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ABSTRACT

Described is the mathematics achievement of 391 elementary education majors at the University of Arizona. Students were administered two subtests of the California Achievement Test: the Mathematics Computation Subtest and the Mathematics Concepts and Application Subtest. The CAT was administered in a non-standard manner. Students were allowed three hours to complete all seven subtests rather than the specified times allotted for each subtest in the test manual. The mean score on the total mathematics section was 82%. The prospective teachers scored higher on the computation subtest than on the concept/application subtest. Correct responses for items ranged from 69% to 99% (mean 85%) on the computation subtest and from 46% to 97% (mean 81%) on the concept/application subtest. Correlations between the scores on the two tests was .83. Results for each of the mathematics tests are analyzed by item for skills and knowledge required. The prospective teachers were more likely to possess knowledge of terms and symbols, and computational skills, than to be able to coordinate and apply this knowledge when solving multi-step problems. Some students had difficulty in transferring algorithms for computing with fractions to analogous algorithms for computing in algebra. (RH)

ELEMENTARY EDUCATION MAJORS PERFORMANCE ON A BASIC MATHEMATICS TEST

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The education and competence of teachers is currently a national concern. Some states have instituted basic skills tests for teacher certification and some colleges of education are requiring basic skills tests at some level of their teacher education program. Mathematics is usually included in any such testing program. Just as the National Assessment of Educational Progress has provided us with some descriptive data of the mathematics knowledge of our country's students, the testing of prospective teachers provides us with a data base of a new population. The purpose of this paper is to describe the mathematics achievement of Elementary Education majors on two subtests of the California Achievement Test: the Mathematics Computation Subtest and the Mathematics Concepts and Application Subtest.

Methodology

A total of 391 Elementary Education majors at the University of Arizona were administered the Total Battery of Level 19, Form C, of the California Achievement Test (CAT) during the 1982-83 academic school year. The CAT was administered in a non-standard manner. Students were given three hours to complete the seven subtests rather than the specific time allotted for each subtest in the test manual. A score of 75% on each of the following sections was required for admission to student teaching: Total Reading, Total Language, and Total Mathematics. This paper reports only the results of the two Total Mathematics subtests: Mathematics Computation and Mathematics

Concepts and Application.

An item analysis was performed on the students' responses to the 85 multiple choice items on the two mathematics subtests. The 40 items on the Mathematics Computation Subtest were classified into a two-dimensional matrix according to the operation to be computed and the set of numbers and type of symbols in the exercise. The 45 items on Mathematics Concepts/Application Subtest were classified into the following content subsets: numeration, properties of operations and relations, geometry, graphing, metric conversion, word problems, algebra, and "other topics". In a number of cases the subsets were not disjoint. The following subsets had some overlap:

1) The geometry subset contains one geometric word problem.

2) The graphing subset contains two items that require reading a graph to solve word problems. One other item in this subset could also be classified as algebra since it involves graphing the solution set to an inequality on a number line.

3) The three metric conversion items are expressed as word problems.

Results

The mean score on the Total Mathematics Section was 82% while the median was 86%. This is due to the positive skewness of the distribution of scores for the Total Mathematics Section which also exists for each of the two subtests. The prospective teachers scored higher on the computation subtest (mean of 85%) than on the concept/application subtest (mean of 81%). This difference between the two subtests is also evident in the range of the percentages of correct responses to individual items. Correct responses for items ranged from 69% to 97% on the computation subtest and from 46% to 97% on the concept/application subtest. Table 1 contains a summary of test statistics for both subtests.

A Pearson Correlation Coefficient was computed between students' scores on the Mathematics Computation Subtest and the Mathematics Concept/Application Subtest using the SPSS computer package. The correlation coefficient was 0.8294 and was significant at the 0.001 level.

Mathematics Computation Subtest

The percentages of correct responses to the computation items was organized in a two-dimensional matrix according to 1) the operation computed and 2) the set of numbers and types of symbols in the exercises. This data is reported in Table 2. The operations tested were: addition, subtraction, multiplication and division. The sets of numbers and types of symbols operated on were: positive rational numbers expressed as decimals and as fractions; integers; and simple algebraic terms. When the 40 items were placed in the 20 cells of the matrix only one cell was empty. The other 19 cells contained one to five test items.

