DOCUMENT RESUME

ED 291 492 PS 017 164

AUTHOR Bentley, Prudence A.
TITLE Defense of the Child's-Play Method for Teaching the Place Value Notation Concept to Elementary and Preschool Children.
PUB DATE 87
NOTE 32p.; For related documents, see PS 017 161-163.
PUB TYPE Viewpoints (120)

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS Arithmetic; *Cognitive Structures; Constructed Response; Costs; Early Childhood Education; *Elementary School Students; *Error Patterns; Instructional Materials; Mathematics Anxiety; Mathematics Instruction; Models; *Place Value; Play; *Preschool Children; Schemata (Cognition); *Teaching Methods; Validity
IDENTIFIERS *Cognitive Conflict; Rule Application

ABSTRACT

The objective of this paper is to explain the need for and defend the sufficiency of the child's-play method of teaching the place value notation concept to preschool and elementary school children. Discussion first illustrates error patterns of school children in the use and interpretation of place value notation, arguing that the errors reflect counting principles learned in preschool and that such errors are the natural consequence of children's assimilation of information concerning digit symbols into their cognitive structures. Subsequent discussion suggests that future research will show that the inadequacy of the preschool principles is a major, if not the root, cause of mathematics anxiety in children, and that the harmonious integration of the preschool counting principles with number representation rules will lessen, perhaps eliminate, math anxiety. Discussion then explores the problem of integrating the rule systems, focusing on cognitive conflict and the task of instruction. The extensive concluding section presents five arguments supporting the child's-play method; these center on: (1) the simplicity of the method and the need for that simplicity; (2) the completeness of the concrete demonstrations and the sufficiency of the confirmations afforded by them; (3) the method's connection to a proven model; (4) evidence of the method's value from preliminary findings; and (5) minimal cost of the materials. (RH)

Prudence A. Bentley 296 Bald Hill Road Ithaca, N. Y. 14850 (607)272-0675

IV Defense of the Child's-play Method for Teaching the Place Value Notation Concept to Elementary and Preschool Children

ABSTRACT

In this final article of the four-part series Bentley focuses on two comments by Gelman and a conjecture by Ginsburg. She explains the relevance of these to the problem of communicating the place value notation concept to elementary school children. Gelman's comments pertain to what preschoolers can and cannot do. Ginsburg's conjecture concerns the genesis of the <u>gap</u> between school children's informal knowledge and formal written procedures: "The gap may originate because instruction does not devote sufficient attention to integrating formal written procedures with children's already existing and relatively powerful informal knowledge."

Ginsburg suggests that school children's errors in written work reflect oral counting language. Bentley agrees but argues that the pattern of errors also indicates that school children spontaneously generate how-to-represent rules which are consonant with their preschool how-to-count principles. She concludes, therefore: (1) that Ginsburg's conjecture is correct concerning the origin of the <u>gap</u>; and (2) that the task of instruction is to address the points of conflict and facilitate resolution. She reasons that resolution requires instructional materials and procedures which clearly and verifiably demonstrate that between the seemingly conflicting rule systems (the children's rules versus place value rules) there is actually continuous, uninterrupted harmony. Verifiable demonstration of continuous harmony is the purpose of the child's-play constructions. Bentley presents five arguments in defense of her method.

IV Defense of the Child's-play Method for Teaching the Place Value Notation Concept to Elementary and Preschool Children

A. Introduction

Two comments by Gelman and a conjecture by Ginsburg are the focus of this final paper in the four-part series. Gelman's first comment concerns what preschoolers can and cannot do:

> Information about what tasks a preschooler can perform would make it easier to determine what makes him unable to perform others. (1)

Ginsburg's conjecture concerns the gap which he has observed between school children's informal knowledge and their written work. My interest is in the gap as it is reflected in school children's use and interpretation of place value notation, base ten. Ginsburg writes:

> The gap may originate because instruction does not devote sufficient attention to integrating formal written procedures with children's already existing and relatively powerful informal knowledge. (2)

I examine error patterns cited by Ginsburg in the light of the preschool how-to-count principles described by Gelman. I find that knowledge of the preschool principles enables explanation and prediction of the typical errors. Therefore, I conclude that Ginsburg's conjecture is correct.

The pattern of errors indicates that school children spontaneously generate how-to-represent rules which are consonant their preschool how-to-count principles.

This confirms the second comment by Gelman:

> What emerges after a child has had some training is the joint result of the training and whatever interpretative structures the child brought to the training. (3)

The rules governing place value notation pose problems to school children because they conflict with certain of the pre-school how-to-count principles. The task of instruction, therefore, is to address the points of conflict and facilitate resolution. Resolution requires instructional materials and procedures which clearly and verifiably demonstrate that between the seemingly conflicting rule systems there is actually continuous, uninterrupted harmony. Demonstration of continuous harmony is precisely the goal in view and the purpose served by the series of child's-play constructions described in article #3. The objective of this paper is to explain the need for and defend the sufficiency of the child's-play method of teaching the place value notation concept to elementary and preschool children.

