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ABSTRACT

The purpose of this study was to assess children's and adults' quantitative understanding of fractions in a new and potentially definitive way, and to determine whether this understanding was related to fraction computational skill. Thirty-seven seventh graders and 32 adults (college students) participated in the study. A computation task was used to classify subjects as "good" or "poor" at fraction computation. An estimation task was used to classify the same subjects as "good" or "poor" at understanding fraction size. If such understanding is important to learning or remembering fraction computation rules, subjects would be expected to be "good" or "poor" at both. The study indicated that a quantitative notion of fractions was only one of the variables determining fraction computational skill. (RH)

Understanding Fractions as Quantities:

Is it Related to Fraction Computational Skill?

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Understanding Fractions as Quantities:

Is it Related to Fraction Computational Skill?

It is commonly believed that students find the rules for manipulating fractions meaningless and arbitrary mainly because they lack a quantitative notion of fractions (e.g., Carpenter, Coburn, Reys, & Wilson, 1976; Post, 1981). That is, they seem to forget or not know that the fractions they are manipulating represent quantities. Surprisingly, there is no direct evidence for a relationship between such fraction understanding and fraction computational skill. Previous work has tended to focus separately on one or the other aspect of students' difficulties with fractions. There is also some question as to how to properly define and assess a quantitative notion of fractions. The usual tasks, for example, those involving locating fractions on a number line (e.g., Larson, 1980; Behr & Bright, 1984), ordering two or more fractions (e.g., Behr, Wachsmuth, Post, & Lesh, 1984; Post, Wachsmuth, Lesh, & Behr, 1985), and generating equivalent fractions (e.g., Hunting, 1984) have yielded different and sometimes conflicting results, and no one task seems more definitive than the rest.

The purpose of this study was to assess children's and adults' quantitative understanding of fractions in a new and potentially definitive way, and determine whether this understanding is related to fraction computational skill. A computation task was used to classify subjects as "good" or "poor" at fraction computation. An estimation task was used to classify the same subjects as "good" or "poor" at understanding fraction size. If such understanding is important to learning or remembering fraction computation rules, we might expect subjects to be either "good" at both or "poor" at both.

Functional measurement methodology (cf. Anderson, 1981, 1982), borrowed from experimental psychology, was applied to the problem of assessing understanding of fractions as quantities. Subjects were asked to estimate the size of fraction symbols on a zero-to-one scale. Estimates were conceptualized in terms of the integration of two pieces of information, size of the numerator and size of the denominator. Main interest was in determining the algebraic structure of the information integration as revealed in the overall pattern of a set of estimates. By definition, estimates of fraction size should show the pattern of a dividing rule,

estimate = numerator / denominator,

even if they are not precisely accurate. This normative rule provides a meaningful and useful way to define correct understanding, and a base for interpreting difficulties. Estimates by subjects with incomplete or incorrect understanding of fraction size should deviate from the dividing rule, and the nature of the deviation should mirror the nature of the difficulty.

An area estimation task was included to validate subjects', especially children's, understanding of task requirements and use of the response scale. The same subjects were presented with partially shaded circles and asked to estimate the amount shaded. No understanding of fraction size was required, only a perceptual response to shaded area and appropriate use of the response scale. Thus, area estimates by all subjects, regardless of understanding of fractions, were expected to show the correct pattern.

Method

Fraction environment. Fifteen proper fractions served as quantities for the estimation tasks, and as a pool for creating problems for the computation task. The fractions were generated from the 3 x 5, numerator x denominator factorial design shown in Figure 1. Numerators were 1, 2, and 3. Denominators were 4, 6, 8, 10, and 12. Each point in Figure 1 represents the value of one of the resulting fractions. Note that the curves form a linear fan. This pattern graphically illustrates the normative rule for fraction size, numerator/denominator.

