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AUTHOR Narode, Ronald
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ABSTRACT

In the past 15 years, educators and education researchers have increasingly emphasized the importance of cognitive processes for effective education in mathematics and science. This may be characterized as a shift away from the epistemology which treats knowledge as a compilation and categorization of facts toward the notion that knowledge is the conscious construction of meaning. This document reviews the literature dealing with metacognition in mathematics and science education. It focuses on three major identified variables: (1) "person variables," which are performance-relevant characteristics of the information processor; (2) "task variables," which are performance-relevant characteristics of the memory task or problem; and (3) "strategy variables," which are potential solution procedures. Each of these variables is explored and examples of problems are provided. (TW)

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METACOGNITION IN MATH AND SCIENCE EDUCATION

Ronald Narode

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In the past fifteen years, educators and education researchers have had a increasingly emphasised the importance of cognitive processes for effective education in mathematics and science. This may be characterized as a shift away from an epistemology which treats knowledge as a compilation and categorization of facts toward the view that knowledge is the conscious construction of meaning. In short, instead of teaching students what to think, they must be taught how to think. (Lochhead & Clement, 1979). The following sub-tasks are necessary prerequisites for a legitimate realization of the stated objective:

1. The thought processes of accomplished learners and problem solvers must be analyzed to document effective skills and strategies.

2. The thought processes of novice problem solvers must also be studied to discover which skills may be missing or inhibited and for which reasons.

3. Implications must be drawn from steps 1 and 2 to facilitate an effective educational pedagogy which transforms novice thinkers into expert thinkers.

Among the many cognitive processes observed in expert problem solvers, attention has recently focused on a type of cognition termed "metacognition". As the etymology of the word suggests, metacognitions are thoughts about thought, knowledge about one's own cognitions. Metacognition is also referred to and may be categorized as beliefs about ourselves as cogitators, and about cognitions themselves. Metacognitive thought is evaluative (cognitive) of ongoing thought processes as well as of the abilities of the thinker and the task at hand. As will be argued in this paper, metacognition is not simply more cognitive processing, such as selecting and organizing data, using inductive and deductive reasoning to form hypotheses, searching for and constructing relevant concepts, heuristics and algorithms, representing quantitative relationships, etc. Instead, metacognition provides a qualitatively different framework than the cognitive frameworks usually employed in the study of learning.

While several educational researchers (Flavell 1976; Lesh, 1982; Silver, 1984; Schoenfeld, 1985) have offered taxonomies of metacognitive skills distinct from the cognitive processes normally ascribed to problem solving, two of these taxonomies appear most useful. Flavell's taxonomy examines metacognition with respect to person, task and strategy variables, while Schoenfeld divides metacognitions into belief systems and control systems. This paper attempts to join these two taxonomies to offer a more encompassing conceptualization of metacognition: a conceptualization with important pedagogical implications.

METACOGNITION AND MATHEMATICS

Although the literature in the psychology of mathematics learning has dealt mainly with the cognitive processes intrinsic to mathematical performance, much attention has recently shifted to the importance of metacognition (Lesh, 1982; Silver, 1982; Garofalo and Lester, 1985;

Schoenfeld, 1985). Research into the cognitive processes involved in mathematical problem solving has been greatly influenced by Polya's (1957) four-phase model which does not explicitly mention metacognition. His model examines a multitude of problem-solving heuristics attached to each of the four phases: understanding, planning, carrying out the plan, and looking back. While this heuristics paradigm has served the research community well for many years, it is insufficient to an adequate analysis of problem-solving behaviors which involve rational number concepts, early number concepts and geometry concepts (Lesh and Landau, 1983). Another consequence of the influence of Polya's paradigm is the great amount of research which has focused on instructional treatments which train students to memorize task-specific and general heuristics in an attempt to improve their problem solving. Lester (1985) suggests that many of these instructional attempts have failed because the development of heuristic skills was overemphasized with little or no attention given to the managerial skills necessary for the proper selection of those same heuristics.

