DOCUMENT RESUME

ED 290 626 SE 048 857

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TITLE	Pair Problem-Solving and Metacognition in Remedial College Mathematics.
SPONS AGENCY	EXXON Education Foundation, New York, N.Y.
PUB DATE	Oct 85
NOTE	15p.
PUB TYPE	Reports - Evaluative/Feasibility (142) -- Reports - Descriptive (141)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	*College Mathematics; College Students; Group Activities; Higher Education; Interviews; Mathematics Curriculum; Mathematics Education; *Mathematics Instruction; *Metacognition; *Problem Solving; *Remedial Mathematics; *Teamwork
IDENTIFIERS	Mathematics Education Research

ABSTRACT

This paper presents the thesis that pair problem-solving serves as an aid to students in developing their metacognitive skills. Metacognition has been defined as knowledge and cognition about cognitive objects and as any kind of monitoring. The ability to monitor one's thoughts and actions is greatly facilitated through the pair problem-solving method. Briefly, the method calls for one student to listen to another student solve a problem aloud. The listener may ask questions so as to understand each step and all the reasonings of the solver's solution. Modeled after the clinical interviews of Piaget, students learn self-monitoring skills while learning math concepts. Examples of verbal protocols from college freshmen illustrate the emergence of metacognition during clinical interviews and during pair and group problem-solving in class. Finally, a description of a remedial math program at a major university provides practical applications of metacognition in the classroom. (Author/PK)

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Research for this paper was supported in part by a grant from the Exxon Education Foundation.

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PAIR PROBLEM-SOLVING AND METACOGNITION IN REMEDIAL COLLEGE MATHEMATICS

ABSTRACT

This paper presents the thesis that pair problem-solving serves as an aid to students in developing their metacognitive skills. Flavell defines metacognition "as knowledge and cognition about cognitive objects" and as "any kind of monitoring" (Flavell, 1980). The ability to monitor one's thoughts and actions is greatly facilitated through the pair problem-solving method of Whimbey and Lochhead (1979). Briefly, the method calls for one student to listen to another student solve a problem aloud. The listener may ask questions so as to understand each step and all the reasonings of the solver's solution. Modelled after the clinical interviews of Piaget, students learn self-monitoring skills while learning math concepts. Examples of verbal protocols from college freshmen illustrate the emergence of metacognition during clinical interviews and during pair and group problem-solving in class. Finally, a description of a remedial math program at a major university will recommend practical applications of metacognition in the classroom.

Metacognition

Very broadly speaking, metacognition may be defined as knowledge about anything cognitive. Reflection about one's own thoughts must be viewed as an absolutely essential feature of human cognition generally. Anytime that a person realizes that they made a mistake they are reflecting on a prior thought and so engage in metacognitive thinking. Indeed, we often make judgements as to the thoughtfulness of another person. As children, most of us were taught to think before we speak. As adults our ability to manage our lives depends largely on our reflective capacities and on our willingness to critically evaluate beliefs, events past and future, and our intentions. Most importantly, we are expected to learn from our mistakes.* One of the first researchers to elucidate and disseminate the concept of metacognition is Flavell who defined the term in 1976: Metacognition refers, among other

* Although the language used in this paper is current to within the past ten years the concepts may be traced far back in human history. One interpretation of "The Fall" in Genesis may be that the allegory of the eating of the fruit of the "Tree of Good and Evil" is an ancient testimony to early man's recognition that wisdom comes from self-reflection. Without reflection their is no sin, no shame and no ethics. The penalty inflicted on Adam and Eve was not simply the pain and suffering of living in the world, for animals also experience the discomforture of life. The point of Genesis is that what distinguishes humanity from the rest of creation is that we reflect on our lives so that we know our pain, we know our suffering and we reflect on our thoughts and on our actions. The authors of Genesis chose to document our most significant metacognition, our awareness of our own mortality. Since Adam and Eve were banished from the Garden before eating from the "Tree of Life" we can only infer that they were mortal but did not know it until they ate from the "Tree of Knowledge" whereupon they learn "for dust thou art, and unto dust shalt thou return." Only upon reflection do we come to know our own mortality. To the ancient Hebrews metacognition was not only a human characteristic, it was also a feature of divinity: And the Lord God said, "Behold, the man is become as one of us, to know good and evil..."

things, to the active monitoring and consequent regulation and orchestration of those processes in relation to the cognitive objects or data on which they bear, usually in the service of some concrete goal or object. (p.232)

