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ABSTRACT

This paper, which analyzes the evaluative aspect of affectivity, argues that the functional internalization of organic evolution for the formation of the intellectual system of human beings required that some means of selecting behavioral variations be devised, and that the selective mechanism devised was the affective system. From this perspective, it becomes possible to relate Pugh's (1977) conception of value-driven decision systems and his account of affective mechanisms to Cellerier's (1979a, 1979b) analysis of pragmatic and epistemic transformations. Tenuous as the resulting model may be, it gives a picture more complete than other models of the role affective evaluations play in adapting action and in constructing knowledge. In a preliminary way, the model also allows the bringing together of empirical data on problem solving, on decision making, and on the evolution of scientific thought. Further study is needed to determine whether these beginnings can mature into a full-blown theory in which other aspects of affectivity will be included. (RH)

Values and Affectivity from a Piagetian Point of View

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> I have called this principle by which each slight variation, if useful, is preserved, by the term Natural Selection, in order to mark its relation to man's power of selection.
>
> Charles Darwin(1964/1859)[2]

> The biological significance of mind is a necessary condition of scientific psychology.
>
> Lev Vygotsky
> (Zinchenko & Davydov, 1985)

In a thoughtful paper "On the Nature and Function of Emotion," Klaus Scherer (1984) directs attention to several important issues that will serve as starting points for this analysis. To begin with, says Scherer, Plato's classification of mental activity into cognitive, emotive, and conative categories is often accepted uncritically. Second, the history of psychology seems to cycle between periods when man is conceived as being basically irrational and emotional[3] and periods when he is conceived as an

2. For historic texts, the original date of publication is given after the date of the edition actually used.
3. Scherer starts off using the term <u>emotion</u> as a rubric for affective phenomena of every sort. Later, he distinguishes emo-

egg-headed creature of reason. Although currently there are indications of a shift away from the rational view inherent in behavioral and cognitivist approaches, renewed interest in emotion, according to Scherer, has so far failed to move us in the direction of more fruitful views of human mental life. One reason for this is that emotion theorists often attempt to deal with individual components of affective reactions rather than to treat them within a wholistic framework embracing the full complexity of the affective system. Another reason is that investigators continue to be divided on three important issues: the function of emotion, the definition of emotion, and the role cognition plays in emotional phenomena.

This paper will begin by reviewing the remedies Scherer proposes and then focus on his rather general functional argument relating to evaluation. Its aim will be to provide a foundational analysis and provisional model linking affective evaluation to cognitive activity in specific ways. Finally, it will relate the conception that results to recent work in problem solving, mathematics instruction, and philosophy of science in order to demonstrate its import and plausibility.

The Scherer Model

In contradistinction to interrupt theories, Scherer sees emotion as an important tool for mediating interchanges between

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tions from affective processes in general as has been done historically for compelling reasons (Fraisse, 1967).

organisms and their environments. Viewed from that perspective, emotion's functions appear to be threefold. First, emotion is used to evaluate situations relative to the organism's needs. Second, it is used to prepare physiological and psychological responses. And third, it aids in communicating reactions, states, and intentions to other individuals. With regard to the first two functions, the phylogenetic evolution of emotion allowed organisms to go beyond purely automatic responses by providing ongoing evaluation of the salience of stimuli and by discriminating adaptive from maladaptive reactions. With regard to the latter function, the intentional organization of behavior made possible by emotion becomes evident to others, as do emotional expressions. Taken all together, external manifestations of this sort contribute importantly to social interaction.

Tentatively, then, Scherer is able to link specific functions to the complex list of emotional components on which he believes modern psychological thought converges. In effect, the function of environmental evaluation results from the cognitive stimulus processing component of emotion, the function of system regulation derives from the neurophysiologic process component, the function of preparing action relates to motivational components, communicating intentions is linked to the expressive component, and reflection and monitoring are accomplished by the subjective component.

Scherer goes on to distinguish affective processes, a generic term for psychological phenomena exhibiting certain general organic and psychological features, from clearly delineated, intensive patterns of affective processes to be called emotions. He believes this distinction can be used to dissolve disagreement about what should properly be termed emotion. Using facet theory, he then sketches out a system for describing affective processes that would allow complex blends of affective components to be identified and the enormous number of affective states to be sorted out. And he ends by presenting a process model of stimulus evaluation that lends itself to theorizing about complex mixtures of emotions.

Scherer's definitions, the functions he assigns emotion, his scheme for describing affective processes, and his process model of stimulus evaluation form a natural and coherent approach to the problem of affectivity. At the same time, the evaluative implications of his model lie scattered across affective theory like broken clam shells on a deserted beach, lifeless fragments of related species to be gathered, classified, and integrated into a coherent theory. In what follows, therefore, I shall attempt to make clear what the basis of affective evaluation might be, to indicate how "environmental factors" are evaluated, and to elucidate the way in which affective processes figure in cognitive processes in general. If successful, I believe an analysis of this kind may be able to go beyond Scherer's inventory and attain something approaching an explanation.

The Biological Necessity of Feeling

Let us begin by recognizing that it is more than coincidental that both values and emotions are concepts meaningful only from the point of view of living systems. It makes no sense to speak of a mountain's morals or a river's anger; only intelligent animals have feelings. Our task, then, is to establish some biological basis for evaluation and to link that basis to intelligence. Only then will affective phenomena take on the necessity inherent in explanation.

Simply conceived, living systems establish and maintain their organization by means of compensations.[4] Energy used up in one way or another is restored by eating, sodium diffusion into a cell is counteracted by active transport out, an infant's helplessness is made up for by a mother's caring, etc. In short, if the concerted actions of an organism's subsystems reverse thermodynamic disorganization, the organism lives. In that sense it is "preferred to" or "selected over" or "valued more" than other systems. Thermodynamic "selection" is, therefore, the bedrock of organic evolution. That it eventually gives rise to systems using "organic" or internal mechanisms of selection does not alter the fact that it constitutes the basis of Darwin's selective principle (Depew & Weber, 1985; Brooks & Wiley, 1986).

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4. Prigogine (1980), Prigogine and Stengers (1984), and Brooks and Wiley (1986) have provided elaborate and somewhat diverse expositions of the basic idea presented here. Prigogine and Stengers working on nonequilibrial thermodynamic systems demonstrate the possibility of stabilizing dissipative systems far from thermodynamic equilibrium. Such systems have many of the properties found in living organisms. Brooks and Wiley interpret the existence of living systems as an instance of their newly formulated entropic theory. Both views are consistent with the global depiction given here.

Such considerations only make clear, however, the way in which material organic systems and some primitive forms of automatic behavior are evaluated. They do not provide any understanding of why an *affective* evaluative system would be needed. In fact, thermodynamic selection has resulted in an incredible array of organisms where feelings play no part. Can one say, therefore, that the evaluation inherent in thermodynamic stabilization corresponds functionally to affective evaluation? Do entropic considerations really determine what wine we choose for dinner? Is it really possible to relate this account of biological selection to Scherer's notion that affective processes allow organisms to discriminate adaptive reactions from maladaptive ones?

Certainly such assertions cannot be taken on faith as Scherer seems to do. It is a very long way from thermodynamic processes globally evaluating behavioral responses in terms of an organism's survival to psychological processes estimating a particular response's "survival value" in terms of feeling. Yet, lacking any other basis for human values, compelling theories of affectivity must bridge this gap.

Oddly enough, the first piling upon which such a conceptual span might be constructed comes from Piaget, a thinker not noted for his concern with feelings. Piaget's idea that intelligence is essentially an internal, psychological reduplication of organic evolution leads immediately to the need for internal mimicking of thermodynamic selection. While Darwin's theory might, by itself, suffice to explain the development of reflexes or instincts in the form of inborn, automatic patterns of behavior, it cannot account for the acquisition and extinction of the simplest habits and is even more impuissant where intentional phenomena are concerned. This is because habits and all higher levels of behavior are selected individually. The organism does not have to die to rid itself of a noxious habit, and thermodynamic selection provides for nothing else. Theories of habit formation always invoke some internal mechanism of selection and, significantly, they always invoke some sort of feelings. That much was clear when Plato (Hubbard & Karnofsky, 1982/4th century B.C.) wrote Protagoras, it was evident in Thorndike's "law of effect" (Nuttin, 1963), and it was obvious in Skinner's (1984) notion of "operant conditioning." Even modern cognitivists, although they seldom mention affectivity (Zajonc, 1980) and do not recognize the fact explicitly, have not got round the basic intuition that affective processes determine the "life" and "death," the "selection" of behaviors.[5] But no one knows exactly how.