Historically, research into metacognitive processes follows cognitive processes research, although one could argue that both may be classified as cognitive. Lesh (1983) suggests that, psychologically speaking, metacognitions and cognitions are distinct but occur together through parallel processing involved in most types of problem solving. Cognitions involved in the solution of a problem are simultaneously monitored by metacognitions.

Cognitive processes consist of the mental structures, concepts, and heuristics that are thought during the attempted solution of a particular problem: the problem-at-hand. These ideas may occur as analogies, memories of similar or previously solved problems, algorithms, images, verbal patterns, and any of a myriad of symbolic representations.

Metacognitive thought consists of thoughts about the cognitive processes in working memory. Generally, such thoughts are reflections about current ideas with their possible rejection or affirmation. Such thinking may evaluate the efficacy of a particular line of thought, heuristic or algorithm. Sometimes the reflections may consist of memories of having had difficulty with a problem similar to the one under consideration. The belief that one can or cannot solve such problems or even learn the necessary skills to do so, is yet another reflection about one's cognitive processes.

The cognitive processes which build conceptual and procedural models in problem-solving are distinct from thought which monitors and evaluates these processes and from the beliefs which bear directly on these processes. Metacognition is by definition, cognition about cognitive processes. According to Flavell (1976):

> "Metacognition refers to one's knowledge concerning one's own cognitive processes and products or anything related to them, eg., the learning-relevant properties of information or data. Metacognition refers, among other things, to the active monitoring and consequent regulation and orchestration of these processes in relation to the cognitive objects on which they bear, usually in the service of some concrete goal or objective. (p. 232)

From Flavell's description, two distinct aspects of metacognition arise: (1) previous knowledge of cognitive processes, their products and the beliefs which relate to such processes, and (2) the monitoring and control of the cognitive processes themselves. Flavell and Wellman (1977) further refined a taxonomy for metacognition in their study of memory performance. They identified three major variables: "Person Variables," which are performance-relevant characteristics of the information processor; "Task Variables." which are performance-relevant characteristics of the memory task or problem; and "Strategy Variables" which are the potential solution procedures. According to Flavell, these variables are interrelated.

Although this taxonomy was developed for the analysis of meta-memory [see Flavell (1978) for a thorough review of relevant memory research] it has proven useful to mathematics education researchers in their attempts to understand the function of metacognition in mathematics problem solving (Lester, 1985; Garofalo and Lester, 1985). As will be shown, both "person" and "task" variables fall largely under the more general rubric of "belief systems" a la Schoenfeld (1983), while the "strategy" variables refer to the need for management and control in the solution of mathematics problems.

Belief Systems in Person and Task Variables

Person Variables

With regard to mathematics, much has been written about the affective factors which influence mathematical performance (Clement, Narode, Rosnick, 1981; Fenema E., & Sherman, J. 1977; Tobias, 1978; Bassarear, 1986). Most of these factors: motivation, anxiety, attitude and past experience, reflect the learner's self-image as a mathematics problem solver. Many affective factors may be described as or traced from the student's beliefs about themselves as problem solvers and learners. Their beliefs may vary in accuracy and effect. A student who believes he is not good at computation may deliberately work more carefully and accurately than a student who believes she is proficient at computation and works too quickly to assure accuracy. The beliefs which underlie subsequent behavior (and the unobserved cognitive activity necessary to produce the behavior) are referred to as metacognitions. As a thought which is reflective of a cognitive ability, it may influence the quantity and quality of cognitive activity. The following example of a person variable metacognition illustrates the power that beliefs exert over the cognitive abilities of many students. Schoenfeld's (1985) conjecture that one reason for the failure of many students to use mathematical argumentation (proofs) has to do with the prevalent belief that only "geniuses" can think mathematically:

> "Only geniuses are capable of discovering or creating mathematics. First corollary: If you forget something, too bad. After all, you're not a genius, and you won't be able to derive it on your own. Second corollary: Accept procedures at face value and don't try to understand why they work. After all, they are derived knowledge passed on 'from above'." (p. 372)

This belief, about the abilities of the problem solver in comparison to the perceived ability of the mathematician, illustrates the rather negative perception that students have of themselves as mathematics problem solvers. Given the above stifling metacognition, a student's cognitive processes may never have the opportunity to engage in learning for fear of failure.

