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ABSTRACT

This "letter to teachers" suggests reasons teachers should be interested in research findings. Research findings in several areas are summarized and interpreted so that they are useful to teachers of elementary school mathematics. Among the areas that are highlighted are: (1) time allocation; (2) low achievement; (3) active teaching; (4) use of materials; (5) calculating and computing; (6) broadening the curriculum; (7) communicating about mathematics; (8) using what children know; (9) learning from each other, and (10) providing equal access. (PK)

RESEARCH ON INSTRUCTION IN ELEMENTARY SCHOOL MATHEMATICS

A LETTER TO TEACHERS

ERIC Clearinghouse for Science, Mathematics and Environmental Education

Information Bulletin No. 3, 1987

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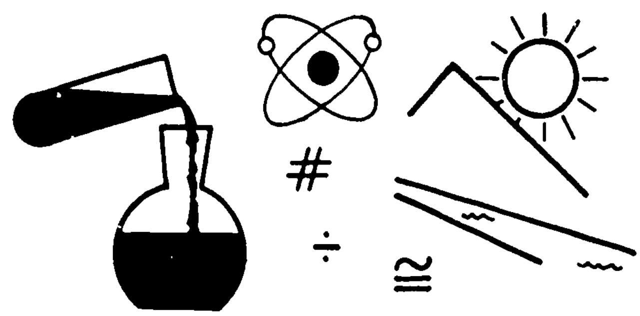

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RESEARCH ON INSTRUCTION IN ELEMENTARY SCHOOL MATHEMATICS: A LETTER TO TEACHERS

Dear teacher of elementary school mathematics:

Like many teachers, I once believed that research had little if anything to say to classroom teachers. Somewhere along the way, however, I found an idea from research that was meaningful to me . . . and then another idea . . . and another. Research doesn't provide all the answers, certainly. There are huge gaps in what we know: it resembles a jigsaw far more than an encyclopedia. Yet the missing pieces are being filled in, although slowly. The effort to interpret what the existing research indicates for classroom practice, and make more sense of the process of teaching, is one with which we all struggle.

The acceptability of research still faces a double-sided problem:

(1) Teachers tend to want research to agree with "common sense" or "conventional wisdom." We must, in short, believe that what research indicates is true, that it reflects what goes on in our classrooms and therefore can be applied in our classrooms.

(2) Nevertheless, if research agrees with common sense, then many teachers do not consider it worthwhile, since "we knew it all along."

Yet some of the findings of this research that agree with common sense can be very exciting to teachers interested in learning more about causes and effects between their actions and children's behaviors. Other research provides new insights on why children learn, or fail to learn — with clues on how we can help them to better.

I'd like now to lay out some of these research findings for you. The headings indicate only the starting focus of each [illegible]n. Skim or read more carefully: choose the method that suits your time and interest.

Starting the Year

Earlier, I noted that some research findings are interesting because they confirm what we have long known or been told — but now that knowledge moves from the category of "folklore" to "evidence." For instance, consider some of the findings from research on effective teaching.

The first three weeks of the school year appear to be crucial in determining how well children will achieve for the remainder of the year. Probably we all believe that it's important to "get the school year off to a good start." This means many different things to different teachers, of course, but among those things are usually the ideas of developing with the class patterns of orderliness, a knowledge of what the rules of expected behavior are, and a sense of the meaningfulness of instruction. For elementary teachers, of course, such ideas pertain to every subject area, but let's consider them just in relation to mathematics instruction.

There is evidence that mathematics achievement at the end of the school year is related to the degree to which teachers establish good control in the very first week of the school year. It's not too late to expect good achievement if control is not established until the second week — but if it isn't established by the third week, achievement during the entire remainder of the year will probably suffer.

Of course, you might say: without control of the class, it's not logical to expect children to learn. We have to have their attention before we can teach them. Without control, time is wasted on disciplinary actions. With control, learning is the focus of attention for both students and teacher, and thus both teaching and learning can proceed smoothly. Yet never before have we had evidence that the importance of establishing good control of the class during the first weeks of school is as important as we've heard. It gives us the little extra incentive to focus on establishing control early, and put good management ideas into practice.

