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ABSTRACT

This study examined student growth in mathematical problem solving and relationships between teachers and changes in class problem-solving performance over a school year. The sample for the study involved 24 junior high school mathematics teachers, 119 mathematics classes, and more than 2,500 students. Classes were tested using a 10 item problem-solving test at the beginning of the school year and at the end of the school year and increases in problem-solving performance were computed. Classes differed substantially in the amount of increases in performance. The five teachers with the largest class mean gains (highs) were compared with the five teachers with the lowest mean gains (lows). The high teachers taught 22 classes and 100 percent of these classes gained more than was statistically predicted from pretest scores. For the low teachers, 75 percent of their classes gained less than was predicted. Thus, the teacher, and indirectly teaching behavior, seems to be an important factor in increasing student performance in the domain of mathematical problem-solving. (Author/PK)

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Abstract

This study examined student growth in mathematical problem solving and relationships between teachers and changes in class problem-solving performance over a school year. The sample for the study involved 24 junior high school mathematics teachers, 119 mathematics classes, and more than 2500 students. Classes were tested using a 10 item problem-solving test at the beginning of the school year and at the end of the school year and increases in problem-solving performance were computed. Classes differed substantially in the size of their increases in performance indicating that much more problem-solving learning takes place in some mathematics classrooms than in others. Teachers were associated with both the size and consistency of these gains. The 5 teachers with the largest class mean gains (Highs) were compared with the 5 teachers with the lowest mean gains (Lows). The high teachers taught 22 classes and 100 percent of these classes gained more than was statistically predicted from pretest scores. For the low teachers, 75 percent of their classes gained less than was predicted. Thus, teacher, and indirectly teaching behavior, seems to be a powerful factor in increasing student performance in the domain of mathematical problem solving. Naturalistic study of teaching practice and teacher beliefs of high performing teachers is an important direction for future research.

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One would be hard pressed to find a period in the history of mathematics education where attention to problem solving in some form was not vigorously advocated for all. Dower (1928) indicated that the "mastery of the fundamental facts and processes is not the ultimate end of arithmetical instruction," but rather life demands that students have the ability to interpret, comprehend, and solve quantitative problems that arise in everyday situations. According to Reeve (1940), even a reduced program of mathematical study should emphasize problem solving and modes of thinking. Similar recommendations are found in more recent literature.

In its Agenda for Action (1980), the National Council of Teachers of Mathematics (1980) recommends that problem solving be the focus of all school mathematics. This position recognizes that the value of mathematics accrues from being able to apply it to problem-solving situations. Begle (1979) points out that "The real justification for teaching mathematics is that it is a useful subject and, in particular, that it helps in solving many kinds of problems."

Considering the long history of recommendations for improved classroom problem-solving instruction, it is surprising that most research has focused on individual students, usually in laboratory settings, rather than on actual classroom teaching and learning. Grouws (1985) points out that there is a dearth of studies of the teacher's role in and contribution to student learning of problem solving. Furthermore, there appears to be a lack of financial investment in instructional research on various mathematical topics (Good & Biddle, 1988).

Another reason for the paucity of classroom research in this area may be that a variety of perspectives on how problem solving should be taught have been advocated in recent years. These can be roughly classified into five categories (Kilpatrick, 1985): osmosis, memorization, imitation, cooperation, and reflection. Most of these recommendations, however, stem from logical analysis, personal experience, or current ideas about student learning rather than from systematic study of classroom practice. However, there have been some successful attempts to improve problem solving instruction (e.g., Charles & Lester, 1984).

Perhaps another reason for the lack of naturalistic classroom problem solving research is a pessimistic view that suggests that problem solving is rarely taught, that such instruction is of poor quality, and that students' problem-solving performance is dismal (Lester, 1985). Indeed there is empirical evidence that problem-solving instruction in some classrooms is relatively uninspiring (Burns & Lash, 1984). Based on case studies, Stake and Easley (1978) strongly suggest that the classroom environment may have to be drastically changed if we wish students to develop logical thinking skills and improved problem solving ability.