Lester and Garofalo (1982) have found further evidence of person variables in their study of third and fifth graders:

* Both third and fifth graders believe that proficiency at computation depends almost entirely on the amount of time spent practicing computation.

* Fifth graders thought that their teachers can make a difference in their ability to perform computation, while third graders think their teacher makes no impact on this skill.

* Both third and fifth graders believe they should take their time performing computations, otherwise they are likely to make mistakes.

As mentioned earlier, there is considerable overlap between the different variables. The first observation reflects the students' belief about a specific task-- computation. It also reflects a belief about the students' ability in computation solely as a result of time spent practicing the skill. Nevertheless, it is clear that all three observations illustrate beliefs about the person as knower, which have direct impact on an area of mathematical skills instruction.

Task Variables

According to Flavell, another set of variables which are considered to be metacognitive are task variables. These are beliefs about a particular task which affect the form of cognitive processes brought to bear on the task. Memory research indicates that beliefs regarding certain tasks, such as memorizing a short story, elicit different strategies in subjects. Some subjects attempt to memorize verbatim while others choose to paraphrase while relating the gist of the story. [Flavell, J. & Wellman, H., 1977]. Garofalo and Lester (1982) advocate the usefulness of the categorization of task variables when applied to metacognitive beliefs about specific mathematical tasks. For example, they found that both third and fifth grade students believed that math word problems are more difficult than computation problems. The same students also believe that solutions of verbal problems depend largely on the selection of a "key word" which would suggest the one necessary arithmetic operation necessary for the solution. Unfortunately, this is all too often the case, and is probably a result of syntactic matching of words to mathematical operations which is prevalent in most algebra textbooks. (Rosnick, 1980).

The role of "key words" has been corroborated in studies of college students. Although the students were proficient with algebraic symbol manipulation they experienced major difficulties solving simple algebra word problems which involve the translation from words to equations (Clement, 1982) and from equations to words (Lochhead, 1980). The following is an example of a problem whose "key word" solution leads to a "reversed" equation. From Clement, (1982):

Using the letters 'S' for the number of students and 'P' for the number of professors, write an equation which states the following relationship:

"At a certain university, there are six times as many students as there are professions."

The typical wrong answer is 6S = P. Approximately 37% of college freshmen in a calculus course gave some version of this reversed equation [the correct equation is S = 6P.] One of Clement's theories for the frequent occurrence of the reversal error states that students perform a direct word-to-symbol match. The words "times" and "are" serve as key words which get translated respectively as "multiply" and "equals". Thus:

6 x	S	=	P
6 times	Students	are (the)	Professors

Many students interpret their reversed equation saying "For every six students there is one professor," and they can draw a picture to illustrate this relationship. As Clement (1982) and Rosnick (1981) point out, although students exhibit a semantic understanding of the given relationship, they nevertheless have deep misconceptions about the meaning of variables, the role of coefficients and about the meaning of the equal sign. In their emphasis on the role of conceptual frameworks, the researchers may have overlooked the presence of a confounding metacognitive task variable, the belief in the key word strategy. Further analysis of this example will serve to distinguish between cognitive and metacognitive activity.

The protocols of students explaining their reversed equation indicate the following cognitive misconceptions:

1) Students misinterpret the letters 'S' and 'P' as labels for "a student" and "a professor" rather than as variables symbolizing a quantity of students and a quantity of professors.

2) Coefficients are misinterpreted as adjectival modifiers as when '6S' represents six students and (1)P represents one professor. [note that most subjects remark that, "In algebra, the "1" is implied."] This misconception misses the implied operation of multiplication in the use of coefficients.

3) The equal sign has no relationship to numerical equality. Instead, it is used as a conjunctive in the sense that one group, the students, "goes with" another group, the professors.

Clement and Rosnick suggest that algebra translation tasks in general, and reversal errors in particular, may be improved with instructional emphasis on the above concepts coupled with an operational interpretation of mathematical functions, as in generating tables of data from the hypothesized equation and checking it with a semantic understanding of the qualitative relationship between variables.