Spending Time

Clearly related to the above idea is time: mathematics achievement, like other content area achievement, is related to the amount of time children spend on task. When time is wasted, whether on disciplinary actions, on delays in passing out needed materials, on interruptions from outside the classroom, or on maintaining lesson pace and flow, that means time is lost from opportunity to learn. As data from the Second International Mathematics Study indicated, opportunity to learn is a prime determinant of achievement: clearly, if children have not been introduced to a topic, or given sufficient time to absorb it, they cannot achieve mastery of it.

Promoting Low Achievement

Many teachers appear to be unaware that they are promoting low achievement for a number of their pupils. Consider how teachers tend to treat low achievers — very differently from the way they treat high achievers. For instance, low achievers are:

- seated farther from the teacher or in a group — sometimes where the teacher seldom looks (even though we know how vital eye contact with all students is)

- given less attention in academic situations (and more attention for misbehavior — thus encouraging them to seek attention through misbehavior)
- called on less often, thus providing them with fewer chances to participate
- given less wait-time (that is, the time between the teacher's question and the move to a second pupil for a response)
- provided with fewer clues and asked fewer follow-up questions when they answer incorrectly
- praised less frequently after correct answers
- praised more for marginal or inadequate answers
- given less accurate and less detailed feedback — less frequently
- required to do less work and put forth less effort
- interrupted more frequently

Thus, in numerous ways, low achievers are being "told" that they don't "count" for as much as faster achievers, that they aren't **expected** by the teacher to achieve (and certainly not to achieve as much as others), and that they really aren't worth listening to. They are expected to fail or do poorly — and they do.

There is also evidence on how teachers interact with students. When a group of teachers was told that some pupils were "bright" or "smart", they talked to and with them in a friendlier, more encouraging, and more accepting manner than they did with supposedly less endowed pupils. And as these pupils responded by achieving better (even though they were actually not "brighter" or "smarter"), the teachers' interaction with them became increasingly positive: teachers spent even more time talking with them.

Remembering They're Different

Other research has indicated that, when they were asked, teachers had different perceptions and expectations for classes at different ability levels. But the behaviors of these teachers did not reflect this. Virtually no differentiation in instruction was found, in either content or methods, even when teachers had verbalized that the students were different and had different mathematical needs. They used the same patterns of instruction for both higher- and lower-ability classes.

What **does** it make sense to do with low achievers? First and foremost, expect them to learn — and communicate this to them. Check over the list of behaviors teachers exhibit toward low achievers, and guard against using them and thus communicating to low achievers that you don't think they are "worth" as much as other learners. Plan realistically to meet their needs, both in terms of content and in the ways you teach them and expect them to learn. And consider the findings of yet other research, which indicates that low achievers profit from brief periods of intensive instruction, interspersed with opportunities to practice. It is very difficult to have low achievers (and other students, for that matter) spend extended periods of time on seatwork or similar forms of practice: they get tired, and bored, and noisy, and disruptive. Remember how difficult you find it to sit in long meetings — and plan the day so quiet activities will be interspersed with activities requiring movement. They need to be interested and active in the process of learning if it is to occur.

Different children begin from differing perceptions and knowledge bases, they have differing learning styles, they are motivated by different factors. Children who do not see the value of schooling, much less the value of mathematics, need alternative approaches. The teacher must do more than give lipservice to the idea that "individual needs must be considered": instruction must change to reflect the fact that we are aware of these individual needs.

Ingredients for Being More Successful

Teachers who are more successful at teaching have at their command a large number of strategies and techniques. Where the poorer teacher tends to follow the same pattern every day, the better teacher makes it a point to vary lessons. Why, mathematics can be exciting and something to be looked forward to, rather than a dull routine!

Successful teachers try to get their pupils involved in the lessons. An active teaching model includes such elements as these:

- hands-on use of materials
- small-group instruction
- consistent feedback
- systematic correction of errors
- reinforcement
- cumulative review of major skills and ideas
- rapid pacing of lessons — not so fast that pupils can't follow, but fast enough so they retain interest

Moreover, more effective teachers:

- state concern for achievement more often
- give more academic encouragement
- are more receptive to student input
- expect students to learn — and let them know it
- monitor pupil behavior carefully and react quickly to stop inappropriate behavior
- are more in touch with pupil needs, anticipating difficulties children might have
- give directions more clearly

