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ABSTRACT

This digest suggests that research findings can provide teachers with verification and clarification of how children learn mathematics, and thus how to teach mathematics more effectively. Three topics are discussed: (1) number concepts; (2) problem solving; and (3) manipulative materials. (PK)

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MATHEMATICS LEARNING IN THE ELEMENTARY SCHOOL

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MATHEMATICS LEARNING IN THE ELEMENTARY SCHOOL

Through the years, the findings of clusters of research studies on some topics have provided teachers with verification and clarification of how children learn mathematics, and thus how to teach mathematics more effectively. Studies published from 1986 on three topics provide an example of such clusters.

Number Concepts: The Base for Arithmetic Work

How children develop number concepts has been the focus of much exploratory work, with particular attention to ways children learn to count. Less attention has been given by researchers to place value, yet we know that children frequently demonstrate poor understanding of the principles of our base-ten system. Understanding of place value requires children to coordinate knowledge of numerals with conceptual understanding of the numbers represented by those numerals. Continuing work by other researchers, Ross described stages in the development of children's understanding of place value. She used six tasks in which children were asked to identify the number of objects represented by each digit in a two-place numeral. Among the 15 children at each grade level, only seven fifth graders, seven fourth graders, two third graders, and no second graders were successful on all six tasks; 28 children failed to demonstrate convincingly any understanding of the tens digit.

To be successful on more than two of the tasks, children have to understand part-whole relations reflected by success on word problems and on logical classification tasks. To be successful on more than four of the tasks, children have to be able to:

- count rationally by tens,
- identify the tens and ones places,
- partition collections into a tens part and a ones part, and
- conserve the grouped number.

Children need help in developing the knowledge and concepts for each of these prerequisites.

Because of their high performance on international tests of mathematics achievement, the way Japanese children are taught mathematics is being studied. Among the differences between the way North American children and Japanese children approach numeration is the attention given in Japan to 5 as a key number. Yoshida and Kuriyama found that children in kindergarten appeared to represent numbers to 5 as an "anchor" for the numbers from 6 to 10. Many children could grasp the concept of fiveness more readily than they could that of tenness, and therefore were able to make better progress in learning. They thought of 7, for instance, as 5 and 2 more; of 9 as 5 and 4 more: they were thus using counting on to help them attain increased understanding and recall. Five served as a base to aid them mentally.

Problem Solving: The Difficulty Continues

Data from each of the four national assessments of mathematics have indicated that problem solving continues to be a major point of difficulty in the curriculum. In an analysis of elementary school mathematics textbooks from 1984, McGinty, Van Beynen, and Zalewski found that the number of problems is only one-third of the number appearing in textbooks from 1924 or 1944, while the number of "drill problems" or exercises has increased. Some evidence from a 1985 study indicates that the number of problems and their variety has increased during the 1980s, but it remains a factor in teaching problem solving: the number of problems children meet affects their problem-solving ability. Teachers may need to add more problems in their instruction, as well as focus children's attention on ways to go about solving problems.

Having children write problems is one way to increase the number of problems they meet — and it can enhance problem-solving performance, according to a study with fifth graders by Malina which is supported by earlier research. One group had five lessons on how to write problems for each operation, while another group spent an equivalent amount of time solving problems like those the other group was being taught to write. Writing problems not only got children personally involved, but also helped them to focus on needed and unneeded information in problems.

The strategies children in grades 1-3 used to solve multiplication and division word problems were studied by Kouba. Children solved the problems in a variety of ways, with strategies reflecting the semantic structure of the problems. Direct modeling was used by most successful children in grades 1 and 2, while transitional counting strategies were used by some second-grade and a few third-grade children and recalled number facts were used by most successful third-graders. Little use was made of repeated addition and almost no use of repeated subtraction. Children who were successful on multiplication and division problems were, however, also successful on addition and subtraction problems, although 15 percent of those successful on addition and subtraction problems could not solve multiplication and division problems. This indicates that strategies which are successful for solving addition and subtraction problems are necessary but not sufficient for solving multiplication and division problems. Teaching the links between direct modeling strategies and the higher level strategies seems necessary, as well as the relationship between repeated addition and multiplication and between repeated subtraction and division.