The elementary education majors were most successful when computing with decimals (91% correct responses) and least successful when computing with fractions (79% correct responses) and simple algebraic terms (73% correct responses). Of the four operations, students made more errors on the division exercises than on the other three operations.

Decimals. There was little variation in the percent of correct responses to addition (90-97%), subtraction (92-99%), and multiplication (89-95%) of decimal exercises. The range on four of the five division of decimals exercises was 83% to 89%; on the fifth problem 77% responded correctly. This latter exercise required the division of a decimal (0.ab) by ten; 11% of the subjects selected a number equal to the dividend as their answer. This would seem to indicate that these students do not routinely evaluate the reasonableness of their answers.

Fractions. The students were more consistent in their level of

correct responses to the two multiplication of fractions exercises and to the three division of fraction exercises than to the two addition of fraction exercises and two subtraction of fraction exercises. The results for the nine fraction exercises are shown in Table 3. There are some similarities in the difficulty level and structure of exercises A and C, and exercises B and D. Exercises A and C were easier with 90% and 89% correct responses compared to exercises B and D with 75% and 71% correct responses. Exercises A and C involved adding or subtracting two proper fractions with unlike denominators, in both cases the answer was a proper fraction that did not have to be reduced if the least common denominator was used. Exercises B and D involved adding and subtracting mixed numerals in which regrouping was involved. In order to select the correct response for the addition problem one must first change an improper fraction to a mixed numeral and "carry" or regroup a one. "None of the above" was the most frequent incorrect response--it was chosen by 16%. This could be due to the absence of the improper fraction in the correct response? When faced with a subtraction problem where the fraction in the minuend was less than the fraction in the subtrahend, 29% selected an incorrect response. Ten percent of the students chose a distractor that could be arrived at by subtracting the top fraction from the bottom fraction, correctly subtracting the whole numbers, and reducing the answer to lowest terms. Another 12% selected "None of the above". In the addition and subtraction exercises, more errors were related to mixed numerals/improper fractions than to equivalent fractions.

Integers. The test contained only four exercises involving computation with integers--one for each operation. The most common error made in multiplication (15%) and division (18%) was selecting the incorrect sign for the answer. In subtracting a negative number from a positive number 14% disregarded the negative sign and subtracted as if the numbers were whole

numbers. Both types of errors were made in the addition exercise containing one negative and two positive numbers: 3% subtracted correctly but selected the incorrect sign and 4% disregarded the negative sign and added as if the numbers were whole numbers.

Simple algebraic terms. Four of the five exercises involving computation of simple algebraic terms contained fractional terms. Whereas 90% of the students could correctly add and subtract fractions with unlike denominators, only 73% could transfer that knowledge to adding and subtracting fractions with a variable in the numerator. Eighteen percent of the students added the denominators and either added or multiplied the numerators. Due to the nature of the distractors on the subtraction item it is difficult to interpret the errors.

The two multiplication items elicited similar incorrect responses from students. Twenty-five percent of the students responded to item C in Table 4 with the product of the numerators over the like denominator. In this case they seemed to be confusing the multiplication algorithm with the addition/subtraction algorithms, yet only 3% made a related error when multiplying with fractions. The most common error made on exercise D was to find the product of the whole numbers, m and n, and then to affix the variable y. This error was made by 28% of the students. There is a great deal of similarity in these two incorrect responses. When taken together, the incorrect response to item C no longer appears to be related to confusing the rules of the algorithms for computing with fractions. An explanation that takes into account the errors to both mutiplication exercises is that these students are using the variable as a label in the manner that a young child would when adding 2 apples and 3 apples and getting 5 apples. To complicate the issue, students were more successful on the related division exercise (See Table 4, E). This was the only instance of a division exercise being easier

than a related multiplication exercise. The most common incorrect response to exercise E was ny. It was made by 10% of the students.