The main features of this very broad definition of metacognition are the acts of monitoring and regulating our development and selection of cognitive objects. In the information processing and artificial intelligence lingo this ability is referred to as the "executive." The concrete goal of interest is problem-solving which according to Brown (1978) is facilitated by the executive in the following manner:

> (1) predict the system's capacity limitations; (2) be aware of its repetoire of heuristic routines and their appropriate domain of utility; (3) identify and characterize the problem at hand; (4) plan and schedule appropriate problem-solving activities; (5) monitor and supervise the effectiveness of those routines it calls into service; and (6) dynamically evaluate these operations in the face of success or failure so that termination of strategic activities can be strategically timed. These forms of executive decision making are perhaps the crux of efficient problem solving because the use of an appropriate piece of knowledge or routine to obtain that knowledge at the right time and in the right place is the essence of knowledge. (p.182)

I would add that the process of identifying and characterizing the problem and generally all of these metacognitive skills occur most effectively in the context of discussion. The communication of ideas is a sure attempt at presenting a problem or concept to another but more importantly it is the representation of an idea to the thinker him/herself. Metaphorically, in order to see a reflection one must first be able to see. Similarly, for someone to reflect on their thoughts they must first make their thoughts manifest--that is, they must represent their thoughts, usually with words, but also with pictures, diagrams, equations, graphs, music, art, facial and bodily expressions, etc. For most people language is the preferred and most readily accessible means of expression. No sooner than we begin to describe our ideas do we evaluate them, alter or elaborate them and effectively monitor ourselves with a new awareness; the self-consiousness which asks questions like: Does my argument make sense? How can I say what I mean? Will this particular method of solution help me? Is this problem clear? and many other such questions which cause us to reflect and to analyze what we think. Even when the listener is passive and non-participatory the speaker will engage in metacognitive activity that will reveal and enhance his own problem-solving strategies and understanding. This is most evident in the clinical interviews of developmental cognitive psychologists who follow that research methodology pioneered by Piaget. An analysis of a transcript of such an interview illustrates the emergence not only of metacognitive thought but also of metacognitive theory in the solution of an algebra problem by an undergraduate.

Metacognition in the Clinical Interview

The following transcript provides us insight into the self-monitoring activities of an undergraduate engineering major from an introductory computer programming course. As in most clinical interviews for cognitive research the interviewer is a passive listener attempting not to influence his subject's thought processes but to record them with as little interference as possible. These are not the thoughts of a problem-solver working quietly by himself. Instead, this is a subject aware of the object of the interview and responsive to queries for more information. As such it provides an excellent example of the types of metacognition that can surface in an on-the-spot communication of a problem solution.

1 I: If you could write a program to represent that statement, uh, using the letters I guess -- C and E -- just read the statement out loud.

2 S: Ok -- there are eight times as many people in China as there are in England, um, the program would, um -- so the equation (writes 6C = then puts an "X" next to it). (Writes 8C = E) Number this one -- (Puts 2 next to 8C = E and 1 next to 6C =). Ok? Um, the same -- the program structure is exactly the same as the last one -- um --(pause) (Draws brackets and writes:) Header

```
Decl
Statements

READ
```

It appears as though writing an equation representing the given relationship is a warm-up heuristic prior to actually writing the program. Even this strategy is subject to inspection and correction as the student changes his initial expression "6C =" to "8C = E." He continues with the realization that the problem is not well defined in that he does not know which is the input and which is the output variable.

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S: There's a part of the problem that is not stated here in the sense that, we should just realize that, if you are not given whether you are gonna input the number of people in England or the people in China, Ok, so what I would do is then write a program which would deal with both.

3 I: Well, let's just do one -- why don't we say that we will input the number of people in China, Ok.

4 S: (Writes:)

```
READ (C)
E = C/8.0
WRITE (E)
STOP
END
```

5 I: Ok, and how did you know how to write each of those lines?

6 S: Um, this one [points to READ (C)] I know you have to input given the factor of an eighth used for the number of people in China, um, we have to calculate E, um (pause) -- I realize I made a mistake in the equation, um --

7 I: What are you looking at?

8 S: This one is wrong, um [points to 8C = E (Eq. 2)] -- I should --

Notice that the student confidently writes a program that contains the correct but reversed equation from the the one he originally wrote. Not until he verbalized his solution did he become aware of his error. Further requests for cognitive information from the interviewer will prompt the student to describe his thoughts retrospectively.