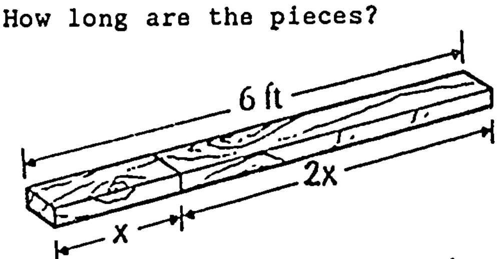
The observations listed above are correctly termed "misconceptions" which are improper conceptual models which serve as elements in the broader context of performing a mathematics translation task. In Piagetian terms, they are structures upon which operations and transformations are executed in the cognitive processes of problem solving. This cognitive activity is characterized as a direct manipulation of conceptual models which are elements of a larger task.

Metacognitive activity is inferred from the observation that students who make the reversal error often do so from a belief that translation tasks from math word problems may be solved using a key word matching strategy. The students' judgement is not merely an overly generalized classification of a type of math problem; rather, the judgement stems from their knowledge of previous learning tasks, i.e. knowledge about their knowledge of a task.

While it would be useful to know the causal interaction between metacognitive and conceptual-cognitive activity, no formal studies have as yet examined the phenomena. With regard to the above example, one may hypothesize that the metacognitive belief in the generalizability of the key word strategy may lead to misconceptions of variables, coefficients and equality. The difficulty may lie in textbooks which stress the key word matching heuristic while devoting little or no attention to the development of these critically important conceptual elements. Consider the following examples from three popular introductory mathematics texts:

From: Keedy & Bittinger, 1985, p. 135.

Example 1: A 6-ft board is cut into two pieces, one twice as long as the other. How long are the pieces?



The picture can help in translating. Here is one way to do it.

Length of one piece plus length of other is 6

$$x \quad + \quad 2x \quad = 6$$

From Beavers, 1985, p. 271.

Example: The enrollment of a college in Pennsylvania increased from 25,000 to 28,000. The increase is what percentage of the original enrollment?

KEY PHRASE	The increase	is	what percent	of	the original enrollment?
	3,000	=	%	x	25,000

  28,000
- 25,000
  3,000 increase

From: Whimbey & Lochhead, 1981, p. 320.

17. Paul's income equals three times Jeff's income.

$$P = 3 \times J$$

or

$$P = 3J$$

Given the algebra and word problem solving experiences of most students there can be little doubt as to the source of metacognitive beliefs. It may be easier for them to adapt conceptualizations of variables, coefficients, etc. to their chosen heuristic than it is for them to change their beliefs about their prior knowledge. This may account for some of the resiliency of misconceptions in the presence of instruction.

Not all misconceptions are concommitant with metacognitions. A substantial literature of misconceptions in physics reveal widespread physical belief systems which are reminiscent of Aristotelian physics. Minstrell and Stimpson (1986) have documented the belief among high school students that a force is necessary to keep an object in motion because things have a "natural tendency" to be at rest. Lochhead (1983) reported that 80% to 90% of college physics students believe that a projectile thrown upward will have a positive acceleration for awhile, will stop for awhile, and then will have a negative acceleration. Trowbridge and McDermott (1980) found that students confuse the rate of change of a quantity with the quantity itself. Clement (1982) has indicated that a number of misconceptions in physics are extremely resilient and resistant to classroom instruction. His recent efforts (Clement, 1986) have been geared toward developing "bridging techniques", via analogies, to overcome certain kinesthetic beliefs about forces which lead students astray in physics problem solving. For example, the mistaken belief that a table exerts no upward force on a book resting on it is related to the students' experiences of a force being a visible push or pull as evident in a compressed spring which pushes up on a hand that pushes down on it. If the student can make the analogy of the compressed spring fit the situation of the book on the table, the initial mistaken belief may be overcome. Because these physical beliefs are about the "way the world works" rather than about objects of cognition, they are properly termed misconceptions, and not metacognitions.

Only when the physical belief becomes a conscious thought on which the thinker reflects, can we say that metacognition plays a role.

