DOCUMENT RESUME

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ABSTRACT

Alaska's sex equity law requires school districts to establish written procedures for: (1) the biennial training of certificated personnel in the recognition of sex bias in instructional materials and in instructional techniques which may be used to overcome the effects of sex bias; (2) the biennial training of guidance and counseling staff in the recognition of bias in counseling materials and in techniques which may be used to overcome the effects of sex bias; (3) the review of textbooks and instructional materials for evidence of sex bias; and (4) the replacement of materials found to exhibit bias. The Alaska Department of Education developed this module for use by local school districts with the intention that district personnel with a minimal amount of experience could conduct an equity inservice focused on issues related to the teaching elementary level mathematics. Activities in this mathematics module deal with a review of research, teacher awareness of gender bias, mathematics anxiety, strategies for problem solving, and career awareness. Focused on the relevance of math to future career choices. Black and white line drawings illustrate portions of the module and 11 handouts are included. (PK)

EQUITY IN MATH

A TRAINING MODULE

Developed by
Alaska Department of Education
Office of Curriculum Services
and
Anchorage School District
Community Relations Department

Funded by
Title IV Sex Desegregation
Technical Assistance Grant
and
Anchorage School District

Equity in education
THE ALASKA PROJECT

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ACKNOWLEDGEMENTS

The development of this Math Equity module has been a collaborative effort requiring the cooperation of Anchorage School District personnel and the Alaska State Department of Education. Those who worked on the development of this module are:

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October, 1987

INTRODUCTION TO THE MODULE SERIES

Alaska's sex equity law, which prohibits sex discrimination in public school education, was passed by the Alaska Legislature in 1981. The law has been cited as one of the strongest state sex discrimination laws in the nation. This is in part due to the fact that the regulations require school districts to establish written procedures:

1. for the biennial training of certificated personnel in the recognition of sex bias in instructional materials and in instructional techniques which may be used to overcome the effects of sex bias;

2. for the biennial training of guidance and counseling staff in the recognition of bias in counseling materials and in techniques which may be used to overcome the effects of sex bias;

3. for the review of textbooks and instructional materials for evidence of sex bias; and

4. for the replacement or supplementation of materials found to exhibit bias.

Since the implementation of these regulations, referred to as Chapter 18, many school districts have relied on the Department of Education to provide them with on-site inservice training in the area of sex discrimination. Recognizing that local school districts need their own cadre of equity trainers as well as materials, the Department of Education utilized Title IV funds for the development of a series of equity modules. During the summer of 1986, educators within Alaska developed six modules, relating directly to curriculum content areas, that are now available to all Alaskan school districts. The modules were developed and written in such a fashion that district personnel with a minimal amount of experience could conduct an equity inservice.

Modules which have been completed include:

Women in American History (Elementary)
Women in American History (Secondary)
Language Bias (K-12)
Science (K-6)
Mathematics (Elementary)
Computer Equity (K-12)
Foreign Language
Fine Arts (Elementary)

The Department is continuing the development of modules in other curriculum areas.

The Department of Education is committed to helping school districts comply with the regulations outlined in Chapter 18.

School district personnel using the modules are requested to complete the evaluation sheet and return it to the Department of Education. This information will be used to update and improve the modules.

TITLE: MATH EQUITY

PURPOSE: To emphasize to educators the many continuing aspects of bias and discrimination in our society in order to provide continuous positive change in the educational environment.

GOAL: To provide educators the opportunity to focus on sex equity issues related to the teaching of mathematics.

LEARNING OBJECTIVE	METHOD	TECHNIQUE	TIME	ACTIVITY	RATIONALE	RESOURCES NEEDED
1) Participants will become acquainted with each other and Trainer, also discuss and receive clarification on the intent of inservice and clarification on purpose of the activities.	Large Group (Small Group Optional)	Information Giving Question/ Answers	15 minutes	Introduction of facilitator to participants and agenda sharing.	To allow for open questioning and reviews of goals and objectives.	Name tags Handout #1 (Optional) Tape Markers Flip Chart or Newsprint
2) Participants will identify need(s) for teaching math in framework of educational equity.	Large Group	Mini-lecture. Group Viewing of videotape.	45 minutes	Research Review and <u>Multiplying Options, Subtracting Bias</u>	To recognize need for educational equity in teaching mathematics.	Handout #2 Flip Chart Marker Video Tape Player Monitor Video: <u>Multiplying Options, Subtracting</u> Bias
3) Math Questionnaire	Large Group		10 minutes	Questionnaire	To gather information on students feelings about math.	Handout #3 A-D Handout #4
4) Participants will be able to identify examples of gender bias.	Large Group	Group Discussion	20 minutes	<u>Gender Bias – Teacher Awareness</u>	To heighten awareness of the teacher's role in combatting gender bias.	Flip Chart Markers

BREAK

LEARNING OBJECTIVE	METHOD	TECHNIQUE	TIME	ACTIVITY	RATIONALE	RESOURCES NEEDED
5) Participants will be able to identify at least four ways to avoid math anxiety while teaching mathematics.	Large Group	Mini-Lecture	30 minutes	Math Anxiety in the Classroom	To heighten awareness of nature of math anxiety and how it impedes learning of mathematics skills.	Flip Chart or Newsprint Markers
6) Participants will be able to identify at least 3 ways of utilizing problem solving strategies in their classrooms.	Individual or Small Group	Group Discussion	30 minutes	Strategies for Problem Solving	To identify problem solving traits and focus on teaching to encourage such skills	Handout #5
7) Participants will receive hands-on practice of sample problem-solving activities	Small group	Hands-on activities	20 minutes	Problem solving for Elementary and Secondary Students	To provide first hand experience with problem solving activities	Handouts #6A-F #7A-E
8) Participants will develop and conduct one career awareness activity from handouts.	Small and Large Group	Group Discussion	30 minutes	Elementary Career Awareness and Wrap Up	To focus on specific math-related activities that emphasize career awareness.	Pencils Dice #9 #10 Handouts #8
9) Evaluation	Large Group		5 minutes	Summary Evaluation of Inservice	To provide an avenue for feedback to the trainer.	Handout #11

MODULE CONTENT SHEET

TITLE:	Mathematics Equity
CONTACT TIME:	3 1/2 hours
TARGET AUDIENCE:	Elementary and Secondary Teachers

HANDOUTS: Copy prior to workshop:

#1	Overall Design and Purpose of Lesson
#2	Research Related to Girls in Math
#3 A-D	Mathematics Questionnaire
#4	Resource List
#5	Strategies for Problem Solving
#6A-F	Elementary Problem Solving Activities (To be used with elementary audiences)
#7A-E	Secondary Problem Solving Activities (To be used with secondary activities)
#8	Elementary Awareness: Job Sort
#9	Additional Career Awareness Activities
#10	Odds on you
#11	Evaluation

MATERIALS:

Name tags
Markers
Pencils or Pens
Paper
Video - Multiplying Options, Subtracting Bias
Videotape Recorder and Monitor
Dice
Flip chart or Newsprint
Tape
Scissors
Toothpicks
Yarn

NOTE TO TRAINER: Read and be familiar with the research section at the back of the module prior to the beginning of the workshop. Secure the following video...Multiplying Options, Subtracting Bias. Please request the video through the Sex Equity Coordinator, Department of Education, Juneau, 465-2841.

OVERALL DESIGN AND PURPOSE FOR SESSION

MATH EQUITY

PURPOSE:

1. To provide educators with increasing awareness of sexual bias in teaching math, and to offer techniques/activities addressing these biases.

2. To provide tools to promote the participation of women in mathematics, and to encourage young women to continue math courses throughout their schooling.

3. To offer materials and strategies for educators designed to increase students' confidence and competence in doing math and to relate the usefulness of math to future career choices.

AGENDA

Time	Activity
15 minutes	Introductions/Agenda Sharing
45 minutes	Research Review and Videotape
10 minutes	Math Questionnaire
20 minutes	Gender Bias - Teacher Awareness
15 minutes	BREAK
30 minutes	Math Anxiety in the Classroom
30 minutes	Strategies for Problem Solving
45 minutes	Career Awareness and Wrap Up
5 minutes	Evaluation

INTRODUCTIONS AND AGENDA SHARING

PURPOSE: To share with participants who you are; to establish a climate where people feel included; to set norms; and to share with participants your expectations about the purposes and agenda for this training session.

GROUP SIZE: 10 to 30 people

TIME REQUIRED: Approximately 15 minutes

MATERIALS:
Name tags
Handout #1 (or copy onto flip chart)
Newsprint or flip chart
Markers
Tape

ROOM ARRANGEMENT: Large group setting, informal

PROCEDURE: (Individual trainers have their own style of introducing a workshop. These are some suggestions and rationale for choosing to do certain things.)

1. Trainer will have participants introduce themselves to each other.

   Optional: Have each person in room introduce self. If you are working with staff from more than one school, you may wish people to say what school they are from and their position.

   Optional: We have found name tags help us associate names with faces. It also helps participants if they are not all from the same school.

2. a. Trainer gives background of the inservice - tells where it was developed and shares how it came to be offered to that school (or district or group). The Equity in Math was developed during the summer of 1985 by two educators working for the Community Relations Department of the Anchorage School District and revised in the summer of 1986 for statewide applications and distribution.

b. Trainer also points out that Chapter 18, Alaska's State Equity Regulations, requires biennial training of staff in the areas of sex bias and sex role stereotyping. This inservice satisfies that part of the law which mandates inservice training, under Chapter 18, for teachers.

3. Trainer distributes Handout #1 or goes over agenda on flip chart.

4. Trainer asks for clarification questions or concerns. Example:

   "What do you expect from the workshop?"
   "Is there anything confusing about the agenda?"
   "Do you have any concerns?"

5. Trainer will post this pre-written goal.

   Goal: To promote mathematical learning for both females and males and to emphasize to educators the many contributions that women have made in the field of mathematics.

OVERALL DESIGN AND PURPOSE FOR SESSION

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AGENDA

Introductions and Agenda Sharing

Research Review and Videotape

Math Quetionnaire

Gender Bias - Teacher Awareness

BREAK

Math Anxiety in the Classroom

Strategies for Problem Solving

Career Awareness and Wrap Up

Evaluation

RESEARCH REVIEW AND VIDEOTAPE

PURPOSE: To recognize need for educational equity in teaching mathematics by making participants aware of current research.

GROUP SIZE; 10-30

TIME REQUIRED: 45 minutes

MATERIALS:
Handout #2
Flip chart
Marker
Video: Multiplying Options, Subtracting Bias
Video Tape Player
Monitor

ROOM ARRANGEMENT: Large Group

PROCEDURES:

1. Prior to workshop, Trainer reads and becomes familiar with the research summary, "Research Related to Girls and Math."

2. Trainer distributes Handout #2.

3. Trainer allows 10 minutes for group to read Handout #2. While participants are reading Handout #2, the Trainer writes on the flip chart:

   Socialization and Biological Differences

   At the end of ten minutes the Trainer asks the participants to volunteer information on what the article says regarding how socialization and/or biological differences affect girls' math abilities.

4. Trainer shows video Multiplying Options, Subtracting Bias.

Research Related to Girls and Math

Research in the area of girls and math falls within two distinct categories: research that focuses on the process of socialization and its subsequent effect on math performance, and research aimed at discerning biological gender differences related to math ability and performance. Current instruments and measurement techniques are not precise enough to accurately measure the very small biological differences that may exist. More importantly, research on biological differences is inconclusive. There is however, a plethora of research that illuminates and supports the concept that real and observable socialization processes can and do affect girls' performance in math and science.

The Three Domains of Socialization

Research examining socialization can be categorized into three major domains. The physical domain deals with the physical world and its properties. The social domain encompasses our socio-cultural existence. The perceptual domain, is the by-product of the interaction between the physical and social domains. It is more abstract than the other two, less straightforward and generally more difficult to change.

The effects of the perceptual domain are often only recognized over time, as the interactions of the physical and social domains become more integrated and pronounced within individuals. These variations become differing cognitive styles, which effect how we learn. When examining the issue of girls and math, we must explore the pervasiveness, complexity, and realities of the perceptual domain and its relation to gender, math motivation and performance.

The Perceptual Domain

Each child's perceptual orientation evolves as her or his social and physical domains interact. Usually girls are more often reinforced to interact on a social level. The intensity of this social interaction increases a girl's ability to interact with, and be familiar with the social realm. Her knowledge of social rules often becomes her primary cognitive style. If math classes ignore interactive motivation, many girls then feel uninvolved.

Within the classroom setting, teacher expectations can affect the performance and motivation of female students. Trowbridge et al., (1981) found that boys in school are "valued for thinking logically, independently, with self-confidence, and an appropriate degree of risk-taking. Girls, however, are valued for their emotional expressiveness, sensitivity to others, dependency, and subjective thinking." Research also suggests that teacher praise, in terms of math performance, is perceived differently by boys and girls. Eccles-Parsons, et al. (1982) found that, "...praise was related to (high) self-concept of ability for boys only." When efforts were made to encourage and praise boys and girls equally, boys were more likely to equate praise as positive reflections on their ability and subsequent self-concept. Girls were more likely to attribute praise to luck. They then were more likely to question their abilities as math achievers, resulting in a general lack of confidence which often affected their math course selection (Fennema and Sherman, 1977).

The Physical Domain

Developmental differences in the physical domain can also lead to disparities within the classroom. Interests and style of play for boys may help them achieve success in math and science. Boys watch more TV science shows, read more books, magazines, and newspaper articles on science, and work more with science projects and hobbies (NAEP, May 1978). Female difficulties with spatial visualization may be the result of less knowledge and experience with manipulative materials (Fennema and Sherman 1977). Skolnick, et al. (1982) also support this observation: "the factors most critical to the development of spatial visualization skills are experience with manipulative materials such as constructing and examining three-dimensional structures, graphing, and modeling." These skills are also important within the classroom, where girls demonstrate more difficulty working with science equipment and apparatus (Kahle 1983). These factors then tend to be mutually reinforcing, hampering the development of strong spatial visualization skills and other opportunities to nurture interests related to math and science.

Peer Group Factors

Social and peer group factors within the school also mold the child's perceptions about her/his skills and desire to participate in math or science. Vockell and Lobonc (1981) found that girls enrolled in public schools selected subjects traditionally viewed as "masculine" such as calculus, chemistry, and physics less often than males, in spite of equal abilities. Enrollment patterns can even affect a girl's self-concept in relation to math and science. Skolnick, et al. (1982) observes: "Typically, a girl who wishes to pursue advanced science cou' :s finds her fear that 'girls don't become scientists' reinforced clearly by the ratio of boys and girls in the classroom."

Family Factors

Parent expectations can play a major role in the development of a child's self-concept of math ability. Eccles, et al. (1982) found that the most dramatic differences between parents of girls, and parents of boys, were their estimates of how hard their children had to try, to do well in math. She found that parents of girls consistently perceived the effort of their daughters to be more difficult than the efforts of sons. Eccles also states, "parents of sons thought that advanced math was more important for their child than parents of daughters." Similarly, she found that, "children's attitudes were influenced more by their parents' attitudes about their abilities than by their own past performances."

Additional research by Jacobs and Eccles (1983) focused on the impact of media reports on parents' perceptions of gender differences and math ability. Exposure to media reports...(which reaffirmed the genetic difference concept) had its largest impact on mothers of daughters and fathers of sons." These reports reinforced parents' own stereotypic beliefs concerning gender and math ability. For mothers it provided a legitimizing of their own math difficulties and tacit approval of their daughters' difficulties in math. Fathers of sons stereotypical impressions were also reaffirmed toward the belief that boys are better than girls in math.

Usefulness of Math

Pedro, Wolleat, Fennema and Becker (1981) found that usefulness was second only to prior performance as a predictor of future math enrollment. Armstrong (1981) found students considered usefulness as the most important reason in deciding to continue to enroll in math classes, while Eccles (1983) found that when compared to boys, girls believed that math was of less value. Students must see the usefulness of math in order to continue studying it.

Attributional Style

Attributional style is the way individuals explain academic success or failure. A student who attributes failure to lack of skill or ability will have little reason to expect future success from the same amount of effort. Concommitantly, attributing failure to lack of effort encourages the possibility of success in the future, because it is within the control of the individual. Wolleat, Pedro, Becker, and Fennema (1980) found that males attributed their success more strongly to ability, whereas females tended to attribute their success more strongly to effort. In terms of failure, girls were more likely to attribute their shortcomings to lack of ability or difficulty of task. At all levels of achievement, females were more likely to attribute their succcess to effort, and as their performance level increased, the degree to which they attributed their work to effort also increased. Contrarily, as achievement increased for males, the extent to which they attributed their success to effort decreased. Similarly, Eccles (1983) found that females were more likely to attribute their successes to effort and their failures to lack of ability.

We must take the socialization research into account when we devise instructional strategies for teaching math equitably. Differences in perceptual orientation, cognitive style, and motivation are all important to understand when designing a math curriculum that will meet needs of female and male students.

– Dave Dugger

QUESTIONNAIRE

PURPOSE: To gather information from students regarding their feelings on mathematics.

GROUP SIZE: 10-30 minutes

TIME REQUIRED: 10 minutes

MATERIALS: Handout #3 A-D
Handout #4

ROOM ARRANGEMENT: Large Group

PROCEDURES:

1. Trainer distributes Handout #3 A-D to participants and suggests to teachers that they might want to use this questionnaire in their own classrooms to gain information on their students' attitudes regarding mathematics. Primary teachers will probably want to survey their students orally.

2. Trainer encourages teachers to send their responses as teachers as well as a sampling of their student response to:

   Anchorage teachers: Anita Robinson
   Administration Building
   Anchorage School District

   Districts outside of Anchorage:

   Sex Equity Coordinator
   Department of Education
   P. O. Box F
   Juneau, AK 99811

   NOTE TO TRAINER: Information could be compiled and be made available to participants and other trainers at a later date.

3. Trainer distributes Handout #4. Resource list and asks if anyone has other resources they would like to share with the group.

MATHEMATICS QUESTIONNAIRE

This questionnaire was designed to be read aloud, with students answering on their own answer sheets (anonymously).

GRADE LEVEL ____________________

MALE ________________ FEMALE __________________

1. Do you like math?

2. If you don't like math now, did you ever like it?

3. Are you good in math?

4. Do you think girls can do math as well as boys?

5. Elementary student question:

   Will you need math when you grow up?

   Secondary student question:

   Will you need math when you graduate?

SECONDARY STUDENTS ONLY

6. If you dislike math now, what grade did you start disliking math?

7. Would you take the next higher level math class if your friends didn't take it?

Other questions teachers may want to ask for their own information.

A. Are you afraid to ask your teacher for help when you don't understand your math assignment?

B. Do you think math would be more fun if you could share ideas about how to solve story problems?

C. Do you think there is only one way to solve a math problem?

D. What's the hardest thing for you to do in math?

And so on!!

ELEMENTARY MATH QUESTIONNAIRE
STUDENT ANSWER SHEET

Grade Level ________________

Male ______ Female ______

1. ______ Yes ______ No
2. ______ Yes ______ No
3. ______ Yes ______ No
4. ______ Yes ______ No
5. ______ Yes ______ No

SECONDARY MATH QUESTIONNAIRE
STUDENT ANSWER SHEET

Grade Level ____________________

Male _______ Female _______

1. _______ Yes _______ No

2. _______ Yes _______ No

3. _______ Yes _______ No

4. _______ Yes _______ No

5. _______ Yes _______ No

6. Grade 1 _____
   Grade 2 _____
   Grade 3 _____
   Grade 4 _____
   Grade 5 _____
   Grade 6 _____
   Grade 7 _____
   Grade 8 _____
   Grade 9 _____
   Grade 10 _____
   Grade 11 _____
   Grade 12 _____

7. _______ Yes _______ No

STUDENT QUESTIONNAIRE RESULTS COMPOSITE

Date questionnaire given ______________________________

Grade level (elementary) __________

or

Course (secondary) ______________________________

Total number of males surveyed ____________

Total number of females surveyed ____________

	FEMALES		MALES	
	YES	NO	YES	NO
1.	______	______	______	______
2.	______	______	______	______
3.	______	______	______	______
4.	______	______	______	______
5.	______	______	______	______

SECONDARY ONLY

6.	Grade 1	____	Grade 1	____
	Grade 2	____	Grade 2	____
	Grade 3	____	Grade 3	____
	Grade 4	____	Grade 4	____
	Grade 5	____	Grade 5	____
	Grade 6	____	Grade 6	____
	Grade 7	____	Grade 7	____
	Grade 8	____	Grade 8	____
	Grade 9	____	Grade 9	____
	Grade 10	____	Grade 10	____
	Grade 11	____	Grade 11	____
	Grade 12	____	Grade 12	____

7.	______	______	______	______

RESOURCE LIST

Recommended Resources on Girls and Math

Research

Armstrong, J.M. "Achievement and Participation of Women in Mathematics," Journal for Research in Mathematics Education 12 (1981): 356-372.

Eccles-Parsons, J., et al. "Socialization of Achievement Attitudes and Beliefs: Classroom Influences," Child Development 53 (1982): 322-354

Eccles-Parsons, J., et al. "Socialization of Achievement Attitudes and Beliefs: Parental Influences," Child Development 53 (1982): 310-321

Jacobs J. and Eccles-Parsons, J. "Science and the Media: Benbow and Stanley Revisited," University of Michigan, 1983 (report).

Pedro, J. P., Wolleat, P., Fennema, E., and Becker, A.D. "Election of High School Mathematics by Female and Males: Attributions and Attitudes," American Educational Research Journal 18 (1981): 207-218

Curriculum Materials

Burns, Marilyn. Math for Smarty Pants. Palo Alto: Creative Publications. Grades 4-8. $6.95. Stories, activities, problem solving in this book and several others in this series.

Downie, D., Slesnick, T., and Stenmark, J. Math for Girls and Other Problem Solvers. University of California-Berkeley: 1981. Elementary school level curriculum activities. Emphasis on problem solving activities: logic and patterns; breaking set; creative thinking, estimating, and observing; and spatial visualization. $7.50 Order from EQUALS. See Organizational Resources.

Fraser, Sherry, et al. SPACES: Solving Problems of Access to Careers in Engineering and Science. Palo Alto, CA: Dale Seymour Publications, 1982. Activities for students in grades 4-10, about scientific careers; problem solving skills; attitudes toward math; spatial visualization skills; and work with mechanical tools. $10.00. Order from EQUALS. See Organizational Resources.

How High the Sky? How Far the Moon? An Educational Program for Girls and Women in Math and Science. Newton, MA: WEEA Publishing Center, 1982. Short exercises for K-12 students on problem solving skills and career information. $14.50. Order from WEEA. See Organizational Resources.

Kaseberg, A., Kreinberg, N., and Downie, D. EQUALS, a program to promote the participation of women in mathematics. University of California-Berkeley: 1980. Describes the EQUALS Teacher Education Program, designed to promote sex-fair mathematics instruction and counseling. Provides methods, and materials for K-12 use, bibliographies.

Organizational Resources

EQUALS
Lawrence Hall of Science
University of California
Berkeley, CA 94720
(415) 642-1823

Women's Educational Equity Act
Publications (WEEA)
Education Development Center
55 Chapel Street
Newton, MA 02160

GENDER BIAS - TEACHER AWARENESS

PURPOSE: To heighten awareness of the teacher's role in combating gender bias.

GROUP SIZE: 10-30

TIME REQUIRED: 20 minutes

MATERIALS: Flip Chart
Markers

PROCEDURES:

1. Trainer leads group discussion using participants' questions and comments from the video and discussion of the research from the previous exercise or use the following:

   "What examples of gender bias have you seen from parents or in textbooks?"

   "What techniques do you use in your classroom to alleviate discrimination between sexes?"

   "How does peer pressure affect students' decisions to take higher level math courses?"

   "Did you ever encounter a teacher during your own school years who demonstrated gender bias?"

2. Trainer may wish to write comments on flip chart and then summarize responses.

   Trainer then perhaps a ten minute break would be in order.

MATH ANXIETY IN THE CLASSROOM

PURPOSE: To give participants information on the nature of math anxiety, an awareness of its detrimental impact on women's future choices, and specific tips to combat math anxiety in the classroom.

GROUP SIZE: 10-30

TIME REQUIRED: 30 minutes

MATERIALS:
Transparency
Overhead Projector
Flip Chart or Newsprint
Markers

ROOM ARRANGEMENT: Large Group

PROCEDURES:

1. Prior to workshop, Trainer reads and becomes familiar with Math Anxiety in the Classroom by Wade H. Sherard. Present a few of the underlined portions to motivate brainstorming. (Remember that research shows classroom experiences, i.e. teachers, are a powerful influence on students' math attitudes. It is essential for teachers to be supportive, flexible, and patient with students who are anxious about math.)

2. Trainer allows ten minutes for participants to brainstorm list of classroom strategies to avoid math anxiety. Depending on group size, you may wish to break up into smaller groups and then come together to summarize. Each group should be provided with markers and sheets of newsprint. With small groups an overhead transparency could be used to record ideas.

3. Have each group tape newsprint to the wall. The ideas should be reviewed and discussed.

   You may want to add and discuss any of the following ideas:

   Record and Accept Feelings: Encourage students to keep a math journal. In it, they can record their own thoughts and feelings about math assignments and tests. If the feedback is negative, you could ask them to make affirmations about their ability, rather than self-defeating statements. Give them examples of positive thinking about math ability such as, "This seems hard, but if I do it one step at a time I can figure it out."

