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ABSTRACT

This document is the third of three related volumes. They present the rationale, background, and framework for a comprehensive monitoring system being developed for the National Science Foundation. The system is being designed to gather information about the effects of national, state, and local policy actions designed to change the teaching and learning of mathematics in the schools of America. The papers included were produced by project staff, commissioned, or reprinted from previous works. Expert reviews and critiques of sets of papers are included. In this volume, papers are presented which address issues and concerns that grow out of the argument that the learning of mathematics happens in a social institution, the school, and that instruction is directed by an adult human, the teacher. The papers: (1) situate the mathematics reform movement in the social reality of schools; (2) examine the social myths surrounding the notions of curricular reform; and (3) consider teacher beliefs about professionalism and perceptions about mathematics. The final section summarizes the entire monograph, reflects on what has been said, and draws implications for future work. (PK)

Program Report 87-3

THE MONITORING OF SCHOOL MATHEMATICS: BACKGROUND PAPERS

Volume 3: Teaching and Future Directions

Edited by Thomas A. Romberg and Deborah M. Stewart

Report from the School Mathematics Monitoring Center

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Wisconsin Center for Education Research

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PREFACE

This set of papers, published in three volumes as a monograph of the School Mathematics Monitoring Center, presents the rationale, background, and framework for a comprehensive monitoring system being developed for the National Science Foundation. The system is being designed to gather information about the effects of national, state, and local policy actions designed to change the teaching and learning of mathematics in the schools of America.

To build the monitoring system three assumptions were made. First, as a society we are involved in a major economic revolution. This revolution, addressed in Chapter 2, directly affects mathematics, its use, and what is deemed fundamental. As a consequence we believe "that most students need to learn more, and often different, mathematics" (Romberg, 1984, p. xi). Second, in spite of the changes in school mathematics inherent in the first assumption, we believe that there is general concensus about the goals for school mathematics and about the kinds of changes needed to achieve those goals. Thus, to develop the framework for the system one must begin with an understanding of those goals and the ideas on which they are based. Only then can indicators be developed to see whether the goals are being reached. Third, the policy actions with respect to the specific goals set for school mathematics must be consistent with the more general educational goals for a free and democratic society.

The need to monitor changes in school mathematics was proposed at two conferences. The first was organized by the Conference Board of the Mathematical Sciences (the New Goals Conference, CBMS, 1984), and the second by the National Council of Teachers of Mathematics, the U.S. Department of Education, and the Wisconsin Center for Education Research (School Mathematics: Options for the 1990s, Romberg, 1984). One conclusion from both conferences was that information about the nature of proposed changes and their effects on schooling practices was needed. During the past 25 years the federal government has invested considerable funds to change the teaching and learning of mathematics in America's schools, and today it is in the process of funding several new projects. Unfortunately, evidence of the impact of past dollars on classroom instruction is lacking. The special evidence that exists was unsystematically gathered and is incomplete. As new monies are spent and programs developed, it is crucial that a systematic plan be adopted to gather information about the effects of these planned changes.

During the past year the staff of the Monitoring Center prepared a series of papers, commissioned additional papers, convinced some authors to allow us to reprint a paper they had recently prepared, and asked a few nationally recognized experts to review and critique sets of papers. In all we have collected some 30 papers that address the issues of a new world view, what is fundamental in mathematics, what implications recent research in psychology or sociology has for school mathematics, etc. The intent of gathering these papers was to assist the staff of the project in the design of a monitoring system for school mathematics. However, since they comprise a review of the current thinking about schooling by a number of noted educators, we have chosen to publish them in this three-volume monograph so that others may have access to this information.

The first volume addresses the need for a monitoring center, the new world view, and what is now considered a fundamental for students to know about mathematics. In the second volume the implications of psychology to the learning of mathematics is addressed, and the problems of assessing learning based on both the new mathematical fundamentals and our knowledge of learning is examined. The final volume is comprised of papers that are based on current sociological notions about schools and how that knowledge affects the role of teachers and instruction in classrooms.

SCHOOLING, TEACHERS, AND TEACHING

This section of the monograph includes five chapters. They address issues and concerns that grow out of the realization that the learning of mathematics happens in a social institution--the school--and that instruction is directed by an adult human--the teacher. Chapter 24, written by the educational sociologist Thomas Popkewitz, situates the mathematics reform movement in the social reality of schools. Chapter 25, prepared by Catherine Cornbleth, another sociologist, examines the social myths surrounding the notions of curricular reform. Two issues about teachers and their beliefs--professionalism and perceptions about mathematics--are examined by the Australian scholars Allan Pitman and John Conroy in chapters 26 and 27. The section concludes with an analysis of teaching derived from the new world view presented earlier. This chapter was prepared by the Center staff.

Chapter 24

INSTITUTIONAL ISSUES IN THE MONITORING OF SCHOOL MATHEMATICS

Thomas S. Popkewitz

How might the monitoring of educational reforms in mathematics be approached? What features of schooling and teaching should be given priority in the discussion of curriculum practices? One approach considers these questions from a critical sociological perspective. To participate in schools is to participate in a social context that contains standards of reason, rules of practice, and conceptions of knowledge. The problem of inquiry is to understand how subject matter is realized within schools' social arrangements and ideological configurations.

School curriculum is not immediately evident when we examine the daily flow of classroom events. Although we often assume there exists an objective, common curriculum for all students, the social contexts in which mathematics education is realized are not the same for all children. Children from different social, cultural, and economic backgrounds respond differently to school work. In addition, teachers bring their own cultural and social understandings to bear on the daily routines and practices of teaching. Occupational ideologies about curriculum design, classroom organization, and definitions of achievement infuse biases into the teaching of subject matter.

The problem of school reform can be viewed in relation to dilemmas, issues, and values in the social processes of schooling. In part, reform is a response to a contradiction between the hope of a common school and the happening of differentiation.[1] The language of reform is a potent symbol that directs attention to a transformation of existing social patterns in the face of social crisis and strain. Reform rituals and rhetoric create images of institutions as progressive and benevolent. The language of reform, however, needs to be juxtaposed to school practice in the everyday world. Ironically, reform activities may have little to do with change. More often than not, what is conceived of as change in mathematics may be limited to mere motion and activity; such change actually legitimizes existing institutional patterns

[1] The distinction between hope and happening is drawn from the essay by V. Lundgren, Between hope and happening, Geelong, Australia: Deakin University, 1983.

rather than altering the substantive values, knowledge, and effects of schooling.[2]

This chapter focuses on the contradictions between hope and happening. Researchers argue that the problems of quality, equity, and innovation in mathematics education cannot be considered in isolation from the institutional process of schooling. To understand this relation, the social conditions in which teaching, learning, and reform take place are explored. In particular, this essay will focus on three questions:

1--What social and cultural issues underlie the institutional patterns of schooling?
2--What are the assumptions and implications of curriculum languages for teaching mathematics?
3--What do we mean by change and reform? How do educational change and reform practices illuminate or obscure the social conditions in which school knowledge is produced?

These questions are posed in this manner to redirect attention to how we think about achievement, success and failure in mathematics education. There is nothing "natural" about the patterns of school work or the traditions and customs that we have come to associate with its institutional arrangements. Schools are socially constructed, sustained, and renewed through the actions of people. These constructions are never neutral: schooling is an imposition of certain patterns from the array of those possible at any one time. Consideration of the assumptions, implications, and consequences of mathematics education in its institutional context is an important element of the monitoring issue.

This essay draws on "critical sociology." The scholarship is European in origin, and it seeks to develop a method of inquiry that involves an interplay of sociology, social philosophy, and history. It is assumed that the doctrines and schemes for educational reconstruction are, in all instances, transformed once they enter the world of practice. Our noblest hopes about schooling must always be juxtaposed against the contradictions of our social institutions. The science and mathematics curriculum reforms of the late 1960s, for example, were never introduced into the schools in ways that were intended. Unanticipated consequences also occurred in the American comprehensive high school; it was designed to eliminate class bias, but new tracking and differentiation strategies developed to maintain inequities and discriminations existing in the larger society.

Consideration of unanticipated consequences involves more than understanding behavioral differences in the implementation of reforms, such as whether new programs are created or more students

[2]For discussion of the assumptions and implications of models for change, see Popkewitz, 1984, ch. 6.

take a course of study. Research is to examine the assumptions, implications and consequences of the ongoing social life of schooling. Instruction is always bounded within institutional structures that contain standards by which a society and its individuals are to judge themselves. Attention is given to how the content and methods of mathematics instruction are interpreted through the everyday language, common sense assumptions, and practices that organize the world of schooling.

1--What Social and Cultural Issues Underlie the Institutional Patterns of Schooling?[3]

I would like to pursue the problem of mathematics education by considering more directly the institutional context of schooling. To understand this context, we need to consider what is learned in schooling as more than mere subject matter. Form and content interrelate to maintain the assumptions and presuppositions by which students orient themselves to situations and make sense of their academic activities. Underlying classroom organization and curriculum standards are dispositions toward how one is to talk, think, see, and feel toward the world. These dispositions differ for teachers and students, and among all students. School mathematics, from this perspective, is not merely the incorporation of new practices, but involves the ier in which mathematics is realized in the social patterns of schooling.

How can we think further about the institutional rules that underline mathematics teaching? One approach is offered by Michel Foucault, a French social philosopher and historian. Foucault (1973) suggested that there are fundamental codes of culture that underlie a society at any one time. These codes govern the society's discourse, its exchanges. its techniques, its values, and the hierarchy of its practices. The codes become a "regime of truth." They shape and fashion what can be said and what must be left unsaid, the types of discourse accepted as true, and the mechanisms that make it possible to distinguish between truth and error.

Codes of culture are illuminated by an examination of discourse. In the realm of discourse, Foucault is interested in more than the rules and structure of grammar. Our signs, gestures, routines, and behaviors carry rules about what is to be considered normal, reasonable, and legitimate. Discourse sets for people the conditions by which events are interpreted and by which an individual is located in a dynamic world. Power relations are embedded in codes of culture, Foucault continued; the notion of power relates not to ownership, but to an understanding of changing social relations and innumerable vantage points from which power is exercised.

[3]For a more general discussion of the concept of institution and the problem of reform, see Popkewtiz, 1979, 1983.

Distinguishing between the surface and underlying meaning of schooling clarifies the importance of the institutional "codes" of schooling. The surface layer of meaning is provided by publicly accepted criteria or standards by which people judge success or failure. Writing a lesson objective, doing microteaching, or working in a team-teaching situation might provide such a public criterion. The underlying layer of meaning directs attention to the assumptions, presuppositions, and "rules of the game" that give plausibility and legitimacy to these criteria. That is, as students are involved in learning mathematical knowledge, they are also being taught appropriate forms in which to cast that knowledge.

The underlying layer of meaning is illustrated by a mathematics lesson observed in an American innercity elementary school (Popkewitz et al., 1982). The students were black, from families of the industrial poor and unemployed.[4] The public purpose of the lesson was to help students learn subtraction; the teacher wrote a lesson plan, constructed materials, and evaluated according to the previously stated objectives. The lesson was justified for different reasons: subtraction is an important element of a mathematics curriculum, and future lessons depend upon acquiring the presented knowledge. During the lesson, the teacher explained elements of subtraction, and students worked with textbooks and ditto sheets.

These surface meanings of the lesson developed in a context of institutional patterns, beliefs, and justifications that constitute the underlying meanings. Rules and procedures exist prior to the mathematics lesson. These rules provide direction to how we should think and act towards the ways children learn and develop, the nature of knowledge, and the patterns of the social order and control of schooling. That teachers do not follow "rational" procedures in teaching is well documented (McCutcheon, 1981). Only partially studied are the relationships of teacher practices to institutional rules and values. In the arithmetic lesson mentioned above, teaching was based on a "deficit" model of learning. Mathematics was viewed as having a fixed and unyielding definition of that with which teachers are to fill the minds of students, reflecting what Paulo Freire (1970) referred to as a "banking conception of education."

Subject matter, however, was only one part of lesson content; the lesson carried social messages that were as important as any cognitive considerations. The introduction to the lesson involved a discussion that focused on the children's academic failures. The discourse reflected the teacher's feeling that the welfare status of the children made it likely that they possessed undesirable "traits" that needed to be overcome before any achievement could be obtained. Much of the classroom interaction was related to the

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[4]The discussion is based on a composite of a variety of lessons that occurred over time.

teacher's belief in the cultural and personal pathology of the children rather than to any textbook notion of "learning."

In the arithmetic lesson, we not only find references to social circumstance but also one of the possible definitions of knowledge and childhood available in the processes of schooling. The lesson on arithmetic was based on a deficit model of learning. In other contexts, one might identify a constructive view, considering knowledge as emerging out of participation in community problem solving. Learning of arithmetic facts is not considered as an end in itself, but in relation to the processes of interaction and responsibility found in the classroom. A view different from both constructivist and deficit models is a social psychology model, in which a dialectical relation is posited among culture, social setting, and the development of the mind (Vygotsky, 1978).

From the different views of cognition emerge contrasting views of society and polity (Popkewitz, 1983c). To treat a child as having a deficit is to define knowledge atomistically; the individual is an essentially receptive, reflective organism whose qualities are shaped by the environment over time. The epistemology also entails a political theory: the individual is denied the role of actor in the creation of history and culture. Social life is defined as fixed and unyielding to intervention. In contrast, a focus on negotiation as a way to consider learning gives value to community and self as integrally related and mutually reinforcing. The idea of negotiation, however, is not neutral. It emphasizes an early twentieth-century liberal view of individuals who work collectively to improve their world. The liberation also has its own limitations: it posits a concern exclusively with the present, obscuring the place of history in fashioning consciousness itself. That is, it ignores the ways in which categories such as "learning" or "individualization" were socially constructed as particular social groups struggled to give definition to cultural, economic, and educational affairs (Braudel, 1980; Napoli, 1981).

Differing views of cognition and society do not exhaust the possibilities of the relationships among pedagogy, psychology, and political theory. They illustrate what may seem to be simple acts of classroom planning or management but which may, in fact, contain profound and complex principles of authority, legitimacy, and power relations.

We can begin to understand from the arithmetic lesson that the form and content of schooling are interrelated; they not only channel thought and action, but reinforce and legitimize social values about authority and control. While mathematics instruction often focuses upon the logical characteristics of the subject-matter content, it is the ideologies embedded in content selection and its realization in teaching that is the learning of schooling. The achievement of schooling is giving direction to social thought and the formation of intelligence both for those who succeed and for those who fail. The banking concept posits knowledge as external to individuals and controlled by those who have power to define and categorize social reality. The social interactions of classrooms reinforce that notion of power by suggesting that failure to learn is a personal, not institutional, failure.

The social patterns in which knowledge is formulated have important implications for the conduct of mathematics education. While mathematics is viewed as a universal language in which the logic of relations to an answer becomes paramount, the institutional processes of schooling compel us to consider the mathematics curriculum as a "social text." The text is read not as an abstraction of numbers but as a part of social processes in which the numbers are given meaning in relation to human practices. The sequences given to lessons, the examples used to explain a concept, and the social/psychological theories of children's growth embody epistemological and political theories about the nature and character of our world. Cultural studies of schooling indicate that mathematics programs are not merely added on to existing systems but become a part of the system that not only affects other variables but also acts on the curriculum itself (Donovan, 1983; Stephens, 1982).

INSTITUTIONAL DIFFERENTIATION IN SCHOOLING

The importance of social processes in defining the meaning and implications of school mathematics can be considered further by focusing on the rituals of homogeneity and differentiation in schooling. The organization of school mathematics implies that there is a unified, universal pattern of behavior and meaning that underlies experience. Everyone is expected to go to school, to be treated equally and objectively in learning school subject matter, and, if differentiation occurs, it is expected to be the result of merit rather than ascribed characteristics of individuals. In most parts of the world, pupils are taught mathematics and science in ways that suggest a homogeneity of practice and consensus of purpose. The problem of what is fundamental and what is not concerns emphasis of content or increased technology, such as calculators or computers, "to insure that every student becomes familiar with these important processes" (see, e.g., Conference Board of the Mathematical Sciences, 1982).

While the rituals and ceremonies of schooling create an illusion of homogeneity, the actual social transactions in schools represent differentiation in what is taught and learned (McLaren, 1986; DeLone 1979). Rather than one common type of school, there are different forms of schooling for different people. These different forms of schooling emphasize different ways of considering ideas, contain different social values, and maintain different principles of legitimacy and forms of control. Let me provide two examples, one historical, one contemporary. Each enables us to explore the issues of knowledge and social differentiation.

Mathematics in the Formation of the American School

The public rhetoric about mathematics education suggests that the organization of knowledge gives emphasis to universal values of learning. Yet when considered historically, the actual construction of curriculum has reference to different social values. Stanic's (1987b) discussion of the emergence of mathematics education in American schools at the turn of the century places two types of instruction as central to the debate about curriculum purpose and organization.[5] One concerned teaching children how to think and reason properly. This focus assumed that public school mathematics would provide the mental discipline and character appropriate for eventual leadership in social and economic institutions. The conception of mathematics education was elite and related to those who would go to college. A second curriculum orientation focused on functional requirements for those who would never go to college. Mathematics education was to provide practice for managing everyday life, such as using arithmetic for household budgets. For each type of instruction, research programs developed to justify and organize teaching, and in the process the issues of social differentiation were translated into scientific questions of individual development and learning.

The social importance of the different curriculum approaches has at least two dimensions. First, the differentiation in the work and knowledge of schooling represents different sensibilities and awareness necessary for access to positions of privilege and status in society. The social organization of schooling transmits the cultural and social awareness appropriate for a society that has different roles, status positions, and occupational tasks. Blue-collar workers, shopkeepers, and scientists do not need, on the surface, the same knowledge or sensitivities to perform successfully in society.

Second, the differentiation in schools represents larger strains and struggles that not only reproduce culture but are dynamic elements within social structure. While knowledge differentiation may seem functional in society, the actual organization of school work may involve interests and dispositions in conflict with the functional requisites of the larger system. The civil rights movement of the 1960s and the feminist movement of the past decade imposed pressures not only on the teaching of mathematics but also on the ways in which mathematics was to be valued in the hierarchy of forms of human knowledge to be transmitted in school (Gilligan, 1982). The definition of school knowledge, from this perspective, is not administrative, technical, or behavioral, but given as subject matter interrelating with social and political interests in schooling.

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[5] Also see Stanic, chapter 4 (1987a).

Mathematics Teaching as the
Social Organization of Knowledge

Differentiation in contemporary schooling provides a second example of how curriculum is rooted in issues of social and cultural production. The issue of differentiation is reviewed in a study of a reform program in elementary schools (Popkewitz et al., 1982). The program was to introduce a management system for individualizing instruction in American elementary schools. Six schools were studied in depth to understand how the technologies of the program were realized in an institutional context. Three patterns emerged for realizing the reform technologies; each contained different assumptions and implications for the work and knowledge of schooling.

If we focus on the mathematics and science taught as one aspect of life in school, we can illuminate the differences in the meaning of practice. Teachers in three of the six schools taught mathematics as a maze of facts, and science as a body of predefined tasks and facts. In another school using the same reform program, mathematics involved a personal playfulness with numbers, and science involved a tentativeness and skepticism towards the phenomena of the world. The first three schools we called technical, the latter constructive.

In a third category of schooling, called illusory, there were regular periods of instruction, textbooks, and activities to symbolize content instruction. Examination of the social patterns of conduct, however, revealed an emphasis on the rituals of teaching and learning without much follow through. While children sat at their desks, they were taught little or no mathematics. The forms of school work and knowledge had purposes other than teaching a subject's content. The discourse in which teaching and learning took place established a moral basis by which children could be socialized to act, juxtaposing the perceived pathologies of the children's community life (unemployed, uneducated parents; single parent) with the teacher's image of correct values. The rituals of illusory schooling established the importance of the categories of school knowledge that students needed to learn for later success, but at the same time provided little or no teaching in the subject matter.

While institutional conditions worked against student success, student failure was defined as personal, the result of inadequacies of personality and community. The ongoing activities, materials, and physical arrangement of the schools conveyed institutional regularity, competence, and benevolence, much of this legitimacy established through incorporation of a language of bureaucracy.

THE SOCIAL PREDICAMENT OF SCHOOLING

The institutional differences in the work and knowledge among schools involves not "good" or "bad" teaching or administration but a complex set of dynamics related to the social predicaments of schooling. Schools exist in societies that are increasingly differentiated in occupational skills, cultural sensitivities, and social awarenesses. The differentiation compels educators to consider certain demands on the school mandate. At one level, educators are asked to respond to a variety of social and cultural issues, ranging from family background problems of drugs, child abuse, and teenage pregnancy. School curricula also are to solve material problems by giving priority to certain economic/cultural forms, such as teaching scientific and technological knowledge and rational thought (see National Science Board Commission on Precollegiate Education, 1983).

The social predicament emerges when we consider that schooling is to respond to social issues resulting from factors over which educators have no control. Further, science and mathematics education must confront the social realities that the sensitivities and awareness associated with scientific knowledge are not readily accessible to all students. The preferred form of mathematics education continues to be those notions drawn from elite strata of society. The "new" science and mathematics of the American curriculum reforms of the 1960s and the computer literacy of the recent reform reports emphasize a tentativeness toward ideas and the development of interpersonal skills (Popkewitz et al., 1986). This code of discourse, however, is not universal or natural to all in society. Mathematics and science are based on a linguistic structure that places value on the "problematic" quality of knowledge, emphasizing individual autonomy and responsibility through the use of words and mental images.[6]

The style of communication in science and mathematics, while different in discipline orientation, responds to the social patterns found in the professional strata of society, where work depends on the ability to play with words and communication (Bernstein, 1977). Alvin Gouldner (1979), a sociologist, has pointed to contradictions of scientific knowledge that emerge from the social location. On one hand, the discipline-centered knowledge may lead to greater understanding. It may also serve processes of social production for the professional strata in contemporary society.

Responses to the Social Predicament

To return to an earlier example, the different institutional patterns of technical, constructive, and illusory schooling can clarify social predicaments of schooling. The behavior, language and beliefs that characterize the classrooms studied were related to larger social conditions and tensions that filtered into classroom practices. There was a clear relationship between the emphasis on personal autonomy and a problematic view of knowledge

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[6]An interesting discussion of the social conditions of being a mathematician; see Ulam, 1976.

found in the constructive school and the professional occupations of the parents in the school community. Illusory schooling is a response, in part, to teachers' perceptions that the requisite dispositions for schooling were lacking in the children who came from the poor communities. Two of the three technical schools were populated by students from families of blue-collar or service occupations. Teachers' and administrators' perceptions of the social and economic location of communities were used to define a school mandate to teach the functional skills necessary for the anticipated later occupations of the pupils. A third technical school involved a different social situation. Located in an affluent business community where a single church dominated the social, cultural and political infrastructure of the community, the hierarchical style of work and principles of authority related social and cultural beliefs with religious ones.

The social predicament of the schools also reflected certain dynamics of American life. The constructive school involved conflict between the teachers' perceived mandate from the specific community of the school and the district's administrative efforts to introduce consistency and standardization in textbooks and procedures across all schools in the district. Stressing the importance of standardization and efficiency is one approach professionals use to create symbols of competence and status. The district-wide efforts to standardize restricted the options available within the school and brought open debate about the role of a professional teacher. The social conditions in which the illusory school and one technical school operated also illustrate the social predicament.

One of the technical schools had many characteristics similar to those of the illusory schools, but differences in its institutional rules can be understood in relation to geographical and historical considerations. The technical school was located in a southern American rural community whose people (white and black) had lived there for many generations. This history produced a feeling of community obligation and responsibility and a familiarity between the people in the community and the staff. The illusory schools, in contrast, were located in urban neighborhoods where there was no sense of history or community. A sense of intergenerational caring and communal obligation found in the community of the technical school was lacking in the illusory schools.

Elements of the social predicament do not emerge as carefully articulated arguments or forcefully documented concerns. Cultural and professional expectations provide background assumptions for schooling and are absorbed into the discourse and practices of the school in a variety of ways. Classroom practice reflects both school traditions and sociocultural values, although they do not always coincide and can involve conflict. Programs are interpreted, modified, and used in relation to professional ideologies with traditions of schooling that make the ongoing patterns seem natural and benign.

The social predicament of the school also involves school traditions that give symbolic coherence and reasonableness to school practices. The public belief that social institutions should be rationally organized, for example, has led to the development of administrative theories about school organization, curriculum, and evaluation. The administrative theories give rise to the development of technologies such as behavioral objectives, criterion-referenced measures, and competency testing. The theories and technologies posit a way of acting that defines the scope and possibility of schooling. In many instances, the curriculum defines experience in science and mathematical knowledge as crystallized products of research rather than forms of inquiry. The activity of science becomes crystallized, defined as a fixed entity to be taught through direct instruction.

Mathematics as a Category of Schooling

Within this context of social predicament, we can return to the question of the social codes that underlie the teaching of mathematics.[7] At least three dynamics of the institutional pattern of school that have little to do with conventional definitions of learning give focus to the subject matter.

First, mathematics instruction gives symbolic reference to the scientific and technological base of society. Mathematics can be viewed as representing the hope and challenge of an industrial and communication-based society. Its cognitive character signifies enlightenment, in which a rational and scientifically based society will bring progress in a material and social world. The enlightenment belief introduces a second and related dynamic to mathematics' status as a preferred category of understanding: mathematics is to be recognized as of value even for those who do not take the course. The curriculum category establishes legitimacy for those experts who have acquired the knowledge, modes of interpretation, and occupations in positions in which mathematical knowledge is made a part of a professional mandate. In part, the presence of the curriculum category carries the status differentials, social divisions, and hierarchies found in the work in society.

A third social implication is the dual quality of mathematics in the construction of reality. Mathematics can enable us to understand relationships and guide interpretations in ways not available in other discourses. In this sense, it provides a form

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[7] I must again emphasize that my focus is not on the internal structure of mathematical knowledge. Rather, the concern is about the manner in which mathematics becomes part of a public discourse about knowledge. The latter focus gives attention to how disciplinary knowledge enters into public institutions in a manner that has different implications from those intended in the formal discussions of educational purpose and goals.

of knowledge that transcends our immediate situation and experiences. But the language of mathematics can also obscure and mystify our social conditions. It can refocus attention on our world in a way that deflects attention from how social patterns are humanly constructed. As mathematics is used to explain political elections, profit debit, budgets, demography, and so on, the numbers become a reality in and of themselves. The historical manner in which people create institutions is buried in a presentation of knowledge that seems to express only the relationships between numbers. Social practices are made to seem beyond human intervention and individual agency. As a result, subjects become objects, and purpose and will become irrelevant to the constructions of social life.

The possibly contradictory meaning of mathematics has little to do with the internal logic of the discipline and more with the social uses of knowledge in a complex and differentiated society. The social function of mathematics is a general public issue of science and the secularization of our world.

2--What are the Assumptions and Implications of Curriculum Languages for the Work and Knowledge of Disciplined Thought?

The issue of the language of mathematics deflecting attention away from the constructed quality of social reality is a central issue in the construction of school curriculum. Theories of pedagogy refocus the dynamic and communal qualities of mathematical knowledge through linguistic inventions that make that knowledge seem objective and natural. Knowledge in schooling is conceptualized as specific qualities of learning, steps or stages of scientific problem solving or formal mathematical equations or concepts. The focus on logical or psychological qualities obscures the interplay of the communal/craft quality of mathematics. The conduct of science involves social patterns. Interwoven with personal skills and individual creativity are community patterns and norms that provide direction and self-correcting mechanisms for the generation of knowledge. The social and personal dimensions of science are lost in curriculum design. Disciplinary structures are made to seem to have no historical context or social interest (Popkewitz, 1977). The practices of inquiry appear as logical stages or natural processes of development (see Popkewitz, 1983b).

To consider how mathematical knowledge is expressed in pedagogical theory and practice, an analogy to certain elements of scientific knowledge can be explored. While recognizing that there are substantive differences in the way mathematicians and scientists work (Hagstrom, 1965), the sociology of science can illuminate certain elements that need attention in considering disciplinary practices. Five observations about science are relevant to the problem of curriculum design.

First, there is not a single method of inquiry, but many methods that scientists create as they confront the problems of their disciplines. Methods are influenced by communal standards, craft skills, and imagination. Sociologists and science historians have continually focused on science and mathematics as methods of inquiry that produce knowledge. The methods of inquiry are not carefully laid out prescriptions for action, but an interplay of orientations, dispositions, and conceptual lenses that combine to give direction to knowledge production. Some sociologists have argued that scientific creativity and imagination are best understood in relation to communal standards of recognition that bestow objective validity upon the particular results of research.

Second, the concepts of science are both answers and questions. Concepts are answers in that the categories create boundaries by which scientists are to think about phenomena. But they are also question-provoking words, suggesting that there are unknowns, mysteries, and ambiguities in the world that need exploration. The heuristic sense of concepts is central to the development of scientific and mathematical knowledge.

Third, many concepts are the subject of constant debate and exploration. For example, the relation of order and randomness is an element of current debate, having implications for the nature of geometric form and causality in science (Crutchfeld et al., 1986). At the cutting edge of science and mathematics is a conceptual playfulness and a competition among colleagues to generate knowledge. Such playfulness, skepticism, and competition are central dynamics of inquiry.

Fourth, concepts are at once affective and cognitive expressions about the world. Concepts contain root assumptions, such as that our social affairs work as a machine, an organism, or dialectically. These values are reflected in the words of biology, sociology, and educational psychology. Social science concepts, in particular, develop as responses to issues of social transformations and form part of a political agenda to respond to change. (For an analysis of this issue, see Popkewitz, 1984). Some concepts make the world seem harmonious and stable, giving legitimacy to existing social institutions; other concepts focus on contradictions and flux, attending to the dynamics that underlie the tensions of change and continuity. While mathematics concepts may not have the same metaphoric quality as those of science and social science, concepts in mathematics do presuppose relations and causal networks; mathematical models tend to emphasize linear, rather than dialectical, relations. Further, the concepts and relations expressed in mathematics are often used to explore human problems, thus providing a horizon by which possibilities are to be framed.

Fifth, science and mathematics have both internal and external influences on knowledge growth. While school history textbooks focus on the accumulation of "facts" and technological development as a reason for the importance of contemporary science, external factors such as industrial growth and demands of state have produced "epistemic drift" (Elzinga, 1985). The current industrial and military uses of computers and the industrial funding of genetic engineering has influenced the problems and theory development in these fields.[8] Many of the current reforms in science, technology, and mathematics contain practices that have a narrow utilitarian conception of the disciplines (Ralston, 1986). One result of the reforms is a concern with excellence rather than with the relationship of excellence to equity (Dickson, 1984; Popkewitz & Pitman, 1986).