B. Error Patterns of School Children in the Use and Interpretation of Place Value Notation

The error patterns cited below are typical of elementary school children. All the citations are from Ginsburg's work. (4)

Rebecca (age 7-3) wrote the following:

203 for twenty-three
305 for thirty-five
503 for fifty-three

Rebecca read 53 as five, three. She read 503 as fifty-three.

Joe (age 11) wrote the following:

ʳ; for fifty-six
3 for three
472 for four hundred seventy-two
600,023 for six thousand twenty-three
710,0085 for seventy-one thousand eighty-five

Ginsburg reported that Paul (age 5-1) could count to a million but could not write any number except 1. He suggests that the errors are a reflection of oral counting language. If we partition the numerals with oral counting language in mind, it is easy to understand their significance to the children. Partitioned thus: 20-3 is easily interpreted as twenty-three, and 6000-23 as six thousand twenty-three. The systematic quality of these errors seems to indicate that a rule of oral language motivated them.

However, I believe we can also see in these errors a reflection of the preschool how-to-count principles.

C. The Role of the Preschool Principles in Causing the Error Patterns

Gelman describes five principles: (1) the one-one principle; (2) the stable-order principle; (3) the cardinal principle; (4) the abstraction principle; (5) the order-irrelevance principle. (5)

The one-one principle includes the component of distinct tags. Rebecca's 203 (twenty-three) and 305 (thirty-five) reflect application of this rule. The zeros are applied as distinguishing markers in accordance with the component of the one-one principle which (1) requires a system of distinct and unique tags and (2) forbids re-use of tags in any particular counting procedure.

Rebecca read 53 as five, three. The same rule obtained as Rebecca interpreted this place value numeral. The order-irrelevance principle negates the possibility that Rebecca would assign a different significance to 5 on the basis of its position relative to the 3. Fifty-three must be written 503 in accordance with the unique tagging component of the one-one principle. The zero enables the child-writer to mark the difference between five and fifty; the zero-marker enables the child-interpreter to discriminate the difference between fifty and five.

Joe's errors are not different in kind from Rebecca's. His writing 600,023 to designate six thousand twenty-three reflects perseveration of the unique tagging component of the one-one rule. Zeros enable discrimination of the cardinal number, six thousand. Joe had learned the place value system for representing numbers up to three figures. Beyond that, his efforts to use notation reflect application of the preschool how-to-count principles.

It is interesting to notice that the conventional oral counting scheme with its stable-order list of name tags offers no conflict to the preschool how-to-count principles. There is a unique tag (count word) for every count and a stable order for the tags. The tags (count words) do not lose their unique significance if the order of name tags is scrambled. Five year old Paul's mastery of the oral counting scheme to one million is not, then, so surprising in as much as it fits so compatibly with the preschool how-to-count principles. Neither is it surprising that representing count by means of conventional symbols is not among his accomplishments. What, after all, do arbitrary digit configurations have to do with the application of how-to-count principles?

The symbol "1" bears an analogical resemblance to finger pointing, the common first step in conducting one-to-one counting; a single stroke of a pencil produces the symbol configuration "1"; the form "1" is just one symbol, no more, no less. Perhaps this explains why Paul could and did write the symbol "1". Since no similar observations can be made concerning other digit symbols, perhaps Paul's sense of what is true could find no reason to adopt them. "2" is not two forms; "3" is not three forms; a single continuous form of action produces both "2" and "3". There is no matching of digit symbol and count except in the case of the symbol "1". Indeed, attaching count-word names to digit symbols could appear as a senseless act to a counting child.

If we put intelligence under the governance of the preschool principles, private thought might express itself as follows: "My how-to-count principles are my own. I figured them out myself. These 'squiggles' (Dr. Sinclair's term in conversation, 1979) which adults call by count-word names have nothing to do with my counting principles. That oral counting scheme, however, is another matter altogether. It fits perfectly with my counting principles. It is useful to me: I can go on and on doing interesting things with my counting principles because that oral counting scheme 'understands' them. In a manner of speaking, the oral counting scheme and my how-to-count principles speak the same language."

If children base how-to-represent rules on their how-to-count principles, they could be well satisfied by the oral counting scheme for extending their oral representation of count. (Apparently Paul was satisfied.) Furthermore, they could be equally satisfied to carry this oral pattern out visually when they employ digits to represent count. Children's use of zeros indicates that this is, in fact, what is taking place. Zeros serve as markers which enable extension of the how-to-count principles into the realm of visual representation. Children's systematic use of zeros (as markers) in representing count is continuously compatible with the application of the preschool how-to-count principles in conducting oral one-to-one counting.