Use Precise Language: Explain the language used in mathematics in precise, understandable terms. (How would you define a circle?) Make sure students have mastered a concept before expecting them to memorize procedure. Always start explanations at the level of math where the students have comprehension and confidence.

Allow Non-Threatening Experiences: Set up classroom problem solving experiences that allow discussion and experimenting with no consequences for wrong answers. Credit could be given for participating in the search for solutions, as well as for correct answers. Be flexible with grading methods.

Encourage Cooperation Between Males and Females: Use cooperative learning with boys and girls depending on each other and working together to solve problems. Don't foster math competition between the sexes. Reward mixed male and female teams as well as individuals.

Be Sensitive: Give help when needed and avoid insensitive behaviors with pupils. These include:

1. Using math as a punishment for misbehavior.
2. Giving assignments that are too difficult and lengthy.
3. Humiliating students who don't understand by asking them to explain or work a problem in front of the class.
4. Making negative remarks about questions students ask like, "You should know how to do that by now!" or "That's an easy problem."

Other Ideas?

MATH ANXIETY IN THE CLASSROOM

Wade H. Sherard

Math anxiety is currently the object of considerable discussion and debate, since many educators are now recognizing math anxiety as a hindrance to learning mathematics. Math anxiety can be described as a fear of mathematics or an intense, negative emotional reaction to mathematics. Its effects on people are quite varied. Some people may have such acute anxiety about mathematics that they avoid mathematics at any cost, with the ultimate effect of handicapping themselves both in their everyday lives and in their employment opportunities.

What can classroom teachers do to contend with students who have math anxiety? What can they do to prevent math anxiety from development in their students?

Researchers and practitioners are actively trying to find ways of relieving math anxiety. Some feel that an approach to this problem is more effective if it combines counseling with a program of mathematics instruction. Others, also emphasizing a psychological approach, advocate behavior modification procedures for decreasing anxiety toward mathematics. Although both approaches have merit, most teachers, unfortunately, lack sufficient training in counseling to use them.

Teachers untrained in counseling techniques, however, need not feel helpless in dealing with the math-anxious student in the classroom. Practical suggestions for dealing with this problem are evolving from research studies and clinical programs, and good, sound teaching procedures are always helpful. The following suggestions and procedures can be used effectively by any teacher.

Avoid Sex Role Stereotyping of Mathematics as Male Domain

Evidence points to the fact that math anxiety can be nurtured by the perception of mathematics as an activity which is more appropriate for males. This view of mathematics as a masculine subject is a sociocultural phenomenon. It usually appears during adolescence, with males stereotyping mathematics as male domain more strongly than females. This perception can cause females to avoid success in mathematics. Furthermore, it can cause females to choose not to enroll in mathematics courses beyond the minimal high school requirements, thus creating lower achievement for females in mathematics. This avoidance of mathematics courses in high school may contribute to math anxiety, since the results of a study by Betz indicated the existence of a moderately strong relationship between math anxiety and number of years of high school math.

Sex role stereotyping of mathematics as a male domain has other effects in the classroom. Teachers tend to treat male and female students differently. They interact more often with males than with females, and they reinforce in both females and males behaviors deemed appropriate for their sex, which may include behaviors related to the learning of mathematics. Ernest found in his survey that half the teachers expect their male students to do better in mathematics wh le none of them expect female students to do better. Such differential expectations on the part of teachers may lead to differences in performance by students.

Make Students Aware of the Everyday Usefulness of Mathematics, Especially for Life Plans and Careers

To help decrease the avoidance of (and anxiety about) mathematics, students need to learn the relevance of mathematics to their daily lives. Equally important, they also need to understand that mathematics is useful in many professional and technical careers and that it is required as a part of many programs in higher education. Males generally perceive mathematics to be more useful to them than do females. Ernest conjectures that males take more mathematics, not because they like it any better than females, but because they see it as necessary for future occupations.

Fox concludes that girls perceive mathematics as less important for their futures than do boys, and that girls' perception of the usefulness of mathematics appears to be related to their perception of mathematics as a male domain and to their beliefs about appropriate careers for women.

To help prevent students from avoiding mathematics, teachers should:

a. Provide students with sound counseling about the mathematics requirements of courses and programs of study in higher education.

b. Provide students with counseling about the usefulness of mathematics in careers. Work with school counselors in career awareness programs as an intervention step.

c. Encourage all students (especially females) to take more mathematics courses. Such encouragement is particularly important when parents or counselors are having a negative influence on students' willingness to continue their study of mathematics.

d. Relate mathematics to the students' everyday lives so that they will see mathematics as meaningful and relevant.

Help Students to Develop Self-Confidence in Their Ability to do Mathematics

Many students who have math anxiety admit to having no confidence in their ability to do mathematics and thus they develop negative attitudes toward mathematics. Armstrong suggests that teachers need to help instill confidence and enjoyment in their students. She feels that students' feelings toward mathematics are largely a reaction to mathematics teaching, the mathematics program, and classroom activities.

Researchers have found that females are significantly less confident of themselves in mathematics than males and that as early as the sixth grade girls express less confidence than boys in their ability to do mathematic .

Counselors report a variety of ways in which math-anxious people exhibit a lack of confidence. Kogelman and Warren write that math-anxious people may make statements such as "I knew I couldn't do math," or "I don't have a math mind," or "Everybody knows what to do except me." Tobias reports that math-anxious people do not trust their own intuition...that if an idea occurs to them, then it must be wrong.

Classroom teachers can take positive actions to help their students develop and increase confidence in their ability to do mathematics:

a. Constantly express confidence that everyone can do the mathematics. Do not compromise expectations for students. Teachers usually get from their students what they expect from them.
b. Encourage students to trust their intuition and their first impressions in doing mathematics.

Students often work a problem correctly and then cross out or erase the solution and replace it with an incorrect one. Kogelman and Warren suggest that when this happens, the teacher should cross out the incorrect solution and give the student credit for the original one in order to encourage the use of intuition. To further encourage the use of intuition they recommend that

> ...math teachers not ask students to explain how they got an answer when the answer is right...When someone offers the correct answer we then ask if anyone else can explain it. The person who gave the answer may offer an explanation if he or she wishes, but it is not necessary to do so.

Students often develop difficulties with mathematics because their <u>pattern of learning is memorization without understanding</u>. Smith describes this situation graphically:

> Something happens early in the school experience when the need for getting an answer and getting it quickly seems to scar many children, forcing them to establish what we call a "school survival strategy," whereby at an early age he/she learns that in order to get by - that is, gain the teacher's approval - it is necessary to <u>memorize, never question, and therefore never understand</u>. This pattern of learning, once established, continues through the grades, frequently with a good bit of success as measured by teachers' marks, until a time when understanding becomes imperative ....and then the whole facade collapses.

Hilton writes that a desire to know should not be severed from a desire to understand and that the student should not depend upon memory as a substitute for understanding.

Classroom teachers should stress the learning of mathematics with understanding and with the synthesis of related facts and ideas. They should ensure that students have the necessary prerequisite skills before being introduced to a new concept in mathematics. In teaching remedial courses teachers should begin the course at the point in the students' mathematical development where they have some understanding and confidence.

Concentrate Especially on Problem Solving, Spatial Skills, and the Language and Symbolism of Mathematics as Important Aspects of the Mathematics Curriculum

Solving word problems is a basic and critical part of the mathematics curriculum for all students, math-anxious or not. As a result of her work with math-anxious people, Tobias feels that word problems are at the heart of math anxiety. Inadequate problem-solving skills can cause students much frustration, tension, and anxiety. Consequently, learning to confront word problems and studying methods for solving them can be effective in helping people to cope with math anxiety.

Spatial skills are also an important part of the mathematics curriculum. It appears that spatial visualization skills are important to the learning of mathematics and that students' spatial ability does correlate with math achievement. However, the whole area of spatial abilities is currently under study, especially the relationship of spatial visualization to sex-related differences in mathematics achievement, making it difficult to draw firm conclusions from available research.

Learning the language and symbolism of mathematics is crucial to effective mastery of mathematics. The language of mathematics can be full of ambiguity for beginning students. Mathematical terms having different meanings in ordinary English can create linguistic confusion and increase insecure feelings about mathematics. Confusion about the meaning and usage of mathematical symbols can lead to serious errors in mathematics. And reading skills for mathematics, such as eye movements, rate of reading, and symbolism, are quite different from those for ordinary prose.

Teachers need to give special emphasis to teaching problem solving, spatial skills, and the language of mathematics.

a. Make problem solving an integral part of instruction in mathematics; do at least one or two problems in class every day.
b. Concentrate on teaching methods for solving word problems. The suggestions of Polya for teaching problem solving are helpful.
c. Develop students' skills in spatial relations. Provide practical exercises that require various spatial skills. Show how spatial skills are used in everyday situations as well as in mathematical situations
d. Teach the vocabulary of mathematics, giving special attention to words used differently in ordinary English.
e. Motivate the meaning and use of mathematical symbolism.
f. Teach students how to read mathematics, giving special emphasis to reading skills for mathematics which differ from those for ordinary English.

Avoid Teaching Behavior Which is Inflexible or Excessively Authoritarian

Many people with math anxiety have erroneous impressions about mathematics. They feel that it is authoritarian, rigid, and not creative; that there is only one right answer to a problem. An overemphasis on obtaining "exact right answers" can create excessive pressure on students and, according to Tobias, can come to represent authoritarianism, competitiveness, and painful evaluation. Authoritarianism in mathematics can breed anxiety and distaste, especially when students are told what to do, that it works, and that they will understand it later.

Teachers should not be rigid and inflexible in their teaching behaviors and should not give students the impression that mathematics is an authoritarian subject. The following measures can be helpful:

a. Encourage creativity in problem solving. Solicit different ways of doing a problem to demonstrate creativity in mathematics. Give recognition for an unusual or insightful solution to a problem.
b. Avoid excessive emphasis on the correctness of the final answer to a problem. Be generous in awarding partial credit for using correct methods and reasoning; identifying a correct method for solving a problem should be at least as important as obtaining the final answer.
c. Teach estimation and approximation skills. Encourage good, intelligent guessing as an appropriate problem-solving skill. Use reality-based problems that do not have exact answers in neat, closed forms.
d. Avoid dismissing a wrong answer to a question; try instead to determine the question that the student was answering. Students often give the right answer to the wrong question.
e. Ask students to respond to each other's questions before assuming the role of the teacher as the final source of authority.

Be Aware of the Possible Negative Effects That the Process of Testing May have on Mathematics Attitudes and Anxiety

Relying on tests as the only means of assessing a student's progress in mathematics may lead to inaccurate evaluation. By over-emphasizing testing, teachers may "teach to the tests" so that students learn only what they think they will be tested on. "Most tests are administered under conditions which violate almost every principle for effectively doing mathematics; thus they themselves generate anxiety and establish a stable association between education and anxiety."

Rounds and Hendel suggest a possible relationship between the term mathematics anxiety and testing. The results of their study indicate that mathematics anxiety is less a response to the evaluation of mathematical skills and that moderate-to-high relationships exist between mathematics anxiety measures and measures of test anxiety and mathematics attitudes.

Teachers need to consider carefully their own use of tests and testing procedures, since they may contribute to math anxiety.

a. Avoid placing an overemphasis on tests as a means of evaluating mathematics achievement. Use alternative means to supplement and broaden the evaluation process, such as homework or classwork assignments, direct observation of learning activities, reports, and outside projects.
b. Provide ample time for working a test. Avoid giving timed tests which create excessive pressure on students.
c. Consider testing for mastery and allowing students to retest when necessary, as evaluation procedures.

Avoid Insensitive Behaviors in Teaching Procedures

People with math anxiety can often trace their negative feelings about mathematics to the insensitive actions of a particular teacher. Fox reports that the impact of teachers upon students seems to be most powerful at the extremes of attitudes, that a very bad experience with a teacher or a very positive experience with a supportive and encouraging teacher may be influential. Donady and Tobias have also found that extreme attitudes toward mathematics are often ascribed to the attitudes of a particular teacher.

Teachers need to be very careful not to involve themselves in insensitive behaviors with their students:

a. Avoid making condescending remarks like:

   "You should know that."
   "It's obvious."
   "That is an easy problem."
   "That's a stupid question."

b. Never use math as a punishment, for example, making an excessively long assignment in order to punish misbehavior.
c. Do not refuse to answer legitimate questions or to give help when it is needed.
d. Never create humiliating experiences for students such as forcing them to go to the chalkboard and reveal before their peers their lack of knowledge or inability to work a problem.
e. Do not make assignments that are unreasonable in length or level of difficulty.

Provide a Relaxed, Supportive Classroom Atmosphere

A basic principle of good teaching is to create a classroom atmosphere conducive to learning. Being relaxed and supportive when interacting with students can decrease tensions, alleviate fears, and help create the proper atmosphere. To be effective, teachers should:

a. Give students encouragement; express confidence in their ability to learn mathematics.
b. Encourage students to ask questions and to participate in class discussions. Be willing to accept "dumb questions" and "off-base" remarks.
c. Provide students with many opportunities for successful experiences in doing mathematics. There is truth in the adage that nothing succeeds like success.
d. Provide students with interactive instruction and ample feedback.
e. Be patient, receptive, supportive, relaxed, and understanding. Show interest in students as fellow human beings.

Finally, classroom teachers must be aware of their own feelings about mathematics. Since teachers do influence the development of their students' attitudes toward mathematics, it is particularly important that they themselves enjoy mathematics and that they show interest in and enthusiasm for mathematics.

STRATEGIES FOR PROBLEM SOLVING

PURPOSE: To examine various traits and skills required to become adept at problem solving, and to discuss math teaching techniques that nurture this ability.

GROUP SIZE: 10-30

TIME REQUIRED: 30 minutes

MATERIALS: Handout #5

ROOM ARRANGEMENT: Individual or Small Group

PROCEDURES:

1. Present Handout #5 on strategies for problem soiving. Allow five minutes for participants to read the handout. Ask for other strategies not mentioned. Spend brief time exchanging ideas on how to incorporate problem solving into the regular curriculum.

   Some ideas you may want to mention:

   a. Give a short challenging problem during first or last 5 minutes of the class.

   b. Give one problem each week that may need outside resources to solve.

   c. During class discussion, have someone other than the person who gave the answer volunteer the explanation.

   d. Use manipulatives.

2. Trainer ends session by saying that participants are now going to have some hands-on experience with problem solving activities.

   NOTE TO TRAINER: If working with elementary teachers, go to the section titles "Problem Solving for Elementary Students. If working with secondary students turn to the section titled "Problem Solving for Secondary Students.

Summary of the National Council of Supervisors of Mathematics

Position Paper on Basic Mathematical Skills

Mathematics supervisors are concerned that, as a result of the "back-to-the-basics" movement, today in many schools there is too much emphasis on computation and not enough stress on other important mathematical skills. To respond to this trend, the National Council of Supervisors of Mathematics (NCSM) set up a twelve-member task force to write a position paper on basic mathematical skills. The position paper was first written in July, 1976, and later revised on the basis of ideas from supervisors through the country.

The position paper urges that we move forward, not "back" to the basics. The skills of yesterday are not the ones that today's students will need when they are adults. They will face a world of change in which they must be able to solve many different kinds of problems. The NCSM position paper lists ten important skill areas that students will need.

Problem Solving: Students should be able to solve problems in situations which are new to them.

Applying Mathematics to Everyday Situations:
Students should be able to use mathematics to deal with situations they face daily in an ever-changing world.

Alertness to Reasonableness of Results:
Students should learn to check to see that their answers to problems are "in the ball park."

Estimation and Approximation:
Students should learn to estimate quantity, length, distance, weight, etc.

Appropriate Computational Skills:
Students should be able to use the four basic operations with whole numbers and decimals and they should be able to do computations with simple fractions and percents.

Geometry:
Students should know basic properties of simple geometric figures.

Measurements:
Students should be able to measure in both the metric and customary systems.

Tables, Charts, and Graphs:
Students should be able to read and make simple tables, charts and graphs.

Using Mathematics to Predict:
Students should know how mathematics is used to find the likelihood of future events.

Computer Literacy: Students should know about the many uses of computers in society and they should be aware of what computers can do and what they cannot do.

STRATEGIES FOR PROBLEM SOLVING

AN ESSENTIAL ELEMENT IN PROBLEM SOLVING IS THAT THERE IS NO OBVIOUS PATH TO THE SOLUTION.

Problem solving is a set of strategies. Consider this list of strategies. A quick examination of this list will reveal techniques for solving problems at many levels and in a variety of situations. The traditional method of teaching mathematics (explain a method, do some examples, give some exercises) does <u>not</u> directly teach these problem solving strategies.

<u>BEGINNING STRATEGIES</u>:

... Determine what the problem is - paraphrase it.
... State what is known.
... Analyze whether information is sufficient, deficient, superfluous.

<u>SOLUTION STRATEGIES</u>:

... Guess and check.
... Organize data.
... Eliminate possibilities.
... Assume an answer and find a contradiction.
... Reduce to a simpler problem and generalize.
... Use a model, manipulative, or picture.
... Look for patterns.
... Break set - consider many possibilities.
... Work backwards from final result.

<u>GENERAL STRATEGIES</u>:

... Work with partner, brainstorm, ask questions.
... Value persistence and patience.
... Sleep on it, if you're stuck.

According to Whimby, Piaget, and a host of other educators and psychologists, a good problem solver is active physically and mentally. A list of problem-solving characteristics would include:

... drawing, visualizing, using props

... approaching the problem in a systematic or logical way

... breaking out of a thinking rut or set

... using accuracy, especially in observation

... visualizing and mentally manipulating objects in space

... having confidence, willingness to risk trying new ideas

Identification of these characteristics gave rise to the following problem solving strands:

A. Using Logic, Strategies, and Patterns: Activities in this strand focus on systematic problem solving. Some fundamental and general techniques are introduced in a recreational format. Solutions to fantasy logic problems are sought, pattern-finding activities are presented, games are played, and appropriate strategies are identified.

B. Breaking Set: These activities present a seemingly impossible problem to the student. The apparent solution path is almost always incorrect. Solution requires being open to non-obvious processes. With experience, as well as exposure to simpler models of the same problem students can succeed with these difficult problems. Activities include topology, rope puzzles, mystery stories, number patterns, and classic math problems.

C. Creative Thinking, Estimating, and Observing: Solving problems without a predesignated right answer leads students to use ingenuity. Problems simulate the real world in which one must seek and evaluate solutions to real problems. The activities require inventing, pretending, building, and experimenting.

   Students are also encouraged to use observation and communication skills to help define and clarify problems and facilitate the solution process. Activities involve giving detailed directions or description, constructing models, participating in non-verbal group constructions, and observing events.

D. Spatial Visualizing: Solutions to problems often are found through pictorial representations or through the construction of three-dimensional models. Sometimes, however, models cannot be constructed, in which case, the ability to analytically examine a visual representation of the problem is necessary. To develop this skill, students can examine optical illusions, symmetry in art, paper folding, rope configurations, and the effects of reflections and rotations.

Why Problem Solving?

Within the mathematics component, EQUALS places primary emphasis on problem solving. In its broadest sense, problem solving is the process of applying previously acquired knowledge to new and unfamiliar situations. It is a set of strategies that can be useful in any decision-making position or situation. As such, it is essential to intelligent participation in society.

EQUALS focuses on problem solving because of the apparent sex differences that have been reported in tests of problem solving abilities; the usefulness of problem solving both in preparing for and in pursuing careers; and the role of problem solving in the mathematics curriculum.

Sex Differences in Problem Solving

Girls have fallen behind boys in some problem solving skills as early as the sixth grade. In the 1978 California Assessment Program, girls consistently scored better than boys in basic arithmetic computation; however, in all areas where multiple-step reasoning was involved, boys scored consistently higher than girls. This pattern continued in the twelfth-grade results, where boys surpassed girls on algebraic word problems, even when math background was held constant.

The California Mathematics Assessment Advisory Committee "refrained from speculating on the causes of these sex differences. It was felt that if further more intensive studies confirm these patterns, these findings may have some far-reaching implications for the design of instructional programs and teaching methods, especially at the elementary school level."

Whatever the causes for these sex differences, the EQUALS staff believes that, if females demonstrate a weakness in this area, we should pay attention to it in the curriculum. An emphasis on problem solving will strengthen all students' mathematics competence and may particularly improve females' performance.

Problem Solving and Careers

Because young women are more likely to continue with mathematics when they perceive it to be useful, EQUALS has emphasized the role of problem solving in a wide variety of careers. In the following quotations, artists, scientists, and business people cite problem solving as a critical element in their work.

"When there is a problem, the first job is to find out what the problem really is. You have to go below all the surface indications..."

"So often, when you try to get down to fundamentals, you end up with something that is your own individual problem, or one in which you are in some way intimately involved. That is why you must first find the real problem and state it clearly before you begin to look for your resources. Then you try to find the resources and evaluate what they have to offer."

(Lillian M. Gilbreth, Management Consultant and Industrial Engineer, in J. Mattfield and C. Van Aken, Women and the Scientific Professions, MIT Press, 1986, p. 222.)

"And now at last, after working in two cultures in which I had thought I was finding nothing really relevant to the problem with which I had come to the field, Tchambuli was providing a kind of pattern - in fact, the missing piece - that made possible a new interpretation of what we already knew."

(Margaret Mead, Blackberry Winter: My Earlier Years, Pocket Books, 1972, p.234.

"The process of making the chalices was the most challenging technical endeavor I have ever encountered; it brought all my ability and training into play. The approach to problem solving on the Project was very different from anything in my previous experience. I learned to be more organized and attentive, to use professional resources in the community, and to work noncompetitively with other women."

(Kathy Erteman, ceramicist and sculptor, as quoted in Documentation Display, Judy Chicago, The Dinner Party, Doubleday Anchor Books, 1979.)

Problem Solving and the Curriculum

Problem solving does not fit into arithmetic, algebra, geometry, or trigonometry as a single topic. Because of this, and a rising emphasis on traditional basic skills, problem solving is often omitted from these courses at a time when it belongs in all of them.

Normally, any mention of problem solving is closely linked with word problems. However, problem solving in mathematics means more than the single-step word problems usually found in textbooks. In fact, one leading textbook tells the student which specific skill to use: "Multiply to find each answer." In other words, the student does no problem solving. An essential element in problem solving is that there is no obvious path to the solution. If any set procedure or algorithm leads directly to the solution, then no problem solving has been done - just exercises completed. Naturally, there are several factors that determine whether the path to the solution is obvious to the problem solver - her age, experience, and mathematical competence.

As mentioned earlier, problem solving is a set of strategies. Consider this list of strategies developed by EQUALS participants. A quick examination of this list will reveal techniques for solving problems at many levels and in a variety of situations. The traditional method of teaching mathematics (explain a method, do some examples, give some exercises) does not directly teach these problem solving strategies.

Beginning Strategies:

Determine what the problem is...paraphrase it.
State what is known.
Analyze whether information is sufficient, deficient, superfluous.

Solution Strategies

Guess and check.
Organize data.
Eliminate possibilities.
Assume an answer and find a contradiction.
Reduce to a simpler problem and generalize.
Use a model, manipulative, or picture.
Look for patterns.
Break set...consider many possibilities.
Work backwards from final result.

General Strategies

Work with a partner, brainstorm, ask questions.
Value persistence and patience.
Sleep on it, if you're stuck.

PROBLEM SOLVING FOR ELEMENTARY STUDENTS

PURPOSE: To acquaint participants with problem solving activities for use with their students.

GROUP SIZE: 10-30

TIME: 20 minutes

MATERIALS: Handouts #6 A-F
Refer to individual handouts for other materials.

ROOM ARRANGEMENT: Divide into groups by grade level or small groups.

PROCEDURE:

1. Trainer announces that participants are now to break up into small groups either by grade level or interest area.

2. Trainer explains that in a few moments one member from each group will come up to examine problem solving activities that can be used in a classroom situation.

3. Trainer allows time for one person from each group to come to the table and choose a problem solving activity.

4. Trainer asks if there are any questions and allows participants 15 minutes to do one of the problem solving activities.

5. At the end of 15 minutes, Trainer has one person from each group briefly summarize the problem solving activity for the rest of the participants.

Special Instructions - Elementary

a. Hexomino Designs: 15 minutes
Hand out directions and hexominos to each participant. So that all participants can leave with their own copies, run off extra sheets of hexominos that will be used during the activity time. Perhaps one copy could be given to every three people and one pair of scissors could be shared at each table.

b. The Man in the Pit: 10 minutes
Problem can be read aloud or written on board.

c. Double Design: 15 minutes
Use blank paper and have participants u › basic shapes in their drawings.

d. Real Life Story: 5 minutes

e. Balloon Ride: 15 minutes
Each set of partners will need 10 toothpicks for this activity.

f. People Sorting: 15 minutes

PROBLEM SOLVING ACTIVITY - ELEMENTARY

HEXOMINO DESIGNS

TOPIC: Spacial Visualization
Manipulative

PURPOSE: The challenge is to visualize the cubes with the letters MATH showing around the sides.

