are desperately in need of collections of questions and problems that, sensitive to the obstacles that arise in students' minds, lead the student through the difficulties and subtleties in thinking and reasoning that he must face and overcome. We need questions that challenge his curiosity and ability to perceive relationships but that he can encompass and deal with successfully a reasonable fraction of the time...Above all, we need questions and problems that, gently and gradually, lead the student into extending, inventing, perceiving questions of his own."

Arons' point is not that the questions are bad but rather that they are inappropriate for the students' current level of reasoning. To generate more suitable questions we first have to understand how the students work problems. Cognitive research (cf. Larkin, Clement, Simon & Simon)[8,9,10] contradicts the notion that student thinking is similar to that of physics teachers, just less practiced. The methods students use to solve a problem often bear no relationship to those intended by the problem's author. If we are to write effective questions we need to know how beginning students think and how their reasoning develops over time. Again we quote Arons:[7]

> "In other cases, eager authors have generated rather more interesting problems, but wittingly or unwittingly, they have written for the eyes of their colleagues rather than for students, and the results are problems far beyond the readiness or immediate comprehension of the students being addressed."

Questions and problems in physics are instructionally good to the extent they satisfy two criteria implicit in the quotations above: (1) they reflect the intellectual style of the scientific community and (2) they offer students a genuine opportunity to extend the boundaries of their intellectual competence. Content-specific memorization, algebraic manipulation, and formula sifting are frequently the principal means whereby students attempt to deal with and understand physics. Many textbook problems and examination questions are solvable by application of specific algorithms.

The two questions below are not atypical:

1. An analysis of projectile motion shows the range to be $R = \frac{V_o{}^2}{g}\sin 2\theta$. What is the initial speed V_0 of a projectile fired at an angle $\theta = 45^\circ$ such that its range is 800 meters?

2. A grinding wheel starts from rest and acquires an angular velocity of 15 $\frac{rad}{sec}$ in 3 sec. The angular acceleration of the wheel is:

   a. $15\frac{rad}{sec^2}$; b. $3\frac{rad}{sec^2}$; c. $45\frac{rad}{sec^2}$; d. $5\frac{rad}{sec^2}$; e. $0.2\frac{rad}{sec^2}$

Correct answers to questions of this type need not reflect an understanding of the underlying physics. The first problem is straight algebraic substitution. The second requires a choice, but that choice can be made solely on the basis of units analysis. Such questions can mislead students in their quest for understanding what the subject is all about because they confirm the notion that it is mostly formula sifting.

On the other hand, the following problem is hardly better:

3. A ball of mass 0.5kg is thrown $40^m/sec$ at Carl Yastremski who, alas, flies out. The ball is caught 100m away (at the same

height that it was hit). The ball stays in the air for 7 seconds. At what speed did the ball leave his bat?

It has been carefully designed so as not to conform to any of the commonly memorized trajectory formulae. But the resulting complexity puts it beyond the reach of any but the best introductory students. Yet there is a sense in which problem 1 appears more sophisticated than 3, and we suspect many novices might feel it is a better test of physics knowledge.

We can only be certain that a particular problem is an instructionally useful physics problem if we know how students in fact solve it. However, in a large lecture course it is difficult to determine the instructional or learning implications of correct or incorrect solutions to isolated problems. One method of obtaining such information is to use paired questions. Figures 1 and 2 show two questions given on the same examination to 300 students. Figure 3 shows the correlated performance of approximately 100 randomly selected students. It is clear in this example that flexible understanding results from mastery of problem 2, but not problem 1. Most of those students who scored over 50% on question 2 also scored high on question 1. However, success on question 1 does not predict success on question 2. This is true in spite of the fact that most physics texts discuss projectile motion after the material of problem 2. Inconsistent performance of this type is evidence of learning strongly coupled to situational particulars. This is characteristic of Piaget's[2] concrete operational or Bruner's[3] ikonic student. Their learning is dependent upon particulars of specific situations. Another example is shown in figure 4. This problem was placed on a final examination in a large course. The intent was to see the extent to which a problem forces students to use inappropriate analysis techniques. An earlier question on the exam had asked students to write an equation appropriate to a quadratic function. Student performance on this was excellent. Nevertheless, on the energy problem, far fewer students wrote a correct equation. The second part of the energy problem gave even more discouraring evidence. 15-20% of the students used $mgx_1 = \frac{1}{2}mv^2 + mgx_2$ in an attempt to solve it.

To write a question pitched at the appropriate level requires detailed knowledge about the students. Recently, one of us (J.L.), placed the following question on the final exam in a course which stressed graphical methods for physics:

> Ed K. is driving a car at 60 miles per hour (88 ft/sec) when he suddenly notices that exam-crazed U. Mass students have constructed a 10,000 lb ice cream sundae in the middle of the road 200 ft ahead. If Ed takes ½sec to react and step on the brakes how fast will he be going when (if) he hits the sundae? The car can decelerate at 20 ft/sec^2.

In the course only one kinematical equation had been discussed:

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$

It was assumed that students would graph velocity vs. time and by examining the area under the graph construct an equation relating the unknown time to the known distance. From this they could calculate the time and from that determine the final velocity. Viewed this way, the question requires that students bring concepts to the problem, and it does not prompt their activity. Unfortunately, several students had looked up and memorized the equation $V_f^2 = V_i^2 + 2ax$. For them the problem required little more than simply selecting the correct formula and executing it. The given information, velocity, distance, and acceleration guided the students' action. Even more discouraging, those who attempted the graphical approach often were overcome by the complexity of the problem. Thus in the end the question rewarded the type of formula plug-in approach it had been designed to discourage. It is our impression that this type of confusion between the intent of a question and its actual affect is disturbingly common even among questions that have been carefully thought out.

As our examples have shown, students have a preference for, and ample opportunity to use, formula manipulation in dealing with the subject of physics. We have also tried to show the extent to which even their success can be misleading in terms of mastery or growth. Pedagogically the situation is even more serious. The inflexible character of the topic specific, formula centered, intellectual style is at odds with both the discipline of physics and the developmental growth requirements of the student.