Computation task. Subjects were asked to "think aloud" as they solved eight two-fraction arithmetic problems. The problems were created according to the 2 x 2 x 2, arithmetic operation (addition or multiplication) x denominator relation (same or different) x fraction type (unit or nonunit) design shown in Table 1. For each subject, fractions were randomly selected from the appropriate subset of the fraction pool, subject to the constraint that the resulting set of eight fraction pairs utilized 13 of the 15 different fractions. The order of the eight problems was then randomized. Subjects were provided with paper and pencil, and the entire task was videotaped.

Fraction estimation task. Subjects were asked to make graphic ratings of the size of 15 fraction symbols. Fractions were presented one at a time on a computer screen. A 16 cm horizontal line served as the response scale (see Figure 2). Left and right ends were labelled "0" and "1", respectively, so that the response scale was essentially an unpartitioned number line from zero to one. Subjects responded to a given fraction by moving a short vertical line, positioned at the zero end, along the response scale until they thought its position corresponded to the displayed fraction size. There were 99 possible positions on the response scale, corresponding to numerical responses of 0.01 to 0.99. Order of presentation of the 15 fractions was randomized separately for each subject for each of three replications.

Area estimation task. This task was the same as the fraction estimation task except that the 15 fractional quantities were presented as partially shaded circles, and the ends of the response scale were labelled with unshaded and shaded circles representing zero and one, respectively (see Figure 3).

Order of tasks. Subjects completed the computation task, fraction estimation task, and area estimation task, in that order, in a single session.

Session length varied from 30 to 60 minutes.

Subjects. Thirty-seven seventh graders and 32 adults participated in the study. Children were recruited through newspaper ads and flyers. They were currently or had just completed reviewing fractions in school. Adults came from two different subject pools in order to maximize the chances of obtaining a range of adult fraction computational skill. First-semester students currently enrolled in a mathematics course were recruited from an introductory psychology course. Fifth- and seventh-semester students who had not taken a mathematics course since high school were recruited from upper division communication studies courses.

Results

Classification of computational skill. Subjects were classified as "good" at fraction computation if at least seven of their eight solutions were scored as correct; otherwise, they were classified as "poor". This classification scheme ensured that a subject who, for example, added across numerators and across denominators on problems involving addition of fractions with different denominators, but got all other problems correct, would be classified as poor. Using this scheme, 19 children and 18 adults were classified as good at fraction computation, and 18 children and 14 adults as poor.

Rules for fraction size. Figure 4 shows the five observed rules for fraction size. Each panel presents the mean estimates of one subject. Figure 4A shows the normative, numerator/denominator rule. Figure 4B shows a numerator-only rule. Estimated fraction size varied directly with numerator size, but denominator size had no effect. Figure 4C shows an analogous denominator-only rule. Estimated fraction size varied inversely with denominator size, but numerator size had no effect. Figure 4D shows a "neither" rule. Neither numerator size nor denominator size was

These results are important for understanding the behavior of subjects whose estimates of fraction symbols in the previous task deviated from the numerator/denominator rule. These deviations were interpreted as indicating a problem in understanding fraction size. However, one might argue that these subjects simply did not use the response scale appropriately. The appearance of the normative linear fan pattern in their estimates of shaded area provides a strong case against this argument, since this pattern depends on appropriate use of a very similar response scale.

Conclusions

Two aspects of the results are of interest here. The first has to do with the present information integration analysis of correct and incorrect understanding of fractions as quantities. Estimates of 54 subjects showed a numerator/denominator rule and, on this basis, were said to reflect correct understanding of fraction size. It should be noted that these estimates were not precisely accurate. Mean percent error ranged from 3 to 34 percent. Estimates of the remaining 15 subjects showed deviations from the normative dividing rule, and those of 9 could be exactly described and tested. The various incorrect rules reveal both the nature and variety of students' difficulties in understanding fraction size.

The present rule descriptions of correct and incorrect understanding of fraction size are not new. In fact, they differ little from those inferred by Behr et al. (1984) from children's explanations for their response to a two-fraction ordering problem. The present experimental methods, however, enable clear and convincing demonstrations of rule usage and precise tests of fit. Thus, they give new force to ideas that have appeared in previous research.