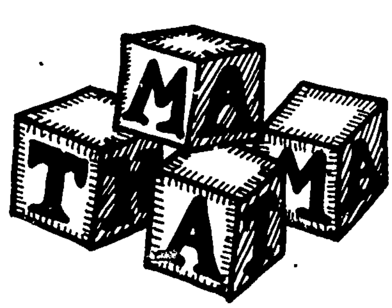
GROUP SIZE: Small Groups
Individuals

MATERIALS: Hexominos
Pencils
Scissors

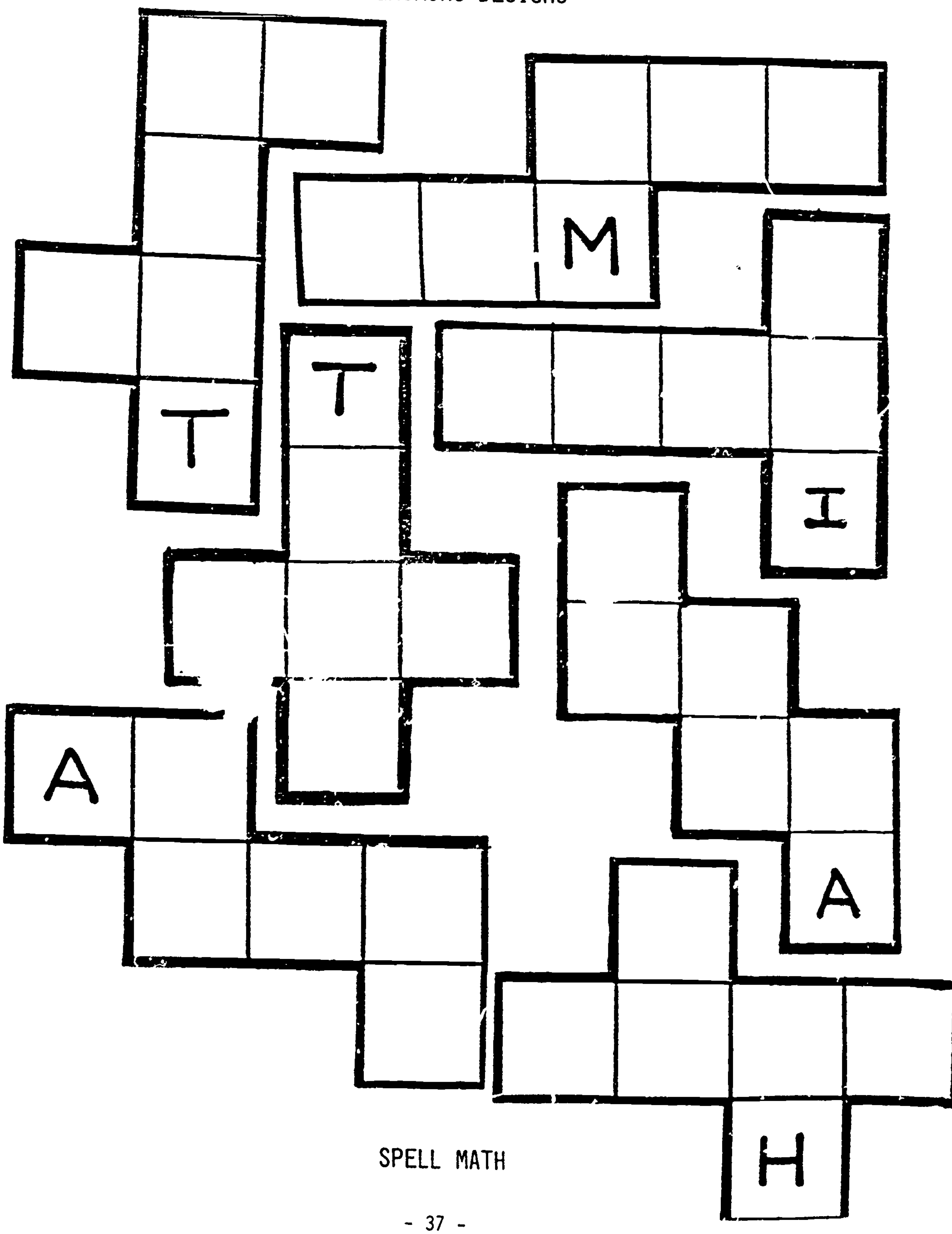
ACTIVITY: Have students fill in the missing letters to spell MATH around the sides of each cube. The top and bottom of the cube should be blank and letters should be facing the right direction. Cubes can then be cut out and folded to check correct placement of letters

ADDITIONAL ACTIVITY: Encourage students to find other hexomino designs that will fold into a cube.

Give points for getting the letters in the right squares even if they are upside down or backwards.



HEXOMINO DESIGNS



SPELL MATH

PROBLEM SOLVING ACTIVITY - ELEMENTARY

THE MAN IN THE PIT

TOPIC: Open-ended problem solving
Creative thinking

PURPOSE: When presenting this problem, if teachers emphasize the fact that there is no right answer and that the purpose is to find any solution that works, students will have the opportunity to use their imagination with little risk or fear of judgment. An open-ended problem should be versatile enough to allow each child to use her knowledge, however limited or extensive.

GROUP SIZE: Individuals or small groups (3 - 5)

MATERIALS: Pencils
Paper

ACTIVITY: Present the problem:

A man, wearing only his hiking shorts, got lost during a summer hike and fell into a pit. The pit was circular. It was about 30 feet across and 20 feet deep. The man found that all the walls of the pit were hard stone, very smooth, and straight up and down, so there were no hand-holds or foot-holds for climbing. The bottom of the pit was also hard as stone. A stream of water kept flowing over the edge of the pit and ran down one wall. Even behind the waterfall the wall was smooth and straight up and down. After reaching the bottom of the pit, the water then disappeared into a small hole in the floor of the pit. The pit was completely empty except for three things:

1. Exactly in the middle of the pit was a tree growing straight up.
2. Near the tree was a loose flat rock.
3. Two boards were also near the tree.

Figure out how the man escaped from the pit.

Have each individual or group share its solution.

ADDITIONAL ACTIVITY:

Other problems and questions for open-ended solutions:

... Have you ever seen a ship inside a bottle? How do you think it got there?

... You are locked in a room. It's raining outside and there's a leak in the middle of the ceiling. There is nothing in the room except an old dirty coffee cup and a hole in the middle of the room. The hole is a little bit larger in diameter than a ping pong ball and longer and narrower than your arm. If you can get the ping pong ball out of the hole, you can come out of the room.

PROBLEM SOLVING ACTIVITY - ELEMENTARY

DOUBLE DESIGN

TOPIC: Communication
Spatial visualization
Giving and following detailed directions

PURPOSE: The challenge is to duplicate an unseen line drawing by following verbal directions. Participants must not only be able to give precise directions, they must listen carefully. This activity can also provide an introduction to scale drawings.

GROUP SIZE: Groups of 2 - 4

MATERIALS: Graph or blank paper
Pencils, pens or crayons

ACTIVITY: Each student draws a design on her own piece of graph paper. Designs should follow the lines on the paper. Encourage students to begin with simple designs.

Have students pair up. One student describes her picture while her partner attempts to reproduce it on a blank sheet of graph paper. Only verbal directions are allowed.

Once the drawing has been described and the copy made, students compare designs. How close are they? What additional information would have been helpful?

Now have the students change roles.

ADDITIONAL ACTIVITY:

This activity can be done on blank paper rather than graph paper.

Have one partner draw a squiggly line on a blank piece of paper. The other partner should be blindfolded. The "seeing" partner places the paper in front of the blindfolded person and gives her a colored pencil. She then gives directions to the blindfolded person to trace over the squiggly line.

Have one partner arrange geometric shapes on blank paper. The other partner is given the same shapes to use. Then, the student who made the design gives specific instructions to his partner to help him duplicate the design without looking. Partners compare their designs.

PROBLEM SOLVING ACTIVITY - ELEMENTARY

REAL LIFE STORY

TOPIC: Open-ended problem solving
Logic
Real life simulation

PURPOSE: This activity gives students a chance to make the most realistic and logical choice from numbers given.

GROUP SIZE: Whole class

MATERIALS: None (can be put on the board)

ACTIVITY: Students read the following story. Then they choose numbers from those given to fill the blanks in the most realistic manner.

---

20 2 5 14 1

Mr. and Mrs. Jones checked into a motel and got a room for ______. They walked across the street to have dinner, which cost $ _________. They gave the clerk a _______ dollar bill and received change of one ______ dollar bill and $ _______.

ADDITIONAL ACTIVITY: Have the students make up their own stories to share with their friends. Try making the stories more difficult by using numbers in different situations that would involve multiplication, division, or fractions. Any answer is correct that can be justified.

PROBLEM SOLVING ACTIVITY - ELEMENTARY

BALLOON RIDE*

TOPIC: Patterns
Strategies
Problem solving

PURPOSE: Strategy games encourage students to find patterns and techniques such as breaking a problem down into a simpler one or working backwards.

GROUP SIZE: Two people

MATERIALS: Toothpicks
Balloon Ride gameboard

ACTIVITY: Set up the gameboard with 10 toothpicks connecting the bottom of the balloon with the ground. The object is to get a free ride by removing the toothpicks according to a strategy.

Read the story to students:

Balloon Ride

The Hot Air Balloon is coming to town. Free rides will be given to anyone who can cut the last tie rope holding the balloon to the ground. Here are the rules:

... 10 tie ropes hold the balloon to the ground.
... Two people take turns cutting ropes. Each person can cut 1 or 2 ropes on a turn.
... Whoever cuts the last rope gets a free ride.

Can you figure out how to get the free ride every time?

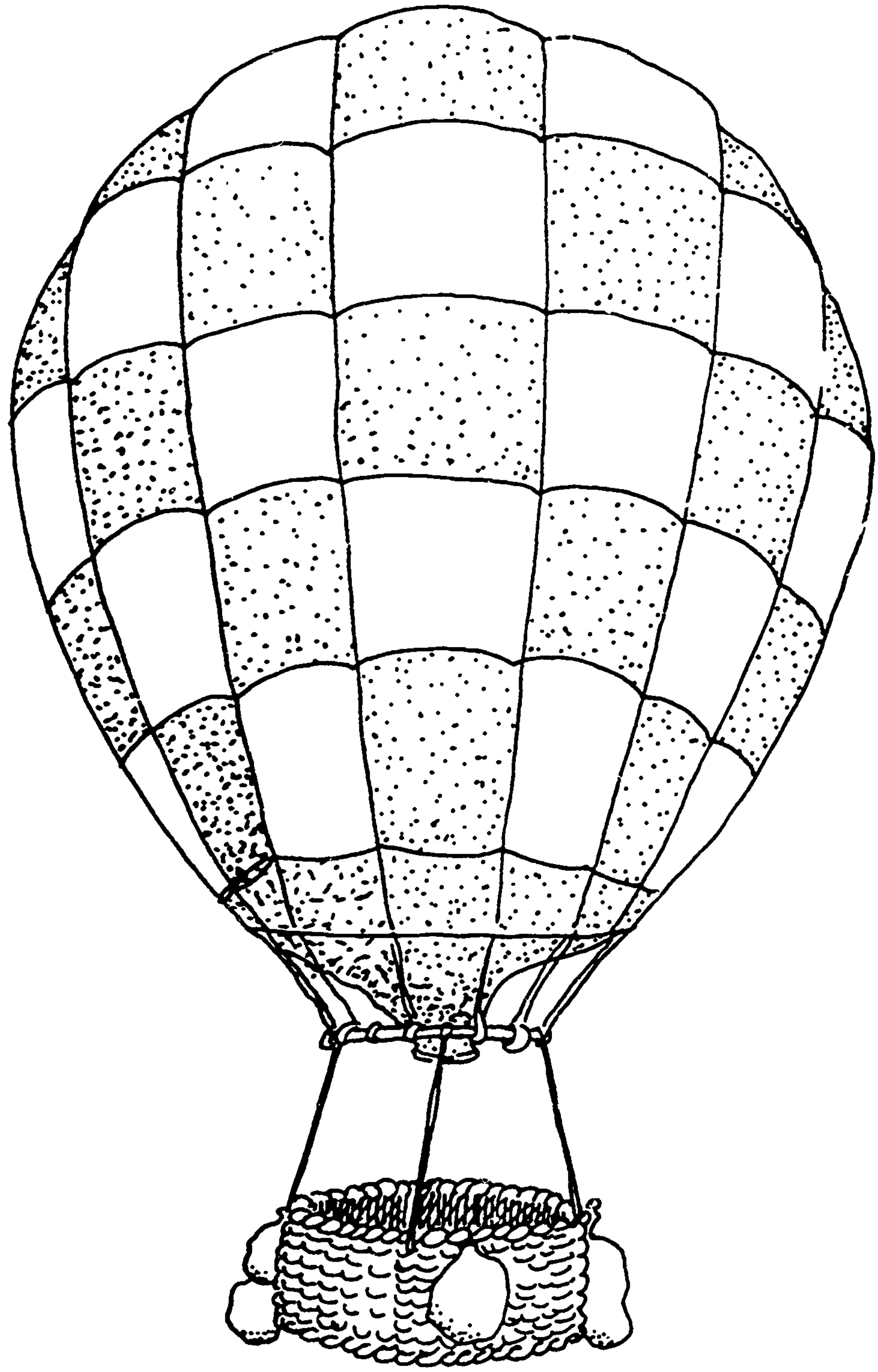
ADDITIONAL ACTIVITY: Balloon Ride can be extended by increasing the number of tie ropes and also by increasing the number of ropes each player may cut. For example:

... 21 tie ropes; players may cut 1, 2, or 3 ropes at a time.
... 40 tie ropes; players may cut 1, 2, 3, or 4 ropes at a time.

Would you rather be the first or second player for each of these?

*This is one of many versions of the ancient Chinese game of NIM.

BALLOON RIDE



PROBLEM SOLVING ACTIVITY - ELEMENTARY

PEOPLE SORTING

TOPIC:

Logic
Sorting
Attribute identification

PURPOSE:

This activity introduces the concept of an attribute and sets the stage for development of logic skills. Everyone gets involved.

GROUP SIZE:

Whole class

MATERIALS:

One large loop of yarn

ACTIVITY:

Put a large loop of yarn on one side of the room. Name a characteristic such as "wearing something green" and have all those who fit that description stand inside the loop. Repeat the activity with several other characteristics such as: having green eyes, wearing a belt, liking dogs. Let the students think of some of the characteristics.

Think of a secret characteristic. Without telling the characteristic, the leader has each person with that characteristic step inside the loop. The challenge is for the group to guess the mystery attribute.

Ask the group what attributes are shared by everyone in the room.

ADDITIONAL ACTIVITY:

Seated in a circle, ask each person to name an attribute she has that is different from anyone else in the group. Point out when appropriate that to say, "I have glasses," might not be enough to distinguish one person from everyone else. Several people may be wearing glasses.

Have one student leave the room. The group sorts itself according to any rule chosen by the group. The "Outsider" comes in and tries to guess the rule.

PROBLEM SOLVING FOR SECONDARY STUDENTS

PURPOSE: To acquaint participants with problem solving activities for use with their students.

GROUP SIZE: 10-30

TIME: 20 minutes

MATERIALS:
Scissors
Toothpicks
Overhead projector, blackboard or flip chart
Paper
Handouts #7 A-E

ROOM ARRANGEMENT: Divide into groups by grade level or small groups.

PROCEDURE:

1. Trainer announces that participants are now to break up into small groups either by grade level or interest area.

2. Trainer explains that in a few moments one member from each group will come up to examine problem solving activities that can be used in a classroom situation.

3. Trainer allows time for one person from each group to come to the table and choose a problem solving activity.

4. Trainer asks if there are any questions and allows participants 15 minutes to do one of the problem solving activities.

5. At the end of 15 minutes, Trainer has one person from each group briefly summarize the problem solving activity for the rest of the participants.

   Special Instructions - Secondary

   a. The Man in the Pit: 10 minutes

   b. Weekly Problems: 15 minutes
   These may take longer to solve, but limit the class time on them. Present only one problem on overhead or board and give participant time to complete. Discussion should follow as to their answer and method of attack.

Answers:

1. 4 19/24 times around
2. 66 2/3 or 66.67 kph
3. 6
4. 16

c. 5 Minutes Problems: 10 minutes per problems
Allow 5 minutes for working the problem out and 5 for discussion. Present only one problem at a time on an overhead or board. Discussion should follow as to their answer and method used to solve. Answers may vary, but should be correct with support.

1.	70	8.	5
2.	1 hour	9.	mumpet
3.	64	10.	39 days
4.	100	11.	171
5.	RR	12.	625
6.	4	13.	←
7.	48		

d. Math Maps: 15 minutes
The handout has two problems on it. Cut along solid line and then you will have two examples to use.

e. Double Design: 15 Minutes
Use blank paper instead of graph paper and tell participants to use basic shapes in their drawings.

PROBLEM SOLVING ACTIVITY - SECONDARY

THE MAN IN THE PIT

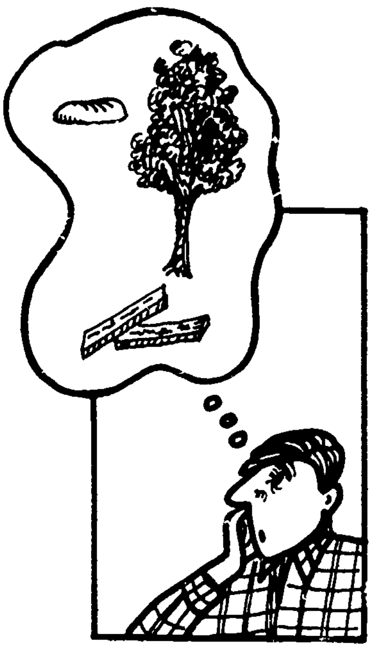
TOPIC: Open-ended problem solving
Creative thinking

PURPOSE: When presenting this problem, if teachers emphasize the fact that there is no right answer and that the purpose is to find any solution that works, students will have the opportunity to use their imagination with little risk or fear of judgment. An open-ended problem should be versatile enough to allow each child to use her knowledge, however limited or extensive.

GROUP SIZE: Individuals or small groups (3 - 5)

MATERIALS: Pencils
Paper

ACTIVITY: Present the problem:



A man, wearing only his hiking shorts, got lost during a summer hike and fell into a pit. The pit was circular. It was about 30 feet across and 20 feet deep. The man found that all the walls of the pit were hard stone, very smooth, and straight up and down, so there were no hand-holds or foot-holds for climbing. The bottom of the pit was also hard as stone. A stream of water kept flowing over the edge of the pit and ran down one wall. Even behind the waterfall the wall was smooth and straight up and down. After reaching the bottom of the pit, the water then disappeared into a small hole in the floor of the pit. The pit was completely empty except for three things:

1. Exactly in the middle of the pit was a tree growing straight up.
2. Near the tree was a loose flat rock.
3. Two boards were also near the tree.

Figure out how the man escaped from the pit.

Have each individual or group share its solution.

ADDITIONAL ACTIVITY: Other problems and questions for open-ended solutions:

... Have you ever seen a ship inside a bottle? How do you think it got there?

... You are locked in a room. It's raining outside and there's a leak in the middle of the ceiling. There is nothing in the room except an old dirty coffee cup and a hole in the middle of the room. The hole is a little bit larger in diameter than a ping pong ball and longer and narrower than your arm. If you can get the ping pong ball out of the hole, you can come out of the room.

PROBLEM SOLVING ACTIVITY - SECONDARY

WEEKLY PROBLEMS

TOPIC: Creative problem solving
Logic

PURPOSE: This activity is designed to allow students to experiment and/or use outside resources over a longer period of time.

GROUP SIZE: Individual or small group (2 - 3)

MATERIALS: Pencil
Paper

ACTIVITY: These are examples of problems that could be handed out one per week, with solutions discussed the following week:

1. Beth and her brother have agreed that each will mow half of their backyard, which is a 60 foot by 72 foot rectangle. The mower cuts a 2-foot-wide path. If Beth starts at a corner and mows around and around the yard toward the center, how many times will she have to go around before she has mowed her half?

2. A test track is 2 kilometers around. A driver drove one lap at 40 kilometers per hour. How fast must the driver drive the second lap in order to average 50 kilometers per hour for the two laps?

3. When 5 is raised to the 999,999th power and then divided by 7, what is the remainder?

4. A snail climbs 3 cm each day. It slips back 2 cm each night. How many days does it take to climb an 18 cm curb?

PROBLEM SOLVING ACTIVITY - SECONDARY

5 MINUTE PROBLEMS

TOPIC: Creative problem solving
Logic

PURPOSE: This activity is designed to force students to think in a short period of time.

GROUP SIZE: Individual or pairs

MATERIALS: Pencil
Paper

ACTIVITY: These are examples of 5 minute problems. They can be given the first part of the class with discussion following:

1. Divide 30 by 1/2 a. id 10. What is the answer?

2. If a doctor gave you three pills and told you to take one every half hour, how long would they last you?

How many quarter-inch cubes does it take to make an inch cube?

4. A book has its pages numbered starting with 1. If the digits of the page numbers from each page were separated and placed in a pile, there would be 192 digits. How many pages does the book have?

5. In one bag there are two red marbles; in another bag there are two blue marbles. In a third bag there is a red and a blue marble. Each bag is labeled RR, BB and RB and is <u>incorrectly</u> marked. You randomly select a bag from which to draw a marble. You draw a red marble from the bag marked RB. What should be the correct label for the bag?

6. If A * B is the sum of the digits in the product of A and B (i.e. 8 * 3 = 6), then 4 * 4 * 4 * equals _______. (Perform * from left to right.)

7. A cube whose sides measure 8 cm was constructed from cubes whose sides measured 1 cm. The large cube was painted black on all faces except the bottom. How many of the unit cubes had exactly two faces painted black?

8. If 5 = 1/5 and 5 = 2/5, then 5 x 5 + 10 x 5 equals ______.

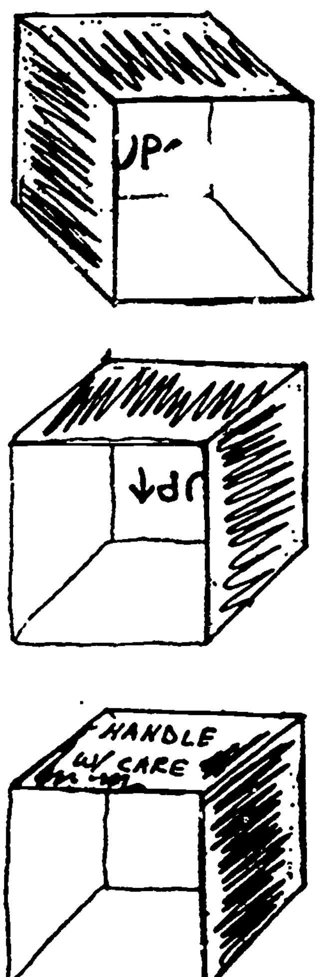
9. A thumpet weighs the same as the combined weights of a humpet and an umpet. An umpet weighs the same as a thumpet less a mumpet. Therefore, a humpet weighs the same as what one object?

10. A water lily doubles itself in size each day. From the time its first leaf appeared to the time when the surface of the pond was completely covered took forty days. How long did it take for the pond to be half covered?

11. The number .232232223... is an irrational number. How many total twos precede the 18th 3?

12. What is the sum of the first 25 odd numbers?

13. What's up? We've shown you which way is UP in two of these three views of the same box. Can you figure out which way is UP in the third?



HANDOUT #7-D

PROBLEM SOLVING ACTIVITY - SECONDARY

MATH MAPS

TOPIC: Spatial visualization

PURPOSE: The challenge is to arrange the numbers in sequential order 1 - 8.

GROUP SIZE: Individual

MATERIALS: Handouts

ACTIVITY: Each person is to only fold the handout on the dotted lines. Some sections may be upside down.

Hints: Mark the sections on both sides of the map.

... Look for patterns: The 4 section must lie between the 3 and 5 sections.
... Don't limit yourself to the obvious: The sections won't usually look like pages in a book. You may need to peek inside to see the section numbers when the map is folded.
... Take a break if you get stuck: Sometimes your mind can find a solution overnight while you sleep.

ADDITIONAL ACTIVITIES: Design your own math maps. Will any arrangement of numbers work?

1	4	2	3
8	5	7	6

1	8	7	4
2	3	6	5

8	7	2	5
1	6	3	4

1	8	2	7
4	5	3	6

A.

1	2	3	4
8	7	6	5

B.

1	2	7	8
4	3	6	5

PROBLEM SOLVING ACTIVITY - SECONDARY

DOUBLE DESIGN

TOPIC:

Communication
Spatial visualization
Giving and following detailed directions

PURPOSE:

The challenge is to duplicate an unseen line drawing by following verbal directions. Participants must not only be able to give precise directions, they must listen carefully. This activity can also provide an introduction to scale drawings.

GROUP SIZE:

Groups of 2 - 4

MATERIALS:

Graph or blank paper
Pencils or pens

ACTIVITY:

Each student draws a design on her own piece of graph paper. Designs should follow the lines on the paper. Encourage students to begin with simple designs.

Have students pair up. One student describes her picture while her partner attempts to reproduce it on a blank sheet of graph paper. Only verbal directions are allowed.

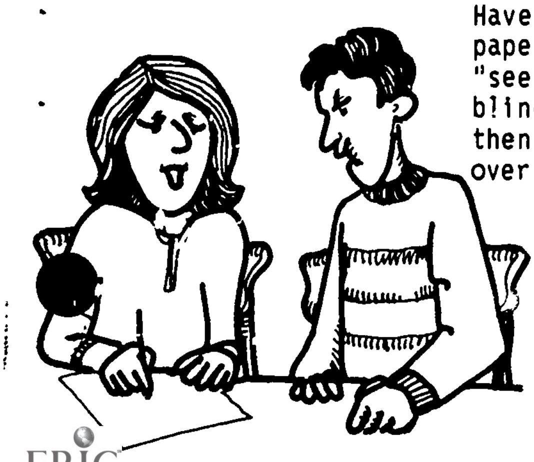
Once the drawing has been described and the copy made, students compare designs. How close are they? What additional information would have been helpful?