A provisional answer and, therefore, the second piling upon which our biological-psychological bridge will be constructed has been developed by George Pugh in his book The Biological Origin of Human Values. In that work, Pugh argues from principles of design for the necessity of evaluation in any system capable of making complicated choices. The striking thing about people, according to Pugh, is that they make good decisions about what behaviors will prove adaptive when the knowledge they have avail-

5. See for example John Anderson's (1980, pp. 264-268) discussion of the value subject's place on similarity in his discussion of "Similarity as a Basis for Selecting Operators."

able is woefully inadequate, when their computing resources are severely limited, and when the time available is all too short. Concretely, human beings decide daily where to invest their money, whether to marry and reproduce, what car to buy, which way to vote, etc. In none of these cases could any amount of computation insure success. Both practically and theoretically, it is impossible to make such choices by developing and exploring every possibility. In fundamental ways, therefore, man is not and cannot be wholly rational in either the logical or praxeological senses of that word.[6] Rather, he is an approximator, a make-doer of remarkable skill who includes among his complex repertoire of decision strategies a middling aptitude for rational deciding that can be applied in fairly simple situations. For the most part, however, values drive the human system, and those values

6. The word rational is ambiguous. Sometimes it is used to imply both possibility and necessity. For example, it is rational to conclude that if one of John's four cats died (and nothing else has changed), he now has three. In this case, there are no other possibilities. At other times, however, rationality implies possibility without necessity. In the instance where a person wishes to start his car but the battery is dead, it would be possible (and in that sense rational) to have someone "jump" it; but it is in no way necessary. He could equally well (and with equal rationality) install a new battery or push the car to start it. But even praxeological rationality is not possible in every case. Many human decisions have to do with situations where the rationality of the goal itself may be in question, where ways to reach the goal may be unknown, or where once some action has been accomplished one is still uncertain whether one is any nearer to the goal. Take, for instance, decisions about foreign aid or child rearing. Can one really know whether providing the Contra's with arms has or will promote the security and well-being of our nation? Can one be sure that teaching Johnny Latin or having him play team sports will produce desired qualities in the man? To understand the complex web of meanings inherent in these different uses, it is necessary to relate success, necessity, and heuristics to one another within the teleonomic framework. How this can be done will be clearer at the end of the article than it can be here.

relate to feelings, not to reason. Pugh explains why, even so, this leads to adaptation.[7]

The third and final pillar upon which our cognitive-affective span will be constructed comes from Guy Cellérier's integration of Piaget with American cognitive science and, therefore, Pugh. His model is helpful because it solves two problems. The first has to do with the lack of any temporal or teleonomic dimension in Piaget's "knowledge structures" (Inhelder & Piaget, 1979), making it impossible for them to explain adapted actions or the affective evaluations that shape them. The second relates to the inability of Pugh's value-driven decision systems to account for how knowledge is acquired. In consequence, some means of knowledge acquisition must be appended to Pugh's hypothesis if we are to approach the human situation.

To solve these problems, Cellérier delineates what kinds of knowledge are needed, he invokes a control system that evaluates action possibilities drawn from knowledge in order to invent adapted actions, he adds a memory for the procedures that are invented, and he posits that knowledge is constructed by integrating these pragmatic building blocks. His model, therefore, illuminates both what he calls "pragmatic" and "epistemic transformations" as well as the evaluative function that connects the two. In order to grasp all of this more clearly, we must examine each constituent more carefully.

7. Pugh, of course, as has the author elsewhere (Brown & Weiss, 1987), argues that there are "intellectual values" and that these also are manifest as feelings. My purpose here being to stress the importance of the irrational qua emotional rather than to illuminate the "rationality" of emotion, I have not entered into this troubling proposal. It is discussed later in the paper.

Piaget's Conception of Intelligence

As is well known, Jean Piaget decided while still an adolescent to create a biological explanation of knowledge. In 1936, he laid out the relationships he envisioned between biological and intellectual domains (Table 1; Piaget, 1952/1936).

(TABLE I ABOUT HERE)

Essentially, he conceived intelligence as a biological adaptation functionally organized around providing a new system of adaptation. Whereas organic evolution proceeds through variation of individuals, thermodynamic selection, and reproductive transmission to succeeding generations, intelligent adaptation proceeds through variation of mental structures, psychological selection, and communicative transmission.[8] With the advent of intelligence, then, a maladapted behavior or idea can be eliminated without sacrificing the organism as a whole. By making it possible for individual psychological structures rather than entire organisms to evolve, the psychological system renders adaptation more rapid and more flexible.

8. To be sure, Piaget (1980) strongly criticized neo-Darwinian theories of evolution based on principles of variation and selection. Faithful adherence to his conception of adaptation in terms of the equilibration of assimilation and accommodation adds nothing at this point, however, and creates an added burden for the reader. For that reason, Piaget's terminology has been avoided.

The table, however, contains several problems that are not remedied in Piaget's later work. To begin with, although Piaget insists that action is the basis of all knowledge, the table does not make clear how action and knowledge are related. Categories related to action, i.e., goal and means as well as ideal and value, are linked to internal organization; but nowhere in Piaget's own work, then or later, does one find any clear statement of how internal organization results in adapted action. Similarly, space and time, objects and causality are categories used to understand the external world, but by themselves they do not produce adapted action on it. Even when Piaget (1974) much later takes up the relationship of action to understanding, he does not make clear how action structures are constructed. Nor when, still later, he relates goals and means to the laws of equilibrium in the treatise on equilibration (Piaget, 1985) does he provide an explicit model of how knowledge and action are related. Even in his final paper where he explains that action structures differ from knowledge structures in that they have teleonomic and temporal direction (Inhelder & Piaget, 1979), he only reemphasizes the distinction without saying exactly how the two relate. In consequence, Piaget's theory of how knowledge is constructed out of action remains seriously incomplete.

The same is true with regard to values. In Origins of Intelligence (1952/1936) where Table 1 appears, Piaget says nothing about the role values play in adapting action. He only includes the idea within the table. When later he identifies values as the diachronic aspect of affectivity and denies the latter a direct role in forming knowledge structures (Piaget, 1981a/1953-54), he does not make the procedure-structure distinction (Inhelder & Piaget, 1979). Nor does he appreciate the specific role values play in formulating action. He does, however, recognize at least intuitively that values play a role in what Cellérier later terms the "pragmatic transformation," i.e., the construction of action out of knowledge.[9]

What we take from Piaget, then, are the notions that intelligence is a system for evolving adapted action, that knowledge structures differ fundamentally from their procedural counterparts, and that the latter owe their special character, even their very evolution, to teleonomic and temporal asymmetries produced by affective evaluation. What we cannot distill from Piaget's rich ferment is an exact model of the relationships involved or a clear idea of how affective values guide adapted action.

Pugh's Value Driven Decision System

In his book The Biological Origin of Human Values, George Pugh (1977) argues, like Piaget, that the purpose of human intelligence is to evolve adapted action and that knowledge structures are necessary to achieve this goal. Pugh, however, makes the relationship of knowledge to action clearer by elucidating the role evaluation plays. His point of departure is the realization that

9. For a more complete discussion of these issues, see Brown & Weiss, 1987.

in designing artificial systems used to decide complicated courses of action, it is, in practice, always impossible to include enough knowledge or enough calculating resources to examine possibilities exhaustively and make optimal solutions. That is not how computer programs operate, and by extension, Pugh concludes that it is not the way people decide complicated questions either. Artificial decision systems, once a certain level of complexity is reached, all converge on the value-driven design. Such systems use limited numbers of operators to generate large numbers of action possibilities from a limited knowledge base. This greatly reduces the amount of knowledge that is needed. At the same time, such systems use a value structure to evaluate action possibilities generated from the knowledge base in order to reduce the number of possibilities to be examined.[10] By avoiding

10. Pugh differs from other cognitive scientists in his explicit recognition of the role evaluation plays and in his equation of evaluation with affective processes. Most mainstream workers in this area invoke heuristics without recognizing that they inscribe a value system or relate to feelings, or they speak of evaluation as if it were a rational, quantitative affair achieved through some exact algorithm where emotion plays no part. For example, the index to Kahneman, Slovic, and Tversky's Judgment Under Uncertainty: Heuristics and Biases (1982) contains no references to affectivity, emotion, evaluation, feeling, motivation, or values, although many of the articles are filled with affective terms. Similarly, the index of Human Problem Solving (Newell & Simon, 1972) contains no references to affectivity or feeling, while emotion, evaluation, motivation, and values are all included either without reference to affectivity or in extremely peripheral ways. For example, emotion is mentioned twice, the first time to label aberrant traces in a cryptarithmetic problem-solving task, the second in a survey of other work in information processing (Abelson's "hot cognitions"); evaluation is referenced many times and is related to heuristic search but is not considered an affective or emotional affair; the several references to motivation are aimed at showing that the problem-solving subject may be considered to be rational; and the few references to value have to do with the values that attributes assume.

exploration of actions that fail to produce progress in the right direction or, in other words, by pursuing only "good" alternatives, such systems economize on time and calculating resources needed for their operation.

The trick, of course, is to know the bad from the good and, says Pugh, the most difficult of the programmer's tasks is to design the value system. For artificial systems, this is done by trial and error, by comparing the computer's decisions to the values inherent in the designer's goal. Value structures that do not make the machine's decisions converge on the designer's wishes are changed until they do. However, even using this procedure, it is seldom possible in complex systems to incorporate the designer's value structure in full detail. Almost always "surrogate values" that approximate decisions the designer would make himself must be employed.