The second aspect of the results concerns the relationship between fraction understanding and fraction computational skill. The results suggest a complex relationship. Incorrect rules for fraction size appeared only for subjects who were poor at fraction computation and not at all for subjects who were good. This suggests that an understanding deficit may underlie or lead to computational difficulties, as is commonly believed. However, not all subjects with poor computational skill lacked an understanding of fraction size. Indeed, 42 and 86 percent of poor chiildren and adults, respectively, showed correct understanding. The implication is that many students use incorrect, often meaningless, rules for manipulating fractions even though they understand the meaning of the symbols they are manipulating. The picture that emerges is a quantitative notion of fractions is only one of the factors determining fraction computational skill.

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Table 1

Fraction Computation Problem Types

Arithmetic Operation	Denominator Relation	Fraction Type	Example
Addition	Same	Unit	1/4 + 1/4
		Nonunit	2/12 + 3/12
	Different	Unit	1/8 + 1/6
		Nonunit	3/4 + 2/10
Multiplication	Same	Unit	1/6 x 1/6
		Nonunit	3/8 x 2/8
	Different	Unit	1/10 x 1/12
		Nonunit	3/10 x 2/4

Table 2

Observed Rules for Fraction Size

Fraction Size Rule	Children Good[a]	Children Poor	Adults Good	Adults Poor	TOTAL	Estimation Accuracy[b]
Num/Denom	16	8	18	12	54	13
Num-only	–	3	–	–	3	73
Denom-only	–	1	–	–	1	31
Neither	–	3	–	–	3	130
Whole No.	–	2	–	–	2	154
Not Class.	2	2	–	2	6	138
TOTAL	18	19	18	14	69	
Accuracy	35	74	11	17		

[a] Fraction computational ability as determined by a pretest.

[b] Mean percent error, calculated for each subject and averaged over subjects.