In contrast, the language of curriculum tends to transform the social, cultural, and political into a crystallized form. In part, the concepts of schooling are treated as objects with fixed parameters for children to internalize. Often, concepts are "things" to be mastered. Yet, concepts are neither fixed nor neutral to our affairs. The manner in which disciplined knowledge is brought into school, however, gives emphasis to consensus and stability. A result is a crystallization of the knowledge, methods, and values of science (Popkewitz, 1977b, 1983b).

The decontextualization of interest can be found at a variety of layers of educational discourse. The structure-of-knowledge argument in curriculum, for example, defines concepts and generalizations as "things" to learn. Concepts are treated as objects whose definitions children are to internalize. The resulting curriculum emphasizes moral and political values under the guise of teaching science and social sciences, while simultaneously asserting a technical neutrality of science (Popkewitz, 1983b). The state of flux surrounding concepts and the debate among competing paradigms of inquiry are ignored.[9] In many ways, the current interest in constructivist psychology maintains an uneasy alliance with behaviorist notions of knowledge: the content of learning is fixed, the processes of discovery defined by constructivist notions concerning various routes to mastery.[10]

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[8]Internal disciplinary values also influence careers in science. These may refer to the value of "basic" vs. "applied" work. Disciplinary values may conflict with the external pressures that move research in certain directions. While mathematicians tend to claim that their discipline is unrelated to practical problems, the current concern for "discrete" mathematics illustrates elements of the field that express interest in utility, having influence on the standards and career developments (Ralston, 1986).

[9]See Popkewitz (1977); Sloan (1983) discussed how technical notions of science become part of our political and educational discourse. Curriculum makes arguments of armament seem "merely" those of efficiency and effectiveness rather than of ethnical and moral issues.

[10]Cobb (1986) discussed the epistological difficulties of constructivist traditions in mathematics education.

Decontextualized discourse is apparent in the general discussion of reform. School reforms focus on the individual's ability to learn, instructional changes that increase learning effectiveness, and standards for evaluation that maintain the assumptions of possessive individualism. Children are to "learn" more or better, teachers are to be held "accountable," and so on. When competency in science or mathematics is discussed, its meaning is related to a narrow and restricted range of experience tied to testing. Teacher education reforms also express theories that assume a universality of experience. Reform is structured as better communication between university and school, more practical experience, and improving the relation between theory and practice. Discourse focuses on procedures, administration, and efficiency; it is assumed that such discourse involves no power arrangements or interest.

It is within this problem of decontextualization of knowledge that we can explore a social function of psychology in pedagogical inquiry. The use of psychology in education suggests teaching is objective and technical; evaluation is based on efficiency.

While Americans have assigned priority to psychology as a discipline of curriculum and evaluation, the practices of inquiry are borrowed from a discipline created for purposes other than the understanding and transmission of mathematics and science. The history of American educational psychology involved the development of an academic discipline concerned with the successful adjustment of the individual to the environment (O'Donnell, 1985; Napoli 1981). American psychology had twin tasks: it was to help mitigate the crisis of religion as late nineteenth-century theology confronted evolutionary theory, and it was designed to disseminate and advance a practical knowledge in an emerging industrial nation. The utilitarian focus had little to do with science itself, leading one historian of psychology to conclude that by the early twentieth-century "psychology in general would flourish neither as a mental discipline nor as a research science but as the intellectual underpinning and scientific legitimator of utilitarian pursuits, especially in the field of education" (O'Donnell, 1985, p. 37). The practical concerns of psychology gave focus to a discourse about schooling that was functional in nature and objective in method, and that transformed moral, ethical, and cultural issues into problems of individual differences.

This argument about psychology is not about the discipline as a form of inquiry in educational practice. Rather, it is about the difficulties of borrowing methods of inquiry without considering their historical origins and epistemological implications for the problem at hand. It is also to argue that a curriculum methodology and monitoring approach about mathematics or science should focus on methods that create lenses for considering the communal/craft qualities of disciplinary knowledge and the institutional processes of schooling.

3--What Do We Mean by Change and Reform? How Do Educational Change and Reform Practices Illuminate or Obscure the Social Conditions in Which Schools Produce Knowledge?

In the previous discussion, I focused on certain dimensions of institutional life. Let me now proceed to the last question to be addressed. It can be rephrased in light of the previous discussion: To consider curriculum change, how might the institutional dynamics of schooling be considered? What notions of change illuminate the social complexities that inform the teaching of mathematics?

To focus on the issue of change or reform, we must return to two assumptions that frame the discussion: First, that which is defined as the curriculum of mathematics has more to do with the social history and imperatives of schooling than with the patterns of work, dispositions, and knowledge found in the scholarly community of mathematics. Second, the conduct of educational research is itself embedded in social contexts and contains values about knowledge and schooling. For example, the task of monitoring contains conceptions of progress as well as descriptions of current status. The prescriptive/descriptive quality of science is one we rarely consider, but it is an irony that linguistic and semiotic scholars have continually brought to our attention. Science is an abstraction of reality through the use of language; the language of science enables us to categorize and classify events in ways that involve predispositions toward those solutions seen as appropriate. Strategies for collecting data about children's or teachers' performance, for example, create boundaries about what is important and how it should be considered. The power of science to understand, and the limitations of the boundaries it creates for considering human possibilities, are always with us.

Let me provide an example. Often we collect information about student achievement or the "effects" of teacher inservice programs. The acts of data creation/retrieval assume the likelihood of at least two related outcomes. First, variations will lead to conclusions about what should be modified. Existing research about the relation of teacher praise and student achievement, for example, leads to recommendations that teachers build more positive reinforcement into their lessons. Positive relations between school leadership and achievement, or "expert" teachers, produce similar recommendations as educators consider how to create effective schools (Berliner, 1986).

A second outcome of research, and to my mind one more important for policy questions, is that inquiry adopts prior assumptions about how the world is organized in the paradigms that guide our research models of the ongoing relations of the world. The praise/achievement example presupposes a positivistic notion of the world, defining the world as a system of discrete and separate things. Change, from this perspective, is additive. The increased quantity of one variable is supposed to influence directly the outcome of the other. To borrow from phenomonology, the only motive that is considered is the "because of"; that is, prior behavior induces some change in current situations to bring about a more desirable outcome.

At least three views of change in mathematics education can be identified and their values explored: (1) a purposeful-rational view in which research/change processes are designed to move the world closer to a prior schema or model; (2) an evolutionary model of change in which there is a slow and steady movement to change elements of a social system; (3) a dialectical view of social systems. The problem is not one of progress, as we traditionally consider that notion, but of illuminating those elements that hinder or limit human possibilities.

These three views have little to do with the specific techniques of inquiry that are used, be they survey, tests, or qualitative approaches. The three possibilities or models of change do not seek to exhaust the possible ideal types, but suggest that strategies of monitoring involve prior questions about the nature of the social world which act on and have implications for understanding the problem of mathematics education itself.

PURPOSEFUL-RATIONAL MODEL

One view of change is a rational model in which there is thought to be an isomorophic relationship between the model and the world. This is evident where people believe flow charts of change or the stage models of reform coincide with the dynamics of our ongoing real world. The problem of change focuses on following a rational, orderly sequence to implement some identified goal that is presupposed by the system. A purposeful-rational order is assumed. There is a definition of the world as logically ordered and rationally controllable through administrative changes in the organization of daily life. Change involves delineating each step in a logical and orderly sequence. The order of the model of change is believed to be universal to all situations, institutions, and organizational purposes. This model assumes that, if people follow its stages to implement reform and are careful not to fall into the pitfalls of organizational resistance, the outcome will be the success of the proposed reform.

Institutionalization, from this perspective, is the use of the reform program or strategies after the initial stages of dissemination and implementation are concluded. The strategies of change become, more often than not, questions of quantity: Are there more computer programming courses than before? Is student achievement higher in mathematics than in previous years? Do teachers and administrators feel more satisfied and more professional? The isomorphic quality of the model is contained in the assumption that the world around us can fit into the stages or sequences identified in the model.

In fundamental ways, the isomorphic model contains a one-dimensional conception of social existence. The "noise" of cultural and social interactions, the complexities of causation that involve nonrandom practice and relational dimensions as part of the social order, and the role of human purpose are lost. Practices of reform are made independent of the nonrational elements of politics, the ambiguities of social affairs, and s. .uctural conditions that provide a background by which choices are seen as relevant and reasonable.

This view of the world is a reification of human existence itself. What is essentially a language of metaphor to enable us to suppose that things are "like this" or "like that" becomes what is and should be. The isomorphic model works against change as it focuses on the facades of social life and crystallizes the status quo. The iconic visions are made literal and empirical attributes of reality.

EVOLUTIONARY MODEL

Related to the isomorphic model is a second view of change that ascribes an evolutionary quality to social organizations. In this "process" model, the problem is to guide the evolution of the system. Strategies of evolution may involve the invention of a new element in staff organization, such as special career incentives to ensure greater professional esprit de corps. The task of change is to devise a way of helping teachers evolve working relations that incorporate the new into the old patterns. Sometimes this is labeled a problem-solving approach. A local staff considers issues and problems, devises strategies of change through inservice programs, and implements the change strategies to bring forth the solutions. An assumption is that the incorporation of the new program will regulate the system, making it healthier and more progressive.

This evolutionary view of change involves certain assumptions that need clarification. In rejecting the mechanical view of the isomorphic model, an assumption of organism is accepted (Nisbet, 1976). The analogy to an organism involves certain assumptions: (1) Change has directionality, that is, there is a trend or longitudinal shape to movement. (2) Growth of an organism is cumulative, i.e., what may be seen at any given moment is the cumulative result of what has gone before it in its life. (3) Developmental change is irreversible; change has stages and these have genetic as well as sequential relations to one another. And (4) there is purpose to growth. St. Augustine saw purpose in the human drama that was transhistorical and spiritual. Purpose, for Marx, was entailed in the struggle toward a classless and just society. In its modern form, progress implies a belief in rational understanding, a possibility of deducing generalizations that remain valid for some time and, to a degree, a determinism in our social conditions. The ability to impose an ever-increasing control over both the natural and social environment is made central to social and moral life.

The difficulty of a structural-functional view of change is its concern with harmony, consensus, and stability; that is, change is explained through focusing on the functional interdependence of the system. The relation to existing structures is stressed. Further, time is identified with social change. One presupposes that one can take a snapshot of a social system over time to reveal structure as one does of the architecture of a building. This assumption is misleading. Social systems have patterns of social relations that are inseparable from their continual reproduction over time. It is like redesigning a floor layout without having focused on changes in a building's structural arrangements; one needs first to posit a theory of structure that can distinguish the stable elements from those which are in flux.

The problem of adaptation related to evolution and function also is filled with ambiguity. Not all adaptations are functional or related to structure. Recent archaeological and anthropological evidence raises serious questions about adaptation as a way of explaining differences or progress. Darwin's view of the effect of variational evolution on group change raises questions about adaptation as a metaphor for social theory. When adaptation is observed in a species, it can be explained by the differential survival and reproduction of variant types being guided and biased by their differential efficiency or resistance to environmental stress and dangers. But any use of differential survival and reproduction, even when it has nothing to do with the struggle for existence, will result in some evolution, not just adaptive evolution. The evolutionary model in social theory becomes a form of Panglossian biology, confusing the idea of Darwin that all adaptation is a consequence of variational evolution with an assumption that all variational evolution leads to adaptation.

DIALECTICS AND CHANGE

The limitations of evolutionary social theory direct attention to a third notion of change, a concern with dialectics. Here, there is an assumption that all social processes involve an interaction between that which seems to be in opposition. Stability is always juxtaposed to change and social transformation, tradition with dynamics. Further, the interaction of tradition and transformations in social conditions produces changes in quality as well as quantity. Having more computers involves considering not just more use of technology, but also the social relations produced as material conditions of schooling are altered. Finally, the dynamics of a system are not orderly and linear; intervention does not ensure progress. The industrial revolution produced more material goods and more worker control over leisure time, yet the peasant of the Middle Ages had more "holidays" and leisure time. The Civil War freed slaves in the South, but new forms of discrimination and racial bias were created by the turn of the century with consequences as serious as those of slavery. The development of mass education provided greater attention to individual merit and access to material success, while also providing more effective means for social reproduction and control in times of social and cultural stress.

These examples illustrate the complexities and unforeseen consequences of social action that must be attended to when considering issues of monitoring. To consider movement in a social system requires attention to the interaction of "contexts" in schooling. One notion of context is the particular time and space in which social action occurs. We assess teachers' attitudes about curriculum or observe particular practices in schools. But in our desire to take into account the "environment," we often ignore a broader notion of context. The most trivial exchange of words in a classroom implicates the speaker in a long-term history of the language in which the words are formed and, at the same time, in the continuing reproduction of that language. When teachers talk about children as learners, mathematics as a subject matter of school, or teaching as a specific series of pedagogical acts, their words contain assumptions about structure, function, agency, and knowledge that have developed in the past and have become a part of common sense language (Stanic, 1987b; O'Donnell, 1985).

The present contains structures of social patterns, such as ideologies of individualism, notions of science as progress, beliefs about mathematics as producing social and material advancement. The difficulty of identifying change in school mathematics lies in considering what elements of our discourse are being produced by current interactions and what elements are derived from contexts that existed outside of schooling and prior to our participation in these social affairs. It is those interactions of "context" that underlie the twin motifs of change and stability. To introduce new mathematics courses involves realizing the program in a historical system that includes teacher behavioral patterns, cultural norms of the classroom and school organization, and the social conditions outside the school that interrelate to produce knowledge.

Conclusions

This essay has focused on three questions about mathematics education in the social context of schooling. In each question, analysis considered the complexities of curriculum in an ongoing, cultural world in which there are unequal social relations and different interests. What is transmitted as mathematical knowledge, it was argued, may have little to do with the disciplinary standards, expectations, and understandings associated with the field of mathematics; rather the subject matter is shaped and fashioned by institutional imperatives and values that underlie schooling.

The problem of monitoring involves identification of a method to consider the complexities of schooling as a social institution. Movement and change are not linear, sequential, or directional processes. Our social conditions contain a host of elements that interact in ways that are never fully specified, predetermined, anticipated, or willed. Further, to add "new" elements to situations is to add to the dynamics of those situations in ways that qualitatively and quantitatively change them. To describe these relationships in more conventional language, all elements of a situation are, at the same time, independent and dependent variables. Each element is modified as it enters into a social situation in ways that alter not only its relationship with other elements, but also its own internal relations.

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Chapter 25

THE PERSISTENCE OF MYTH IN CURRICULUM DISCOURSE

Catherine Cornbleth

Myth is integral to modern as well as ancient societies. In contemporary curriculum and school discourse, it is not uncommon to find disparaging reference to this or that as myth, as if the label were sufficient to discredit the belief and accompanying practice by associating them with times past, fantasy, or false consciousness. Yet they persist, shaping and reflecting school practice. Of particular interest here is the holding power of myth. How might we account for the persistence of myth in curriculum discourse and practice?

After briefly examining the sociohistorical nature and role of myth, I draw on earlier work (Cornbleth, 1985a) to sketch three prevailing myths that I find especially problematic. They are the myths of thinking skills, the right answer, and stages and styles of cognition and learning. Although primarily associated with curriculum and schooling, these myths can be seen as manifestations of more encompassing themes in U.S. culture and its institutions, related to what Berger, Berger, and Kellner (1973) called modern technological consciousness. Against this background, I then explore the persistence of these myths in curriculum discourse, with a focus on the social function of myth and its role in mathematics education. My purpose is to explain the appeal of such myths, with the assumption that such understanding can prompt the modification of old myths or the creation of new ones more compatible with the professed goals of schooling in democratic states, for example, to promote the acquisition of valued knowledge and to contribute to social justice.

Those who would reform curriculum might well attempt to better understand what is to be reformed, how it came to be, and why it persists (Reid, 1978). That understanding necessitates probing beneath surface appearances and questioning prevailing language, assumptions, and practices, including sustaining myths. Making problematic what has been taken for granted in curriculum creates possibilities for reform.

FROM TRUE STORIES TO SCIENTIFIC FABLES

Myth is a widely held belief with tenuous connection to pertinent evidence or circumstance. Sometimes myth is elaborated as a theory or story, sometimes it is simplified as a proverb or slogan. In ancient societies, myths were "true" stories about historical origins as distinguished from false stories or legends

(Eliade, 1963). Today, we tend to think of myth as false belief and to assume that we have rid ourselves of such vestiges of a prescientific age. All cultures, however, have their guiding myths, although we may not think of our own myths as such (Lakoff & Johnson, 1980). Like the ancients, we see these beliefs as "truths"--as common sense, as empirically established fact, or as natural law (Toulmin, 1982). Myth is an integral, often unexamined part of contemporary culture, a culture that includes curriculum discourse.

Ancient myths were historical and particularistic, a rich narrative of "real" events in times past. Most often, they concerned the role of supernatural beings in creating or bringing about events and institutions. The history sustained through myth was considered sacred because it was the work of supernaturals. The myths provided models for human behavior and gave "meaning and value to life" (Eliade, 1963, p. 2). In primitive societies, according to Malinowski (1926),

> myth expresses, enhances, and codifies belief; it safeguards and enforces morality; it vouches for the efficiency of ritual and contains practical rules for the guidance of man. Myth is thus a vital ingredient of human civilization; it is not an idle role, but a hard-worked active force. (p. 19)

Knowledge of cultural myths also carried social and political import insofar as such knowledge of the origins of phenomena suggested the ability to control them.

Modern myths, in contrast, tend to be abstract and transhistorical, such as general principles of learning, for example.[1] Through repetition and reification, the abstraction then comes to be treated as "real" or natural (Barthes, 1957/1972; Berger & Pullberg, 1965). Psychological principles of learning become <u>the way people learn</u> or <u>learning</u> itself, and students who do not observe these principles are typically considered deficient or recalcitrant. In some instances, myths can become self-fulfilling, as when the presumably deficient or recalcitrant student is treated as such and begins to respond in kind, thus confirming the teacher's expectation and perpetuating both the myth and the student's nonlearning.

A number of contemporary myths seem to have their origins in descriptive or explanatory metaphors, that is, in efforts to understand new or puzzling phenomena in terms of other, more familiar ones (Lakoff & Johnson, 1980). Problems arise when metaphors (e.g., the mind is a machine) are taken literally and

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[1]Also, whereas ancient myths tended to be anthropomorphic, witness Zeus, for example, the myths of the twentieth century are more likely to be "mechanomorphic" (Toulmin, 1982, p. 24).

thus turned into myth (Hesse, 1980). Conceptualizing the mind in terms of a machine, for example, makes the nonobservable phenomenon manageable and offers a heuristic for further inquiry. While highlighting some aspects of the mind, the machine metaphor obscures others. To treat the mind as if it were a machine is to abuse the metaphor and distort the concept. It is to engage in reification and consequent mystification, which yield conceptual absurdity and similarly absurd and often dehumanizing practice. An irony of such reification is that something that has been reified (e.g., the machine mind) cannot exist on its own as an entity apart from the human activity that created and maintains it (Berger & Pullberg, 1965); yet, we often act as though we are necessarily constrained by our own creations. The mythic reification of belief and accompanying practice is a prevalent but not inevitable aspect of human existence and institutions.

The abstractions and metaphors that give rise to myth in modern society tend to be distinguished by claims to a scientific character. Claiming a basis in science is intended to, and often does, bestow legitimacy given the awe in which science is held in contemporary Western societies. Scientific myths, according to Toulmin (1982), are created when scientific concepts or scientifically obtained data are employed for other than scientific purposes or to answer nonscientific questions. They often result from overinterpretation, such as generalization beyond the bounds of available data. The link with science, however strained, lends an aura of authority. A prime example is the belief in some kind of social or cultural evolution; another is the assumption that observed right-left brain differences should be accommodated in curriculum. The identification of differences does not mean, in any scientific sense, that they should be responded to in any general or particular way.

Related to the abstract quality and scientific veneer of modern myth is the appearance of universalism, in which the creations of particular social and historical circumstances are decontextualized and made to seem universal. Myth "transforms history into nature" (Barthes, 1957/1972, p. 129), thus "giving an historical intention a natural justification, and making contingency appear eternal" (p. 142). For example, bourgeois representations and "relations between man and the world" have become "'normalized' forms" such that "bourgeois norms are experienced as the evident laws of a natural order" (Barthes, 1957/1972, p. 140). Psychological constructs (e.g., learning, skill) have been similarly decontextualized and made to seem universal and natural. The myth of thinking skills, for example, can be seen as a manifestation of a Western middle-class world view and psychology made to appear universal.

Myth serves multiple, interrelated social functions: it explains phenomena and directs action, justifies particular interests or practices, dramatizes ideals, and provides cultural cohesion. Myth offers both description and prescription, an account of things and a model for action, be it a myth of rainmaking or a myth of stages and styles. Even when expectations remain unfulfilled, myth and its associated practices tend to prevail. We are, perhaps unavoidably, creatures of myth insofar as human experience encompasses much that cannot be explained by other means.

The justification function of myth is closely related to its explanatory role. To explain and direct action not only makes something plausible but also legitimizes it. The myth of stages and styles, for example, is seen as explaining observed differences among individuals and calling for differentiated treatment of students. Different learning opportunities and outcomes are thus justified by the myth. (That the myth labels more than it explains and diverts attention from efforts to teach "slow" or otherwise "different" students is considered in the next section.) Justification of particular interests or practices via myth also serves to obscure or mask others. The individual focus of the stages and styles myth, for example, deflects attention from the influence of culture and social structure on individuality.

Myths also serve as ideals that orient a culture. For example, in the United States, "We hold these truths to be self-evident: that all men are created equal, that they are endowed by their Creator with certain unalienable rights, that among these. . . ." Myth thus inspires and guides individual and collective action.

Finally, myth provides cultural cohesion. By offering meaning, legitimacy, and ideals, myth perpetuates a shared corpus of belief and thus provides a reassuring degree of security. That myth seems to be pervasive and indispensible to society, however, ought not lead to unquestioning acceptance of particular myths as inevitable. Just as myth is socially constructed, so it can be reconstructed.

ILLUSTRATIVE MYTHS

The field of mathematics education is rich with specific examples of the general metaphorical, prescriptive, reifying and legitimatizing functions served by myths. Some myths arose from a dearth of empirical evidence regarding the nature of understanding and how it might be fostered in the classroom. For example, emphasis on rote learning reflected a belief that most students were incapable of understanding mathematical concepts. On the other hand, the "new math" of the 1960s assumed that meaningful instruction would take its organization from the structure of mathematics itself, and that understanding would be achieved through an earlier presentation of concepts.

Ability grouping seemed so logical that the affectivity of lower achievers and the social-cognitive benefits of classroom interaction among diverse students were ignored (Johnson & Johnson, 1975; Schmuck & Schmuck, 1979). It also seemed reasonable to excuse computational errors as "careless mistakes"; such a magnanimous gesture, however, left systematic errors, or so-called "buggy" algorithms, undiagnosed (Brown & Burton, 1978). And only within the last decade have we come to recognize the debilitating effect of the input-output industrial production metaphor on mathematics instruction. This metaphor likened children to raw material and an accompanying assumption in mathematics education, that children come to school with no mathematical knowledge, precluded investigation of the rich informal mathematical systems they had invented for themselves (Ginsburg, 1977; Resnick, 1976).

Finally, some myths arose from an effort to oversimplify and find a panacea to handle the complex web of problems surrounding mathematics education: "Our students will certainly learn math better if our teachers know more mathematics!"

The three myths that are further examined here reflect and contribute to the pervasiveness of technological consciousness in curriculum discourse and practice.[2] The myths of thinking skills, the right answer, and stages and styles are powerful, widely held, and considered sacred by many educators and laypersons. They are promoted by experts and institutionalized in curriculum and schooling. Yet, in one way or another, these myths contradict such democratic ideals as equity and enlightenment or subvert attempts to realize them.

The Myth of Thinking Skills

This is a double-barreled myth, one aspect of which asserts that thinking is composed of a number of discrete cognitive skills or steps. The second asserts that these skills or steps are content-free or generic. Together, they direct us to teach "thinking skills" apart from thinking subject matter, as can be seen in curriculum and methods texts that offer separate skill-development chapters and teaching sequences with little or no attention to their integration with one another or relation to subject matter. The problems of the first part of this myth are largely conceptual; the weaknesses of the second are both conceptual and empirical.

Skill A + B + C = Thinking

There are at least three interrelated problems with the position that thinking consists of the sum of any list of skills or steps. Without some conception of the nature of thought, we cannot predict whether such skills or steps might add up to thinking, or something else, or nothing at all. A second problem is that lists

[2]Elaboration of the arguments and evidence provided here can be found in Cornbleth (1985a, pp. 18-32).

and steps imply that thinking is--or should be--linear, which is rarely, if ever, the case. Instead, thinking tends to be recursive. A sequence of skills or steps is not a logic-in-use but a restructured logic (Kaplan, 1964) that misrepresents what occurs while one is thinking. Such an approach also ignores the dispositional quality of thought.

The third and more serious problem concerns the reductionist assumption that thinking can be cut into pieces and reassembled without damage. This reductionist assumption destroys the substance and spirit of thought; it is impossible to dissect a live frog. Further, when thinking is conceived as raising and pursuing questions about the ideas one encounters, the questions raised and the means needed to pursue them will depend to a large extent on the situation, including the ideas encountered, the social context of the encounter, and the prior knowledge and values of the questioner. Even given a general orientation or disposition to think critically, particular aspects of thinking are situation dependent and cannot be predetermined or specified out of context (see Mishler, 1979).

If thinking consisted of a number of skills or a series of steps, one might expect considerable agreement about the constituent skills or steps. However, there appears to be little or no such agreement; there is not even consensus about what constitutes a skill (Beyer, 1984a). One might also expect that students who had mastered a given set of skills would demonstrate sound thinking. To my knowledge, however, no such evidence exists. Students who have mastered a skill have done just that; it does not necessarily follow that the skill will result in, or be used in the service of, thinking. It is also conceivable that preoccupation with skills might dull one's critical capacity.

No list of skills is either necessary or sufficient for thinking. Any or all possible skills, however, may contribute to thinking in a given case. This position is similar to Dewey's (1910/1933) description of the interdependent phases of reflective thought as dependent on, but not equivalent to, diverse skills such as observation and inference.

Thinking cannot be reduced to a universally applicable formula of skills or steps to follow except by conceptual or empirical alchemy. The apparent rationality of specifying thinking skills and sequencing them in a logical or psychological order for instruction is appealing. But that rationality is illusory. Analytically distinguishable elements are interdependent in practice, and logical or psychological order bears little resemblance to the organization of practice. Despite claims to the contrary (Beyer, 1984b), there is no reason or evidence to suggest that further efforts to specify and sequence supposedly generic skills will be any more productive than previous ones.

Drawing on Aristotle, Wiggins (1978) made a similar case with respect to "practical reasoning." Accord[illegible] to Aristotle, "the openness, indefiniteness and unforeseeability of the subject matter of praxis" resists codification. Wiggins (1978) suggested that those who seek "a system of rules" hope "to spare themselves some of the agony of thinking and all of the torment of feeling and understanding that is actually involved in reasoned deliberation" (p. 150). From a psychological perspective, Shulman and Carey (1984) have also argued against reductionism and the assumption of universality with respect to reasoning and rationality. Formulaic approaches to reasoning and thinking are increasingly eschewed in both philosophical and psychological work (see McPeck, 1981, for a particularly detailed and devastating critique).

In terms of mathematics, educators once looked hopefully to programs purporting to teach generic thinking skills for transfer to mathematical problem solving. Bloom and Broder (1950) taught problem solving to college students with apparent success, but they cautioned that "a certain amount of background in the subject was indispensible" and that methods themselves "could not serve as a substitute for the basic knowledge of the subject matter" (pp. 76-77). Nevertheless, their success, together with the fact that large corporations had been using creativity training programs with their employees since the 1930s, inspired experimentation with several productive thinking-skills programs during the 1960s. In general, the success of these programs was limited to improved performance on the same types of problems employed in the lessons, with little evidence of transfer to dissimilar or more complex problems (Mayer, 1983).

Recent research in the field of cognitive psychology has involved the analysis of performance on complex mathematical tasks. By constructing computer simulation models of successful problem solving, researchers have come to a better understanding of the types of implicit knowledge that make successful performance possible. Work such as Brown and Burton's (1978) analysis of subtraction, Riley, Greeno and Heller's (1978) analysis of word problems in arithmetic, and Greeno's (1978) analysis of high school geometry proofs has all pointed to the need for mathematics-specific knowledge and strategies.

Skills and Strategies Are Content Independent

The second part of the thinking skills myth follows from the first. If thinking consists of a number of skills, the underlying assumption is that these skills are independent of the subject matter content to which they might be addressed. Thinking skills and strategies (patterned combinations of skills) are assumed to be knowledge free or generic and transferable from one situation to another. However, the evidence regarding transfer is mixed at best (Belmont & Butterfield, 1977). There are no supervening skills that can replace substantive knowledge of the field in question.

Increasing evidence indicates that the development of thinking and associated skills is highly knowledge dependent (Glaser, 1983;

Greeno, 1980; Voss, in press). In other words, we cannot think about ideas we encounter unless we know something about the area in question. With the possible exception of metacognition, skills that contribute to thinking tend to be domain specific.

The conceptual argument for the domain specificity of thinking is well presented by McPeck (1981), who draws particularly on the earlier work of Toulmin (1958). The knowledge in which thinking is embedded is more than empirical and conceptual. The various fields of knowledge have different logics or modes of reasoning. While not mutually exclusive, the standards of judgment for what counts as sound knowledge and argument differ from one subject area to another. What constitutes good reason and evidence for belief in history differs from that in economics, law, and chemistry. "Just as there are different kinds of knowledge, so there are different kinds of reasons, evidence, and modes of justifying them" (McPeck, 1981, p. 23). The domain knowledge that is crucial to thinking is procedural and normative as well as empirical and conceptual. Consequently, the separation of subject matter content and thinking process is arbitrary and misleading.

Because each area of knowledge has its own distinctive logic, the criteria or expectations for thinking vary from one area to another. The legitimacy of these differences has been well argued by Toulmin (1958). Toulmin's additional argument that the natural sciences are distinguished "not by the types of objects with which they deal, but rather by the questions which arise about them" (1977, p. 149) seems equally apropos of the social sciences. There are also intrafield differences in questions raised and acceptable grounds for belief, such as those between behavioral and cognitive psychology. The differences in questions raised and means of pursuit between and within fields point to a further distinction that cuts across subject matter or disciplinary boundaries. This is the paradigmatic distinction. A paradigm is a world view or framework of knowledge and belief through which we "see" and investigate the world. Scientific paradigms consist of working assumptions about the world and how it is to be studied, understood, and acted upon, i.e., interrelated concepts and values, questions, procedures, and actions. Commitment to a particular scientific paradigm involves affiliation with a community of scholars who share, sustain, and shape the paradigm.