Not all beliefs about tasks are detrimental. The following task variables, identified by Silver (1982), have also been shown to have an important effect on problem solving in mathematics:

* The belief that there is usually more than one way to solve a problem
* The belief that two very different methods of solution can have the same correct results.
* The belief that there exists a most concise method in solving or presenting the solution to a problem.

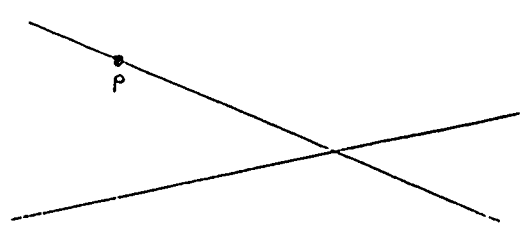
These are just a few of the beliefs which can aid a student in learning mathematics and in solving problems.

Strategy Variables: Metacognition as Monitoring and Control

The instruction of heuristics and algorithms in mathematics is rendered useless if it is not accompanied by the metacognitive skills needed to select, implement and evaluate these strategies. Stated another way, the question which motivates the study of metacognition in mathematics and science is, "What are the psychological processes that will enable students to use ideas, that they in fact have but cannot or do not use?".

To illustrate, Schoenfeld (1983) reported on the reasoning processes of a dozen pairs of students as they worked the following problem:

> You are given two intersecting straight lines and a point P on one of them, as in the figure below. Show how to construct, with straightedge and compass, a circle that is tangent to both lines and that has the point P as its point of tangency to one of them.



The subjects were college freshmen. Some had one semester of calculus and all had high school geometry. Not one of the twelve pairs of students managed to solve the problem, and all pairs proceeded by trial and error. Only one pair attempted to justify their solution mathematically, although all were satisfied with their solutions. What was most disturbing to Schoenfeld was that although the students failed to solve the problem they had all the prerequisite knowledge for the necessary mathematical argumentation. Reflecting on the phenomenon in a later paper, Schoenfeld (1985) wrote:

> "After they finished the problem session, I asked them to show that the points of tangency on the circle were equidistant from the vertex angle--- points that the students had conjectured and relied upon without proof. All pairs managed to do so, usually within 5 minutes. Thus, they had the means to solve the problem easily within their reach, but did not call upon them. They did not even think to call upon them!" (p. 371)

Mathematical knowledge and conceptual understanding are not enough to insure that students will know how and when to use such knowledge. Getting an idea i :o a student's head is insufficient. The student must also learn which ideas are most appropriate and when to use them. Similarly, Lesh (1982) wrote of his own research "... our goal is to identify processes, skills and understanding that will enable average ability students to use ideas that they do have, but which may be based on unstable conceptual models." Lesh's theory of metacognition will be treated later. For now, consider some of the primary features of monitoring and control which are the main descriptors of metacognition.

One of the clearest descriptions of the monitor and control features of metacognition comes from the fields of information processing and artificial intelligence. In the parlance of this group, these skills are referred to as the "executive". The following elucidation of the role of the "executive" comes from Brown (1978):

> (1) Predict the system's capacity limitations; (2) be aware of its repertoire of heuristic routines, and their appropriate domain of utility; (3) identify and characterize the problem at hand; (4) plan and schedule appropriate problem-solving activities; (5) monitor and supervise the effectiveness of these routines it calls into service; and (6) dynamically evaluate these operations in the face of success or failure so that termination of strategic activities can be strategically timed. These forms of executive decision making are perhaps the crux of efficient problem solving because the use of an appropriate piece of knowledge or routine to obtain that knowledge at the right time and in the right place is the essence of knowledge. (p. 182)

While much of the metacognition research, especially the early research, was in the area of memory and retrieval, almost all of the data is in the form of clinical interviews. This feature seems predominant in the literature on problem solving [Clement (1984), Lesh (1984), Schoenfeld (1983, 1985)].