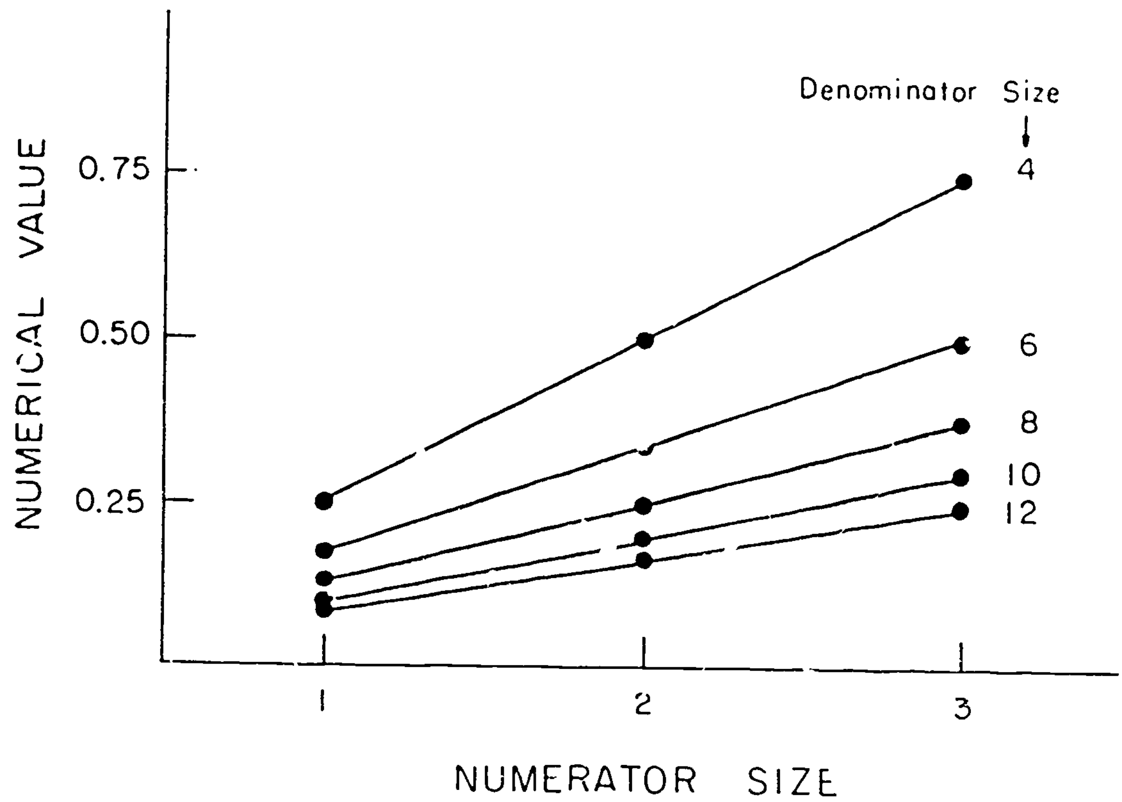

Figure 1. The normative rule for fraction size: numerator/denominator.

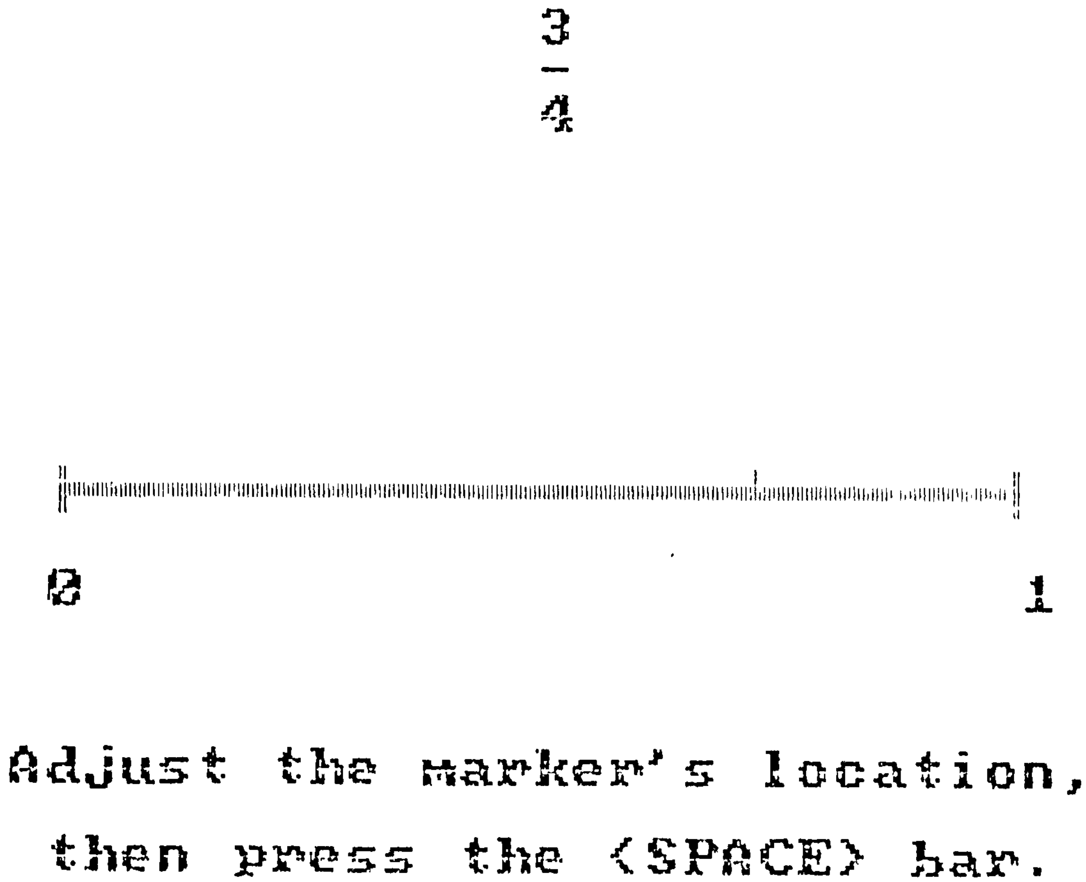

Figure 2. A fraction estimation trial.

