DOCUMENT RESUME

ED 294 766	SE 049 171

AUTHOR	Cauley, Kathleen M.
TITLE	Children's Misconceptions about the Multidigit Subtraction Algorithm.
PUB DATE	88
NOTE	10p.; Paper presented at the Annual Meeting of the American Educational Research Association (New Orleans, LA, April 5-9, 1988).
PUB TYPE	Reports - Research/Technical (143) -- Speeches/Conference Papers (150)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	Basic Skills; *Computation; Educational Research; *Elementary School Mathematics; Grade 3; Grade 4; Grade 5; *Intermediate Grades; *Mathematics Achievement; Mathematics Curriculum; Mathematics Education; Mathematics Instruction; Mathematics Skills; *Misconceptions; *Subtraction

ABSTRACT

This study probed children's reasoning about both correct and incorrect but plausible statements of hypothetical children concerning the concepts of subtraction, with the intention of examining misconceptions in greater depth. Eight third graders, 14 fourth graders, and 14 fifth graders were interviewed individually to assess their understanding of regrouping in multidigit subtraction. The results illustrate the firmness of children's misconceptions about the regrouping procedure in particular and place value in general. (PK)

CHILDREN'S MISCONCEPTIONS ABOUT THE MULTIDIGIT SUBTRACTION ALGORITHM

Kathleen M. Cauley

Virginia Commonwealth University

Paper presented at the Annual Meeting of the American Educational Research Association (New Orleans, LA, April, 1988).

Supported by the Faculty Grant-in-Aid program of Virginia Commonwealth University.

Children's Misconceptions about the Multidigit Subtraction Algorithm

Kathleen M. Cauley

Virginia Commonwealth University

Objectives

Hiebert and Lefevre (1986) observe that it is not yet possible to specify how links between procedural and conceptual knowledge are established, nor how instruction can facilitate these linkages. To that end it is important to understand the reasoning of mathematically able children and especially their common misconceptions. This study probes children's reasoning about both correct and incorrect but plausible statements of hypothetical children concerning the concepts of subtraction to examine misconceptions in greater depth.

Theoretical Framework

Children's developing mathematical knowledge is an especially interesting question because the research indicates that the relationship between procedural knowledge (knowledge of step-by-step rules or algorithms) and conceptual knowledge (bodies of information with rich, elaborate relationships) appears to change with age (Hiebert and Lefevre, 1986). It seems clear that before formal mathematics instruction, children's conceptual and procedural knowledge are closely related (Gelman & Meck, 1983, 1986; Baroody & Ginsburg, 1986; Carpenter, 1986; Carpenter and Moser, 1984). Once children begin formal instruction in arithmetic, however, conceptual and procedural knowledge diverge and seem to develop independently. Many children learn to manipulate symbols, or follow procedures acceptably well, but they have little knowledge of the meaning of

Supported by the Faculty Grant-in-Aid program of Virginia Commonwealth University. Presented at AERA, April, 1988.

the symbols, the procedures, or their final problem solution (Hiebert, 1984; Hiebert and Lefevre, 1986; Cauley, 1986; Resnick, 1986, Resnick & Omanson, 1986).

It is important for conceptual and procedural knowledge to be linked, Hiebert and Lefevre (1986) argue, because the relationship between the two is necessary for full competence in mathematics. Yet, in spite of this importance, Hiebert and Lefevre (1986) maintain that the building of relationships among conceptual and procedural knowledge is a difficult problem. One of the reasons cited is that knowledge acquisition appears to be context bound and compartmentalized. For example, Resnick and Omanson (1986) extensively trained students who employed "buggy" subtraction algorithms with a procedure that concretely mapped each step of the borrowing algorithm with blocks. Students continued training until they perfected it with five types of problems, including ones that elicited their "bug," before the physical manipualtion of blocks was faded out. Yet, at the delayed posttest, only 20% of the students had complete understanding of the mathematical concepts relevant for subtraction, and only those students had eliminated their bugs. What they had learned in one context did not transfer to other situations. Cauley (1987) found that even children who were procedurally proficient with multidigit subtraction were notably lacking in their understanding of the values that were exchanged during regrouping and the conservation of the minuend. Subtraction is one of the most basic arithmetic operations. Problems here will only be compounded during later mathematics learning.