Student performance data also indicate that there is a need for substantial improvement in how well students solve problems. Data from the Third National Assessment of Educational Progress (Carpenter, Matthews, Lindquist, & Silver, 1984) show that the majority of students at all age levels have difficulty with any nonroutine problem that requires some analysis or thinking. One conclusion from the Second International Study of Mathematics Achievement (McKnight et al., 1987) is that U.S. students are well below the international average in such areas as problem solving. In summary, there are good reasons not to be satisfied with problem-solving instruction in many classrooms and to be disappointed with students' problem-solving performance in general.

The preceding notwithstanding, it may be that this bleak picture of mathematics instruction is too generalized. That is, there certainly are some students who are becoming excellent problem solvers. For example, selected, talented U.S. students did well in the recent international competition sponsored by the International Commission on Mathematics. There probably are also entire classrooms in which most students are making substantial progress in developing their problem-solving ability. Of the more than 250,000 mathematics teachers in the U.S., there likely are some who foster large increases in student problem-sol ing ability in every class they teach. Due to an increased emphasis on problem solving, some mathematics education programs and school districts are probably relatively successful in helping classroom teachers to become more effective problem-solving teachers. There is a need for objective evidence on this latter point, however, and careful study of the classrooms and students involved. Particularly needed is attention to the quality of instruction. Recent research (e.g., Good & Grouws, 1987) suggests that under certain conditions, assisting teachers to attend to the quality of the development component of their lessons can positively influence student performance in areas requiring higher-order thinking.

The purpose of this paper is to discuss data from the first stage of a research project that examines a large sample of teachers and the effects of their instruction on the problem solving ability of their students across an entire school year. We wanted to see if variation in students' learning of problem-solving skills was consistently associated with classroom instruction.

The Study

To study problem-solving performance one must first carefully define problem solving. Silver and Thompson (1984) point out that readers of the problem-solving literature are confronted with a wide variety of definitions of problem solving. There is agreement, however, that what constitutes a problem is idiosyncratic; that is, what represents a problem for one student may not be a problem for another student. There also is a consensus that a problem involves a situation where something is to be found or shown and the way to do this is not immediately obvious to the solver. However, as Shulman (1985) has suggested, when analyzing problem solving it may be beneficial to specify some of the details concerning the kind of problem solving under analysis rather than argue about whether something is or is not "real" problem solving.

Problem Solving Measure

In spite of the importance ascribed to problem solving over the years and the common elements in most definitions, there is little agreement on how to measure problem-solving ability (Schoen & Oehmke, 1980). Charles, Lester, and O'Daffer (1987) have suggested alternative means of assessing problem solving and correctly emphasized that the evaluation procedures must take into account the purpose of the assessment.

The problem solving assessment in the current study was for screening purposes. A ten-item paper and pencil instrument composed of a variety of situations of varying difficulty that would constitute problems for most students in the sample was developed. The test was designed to involve more than simple word problems that would only be routine exercises for students, yet not be so challenging that it would be beyond the capacity of most seventh- and eighth-grade students. Table 1 displays for each item a

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Insert Table 1 about here

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description, point biserial coefficient, item difficulty, and standard deviation based on all students taking the pretest administration of the instrument (N = 2540). The test items were written to meet given descriptive criteria (see Table 1) and to span a continuum from relatively simple verbal problems to nonroutine problems. The following problem from the instrument (# 9) reflects a problem of the nonroutine type.

> How many times must the number keys on a typewriter be hit in order to type page numbers on a paper that has 124 pages?

None of the problems involved just two numbers that had to be simply combined with an arithmetic operation to obtain a correct solution. A mathematics educator outside the project staff classified items by descriptive category, and these data confirmed our original judgments. The KR-20 coefficient of internal consistency for the instrument was .73 (N = 2540).

Sample

The sample was composed of 24 teachers who taught mathematics in one of the six junior high schools in a large midwestern school district. This volunteer sample represented about 80 percent of the junior high mathematics teachers in the district. These teachers taught 119 classes composed of more than 2500 students. All students at grade 7 and grade 8 in the school district took one of the following courses each year: Math 7, Advanced Math 7, Math 8, PreAlgebra, or Algebra.