From other observations during mathematics lessons comes evidence that teachers are more effective when they:

- provide many examples
- ask a large number of questions and give opportunities for pupils to participate actively in the lesson
- pose more questions to the entire group, thus increasing pupils' opportunities to respond
- ask more process questions, calling for an explanation or discussion
- ask more new questions after correct answers have been given, to encourage students to verify their answers.
- encourage students to ask questions — to clarify points, to pursue a puzzling idea that's occurred to them, to request help
- provide feedback as soon as possible after an answer or action has happened
- divide seatwork into smaller assignments
- provide for continuous student practice of new and already taught material

From other research come findings that learning is promoted by identifying what it is you are trying to teach and what you want the children to remember. Such emphasis at the beginning of the lesson, plus summarizing by the end, promotes achievement. Many teachers have found that it helps to talk over with their students various ways of remembering.

Experiencing the Concrete

Research findings from a number of studies indicate that lessons in which materials were used are more likely to promote achievement than lessons in which materials could have been used, but were not. Teacher preparation programs have done a remarkable job of making teachers aware that the correct answer to the question, "Should you use manipulative materials?" is "YES!". Yet survey after survey indicates that actual classroom use of materials is low.

Why? Sure, it's difficult to manage the use of materials: but it's not impossible. It takes time to collect materials, to organize them, to establish ways in which they can be efficiently distributed, used, and collected, to keep children's attention focused on their use. But the expenditure of time is worth it. Many teachers spend a comparable amount of time preparing worksheets, without the same degree of payoff in achievement results. Perhaps fear is a factor: fear that children will go "out of control." With worksheets, they are quiet; with manipulative materials, they become involved, talk, share information. Yet we know that quietness is not a reliable criterion for learning. Certainly, there is a noise level in a classroom which is prohibitive to learning, but some degree of noise, provided children are involved in a meaningful activity, and learning while they do so, is likely to be a positive factor. Children learn better by doing than by being told. Unfortunately, in too many of our classrooms, "teacher telling and children doing what the teacher has just told them" is the primary learning mode. Children are given few opportunities in which to explore.

Incidentally, research also indicates that the use of manipulative materials is a characteristic of "better" teachers (that is, those teachers whose children's achievement is higher). Yet teacher after teacher uses only the textbook and the chalkboard. The textbook is the primary determinant of the curriculum that is presented to our students, used day after day to the exclusion of other activities which might develop greater understanding of mathmatical ideas than the printed page alone can do.

This mention of printed materials recalls to mind another research finding we need to consider. As we use materials and pictures and symbols, many teachers forget to relate them. Most children need help in building bridges between these stages. They must come to see that the work they do with materials is then expressed in pictures and in symbols. Many children view these three stages as separate, unrelated learning activities: they need to see the connections between and among them.

Calculating and Computing

Speaking of materials that should be used but often aren't brings us to consider the calculator. Research from over 100 studies indicates that the use of calculators (a) promotes achievement, (b) improves problem-solving skills, and (c) increases understanding of mathematical ideas. Yet teachers have continued to behave as they did in 1975, when calculators first became widely and inexpensively available: they refuse to use them, under the guise of equity ("not all children have them") or "rotting the mind" ("they forget how to do work with paper and pencil"). If not all children have them, then actions such as those taken by the public schools in Chicago or by the state of Connecticut are possible: buy calculators for all children. As for "rotting the mind," the evidence indicates that this is simply not true: children both learn and retain better when they use the calculator. They have scored higher on test after [illegible] with paper and pencil even though use of calculators replaced much of their work with paper and pencil in daily classroom practice. They have learned basic computational facts better through using calculators even when learning the basic facts was not the objective of their work with calculators.

Avoiding all paper-and-pencil computation, however, is not the goal in any school: taking into consideration that in today's world children need to be able to do more than compute with paper and pencil is the goal. Thus children need to become proficient in mental computation and in estimation, both of which are used with far greater frequency than paper and pencil computation in the real world of both children and adults. And they need to use calculators, both as a means of learning key mathematical ideas (such as place value) and as adults use them, to do the more difficult computations. It has been strongly recommended by such groups as the National Council of Teachers of Mathematics that computations beyond two places — that is, computations such as $345+587$, or $496-208$, or 569×876, or $345/17$ — should be done with calculators.