Thinking, then, varies with the domain investigated and the paradigm adopted. To argue that thinking is neither absolute nor universal is not to suggest that it is either idiosyncratic or individual. Thinking is a social as well as a cognitive activity, shaped by the setting and the norms of the community in which it occurs, e.g., Marxist, empirical-analytic. The complexity and pluralism of the knowledge underlying thinking renders generic thinking skills illogical and impotent.

The empirical evidence for the domain specificity of critical thinking and associated skills and strategies comes largely from recent studies of reasoning and problem solving in math, science, and social science. (Glaser, 1983, and Voss, in press, provide excellent reviews.) This research shows that, while general skills and strategies can be identified, they are relatively weak and useful only when one does not know much about the field or problem in question.

As inferred cognitive or intellectual processes, skills are important to thinking, but they are not a proxy for thought. By misrepresenting thinking, the "skill A + B + C" part of the myth of thinking skills tends to mislead curriculum designers, teachers, and teacher educators into fragmenting thought and teaching skills in lieu of thinking. The generic part of the myth tends to mislead further by focusing attention and effort on weak rather than strong skills and strategies. Yet the myth persists, as evidenced by the attention to thinking skills in many curriculum and methods texts and the numerous generic thinking-skills materials for classroom use.

The mathematics community has frequently been cautioned to distinguish between knowledge and the record of knowledge (Lindquist, 1984; Romberg, 1983; Romberg & Carpenter, 1986). Romberg (1983) elaborated on this important distinction:

> A person gathers, discovers, or creates knowledge in the course of some activity having a purpose. This active process is not the same as the absorption of the record of knowledge--the fruits of past activities. When the record of knowledge is mistakenly taken to be knowledge, the acquisition of information becomes an end in itself, and the student spends his time absorbing what other people have done rather than having experiences of his own. (p. 122)

It now appears that the myth of thinking skills in mathematics was rooted in the confusion of knowledge and the record of knowledge. Research has established the existence of strong ties between mathematics content understanding and the usability of mathematical ideas (Behr, Lesh, Post, & Silver, 1983; Lesh, Landau, & Hamilton, 1983). Usability appears to develop concomitantly with mathematical concepts that are cognitively modeled and represented in a meaningful way. This implies that rote learning does not encourage mathematical thinking.

Early in the century, when Thorndike's stimulus-response bonding theory was applied to mathematics, the simplest components of every topic were isolated, viewed as ends in themselves, and mastered by rote. Mathematics teachers were perceived as marketing specialists; their role was to provide students with simple packaging and step-by-step procedures. For the student, "doing" mathematics meant algorithmically manipulating a notational system devoid of conceptual underpinnings. This view of mathematics became so inveterate that it bucked progress even when the "new math" revolution tried to reinject meaning into the curriculum. For example, it was thought that using bases other than ten would add meaning to the decimal system, but teachers were so entrenched in their perceived roles that many of them promptly developed algorithms for adding and subtracting in other bases and made flash cards for drill!

Indeed, mathematics education has paid a high price to learn the folly of dissecting mathematics and making a fetish of algorithms. We have learned the hard way that mathematical thinking is possible only by "experiencing" math in Dewey's (1916) sense, that is, by actively making concepts a part of one's own cognitive organization.

The Myth of the Right Answer

According to this myth, a correct student response to a teacher or textbook question is evidence of student thought and learning. Right answers are sought by teachers and rewarded with the praise and points that add up to good grades. Correct responses tend to be valued regardless of the question or the means by which students produce them. But right answers do not necessarily demonstrate student learning; students may simply be repeating what they already know. Nor do right answers necessarily demonstrate subject matter knowledge or thought. They may be the result of a lucky guess, successful evaluation of "what the teacher wants," or other survival strategies (see Anderson, Brubaker, Alleman-Brooks, & Duffy, 1984). The right answers valued by this myth typically are responses to simple knowledge recall or memory questions. The myth of the right answer tends to encourage rote memorization rather than thoughtfulness.

Rote memorization also can be seen as a myth. The memorization myth asserts that information must be acquired (i.e., memorized) before one can use it for thinking or other purposes, and that rote memorization is an effective means of information acquisition. Learning by rote refers to rehearsal, drill, or "saying it over," none of which necessarily involve knowledge comprehension, integration, or application. Once again, the evidence does not support the claim. Instead, we find that rote memorization usually is an inefficient means of information acquisition and that it does not provide a functional knowledge base for further thought (Simon, 1980) or action.

How information is elaborated and organized in memory appears to affect its accessibility and usefulness. Rote elaboration strategies do not foster the organization and integration of information that enables retrieval and use in new situations. According to Simon (1980), "rote memorization, as we all know too well, produces the ability to repeat back the memorized material but not to use it in solving problems" (p. 87). The same applies to concepts, procedures, and other kinds of knowledge. Unless appropriately interrelated in memory, they are not likely to be useful in problem solving or other kinds of thinking (Norman, Gentner, & Stevens, 1976). Thus, the myths of the right answer and rote memorization foster an illusion of knowing that subverts construction of a functional knowledge base.

In mathematics, the myth of the right answer is sustained primarily by the manner in which student progress is assessed. From in-classroom testing to national assessments, low-inference responses measure student mastery of behavioral objectives. The processes and cognitive outcomes of learning are largely ignored. Consequently, we see a "backwash" effect (Walkden & Scott, 1980), in which students acquire by rote only those mathematical facts and strategies that they consider most likely to lead to successful performance on their tests.

As in most critical issues in mathematics education, things depend on things that depend. The assessment issue is not a discrete problem with a simple answer. Lindquist (1984) described the dilemma:

> Text publishers include items because they are covered on tests, test makers include items because they are in texts, texts include content because a state requires it, mathematics educators train teachers to teach what is in the text, and on and on. (pp. 607-608)

Reform will necessarily be a task of great magnitude, but in the midst of widespread concern with the cognitive organization of knowledge and the processes by which it is brought to bear on mathematical problems, it is clearly an "outdated notion that one can assess the learning of mathematics solely in terms of the ability to produce correct answers" (Romberg & Carpenter, 1986, p. 869).

The Myth of Stages and Styles

This, too, is a composite myth, encompassing various beliefs about the existence and implications of stages of cognitive development and styles of cognition or learning. Whereas "stages" usually refer to presumed abilities of one sort or another, "styles" usually refer to preferred modes of information processing or learning, i.e., cognitive and learning styles. There are similar problems with stage and style forms of differentiation and categorization of students. Both tend to have a negative effect on expectations regarding students' abilities or dispositions to learn and their "readiness" to develop desired knowledge or reasoning skills. They might also negatively affect expectations regarding whether and how something can be taught. For example, if critical thinking is assumed to be dependent on having reached the formal operations stage in Piaget's hierarchy, then one would not expect students assigned to the concrete operations stage to be capable of critical thought or of learning how to think critically.

There seems to be a "catch-22" in this line of reasoning that limits the possibilities of teaching and learning. If students have reached the appropriate stage, they can do it and either could benefit from instruction or do not need it; if they have not, they cannot do it and are not "ready" to be taught. Yet, there is evidence of instruction "bringing children who were not even on the verge of acquiring a particular logical form of thought to complete mastery of tasks that supposedly require that form of thought" (Case, 1981, p. 144).

Beyond questions of teaching and learning, there are questions about the consistency of developmental stages, specifically the assumption that an individual at a particular stage functions at that level across tasks and subject areas. According to this interpretation of stage theory, one would not be at a formal operational level in one domain, say history, and at a concrete operational level in another, say mechanics. The empirical evidence, however, is to the contrary: "Careful reviews of the research literature do not support the picture of homogeneity of cognitive activities at particular ages assumed by the Piagetian stage model" (Estes, 1978, p. 13; also see Case, 1981; Mandler, 1983).

Task-relevant knowledge, rather than developmental stage or capability, may better explain identified age-related differences in students' reasoning (Chi, 1983; Chi & Glaser, 1983; Ortony, 1980). For example, how well students categorize a given set of items depends not simply on their stage of development or "categorizing ability," but also on their knowledge of the items and the kinds of categories into which they might be sorted (Chi, 1983). Ortony (1980) suggests a similar interpretation of studies of children's comprehension of metaphor, many of which did not control for "world knowledge." Prior content knowledge is also important to problem solving. Whether and how a problem is solved are dependent on the knowledge one brings to the situation (Chi & Glaser, 1983). In sum, it increasingly appears that "changes in such [domain] knowledge may underlie other changes previously attributed to the growth of capabilities and strategies" (Siegler & Richards, 1982, p. 930).

As with the stages myth, there are contrary data and alternative explanations of observed differences attributed to cognitive or learning style. A major challenge comes from studies showing that styles are context dependent, i.e., they vary with the learning task (Laurillard, 1979). Given more than one kind of task, students demonstrate more than one style. Interestingly, many studies purporting to show individual differences in cognitive or learning style employed only one task or one kind of task. There was no opportunity for students to demonstrate task-related differences. So-called style seems to be less an inherent characteristic of individuals than a function of contextual factors such as the nature of the task, subject-matter content, teaching mode, and expected kind of test. The evidence strongly indicates that identifiable styles do not exist in the individual, but, if at all, in the interaction of individual and situation (McConkie, 1977).

Stages can be imposed on anything that changes over time; styles can be imposed on anything that varies. The danger in categorizing students according to their presumed developmental level or their presumed learning or cognitive style is twofold; it lies both in the category labels and in the school practices that are based on such labeling. First, the labels are neither neutral nor helpful to teaching or learning. Not only do they obscure the individual, but they tend to be explanatory fictions whose undesirable social consequences have been well noted (Apple, 1975). Pedagogical consequences include negative teacher expectations for many students and the formal or informal tracking of students with the concomitant denial of opportunity to those deemed unready or incapable of certain kinds of learning.

Curriculum discourse incorporates and perpetuates the myths of stages and styles as evidenced in texts and methods courses. Instead of assuming that students <u>are</u> "this way" or "that way" as an inevitable consequence of their inherent cognitive or learning style, consider the possibility that they have or have not learned to approach given tasks in particular ways. If students have not yet developed the requisite knowledge and reasoning skills for critical thinking, for example, opportunity and instruction should be provided, not denied.

During the 1960s and 1970s, a great volume of mathematics education research sought to validate Piaget's stages of logical operation and to determine the effect of instruction on a child's progression through those stages. Research is now focused more on describing how children organize what they already know, with the ultimate goal of matching instruction to their existing cognitive structures. "Stage of development" refers more to the complexity of a child's cognitive framework: how well (s)he becomes oriented to a problem, represents it, and connects it with existing knowledge; how well (s)he organizes a plan of action and executes it; how well (s)he regulates and assesses decisions and outcomes (Lester, 1985).

Cognitive research on early addition and subtraction concepts (Carpenter, Hiebert, & Moser, 1983; Carpenter & Moser, 1983; Riley, Greeno, & Heller, 1983) has shown that children come to school with an understanding of addition and subtraction sufficient to have created their own solution strategies to word problems. They rely heavily on semantic structure, are capable of transforming problems to equivalent forms with which they feel more comfortable, and are particularly adept at mathematical modeling. Research in geometry with older children has shown that their initial geometric cognitive structures do not correspond to primitive mathematical structures (Moyer, 1978) and that they make "many mathematical judgments using qualitatively different methods than those typically used by adults" (Lesh, 1976, p. 186). These results challenge mathematics educators to develop a curriculum that reconciles mathematics content and instruction with children's pre-instructional mathematical development and subsequent construction of knowledge during instruction.

From Skill in Thinking to Thinking Skills

Examining in historical context the emergence of the myth of thinking skills and its incorporation in curriculum discourse and practice serves two purposes. Such an approach illustrates how myth emerges in professional education discourse; it also illustrates how education myths are related to broader cultural themes. What follows is, necessarily, a partial and retrospective account. Although it is neither comprehensive nor causal, it can be seen as plausible and suggestive of further lines of inquiry.

The language of thinking skills and its underlying assumptions are commonplace in contemporary curriculum discourse. Exceptions include some of the philosophically oriented literature and discourse within a critical perspective. These exceptions have been overshadowed by the predominantly psychological and technical orientation of curriculum. This psychological orientation, behavioral and now increasingly cognitive, together with the social efficiency and scientific management movements earlier in this century, and the anti-intellectualism that has characterized U.S. history and society for more than two centuries, seem to have been major factors in the transformation of skill in thinking to thinking skills.

At the turn of the century, the prevailing language was one of skilled performance, where skill referred to proficiency at a task, usually a physical activity viewed as practical or useful, as in the case of the skilled worker. By the 1920s, there is reference in the literature to skill subjects such as reading, arithmetic, and writing, earlier called "common branches" and later "basic skills." It is not until the 1950s that intellectual activity and thought are referred to as thinking skills. The 1940s appear to have been a decade of transition, and the 1950s one of consolidation, such that by the 1960s thinking skills were largely taken for granted.

Since the 1920s, social efficiency by means of "scientific management" has been a watchword of U.S. schooling (Callahan, 1962). The 1980s language of excellence and effectiveness can be seen as a restatement of the social efficiency theme. It is assumed that measurably effective teaching and schooling practices can be identified and implemented to yield more and better informed individuals who, collectively, will strengthen and improve society at large.

Measurement and task analysis have been key features of the scientific management that is to provide efficiency. Earlier in this century, measurement took the form of individual aptitude and achievement tests, teacher rating scales, and school surveys, resulting in what Rugg decried as "an orgy of tabulation" (1926, p. 71). Subsequently, these measurement forms have been augmented by behavioral observation and quantification, and extended by sophisticated computers that make feasible larger data bases and more complex analyses. Measurement was to enable determination of efficiency and, later, accountability, such that less efficient practices could be identified and eliminated in favor of more efficient ones.

Psychometric technology required task analysis, that is, the decomposition of complex acts into presumably discrete elements amenable to quantification (cf. Frederiksen, 1984). Task analysis also contributed to scientific management by facilitating the prespecification and control of curriculum and teaching that was to enhance efficiency. It was assumed not only that tasks such as reading, thinking and teaching could be meaningfully decomposed, but also that the identified elements could be best taught and learned one at a time and then assembled to create the desired whole.

Illustrative of these assumptions and practices is the curriculum work of Bobbitt, Charters, and later Tyler, a student of Charters. Bobbitt, a leading advocate of scientific management in education, and Charters adopted a task analysis approach to curriculum construction. For example, in his curriculum books, Bobbitt emphasized the precise specification of "particularized" objectives derived from activity analysis as the central task of curriculum construction. Appropriate learning experiences and means of evaluation, he believed, would routinely follow from such analysis and specification. The "Tyler rationale" with its "production model" of teaching and learning (Kliebard, 1975, p. 45) and behavioral objectives can be seen as an extension of this earlier work, as can more recent technocratic conceptions and "rational management" models of curriculum construction (Cornbleth, 1985b).

Thus, the demands of curriculum, teaching, and particularly evaluation for specification of what was to be taught, learned, and measured strongly influenced the redefinition of "skill in thinking" to thinking skills. It is more difficult to observe and measure thinking or its products, for example, than one's response to a multiple-choice test item that purports to measure a component element or skill, such as determining whether a conclusion logically follows from given premises (cf. Dressel & Mayhew, 1954; Smith & Tyler, 1942). Reporting on the Cooperative Study of Evaluation in General Education, Dressel and Mayhew (1954) noted that available definitions of critical thinking

> all proved too abstract for evaluation or teaching purposes. The only recourse was to define critical thinking as the sum of certain rather specific behaviors which could be described and which could be inferred from student acts. (p. 37)

By the 1950s, "the only recourse" was to cast thinking as skills to satisfy psychometric demands.[3]

Dissection of thought into presumably discrete skills occurred in the political context of demands for accountability and efficiency, supported by psychologists and professional measurement specialists who promised the desired data and outcomes. This context seems to have been forgotten, while thinking as skills has come to be taken for granted as the correct interpretation or the natural state of affairs. The decontextualization of the transformation, and the subsequent recontextualization of thinking as skills in curriculum discourse, tends to obscure the political and professional interests that contributed to and continue to benefit from the transformation (e.g., standardized test developers and publishers, education policymakers).

In addition to the fragmenting of education evident in the measurement and task analysis efforts associated with social efficiency and scientific management, the pervasive anti-intellectualism of American society contributed to the redefinition of thinking as skills. While Americans traditionally have been anti-intellectual (Hofstadter, 1963), skillfulness has been an integral aspect of American traditions and values. Because skill is useful and practical, it is highly valued in the American context. Intelligence and thought have gained acceptance in this milieu in part by transformation to a collection of skills; making the intellectual technical has made it acceptable. Thinking as skills is manageable; it is compatible with prevailing values and functional within curriculum and schooling. The skills language implies the utility of thinking. Not only are skills socially useful, but they can be specified in ways that enable efficient measurement, and sequential arrangement for teaching and learning. As a psychological construct, skills became a pedagogical imperative. Interpreting the skills construct literally, accepting thinking skills as "real," reified an abstraction and contributed to the creation of a myth with considerable holding power.

The myth of thinking skills entered into curriculum through the medium of professional discourse. Journals, books, and school materials gradually adopted the skills language and assumptions. For example, in 1940, the National Council for the Social Studies (NCSS) published a bulletin entitled <u>Selected Items for the Testing</u>

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[3]A related development contributing to the redefinition of thinking as skills was the refinement of mental measurement and statistical analysis techniques, especially the work of Thorndike (e.g., his 1904 <u>Theory of Mental and Social Measurements</u>), Terman (the 1916 Stanford-Binet intelligence test with its multiple subtests), and Thurstone (his work in factor analysis in the 1930s). By the 1930s, a differentiated rather than a unitary view of intelligence predominated, compatible with a component skills view of thinking.

of Study Skills. In 1957, a third edition was published as Selected Items for the Testing of Study Skills and Critical Thinking (Morse & McCune, 1957). In their preface, the authors noted increased attention to the teaching and testing of critical thinking since the American Council on Education's 1950-53 Cooperative Study of Evaluation in General Education (Dressel & Mayhew, 1954) and the twenty-fourth NCSS yearbook, Skills in Social Studies (Carpenter, 1953). The language of the 1957 edition is mixed with respect to thinking and skills. One finds reference to the "skill of thinking critically and clearly" (p. 3), "critical skills" (p. 8), and "skills of critical-mindedness" (p. 25). With the fifth edition (Morse, McCune, Brown, & Cook, 1971), the language is unequivocally the language of critical thinking skills. The multiple editions of this bulletin indicate its wide appeal and potential impact on curriculum and testing in social studies education. The changing language can be seen as reflecting changes in the larger education community and responding to compelling social mandates.

In the mathematics classroom as much as in any other, thinking has been misidentified as a skill. We hear elementary teachers speak of addition and subtraction "facts" and multiplication "tables"--hard, cold information that is not to be reflected upon, but recited quickly and accurately. They conduct "drills," practice with "flash" cards and give "time tests" to ensure automaticity. Of course, automaticity is desirable to free a child's working memory for more complex activities (Case, 1978), but as Lesh (1985) pointed out, many teachers tend "to turn a 'means to an end' into an 'end in itself'" (p. 324).

While educators now encourage the view that to learn mathematics is to learn to solve problems, some still contend that students should learn a set of skills and then apply them to solve problems (Gagné, 1983). This bottom-to-top hierarchical approach, which implies that proficiency in a sequence of preparatory skills will eventually result in thinking, encourages a thinking-as-a-skill mentality.

Skill in thinking, however, is not an extinct notion. Current problem-solving research promises to reinstate a focus on skill in thinking with the notable difference that it will be content-based reasoning (Resnick, 1987). This research hypothesizes that problem-solving performance will improve if students can be made to reason and to think systematically about their thinking. Whether or not students become skillful thinkers is, in turn, thought to be dependent on their belief systems, and a special subclass of metacognitive research now concerns itself with students' beliefs about mathematics, both affective and epistemological (Schoenfeld, 1983a, 1983b).

MYTH PERSISTENCE

Examination of the emergence of the myth of thinking skills and its entry into curriculum discourse suggests why such myths persist despite contrary evidence, alternative interpretations, and abuse in practice that contradicts professed goals of schooling. One reason is that myth becomes such an integral part of day-to-day activities that it is taken for granted. No longer seen or heard, it is unlikely to be questioned, challenged, or changed. A second, related reason for the appeal and persistence of a particular myth is its congruence with broader cultural themes or aspects of consciousness. Such myths "make sense" in their cultural context and may be either taken for granted or overtly cherished. Further reasons for myth persistence are related to the social functions of myth noted earlier: to explain phenomena and direct action, to justify particular interests or practices, to dramatize ideals, and to provide cultural cohesion.

Like the myth of thinking skills, the myths of the right answer and stages and styles have become integral to curriculum language and activity. These beliefs and their accompanying practices are so taken for granted that many educators would be astonished to find them questioned or cast as myth. The embeddedness of myth in daily practice renders it barely visible; the visibility of myth is further diminished to the extent that the myth is congruent with overarching aspects of consciousness.

Berger et al.'s (1973) conceptualization of modern consciousness is helpful in integrating and understanding the persistence of myth. The three myths of concern here can be seen as manifestations of a particularly modern consciousness whose primary sources and carriers are technological production and political bureaucracy. Berger et al. define consciousness as the historically and socially located and constructed meanings formed in people's interactions with each other and their institutions. This shared consciousness or "symbolic universe" encompasses interrelated cognitive and normative dimensions, an organization of knowledge and an orientation toward knowledge and action. The organization of knowledge and orientation derived from technological production and political bureaucracy carry over into other areas of life, including education, to form modern technological-bureaucratic consciousness. This "symbolic universe of modernity," consisting of a "network of cognitive and normative definitions of reality," provides a common "frame of reference shared by most members of a society" (pp. 108-109).

Among the major themes of the symbolic universe of modernity delineated by Berger et al. (1973) are functional or technical rationality, componentiality, makeability, and progressivity derived from technological production; and orderliness and taxonomization derived from political bureaucracy. Each of these themes can be seen to make the myths of thinking skills, the right answer, and stages and styles plausible in the curriculum context.

Technical rationality represents the generalization of an engineering mentality to the manipulation of cognitive, social and material objects. It carries assumptions of machine-like functioning, reproducible linear process (e.g., interchangeable parts in assembly-line production), and measurability of output. Technical rationality also depends upon the assumption of componentiality, i.e., that "everything is analyzable into constituent components, and everything can be taken apart and put together again in terms of these components" (Berger et al., 1973, p. 27). These components are seen as "self-contained units" that are "interdependent in a rational, controllable and predictable way" (p. 27). Clearly, both thinking and curriculum have been deeply affected by the widespread adoption of technical rationality in American life, and not surprisingly a skills approach to teaching thinking has been incorporated into curriculum discourse and practice.

The theme of makeability follows from technical rationality and componentiality. It refers to a "tinkering attitude" and a "problem-solving inventiveness" (Berger et al., 1973, p. 30) that seeks to maximize output or results, usually on criteria of quantity or cost effectiveness. Progressivity refers to "an 'onward and upward' view of the world" (p. 113) that expects and favors continuing change and improvement. It can be seen as underlying and reinforcing the makeability theme and contributing to the plausibility of the myths of thinking skills and developmental stages.

From political bureaucracy, modern consciousness derives the complementary themes of orderliness and taxonomization. Bureaucracy creates and maintains order and predictability through rationalized procedures, i.e., normal channels. Bureaucratic systems of procedures are typically "based on a taxonomic propensity" (Berger et al., 1973, p. 49) similar to but more arbitrary than the componentiality of technological production.

> Phenomena are classified rather than analyzed or synthesized. The engineer puts phenomena into little categorical boxes in order to take them apart further or to put them together in larger wholes. By contrast, the bureaucrat is typically satisfied once everything has been put in its proper box. (p. 49)

The myths of the right answer and of stages and styles can be seen to persist in part as a consequence of their congruence with these bureaucratic themes of modern consciousness. They also gain plausibility from themes more specific to the U.S. experience.

The myth of the right answer also is sustained by its compatibility with a Christian religiosity. Compatibility is evident in the shared assumption of known "truth" residing in a sacred text (Bible, textbook) as interpreted by authorized experts (clergy, teachers). Further, the desired truths are to be learned by repetition (catechism, drill and practice). Whether in church or in school, youth are to accept, remember, and recite the right answers as defined by their elders.

The myth of stages and styles reflects a particular liberal ideology of individualism, as well as the general modern consciousness. Its assumptions of discrete categories and linearity (e.g., concrete operations are distinct from and precede formal operations) are consonant with technical rationality, componentiality, and taxonomization. The assumption that individuals have a particular stage or inherent style reflects not simply attention to individuality but a "possessive individualism" which assumes individuals to be the proprietors of their capabilities and characteristics, owning them much as one might possess an automobile or a green sweater (Popkewitz, 1983, 1985). It is an American conception of the individual that can be traced to early Protestant beliefs in progress, liberal democratic political theory (e.g., the seventeenth-century political philosophy of Locke and the eighteenth-century political economy of John Stuart Mill), and the ideology of laissez-faire capitalism. In the twentieth century, it finds expression in psychological approaches to measurement and task analysis intended to further scientific management of schooling for purposes of social efficiency. The compatibility of this conception of individualism with technical rationality and other themes of modern consciousness helps to sustain each and to give credence to the myth of stages and styles.

The social functions of myth also help explain its persistence. Insofar as particular myths exemplify cultural themes, they not only derive support from those themes but serve to dramatize them as cultural ideals and to orient individual and collective action toward their realization. Thus, myth becomes an integral, sustaining element of everyday life. It is difficult to imagine, for example, curriculum and teaching stripped of the myth of the right answer. What would school classrooms be like without the recitation and the worksheet?

The cultural cohesion provided by myth is closely related to myth's orienting function. Shared ideals and beliefs offer a feeling of community and a comforting measure of security in that community. Myth also serves to promote the interests of particular subgroups--professional educators in this case. Myths such as those examined here contribute to professional identity and claims to expertise by providing a language incomprehensible to most lay people and by suggesting that professionals possess the knowledge and capacity to deal with the problems they have identified. Educators are the official proprietors and interpreters of myths to which the uninitiated have neither direct access nor the right of appeal. Participation in myth-based ritual fosters group solidarity; the myths become symbols of group membership and grounds of moral legitimacy (Durkheim, 1964).

Myth also persists in curriculum discourse and practice because it provides an illusion of explanation that serves to direct action and thereby to justify particular social interests and professional practices. The descriptive-prescriptive function of myth has already been touched on. The description that myth offers is often taken as explanation, as if to name or label a phenomena such as cognitive stage or style is to comprehend and perhaps to control it. On close examination, however, the illusion of understanding disappears and the tautology emerges. The myth of thinking skills, for example, explains neither the nature nor the development of thought.

The labels and their mythic elaborations do direct practice; one example is the admonition that teachers adapt their teaching and/or expectations for learning to the identified stages and styles of individual students. Myth thus justifies school practices such as ability grouping or tracking; it makes differentiation of students and curriculum seem reasonable and appropriate (Popkewitz, 1985). Myth is perpetuated in part because it supports the political, economic, or cultural interests of socially dominant groups, justifying what might otherwise be viewed as discriminatory practices. Myth thereby diverts attention from different problems of curriculum and teaching or provides excuses for not attempting to resolve them.

The influences of technological production and political bureaucracy on modern consciousness explain the persistence of many myths in the domain of mathematics. One of the most poignant demonstrations of the effects of a technical rationality was a study by Anyon (1983), in which she compared the interpretations of "work" in schools serving four different social classes. In the mathematics classes in those schools, both the processes and the products of student-teacher interaction showed four distinct philosophies of work.

In a working-class school, instruction in long division followed a show-and-tell approach. Concentration on the division algorithm led Anyon to conclude that "work is following the steps of a procedure. The procedure is usually mechanical, involving rote behavior and very little decision-making or choice" (p. 149).

In a middle-class school, students were encouraged to think and to make decisions in the course of applying the division algorithm, but the betraying attitude was that work meant getting the right answer: "I want you to make sure you understand what you are doing--so get it right." Anyon explained the middle-class notion of work: "If one accumulates enough right answers, one gets a good grade. One must follow directions in order to get the right answers, but the directions often call upon some figuring, some choice, some decision making" (p. 153).

In an affluent professional school, work was "a creative activity carried out independently" (p. 155), while in the most economically exclusive school, prolonged discussion and analysis, as well as the application of principles to new situations, led Anyon to conclude that work was "developing one's analytic intellectual power" (p. 159).

Anyon's study highlights the pervasive and debilitating nature of the input-output mentality and suggests that curriculum reform will be facilitated by heightening public awareness of the implications of the "information age" (Zarinnia & Romberg, 1987).

The demands of a bureaucratic mentality for orderliness, normalcy and predictability also have invaded the mathematical domain. A demand for uniformity was, until recently, a contributing factor to the low-priority status of individual differences research. Consider, for example, Gagné's (1962) cumulative learning theory as applied to mathematics. The underlying principle in his hierarchy-based instruction was that individual differences were explained by individual momentary states of knowledge and skill with respect to a given task (Resnick & Ford, 1981). It was thought that students with the same level of prerequisite knowledge and skills would show little variability in their mastery of successively more difficult mathematical tasks. Due to considerable time differences in individual patterns of mastery (Resnick, Wang, & Kaplan, 1973), the predictability and reduction of variability in performance expected from the learning hierarchies were not realized.

Cognitive research is concerned with areas where bureaucrats fear to tread. Consequently, since the 1970s, the mathematics education community has been more accepting of irregularity, unpredictability and individuality. Individual differences researchers see computer simulation models of student performance, such as those developed by Briars and Larkin (1984) and Riley, Greeno, and Heller (1983), as powerful methodologies for investigating differences in the ways more- or less-able students solve problems and for identifying potential implications for planning optimal instruction (Threadgill-Sowder, 1985).

RECONSTRUCTION OF MYTH AND CURRICULUM

To relate the myths of thinking skills, the right answer, and stages and styles to modern consciousness, cultural themes specific to the U.S. experience, and the historical-social functions of myth is to begin to understand the social construction and persistence of myth in curriculum discourse and practice. If curriculum is to be reconstructed in ways that affirm cultural ideals, the limits of both our conceptions of curriculum and our guiding myths must be recognized and challenged. While socially functional in some respects, myth is also preemptive. Myth serves to obscure as well as to illuminate, to impede as well as to generate thought and action. Reconstruction of myth and curriculum requires moving beyond prevailing conceptions to consider or create alternatives.