Now have the students change roles.

ADDITIONAL ACTIVITY:

This activity can be done on blank paper rather than graph paper.

Have one partner draw a squiggly line on a blank piece of paper. The other partner should be blindfolded. The "seeing" partner places the paper in front of the blindfolded person and gives her a colored pencil. She then gives directions to the blindfolded person to trace over the squiggly line.



ELEMENTARY CAREER AWARENESS

PURPOSE: To tie together numerous aspects of equi+ · in teaching math and to focus on career awareness activi..es to illustrate relevance of math in the future.

GROUP SIZE: 10-30 people

TIME REQUIRED: 30 minutes

MATERIALS: Handout #8
Handout #9

PROCEDURES:

1. Prior to workshop, Trainer should read Trainer Instruction Sheet, Career Awareness.

2. Trainer asks for suggestions on how participants incorporate career awareness into their everyday curriculum. Responses might include:

... Make bulletin boards focusing on specific careers which need math skills.
... Invite guest speakers (see Research Section for appropriate suggestions).
... Assign research projects/reports pertaining to career qualifications.
... Have student write or draw what they think a day of their life will be like when they are 30.

3. Trainer list suggestions on board to flip chart.

4. Trainer distributes Handout # Job Sort Game.

5. Trainer allows five minutes for participants to review "Job Sort" and then asks how the game might be adapted for various grade levels.

6. Trainer distributes Handout #, Career Awareness Activities as a closure to this activity.

CAREER AWARENESS

Young people receive most of their career information from television, movies, and magazines. Their knowledge of the working world is limited to the glamorous, highly visible, or traditional occupations portrayed in the media, and the work their parents or friends do. As a result, young people perceive very limited choices for future careers. Women are portrayed by the media in the traditional roles of mother, teacher, nurse, waitress, or secretary—images that reinforce the belief that women perform only a few roles in society.

When EQUALS participants asked their students to describe a typical day in their life at 30, most students chose very traditional roles. A third of the females indicated nontraditional work, yet almost half of these nontraditional choices were veterinarian. Nearly a third of those mentioning work indicated a job in fields that actually employ only 1% of the work force, such as model, professional athlete, fashion designer, and entertainer. However, many teachers reported that their students also saw the future in very negative terms—work was seen as little more than the drudgery needed to earn enough money to enjoy life on the weekend. Finally, teachers remarked on the low level of students' understanding of the connection between education and future careers. In particular, students have little understanding of the math required for various careers.

The goal of the EQUALS career component is to widen options and improve access to careers through mathematics and science education. By relating the study of math to future careers, teachers can answer the question, "What do I need math for anyway?" and provide students with information that may help them to consider new career options.

The EQUALS career component consists of activities, materials, and strategies that are discussed in the following sections.

One way to expand career options for young people is to introduce them to role models. Women role models provide valuable information, insight into the world of work, and an opportunity for all students to learn about occupations other than those to which they are normally exposed.

Role Models

The most effective strategy in an EQUALS session on nontraditional careers is a panel discussion by three or four women working in a male-intensive field. Each panel member describes her job and a few details about her background. Workshop participants then ask questions of the panel members. Small, informal group discussion with individual panelists can be an extremely valuable follow-up to the large group discussion. Lunch is an ideal time for these discussions.

It is important to include women role models who are in jobs that do not require college training. Women representing apprenticeship programs, the skilled trades, and technical fields are popular panelists. The most frequent suggestion previous EQUALS participants have made about improving workshops is to increase the representation of noncollege-bound, nontraditional careers on the role model panels.

As a required follow-up to many workshops, EQUALS participants have invited role models to visit their classrooms or schools. EQUALS participants have suggested several ways to find role models; however, the workshop leader should be prepared to help participants find role models to speak to their students.

Parents of students may be working in nontraditional jobs or may know someone at their place of work who would be a possible role model. They may also be aware of women in nontraditonal trades or professions outside their places of work or among acquaintances.

The yellow pages of phone directories list most professional and trade organizations, as well as individuals in these careers. The organizations have up-to-date membership lists and may have names of their members who are interested in speaking to groups. Calling local industries and businesses is another approach. Contacting police departments, fire departments, and union halls is helpful in locating people in nontraditional roles. Many schools or school districts have career specialists who may be able to locate speakers or role models for the teacher.

There are some variations on the role model theme which can be quite effective in expanding young people's career options. One teacher told her students that they would be having a pilot visit their class. The next day, their female principal, who was a licensed pilot, came to talk to them about a side of her life that was unknown to them. Often, this type of avocation can be very helpful in breaking down stereotypes and requires very little work on the part of the teacher in making arrangements.

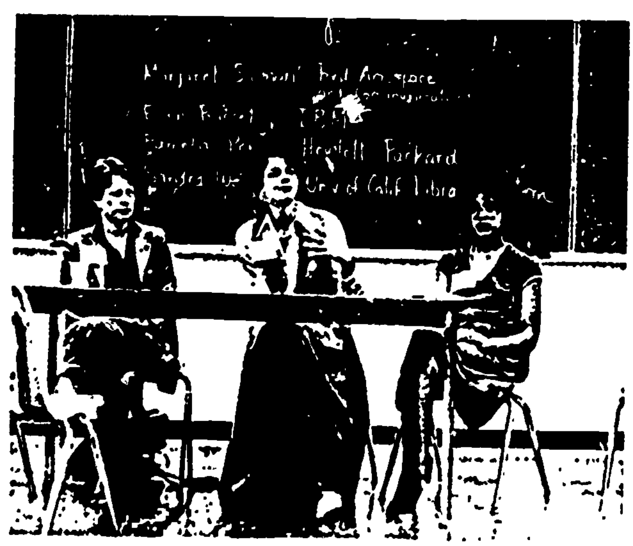
Matching work schedules with class times is difficult, particularly for people working in the trades or apprenticeships. Quite successful role models have been obtained from vocational school trainees. Recent alumnae make ideal role models; they are enthusiastic about their future and can offer students a new perspective. Seniors or graduate students at local colleges can also be effective role models.

Once the role models have agreed to visit, the teacher needs to do some planning so that the students will gain the most from the experience. This might be done by having students brainstorm questions before the visit. These questions can be reproduced for students' reference while the role model is present. The teacher might give the role model copies of these questions ahead of time. Some questions you might share with the role model or add to those that your students suggest are provided. Remember that the students' own questions are the most important.

What were you like when you were our age?
- Did you like school?
- What was the most useful subject(s) you took in school? How did you feel about math?
- What influence did your parents have on your decisions?
- Did you think you would be working in a job like this when you were our age?

How did you get your job?
- How did you decide upon this job?
- What preparation, work experience, or training was especially important?

What is your job like?
- Describe a typical day at your job.
- What satisfactions or rewards do you get from your job?
- What problems do you have in your job?
- How do you use math in your job? Are computers important in your job?
- Do you plan to "move up" in your field? What does it take?
- What other jobs could you do with your experience or training?
- What are typical salaries in your field? Do you earn as much money as your male counterparts?
- What is it like to be a woman on this job? Are there other women?
- What suggestions do you have for people our age who might be interested in your field?



Some EQUALS participants bring a panel of role models for a special school assembly or a pooling of three or four classes. The advantages of having a panel is the potential for interaction among panelists and the variety of nontraditional careers that can be represented. One interesting panel included a civil engineer, an electrician's apprentice, and a computer scientist.

Role models may be brought in for a class and then stay for lunch and discussion. You might open the discussion to all students and announce it school-wide. To increase parent awareness, consider having a role model panel during a parent night. The panel can be made up of parents in nontraditional jobs, which would give them one "experience" as a role model before speaking to students. On the other hand, outside speakers are often eager to talk to parents about nontraditional careers and can be very effective.

Expanding student and teacher awareness of career options is, of course, the desired outcome from interaction with role models. For many educators, role model panels provide a first opportunity to meet or talk with a woman working in a nontraditional field. Because a number of teachers are having to think about career changes, many are acutely interested in what fields offer good employment opportunities. In encouraging this self-interest, EQUALS helps teachers become more enthusiastic about communicating the same information to students.

JOB SORT

The JOB SORT deck consists of 54 cards picturing occupations, work locations, and activities people do in their occupations. JOB SORT illustrates the variety of activities involved in most jobs, and the number of different places that people with the same occupation can work.

The illustrations show many women and minorities in non-traditional careers. Research evidence indicates that young children often identify preferred occupations merely on the basis of the sex-role associated with the occupation. JOB SORT cards are intended to counter the limitations that traditional sex-role stereotyping places on young children's career preferences.

JOB SORT cards are a starting point for an on-going career awareness program for young children. Students can make additional JOB SORT cards by drawing or using illustrations from magazines. Another activity might include finding pictures of people at work, describing the occupation shown, where it is being done, and what activities are implied.

With older students, many other aspects of a job can be considered: wages and salaries; vacation time; satisfactions and rewards; educational requirements (importance of math and science courses); and personal characteristics needed for the job.

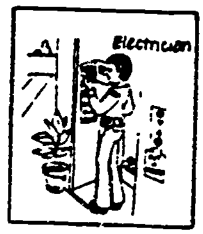
A number of rummy-type or matching games can be played with the JOB SORT deck. Two descriptions have been included to give you an idea of how the cards might be used. Both games, JOB SORT I and II, are noncompetitive. Students themselves will have to determine if a story or description of a "Job Set" is reasonable. No attempt is made to have a "winner" in the games. These and other activities with the JOB SORT deck should draw upon students' own observations and knowledge as much as possible.

You are encouraged to make up your own rules to fit the level of your students or the objectives you wish to meet. We are interested in collecting games and activities using the JOB SORT cards and would appreciate your sharing them with us.

JOB SORT I, Making "Job Sets" 1-4 players

Sort the *occupation* cards, the *location* cards and the *activity* cards into separate piles.

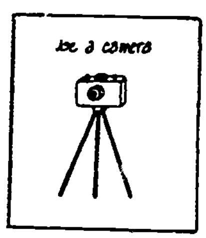
Each player chooses one occupation card for the game. Set the remaining occupation cards aside.

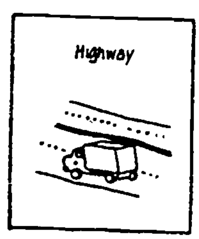


geologist first player

electrician second player

Shuffle the other two piles separately and place each face down in the center of the playing area. The first player draws the top card from each pile and tries to describe a "Job Set" with her occupation card.





Geologist

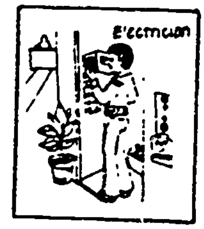
Use a Camera

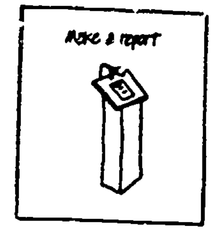
Highway

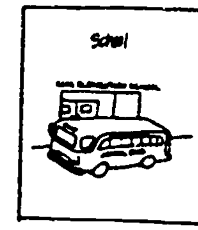
The *geologist takes a picture* of the rock slide along the *highway* so she can help the highway department fix the road.

If she can describe how a worker in her occupation uses the activity at the indicated place of work, she has a Job Set. She then keeps the two cards. If she cannot describe a Job Set, she discards the two cards and draws once more.

The next player now draws the top cards from the *location* and *activity* piles and describes a Job Set.







Electrician

Make a Report

School

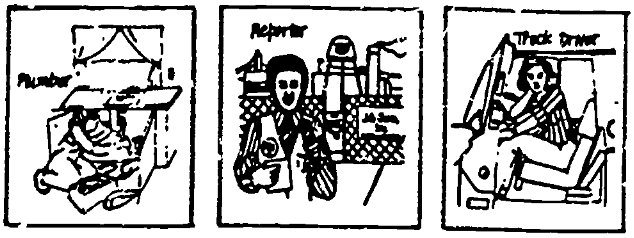
The *electrician makes a report* to the principal about the *school's* electrical service.

Play continues until the location cards and the activity cards are all drawn.
cards are all drawn.

Objectives:

Gain familiarity with various occupations.
Gain practice in oral communication skills.
See adults pictured in non-traditional occupations.
Observe or become aware of how occupations involve a variety of activities.

Remove the 18 occupation cards from the JOB SORT deck. Shuffle the occupation cards and place three face up in the playing area. Set aside the remaining occupation cards.



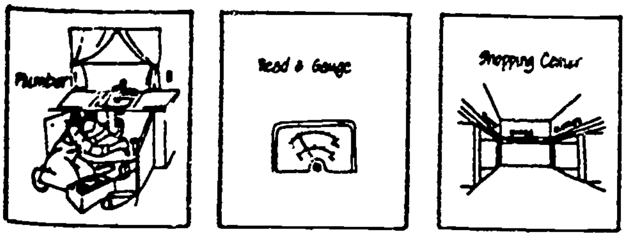
Plumber   Reporter   Truck Driver

Shuffle the activity and location cards together. Deal six to each player.

Player to the left of the dealer chooses two of her cards that, with one of the occupations in the playing area, makes a complete "Job Set" (occupation, location, activity). She plays her two cards by the occupation and gives a description or story of the job.

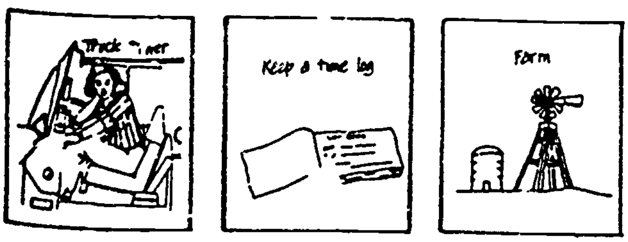
Examples:

The *plumber reads a gauge* on the fire sprinkler system in the *shopping center*.



Plumber   Read a Gauge   Shopping Center

The *truck driver keeps a time log* of her driving hours while hauling cattle from the *farm*.

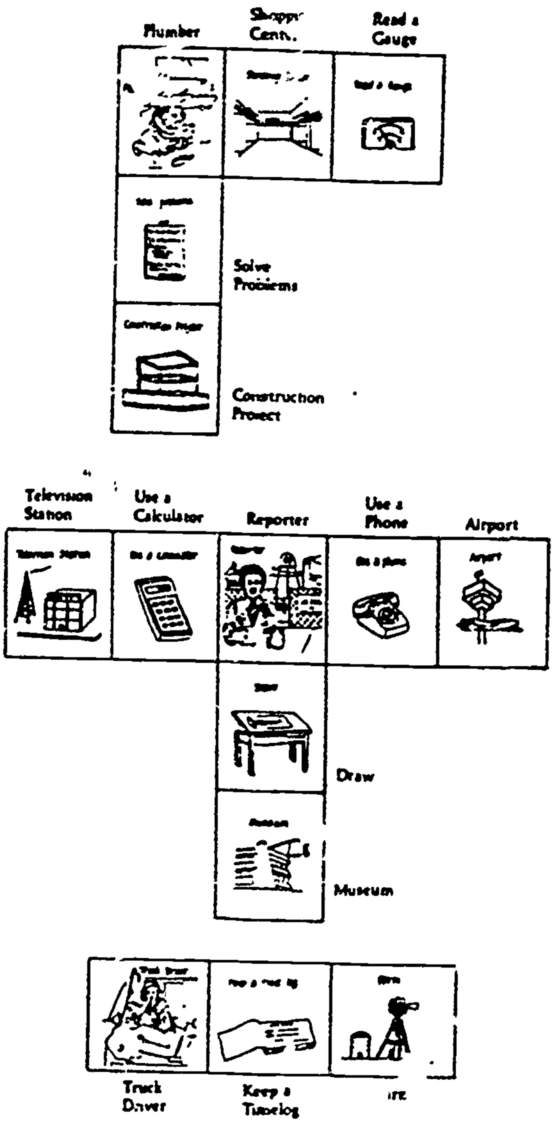


Truck Driver   Keep a Timelog   Farm

The next player chooses two of her cards to match with one of the three occupations. She may play on any of the sides of the occupation cards that remain empty. If she cannot play, for any reason, she draws two more cards and tries to play again.

Play continues in turn. After one player has played all her cards, each remaining player gets one more turn.

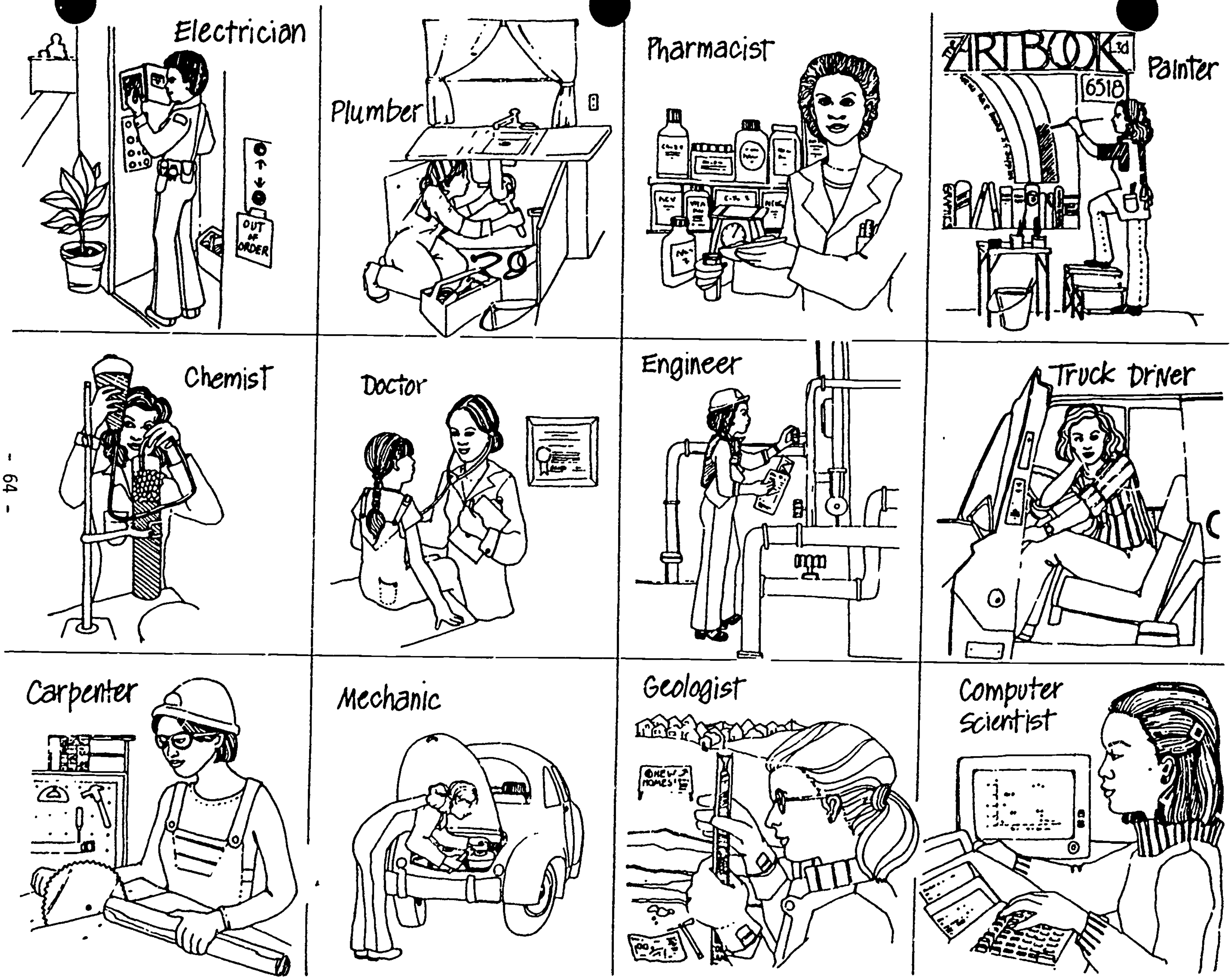
Playing area after six plays:



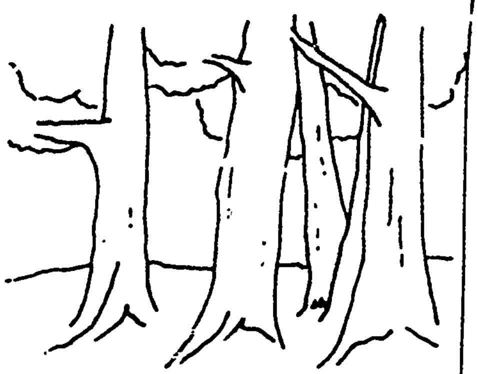
Objectives: (in addition to those of JOB SORT I)
Visualize occupations in diverse environments.

JOB SORT CARDS

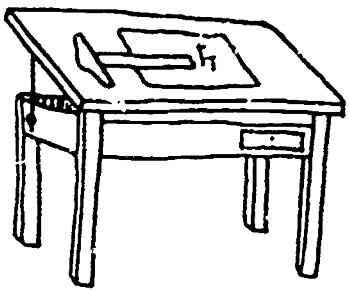
NOTES:


Electrician
Plumber
Pharmacist
Painter
Chemist
Doctor
Engineer
Truck Driver
Carpenter
Mechanic
Geologist
Computer Scientist