The parsing of protocols is considered the single most useful tool for metacognition research. While methods have been developed for this analysis as a short-term goal, the long-term goal is to search for a correlation between problem-solving success and metacognitive activity (Hart & Schultz, 1985). The following analysis of a clinical interview is offered as one example of the type of data and analysis central to the research. Although the interview was conducted with the purpose of identifying the cognitive structures employed in the solution of a computer programming problem, it nevertheless offers insight into the self-monitoring activities of an undergraduate engineering major from an introductory computer programming course. The question used is due to John Clement who is also the interviewer.

1 I: If you could write a program to represent that statement, uh, using the letters, I guess, -- C and E --, just read the statement out loud.

2 S: Ok -- there are eight times as many people in China as there are in England, um, the program would, um, -- so the equation (writes 6C = then puts an "X" next to it). (Writes 8C = E) Number this one -- (puts 2 next to 8C = E and 1 next to 6C =). Ok? Um, the same -- the program structure is exactly the same as the last one -- um -- (pause) (Draws brackets and writes):

```
Header
Decl
Statements

READ
```

It appears as though writing an equation to represent the given relationship is a heuristic used prior to actually writing the program. Even this strategy is subject to inspection and correction as the student changes his initial expression. "6C =" to "8C = E." This mistaken equation represents an error in translation tasks which is common and non-trivial (Clement, 1986) The subject continues with the realization that the problem is not well-defined in that he does not know which is the input and which is the output variable.

3 S: There's a part of the problem that is not stated here in the sense that, we should just realize that, if you are not given whether you are gonna input the number of people in England or the people in China, Ok, so what I would do is then write a program which would deal with both.

4 I: Well, let's just do one -- why don't we say that we will input the number of people in China, OK.

5 S: (writes:)

```
READ    (C)
        E = C/8.0
        WRITE (E)
        STOP
        END
```

6 I: Ok, and how did you know how to write each of those lines?

7 S: Um, this one, [points to READ (C)] I know you have to input given the factor of an eighth used for the number of people in China, um, we have to calculate E, um (pause) -- I realize I made a mistake in the equation, um --

8 I: What are you looking at?

9 S: This one is wrong, um [points to 8C = E (Eq. 2)] -- I should --

The student confidently writes a program that contains the correct but reversed equation from the one he originally wrote. Not until he verbalized his solution did he become aware of his error. Further requests for cognitive information from the interviewer will prompt the student to describe his thoughts retrospectively.

10 I: How can you tell it's wrong? What did you just think of there?

11 S: Well, I realized that I wrote it right in the program and it's different than the one I wrote up there, so that I would read, oh, I would change it. (Put an X next to 8C = E.)

12 I: Ok -- that's interesting -- what convinces you that the program is right?

13 S: The fact that I know there's more people in China than there are in England and in the equation the E would end up being 8 times greater than the C, which is not true, Ok. [Writes E = C/8]

14 I: ...in the second line of the program, what were you thinking in order to write the second line there, when you wrote it? (Pause) Do you remember?

15 S: Just that E had to be a smaller number than C.

The subject has identified the key qualitative understanding which made his solution possible -- namely that there are more people in China than there are in England. He next affirms his qualitative understanding and attempts to unravel the reasons for his mistake by formulating a theory as to why "somebody would make that error".

16 I: You're pretty sure that's what you were thinking when you wrote it? Yeah? Ok.

17 S: In fact I think that this (points to 8C = E) was in my head in that form (points to E = C/8) and it just got written down that way -- wrong -- I don't think I ever had it conceptually that E was bigger than C -- just that it got written wrong because I didn't even think about rewriting it (points to line 2 in program) -- I just thought of the way to write it, yeah.

18 I: So when you first read the problem before you wrote equation 2 there, you immediately realized there were more people in China? (Nods) But this is confusing -- a lot of people do this, that's why we're interested -- but when you write down 8C = E -- what do you think you are working from there, um, when you make that error?

19 S: Hmm -- I don't know why somebody would make that error, um, in terms of -- except that maybe, you're thinking like, um -- you are conceptualizing that C is 8 times larger than E, um, and so you associate the 8 and the C somehow in your mind perhaps, but, ok, I think the knowledge that C is 8 times larger than E is like, I didn't have any trouble conceptualizing that, it's just getting it written down accurately, right.