This is not to call for the exorcism of myth. To do so would be futile, for myth is integral to social life. Myth probably is inevitable, and not all myths are pernicious. In curriculum, our myths need not obscure social conditions that impede school learning by psychologizing or otherwise masking them. Neither need they foster practices contrary to espoused purposes. Instead of limiting students' opportunities, our myths might expand possibilities for all students and make more accessible the means of their attainment.

The particular myths examined here are neither neutral or inevitable. They have been created out of U.S. history and culture and the particular experiences and interests of professional educators and researchers within that context. As myths become incorporated into the everyday language and practice of curriculum, however, their original purposes and underlying values are gradually lost, and the myths acquire a universal quality that tends to deter challenge. We ought not to be so intimidated.

Consider, for example, possible alternatives to prevailing technocratic conceptions of curriculum and rational management models of curriculum making (Cornbleth, 1985b, 1985d). Typically, curriculum is taken to be a tangible product such as a document or plan to guide classroom teaching and learning, and curriculum making is seen as a technical project in which one follows a predetermined set of procedures to develop the curriculum product. The step-by-step development procedures imply that curriculum is composed of discrete components (e.g., objectives, subject matter, content, materials) that can be separately constructed, often in a linear sequence, and then assembled to make a coherent curriculum product. The procedures, intended to provide rational, i.e., efficient, management and control of development resources and activities once curriculum policy decisions have been made by others, are assumed to be value neutral, and without social, political, or ethical consequences. Curriculum making, thus conceived as a technical project of efficiently managing resources to produce a tangible product, gives the appearance of being scientific and conveys images of efficiency, effectiveness, and benevolence.

While questions have been raised regarding the viability of the technocratic conception of curriculum and rational management models of curriculum making, as well as their underlying assumptions of objectivity or value neutrality (Cornbleth, 1985b), a technical project view of curriculum persists largely for the same reasons as the previously discussed myths. It is compatible with modern consciousness and U.S. experience, and it is socially functional in that context. Not surprisingly, a technocratic approach to curriculum and myths such as thinking skills, stages and styles, and the right answer are mutually reinforcing.

An alternative conception of curriculum that is less supportive of such myths is a social process view (Cornbleth, 1985b). Here, curriculum construction is conceived as an ongoing social activity that is shaped by various contextual influences within and beyond the classroom and accomplished interactively by teachers and students. Curriculum is not a discrete, tangible product but the actual, day-to-day classroom interactions of students, teachers, ideas, and materials. The curriculum is the curriculum-in-use. Planning documents are seen as one aspect of the context that shapes curriculum-in-use.

In contrast to the technocratic view, a social process approach does not separate curriculum policymaking, construction, and implementation. Instead of assuming or prescribing a linear sequence of discrete events, a social process view assumes a dynamic, interactive relationship among policy, planning, enactment, and their sociocultural context, i.e., curriculum is constructed and reconstructed in practice. A social process conception of curriculum and its construction is less hospitable to the myths of thinking skills, stages and styles, and right answers than is the technocratic view. A social process conception would be supportive of constructivist notions of thinking, individuality, and learning, i.e., reconstruction of persistent myths.

The reconstruction of myth, however, is neither simple nor straightforward. Not only does it require modification of curriculum beliefs and accompanying practices, but also concomitant modification of the institutional conditions that sustain modern consciousness. Such change is not without precedent, and countermodernizing trends are now evident in other institutional sectors; examples include modifications of assembly line and business management practices and the revival of "wholistic" medicine. The challenge is to cultivate skepticism and a critical stance that renders both myth and institutional arrangements fragile and susceptible to reconstruction.

Meaningful instruction in mathematics, that is, instruction that promotes an understanding of mathematics deep enough to make mathematical principles functional in other disciplines and nonacademic settings, has been a highly illusive commodity. While computer simulation models have become increasingly effective in providing knowledge regarding the components of successful performance on certain tasks used in mathematics instruction, they are no panacea. It is becoming clear that we must look beyond the confines of the mathematics education discipline for guidance in extending successful performance beyond the walls of the classroom. In chapter 19, Greeno (1987) has done just that. In a review of current literature from several fields, he cited studies destined to provide some perspective on what constitutes meaningful instruction in mathematics.

Greeno identified three instructional environments that have been found to be conducive to the active construction of knowledge. These include (1) collaborative settings in which teachers and students cooperate to construct meaning, (2) situations in which teachers first model activities and then act as guides while the students are engaged, and (3) situations that encourage the exploration of ideas, such as generating and testing hypotheses.

To provide a link, then, between mathematical knowledge gained in the classroom and reasoning in noninstructional settings, he indicated that studies from the field of cognitive anthropology, which focus on reasoning in practical settings (situated reasoning), are of exceptional interest. Ultimately, it appears that this type of interdisciplinary communication and cooperation will affect the intraclassroom communication of mathematics so that classroom processes will, in turn, affect the application of mathematics across all disciplines and settings.

POSTSCRIPT
by Monitoring Center Staff

Cornbleth's triad of myths is rooted in and perpetuated by such strong sociopolitical, economic, and cultural themes that its influence shows no respect of disciplinary boundaries. The very myths that divert attention from critical issues in social studies curricula have, at times, masked a famine in empirical support for certain practices in mathematics education.

It would be grossly inaccurate to imply, however, in a current state-of-the-art assessment, that mathematics education is steeped in myths that preclude lucid discourse. Even before World War II, mathematics educators envisioned the rapid technological changes that were imminent, as well as the role that mathematics education would play in world developments. During the last 20 years, the mathematics education community has been empirically supplanting inappropriate maxims, intuitions, opinions, and assumptions concerning every component of the teacher-mathematics-student synergy and suggesting new options for curriculum reform.

What is or should be the impact, then, of Cornbleth's perspectives on curriculum reform? She reminds us that curriculum reform is an endeavor that encompasses more than the reworking of an intended outcome. The implemented and achieved mathematics curricula will necessarily be poor matches to the intended curriculum if the "hidden" curriculum is ignored. Myths, personal belief and values systems, affect, and mathematical epistemology--all nebulous but real phenomena--will undermine the best-laid plans.

In essence, Cornbleth's assertions constitute a consciousness-raising effort. Unless our inevitable technological consciousness is tempered by a student, teacher, community, and national cognitive-affective consciousness, plans will go awry.

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Chapter 26

TEACHER PROFESSIONALISM

Allan Pitman

The current discussions concerning the future of schooling in the United States embrace a range of issues, including the content of curriculum, length of the school year and day, number of years of schooling, and a set of issues involving the definition of the work of a teacher. This chapter focuses on these teaching issues and, in particular, on those aspects of the job of teaching that relate to notions of professionalism.

The discussion is structured as follows. First, the variety and ambiguity of meanings associated with the common usage of the term "professionalism" will be considered. Second, the question of what might define the profession of teaching, and more specifically of mathematics teaching, will be addressed. This discussion concludes with an attempt to outline the "job specifications" for such a profession. Third, the current ways in which the work of mathematics teaching is defined in this country will be described and contrasted with the earlier description of a profession. Fourth, the variables associated with the definition of the profession of mathematics teaching will be identified as a mechanism for monitoring change in teaching.

THE AMBIGUITY OF "PROFESSIONALISM"

> In other professions in which men engage, the army, the navy, the church and the stage . . . (Gilbert, 1888)

The difficulty with the common use of the term "teacher professionalism" lies in its inability to distinguish between the behavior of individuals within an already defined occupation and the behavior of members of an occupation considered to be a profession. This problem becomes evident in reference to the dictionary definitions of "profession," "professional," and "professionalism." It will be seen that the notion of a professional can be defined fairly independently of the idea of a profession.

The Oxford English Dictionary (1981) allows one to trace the notion of a profession to its religious roots, where it is defined as a vocation to which one makes profession. The religious orders can be considered a historical benchmark in the search for characteristics that differentiate professions from other occupations. On the other hand, a person may be professional in sport simply by accepting money to engage in it as an occupation.

Professionalism carries both "profession" and "professional" as stems.

Let us look first at the definitions and etymology of "profession." Its roots are traced to the latin professionem, "a business or that which one publicly avows." Uses of the term "profession" in reference to "a particular order of monks, nuns, or other professed persons" is traced by the dictionary to Chaucer in 1386, and its application to medicine was first noted in use by Copland in 1541. By 1581, Pettie referred to the bearing of arms as a profession. Addison, in 1711, talked of "The three Great Professions of Divinity, Law, and Physick." This line of reference leads to the dictionary's definition.

> III. 6. The occupation which one professes to be skilled in and to follow. a. A vocation in which a professed knowledge of some department of learning or science is used in its application to the affairs of others or in the practice of an art founded upon it. Applied spec. to the three learned professions of divinity, law, and medicine; also to the military profession.
>
> b. In wider sense: Any calling or occupation by which a person earns his living. (Oxford English Dictionary, 1971, pp. 1427, 1428)

The second part of this definition notes a "wider sense" and contains the nub of the ambiguity in the current discussion about professionalism. In elaboration, the dictionary remarks that it is:

> Now usually applied to an occupation considered to be socially superior to a trade or handicraft; but formerly, and still in vulgar (or humorous) use, including these. (p. 1428)

This confusion about what we mean by a profession pervades our interpretation of professional behavior and of professionalism. Both the adjective and noun uses of "professional" bear investigation. The adjective is first referred to in an obsolete usage: "pertaining to or marking entrance to a religious order." Definitions regarding the practitioner include:

> 3. Engaged in one of the learned or skilled professions, or in a calling considered socially superior to a trade or handicraft.
>
> 4. That follows an occupation as his (or her) profession, life-work, or means of livelihood, as a professional soldier, musician, or lecturer; spec. applied to one who follows, by way of profession or business, an occupation generally engaged in as a pastime; hence used in contrast to amateur, as in professional cricketer . . . b. Of play, sports, etc.: Undertaken or engaged in for money,

or as a means of subsistence; engaged in by professionals (as distinct from amateurs).

5. That is trained and skilled in the theoretic or scientific parts of a trade or occupation, as distinct from its merely mechanical parts; that raises his trade to the dignity of a learned profession. (p. 1428)

The range of definitions offered for "professional" as a noun covers the gamut from

B. (substantive) 1. One who belongs to one of the learned professions; a professional man

to

2. One who makes a profession or business of any occupation, art, or sport, otherwise usually or often engaged in by amateurs, esp. as a pastime. (p. 1428)

It is not surprising that by the time the dictionary reaches "professionalism" the definitions are becoming very broad:

1. Professional quality, character, method, or conduct; the stamp of a particular profession.

2. The position or practice of a professional as distinguished from an amateur; the class of professionals. (p. 1428)

It is important to take note of these meanings because they direct attention to the debate about whether individuals should be reformed to better meet the tenets of an assumedly adequate occupation, or whether current definitions of the job of teaching adequately permit it to be described as a profession.

The religious orders can be characterized by a set of quite particular features. They claim access to a specific body of knowledge. They are restrictive in their membership, and they maintain control over entry to their ranks. They claim the existence of a greater value in their work. They look to their own leadership for direction, to the point of oaths of obedience. In the other occupations generally thought of as professions, these characteristics have been not only adopted but codified. It is instructive to compare these characteristics with the variables of professional organizations identified by Hall (1969):

<u>The use of the professional organization as a major reference.</u> The structures of those occupations generally perceived of as professions include strong national organizations or networks of organizations. These bodies assume a central role in the conduct of their respective occupations, not only in an "industrial" sense but also in matters of ethics and the policing of standards of entry and performance.

Belief in public service. A belief held both by practitioners and the larger public is that the accepted professions provide a needed service to society not available from other occupational groups.

Autonomy. Members of accepted professions have within their job definitions the right and responsibility to make significant decisions relating to their practice. It is important to note that this element is autonomy, and not license.

Sense of calling to the field. A belief prevails that practitioners within professions receive intrinsic satisfaction from their work based on a sense of the moral and practical worth of the task.

Self-regulation. The locus of control of the accepted professions rests significantly within the occupational network. Boards of registration control national examinations, admission and expulsion of members, and contribute significantly to the establishment of the law as it relates to practice.

Each of these characteristics, with the exception of autonomy, bears a close resemblance to the characteristics of the religious orders described earlier. It is worthy of note that Hall failed to include in his set of variables the claim of access to a specific body of knowledge and the codification of these "professional" characteristics.

How could the occupation of teaching be characterized in terms of knowledge base and occupational structure? Among the documents associated with the current debate about the state of education in the United States, several touch on the issue. The Holmes Group (198_) and the Carnegie Forum (1986) are the most specific, but a number of the earlier reports also expressed positions with respect to the appropriateness of current definitions of the world of teachers' work. In particular, Sizer (1984), Goodlad (1984), and Romberg (1984) presaged a number of the concerns expressed in the two later reports.

In general, Sizer (1983) argued, the first wave of reports in the current reform movement served to legitimize traditional, back-to-basics trends. The most well known of these documents are A Nation at Risk (National Commission on Excellence in Education, 1983), Making the Grade (Twentieth Century Task Force, 1983), and Academic Preparation for College (College Board, 1983). One of the themes that Sizer identified in the 1983 reports is that of authority. In his discussion, Sizer (1983) noted the paradoxical nature of this theme.

> Paradoxically, however, the fresh authority specified in many reports does not reach these instructors--something that one might expect to happen if we wish to raise teachers' status and make the role autonomous enough to attract strong candidates. On the contrary, the new

> authority flows primarily from regulation of courses required for the high school diploma, of examinations controlled by outside authorities, of precise daily regimens, and yearly calendars set by people outside the classroom or school building.
>
> Even more ironic is the fact that there is an opposite trend in professive American business--devolved authority. While many commentators say they support building level decision making, the effect of their total package is to hobble the individual teacher even more. (p. 5)

The locus of control, Sizer (1983) observed, is moved firmly to the states by a succession of reports which themselves have their roots in the political rather than the academic world.

Goodlad (1984) also decried the tendency of the states ". . . to focus on principals, teachers, and individual schools in their efforts to assure accountability" (p. 274). Goodlad advocated district accountability based on decentralization of authority and responsibility to schools within a framework which assures equity and accountability. "The school, then, would become largely self-directing" (p. 276).

The set of ten recommendations of the School Mathematics: Options for the 1990s conference (Romberg, 1984) included two that placed the responsibility for coordination of mathematical curricular reform squarely with teachers and their professional organizations. The conference concluded that the establishment of school mathematics committees at school or district level should provide a mechanism linking teachers with colleagues at every level; in addition, the committees should deal with research findings, teacher isolation, parental concerns, and college expectations. Professional organizations were charged with the responsibility for providing leadership through a steering committee for mathematics education; the point was explicitly made that the role is one for the professional groups, and not for government.

A common response to the perceived crisis in education has been to legislate improvement. The range of enactments covers specification of the minimum hours to be devoted to key subjects in the curriculum (Wisconsin), increments in the minimum number of years of mathematics that children must study (Pennsylvania), and specification of teaching practices (Tennessee).

A PROFESSION OF MATHEMATICS TEACHING

This section addresses the question of whether the occupation of teaching can be defined in such a way that it qualifies as a profession, or whether, in Etzioni's (1969) terms, it can only aspire to status as a semiprofession. Whereas a profession (or an occupation that aspires to that status) holds no impediments within its definitions that would prevent its being restructured to satisfy all of the criteria demanded, a semiprofession contains within the core work itself definitions that would preclude full satisfaction of the definitional requirements. Teaching and nursing are two occupations that, Etzioni argues, cannot aspire to the rank of a profession. This discussion concluded, however, that it is possible to conceive of teaching as a profession, and suggestions were offered about what the work of a member of that profession might involve.

The occupation of teaching might be viewed as a profession if it is defined according to the characterization discussed above, given the broadening of the definition beyond the original clergy, medicine, and law. In this case, one must be able to identify teaching "professed knowledge" and to demonstrate the ways in which occupational practice satisfies the suggested criteria.

A useful insight into the issue of teacher knowledge is provided by Shulman (1986), who established classifications of "content knowledge" and "forms of knowledge." He identified three categories of content knowledge: subject matter content knowledge, pedagogical content knowledge, and curricular knowledge. The three forms of knowledge he labeled propositional, case, and strategic.

The content knowledge characterization is important in that it focuses on the interrelation of subject matter content and the creation of situations in which teaching and learning related to that subject matter take place. A vital aspect of this relationship is the explicit recognition of the generalization of knowledge bases beyond the specific lesson in two directions. The first of these is in a depth of knowledge in the subject matter of the lesson beyond the immediate content. The second is in the knowledge of how the specific lesson is located within a sequence of educational experiences to which the students are subjected.

Shulman's forms of knowledge direct our attention to the fact that knowledge is structured in ways that differ from one individual to another. Further, the knowledge-in-action that informs practice provides the dynamic link between content and teacher behavior.

A consequence of this set of definitions is that attention is drawn to two distinct elements of the differentiation of knowledge structures held by members of a "profession," and the social structures that permit the term for some occupations but deny it to others. As Shulman pointed out, the bifurcation of knowledge about content and pedagogy is not a fundamental necessity, as the historical definitions of university activities from the Middle Ages demonstrate.

It is the purpose of this chapter to concentrate on the characteristics that might define the profession of teaching mathematics. The body of knowledge is here defined as the intersection of pedagogy, cognitive theory, and mathematics: the interplay of teacher, student, and content. This knowledge in combination is unique to the practice of mathematics teaching. Any single element is essential but not unique to the task. In many respect, the arguments presented here complement Romberg's (1987) proposition that "the teaching of mathematics has an inescapable social context" (p. 31), and that it is possible within the cultural and social structures of the United States for teaching to be transformed into an accepted profession.

The application of this knowledge as service (Hall, 1969) in the form of pedagogical practice represents its "application to the affairs of others" (Oxford English Dictionary, 1971, p. 1427). Further, the pedagogy defines a form of practice distinct from that of others bearing either mathematical or psychological knowledge.

The cognitive theory informs the practice: without a theoretical framework about learning--albeit an incomplete one--pedagogy cannot be rationally structured. Over the years we have seen a succession of learning models, including the humanists' image of the brain as a muscle, and more recent theories based on developmental psychology and on information theory. It is not important whether the cognitive theory is "correct"; what matters is that it has general support in the occupation and is generally supported by empirical evidence.

Mathematics is the content of the interaction between teacher and pupil, and it is what differentiates mathematics teachers from other teachers. It is not unreasonable to suspect that the content will relate in particular ways to cognitive processes, and hence inform particular pedagogical practices. It is, however, a moot point whether this should suggest a separate "profession" or a specialization within a generalized profession of teaching.

Thus, the three elements combine to support a unique occupational definition in terms of a body of knowledge and its application to others. How then can this be made consistent with the possibility of a profession of teaching in light of Hall's five criteria? Currently the definitions of a teacher's work vary from one district to another and from state to state. Definitions here differ from those accepted in other countries. In some instances, these definitions come close to describing the work as professional; in others, the term would be inappropriate.

Let us now consider a possible profession of teaching in terms of Hall's five criteria.

The use of the professional organization as a major reference. A range of professional organizations are potentially capable of acting as major referents. The national teacher organizations, AFT and NEA, might provide general teacher professional foci with respect to dissemination of information, provision of coherence of occupational identity, and a political identity in negotiation and public relations. The specialist organization, NCTM, is in a position to provide a complementary focus for mathematics teachers.

All of these organizations already have networks of state and local affiliates providing in varying degrees publications and activities for members.

Belief in public service. While this criterion appears to relate to the orientation of the individual member, there is a sense in which it is applicable to the occupation as a group. The application of knowledge through pedagogy provides the link whereby the practice of teaching constitutes an act of public service. The community perception that practice of an occupation involves the provision of a public service is an integral component of the relationship between the professional and the client--in this case, the pupil, parent, or school board member.

Autonomy. A differentiation is made here between autonomy and license. Within the definition of the occupation based on field of knowledge and its application to other occupations, a group of teachers can be given responsibility for the work that they do. Teachers who are given responsibility for their decisions and actions also achieve a degree of occupational autonomy. In every teaching job there is some measure of autonomy, however curtailed: the "black box classroom" metaphor attests to the inevitability of this. Legislation such as that which established Tennessee's career structure rejects this position, however, and tries to eliminate autonomy in pedagogy.

Sense of calling to the field. The practitioner is said to demonstrate a sense of calling if the motivation for, and the rewards of, practice are identified as intrinsic. In other words, extrinsic rewards such as remuneration are not dominant sources of motivation. If teaching is not yet a profession, educators must identify those occupational definitions that need to be transformed in order to engender such intrinsic motivation.

Self-regulation. It is conceivable that the occupation could effectively regulate its membership and standards of competence and behavior. National and state bodies, possibly newly constituted, could establish and maintain standards of entry and codes of conduct. National and state registration, recognized by all authorities, would need to be placed in the hands of boards that represent both the profession and the community. These boards would also define acceptable standards of professional education and training, and would set and administer codes of conduct.

Teacher competency evaluation within this framework would become the responsibility of the occupational membership. Evaluation would become the task of peers within the profession rather than the role of administrators who would be perceived as outside the profession. This is not to preclude the idea of principals becoming more closely integrated into the occupational group of teachers, but to suggest that the responsibility would not be theirs alone.

What, then, might a profession of mathematics teaching look like to one of its members?

It would be one in which he or she would be expected to keep abreast of developments in teaching and in mathematics. This expectation would be translated into an understanding on the part of clients and employers that the work of a teacher extended beyond the classroom. Teachers would be expected to attend professional conferences and seminars, offered under the aegis of professional organizations.

As a member of the profession of teaching, he or she would be viewed as the provider of a valued service to the public, a service that could not be attained from any other occupational group.

As a professional, the teacher would be held responsible for the selection and teaching of the curriculum content. The sequencing of content, not only within one unit of a child's career in mathematics but throughout the years of schooling, would be the responsibility of a school faculty as a group and each teacher individually. Professional autonomy would create a demand for coherence at the departmental level.

The profession would be one in which the teacher was seen to have a commitment that transcended considerations limited to extrinsic motivations such as monetary rewards. This does not suggest that salary is not a legitimate concern, but teaching professionals would experience a sense of calling related to the intrinsic belief that the task is worth doing.

Teachers would be governed by codes of conduct and competence determined and enacted by the profession. Promotion would involve assessment by peers and would be directed toward positions involving greater responsibility for the curriculum and pedagogical stances of the school in which the professional worked, and a concern for the guidance of new members. The differentiation between principals and their teachers would most likely be weakened as the role of principal could become one of educational leadership, based on promotion through teaching and curricular responsibility. General administrative responsibilities could rest with school managers.

CURRENT DEFINITIONS OF TEACHERS' WORK

The theme of this section is that the historical emphasis on scientific management of education has profoundly influenced the American schooling system. Its effects manifest themselves in the objectives movement, in the production model of schooling, in the dominance of summative testing, and in a restricted conception of the work of a teacher with regard to the generation and control of curriculum. The metaphor of school as factory is pervasive.

A review of notions of accountability and of employment conditions of teachers leads to the conclusion that the relevant metaphor for teaching is the work of a factory foreman. Such a model can be traced through the writing of Bobbitt (1912, 1913, 1922, 1924), Charters (1922), and Charters and Warples (1929) to much of the discussion in the reform documents of the 1980s.

These documents vary in content in significant ways; some such as the Carnegie Forum paper contain a critique of current notions, while others do not question the metaphor.

An implicit assumption underlies the work of those who subscribe to the process-product industrial model of schooling. It relates to the nature of a teacher's work. Whether one takes the approach of Bobbitt (1922) in his Los Angeles recommendations, or of Charters and Warples (1929), or of planners of courses such as the PSSC physics and BSCS biology, the objectives are determined by experts. The teachers, as clients of the developers, have little control over the corpus of their work--the curriculum. If the children are the rank and file, the teacher is reduced to the position of foreman, with no control over or responsibility for determination of the process, but only for its faithful implementation. This view makes the notion of teaching as a profession very difficult to sustain.

The decontextualized and depersonalized nature of the industrial metaphor is overt in Bobbitt's work, but less so in that of other writers. It is significant, however, that Bobbitt could articulate the pervasive metaphor in the language of plant, working efficiency, elimination of waste, and the processing of raw materials into finished products (Bobbitt, 1912, pp. 260-269). It is within the context of this metaphor that the objectives movement is to be best understood. The structure of objectives such as that suggested by Mager (1962) follow closely the model suggested by Bobbitt (1913):

> At the end of a year's work, an eighth grade child shall be able to copy figures in pencil on paper at the rate of 117 characters per minute. (p. 21)

The characteristics of terminal behavior, important conditions, and acceptable performance criterion specified some fifty years later by Mager are evident in this example.

The practical outcome of this metaphor can be seen in the definition of teachers' work: Romberg (1985) has concluded that in the United States, the job of teachers is

> to assign lessons to their class of students according to some schedule, explain the rules and procedures of each lesson, judge the actions of students during the lesson, and maintain order and control throughout. (p. 5)

Both Romberg (1985) and Shulman (1986) noted the disappearance of subject content from the lists of essential specifications for effective teaching and many of the attempts to evaluate teaching. The heavy reliance on commercial texts to provide both curriculum and content knowledge has served to impede the development of teaching as a profession.

In the last five years there has been a growing public argument that the quality of teaching practice in the United States is not of a satisfactory standard. A Nation at Risk (National Commission, 1983) and the plethora of reports that followed it all leveled criticism at the teaching force. Subsequent legislative actions tended to reinforce this perception of an incompetent work force by imposing ever more restrictive limits on the decision-making roles of teachers, by supporting the use of competency testing to evaluate teachers, and by moving to control curriculum.

A competing movement argues that, by establishing the occupation of teaching as a profession, a higher caliber of person can be attracted. The Carnegie Task Force's (1986) report, for example, suggested initiatives that echo the Flexner Report (1910) on medicine. Attention is directed to pre-employment training standards, to remuneration, to supply of support staff, and to control of the work in schools. It is possible that the strategies outlined in this report could form a mechanism for moving toward the establishment of teaching as a profession, albeit one unabashedly based on the processes employed to codify the practice of medicine in the United States in ways seen as consistent with the maintenance of the occupation as a profession.

In summary, then, it is argued here that the pervasive view of schooling in the United States is one based on an industrial factory metaphor. This perception has led to a conceptual differentiation between process and product in schooling, with the "individualization of education" as the process by which differentiated raw materials are treated. The quality control methods are normative testing, and the role of the teacher is limited in significant ways to that of factory foreman.

MONITORING POSSIBLE CHANGE

In contrasting current definitions of teachers' work with a professional ideal, certain variables can be identified that may be subject to change. These variables are associated with the definition of the occupation and with the behavior of individuals within the job specification--that is, with both "profession" and "professional."

The tentative listing that follows is presented in two parts, the first dealing with variables associated with the definition of the occupation and the second with members' behaviors and attitudes within a contemporary occupational context. To further systematize the listing, the categories suggested by Hall's (1969) scale have been used. It should be noted, however, that their application here is broader than that intended by their author.

It was deemed most important to draw attention to the fundamental dichotomy between objects of measurement: the occupational definition and the practitioner's definition. As a consequence, no attempt has been made to carefully define the variables operationally or to impose levels of measurement upon them. These two tasks are most appropriately the work of those charged with monitoring such changes as do occur.

Variables Associated with the Definition of the Occupation

Professional Organization as a Major Reference

It is obvious that, if a professional organization is to become a major reference, it must have an infrastructure sufficient to enable interaction between it and the members of the occupation. Thus, the existence of local affiliates of professional organizations such as the National Council of Teachers of Mathematics (NCTM) and, more broadly, the National Education Association (NEA) and/or American Federation of Teachers (AFT) are key indicators.

The activity of these local or regional affiliates might be monitored through observation of the frequency of meetings and the rates of attendance. Substantive content of meetings could be categorized. Frequency of publication of professional materials by the national and regional affiliates in the form of journals, newsletters and bulletins is another indicator of activity.

Activity of national professional organizations in representing the interests of the occupation directs attention to the use of the professional bodies as major referents for interest outside teaching. Frequency of meetings and their attendance (conferences, etc.), and the substantive content of the meetings offer insight to the ways in which the organization is providing a professional focus, both intellectually and politically, for its members. Such information also would track the organizations' involvement in national policy making as representatives of the occupation. Lobbying on behalf of the occupation and the substance of the organizations' involvement in policymaking should be monitored.

Belief in Public Service

Public statements of codes of ethics by national, regional, and local professional organizations and contractual statements of employment will yield information pertinent to the degree to which the work is seen as being of public service.

Occupational Autonomy

Terms of employment, as they are defined through state legislation and local contractual arrangements, spell out teachers' responsibility for curricular content in range and sequence, for selection of curricular knowledge along with its divisions and allocation of time, and for pedagogy, its types and range. Also, the capacity of teachers to step outside their classrooms to participate in other professional activities is central to the definition of their occupational autonomy.

Sense of Calling

Prioritizing job characteristics in advertising for the occupation offers insight into the extent to which the occupation can draw on the intrinsic worth of the work, rather than on external rewards unrelated to the task, in attracting recruits. Industrial attractions based on payment, hours of employment, and length of the working year can be juxtaposed against intrinsic rewards based on a sense of satisfaction and the social value of the task.

Self-regulation

Standards of conduct can be codified either from within the occupation or by outside forces. The extent to which the locus of control rests with teacher organizations and representative boards rather than with legislatures is a critical measure of self-regulation. Further, the specificity of regulation with regard to pedagogy and associated classroom behaviors relates both to self-regulation and to the assumption of individual autonomy. Regulation of entry and maintenance of membership through the registration of teachers would define national and regional loci of control.

Variables Associated with Individual Behaviors

Professional Organization
as a Major Reference

Membership of NCTM and its affiliates, and of NEA or AFT and affiliates, would provide an initial coarse measure of the extent to which individuals look to the professional organizations as points of reference. Rates of participation in local affiliate meetings and rates of readership of professional materials originated by the organizations and their local affiliates would provide data about the quality and quantity of information sought from the organizations.

Belief in Public Service

The public's perception of teachers and teachers' views of themselves and their occupation as the provider of a unique service to society might be measured by survey and interview.

Occupational Autonomy

The scope of the occupational task as defined by the public and by teachers offers evidence of the general status of the occupation and its specific status in local contexts. The evaluation of teacher performance and the beliefs of individual teachers will yield direct information about the definition of the work of teachers from the perspective of practitioners.

Sense of Calling

Teachers' explanations of why they choose to remain in the occupation, or choose to leave it, provide evidence of the degree to which the intrinsic value of the work is the prime consideration in the seeking of reward in the occupation.

Self-regulation

The expression of teachers' beliefs about the appropriate forms of occupational control of standards, behavior, pedagogy, and content offers insights into the perceived extent of self-regulation and its distance (if any) from that seen as ideal.