Rebecca's and Joe's errors in the use of place value notation are typical of elementary school children. Such errors are the natural consequence of children's assimilating information concerning digit symbols into their already existing cognitive structures.

> What emerges after a child has had some training is the joint result of the training and whatever interpretative structures the child brought to the training. (3)

D. The Inadequacy of the Preschool Principles and the Consequences of that Inadequacy

The children's how-to-count, how-to-represent rule system is comprehensible; unfortunately, it is neither efficient nor acceptable. Also unfortunate for the children is the perseveration of a rule system which dooms them to commit errors when it comes to written procedure. Gelman's comment is relevant to this problem:

> Information about what tasks a preschooler can perform would make it easier to determine what makes him unable to perform others. (1)

As educators we need to realize that children making the typical errors cannot perform accurately on tasks involving place value notation because their efforts are founded upon and guided by the preschool how-to-count principles. Without specific enhancements the preschool principles are inadequate procedural tools for the tasks which the place value convention requires of school children. The basic inadequacy of the preschool principles as procedural tools is the reason, I believe, that the gap observed by Ginsburg is initiated.

> There often exist gaps between children's informal knowledge and their written work. In general children are uncomfortable with written work and botch it up. At the same time they may possess an impressive informal understanding of the same concepts. Even children with severe learning problems may have unsuspected informal strengths. *The gap may originate because instruction does not devote sufficient attention to integrating formal written procedures with children's already existing and relatively powerful informal knowledge.* (2; italics mine)

I conjecture that "the already existing and relatively powerful informal knowledge" which requires integration is the set of preschool how-to-count principles. The place value notation symbol system is the major tool we use in teaching basic math skills to children. I conjecture that future research will prove that the fundemental inadequacy of the preschool principles as procedural tools for dealing with the place value symbol system is a major if not the root cause of math anxiety in children. I conjecture further that when the problem of integrating these principles is harmoniously resolved, there will be a significant lessening, and perhaps elimination, of math anxiety in children.

E. The Problem of Integration: Cognitive Conflict and the Task of Instruction

Underlying school children's persistent difficulty in gaining mastery over the place value convention and its notation is the problem of integrating rule systems which are at odds with each other: viz. the children's how-to-count, how-to-represent rules versus the rules governing place value notation. As educators we need to understand that the children's sense of a conflict of rule systems actively opposes integration, and that this sense of conflict persists over time. Recall that Joe (age 11) had mastered the place value method for representing numbers up to three figures. Thereafter, he relied on the more dependable (for him) how-to-count, how-to-represent method and wrote 600,023 for six thousand twenty-three.

> Children learn certain rules for small numbers but then, on encountering large ones, need to learn once more what are essentially the same rules. In a similar fashion, children who can do addition with two digit numbers need to learn it over again for three, even though the process is essentially the same in the two cases. (6)

There are several differences between the children's how-to-count, how-to-represent rules and the place value system. The first of these is the difference of simplicity and complexity. Both the preschool principles and the how-to-count, how-to-represent rules are governed continuously by the principle of one-to-one correspondence. This principle governs place value notation only as far as ten. Thereafter place value notation achieves its significance by imposing a 1:10 ratio structure and progression on one-to-one correspondent counting.

The introduction of ratio structure and progression brings with it complex requirements which violate the children's entrenched rule system. Two of the preschool principles conflict with those governing place value notation. The order-irrelevance principle forbids assigning differing values to digits based on their relative position. The one-one principle with its unique tagging requirement forbids recycling of digits (there must be a unique visual tag, i.e. symbol, for each unique count word).

The task of instruction, therefore, is to address these points of conflict and resolve them. Resolution requires instructional materials and procedures which clearly and verifiably demonstrate that between the seemingly conflicting rule systems there is actually continuous, uninterrupted harmony. Demonstration of continuous harmony is precisely the gc l i.. view and the purpose served by the series of child's-play constructions described in article #3.

F. Five Arguments in Support of the Child's-play Method of Teaching the Place Value Notation Concept Base Ten, to Elementary and Preschool Children

The five arguments I present in support of the child's-play method are these: (1) The simplicity of the method and the real need for that simplicity; (2) The completeness of the concrete demonstrations and the sufficiency of the confirmations afforded by them; (3) The child's-play method follows closely a proven model; (4) Preliminary findings give evidence of potential value; (5) The minimal cost of the cnild's-play materials.

1. The Simplicity of the Method and the Real Need for that Simplicity

The child's-play method rests upon a single simple principle: the one:one relation. This principle continuously undergirds the method and guides the focal attention of the active, perceiving children. While the place value notation concept also rests upon this principle, this fact is obscured by the rules which govern place value notation. Place value notation reflects a one:ten relation. Place value notational procedure is further complicated by rules which require: (1) that the 1:10 relation be imposed successively in progressive multiples (1:10:100:1000 and so forth); (2) that this progression be designated in a right to left direction by digit symbols set in fixed relative horizontal positions (ones, tens, hundreds, thousands and so forth); but (3) that the place value numerals must be read for meaning in the opposite (left to right) direction (thousands, hundreds, tens, ones).