If we are to avoid these difficulties, then we must, as Reif suggests in his contribution to this issue, describe both to ourselves and to our students explicitly what type of performance we are seeking. The naive interpretation of Reif's dictum would have us break down each difficult concept into finer and finer components until students were able to master each part. In our experience students are capable of learning almost any well defined operation if the need to do so is clearly indicated and fully reflected by the grading system. They can already do an amazing job in memorizing formulas! But this type of mastery rarely results in flexible knowledge; that is, what students have learned is often rigidly specific to the original details and reflects little understanding.

A less naive interpertation of Reif suggests that we must be explicit in our desire for flexible knowledge; for learning that is transferable to unfamiliar situations. Here lies a paradox; we must be explicit about our desire for students to learn in such a way that they can use their knowledge in novel situations. But if these situations are to be truly novel some details must remain unexplicit. To be a test of understanding a problem cannot explicitly tell the students how they are to solve it. In the next section we suggest solutions to this paradox.

A Framework for Constructing Questions

Fig. 5 is the result of our attempt to impose some pattern on the ways in which students approach problems. The idea of classifying questions according to the type of cognitive activity they require is not new. Perhaps the best known method is Bloom's taxonomy.[11] What we have added to the traditional categories is a second dimension: namely, the way in which the student acts on the question.

The three categories on the horizontal axis describe the level of conceptual sophistication required by the content of the question. An algorithmic operation is a procedure which can be carried out on a collection of objects: labeling something with a name, adding two numbers, looking up sin X in a table. This type of activity can be thought of as manipulating objects or symbols. It does not involve the creation of new objects or symbols. A representation (literally "repeat presentation") involves a procedure for symbolizing objects or their interrelationships. The emphasis here is on preserving certain critical features of the original set: assigning numbers or letters to lengths, describing velocities with vectors, plotting points on a graph. The activity is something like copying but can involve the creation of new objects or symbols that stand for old ones. A transformation (literally a change in form) is a procedure for symbolizing objects or relationships in such a way that certain aspects are <u>not</u> preserved. Here the emphasis is on deliberately discarding information in order to generate new information: making a two dimensional cross section of a three dimensional object, differentiating a function.

While all actions can in some sense be conceptualized in any of our three categories, we are interested in how the activities are consciously conceptualized by the actor undertaking them. One may calculate the derivative of a polynomial without consciously recognizing that the new function has a new form and that it represents a conceptually distinct though related quantity. One may start out with work and distance and end up with force, a transformation, but one may also manipulate two algebraic symbols W and L and call the resulting quantity F. This latter activity is simply an algorithm.

The three categories on the vertical axis refer to the way in which the student acts on the question. Execute refers to what are essentially reflexive activities. These are relatively automatic, mindless, though by no means unimportant actions. They can be conceptualized as carrying out a prescribed series of steps as in a simple computer program. The select category refers to situations in which the student makes a choice among several known possibilities. These require some thought and judgment, especially when the choices are not explicitly listed. There is a need for the student to cast the problem into some context, to apply additional knowledge which can elaborate beyond the content explicitly given. If the question carries in it all the selection rules necessary for the choice, then we would classify it as an execution problem, and not a selection.

Finally, the construct category refers to those situations in which the student exhibits true creativity. Here the student creates a new object or operation. But questions such as, "construct a perpendicular bisector to the line..." normally would not be examples since they are asked after that procedure has become a well practiced algorithm. The circumstances which determine whether a question involves construction depend crucially on the current state of the student's knowledge. It is extraordinarily difficult to write questions which elicit this behavior since they cannot be composed without a detailed understanding of the student's knowledge and skill.

The matrix of fig. 5 resulted from a casual exercise concerning student difficulties in dealing with problem situations on several levels. The idea was to present a single SITUATION asking students questions in order to inventory thei· ability to function at different cognitive levels. The words SEEING, REPRESENTING, and ACTING were selected to identify three types of cognitive functioning making them as explicit as possible.

The SITUATION of fig. 6 is a picture from Halliday and Resnick[12] at the end of a chapter on statics. Eight students volunteered to answer these questions. All satisfactorily answered questions concerning number of rungs and free body diagrams. The startling result was that six of the students answered the last question as $h = \frac{1}{2}(AE)(AC)$! Only two students made any attempt to determine the height of the ladder in terms of the presumed height of the man. Even these two evidenced an uncertain feeling. At best they thought the question tricky, having no place in a physics course since the nature of the answer could at best be tentative, i.e., what if the man was a midget or a professional basketball player?

Readers will recognize the incorrect student answer for the height as an algorithm concerning the area of triangles. It is as though the SITUATION prompts the use of a remembered formula concerning triangle, because one SEES a triangle.

The same SITUATION was subsequently given to several different students, the letters A through E being deleted. Here no one used an algorithm concerning triangles, but all were vehement in their unwillingness to "bring something" to the question of tallness.

What these results suggest is that students expect the answer to be in the qestion. They manipulate the data in the question with algorithms associated with the situation but they have no mechanism for placing the question in a context. Several years ago we made the mistake of including on the final exam a statics question concerning a monkey and a ladder. Earlier in the course a monkey had been involved in a dynamics question. A full twenty five percent of the students treated the statics question as though it were about dynamics, probably because of the monkey.

Most students only feel comfortable with activities which can fit in the upper left hand cell of figure 5. Our goal as teachers has been to move them towards the lower right hand cell. In the next section we will provide two examples of ways in which we have tried to do that. Our techniques have been simple and they do not require a detailed knowledge of cognitive psychology. However, our own knowledge of the subject has helped in two ways. First it has made us sufficiently interested in student thought to observe and categorize it. Second, it has made us sensitive to the ambitious nature of our undertaking and, we hope, a bit more patient with our students.

Constructing the Transformation

In order to move students out of the upper left hand corner of our conceptual matrix, R.L.G. developed a series of exercises which would as explicitly as possible make students aware of the type of reasoning expected from them. Over two semesters some 25 SITUATION sheets were written for the 70-80 students in two recitation sections of the calculus based course for engineers. These were accompanied by frequent references, both in lecture and in recitation on the need to go beyond algorithmic manipulation of information and to begin to think in terms of transformations. The first problem (figure 7) was given six weeks into the course; the results from 80 students are shown in figure 8.

Two features of the data deserve mention. Nearly a quarter (23%) of the students showed evidence that they did not understand the meaning of slope or velocity though they were able to correctly determine it in the second stage of the problem. Many correctly wrote $x = mt + b = {}^{5}/2t + 20$ but reverted to $M = \frac{x}{t} = \frac{45}{8}$ when asked to transform position to obtain velocity; i.e., they reverted back to the mode in which they were most comfortable -- SEEING.