CONCLUSION

The current discourse concerning professionalism in the teaching occupation conceals an ambiguity that allows conflicting solutions to be offered to the problem of schooling in the United States. By differentiating between characteristics relevant to the definition of the occupation and those relevant to the behaviors of practitioners within the current definitions, a set of variables has been identified which would permit the tracking of transformations in the work of teachers. Further, a case was made that current metaphors of teaching and schooling will need to be replaced if the occupation is to attain the status of those traditionally thought of as professions.

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Chapter 27

MONITORING SCHOOL MATHEMATICS: TEACHERS' PERCEPTIONS OF MATHEMATICS

John Conroy

This chapter deals with issues that are fundamental to questions concerning the need to develop a system to monitor the health and progress of school mathematics; namely, issues related to the underlying beliefs and values held by teachers concerning the nature of mathematics and how these might change over time. The issues are treated under four headings:

1. the need to acknowledge the importance of teachers' perceptions of the nature of mathematics;
2. matters associated with assessing teachers' perceptions;
3. recent considerations related to teachers' perceptions;
4. changing teachers' perceptions.

It will be argued that the success of any action aimed at influencing the health and progress of school mathematics will depend on the relative congruence between the perceptions of the majority of teachers concerning the nature of mathematics, and the perceptions of those who initiate the changes.

THE NEED TO ACKNOWLEDGE THE IMPORTANCE OF TEACHERS' PERCEPTIONS OF THE NATURE OF MATHEMATICS

One of the four sets of variables identified by Dunkin and Biddle (1974) as influencing classroom events is labeled the presage variables. These variables include teacher formative experiences, teacher training experiences, and teacher properties. Similarly, Van Fleet (1979, reported in Brown & Cooney, 1982) suggested that teachers acquire their knowledge and beliefs about teaching through three somewhat related processes:

enculturation, that is, contact with a wide range of teachers at school;

education, that is, direct learning through their own teaching experience and from interaction with colleagues;

schooling, that is, teacher education programs, removed from the school environment.

Teachers' perceptions, beliefs and values concerning the nature of mathematics, mathematics education, change related to school mathematics and schooling fall into the presage-variables category. Dunkin and Biddle's (1974) analysis of previous research on teaching reveals that, for a variety of reasons, presage variables have not been highly successful in controlling and predicting classroom events. These authors believe that two classes of presage variables that may be more effective include (a) formative experiences that "have left a significant impact on teachers or . . . cause continuing differential responses to teachers among pupils" and (b) the wide range of teacher coping behaviors that are "subject to beliefs held by the teacher concerning the curriculum, the nature and objectives of the teaching task, expectations for pupils and norms concerning appropriate classroom behavior" (p. 412).

We could assume, therefore, that investigators of the teacher perceptions addressed in this monograph would give some reasonable indication of how a teacher might be expected to behave in the mathematics classroom, and that, if we wish to see certain changes in teachers' behaviors, we must help teachers change whatever existing perceptions will most likely influence those behaviors.

There are indications that teachers' perceptions of the nature of mathematics and mathematics education influence their pedagogical repertoire (Davey & Nesbit, 1979) and govern their teaching design, choice of approaches and activities, and general orientation to teaching (Bell, Costello, & Kuchemann, 1983). Bishop and Nickson (1983) observed that

> teachers' perceptions of mathematics clearly are a vital constraint in the complex classroom situation in which they work. These perceptions inevitably interact with, and affect other, teacher characteristics which further constrain the teaching/learning situation, and outcomes in terms of mathematical learning. (pp. 43-44)

Similarly, Thompson (1984) found that

> If teachers' characteristic patterns of behavior are indeed a function of their views, beliefs, and preferences about the subject matter and its teaching, then any attempt to improve the quality of mathematics teaching must begin with an understanding of the conceptions held by teachers and how these are related to their instructional practice. Failure to recognize the role that the teachers' conceptions might play in shaping their behavior is likely to result in misguided efforts to improve the quality of mathematics instruction in schools. (p. 106)

Texts and curriculum materials also reflect the perceptions of the nature of mathematics, of mathematics education, and of changes in school mathematics held by their authors. The extent to which a mathematics teacher is committed to the use of specific texts or materials determines the extent to which the authors' perceptions are adopted and perpetuated. Alternatively, the teacher's choice of materials, methods, and approaches may be determined not so much by the actual perceptions of textbook authors or curriculum developers as by the teacher's (mistaken) beliefs about the perceptions of such authors, based on the teacher's own bias (Smith, 1983).

Even more crucial can be students' perceptions of mathematics acquired in high school. On the one hand, student attitudes and their willingness to enroll in advanced mathematics courses may be affected if tension exists between the students' and their teachers' perceptions of mathematics (Skemp, 1979, in Bishop & Nickson, 1983). On the other hand, and more insidiously, student teachers at graduation may recognize discrepancies between their current perceptions of the nature of mathematics and their remembered but limited perceptions of high school mathematics, viewing the latter as comfortable and satisfying. Will teachers so new to the profession be sensitive to the need for change in school mathematics, or will they be content to perpetuate outdated but familiar practices and restricting procedures (Galbraith, 1984)?

Interactions between and perceptions of the nature of mathematics and attitudes toward mathematics held by teachers and pupils are both important and complex. It is argued here that perceptions of mathematics help form attitudes, rather than vice versa. Thus, the perceptions and attitudes of teachers (and also of significant others) act on students' perceptions and attitudes, shaping learning effectiveness and influencing decisions about further mathematics studies and future careers. These interrelationships are illustrated in Figure 1.

This diagrammatic representation demonstrates the dominant influence of perceptions in this chain of dependencies. Therefore, in planning change, not only must perceptions be taken into account as prime targets, but it must also be recognized that change that involves a chain process requires a considerable time lag before clear results can be achieved and observed. It would be a mistake to pursue curriculum change in the belief that teacher perceptions will change simply through the adoption of new texts, materials, and the like. It seems unrealistic within the model proposed to assume that teacher perceptions are flexible and easily malleable; consequently, changing teacher perceptions to agree with the demands and emphases of some accepted view of the nature of mathematics and of mathematics instruction must be a top priority when curriculum change is to be implemented.

As indicated here, the issue of teachers' perceptions of the nature of mathematics, of mathematics education, and of change in school mathematics is essential to the general health and progress of school mathematics. There is a need first to measure these perceptions; how must this be done? Secondly, there is the need to know what information exists about teachers' perceptions, such as the nature of those perceptions, and the effect of any lack of

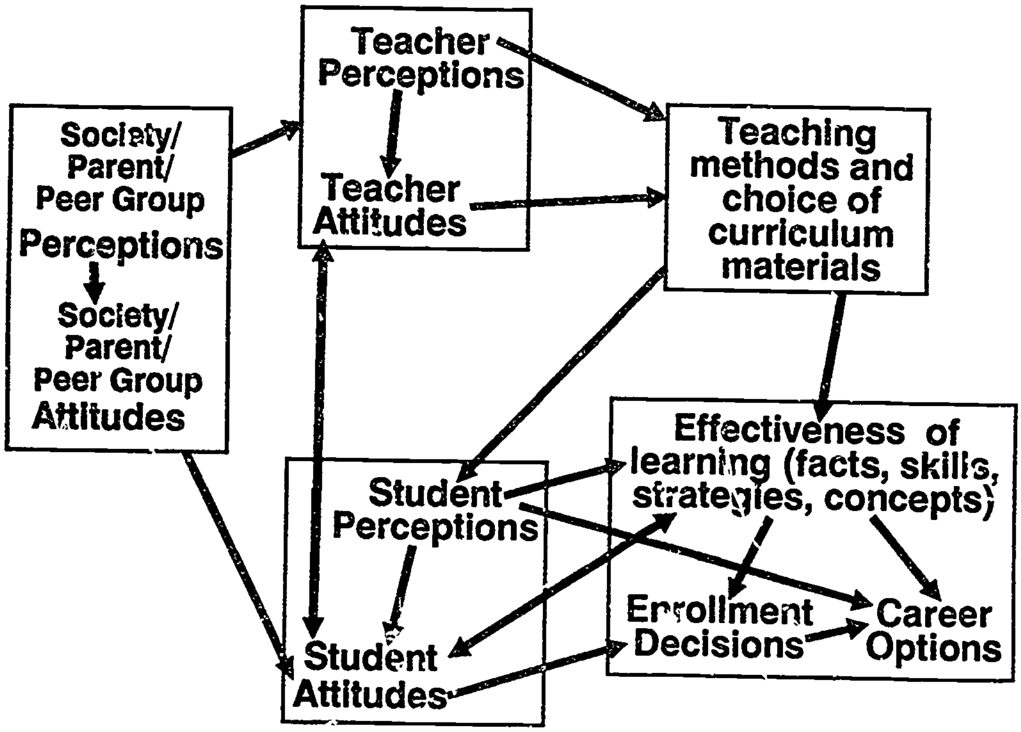

Figure 1. The place of perception in the mathematics education process.

congruence between teachers' perceptions, the perceptions of students, and of those engaged in instituting change. Thirdly, if such discrepancies between perceptions do exist, how are changes to be achieved? What kinds of inservice education have been or will be most effective in bringing about changes of the desired kind, in the desired direction, and of the desired degree? How long must teacher inservicing procedures be extended to be successful and lasting?

ASSESSING TEACHERS' PERCEPTIONS

If we wish to measure some property of a person or object, it is essential to possess a clear idea of what that property is so that it can be recognized in the first instance, and so that an appropriate instrument can be selected to measure it in the second. Writers have variously used the terms attitudes, beliefs, conceptions, perceptions, values, views and the like to describe the teacher property we are addressing. Although there may be clear links, as in the case of attitudes and perception, we would want to rule out attitudes as a descriptor of the teacher characteristic under consideration because this term suggests affectiveness, implying such semantic differentials as like/dislike, easy/difficult. Also, we would reject the term view as suggesting an unfounded opinion or personal bias. Perception of the nature of mathematics is used here in the sense of a firm (though not necessarily sound) belief as to the nature of mathematics, which in turn and in varying degrees might determine behaviors, attitudes, and values.

Initially, measurement of teachers' perceptions tended to be by means of some form of inventory to determine level of agreement with the idiosyncratic views of a particular author. Such were the two instruments used by Angelo (1970). He dichotomized the high school curriculum into "why-topics," dealing with structure, logic and proof, and "how-topics," dealing with computation and symbol manipulation. He then devised a checklist of 14 topics belonging to each of these categories and asked respondents to rank each topic on a scale from 0 to 5 according to its relative importance in their perception of the nature of mathematics. Angelo's second instrument used the notion of forced choice items. To each of 16 mathematical problems, two alternate responses were attached, both equally correct. One, however, was a "why" response; the other, a "how" response. Respondents selected the preferred "best" response. The two measures could be used as indicators of consistency between perception and preferred action. Angelo's technique may be criticized on the grounds that it provides a rather personal and, therefore, a limited view of the nature of mathematics.

Scales developed by Collier (1972) were favored by several subsequent investigators. These inventories consisted of 20 items, and responses were recorded on a Likert scale according to the way in which respondents perceived their beliefs about mathematics and mathematics instruction on a formal/informal continuum.

A similar instrument using a 5-point Likert scale and a similar continuum had been used by Rettig (1971, reported in Scheding, 1981), who attempted to validate the items by asking a group of college professors to indicate which items they believed "would discriminate." Items were then piloted on secondary mathematics teachers, and those items were retained that were able to discriminate between respondents whose total scores were in the top or bottom 35%.

Harker (1979) appears to have been the first to approach this issue critically, to express a need to provide a theoretical framework around which measurement procedures might be built, and to establish suitable criteria for the development of an instrument. Because investigations of teachers' perceptions of the nature of mathematics were so rare, Harker surveyed the literature of science education on the issue of perceptions. She found limitations of four kinds among instruments used in this field: those related to content, to models, to global scores, and to closed-choice response questionnaires.

Typically, inventories included a mix of items related to attitudes to and ethics of science, as well as to the nature of science. Resulting total scores were less than meaningful.

A frequent problem was defining what is the "correct" or "agreed" view of the nature of a discipline. Harker's view was that provision must be made to represent various models from the literature of the philosophy of science/mathematics, and to allow for views held by students that might lie outside this range.

Global scores, such as those referred to in the first point, above, hide much of the data's detail and can conceal differences in the patterns of responses among individuals whose total scores are similar.

Closed-choice response questionnaires tend to prompt individuals into giving preconceived answers. Instruments using this technique tend chiefly to be concerned with the degree to which respondents agree with a single model of the nature of the discipline. Harker sees more value in trying to gain some image of the particular model held by the individual.

Finding no suitable instrument or strategy for investigating perceptions of the nature of mathematics, Harker suggested two major principles for their construction. First, contrasting models must be recognized. Second, free-response questions must be included to find the respondent's own view of the nature of mathematics; items would be content analyzed. One method would simply ask respondents to write for, say, ten minutes on "What is mathematics?" Harker (1979) used this technique as part of an unreported study, as did Leder (1979).

Scheding (1981) chose to build on Harker's ideas and attempted also to establish a theoretical foundation for his study. He identified four views of the philosophical foundations of mathematics among mathematicians and philosophers of science: the formalist, intuitionist, logicist, and platonist views. He rejected each of these as the definitive position because mathematicians generally acknowledge that none provides an entirely satisfactory foundation. None agrees about which provides the correct view, and many mathematicians decline to profess allegiance to any one particular philosophical position. It was not reasonable, therefore, to design a survey based on one of these approaches and expect lay teachers to express clear ideas on matters that were still disputed by professionals. Instead, he selected approximately 12 core beliefs on which there is no substantial disagreement among mathematicians and condensed these into seven facets of the nature of mathematics:

1. The nature and attributes of mathematics systems; mathematics as an organized body of knowledge; the generality of mathematics.

2. The nature and attributes of proof; the roles of deduction and induction in mathematical discovery and proof.

3. The role of insight and intuition in the work of the mathematician.

4. Beauty in mathematics, the aesthetic value of mathematics, mathematics as a creative art.

5. The relative importance of massive or complex numerical calculations and abstract or symbolic thought in the work of the mathematician.

6. The relationship of mathematics to the real (physical) world; the degree to which applications of mathematics are mathematics.

7. The status of the foundations of mathematics; the existence of differing views of the nature of mathematics. (Scheding, 1981, p. 118)

On the basis of these seven facets, Scheding produced an inventory of some 50 items, approximately one-third of which were derived from Rettig or Collier (1972). He circulated the inventory (first as a restricted pilot and, later, as a more widely published draft) to university mathematicians in the U.S. and Australia. Items finally selected were thus given content validity in that mathematicians generally agreed or disagreed that the items reflected some aspect of the nature of mathematics. Items were also clustered to represent a position on one of the seven facets. A high score on any one facet or on the inventory as a whole would thus reflect a view that was in particular or general agreement with professional mathematicians.

Thompson (1984) took a quite different approach and used the case study method. Three junior-high-school mathematics teachers were observed teaching a mathematics class every day for four weeks. During the last two weeks, daily interviews lasting about 45 minutes were held after each lesson. This procedure was adopted to familiarize the researcher with the social context in each case and to enable independent inferences to be made about each teacher's perceptions before these inferences became colored by direct input from the teachers. Inferences about beliefs based on observation of instruction could then be tested against teachers' own professions of their beliefs. Lessons were audio-recorded for later analysis and for use as a point of reference during interviews. The interviews, generally related to events occuring during the day's lesson, also were audio-recorded for later analysis.

During the study, each teacher also was asked to complete six writing tasks. Five items asked teachers to express their views about (a) the relative importance of certain instructional goals,

(b) the relative importance of certain instructional objectives, (c) the relative importance of certain instructional practices, (d) the reasons for student failure, and (e) information used in judging their own teaching effectiveness. The sixth task was to complete an instrument comprising six bipolar dimensions for describing mathematics. Thompson's analysis consisted of a comparison of the teachers' professed views about the nature of mathematics against their instructional behaviors. A cross-sectional analysis of the teachers' conceptions was used to explain key differences among them.

Erlwanger (1974, reported in Bell et al., 1983) also used the case study method in an attempt to clarify understanding of children's perceptions of mathematics. He sought to describe the general direction and trend of each child's thinking underlying his/her observable behavior. Another study of student perceptions was made by Preston (undated, also reported in Bell et al., 1983). Apart from the mention that factor analysis techniques were used, no other detail of Preston's method was available.

To sum up, the majority of studies over the past 20 years have used some form of inventory to gain information about teachers' and students' perceptions of the nature of mathematics and mathematics instruction. Three difficulties have been associated with this technique:

selecting and validating items to represent a generally agreed on view of the nature of mathematics;

developing scales to take into account the multifaceted nature of mathematics; and

allowing for the expression of alternative views about mathematics.

Some studies have addressed (c) by inviting answers to the open-ended question, "What is mathematics?" Some have used a case study approach. Not until more research is generated can there be an assurance that there exist sound, adequate, and varied tools of investigation.

The next section examines specifically some of the theory and research dealing with perceptions of mathematics. This includes perceptions among teachers; perceptions among students; perceptions among teachers and their students, and the match between the two; the consistency between teachers' professions of belief about the nature of mathematics and their observed classroom behavior; and the stability of perceptions and their susceptibility to change.

REVIEW OF LITERATURE RELATED TO TEACHERS' PERCEPTIONS

Considerations of perceptions may be concerned not only with <u>what</u> perceptions teachers hold about the nature of their discipline, but also with the <u>degree of correctness</u> or <u>closeness of</u>

agreement of their views with some currently accepted view. Perceptions may also be regarded as single-faceted or multifaceted, and may be treated from a defined or open-ended perspective. We have already made clear that we are not dealing with teachers' attitudes toward mathematics, though these are closely influenced by perceptions. Bowling (1976, quoted in Scheding, 1981) pointed out that attitudes about and perceptions of the nature of mathematics are distinct and different entities. There is, indeed, a growing volume of literature dealing with affective issues in the field of mathematics education; there is considerably less dealing with perceptions. In discussing perceptions, we are concerned with the beliefs held by an individual about the nature of a specified area of knowledge or behavior. Inevitably, this discussion will incorporate questions of values in the sense that there is an implication that teachers' beliefs influence the ideas and behaviors they will favor and emphasize, and (consciously or unconsciously) regard as acceptable in their students' mathematical activity.

In brief, research suggests that teachers' perceptions or beliefs tend to be regarded as a presage given; their formation tends to be addressed in general terms only, and they tend to be regarded as unsusceptible to change. They are, however, becoming recognized as powerful determinants of teacher behavior. The following survey does not claim to be exhaustive. It presents several investigations important to the issue of monitoring the health and progress of school mathematics.

Angelo's (1970) study compared the perceptions of the nature of mathematics held by college faculty and their freshmen students. It will be recalled that two instruments were used (a checklist and a forced-choice inventory). The results showed that distinctly different perceptions were held by each group: faculty rated topics dealing with structure, logic, and proof significantly higher than did freshmen, while freshmen rated topics dealing with computation and symbol manipulation significantly higher than did faculty.

Van de Walle (1972) attempted to determine the influence of different perceptions of mathematics on students' mathematical learning (that is, student computational ability and comprehension of mathematical concepts) and on their attitudes toward mathematics. He developed a 16-item "perceptions of elementary school mathematics" instrument using a formal/informal continuum based partially on the Collier and Rettig (referred to in Scheding, 1981) scales. A "formal" perception regarded mathematics as "a rigid set of memorized rules, facts and procedures"; an "informal" perception saw it as "probing, creative and involving definite aspects of originality and trial and error." The instrument was given to third-grade and sixth-grade teachers.

Van de Walle found that third-grade teachers with informal perceptions of the nature of mathematics were associated with student comprehension of mathematical concepts if those teachers also had a positive attitude about mathematics. If their attitudes were negative, however, they were associated with student computational ability. Students in third grade performed higher on all three measures if they were taught by teachers with informal perceptions. No significant relationships were found between sixth-grade teachers' perceptions and their pupils' math performance.

In a study by Foster (1977), student teachers entering college for their initial training were presented with the two inventories developed by Collier (1972) to evaluate beliefs about mathematics and beliefs about mathematics instruction. Foster found that, on entry, the amount of mathematics studied at school significantly influenced how students viewed mathematics. Those who had studied mathematics for 12 years or more prior to coming to college saw mathematics as a flexible discipline that encouraged creative and divergent thinking; those who had studied it for 10 years or less saw mathematics as a rigid, inflexible discipline that encouraged convergent thinking. With regard to mathematics instruction, both groups leaned slightly toward an informal view favoring a guided discovery approach to mathematics teaching and learning.

Harker's (1979) study, previously referred to, is an important one. While it does not report research findings as such, it raises a number of highly pertinent issues.

> Although there is a considerable volume of research literature on students' responses to the question What is science? there is little comparable data for mathematics. Such information can provide the basis for new curriculum development or can be used to assess the effectiveness of new curricula in developing children's perceptions.
>
> A study of instruments used in surveys of perceptions of the nature of science are open to a number of criticisms related to content, models of science, scoring and question format. (See the previous section.)
>
> Key factors are considered for designing an instrument to measure perceptions (also considered in the previous section).
>
> Needed areas of research are listed: (i) investigations of perceptions of students at various levels; (ii) longitudinal studies of changes in perceptions; and (iii) analysis of curricula and texts to determine the models of mathematics on which they are based.

Leder (1979) divided her study into two parts: an initial open-ended question that asked teachers to write a brief account of the best way to teach mathematics to students having difficulties, and a structured questionnaire based on Collier's (1972) instruments that measured beliefs about mathematics and mathematics instruction. Leder's results generally agreed with Collier's, namely, that elementary teachers hold more formal views about mathematics than they do about mathematics instruction, and the more formal beliefs are held by teachers with lower mathematical backgrounds. (Formality refers to an approach that is highly structured; informality refers to a view that is flexible and individually oriented.) Leder suggested that the informal views held by teachers with stronger mathematics backgrounds could be explained by their having a richer appreciation of mathematics and so being able to exercise greater flexibility in their teaching. Galbraith (1984) questioned whether this relationship need apply to teachers who are unable to resolve conflicts between their perceptions of mathematics at the college level and at the high school level. Leder's study did not consider how teacher beliefs related to actual classroom practice.

An important study by Scheding (1981) was built on a number of prior investigations into teachers' perceptions: Howard (1942, cited in Scheding, 1981), Angelo (1970), Rettig (1971, cited in Scheding, 1981), Van de Walle (1972), Collier (1972), Bowling (1976, cited in Scheding, 1981), Leder (1979), and Harker (1979). Only one of these (Harker, 1979) attempted to establish any theoretical foundation for the study of perceptions. Scheding's research (a) provided statements otherwise unavailable about the nature of mathematics with which mathematicians are known to be in agreement or disagreement; (b) provided data on the extent to which teachers' and prospective teachers' views of the nature of mathematics are in agreement with those of mathematicians; and (c) enabled analysis to be carried out in areas where views are in least agreement.

Using the inventory described in the previous section, Scheding surveyed elementary and secondary teachers and prospective teachers in Colorado and in New South Wales, Australia to determine the degree to which their perceptions of the nature of mathematics agreed with those of mathematicians in each country.

Analysis of his seven facets of mathematics revealed that, on facets 1 (attributes of mathematical systems), 4 (beauty in mathematics), and 6 (mathematics and the real world), teachers scored higher than prospective teachers. Secondary mathematics teachers scored higher than elementary teachers on all facets and on the total scale. Despite these differences, <u>all</u> respondents' scores were in the "correct" half of the total scale; that is, they tended to be in relative agreement with the perceptions held by mathematicians. Even so, Scheding found that one in five secondary mathematics teachers and one in three elementary teachers disagreed with mathematicians on facet 5 (massive calculations and abstract thought); one in five elementary teachers disagreed with mathematicians on facets 3 (the role of insight and intuition) and '; and one in nine elementary teachers disagreed with mathematicians on facet 7 (differing views of the nature of mathematics). Between American and Australian teachers, the only difference was on facet 6.

Scheding pointed to the undesirability of any teacher holding an "incorrect" view of any whole facet of the nature of mathematics. It would seem, however, that secondary mathematics teachers needed to place more emphasis on the importance of abstract or symbolic thought relative to massive or complex numerical calculations (facet 5). A significant minority of elementary teachers, on the other hand, appeared to have deficiencies in perceptions on four facets out of the seven: the role of insight and intuition, beauty in mathematics, the importance of abstract thought relative to large calculations, and the differing views of the nature of mathematics.

Analysis of responses to individual items suggested that many elementary teachers have a limited view of the nature of mathematics. Their differences from the "correct" view may, in fact, influence their students' views of the nature of mathematics in such areas as the use and importance of rote-memorized rules and formulae, the need for undefined terms, the need for systems to be related to real objects, the place of deduction in mathematical investigation, the importance of intuition, the notion of elegance, the universal properties of mathematical systems, and viewing mathematics narrowly as a study of numbers.

Scheding acknowledged certain limitations to his study. His definition of "correctness" of perception in terms of respondents' conformity with the perceptions of a group of mathematicians is not the only way to reach a definition. Secondly, since the set of perceptions were obtained solely from academic mathematicians, those perceptions may present bias. Thirdly, the inventory items might have been extended to include other matters (for example, computers) in relation to mathematics. Finally, there may be additional facets of the nature of mathematics to consider.

Thompson's (1984) case-study research on three junior-high-school teachers is also interesting. He concluded that

> teachers' beliefs, views, and preferences about mathematics and its teaching, regardless of whether they are consciously or unconsciously held, play a significant, albeit subtle, role in shaping the teachers' characteristic patterns of instructional behavior. (pp. 124-125)

For example, differences in teachers' prevailing view of mathematics accounted for contrasts in emphasis on such matters as the appropriate locus of control in the teaching process, what constituted evidence of mathematical understanding in students, planning and preparing for instruction, flexibility in teaching, and the overall cognitive goals and objectives of mathematics instruction. The degree to which their belief systems were integrated and the degree to which they reflected their actions in relation to beliefs, students, and subject matter were additional, general characteristics that differentiated teachers.

> [The] study found that the teachers' conceptions are not related in a simple way their instructional decisions and behavior. Instead, the relationship is a complex one. Many factors appear to interact with the teachers' conceptions of mathematics and its teaching in affecting their decisions and behavior, including beliefs about teaching that are not specific to mathematics. (Thompson, 1984, p. 124)

Erlwanger's (1974, quoted in Bell et al., 1983) detailed case studies of a small group of children led him to believe that their underlying conceptions of the nature of mathematics were expressed in their ideas, beliefs, emotions, and views about mathematics and learning mathematics. He regarded these conceptions as comprising a stable, cohesive and complex system, varying considerably, however, from child to child.

Preston's (undated, quoted in Bell et al., 1983) factor-analytic study of high school students in tenth grade found that boys scored significantly higher than did girls on two factors: mathematics in an open-ended, intuitive, and heuristic context; and commitment, interest, and application to mathematics. In addition, students in urban schools saw mathematics in a restricted and predictable context as compared with suburban and rural students. They also scored lower on the commitment/interest/application factor than did suburban and semirural students; rural children scored lowest on this factor. Of special interest from the point of view of curriculum change was the fact that students enrolled in a particular modern mathematics course differed significantly from the norm in perceiving mathematics as having a wide contextual application, in their flexibility in problem solving, and in their developed sense of intuition. These same students, however, showed a disappointing, significantly low score on commitment and interest.

Although the number of empirical studies is not extensive, various researchers have identified teachers' perceptions as being of vital interest. Bishop and Nickson (1983) surveyed research related to the social context of mathematics education and in several places remarked on the limited perceptions of mathematics among elementary teachers. They quoted a study by Ward (1979) that suggested that the limited mathematical background of elementary teachers diminishes their confidence in their ability to deviate from a narrow, factual path or to adapt curricula or methodologies to specific needs. Bishop and Nickson (1983) regard this limited perception of (as well as knowledge of and attitude toward) mathematics and mathematics curricula as an outstanding problem among elementary teachers. In referring to a study of elementary teachers by Bennett (1976), these authors commented:

> It is commonplace that the hierarchical nature of mathematics can easily impose a rigid structure on the way the subject is taught. It may too easily be accepted as "given" and tend to constrain teachers to

> present content in a particular order as well as in a particular manner. . . . (at elementary) level, a mechanical view of the nature of mathematics is likely to result in the teacher acting as a purveyor of mathematical facts, with pupils performing repetitive tasks in a somewhat passive manner. On the other hand (elementary) teachers who are aware of, and able to identify, the processes in the formation of mathematical concepts are likely to approach the teaching of the subject in quite a different way. This perception of the nature of mathematics clearly will result in a more varied classroom atmosphere, characterized in some degree by activity and inquiry. (pp. 62-64)

Bishop and Nickson also identified a report of the Royal Society (1976) that saw dangers, too, for high school mathematics teachers who perceive their subject as inherently possessing narrowly conceived academic standards. They concluded:

> Teachers' perceptions of mathematics clearly are a vital constraint in the complex classroom situation in which they work. These perceptions inevitably interact with, and affect, other teacher characteristics. (p. 43)

Other writers point to the importance of teachers' perceptions on the quality of experiences in the mathematics classroom. Nickson, writing in Cooney et al. (1985) stated:

> Clearly, whatever our perceptions may be, these will determine our beliefs and will provide a rationale for choices and actions and, ultimately, the kinds of teaching situations that are created in the classroom. If teachers are to meet curriculum developers on equal terms . . . , this step of identifying and clarifying beliefs and perceptions of their subject becomes even more important. . . . (Not) the least of (the benefits which follow) will be the examination of constructs used which may freeze teachers in a particular stance in their approach to the teaching of mathematics. (p. 29)

The Research Advisory Committee of the National Council of Teachers of Mathematics (1984) pointed to the prime need for research into teachers' conceptions of their role in the classroom, and the contribution research in this area might make to the design of teacher education programs and knowledge concerning teacher effectiveness. The committee posed three questions: When do teachers form these conceptions about mathematics and their roles as teachers? How strongly are these views held? How might these views be changed or influenced?

Brown and Cooney (1982) regarded the study of teachers' beliefs about teaching, about mathematics, and about how students learn as crucial to an understanding of the types of decisions teachers make, and as of equal importance as the striving to understand how children learn mathematics.

In summary, there appear to be four points that can be extracted from this brief survey of literature.

1. Investigation of the character of teachers' perceptions of the nature of mathematics suggests that perceptions are multidimensional (Scheding, 1981).

2. The length of time given to the study of mathematics and the depth of study will affect the flexibility of teachers' perceptions of the nature of mathematics (Bishop & Nickson, 1983; Foster, 1977; Leder, 1979; Scheding, 1981). Galbraith (1984) would add: "Up to a point."