Subjects

Eight third graders, 14 fourth graders and 14 fifth graders who were identified as procedurally proficient in multidigit subtraction were interviewed. These students represented a wide range of mathematical ability with scores on the Scott Foresman Comprehensive Achievement Test ranging from the 34th percentile to the 99th percentile with a median of 80.

Procedure

Students were interviewed individually in a quiet room of the school for 10 to 30 minutes to assess their understanding of regrouping in multidigit subtraction. In the first part of the interview students solved two subtraction problems and answered questions which probed their understanding of the values exchanged during regrouping and their knowledge that the value of the minuend was conserved. The following questions were asked for the first problem "426 - 184" (all problems were written in vertical form for the children):

a. Can you tell me, why you crossed out this number (the 400)?
b. How much did you borrow (take away, etc.) from that column? How can you tell?
c. Probes (asked for the other two possible responses): Some kids say they took 100 away from that column. Do you think they're right or do you think they're wrong? Why? Repeat question c using 10 and 1 as appropriate.
d. What did you do with what you borrowed (took from this column, etc.)?
e. How much did you put in this column (tens)?
f. Probes (asked for the other two possible responses): Some kids say they put 100 in that column. Do you think they're right or do you think they're wrong? Why? Repeat question f using 10 and 1 as appropriate.
g. Before you borrowed you had 426, and now you have all of this (circle the minuend and all borrowing marks). Do you still have 426 here, or do you have more or less? Why? How much do you have?

The same questions are repeated for problem 2 (824 - 495) but the probes c and f are eliminated.

Results and Conclusions

First, the results indicate that only three of the thirty-six procedurally proficient students spontaneously argued that they exchanged 100 during regrouping for the first problem, "426-184." The most common alternative, offered by 12 students, was that they borrowed a one because "four minus one equals three" and put a ten in the tens column because "ten plus two equals twelve." The next most common alternative, offered by 10 students, was that ten was exchanged, arguing in various ways that they "need a ten to put with the two to make twelve." The eleven remaining students gave a variety of answers, including "don't know."

The succeeding probes (varied according to the student's initial, spontaneous response) demonstrated that an overwhelming number incorrectly rejected the possibility that they added 100 to the tens column. Of the 31 students who were asked if they could be borrowing 100, 13 (40%) correctly said that it could be right, "because it's the 100's column. Fourteen (52%) said that it could not be 100 for reasons like:

"It's really a 1, but you're putting a 10 with the 2;"

"You don't need 100 to make 10;"
"You can't take 100 away."

Of the 32 students who were asked if they could be putting 100 in the tens column, only one said that it could be right. The justifications of students who rejected the possibility included:

"You would have thousands (or 102, or 200, or 112);"

"You only need a 10."

While 13/31 students agreed that it was possible that 100 was borrowed from the hundreds column, only 4 ultimately adopted that response following the probes.

Students were equally clear about their rejection of the possibility that a value of one was exchanged. Most of the students, however, justified that response incorrectly. Twelve of the 19 students gave incorrect justifications such as:

"because you borrowed a 10"

"Because it would make this a 3 instead of 12."

Only four students correctly justified their response with reasons such as:

"because you borrowed from the 100's"

"it would be 399 in the hundreds place."

Of the 33 who were asked if they could put a one in the tens column, 3 said no and gave a valid reason. Twenty-four other students also said that one could not be put in the tens column, but they gave an invalid reason such as:

"because it would make this a three instead of twelve;"

"because you can't put a one in the tens column."