Duncan described what fifth graders say and do when solving verbal mathematics problems in small groups. Each of three four-member groups was asked to reach a consensus on the solution of some problems. The children displayed some general patterns of behavior:

- the manner in which the groups approached and effectively isolated the contextual elements of a verbal problem,
- the propensity of groups to change the mode in which a problem is represented by using manipulatives, diagrams, tables, and other physical displays, and
- the manner in which groups monitored the course of problem solving and reached consensus on solution proposals.

Bebout investigated the effects of teaching children to represent addition and subtraction verbal problems with number sentences directly reflecting the structure of the problem. Children were taught to write canonical number sentences (such as $7+5=?$ or $8-3=?$) and noncanonical number sentences (such as $9+?=11$ or $?-7=8$) to reflect the informal strategies they used to solve problems with concrete objects. Before instruction, nearly 60% of the first graders solved problems that could be directly represented with canonical number sentences; less than 40% solved problems that could not be directly represented with canonical number sentences. After instruction, over 60% solved all classes of problems.

Similarly, before instruction, less than 25% wrote correct number sentences for problems that could not be represented directly by canonical number sentences. After instruction, nearly 80% wrote correct number sentences for all basic problem types. Thus, after instruction children wrote either canonical or noncanonical sentences that directly represented the action described in the given problems.

Earlier research indicated that before instruction students were most likely to write sentences modeling the action described in a problem, whereas after instruction they wrote sentences representing the solution means, without attention to the action described. With careful development, they can be taught to write sentences directly representing the action. This can be helpful as they proceed in mathematics.

Manipulative Materials: Use Them!

Grounding concepts in concrete experiences and careful checking for understanding seem vital, according to Katterns and Carr, as it has to many other researchers and teachers.

Working with students in grades 5-7, Beattys reported that significant differences favored the group using manipulatives in learning area measurement, compared with a group just using the textbook. Manipulative-using students could solve area measurement problems that they were unable to solve prior to the use of manipulatives. This compares to textbook students' success rates of 18% for grade 5 and 30% for grade 6.

The amount of teacher-pupil interaction may be a deciding factor: Blalock reported that pupils in kindergarten achieved equally well using manipulatives or worksheets when lessons involved equal amounts of teacher-pupil interaction. This agrees with the conjectures of others that perhaps the primary effect of manipulative materials is to enhance the amount and quality of communication about mathematics.

Finally . . .

All of these findings point to a major point: the teacher must make many decisions about how and when, as well as what, to teach the particular students in his or her classroom. Teaching is a decision-making process: research findings about how children learn can provide aid in helping teachers as they make decisions.

REFERENCES

Beattys, Candice B. *The Effects of Instructional Approach and Spatial Ability on the Learning of Area Measurement in Intermediate Grades.* Dissertation Abstracts International 46A: 2604-2605; March 1986.

Bebout, Harriett C. *Teaching Children to Symbolically Represent Addition and Subtraction Verbal Problems with Number Sentences That Represent Problem Structure.* Dissertation Abstracts International 47A: 1228; Oct. 1986.

Blalock, Frances V. *Effectiveness of Manipulation in the Development of the Concept of Place Value in Kindergarten Children.* Dissertation Abstracts International 47A: 1587-1588; Nov. 1986.

Duncan, James E. *The Heuristics Utilized by Fifth Grade Students in Solving Verbal Mathematics Problems in a Small Group Setting.* Dissertation Abstracts International 46A: 2916-2917; April 1986.

Katterns, Bob and Carr, Ken. "Talking with Young Children about Multiplication." *Arithmetic Teacher* 33: 18-21; April 1986.

Kcuba, Vicky L. *An Investigation of Children's Solution Strategies for One-Step, Grouping and Ratio Forms of Multiplication and Division Word Problems.* Dissertation Abstracts International 47A: 112; July 1986.

Malina, Eva A. S. *Problem Writing as a Means of Enhancing Problem Solving Performance.* Dissertation Abstracts International 47A: 2006-2007; Dec. 1986.

McGinty, Robert L.; VanBeynen, John, and Zalewski, Donald. "Do Our Mathematics Textbooks Reflect What We Preach?" *School Science and Mathematics* 86: 591-596; Nov. 1986.

Ross, Sharon H. *The Development of Children's Place-Value Numeration Concepts in Grades Two Through Five.* Dissertation Abstracts International 47A: 819: Sept. 1986.

Yoshida, Hajime and Kuriyama, Kazuhiro, "The Numbers 1 to 5 in the Development of Children's Number Concepts." *Journal of Experimental Child Psychology* 41: 251-266; April 1986.

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