3. Teachers' perceptions affect the nature and quality of the learning experiences they provide and their responses to curriculum change (Bishop & Nickson, 1983; Brown & Cooney, 1982; Cooney, Stephens, & Nickson, 1985; Thompson, 1984; Van de Walle, 1972).

4. Discrepancies may exist between perceptions of mathematics held by teachers and their students (Angelo, 1970).

THE SUSCEPTIBILITY OF PERCEPTIONS TO CHANGE

Ample evidence exists that teachers' perceptions have a significant influence on the way in which they approach the teaching of mathematics, and on the perceptions of the students they teach. It has also been emphasized that knowledge of the nature of teachers' and students' perceptions is vital information for curriculum developers in terms of curriculum implementation and evaluation. In this connection, it is also crucial to know how amenable teachers' and students' perceptions are to change. The evidence on this point conflicts.

Angelo's (1970) study provided a cross-sectional view of freshmen's and seniors' perceptions of mathematics. While it may be questioned whether or not cross-sectional studies provide a valid measure of change, the average rating by seniors of mathematics as a subject dealing with logic, structure, and proof was significantly higher than the rating by freshmen; that is, ratings by students who had had instruction in mathematics shifted to become more similar to views held by faculty.

Foster (1977) carried out a longitudinal study of college students planning to become elementary school teachers, divided into two groups: those with a limited background in mathematics and those with an extensive mathematics background. As they entered college, he tested both groups on their attitudes toward mathematics and their attitudes toward mathematics instruction, using Collier's (1977) scales. The students with the greater background in mathematics possessed a significantly more informal attitude toward mathematics than those with limited mathematics backgrounds, while both groups held an informal view of mathematics instruction.

The students were tested again at the end of their first year, during which they had completed a one-year background course in mathematics and three school practicums. Twelve months later, having completed an additional one-year course in mathematics curriculum and instruction emphasizing a guided discovery approach, they were given the Collier tests a third time. After the third testing, that is, after having completed courses in both mathematics and mathematics instruction, both groups showed a significant increase in the belief that mathematics is a flexible discipline that encourages creative and divergent thinking. In addition, both groups showed a significant change in their beliefs about mathematics instruction, favoring the guided discovery approach over more informal methods. At the end of the two years, the differences in attitudes between the two groups had narrowed considerably on both scales, and Foster proposed that the groups be tested after they had completed a period of full-time classroom teaching to check for possible further changes in attitudes.

Scheding's (1981) investigation included an experimental treatment of student teachers in training. An instructional module was given to two groups of elementary teachers in the second and third years of their preservice training at an Australian college. The instruction involved five to six hours of class-time activities that were, in essence, aspects of the work of a professional mathematician.

One group was tested three days after the end of the treatment; the other was tested six weeks later. A comparison of group means was used to investigate whether or not the treatment had affected the prospective teachers' views of the nature of mathematics and if these views persisted over time. There was no evidence to suggest that the experimental treatment had any effect on the prospective teachers' perceptions as measured by Scheding's instrument. The author felt that insufficient time had been given to the experimental treatment for it to have had a marked effect. It may be questioned also whether there might be alternative treatments that might have differing degrees of effectiveness.

The evidence is scant, consisting only of a single study on each of three categories: cross-sectional, longitudinal, intervention. Each of the investigations related to students in training; none followed these individuals into the classroom. No studies were traced that explored attempts to test whether or not it is possible to influence the perceptions of teachers in the classroom and, if so, under which circumstances.

> It would be interesting and very helpful then to know not only the nature of teachers' conceptual systems as they come to us but also how rigidly the systems are held and

> the extent to which they are modifiable. Such understanding is basic to the bulwark of our professional lives: the education of mathematics teachers. (Brown & Cooney, 1982, p. 17)

There are questions yet to be answered. Can longitudinal studies help us to determine the levels at which students begin to form perceptions of the nature of mathematics (Harker, 1979) and are these indeed stable (Erlwanger, 1974, quoted in Bell et al., 1983) or do they vary over time or with class teachers (Thompson, 1984)? What instructional treatments are effective in changing teachers' perceptions of the nature of mathematics and of mathematics instruction? Do teachers' perceptions interact with contextual factors such as grade level, students' abilities, and mathematical content taught (Thompson, 1984)? What relationship exists between teacher reflectiveness and consistency of beliefs, and between these and classroom behaviors (Thompson, 1984)? To what extent does mismatch between teacher and student perceptions create tensions that affect student attitudes, effective learning and decisions about further mathematical studies?

> As more is learned about teacher conceptions of mathematics and mathematics teaching, it becomes important to understand how these conceptions are formed and modified. Only then will the findings be of use to those involved in the professional preparation of teachers, attempting to improve the quality of mathematics education in the classroom. (Thompson, 1984, p. 126)

Clearly the importance of teacher perceptions of the nature of mathematics, of the nature of mathematics instruction, of the changing nature of mathematics education and schooling has been seriously underestimated. It is essential for the health and progress of school mathematics that future action directed toward curriculum change and teacher development must place the highest emphasis on the careful study and monitoring of perceptions.

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Chapter 28

EPISTEMIC TEACHING OF SCHOOL MATHEMATICS

E. Anne Zarinnia, Susan J. Lamon & Thomas A. Romberg

Introduction

To improve the quality of education in the United States, we will need more "outstanding new teachers in the years to come" (Boyer, 1986, p. xviii). The typical teacher is a middle-aged white female with a master's degree who has worked in the school district for about 15 years. She works at school just over seven hours daily and an additional couple of hours at home (Feistritzer, 1986). She has approximately 23 students (Feistritzer, 1986) in a self-contained classroom; her mathematics period typically lasts 43 minutes, and about half is spent on written work; instruction is directed toward the whole class; and a single textbook is used, being followed fairly closely as a source of problem lists. Other than the problems, students read no more than one or two pages (National Advisory Committee on Mathematical Education [NACOME], 1975, p. 77). Essentially,

> [her] job is to assign lessons to [her] class of students, start and stop lessons according to some schedule, explain the rules and procedures of each lesson, judge the actions of students during the lesson, and maintain order and control throughout. (Romberg, 1985, p. 5)

Romberg (1985) suggested that, in such situations, the teaching of mathematics becomes a procedural or managerial routine, often performed without care or reflection. In the context described, too many feel obligated to cover the book and too few see student learning and use of mathematics as the primary goal. Furthermore, teachers have an inadequate academic background in mathematics and insufficient opportunity for sharing information with colleagues. Under these conditions,

> it is hard to argue that teaching is really a profession. The teacher's job is related neither to a conception of mathematical knowledge . . . , nor to an understanding of how learning occurs, nor to knowing the likely outcomes of various instructional actions. [Thus,] research on learning and teaching has little relevance because the judgements and decisions being made are not about learning but about management. (Romberg, 1985, p. 3)

A typical assumption is that to improve teaching, one must improve teachers. However, an alternative view is that to improve teachers, one must improve teaching. This paper addresses one small part of that issue by conceptualizing the job of teaching mathematics to children as an epistemic process.

Changed Demands on Teachers

Expectations and Assumptions

> Paradoxically, teachers are the butt of most criticism, yet singled out as the one best hope for reform. . . . Much is at stake, for American students' performance will not improve much if the quality of teaching is not much improved. (Holmes Group, 1986, p. 3)

Teachers, however, are not the sole answer to reform. The educational system in a society is the means whereby expectations, values, knowledge, and standards of behavior--the vehicles for self-perpetuation of the society--are instilled in the young. "Education isn't something that any individual does. Education is a process, an organic process that involves the complex, dynamic interaction of multiple components" (Futrell, 1986, p. 6). Nevertheless, teachers are vital intermediaries in the process, and the current reform movement, unlike the Coleman Report (Coleman et al., 1966), attaches great importance to the crucial role of teachers in the education of children.

In chapter 2 it was argued that the perceived problem in education stems less from a decline in standards than from a change in expectations. In other words, the values have changed and so standards of judgement have also changed (cf. Popkewitz, 1984). Instead of needing adults able to function smoothly and effectively in a stable economy geared to mass production, society now needs people who are sufficiently independent in their thinking to adapt, innovate, and invent. Therefore,

> restructuring teaching requires, before all else, reflection on the purpose of that reconstruction. . . . The imperative for reconstruction derives from the need to better prepare students to adapt and improvise successfully in the face of relentless change, and relentless . . . retreat from established orthodoxies. We must help students become mentally agile, emotionally resilient, and intellectually adventurous. We must help them learn to be learners. (Futrell, 1986, p. 5; cf. Greene, 1982)

The Holmes Group (1986) described the naive view of teaching as "'passing on' a substantive body of knowledge. . . . 'planning, presenting, and keeping order.'. . . The teachers' responsibility basically ends when they have told students what they must remember to know and do" (pp. 27-28). This succinct, if simplistic, characterization described a perception of the role and work of teachers at the height of Industrial Age, which stemmed from

(1) an objective view of truth
(2) an iconic, formalistic view of knowledge;
(3) the notion of the learner as a product;
(4) a stimulus-response view of learning; and
(5) the need to efficiently prepare the majority of students to fit smoothly into a mass-production economy.

Each of the traditional assumptions on which the industrial age view of teaching was based has changed. Truth is being regarded as a social construction (Ziman, 1978); the prevailing view of knowledge is constructivist rather than formal (e.g., Lochhead, 1979a; Rescher, 1979; von Glaserfeld, 1983); the learner is seen as an active participant rather than a product; psychology has progressed beyond behaviorism to cognitive science and models of how information is processed and knowledge constructed; and the need is for people who can continue to learn in order to adapt to rapidly changing circumstances and to produce new knowledge. In other words, values have changed and, consequently, notions of effective teaching have also changed:

> Competent teachers are careful not to bore, confuse, or demean students, pushing them instead to interact with important knowledge and skill. Such teachers interpret the understandings students bring to and develop during lessons; they identify students' misconceptions, and question their surface responses that mask true learning. (Holmes Group, 1986, p. 29)

The Holmes Group (1986) addressed the importance of the affective aspect of teaching, especially as it contributes to motivation and cognition as a secure context for the intellectual risk-taking of conjecture. It also took account of students' existing and developing knowledge structures and pointed to the need for a diagnostic approach to students' misconceptions. The Carnegie Forum echoed the position of educational leaders in Britain and the Soviet Union (see, for example, Department of Education and Science (DES), 1985; Markova, Orlov, & Fridman, 1986) on the task of teaching when it stressed that

> students must be active learners, busily engaged in the process of bringing new knowledge and new ways of knowing to bear on a widening range of increasingly difficult problems. The focus of schooling must shift from . . . the passive acquisition of facts and routines to the active application of ideas to problems. (Carnegie Forum on Education and the Economy, 1986, p. 25)

<u>Redefinition of Literacy</u>

Both the Holmes and the Carnegie reports are a response to a change in values that imply a changed definition of literacy.

Minimal literacy acquired early in life through passive methods is no longer enough. The personal, national, and global problems to be faced and solved require adults who are able to continue to learn and adapt because they have confidence in their own personally created and firmly founded understandings (cf. Committee of Inquiry into the Teaching of Mathematics in Schools (CITMS), 1982). Radical change in the definition of literacy is epitomized by demands for universal computer literacy (e.g., Luehrman, 1981; Monakhov, 1986; "Second Stage Literacy," 1986). Succinctly, the situation requires that the industrial notion of literacy as conditioning for predictable reaction be revised to the notion of literacy as the capability for innovative adaptation.

This need for literacy to support continued learning draws attention to the Resnicks' (1977; cf. Freudenthal, 1973a) distinction between high and low literacy, the consequence of which was characterized by Venezky, Kaestle, and Sum (1987) as "a growing danger of a bifurcated workforce mirrored by a bifurcated educational system" (p. 52). Under the industrial model, teaching the low-literacy majority was essentially a matter of acculturing and training children to fit a system that required punctuality, obedience, predictable performance, and minimal literacy (see Anyon, 1981; Havighurst, 1966). There are obvious connections between this model, stimulus response theory, and teaching and evaluating according to behavioral objectives. At best, this industrial model of teaching amounted to Skemp's (1979) notion of instrumental learning, the acquisition of a repertoire of fixed plans.

Present demands, by contrast, require a high literacy model for all students. For example,

> No student except the severely handicapped should be able to choose courses in a way which, before grade 11[,] makes it impossible for him/her to move to a college preparatory curriculum. (CBMS, 1983, p. 11)

A clear implication is that all students should enter the eleventh grade with a high-literacy preparation in mathematics that is capable of supporting continuing education. This statement clearly rejects the notion of literacy as minimal training and replaces it with the ability to continue learning (cf. Carnegie Forum on Education and the Economy, 1986).

People only begin to learn when they go beyond what they are taught and teach themselves, reflectively taking initiatives (Greene, 1982, p. 32). Thus, high literacy is constructive, not passive, because it necessitates relational understanding, which entails "building up a conceptual structure (schema) from which its possessor can (in principle) produce an unlimited number of plans" (Skemp, 1976, p. 6). It is a dynamic concept of literacy as supporting quest, accommodation to new environments or changed conditions, and the active creation of knowledge. High literacy has an empowering quality (Greene, 1982) and political implications

(Hirsch, 1983) but is implicit in the repeated demand for adults who can think, learn, and solve problems.

However, the existing educational system is a tightly coherent ecosystem (Futrell, 1986), which is highly efficient at the function for which it was intended: mass low-literacy. A change to mass high-literacy supporting adults who can learn, adapt, innovate, and create knowledge is unlikely unless the entire structure is reconceptualized around that purpose. Thus, the crux of reform is establishing an equally coherent ecosystem for high literacy, including notions of teaching for high literacy.

Research on Teaching: Some Problems

Discussing the pivotal role of teachers, Futrell (1986) pointed out that they need the support of the research community to conceive and implement reform. However, most research on teaching has been conducted from the perspective of achieving universal low literacy with the greatest efficiency. Thus, the prevailing model of effective instruction in the United States (e.g. Rosenshine & Stevens, 1986) is predicated on methods of assessment sensitive to the objectives of low, rather than high, literacy. In addition, although the hazards of the research on which the model is based have long been clear (Berliner, 1975), highly respected practitioners have continued to base findings on it (e.g., Berliner & Rosenshine, 1976). The unfortunate result is that the enormous body of research on teaching offers inadequate guidance.

The problem is exacerbated by the normal tendency for people to apply new technology, whether intellectual or practical, first to old tasks (Joseph, 1984). To illustrate: cognitive researchers --spurred by the demands of the artificial intelligence community, and focusing on the structure of the knowledge base--have attempted to define the distinguishing attributes of experts and novices. For example, experts have a better structured and richer base than novices (Larkin, 1985). When this approach was adopted and applied to instruction, expert teachers were defined as those whose students had learned effectively, and their instructional behaviors were documented and compared to the behaviors of novice teachers (Leinhardt & Greeno, 1986; Leinhardt & Smith, 1985).

Unfortunately, the definition of students who had learned effectively was tied to their performance on standardized tests of achievement, which stress fragmented computation skills rather than the higher order thinking and metacognitive skills essential to the empowerment of learning. The methodology ignored Fey's (1969) observation that

> student achievement on some particular standardized test is a grossly inadequate measure for teaching success. More comprehensive diagnostic assessments of student outcomes must be developed and used. (p. 82)

By using standardized tests, the process provided a narrow definition of expertise. The teachers selected as models were expert at the direct teaching of attributes measured by standardized tests.

In addition, the expert strategies presented for emulation illustrated expository presentation of the record of knowledge in the shortest possible time by writing a definition on the blackboard and demanding choral recitation (e.g., Leinhardt & Greeno, 1986, p. 89). In this respect, Scheffler (1973) argued that passing on knowledge (cf. Holmes Group, 1986) to children through words is a myth, because all that words transmit is information on which to base beliefs. Knowledge, by contrast, requires insight (see Hatano & Inagaki, chapter 20), which students can only create by testing the predictive power of their beliefs. Furthermore, public validation of their knowledge requires them to advance reasonable arguments in its support (see Hatano & Inagaki, chapter 20; Ziman, 1978). Embodied in this vision of students' knowledge as substantiated beliefs are an emphasis on innovation as opposed to reproduction and a view of teaching and learning as involving confrontation, deliberation, and judgement. They are not processes for changing a raw material but an effort to coopt children from private to public life (Scheffler, 1973). One inference is that the model of teaching as the distribution of knowledge through exposition, "teaching as telling" (Goodlad, 1983), needs to be changed.

Literacy as Human Potential

Generalization of the prevailing model of effective low-literacy instruction to present purposes is suspect. The high-literacy characteristics being sought closely resemble those traditionally outlined as desirable for gifted pupils (e.g., Wright, 1983). For instance, demands for excellence (Task Force on Education for Economic Growth, 1986), accomplishment to the highest levels of ability (National Commission on Excellence in Education, 1983), problem solving (National Council of Teachers of Mathematics, 1980), and a commitment to future learning (Carnegie Forum on Education and the Economy, 1986) are the focus of both reform documents and of gifted and talented literature (cf. Feldhusen, 1985; Torrance, 1986; vanTassel-Baska, 1985; Wright, 1983). Unfortunately, research on teaching gifted and talented children offers little help for working toward mass high literacy in mathematics, because the predominant strategy in teaching mathematics to gifted children has been radical acceleration of the mathematically precocious (Stanley, Keating, & Fox, 1974), a compaction of the traditional curriculum for a tiny minority. It says little about the effectiveness of such teaching strategies as mentoring, self-direction, or lateral thinking for teaching mathematics to all students.

High literacy, concern for the education of the gifted and talented, and the prevalent emphasis on life-long learning all have in common the notion of human potentials. High literacy and gifted and talented education represent an effort to realize the potential capacities of a selected minority. Life-long learning, by contrast, stresses the "capability to become" (Scheffler, 1985, p. 58) and seeks to empower the expansion of potentials in the majority.

> Potentials and their realizations are not isolated and discrete but intricately linked to one another. A girl who is potentially good at mathematics becomes a different person with actual achievement of mathematical skill. New potentials arise with realization of the old; ways of thinking about related topics are now open to her, that were formerly closed. New feelings of confidence may contribute to potentials for other sorts of learning as well. Realization, in short, is prospective and not merely retrospective. It gives rise to fresh potentials at the same time that it consummates earlier ones. (Scheffler, 1985, pp. 11-12)

Scheffler's position is indirectly supported by Schrag's (in press) conclusions about higher order thinking: "Identification of level of thinking . . . seems inescapably relative to the resources of the thinker" (p. 35). This challenge to traditional assumptions is reinforced by Volovnikova's (1984) assertion that practice suggests that younger students are capable of a high degree of abstraction and generalization.

When seeking an alternative model around which to restructure teaching, one of the traditions challenged (see Romberg & Price, 1983) is the myth that children have an inherent and fixed set of talents which place prior limits on any teaching (Simpson, 1985). This a priori philosophy was consistent with a society that anticipated little change and expected the educational process to sort and guide children into an occupation appropriate to perceived ability. By contrast, a society demanding maximum fulfillment of students' capabilities (National Commission on Excellence in Education (NCEE), 1983) and the capability to constantly learn in order to adapt requires emphasis on children's learning processes rather than society's sorting process. Strategies of selection need to be replaced by strategies of instruction (Resnick, 1984).

Effective Teaching: Some Considerations

Teaching and Learning

> The overall goal is finding ways to construct a demanding school curriculum, informed by the results of psychological studies, which will lead Soviet children forward in their intellectual growth and development in the setting of the common school. (Szekely, 1986, p. 5)

The implication of Szekely's remarks is that curricula should be constructed in the light of research on understanding (cf. Bell, O'Brien, & Shiu, 1980; e.g., Bell, 1982a; 1982b). This echoes Gage's (1964) argument that theories of learning should be transformed into theories of teaching. However, any inference that theories of learning have not typically been transformed into theories of teaching is implausible in view of the many programs based on meticulously constructed behavioral objectives (e.g., Lindvall & Bolvin, 1976). The fundamental problem of teaching stems rather from the ossification of instructional patterns based on outdated learning theory and notions of fixed talent.

Recent research in cognition provides a picture of learning that is quite different from the assumptions of traditional classrooms, suggesting that children construct knowledge rather than simply absorbing what they are told (Romberg & Carpenter, 1986). The overriding purpose of schools now should be for children to learn how to learn, which may be restated as to learn how to create knowledge. Thus, Gage's (1964) argument needs to be reconsidered. Rather than separating theories of teaching from theories of learning, we should reunite and integrate them. The original distinction served the invaluable purpose of contrasting the growth of knowledge about the learning process with the rigidity of instruction, but continued separation can only perpetuate incoherence between models of teaching and revised views of the purpose of schooling.

> Historically, the pendulum of pedagogical innovation has swung between extremes of "structured instruction" (teacher as source of all knowledge) and "unstructured learning" (student obtains information via trial and error experience). To maximize the effectiveness of the new instructional technologies, much of the curriculum may be taught through "structured learning." In this approach, the pupil discovers knowledge within an organized, information-rich environment. The challenges faced are sequenced and tailored to individual needs, with help from the teacher or technological device available as required. (Dede, Zodhiates, & Thompson, 1985, p. 95)

Developments in technology and cognition are perturbing traditional assumptions and routines. Note that the model of effective teaching as described by Rosenshine and Stevens (1986) assumed a technology of print and voice distribution as well as a model of knowledge. Clearly, investigation of the use of computers for tasks that require intelligence and application of the findings to instruction has inevitably forced a close examination both of how learning takes place and of how teaching promotes the process of learning. The result is that technological developments are acting as a catalyst (cf. Olson, 1976) for integration. Integration is likely to be furthered by consideration of some implications for teaching of the accumulating knowledge about learning, especially the learning of mathematics.

Intrinsic Difficulties of Mathematics

Most important is the realization that mathematics is a very difficult subject for children to learn (Hart, 1981) because, at even a simple level, it piles abstraction on top of abstraction (von Glaserfeld, 1983). The essence of mathematics is the creation and manipulation of structures, which are inescapably "abstractions of extreme generality" (Fischbein, 1973, p.227). It is especially difficult when the original structure abstracted is a conception rather than a perception, as is the case with intensive quantities (Kaput, 1985, 1986b; Gray, 1979). Freudenthal (1983a) observed that "our mathematical concepts, structures, ideas have been invented as tools to organize the phenomena of the physical, social and mental world" (p. ix). Thus, it is important for teachers to contrive experiences in which children empirically derive new structures from practical perceptions (Fischbein, 1973, p.227) so that their mathematical understanding is not occluded by unnecessarily precipitate abstraction.

Notations, whether musical or mathematical, have no value unless to unequivocally describe a commonly held construct. That is the minimal definition of literacy. Contrary to the notion that mathematics is an unambiguous language (CITMS, 1982, p.1; Ziman, 1978), it is open to multiple misconceptions on the part of a novice. First, like any other language, it is contextual. Second, it is open to homonymy and synonymy (Adda, 1982). In normal language, the concepts involved can be distinguished because they are either derived from common experience or embedded in recognizable context; the unreasonable power of mathematics lies in the very abstraction and sparseness of its notation. The result is that the notation of mathematics is laconic; the deep structures of the mathematics are not necessarily revealed by the surface structures of the notation (Skemp, 1982).

In addition, the mathematical alphabet is open ended. New characters are devised when necessary to the development of the field. The emphasis on mathematical notation captures in microcosm the problem of teaching mathematics from the perspective of learning: Knowledge is a matter of personal construction; language is a matter of assigning names to concepts; communication entails assigning the same name to a commonly held concept; and literacy is a matter of using an agreed upon notation to describe a particular construct.

The inherent abstraction of mathematics is compounded for children by the traditional instructional emphasis on notation as opposed to knowing (Dionne, 1983). Traditionally, mathematics instruction has reversed the process whereby language is acquired --as concept, then as word, and finally as written notation-- proceeding straight to the formal notation as the most efficient means of delivery (Whitney, 1985). "Our children are expected to move straight to the contracted symbolism" (Howson, 1983, p. 569). In other words, the abstraction of mathematics has been emphasized by formal exposition in its notation. One result is to stress formulas, thereby sustaining a static view of the subject and inhibiting the growth of formal operations (Gray, 1979, p. 224; cf. Woodrow, 1982).

The dilemmas of children learning mathematics are comparable to those of adults studying a foreign language with an unfamiliar alphabet, concentrating on the vocabulary of an unfamiliar and abstract domain. Children need to have active and direct experience with the deep structures of situations to arrive at the surface structure of the notation (James & Mason, 1982).

Teaching for Long-Term Understanding

The intrinsic difficulties of mathematics and the problems created by neglecting meaning for notation--coupled with the demand for knowledge that will transfer to new situations--have made it essential to teach in such a way as to encourage long-term understanding based on well-founded knowledge structures. The capability to resume learning at any time to adapt to change requires a framework of support from which an understanding can be reconstructed, even though the details of its first introduction may have been forgotten (Bell & Purdy, 1985). Such frameworks of understanding underlie both further understanding and the crucially important ability to transfer knowledge and strategies to new situations (cf. Greeno, 1983).

Greene's (1982; cf. Volovnikova, 1984) stipulation about literacy--that learning begins when people go beyond what they are taught and teach themselves--implies that childrens' knowledge should be structured, that the child should be conscious of that structure, of the means whereby it was created and, hence, of strategies for its extension. Kieren (1983) argued that formal mathematical knowledge must be built on the informal and intuitive knowledge of the person (cf. Lampert, 1986a). Deliberate knowledge building requires mutual accommodation between the psychological, the mathematical, and the intuitive (Kieren, 1983), and a reconnection between informal, intuitive ways of knowing and the planned development of mathematical knowledge, which is the goal of teaching. Unfortunately, the school's emphasis on teaching computational and procedural knowledge effectively separates children's intuitive perceptions from any principled knowledge (Lampert, 1986b).

A strong coherence of opinions has emerged that points to the following as intrinsic to teaching for the construction of readily accessible mathematical knowledge structures capable of supporting further learning:

1. Didactic situations in which structures are developed or reinvented in the course of realistic involvement with phenomena (Bell & Purdy, 1985; Biggs, 1985; di Sessa, 1979; Freudenthal, 1983a; Greene, 1982; Lesh, 1985; Pollak, 1986; Piaget, 1973a; Wheeler, 1978; Whitney, 1985). The situation serves as both source

of conceptual structures and field of application (Treffers & Goffree, 1985).

2. Diversity of expression, using different representational notations, formal and informal, according to preference and purpose (Bell, 1985; di Sessa, 1979, p. 256; Kaput, 1979, 1986a; Lampert, 1986b; Lesh, 1985; Lesh, Landau, & Hamilton, 1983; Janvier, 1987; Skemp, 1979, 1982).

3. Dialogue, essential to the processes of knowledge construction (Ziman, 1978) and teaching and learning (Bell, Pratt, & Purdy, 1986; Biggs, 1985; Brown & Campione, 1987; Greene, 1986; Hatano & Inagaki, chapter 20; e.g., Lampert, 1986b).

4. Diagnosis, a means to ascertain obstacles to learning and to plan tactics for teaching (Bell, 1982a, 1982b; Bell, Swan, Onslow, Pratt, & Purdy, et al., 1985; Freudenthal, 1983b; Romberg, 1974).

5. Devolution, deliberate transfer of control to the student (Biggs, 1985; Brown & Campione, 1987; Greene, 1986; Kieren, 1983; Love, 1983; Markova, Orlov, & Fridman, 1986; Piaget, 1973b).

6. Dissimilarity rather than homogeneity in the size and composition of student groupings (Biggs, 1985; Brown & Campione, 1987; Freudenthal, 1978/80; cf. Lampert, 1986b).

Structures from Situations

The most obvious reason for the importance of situations is the demand for problem solving ability--a goal, a process, and a basic skill. "The ability to solve problems is at the heart of mathematics" (CITMS, 1982, p. 73) and teachers should do everything in their power to develop a student's capability (NCTM, 1980). However, problem solving as it is commonly construed in school mathematics consists typically of bare bones prose statements of preceding exercises in calculation that bear little relationship to real problems (Whitney, 1985) and are consequently insufficient to engender that level of "at-homeness" (CITMS, 1982, p. 11) with mathematical knowledge essential to coping with practical mathematical demands. Traditional teaching emphasizes practice in notational transformations, solution algorithms, or the skilled use of heuristics for solving problems as necessary precursors to coping with a problem. It ignores the fact that knowledge emerges from the problems rather than the other way around. Hence Piaget (1973a), Thom (1973), and Freudenthal (1973a) all stressed the importance of reinvention in mathematical education.

Despite stress on the direct teaching of heuristics (e.g. Schoenfeld, 1979), there is an increasing consensus that improvement of the capability to learn is inseparable from the specific domain of application. Artificial intelligence (Feigenbaum, 1984), comparison of expert and novice performance (Larkin, 1985), and work with the mathematically precocious

(Stanley, Keating, & Fox, 1974) all suggest that content knowledge is important to expert functioning. Understanding in a complex domain requires a great familiarity with its connections (Rissland, 1985); "good thinking almost always involves articulation between knowledge and strategies" (Pressley, 1986, p.144; cf. Chi, 1985; Larkin, 1979). The purpose for which knowledg was created and the process by which it was acquired are as essential as the formal structure of the ideas (di Sessa, 1979), because the mathematical meaning of the parts of a situation is often derived entirely from the situation of which they are a part (Lesh, 1985). More specifically, learning to solve problems embedded in a situation is important because "effective thinking is the result of conditionalized knowledge--knowledge that becomes associated with the conditions and constraints of its use" (Glaser, 1984, p. 99).

This kind of learning is most likely to emerge in a problem solving situation (Bransford, Franks, Vye, & Sherwood, 1986), suggesting that, instead of the traditional expectation that skill in computation should precede word problems, experience with problems develops the ability to compute (Carpenter & Moser, 1983). Thus, present strategies for teaching mathematics by first teaching skills and then exposing children to stylized application problems need to be reversed; knowledge should emerge from experience with problematic situations.

It is useful to characterize a problematic situation. Situation is here intended to mean a position with respect to conditions and circumstances. A genuine problem is a situation in which, for the individual or group concerned, one or more appropriately analogous ordering structures have yet to be developed. In devising problematic teaching situations, structures capable of simplifying complexity by lending order should not be immediately obvious. In other words, the situation should be complex enough to offer challenge but not so complex as to be cooperatively insoluble by the group of students (cf. Biggs, 1985; Markova, Orlov, & Fridman, 1986). The situation should also be epistemic (cf. Vergnaud, 1983); it should parallel the kind of phenomena for which mathematical structures have been typically created (Freudenthal, 1983a). Thus, pupils need to experience the phenomena for which such concepts, structures, and ideas were created. Such situations would guide the learning of mathematics but leave open the possibility for genuinely new knowledge to emerge (cf. Tall, 1980). In "didactical phenomenology" (Freudenthal, 1983a, p. 10), the experiences with phenomena are aimed directly at the development of mathematical structures from intuitions acquired in the course of realistic project work (e.g., Biggs, 1985).