Summary. Thirty-four percent of the elementary majors made less than three errors on the computation subtest. These students have well developed computational skills across the four operations with the diverse numbers and symbols tested. Another third made 3 to 6 errors on the computational exercises, i.e., 65% of the students scored at least 85%. Therefore, most of the errors were made by 1/3 of the students. These students could benefit from further instruction in mathematics with an emphasis on meaning and evaluating the reasonableness of their responses. The latter is definitely based on estimation skills with the diverse sets of numbers used when computing, not just whole numbers.

Mathematics Concept/Application Subtest

The 45 items on the concept/application subtest were classified into seven content subsets. Since students had varying success with computing with the diverse sets of numbers and symbols on the computation subtest, the 45 items were also classified according to the set of numbers and type of symbols used in the item. The sets of numbers and type of symbols used in these items were: whole numbers, integers, rational numbers expressed as fractions and as decimals, percents, variables and "other". The "other" category contained one item that involved changing a decimal to a fraction, four geometry items that contained letters as labels for geometric entities, and one item that contained no mathematical symbols or numbers. If an item contained an integer or fractional expression and a variable it was placed in the variable subset.

The percentages of correct responses to items on the Mathematics Concept/Application Subtest are reported in Table 5. No set of numbers/symbols was overall more difficult for the students. There were too

few items containing integers to consider the mean of 91% as being an indication that students were more successful with integer concepts than with concepts and applications associated with the other five subsets of numbers/symbols. The means of the other five numbers/symbols subsets ranged from 76% to 82%.

There was more variation within some content subsets than there was between the means for the content subsets. Students responses to the individual items comprising each content subset are described below.

Other topics. Most of the items in the "other topics" subset test students' ability to apply or recognize basic definitions of terms and symbols. The elementary education majors were the most successful with this subset of items. They were familiar with the meaning of exponents, the general form of a fractional name for one, and the intersection of sets. They could change a decimal to a common fraction, find the least common denominator of two fractions and complete a series. They were less successful on exercise H (78%) than on a similar addition of integers exercise (89%) on the computation subtest. Both items involved adding integers with unlike signs; a major difference in the two items is the type of response expected. On the computation exercise all of the choices were integers, on exercise H the choices were addition or subtraction expressions. Seven percent of the students selected a response to -10 + 8 as being equal to -8 + 10 (-10 + 8 and -8 + 10 are structurally similar to the expressions in the actual item). This error appears to be a misapplication of the commutative property. Nine percent of the students selected responses equal to -2 rather than +2.

Properties of operations and relations. Whereas most students could successfully select a term to complete a general statement of the associative property, 25% could not similarly complete a specific example of the distributive property (See Table 7, exercises A and D). Exercises B and

C testing the transitive property of a relation were structurally equivalent. They were both of the form "X is related to Y, and Y is related to Z. Select the true statement." with the correct response being "X is related to Z." This would seem to indicate that some students are not familiar with the transitive property of relations and responded to these items based on their understanding of the specific relation.

Graphing. Four different types of graphs were tested. The results are reported in Table 8. Students were most successful with associating an ordered pair with a point on a coordinate system. They could also extricate information from a bar graph that was needed solving a simple word problem.

Only 71% of the students could select a number line graph of the solution set of an inequality. Even though the item specified that the variable was a whole number, 8% selected a response that included some negative integers.

Students lack of success on the circle graph item (See Table 8, exercise E) seems to be more related to finding a percentage than to reading the graph. Ten percent of the students selected a response of the form a% where a is the amount of time shown for the specified activity.