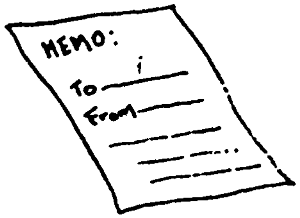
Outdoors



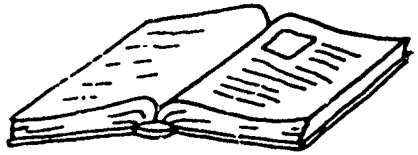
Draw



Write

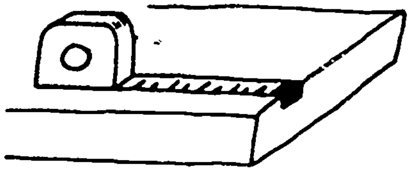


Read

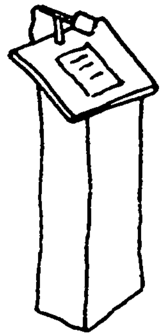


Keep a time log

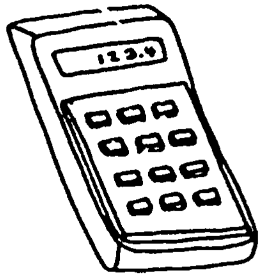
Measure



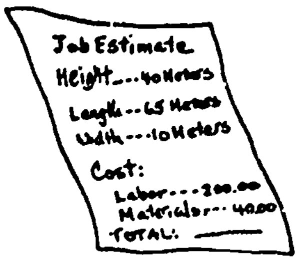
Make a report



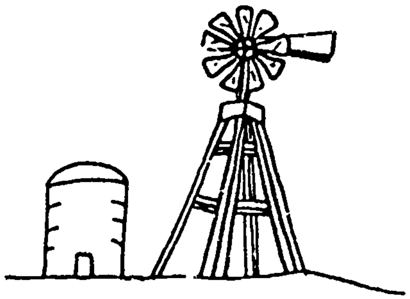
Use a calculator



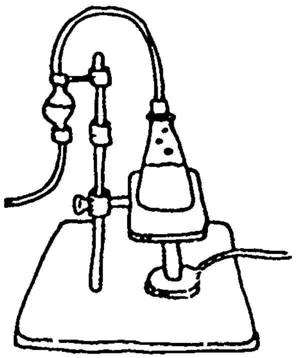
Use numbers



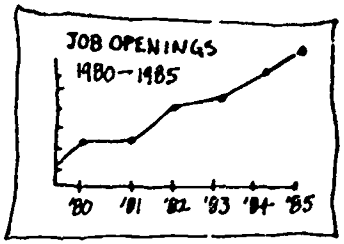
Farm



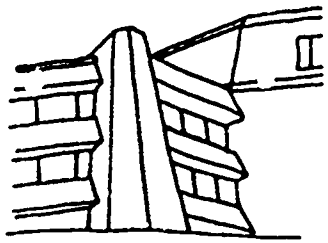
Laboratory



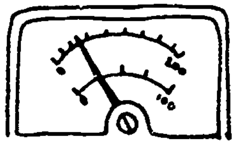
Read a graph



Museum

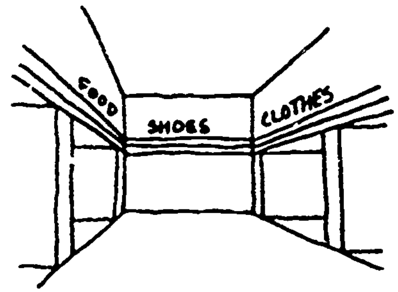


Read a Gauge

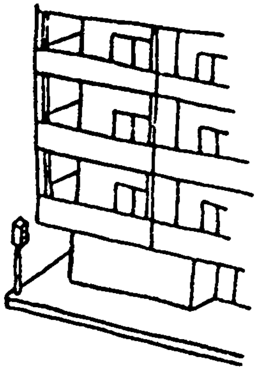


Hospital

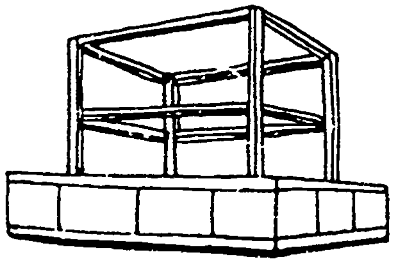
Shopping Center



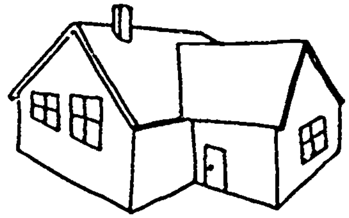
Apartment Building



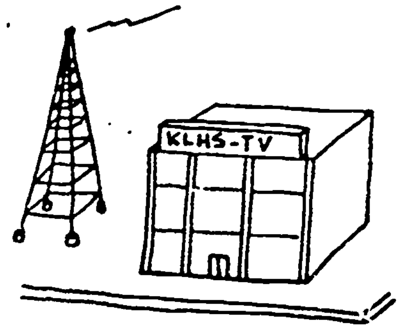
Construction Project



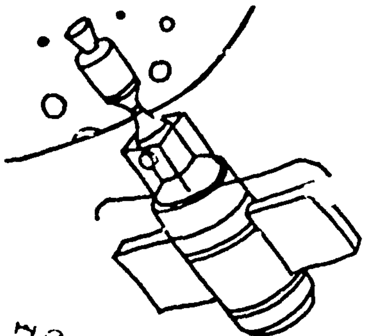
House



Television Station

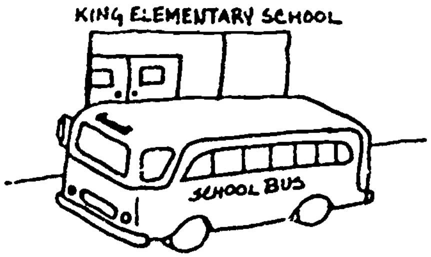


SkyLab

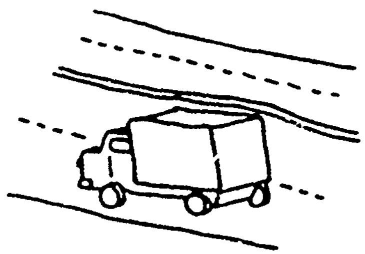


Bank

School

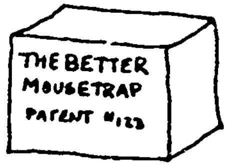


Highway

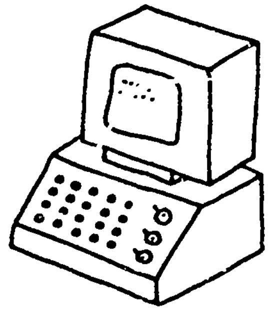


Use a phone

Invent

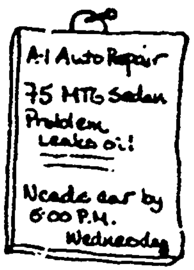


Use a computer

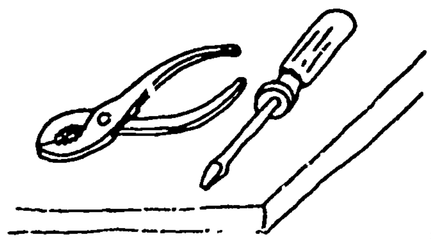


Think of ideas

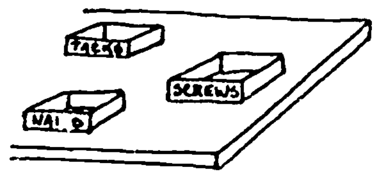
Solve problems



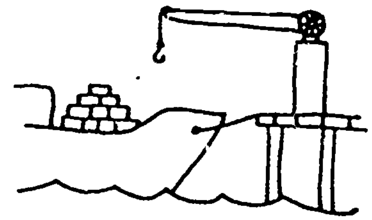
Use tools



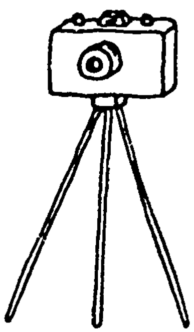
Sort Objects



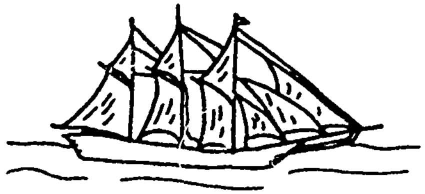
Port



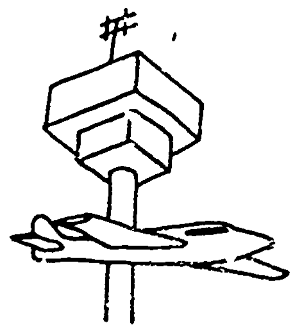
Use a camera



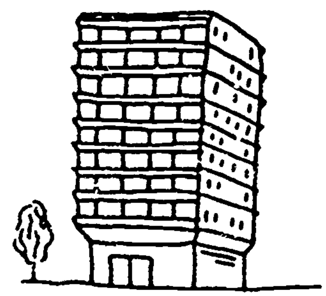
On the Ocean



Airport



Office Building



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CAREER AWARENESS ACTIVITIES

A DAY IN MY LIFE

Grades K - 3

Ask your students to: "Draw a picture of yourself when you are grown up and at work." Have each student dictate to you some statements about their picture. Record their statements below their picture.

Grades 4 - 6

Ask your students to: "Imagine you are 30 years old. Describe a typical Wednesday in your life." Have a class discussion for fun allowing anyone who wishes to do so to read their essay orally. Follow-up activities may include drawing a picture to go with their thoughts or doing a report on the career they discussed.

COLLAGE

Encourage students to collect and bring to class pictures of women working. Challenge them to bring in pictures of women working in occupations that are unusual for women. Let them make a collage with their picture collections. Of course this activity could be done looking for men working in non-traditional careers as well. Some students may be able to bring in real snapshots of people they know who have non-traditional careers.

THIS IS YOUR LIFE

Give each student eight squares of paper. They label the first one 0 - 10 years, the second 10 - 20 years, and continue to 70 - 80 years. On each square of paper the student attempts to describe or predict what her life will be like during those years. Both words and pictures can be used.

Once everyone has finished their time line, have them share a significant event in each decade with the rest of the group. Time lines then can be hung on the wall. Class discussions may focus on how lives would be different if students had chosen different careers. Remind students that 9 out of 10 women work and that women work an average of 22 years during their lifetimes.

CAREER AWARENESS AND WRAP UP

PURPOSE: To tie together numerous aspects of equity in teaching math and to focus on career awareness activities to illustrate relevance of math in the future.

GROUP SIZE: 10-30 people

TIME REQUIRED: 30 minutes

MATERIALS: Handout #10
Dice - enough for 2 per group

PROCEDURES:

1. Trainer distributes Handout #10
2. Trainer summarizes the Introduction and rules of the game.
3. Trainer asks if there are any questions and then instructs each group to begin playing the game.
4. At the end of 30 minutes Trainer asks groups to stop playing.
5. Trainer asks how people felt as they played the game.
6. Trainer asks how participants might use this game in their classroom.

ODDS ON YOU*

Introduction

Our lives are filled with decisions. Some seem very important at the time but have little lasting effect. Others do not seem important at all and yet may have a major impact on our lives. *Odds on You* highlights some important decisions or turning points in your career development. The activity is not intended to predict your future life, but by starting with your academic goals and experiences, you might get an idea of what some possibilities are for your near future.

Odds on You uses a mathematical model. Mathematical models are common in fields such as business, economics, urban planning, science, and medicine. With the growing use of computers, mathematical models are becoming more common in other fields as well.

An example of a mathematical model:

Suppose you work as a buyer for a shoe store. It is time to order the spring shoe selection. Several styles are available in sizes 4 to 10. Should you buy 100 pairs of each size? Why or why not? If you wear a common or average size, think back to how hard it is to find sale items that fit you.

It is anticipated that some will answer that 100 pairs of each size is a good order. Others will, correctly, argue that the number of people wearing each size is not the same, and that relatively large quantities of middle sizes (6,7,8) and very few of the other sizes (4,5,9,10) should be purchased. A good model will predict the number of shoes of each size the buyer needs to purchase.

To give a realistic view of what can happen to you and other students after high school, all decisions in *Odds on You* (those *you* make in real life) are left to chance (rolling of dice). The outcomes of these chance decisions are, however, based on statistics about young people. If you are female, there is one chance in ten that you will become pregnant during the ages 12 to 18. The outcomes in the "Cast Your Fate to the Wind" section reflect this statistic. If you are male, there is over a 90% chance that you will be fully employed during most of your life. If you are female and over 16 years of age, there is a 50% chance that you will be working at any given time. A woman can expect to work an average of 22.9 years. These are the types of data from which the *Odds on You* model was developed.

Special Notes to Teachers or Workshop Leaders

Thousands of statistics are available on what happens to young people as they pass through high school, in post-high school opportunities, and eventually, to the job market. Many of these statistics are surprising, even shocking. Endless lists of numbers turn many young people off. This activity places students in a position of experiencing the statistics. They may drop out of school, get pregnant, and experience failure in getting a job, or they may take substantial math, get professional training, and become a highly paid specialist. The possibility of these outcomes occurring is based on the statistics describing what young people actually do with their lives.

The introduction will give you additional background about the *Odds on You* activity. Some of the background should be shared with your students or participants before you begin the activity. Of particular importance are three items:

1. The activity reflects the decisions made by young people during the ages of 14-24.
2. The activity is a mathematical model of a real situation.
3. Participating in the activity as a member of the opposite sex is intended to give young people a better idea of the choices and outcomes available to their brothers and sisters or their girl or boy friends. *Encourage* students to look upon this aspect as a very important part of the activity.

Students should be allowed to work through the activity in small groups of three or four students. They should be encouraged to help each other and to discuss their results as they go along. Each student will need one pair of dice, a copy of the *Odds on You* pages, and a record sheet.

The activity ends with a salary determination. This should not be a young person's ultimate goal but, with the realities of inflation and the necessity for people to work today, a young person might as well work in a job that gives both satisfaction and a reasonable income.

*[Deve]loped by Alice Kaseberg

ODDS ON YOU: COULD THIS BE YOUR LIFE?

Use this page to record your results.

TALLY EXPERIENCE POINTS HERE from Sections 3, 4, 7, 9, 11

1. Sex: Male ____ Female ____

2. Parents' income:
   Employed mother ____
   Employed father ____
   Total ____

3. Your income during high school: Do you work? ____ Annual Income ____

4. Your high school education:
   A. Graduate ____
   B. Math category ____
   C. More math? ____
   D. Electives
   (1) ____
   (2) ____

5. Cast your ? to the wind: Married? ____ Pregnant? ____

6. Post high school: Circle your next step.
   Armed forces · Vocational school · Out of labor force
   Community college · Job market, Type ____ · College

7. Community college training ____

8. College:
   A. Major: requires calculus · requires no calculus
   B. Graduate? ____
   C.D., Out of labor force ____ Armed forces ____ Job market, Type ____
   Further degree? ____

9. Armed forces ____

10. Out-of-labor-force status ____

11. Vocational training ____

12. Job Market: Type I Type II Type III
    A. Delay in finding work ____
    B. Kind of job ____
    C. Salary ____

Are you satisfied with how chance decided your fate? ____

What decisions made with the dice in this game can you make for yourself?

____

____

____

You have probably already made several decisions about your life. If you have time, go back through the activity and make your own decisions without the dice. Use the dice for decisions from Section 6 to the end. Do you now come out with a more satisfactory job and salary?

ODDS ON YOU

Go through each section in order unless directed to skip. Keep track of your results on the "Could this be your life?" sheet.

1. Sex: Roll 1 die. Even number you are female, odd number a male.

2. Parents' income:

Mother:	Roll 1 die.	Father:	Roll 1 die.
1-2	she is not employed.	1	he is not employed.
3	roll again.	2	he is not in the family unit.
4-6	she is employed.	3-6	he is employed.

If either or both parents are employed, roll two dice and sum. Use this scale to determine the annual income for each employed parent. Use the same roll for both incomes.

Father: $2,000 x sum of dice.
Mother: $1,000 x sum of dice.

3. Your employment during high school:

Female:	Roll 1 die.	Male:	Roll 1 die.
1-2	employed	1-3	employed
3-6	not employed	4-6	not employed

If you are employed, roll two dice and sum Then calculate the annual income.

Employed female: $300 x sum of dice = annual income.
Employed male: $480 x sum of dice = annual income.

Bonus: if sum of dice was over 8, collect experience points: 100 if female, 200 if male.

4. Education in high school. Do sections A, B, C, and D unless directed elsewhere.

4A. High school:	Roll two dice and sum.
2-3	Graduate, top 8% of class (50 experience points)
4-8, 10-12	Graduate
9	Drop out of high school. Go directly to Section 5.

4B. High school math: Roll two dice and sum to determine your math experience.

Female:		Male:	
11	No math	12	No math
6, 10	General Math	7	General math
8, 9	Algebra I	5, 9	Algebra I
2, 7, 12	Geometry	3, 6	Geometry
4, 5	Algebra II	8, 10, 11	Algebra II
3	Calculus or 4th year math (100 experience points)	2, 4	Calculus or 4th year math (100 experience points)

4C. High school math: Your determination to continue in math depends on many factors. See if you have any special reason to take more mathematics. Roll two dice and sum.

Female:

2	A teacher encourages you in junior or senior high.	→ Repeat Section 4B and take the higher math of your two tries. Then go on to Section 4D.
3	You took Algebra in the eighth grade.	
4	You enjoy math.	
5	You have a clear career goal.	
6-12	No reason to take more math	Go on to Section 4D.

Male:

2, 3	Your parents encourage you.	→ Repeat Section 4B and take the higher math of your two tries. Then go on to Section 4D.
4	You have a career goal.	
5	You are good at math.	
6	Your parents expect you to take math	
7-12	No reason to take more math.	Go on to Section 4D.

4D. High school electives: Roll two dice and sum. Select first elective based on this roll.

Female:

2 Computer Programming (200 experience points)
3,5,9,11 Typing, Bookkeeping, Accounting (50 experience points)
4, 10 Art, Journalism, Music (25 experience points)
6-8 Home Economics (25 experience points)
12 Automotive, Drafting, Welding, Woodshop (150 experience points)

Male:

2-4 Typing, Bookkeeping, Accounting (100 experience points)
5, 10 Computer Programming (150 experience points)
6-8 Automotive, Drafting, Welding, Woodshop (100 experience points)
9 Art, Journalism, Music (25 experience points)
11, 12 Home Economics (25 experience points)

Roll again and select a second elective. Record your experience points.

5. Cast your fate to the wind: Roll two dice and sum.

Female:

2-4 Get married (Go directly to Section 10).
5 Get pregnant (Go directly to Section 10).
6-12 Go on to Section 6.

Male:

2-3 Get married (Go directly to Job Market, Section 12 as Type I).
4-12 Go on to Section 6.

6. Post high school. Roll two dice and sum. Find out what you do after high school based on the appropriate math category determined in Section 4B.

If your parents and your together earn over $28,000 per year, take an extra roll and choose the result you prefer within your math category.

A. High School Dropout

2-3	Get G.E.D. (Go to Section 6C)
4	You are out of the labor force, (Go to Section
5-9	Go to the job market, Type I. (Go to Section .
10-12	Go to armed forces. (Go to Section 9)

B. No math

2-3	Go to armed forces. (→9)
4	You are out of the labor force. (→10)
5	Go to vocational school. (→11)
6-10	Go to job market, Type I. (→12)
11-12	Go to community college. (→7)

C. General Math or Algebra I

2-5	Go to community college. (→7)
6-7	Go to job market, Type I. (→12)
8	Go to armed forces. (→9)
9-10	Go to vocational school (→11)
11	You are out of the labor force. (→10)
12	Go to college. (→8)

D. Geometry or Algebra II

2-5	Go to college. (→8)
6-8	Go to community college. (→7)
9-10	Go to job market, Type I. (→12)
11	Go to vocational school. (→11)
12	Go to armed forces. (→9)

E. Calculus or 4th Year Math

2	Go to job market, Type I. (→12)
3-9	Go to college. (→ 8)
10-12	Go to community college. (→7)

7. Community college: Roll two dice and sum.

Female	Male	
2-5	2-5	Take college credit courses, transfer to college in two years. Go on to Section 8.
6-8	7	Take vocational training courses, no additional math. Go to job market, Type I. (200 experience points) (→12)
9	6	Take math missed in high school and continue in college credit courses. Go on to Section 8.
10	8-10	Take math and vocational training courses. Go to job market, Type II. (300 experience points) (→12)
11-12	11-12	Go to job market, Type I. (numerous reasons) (→12)

8. College

8A. College major: Roll two dice and sum. Use your high school math category. Note: In many universities, up to 75% of all possible majors require calculus, including science, economics, business, engineering, and pre-medicine. Traditionally non-calculus majors (librarianship, music, elementary education, literature, and history) are being strongly influenced by computers and, hence, mathematics.

	Female	Male	
General Math	2-11	2-11	Major requires no calculus.
or Algebra I	12	12	Major requires calculus.
Geometry or	2-10	2-9	Major requires no calculus.
Algebra II	11-12	10-12	Major requires calculus.
Calculus or	2-9	2-8	Major requires no calculus.
4th year math	10-1.	9-12	Major requires calculus.

Bonus: If you took high school Algebra II or beyond, take another roll of the dice and see if you can get into a calculus major.

8B. College graduation: Roll two dice and sum.

Female	Male	
4-7	2,4-7	Did not graduate. (→8C below)
2,3,8-12	3,8-12	Graduate. (→8D below)

8C. Did not graduate: Roll two dice and sum.

Female	Male	
2-4,7-12	2-8,10-11	Go to job market, Type I. (→12)
5-6	12	Out of labor force. (→10)
----	9	Go to armed forces. (→9)

8D. You graduate! In Section 8A, you determined whether your major needed calculus. Use the major now to find out what you do after college.

Major required calculus:

Female	Male	
2-7,10	2-7	No further degree. Go to job market, Type III. (→12)
8,12	8-10	M.A., Ph.D., or professional degree. Go t ob market, Type III. (→12)
9	11	No further degree. Out of labor force. (→10)
11	12	M.A., Ph.D. Out of labor force. (→10)

Major required no calculus:

Female	Male	
2-4,7	2,3,8,9	No further degree. Go to job market, Type II. (→12)
5-6	10	No further degree. Out of the labor force. (→10)
8,9	4-7	M.A., Ph.D., or professional degree. Go to job market, Type II. (→12)
10-12	11-12	M.A., Ph.D. Out of labor force. (→10)

9. Armed forces: Roll two dice and sum.

Female	Male	
2-7	2-6	Stay initial enlistment period (3-4 years). (200 experience points) Go to job market, Type I. (→12)
8-10	7-9	Re-enlist, 3-4 years. (250 experience points) Go to job market, Type II. (→12)
11-12	10-12	Stay 20 years and retire with a pension of $14,832/yearly. Go to questions at end of record sheet.

10. Out of labor force: Roll two dice and sum.

Female	Male	
6-8	4-8	Go to job market, Type I, at least 25 years of your life (Type II if you have calculus). (→12)
4-5	3,9	Unemployed, not eligible for compensation. This is your life, well past the age of 24. What are your options now? Go to questions at end of record sheet.
2,3,9-12	2,10-12	Other unpaid positions. What might these be? What are your options now? Go to questions at end of record sheet.

11. Vocational school or apprenticeship training: Roll two dice and sum.

Female	Male	
4-5	2	Service training (200 experience points)
6-8	3	Clerical training (200 experience points)
3	10-12	Fire or police protection (300 experience points)
2	4,6	Mechanic or repair (300 experience points)
9-10	9	Health occupations (300 experience points)
11	8	Machining, printing, industrial (300 experience points)
12	5,7	Electrical, carpentry, plumbing (300 experience points)

Now, go to job market, Type I. (→12)

12. Job market: First, you need to find out how long it takes you to get a job (12A). Then you will use your Type I, Type II, or Type III in the job category section (12B).

12A. Delay in finding a job: Roll two dice and sum.

School Drop-Out

Female	Male	Delay
2-7	2-6	1 to 4 weeks
8-9	7-8	5 to 14 weeks
10	9,11	15 to 26 weeks
11-12	10,12	more than 27 weeks

High school graduate

Female	Male	Delay
2-6	2-6	1 to 4 weeks
7-8	7-8	5 to 14 weeks
9-10	9-10	15 to 26 weeks
11-12	11-12	more than 27 weeks

Education beyond high school

Female	Male	Delay
2-6,9	2-6,10	1 to 4 weeks
7,8,10	7-9	5 to 14 weeks
11-12	11-12	15 to 26 weeks

For every 300 experience points, cut 4 weeks off delay time in finding a job.

12B. Jobs: If you are Type I and have 400 experience points, go on to Type II.

Type I: Roll two dice and sum.

	Female	Male
Clerical (secretary, clerk)	4-6	4
Service Work	10-12	10
Professional, Technical	8	6
Operative (machine op., drivers)	9	7
Sales	7	3
Managers, Administrators	2	8
Laborers	--	11
Craft Workers	--	5,9
Other	3	2,12

Type II: Roll two dice and sum. (Includes educators)

	Fe...ale	Male
Clerical	8-9	---
Service	2-4	3
Professional, Technical	6-7	6-7
Operatives	11	4-5
Managers, Administrators	5	8-9
Sales	10,12	2
Craft Workers	---	10-12

Type III: Roll two uice and sum. Professionals (Excludes educators)

	Female	Male
Engineers	12	9-12
Physicians	2,3	8
Other (Lawyer, veterinarian, C.P./.., M.B.A.)	4-11	2-7

12C. Salary

Salary is determined by your training, your experience, and your education. These salaries represent national average *starting* salaries (1977)

	Type I		Type II	
	Female	Male	Female	Male
Clerical	$7,400	$11,900	$8,300	$13,100
Service	5,600	10,000	6,000	10,500
Professional, Technical	9,300	13,400	10,900	16,800
Operative	6,500	11,500	6,500	11,500
Sales	5,300	12,700	7,000	16,500
Managerial, Administrative	8,700	14,400	13,100	20,600
Laborer	---	9,700	---	---
Craft Worker	---	13,100	--	14,800
Other	7,150	12,300	---	---

	Type III	
	Female	Male
Engineer	$16,000	$19,900
Physician	19,000	25,000
Other	12,000	16,800

SUMMARY ACTIVITIES

Recording Information

As students finish *Odds On You*, they should record the indicated information on charts (shown below) placed on an overhead or a blackboard. This provides a quick visual comparison of results.

	Female	
High School Math	Experience Points	Salary

	Male	
High School Math	Experience Points	Salary

It is possible to change the outcomes in real life. Women do not have to settle for smaller salaries. Critical areas that can help include:

1. Mathematics taken in high school — Taking more math expands job options.
2. Elective choices in high school or post-high school education — Taking computer education or skill building courses expands job options (see which courses give experience points in Section 4D).
3. Recreational activities — Many activities provide opportunities for learning skills and developing the ability to work with people. These help in getting a job.
4. Type of training or college major selected — Some very popular college majors provide little employment opportunity. Some types of vocational training offer excellent job opportunities.
5. Working in part-time jobs during the educational years — Part-time jobs should require considerable learning or on-the-job training for skills usable in future jobs.
6. Taking a nontraditional job — The larger salaries are in fields not ordinarily entered by women.

Discussing Probability

To better understand the exact probabilities of the outcomes, students should work through the What Are The Odds page. Then, the dice outcomes in the *Odds On You* activity can be converted into probabilities. This will give students a better idea about the relative likelihood of their taking certain math courses, going to college, etc. This is best accomplished in small groups where interaction about the probabilities can arise naturally. It is *not* recommended that students be assigned to convert the dice outcomes to probabilities as an individual task.

Students should review how their individual outcomes compare with the overall range of outcomes on any given roll of the dice. Was their outcome typical of other students?

To explore theoretical versus experimental probabilities, a total class summary of results can be compared with the given probabilities for a selection of the sections.

EVALUATION

PURPOSE: To solicit feedback from participants on the content and organization of the workshop.

GROUP SIZE: 10 to 30

TIME REQUIRED: 5 minutes

MATERIALS: Handout #11 (or school district's evaluation tool)

PROCEDURE:

1. Inform participants that they now will have an opportunity to evaluate the workshop.

2. Distribute Handout #11 and allow 10 minutes for participants to complete evaluation.

3. Ask participants to place evaluations on a table as they leave the workshop.

4. While participants are completing their evaluation forms, it would be a good time for the Trainer to complete the Trainer's Module Evaluation found at the end of this module. Once completed, please return to:

   Sex Equity Coordinator
   Department of Education
   P.O. Box F
   Juneau, AK 99811-0500

   In Anchorage send the completed form to:

   Anita Robinson
   Community Relations Department

   Thank you!

Anchorage School District Trainers substitute ASD evaluation form for this page.

WORKSHOP EVALUATION

I. How would you rate this workshop in the following areas?

(Please circle the most appropriate rating)

		Very Clear				Not Clear
A.	Objectives were made clear.	1	2	3	4	5
		To a great extent				Not Met At All
B.	Objectives were met.	1	2	3	4	5
		Great Value				No Value
C.	Information was of practical value.	1	2	3	4	5
		Most Relevant				Not Relevant
D.	Handouts/materials were relevant to my present needs.	1	2	3	4	.
		Highly Effective				Not Effective
E.	Presentation was effective.	1	2	3	4	5

II. Circle one of the following ratings which best describes your feeling about this workshop in comparison to others you have attended?