Recommendations for emphasis on situations as effective contexts for children to develop the habit of identifying patterns and creating mathematical structures are becoming general (e.g., Bell & Purdy, 1985; Berthelot, 1985; Biggs, 1985; Bransford, Hasselbring, Barron, Littlefield, & Goin, 1987; DES, 1985; Freudenthal, 1983b; Love, 1983; Pollak, 1986; Vergnaud, 1983;

Volovnikova, 1984; Whitney, 1985). Freudenthal's (1983a) terminology, "didactical phenomenology of mathematical structures," placed emphasis on the classroom approaches most appropriate to the development of conceptual structures by children. To teach the mathematizing of situations, teachers need to create suitable contexts; the most abstract mathematics needs the most concrete contexts.

The situated teaching of mathematics involves more than provision of manipulatives or the use of realistic problems and applications after first teaching a sequence of skills necessary to solution of the problem. It requires, by contrast, realistic and complex situations which are susceptible to simplification through mathematizing. Mathematizing (e.g., Freudenthal, 1973a; Love, 1983) is "an organizing and structuring activity by which acquired knowledge and abilities are called upon to find out still unknown regularities, connections and structures" (Treffers & Goffree, 1985, p.109).

Succinctly, mathematics is about patterns (Biggs, 1985). Instead of instructing children in notations and expressions designed to represent patterns that others have agreed on (without ever discussing either "others" or the process of agreement), teachers need to provide situations in which the children themselves create the patterns (cf. Perkins, 1986) and then describe the structures according to the most comfortable means.

As long as the situations are familiar, conceptions are created from objects, events, and relationships in which operations and strategies are well understood. From this understanding, students form a framework of support which can be drawn on in the future, when rules may well have been forgotten but the structure of the situation remains embedded in memory, a foundation for reconstruction (cf. Brainin, 1985). Rules are pointless without understanding the situation which contains them (Bell & Purdy, 1985):

> There is no point in attempting to teach rules for manipulating negative numbers until pupils have an understanding of those situations which contain them. Moreover, when these situations are understood, they are available as reference situations which give meaning to directed number manipulations, so that any difficulties arising subsequently with the numbers can be treated by putting them into a context. For this reason, the contexts used must be familiar ones and ones which will continue to be in use in their own right. (p. 16)

Situations should be sufficiently simple to be manageable but sufficiently complex to provide for diversity in approach. They should be amenable to individuals, small groups, or large groups; involve a variety of conceptual fields; and be open to the methods to be used. The Voyage of the Mimi, an integrated television/text/software series (Storey & Julyan, 1985), and the macrocontext (Bransford, Hasselbring, Barron, Littlefield, & Goin, 1987) based on Raiders of the Lost Ark, a popular movie, both illustrate efforts to design teaching and learning around complex situations. Except for their video format and comparability to experiences with which children have a vicarious acquaintance through television, neither could be described as a familiar context. Despite this, both adopted a coherent story format (see Romberg & Tufte, chapter 15) and did help students formulate key questions, comprehend the problem, estimate the reasonableness of results, and link different problems together as a means of progress towards a major goal. One unplanned result was that students were observed spontaneously transferring the strategies developed to problems of their own making (Bransford, Hasselbring, Barron, Littlefield, & Goin, 1987). Situations are also the instructional units for the television mathematics series Square One (Children's Television Workshop, 1987).

Emphasis on knowledge developed from a situation is comparable to the in-basket strategy (Frederiksen & Pine, in press) adopted by business schools. The common interest in problematic situations as a strategy of teaching and learning derives from research showing that knowledge acquired in a problem situation is more likely to transfer to new situations (Bransford, Franks, Vye, & Sherwood, 1986), perhaps because resultant learning is saturated with experience (Vygotsky, 1962). Hence, the Russians are advocating study situations as the minimal structural unit of a lesson and have prepared universal conversion of schools to specially equipped classrooms for all subjects (Volovnikova, 1984). The British, adopting a measurement-driven instruction strategy (see Popham, 1987), have required 20% of the grade in the new general certification of secondary education to be for investigation and project work.

Multiple Representation

> The root phenomena of mathematics learning and application are concerned with representation and symbolization because these are at the heart of the content of mathematics and are simultaneously at the heart of the cognition associated with mathematical activity. (Kaput, 1983, p. 61)

The distinction between understandings and their representation (Lesh, Landau, & Hamilton, 1983) is important to both the teaching and the creating of mathematics. The notation of mathematics is so powerful that it must be seen separately from content as a sophisticated tool (Byers & Erlwanger, 1984; Skemp, 1982). "The approach to symbolization can probably be more profound if the use of symbolic systems is seen within a problem-solving or situation handling process" (Vergnaud, 1987, p. 231). It is naive and even dangerous to expect children to immediately use notation for the purpose of creating mathematics.

Children need to learn that "there are many ways to represent the same knowledge; the one chosen depends on our preferences and purposes" (Rissland, 1985, p. 109). In other words, representation is an active process and is distinctly problem-specific. Children may use several kinds of representation in attacking any given problem, some representations being closely tied to specific procedures, and solutions to particular problems being facilitated by the most relevant representation (Lesh, Landau, & Hamilton, 1983). They need to proceed from perceiving patterns and qualitatively imposing order on a situation, to informal articulation of those patterns, and ultimately to their expression in formal notation. In this process they will experience the convenience and power of notation for communicating and manipulating mathematical ideas (Skemp, 1982).

> It is damaging to pupils' mathematical development if they are rushed into the use of notation before the underlying concepts are sufficiently developed and understood. (Department of Education and Science, 1985, p. 10)

Although it is undesirable to rush children into formal notation, insights from written language suggests that considerable experience in describing and writing about the patterns observed or created is part of the iterative process of thinking, describing, and reflecting; this process is essential to the emergence of knowledge structures. Thus children need to be free to express their ideas in multiple ways (cf. Kowszun, 1986). However, "results of all investigations should be communicated in forms becoming progressively more abstract: by picture or diagram, in words, by a table, by a graph, or by an algebraic relation" (Biggs, 1973, p. 221). With experience, children naturally gravitate toward formal mathematical language (Biggs, 1985), perhaps because "the great benefit of symbolism is that it allows us to perform mathematics without thinking . . . . Moreover, good symbolism can actually suggest new results, prompt new mathematics" (Howson, 1983, p. 571).

One of the impacts of technology on the learning and teaching of mathematics is to permit multiple representation, both concurrently and recurrently. This flexibility may permit children to use their most comfortable mode of expression while compelling attention toward underlying structure by virtue of the simultaneous visualization of the same idea in alternative formats (Kaput, 1986a). This translation of a concept from one representation to another is crucial to Lesh's (1985) notion of stable understanding in mathematics. Kaput (1986a), for example, asked whether dynamically linked representations, "in which students could see the results of manipulations in one representation ramify across other simultaneously visible representations in a multiple window environment" (p. 12), would facilitate skill in translating between different modes of representation. Another possible effect would be to increase the likelihood of inductive inference, which is both based on and a source of pattern recognition (Watanabe, 1985).

Unfortunately, even the best of existing software, although conceptually exciting (e.g., Kidder, 1985), remains technically crude. It begs for implementation on more powerful equipment. Extant versions reduce the cognitive load on the student by performing the computation (e.g., Schwartz & Yerushalmy, 1983) or the mechanical construction (e.g., Schwartz & Yerushalmy, 1985) to help the child concentrate on the conceptual structures involved. Implementation on more powerful equipment would enhance the level of intuitive facility. In efforts toward alternative representation, two key features are important in mathematics as in word processing: transparency and appropriateness. Transparency of representational system is needed for reflective activity regardless of the mode of use, while appropriateness of notational device to purpose may promote the search for solutions (Lesh, Landau, & Hamilton, 1983).

Dialogue

The prevailing model of effective teaching (e.g., Rosenshine & Stevens, 1986) reflects the traditional view in which teaching as the telling of knowledge consists of monologue to a large and captive audience. Control is maintained through means ranging from instilled standards of courtesy to the negative inducement of unpleasant consequence. In retrospect, telling and choral recitation (e.g., Leinhardt & Greeno, 1986) reflect a persistence of the technology of the oral tradition, in which knowledge was the purview of a few and selected segments were distributed orally according to perceived group needs. The oral tradition was a product of an illiterate population, a view of knowledge as the record of knowledge, and the scarcity of artifacts of that record. However, a perceived need for all to participate in and adapt to a knowledge-based economy, widely available artifacts, and a constructivist emphasis in mathematics suggest that teaching as telling is now anachronous.

The oral tradition of teaching as telling was perpetuated by the textbook and could easily be continued with computer-assisted instruction. However, computer-based technology has acted as a catalyst to collaborative argumentation as well as to conceptual manipulations and multiple representation. One of the unforeseen side-effects of using computers in the mathematics classroom was that the need to share the technology encouraged the kind of domain-related discussion, argumentation, and cooperation in the pursuit of knowledge that would never have been permitted in a text-based classroom. In addition, microworld, tool, and simulation software has broken away from the oral tradition of teaching as telling in favor of constructive and investigative approaches. Some software has enabled students to maintain a ledger-sheet of explorations for reflection on and justification of their ultimate structures (e.g., Schwartz & Yerushalmy, 1983). This kind of trace promotes both group discussion and individual metacognition about the way a particular piece of knowledge has been constructed.

An ongoing dialogue between teacher and child, child and teacher, and child and child is essential (Cazden, 1986) for

1. individual and cooperative creation and validation of knowledge (Biggs, 1985; Ziman, 1978);

2. making the process of knowledge creation visible for the purposes of emulation, metacognition, and the diagnosis of obstacles to learning (Bell, 1985; Biggs, 1985; Brown & Campione, 1987)

3. conflict discussion, probing a situation by jointly seeking explanations and challenging faulty notions (Bell, Pratt, & Purdy, 1986), pushing children to the point of frustration (Mason, 1986) and cognitive conflict (Festinger, 1957) to deliberately perturb and promote the growth of their knowledge structures.

4. scaffolded instruction (Palincsar, 1986), creating cooperative environments that help a child achieve a goal beyond the reach of unassisted effort but attainable through the temporary, supportive framework of collaboration with peers and teacher.

In other words, teachers need to encourage students to reflect on and argue their intuitions; such activity is essential to the development of mathematical attitudes. Furthermore, dealing with one's own mathematical activity as a subject matter is a step toward a higher level of activity (Freudenthal, 1983a), tantamount to the metacognitive strategies urged by cognitive strategy instruction. Encouraging students to explain how they "know" and to persuade and convince others of that knowledge is an important ingredient of mathematical expertise which is fundamental to the notion of proof. It requires written logs of accumulating information that supports a belief, reflection on and discussion of that record, and a more formal report to present beliefs and grounds (e.g., Biggs, 1985; cf. Toulmin, Rieke, & Janik, 1979).

Contemplating extended student dialogue, Lochhead (1979b) made the following points:

> 1. Teachers talk too much. . . . [they] "must learn to shut up and listen." One of the best ways to do this is to encourage serious dialogue between students.
>
> 2. What at first appears to be poor performance may on closer examination turn out to be impressive work. . . .
>
> 3. During learning, performance may temporarily decrease; only over the long term is there a steady increase. . . .
>
> 4. Students can construct mathematical concepts and formulas if given time to do so. Most current instruction systematically denies students this opportunity. . . .

5. The correct use of terminology is a natural consequence of understanding but does not result from skill in the correct use of terminology. . . .

6. Promoting constructive dialogue is not easy. It requires a thorough understanding of the student's stage of development and a deep understanding of the material to be taught. (p. 177)

Diagnostic Teaching

Exploring situations embodying a variety of mathematical questions, each of which may be addressed by different methods and described in different ways, places rigorous demands on teachers' ability to flex with the requirements of the children. They must not only have an intensive understanding of their subject "but must understand in a pedagogically reflective way; they must not only know their own way around a discipline, but must know the 'conceptual barriers' likely to hinder others" (Hawkins quoted in McDonald & Naso, 1986, p. 8).

If students are to be free to identify and pursue their own questions, as is implied by Greenes's (1982) conception of literacy, it is important for teachers to be able to diagnose (Romberg, 1974; Bell, 1985) the condition of a student's cognitive structure in order to help them identify questions by which they will be challenged but not defeated. The whole strategy of cognitive conflict (Bell, Pratt, & Purdy, 1985; Festinger, 1957) is inherently dependent on diagnosis. However, it is crucially important to distinguish diagnostic teaching from diagnostic testing and the use of diagnosis in mastery learning.

Diagnostic testing takes a normative approach and asks, How does the individual compare with peers on key indices of achievement and ability? (Ginsberg, 1983, p. 248); such diagnostic tools are rarely explicitly identified that way. The purpose of The Algebra Readiness Test (Mathematics Diagnostic Testing Project, MDTP, 1986), for example, is described as "to identify areas in the seventh and eighth grade curriculum which need strengthening so that students have a better chance of succeeding in the beginning algebra course" (p. 1). The test is intended to be given three-quarters through a one-year prealgebra class to identify those needing alternative treatment of weaker topics, or summer school remediation. However, the third use MDTP (1986) gave is that "teachers and counselors could use test results along with other factors to assist them in advising students about their next mathematics course. . . . [They should] see a separate article in this newsletter concerning cutoff scores" (p. 2).

This "cutoff" application represents the most pernicious and yet most common use of diagnosis in school mathematics; it is a sorting mechanism in much the same way as the prerequisite of algebra has been used to select students for high school computer programming classes: "We have to keep the dirtballs out some way" (Personal conversation with the head of a high school mathematics department, 1985). The cutoff application reinforces the view of mathematics as a social filter (Davis & Hersh, 1986). This use flies in the face of both reasonable human behavior and demographic trends in the United States (see Steen, 1986).

In essence, diagnostic testing is a terminal perception of diagnosis that more closely resembles an autopsy; it has little to do with cognition (Ginsberg, 1983). Diagnosis, by contrast with autopsy, is a matter of identifying a disease from its symptoms and conducting an investigation into the origin or nature of the condition, usually in an iterative cycle of diagnosis and treatment until the condition is resolved (Romberg, 1974). Diagnosis that understands why a mistake has been made (Freudenthal, 1983b) must prevail if children are to create sound mathematical structures.

Mastery learning relies on an iterative process of formative evaluation via diagnostic progress tests. The assumption is that all students can learn the basic curriculum, some just take longer. Time is the key variable. Teaching for mastery depends on clear instructional objectives and definition of units of mastery through rational task analysis. In other words, it relies on the scientific management of a rationally analyzed segment of the record of mathematical knowledge; consequently, it is antithetical to the critical thinking (Sternberg, 1986) involved in a constructive approach to mathematics.

If learning were a matter of teaching the record of mathematical knowledge, then diagnosis could be restricted to identification of segments not acquired. In fact children might do as well, if not better, to stay home and memorize a two-volume encyclopedia. However, learning requires making sense of things and creating meaning, establishing relationships between new information and what is already known. Difficulties arise from failure to relate the new to what is already known, deficiencies in knowledge, and incorrect elements in the knowledge possessed (Simpson, 1985). These difficulties may lead to distortion of the new material in order to fit it into existing structures; alternatively, the old framework may be modified or abandoned and a new one created (Bell, 1982a; Kintsch, 1986).

Bell, O'Brien, and Shiu (1980) described diagnostic teaching as emerging from research on understanding. Its central problem is that any assessment aimed at a spectrum of understanding in mathematics on which a student scores an acceptable percentage offers a misleading picture of that student's understanding (Whitney, 1985). It may well conceal serious misconceptions, which are carried on and become obstacles to subsequent learning. Therefore,

> we need to take the diagnosis and treatment of pupils' mistakes far more seriously than we do--in fact, in general we need to do less direct teaching and concentrate on finding

> out what they think in relation to the problems on hand, discussing their misconceptions sensitively, and giving them situations to go on thinking about which will enable them to readjust their ideas. (Bell, 1982a, p. 7)

Consequently, diagnostic teaching is directed less toward assessment than toward a strategy for teaching in a way that contributes to development of children's understanding. By definition, it requires starting from the structure of the child's knowledge. The information on which to base diagnosis may be acquired through observation, questioning and conversation, and reflection on the products of student activity. A key strategy is to prompt children to develop their knowledge structures by deliberately provoking predictive inadequacy. Perturbation of existing structures is accomplished through critical tasks that embody known misconceptions (Bell, 1982a, 1982b, 1985; Bell, Pratt & Purdy, 1986; Bell & Purdy, 1985; cf. Hatano & Inagaki, chapter 20).

Succinctly, there are two key issues demanding diagnostic strategies: the crucial importance of complex epistemic situations, and progress through controlled cognitive conflict (Festinger, 1957) initiated by conflict-discussion (Bell, Pratt, & Purdy, 1986, p. 3). "To promote learning, it is important to focus on controlled changes of structure in a fixed context . . . or on deliberate transfer of a structure from one context to another" (Bell, 1985, p. 72).

Devolution and Dissimilarity

> No real intellectual activity could be carried on in the form of experimental actions and spontaneous investigations without free collaboration among individuals--that is to say, among the students themselves, and not only between the teacher and the student. Using the intelligence assumes not only continual mutual stimulation, but also, and more importantly, mutual control and exercise of the critical spirit. The workings of logic are, in effect, always "cooperations." (Piaget, 1973b)

The oral tradition of teaching as telling (Adler, 1982, p. 51; Goodlad, 1983) created a clear distinction between learning and teaching and, therefore, between the corresponding responsibilities and authorities of teachers and students. The teacher was an absolute ruler who controlled truth. Thus, collaboration in the traditional school was strongly discouraged (Piaget, 1973b). Integrating theories of teaching and learning would suggest that, if children are to take an active involvement in their own learning, these social duties and relationships also need to be integrated. There are indications that the functional relationships of students and teachers are already under review (e.g., Hedin, 1986; McDonald & Naso, 1986).

Use of technology in the classroom has prompted insights into the role of teachers in promoting learning. One consequence of addressing the use of computers as teaching tools (McDonald, 1985), for example, was the insight that the process of applying technology in the schools throws into relief the conditions required to stimulate learning on the part of teachers (McDonald & Naso, 1986; cf., Watts, 1985). By analogy, the process also throws light on the conditions required for children to learn.

> Both [teachers and children] construct new knowledge by noticing and analyzing experience, by filtering this experience through an interpretive network of previously learned concepts, and by readjusting this network in the light of new knowledge. . . . [This] requires intensive engagement with phenomena, sufficient time for reflection, encouragement to risk new thinking, and support from an experienced teacher who can point out discrepancies, pose questions, and guide the learner's thinking by means of a senses of the discipline's core structure. (McDonald & Naso, 1986, p. 2).

Implementing technology in the classroom gave teachers an officially approved opportunity to experiment and tinker. It caused thinking about the process of learning, whether through a fresh study of their own subject or through a fresh perspective on students' learning. It softened the barrier between what students do and what teachers do; both were collaborating to learn. In consequence, teachers modelled the learning process for students while they gained fresh insight on the teaching process by watching students learn. Three points about the conditions for teachers' learning have implications for their helping students learn (McDonald & Naso, 1986):

1. They must be partners in innovation; a critical partnership is needed between teachers, administrators, students, parents, community, university, and computer industry.
2. They need time to learn; time to reflect, absorb discoveries and adapt practices.
3. They need collegial advisors rather than supervisors; advising is a partnership.

The stress on time in the learning process was also strong in Watts's (1985) study of how teachers learn and was related to teachers exercising their own discretion and taking responsibility for their own curriculum. Those who were most self-directed were most insistent in their plea for time. Teachers' demands for time were closely related to the issues of empowerment, control, and responsibility, suggesting that an integrated theory of teaching and learning will need to address the same issues. Other problems of teachers may well be problems for children, and vice versa. For example, the rule-governed activity of direct teaching isolates and constricts the communication of both child and teacher. Each needs a collaborative environment (Holly, 1983).

The experiences of teachers and children using computers in the classroom (McDonald & Naso, 1986) are remarkably analogous to the model of reciprocal teaching (Brown & Campione, 1987). This takes place in a cooperative learning atmosphere in which each member of the group takes a turn in leading the discussion, questioning, clarifying, summarizing, and predicting. "The goal is joint construction of meaning" (p. 11). As students become more proficient, the teacher gradually fades out of the dialogue after modelling strategies and problems and overcoming difficulties. The result is to transfer power to the student and to model the process of creating meaning through reflection by externalizing the process (e.g., Biggs, 1985; cf. Confrey, 1985; Freudenthal, 1980; Wheeler, 1978).

One difference between reciprocal teaching and using computers in the classroom is that the transfer of teaching is deliberate but artificial in reciprocal teaching but occurs spontaneously during efforts to use computers in the classroom. Some children develop a profound involvement with the technology or the software, spend considerable time on it, and know more than anyone else in the group, including the teacher. Also, in many cases, both teachers and students are novices; thus, the creation of knowledge is a genuinely cooperative endeavor. Epistemological authority is redefined, the consequence of which is to redefine social authority and personal responsibility (Kaput, 1986a; cf., Pollak, 1986; Skovsmose, 1985). Cooperation creates a setting in which novices

> can contribute what they are able and learn from the contributions of those more expert than they. Collaboratively, the group, with its variety of expertise, engagement, and goals, gets the job done. (Brown & Campione, 1987, p. 17)

The devolution of authority and cooperative participation by dissimilar students result directly from, and contribute to, an intense cognitive motivation. Both Skemp (1979), discussing cognition, and von Glaserfeld (1983), discussing mathematics, arrived at similar conclusions regarding motivation (cf. Markova, Orlov, & Fridman, 1986):

1. Progress is achieved through reflective activity requiring goal orientation and intense motivation.

2. The process of reflectively constructing one's own knowledge is a powerful source of satisfaction (cf. Freudenthal, 1978/1980).

> The issue of motivation is central to the practicalities of today's classroom. . . . strengthening the link between instruction and life, shaping in schoolchildren the techniques and modes of self-education and the independent acquisition of knowledge. . . . The schoolchild's orientation toward the modes of knowledge acquisition is directly related to the shaping of self-education techniques. (Markova, Orlov, & Fridman, 1986, p. 8; cf. Dede, Zodhiates, & Thompson, 1985)

Succinctly, in genuine efforts to create knowledge, important points emerge about the kind of teaching that would encourage it: the responsibility for teaching must be gradually transferred to those learning (Greene, 1982; Whitney, 1985); the group taught needs to be diverse in its talents and abilities so that the scaffolding of collaboration is possible (Freudenthal, 1980). To borrow from Kaput's (1986a) characterization of a computer-based learning env ronment, teaching is "organized around the student as an active agent using a potent tool for the investigation of important mathematical ideas" (p. 19).

Summary

The argument of this paper is that the teaching of mathematics has an inescapable social context that requires the redefinition of mass literacy from basic training to the universal empowerment of human potential. Given the goal of long-term understanding in support of life-long learning, a radically different, but strongly supported, picture of school mathematics teaching was proposed. Succinctly, teaching for long-term learning and the development of knowledge structures requires teachers to create epistemic situations in which children can explore problems, create structures, generate questions, and reflect on patterns. It requires teachers with the academic and pedagogical knowledge to provide for flexible approaches, encouraging informal and multiple representation while fostering the gradual growth of mathematical language. It requires:

1. teachers who can diagnose difficulties and devise questions to promote progress through cognitive conflict;

2. teachers who, while having everything else expected of them, can maintain a collaborative atmosphere leading to students' increasing independence;

3. careful reflection on strategies for cognitive motivation;

4. recognition of the epistemic, cognitive (Skemp, 1983), and social commonalities of teaching and learning.

Conclusion

It is both humbling and salutary to read Edith Biggs (1985): humbling because her work is a tightly coherent picture of all that has been outlined, yet was conducted some 20 years ago; salutary because it demonstrates the attainability of improved teaching of mathematics to all children.

Biggs's (1985) project originated in her efforts to understand the problems of teachers trying to introduce new content and adopt a new style of teaching mathematics. She pursued the same goals working with slow and able children between the ages of 5 and 13 as she had in inservice work with teachers, feeling that they should

1. enjoy mathematics;

2. become confident in their ability to learn mathematics through planned activities;

3. discuss their findings with peers and their teacher;

4. understand the mathematical concepts they use;

5. see mathematics in action and realize why they need to memorize number facts and carry out calculations; and

6. appreciate that mathematics is concerned with patterns and the communication of patterns in simple language.

> I found that it was possible for pupils to learn mathematics by means of investigations (sometimes using material) and questioning, rather than by demonstration and practice, the method by which I myself was taught at school. The immediate positive effect of giving pupils an opportunity to investigate a new topic and to develop their own solutions (in other words giving them more responsibility for their own learning of mathematics) took me completely by surprise. They developed an enthusiasm for mathematics and no longer asked: "Why do we have to learn mathematics? Will it be any use to us when we leave school?" and (worse) "Is it in the syllabus?" Although I was introducing new material no one checked it against the examination syllabus. Even those pupils who had previously shown no interest made great efforts to arrive at original solutions. For the first time they began to develop investigations further and to create mathematics for themselves. (p. i)

> When . . . I worked with children at the primary phase, I found that here, too, children's interest was captured if they were given real problems to solve and the necessary material was available to help them arrive at a solution. By means of carefully structured activities and discussion, they were able to acquire and understand mathematical concepts. Neither did they need to be shown methods of calculation; these, too, could be developed through activities and questioning, provided that the children had an adequate knowledge of number facts. (p. ii)

The picture of mathematics teaching described by Biggs (1985) points to the enthusiasm and intense involvement with the discipline which is possible for children with widely varying abilities. It illustrates the parental cooperation that can be tapped in pursuing mathematical investigations beyond the classroom, a different quality of experience and relationship from the traditional notion of homework as practice in algorithms learned in school. The children's efforts led to an internal integration of mathematics, an automatic involvement in data handling, multiple ways of representing information, and a mathematical attitude of mind in presenting and arguing their findings. It also led almost inevitably to interdisciplinary efforts with other subjects (cf. Pollak, 1986) and independent, self-generated, and often collaborative efforts beyond the tasks assigned, sometimes beyond school.

However, problems as well as opportunities became obvious (Biggs, 1985). Beliefs of both children and teachers about mathematics need to be changed (Confrey, 1985; Whitney, 1985). Children automatically assess themselves: "Many children were so lacking in confidence that they always expected to be wrong" (p. 34). It was important to convince some children that their achievement could be measured by their understanding of mathematical concepts as well as by the correctness of their written calculations. Both children and teachers need to accept that practicing written calculations which children do not understand is a waste of time (Biggs, 1985). Both need to accept that mathematics can involve activities other than the five-step lesson, materials other than blackboard and textbook, subjects in addition to mathematics, and places other than school.

Unanticipated classroom inquiry can create a level of uncertainty that can cause some teachers to reconsider the "network of related ideas that constitutes their subject" (p. 8), while causing others to retreat to the traditional linear curriculum. This problem highlights the need for teachers to be mathematically powerful; the flexibility Biggs displayed in seizing teachable moments and allowing children to direct the trend of investigations was enabled by her deep understanding of mathematics. The problem also draws attention to the strictly linear character of the traditional textbook. As children pursue their investigations, their path through the topic (cf. Jackson, 1984) may not match that in the text. Skipping to another chapter in a text dependent on a linear argument may provide inadequate support, suggesting both the need for hypertext (Jonassen, 1986) and for a far more flexible interpretation of text as resource material (cf. Confrey, 1985).

Biggs worked with groups of fewer than 10 children. The kind of teaching she illustrated requires time for both students and teachers to reflect but also intense concentration, which is difficult to manage in larger groups. Schools as they are now organized give teachers little time to reflect and insist on groupings that are more typically around 25. Such large groups make it difficult to conduct or find time for the level of dialogue required. Talking to each child in a class for 10 minutes a month (Klausmeier, Jeter, Quilling, Frayer, & Allen, 1975); conducting diagnostic assessment with practical materials (Joffe, 1985), would take weeks with existing staffing and scheduling arrangements.

Consider the demands of teachers seeking time to spend on students (Watts, 1985):

> I'd have a secretary to do all the nitty-gritty of taking roll and recording the grades, all of the things that don't require you to think . . . and I'd have two aides. (p. 25)

> I'd buy time. I'd just make my own schedule and nobody would bother us and we'd do whatever was important to do. (p. 27)

Modifications to the traditional patterns of teaching such as smaller groups, the training and ongoing professional development of teachers, and alternatives to textbooks are most frequently addressed from the aspect of cost. However, as Freudenthal (1983b) has pointed out, "Observing and understanding the individual child is not expensive. What is really expensive is wasting the vast resources of human experience" (p. 2). Furthermore, the issues go beyond cost to control and organization of the school day, district resources, hiring practices. In fact, both Biggs's (1985) work and the consideration of the teacher as a learner (McDonald & Naso, 1986) reveal some of "the deep and pernicious regularities in the culture of schooling" (p.12) that obstruct learning and that will, unless challenged, continue to impede the learning of mathematics.

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REFLECTIONS AND DIRECTIONS

This single chapter was originally prepared both as a concluding chapter for the monograph and as the basis for the proposal to the National Science Foundation for the refunding of the School Mathematics Monitoring Center. The papers in the three volumes make it clear that we are currently involved in a social revolution and that, if we are to monitor progress toward reform in school mathematics that complements the needs of students to be productive citizens in the world of tomorrow, the monitoring must reflect the reform movement; however, the Foundation chose not to refund the Center. This final chapter is presented in the form originally prepared with the expectation that many of the next steps being suggested will be taken in the near future.

Chapter 29

WHERE DO WE GO FROM HERE?

Thomas A. Romberg

INTRODUCTION

The previous 28 chapters in this monograph were written, commissioned, or reprinted so that the ideas underlying the rhetoric of the current reform movement in school mathematics could be understood. The purpose of this chapter is to summarize what has been learned, to reflect on that knowledge, and to draw implications from that knowledge for the future work necessary for the development of a comprehensive monitoring plan.

The general purpose for establishing SMMC was to develop a monitoring plan for the National Science Foundation. The plan was to specify cost-effective means of gathering information about the impact of the current reform efforts with respect to school mathematics. Such a monitoring plan was deemed important so that policymakers make informed decisions. It was determined that there were four assumptions upon which the work of the SMMC has been based, eight anticipated changes in school practices, and ten tasks that need to be accomplished. This summary is organized with respect to these assumptions, changes, and tasks.

ASSUMPTION 1. There is general agreement within the mathematics education community about the objectives of the reform movement.