With all of this in mind, Pugh reasons that, faced with building a decision system that could mimic organic evolution, the genomic system, too, was forced to use a value-driven system and that the values inherent in the system are surrogates for survival value. To do otherwise, i.e., to build a machine that could exactly calculate the survival potential of action possibilities, would involve incredible amounts of knowledge and computational machinery. Even nature's resources were outstripped. Consequently, what has evolved is an organism with a set of primary values evaluated against the thermodynamic axiology imposed by biological selection. This means that the hedonic system of the Greeks, at least in its most primitive manifestation, is a system of innate feelings used to approximate survival value. Hedonic feedback either "reinforces" or "extinguishes" activities according to some surrogate heuristic for estimating survival and allows adapted actions to evolve.

Were this the whole story, however, it would not be very interesting. While it is easy enough to understand the adaptive value of a newborn infant uttering distress cries if it is cold or hungry or ejecting acid solutions from its mouth, it is a long way from such elementary evaluations to a critic's enchantment with a singer, to Dirac's fascination with his equations' beauty, or to a right-to-lifer's disapproval of abortion. Pugh addresses this problem in two ways.

To begin with, says Pugh, even in simple computer systems, the values structures necessary to make reasonable decisions are a good deal more complex than most people realize. Multiple values are often necessary and these must be made to vary temporally in intensity. So the value system with which the human infant starts is probably much more complex than we have ever dreamed and almost certainly includes intellectual and social values as well as values relating to simple physiological needs. From this point of view, it is not so strange that formal beauty or moral convictions are related to survival.

Secondly, says Pugh, value driven systems, although they cannot modify their primary value structure, can elaborate secondary values in the form of likes and dislikes, short-cuts, rules of thumb, wise saws and sayings, fashions, esthetic or moral systems, etc., to help them approximate primary values more easily and effectively. Consequently, it is not only possible but it is also to be expected that extremely complicated secondary value systems will evolve through the functioning of the primary system. That being the case, it is less difficult to understand one's feelings faced with Caballé's perfection. From Pugh's perspective, such emotion derives through myriad layers of elaboration from primary values dictating the positive value of others' voices, of symmetries and patterns, of social interaction, and so forth.

It should be noted that the uniqueness of Pugh's interpretation does not lie in his recognition that optimal decision strategies are impossible. Cognitive scientists in general accept that fact. What Pugh adds that is original is explicit recognition that the main heuristic system employed in humans is affective. People do not make most of their decisions by "minimizing distances in problem space," by consciously or unconsciously calculating probabilities, or by employing any other of the host of quasi-rational criteria invoked by modern workers. Not only is it arguable that many of the criteria identified by decision scientists are known to the deciding subject through the medium of feelings, but it is also demonstrable that many of our decisions make no pretense to rationality at all. What modern decision science seems blind to is the fact that feelings are reasons, and in most instances, the only ones available. Nor can it see that these "reasons of the heart" have a pragmatic logic of their own.

While Pugh has much more to say about the human value system and how it relates to affectivity, this brief sketch is sufficient for the present. What we take from his account are the ideas that human feelings are the subjective aspect of evaluative activities, that such activities constitute heuristic devices for reducing search spaces in formulating action, that all evaluation derives from a complex system of inborn values surrogate for adaptive value, and that much of mental development has to do with the construction of secondary value systems aimed at making the estimation of primary values simpler when dealing with ever more complicated problems. What we cannot draw from Pugh is any inkling of how the knowledge structures used to generate action possibilities are constructed. For that reason, we turn to Cellérier.

Cellérier's Epistemic and Pragmatic Transformations

Cellérier rejects the notion that Piaget's genetic psychology and American cognitive science are incompatible, arguing instead that they only approach the problem of how knowledge is used to organize adapted action from different points of view. On the one hand, Piaget's structuralism focuses on the "epistemic transformation," or on how knowledge structures are derived from action. On the other, cognitive science focuses on the "pragmatic transformation," or on how action is constructed out of knowledge. For Cellérier, both questions are legitimate as is the further question of how to resolve them on the basis of a single model.

Cellérier begins his synthesis by recognizing that both cognitive scientists and Piagetians study "the psychological subject" and that subject's behavior[11] is organized through cyclic interaction of knowledge and action, converging gradually on practical or intellectual success. In such interaction, initial assimilation to existing knowledge structures produces understanding of the situation requisite to establishing a goal-means structure and planning action directed toward the goal. Any action undertaken produces a new situation that must be assimilated in turn, new understanding or meaning established, a new goal-means structure imposed, and new action implemented. If the aim is simply to succeed in some practical task, the cycle stops when the goal is reached. If the aim is to understand or explain, it continues working out alternative solutions and integrating successful and unsuccessful schemes with one another. That much is the same for Piaget and Pugh.

Now, Cellérier argues, each assimilatory framework set up to understand the initial situation or the results of action constitutes a transitory model, but not just a model of the environment as cognitivists suppose. Rather, such frameworks model "adaptive interactions" representable by <u>production rules</u> of the form: If some external situation is detected and if some motivational

11. Piaget, of course, objected to the term behavior because of its suggestion of behaviorism, a doctrine he thoroughly rejected. Since his own term, conduct, sounds stilted and carries no specific English meaning, I use behavior to designate all things mental.

state exists, then execute some action or pass to a certain other state or both. Realizing this puts Cellérier in position to argue that adaptive interactions can be organized as production systems and are therefore programmable. This answers Moore and Newell's (1974) objection that Piaget's explanatory concepts have no substance because they are not "effective procedures." At the same time, it indicates features of existing cognitivist conceptions that must be remedied if one wishes to approach the human situation.

Perhaps the most striking thing about these transitory models is that they require three separate kinds of knowledge. Understanding the external situation, whether it is a world of "real" or logicomathematical objects, requires Piaget's operatory knowledge; recognizing motivational states and thereby establishing a goal-means framework requires what Cellérier calls axiological knowledge, a knowledge form that usually goes unnoticed; and knowing how to perform some action or pass to some other state requires pragmatic knowledge indicating how to transform the situation given. But these knowledge forms are not of equal rank. Operatory and pragmatic knowledge are functionally subordinated to axiological knowledge which alone determines the pertinence of actions and situations by setting up implicative relationships between means and ends. When values, goals, and means are conscious, these relationships have to do with intentional finality and knowledge and are explicit. When they are associated with "needs," i.e., with evaluative functions unaccompanied by corresponding conscious conceptualizations, they only constitute an aspect of implicit know-how.

A second striking feature of transitory models is that they do not spring from knowledge, however many kinds there are, without the activity of a control system that is itself subject to a value system. In other words, knowledge itself is is not sufficient to understand a situation, to establish goals, or to know how to perform various transformations. Something must select transformations from among the numerous possibilities made available by knowledge and coordinate them into acts either of comprehension or action. Something must make transformations understood and valued as means lead to the states or situations understood and valued as goals. This is the nature of control.

Having identified adaptive interactions as the basic subject matter of both Piagetian and cognitivist psychology and having teased out their general features, Cellérier then uses them as a Rosetta stone to establish correspondences between Piagetian and cognitivist conceptions. With minimal modification, Piaget's "schemes of action" and cognitive science's "procedures" can be brought together under the rubric of pragmatic knowledge. With a bit more difficulty, cognitive science's environmental models can be transformed into Piaget's empirical and pseudoempirical structures requisite for understanding. With substantial imagination, the Piagetian concept of "groupements of qualitative values" and the complex value structure implicit in Newell and Simon's "heuristics," can be subsumed under axiological knowledge. And finally, with a giant leap of creative insight, Newell and Simon's recursive method for analyzing means and ends and Piaget's optimizing equilibration can be interpreted in terms of the "problem of control" posed on pragmatic and epistemic axes respectively.

By themselves, however, such correspondences do not lead to an integrated model. Piaget oscillates between conceiving schemes as procedures and knowledge structures, when the two are irreducible to one another; the "effective procedures" of cognitive science have no psychological content. Neither environmental models nor Piaget's empirical and pseudoempirical structures furnishes all the knowledge that is needed. Neither theory explicitly recognizes the role axiological knowledge plays. And while in both cases, the control structure represents task-independent extremalizing mechanisms, they operate on different axes. In the pragmatic transformation, they act to minimize the _distance_ between starting point and goal. In the epistemic transformation, they act to maximize both the _invention_ of new knowledge structures and the _conservation_ of old. Consequently, Cellérier feels forced to create a model of his own.