School children are well ready for instruction in the place value notation concept as far as the one:one rudiment is concerned. But they are totally unprepared for the complexities of the 1:10 ratio structure and progression. Where application of the ratio component is required, we begin to see the error patterns documented by Ginsburg, and, I conjecture, the commencement of cognitive distress which begins as befuddlement, develops into frustration, and matures into math anxiety. I have witnessed this painful pattern year after year in the public classroom.

The child's-play method prepares children to deal with the complexities of place value notation. Indeed, the method makes these complexities child's play. The one:one principle continuously guides the children's focal attention as the 1:10 ratio structure and progression is defined, displayed, and confirmed by structures and manipulations which are comprehensible in terms of the how-to-count principles and compatible with the how-to-count, how-to-represent rules.

The preeminent role of the one:one principle in explicating and simplifying the complexities of the place value notation concept is detailed below:

1. First the children individualize the cups (one:one) by constructing the contiguous lineup. One by one they handle

each of the 100 cups in the cup game, each of the 1000 cups in the number towers model.

2. Second they impose (one:one) successive (one after one) 1:10 organizations on the cups.

3. Third, the filled frame is emptied (one:one). One by one cups are removed from the filled frame and stacked (one:one). Stacking creates a singular new form (one:one:one; child:stack: placement of stack). Because the cups are stacked, they require as stacks no more space horizontally than single cups.

(This strategy is the key to the simplicity of the child's-play method. It enables definition of ratio structure and progression in terms of the one:one principle. It defines place value notation in terms of the one:one principle and imitation. The most puzzling place value symbol for young children would probably be "1111"; this numeral is transparently easy to interpret in terms of the concrete model, number towers. Place value notation counts stacks; the number towers model teaches how to tell one stack from another by involving the children in successive procedures which construct them. One might almost say the children <u>reinvent</u> (7) the concept by exercising control over the materials used to display and confirm the component relations of the place value notation concept, base ten.)

4. The one:one principle continues to give continuous, stable support as the rudimentary counting schemes (by ones to ten; by tens to one hundred) are introduced and practiced. The ten-count game performs a vital bridging role to the process of integration. A key strategy puts the children in controll of organizing count the place value way. In the cup game

and the number towers model llness of the triangular frame triggered reverse imitation (emptying and stacking). In the ten-count game the child, rather than the container, determines fulness. The children agree that a cup with ten tokens in it is full; then they themselves impose this rule and apply appropriate successive count-word names. Thus 1:1:10 and 1:10:100 is defined by counting and demonstrated by concrete manipulations. Both one by one counting and ten by ten counting rest upon the steady application by the children of the one:one principle.

5. Finally, errorless manipulation of the flip-card counter in coordination with one-to-one correspondent counting displays place value notation, base ten accurately and continuously, one:one. At the same time it also tracks step by step and summarizes the entire construction of the number towers model. After the number towers materials have been put away, the flip-card counter remains a permanent summary representation of the number towers construction procedures.

(2) The Completeness of the Concrete Demonstrations and the Sufficiency of the Confirmations Afforded by Them

In the introduction to this article I stated that the rules governing place value notation pose problems to school children because they conflict with certain of the preschool how-to-count principles. The task of instruction, therefore, is to address the points of conflict and facilitate resolution. I suggested that resolution requires instructional materials and procedures which clearly and verifiably demonstrate that between the seemingly conflicting rule systems there is actually continuous, uninterrupted harmony.

In sections B-D I explained the relation of the preschool how-to-count principles to the error patterns documented by Ginsburg. In article #3 I discussed the child's-play constructions as a means of introducing the place value notation concept to children for the first time; in the section above I have detailed the simplicity of the method. Now let us look at the child's-play method in terms of its potential helpfulness to children like Rebecca, age 7, or Joe, whose errors at age 11 still reflected the perseverant influence of the preschool principles. Of what practical remedial use is the child's-play method? How, for example, does the child's-play method address children's need for zero markers?

In the course of construction Rebecca's numbers are given concrete illustration. It takes but two triangular frames to give concrete illustration to twenty-three, thirty-five, or fifty-three. The number towers model addresses the problem of zero markers concretely. The primary version of the flip-card counter would be helpful to Rebecca. As a teacher, I would suggest involving Rebecca (and perhaps her whole class) in manipulating the flip-card counter in coordination with one-to-one oral counting. My recommendation is that the flip-card counter be introduced following rather than preceding the cup game. The two-handed coordination of flipping back the numeral cards in the units position while fliping forward the next single card in the tens position is representative of the interruption to construction occasioned by the emptying-stacking procedures of the cup game and the number towers model. Twenty on the number towers model appears as two stacks in the tens position and an empty units position. Twenty-three appears as two stacks in the tens position and three single cups in the units position. The zero is signified as a step of procedure in the succession of steps leading to the display of twenty-three.