This assumption was based on the discussion, recommendations, and activities of several professional groups within mathematics education during the past decade. For example, there have been numerous calls for reform in school mathematics (e.g., Nation at Risk, National Commission on Excellence in Education, 1983; or Educating Americans for the 21st Century, National Science Board Commission, 1983) followed by responses by the Conference Board of the Mathematical Sciences (CBMS, 1984) and the Department of Education (Romberg, 1984). These and other responses have been comparable and consistent. In fact, the validity of this assumption was verified by Pollak (1986) who, when concluding a recent MSEB conference on reform in school mathematics, stated: "One of the first points that struck me . . . was the degree of agreement about mathematics education the outside world perceives in the mathematics education community" (p. 1). There is agreement on the basic principles and direction of the reform movement; consensus on the details is now being negotiated. Based on this assumption two tasks were specified:

Task 1. This task involved reviewing and examining the main aspects of the reform movement; this was necessary in order to understand the rhetoric of reform and to describe the ideal mathematics curriculum and classroom envisioned by the reform movement. This vision is to be the benchmark for SMMC against which one can judge the degree of change from current practice toward that goal. This review was also necessary in order to clarify the traditions and values being challenged which would impede efforts at reform.

The main product of this task is this monograph. In addition, 4 working papers (see page 147) and 6 related papers (see Appendix for a listing) have been prepared. This review's contribution to an understanding of the reform movement can be summarized under four topics: the Information Age, aspects of reform, mathematical literacy, and shift in emphasis.

The Information Age refers to the realization that the reform movement in school mathematics is a single element of an economic and social revolution (see chapters 2, 18, and 28). Mathematics educators tend to restrict their focus to the changes in mathematics and the technology of mathematics, such as the need for calculators, computers, the addition of discrete mathematics, statistics, and problem solving. This review has forced us to see those changes from a much broader perspective and to think in systemic terms with respect to the reform movement. The demands for change in school mathematics are symptomatic of the fact that it is not just the structure of the economy that is changing but a whole cohesive system that is passing and being replaced. A new basis for cohesion is slowly emerging to replace that centered around industrial processes and Newtonian physics. The level and pattern of disruptions can no longer be regarded as minor or as a temporary aberration. Because reform in mathematics education is embedded in a societal transition to an Information Age, the monitoring of mathematics and the linking of information collected to policy issues must be addressed in the context of the new age. In fact, the real challenge will focus on the traditions which have emerged based on an industrial model of schooling (see chapters 2, 24, and 26).

Three aspects of reform underlie most of our recommendations. First, much reform centers around a revolution in communications technology that is a response to the accelerated development of new scientific and technical knowledge and the impacts thereof (see chapter 2). Thus, conceptions of knowledge and the theory and technology of communication are crucial to the reform, generating problems, tools, and sources of analogy. The second aspect derives in large part from the first; psychology now regards learning as a constructive rather than an absorptive process and focuses on multiple intelligence (see chapter 12). Analogies for the working of the mind are drawn from the technology of information processing and communication networks. Thus, knowledge is something created by the individual and, through communication, validated by a community. The level of a child's understanding is a matter of the density of his or her network of interconnecting knowledge. The third aspect of the reform is sociological; it is clear that knowledge is differentially distributed in schools in ways that disenfranchise many, especially minorities (see chapters 7, 24, and 25). Those minorities are expected to comprise 30 percent of the population before the century closes. Thus, the cost of a system that deliberately differentiates has become too high, both economically and socially. Historically, these three aspects were intertwined in an industrial model of schooling; in the industrial economy, it was cost effective to educate a few and train the rest. Now that global and individual adaptation and problem solving are required, however, all need to be educated to think. Therefore, the patterns related to the old economy are no longer acceptable or effective.

Mathematical literacy is our description of the goal we have identified for the reform movement in school mathematics (see chapter 28). The focus of schooling must shift from the passive acquisition of facts and routines to the active application of ideas to problems. Minimal literacy acquired early in life through passive methods will no longer suffice. The personal, national, and global problems to be faced and solved require adults who are able to continue to learn and adapt because they have confidence in their own personally created and firmly founded understanding. The situation requires that the industrial notion of literacy as conditioning for predictable reaction be revised to the notion of literacy as the capability for innovative adaptation.

This need for literacy to support continued learning draws attention to Resnick's (1987) distinction between high and low literacy. Under the industrial model, teaching the low-literacy majority was essentially a matter of acculturating and training children to fit a system that required punctuality, obedience, predictable performance, and minimal literacy. There are obvious connections between this model, stimulus response theory, and teaching and evaluating according to behavioral objectives.

Present demands, by contrast, require a high literacy model for all students. For example, "no student except the severely handicapped should be able to choose courses in a way which, before grade 11 [,] makes it impossible for him/her to move to a college preparatory curriculum" (Conference Board of the Mathematical Sciences, 1984, p. 11). A clear implication is that all students should enter the 11th grade with a high-literacy preparation in mathematics capable of supporting continuing education.

The reform involves a shift in content emphasis in the teaching of mathematics rather than a shift in content to be covered. While a few mathematical topics which are not now in the school mathematics curriculum will be emphasized (e.g., statistics, mathematical modeling), most of the topics in the curriculum will remain. However, rather than stressing procedural competence, conceptual understanding must be emphasized (see chapters 15 and 28). This implies that the curriculum should include a richer array of problem situations, an emphasis on the language (terms and symbols) used to represent situations (including multiple representations), practice in the structure of algorithms, and more work on the validation of assertions (both within a symbol system and to problem situations).

Given this understanding of the roots of the reform movement the first four (of eight) anticipated operational changes in school mathematics outlined in chapter 3 (page 64) are essential if the reform is to be accomplished. These are:

1) changes in course content and structure,

2) changes in course requirements,

3) changes in the sequencing and segmenting of mathematical topics, and

4) changes in the use of technology.

Mathematics is a growing, dynamic discipline that is undergoing profound changes which must be reflected in the school mathematics curriculum. The changes in school mathematics listed above are operational changes largely based on what is happening within mathematics (although changes in conceptions of learning and access to knowledge cannot be ignored). Chapters 6 to 11 provide a sample of the thinking of several noted scholars on this topic and a conceptual basis for these changes in school mathematics, while chapters 12 and 15 outline the psychological implications related to these specific changes. The details of these four changes are now being established. In fact, three professional groups currently are preparing outlines for school mathematics which will provide details with respect to these four changes. These projects are:

1) The NCTM Standards Commission (Romberg, 1987), which is articulating standards in school mathematics for today's schools;

2) The MSEB Frameworks Task Force (Ralston, 1987), which is articulating visions for the near future and curricular frameworks for achieving them; and

3) Project 2061 (AAAS, 1985) which is defining long-term goals and visions.

The initial task needed is to summarize the reports of these three groups and to identify major mathematical domains which will receive significant and differential emphasis as a result of these proposed changes.

Task 2. This task required the development of a causal model of schooling practice which would reflect the key variables assumed to be important in the vision identified in Task 1. In chapter 3, a preliminary model was described; in chapter 5, Shavelson, Oakes, and Carey prepared a thoughtful critique of that model. Three aspects of their analysis merit comment. First, whether the model is called "causal" or "logical" is a matter of semantics, not intent. To assuage their concerns about interpretation, we will change our label and now refer to our model as a "structural model." Second, their distinction between "direction of change" and "tactical" criteria upon which to base a model is important; however, they should not be seen as competing perspectives. Rather, they should be seen as complimentary in that they meet different purposes. Third, comparing our model with others suggested that the following be more fully considered:

- Instructional time. It was mentioned as a proxy for "pedagogical decisions" and is important.

- The school as a separate level of analysis. This is explicit in our sampling design (where a high school with its feeder schools is the sampling unit), but was only implicit in the model.

- The multiple levels of policymakers influencing educational practice. The policy environment variable in our model is inadequate. Also, it is true that our model was deliberately designed to address one set of policy questions--the degree to which a reform movement, supported by NSF, is occurring in schools.

- The multiple directions of causal, conceptual, or logical relationships among components of the educational system. In the diagram of our model (page 76) most of the arrows are unidirectional because they reflect time-ordered events over a short time span (as is customary in sociological models). However, in complex systems over longer time frames, the relationships between variables are bidirectional.

- Cultural and environmental influences on schooling. "Social context" is a variable in the model. However, given Steen's (1986) views on the projected demographics of school mathematics, we must highlight this set of variables in a revision of the model.

As important as these comments about our structural model are, several items from other chapters also should be brought to bear on any revision of the model:

- Outcomes are important and our means of assessing knowledge outcomes needs to be based on an approach which validly captures the maturity of a student's thinking in a mathematical domain (see chapters 17 to 21).

- Intrinsic motivation should be a major aspect of "pupil pursuits" (see chapter 13).

- Intuition as an aspect of mathematical reasoning must be included in the model (see chapter 14).

- The form of text materials must be included as a component of the "content" variable in the model (see chapter 15).

- "Attitudes" must include both attitudes toward mathematics and mathematical attitudes (see chapter 28).

In summary, the structural model posed in chapter 3 is generally satisfactory. Some revisions are warranted as a result of the review about reform. However, most of the suggestions involve the identification of components within existing variables in the model and the specification of measures (proxies) for each of the model variables. This revision, clarification, and operationalism of the model is a task that also needs to be completed.

ASSUMPTION 2. The cost-effective indicators of change toward those objectives should be developed by combining extant data with measures which are to be developed.

This assumption was based on the belief that, for the structural model to be operational, one or more measures would be needed for each variable in the model. We assumed that new measures for many of the variables would not need to be developed, although the nature of the reform movement would necessitate the development of some new measures. Unfortunately, the review suggests that more new measures are needed than we had originally anticipated.

Task 3. This critical task was to specify for student outcomes a new content framework, reflective of the anticipated reforms, that can be used to validate tests and other assessment procedures. This task reflects change 5 specified in chapter 3.

Changes in methods of assessment were addressed in chapters 17 through 23. It is now clear that to assess an individual's maturity with respect to a particular mathematical domain, a new assessment strategy is truly needed. The approach we have adopted is the "domain knowledge strategy."

We began this task by first delimiting the specification of knowledge outcomes to those which might be expected of students at grades 5, 8, and 11 (to correspond with NAEP and many state assessments). Second, we decided to concentrate our efforts initially on grade 8. Third, we identified mathematical topics where we expect there will be major changes in emphasis as a result of the reform movement. At grade 8, these probably will include: ratio and proportion, directed numbers, exploratory data analysis, transformational geometry, and linear algebra. The final list will be based on the review of group reports. Finally, for illustrative purposes, we decided to build a description of the domain related to ratio and proportion.

The domain knowledge strategy is based on Gerard Vergnaud's notions about "conceptual fields" (1982) (also see chapter 20). His notions are based on the philosophic premise that the power of mathematics lies in the fact that a small number of symbols and symbolic statements can be used to represent a vast array of different problem situations. Thus, if a set of symbols represents a closely related set of concepts, referred to as a "conceptual field," then this monitoring framework should allow one to determine the degree of knowledge a student or group have acquired with respect to that domain.

Such fields are derived for a specific mathematical domain as indicated below. It should be noted that in the first three steps, only the formal aspects of a mathematical system are considered.

1. The symbolic statements which characterize the domain are identified.

2. The implied task (or tasks) to be carried out is specified.

3. One identifies the rules (invariants) which can be followed to represent, transform, and carry out procedures to complete the task.

4. One identifies a set of situations that have been used to make the concepts, the relationships between concepts, and the rules meaningful.

The results of following the above steps yields a map (a tightly connected network) of the domain of knowledge. It is this map we are using as the framework for assessment.

For example, the domain of ratio and proportion has been specified in some detail by following this approach (Zarinnia, Lamon, & Webb, 1987). This domain is characterized by the symbols a/b, $a:b$, $a/b = c/d$ and $y = ax$. The implied tasks that are carried out include situations where comparisons are made involving one or more ratios; a ratio is given and some other quantity is to be found; three of four quantities of a proportion are known and the fourth quantity is unknown; or a set of ordered pairs are given and the constant of proportionality is the unknown. The rules that apply to represent, transform, and carry out procedures to complete the tasks are the basic facts; symbolic transformations such as $a:b \rightarrow a/(a+b)$, $\sin a / \cos a = \tan a$, $a\% = a/100$, and $a/b = [?]d \rightarrow [?] = a\,(d/b)$; and computational algorithms. The set of situations that make the concepts, the relationships among the concepts, and the rules meaningful are numerous and include those involving comparisons, extensive/intensive quantities, and others.

Five properties of conceptual fields make them particularly attractive in light of the reform movement currently in progress in mathematics.

1. The set of rules (invariants) is more extensive and less hierarchically ordered than has been typically considered. This is important, since with the advent of calculators and computers, we no longer must emphasize the most efficient rules for finding answers.

2. The set of situations which give meaning to the concepts and rules are considered equally as important as learning to follow the procedural rules. In fact, it is assumed that these situations are the means by which the procedural rules are understood and learned. Too often in the present curriculum, only a single situation is briefly used to introduce the mathematical symbols and rules for a domain. Instruction has emphasized students' proficiency at following a few procedures. This domain knowledge framework assumes that a richer set of situations that reflect common frames of experience should be included. The assumption is central to the approach proposed--the same symbols can be used to represent lots of seemingly dissimilar situations.

3. While it is true that <u>all</u> situations for which the formal mathematical domain could be used can never be considered (new situations are always arising), students should understand that the mathematical symbols and rules are not idiosyncratic to one situation. Furthermore, the situations that should be included are those from which the development of the particular symbols and rules arose, and those where they are commonly used.

4. A major advance has been accomplished by classifying the problem situations with respect to four properties of most problem situations: semantic structure; quantity (type and size of numbers); form of the information (verbal, pictorial, etc.); and familiarity of the setting.

5. Psychologists and mathematics educators during the past decade have found that this framework has proven to be very valuable. In particular, while a conceptual field is a network and, as with all networks, there are multiple possible paths for traversing the network, it has been possible to describe a natural progression of situations that give meaning to the formal mathematics. The description is based on a combination of the four properties listed above as they are related to the symbols, relationships, and invariants of the system. This description also seems to match both observed developmental trends and constructivist notions about how information is structured and restructured. Also, it is this description of the progression which we plan to use for monitoring purposes.

For assessment purposes, the exercises posed to students are of two types: some are wholly in the formal mathematical system to ascertain whether students know the symbols, relationships, and rules; and others are given as problem situations that reflect the hypothesized progression described above. Exercises are administered in a combination of ways. Standard group testing can be followed, but individually administered performance tests and small-group cooperative tests also are necessary. What is important is that the administration procedure is appropriate for the particular situation. The form of the responses must also vary. In particular, since often one is interested in the strategy a student has used to work a problem, an open-ended response form may be appropriate. The rules one uses to judge responses are complex. They must include both the correctness of a response and the appropriateness of the strategy used for the particular problem situation. Finally, the aggregation of responses may involve estimating the progress of individuals in the domain. Then data from those individuals would be combined to estimate the development of a group with respect to that domain. On the other hand, for monitoring purposes the aggregation would most likely be done directly for groups since different individuals may respond to different tasks. However, in either case, the aggregation will not be a simple sum but a Boolean combination of information.

For example, the domain knowledge indicator system (Figure 1) for gathering information about children's developing knowledge about ratios originated in a review of known difficulties in proportional reasoning. Children develop this reasoning through experience with the situations that give it meaning. Therefore, their difficulties were classified according to the way situations are encountered by children (presentation); the structural properties of situations (elements, quantities and relationships), and the aspects involved in the children's re-presentation of the situation (von Glaserfeld, 1985) (the representation). Secondary and tertiary nodes in the network developed from the epistemic indicator system provide for such things as problems posed to a group and representations varying from informal or pictorial to geometric or algebraic.

One attraction of the approach is that it intertwines the logical and the psychological in a single, tightly knit structure. Second, a wealth of interrelated information is conveyed when outcomes in proportional reasoning are described according to an entailment network developed from the system. Third, although devised for ratio and proportion, the epistemic indicator system generalizes to other mathematical topics. Thus, a fourth possibility emerges, that of <u>indicator systems</u> for domain knowledge.

> An indicator system is more than just a collection of indicator statistics about a complex phenomenon. . . . [It] measures <u>distinct</u> components of the system of interest [and] also provides information about how the individual components work together to produce the overall effect. In other words, the whole of the information to be gained from

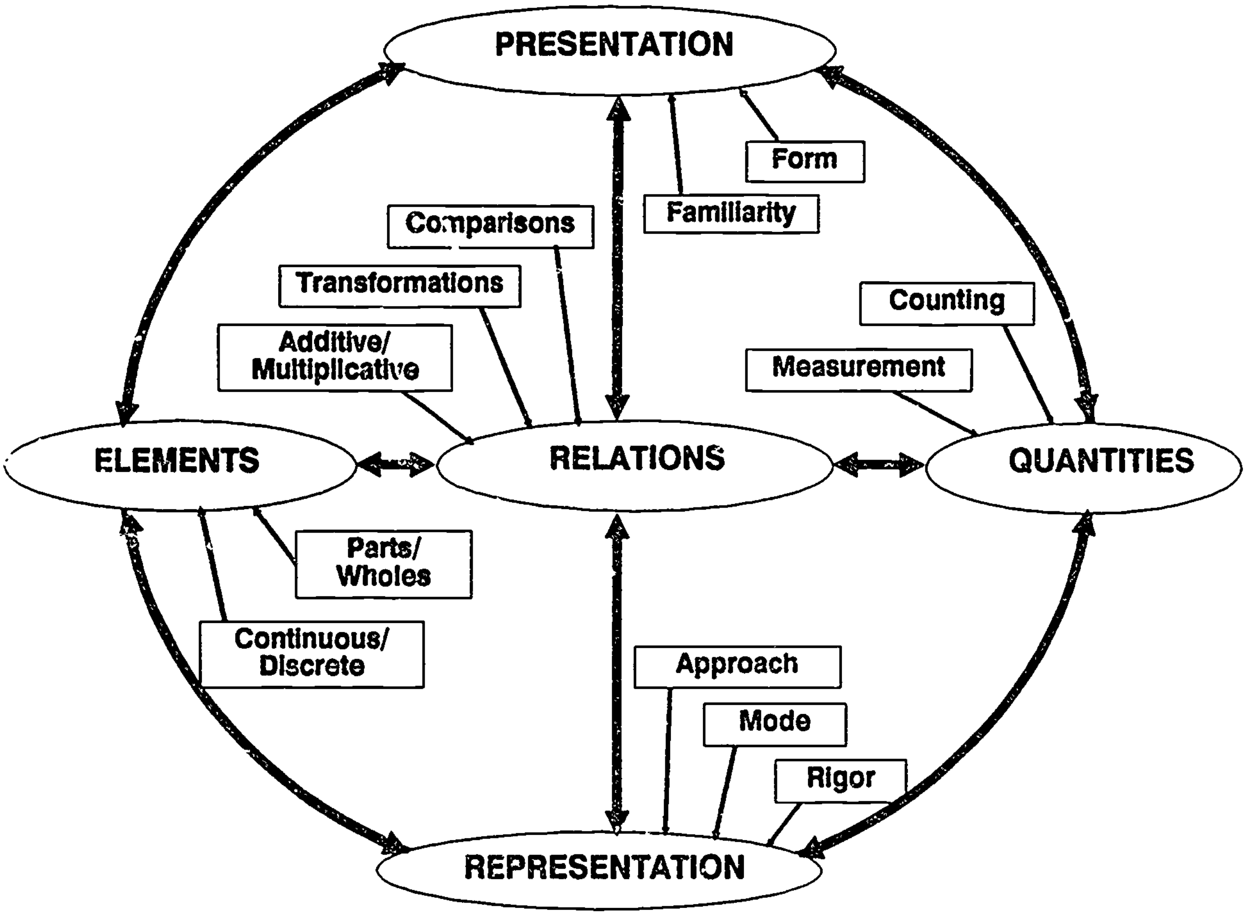

Figure 1. The Entailment Mesh for Ratio and Proportion
(Adapted from Zarinnia, Lamon, & Webb, 1987)

a system of indicators is greater than the sum of its parts. (Oakes, 1986, p. 7)

Consequently, work with the epistemic indicator system developed for ratio and proportion suggested a model and an overall strategy for domain knowledge indicator systems that focuses on conceptual fields that are of strategic importance to the reform of school mathematics.

Ratio and proportion is a fortuitous example of the strategy. First, the understanding of elements and quantities and the additive and multiplicative strategies children take to them are especially informative about their understanding of a whole range of mathematics and associated symbol systems, includir~ the geometric, the algebraic, and the numeric. Also, chilr ren's abilities to decipher, decode, translate, transform and re-present are used with problems in this domain. Second, the field of proportional reasoning is particularly important because its ideas are crucial to the learning of the most basic aspects of science. Third, although science educators demand such an understanding by the age of 13, children find it very difficult. That difficulty varies, however, from country to country. Fourth, little research has been conducted on the potential of elementary children on this topic. In summary, it is a crucial and difficult field in which children typically do poorly but which is likely to be sensitive to serious efforts to improve children's understanding of mathematics. Thus, a more sensitive tool for assessing student knowledge is also likely to be appropriate for examining issues of content and, in consequence, a more sensitive instrument for devising policy with respect to teaching, curriculum, materials, technology and tests.

Note that the framework for assessing student understanding has strong implications for independent variables of content: instructional materials, tests, curriculum guidelines, and technology. It suggests, for example, that instructional materials need to provide a rich variety of familiar and unfamiliar settings in their presentations, communicated in a variety of symbol systems. It suggests that strategies for assessment, such as multiple-choice questions, that fail to provide for student representation produce inadequate information about a student's understanding. It implies that curriculum guidelines should address all of the nodes and prompts consideration about the intelligent use of technology, as in windowing software for multiple representation, for example. All of these considerations impinge on the independent variables of teacher knowledge and professional responsibilities to the extent that content is seen as influencing both. Furthermore, of the three, content is the most susceptible to the influence of policy.

We now believe that the "domain knowledge" strategy which has been described will yield for a particular mathematical domain (such as ratio and proportion) a valid index or scale to measure a group's knowledge or maturity with respect to that domain. The expectation is that such a scale would be more valid than the conventional scales derived from the "content-by-behavior matrix" approach because it reflects a more integrated and coherent picture about knowledge.

A primary effort should be to identify a small number of important conceptual domains in mathematics and to follow the procedures described above to specify a map for each domain.

Task 4. This task was to examine extant test items and evaluate the utility of those items in light of the new content framework. This task is seen as part of a cost-effective strategy for monitoring. Redundancy in data gathering from schools would be avoided to the extent that existing data can be used for the monitoring.

For example, the Second International Mathematics Study (Crosswhite et al., 1986) administered 180 items to eighth-grade students. SIMS classified 9 of these as ratio and proportion. However, the classification used by SIMS was not always clear, and in using the ratio and proportion entailment mesh, we found that more than 20 of the 180 addressed proportional reasoning. The additional items had been classified by SIMS as percent, congruence, similarity, or representation of data and probability.

Analysis of the 9 items classified by SIMS and a sample of the other proportional reasoning items makes it clear that nearly all of the items were presented in formal mathematical prose. Only a few items included a graph or picture, and none used these alternatives as the primary form of presentation. In all cases, the students were required to work individually, never in a group requiring interaction with others to understand and derive an answer to the item.

Initial application of the entailment mesh for ratio and proportion to eighth-grade items from SIMS puts the assessment information in perspective by identifying limitations, areas of balance, and additional possible information embedded in other items. The mesh showed that the set of items included, for example, a balance of intensive and extensive quantities. However, there is no variation in representation. All require calculation of a precise answer in formal mathematical symbols. Students are not asked to estimate or approximate, to derive an answer while working as a group, or to provide verbal explanation. A main reason for the restriction is the multiple-choice format, which requires no representation from the student other than selection among choices (Price, Zarinnia, & Day, 1987).

Although several prospective epistemic coding forms have been considered, the dependence of coding on the indicator system resulting from Task 3 has become evident. For example, it is already clear that most existing tests have very few items which assess ratio and proportion--and those that do address only limited aspects. Thus, gaps in extant tests are apparent. Beyond that, the coding framework will provide us with a benchmark against which to compare measures that will be developed in Task 5.

Four working papers were written to summarize progress of the School Mathematics Monitoring Center on Task 4. They are:

1. The domain knowledge assessment strategy by T. A. Romberg.

2. Ratio and proportion literature review by S. J. Lamon, E. A. Zarinnia, and N. L. Webb.

3. Entailment mesh for ratio and proportion by E. A. Zarinnia, S. J. Lamon, and N. L. Webb.

4. Progress report on a plan to appraise coverage of ratio and proportion in extant tests by G. G. Price, E. A. Zarinnia, and R. Day.

Task 5. Here we planned to develop new measures of outcomes related to the content framework which can be combined with existing data. It was assumed that as a result of completing Task 4, gaps would be found; new measures were to be developed to fill those gaps. For example, in order to describe the conceptual domain for ratio and proportion, we have collected and written several test items to reflect the situations, use of symbols, etc. This is now seen as a much larger task than originally planned. Thus, a major effort should be to develop a set of exercises (items) related to each domain.

Task 6. Here we were to develop a scaling procedure for the aggregation of outcome information. From the work of the previous tasks, we assumed that several types of information would be gathered (not only correct responses, but strategies, errors, etc.) with respect to specific content domains in mathematics. This information will then be organized into scales. At this date, we have outlined the basic strategy for developing scales.

1. Identify a small number of important conceptual domains in mathematics and following the procedures described above and identify a map of each domain.

2. Develop a set of tasks which are related to each domain.

3. Administer the tasks to groups of students.

4. Logically combine information based on response to the task to yield a score.

5. For each group construct a score vector over all of the domains (e.g., School 1 ($\underline{X}_1$, $\underline{X}_2$, . . ., $\underline{X}_n$) where $\underline{X}_i$ = score for a domain).

Task 7. Finally, measures need to be identified or developed to be used to create indicators for the other key variables in the structural model. We have started to identify reasonable measures. For example, the professionalism of teachers was addressed in chapters 26, 27 and 28. It is obvious that there is growing concern that teachers might not participate in the revolution. Teacher ownership of the principles and procedures of the reform effort undoubtedly will be a key as to whether the reform movement will succeed. To develop an indicator for professionalism, one should draw heavily on the experience of the Documentation Project for the Urban Collaborative Projects for the Ford Foundation (see Romberg, Webb, Pitman, & Pittelman, 1987). In that project, a survey for "teacher professionalism" was developed. This survey should be used as a first approximation to an instrument for the monitoring plan. Another example is related to methods of teaching. This was alluded to in several chapters and directly addressed in chapter 28. The role of the teacher during instruction must change from director to mentor or guide. The details of this shift in the way the job of teaching is defined will be difficult to monitor. Thus, one task should be to examine methods of gathering reliable data on this shift in teaching.

ASSUMPTION 3. That a reasonable monitoring plan would involve repeated measures gathered from a small sample of schools.

This assumption led to Task 8, which was to specify a reasonable sampling plan to be used to systematically gather and store the information identified in the previous tasks. Our proposed strategy was based on the belief that the coherence of a mathematical program based on the reform goals would best be portrayed by studying the programs in a sample of high schools and their feeder schools (the sampling unit). An on-line data base system would be established to collect data periodically (weekly, monthly, by semester or year) on the key indicators developed for the structural model, depending on the index. It was proposed that our sampling strategy follow this "Nielson" model (i.e., a small number of high schools).

ASSUMPTION 4. That a usable data base can be created which would be useful for mathematics educators and educational policymakers.

Task 9. To pilot test the instruments and data gathering techniques.

It should be noted here that the policy environment has not been directly addressed in this monograph. Because of the political emergence of MSEB into this policy arena, they should take responsibility for this task.

Task 10. To outline a reporting procedure to inform NSF and others about the progress of the reform movement.

Both technical reports and brief reports should be produced for NSF related to all of these tasks. The technical reports will be lengthy documents containing the details of a particular analysis (e.g., the coverage of proportional reasoning on the SIMS test for grade 8). The audience of such reports would be scholars and researchers interested in the issue. Such reports also will serve as the backup documentation for the more widely distributed brief reports.

The briefs would be summary statements, succinct and digestible in style and format, prepared specifically for the policy community. They would address particular policy questions and concerns, using information derived from one or more of the technical reports. More specifically, the reports would inform policymakers on the issues, problems, strategies, and results. They would provide background information and draw attention to significant correlations between key variables and outcomes. This kind of information would be desirable regarding both domain knowledge outcomes and the strategies adopted at various policy levels.

SUMMARY

In 1985, the National Science Foundation funded this project to plan and establish a <u>School Mathematics Monitoring Center</u> designed to gather information about the impact of the reform movement on school mathematics. This review was carried out to identify the scope of activities needed to achieve the Foundation's intent.

> Parents, policymakers, professionals, and other taxpayers need timely, factual information on what students know and can do. . . .
>
> The continuing wave of school reform across our nation makes this need ever more urgent. Communities, schools, school districts, and states are in the midst of momentous decisions that will affect the quality of American education.
>
> Decision makers need to see the facts clearly. (Alexander & James, 1987, p. vii)

The underlying rationale associated with monitoring is that the society has goals, articulated by policymakers, and that the purpose of monitoring is to provide supporting information.

The basic strategy of SMMC hinges on the variables outlined in the structural model for monitoring; particularly noteworthy is the association of the key variables, especially the independent variable of policy, and the dependent variables of outcome. The sine qua non is valid information about domain knowledge, which is to be gathered using a <u>domain knowledge indicator system</u>. That information is to be correlated both with key variables in the structural model and with data from the policy framework. It is difficult to detect association among policy, instrument, and outcome; valid and sensitive measures are essential to identification of meaningful correlations. Their cost would be prohibitive if applied to the whole of school mathematics. However, if restricted to strategic topics important to the reform, they should be cost-effective indicators of changed outcomes, more susceptible to correlation, and more informative to policymakers.

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Appendix

Other papers from the School Mathematics Monitoring Center

1. The Content Validity, for School Mathematics in the U.S., of the Mathematics Subscores and Items for the Second International Mathematics Study. (Prepared for the Committee on National Statistics, National Research Council of the National Academy of Sciences, October 1985.)

2. Mathematics for Compensatory School Programs. (Prepared for the Conference on Effects of Alternate Design in Compensatory Education, June 17-18, 1986.)

3. Elementary School Mathematics: The IGE Legacy. (Prepared for Elementary Education.)

4. Measures of Mathematical Achievement: Problems and Influences. (Prepared for the National Conference on the Influence of Testing on Mathematics Education, June 27-28.)

5. National Assessment of Mathematical Performance. (Prepared for Study Group on National Assessment, August 1986.)

6. Reform in School Mathematics. (Prepared for the 1986 National Forum on School Science, November 15-18, 1986.)