This student exhibited many metacognitive skills: interpretation of the problem and the relationships within, selection of heuristics, re-examination of previous work, resolution of conflicting ideas, further qualitative assessment of the problem to check his solution, and an explanation of his error in the form of a general metacognitive theory about errors of that type, which was not unlike the theories of Clement, Lochhead, & Soloway (1980).

These researchers attempted to ascertain why a programming context decreased the reversal errors frequently found in the translation of English sentences into mathematical equations. Of the five hypotheses generated three involve the cognitive features of computer programming: (1) unambiguous semantics, (2) explicitness of syntax, (3) active input/output transformation.

The remaining two hypotheses appear metacognitive in aspect: (4) the practice of debugging programs, (5) decomposing a problem into explicit steps. This study indicates that certain metacognitive skills are common features of training in computer programming while they are not as common in typical algebra courses.

Even in problems which contain no mathematics whatsoever, Clement (1984) concluded that engaging in a cycle of conjecture, evaluation and self-correction is a basic problem-solving skill which serves as a prerequisite for more advanced problem solving in mathematics and science. Perhaps the most telling feature of clinical interviews analyzed from the perspective of metacognitive processes is the absence of those processes in failed problem solutions. [Schoenfeld (1985), Lesh (1985)]

Lesh (1983) sees in his analysis of problem-solving protocols two levels of cognition. First-order cognition is the construction and coordination of conceptual models with which the problem solver makes sense of his world. Lesh's definition of such conceptual models consists of four independent components:

(a) within-idea systems: organization and relational systems imposed on the environment by the thinker.

(b) between-idea systems: relationships between ideas.

(c) representational systems: symbol systems with networks of translation and transformation between representations.

(d) systems of modeling processes which contribute to both the development and usability of the first three components of the model.

According to Lesh, metacognition is second-order cognition which treats as its object of cognition all of the above components of the problem solver's conceptual model. He argues that solutions to problems are "constructed by gradually refining, integrating and adapting unstable systems (i.e., poorly coordinated conceptual models). From this perspective the most important metacognitive events occur at "reorganization" points when a redirection occurs in a solution path. Furthermore, Lesh interprets the absence of metacognitive activity (eg., planning, monitoring, assessing) in the solution attempt as one important probable cause for failure in problem solving.

However, Lesh does not wish to imply that constant and continuous metacognition will aid the problem solver in all contexts. Unstable conceptual models are most debilitating at precisely the points of reorganization at which metacognition comes into play. Lesh's protocols indicate that success can be facilitated by explicitly avoiding paying too much attention to process and detail. For this reason, he hypothesizes two orders of metacognition: first-order metacognition; monitoring, planning and assessing conceptual model components, and second-order metacognition which treats as its object the features and uses of first-order metacognitions. An example of second-order metacognition would be a conscious decision to allow an incubation period to permit a "settling time" for ideas to stabilize. Another example, with an opposing perspective but with the same goal, is the conscious decision to engage in "brainstorming" which encourages the creation and sharing of ideas without concurrent evaluation. It appears that expert problem solvers demonstrate the ability to select from among a host of metacognitive strategies those which appear most useful at the time.

CONCLUSION: DEVELOPING A "METACOGNITIVE ORIENTATION"

While the long-term goal in metacognition research is to show a correlation between problem-solving success and the attendant metacognitive activity, there is yet much uncertainty as to the success of the endeavor. Several studies indicate that the amount of metacognitive activity is not necessarily a predictor of problem solving success. Hart and Schultz (1985) found that expert mathematics problem solvers may demonstrate little metacognitive activity and yet achieve a quick and correct solution to a problem which a novice problem solver could not do correctly although extensive metacognitions were evident. Simon and Simon (1978) report similar findings with expert and novice physics problem solvers. Furthermore, contrary to Flavell, some memory researchers found that even good students know very little about mnemonic techniques that they may or may not use to better retain information studied. (Dansereau, D. et al, 1975)