Similarly the appearance of the place value numeral "20" is a step of procedure in the course of counting to "23" using the flip-card counter. If Rebecca wonders where the zero marker is, she has only to peek beneath the "3" in the units position to find it.

It would probably be clarifying for Joe to set the flip-card counter by stages (following experience with the number towers model). Counting by thousands and then by ones to display 6,023 would be an efficient way to draw attention to the simplicity and conciseness of the place value method. Repeated experimentation and exploration of the flip-card counter would be very helpful to uppergrade students like Joe because there is no need to abandon or set aside the preschool principles. Only time and volitional opportunity is needed for the children to confirm the truth of the relationships given obvious one:one display by the child's-play construction and manipulation procedures.

(3) The Child's-play Method Follows Closely the Pattern of a Proven Model

The behavioral accomplishments of the twenty-four month old youngsters are practical evidence that the environmental setup of the Geneva research plan was an ingenious pedagogical design. The setup prompted child-directed exploration, displays of relation, and systematic confirmations. The ultimate result was tnat the organizational schemes which were systematically confirmed were also internalized. Proof of internalization was evidenced by the reappearance of the schemes. When they reappeared, the schemes functioned as procedural tools in solving new problems.

Since no one told the Geneva children what they must do, I reasoned that the environmental setup motivated interest, and interest sparked initiative. Thereafter a didactics of circumstances and events was the children's teacher. In article #1 I theorized as follows:

> The child who grasps objects and touches them to his body ceases to be a passive receiver of perceptual information and becomes an actor-controller of circumstances and events. As such he becomes the creator of information over which he has partial control. The relations and transformations to which he/she subjects objects are didactical circumstances and events which mediate information to him/her via his/her perceptions. The young child who exercises control over objects and procedures and perceives the consequences of his control also detects constants and invariants which pertain to the objects and to the form of action he/she imposed.

The constants and invariants which are discriminated by the perceiving child are registered cognitively in a one:one relation (self:information, one:one). It is intrinsic to the human design to perceive as one being; the intrinsic coordination of the perceptual system makes possible confirmation motorically of information perceived visually.

Hands, eyes, and form of action were brought to bear in the Geneva children's confirmation procedures. Sequence F was a culminating event of confirmation.In article #2 I suggested that th.s event was an external response to an internal need. I conjectured that the children were cognitively stressed by the regular dissolution (out of their sight) of every system of order they imposed on the objects, and that it pleased the children to conduct the culminating event because it was an integrative experience. I suggested that reviewing all the organizational procedures in continuous, uninterrupted succession enabled the children to treat the set of procedures as a whole. This event put the children in continuous control of both order and dissolution. Therefore, the review was integrative. The perception of integrated wholeness and the experience of continuous control quenched the sense of cognitive distress. Further practice of organizing the objects ceased because there was no longer any internal need. Following the culminating integrative event, the organizing practices ceased abruptly. I suggested that this cessation marked the point in time when the organizing procedures were internalized as schemes. When the internalized schemes reappeared, they functioned as procedural tools to solve new problems.

When the flip-card counter is manipulated in coordination with one-to-one oral counting (or any other counting pattern one wishes to implement), it also tracks one:one the revelatory procedures of the cup game, the number towers model, and the ten count game. Therefore, errorless manipulation of the flip-card counter can be expected to perform for school children the integrative role which the culminating sequence F event performed for the Geneva children. Furthermore, we can expect that all the displays of relation which have been systematically practiced and perfected will also be internalized as schemes. Finally, we can expect these internalized schemes to function effectively as procedural tools which enable school children to solve problems involving the application and interpretation of place value notation, base ten.

4. Preliminary Findings give Evidence of Potential Value

It is good to have the Geneva model to follow. The question is, will the child's-play method achieve similar dramatic results? We must not forget that the fourteen-month-long course of procedures conducted by the Geneva children was entirely a voluntary event. Can we expect similarly sustained attention to the strategies suggested for displaying the place value notation concept? Will children voluntarily initiate the revelatory procedures? Will their interest be sustained long enough to demonstrate, confirm, and internalize all the component relations of the place value notation concept which the method invites children to notice? An instructional plan which may seem right in theory and look superb in the adult's plan book may nevertheless fail miserably when presented in the interactive environment of children. Needless to say, I have been very curious to discover whether children would find the materials and procedures interesting. Following are some anecdotal records:

(a) Ronnie, Age 3

A young mother and her three year old son called on me at my home. To keep the young child entertained while his mother and I conversed, I asked him if he would like to help me "measure" my rug. I set down an interesting little case containing one hundred paper cups, and invited Ronnie to see if he could figure out how to open it. After Ronnie opened the case, I gestured explaining that I wanted to measure the rug with cups and asked if he'd like to help me. I lined up a few along the border.