His basic strategy is to begin with an existing model of the pragmatic transformation and then modify it to include the epistemic transformation by adding memory, buttressing hierarchical with heterarchical principles of control, and reformulating the notion of procedure. The model he begins with is Newell and Simon's (1972) General Problem Solver (GPS). Briefly, GPS operates on problems where "operators" are used to transform "objects." Various goals are possible, and methods relevant to each goal are needed. Since problem spaces are relatively complex, heuristic methods must be used instead of methods of exhaustive calculation. Given all that, the control structure of the program is quite simple: the goal inherent in a problem presented to the system is first evaluated to see whether it is worth achieving and whether success seems likely; if that is the case, one of the methods relevant to that goal is selected; if it leads to success, the program stops; if it fails, the loop is repeated, each time selecting another method until all pertinent methods have been exhausted. If necessary, subgoals may be invoked and the steps above applied to them (Newell & Simon, 1972, pp. 416-420).

Figure 1 about here

As can be seen in Figure 1, the methods employed by the logic-problem version of GPS are provided to the system in the form of a table connecting operators with various differences among objects. Rules prescribing what differences are most important also are provided. The control structure uses these components to perform means-end analyses by oscillating among goals, the kinds of transformations needed, and operators that perform those transformations. For example, instructed to change L1: R.(~P -> Q) into L0: (Q v P).R, GPS produces the trace shown in Figure 2. Since no operator can directly transform the first expression into the second, a subgoal must be introduced (goal). Since differences between expressions are more important than differences between subexpressions (see "Criteria of Progress" in Figure 1), they are worked on first. The only difference at this level has to do with the positions of the variables. They must be changed if the two expressions are to become identical (function). The program will try, therefore, to put the variables L1 into the same position as they are in L0. As it happens, operator R1 affects position, so it is chosen (method). By proceeding in this way, the program arrives at a solution.

Figure 2 about here

The most glaring difference between GPS and intelligent beings is that, like Athena, GPS sprang fully armed from the head of Zeus. The knowledge it possesses is the knowledge it will have forever. Not only does it not improve its understanding of new problems or augment the values inscribed in its heuristics, but it does not even expand its repertoire of procedures. Once a problem has been solved, it forgets what it has done and begins all over. Given the same problem two times running, it always starts from scratch. Cellérier is forced, therefore, to add a memory.

Linked to a control structure that invents schemes or psychological procedures from previous knowledge, this would account for the pragmatic transformation. Procedures worked out by the GPS control system are retained so that the next time the situation is encountered the system knows exactly what to do. But this does not account for how operatory or axiological knowledge is developed. Cellérier deals with the first in the following manner.

Procedures are adapted by accommodating to external environmental conditions and at the same time meeting internal conditions imposed by the goal-means structure. If the first is not done, action is ineffective; if the second fails, it is unsatisfying. With the advent of an internal environment produced by memory, a second universe of action becomes possible. In Cellérier's view, it is action on this internal world that underlies the epistemic transformation and provides the basis for true understanding.

> Our hypothesis is precisely that at the microevolutive (problem solving) level where we are situated, how problems are understood is captured in part by the following formulation: construct a sequence of epistemic transformations reconstituting the practical model, the implicit know-how corresponding to success, in terms of an operatory model the components of which arise from knowledge, i.e., from the explicit and consciously conceptualized operations of the subject. This formulation ("to succeed in understanding") involves the same equilibratory cycle, but acting at a different logical level. In a way, it is no longer a matter of tracing an action path through problem space starting from elementary displacements but of constructing a thought geom-

etry of paths by generating the set of all paths possible (Cellérier, 1979a, p. 99.)[12]

Operatory knowledge, then, derives from accommodating an action scheme to conditions imposed by an "external" environment of possibilities and at the same time meeting conditions "internal" to the goal of reconstituting the practical procedure. In the process, actions must be chosen in terms of what is possible within the world of schemes as well as what is necessary to effect the reconstruction. Finally, remembering these internal actions unfolding in a space of possibility completes the epistemic transformation. Although highly abstract and difficult to study,[13] this conception provides a preliminary understanding of how operatory knowledge is invented and conserved. It does not, however, shed any light on the axiological transformation, and, in fact, that issue is only partially considered by Cellérier.

What he does say is that axiological knowledge is used both to establish the "pertinence" of pragmatic and operatory knowledge, and to determine the value of transformations vis-a-vis some goal. This means that as a situation is interpreted by assimilation into operatory structures, the various elements identified are evaluated and motivations or goals determined. Then, once this teleonomic structure is set up, axiological knowledge

12. Excerpts from references given in French have been translated by the author.
13. Although certain aspects have been extensively considered in the Genevan School's work on abstraction (Piaget, 1977), generalization (Piaget, 1978), and the construction of possibilities (Piaget, 1981b).

is used to evaluate means relative to the goal. Cellérier describes how all of this transpires in a complicated way:

> The axiological dimension is introduced by the definition of a directed difference between the possible values of a given parameter. Over the multidimensional adaptive surface representing the problem space, i.e., the set of possible situations that may be generated by combining operators, the vector of differences or "distance" between two situations, alpha and beta, is used to construct an evaluative function the optimum of which corresponds to a null distance (alpha = beta). To reduce this distance is both to reach a situation evaluated as better from the axiological point of view and to approach the goal within the teleonomic framework associated with it . . . . Certain characteristics of the vector, e.g., its axis, its direction, its intensity, etc., are used to preselect activity. Only operators acting along the proper axis and in the right direction are chosen. Finally, the components of the vector, the different parameters, are ordered according to their importance which introduces a hierarchy into the system's different value scales . . . . This "constituted structure" is furnished prefabricated to the cycle coordinating goals and means which is invariant and independent of the various contents that are presented to it. Thus, the cycle accepts the definition of a problem in the general invariant form: "transform the initial situation into the terminal situation" which takes on the

status of goal. The solution has the general form of a composition or ordered sequence of operators the argument of the first of which is the situation alpha and the value of the second of which is the situation beta (Cellérier, 1979a, p. 96).

If we return to Figures 1 and 2 for some concrete help in understanding Cellérier on this issue, we note that, with reference to GPS, the directed difference of which he speaks is embedded in the Criteria of Progress central to the program. These criteria establish direction by focusing on logical differences between expressions rather than, say, variations in typeface. Second, they rank order the logical differences considered relevant by the program in terms of which should be worked on first, which second, etc. Finally, they allow the results produced by operators to be evaluated in terms of whether the difference is diminished or disappears or whether an even more difficult difference is produced.

As an example, consider steps 6 through 10 in Figure 2. In attempting to transform L2, i.e., (~P -> Q).R, into L0, i.e, (Q v P).R, the program sets up the subgoal of reducing the differences in the binary connectives "->" and "v" in the left hand terms of both expressions. It chooses to do this--it is most interested in doing this--at this time because the Criteria of Progress (Figure 1) prescribe that differences in binary connectives are more important than differences in position. From that point of view, the thing to work on first is the difference chosen rather than the difference in variable position between the two expressions or the difference in sign of variable P (a lower level component). Having decided all of this, operator R5, i.e., A v B <--> ~(~A . ~B), is selected using the table of connectives because it is the first rule that operates on the difference targeted. However, because the conditions for applying that operator are not met, it is necessary to transform L2 so that the operator may be used (step 9). This would require that the in L2 be transformed into either "v" or "." . To do this, however, is just as difficult as doing steps 6 and 7, so R5 is rejected and the program goes on to R6.

While all of this is helpful in understanding how evaluation takes place in Cellérier's revamped system, it does not elucidate how axiological knowledge is constructed. As Cellérier points out, the value structure written into the Table of Connectives and Criteria of Progress was worked out by Newell and Simon, not by the program itself; and the program has no power to change it. Nor are the values inherent in the system of control subject to construction. In fact, Cellérier specifically characterizes this cycle as invariant and content-independent. From beginning to end, it values activity of a single sort: set up a goal-means framework for transforming alpha into beta. While "reduce" and "apply" goals may be invoked to achieve this end, no other activity has any value to it. And finally, the values in the knowledge base and in the control structure were matched apriori by the programmers. Consider, for example, what significance the Criteria of Progress or the Table of Connectives would have in a program whose content-independent "motivation" was to rewrite formulas in Greek. Obviously, they would be irrelevant.

We must, therefore, look once again to memory to explain the axiological transformation. To begin operating, a system of the sort envisioned by Cellérier must have available a list of operators linked to specific features of a problem space along with some heuristic rules. Such knowledge plus a control structure allows the system to invent new procedures and, thus, makes the growth of knowledge possible. In addition to the values embedded in what is given, new values are created by the decisions of the system. For example, in Figure 2, Steps 5-10, **R5** is bad, **R6** good, under the goal-means framework of that specific problem. Insofar as the procedure is remembered, this axiology is preserved as well because it is embedded in the list of operators devised to solve the problem. Such values are, however, specific to the goal-means framework and are derivatives of a general value structure. It is not clear how Cellérier's revised model would account for the growth of general evaluation principles or even if he thinks such principles evolve. Perhaps they are abstracted from the values inherent in remembered schemes by some process akin to Piaget's "intellectualization of feelings," but that is only a conjecture. Since, apparently, Cellérier's model cannot be taken any further, let us attempt to integrate it with Pugh's.