Nevertheless, the metacognition paradigm remains useful. Hart and Schultz (1985) conjectured that the specific problem asked of expert problem solvers must be challenging enough so that it is in fact a problem for them. It is assumed that a truly challenging problem requires demonstrated metacognitive activity for its solution. The implication here is that simple problems can be solved with little evidence of metacognition due to the speed with which the solution is attained. Similarly, Silver, Branca and Adams (1980) stated:

"It may be that the metacognitive behaviors of experts become so integrated into problem-solving routines that they become difficult to observe, or it may be that the metacognitive analyses are only needed at certain points in the development of expertise. Once a sufficient level of expertise has been generated, the metacognitive aspects of thinking assume a secondary role, with technique and execution assuming priority." p. 216

Although problems with the research have surfaced at this early stage in its development, there are yet some very compelling reasons to continue with the program.

1. Attention is focused on an aspect of problem solving which appears differentially in expert and novice problem solvers.

2. Cognitive process research is further delineated into areas whose research methodologies may differ. The number of researchers in the field may also increase as specialties develop.

3. Many educational pedagogies [Whimbey & Lochhead, 1980; Belmont, Butterfield and Ferretti, 1982; Novak & Gowin, 1984; Confrey, 1984] have integrated metacognitive skills training into the curriculum on the assumption that such skills are useful or even necessary for learning.

Perhaps the most significant pedagogical contribution for the development of metacognitive skills is the instilling of an awareness that one needs to take conscious, planful action when learning mathematics and science, and in problem solving. While Flavell (1981) considers such an awareness to be related to metacognitions relevent to task variables, Taylor (1983) argues convicingly that awareness is prerequisite to the development of metacognitive knowledge. She contends that the process of becoming aware, while problematic in its analysis, is most likely learned when it is encountered in others. Conversely, if the awareness process is seldom encountered, children may exhibit little evidence of having a metacognitive awareness.

> "The contention here is that metacognition skills, especially those related to school learning, may be difficult to develop because the strategy involved in successful task performance is usually not available to the child either through observation or direct instruction. We can't readily observe what people do to help themselves remember, nor do children frequently encounter adult efforts to make these strategic processes explicit to them." (p. 272)

Taylor argues for what she terms a "metacognitive orientation", where the teacher models metacognitive behavior and devises activities for students to exhibit metacognition themselves. Some of the activities she suggests are: ask students to estimate their performance on a task, check their performance against their predictions, keep records of the accuracy of their predictions, compare a variety of study strategies, compare their actual performances to their estimates and to an externally set level of adequate performance. These activities are general so that they may be applied to any curriculum, thus accomplishing two goals: helping students to learn curriculum and also to learn metacognitive skills relevent to learning strategies, to various kinds of tasks and about themselves as learners.

The pair problem solving method of Whimbey and Lochhead (1980) requires metacognitive activity as a continuing process in the solution of problems. Fashioned after the clinical interviews of Piaget, pair problem solving requires the problem solver to read and solve a problem aloud to a listener. The listener may stop the solver at any time to ask for clarification if he or she did not understand a step in the solution. At no time should the listener take over the solution of the problem or suggest ways to solve the problem. By keeping the thinking process verbal the solver also listens to him/herself and monitors along with the listener, the solution with all reasons, dead ends, conjectures, doubts, etc. The richness of the reasoning process is manifest in the ensuing interview. By exchanging roles students learn not only how to solve problems but also how to communicate their ideas effectively to a listener. They also learn how to listen and ask questions that will clarify for themselves someone else's ideas.

As can be seen from the above commentary metacognitive skills need not be yet another curricular item. By teaching content and concepts through pair problem solving metacognitive awareness is modelled on a daily basis, and students develop a metacognitive orientation towards each learning task. The role of the teacher is more that of a facilitator rather than a lecturer. Occasionally the teacher may solve a problem aloud, sometimes spontaneously, while the students ask questions to clarify their understanding of their teacher's thought processes. More often, the teacher listens to student solutions and asks questions to clarify his/her understanding of the student's ideas. Generally both the student and the teacher learn from the interaction, and while content knowledge may elude the student's long term memory, hopefully the metacognitive orientation will remain.

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