Procedure

Teachers administered the problem-solving instrument to each of their seventh- and eight-grade mathematics classes during the same three day period in early October. Teachers used standardized directions to give the test and students had the entire class period to complete the test. During the school year each teacher was observed on a regular basis. Teachers administered the problem solving instrument to each class again in early May during a two day period.

Data Analysis

For each class that had a complete data set pretest and posttest problem-solving mean scores were computed. Linear regression on these pairs of means was used to generate a linear equation that predicted class posttest mean scores based on class pretest mean scores. A residual class mean (posttest score minus predicted score) that indicated gain beyond that predicted was then computed for each class.

Results

Student Performance

The mean student score on the problem solving pretest was 4.8 problems correct (s.d. = 2.3); the posttest mean was 5.8 problems correct (s.d. = 2.3). Thus, on the 10-problem test, students scored an average of one more problem correct on the posttest than they did on the pretest. Table 2 shows the pretest and posttest means for each test item. The greatest growth in

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Insert Table 2 about here

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performance was on items 2 and 7, and the smallest increase was on item 9. Item 2 was a two-step problem that required students to subtract two numbers and then subtract that result from a third number. Item 7 was classified as a diagram-type problem because comprehension of the problem was greatly facilitated by making a drawing or sketch. The problem involving the least improvement was a nonroutine problem (#9) in the sense that it only included one number so immediately performing an arithmetic operation was not possible.

There were substantial differences in performance gains among classes. The smallest pretest to posttest gain for a class was -1.06, and the class with the largest gain had an increase of 2.92. The residualized mean scores for these classes were -2.01 and 1.95, respectively.

Teacher Data

Class residual means were aggregated by teacher and a mean of these means was completed. Table 3 displays these means by teacher in rank order from

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Insert Table 3 about here

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largest to smallest. The largest teacher mean was .80, which implies that on average each of these teacher's five classes gained .80 more than was predicted. For example, in one of these classes the mean pretest score was 5.46 and the associated predicted posttest score was 6.38. The actual mean posttest score was 7.21, giving a residualized mean of .83 for this class.

As an indication of the stability of teacher means, a proportion was examined. For each teacher, the number of classes with more than expected gains was compared to the total number of classes (see Table 3). There were 22 classes taught by the five highest teachers. In 100 percent of these classes the class mean exceeded the posttest mean predicted by the pretest mean scores. In contrast, there were 20 classes taught by the five lowest teachers. In 15 of these classes, or 75 percent, the class mean did not reach the level predicted by the pretest results.

Discussion

In considering the implications of the data from this study it is important to remember that it was a naturalistic study and thus a number of factors could not be controlled. For example, not every class in the sample used the same textbook, but we did insure that every type of class (e.g., Math 7, Math 8, and so on) did use the same textbook. Not every teacher taught the exact same mix of classes but we did examine the sample with this in mind and found that generally teachers taught a variety of classes (e.g., no teacher taught just prealgebra classes). Further, not all classes had students of exactly the same ability. Adjusted posttest scores were used to attempt to statistically account for such differences. In summary, we were sensitive to the potential importance of such factors and found no systematic patterns that would seem to bias the findings we have reported. The relatively large sample used in the study should also tend to diminish the importance of some factors that might otherwise have been of concern. Further, in consideration of the nature of the study we have also focused on reporting patterns of results rather than the results from a single teacher or class.

With the preceding qualifications in mind, the data from this study lend support to several generalizations. There is important variation in teachers' effects on student problem solving performance in junior high school mathematics classes. In particular, within extant practice there are teachers who in a given school year obtain substantially better problem-solving results than would be predicted. It is important that these favorable outcomes occurred in <u>every</u> mathematics class the teacher had responsibility for that school year. This argues against the effects being random in nature or attributable to a particular type of class or a particularly suitable combination of teacher and class type. Thus, the data support the contention that some teachers offer effective problem solving instruction and that what these teachers do in instruction merits research.