Ronnie lined up all the rest by himself with no further help from me.

When Ronnie had placed the last cup he stood up, spread his arms wide and surveyed all he had accomplished. "Look," he announced. Gesturing broadly he continued, "Dat. All dat, I did." We then continued the game to completion.

I didn't see Ronnie again for several weeks. One evening he arrived at my door with his two older brothers, both parents, and his aunt and uncle. His first word to me was "Cups."

"Cups!" he repeated authoritatively, as though I was supposed to know what he meant. It took me a minute to realize that he wanted to play the cup game again, this time with his brothers, aged 5 and 7. The boys played the game to completion, all helping to make the line-up, and taking turns filling and stacking. Ronnie was the major instructor.

(b) Amar, Age 3

During a vacation period Amar met me at a neighborhood pre-school accompanied by his ten year old brother, Amit. Amar came to play the cup game; Amit kindly consented to run the video camera for me. The videotape documents Amar's sustained interest in conducting all the procedures of the game despite comings and goins and deprecating comments of an older child (who declined invitation to join us) as well as noises of wild play and high screaming in the background.

During the same vacation period Amit also videotaped Amar playing the ten-count game. After counting tokens and lidding the first few cups, Amar flung his hands up and triumphantly announced in his own words the rule the game is designed to illustrate.

(c) Robert and Nicky, age 5

During vacation Robert and Nicky met me at the same preschool room. Other children were present part of the time. Again Amit was the camera man. We played the cup game to completion with some interesting and significant differences. Robert and Nicky were two years older than Ronnie and Amar.

The line-up manipulations by the older children were quicker and appeared to be more authoritatively conducted; the filling of the triangular frame was more independently and meticulously accomplished. Stacking the first time was approached with such vigor by Robert that there were several collapses. This elicited high giggles from a younger observer.

After three stacks had been formed and the frame was being filled for the fourth time, Robert announced,

"Pretty soon we're going to have four piles, and then we can put them all on top of each other."

Nicky chimed in, "And then we can keep piling more and more and more."

Robert continued, "And you know what? And put more and more. Let's get more and more and more (points to each of the four stacks successively) and put more on the piles and then put them all together (frames tall imaginary structure with his hands and forearms), and then we'll have one big statue. How about that?!

And get all the cups all around the house--"

Nicky chimed in again, "And build one even bigger than you, Mrs. Bentley."

(It is interesting and significant to notice that Robert and Nicky were already anticipating a time of consummation when only three or four stacks had been formed. This incident gives me reason to pause and wonder: Could it be that the Geneva youngsters were not so much stressed by regular dissolutions of their orderings as they were eager to prove their consummate control over the disarray? Consummate control by the Geneva youngsters was evidenced as the continuous, uninterrupted demonstration of successive events of control. It was the children's "grand finale" signaling the end of organizing the disarray.)

Nicky and Robert conferred about the construction idea and agreed that they had thought of it together. (This too is a significant response. The boys had taken the filling-stacking concept on as their own and were spontaneously extending application of the concept at first displayed to them.)

We played the game to completion. As I was driving Nicky home, we discussed the idea of building a tall "statue". Nicky mused from the back seat, "It'll pro'bly take a hundred cups. We'll need a million hundred cups."

I suggested that if he and Robert wanted to do it, we might ask some friends to help; we could build it at my house, I offered, because the ceiling is very high.

"I'll make a design," Nicky commented.

I asked Nicky if he could draw a picture of his plan. (I theorized in my mind that a picture drawn by Nicky prior to construction would be an indicator of what he may have internalized already after playing the cup game just once.)

We convened at my home about a week later. Nicky not only brought a picture for me but several others for the friends who would help us. Nicky's picture-plan is shown in Fig. 1. Nicky's explanatory comment was this: "Someone is on the ladder and he's putting a cup on the tower of cups." The ladder is shown stretching from the living room floor in my home to the second floor balcony. Notice that the tower is made of distinct segments.

With the help of several young friends and a couple of parents we played the cup game to completion using one thousand cups. In keeping with Nicky's picture-plan, we set up a step ladder. At the end of the game the children took turns climbing the ladder in order to touch the top cup.