The other aspect implicit in the data is what the outcome might have been had the intercept in the original problem been zero. Quite likely a very large fraction of the students would have answered all the questions correctly. The instructor would have been tempted to assume comprehension on their part and successful teaching on his.

The tendency of students to revert back to a SEEING type response when confronted with an ACTING type question has been referred to by Bruner[13] as "perceptual seduction". The exercise in figure 9 was used with 35 of the original 80 students to seek further evidence for this dominance of the perceptual over the conceptual. Seven students gave the answer t = 4 seconds. Several still quoted a slope of 2 in the REPRESENTING part in spite of considerable class discussion of that error on the SITUATION of figure 7.

These results can be summarized by saying that students, especially those in difficulty, resist go.. g beyond the information given.[14] Physics teachers have realized for a long time that multiple step problems are hard. But that description is inadequate; it is the qualitative nature of the steps rather than the number or their sequential character that makes them hard. Students who have only recently become capable of formal thought tend to be perceptually bound. They look for solutions within the problem specifics rather than by bringing general prin .ples to bear. Problems such as those given at the beginning of this paper encourage the former activity rather than help students to move beyond it.

A more recent example of paired examination questions is shown in figure 11 The correlated scores of 54 students in a large lecture course are shown in figure 12 These students received explicit instruction in "going beyond the information given." Problem 1 of this pair is conventional. Problem 2 is conceptually identical to problem 1 except for the numerical value of the acceleration. Problem 2 is not conventional in that students are given v as a function of x and asked questions about time t, force F, etc. not obtainable by simple arithmetic operations with numbers given. The student must bring something to the problem.

Comparison of figure 2 and figure 12 shows overall improvement in performance where comprehensive understanding is being tested. In particular, the data confirms the notion that teaching for flexible understanding reduced the number of students who perform poorly on both paired questions and shifts many students into the quadrant representative of doing well on both questions. The fact that all but one student in figure 12 is in the upper half of the graph is a desirable outcome of instruction. Still more encouraging is the fact that some students actually recognize the relationship between paired questions. They become aware of the many ways they are able to deal with a single situation.

One further comment concerning problem 2 of this pair of questions. J.L. described earlier in this paper how students used $V_f^2 = V_i^2 + 2ax$ to solve a problem presumably not designed to elicit its use. Very few students used this handy formula to solve problem 2 even though it is entirely appropriate. Solutions written by students on exam papers showed their attempt to use a variety of conceptual formulations in solving the problem and in particular their attempts to determine consistent solutions from different points of view.

A second approach to moving students beyond rote algorithms has been developed by J.L. This approach has involved a rather substantial modification of the curriculum in the form of a prephysics course in problem solving[15]. The goal of this course has been to have students practice transforming information between three representations: graphs[16], English, and equations. Figure 13 illustrates the types of problems used in this course. Throughout the semester there is a constant attempt to deny students access to the traditional algorithms. However, as we indicated earlier this effort is not always successful.

A critical feature of the course is that students are required to write out detailed verbal descriptions of how they solve each problem. This discourages mindless execution of algorithms since even the weakest students find that approach unsatisfactory once they come to writing it up. However, this is also the major disadvantage of the approach since grading papers is escessively time consuming. Until that problem can be solved the method will have only limited application.

Evaluation

In both approaches students showed significant progress in their ability to solve the particular types of problems we gave them. Figure 10 is an example from R.L.G.'s course, late in the second semester when Faraday's law was the topic of interest. The SITUATION of figure 10 is simply figure 9 with a relabled graph. The rate of change of magnetic flux was now understood as operationally similar to the rate of change of position. The general abstract meaning of slope, intercept, etc. were now more firmly held by students precisely because they could see the utility of such conceptions quite apart from situational specifics. The differences between situation specifics were now not so successful in inhibiting the use of conceptual similarities. This is the essence of moving from concrete operational to formal operational in the Piagetian sense. Student performance on figure 9 showed considerable improvement. While a few students still made errors, all students recognized the overlap in the two problems and all had begun to distinguish between SEEING, REPRESENTING, and ACTING. The data in figure 14 shows how student performance improved in J.L.'S prephysics course. On these specific types of problems students were significantly better than comparison groups two to three years their senior. However, we have no evidence that success on these problems transfers to an overall improvement in physics ability. Data on that issue will not be available until after these students have completed the two semester physics sequence. On the other hand, there is evidence which suggests that students who worked on the SITUATION sheets were able to transfer their skills. Their average grade on the course final was significantly higher than the average for all the other students in the course. Unfortunately, this data does not control for differences in the recitation instructors so the inference needs further support.

Some Unsolved Problems

One prediction of Piaget's theory is that people can only learn new concepts if they already possess the necessary building blocks. When they do not they will assimilate what they are told to "inappropriate" structures and may remain unaware of the discrepancy. Thus no matter how well we prepare an instructional sequence we will fail if the students are not ready for it.

One example is startling because of its simplicity: the concept of acceleration. While beyond the grasp of the concrete thinker, it should nevertheless be relatively easy for anyone skilled in formal operations. It rarely appears as a source of serious confusion in standard physics questions. However, when students are asked to graph acceleration as a function of time they usually experience difficulty.

The data in figure 13 is a composite of that collected from several schools. It shows that approximately 90% of the students who have had the mechanics portion of a calculus based course in introductory physics are unable to select the correct acceleration graph.

Surely this cannot be all that serious a confusion. If students are given training in drawing graphs and especially in sketching the shape of derivative graphs then the confusion should vanish. Their understanding can be further strengthened by explicit instruction concerning the difficult aspects of acceleration; one might for example repeatedly point out cases where the velocity is zero but the acceleration is not. This can be coupled with extensive laboratory experience and appropriate illustrations of F = ma. These were precisely the type of activities covered in prephysics.