Think about when **you** turn to a calculator. If you find it more plausible to use a calculator for certain computations, why is it that children should toil far beyond this level — and not have time to do more mathematics as a result? School should not be the place where you will learn what you never will use except in school.

All of these positive statements urging the use of calculators are made with the understanding that there are, of course, times when you do not want the children to use calculators. When mastery of basic facts or algorithms is the goal, for instance, you do not want to have children use calculators to attain the answers.

As for computers, the picture is definitely different. We as a society believe that children should learn to interact with computers. There is no emotional reaction opposing their use. The research evidence has long indicated their potential for helping children to use mathematics, and more recent evidence from the vast amount of work going on with microcomputers substantiates the fact that children can learn from computers, both by using software and by programming.

Fortunately, the software available for use in elementary school mathematics is increasing in both quantity and quality. For many years, drill and practice programs were pervasive. Many schools used them, and continue to use them — even though the evidence from almost 30 years of work with such programs indicates that children learn from working with the teacher as effectively as they do from the computer. Today, however, software that approaches problem solving, geometry, and other mathematical ideas from new perspectives is enabling us to provide children with more worthwhile experiences for their computer time.

Broadening the Curriculum

The curriculum also needs to be expanded beyond the focus on computation because of the same factor that was named in connection with calculators: usefulness. One cannot pick up a newspaper or magazine or watch television without seeing statistics in use —from graphing to probabilities. Yet a range of probability and statistics topics remain unknown in most schools. The key reasons given include "we don't have time; we must teach computation."

And then there is geometry. Geometry is another lost topic: little more than recognition of common shapes is taught in most classrooms. Yet we know from research that our children are seriously hampered later (for instance, in secondary school geometry) by their lack of understanding of geometric ideas — whether it be in fitting objects into given spaces, considering objects from different perspectives, or proving that something is true or not true.

In 1987, the National Council of Teachers of Mathematics circulated the working draft of *Curriculum and Evaluation Standards for School Mathematics*, in order to obtain reactions from a broad spectrum of teachers and others interested in education before a final draft is released in 1989. Standards for the elementary school curriculum are presented for a broad range of topics in addition to concepts and operations for whole numbers, fractions, and decimals. Among these topics are estimation, number sense, numeration, measurement, geometry, statistics, probability, patterns, relations, and functions. This document is in many ways a logical successor to *An Agenda for Action: Recommendations for School Mathematics of the 1980s,* released by the Council in 1980. The Standards also build on the earlier list of "Basic Mathematical Skills" prepared by the National Council of Supervisors of Mathematics — which is also being revised to reflect needed curricular change. Both of these reports have the potential for having an impact on what you teach and how you teach it. Familiarizing yourself with their contents would appear to be a logical first step, so watch for each to appear.

Problem Solving as a Way of Teaching

Problem solving is the focus of the curriculum, as the NCTM's *Agenda for Action* indicated. Indeed, lack of ability in problem solving is behind the complaints of many parents — rather than computational skill. Yet data from state and national assessments indicate that children perform well on computational examples, especially with whole numbers — until the computations are obscured in word problems. It is crucial that children learn to solve problems: it is the primary reason behind mathematics instruction in schools. The first writers of textbooks in colonial days knew this, and provided myriad problems mirroring real-life situations. Current textbook writers are attempting to provide problems that mirror the kinds of situations children (and adults) meet in today's world.

Research indicates that before they come to school, young children can solve a variety of problems — usually by using counting or by modeling the problem situation with concrete objects. Yet very soon after they begin school, they "lose" some of this ability — as they attempt to follow the procedure their teacher has demonstrated, as they attempt to do with paper and pencil algorithms what once seemed natural with counting and materials. They perceive of school mathematics as a "follow the leader" game, rather than a natural activity in which you use what you know to solve a problem. The imperative need to **reason**, inherent in problem solving, seems to have been lost.