(d) Christopher, Age 3

In my study I have a sequence of pictures showing Chris playing the ten-count game with me. To initiate the game I displayed a covered basket and asked Chris to go upstairs and get me ten paper cups. At that time I thought it was necessary for a child to be able to count accurately to ten in order to play the game. (I no longer think so.) In fact I made up the game for Christopher because I had been told he could count accurately to ten. I thought it important that he be introduced to the next simplest scheme (viz. ten to one hundred) so that he might from the begin-

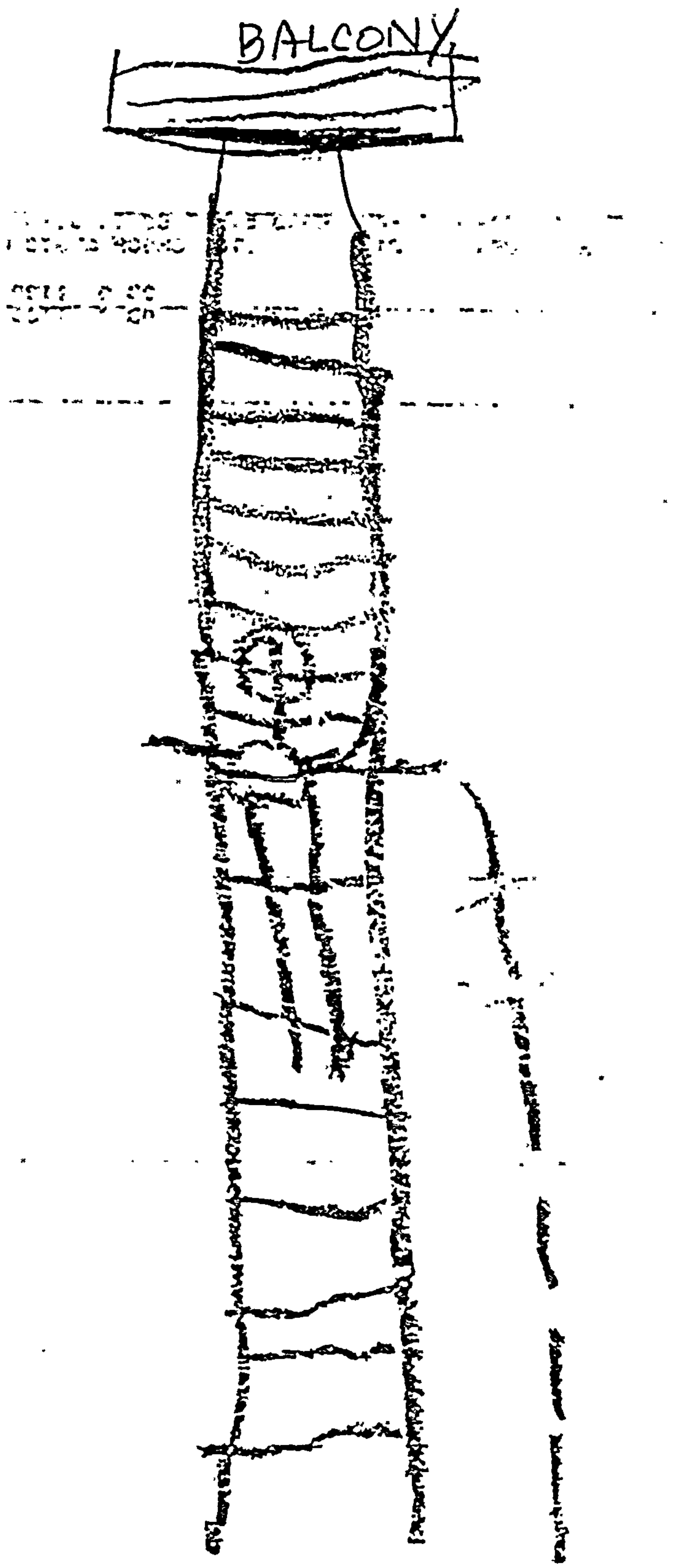

Figure 1.

ning enjoy an integrated set of count schemes which would form an adequate preparation for the introduction of place value notation. About two months earlier I had played the cup game with Christopher.

Chris returned from upstairs with exactly ten cups. He then chose exactly ten poker chips and counted them into my hand. He counted nuts successively into cups until all the cups had been lidded. I started to make a place for the lidded cups using three pipe cleaners. (This is unnecessary; I don't do this anymore.) Christopher interrupted me to ask, "Do you want me to get some tape to make a fri-angle?" I mention this because his question indicates that the triangular method of dealing with paper cups had been internalized by Christopher from the previous cup game experience. I had left the set of one hundred cups with Chris before. Using ten cups from this original set promoted, I believe, a natural sense of continuity.

(e) Christopher, Age 5

When Chris was five, we built the flip-card counter together. Chris had not yet entered school and had not yet been formally instructed regarding rules for place value notation. Chris took obvious pleasure in striving to master the two-handed coordination which makes possible simultaneous display of the place value numeral "10" as the count of "ten!" is enunciated.

(f) Concluding Remarks

Perhaps these anecdotal records will pique the interest of some educators and concerned researchers. I hope so. It seems to me that these anecdotal records are preliminary findings which indicate that the child's-play method has practical as well as theoretical value. The child's-play constuctions are a viable means of addressing the problem of integration; they could be the means of eliminating the gap between school children's informal knowledge and formal written procedures which require children to apply or interpret place value notation, base ten.