Last fall, students in the prephysics course were selected to have average or above mathematical preparation and math SAT scores of 550 to 650. Figure 13 shows that while at the end of the course these students were substantially above average in their ability to select the correct velocity vs. time graph they showed little improvement in ability to select the correct acceleration graph. This was after one intensive semester specifically focused on developing that concept! To quote Fuller's[17] description of his own efforts to teach forces and their components, "Never have so many students spent so much time on such a small concept and shown so little mastery of it." We account for this discouraging performance by recognizing that unlike velocity or distance, acceleration can only be understood as an abstraction, i.e., a transformation of the velocity function. Clearly, the course was only partially successful in reversing twenty years of perceptually dominated learning and students were not yet fully capable of conceptual learning.

Cognitive theory has not yet provided the insight needed to overcome these difficulties. But it has shown us how to look for them; and how to isolate and study them. It may be that a full understanding of some concepts such as acceleration is not feasible in the first semester. If that is so we ought to know it and then work to better understand why that is the case.

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Problem 1 Standard Exam Question

1. A cannon placed on a hilltop fires a projectile at an angle of 30^0 above the horizontal with a muzzle velocity of 180 m/sec. The projectile is seen to strike its target in the valley below 20 seconds later. Neglect air friction and use $g = 10\ m/sec^2$.

   a) (5 pts) What are the horizontal and vertical components of the velocity?

   b) (5 pts) What is the horizontal displacement for the entire trajectory?

   c) (5 pts) What is the height of the cannon above its target?

   d) (5 pts) What is the vertical component of the final velocity?

   e) (5 pts) What is the speed of the cannon ball just before impact?

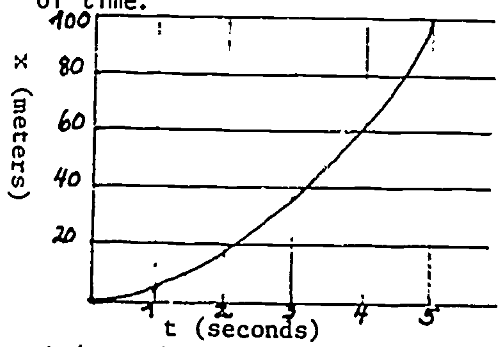
Figure 1

Problem 2 Question Requiring Active Mode

2. All of the questions below concern the motion of a body moving in one dimension. Parts a) to d) refer to body A and parts e) to g) refer to body B.

Body A

The following is a graph of the position of object A as a function of time.



a) (4 pts) Write an algebraic expression for the position of A as a function of time. Include any constants as numbers.

x(t) =

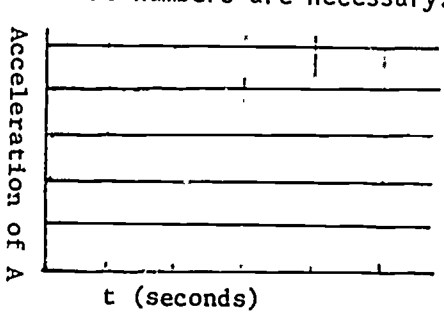
b) (4 pts) What is the average acceleration of A during the time interval t = 0 to t = 3 sec?

$\bar{a}$ =

c) (4 pts) What is the velocity of A at t = 4 sec?

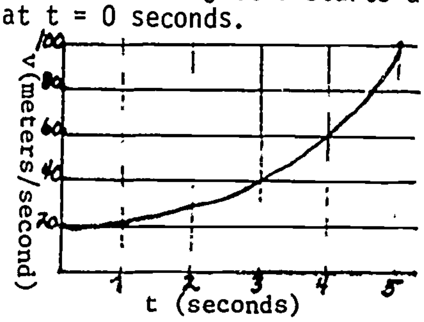
v(t = 4 sec) =

d) (4 pts) Sketch the graph of acceleration vs. time for body A. No numbers are necessary.



Body B

The following is a graph of the velocity of object B as a function of time. Assume that object B starts at the origin at t = 0 seconds.



The algebraic expression for this graph is $v(t) = 20\ m/sec + 4\ m/sec^3 t^2$

e) (3 pts) What is the average acceleration of B during the time interval t = 0 to t = 3 sec?

$\bar{a}$ =

f) (3 pts) Where is the object B located at t = 4 sec?

x(t = 4 sec) =

g) (3 pts) Sketch the graph of acceleration vs. time for body B. No numbers are necessary.