We know from research that it is important that children learn to do such things as the following if they are to become good problem solvers:

- solve many, varied problems which involve more than practice in applying a known procedure: they must tackle problems that involve thinking, not merely recalling.
- learn how to use a variety of problem-solving strategies, to increase their options when attacking problems
- estimate answers and then check to see if the answers they obtain are reasonable
- look back when they have finished and talk about what they did, what they might do, what worked, what didn't work — and why, or why not

Children need to develop a repertoire of approaches to solving problems: instruction on such strategies as acting it out, making a drawing, looking for a pattern, and making a table can lead to higher achievement and to greater flexibility in solving problems. Along with this approach, research has indicated that discussing the nature of a problem, strategies used in solving the problem, and the reasons why the problem was solved in this way are very important. We want children not just to know "what to do," but also why the approach works — so they are better able to solve other problems.

But we must not forget that problem solving should be a way of teaching mathematics. A questioning approach of "how could we go about this" and "what do you think is a way of finding the answer" makes the learner an explorer of mathematical ideas. This is the same method elementary school teachers very often use in teaching reading, social studies, and other curricular areas. For instance, in reading, it is common to hear a teacher ask, "What word makes sense in that sentence?" The children are involved in finding, or reasoning their way toward, an "answer." Teachers need to apply such a method in teaching mathematics, rather than treating it as an arbitrary set of facts and skills and procedures to be mastered.

Using What the Children Know

We need to help children make use of the knowledge they bring with them to the classroom — and we need to teach with the knowledge that children carry many mathematical ideas with them into every mathematics lesson. The research evidence indicates that some of those mathematical ideas that they have discovered informally, either out of school or in school, are correct, and some are not. If we have an understanding of what their misconceptions are likely to be, we can teach in such a way that these misconceptions are replaced with correct conceptions. We need to understand that some of the informal procedures children use are as good as — and some are better — than those we teach from a textbook.

We will fail to help many children if we do not realize that what we are trying to teach them must be "attached" to what they have already assimilated, even if that learning is incorrect (or perhaps especially if that learning is incorrect). We must help them to make connections with what they already know, and help them to correct mistaken ideas. During this process, we must continuously take into account their developmental levels, which affect what they learn so strongly.

It has already been noted that many children view mathematics as a collection of individual facts and skills that must be memorized. This is one of the things that makes learning mathematics so difficult for so many children. Seeing the interrelationships of topics and ideas helps them to connect the individual entities into something that makes sense. And that is what teaching mathematics is all about — helping children to build a structure of mathematical ideas. The interrelationships range from building connections between addition and subtraction, to teaching about integers as an extension of the system of numbers they already know or grasping the idea that geometry and numbers are related.

Research has long indicated the importance of developing meaning: it promotes understanding and facilitates retention and transfer. The use of manipulative materials and real-world applications illustrates one aspect of meaningfulness. Helping children to realize that there is a reason why the multiplication algorithm is set up as it is, or why things viewed from one perspective look different than from another perspective, also involve meaningfulness. This is mathematical meaning, as important as social meaning to young children — and even more important as they continue in their study of mathematics.

Talking About Mathematics

In a variety of studies, the importance of communication is highlighted. Children must become involved in discussions of what to do, in arguments about why what they propose is sensible, and in verification of their accuracy. They must go far beyond giving answers to such questions as "What is the difference between 9 and 3?" by verbalizing their thinking about what the answer might

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Patricia E. Blosser
Bulletin Editor

be and why they think so — and how they can convince others that they are right. Mathematics should involve the joy of conjecturing and finding out you're right — and this comes about through discussions and talking about mathematics.

Learning from Each Other

In many classrooms today, particularly as grade level increases, children work alone on mathematics. There are, of course, times when this is appropriate — but research indicates that there are many other times when children benefit from working together on mathematics. Peer teaching (that is, two children working together) and cooperative learning groups (where the group shares in the "reward" of progress and attainment) are only two instances of ways in which children gain from group work. The argument that only children with the same ability or achievement levels can work together has been negated by many teachers: children can learn from each other despite, and because of, their differences.

Providing Equal Access

The evidence indicates that many teachers treat girls differently than boys: they encourage boys to explore and enjoy mathematics more. We need to watch that we encourage girls as well as boys, and that we search to reach the child with emotional problems, and that we diligently avoid applying "truisms" to children from differing races or cultures. Moreover, it means that all children — even low-achieving children — should be introduced to the full range of mathematical topics. They need to know about geometry and probability and estimation and a host of other mathematical ideas, just as other children do: in short, they need more than computation. At the very least they will know that mathematics means more than computation. And some of them will cause surprise by their learning of those other topics!

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