Metric conversion. The percentage correct for each metric conversion exercise is reported in Table 9. The principal skill being measured by these three items is the ability to use a table of equivalent measures. Most of the students were successful in converting from one unit of measurement to another when they were given a statement of the equivalent measures (See exercise A). One-fourth of the students selected incorrect responses when required to 1) use the information in a table of measures to arrive at the relationship of two metric units (exercise B) and 2) evaluate the four choices in terms of the three equivalencies for the unit that are

given (exercise C). Most of the incorrect responses to exercises C and D involved place value. Computationally, exercise C is simpler than exercise B. A logical approach would be to multiply 1/n times 10, 100, or 1000 for each equivalent measure and then find a match in the list of possible responses. The inability to devise such a strategy seems to indicate a lack of test-taking skills on the part of the students. Since all responses were expressed as decimals, including two whole numbers, it also indicates that these students either do not evaluate the reasonableness of their responses or lack the underlying understanding of decimals for doing so.

Numeration. When asked to identify the numeral that has the greatest value in the hundredth's place, 34% of the students selected the numeral with the greatest value in the hundreds place. Are students unfamiliar with the names of the decimal places? Students' level of correct responses (91%) to exercise B described in Table 10 would seem to indicate that they are familiar with the location of the tenth's place. Thus, it can reasonably be inferred that the students could also identify the hundredth's place. One explanation for over half of the students incorrectly reponding to exercise E is that they incorrectly read or understood the question. Perhaps some read only the first part of the question and selected the numeral with the greatest value which also happened to be the numeral with the greatest value in the hundred's place. That students comprehend what is meant by "greatest place value" is evident by their correct responses to exercise A (97%).

Students were more successful in selecting a standard numeral for a number expressed in scientific notation than in expressing a whole number in expanded notation where the value of the places was denoted by powers of ten. Seven percent of the students selected a response to the expanded notation exercise (Table 10, exercise D) in which each exponent was off by a + 1.

Geometry. Students were consistent in their level of correct responses (81% - 88%) to five of the geometry exercises (See Table 11). Exercises A - D involve associating a geometric term with a specific instance and Exercise E involves recognizing the formula for the area of a triangle. More students recognized the formula for the area of a triangle than could recall the formula for the area of a parallelogram.

A little over half of the students could solve a geometric problem that required 1) basic knowledge about the relationship of angles in a parallelogram--adjacent angles are supplementary, hence their sum is 180 degrees--and 2) how to set-up and solve an equation of the form $ax + x = 180$, where a is a constant.

Word problems. The seven word problems are identified in Table 12. Students were most successful in solving a one-step problem that required finding a square root.

Three of the problems, Exercises B, C, and F, involved percent. Finding a percent of a number (exercise B, 87%, and exercise C, 92%) was easier than finding the percent of discount (72%). Exercise 1 in Table 8 similarly required finding the percentage of one number to another, and the percent correct was 64%.

The proportion and rate problems (exercises D and E) were of equal difficulty. The multi-step ratio problem was the most difficult of this subset.

Algebra. The results for the six algebra exercises are reported in Table 13. Exercises A, B, and C, tested students' ability to solve linear equations. The students were most successful in solving exercise A after first substituting specifc values for two variables. Exercises B and C differed from Exercise A in that they contained rational expressions. The added difficulty of these two exercises can probably be attributed to the

students' inability to transform the equations so that they no longer contained rational expressions.

The two most difficult items (exercises E and F) in this section both involved factoring, i. e., the application of the distributive property. Thirty-two percent of the students selected a response of the form 12(m + n) as being equivalent to an expression similar to 3m + 9n. Two incorrect responses to exercise F were each selected by almost a fourth of the students. An incorrect response of the form $a^2 - b^2$ was selected by 24% and another of the form 2a - 2b by 22%. One-fourth of the students also had difficulty with the distributive property when no variables were involved (See Table 7, exercise D).

Summary. The exercises that were the easiest for the students involved simple recognition or one-step procedures for determining the correct response. However, some exercises of the same type were more difficult, suggesting that some content is less familiar to students. Specific topics in the latter group include the distributive property, transitive property of relations, solving and graphing inequalities, one-step rate and proportion word problems, finding a percentage, and algebraic computation.