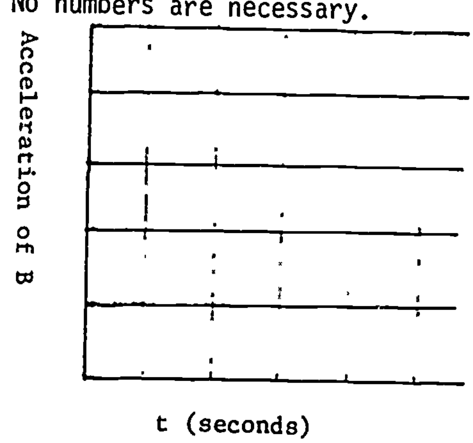


Figure 2

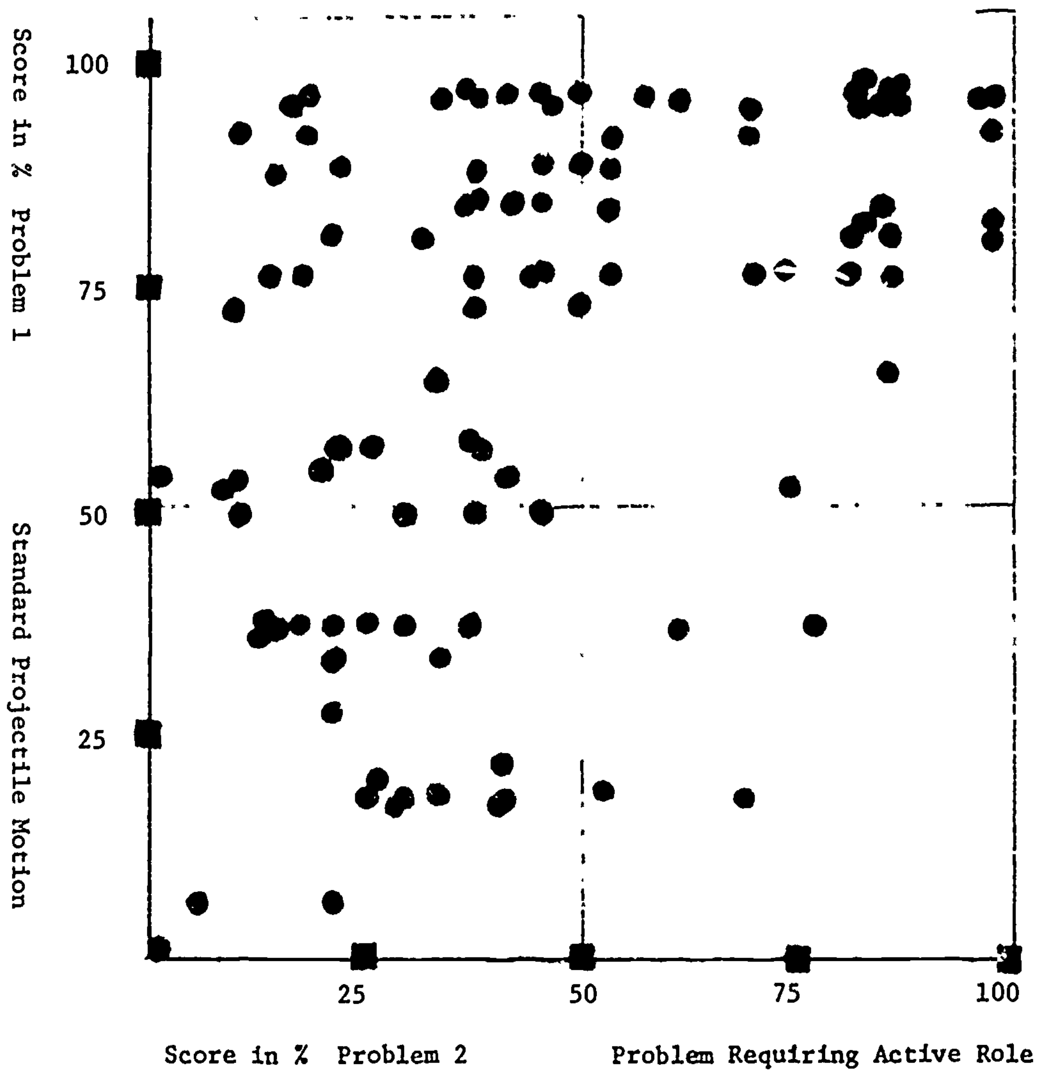
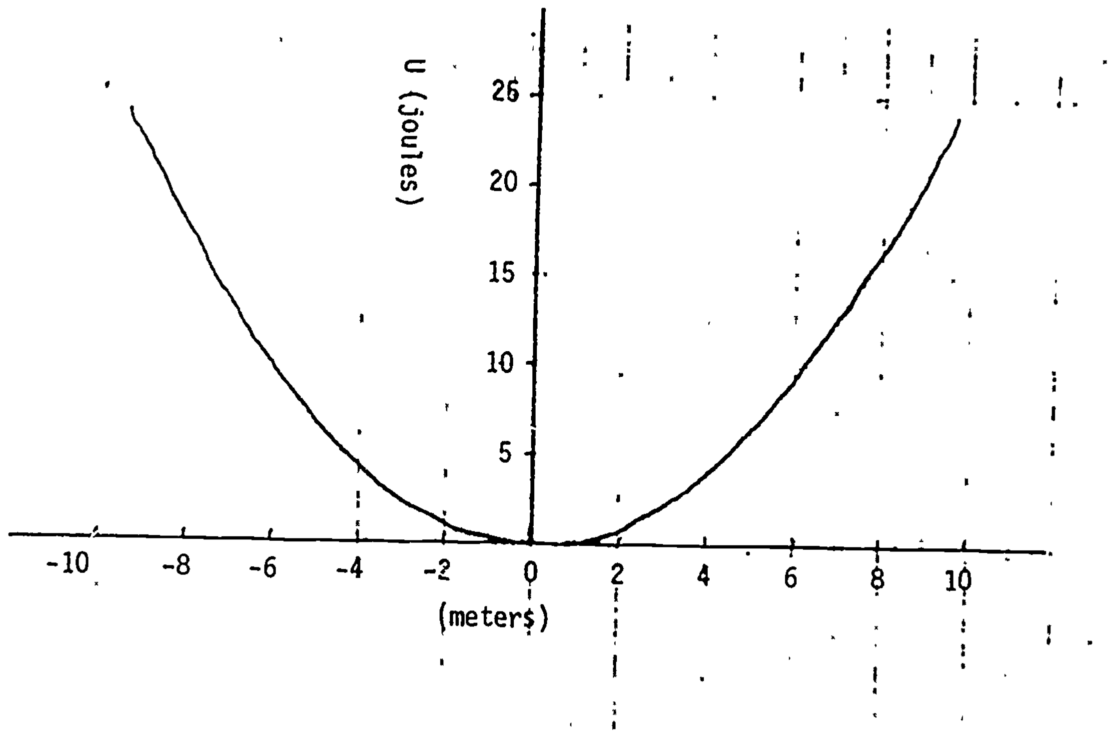

Score in % Problem 1
Standard Projectile Motion
100
75
50
25
25
50
75
100
Score in % Problem 2
Problem Requiring Active Role

Figure 3

5. An object is subject to conservative forces such that the potential energy of the object as a function of position is as shown below. If the object is placed at rest at x = +6m what will be the velocity of the object at x = -4m? The mass of the object is 0.5kg.



(b) Write an equation for U = f(x)

(c) What is the acceleration of the object at x = +4m?