8 I: How can you tell it's wrong? What did you just think of there?

9 S: Well, I realized that I wrote it right in the program and it's different than the one I wrote up there, so that I would read, oh, I would change it. (Puts an X next to 8C = E.)

10 I: Ok -- that's interesting -- what convinces you that the program is right?

11 S: The fact that I know there's more people in China than there are in England and in the equation the E would end up being 8 times greater than the C, which is not true, Ok. [Writes E = C/8]

12 I: ...in the second line of the program, what were you thinking in order to write the second line there, when you wrote it? (Pause) Do you remember?

13 S: Just that E had to be a smaller number than C.

The subject has identified the key qualitative understanding which made his solution possible--namely that there are more people in China than there are in England. He next affirms his qualitative understanding and attempts to unravel the reasons for his mistake by formulating a theory as to why "somebody would make that error."

14 I: You're pretty sure that's what you were thinking when you wrote it? Yeah? Ok.

15 S: In fact I think that this (points to 8C = E) was in my head in that form (points to E = C/8) and it just got written down that way -- wrong -- I don't think I ever had it conceptually that E was bigger than C -- just that it got written wrong because I didn't even think about re-writing it (points to line 2 in program) -- I just thought of the way to write it, yeah.

15 I: So when you first read the problem before you wrote equation 2 there, you immediately realized there were more people in China? (Nods) But this is confusing -- a lot of people do this, that's why we're interested -- but when you write down 8C = E -- what do you think you are working from there, um, when you make that error?

16 S: Hmm -- I don't know why somebody would make that error, um, in terms of -- except that maybe, you're thinking like, um -- you are conceptualizing that C is 8 times larger than E, um, and so you associate the 8 and the C somehow in your mind perhaps, but, ok, I think the knowledge that C is 8 times larger than E is like, I didn't have any trouble conceptualizing that, it's just getting it written down accurately, right.

The student's language betrays his embarrassment at having made an error but as evidence in the course of the protocol he has little to be ashamed of. He exhibited many metacognitive skills: interpretation of the problem and the relationships within, selection of heuristics, re-examination of previous work, resolution of conflicting ideas, further qualitative assessment of the problem to check his solution, and an explanation of his error in the form of a general metacognitive theory about errors of that type (which was not unlike our own theories). (Clement, Lochhead, Soloway, 1980)

We will never know how much metacognition would have occurred had the student not been questioned in the interview. Perhaps he would have written the initial wrong equation and then the correct program without ever recognizing the descrepancy or becoming aware of his thought processes. However, we can be certain that his verbalization provided numerous insights to him and to us.

Mathematical Understanding Though Pair Problem-Solving

I would like to begin this section by agreeing with Schoenfeld's (1985) three assertations relevant to mathematical understanding:

> 1) Metacognitive skills and a "mathematical epistemology" are essential components of competent mathematical performance.
>
> 2) Most students do not develop very many metacognitive skills or a mathematical epistemology to any degree, largely because mathematical instruction focuses almost exclusively on mastery of facts and procedures rather than "understanding;" these are basic causes of students' mathematical difficulties.
>
> 3) It is possible, although difficult to develop such skills in students. (in Silver, p. 361)

As Schoenfeld correctly points out, most students haven't the slightest notion as to what mathematics is about. For almost all of our students in remedial math finding the "right formula" is a preconception which borders on obsession. They have no idea how such "formulas" are arrived at and consider most problem solutions as "tricks." That their former education has promulgated these notions is unfortunate indeed. The only way for these students to discover mathematics is to engage in math problem solving while monitoring their thoughts so as to understand each step in their solution--the steps back as well as the steps forward.

The only statement with which I take issue is that regarding the difficulty of the task of developing metacognitive skills in our students. The pair problem-solving method of Whimbey and Lochhead (1979) is developed to accomplish this task almost naturally provided the instructor is well trained. The task is not for the teacher to develop the skills in our students but rather for the students to develop metacognitive skills for themselves. According to the methods of Whimbey and Lochhead, the best teacher is the one who doesn't teach. Instead the teacher models the role of a clinical interviewer so that the students see what is expected of them when they engage in pair problem solving.

This type of instruction, or rather non-instruction, requires the students to learn to become clinical interviewers themselves. They work in pairs--one student solves a problem while the other listens carefully asking for the other's thoughts, reasonings, and elucidations until they both follow clearly what has been said and solved. They then exchange roles and work another problem. Obviously the problems used play a crucial role in this interchange. They are word problems chosen to challenge the students without being too frustrating. They must be conceptual in that their solution will lead to an exploration of some math concept where success does not depend upon the application of rote algorithms.