1 One of the Best

2 Better Than Most

3 About Average

4 Weaker Than Most

5 One of the Worst

What were the strongest features of the workshop? ______________________________

____

What were the weakest features of the workshop? ______________________________

____

TRAINER'S MODULE EVALUATION

TRAINER NOTE: Now that you have completed the workshop, please take a moment to complete the following evaluation. Your input will be of vit.. importance as the moaules are refined to meet the needs of teachers.

YOUR NAME: (optional) ______________________________

NAME OF MODULE: ______________________________

WHERE PRESENTED: ______________________________

NUMBER OF PARTICIPANTS: ______________________________

I. Trainer Instruction Sheet

A. Were trainer instructions clear and precise? _____ YES _____ NO

If no, please state page number and problem area: ______________

____

Other comments: ______________________________

____

B. Was the format of the Trainer Instruction Sheets easy to follow?

_____ YES _____ NO

II. Participant Activities

A. Which activity did the participants appear to get the most out of?

____

B. Are there any activities that you feel need to be eliminated or replaced? If so, please identify. ______________

____

C. Was the timing allocated for activities appropriate?

_____ YES _____ NO

D. Overall, do you feel this module raised the participants' awareness of sex bias?

____

RESEARCH SECTION

EQUALS

EQUALS is a program that focuses on a specific educational problem: math avoidance among young women. Its purpose is to ultimately bring about change in the occupational patterns of working women.

"...nine out of ten girls in school today will work. So long as they are reared to believe that their life's career will be that of housewife and mother, they will be doomed to discovering too late that they must enter the work force at a low-level job, with low reward." (Farley, 1979)

Traditionally, women students have been expected to excel in arts and languages and men in mathematics and the sciences. The enrollment patterns of high school students throughout the country support the premise that advanced mathematics and science courses are for men only.

Although it had been debated and discussed within the scientific community for at least a decade, this pattern was so firmly entrenched that it was not questioned by the public until 1973. At that time, sociologist Lucy Sells demonstrated the consequences of inadequate mathematics preparation for women.

Sell's findings (1973) indicated that substantially more men than women had taken the necessary high school mathematics to enter the calculus sequence required for 60 of the 104 undergraduate majors at the University of California, Berkeley. The remaining 44 majors for which no calculus was required were in the arts and humanities, education, and foreign languages - traditionally female fields of study leading to limited employment opportunities.

Regardless of their educational training, however, women hold a disproportionate share of clerical and service jobs and are severely underrepresented in the scientific, technical, and managerial positions that command high salaries and opportunities for advancement. For example, the earnings gap between the sexes has been steadily increasing at the same time that the percentage of women entering the work force is growing.

EQUALS works to increase the mathematical preparation of women students, thereby improving their opportunities for movement into predominantly male fields of study and work.

THE NEED FOR EQUALS

Why do young women avoid mathematics?

Research demonstrates that girls are as gifted as boys in mathematics, but, as they progress through the elementary and secondary grades, young women lose interest in studying math.

Recent studies have examined the reasons for differences in math participation between young men and women (Armstrong, 1979; Casserly, 1979; Fennema and Sherman, 1978; Fox, 1979; Haven, 1972; Kirk, 1975; Levine, 1976; Stallings, 1979). It appears that young women have a variety of negative attitudes toward mathematics, including lack of interest, lack of confidence in their ability to do well, inability to see the relevance of math to their present or future interests, and acceptance of the stereotypic belief that math is an inappropriate pursuit for females.

To facilitate a change in these attitudes, the EQUALS training uses three components: (1) awareness of the obstacles that math avoidance creates; (2) activities that foster positive attitudes toward and increased competence in mathematics; and (3) materials and strategies that promote interest and motivation in those nontraditional occupations requiring mathematical literacy.

How do negative attitudes toward math develop?

Young women may be influenced by the kinds of activities and games provided by parents. In a small sample of parents of gifted children, Astin (1974) found that boys were more likely to receive science kits, microscopes, or telescopes than girls. In a Lawrence Hall study, Sneider (1979) questioned 139 high school students and learned that 44% of the girls, but only 23% of the boys, had never before used a telescope. Kirk (1975) discovered that gifted young women who were "science bound" were significantly more interested in puzzles and problem solving than were those who were not science-bound.

Thus, EQUALS provides activities that stimulate inquiry and investigation and promote curiosity in problem solving - so essential to the development of mathematical and scientific interests and abilities.

How does the school environment affect women?

Teachers, counselors and peers may perpetuate stereotypic beliefs that women don't need to study math, that boys are better than girls at math, and that women couldn't or shouldn't become scientists or technicians. Nearly half of the elementary and secondary school teachers that Ernest (1975) questioned were convinced that boys are better at math than girls. Many of the talented high school women studied by Casserly (1975) reported having been discouraged from enrolling in advanced science and math courses by their guidance counselors. In some instances, the counselors felt that advanced math wasn't necessary for college, in others, that it was superfluous for girls, perhaps even damaging if it might lower grade point averages.

What is the long-term effect of math avoidance?

Sells termed mathematics the "critical filter" when she described the effects of math avoidance. We can see this filter in operation when we examine current employment patterns. Working women are concentrated in clerical and service jobs, or work as teachers, nurses, or social workers earning approximately 58% of the wages earned by working men.

One way to close the earnings gap between the sexes is to ensure that young women receive the mathematical, scientific, and technical training in secondary school that will encourage their full participation in all fields of employment.

In 1979, women were 3% of the engineers, 3% of the architects, 11% of the physicians, 15% of the life and physical scientists, but 83% of the elementary school teachers. Furthermore, women were 3% of apprentices in the skilled trades, but 99% of the secretarial work force. The need for EQUALS has arisen because female students throughout the country graduate from high school with inadequate mathematics preparation. The effect of this has been to limit severely the number of future educational and occupational opportunities available to young women.

3

*Mathematics and Sex**

Men are not free to avoid math; women are.

In a major address to the American Academy of Arts and Sciences in 1976, Gerard Piel, publisher of *Scientific American*, cited some of the indicators of mathematics avoidance among girls and young women. "The SAT record plainly suggests that men begin to be separated from women in high school," he noted. "At Andover [an elite private high school] 60 percent of the boys take extra courses in both mathematics and science, but only 25 percent of the girls. . . . By the time the presently graduating high school classes are applying to graduate school," he concluded, "only a tenth as many young women as men will have retained the confidence and capacity to apply to graduate study in the sciences."[1]

Some other measures of mathematics avoidance among females are these:

Girls accounts for 49 percent of the secondary school students in the United States but comprise only 20 percent of those taking math beyond geometry.

*"Mathematics and Sex" is the title chosen by John Ernest for his important essay on the problem. The essay was published by the *American Mathematical Monthly* and reprinted by the Ford Foundation in 1976.

The college and university population totals 45 percent women; yet only 15 percent of the majors in pure mathematics are women.

Women make up 47 percent of the labor force and 42 percent of those engaged in professional occupations. Yet they are only 12 percent of the scientific and technical personnel working in America today.

Are these data simply evidence of individual preference, or do they represent a pattern of math avoidance and even math anxiety among women? We know that there are differences in *interest* between the sexes. What we do not know is what causes such differences, that is whether these are differences in ability, differences in attitude, or both; and, even more important, whether such differences, if indeed they exist, are innate or learned.*

Most learning psychologists doing research today are environmentalists; that is, they tend to be on the "nurture" side of the nature-nurture controversy. Most of them would therefore not subscribe to the man on the street's belief that mathematics ability is just one of those innate differences between men and women that can neither be ignored nor explained away. Yet even the most recent research on sex differences in intelligence accepts the fact that performance in math varies by gender.[3] Because this is assumed to be natural and inevitable (if not genetic in origin) for a long time the causes of female underachievement in mathematics have not been considered a promising area for study

*This chapter draws particularly on the work of Elizabeth Fennema, Julia Sherman, and Lynn Fox, who were commissioned by the National Institute of Education in 1976 to review critically research on mathematics and women. Also important is the work of John Ernest, Lorelei Brush, and Lynn Osen. Michael Nelson has prepared an exhaustive annotated bibliography on all aspects of math ability and disability, including a chapter on mathematics and sex. See the reference section for details about these works.[2]

and certainly not an urgent one.

But recently, as women began to aspire to positions in fields previously dominated by men, this attitude began to change. The women's movement and the accompanying feminist critique of social psychology can be credited, I believe, with the rise in interest in mathematics and sex and with the formulation of some important new questions. Do girls do poorly in math because they are afraid that people (especially boys) will think them abnormal if they do well, or is it because girls are not taught to believe that they will ever need mathematics? Are there certain kinds of math that girls do better? Which kinds? At what ages? Are there different ways to explain key concepts of math that would help some girls understand them better?

One example of a new approach is the initially informal survey undertaken by John Ernest, a professor of mathematics at the University of California at Santa Barbara. In 1974, Ernest volunteered to teach a freshman seminar about Elementary Statistics. Believing that these notions are best learned in a concrete situation, he turned his seminar into an investigation of the relationships, real and imagined, between mathematics and sex. His students fanned out into neighboring junior and senior high schools to interview teachers and students about girls' and boys' performance in mathematics. The results of their inquiry were nearly always the same. Both boys and girls, they were told, have a fair amount of trouble doing math and most of them do not like the subject very much. The difference between them is this: boys stick with math because they feel their careers depend on it and because they have more confidence than girls in their ability to learn it.[4]

Ernest augmented the report with a survey of other

people's research on mathematics and women and sent it off in the fall of 1975 to be published by the *American Mathematics Monthly*, a journal read mostly by mathematicians. The article was, however, considered so important that the Ford Foundation pre-printed it as a brochure and mailed it to 44,000 educators around the country. Partly in response to the interest sparked by Ernest's article and partly to the spread of other new research on women and math, in summer 1976 the National Institute of Education commissioned three experts in the field to review critically all the research that had been done so far and to propose new research priorities for the agency.

In one of those reports, Elizabeth Fennema, a professor of curriculum and instruction at the University of Wisconsin-Madison, concluded, in agreement with Ernest's findings, that the problem may not be so much one of discrimination or of differences in ability, but rather one of *math avoidance* on the part of women and girls. Whatever the other reasons might be, not taking math in the eleventh and twelfth grades would surely affect females' math performance later on. Fennema writes: "The problem with girls is not the ability to learn math but the willingness to study math." She bases her conclusion on several recent studies of her own, including a close inspection of math enrollments in Wisconsin high schools. In one of these, at the twelfth grade level, 45 percent of the boys but only 29 percent of the girls were still taking math. Fennema suggests that if four years of high school math were required of all high school students, we would go far toward eliminating the problem of differences in math performance between the sexes.[5]

As Elizabeth Fennema well knows, this is not a realistic solution. Some number of girls (and boys) cannot be pressed or cajoled into taking more math. Still, her argument is worth bearing in mind as we proceed in this analysis of sex differences because girls might do as well as boys if they took the same amount of math. We do not know whether they would because several past national studies of math achievement showing lower scores for girls did not consider that girls take fewer courses in math than boys. A new study, the National Assessment of Performance and Participation of Women in Mathematics, begun in the fall of 1978, will attempt to correct this lack by noting carefully the amount and nature of math previously studied.[6]

In fact, math avoidance is not just a female phenomenon. Most people of both sexes stop taking math before their formal education is complete. Few people become mathematicians and many very smart people do not like math at all. Thus, "dropping out" of math is nearly universal, and is by no means restricted to girls and women. From this perspective, girls who avoid math and math-related subjects may simply be getting the message sooner than boys that math is unrewarding and irrelevant, but boys will also get that message in time.

A recent survey of attitudes toward math among ninth and twelfth graders demonstrated this point very well. Although ninth grade girls had a more negative attitude toward math than ninth grade boys, by the twelfth grade boys had caught up. The researcher concluded that by age 17 a majority of all students have developed an aversion to math, which is tragic but certainly not sex-related.[7]

What then is gender-related? What can we say with certainty about mathematics and sex?

Performance and "Ability"

Since innate "ability" can be measured only by performance on some test, those of us who are interested in sex differences in mathematics are forced to look at the results of tests given to boys and girls to measure their math achievement at different points in their lives. Not all of the tests have been of high quality, but some of the most widely quoted assessments have been carefully done and most have surveyed large, national populations. Yet, even when sex differences are found, they do not necessarily imply sex differences in "mathematical intelligence" or even in aptitude for math.

There are many reasons to be cautious. One is that tests of math performance, however well designed, are still tests. Therefore whatever people bring to a test—test anxiety, math anxiety, or hostility to math—will interfere with their performance. Thus even the best national assessments of girls' and boys' mathematical performance may not reveal as much as we think they do.

Take Project Talent, for example. Completed in 1960, it surveyed thousands of boys and girls. Yet, for all its care in sampling and testing, girls were probably compared to boys who had taken more years of math. It should come as no surprise that by grade twelve, males significantly outperformed females.

Even where greater care has been taken to compare boys and girls with similar mathematics backgrounds, the conclusions reached are not always qualified by other factors or warranted by the degree of difference found. The College Board and National Merit administrators report a consistent pattern of male superiority in math, but do not adjust for the fact that their populations may already be specializing in a field. In the National Assessment of Educational Progress, done in 1972–73, the researchers found some differences but far from enough to warrant their conclusion that the "advantage displayed by males, particularly at the older ages, is overwhelming."[8]

Besides, there is contrary evidence. When Elizabeth Fennema and Julia Sherman compared boys and girls in four Wisconsin high schools in 1974–75, they located disparities in math achievement in only two of the four schools and found only minimal differences at that. Taken alone, this finding is not significant. The important difference between their results and others' probably has to do with careful comparison of boys and girls having similar math backgrounds. It is also possible, as Fennema and Sherman point out, that sex differences in math performance are diminishing—hence, the more recent the research the fewer the differences. Nevertheless, their results must be dealt with.

Although it is tempting to draw conclusions from global comparisons of boys' and girls' performance on math achievement tests, we may not be asking the right question. We know that the critical factor in ability to learn mathematics is intelligence and that "male" and "female" intelligence are very much the same, or certainly more alike than they are different. Thus, we are forced to overemphasize the small differences that show up. In trying to learn why some people do better at math than others, some researchers set the math achievement tests aside and look, instead, at the differ-

ent dimensions of math ability: number sense, computation, spatial visualization, problem-solving skills, and mathematical reasoning. This might enable us to find out exactly where those differences in achievement are and greatly improve our chances of doing something about them.

Popular wisdom holds that females are better at computation and males at problem solving, females at "simple repetitive tasks" and males at restructuring complex ideas. However, since experts cannot even agree on what these categories are, still less . . how to measure them, we have to be careful about accepting sex differences in "mathematical reasoning" or "analytic ability" as reported by the researchers in this field. It is fascinating to speculate that there are "innate capacities" to analyze or to reason mathematically, but these qualities have simply not been found.

What then do we know? As of 1978, are there any "facts" about male-female differences in mathematics performance that we can accept from the varied and not always consistent research findings? Possibly not, since the field is so very much in flux. But at least until recently, the "facts" were taken to be these:

Boys and girls may be born alike in math ability, but certain sex differences in performance emerge as early as such evidence can be gathered and remain through adulthood. They are:

1. Girls compute better than boys (elementary school and on).
2. Boys solve word problems better than girls (from age 13 on).
3. Boys take more math than girls (from age 16 on).
4. Girls learn to hate math sooner and possibly for different reasons.

One reason for the differences in performance, to be explored later in this chapter, is the amount of math learned and used at play. Another may be the difference in male-female maturation.* If girls do better than boys at all elementary school tasks, then they may compute better only because arithmetic is part of the elementary school curriculum. As boys and girls grow older, girls are under pressure to become less competitive academically. Thus, the falling off of girls' math performance from age 10 to 15 may be the result of this kind of scenario:

1. Each year math gets harder and requires more work and commitment.
2. Both boys and girls are pressured, beginning at age 10, not to excel in areas designated by society as outside their sex-role domain.
3. Girls now have a good excuse to avoid the painful struggle with math; boys don't.

Such a model may explain girls' lower achievement in math overall, but why should girls have difficulty in problem solving? In her 1964 review of the research on sex difference, Eleanor Maccoby also noted that girls are generally more conforming, more suggestible, and more dependent upon the opinion of others than boys (all learned, not innate behaviors).** Thus they may not be as willing to take risks or to think for themselves, two necessary behaviors for solving problems. Indeed, a test of third graders that cannot yet be cited found girls

*Girls are about two years ahead of boys on most indices of biological maturation throughout childhood

**This is confirmed by Susan Auslander of the Wesleyan Math Clinic, whose "Analysis of Changing Attitudes toward Mathematics" (1978, unpublished) found that females place more value on outside opinion of success in mathematics than males.

nowhere near as willing to estimate, to make judgments about "possible right answers," and to work with systems they had never seen before. Their very success at doing the expected seems very much to interfere with their doing something new.

If readiness to do word problems, to take one example, is as much a function of readiness to take risks as it is of "reasoning ability," then there is more to mathematics performance than memory, computation, and reasoning. The differences between boys and girls—no matter how consistently they show up—cannot simply be attributed to differences in innate ability.

Still, if you were to ask the victims themselves, people who have trouble doing math, they would probably not agree; they would say that it has to do with the way they are "wired." They feel that they somehow lack something—one ability or several—that other people have. Although women want to believe they are not mentally inferior to men, many fear that in math they really are. Thus, we must consider seriously whether there is any biological basis for mathematical ability, not only because some researchers believe there is, but because some victims agree with them.

The Arguments from Biology

The search for some biological basis for math ability or disability is fraught with logical and experimental difficulties. Since not all math underachievers are women and not all women avoid mathematics, it is not very likely on the face of it that poor performance in math can result from some genetic or hormonal difference between the sexes. Moreover, no amount of speculation so far has unearthed a "mathematical competency" in some tangible, measurable substance in the body. Since masculinity cannot be injected into women to see whether it improves their mathematics, the theories that attribute such ability to genes or hormones must depend on circumstantial evidence for their proof. To explain the percent of Ph.D.'s in mathematics earned by women, we would have to conclude either that these women have different genes, hormones, and brain organization than the rest of us; or that certain positive experiences in their lives have largely undone the negative influence of being female; or both.

In her wide-ranging and penetrating review of the research on biological causes of mathematics ability, Julia Sherman captures the right historical perspective when she reminds us that as recently as a hundred years ago, it was assumed that because men are larger and heavier their brains must be larger and heavier, too.[9] When it was finally demonstrated that brain weight is irrelevant to brain power, the search for the causes of male intellectual superiority turned to secondary sexual differences: the incidence of gout in men, owing to excess uric acid, and the draining off by women's wombs of the "life forces" that fed the brain.

Sherman concludes from her review of the uric acid controversy (which still rages in some circles), that there is no likely connection between gout and genius, although excess uric acid, being the result of excess stress, could correlate with high activity and intelligence without causing it. As for the womb-brain controversy, historians and women students themselves have long since put this fantasy to rest. The fear was not so much that the brain would be starved by the womb as

that the womb would be starved by the brain, and that on the eve of her manifest destiny, America would be populated by a generation of "puny men."

At the root of many of the assumptions about biology and intelligence is the undeniable fact that there have been fewer women "geniuses." The distribution of genius, however, is more a social than a biological phenomenon. An interesting aspect of the lives of geniuses is precisely their dependence on familial, social, and institutional supports. Without schools to accept them, men of wealth to commission their work, colleagues to talk to, and wives to do their domestic chores, they might have gone unrecognized—they might not even have been so smart. In a classic essay explaining why we have so few great women artists, Linda Nochlin Pommer tells us that women were not allowed to attend classes in art schools because of the presence of nude (female) models. Nor were they given apprenticeships or mentors; and even when they could put together the materials they needed to paint or sculpt, they were not allowed to exhibit their work in galleries or museums.[10]

Women in mathematics fared little better. Emmy Noether, who may be the only woman mathematician considered a genius, was honored (or perhaps mocked) during her lifetime by being called "Der Noether" ("Der" being the masculine form of "the"). Der Noether notwithstanding, the search for the genetic and hormonal origins of math ability goes on.

Genetically, the only difference between males and females (albeit a significant and pervasive one) is the presence of two chromosomes designated "X" in every female cell. Normal males have an "X-Y" combination. Since some kinds of mental retardation are associated with sex-chromosomal anomalies, a number of researchers have sought a link between specific abilities and the presence or absence of the second "X." But the link between genetics and mathematics is simply not supported by conclusive evidence.

Since intensified hormonal activity begins at adolescence and since, as we have noted, girls seem to lose interest in mathematics during adolescence, much more has been made of the unequal amounts of the sex-linked hormones, androgen and estrogen, in females and males. Estrogen is linked with "simple repetitive tasks" and androgen, with "complex restructuring tasks." The argument here is not only that such specific talents are biologically based (probably undemonstrable) but also that such talents are either-or; that one cannot be good at *both* repetitive and restructuring kinds of assignments.

Further, if the sex hormones were in any way responsible for our intellectual functioning, we should get dumber as we get older since our production of both kinds of sex hormones decreases with age.* But as far as we know, hormone production responds to mood, activity level, and a number of other external and environmental conditions as well as to age. Thus, even if one day we were to find a sure correlation between the amount of hormone present and the degree of mathematical competence, we would not know whether it was the mathematical competence that caused the hormone level to increase or the hormone level that gave

*Indeed, some people do claim that little original work is done by mathematicians once they reach age 30. But a counter explanation is that creative work is done not because of youth but because of "newness to the field." Mathematicians who originate ideas at 25, 20, and even 18 are benefiting not so much from hormonal vigor as from freshness of viewpoint and willingness to ask new questions. I am indebted to Stewart Gillmor, historian of science, for this idea

us the mathematical competence.

All this criticism of the biological arguments does not imply that what women do with their bodies has no effect on their mathematical skills. As we will see, toys, games, sports, training in certain cognitive areas, and exercise and experience may be the intervening variables we have previously mistaken for biological cause. But first we must look a little more closely at attitude.

Sex Roles and Mathematics Competence

The frequency with which girls tend to lose interest in math just at puberty (junior high school) suggests that puberty might in some sense cause girls to fall behind in math. Several explanations come to mind: the influence of hormones, more intensified sex-role socialization, or some extracurricular learning experience boys have at that age that girls do not have. Having set aside the argument that hormones operate by themselves, let us consider the other issues. Here we enter the world of attitudes, as formed by experience and expectation.

One group of seventh graders in a private school in New England gave a clue to what children themselves think about this. When visitors to their math class asked why girls do as well as boys in math until sixth grade but after sixth grade boys do better, the girls responded: "Oh, that's easy. After sixth grade, we have to do real math." The reason why "real math" should be considered accessible to boys and not to girls cannot be found in biology, but only in the ideology of sex differences.

Parents, peers, and teachers forgive a girl when she does badly in math at school, encouraging her to do well in other subjects instead. " 'There, there,' my mother used to say when I failed at math," one woman remembers. "But I got a talking-to when I did badly in French." "Mother couldn't figure out a 15 percent tip and Daddy seemed to love her more for her incompetence," remembers another. Lynn Fox, who has worked intensively in a program for mathematically gifted teenagers who are brought to the campus of Johns Hopkins University for special instruction, finds it difficult to recruit girls and to keep them in her program. Their parents sometimes prevent them from participating altogether for fear it will make their daughters too different, and the girls themselves often find it difficult to continue with mathematics, she reports, because they experience social ostracism. The math anxious girl we met in Chapter Two who would have lost her social life if she had asked an interesting question in math class, was anticipating just that.

Where do these attitudes come from?

A study of the images of males and females in children's textbooks by sociologist Lenore Weitzman of the University of California at Davis, provides one clue to why math is associated with men and boys in the minds of little children.*[11] "Two out of every three pictures in the math books surveyed were of males, and the examples given of females doing math were insulting and designed to reinforce the worst of the stereotypes," she reports (Fig. III-1 and III-2).

*The study, done in 1975, reviewed textbooks in current use in the California school system. Eight thousand pictures were sorted and coded by sex, race, and age of the persons in the picture; also by what they were doing. It is available as a slide show from the National Education Association in Washington, D.C. The pictures included here are composites, typical, but not taken from any one book.

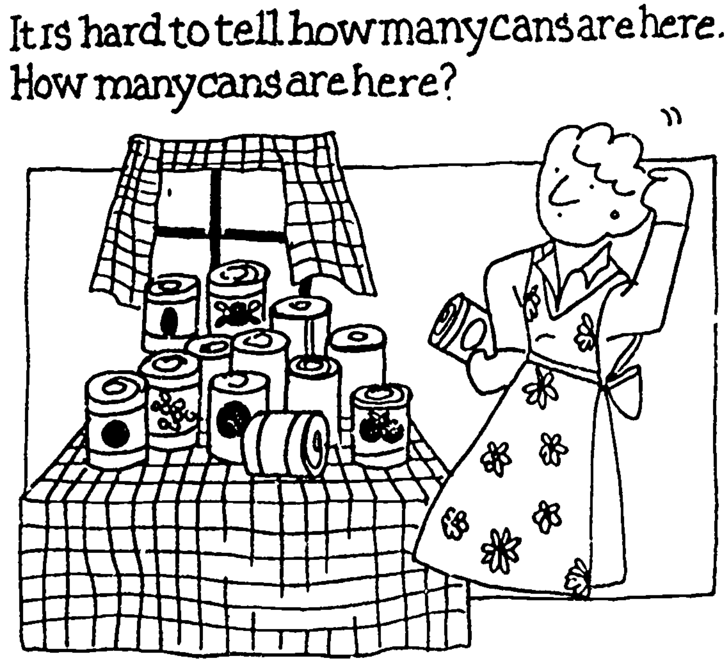

FIG. III-1

Weitzman comments: "It seems ironic that housewives who use so much math in balancing their accounts and in managing household budgets are shown as baffled by simple addition."