Figure 4

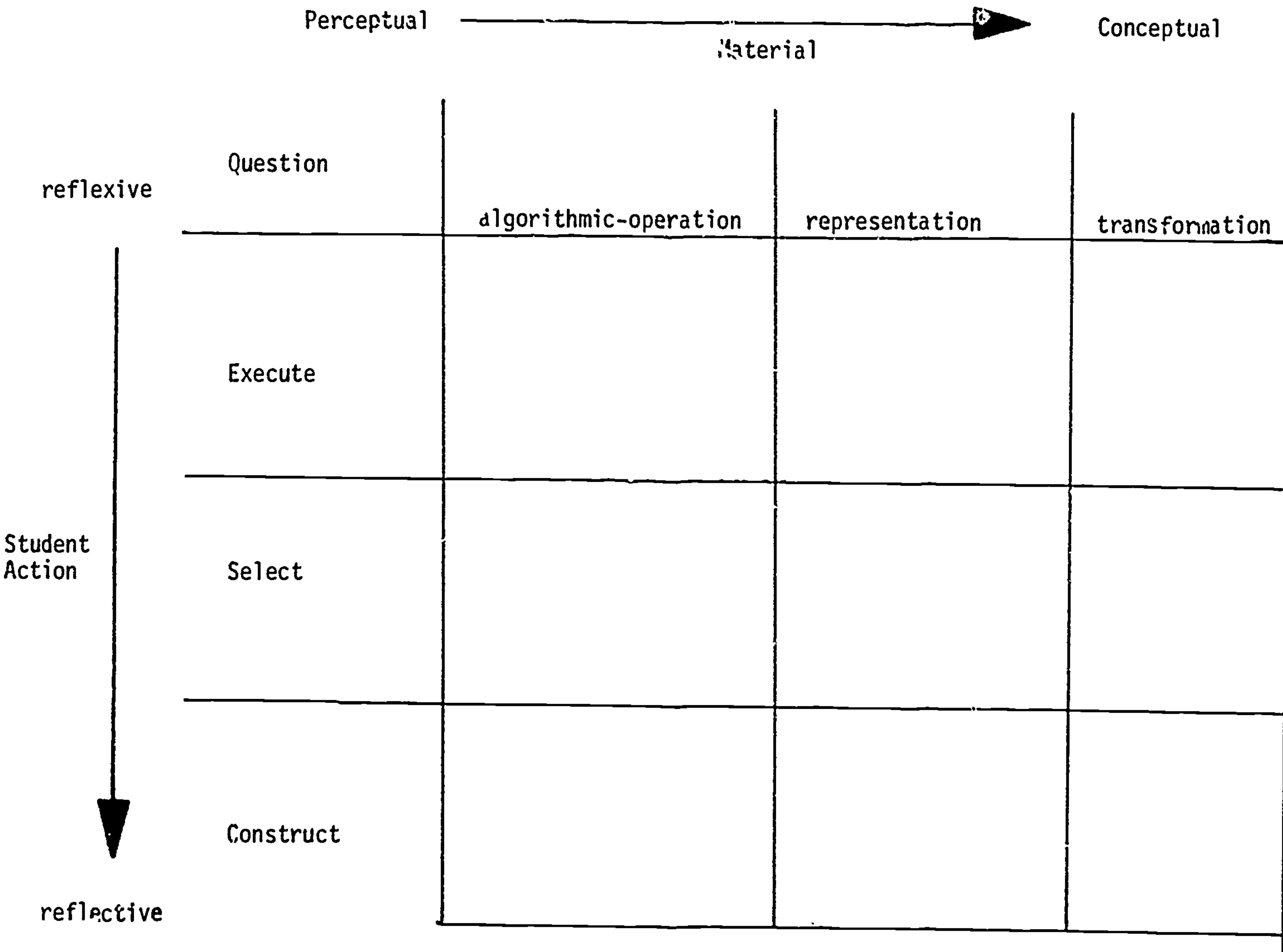
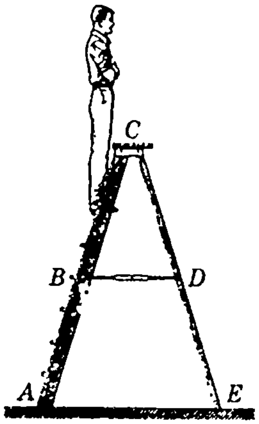

Perceptual
Material
Conceptual
Question
reflexive
algorithmic-operation
representation
transformation
Execute
Student
Action
Select
Construct
reflective

Figure 5

SITUATION

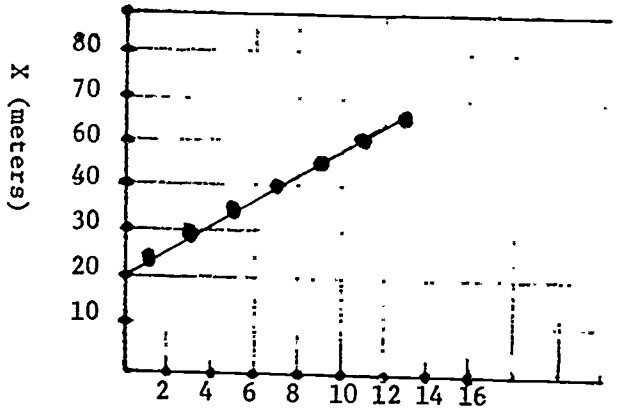


SEEING	REPRESENTING	ACTING
How many rungs does the ladder have?	Draw a free body diagram showing the forces acting on the man who is at rest.	How tall is the ladder?

Figure 6

SITUATION

The graph shown is data for the location (x) of a car as a function of time (t). The location is given in meters and the time in seconds.



SEEING	REPRESENTING	ACTING
What is the location of the car at t = 8 seconds?	Write an equation representing the data of the graph. Make sure that the equation is specific to this graph.	Given that speed is the rate of change of location, what is the speed of this car at t = 8 seconds?

Figure 7

The data below represent a total of 80 students in Sections 1 and 2

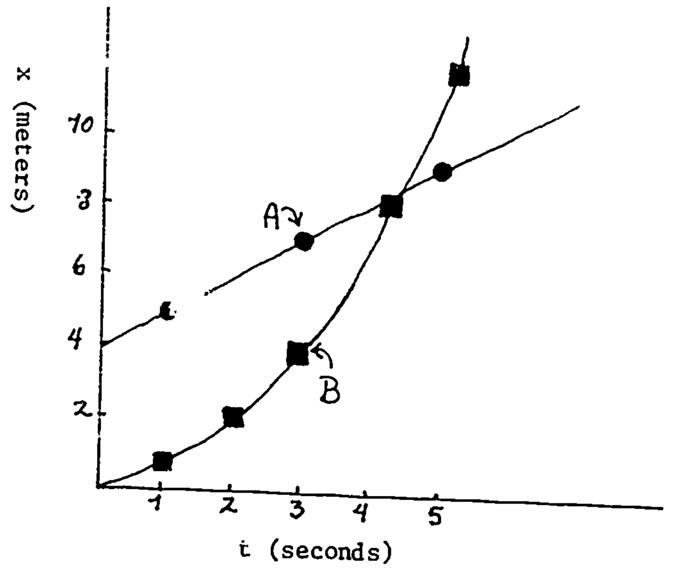
	% Response
I. Seeing	
1. Correct answer	97
2. Incorrect	3
II. Representing	
1. Correct answer	45
2. Incorrect because of slope and/or intercept	28
3. Incorrect with no redeeming features	5
4. No answer given	22
III. Acting	
1. Correct answer	22
2. Correct answer but inconsistent with II	4
3. Wrong answer but consistent with II	7
4. Wrong answer and inconsistent with II	21
5. Incorrect with no redeeming features	20
6. No answer given	26

Figure 8

SITUATION

The graph to the right represents the position x of two objects as a function of time t. The dots represent object A and the squares represent object B.