In the remedial math course taught at the University of Massachusetts, Amherst, pair problem solving is the mode of instruction. Typically twenty percent of class time is allotted for the instructor to answer questions or lead brief discussions on some topic of the day. For the remainder of the class, students gather into pairs or sometimes groups of three or four to work on word problems aimed at conceptual understanding. One instructor and one undergraduate assistant circulate among the groups listening and asking questions but not answering them. This is a very important aspect of the classroom; the teacher must not be seen as the know-it-all, the authority, the repository and paymaster of truth--mathematical or otherwise. Math is problem solving, and it is problem solving without clear directions, ready-made formulas, truth from "on high" or answers in the back of the book. Doing math means mucking around looking for useful approaches to the problem testing and evaluating ideas and creating new ones. (Lakatos, 1976) Any worthwhile mathematical epistemology must include an appreciation for the full range of metacognitive thinking involved in the solution of problems using mathematics. To do mathematics requires thinking about one's thoughts, and by doing mathematics I mean understanding math concepts. For example, compare the level of understanding evidenced in the correct solution to each of the following two problems:

a) 3/4 of his income went towards paying for school and 1/2 of that was spent on tuition. What fraction of his income was spent on tuition?

b) $3/4 \times 1/2 = ?$

While it is useful to be able to multiply two fractions, it is far more useful to be able to solve problems like type 'a'. In fact, solving problems without the fraction multiplication algorithm may require an understanding of fractions far superior to what is required for rote multiplication. Obviously we'd like our students to be able to do both but the discussion generated in the group solutions of word problems often leads the students to recognize a multiplication-type problem if not to the discovery of the algorithm itself.

Most importantly, the students need to develop a mathematical epistemology which does not depend upon outside authority. To this end the instructor must be careful not to interfere in a way which may deprive the students of their discoveries. Rather, the instructor must alway ask the student to describe what they were thinkin t the problem, what the problem is asking, what the problem states, what pictures or diagrams can they draw, what their partners think, etc. Both the student and the instructor reflect on the ideas suggested by the student. If the instructor can restrain him/herself from working the problem, the student will usually succeed, perhaps after much frustration. Most of our students see the advantage to solving the problem themselves and feel compensated for their efforts with understanding.

It is far more difficult for a student to learn metacognitive skills from a teacher who shows them how to do a problem than from one who does not. For most teachers it is far more difficult to listen to the student and to ask helpful questions that are not so leading as to make the student feel like they haven igured the problem out themselves than to simply show-and-tell. We train all of our instructors to be clinical interviewers and classroom managers in addition to being good problem solvers with an understanding and appreciation of the complexities of what most people regard as the simplest of math concepts. Although we teach basic mathematics many of the problems we ask (and also solve) are difficult even for us. Monitoring our own thoughts, and each other's, builds our metacognitive skills and teaches us patience for our students.

It is also useful to record ideas on paper while solving a problem. We often require our students to write their thoughts as they thi them; then to read them and study their thought processes. We call such assignments "thought-process protocols", and we grade them without consideration of grammar or composition but mainly for thoroughness. They may be from 300 to 600 words and include pictures, diagrams, charts, or equations. Basically, anything goes so long as the protocol genuinely documents their thinking.

While we consider such assignments useful, there is still room for improvement and for more research into the benefits of the method.

Conclusion

The effectiveness of a remedial math program rests on the ability of the student to formulate a better understanding of the type of thinking that mathematics entails. They need to develop a mathematical epistemology that goes beyond the rote memorization of formulas matched to sample problem-types. Conceptual understanding must be seen as the goal of math instruction. This can be realized only through an articulated attempt to solve word problems.

The self-conscious reflection about one's own thoughts and ideas is the essence of metacognition. It is a necessary condition for effective problem solving, and it is facilitated through oral and written communication of thinking as-it-happens. The clinical interviews employed in the research of cognitive processes is perhaps the best model of metacognitive thinking available. Teaching students to work in pairs; reasoning aloud and interviewing each other so as to understand the thought processes of the problem solver, is one effective means to developing metacognition and conceptual understanding in mathematics. Instructors must also allow their students to explore and describe their own ideas and to convince themselves, their peers, and their teachers of the effectiveness of those ideas. Ultimately, the focus of instruction is on the process of gaining knowledge rather than on the objects of knowledge.

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