5. The Minimal Cost of the Child'-play Materials

What does it cost to implement the child's-play method? I have spent about thirty-five dollars on the materials I have assembled to play these games with children. Following is a list of my expenditures:

1000 paper cups	$12.90
masking tape	1.50
clear plastic for sheaths	3.50
piercing device	3.95
set of numeral stamps (0-9)	11.00
stamp pad	2.15
4 rings (I use chicken leg bands)	.20
paper for flip-card counter (advanced)	.10
100 tokens (I use hickory nuts)	free
10 poker chips (from the attic)	free
Total	$35.30

G. Acknowledgements and Suggestions for Future Research

The difficulty of communicating the place value notation concept for children's discovery has intrigued my interest and spurred my creative efforts for twenty-five years. My fascination with this concept began in graduate school when Dr. Harrison Geiselmann explained it briefly to my class. At that time "discovery learning" was in vogue, and I looked forward with great eagerness to helping my first class of third graders discover the place value notation concept. I was both stunned and fascinated by the problem this concept seemed to pose to my students.

Now I wish to express my grateful indebtedness to those who have helped me to resolve this enigma: Dr. Hermine Sinclair, the Geneva research team, Dr. Rochelle Gelman, Dr. Herbert Ginsburg and those whose names and writings are listed in the statement of references. Of course, I wish to thank also my many students and particularly the children whose names appear in this article: Ronnie and his brothers, Amar, Amit, Christopher and Laurie (about whom I will write another time).

It would be useful, I think, to gather data which might answer the following questions:

1. On standardized test items involving place value notation, do children taught by the child's-play method score higher than children not taught by the child's-play method?

2. Do first graders, who have had experience with the cup game, the ten-count game, and the flip-card counter (primary version), score higher on teacher-made or standardized tests than children who have not had experience with the child's-play constructions?

3. How do the scores of deaf children compare with those of hearing children when: (a) the deaf children have done all (or some) of the constructions and the hearing children have done none; (b) when the hearing children have also done the same constructions as the deaf children?

4. If the number towers model and the flip-card counter are introduced in the upper grades, is there a significant difference in performance on standardized test items before and after the child's-play construction experiences?

5. Three of the child's-play constructions are easily applicable to bases other than base ten (the cup game, the number towers model, and the flip-card counter). After mastering construction procedures for building the number towers model and manipulations of the flip-card counter, are upper grade students (independently or with teacher guidance) able to apply the same strategies to illustrate other bases and create, for example, a base five number towers model and a base five flip-card counter?

6. Do children who have used the child's-play constructions score higher on test items (teacher-made or researcher-designed) concerning bases other than ten than children who have not experienced the child's-play constructions?

7. Are there any noticeable attitudinal or behavioral changes evident during or following work with the child's-play constructions which might indicate a lessening of math anxiety or math avoidant behavior?

8. Are children who have experienced the child's-play con-constructions better able to explain place value notation than children who have not experienced the child's-play method? Does experience with the child's-play constructions make a difference in how many children achieve Ginsburg's Stage III level of understanding written number? (See pages 85-90 of Children's Arithmetic)

I would welcome opportunity to cooperate with researchers and/or schools who may wish to gather data to answer these questions or to test any of the claims or conjectures which I have presented in this series of articles.

H. References

1. Gelman, R. and Gallistel, C. R. The Child's Understanding of Number. Cambridge: Harvard University Press, 1978, p. 9.

2. Ginsburg, Herbert. Children's Arithmetic. New York: D. Van Nostrand Co. 1977, P. 129.

3. Gelman, R. and Gallistel, C. R. The Child's Understanding of Number. P. 8.

4. Ginsburg, Herbert. Children's Arithmetic. P. 79-82.

5. Piaget, Jean. Developments in Mathematical Education. Second International Congress on Mathematical Education. ed. A. G. Howson. New York: Cambridge University Press, 1972.

Descartes, Rene. Rules for the Direction of the Mind. Translated by Laurence J. LaFleur. Indianapolis and New York: The Bobs-Merrill Co., 1961.

Forman, George E. and Kuschner, David S. The Child's Construction of Knowledge: Paget for Teaching Children. Monterey Calif.: Brooks/Cole (A Division of Wadsworth Publishing Co., Inc.), 1977.

Gibson, James J. The Senses Considered as Perceptual Systems. Boston: Houghton Mifflin Co., 1964.

Ginsburg, H. and Opper S. Piaget's Theory of Intellectual Development. Englewood Cliffs, N. J.: Prentice Hall Inc., 1979.

Piaget, Jean. The Child's Concept of Number. New York: The Norton Co., 1965.

Piaget, Jean. The Construction of Reality in the Child. New York: Ballentine Books, 1954.

Piaget, Jean. Genetic Epistemology. Translated by Eleanor Duckworth. New York: W. W. Norton Co., Inc. 1970.

Wren, F. Lynwood. Basic Mathematical Concepts. New York: McGraw-Hill Book Co, 1965.