Answer the questions below about these two objects.



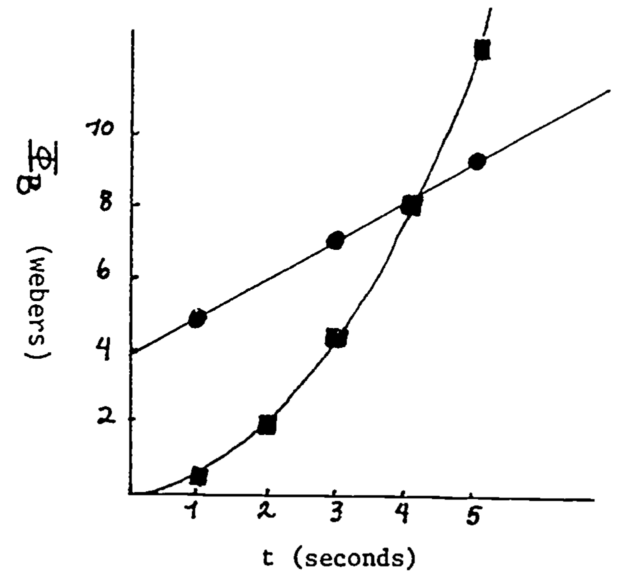
SEEING	REPRESENTING	ACTING
Where is object B at t = 4 seconds?	Write an equation representing the position of object A as a function of time. Any and all constants should appear as numbers.	At what time t do these two objects have the same velocity?

Figure 9

SITUATION

The graph to the right represents the flux of $\vec{B}$ in two cases as a function of time t. The dots represent case A and the squares represent case B.

Answer the questions below about these two cases.

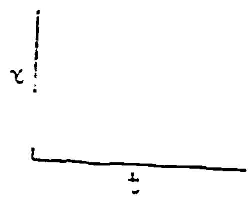


SEEING	REPRESENTING	ACTING
What is the flux of $\vec{B}$ in case B at t = 3 seconds?	Write an equation representing the flux of $\vec{B}$ in case A as a function of time. Any and all constants should appear as numbers.	Suppose the flux of $\vec{B}$ in both cases is associated with the same closed circuit in each case. At what time would the induced emf $\mathcal{E}$ be the same in both cases

Figure 10

1. An object moves according to the equation $x = 2t + [illegible].5t^2$

5 pts. a) Sketch this equation in the graph to the right. No numbers are necessary.

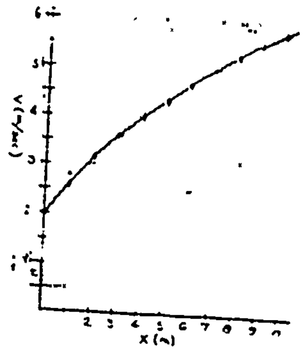


5 pts. b) Where is this object at t = 2 sec?

5 pts. c) Where is this object when it has doubled the velocity it had at t = 0 ?

5 pts. d) What is the acceleration of this object at t = 7 sec ?

2. An object of mass [illegible] is at x = [illegible] at t = 0. The object has <u>constant</u> acceleration [illegible] that its velocity varies with [illegible] as shown in the graph.



4 pts. a) Where is the object when its [illegible] is numerically the same as [illegible]?

4 pts. b) How much work has been done by all forces acting on this object as it moves from x = [illegible] to x = [illegible]?

4 pts. c) How long does it take this object to go from x = 3 to x = 4m?

4 pts. d) What total force must be acting on this object during its motion?

4 pts. e) Write an equation x = f(t) appropriate to specifics of this problem?

Figure 11

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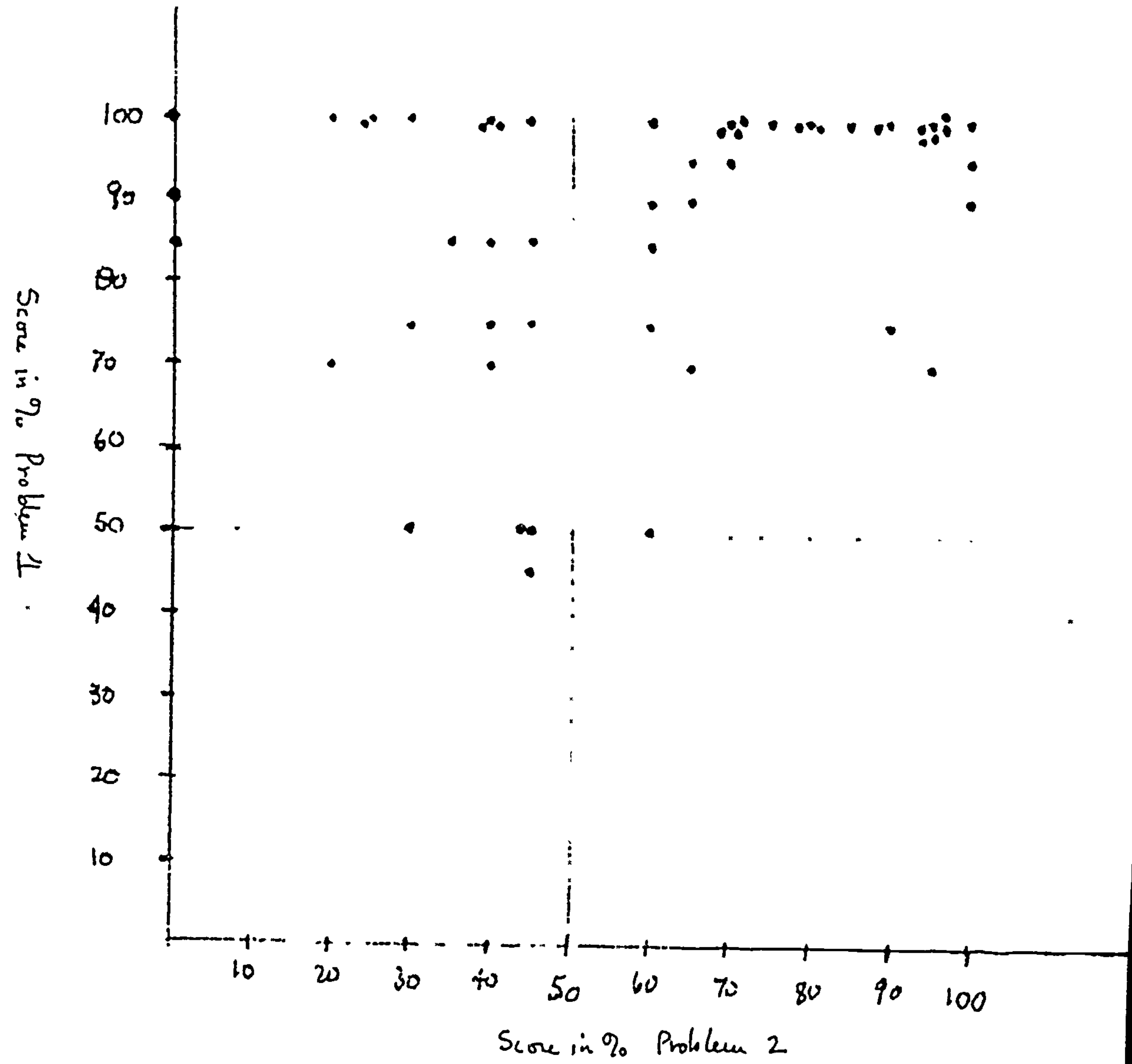