The six most difficult exercises were correctly answered by 48% to 64% of the students. Three of these exercises were word problems (Table 8, exercise E; Table 11, exercise G; and Table 12, exercise G) that required the coordination of information and procedures from diverse areas of mathematics. Exercise E, Table 8, is solved by reading a simple circle graph, changing a number of hours to minutes, and then finding the percentage that one number is of the total. Students are more able to demonstrate each of the skills in isolation, yet future teachers need to be able to successfully select skills and information needed to solve problems.

Conclusions

The results of this analysis of test data provide a description of prospective teachers' mathematics achievement at one university. Do prospective teachers in other teacher education programs exhibit a similar pattern of achievement?

The prospective teachers tested were more likely to possess knowledge of terms and symbols, and to possess computational skills, than to be able to coordinate and apply this knowledge when solving multi-step problems. Some students have difficulty in transferring algorithms for computing with fractions to analogous algoriths for computing in algebra.

The data also point out that there is a range in the mathematics knowledge of prospective teachers. Although 22% of the prospective teachers did not score at the 75% passing level; 34% of the prospective teachers scored between 90% and 100%. Over two-thirds of those tested scored at least 80% on the total test. Thus, the prevalent perception that prospective elementary teachers do not possess basic skills in mathematics is false for over two-thirds of those tested.

Table 1

Summary of Test Statistics

	Computation Subtest	Concept/ Application Subtest	Total Mathematics Section
No. of Items	40	45	85
Mean Score	33.94 85%	36.38 81%	70.32 82%
Median Score	35.61 89%	37.63 84%	72.96 86%
Standard Deviation	5.94	6.50	11.90
Reliability (KR-20)	.886	.874	.933
Standard Error	2.09	2.48	
Range	0 - 40	0 - 45	0 - 85
Number of Scores	391	391	391

Table 2

Percentages of Correct Responses to Items on Mathematics Computation Subtest

Numbers/ Symbols	Operations				
	+	−	x	/	Totals
Rational Nos.					
Decimals	94* (5)**	95 (5)	92 (4)	84 (5)	91 (19)
Fractions	83 (2)	80 (2)	86 (2)	72 (3)	79 (9)
Fraction & Decimal	92 (1)	90 (1)	73 (1)	(0)	85 (3)
Integers	89 (1)	79 (1)	80 (1)	76 (1)	81 (4)
Simple Algebraic Terms	72 (1)	73 (1)	70 (2)	80 (1)	73 (5)
Totals	89 (10)	88 (10)	83 (10)	79 (10)	85 (40)

* Percent of correct responses.
** Number of test items.

Table 3

Results of Computation with Fractions

Exercise	Percent Correct
Addition	
A. Add two proper fractions with unlike denominators that are relatively prime.	90%
B. Add with regrouping three mixed numerals with unlike denominators having common factors.	75%
Subtraction	
C. Subtract two proper fractions with unlike denominators of the form y and yz.	89%
D. Subtract with regrouping mixed numerals with common denominators.	71%
Multiplication	
E. Fraction x Whole Number	90%
F. Proper Fraction x Proper Fraction	82%
Division	
G. Mixed Numeral / Mixed Numeral	75%
H. Mixed Numeral / Mixed Numeral	71%
I. Proper Fraction / Whole Number	70%

Table 4

Results of Computation with Algebraic Terms

Exercise	Percent Correct
A. $y/m + y/n$	72%
B. $y/m - y/n$	73%
C. $m/y \times n/y$	71%
D. $my \times ny$	69%
E. y^n/y	80%

Note: In all exercises, y is a variable and m and n are whole numbers less than 11.

Table 5

Percentages of Correct Responses to Items on Mathematics Concept/Application Subtest

Content	Numbers/symbols							
	Whole Nos.	Integers	Decimals	Fractions	Percents	Variables	Other	Totals
Other Topics	89* (1)**	78 (1)		91 (3)		93 (2)	94 (1)	90 (8)
Properties				71 (1)		87 (2)	94 (1)	86 (4)
Graphing		97 (2)	91 (1)		64 (1)	71 (1)		84 (5)
Metric Conversion	75 (1)		85 (2)					81 (3)
Numeration	71 (1)		80 (4)					78 (5)
Geometry	70 (2)		74 (1)				82 (4)	78 (7)
Word Problems	86 (2)		77 (1)	61 (1)	80 (3)			77 (7)
Algebra						71 (6)		71 (6)
Totals	78 (9)	91 (3)	81 (9)	82 (5)	74 (4)	78 (11)	86 (6)	81 (45)

* Percent of correct responses.
** Number of test items.