"Another feature of the mathematics textbooks," says Weitzman, "is the frequent use of sex as a category for dividing people, especially for explaining set theory" (Fig. III-3).

"When sex is used as a category, girls are told that they can be classified as different," Weitzman believes,

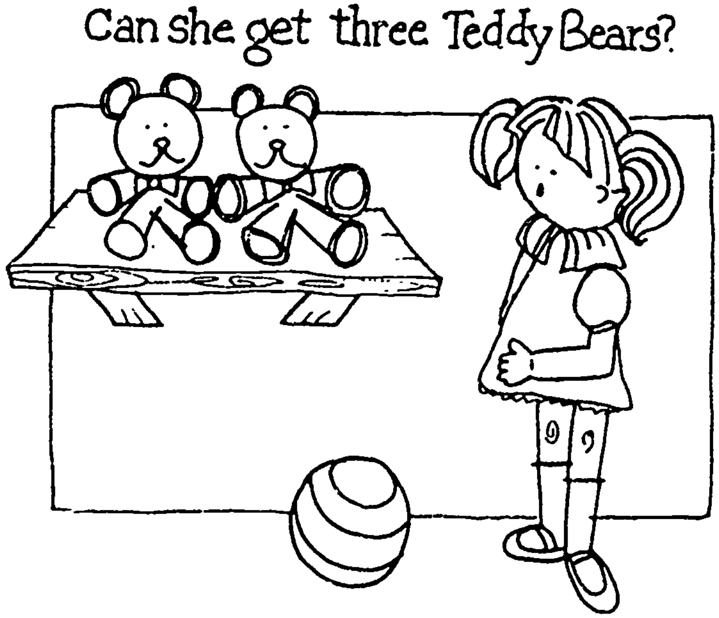

FIG. III-2

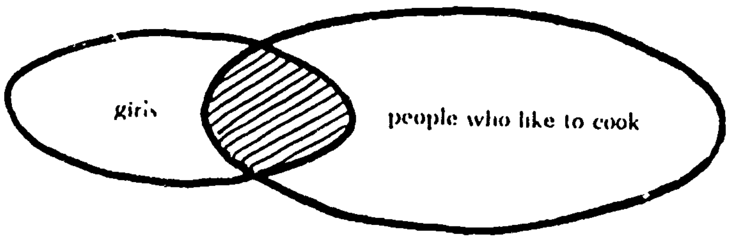

FIG. III-3

"as typically emotional or domestic . . . There is also strong sex typing in the examples used and in the math problems" (Fig. III-4 and III-5).

FIG. III-4

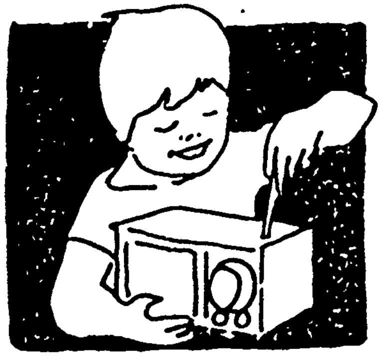

FIG. III-5

"We found math problems," Weitzman writes, "in which girls were paid less than boys for the same work. It would be hard to imagine a textbook publisher allowing this example if a black boy were being paid less than a white boy. Yet it seems legitimate to underpay girls" (Fig. III-6).

In another survey of math textbooks published in 1969, not one picture of a girl was found and the arithmetic problems used as examples in the book showed adult women having to ask even their children for help with math, or avoiding the task entirely by saying, "Wait until your father comes home."*

*As a matter of interest, it is not necessary to use boys or girls or mommies or daddies in arithmetic problems. Holt, Rinehart and Winston and Addison-Wesley publish texts with problems like these: "Sold six fish, bought two more. How many now?"

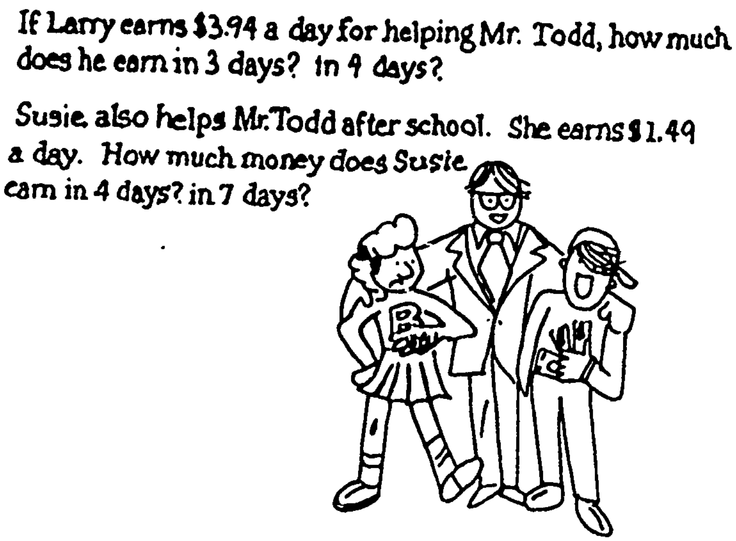

FIG. III-6

Adults remember their junior high school experiences in math as full of clues that math was a male domain. No so long ago, one junior high school regional math competition offered a tie clasp for first prize. A math teacher in another school, commenting unfavorably on the performance at the blackboard of a male student, said to him, "You think like a girl." If poor math thinkers think like girls, who are good math thinkers supposed to be?

The association of masculinity with mathematics sometimes extends from the discipline to those who practice it. Students questioned about characteristics they associate with a mathematician (as contrasted with a "writer") selected terms like rational, cautious, wise, and responsible. The writer, on the other hand, in addition to being seen as individualistic and independent, was also described as warm, interested in people, and

altogether more compatible with a feminine ideal.[12]

As a result of this psychological conditioning, or what Lynn Osen calls the "feminine math-tique," a young woman may consider math and math-related fields inimical to femininity. In an interesting study of West German teenagers, Erika Schildkamp-Kuendiger found that girls who identified themselves with the feminine ideal underachieved in mathematics, that is, did less well than was expected of them on the basis of general intelligence and performance in other subjects.[13]

Thus, the problem of girls' poor performance in math is by no means limited to America, though it is also *not* found in a few countries. "Men in the Soviet Union," writes Norton Dodge, "are so accustomed to women's participating in all fields of study, that the performance of girls is comparable to that of boys in mathematics and physics."[14]

Is there anything about sex roles and mathematics that is not culturally relative?

Interestingly, research shows that intelligence in general (and possibly even mathematical intelligence would fit this model) correlates not with extreme masculinity or femininity but with cross-sex identification. Boys and girls who pursue some of the interests and behaviors of the opposite sex score higher on general intelligence tests and tests of creativity than children who are exclusively masculine or feminine.[15] Girls who resist the pressure to become ladylike and instead develop aggressiveness, independence, self-sufficiency, and tough-mindedness score higher on these tests than more passive, "feminine" girls. And similarly, boys who are "sensitive" score higher than more typical, aggressive boys.

Average boys may feel the need to misbehave in school because they are getting mixed messages from their parents: be naughty on the playing field but quiet and controlled in school. Yet girls are expected to be consistently docile. Another possibility is that children of both sexes who are more intelligent to begin with may find society's rules cumbersome and choose to ignore them. That is, either intelligence breeds behavior that is not typical of the child's sex, or unconventional experience stimulates intelligence, or both.

There is far less data about personality characteristics related to mathematical performance than about personality and intelligence in general. But one recent study compared three groups of women college students, one group higher in verbal abilities than in math, the second group higher in math than in verbal, and the third group about equally competent in both. The study found that the women higher in math ability responded positively to a cluster of attributes considered masculine, such as "logical," "persistent," and "intellectual." But this group also scored high on positively valued feminine attributes, such as "warm," "generous," and so on. The researcher concluded that these women were not rejecting femininity itself, but such low-valued feminine characteristics as dependence and passivity. The women seemed to have a healthy orientation toward the best of both the male and female worlds.[16]

Such a conclusion is an important modification of past studies that found women who do well in math to be "masculine." One such study, linking problem-solving ability in math with a masculine self-image, even went so far as to conclude that nonmathematical men had an image problem. Another early analysis of the autobiographies of women in mathematics concluded that these women either lacked a "typical feminine identification" or were "conflicted" over their female role. Far better and more recent studies of women mathematicians do not explain their success by their masculine or feminine "nature," but find that these women enjoy some real, tangible advantages, among them strong family support.

If sex-role socialization is what one is taught about oneself by others, then we may call what one learns about oneself by oneself "experience." And we must look into the impact of *experience* on learning math.

One appealing theory sounds almost too simple. It is that people who do well in mathematics from the beginning and people who have trouble with it have altogether different experiences in learning math. These differences are not necessarily innate or cognitive or even, at the outset, differences in attitude or in appreciation for math. Rather, they are differences in how people cope with uncertainty, whether they can tolerate a certain amount of floundering, whether they are willing to take risks, what happens to their concentration when an approach fails, and how they feel about failure. These attitudes could be the result of the kinds of risks and failures they remember from early experience with mathematics, because our expectations of ourselves are shaped not simply by what others say but by what we think we can and cannot do.

Street Mathematics: Things, Motion, and Scores

If a ballplayer is batting .233 with 103 times at bat and gets three hits in four times at bat that day, someone watching the game might assume that the day's performance will make a terrific improvement in his batting average. But it turns out that the three-for-four day only raises the .233 to .252. Disappointing, but a very good personal lesson in fractions, ratios, and percents.

Scores, performances like this one, lengths, speeds of sprints or downhill slaloms are expressed in numbers, in ratios, and in other comparisons. The attention given to such matters surely contributes to a boy's familiarity with simple arithmetic functions, and must convince him, at least on some subliminal level, of the utility of mathematics. This does not imply that every boy who handles runs batted in and batting averages well during the game on Sunday will see the application of these procedures to his Monday morning school assignment. But handling figures as people do in sports probably lays the groundwork for using figures later on.*

Not all the skills necessary for mathematics are learned in school. Measuring, computing, and manipulating objects that have dimensions and dynamic properties of their own are part of everyday life for some children. Other children who miss these experiences may not be well primed for math in school.

Feminists have complained for a long time that playing with dolls is one way to convince impressionable little girls that they may only be mothers or housewives, or, in emulation of the Barbie doll, pinup girls when they grow up. But doll playing may have even more serious consequences. Have you ever watched a little girl play with a doll? Most of the time she is talking and not doing, and even when she is doing (dressing, undressing, packing the doll away) she is not learning very much about the world. Imagine her taking a Barbie doll apart to study its talking mechanism. That's not

*An early analysis of the possibility that performance differences between the sexes could arise from differences in games and activities is in Julia Sherman, "Problems of Sex Differences in Space Perception and Aspects of Intellectual Functioning," *Psychological Review*, 1964, 74, pp 290–299

the sort of thing she is encouraged to do. Do girls find out about gravity and distance and shapes and sizes playing with dolls? Probably not!

A college text written for inadequately prepared science students begins with a series of supposedly simple problems dealing with marbles, cylinders, poles made of different substances, levels, balances, and an inclined plane. Even the least talented male science student will probably be able to see these items as objects, each having a particular shape, size, and style of movement. He has balanced himself or some other object on a teeter-totter; he has watched marbles spin and even fly. He has probably tried to fit one pole of a certain diameter inside another, or used a stick to pull up another stick, learning leverage. Those trucks little boys clamor for and get are moving objects. Things in little boys' lives drop and spin and collide and even explode sometimes.

The more curious boy will have taken apart a number of household and play objects by the time he is ten; if his parents are lucky, he may even have put them back together again. In all this he is learning things that will be useful in physics and math. Taking out parts that have to go back in requires some examination of form. Building something that stays up or at least stays put for some time involves working with structure. Perhaps the absence of things that move in little girls' childhoods (especially if they are urban little girls) quite as much as the presence of dolls makes the quantities and relationships of math so alien to them.

In sports played, as well as sports watched, boys learn more math-related concepts than girls do. Getting to first base on a not very well hit grounder is a lesson in time, speed, and distance. Intercepting a football in the air requires some rapid intuitive eye calculations based on the ball's direction, speed, and trajectory. Since physics is partly concerned with velocities, trajectories, and collisions of objects, much of the math taught to prepare a student for physics deals with relationships and formulas that can be used to express motion and acceleration.* A young woman who has not closely observed objects travel and collide cannot appreciate the power of mathematics.

Unfamiliarity with things may also cause a girl to distrust her environment. Since the movement of objects seems not only irregular but capricious, watching things move may not seem to her to be as reliable a way to learn about the world as following the lesson in a book.** A wilderness canoe instructor confirms this as he describes a woman learning to canoe:

> When I start my preliminary instruction, she hangs on to my words, watching me intently. When she gets into the canoe, she mimics exactly what I have done, even if it is inappropriate ... She wants to know how to put the paddle in the water, how hard to pull on it, when to start pulling, where to hold it . . . She makes the operation into a ritual like a dance, becoming increasingly tense (and frustrated) as time goes on . . .
>
> At some point I say to her, "Now look, you are trying to get your feedback from the wrong place. You keep watching me and you should be watching the boat. The boat will tell you what is right and what is wrong. When you do it right the boat does what you want it to.
>
> "I can tell you what to look for, but I can't tell you how it feels. As long as you keep watching me you will get nowhere. Now forget about being instructed. Just go out in the canoe and play around with it and find out what it does."[17]

*In learning physics, unlike math, however, intuitive notions have to be unlearned. for example, heavier objects will not always fall faster So, to some extent, real-life experience may be counterproductive.

**Even the arithmetic games that girls like to play will hardly teach them about the world of natural physical events Monopoly and playing store provide practice in arithmetic fundamentals, but nothing that might suggest some of the more complex phenomena in her environment that mathematics can explain.

The point is that what we get out of an experience, even a good one, may depend on what we have done and learned before. One thing is what you know you *can* do. Andther is what you think you *should* do; and the combination of limited physical experience and negative attitudes toward math may be the principal contributor to females' poorer performance in mathematics.

Conclusion

After surveying the summaries of research in this area and interviewing people who claim to be incompetent at mathematics, I have reached a conclusion. Apart from general intelligence, which is probably equally distributed among males and females, the most important elements in predicting success at learning math are motivation, temperament, attitude, and interest. These are at least as salient as genes and hormones (about which we really know very little in relation to math), "innate reasoning ability" (about which there is much difference of opinion), or number sense. This does not, however, mean that there are no sex differences at all.

What is ironic (and unexpected) is that as far as I can judge sex differences seem to be lodged in *acquired skills;* not in computation, visualization, and reasoning *per se,* but in ability to take a math problem apart, in willingness to tolerate certain kinds of ambiguity, and in careful attention to mathematical detail. Such temperamental characteristics as persistence and willingness to take risks may be as important in doing math as pure memory or logic. And attitude and self-image, particularly during adolescence when the pressures to conform are at their greatest, may be even more important than temperament. Negative attitudes, as we all know from personal experience, can powerfully inhibit intellect and curiosity and can keep us from learning what is well within our power to understand.

An Afterword: Math Anxiety or Math Avoidance—How Can We Tell the Difference?

There is ample evidence that avoidance of mathematics is disproportionate among girls and women, beginning with the eleventh grade and extending through every stage of their educational and professional development. A recent study of women Ph.D. candidates in political science found that of all the factors that might have influenced these women's choice of graduate school (including academic standards, prestige, percentage of women on the faculty, financial aid availability, etc.), only one factor in every case predicted the school the woman selected: whether there was a mathematics or statistics requirement.[18] Was this math avoidance rational or does it indicate severe math anxiety?

It may be, as some argue today, that math anxiety is only forgetfulness, unfamiliarity, and awkwardness in returning to a subject one has not studied for a very long time, that it is not so much a *cause* of math avoid-

ance as an *effect*. Leaving that question for the moment, are there dangers in emphasizing "anxiety" over "avoidance?" Does such a term, particularly when applied to women, imply that women are more impressionable and weaker in spirit than men? Several feminists criticize the anxiety model, pointing out that since the causes of math anxiety lie in "political and social forces that oppress women" and are not wholly psychological and educational in origin, the goal of remediation should not be "the curing of an individual case but the elimination of the conditions that foster the disease."[19]

The identification of mathematics anxiety as a problem for women could become two-edged. Focusing on one more female "disability" may feed the prejudices that already abound in the real world about women and math, women and science, and women and machines. We also have to consider the needs of women who are very competent in math and have a hard time proving this to their colleagues. Finally, we have to contemplate the possibility that attention given to this issue might expose women to exploitation by "math anxiety experts."

Mindful of all these objections, I still argue that excessive anxiety inhibits women more than it does men. If you ask any female how she feels about mathematics, you may find out how she feels about many other gender-related aspects of her life as well. Very bright girls who excel at almost everything in school feel quite comfortable failing at math, not simply because their parents allow it and their peers accept it, but because it provides a solution to the conflicts their brightness creates for them. Rebellious adolescent girls, on the other hand, may actually force themselves to like and do well at math as a way of holding their femininity at bay for a while.

Trying to help one young woman graduate student overcome her intense hostility to math, Stanley Kogelman, co-founder of Mind over Math, heard her say that it was the "logic" and "discipline" of mathematics that she disliked most. Probing to find out where those feelings about "logic" and "discipline" come from, Kogelman concluded that the woman was really disturbed by the fear that she would enjoy the rigorous part of her *own* mind. Mathematics was incidental in her struggle. She was actually in conflict over her own identity.[20]

Do men suffer from math anxiety, and does it intrude as much on their lives as it does for women? Until we have more satisfactory measures of math anxiety and have more math autobiographies from men, we will not have much to say about this. But one recent dissertation study does suggest that although men have math anxiety too, it doesn't trouble them quite as much. The study was done with 655 Ohio State University undergraduates enrolled in a precalculus course. The researcher tested their math anxiety (as best she could with a paper-and-pencil questionnaire) and then compared their anxiety ratings with their final grades. She found, interestingly, that the men's math anxiety scores did *not* correlate with their final course grades nearly as much as the women's anxiety scores correlated with theirs.[21] Perhaps the men went to greater lengths to hide their anxiety even from the researcher. Or perhaps, as the researcher concludes, math anxiety is harder for women to overcome.

My hunch is that the researcher is right. Men have math anxiety too, but it disables women more.[22]

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2

The Nature of Math Anxiety: Mapping the Terrain

A warm man never knows how a cold man feels.
—Alexander Solzhenitsyn

Symptoms of Math Anxiety

The first thing people remember about failing at math is that it felt like sudden death. Whether it happened while learning word problems in sixth grade, coping with equations in high school, or first confronting calculus and statistics in college, failure was sudden and very frightening. An idea or a new operation was not just difficult, it was impossible! And instead of asking questions or taking the lesson slowly, assuming that in a month or so they would be able to digest it, people remember the feeling, as certain as it was sudden, that they would *never* go any further in mathematics. If we assume, as we must, that the curriculum was reasonable and that the new idea was merely the next in a series of learnable concepts, that feeling of utter defeat was

simply not rational; and in fact, the autobiographies of math anxious college students and adults reveal that no matter how much the teacher reassured them, they sensed that from that moment on, as far as math was concerned, they were through.

The sameness of that sudden death experience is evident in the very metaphors people use to describe it. Whether it occurred in elementary school, high school, or college, victims felt that a curtain had been drawn, one they would never see behind; or that there was an impenetrable wall ahead; or that they were at the edge of a cliff, ready to fall off. The most extreme reaction came from a math graduate student. Beginning her dissertation research, she suddenly felt that not only could she never solve her research problem (not unusual in higher mathematics), but that she had never understood advanced math at all. She, too, felt her failure as sudden death.

Paranoia comes quickly on the heels of the anxiety attack. "Everyone knows," the victim believes, "that I don't understand this. The teacher knows. Friends know. I'd better not make it worse by asking questions. Then everyone will find out how dumb I really am." This paranoid reaction is particularly disabling because fear of exposure keeps us from constructive action. We feel guilty and ashamed, not only because our minds seem to have deserted us but because we believe that our failure to comprehend this one new idea is proof that we have been "faking math" for years.

In a fine analysis of mathophobia, Mitchell Lazarus explains why we feel like frauds. Math failure, he says, passes through a "latency stage" before becoming obvious either to our teachers or to us. It may in fact take some time for us to realize that we have been left behind. Lazarus outlines the plight of the high school student who has always relied on the memorize-what-to-do approach. "Because his grades have been satisfactory, his problem may not be apparent to anyone, including himself. But when his grades finally drop, as they must, even his teachers are unlikely to realize that his problem is not something new, but has been in the making for years."[1]

It is not hard to figure out why failure to understand mathematics can be hidden for so long. Math is usually taught in discrete bits by teachers who were themselves taught this way; students are tested, bit by bit, as they go along. Some of us never get a chance to integrate all these pieces of information, or even to realize what we are not able to do. We are aware of a lack, but though the problem has been building up for years, the first time we are asked to use our knowledge in a new way, it feels like sudden death. It is not so easy to explain, however, why we take such personal responsibility for having "cheated" our teachers and why so many of us believe that we are frauds. Would we feel the same way if we were floored by irregular verbs in French?

One thing that may contribute to a student's passivity is a common myth about mathematical ability. Most of us believe that people either have or do not have a mathematical mind. It may well be that mathematical imagination and some kind of special intuitive grasp of mathematical principles are needed for advanced research, but surely people who can do college-level work in other subjects should be able to do college-level math as well. Rates of learning may vary. Competence under time pressure may differ. Certainly low self-esteem will interfere. But is there any evidence that a student

needs to have a mathematical mind in order to succeed at *learning math?*

Leaving aside for the moment the sources of this myth, consider its effects. Since only a few people are supposed to have this mathematical mind, part of our passive reaction to difficulties in learning mathematics is that we suspect we may not be one of "them" and are waiting for our nonmathematical mind to be exposed. It is only a matter of time before our limit will be reached, so there is not much point in our being methodical or in attending to detail. We are grateful when we survive fractions, word problems, or geometry. If that certain moment of failure hasn't struck yet, then it is only temporarily postponed.

Sometimes the math teacher contributes to this myth. If the teacher claims an entirely happy history of learning mathematics, she may contribute to the idea that some people—specifically her—are gifted in mathematics and others—the students—are not. A good teacher, to allay this myth, brings in the scratch paper he used in working out the problem to share with the class the many false starts he had to make before solving it.

Parents, especially parents of girls, often expect their children to be nonmathematical. If the parents are poor at math, they had their own sudden death experience; if math was easy for them, they do not know how it feels to be slow. In either case, they will unwittingly foster the idea that a mathematical mind is something one either has or does not have.

Interestingly, the myth is peculiar to math. A teacher of history, for example, is not very likely to tell students that they write poor exams or do badly on papers because they do not have a historical mind. Although we might say that some people have a "feel" for history, the notion that one is *either* historical or nonhistorical is patently absurd. Yet, because even the experts still do not know how mathematics is learned, we tend to think of math ability as mystical and to attribute the talent for it to genetic factors. This belief, though undemonstrable, is very clearly communicated to us all.

These considerations help explain why failure to comprehend a difficult concept may seem like sudden death. We were kept alive so long only by good fortune. Since we were never truly mathematical, we had to memorize things we could not understand, and by memorizing we got through. Since we obviously do not have a mathematical mind, we will make no progress, ever. Our act is over. The curtain down.

Ambiguity, Real and Imagined

> What is a satisfactory definition? For the philosopher or the scholar, a definition is satisfactory if it applies to those things and only those things that are being defined; this is what logic demands. But in teaching, this will not do: a definition is satisfactory only if the students understand it.
>
> —H. Poincaré

Mathematics autobiographies show that for the beginning student the language of mathematics is full of ambiguity. Though mathematics is supposed to have a very precise language, more precise than our everyday use (this is why math uses symbols), it is true that mathematical terms are never wholly free of the connotations we bring to words, and these layers of meaning may get

in the way. The problem is not that there is anything wrong with math; it is that we are not properly initiated into its vocabulary and rules of grammar.

Some math disabled adults will remember, after fifteen to thirty years, that the word "multiply" as used for fractions never made sense to them. "Multiply," they remember wistfully, always meant "to increase." That is the way the word was used in the Bible, in other contexts, and surely the way it worked with whole numbers. (Three times six always produced something larger than either three or six.) But with fractions (except the improper fractions), multiplication always results in something of smaller value. One-third times one-fourth equals one-twelfth, and one-twelfth is considerably smaller than either one-third or one-fourth.

Many words like "multiply" mean one thing (like "increase rapidly") when first introduced. But in the larger context (in this case all rational numbers), the apparently simple meaning becomes confusing. Since students are not warned that "multiplying" has very different effects on fractions less than one, they find themselves searching among the meanings of the word to find out what to do. Simple logic, corresponding to the words they know and trust, seems not to apply.

A related difficulty for many math anxious people is the word "of" as applied to fractions. In general usage, "of" can imply division, as in "a portion of." Yet, with fractions, one-third *of* one-fourth requires multiplication. We can only remember this by suspending our prior associations with the word "of," or by memorizing the rule. Or, take the word "cancel" as used carelessly with fractions. We were told to "cancel numerators and denominators" of fractions. Yet nothing is being "cancelled" in the sense of being removed for all time. The same holds true for negative numbers. Once we have learned to associate the minus sign with subtraction, it takes an explicit lesson to unlearn the old meaning of minus; or, as a mathematician would put it, to learn its meaning as applied to a new kind of number.