It is important to emphasize strongly at this point that we do not maintain that the instruction taking place in the aforementioned classrooms is good or bad nor that other teachers should emulate it. It is possible that examination of the problem solving gains in greater depth through other procedures (e.g., observation of individuals solving problems, with more attention given to thinking strategies employed and techniques used) will temper the results. It may be that when problem solving is defined differently, the gains may not be as impressive. Similarly it is possible, though not very likely in our view, that the results were obtained using inappropriate instructional methods (such as a focus on rote practice) or at the expense of other important outcomes (such as student interest or attitude). It is important, however, that we now have data to begin to describe the results of contemporary practice. These data are encouraging in that they suggest that careful study of extant practice in mathematical problem solving has potential for making a contribution to our understanding of how student problem solving ability may be improved in classroom settings. Again, we stress that an examination existing practice is but one basis for developing an understanding of problem solving instruction and learning. Empirical studies of student learning and new models and theories of problem solving learning that go beyond extant practice are also needed. Still we believe that an understanding of current practice -- the good, the typical, and the problematic -- is an important part of the dialogue.

Future Research

We have previously mentioned that there is a lack of agreement as to how problem solving should be measured. To conduct the present research we operationally defined problem solving and developed an instrument that reflected a conceptualization of problem solving that was responsive to practicing teachers' views as well as professional recommendations. An important question to investigate is whether the same teachers are effective if different student measures are used. A prediction based on a theoretical position should be made, instruments should be developed, and data to help answer the question gathered. Research of this type is essential in answering the many arguments that exist on both sides of the issue.

Research should also examine the effects of these teachers' beliefs and attitudes on their problem-solving instruction. Do these successful teachers have a greater interest in problem solving than do other teachers? McLeod (1987) suggests that affective considerations in problem solving at both the student and teacher level have not been studied adequately. How teachers view student affective outcomes may also be important. Similarly, a teacher's belief structure may influence what content is emphasized, how it is presented, and in turn, what the students eventually learn (Cooney, 1985). These issues are important in developing a more informed view of the relationship between the teaching of problem solving and problem solving learning.

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Table 1

Problem Solving Test Items by Type, Point Biserial Correlation, Item Difficulty, and Standard Deviation

Problem Number	Problem Type	Point Biserial	Item Difficulty	Standard Deviation
1	Extra Information	.48	.82	.39
2	Two-Step Problem	.55	.41	.49
3	Extra Information	.62	.23	.42
4	Proportion Problem	.63	.39	.49
5	Implied Number	.46	.82	.39
6	Work Backward	.59	.43	.50
7	Diagram	.49	.72	.45
8	Guess and Check	.62	.29	.45
9	Nonroutine	.40	.06	.24
10	Nonroutine	.52	.67	.47

Table 2

Differences in Performance on Problems from the Pretest to the Posttest

Problem Number	Pretest Mean	Posttest Mean	Difference
1	.82	.88	.06
2	.41	.56	.15
3	.23	.33	.10
4	.39	.51	.12
5	.82	.87	.05
6	.43	.56	.13
7	.72	.78	.06
8	.29	.44	.15
9	.06	.10	.04
10	.67	.78	.11

Table 3

Ordered List of Mean Residual Problem Solving Gain Scores

Teacher Number[a]	Number of Classes	Number of Classes Pos. Res. Gains	Mean of Residual Gains
13	5	5 of 5	.80
15	4	4 of 4	.59
9	5	5 of 5	.44
19	3	3 of 3	.38
21	5	5 of 5	.38
6	6	2 of 6	.19
4	5	2 of 6	.07
7	3	2 of 3	.07
12	6	4 of 6	.05
11	5	2 of 5	.02
23	5	1 of 5	-.01
8	6	4 of 6	-.02
14	6	5 of 6	-.03
3	5	1 of 5	-.11
25	5	2 of 5	-.13
2	6	2 of 6	-.14
17	3	0 of 3	-.20
16	6	2 of 6	-.23
22	4	2 of 4	-.24
24	6	2 of 6	-.30
18	1	0 of 1	-.32
10	5	1 of 5	-.43
5	3	1 of 3	-.44
1	5	1 of 5	-.58

[a]The 24 teachers were assigned numbers using the numbers 1 to 25; no teacher was assigned number 20 in this process.