Score in % Problem 1
100
90
80
70
60
50
40
30
20
10
10
20
30
40
50
60
70
80
90
100
Score in % Problem 2

Figure 12

1.) Joan is given an electric train for Christmas. But her parents are cheap and she only gets enough track to make a four foot diameter circle. She soon tires of watching the train go round in circles and decides to look at it in as many different ways as possible. On a single piece of graph paper draw graphs for the following measures of the distance to the moving train.

a. Distance from the center of the circle.

b. Total distance covered along the track.

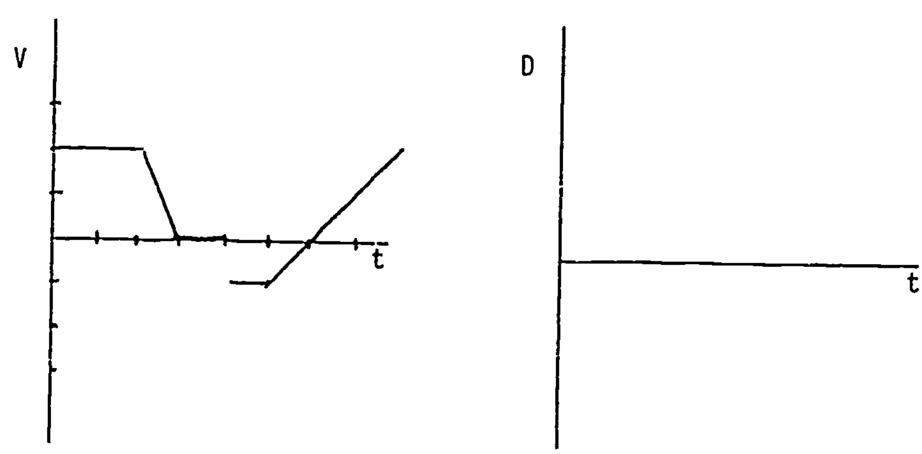
c. Distance from a point on the circle.

d. Distance above (+) or below (-) a diameter.

e. Distance from a point inside the circle (not the center).

f. Distance from a point outside the circle.

2.) Draw the distance vs. time graph corresponding to the following velocity vs. time graph.



3.) Write an equation using the variables C and S to represent the following statement:

"At Mindy's restaurant, for every four people who ordered cheesecake, there were five who ordered strudel."

Let C represent the number of cheesecakes ordered and let S represent the number of strudels ordered.

Figure 13

1. Weights are hung on the end of a spring and the stretch of the spring is measured. The data are shown in the table below.

Write an equation using the letters S and W to summarize the data below.

Stretch S(cm)	Weight W(g)
0	0
3	100
6	200
9	300

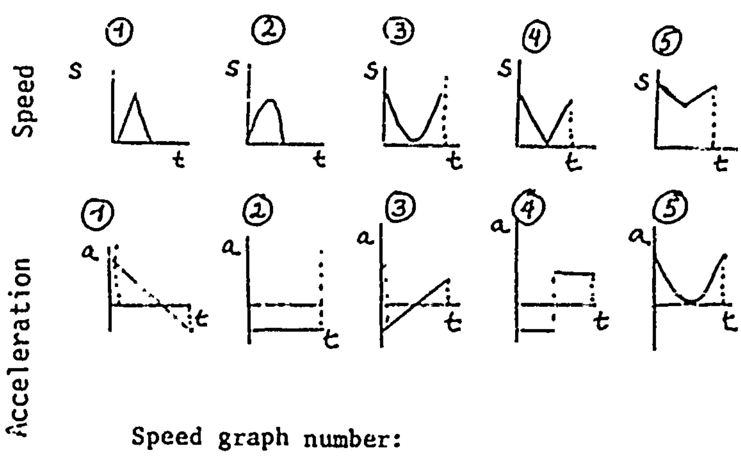
(The term weight was employed incorrectly inorder to make the problem consistent with popular language.)

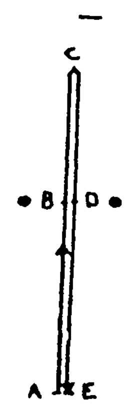
% Correct

prephysics freshmen	2nd yr. engineering
97	66

2. A coin is tossed from point A straight up into the air and caught at point E.

What is the shape of a speed vs. time graph and an acceleration vs. time graph for the coin while it is in the air? Ignore friction.





Speed graph number:

Acceleration graph number:

% Correct

prephysics freshmen	3rd 4th yr. mech. engineering
71	30
21	11

Figure 14