Putting Cellérier and Pugh Together

Pugh, as we saw earlier, considers affectivity to be a surrogate, heuristic apparatus used to work out solutions in the problem space of psychological adaptation. Affective phenomena observable at birth both in the baby's system of preferences and in his emotional expressions are manifestations of a complex system of innate values that has been selected phylogenetically for the consistency of the choices it determines with the choices biological selective mechanisms would make themselves were behaviors genomically determined. Pugh is careful to make the point that this "given" value structure is not based on physiological need alone but includes, alongside "selfish" values of that sort, values based on social and intellectual necessities. While, as Pugh points out, this primary value structure cannot be changed by the functioning of the system, still it can be used as a reference for evolving secondary value structures. Evaluative criteria set up by the functioning of the system should then, insofar as they are successful, approximate primary values in one way or another.

Cellérier, of course, does not speak of affectivity at all, except to mention "needs" in a place or two. Nor does he explicitly connect axiological knowledge or values with the diachronic aspect of affectivity as did Piaget (1981a). For him, values are embedded in the structure of pragmatic and operatory knowledge and in the immutable axiology of the system of control. Although he agrees with Pugh that the psychological system must start with already constructed knowledge provided by the genome, the important question for Cellérier is not the line of demarcation between genomically provided and psychologically acquired knowledge. Rather, it is to understand the computational problem of how any system of control achieves internal and external adaptation. Moreover, Cellérier differs from Pugh in his almost exclusive focus on the values operative in rational deciding.

One way to begin bringing Pugh and Cellérier together is to seek correspondences between the functional components Pugh considers fundamental to any value driven system and the components Cellérier includes in his model. According to Pugh (1977, p. 54), all value-driven decision systems must have the following elements:

1. A data collection procedure to supply information needed to define the environment as it affects action alternatives.
2. A model of relationships in the environment which defines action alternatives and their consequences.
3. A procedure for exploring available action alternatives and estimating their consequences.
4. A method for assigning values to the estimated consequences.
5. A decision mechanism for selecting the alternatives that show the best value.
6. (Optional) Procedures for creation, improvement, and refinement of the model.

It is clear from this list that the first five components relate to Cellérier's pragmatic transformation and that the last corresponds to his epistemic transformation. Proceeding one by one, the first component on Pugh's list can be seen to conform to what Piaget and Cellérier would call perception. Obviously, this notion would have to be extended to include "perception" of information internal to the system (the "refléchissement" in Piaget's (1977) reflective abstraction). Without that, there would be no way to collect information about the internal environment of schemes, making abstraction and generalization, the central mechanisms for constructing operatory knowledge, impossible. Neither Cellérier nor Pugh deals successfully with this issue.

The second component, an environmental model typical of cognitivist conceptions generally, fuses Cellérier's pragmatic and operatory knowledge and is ambiguous about their interactive character. To bring Pugh's models into line with Cellérier's, it would be necessary to specify that all models are interactional, to distinguish pragmatic from operatory knowledge, and to include pseudoempirical knowledge, i.e., knowledge of logicomathematical environments, alongside the empirical knowledge inherent in Pugh's environmental model. Axiological knowledge could then be considered part of Pugh's fourth and fifth components, and Cellérier's requirements would all be met.

Pugh's procedure for exploring action alternatives, i.e., the third component on the list, is a search method invoked without elaboration. Nor such methods mentioned specifically by Cellérier, although they are included implicitly within his structure of control. In either case, as elements of the control structure, search methods are somewhat problematic because in both theories control is conceived relative to formulating intentional behaviors when a considerable part of the behavior for which they must account is unintentional, i.e., instinctual or habitual. Another way of saying this is that, given some goal, the GPS control structure looks for transformations that it can compose into a procedure for moving from starting point to goal. Instincts and habits do not allow such compositions, instead invoking behavioral sequences whole. These sequences either achieve the goal or they do not, but recomposition is not possible. Such considerations create some doubt, therefore, as to the suitability of Cellérier's control structure or Pugh's decision system for representing control in unintentional behavior. In consequence, the search methods of either theory are also brought into question.

Cellérier (1979b, p. 113) is cognizant of this problem and attempts to handle it by distinguishing unintentional actions ("intrarule transfers of control") in the form of hereditary procedures (instincts) and acquired associations (habits) from intentionally structured actions ("interrule transfers of control") and then using "needs," his only affective term, as the evaluation principle in forming associations. Effectively, instincts would be constituted and conserved by the genetic system which acts both as the control system and memory in the broad sense. Such behaviors could be used by a psychological control system and memory both as a table of means and ends for constituting new procedures and for forming new matrices of interactions. Habits, conceived as associations between perceptions and actions, would result from the psychological control system evaluating new interaction rules formed from instinctual elements in terms of whether they satisfy a need. And intentional behaviors would result from the goal-oriented composition of instinctual and associational elements using the evaluative principle of minimizing distances in problem space. As will be seen below, this confuses separate aspects of control having to do with how goals are set and monitored and how transformations are evaluated within a given goal-means structure. While it appears that the problem could be resolved in ways satisfactory both to Cellérier and Pugh, the specifics of such a solution remain to be discovered.

The fourth component in Pugh's list brings up several complicated issues. To begin with, there is the question of just what values are for Cellérier and Pugh. Then one wants to know how values are assigned. And finally, since both authors speak of different levels of evaluation, it is necessary to consider these issues at every level. Only after all of that is done will we have some understanding of the issues surrounding methods for assigning values.

With respect to the nature of values, apparently both Cellérier and Pugh agree that they represent psychological mechanisms for selecting adapted actions and that their ultimate source is biological selection. At the instinctual level, both authors also apparently agree that evaluation is accomplished by the multileveled selective mechanisms operating within the genetic system as well as by ultimate decision in survival terms (thermodynamic selection). However, it is not certain that they see the product as identical.

Pugh, in addition to instinctual behaviors and a central control apparatus, posits that organic evolution provides the organism with specific evaluative "processors" in the form of an affective system. These processors, much like elementary perceptual processors, operate outside the vale of central control and cannot be influenced by it at any level. The input upon which they operate is, as Pugh sees it, basically sensory information. As indicated earlier, this rather limited view reflects his adherence to the idea of a purely environmental model perceived through a sensory apparatus. In order to deal with Cellérier's much more realistic interactive "world," Pugh's notion of environment would have to be expanded to include the internal epistemic universe in addition to the world of the usual senses. Only then could one have feelings about ideas and meanings.

Little is known, of course, about how affective processors determine values, but their output, Pugh contends, is of at least two types. One of these involves direct transfer of information to centers lower in the hierarchy, e.g., metabolic and temperature control centers. Although this may be important to the arousal functions of the affective system, Pugh does not discuss it in detail. The output of central interest to Pugh has to do with the forwarding of evaluative information to the decision system, the central control where action is composed. It is Pugh's belief that such information is experienced in the form of feelings and that its quality and intensity are determined by the processor from which it emanates. Such data, like sensory data processed by perceptual processors and presented to the control system as perceptions, become conditions for productions for both Cellérier and Pugh.

Cellérier's account of all of this is extremely scanty. Unlike Pugh, he is concerned with evaluation at higher levels in the system and only mentions physiological need, the Piagetian euphemism for affectivity, in passing. The crucial issue in comparing the two authors becomes, therefore, how Cellérier's view of rational evaluation would be handled in Pugh's model. Does that model allow for the construction of an evaluative function based on distances in problem space as conceived by Cellérier? It appears the answer would be yes only if Cellérier agreed to include the output of Pugh's affective processors in the information that is fed into the system. As it stands, Cellérier's notion of predetermined axiological knowledge relates to structural features of the system and leaves room for active processes of evaluation only within the system of control. Almost certainly this is incorrect.

If we turn to the issue of how values are determined on various levels, we find other differences between Cellérier and Pugh. Given innate knowledge structures in the form of reflexes and instincts, perceptual processors in the broad sense including proprioceptive devices, etc., Pugh's affective processing system, and a system of control, the subject begins immediately to acquire habits. This much appears to be allowable within either model. There is, however, a problem with Cellérier's account.

In effect, Cellérier envisions habit formation as a process of setting affective goals in the form of needs and evaluating behavior in terms of need satisfaction. He argues that this constitutes an "implicit" version of distance-in-problem-space evaluation that will give place to an explicit version once intentional action becomes possible. The problem with this is that it not only confuses physiological with intellectual needs which are very different but it also confuses the evaluation method used within a means-ends analysis once a goal is set with the external evaluation method used to determine goals.