Knowles Dougherty, a skilled teacher of mathematics, notes:

> It is no wonder that children have trouble learning arithmetic. If you ask an obedient child in first grade, "What is Zero," the child will call out loudly and with certainty, "Zero is nothing." By third grade, he had better have memorized that "Zero is a place-holder." And by fifth grade, if he believes that zero is a number that can be added, subtracted, multiplied by and divided by, he is in for trouble.[2]

People also recall having problems with shapes, never being sure for example whether the word "circle" meant the line around the circle or the space within. Students who had such difficulties felt they were just dumber than everyone else, but in fact the word "circle" needs a far more precise definition. It is in fact neither the circumference nor the area but rather "the locus of points in the plane equidistant from a center" (Fig. II-1).*

A mind that is bothered by ambiguity—actual or perceived—is not usually a weak mind, but a strong one. This point is important because mathematicians argue that it is not the subject that is fuzzy but the learner who is imprecise. This may be, but as mathematics is often taught to amateurs differences in meaning be-

*One student, learning to find the "least common denominator," took the phrase "least common" to mean "most unusual" and hunted around for the "most unusual denominator" she could find. Instead of finding the smallest common denominator, then, she found a very large one and was appropriately chastised by her teacher for misunderstanding the question

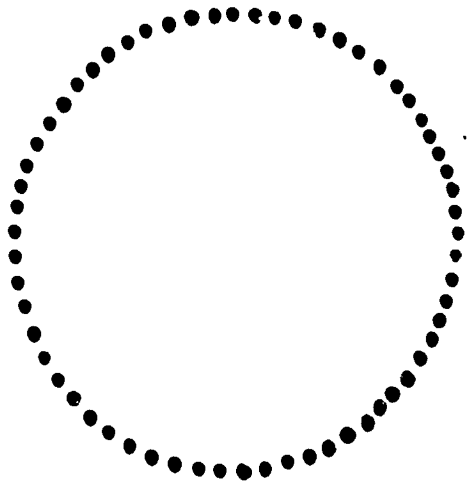


FIG. II-1

tween common language and mathematical language need to be discussed. Besides, even if mathematical language is unambiguous, there is no way into it except through our spoken language, in which words are loaded with content and associations. We cannot help but think "increase" when we hear the word "multiply" because of all the other times we have used that word. We have been coloring circles for years before we get to one we have to measure. No wonder we are unsure of what "circle" means. People who do a little better in mathematics than the rest of us are not as bothered by all this. We shall consider the possible reasons for this later on.

Meanwhile, the mathematicians withhold information. Mathematicians depend heavily upon customary notation. They have a prior association with almost every letter in the Roman and Greek alphabets, which they don't always tell us about. We think that our teachers are choosing X or a or delta (Δ) arbitrarily. Not so. Ever since Descartes, the letters at the end of the alphabet have been used to designate unknowns, the letters at the beginning of the alphabet usually to signify constants, and in math, economics, and physics generally Δ means "change" or "difference." Though these symbols appear to us to be chosen randomly, the letters are loaded with meaning for "them."

In more advanced algebra, the student's search for meaning is made even more difficult because it is almost impossible to visualize complex mathematical relationships. For me, the fateful moment struck when I was confronted by an operation I could neither visualize nor translate into meaningful words. The expression $X^{-2}=\frac{1}{X^2}$ did me in. I had dutifully learned that exponents such as ^{2} and ^{3} were shorthand notations for multiplication: a number or a letter squared or cubed was simply multiplied by itself twice or three times. Trying to translate math into words, I considered the possibility that X^{-2} meant something like "X not multiplied by itself" or "multiplied by not-itself." What words or images could convey to me what X^{-2} really meant? To all these questions—and I have asked them many times since—the answer is that $X^{-2}=\frac{1}{X^2}$ is a definition consistent with what has gone before. I have been shown several demonstrations that this definition is indeed consistent with what has gone before.* But at the time

*While interviewing for this book, I have finally found out that negative two is a different kind of number from positive two and that it was naïve of me to think that it would have the same or similar effect on X. And it does work. If you divide X^3 by X^5 (remember you subtract exponents when you divide) you end up with $\frac{1}{X^2}$. See the following.

$$X^{-2} \equiv \frac{X^3}{X^5} = \frac{XXX}{XXXXX} = \frac{1}{XX} = \frac{1}{X^2}$$

I did not want a demonstration or a proof. I wanted an explanation!

I dwell on the X^{-2} example because I have often asked competent mathematicians to recall for me how they felt the first time they were told $X^{-2} = \frac{1}{X^2}$. Many remember merely believing what they were told in math class, or that they soon found the equivalency useful. Unlike me, they were satisfied with a definition and an illustration that the system works. Why some people should be more distrustful about such matters and less willing to play games of internal consistency than others is a question we shall return to later.

Willing suspension of disbelief is a phrase that comes not from mathematics or science but from literature. A reader must give the narrator an opportunity to create images and associations and to "enter" these into our mind (the way we "enter" information into a computer), in order to carry us along in the story or poem. The very student who can accept the symbolic use of language in poetry where "birds are hushed by the moon" or the disorienting treatment of time in books by Thomas Mann and James Joyce, may balk when mathematics employs familiar words in an unfamiliar way. If willingness to suspend disbelief is specific to some tasks and not to others, perhaps it is related to trust. One counsellor explains math phobia by saying, "If you don't feel safe, you won't take risks." People who don't trust math may be too wary of math to take risks.

A person's ability to accept the counter-intuitive use of time in Thomas Mann's work and not the new meaning of the negative exponent does not imply that there are two kinds of minds, the verbal and the mathematical. I do not subscribe to the simple-minded notion that we are one or the other and that ability in one area leads inevitably to disability in the other. Rather, I think that verbal people feel comfortable with language early in life, perhaps because they enjoyed success at talking and reading. When mathematics contradicts assumptions acquired in other subjects, such people need special reassurance before they will venture on.

Conflicts between mathematical language and common language may also account for students' distrust of their intuition. If several associated meanings are floating around in someone's head and the text considers only one, the learner will, at the very least, feel alone. Until someone tries to get inside the learner's head or the learner figures out a way to search among the various meanings of the word for the one that is called for, communication will break down, too. This problem is not unique to mathematics, but when people already feel insecure about math, linguistic confusion increases their sense of being out of control. And so long as teachers continue to argue, as they have to me, that words like "multiply" and "of," the negative exponents, and the "circles" or "disks" are not ambiguous at all but perfectly consistent with their definitions, then students will continue to feel that math is simply not for them.

Some mathematics texts solve the problem of ambiguity by virtually eliminating language. College-level math textbooks are even more laconic than elementary texts. One reason may be the difficulty of expressing mathematical ideas in language that is easily agreed upon. Another is the assumption that by the time students get to college they should be able to read symbols. But for some number of students (we cannot know how many since they do not take college-level math)

proofs, symbolic formulations, and examples are not enough. After I had finally learned that X^{-2} must equal $\frac{1}{X^2}$ because it was consistent with the rule that when dividing numbers with exponents we subtract the exponents, I looked up "negative exponents" in a new high school algebra text. There I found the following paragraph.

Negative and Zero Exponents

The set of numbers used as exponents in our discussion so far has been the set of positive integers. This is the only set which can be used when exponents are defined as they were in Chapter One. In this section, however, we would like to expand this set to include all integers (positive, negative and zero) as exponents. This will, of course, require further definitions. These new definitions must be consistent with the system and we will expect all of the laws of exponents as well as all previously known facts to still be true.[3]

Although this paragraph is very clear in setting the stage to explain negative exponents through definitions which are presumably forthcoming, it does not provide a lot of explanation. No wonder people who need words to make sense of things give up.

The Dropped Stitch

"The day they introduced fractions, I had the measles." Or the teacher was out for a month, the family moved, there were more snow days that year than ever before (or since). People who use events like these to account for their failure at math did, nevertheless, learn how to spell. True, math is especially cumulative. A missing link can damage understanding much as a dropped stitch ruins a knitted sleeve. But being sick or in transit or just too far behind to learn the next new idea is not reason enough for doing poorly at math forever after. It is unlikely that one missing link can abort the whole process of learning elementary arithmetic.

In fact, mathematical ideas that are rather difficult to learn at age seven or eight are much easier to comprehend one, two, or five years later if we try again. As we grow older, our facility with language improves; we have many more mathematical concepts in our minds, developed from everyday living; we can ask more and better questions. Why, then, do we let ourselves remain permanently ignorant of fractions or decimals or graphs? Something more is at work than a missed class.

It is of course comforting to have an excuse for doing poorly at math, better than having to concede that one does not have a mathematical mind. Still, the dropped stitch concept is often used by math anxious people to excuse their failure. It does not explain, however, why in later years they did not take the trouble to unravel the sweater and pick up where they left off.

Say they did try a review book. Chances are it would not be helpful. Few texts on arithmetic are written for adults.* How insulting to go back to a "Run, Sport, run!" level of elementary arithmetic, when arithmetic can be infinitely clearer and more interesting if it is discussed at an adult level.

Moreover, when most of us learned math we learned

*Deborah Hughes-Hallett is writing a book (W.W Norton, 1978) for adults and college students that starts with arithmetic and brings the reader up to calculus, in two volumes.

dependence as well. We needed the teacher to explain, the textbook to drill us, the back of the book to tell us the right answers. Many people say that they never mastered the multiplication table, but I have encountered only one person so far who carries a multiplication table in his wallet. He may have no more skills than the others, but at least he is trying to make himself autonomous. The greatest value of using simple calculators in elementary school may, in the end, be to free pupils from dependence on something or someone beyond their control.

Adults can easily pick up those dropped stitches once they decide to do something about them. In one math counselling session for educators and psychologists, the following arithmetic bugbears were exposed:

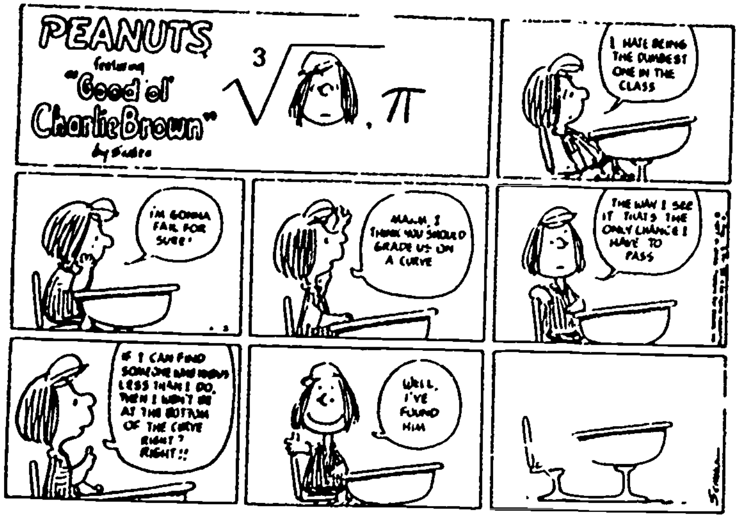
> How do you get a percentage out of a fraction like 7/16?
>
> Where does "pi" come from?
>
> How do you do a problem like: Two men are painters. Each paints a room in a different time. How long does it take them to paint the room together?*

The issues were taken care of within half an hour.

This leads me to believe that people are anxious not because they dropped a stitch long ago but rather because they accepted an ideology that we must reject: *that if we haven't learned something so far it is probably because we can't.*

*See Chapter Six for a discussion of fractions and percents, see Chapter Five for a discussion of the Painting-the-Room Problem. *Pi* can be derived by drawing many-sided polygons (like squares, pentagons, hexagons, etc.) and measuring the ratio of their perimeters to their diameters. Even if you do this roughly, the ratios will approach 3.14

Fear of Being Too Dumb or Too Smart



One of the reasons we did not ask enough questions when we were younger is that many of us were caught in a double bind between a fear of appearing too dumb in class and a fear of being too smart. Why anyone should be afraid of being too smart in math is hard to understand except for the prevailing notion that math whizzes are not normal. Boys who want to be popular can be hurt by this label. But it is even more difficult for girls to be smart in math. Matina Horner, in her survey of high-achieving college women's attitudes toward academic success, found that such women are especially nervous about

competing with men on what they think of as men's turf.[4] Since many people perceive ability in mathematics as unfeminine, fear of success may well interfere with ability to learn math.

The young woman who is frightened of seeming too smart in math must be very careful about asking questions in class because she never knows when a question is a really good one. "My nightmare," one woman remembers, "was that one day in math class I would innocently ask a question and the teacher would say, 'Now that's a fascinating issue, one that mathematicians spent years trying to figure out.' And if that happened, I would surely have had to leave town, because my social life would have been ruined." This is an extreme case, probably exaggerated, but the feeling is typical. Mathematical precocity, asking interesting questions, meant risking exposure as someone unlike the rest of the gang.

It is not even so difficult to ask questions that gave the ancients trouble. When we remember that the Greeks had no notation for multi-digit numbers and that even Newton, the inventor of the calculus, would have been hard pressed to solve some of the equations given to beginning calculus students today, we can appreciate that young woman's trauma.

At the same time, a student who is too inhibited to ask questions may never get the clarification needed to go on. We will never know how many students developed fear of math and loss of self-confidence because they could not ask questions in class. But the math anxious often refer to this kind of inhibition. In one case, a counsellor in a math clinic spent almost a semester persuading a student to ask her math teacher a question *after* class. She was a middling math student, with a B in linear algebra. She asked questions in her other courses, but could not or would not ask them in math. She did not entirely understand her inhibition, but with the aid of the counsellor, she came to believe it had something to do with a fear of appearing too smart.

There is much more to be said about women and mathematics. The subject will be discussed in detail in Chapter Three. At this point it is enough to note that some teachers and most pupils of both sexes believe that boys naturally do better in math than girls. Even bright girls believe this. When boys fail a math quiz their excuse is that they did not work hard enough. Girls who fail are three times more likely to attribute their lack of success to the belief that they "simply cannot do math."[5] Ironically, fear of being too smart may lead to such passivity in math class that eventually these girls also develop a feeling that they are dumb. It may also be that these women are not as low in self-esteem as they seem, but by failing at mathematics they resolve a conflict between the need to be competent and the need to be liked. The important thing is that until young women are encouraged to believe that they have the right to be smart in mathematics, no amount of supportive, nurturant teaching is likely to make much difference.

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Distrust of Intuition

> Mathematicians use intuition, conjecture and guesswork all the time except when they are in the classroom.
>
> —Joseph Warren, Mathematician

> Thou shalt not guess.
>
> —Sign in a high school math classroom

At the Math Clinic at Wesleyan University, there is always a word problem to be solved. As soon as one is solved, another is put in its place. Everyone who walks into the clinic, whether a teacher, a math anxious person, a staff member, or just a visitor, has to give the word problem a try. Thus, we have stimulated numerous experiences with a variety of word problems and by debriefing *both* people who have solved these problems and people who have given up on them, we gain another insight into the nature of math anxiety.

One of the arithmetic word problems that was on the board for a long time is the Tire Problem:

A car goes 20,000 miles on a long trip. To save wear, the five tires are rotated regularly. How many miles will each tire have gone by the end of the trip?

Most people readily acknowledge that a car has five tires and that four are in use at any one time. Poor math students who are not anxious or blocked will poke around at the problem for a while and then come up with the idea that four-fifths of 20,000, which is 16,000 miles, is the answer. They don't always know exactly why they decided to take four-fifths of 20,000. They sometimes say it "came" to them as they were thinking about the tires on the car and the tire in the trunk. The important thing is that they *tried* it and when it resulted in 16,000 miles, they gave 16,000 a "reasonableness test." Since 16,000 seemed reasonable (that is, less than 20,000 miles but not a whole lot less), they were pretty sure they were right.

The math anxious student responds very differently.

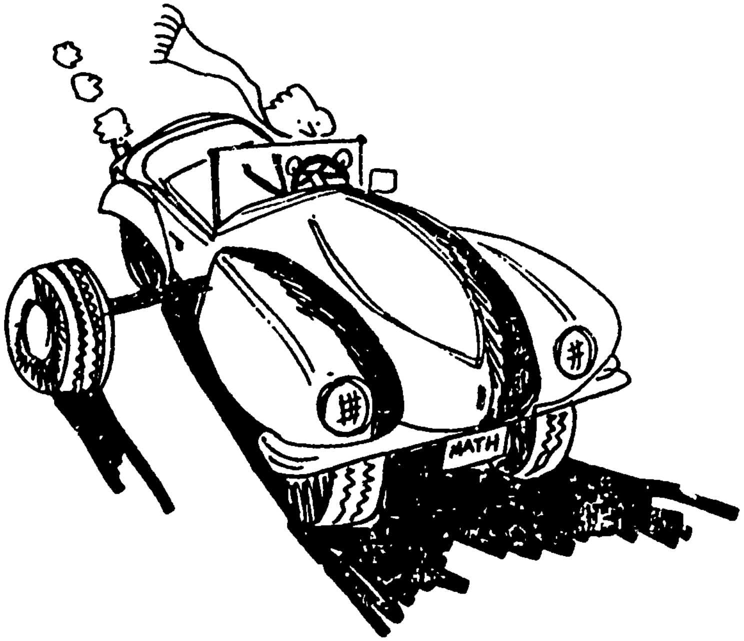

FIG. II-2

The problem is beyond her (or him). She cannot begin to fathom the information. She cannot even imagine how the five tires are used (See FIG. II-2.) She cannot come up with any strategy for solving it. She gives up. Later in the debriefing session, the counsellor may ask whether the fraction four-fifths occurred to her at all while she was thinking about the problem. Sometimes the answer will be yes. But if she is asked why she did not try out four-fifths of 20,000 (the only other number in the problem), the response will be—and we have heard this often enough to take it very seriously—"I figured that if it was in my head it had to be wrong."

The assumption that if it is in one's head it has to be wrong or, as others put it, "If it's easy for me, it can't be math," is a revealing statement about the self. Math anxious people seem to have little or no faith in their own intuition. If an idea comes into their heads or a strategy appears to them in a flash they will assume it is wrong. They do not trust their intuition. Either they remember the "right formula" immediately or they give up.

Mathematicians, on the other hand, trust their intuition in solving problems and readily admit that without it they would not be able to do much mathematics. The difference in attitude toward intuition, then, seems to be another tangible distinction between the math anxious and people who do well in math.

The distrust of intuition gives the math counsellor a place to begin to ask questions: Why does intuition appear to us to be untrustworthy? When has it failed us in the past? How might we improve our intuitive grasp of mathematical principles? Has anyone ever tried to "educate" our intuition, improve our repertoire of ideas by teaching us strategies for solving problems? Math anxious people usually reply that intuition was not allowed as a tool in problem solving. Only the rational, computational parts of their brain belonged in math class. If a teacher or parent used intuition at all in solving problems he rarely admitted it, and when the student on occasion did guess right in class he was punished for not being able to reconstruct his method. Yet people who trust their intuition do not see it as "irrational" or "emotional" at all. They perceive intuition as flashes of insight into the rational mind. Victims of math anxiety need to understand this, too.

The Confinement of Exact Answers

"Computation involves going from a question to an answer. Mathematics involves going from an answer to a question."

—Peter Hilton, Mathematician

Another source of self-distrust is that mathematics is taught as an exact science. There is pressure to get an exact right answer, and when things do not turn out right, we panic. Yet people who regularly use mathematics in their work say that it is far more useful to be able to answer the question, "What is a little more than five multiplied by a little less than three?" than to know *only* that five times three equals 15. Many math anxious adults recall with horror the timed tests they were subjected to in elementary, junior and senior high school with the emphasis on getting a unique right answer. They liked social studies and English better because there were so many "right answers," not just one. Others were frustrated at not being able to have discussions in math class. Somewhere they or their teachers got the wrong notion that there is an inherent contradiction between rigor and debate.

This emphasis on right answers has many psychological benefits. It provides a way to do our own evaluation on the spot and to be judged fairly whether or not the teacher likes us. Emphasis on the right answer, however, may result in panic when that answer is not at hand and, even worse, lead to "premature closure" when it is. Consider the student who does get the right

answer quickly and directly. If she closes the book and does not continue to reflect on the problem, she will not find other ways of solving it, and she will miss an opportunity to add to her array of problem-solving methods. In any case, getting the right answer does not necessarily imply that one has grasped the full significance of the problem. Thus, the right-answer emphasis may inhibit the learning potential of good students and poor students alike.

In altering the learning atmosphere for the math anxious the the tutor or counsellor needs to talk frankly about the difficulties of doing math. The tutor's scratch paper might be more useful to the students than a perfectly conceived solution. Doing problems afresh in class at the risk of making errors publicly can also link the tutor with the student in the process of discovery. Inviting all students to put their answers, right or wrong, before the class will relieve some of the panic that comes when students fail to get the answer the teacher wants. And, as most teachers know, looking carefully at wrong answers can give them good clues to what is going on in students' heads.

Although an answer that checks can provide immediate positive feedback, which aids in learning, the right answer may come to signify authoritarianism (on the part of the teacher), competitiveness (with other students), and painful evaluation. None of these unpleasant experiences is usually intended, any more than the premature closure or panic, but for some students who are insecure about mathematics the right-answer emphasis breeds hostility as well as anxiety. Worst of all, the "right answer" isn't always the right one at all. It is only "right" in the context of the amount of mathematics one has learned so far. First graders, who are working only with whole numbers, are told they are "right" if they answer that five (apples) cannot be divided between two (friends). But later, when they work with fractions, they will find out that five *can* be equally divided by giving each friend two and one-half apples. In fact both answers are right. You cannot divide five one-dollar bills equally between two people without getting change.

The search for the right answer soon evolves into the search for the right formula. Some students cannot even put their minds to a complex problem or play with it for a while because they assume they are expected to know something they have forgotten.

Take this problem for, example,

> Amy Lowell goes out to buy cigars. She has 25 coins in her pocket, $7.15 in all. She has seven more dimes than nickels and she has quarters, too. How many dimes, nickels, and quarters does she have?

Most people who have done well in high school algebra will begin like this: the number of nickels equals x; the number of dimes equals $x + 7$, the number of quarters equals $25 - (2x + 7)$. The total value is $7.15. They won't stop to realize that Amy Lowell must have miscounted her change, because even if all 25 coins in her pocket were quarters (the largest coin she has), her change would total $6.25, not $7.15.*

This is a tricky problem, which is fair, as opposed to a trick problem which is not. But it also shows how searching for the right formula can cause us to miss an

*I am indebted to Jean Smith for this example.

obviously impossible situation. The right formula may become a substitute for thinking, just as the right answer may replace consideration of other possibilities. Somehow students of math should learn that the power of mathematics lies not only in exactness but in the processing of information.

Self-Defeating Self-Talk

One way to show people what is going on in their heads is to have them keep a "math diary," a running commentary of their thoughts, both mathematical and emotional, as they do their homework or go about their daily lives. Sometimes a tape recorder can be used to get at the same thing. The goal is twofold: to show the student and the instructor the recurring mathematical errors that are getting in the way and to make the student hear his own "self-talk." "Self-talk" is what we say to ourselves when we are in trouble. Do we egg ourselves on with encouragement and suggestions? Or do we engage in self-defeating behaviors that only make things worse?

Inability to handle frustration contributes to math anxiety. When a math-anxious person sees that a problem is not going to be easy to solve, he tends to quit right away, believing that no amount of time or rereading or reformulation of the problem will make it any clearer. Freezing and quitting may be as much the result of destructive self-talk as of unfamiliarity with the problem. If we think we have no strategy with which to begin work, we may never find one. But if we can talk ourselves into feeling comfortable and secure, we may let in a good idea.

To find out how much we are talking ourselves into failure we have to begin to listen to ourselves doing math. The tape recorder, the math diary, the self-monitoring that some people can do silently are all techniques for tuning in to ourselves. Most of us who handle frustration very poorly in math handle it very well in other subjects. It is useful to watch ourselves doing other things. What do we do there to keep going? How can these strategies be applied to math?

At the very minimum this kind of tuning in may identify the particular issue giving trouble. It is not very helpful to know that "math makes me feel nervous and uncomfortable" or that "numbers make me feel uneasy and confused," as some people say. But it may be quite useful to realize that one kind of problem is more threatening than another. One excerpt from a math diary is a case in point:

> Here I go again. I am always ready to give up when the equation looks as though it's too complicated to come out right. But the other week, an equation that started out looking like this one did turn out to be right, so I shouldn't be so depressed about it.

This is constructive self-talk. By keeping a diary or talking into a tape recorder we can begin to recognize our own pattern of resistance and with luck we may soon learn to control it. This particular person is beginning to understand how and why she jumps to negative conclusions about her work. She is learning to sort out the factual mistakes she makes from the logical and even the psychological errors. Soon she will be able to recognize the mistakes she makes *only* because she is anxious. Note that she has been encouraged to think and to talk about her feelings while doing mathematics. She

is not ashamed or guilty about the most irrational of thoughts, not frightened to observe even the onset of depression in herself; she seems confident that her mind will not desert her.

The diary or tape recorder technique has only been tried so far with college-age students and adults. So far as we can tell, it is effective only when used in combination with other nonthreatening teaching devices, such as acceptance of discussion of feelings in class, psychological support outside of class, and an instructor willing to demystify mathematics. The goal in such a situation is not to get the right answer. The goal is to achieve mastery and above all autonomy in doing math. In the end, we can only learn when we feel in control.

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This chapter is based primarily on interviews with and observations of math anxious students and adults. These people are not typical of those who are math incompetent. Most of them are very bright and enjoy school success in other subjects, but they avoid or openly fear mathematics.

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