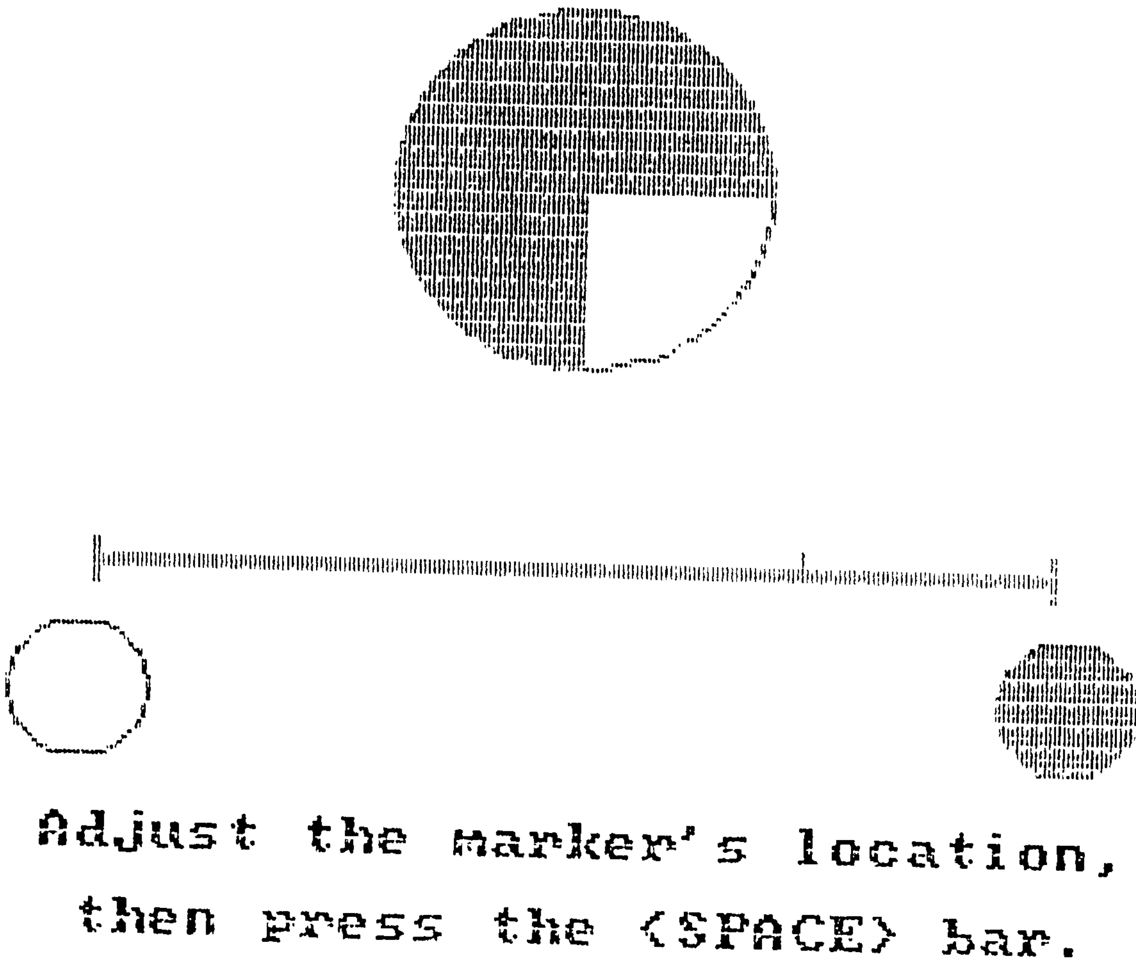

Figure 3. An area estimation trial.