Pugh is much clearer on this point. He holds that although physiological and intellectual needs are both part of the affective system, they represent different forms of values, serve different purposes, stem from separate innate sources, and lead to different elaborative products. The first are instances of "Selfish Values" insuring individual welfare and survival. They arise from specific innate processors, and their secondary elaboration has to do with controlling physiological urges. Examples would be such things as toilet training or dieting in order to be pretty, etc. By contrast, "Intellectual Values" promote the construction of accurate world models. They relate to needs for certainty, for apportioning intellectual effort, for giving up on problems when they prove too difficult, etc. They are manifest in such feelings as curiosity, distaste for ambiguity, and Janet's (1928) activation and termination affects generally. Their secondary elaboration leads to changing intellectual goals and, at the highest social level, to an evolving scientific axiology. Neither of these forms of values is, however, internal to the means-ends analysis or control system. At both habitual and intentional levels, that system will act to minimize distances in problem space or to maximize innovation and conservation in order to achieve whatever goal is set. What selfish and intellectual as well as the social values included in Pugh's model do is to set diverse goals for the means-ends mechanism and evaluate the products of the transformations it determines by processes outside the system. While these processes may in a way be said to minimize distances between points in problem space, they do so outside awareness in a superordinate problem space of biological adaptation. With respect to the control structure itself, however, affective processors are used to determine goals and then provide ongoing evaluation of the effects of action in terms of selfish, social, or intellectual desirability rather than in terms of progress toward the goal. Should an action, however effective in approaching a goal, lead to a result with unforeseen negative affective consequences, the value of the goal relative to which that action's value has been determined is itself called into question. In this way affective mechanisms track the control structure's progress and regulate it by continuing or changing goals.[14]

Pugh's fifth component, i.e., the decision mechanism for selecting best valued alternatives, is part of a central control system where intentional actions are composed. Essentially, Pugh

14. Note the resemblance of this account to Janet's (1928) notion of affectivity as "secondary regulations," and to Piaget's (1981a/1953-1954) idea of "the synchronic aspect of affectivity."

equates central control with the conscious system conceived as a serial processor using input from perceptual, mnemonic, and affective structures to set goals, explore action alternatives, and evaluate which ones are better. As such, it includes Cellérier's control structure but goes considerably beyond it to make decisions on at least two levels, one the level of goal selection, the other the level of means selection relative to that goal. Undoubtedly this system also regulates attention, perceptual centration, etc., but these are subjects left unanalyzed by both Cellérier and Pugh.

Finally, let us only recognize that Pugh's sixth and optional component, i.e., procedures for creating and refining world models, was extensively dealt with in our discussion of Cellérier's epistemic transformation. There is, therefore, no need for further elaboration.

It seems clear from all of this that Pugh and Cellérier need one another. Pugh needs Cellérier in order to incorporate knowledge construction into his model; Cellérier needs Pugh in order to extend his notion of evaluation to goals and to give it a more thoroughgoing affective interpretation. While the model that emerges from their integration is no doubt incomplete and open to objection, it seems to move us closer to understanding the role of affective evaluation in constructing adapted actions and extracting knowledge structures from them. There remains, however, a small piece of unfinished business.

Some Applications

Although the main thrust of this paper has been theoretical, it seems wise to ask, even in a very preliminary way, what empirical ramifications integrating Pugh's and Cellérier's conceptions might have. No studies of our own having yet been undertaken, such speculation must be based on existing data. While the examples below are in no way exhaustive, they do indicate three approaches to which the notions here may contribute.

Genevan Studies of Procedures

Actually, the switch of the Genevan school from studies of structures to the study of procedures preceded Cellérier's theoretical formulation. It was, in fact, officially announced during festivities honoring Piaget's eightieth birthday in 1976 (Inhelder, Ackermann-Valladao, Blanchet, Karmiloff-Smith, Kilcher-Hagedorn, Montangero, & Robert, 1976) and considerable empirical work was done either before Cellérier's (1979a,b) synthesis or before Inhelder and Piaget's (1979) "Procédures et structures." To date, Genevan investigators have produced somewhere around a hundred studies on procedures, so obviously every aspect of their work cannot be considered.

As Alex Blanchet (1987, p. 253), the only Genevan to recognize the affective nature of evaluation, points out, investigations of children's problem solving strategies inspired by Inhelder and Cellérier have focused on the meanings children apply in seeking to achieve some goal. While they have produced better understanding of how goals and means become coordinated, on how ascending and descending control interact, and on how children reconcile different descriptions of an object with one another, they have neglected the role that values play. Our purpose in examining one such inquiry is, therefore, to see whether affective factors have crept undetected into the data.

In a study focusing on relationships of ascending and descending control, Boder (1978) asked children to load a truck with colored blocks so that they could drive along a little road and deposit blocks on loading. They were instructed that the block deposited had to be of the same color as the dock. The truck was of a size that blocks could only be stacked one on top of the other, and the direction of the route was indicated. Since blocks had to be unloaded in top down order and since backing up or turning around were not allowed, it was necessary that the blocks be placed in the truck in reverse order to the order in which loading docks would be encountered. Children from ages four to eight and a half years old were studied.

Boder found that one reaction of children from four to about five and a half years of age (group 1) was to load the blocks in essentially arbitrary fashion. Although children reacting in this way seemed to try to coordinate color, they did so in a peculiar way. When they had a block in hand they appeared to match it to a particular loading dock, but they did not consider the loading docks only once or consider them in order. Consequently, they loaded several blocks for some loading docks and none for others. This resulted not only in a lack of correspondence between the order of the blocks loaded and the order of the loading docks but also in lack of correspondence between the number of the blocks in the truck and the number of loading docks along the road. Boder does not report what happened when mismatches occurred during unloading.

Other children in this group (group 2) understood that it was necessary to organize how the blocks were loaded but still did not organize loading to correspond to unloading contingencies. Usually, these children put the correct number of blocks in the truck and often the correct number of each color. Oddly however, they put all the blue blocks in first, the yellow second, or vice versa, even though this violated the unloading order.

A third group (group 3) indicated progress toward coordinating loading with unloading but without complete success. One child, for example, noticed that somewhere along the route there were two blue loading docks together. He began, therefore, by putting two blue blocks into the truck, explaining, "I start there because there is another blue and that's not hard." Sometimes subjects in this group also reversed the direction in which the truck was going, a violation of instructions, in order to achieve better correspondence.

A fourth reaction appearing in this group and one that persisted into later ages (group 4), was to alternate loading blue and yellow blocks without keeping count of the number loaded or making that number correspond to the number of loading docks. In consequence, both the order of the colors and the number of blocks loaded proved incorrect. Apparently, alternation seemed the most significant aspect of the situation.

In children five and a half to eight and a half, the notion that the loading docks dictate both the number and order of the blocks to be loaded becomes stable, but the need for inversion presents problems. A subgroup of these (group 5) put a block corresponding to the first loading dock into the truck first, a block corresponding to the second dock second, etc. This, of course, puts the block for the first dock on the bottom of the pile and makes it impossible to unload it first. Subjects in this group could not correct their strategy.

Some children who make this mistake, however, come to understand that their error consists in reversing the unloading order and begin to correct it (group 6). This is done by inverting the order of only two or three elements at first, but eventually it leads in certain individuals to complete inversion and success.

Because the task involved is highly intellectual and relatively easy, it might be thought that affective approximating devices would not be needed. That assumption is strengthened by the fact that Boder's analysis has to do with ascending and descending control and makes no mention of values or affectivity. Nevertheless, it is instructive to consider his data, particularly his minute descriptions of the childrens' struggles, from the affective point of view. Despite the paucity of affective description, doing so reveals that even in activity of supposedly so rational a nature, affectivity is at work at every turn.

Three questions need consideration: first, what are the childrens' goals at various stages of activity; second, how do they decide on the methods used; and third, what terminates activity. With regard to the first question, all children, apparently, start off wanting to follow the instruction. It seems unlikely that children would be forced, and no protests are described. True, had children been asked about their motives, they might have given "rational" justifications, but these would take forms such as, "I don't want to be punished," "I want you to like me," or "I just want to," etc., for the simple reason that there is no basis other than feelings on which to decide such questions. Since wanting to comply with the experimenter means accepting the instruction that the blocks are to be carried to the loading docks in the truck and not transported in some other manner, that subgoal now inherits affective value. Children of this age know, of course, that in order to comply with the instruction, the blocks must be loaded before the truck drives along the road, so this activity next gains favor, and they set to work. But this does not mean that affective mechanisms can take a rest. Not only are three forms of correspondence desired, i.e., correspondence of color, number, and order, but the entire motivational framework is constantly open to revision, depending upon reassessments of interest, fatigue, concentration, etc. Why, for example, should group 1 subjects first set the subgoal of matching color? It is not enough to say they do not understand or that knowledge or computational resources are insufficient. These subjects can count and could, therefore, have made similar mistakes by centering on number. The truth of the matter is that affective mechanisms in the form of rather primitive preferences determine the relative salience of these factors. And as Boder points out quite clearly, progress requires that the subject "distance" himself from biases of this sort which are multiple.