Both before and after the probes, most students argued that a ten must be put in the tens column. Before the probes 11 (31%) of the students incorrectly believed that they were borrowing a ten and 27 (75%) incorrectly believed they put ten in the tens column. In addition, of the 25 students who were asked if they <u>could</u> borrow a ten, 9 (36%) reasoned correctly and 16 (64%) reasoned incorrectly.

Following the probes, 16/36 students incorrectly believed that they exchanged 10 when solving problem 2. Ten others also believed that they put a ten in the tens column, but they claimed to have borrowed another number. Altogether, 26 students claimed to borrow 10 after the probes, giving reasons such as:

"ten plus two is twelve;"

"you need a ten to add to the two so you can subtract."

A similar pattern was obtained when students were questioned about the conservation of the minuend. All four of the students who argued that 100 was exchanged also conserved the minuend. Three of the students who claimed that 10 was exchanged also conserved the minuend. Their rationales for conserving included were:

"because you didn't add or subtract any, you just used the numbers you had."

"because if you add 300 + 20 + 6 you get 426;"

"because you took 100 from there and put it there so its still the same."

The rest argued against conservation, giving reasons like:

"You changed the numbers around and made it more;"

"It's 3,126 now;"

"It's 326 now."

Educational Significance

The results illustrate the firmness of children's misconceptions about the regrouping procedure in particular and place value in general. The majority of students firmly believe that the regrouping procedure of the subtraction algorithm cannot involve the exchange of 100 from the hundreds to the tens column for three digit problems like "824-495."

Students' belief that neither 100 nor one can be put in the tens column, that it has to be a ten, could stem from instruction in addition which requires students to "carry" 100 because you can't have more than 90 in the tens column. The regrouping algorithm asks students to violate that rule.

The larger problem, however, appears to be that students learned a task specific concept that they tried to generalize incorrectly to another task, rather than the more general principle of additive compositon which would apply equally well to both situations. As research on the relationship between concepts and procedures progresses, it will be necessary to distinguish between children's acquisition of those task specific concepts and the more general principles.

References

Baroody, A. & Ginsburg, H. (1986). The relationship between initial meaningful and mechanical knowlede of arithmetic. In J. Hiebert (Ed.) Conceptual and procedural knowledge: The case of mathematics (pp. 75-112). Hillsdale, NJ: Erlbaum.

Carpenter, T. (1986). Conceptual knowledge as a foundation for procedural knowledge. In J. Hiebert (Ed.), Conceptual and procedural knowledge: The case of mathematics, pp. 113-132. Hillsdale, NJ: Erlbaum.

Carpenter, T. & Moser, J. (1984). The acquisition of addition and subtraction concepts in grades one through three. Journal for Research in Mathematics Education, 15, 179-202).

Cauley, K. (in press). The construction of logical knowledge: A study of borrowing in subtraction. Journal of Educational Psychology.

Cauley, K. (1987, June). Constructing the logic of arithmetic. Seventeenth Annual Symposium of the Jean Piaget Society, Philadelphia, PA.

Gelman, R. & Meck, E. (1983). Preschoolers' counting: Principles before skill. Cognition, 13, 343-359.

Gelman, R. & Meck, E. (1986). The notion of principle: the case of counting. In J. Hiebert (Ed.), Conceptual and procedural knowledge: The case of mathematics, pp. 29-58. Hillsdale, NJ: Erlbaum.

Hiebert, J. (1984). Children's mathematics learning: The struggle to link form and understanding. The Elementary School Journal, 84, 497-513.

Hiebert, J. & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.). Conceptual and Procedural Knowledge: The Case of Mathematics, pp. 1-27. Hillsdale, NJ: Erlbaum.

Resnick, L. (1986) The development of mathematical intuition. In M. Perlmutter (Ed.) Perspectives on Intellectual Development, Vol. 19 of the Minnesota Symposia on Child Development. Hillsdale: NJ: Erlbaum.

Resnick, L. & Omanson, S. (1986). Learning to understand arithmetic. In R. Glaser, (Ed.), Advances in instructional psychology, Vol. 3, pp. 41-96. Hillsdale, NJ: Erlbaum.