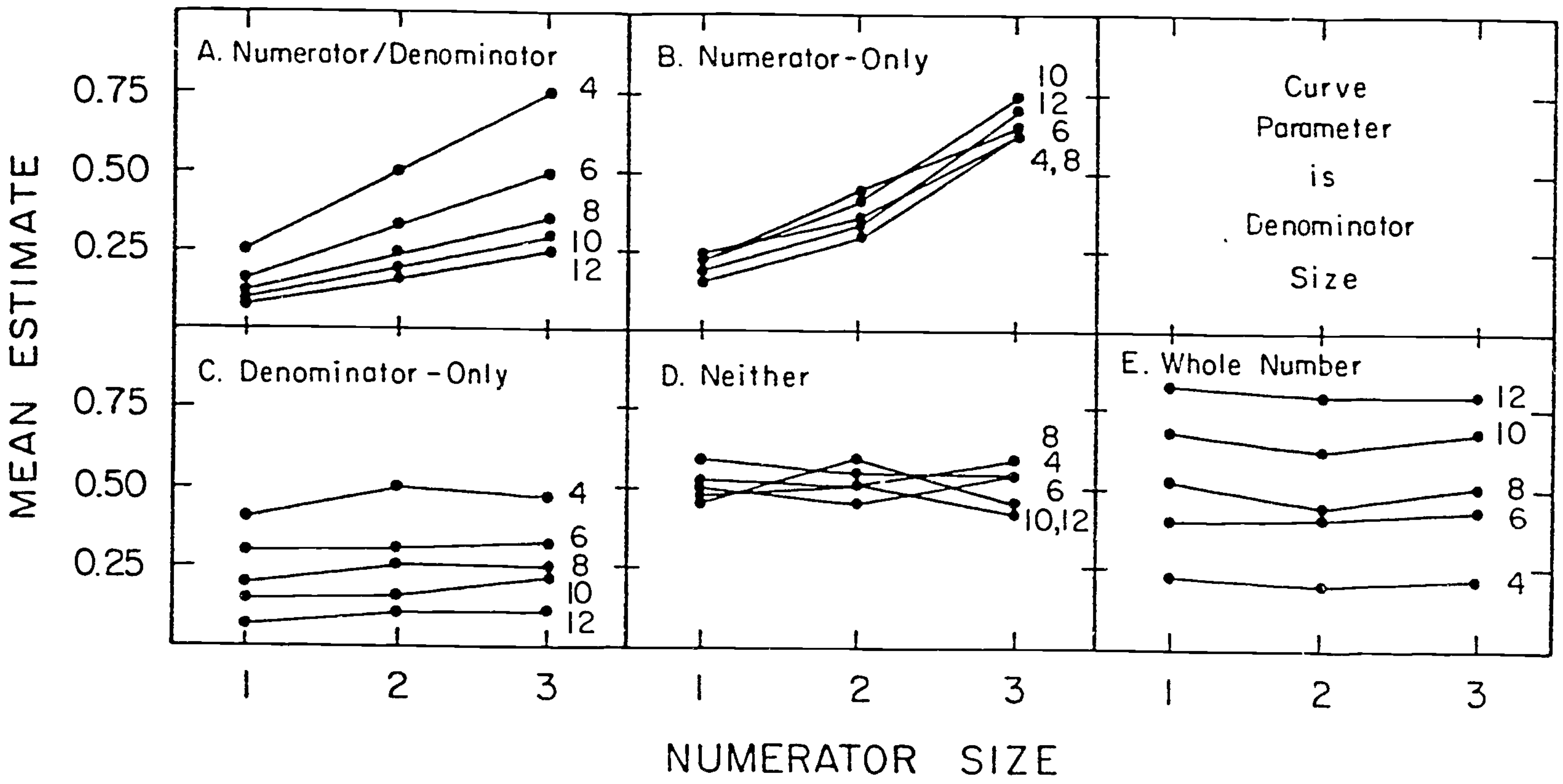

Figure 4. Observed rules for fraction size.