Turning to the second question, i.e., how do subjects decide on the methods used, it seems obvious that Pugh's "intellectual values" guide many of the choices. In general, such values play off the cost of computation in terms of time and effort against the degree of accuracy desired. From that point of view, it is clear that group 2 subjects have opted to minimize effort with their strategy of counting the number of each color needed and loading them into the truck in bunches. What they save in energy is, of course, sacrificed in accuracy, since order is not considered. While in simple tasks where total accuracy can reasonably be required, their error may seem quite stupid, it becomes acceptable in many real life situations. The IRS, for example, fails to collect billions of dollars in taxes each year because of the tremenduous cost of accurate calculation. Faced with millions of complicated tax reports, shortcuts of the sort employed by group 2 children become quite reasonable.

Briefly examining the other groups, it is clear that intellectual values operate in choosing methods at every level. There is space for only two examples. "Vla" in group 3 says explicitly that he put two blue blocks in the truck first because he could see that there were two blue loading docks together and that that part of the problem was easy. And even the highest level subjects who succeed by carefully considering the order of the docks when the road is straight and they can inverse a simple right left order persist in reading right to left after a curve has been introduced reversing the order of the last three docks. In other words, the curve changes the order "yellow, blue, yellow, blue, blue" to "yellow, blue, blue, blue, yellow." Rather than follow the truck's movements in their heads as they did in discovering the straight-road solution, subjects substitute the strategy "load in reverse right-left order," even though doing so leads to error. Again, this reflects affective choices corresponding to Janet's "economy of action," to Piaget's "synchronic aspect of affectivity," and to Pugh's "intellectual values."

Finally, Boder gives little information relative to our third question having to do with the termination of action. Since the majority of his subjects do not succeed and since there is no reason to believe that they are still working on the problem, we must assumed that something turned them off. Either they lost interest, complied with Boder's request that they stop, or gave up for other reasons. No doubt some were extremely frustrated, as witness the five and a half year old who finally figured out that he would have to put the block corresponding to the first dock on top, became distressed about the pseudocontradiction that it is first off the truck but last into it, and concluded that although the block must be in the truck it was impossible to put it on top. In this instance, the exclamation point after his final answer must be read as an affective marker.

Examples from

Decision Science

As mentioned in footnote 10, the index to Kahneman, Slovic, and Tversky's (1982) book, Judgment Under Uncertainty: Heuristics and Biases, contains no references to affective terms of any sort even though article after article refers to affective processes. Let us, therefore, examine one of the articles to see how affectivity comes into play.

In a paper entitled "Facts versus Fears: Understanding Perceived Risk," Slovic, Fischhoff, and Lichtenstein (1982) point out that even when considerable statistical evidence is available as a basis for deciding important protection issues, "facts can only go so far toward developing policy. At some point, human judgment is needed to interpret the findings and determine their relevance" (p. 463). As in so much of this literature, the imprecise use of language is disturbing. Is the reader to understand that there are two deciding agents, that up to a point facts decide issues and after that people do? What are facts, what is judgment, and what is the relationship between the two? As best can be determined, the distinction that the authors wish to make is one between rational decisions based upon probabilities calculated from systematically established information and irrational, "heuristic" decisions based upon probabilities estimated from criteria of another sort. Rather than attempting to straighten all this out, let us simply note that the basic thesis to be supported by the studies cited is that risk assessment is "inherently subjective" and that knowing the limitations of such assessments is "crucial to effective decision making."

The first group of studies cited by the authors have to do with "judgmental biases in risk perception." Essentially, the authors argue that people are ususally forced to make estimates of risk without systematic information and that in doing so they employ "very general inferential rules" that have been brought to light by psychological investigations. For example, people often assume that the liklihood of some disastrous event is directly proportional to the ease with which examples come to mind (availability bias). In support of this contention the authors cite data from three sources: studies on how people estimate the risk of floods; studies on the relationship of earthquakes and sales of earthquake insurance; and studies on the reaction to public discussion of the risks of recombinant DNA research. In the first instance, it has been shown that despite the availability of good evidence to the contrary, people tend to believe that future floods will not exceed in extent or severity the flood most recently experienced. In the second instance, it is known that the sale of earthquake insurance increases sharply after an earthquake occurs and decreases as "memories fade." In the third instance, it was found that initial praise for scientists' announcement of possible risks from recombinant DNA experiments soon transformed into outrage as speculation escalated and scenarios became more "scary."

It is instructive to compare the way Slovic, et al, explain these finding with Pugh's ideas about decision making. In the first instance, Slovic and his colleagues argue that ease of remembering becomes a criterion of probability. This takes on affective significance when one remembers that ease is a feeling and that Pugh links it explicitly to intellectual values. Moreover, this significance is considerably increased by evidence that affective experience influences how memory is organized, so that ease of remembering may well be linked to other sorts of feelings. In the second instance, Sovic, et al, again invoke a principle of memory to explain the findings, i.e., fading memory of an earthquake is responsible for decreasing sales of earthquake insurance. Assuming that they are correct, the question then becomes, "What causes memories to be vivid and then to fade?" There seems little doubt, moreover, that it is not the memory itself that faded. Subjects asked to recall the earthquake no doubt could recall its major features, although some detail would eventually be lost. The fading factor that motivates insurance purchases would much more reasonably be identified with subjects' fear, i.e., with fading feelings. Not only would this make affective evaluation a central feature in the decision process, but it would also illustrate Pugh's principle of "time dependence" as part of any value structure. Finally, Slovic and his colleagues invoke a frankly affective explanation in the recombinant DNA example: they mention "heat" and "scariness" as explanatory principles. They do not, however, go on to interpret affects as heuristic evaluation mechanisms in Pugh's manner. The affective nature of many heuristic decision mechanisms remains implicit in their work.

Although this only touches on the treasure chest of data in Slovic, et al's, paper and in Kahneman, et al's, book that could be reinterpreted from the affective point of view, there is space for only one more example. Its purpose will be to demonstrate the lengths the authors go to to avoid invoking affective principles. It concerns what Slovic and his colleagues call the "it won't happen to me" heuristic. To illustrate this decision strategy, they cite studies showing that most individuals believe themselves to be better than average drivers, to be more likely than average to live past 80, and less than likely to be harmed by the products they employ. Their explanation of these judgments is based upon the notion that people are poor statisticians; it seems more likely that in making such decisions, most people are not statistically engaged. For example, Slovic, et al, suggest that the reason people think they are such good drivers has to do with the fact that "despite driving too fast, tailgating, etc., poor drivers make trip after trip without mishap. This personal experience demonstrates to them their exceptional skill and safety. Moreover, their indirect experience via the news media shows them that when accidents happen, they happen to others." Would it were that people were so reasonable. It seems likely, however, that people base their belief in being good drivers only very partially on such reasons. Vanity, anxiety about death, and numerous other affective factors play into how people assess their driving. Moreover, careful scrutiny of the reasoning behind such decisions would reveal mechanisms of repression, denial, and the like, making explanation much more complicated than poor statistics.

Let us conclude this example by pointing out that despite their resistance to affective language and their limited understanding of affective theory, Slovic and his colleagues appear to agree with the central premise underlying the work of Pugh. This is illustrated by the following excerpt having to do with the technical obstacles impeding rational estimation of how probable serious nuclear reactor accidents:

> The technical reality is that there are few "cut-and-dried facts" regarding the probabilities of serious reactor mishaps. The technology is so new and the probabilities in question are so small that accurate risk estimates cannot be based on empirical observation. Instead, such assessments must be derived from complex mathematical models and _subjective judgments_ (author's italics; p. 486).

The Axiology of Science

As a final illustration, let us turn from studies conducted on the microgenetic time scale to examine an account of historical construction. We do this in the belief that such studies are no less empirical than "experimental" microgenetic studies, although the nature of the data is quite different (Garcia, 1983). We also do it because they illustrate quite well how Pugh's secondary values evolve and how Cellérier's epistemic transformation operates in history.

The text at issue is Laudan's Science and Values. In this work, Laudan is concerned with two puzzles concerning science. On the one hand, there is the puzzle of how, in a social world of such diversity, scientists ever can agree. On the other hand, there is the puzzle of how the disagreement or dissensus increasingly recognized in science can be explained the defining feature of science is adherence to methods producing consensus.

Laudan begins his analysis by considereing the best known theory of how consensus is formed. This he summarizes in what he calls "The Simple Hierarchical Model of Rational Consensus Formation." Essentially, this model holds that disagreement can occur and be resolved on three hierarchized levels. The first level of disagreement is factual, the lowest level in the hierarchy. According to this theory, agreements at this level are settled by evidentiary rules to which all scientists adhere. There are, however, disagreements at a second level which facts do not resolve. These are disputes about the rules of evidence or rules of procedure and their application used to settle factual disputes. Methodological disagreements of this sort, according to the hierarchiacal model of consensus, must be resolved on an axiological basis. The reason this is so is that methods are means of achieving scientific goals and can, therefore, be evaluated in terms of their effectiveness in realizing scientific or cognitive ideals. Here, however, the strategy of resolving problems on one level at the next higher level in the hierarchy breaks down. Axiological disputes, that is disagreements about the aims and goals of science, are either held not to exist on the grounds that all scientists share identical values; or, if they are admitted to exist, they are held to be incapable of resolution. Laudan carefully documents each contention.