Table 6

Other Topics

Exercise	Percent Correct
A. Express a^4 as a x a x a x a.	97%
B. Change a decimal to a fraction.	94%
C. Find the least common denominator of two fractions, the denominators are relatively prime.	93%
D. Select simple probability of an event.	92%
E. Recognize fractional form for 1.	89%
F. Given two sets, select an element in the intersection.	89%
G. Select missing number in a series of rational numbers with an increment of 1/n.	87%
H. Given -8 + 10, select 18 - 16.	78%

Note: The examples used in exercises A and H are similar to those in the CAT.

Table 7

Properties of Operations and Relations

Exercise	Percent Correct
A. Select a missing term in an expression of the associative property of multiplication.	97%
B. Apply transitive property of "taller than" relation.	94%
C. Apply transitive property of inclusion relation.	77%
D. Select the missing number in an expression of the distributive property of multiplication over addition.	75%

Table 8

Graphing

Exercise	Percent Correct
A. Given a point at (x, -y) on graph, select coordinates.	97%
B. Given coordinates (-x,-y), select point on graph.	97%
C. Read bar graph to solve one-step multiplication word problem.	91%
D. Select number line graph of the solution set of an inequality.	71%
E. Read circle graph showing time usage and convert the time of one activity to a percentage of total time.	64%

Table 9

Metric Conversion

Exercise	Percent Correct
A. Simple addition of decimals word problem requiring the conversion of A's to B's. The number of B's equivalent to one A is given. (A and B are different metric units.)	94%
B. Convert A's to D's. The relationship of A's to B's, B's to C's, and C's to D's is given in a table. (A, B, C, and D are different metric units)	75%
C. Find a metric equivalency for 1/n of a metric unit. Three metric equivalencies for the metric unit are given.	75%

Table 10

Whole Number and Decimal Numeration

Exercise	Percent Correct
A. Select numeral in which "n" has the greatest place value. Places vary from thousands to thousandths.	97%
B. Round to nearest tenth.	91%
C. Expand number given in scientific notation.	85%
D. Given whole number expressed in words, select expanded notation of the form $(a \times 10^n)$.	71%
E. Select numeral with greatest value in the hundredth's place.	46%

Table 11

Geometry

Exercise	Percent Correct
A. Select a similar rectangle.	88%
B. Select the bases of a solid figure.	84%
C. Select a ray perpendicular to a line.	83%
D. Select the intersection of two planes.	81%
E. Recognize the formula for the area of a triangle.	81%
F. Find the area of a parallelogram.	74%
G. Find the measure of an angle in a parallelogram, given the ratio of the measures of two adjacent angles.	52%

Table 12

Word Problems

Exercise	Percent Correct
A. One-step problem solved by finding square root.	93%
B. One-step problem solved by finding a given percent of a number.	87%
C. Two-step percent problem, solved by finding discounted price, given percent of discount and list price.	82%
D. One-step proportion problem.	78%
E. One-step rate problem.	77%
F. Multi-step problem solved by finding percent of discount.	72%
G. Multi-step ratio problem.	61%

Table 13

Algebra

Exercise	Percent Correct
A. Substitute values for 2 variables in a linear equation and solve for a third variable.	92%
B. Solve a linear equation when the variable is the numerator of a fraction.	81%
C. Solve a linear equation in which the variable is in the denominator of two fractional terms.	77%
D. Substitute given values for three variables to evaluate a third degree rational expression.	77%
E. Factor a two term expression e.g., $3m + 9n = 3(m + 3n)$.*	53%
F. Select expression for which $(a - b)$ and $(a - b)$ are factors.*	48%

* Expressions are similar to those in the CAT.