Particularly relevant to our purposes are his examples concerning the supposedly unrevisable nature of scientific goals. Reichenbach, he points out, believed that goals in general, including scientific goals, are not rationally negotiable. He quotes Reichenbach as follows: " 'If anyone tells us that he studies science for his pleasure [as opposed to his doing science because he wants to know the truth], . . . it is no statement at all but a decision and everybody has the right to do what he wants . . . . [When we propose an aim for science, we cannot] demand agreement to our proposal in the sense that we can demand it for statements which we have proven to be true' " (p. 49). Similarly, Laudan points out that Popper believed that realism and instrumentalism were both internally consistent accounts of science and that which account one chose "ultimately reduced to an irresolvable matter of taste." Laudan sums up:

> The thrust of such arguments [is that if] a certain set of cognitive ends or values is internally consistent, then there is no scope for a rational evaluation of those aims or for a rationally grounded comparison of those aims with any other (consistent) set. We may or may not like a certain set of goals; we may or may not share them. But these are

emotive matters, quite on a par with other subjective questions of personal or sexual preference (p.49).

Laudan, of course, believes that the unrevisability of goals is false and denies the hierarchical model. In its stead he erects "The Triadic Network of Justification" where theories, methods, and aims form a triangle and influence one another mutually. The relationships envisioned by Laudan specify that theories must be consistent with aims and constrain methods while at the same time being justified by them. Conversely, methods justify theories and realize aims while being constrained by theories and justified by aims. And finally, aims must be in harmony with theories while at the same time both justifying methods and being realized by them. All of this is illustrated in Figure 3.

Figure 3 about here

What makes it possible for Laudan to bend what was formerly a straight line into a triangle is his demonstration that scientists' goals are open to criticism and revision. This destroys the hierarchy. As evidence, he cites many 18th century scientists' realization that the aims of empiricist science, formulated in the wake of Newton, were no longer desirable. Too many phenomena had been discovered that were unamenable to explanations limited to observable entities and processes. Successful theories of electricity, of embryology, and of chemistry had to invoke unobservable factors for progress to occur. During this epoch also, Lesage's chemical and gravitational theories, Hartley's neurophysiological ideas, and Boscovich's conception of matter were all criticized, not on grounds of theory or method, but because they deviated from empiricist axiology, i.e., from ambition or goal of explaining nature in terms of observations.

Laudan identifies two criteria on which such criticism and revision can be rationally based. The first or "utopian" criterion is leveled against goals that, however desirable, cannot be realized or that cannot be known to have been accomplished even should that be the case. Although not cited by Laudan, an example might be the abandonment of Hilbert's search for a finitistic absolute proof of consistency for arithmetic when Gödel's theorem showed that it was essentially impossible to achieve (Nagel & Newman, 1968). The second or "shared archetype" criterion is leveled against scientists who do not practice what they preach, or in other words, who pursue contradictory goals. As an example, Laudan points out that Newton, despite stressing "hypotheses non fingo," had, in fact, used hypotheses. Under attack, Lesage employed this fact to defend his own use of hypothetical entities and to argue against complete observability as a scientific aim. As history has shown, Lesage won; empiricist aims were gradually abandoned.

This brief précis of Laudan's conception of consensus and dissensus in the development of scientific knowledge not only provides an excellent illustration of Pugh's notion of secondary values but also illustrates how values operate in Cellérier's epistemic transformation. Out of primitive intellectual values, i.e., feelings regulating intellectual activity in terms of interest, fatigue, satisfaction with success, disappointment with failure, etc., complex secondary values are constructed. When these are discovered to be inconsistent with primary intellectual values, as in the example of Lesage abandoning empiricism's aspirations, they must be rejected. But the primary values are not themselves open to revision. Scientists will always value intellectual activity and success, will always husband intellectual resources, and will always devalue inconsistency and failure. Their goals as well as their theories and methods must reflect those values. If they do not, they will eventually be corrected, unless the primary value structure, because of its surrogate nature, is inadequate to the problem or unless competing values outweigh the goal of rationality and lead to irremediable disaster before reason can come into play.

Conclusion

Selecting the evaluative aspect of affectivity for analysis, we have argued that the functional internalization of organic evolution to form the intellectual system required devising some means of selecting behavioral variations and that the selective mechanism devised was the affective system. From that perspective it is possible to relate Pugh's conception of value-driven decision systems and his account of affective mechanisms to Cellérier's analysis of the pragmatic and epistemic transformations.

Tenuous as the resulting model may be, it gives a picture more complete than other models of the role affective evaluations play in adapting action and in constructing knowledge. In a preliminary way, it also allows us to bring together empirical data on problem solving, on decision making, and on the evolution of scientific thought. Further study is needed to determine whether these promising beginnings can mature into a full blown theory in which other aspects of affectivity will be included.

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Table 1. (Adapted from Piaget, 1952/1936, p. 9.)

BIOLOGICAL FUNCTION		PSYCHOLOGICAL FUNCTION	CATEGORIES OF REASON
Organization		Regulatory Function	Totality x Relation
			Goal (Ideal) x Means (Value)
Adaptation	Assimilation	Implicative Function	Quality x Class
			Quantity x Number
	Accommodation	Explanatory Function	Object x Space
			Causality x Time

Figure 1. (Adapted from Newell & Simon, 1972, pp. 419 & 431.)

Differences between objects. The differences apply to subexpressions as well as total expressions, and several differences may exist simultaneously for the same expression.

- **delta t** A variable appears in one expression but not in the other. E.g., P v P differs by +t from P v Q, since it needs a Q; P -> R differs by -t from R, since it needs to lose the P.
- **delta n** A variable occurs different numbers of times in the two expressions. E.g., P.Q differs from (P.Q) -> Q by +n since it needs another Q; P v P differs from P by -n, since it needs to reduce the number of P's.
- **delta s** There is a difference in the "sign" of the two expressions. E.g., Q versus ~Q, or ~(P v R) versus P v R.
- **delta c** There is a difference in binary connective. E.g., P -> Q versus P v Q.
- **delta g** There is a difference in grouping. E.g., P v (Q v R) versus (P v Q) v R.
- **delta p** There is a position difference in the components of the two expressions. E.g., P -> (Q v R) versus (Q v R) -> P.

Operators.

R1	A v B --> B v A A.B --> B.A	R7	A v (B.C) <--> (A v B).(A v C) A.(B v C) <--> (A.B) v (A.C)
R2	A -> B --> ~B -> ~A	R8	A.B --> A A.B --> B
R3	A v A <--> A A.A <--> A	R9	A --> A v X
R4	A v (B v C) <--> (A v B) v C A.(B.C) <--> (A.B).C	R10	A, B } --> A.B
R5	A v B <--> ~(~A.~B)	R11	A -> B, A } --> B
R6	A -> B <--> ~A v B	R12	A -> B, B -> C } --> A C

Connections between Differences and Operators. +, -, or x in a cell means that the operator in the column of the cell affects the difference in the row of the cell. + in the first row means +t, - means -t, etc.

	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	R12
delta t			x					-	+	+	-	x
delta n							x	-	+	+	-	x
delta s		x			x	x						
delta c					x	x	x					
delta g				x			x					
delta p	x	x										

Criteria of Progress. All differences in subexpressions are less important than differnces in expressions. For a pair of expressions the differences are ranked: +t, -t, +n, -n, delta s, delta c, delta g, delta p, from most important to least. E.g., delta s is the more important in comparing (P v Q) with R -> Q, while delta c is the more important in comparing P v Q with P -> Q.

Figure 2. (Adapted from Newell & Simon, 1972, p. 420.)

Given L1: R.(~P -> Q)
Given L0: (Q v P).R

Step 1: Goal 1: Transform L1 into L0
Step 2: Goal 2: Reduce delta p between L1 and L0
Step 3: Goal 3: Apply R1 to L1
Step 4: Goal 4: Transform L1 into condition (R1)
Step 5: Produce L2: (~P -> Q).R

Step 6: Goal 5: Transform L2 into L0
Step 7: Goal 6: Reduce delta c between left (L2) and left (L0)
Step 8: Goal 7: Apply R5 to left (L2)
Step 9: Goal 8: Transform left (L2) into condition (R5)
Step 10: Goal 9: Reduce delta c between left (L2) and condition R5
Step 11: Rejected: No easier than Goal 6

Step 12: Goal 10: Apply R6 to left (L2)
Step 13: Goal 11: Transform left (L2) into condition (R6)
Step 14: Produce L3: (P v Q).R

Step 15: Goal 12: Transform L3 into L0
Step 16: Goal 13: Reduce delta p between left (L3) and left (L0)
Step 17: Goal 14: Apply R1 to left (L3)
Step 18: Goal 15: Transform left (L3) into condition (R1)
Step 19: Produce L4: (Q v P).R

Step 20: Goal 16: Transform L4 into L0
Step 21: Identical, QED

Figure 3: (From Laudan, 1984, p. 63)